Coordinated Science Laboratory

A TOPOLOGICAL TECHNIQUE FOR ANALYSIS OF ACTIVE NETWORKS

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ABSTRACT

This paper presents a generalization of Fuessner's method for the analysis of active networks. This method makes it possible to determine system functions of any network in a simple and straightforward manner.

In the usual topological analysis of networks it is necessary to list all possible trees or co-trees and the signs of their tree admittance products as well as determining the set of 2-tree products and their signs for the network function in question. Usually two graphs, a current and a voltage graph, are used to implement this process when dependent sources are present in the network. The method presented here utilizes solely the original topology and the assigned branch current directions to determine these terms and their sign. Justification of the technique is given for the cases of one and two dependent sources present in a network with the summary of the method for each case. The analysis of a network with an arbitrary number of dependent sources is seen to follow directly. Finally an example of a network with two dependent sources is given which illustrates the method.

INTRODUCTION

The early methods of Kirchhoff[4,5], Maxwell[9] and Feussner[2,7,11] demonstrate the practicability and simplicity of topological techniques used in the determination of network functions of passive networks. This paper is concerned with the application of topological techniques to active (nonseparable) networks with the discussion in terms of branch impedances and admittance functions. Specifically, Feussner's method is extended to networks containing dependent voltage sources which depend on current in other branches of the network. By using elementary source transformations, both dependent current sources and dependent voltage sources with arbitrary dependencies can also be treated.

The discussion will be restricted to resistive networks for simplicity and uniformity but applies directly to general R, L, C networks. In fact, networks with mutual inductance can be treated if the coupling is represented by appropriate dependent sources[9]. Also, networks containing only one independent source will be considered since the Principle of Superposition can be applied. Each element will be considered as a separate branch except for voltage sources (both dependent and independent) in series with a passive element.

Since Feussner's method and the Method of Residual Networks after R. Seacat[7] are basic to the discussion, a brief summary of these will be given prior to the main results.

1. Feussner's Method and the Method of Residual Networks

Feussner's method provides an easy means for finding the co-trees of a passive network and since the determinant of the mesh-impedance matrix of a network yields the sum of the co-tree products, this provides a simple way of calculating the denominator of the response functions of the network.

The current response in the k'th mesh of an n mesh network containing a single voltage source in mesh 1 is given by

$$I_k = E_1 \Delta_{1k}/\Delta \tag{1}$$

where Δ is the network determinant and Δ_{1k} is the cofactor obtained from Δ by crossing out the first row and the k'th column.

Calculation of the Denominator

If R_a appears only in terms on the main diagonal of the system determinant (this can always be done by a suitable choice of mesh currents), Δ can be expressed as

$$\Delta = R_a \Delta_a + \Delta_{a'}$$

where Δ_a is the cofactor obtained by crossing out the row and column containing R_a and $\Delta_{a'}$ is the original determinant with R_a equal to zero. Upon inspection of Δ_a it is apparent that it is the determinant of a network N_a which is identical to the original network except that R_a has been replaced by an open circuit. Also $\Delta_{a'}$ is the determinant of a network $N_{a'}$ which is the same as the original network with R_a replaced by a short circuit.

Now N_a and $N_{a'}$ can be considered as new networks and their determinants can be expanded similarly, say with respect to element R_b. Thus

$$\Delta = R_a R_b \Delta_{ab} + R_a \Delta_{ab'} + R_b \Delta_{a'b} + \Delta_{a'b'} \tag{2}$$

This process can be continued until the original network has been re-duced to a set of primitive networks whose system determinants can be found by inspection. These primitive networks, their system determinant, and the topological graph corresponding to the principle determinant are shown in Figure 1. From Figure 1 it is apparent that a graph which is connected but which has no circuits has a determinant equal to unity (see graph (a), Δ_a; and graph (c), Δ_a); a graph which contains a "double short" has a zero deter-minant (see graph (a), $\Delta_{a'}$; and graph (b), $\Delta_{a'}$); and a graph which is sep-arated has a zero determinant (see graph (e), $\Delta_{a'b'c'}$). The source can be shorted without affecting the determination of Δ.

In this manner it is possible and quite simple to find all the terms of the denominator of an admittance function directly. This is a self-checking method which eliminates the tedious cancellation of terms in the solution of the system determinant by conventional techniques.

For passive networks containing inductors and capacitors the Method of Residual Networks due to R. Seacat[7] provides a means of directly calcu-lating the denominator (and numerator) in the form of a polynomial, and yields any coefficient of the polynomial essentially by inspection. (This fact can be used nicely in active networks to ascertain the presence and depend-ence of any degeneracy due to the active elements as well as indicating ordi-nary degeneracies due to all-capacitor and all-inductor loops and cut-sets). In this method the expansion is done in terms of all the inductors and capacitors of the network by replacing each of these elements by switches. Since the ex-pansion is done in terms of all the reactive elements, residual networks con-taining only resistors are left and Feussner's primitive-graph correspondences are used to ascertain the complete coefficient. If a particular L-switch is
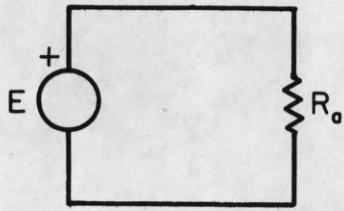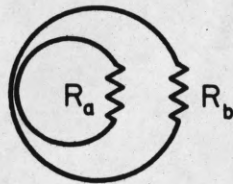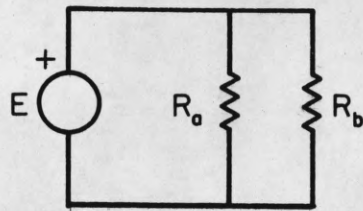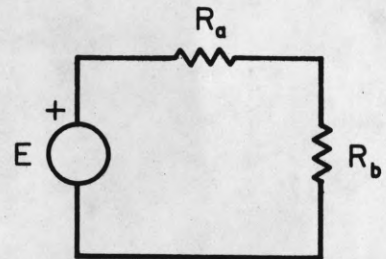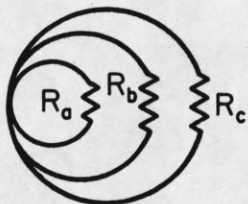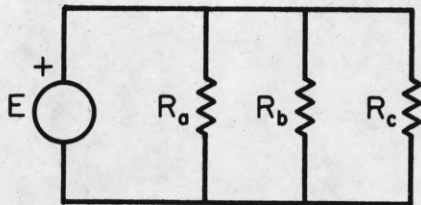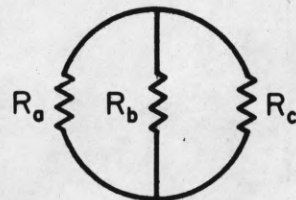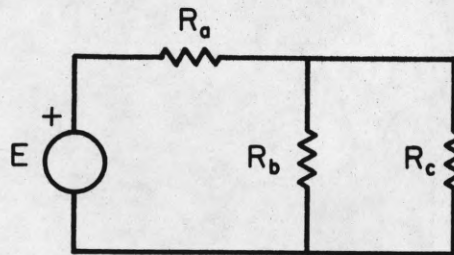
Figure 1

open (L is extremized to infinity), L appears in a product of terms in a coefficient, and doesn't appear if its switch is closed; alternatively if a particular C-switch is closed (C is extremized to infinity) C appears in a coefficient and doesn't appear if its switch is opened. To make this clear, suppose we have a network containing two capacitors C_1 and C_2 and one inductor L_3. The denominator will be of the form

$$= \frac{1}{s^2 C_1 C_2} [s^3 \{C_1 C_2 L_3 \Delta_{1'2'3}\} + s^2 \{C_1 C_2 \Delta_{1'2'3'}$$

$$+ C_1 L_3 \Delta_{1'23} + C_2 L_3 \Delta_{12'3}\} + s\{C_1 \Delta_{1'23'}$$

$$+ C_2 \Delta_{12'3'} + L_3 \Delta_{123}\} + \Delta_{123'}]$$

(3)

where for example $\Delta_{1'2'3}$ is the determinant of the residual network which is obtained by shorting switches C_1 and C_2 and opening switch L_3.

