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STABILITY OF NONLINEAR CIRCUITS WITH PERIODIC INPUTS

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Abstract

A new method is presented by which the stability of nonlinear circuits containing bounded periodic sources can be determined. By stability we mean that the steady state output is unique and periodic with the same period as the input and all transients decay to this unique steady state solution. It is assumed that the first and second derivatives of the nonlinear function exist and are continuous within a certain allowable operating range of the nonlinear element. The first derivative should be positive at the bias point (not always necessary). A sufficient condition for the stability of the above circuit is given in terms of a maximum allowable input amplitude. However, it is shown that, given a certain upper bound on the input amplitude, the requirement that all transients decay to this unique steady state solution is too stringent and results in many computational problems. Therefore, a small perturbation approach is adopted which results in fewer computations and less stringent conditions on the input amplitude. Experimental results indicate that this new approach is much better than previous results.

I. INTRODUCTION

In previous work [1,2] lumped circuits containing one nonlinear element and bounded periodic sources with period T were discussed. More nonlinear elements [1] and the distributed circuit [3] can be handled too. It was assumed that the first and second derivatives of the nonlinear function existed in a certain allowable dynamic range of the nonlinear device; and in some problems it was necessary that the first derivative be positive at the bias point. In the above circuit sufficient conditions were given such that the steady state response would be unique in the space of bounded measurable functions of period T, the period of the sources. This was done by assuming that for very small inputs the circuit is essentially linear, and thus, for very small inputs there exists a unique bounded steady state solution of period T. This assumption is indeed justified [3]. An upper bound on the amplitude of the input voltage was then determined below which there existed a unique steady state solution in the space of bounded measurable function of period T. Unfortunately, it was shown by experiment that the existence of a unique steady state solution in the space of bounded measurable functions of period T does not rule out the possibility of other solutions, such as, a solution in the space of bounded measurable functions of period 2T, i.e., subharmonics. In other words the steady state solution may be unstable. This result has led to the current problem, that is, the stability of the unique steady state solution with period T.

II. STABILITY RESULTS

The circuit under consideration is illustrated in Figure 1, and its Thevenin equivalent is shown in Figure 2. The voltage $e_p(t)$ is bounded and periodic with period T_1, and the voltage $e_s(t)$ is bounded and periodic with period T_2, where T_1/T_2 is rational. Therefore, e(t), the Thevenin equivalent voltage, is bounded and periodic with period T, where T is the least common multiple of T_1 and T_2. We will only consider the case in which the nonlinear element is capacitive, however, the nonlinear resistance or the nonlinear inductance (without hystersis) case can be handled in a similar manner [1].

Due to the aforementioned properties of the nonlinear element we can write [1]

$$v(q) = v(q_o) + \frac{1}{C_\ell} q_v(t) + R(q_v(t)) \quad , \tag{1}$$

where q_o is the charge at the bias point, $q(t) = q_v(t) + q_o$, and

$$R(q_v(t)) = \int_{q_o}^{q(t)} v''(\alpha) \, [q(t)-\alpha] d\alpha \; .$$

Assuming that the d-c bias is turned on first and all bias transients have decayed to zero, an a-c equivalent model in the zero state can be obtained, Figure 3. In previous work [1,2] it was assumed that the steady state solution to the circuit in Figure 3 satisfies the equation

$$q_p(t) = \int_{-\infty}^{t} h(t-\tau)[e_v(\tau) - R(q_p(\tau))]d\tau + \underline{C}[e_v(t) - R(q_p(t))] \quad , \tag{2}$$

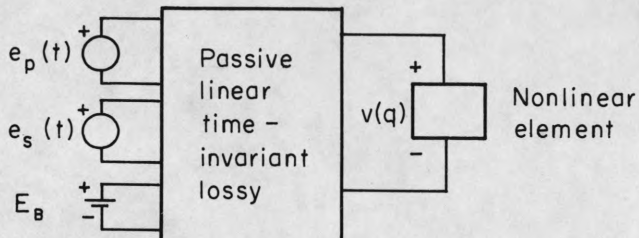

Fig. 1. Nonlinear Circuit with Periodic Inputs

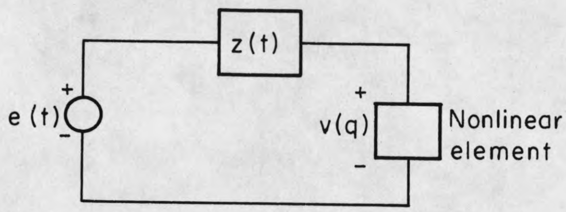

Fig. 2. Thevenin Equivalent Circuit

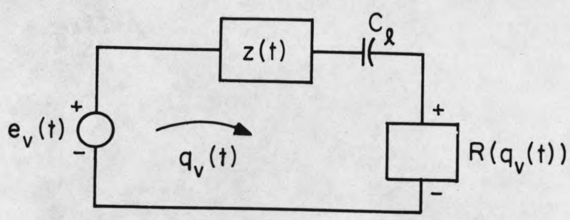

Fig. 3. A-C Zero State Model

where

$$\underline{C} + H(s) = \frac{1}{s}\left[Z(s) + \frac{1}{C_{\ell} s}\right]^{-1} ,$$

and H(s) is the Laplace transform of an absolutely integrable function. It will be assumed that the solution including transients satisfies the equation

$$q_t(t) = \int_a^t h(t-\tau)[e_v(\tau) - R(q_t(\tau))]d\tau + \underline{C}[e_v(t) - R(q_t(t))]. \qquad (3)$$

