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**BOUNDED INPUT -
BOUNDED OUTPUT
STABILITY OF
NONLINEAR TIME-VARYING
RC NETWORKS**

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BOUNDED INPUT-BOUNDED OUTPUT STABILITY OF
NONLINEAR TIME-VARYING RC NETWORKS[†]

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Abstract

It is shown that if a nonlinear time-varying capacitor, whose incremental elastance is finite and greater than zero for all time, is embedded into a lumped linear passive time-invariant RC network, then the network is always bounded input-bounded output stable. In fact, if the input is periodic with period T , then the transient response asymptotically approaches a unique bounded periodic response with period T .

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INTRODUCTION

In the past few years a number of papers have appeared on frequency domain criteria for the determination of the absolute stability or instability of certain nonlinear time-varying systems. This work is well documented in the literature [1-5]. However, relatively little work has been done on the bounded input-bounded output stability of nonlinear time-varying systems. One of the first papers to appear on this subject was published by Leon and Anderson [6]. The authors considered the Thevenin equivalent circuit shown in Figure 1. Application of Kirchhoff's laws results in the equation

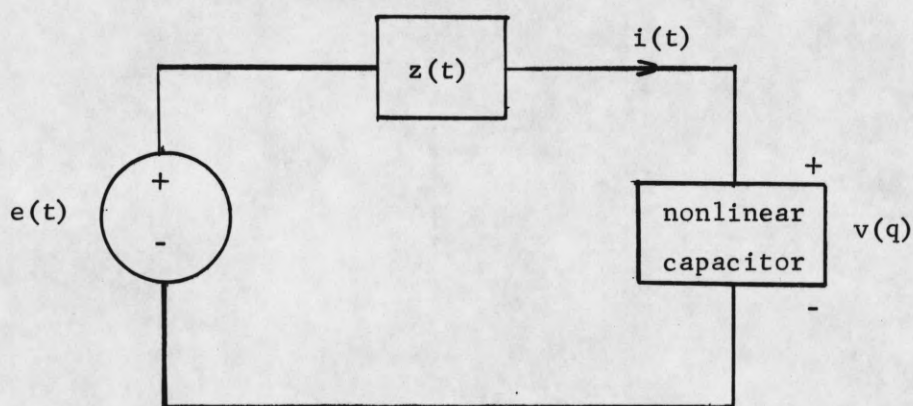


Figure 1. Linear network containing a nonlinear capacitor.

$$e(t) = \int_a^t z(t-\tau) i(\tau) d\tau + v(q(t)), \quad (1)$$

where, $v(q(t))$ is the voltage across the nonlinear element which is assumed to be continuous in q , the first and second

derivatives exist with respect to q , and the first derivative is positive,

$z(t)$ is the impulse response of the linear passive circuit due to a current impulse with $v(q) = 0$,

$e(t)$ is the bounded Thevenin equivalent voltage, and

a is the time at which the excitation first appears.

The authors determine an upper bound on the input amplitude $e(t)$, which is a function of the nonlinearity and the linear network, such that a unique steady state solution exists ($a = -\infty$) which is stable in the sense that all transients asymptotically approach this steady state solution. In the case when $e(t)$ is periodic with period T improved estimates of the upper bound on $e(t)$ were obtained [7]. Unfortunately both of these estimates can become difficult to compute and have no simple graphical interpretation as the frequency domain criteria.

In 1964 V. A. Yakubovich published the first results on a frequency domain stability criterion for forced systems [8]. He considered the following system.

$$\frac{dx}{dt} = Px + q \phi(\sigma) + \underline{f}(t), \quad (2)$$

$$\sigma = r^t \underline{x},$$

where \underline{x} is an n -vector, P is an $n \times n$ matrix, q and r are $n \times 1$ column matrices, and $\underline{f}(t)$ is a vector function bounded in $(-\infty, \infty)$. The scalar function $\phi(\sigma)$ can have isolated points of discontinuity of the first kind. Yakubovich proved that if the following conditions are

satisfied, then the System (2) has in $(-\infty, \infty)$ a unique solution $x_0(t)$ which is exponentially stable in the large. These conditions are:

1. The roots of the equation $\det(P-sI) = 0$ are in the half plane $\text{Re}\lambda < -\alpha \leq 0$
2. The function $\phi(\sigma)$ satisfied the condition

$$0 \leq \frac{\phi(\sigma_1) - \phi(\sigma_2)}{\sigma_1 - \sigma_2} \leq \mu_0$$

where $\mu_0 \leq \infty$, σ_1, σ_2 are points of continuity of $\phi(\sigma)$, and $-\infty < \sigma_1 < \sigma_2 < +\infty$.

3. $1/\mu_0 + \text{Re } W(-\alpha + i\omega) > 0$, where $W(s) = r^t (P-sI)^{-1} q$ is the transfer function of the linear part of the system from the input ϕ to the output $-\sigma$ with $\underline{f}(t) = 0$.
4. If $\mu_0 = \infty$, then also

$$\lim_{\omega \rightarrow \infty} \omega^2 \text{Re } W(-\alpha + i\omega) > 0.$$

The most recent bounded input-bounded output stability result was derived by I. W. Sandberg [9]. He considered the system

$$\underline{g}(t) = \underline{f}(t) + \int_0^t k(t-\tau) \underline{\Psi}[\underline{f}(\tau), \tau] d\tau, \quad t > 0, \quad (3)$$

where $\underline{g}(t)$, $\underline{f}(t)$, and $\underline{\Psi}(\underline{f}(t), t)$ are real measurable N -vector-valued functions and it is assumed that $\underline{g}(t)$ is bounded, $\underline{f}(t)$ is square integrable over any finite range $(0, y)$, $k(t)$ is an $N \times N$ matrix such that $t^p k(t)$ is absolutely and square integrable for $p = 0, 1, 2$, with

$$K(s) = \int_0^{\infty} k(t) e^{-st} dt \text{ for } \sigma \geq 0, \text{ and}$$

$$(i) \quad \det [I + \frac{1}{2} (\alpha + \beta) K(s)] \neq 0 \text{ for } \sigma \geq 0$$

$$(ii) \quad \frac{1}{2} (\beta - \alpha) \sup_{-\infty < \omega < \infty} \Lambda\{[I + \frac{1}{2} (\alpha + \beta) K(i\omega)]^{-1} K(i\omega)\} < 1$$

where $\Lambda[M]$ denotes the positive square-root of the largest eigenvalue of M^*M (* denotes transpose complex-conjugate of M). Also it is assumed that

$$\alpha \leq \frac{\Psi_n(w, t)}{w} \leq \beta \quad (n = 1, 2, \dots, N), \quad \text{for } w \neq 0,$$

$$\Psi_n(0, t) = 0 \quad (n = 1, 2, \dots, N),$$

and

$$\alpha \leq \frac{\Psi_n(w_1, t) - \Psi_n(w_2, t)}{w_1 - w_2} \leq \beta \quad (n = 1, 2, \dots, N)$$

for $t \in [0, \infty)$ and all real w_1, w_2 such that $w_1 \neq w_2$. If the above conditions are satisfied, then $\underline{f}(t)$ is bounded, $\underline{f}(t) \rightarrow 0$ as $t \rightarrow 0$ whenever $\underline{g}(t) \rightarrow 0$ as $t \rightarrow \infty$, and $\underline{g}(t)$ ultimately periodic with period T implies that $\underline{f}(t)$ is ultimately periodic with period T . In the case $N = 1$ the conditions (i) and (ii) have a simple graphical interpretation in the $K(i\omega)$ plane [4].

FORMULATION OF THE CAPACITANCE PROBLEM

In this work it will be shown that a passive lumped linear T.I. (time-invariant) RC network containing a nonlinear time-varying capacitor, whose incremental capacitance is greater than zero, is always bounded input-bounded output stable. The Thevenin equivalent circuit is illustrated in Figure 2.