The term $s^2 C_1 C_2$ which appears in the denominator of this expression will also appear in the same manner in the numerator of all admittance functions and therefore can be disregarded.

Calculation of the Numerator

The underlying theory for the method of calculation of the numerator given here, which is due to Kirchhoff, will be deferred and taken up in the next section in conjunction with the analysis of active networks and only the technique is given at this point.

This calculation involves the concept of a _path current_ which is defined as a current flowing from the source through the network in a single closed loop. The algebraic sum of all path currents through a particular branch equals that branch current. The number of path currents in a network

due to a single source will in general be greater than or equal to the number of independent mesh currents.

Since this concept is best explained by an example, let us calculate the admittance function I_2/E_1 of the network of Figure 2a which can be written as

$$I_2/E_1 = \Delta_{12}/\Delta .$$

In this simple example only one path current flows through the source and branch 2, and in completing its path it flows through R_1, R_3 and R_2. Δ can be expanded in terms of these three elements:

$$\Delta = R_1 R_2 R_3 \Delta_{(123)} + R_1 R_2 \Delta_{(123')} + \cdots + R_3 \Delta_{(1'2'3)} + \Delta_{(1'2'3')}$$

$$(4)$$

Similarly Δ_{12} can be expanded with respect to these same elements:

$$\Delta_{12} = R_1 R_2 R_3 \Delta_{12(123)} + R_1 R_2 \Delta_{12(123')} + \cdots +$$

$$R_3 \Delta_{12(1'2'3)} + \Delta_{12(1'2'3')} \qquad (5)$$

By definition of this method of expansion, it is obvious that the only term which contributes to Δ_{12} is the term $\Delta_{12(1'2'3')}$ which is the residual determinant obtained from the original network by shorting R_1, R_2 and R_3. All of the other terms indicate that one or more of these three resistors have been opened and therefore won't allow this path current to flow. Therefore to topologically determine the terms in Δ_{12} arising from a path current J, open the terminals of the source and short all elements in path J and determine the determinant of the resultant primitive network using Feussner's method (see Figure 2c).
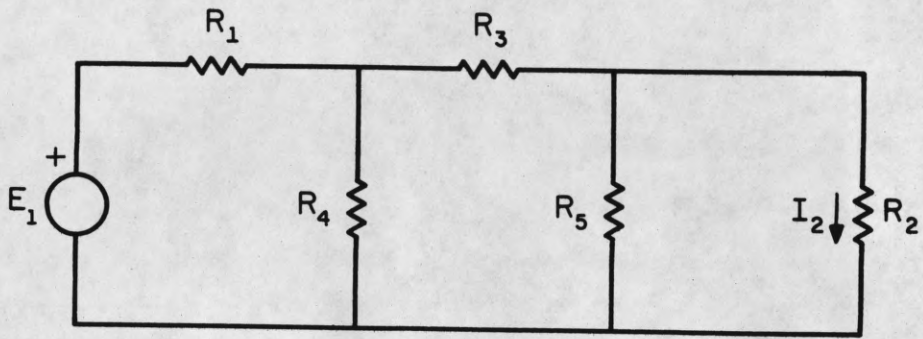
$$I_2/E_1 = R_4 R_5 / \Delta$$

(a)
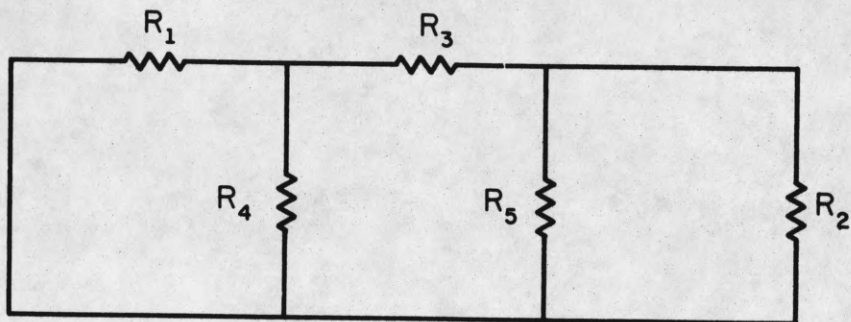
$$\Delta = R_1 R_2 R_3 \left[1\right] + R_1 R_2 \left[R_4 + R_5\right] + R_1 R_3 \left[R_5\right] +$$
$$R_2 R_3 \left[R_4\right] + R_1 \left[R_4 R_5\right] + R_2 \left[R_4 R_5\right] + R_3 \left[R_4 R_5\right] + O$$

(b)
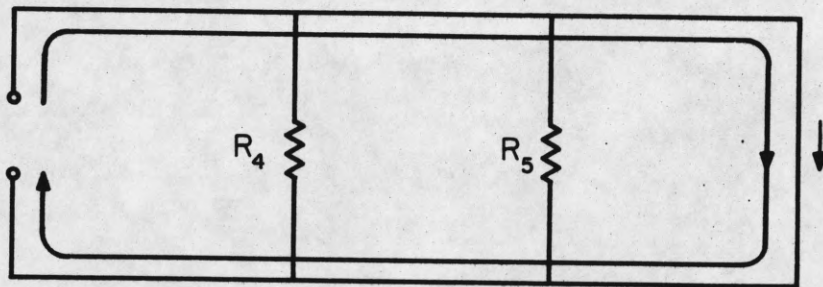
$$\Delta_{12} = R_4 R_5$$

(c)

Figure 2

Thus the expression for any branch current is obtained by algebraically summing the residual contributions of each path current flowing through the branch in question.

So far nothing has been said about the signs of these terms. For uniformity all path currents will be drawn through the source from the negative to the positive terminal. Branch current directions are assigned, where possible, to agree with the direction that a positive E would drive a positive current through a particular branch. Now the contribution of a single path current through the branch in question will be positive if its direction agrees with the branch current direction, negative otherwise.

The determination of path currents using the method of residual networks for passive R, L, C networks follows directly with all the L-switches and C-switches in a path under consideration necessarily being shorted and the determinant of the residual network calculated as above.