In the transient case the Thevenin equivalent voltage, $e_v(t)$, is in general not periodic of period T, since it contains transient terms. For the sake of simplicity we will assume that in the interval $a \leq t \leq \infty$ the Thevenin equivalent voltages in the steady state problem and in the transient problem are equal and periodic of period T. In the Appendix the transient terms of $e_v(t)$ are included, and it is shown that similar results can be obtained.

The problem now is to determine just how large an input voltage $e_v(t)$ can be tolerated such that $q_t(t)$ asymptotically approaches $q_p(t)$ as $t \to \infty$, and such that the nonlinear element is operated in its allowable dynamic range. Taking the difference between Eqs. (2) and (3) we have

$$q_t(t) - q_p(t) = \delta q(t) = M(t) + \int_a^t h(t-\tau)[R(q_p(\tau) - R(q_t(\tau))]d\tau$$
$$+ \underline{C}[R(q_p(t)) - R(q_t(t))] \qquad (4)$$

where

$$M(t) = \int_{-\infty}^{a} h(t-\tau)[e_v(\tau) - R(q_p(\tau))]d\tau .$$

Expanding $R(q_t(t))$ about $q_p(t)$ by means of Taylor's theorem we obtain

$$R(q_t(t)) = R(q_p(t)) + R'(q_p(t))\delta p + N(\delta q) , \qquad (5)$$

where

$$N(\delta q) = \int_{q_p}^{q_p+\delta q} R''(\alpha)[q_p+\delta p-\alpha]d\alpha \ .$$

Because of the assumed differentiability properties of the nonlinear element, the quantity $N(\delta q)$ is well defined in the allowable dynamic range of the device.

The substituion of Eq. (5) into Eq. (4) results in

$$\delta q + \int_a^t h(t-\tau)R'(q_p(\tau))\delta q + \underline{C}\ R'(q_p(t))\delta q = M(t)$$

$$- \int_a^t h(t-\tau)N(\delta q)d\tau - \underline{C}\ N(\delta q) \ . \qquad (6)$$

In previous results [1,2] upper bounds on $q_p(t)$ and $R'(q_p(t))$ were determined for a given input voltage such that the nonlinear element was operated in its allowable dynamic range, and such that there existed a unique steady state solution to Eq. (2) in the space of bounded measurable functions of period T. Thus, in the above sense, we will assume that $q_p(t)$ is known. With the above information it follows that

$$|M(t)| \leq \sup_{\text{all } t} |e_v(t)-R(q_p(t))| \int_{t-a}^{\infty} h(\lambda)d\lambda \ . \qquad (7)$$

Hence, M(t) asymptotically approaches zero as $t \to \infty$.

The usual sequence of iterates is defined as

$$\delta q^{(1)} + \int_a^t h(t-\tau)R'(q_p(\tau))\delta q^{(1)} + \underline{C}\ R'(q_p(t))\delta q^{(1)} = M(t), \qquad (8)$$

and

$$\delta q^{(n)} + \int_a^t h(t-\tau)R'(q_p(\tau))\delta q^{(n)} d\tau + \underline{C}\ R'(q_p(t))\delta q^{(n)} = M(t)$$

$$- \int_a^t h(t-\tau)N(\delta q^{(n-1)})d\tau - \underline{C}\ N(\delta q^{(n-1)}) \ . \qquad (8)$$

The difference between successive iterates is

$$\delta q^{(n+1)} - \delta q^{(n)} + \int_a^t h(t-\tau)R'(q_p(\tau))(\delta q^{(n+1)} - \delta q^{(n)})d\tau + \underline{C}R'(q_p(t))(\delta_q{}^{n+1} - \delta q^n)$$

$$= - \int_a^t h(t-\tau)[N(\delta q^n) - N(\delta q^{(n-1)})]d\tau - \underline{C}[N(\delta q^{(n)}) - N(\delta q^{(n-1)})] \ . \qquad (10)$$