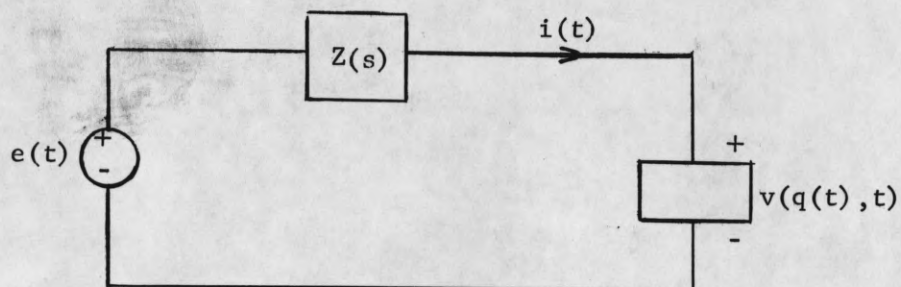


Figure 2. RC network containing a nonlinear time-varying capacitor.

Assumption I: The impedance $Z(s)$ is that of a lumped linear time-invariant passive RC network with all shunt capacitance in $Z(s)$ lumped with the nonlinear time-varying element and $\text{Re } Z(s) > 0$ for $\text{Re } s > 0$. The Thevenin equivalent voltage, $e(t)$, contains all the initial conditions, is assumed to be bounded and measurable, and can be written in the form

$$e(t) = e_p(t) - e_t(t), \quad (4)$$

where $e_p(t)$ is the steady state Thevenin voltage and $e_t(t)$ is the transient voltage which exponentially approaches zero as $t \rightarrow \infty$.

Assumption II: Let us also assume that the nonlinear time-varying element is assumed to satisfy the following condition.

$$\alpha \leq \frac{v(q_1, t) - v(q_2, t)}{q_1 - q_2} \leq \beta, \text{ for all } q \text{ and } t,$$

and where $0 < \alpha \leq \beta$ and $v(q, t)$ is measurable for all t and q .

The integral equation for the above network is

$$e(t) = \int_a^t z(t-\tau) \frac{dq}{d\tau} d\tau + v(q, t)$$

or

$$e(t) = \int_a^t z(t-\tau) \frac{dq}{d\tau} d\tau + \frac{1}{2} (\alpha + \beta) q + \{v(q, t) - \frac{1}{2} (\alpha + \beta) q\}.$$

Solving for $q(t)$ we obtain

$$q(t) = \int_a^t h(t-\tau) [e(\tau) - (v(q, \tau) - \frac{1}{2} (\alpha + \beta) q(\tau))] d\tau, \quad (5)$$

where

$$H(s) = \frac{1}{sZ(s) + \frac{1}{2} (\alpha + \beta)} = \frac{Y_0(s)}{s}.$$

The admittance $Y_0(s)$ is the reciprocal of $Z_0(s)$ and $Z_0(s)$ consists of $Z(s)$ in series with a capacitor of $\frac{2}{\alpha + \beta}$ farads. Hence,

$$\frac{Y_0(s)}{s} = \frac{K_0}{s} + K_\infty + \sum_{i=1}^n \frac{K_i}{s + \sigma_i},$$

where $\sigma_i > 0$ and K_i is real and positive. However, because of the series capacitance we note that $Y_0(s)|_{s=0} = 0$, implies that $K_0 = 0$. Also, the shunt capacitance was removed from $Z(s)$, therefore $K_\infty = 0$. Thus, $h(t) = \sum_{i=1}^n K_i e^{-\sigma_i t}$, for all $t \geq 0$, and we conclude that

(A) $h(t)$ is absolutely integrable, and

(B) $h(t) \geq 0$, for all $t \geq 0$.

This second inequality is a key step in the results which follow.

Mathematical Results for the Nonlinear Time-Varying Capacitor

First let us state the following theorem:

Theorem 1: If Assumptions (I) and (II) are satisfied for the network

in Figure 2, then the network is always bounded input-bounded output stable. The output is unique, and if the input is bounded and periodic with period T , then the output asymptotically approaches the unique bounded periodic response with period T .