2. Extension to Active Networks

We now justify the techniques presented in Section I and extend them to active networks. The network to be considered has b branches, $b_1, b_2, \ldots, b_b$, and n nodes, $a_1, a_2, \ldots, a_m, a_n$ with node a_n to be taken as datum and where $m = n - 1$. Branch b_j will be denoted by j whenever the context is such that no confusion will arise.

We will utilize, in the analysis, the matrix equation[11]

$$\begin{bmatrix} O & A \\ A^T & R \end{bmatrix} \begin{bmatrix} E \\ I \end{bmatrix} = \begin{bmatrix} -I_s \\ E_s \end{bmatrix} \tag{6}$$

where E is the m x 1 node voltage matrix

I is the b x 1 branch current matrix

A is the m x b reduced incidence matrix

R is the b x b branch resistance matrix which for a passive net-work is diagonal; for an R, L, C, M network with dependent sources R is replaced by Z which is not diagonal in general.

Denoting the $(\text{m x b})^{\text{th}}$ order matrix by D we have

$$D = \begin{array}{c} \begin{array}{cccccccc} a_1 & a_2 & \cdots & a_m & & b_1 & b_2 & \cdots & b_b \end{array} \\ \begin{array}{c} a_1 \\ a_2 \\ \vdots \\ a_m \\ b_1 \\ b_2 \\ \vdots \\ b_b \end{array} \begin{bmatrix} O & A \\ A^T & R \end{bmatrix} \end{array} \tag{7}$$

We want to find, topologically, the network junction I_i / E_{b_1} which can be written as

$$\frac{I_i}{E_{b_1}} = \frac{\Delta^o_{b_1 i} \pm \sum_j \mu_j \Delta_{b_1 i_j d_j} \pm \cdots \pm \mu_1 \mu_2 \cdots \mu_s \Delta_{b_1 i_1 d_1 \cdots sd_s}}{\Delta^o \pm \sum_j \mu_j \Delta_{jd_j} \pm \sum_{j<k} \mu_j \mu_k \Delta_{jd_j kd_k} \cdots \pm \mu_1 \cdots \mu_s \Delta_{1d_1 \cdots sd_s}} \qquad (8)$$

where

Δ^o = $|\,\mathbb{D}\,|$ with all μ's in $\mathbb{D}$ (and in the network) set to zero;

$\Delta^o_{b_1 i}$ = $|\,\mathbb{D}_{b_1 i}|$ where row b_1 and column b_i are crossed out in $\mathbb{D}$ and all μ's set to zero;

Δ_{jd_j} = $|\,\mathbb{D}_{jd_j}|$ where row b_j and column d_j are crossed out in $\mathbb{D}$ and all μ-source (dependent source) voltages set to zero except $\mu_j I_{d_j}$; etc.

The ambiguity of signs as well as the signs of terms within each cofactor will be resolved in the sequel.

A. Determination of Δ^o

Δ^o = $|\,\mathbb{D}\,|$ with all dependent sources set to zero, (i.e. shorted) leaving a passive resistive network. Therefore $\mathbb{R}$ is diagonal and $\mathbb{D}$ can be expanded by Laplace's rule to yield the sum of co-tree products of the network. The details of this expansion will be omitted here since the topological determination of Δ^o has been discussed and justified in section I.

B. Determination of $E_1 \Delta^o_{b_1 i}$ and $\mu_j \Delta_{jd_j}$

$\Delta^o_{b_1 i}$ = $|\,\mathbb{D}_{b_1 i}|$ with all dependent sources set to zero in the network and therefore corresponds to the numerator of the network junction I_{i/E_1} for a purely passive network. Without loss of generality it will be assumed that the desired network junction is I_{2/E_1} and the cofactor to be found is Δ_{12}.

It is apparent that the analysis for $E_1 \Delta_{12}$ applies equally well to the denominator term $\pm \mu_1 \Delta_{12}$ where the independent source and all dependent sources

have been set to zero except $\mu_1 I_2$. Thus in general, the analysis to follow applies to the term $\pm \mu_j \Delta_{jd_j}$ which arises from a single dependent source in branch b_j depending on current in branch $d_j = b_k$ for some k.

As indicated above, a dependent source voltage will be written as $\mu_j I_{d_j}$ which means that μ_j effectively has units of resistance. It is assumed for simplicity that R_j and R_{d_j} are nonzero although the theory will hold if either or both are zero. Also the voltage $\mu_j I_{d_j}$ is to appear in branch b_j preceded by a plus sign with the voltage reference signs to be adjusted accordingly.

We expand $|\mathbb{D}_{12}|$ by Laplace's rule by first crossing out row b_1 and column b_2 of $\mathbb{D}$. Using the first m rows of $\mathbb{D}_{12}$ (i.e. all the rows of $\mathbb{A}$) we consider all minors D_α of order m that can be found from the rows and columns of $\mathbb{A}^{(2)}$ which is $\mathbb{A}$ with column b_2 crossed out. There are $\binom{b_j}{m-1}$ possible minors. Since the rows $a_1, a_2, \ldots, a_m$ are the same for all such minors we denote a particular minor D_α by the ordered m-tuple of its column numbers as given in $\mathbb{D}$ taken in order of increasing magnitude. Therefore $D_\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_m)$ where $\alpha_i = b_j$ for some j and is actually the $(m + \alpha_i)^{th}$ column of $\mathbb{D}$; also row a_i of D_α is the ith row of $\mathbb{D}$.

For each minor D_α there is a corresponding algebraic complement $(-1)^\eta D^\alpha$ obtained from $\mathbb{D}$ by crossing out the rows and columns chosen in D_α, where η is the sum of the numbers of the rows and columns contained in D_α. Therefore

$$|D_{12}| = (-1)^{b_1 + b_2} \Sigma (-1)^\eta D_\alpha D^\alpha . \tag{9}$$

Now

$$D^\alpha = |[\mathbb{A}^T]_{(1)} \vdots \mathbb{R}^\alpha|$$

where $\mathbb{R}^\alpha$ is obtained by crossing out row b_1, column b_2 and columns $\alpha_1, \alpha_2, \ldots, \alpha_m$ of $\mathbb{R}$. It is more convenient to work with

$$D^\alpha = \begin{vmatrix} A^{(1)} \\ \hline \mathbb{R}_\alpha \end{vmatrix}$$

where $\mathbb{R}_\alpha = [\ \mathbb{R}^\alpha]^T$ and can be obtained from $\mathbb{R}$ by crossing out column b_1, row b_2, and rows $\alpha_1, \alpha_2, \ldots, \alpha_m$. Applying Laplace's rule to the expansion of D^α considering all minors D_β of order m obtained from $A^{(1)}$ we have

$$D^\alpha = \sum_\beta (-1)^\nu D_\beta R_\alpha^\beta$$

where $D_\beta = (\beta_1, \beta_2, \ldots, \beta_m)$ and R_α^β is the minor of $\mathbb{R}$ formed by crossing out rows b_2, $\alpha_1, \alpha_2, \ldots, \alpha_m$ and columns b_1, $\beta_1, \beta_2, \ldots, \beta_m$. As before, ν is the sum of the numbers of the rows and columns contained in D_β. Therefore

$$\Delta_{12} = (-1)^{b_1+b_2} \sum (-1)^{\eta+\nu} D_\alpha D_\beta R_\alpha^\beta \tag{10}$$

Since D_α and D_β are m^{th} order determinants taken from the reduced incidence matrix and R_α^β is not diagonal in general for arbitrary choices of D_α and D_β, some of the terms in this expansion will be zero. Considering just one term in this summation we have

<u>Proposition 1.</u> $D_\alpha D_\beta R_\alpha^\beta$ is nonzero only if D_α and D_β are m^{th} order minors which both correspond to trees of the network.