Taking the supremum of Eq. (10) we have

$$\{1 - \|R'(q_p(t))\| \ [\underline{C} + \int_o^\infty |h(t)|dt]\} \|\delta q^{(n+1)} - \delta q^{(n)}\|$$

$$\leq [\underline{C} + \int_o^\infty |h(t)|dt] \ \|N(\delta q^{(n)}) - N(\delta q^{(n-1)})\|$$

$$\leq [\underline{C} + \int_o^\infty |h(t)|dt] \ \|N'(\delta q)\| \ \|\delta q^{(n)} - \delta q^{(n-1)}\| \ , \qquad (11)$$

where

$$\|N'(\delta q)\| = \max_{|\delta q| \leq \Delta\bar{Q}} [R'(q_p + \delta q) - R'(q_p)] \ , \qquad (12)$$

and $\Delta\bar{Q}$ must be chosen so that the above iterates converge. Sufficient conditions for the convergence of the iterates are as follows:

(1) $$r_1 = \|R'(q_p(t))\| \ [\underline{C} + \int_o^\infty |h(t)|dt] < 1 \ , \qquad (13)$$

and

(2) $$r_2 = \frac{\|N'(\delta q)\| \ [\underline{C} + \int_o^\infty |h(t)|dt]}{1 - r_1} < 1 \ . \qquad (14)$$

If the above conditions are satisfied, then it follows that

$$\|\delta q(t)\| \leq \left[\frac{\|M(t)\|}{1-r_1}\right]\left[\frac{1}{1-r_2}\right] = \frac{\|M(t)\|}{1-r_1-\|N'(\delta q)\|[\underline{C}+\int_0^\infty |h(t)|dt]} \quad . \tag{15}$$

Note that condition (13) can always be satisfied by making $q_p(t)$ small enough ($R'(q_p)$ is continuous and $R'(0) = 0$), which amounts to restricting the amplitude of $e_v(t)$. Also, condition (14) can be satisfied by restricting $\delta q(t)$ to small enough values (likewise $N'(\delta q)$ is continuous and $N'(0) = 0$). Equation (15) shows that this can be done by making $\|M(t)\|$ small, and from Eq. (7) we see that this merely amounts to again making $e_v(t)$ "small enough."

As one may have guessed by now, the computations involved are very difficult, and in an example which follows shortly it will be shown that these sufficient conditions are too conservative. These two problems led to the development of the small perturbation theory below, where it will be shown that only condition (13) need be satisfied.

III. STABILITY TO SMALL PERTURBATIONS OF THE INPUT

Recall that the steady state integral equation is

$$q_p(t) = \underline{C}[e_v(t)-R(q_p(t))] + \int_{-\infty}^{t} h(t-\tau)[e_v(\tau)-R(q_p(\tau))]d\tau \;.$$

Now let's assume that the input is $e_v(t)-\Delta e(t)$ and the steady state integral equation can be written as

$$q_p(t)+\Delta q_p(t) = \underline{C}[e_v(t)-\Delta e(t)-R(q_p(t)+\Delta q_p(t))] + \int_{-\infty}^{t} h(t-\tau)[e_v(\tau)-\Delta e(\tau)-R(q_p(\tau)+\Delta q_p(\tau))]d\tau \;. \tag{16}$$

Suppose that in the time interval $-\infty < t \leq a$ the input is $e_v(t)-\Delta e(t)$, and for $t > a$ the input is $e_v(t)$. Thus, for $t > a$,

$$q_t(t) = \underline{C}[e_v(t)-R(q_t(t))] + \int_{-\infty}^{a} h(t-\tau)[e_v(\tau)-\Delta e(\tau)-R(q_p(\tau)+\Delta q_p(\tau))]d\tau + \int_{a}^{t} h(t-\tau)[e_v(\tau)-R(q_t(\tau))]d\tau \;. \tag{17}$$

We wish to find conditions on $e_v(t)$ such that $q_t(t)$ asymptotically approaches $q_p(t)$ as $t \to \infty$, in other words, the circuit is stable for small perturbations in the input voltage. Thus, for $t > a$,

$$q_t(t)-q_p(t) = \delta q(t) = \underline{C}[R(q_p(t))-R(q_t(t))] + \int_{-\infty}^{a} h(t-\tau)[-\Delta e(\tau) - R(q_p(\tau)+\Delta q_p(\tau))+R(q_p(\tau))]d\tau + \int_{a}^{t} h(t-\tau)[-R(q_t(\tau))+R(q_p(\tau))]d\tau. \tag{18}$$