Proof: First we will show that the mapping

$$F[q(t)] = \int_a^t h(t-\tau) [e(\tau) - v(q(t), \tau) + \frac{1}{2} (\alpha + \beta) q(\tau)] d\tau \quad (6)$$

is a contraction mapping [10] in the space \mathfrak{L}_∞ for any bounded input, i.e., there exists a unique fixed point, a solution to Eq. (5), in \mathfrak{L}_∞ . We say that $f(t) \in \mathfrak{L}_\infty$ if $f(t)$ is bounded and measurable with norm

$$\|f\| = \sup_{-\infty < t < \infty} |f(t)| < \infty.$$

Let us define a sequence of iterates as follows.

$$\begin{aligned} q^{(0)}(t) &= 0 \\ q^{(1)}(t) &= \int_a^t h(t-\tau) e(\tau) d\tau \\ q^{(n)}(t) &= \int_a^t h(t-\tau) [e(\tau) - v(q^{(n-1)}, \tau) + \frac{1}{2} (\alpha + \beta) q^{(n-1)}(\tau)] d\tau. \end{aligned}$$

The difference between successive iterates is

$$\begin{aligned} q^{(n+1)} - q^{(n)} &= \int_a^t h(t-\tau) [v(q^{(n-1)}, \tau) - \frac{1}{2} (\alpha + \beta) q^{(n-1)} - v(q^{(n)}, \tau) + \\ &\quad + \frac{1}{2} (\alpha + \beta) q^{(n)}] d\tau, \end{aligned}$$

and

$$|q^{(n+1)} - q^{(n)}| \leq \int_a^t |h(t-\tau)| \left| [v(q^{(n)}, \tau) - \frac{1}{2}(\alpha+\beta)q^{(n)}] - [v(q^{(n-1)}, \tau) - \frac{1}{2}(\alpha+\beta)q^{(n-1)}] \right| d\tau.$$

From Assumption (II) we conclude that

$$|q^{(n)} - q^{(n+1)}| \leq \int_a^t |h(t-\tau)| \frac{1}{2}(\beta-\alpha) |q^{(n)} - q^{(n-1)}| d\tau.$$

Taking the norm of both sides, we obtain

$$\|q^{(n)} - q^{(n+1)}\| \leq \frac{1}{2}(\beta-\alpha) \int_0^\infty |h(t)| dt \|q^{(n)} - q^{(n-1)}\|.$$

Thus the Mapping (6) is a contraction mapping provided

$$d = \frac{1}{2}(\beta-\alpha) \int_0^\infty |h(t)| dt < 1. \quad (7)$$

However, from Property (B) we see that

$$\int_0^\infty |h(t)| dt = \int_0^\infty h(t) dt = H(s) \Big|_{s=0}.$$

Thus

$$d = \frac{\frac{1}{2}(\beta-\alpha)}{sZ(s) + \frac{1}{2}(\beta+\alpha)} \Big|_{s=0}. \quad (8)$$

Since $0 < \alpha \leq \beta$ and $\operatorname{Re} sZ(s) \Big|_{s=0} \geq 0$, it is obvious from Equation (8) that $d < 1$. Since the mapping is a contraction, it follows that

$$\|q(t)\| \leq \frac{\|q^{(1)}\|}{1-d}.$$

Actually, we have only shown that there exists a unique solution to Eq. (5) in \mathcal{L}_∞ . In order to show that this solution is stable we must assume that the transient solution $q(t)$ is bounded for all finite t (this is generally a good engineering assumption) and show that $q(t)$ asymptotically

approaches $q_p(t)$ as $t \rightarrow \infty$, where the unique steady state solution $q_p(t) \in \mathcal{L}_\infty$ and is determined from the integral equation

$$q_p(t) = \int_{-\infty}^t h(t-\tau) [e_p(\tau) - v(q_p, \tau) + \frac{1}{2}(\alpha+\beta)q_p(\tau)] d\tau. \quad (9)$$