<u>Proposition 2.</u> $D_\alpha D_\beta R_\alpha^\beta$ is nonzero if and only if D_α corresponds to a tree and contains column b_1, D_β corresponds to a tree containing branch (column) b_2 and the remaining m-1 columns of D_α are identical to the last m-1 columns of D_β in the same order.

Proof: Crossing out row b_1 and column b_2 of $\mathbb{D}$ leaves a submatrix $\mathbb{R}_{12}$ which is not diagonal and has only resistive terms for its nonzero entries. The rows and columns of $\mathbb{R}_{12}$ which are subsequently eliminated appear as columns of D_β and D_α, respectively. Thus crossing out row b_2 and column b_1 leaves a diagonal submatrix whose determinant is therefore nonzero. Similarly for every row eliminated in $\mathbb{R}_{12}$, the corresponding column must also be crossed out if the resulting minor is to remain diagonal and therefore nonzero.

Q.E.D.

Letting $\alpha_1 = b_1$ and $\beta_1 = b_2$ we have

$$D_\alpha = (b_1, \alpha_2, \ldots, \alpha_m)$$

and

$$D_\beta = (b_2, \beta_2, \ldots, \beta_m)$$

where $\alpha_i = \beta_i$ for $i \neq 1$. Also R_α^β is a positive product of $(b-m-1)$ resistances which appear in the tree complements of D_α and D_β.

In the case of a μ-source depending on its own branch current, i.e., $\mu_1 I_1$, a term in the summation is nonzero if and only if D_α and D_β both correspond to trees and all their columns are identical. (This case can be regarded as passive with total resistance $R_1 \pm \mu_1$ in branch b_1.)

Now with these conditions and the fact that the original column numbers have been used to designate D_α we have

$$\eta = [a_1 + a_2 + \ldots a_m] + [(m + b_1) + (m + \alpha_2 - 1) + \ldots + (m + \alpha_m - 1)]$$

$$= \frac{m(m+1)}{2} + m^2 + b_1 + (\alpha_2 - 1) + (\alpha_3 - 1) + \ldots + (\alpha_m - 1) \tag{11}$$

since the column numbers of $\mathbb{D}_{12}$ have decreased by one after column b_2 because b_2 has been crossed out. Similarly

$$\nu = [a_1 + a_2 + \ldots a_m] + [(b_2 - 1) + (\beta_2 - 1) + \ldots + (\beta_m - 1)]$$

$$= \frac{m(m + 1)}{2} + (b_2 - 1) + (\beta_2 - 1) + \ldots + (\beta_m - 1) \tag{12}$$

since row b_1 has been crossed out and therefore the row numbers of D_β^T are decreased by one after row b_1 from their original numbers in $\mathbb{D}$. Thus since $\alpha_i = \beta_i$, $i = 2, 3, \ldots, m$ we have

$$(-1)^{\eta + \nu} = (-1)^{m(m + 1) + m^2 + b_1 + (b_2 - 1) + 2(\alpha_2 - 1) + 2(\alpha_3 - 1) + \ldots 2(\alpha_m - 1)}$$

$$= (-1)^{m^2} (-1)^{b_1 + b_2 - 1}$$

It is apparent that the factor $(-1)^{m^2}$ will appear in every term in both numerator and denominator of the network function and will therefore cancel. However, it is convenient for the rest of the discussion if m is assumed an even number which means $(-1)^{m^2} = +1$. Therefore the term becomes

$$(-1)^{b_1 + b_2} (-1)^{\eta + \nu} D_\alpha D_\beta R_\alpha^\beta = (-1)^{b_1 + b_2} (-1)^{b_1 + b_2 - 1} D_\alpha D_\beta R_\alpha^\beta$$

$$= -D_\alpha D_\beta R_\alpha^\beta \tag{13}$$

Now all that remains is to determine the sign of $D_\alpha D_\beta$. Since $D_\alpha = \pm 1$ and $D_\beta = \pm 1$ it is sufficient to determine the sign of D_β with respect to D_α.

Noting that $D_\alpha = (b_1, \alpha_2, \alpha_3, \ldots, \alpha_m)$ and $D_\beta = (b_2, \alpha_2, \alpha_3, \ldots, \alpha_m)$, when the two trees corresponding to these minors are coupled, a subnetwork is obtained which has exactly one loop containing both branch b_1 and branch b_2. Call this loop, J, and let its branches be labeled $b_1, b_2, \gamma_1, \gamma_2, \ldots, \gamma_p$ where $p \leq m-1$. The branches $\gamma_1, \gamma_2, \ldots, \gamma_p$ appear as columns in both D_α and D_β.

In D_β, add column γ_j $(j = 1, 2, \ldots, p)$ to column b_2 if branch currents I_{b_2} and I_{γ_j} have the same directions around the loop; subtract γ_j from b_2 otherwise. The result of these operations which don't change the value of D_β yields a column in position 1 which is plus or minus column b_1, the leading column of D_α. This first column of D_β equals minus b_1 if and only if I_{b_1} is in the same direction as I_{b_2} around the loop and in this case $D_\beta = -D_\alpha$ and $D_\alpha D_\beta = -1$; otherwise $D_\alpha = D_\beta \cdot D_\beta = +1$. Thus since $(-1)^{b_1 + b_2} (-1)^{\eta + \nu} = -1$, equation 13 becomes $+ R_\alpha^\beta$ if branch currents I_{b_1} and I_{b_2} have the same sense around loop J and is $-R_\alpha^\beta$ otherwise.

Now the method given in Section 1 follows directly from the above and the terms of $E_1 \Delta_{12}$ can be found topologically: open the terminals of E_1 and draw a path current through E_1 and R_2; set all elements in this path to zero (short-circuit them) and find the residual network terms by Feussner's method. The sign of these terms is plus if both I_1 and I_2 are in the same sense around the loop; otherwise the sign is minus. Since branch current directions are assigned in a natural way with respect to E_1 without loss of generality, the sign determination can be simplified. Draw the path current J from $-$ to $+$ through E_1 traversing branch b_2; the sign of the terms is positive if the path current direction agrees with the direction of I_2; otherwise the sign is negative. This eliminates the comparing of directions of the two branch currents I_1 and I_2.

Drawing all possible path currents through the opened terminals of E_1 traversing branch b_2 one at a time, finding the residual terms as above and adding these terms algebraically yields $E_1 \Delta_{12}^o$.

By replacing E_1 with $(-\mu_1)$ we have the denominator term $-\mu_1 \Delta_{12}$ due to a single dependent source $\mu_1 I_2$. This replacement is valid since, by the

convention of assigning branch current directions, $+E_1$ appears in position $d_{b_1 b_2}$ of $\mathbb{D}$ in the determination of I_2 by Cramer's rule whereas a voltage rise across the dependent source in the direction of positive I_1 appears in this same position as $-\mu_1$.