Again using Taylor's theorem to expand $R(q_p(t)+\Delta q_p(t))$ and $R(q_t(t))$ about the time-varying bias $q_p(t)$ we obtain

$$R(q_p(t)+\Delta q_p(t)) = R(q_p(t)) + R'(q_p(t))\Delta q_p(t)+N_1(\Delta q_p(t)) , \qquad (19)$$

where

$$N_1(\Delta q_p(t)) = \int_{q_p}^{q_p+\Delta q_p} R''(\alpha)[q_p+\Delta q_p-\alpha]d\alpha ,$$

and

$$R(q_t(t)) = R(q_p(t)) + R'(q_p(t))\delta q(t) + N_2(\delta q(t)), \qquad (20)$$

where

$$N_2(\delta q(t)) = \int_{q_p}^{q_t(t)} R''(\alpha)[q_t(t)-\alpha]d\alpha .$$

Thus,

$$\delta q(t) + \underline{C}\, R'(q_p(t))\delta q(t) + \int_a^t h(t-\tau)R'(q_p(\tau))\delta q(\tau)\,d\tau$$

$$= M_1(t) - \underline{C}\, N_2(\delta q(t)) - \int_a^t h(t-\tau)N_2(\delta q(\tau))\,d\tau , \qquad (21)$$

where

$$M_1(t) = \int_{-\infty}^{a} h(t-\tau)[-\Delta e(\tau)-R'(q_p(\tau))\Delta q_p(\tau)-N_1(\Delta q_p(\tau))]d\tau . \qquad (22)$$

Note that Eq. (21) is very similar to Eq. (6) except that the terms $M(t)$ and $M_1(t)$ are slightly different, however, both terms go to zero as $t \to \infty$. Thus, the sufficient conditions for a sequence of iterates to converge for Eq. (21) are the same as the conditions for Eq. (6), except that in Eq. (15) $\|M(t)\|$ is replaced by $\|M_1(t)\|$.

Now, from previous results [1,2] it follows that one can always restrict the amplitude of $e_v(t)$ such that condition (1), inequality (13), is satisfied.

From the definition of $N_2(\delta q)$ we see that $N_2(0) = 0$, and $N_2(\delta q)$ is continuous in $\delta q(t)$. Hence, a number $\Delta\bar{Q}$ can be found such that if $\|\delta q(t)\| \leq \Delta\bar{Q}$, then condition (2), inequality (14), is satisfied. However, if $r_1 < 1$, and $r_2 < 1$, then

$$\|\delta q(t)\| \leq \Delta\bar{Q} = \frac{\|M_1(t)\|}{(1-r_1)(1-r_2)} \ . \tag{23}$$

One now merely needs to observe that for a given $\Delta\bar{Q}$, r_1, and r_2, the right side of Eq. (23) can be made equal to $\Delta\bar{Q}$ by making the perturbation in the input voltage, $\Delta e(t)$, small enough (see Eqs. (22), (16), (2)). Thus, the problem now has been reduced to satisfying only condition (13), that is,

$$r_1 = \|R'(q_p(t))\| \, [\underline{C} + \int_0^\infty |h(t)|dt] < 1 \ .$$

Note that this computation is easier than that of Leon and Anderson [3], since we are working with the steady state solution $q_p(t)$. Also, it gives better results as the following example will show.

IV. EXPERIMENTAL RESULTS

Consider the subharmonic oscillator circuit in Figure 4. It is well-known that this circuit can sustain subharmonic oscillations of order 1/2 when driven at twice the small signal resonant frequency of the circuit. Note that the subharmonic state is an unstable state according to our definition of stability. Thus, let's assume that the source frequency is twice the above resonant frequency. The voltage across the varactor diode is

$$v(q(t)) = \frac{-q^2(t)}{4K^2} + \phi \ , \tag{24}$$

where $K = 30 \times 10^{-12}$, and $\phi = 0.5$. Using Taylor's theorem we obtain

$$v(q(t)) = -E_B + \frac{1}{C_\ell} q_v(t) + R(q_v(t)) \ , \tag{25}$$

where

$$\frac{1}{C_\ell} \triangleq \frac{-q_o}{2K^2} \ ,$$

$$q_v(t) = q(t) - q_o \ ,$$

and

$$R(q_v(t)) = -\frac{1}{4K^2} q_v^2(t) \ .$$

Let's assume that the Q of the circuit is 10 and that the varactor is biased such that $C_\ell = 20$ pf. The a-c equivalent circuit is shown in Figure 5. From this circuit we see that

$$H(s) = \frac{1/L}{s^2 + R/Ls + \frac{1}{LC_\ell}} \ , \text{ and } \underline{C} = 0 \ .$$

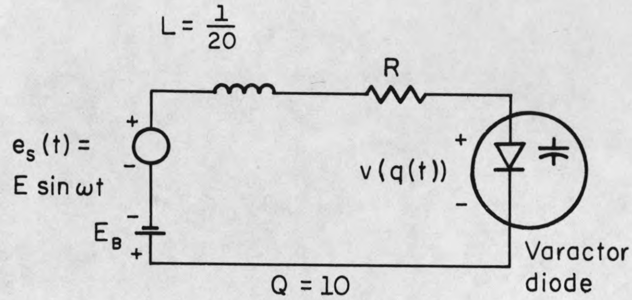

Fig. 4. Subharmonic Oscillator

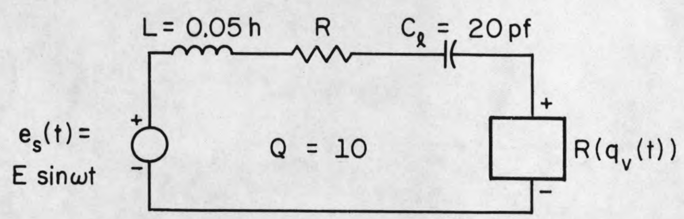

Fig. 5. A-C Equivalent Circuit

Thus,

$$\int_{o}^{\infty} |h(t)|dt \approx C_{\ell} \left[\frac{1 + e^{-\pi/2Q}}{1 - e^{-\pi/2Q}} \right] = 255 \times 10^{-12} \quad . \tag{26}$$