We will now finish the proof of the theorem by showing that the difference

$$\begin{aligned} q(t) - q_p(t) &= \delta q(t) = \int_a^t h(t-\tau) [e(\tau) - v(q, \tau) + \frac{1}{2}(\alpha+\beta)q(\tau)] d\tau \\ &\quad - \int_{-\infty}^t h(t-\tau) [e_p(\tau) - v(q_p, \tau) + \frac{1}{2}(\alpha+\beta)q_p(\tau)] d\tau \end{aligned} \quad (10)$$

is unique and asymptotically approaches zero as $t \rightarrow \infty$.

Equation (10) can be put into the form

$$\delta q(t) = M(a, t) + \int_a^t h(t-\tau) [v(q_p, \tau) - v(q, \tau) - \frac{1}{2}(\alpha+\beta)(q_p(\tau) - q(\tau))] d\tau, \quad (11)$$

and with Eq. (4)

$$\begin{aligned} M(a, t) &= -\int_a^t h(t-\tau) e_t(\tau) d\tau - \int_{-\infty}^a h(t-\tau) e_p(\tau) d\tau + \int_{-\infty}^a h(t-\tau) [v(q_p, \tau) - \\ &\quad - \frac{1}{2}(\alpha+\beta)q_p(\tau)] d\tau. \end{aligned}$$

Since, $h(t)$ is the absolutely integrable impulse response of a lumped system, $e_p(t)$ and $q_p(t)$ are bounded, and $e_t(t)$ exponentially approaches zero as $t \rightarrow \infty$, then it follows that there exists positive constants k_1 and c_1 such that

$$|M(a, t)| \leq k_1 e^{-c_1 t}.$$

Let us define

$$k(t) = \frac{v(q_p, t) - v(q, t)}{q_p - q}, \quad (12)$$

where $\alpha \leq k(t) \leq \beta$.

It follows as before that the mapping

$$Q[\delta q(t)] = M(a,t) + \int_a^t h(t-\tau) \left[k(\tau) - \frac{1}{2}(\alpha+\beta) \right] \delta q(\tau) d\tau \quad (13)$$

is a contraction mapping and

$$\|\delta q(t)\| \leq \frac{\|M(a,t)\|}{1-d}, \quad (14)$$

hence there exists a unique $\delta q(t) \in \mathcal{L}_\infty$. We must now show that since

$\lim_{t \rightarrow \infty} |M(a,t)| = 0$, then $\lim_{t \rightarrow \infty} |\delta q(t)| = 0$. Note that if we can show that $e^{rt} \delta q(t)$ is bounded for any positive r , then it follows that $\delta q(t) \rightarrow 0$ as $t \rightarrow \infty$. Therefore multiplying Eq. (13) by e^{rt} we have

$$e^{rt} \delta q(t) = e^{rt} M(a,t) + \int_a^t e^{r(t-\tau)} h(t-\tau) \left[k(\tau) - \frac{1}{2}(\alpha+\beta) \right] e^{rt} \delta q(\tau) d\tau. \quad (15)$$

First, it is obvious that if we want the above mapping to be a contraction we should restrict $r < c_1$ so that $e^{rt} M(a,t)$ is bounded and $e^{rt} h(t)$ is absolutely integrable. This is always possible from the above formulation of the problem. Now $e^{rt} \delta q(t) \in \mathcal{L}_\infty$ if

$$d_1 = \frac{1}{2}(\beta-\alpha) \int_0^\infty |e^{rt} h(t)| dt < 1. \quad (16)$$