Thus the ambiguity of sign is eliminated in equation 8. That is, the sign preceding a cofactor in either numerator of denominator is + if the cofactor is multiplied by an even number of μ's and is minus otherwise.

C. Determination of $(-\mu_j)(-\mu_k)\Delta_{j d_j k d_k}$ and $E_1(-\mu_k)\Delta_{1 i k d_k}$

$$\Delta_{j d_j k d_k} = \left| \mathbb{D}_{j d_j k d_k} \right|$$

where all but two dependent source voltages, $\mu_j I_{d_j}$ and $\mu_k I_{d_k}$, have been set to zero in the network. There are various configurations for these two μ-sources:

1. both μ-sources in distinct branches, each depending on distinct branch currents;

2. both μ-sources in distinct branches, one depending on current in a distinct branch and the other depending on the branch current through either of the two μ-sources;

3. both μ-sources in distinct branches each depending on current in their own branches or on the current in each other's branch.

This cofactor is zero if both μ-sources are in the same branch since $j = k$ and both appear in the same row of $\mathbb{D}$. Also the cofactor is zero if both depend on current in the same branch since then $d_j = d_k$ and both μ_j and μ_k appear in the same column of $\mathbb{D}$.

Case 1: Again, without loss of generality, we can relabel the branches and consider $\mu_1 I_3$ and $\mu_2 I_4$ and the cofactor Δ_{1324} where rows b_1 and b_2 and columns b_3 and b_4 have been eliminated from $\mathbb{D}$. Expanding Δ_{1324} by Laplace's rule, we have as before

$$(-\mu_1)(-\mu_2)\Delta_{1324} = \mu_1\mu_2(-1)^{b_1 + b_2 + b_3 + b_4} \sum_{\alpha,\beta} (-1)^{\eta+\nu} D_\alpha D_\beta R_\alpha^\beta . \qquad (14)$$

The same reasoning used in Proposition 2 applies to this expansion also and we have

Proposition 3: $D_\alpha D_\beta R_\alpha^\beta$ is nonzero if and only if D_α corresponds to a tree and contains columns (branches) b_1 and b_2 and D_β corresponds to a tree which contains branches b_3 and b_4, the other $m-2$ columns of D_α and D_β being identical.

R_α^β now is a product of $(b-m-2)$ resistors which appear in the complement of D_α and D_β and

$$D_\alpha = (b_1, b_2, \alpha_3, \ldots, \alpha_m)$$

$$D_\beta = (b_3, b_4, \beta_3, \ldots, \beta_m)$$

where

$$\beta_i = \alpha_i = b_j \text{ for some } j \neq 1,2,3,4. \quad \text{Also}$$

$$\eta = \frac{m(m+1)}{2} + (m+b_1) + (m+b_2) + (m+\alpha_3-2) + \ldots + (m+\alpha_m-2) \qquad (15)$$

and

$$\nu = \frac{m(m+1)}{2} + (b_3-2) + (b_4-2) + (\beta_3-2) + \ldots + (\beta_m-2) . \qquad (16)$$

Therefore $(-1)^{\eta+\nu} = (-1)^{m^2} (-1)^{b_1 + b_2 + (b_3-2) + (b_4-2)}$ and since m is assumed even, we have

$$(-1)^{b_1 + b_2 + b_3 + b_4} (-1)^{\eta+\nu} D_\alpha D_\beta R_\alpha^\beta = + D_\alpha D_\beta R_\alpha^\beta . \qquad (17)$$

We again consider the sign of D_β with respect to D_α in the sign determination of $D_\alpha D_\beta$. Because of the characteristics of D_α and D_β, it is apparent that when their corresponding trees are coupled, a subnetwork containing two loops is obtained. Actually more than two loops may exist but there are only two with a single μ-source in each. Both branches b_3 and b_4 may appear in the same loop with a single μ-source but then only one of b_3 or b_4 can belong to the second loop. Branches b_1 and b_2 cannot both appear in two loops, and branches b_3 and b_4 cannot both appear in two loops. These last two possibilities have interesting interpretations when considered topologically as will be seen.

Therefore there are effectively two possibilities to consider for D_α and D_β nonzero:

(a) one loop, J_1, contains branches b_1 and b_3; the second loop, J_2, contains b_2 and b_4. Call this the normal case.

(b) one loop, J_1^p, contains branches b_1 and b_4; the second loop, J_2^p, contains b_2 and b_3. Call this the permuted case.

(a) Normal case: Loop J_1 contains branches b_1, b_3 and $\delta_1, \delta_2, \ldots, \delta_p$, $p \leq m-2$; loop J_2 contains branches b_2, b_4 and $\gamma_1, \gamma_2, \ldots, \gamma_q$, $q \leq m-2$ where $\delta_i = b_j$ for some $b_j \in \{b_4, \alpha_3, \alpha_4, \ldots, \alpha_m\}$, $i = 1, 2, \ldots, p$ and $\gamma_i = b_k$ for some $b_k \in \{b_3, \alpha_3, \alpha_4, \ldots, \alpha_m\}$ $i = 1, 2, \ldots, q$. It is possible that $\delta_i = b_4$ or $\gamma_i = b_3$, not both.

$$D_\alpha = (b_1, b_2, \alpha_3, \ldots, \alpha_m)$$

$$D_\beta = (b_3, b_4, \alpha_3, \ldots, \alpha_m)$$

In D_β add column δ_i $(i=1,2,\ldots,p)$ to column b_3 if the branch currents I_{b_3} and I_{δ_i} have the same sense in loop J_1; subtract it otherwise. The resulting column in the first position of D_β is the same as column b_1, the first column of D_α, if I_{b_3} and I_{b_1} have opposite senses in loop J_1 and is the negative of column b_1 otherwise. Similarly, adding or subtracting column γ_i $(i=1,2,\ldots,q)$ to column b_4 of D_β yields plus column b_2 in this second column position of D_β if I_{b_4} and I_{b_2} have opposite senses around loop J_2 and yields minus column b_2 otherwise.

Thus if the branch current pair I_{b_1}, I_{b_3} have the same direction around loop J_1 and the pair I_{b_2}, I_{b_4} have the same direction around loop J_2 or if both pairs have opposite senses in the respective loops, then $D_\alpha = D_\beta$, $D_\alpha D_\beta = +1$ and from equation 17 the resulting term is $+ R_\alpha^\beta$; otherwise $D_\beta = -D_\alpha$ and the term is $-R_\alpha^\beta$.

Applying the simplification discussed for the one μ-source case, the topological determination follows directly from the above analysis: open the terminals of both μ-sources and draw path current J_1 from - to + through μ_1, going through R_3 and path current J_2 from - to + through μ_2 and going through R_4; set all elements in both paths to zero and calculate the residual terms using Feussner's method. The sign of these terms is positive if J_1 traverses R_3 in the same direction as I_{b_3} and J_2 traverses R_4 in the same direction as I_{b_4} or if both J_1 and J_2 have opposite directions with respect to I_{b_3} and I_{b_4} respectively; otherwise the sign is negative. Drawing all possible path current pairs, J_1 and J_2, finding the residual terms corresponding to each pair and adding these set of terms algebraically yields terms in Δ_{1324}.