Using the rigorous stability conditions (13) and (14) it was found by trial and error (and very tedious calculations) that the maximum input amplitude was obtained when $r_1 = 0.1$ and $r_2 = 0.5/0.9$. Under this condition

$$\|R'(q_p(t))\| = \frac{0.1}{2.55 \times 10^{-12}} \quad , \tag{27}$$

and so,

$$\|q_p(t)\| = .706 \times 10^{-12} \text{ coulomb} \; . \tag{28}$$

With this norm (28) one can now use the steady state results of previous work [1,2] to compute the upper bound on the input amplitude E. In the steady state work

$$d = [\sum_{m=-\infty}^{\infty} |H(jm\omega_o)|^2]^{1/2} \; \|R'(q_p(t))\| \approx \frac{\pi}{\sqrt{8}} C_{\ell} \left[\frac{0.1}{255 \times 10^{-12}} \right] = .0083, \tag{29}$$

hence the norm of the 1st iterate is

$$\|q_p^1(t)\| = (1-d)\|q_p(t)\| = 0.7 \times 10^{-12} \text{ coulomb.} \tag{30}$$

Recall now that $q_p^1(t)$ is just the steady state solution to the linearized problem. One easily obtains that if

$$\max |E| < 0.105 \text{ volts,} \tag{31}$$

then

$$\max |q_p^1(t)| \leq 0.7 \times 10^{-12} \text{ coulomb.}$$

However, r_2 also imposes an upper bound on E. One finds that

$$\|N'(\delta q)\| = \frac{0.5}{255 \times 10^{-12}} , \tag{32}$$

and from Eq. (5) it follows that

$$\|\delta q\| = 3.52 \times 10^{-12} \text{ coulomb} . \tag{33}$$

The norm $\|M(t)\|$ can be computed from Eq. (15) so that if

$$\|M(t)\| \leq 1.41 \times 10^{-12} , \tag{34}$$

then

$$\|\delta q(t)\| \leq 3.52 \times 10^{-12} \text{ coulomb} .$$

Observe that

$$\|M(t)\| = \| \int_{-\infty}^{a} h(t-\tau)[e_v(\tau) - R(q_p(\tau))]d\tau\|$$

$$\leq \| \int_{-\infty}^{a} h(t-\tau) e_v(\tau) d\tau\| + \|Rq_p(t)\| \int_{0}^{\infty} |h(t)|dt .$$

$$\leq \frac{2|E|}{2L\,\omega_d^{\,2}} + 255 \times 10^{-12} \|R(q_p(t))\| \tag{35}$$

If we set Eq. (35) equals to 1.41×10^{-12} coulomb so that inequality (34) is satisfied, then

$$\max |E| = 1.41 \times 10^{-12} - \frac{255 \times 10^{-12} (.706 \times 10^{-12})^2}{4 \times 900 \times 10^{-24}} \; \frac{3 \times 10^{12}}{40}$$

$$= 0.103 \text{ volt} . \tag{36}$$

Hence, from Eqs. (31) and (36) we conclude that if the maximum amplitude of the input voltage is approximately 0.103 volt, then the network is stable.[1]

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[1]On a first guess it was decided to choose $r_1 = 0.25$ and $r_2 = 0.5$ with these values it was found that max $|E| \leq 0.074$ volt was a sufficient condition for stability. Obviously this choice of r_1 and r_2 does not optimize max $|E|$.

Applying the stability results of Leon and Anderson [3] it was found that the sufficient condition for stability is

$$\max |E| \leq 0.088 \text{ volt} . \tag{37}$$

Thus, at the expense of a considerable amount of computation our method results in a 17% improvement on the stability bound. Since Leon and Anderson's results were already much too conservative, we really haven't bought much.

Now let's consider the perturbation method which results in considerably less calculations. Since only inequality (13) need be satisfied, let

$$\|R'(q_p(t))\| \int_0^\infty |h(t)|dt = .99 , \tag{38}$$

then

$$\|R'(q_p(t))\| = \frac{.99}{255} \times 10^{12} . \tag{39}$$

This gives us an upper bound on $q_p(t)$ which in turn can be used to bound E, the amplitude of the input. Thus,

$$\|q_p(t)\| = \frac{.99}{255} \times 10^{12} \ (2K^2) = 7 \times 10^{-12} \text{ coulomb} . \tag{40}$$

Now applying our steady state results [1,2] we find that

$$\max |E| \leq 1 \text{ volt} \tag{41}$$

is a sufficient condition for stability to "small" perturbations. Note that the small perturbation stability theory involves considerably fewer calculations than both of the other stability criteria. Also, the upper bound on the input amplitude is considerably greater for this new approach to the stability problem. Practically speaking, one might ask if the small perturbation method is a good stability criterion. We will attempt to give some justification for the method by the following experimental results.