However, $e^{rt} h(t) > 0$, hence Eq. (16) can be expressed as

$$d_1 = \frac{1}{2}(\beta-\alpha) \left[\frac{1}{sZ(s) + \frac{1}{2}(\beta+\alpha)} \right]_{s=-r} < 1,$$

or

$$d_1 = (1-\epsilon) \left[1 + \left(\frac{2}{\beta+\alpha} \right) (-r) Z(-r) \right]^{-1} < 1, \quad (17)$$

where $0 < \epsilon < 1$ since $\alpha > 0$. We will now show that one can always pick r

small enough such that Inequality (17) is satisfied. We must consider two cases. First the case in which $Z(s)$ has a pole at the origin, Fig. 3. From Fig. 3 it is clear that if one picks $z_1 < -r < 0$, then $Z(-r)$ is

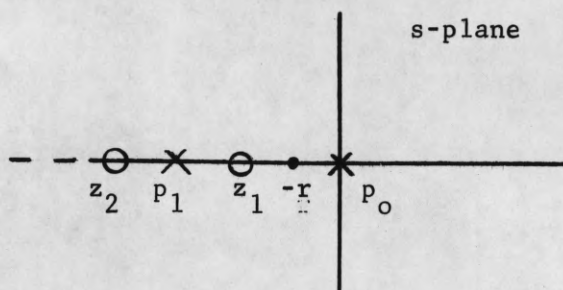


Fig. 3. Pole-zero diagram of an RC impedance with a pole at the origin.

negative. Therefore $-rZ(-r)$ is positive and $d_1 < 1$. In the case where there is no pole at the origin, Fig. 4, then for $p_1 < r_1 \leq 0$ the impedance

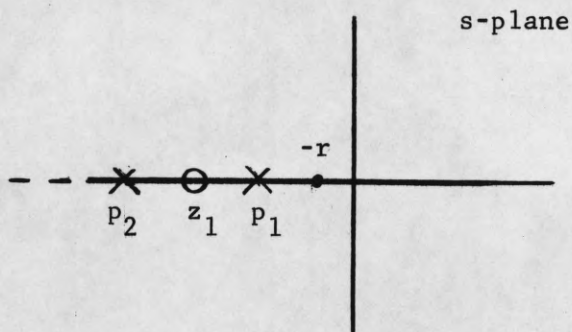


Fig. 4. Pole-zero diagram of an RC impedance with no pole at the origin.

$Z(-r)$ is finite and positive. Thus,

$$-rZ(-r) \begin{cases} \text{negative, } p_1 < r < 0 \\ 0 & , r = 0 \end{cases}$$

and $-rZ(-r)$ is continuous in this region. Hence it follows that we can pick r small enough such that

$$\left(\frac{2}{\beta+\alpha}\right)(-r)(Z(-r)) = \epsilon/2.$$

Substituting this result into Inequality (17), we obtain

$$(1-\epsilon) \frac{1}{1-\epsilon/2} < 1.$$

Inequality (17) is satisfied implies that $e^{rt}\delta q(t)$ is bounded and $\delta q(t) \rightarrow 0$ as $t \rightarrow \infty$. Therefore the solution of $q(t)$ is asymptotically stable. In fact if $e_p(t+T) = e_p(t)$ and $v(q,t) = v(q,t+T)$, then from Eq. (9)

$$q_T = q_p(t+T) = \int_{-\infty}^{t+T} h(t+T-\tau) [e_p(\tau) - v(q_p, \tau) + \frac{1}{2}(\alpha+\beta)q_p] d\tau.$$

Let $\lambda = -T+\tau$, then

$$q_T = \int_{-\infty}^t h(t-\lambda) [e_p(\lambda) - v(q_T, \lambda) + \frac{1}{2}(\alpha+\beta)q_T] d\lambda. \quad (18)$$

Note that Eqs. (9) and (18) are identical. Since the solution is unique, then

$$q_T = q_p(t+T) = q_p(t).$$

The solution is periodic with period T .

CONCLUSION

It follows that the above circuit can never sustain subharmonic oscillations, whereas, subharmonic oscillations occur frequently in nonlinear RLC networks [7]. The above results are being extended to RC circuits containing N nonlinear time-varying capacitors. However, the

above method of proof seems to fail in this case and other methods must be used.

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