This procedure does not necessarily yield all the terms in Δ_{1324} as will be seen in the discussion of the permuted case. Therefore identify all terms obtained from this normal case as Δ_{1324}^N.

(b) Permuted Case: When the trees of D_α and D_β are coupled a sub-network with (at least) two loops is obtained which may associate branches b_1 and b_4 with loop 1 and branches b_2 and b_3 with loop 2. Therefore terms in $\Delta_{13,24}$ can be thought of as arising either from the μ-source pair $\mu_1 I_3$, $\mu_2 I_4$ or from the pair, $\mu_1 I_4$, $\mu_2 I_3$. In this case we write

$$D_\alpha = (b_1, b_2, \alpha_3, \ldots, \alpha_m)$$
$$D_\beta^P = (b_4, b_3, \alpha_3, \ldots, \alpha_m)$$

where the first and second columns of D_β have been permuted to yield D_β^P.

Now the procedure for comparing D_β^P with D_α is the same as in the normal case and the resulting sign of $D_\alpha D_\beta^P$ is exactly the negative of the sign of $D_\alpha D_\beta$ determined in the normal case since some of the columns of D_β have been permuted an odd number of times.

Thus the topological determination of these terms is effected by open-ing the terminals of both μ-sources and drawing a path current J_1^P from - to + through μ_1 traversing branch b_4 and a path current J_2^P from - to + through μ_2 traversing b_3; the rest of the procedure is the same as for the normal case. The result is a set of signed terms which will be denoted as Δ_{1324}^P and the effect of the (odd) permutation is accounted for by multiplying this "cofactor" by -1.

An alternate way of describing the set of terms obtained in this case is

$$- \Delta_{13,24}^P = + \Delta_{14,23}^N$$

and $\Delta_{14,23}^N$ can be determined from the normal case analysis associated with the μ-source pair of $\mu_1 I_4$ and $\mu_2 I_3$. (The sign of $(-1)^{\eta+\nu}$ changes if this idea is used and the two descriptions are seen to be equivalent.)

Thus a μ-path current can be thought of as a loop containing the μ-source and the branch from which it takes its dependence.

The sets of terms found in (a) and (b) constitute all of the terms in the cofactor. In the topological determination of Δ_{1324}, there is a possibility that the same term appears in both Δ^N_{1324} and Δ^P_{1324} but these terms will always cancel since this case arises when b_3 and b_4 appear in both loops J_1 and J_2 and thus appear in both J^P_1 and J^P_2 also with relative current directions remaining the same. This agrees with the analysis of $\Delta_{13,24}$ by Laplace's rule since this term has $D_\alpha D_\beta = 0$ and therefore doesn't appear in the expansion. Thus when both b_3 and b_4 appear in J_1 and in J_2 the resulting term can be neglected by inspection. The similar possibility of b_1 and b_2 appearing in two loops does not arise topologically if we agree that a path current due to one μ-source cannot travel through another μ-source (or E-source) because the terminals of the μ-sources have been opened.

For the dependent source configuration of Case 2, we consider $\Delta_{12,23}$ or $\Delta_{13,22}$ (recall $\Delta^N_{12,23} = - \Delta^P_{13,22}$). D_α now must contain column b_1, D_β must contain column b_3, and the remaining m-1 columns of D_α and D_β must be identical. In Case 3, only $\mu_1 I_1$, $\mu_2 I_2$ need be considered which results in the cofactor $\Delta_{11,22}$ $(\Delta^N_{12,21} = - \Delta^P_{11,22})$. In this case all columns of D_α and D_β must be identical. The analysis for both Case 2 and Case 3 is essentially the same as for Case 1.

Now we can find topologically the network determinant of a network containing two dependent sources:

$$\Delta = \Delta^\circ - \mu_1 \Delta_{13} - \mu_2 \Delta_{24} + \mu_1 \mu_2 [\Delta^N_{1324} - \Delta^P_{1324}] \tag{18}$$

where both $\mu_1 \equiv 0$ and $\mu_2 \equiv 0$ to find Δ°; $\mu_2 \equiv 0$ for the determination of $\mu_1 \Delta_{13}$, etc.

As in the one μ-source case, if $(-\mu_1)$ is replaced by $(+E_1)$ the above analysis applies directly to the terms in the numerator of the branch current I_3 which are multiplied by μ_2. Thus the network function I_{3/E_1} with one dependent voltage source, $\mu_2 I_4$, is given by

$$\frac{I_3}{E_1} = \frac{\Delta_{13}^o - \mu_2 \Delta_{1324}}{\Delta^o - \mu_2 \Delta_{24}} \tag{19}$$

The general case of s dependent voltage sources present in a network is treated in essentially the same manner as for two μ-sources. Consider the topological determination of

$$(-\mu_1)(-\mu_2)\ldots(-\mu_s)\,\Delta_{1d_1 2d_2 3d_3 \ldots s d_s} \tag{20}$$

where $\mu_1 I_{d_1}$, $\mu_2 I_{d_2}$, $\ldots$, $\mu_s I_{d_s}$ are the dependent source voltages. We will assume that $s \leq b - n + 1$ since for a network with n nodes and b branches, the network determinant consists of terms which are products of $b - n + 1$ elements.

To find $\Delta_{1d_1, 2d_2, \ldots, s d_s}$ all s! permutations of the branches $d_i (i=1,2,\ldots,s)$ have to be considered. Since there are $\frac{s!}{2}$ even and $\frac{s!}{2}$ odd permutations, let P_i denote an even permutation if i is even; an odd permutation otherwise. Thus

$$\Delta_{1d_1 2d_2 \ldots s d_s} = \sum_{i=0}^{s!-1} (-1)^i \, \Delta_{1d_1 2d_2 \ldots s d_s}^{P_i} \tag{21}$$

where

$$\Delta_{1d_1 \ldots s d_s}^{P_o} = \Delta_{1d_1 \ldots s d_s}^{N} \; .$$

There are a total of $s! \sum_{i=0}^{s!-1} N_i$ path current combinations that have to be checked where N_i is the number of sets of path current possibilities for the ith permutation of μ-source dependences.

The sign of a term within any of the permutation-cofactors is negative if an odd number of path currents traverse their respective permuted dependent branches in the opposite sense of the assumed branch current direction; otherwise it is positive.

By replacing $(-\mu_1)$ by E_1 we obtain the terms in the numerator function I_{d_1/E_1} which are multiplied by $\mu_2\mu_3...\mu_s$. Thus the complete topological determination of a network function I_{i/E_1} with a $s-1$ dependent voltage sources present is at hand. The proof for the general case is necessarily rather involved and will be omitted here.

CONCLUSION

The method given in this paper provides an easy means of analyzing network functions of arbitrary networks and facilitates the determination of the effect of any circuit element on the network. Specifically the effect of dependent sources can be ascertained immediately and independently without the necessity of calculating the entire network function.

Further investigation in this area would seem to lie in the direction of utilizing the insight and simplicity offered by this method in network synthesis.