In the experimental circuit, Figure 6, the bias is one volt and the pump frequency is approximately twice the small signal resonant frequency of the circuit. A threshold on the pump voltage was determined above which subharmonic oscillations existed, i.e., the circuit was operating in an unstable state. In this section we will compare the upper bound on the input voltage below which the circuit is stable to "small" perturbations with the above threshold for subharmonics. Hopefully, these two bounds will be close together with the subharmonic threshold slightly greater than the stability bound.

First, let us compute the stability bound. The a-c equivalent circuit is illustrated in Figure 7. The stray capacitance C_s is due to the scope and leads and must be taken into account, since it is about the same magnitude as the nonlinear capacitance. From this model we compute

$$\underline{C} + \int_0^\infty |h(t)|\,dt \approx \frac{C_s C_\ell}{C_s + C_\ell} + \frac{C_\ell^{\,2}}{C_\ell + C_s} \left[\frac{1 + e^{-\pi/2Q}}{1 - e^{-\pi/2Q}}\right]$$

$$= 1140 \times 10^{-12}\,, \tag{42}$$

where the small signal Q of the circuit is approximately 50. Again, assuming that

$$\|R(q_p(t))\| \, [\int_0^\infty |h(t)|\,dt + \underline{C}] = 0.99$$

we obtain

$$\|q_p(t)\| = 1.89 \times 10^{-12} \text{ coulomb}\,. \tag{43}$$

The steady state results [1,2] give

$$\|q_p^{\,1}(t)\| = 1.84 \times 10^{-12} \text{ coulomb}\,. \tag{44}$$

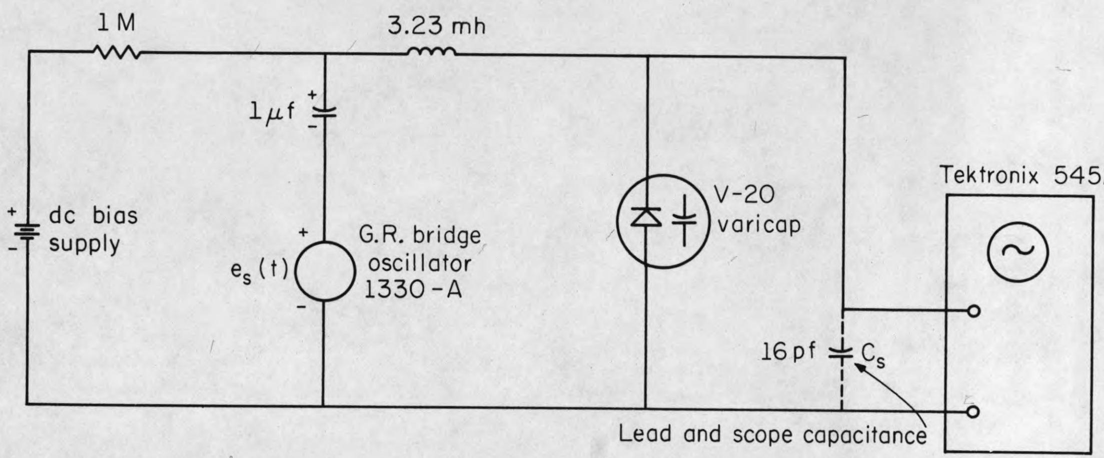

Fig. 6. Experimental Subharmonic Oscillator Circuit Diagram

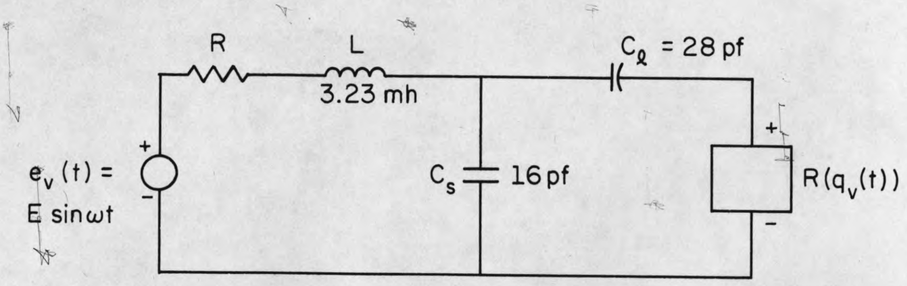

Fig. 7. A-C Equivalent Circuit of the Experimental Circuit

Hence,

$$\max |E| = 0.192 \text{ volts} . \tag{45}$$

In other words, if the amplitude of the source is kept below 0.192 volts, then the network will be stable to small changes in the input voltage.

Let's compare the above bound to the experimental threshold on the source voltage at which subharmonic oscillations commence. In this experiment it was found that at a pump voltage of 0.71 volts rms or $|E| \simeq 1$ volt the circuit went into subharmonic oscillations. If one uses the stability criterion of Leon and Anderson the result is that

$$\max |E| = 0.0613 \text{ volt}$$

is a sufficient condition for stability of the circuit. The following table summarizes the results.

Table I

Voltage	Subharmonic Threshold	Stable to Small Changes in the Input for E below	Stable for all Bounded Inputs for E below
E	1.0volt	0.192 volt	0.0613 volt

Just how small a perturbation in the input voltage is allowed for our stability criterion to be valid depends on r_1. The smaller r_1 is chosen, the larger the perturbation that is allowed. However, it is interesting to note that in the several experimental circuits that were constructed the subharmonic threshold always occurred at a fixed input voltage, regardless of the transient behavior of the circuit. The transient behavior only seems to determine the phase of the subharmonic oscillations with respect to the input. If this is the case, then the perturbation approach to stability of networks is justified even though the transients due to changes in input voltage exceed their allowable bounds.

V. CONCLUSION

The stability criterion of Leon and Anderson was good for nonlinear circuits containing bounded inputs. In this paper it was assumed that the input was bounded and periodic. Using this periodicity property a new sufficient condition for stability was obtained, unfortunately, it was shown by an example that this new stability criterion is computationally more difficult than that of Leon and Anderson and results only in a slightly improved allowable range of operation for the nonlinear device. Experimental results showed that both of the above bounds were too conservative. These disappointing results led to the perturbation approach to stability. In this method the response was assumed to be in the desired steady state, and at some time $t = a$ the input was increased by some small increment. A condition was then determined such that all resulting transients for $t \geq a$ asymptotically approached the desired steady state solution for "small enough increments" in the input voltage. The answer to the question on how small these increments must be depends on the choice of r_1. The computations involved in determining how small this increment must be are very time consuming for the small amount of information they yield. One should merely note that a small enough increment can always be found such that the iterates converge. One would like to conjecture at this point that if the circuit is stable to small perturbations in the input voltage then it will also be stable to large changes in the input voltage. Experimental results indicate that this is indeed the case for the subharmonic oscillator problem, however, more work needs to be done before this conjecture can be extended to other nonlinear phenomena.

APPENDIX

It was mentioned in an early part of the paper that the Thevenin equivalent voltage would not necessarily be periodic, but would contain transient terms. For simplicity it was assumed that the Thevenin equivalent voltage was periodic of period T. It will now be shown that the inclusion of transient terms does not alter the problem significantly.

Again we shall assume that all d-c bias terms have been removed and that the a-c Thevenin equivalent voltage is of the form

$$e_T(t) = \int_{-\infty}^{t} f(t-\lambda)\, e_v(\lambda)\, d\lambda \ , \tag{A.1}$$

where f(t) is the absolutely integrable voltage transfer impulse response function from the terminals of the source to the terminals of the nonlinear capacitance. Substituting Eq. (A.1) into Eq. (17) for the Thevenin equivalent voltage gives

$$\begin{aligned} q_t(t) = \underline{C}\{ \int_{-\infty}^{a} f(t-\lambda)[e_v(\lambda)-\Delta e(\lambda)]d\lambda + \int_{a}^{t} f(t-\lambda)\, e_v(\lambda)\, d\lambda - R(q_t(t))\} \\ + \int_{-\infty}^{t} h(t-\tau)[\int_{-\infty}^{a} f(\tau-\lambda)(e_v(\lambda)-\Delta e(\lambda))\, d\lambda + \int_{a}^{\tau} f(\tau-\lambda)\, e_v(\lambda)\, d\lambda \\ - R(q_t(\tau))]d\tau \ . \end{aligned} \tag{A.2}$$

Likewise from Eq. (2)

$$\begin{aligned} q_p(t) = \underline{C}[\int_{-\infty}^{t} f(t-\lambda)\, e_v(\lambda)\, d\lambda - R(q_p(t))] \\ + \int_{-\infty}^{t} h(t-\tau)[\int_{-\infty}^{\tau} f(\tau-\lambda)\, e_v(\lambda)\, d\lambda - R(q_p(\tau))]d\tau \ . \end{aligned} \tag{A.3}$$