APPENDIX

The following example clarifies the arguments given above and illustrates the simplification effected by this method.
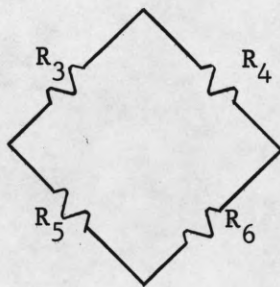
<u>Example</u>
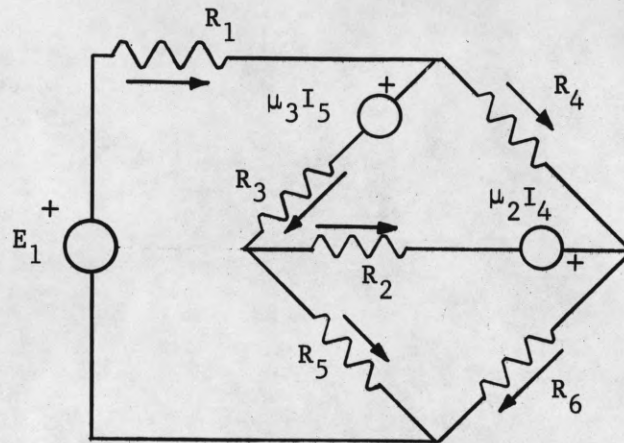
We wish to find the network function

$$Y_{16} = \frac{I_6}{E_1} = \frac{\Delta^o_{16} - \mu_2\Delta_{1624} - \mu_3\Delta_{1635} + \mu_2\mu_3\Delta_{162435}}{\Delta^o - \mu_2\Delta_{24} - \mu_3\Delta_{35} + \mu_2\mu_3\Delta_{2435}}$$
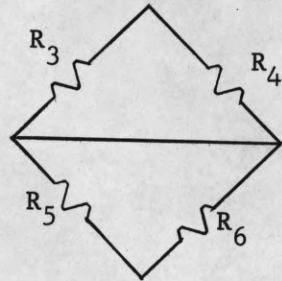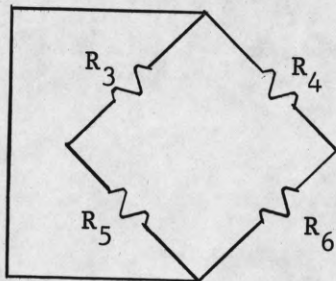
Δ^o:

Expand by R_1 and R_2

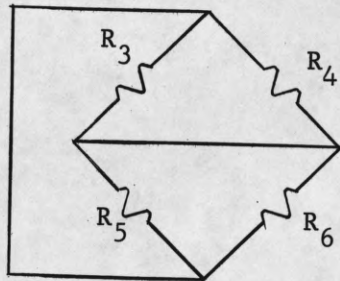$$\Delta^o = R_1R_2\Delta_{(12)} + R_1\Delta_{(12')} + R_2\Delta_{(1'2)} + \Delta_{(1'2')}$$

$$\Delta_{(12)} = R_3 + R_4 + R_5 + R_6$$
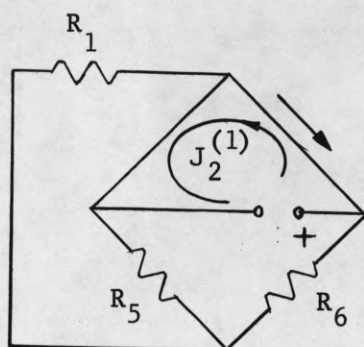
$$\Delta_{(12')} = (R_3 + R_4)(R_5 + R_6)$$

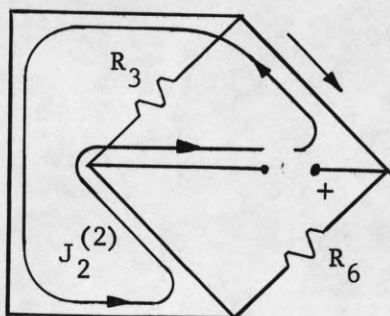$$\Delta_{(1'2)} = (R_3 + R_5)(R_4 + R_6)$$

$$\Delta_{(1'2')} = R_3[R_4(R_5 + R_6) + R_5 R_6] + R_4 R_5 R_6$$

Therefore

$$\Delta^o = R_1 R_2(R_3 + R_4 + R_5 + R_6) + R_1(R_3 + R_4)(R_5 + R_6) + R_2(R_3 + R_5)(R_4 + R_6)$$
$$+ R_3 R_4(R_5 + R_6) + R_5 R_6(R_3 + R_4)$$
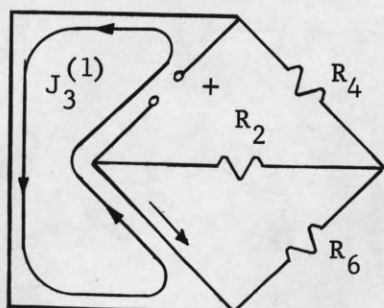
$-\mu_2 \Delta_{24}$:

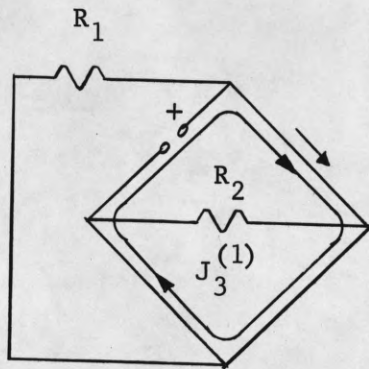$$\Delta_{24}^{(1)} = -[R_1(R_5 + R_6) + R_5 R_6]$$

$$\Delta_{24}^{(2)} = -R_3 R_6$$

Therefore

$$-\mu_2 \Delta_{24} = \mu_2[R_1(R_5 + R_6) + R_6(R_3 + R_5)]$$

$-\mu_3 \Delta_{35}$:

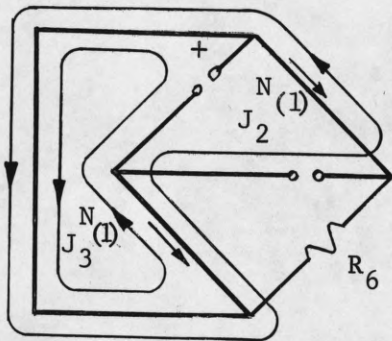

$$\Delta_{35}^{(1)} = -[R_2(R_4 + R_6) + R_4 R_6]$$

$$\Delta_{35}^{(2)} = - R_1 R_2$$

Therefore

$$-\mu_3 \Delta_{35} = + \mu_3 [R_2(R_1 + R_4) + R_6(R_2 + R_4)]$$

$\mu_2 \mu_3 \Delta_{2435}$:

$$\Delta_{2435}^{N(1)} = + R_6$$
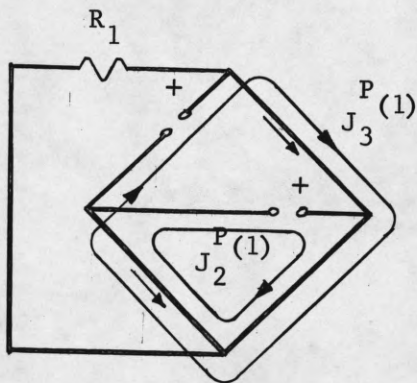
$$(-1)^1 \Delta_{2435}^{P(1)} = (-1)(-R_1) = R_1$$