Taking the difference between (A.2) and (A.3)

$$\delta q(t) = q_t(t) - q_p(t) = \underline{C}\{-\int_{-\infty}^{a} f(t-\lambda)\Delta e(\lambda)\,d\lambda + R(q_p(t)) - R(q_t(t))\} + \int_{-\infty}^{t} h(t-\tau)\{-\int_{-\infty}^{a} f(\tau-\lambda)\Delta e(\lambda)\,d\lambda + R(q_p(\tau)) - R(q_t(\tau))\}d\tau \ , \tag{A.4}$$

or

$$\delta q(t) = \underline{C}\{R(q_p(t)) - R(q_t(t)) - \int_{-\infty}^{a} f(t-\lambda)\Delta e(\lambda)\,d\lambda\}$$
$$+ \int_{-\infty}^{a} h(t-\tau)\{-\int_{-\infty}^{a} f(\tau-\lambda)\Delta e(\lambda)\,d\lambda + R(q_p(\tau)) - R(q_p(\tau)+\Delta q_p(\tau))\}d\tau$$
$$+ \int_{a}^{t} h(t-\tau)\{-\int_{-\infty}^{a} f(\tau-\lambda)\Delta e(\lambda)\,d\lambda + R(q_p(\tau)) - R(q_t(\tau))\}d\tau \ . \tag{A.5}$$

Using Taylor's theorem, as before, we obtain

$$\delta q(t) + \underline{C}\, R'(q_p(t))\delta q(t) + \int_{a}^{t} h(t-\tau)R'(q_p(\tau))\ \delta q(\tau)\,d\tau$$
$$= - \underline{C}\ N_2(\delta q(t)) - \int_{a}^{t} h(t-\tau)N_2(\delta q(\tau))\,d\tau - M_2(t) \ , \tag{A.6}$$

where

$$M_2(t) = \underline{C} \int_{-\infty}^{a} f(t-\tau)\Delta e(\lambda)\,d\lambda + \int_{-\infty}^{a} h(t-\tau) \int_{-\infty}^{a} f(\tau-\lambda)\Delta e(\lambda)\,d\lambda\ d\tau$$
$$+ \int_{-\infty}^{a} h(t-\tau)[R'(q_p(\tau))\Delta q_p(\tau) + N_1(\Delta q_p(\tau))]d\tau$$
$$+ \int_{a}^{t} h(t-\tau) \int_{-\infty}^{a} f(\tau-\lambda)\Delta e(\lambda)\,d\lambda\ d\tau \ .$$

Note that

$$\|M_2(t)\| \leq \underline{C} \; \|\Delta e(t)\| \int_{t-a}^{\infty} |f(\tau)|\,d\tau + \|R'(q_p)\Delta q_p + N_1(\Delta q_p)\| \int_{t-a}^{\infty} |h(\tau)|\,d\tau$$

$$+ \|\Delta e(t)\| \int_{t-a}^{\infty} |h(\lambda)| \int_{t-a-\lambda}^{\infty} |f(s)|\,ds\,d\lambda$$

$$+ \|\Delta e(t)\| \int_{o}^{t-a} |h(\lambda)| \int_{t-a-\lambda}^{\infty} |f(s)|\,ds\,d\lambda \; . \qquad (A.7)$$

However,

$$\int_{o}^{t-a} |h(\lambda)| \int_{t-a-\lambda}^{\infty} |f(s)|\,ds\,d\lambda = \int_{o}^{\frac{t-a}{2}} |h(\lambda)| \int_{t-a-\lambda}^{\infty} |f(s)|\,ds\,d\lambda$$

$$+ \int_{\frac{t-a}{2}}^{t-a} |h(\lambda)| \int_{t-a-\lambda}^{\infty} |f(s)|\,ds\,d\lambda \leq \int_{o}^{\infty} |h(\lambda)|\,dx \int_{\frac{t-a}{2}}^{\infty} |f(s)|\,ds$$

$$+ \int_{\frac{t-a}{2}}^{\infty} |h(\lambda)|\,d\lambda \int_{o}^{\infty} |f(s)|\,ds \; . \qquad (A.8)$$

Hence, if inequality (A.8) is substituted for the last term in equality (A.7), then it follows that $M_2(t)$ asymptotically approaches zero as $t \to \infty$. Also, note that $\|M_2(t)\|$ can be made as small as desired by making $\|\Delta e(t)\|$ "small enough," and thus, $\|\Delta q_p(t)\|$ "small enough." Therefore, comparing Eq. (A.6) with Eq. (21) we see that the problem does not change too significantly when transient terms are included in the Thevenin equivalent voltage source.

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A new method is presented by which the stability of nonlinear circuits containing bounded periodic sources can be determined. By stability we mean that the steady state output is unique and periodic with the same period as the input and all transients decay to this unique steady state solution. It is assumed that the first and second derivatives of the nonlinear function exist and are continuous within a certain allowable operating range of the nonlinear element. The first derivative should be positive at the bias point (not always necessary). A sufficient condition for the stability of the above circuit is given in terms of a maximum allowable input amplitude. However, it is shown that, given a certain upper bound on the input amplitude, the requirement that all transients decay to this unique steady state solution is too stringent and results in many computational problems. therefore, a small perturbation approach is adopted which results in fewer computations and less stringent conditions on the input amplitude. Experimental results indicate that this new approach is much better than previous results.

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