Therefore

$$\mu_2 \mu_3 \Delta_{2435} = \mu_2 \mu_3 (R_1 + R_6)$$

Δ_{16}^{o} :
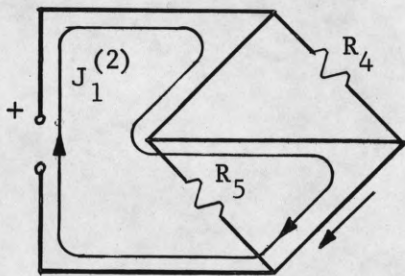
$$\Delta_{16}^{(1)} = R_2(R_3 + R_5) + R_3R_5$$

$$\Delta_{16}^{(2)} = + R_4R_5$$
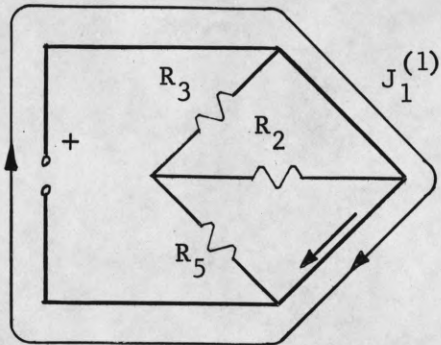
Therefore

$$\Delta_{16}^{o} = R_2(R_3 + R_5) + R_5(R_3 + R_4)$$

$-\mu_2\Delta_{1624}$:
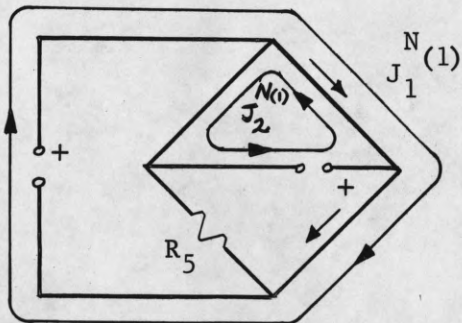
$$\Delta_{1624}^{N(1)} = - R_5$$
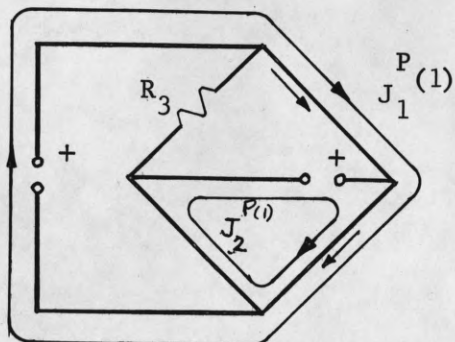
$$(-1)^1 \; \Delta^{P(1)}_{1624} = (-1) \; R_3$$

Therefore

$$-\mu_2 \Delta_{1624} = \mu_2 [R_3 + R_5]$$

$-\mu_3 \Delta_{1635}$:

$$\Delta^{N(1)}_{1635} = - R_2$$
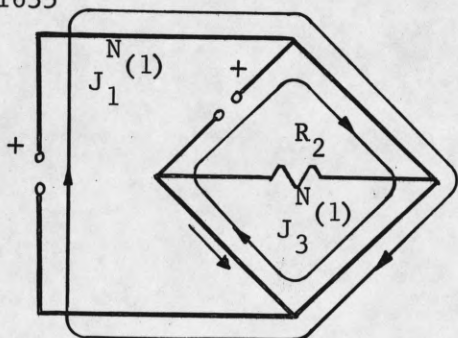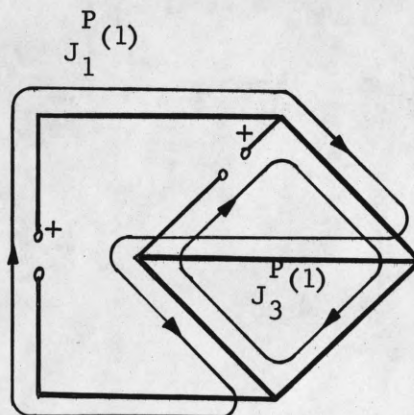
$$(-1) \; \Delta^{P(1)}_{1635} = 0$$

$$-\mu_3 \Delta_{1635} = + \mu_3 R_2$$

$\mu_2 \mu_3 \Delta_{162435}$:

By inspection $\quad \Delta^{N}_{162435} = \Delta^{P_4}_{162435} = \Delta^{P_5}_{162435} = 0$
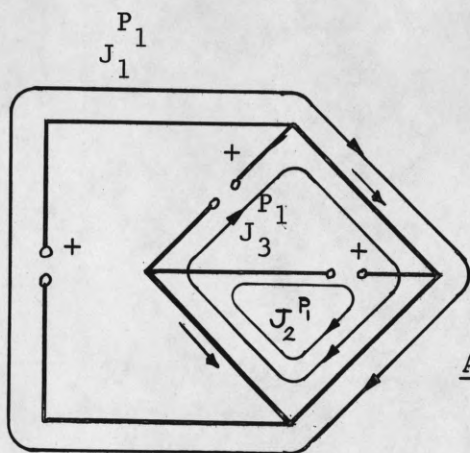
$$(-1)^1 \, \Delta_{162435}^{P_1} = \Delta_{142635}^{N} = (-1)(-1) = 1$$

$$= (-1)^1 \, \Delta_{162435}^{P_3} = \Delta_{162534}^{N} = (-1)(-1) = 1$$

Also

$$(-1)^2 \, \Delta_{162435}^{P_2} = \Delta_{142536}^{N} = (-1)^2(-1) = -1$$

Therefore

$$\mu_2\mu_3\Delta_{162435} = \mu_2\mu_3[1 + 1 - 1] = \mu_2\mu_3$$

Finally

$$\frac{I_6}{E_1} = \frac{R_2(R_3 + R_5) + R_5(R_3 + R_4) + \mu_2(R_3 + R_5) + \mu_3 R_2 + \mu_2\mu_3}{\Delta^0 + \mu_2[R_1(R_5+R_6) + R_6(R_3+R_5)] + \mu_3[R_2(R_1+R_4) + R_6(R_2+R_4)] + \mu_2\mu_3(R_1+R_6)}$$

ACKNOWLEDGMENT

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13. ABSTRACT

This paper presents a generalization of Fuessner's method for the analysis of active networks. This method makes it possible to determine system functions of any network in a simple and straightforward manner.

In the usual topological analysis of networks it is necessary to list all possible trees or co-trees and the signs of their tree admittance products as well as determining the set of 2-tree products and their signs for the network function in question. Usually two graphs, a current and a voltage graph, are used to implement this process when dependent sources are present in the network. The method presented here utilizes solely the original topology and the assigned branch current directions to determine these terms and their sign. Justification of the technique is given for the cases of one and two dependent sources present in a network with the summary of the method for each case. The analysis of a network with an arbitrary number of dependent sources is seen to follow directly. Finally an example of a network with two dependent sources is given which illustrates the method.

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It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as (TS), (S), (C), or (U).

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