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SOME PROPERTIES OF A EULER GRAPH

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SOME PROPERTIES OF A EULER GRAPH

Abstract

Some properties of a Euler graph are found. It is shown that the number of distinct circuits containing an edge in a Euler graph is odd, that the number of paths between any two vertices in a Euler graph is even and that the number of edges in a cut-set in a Euler graph is even.

SOME PROPERTIES OF A EULER GRAPH

A graph is called a Euler graph if each of its vertices has even degree. This type of graph was introduced mainly to solve the famous Königsberger bridge problem. However, little has been known about it although some of its properties are found in, for example, [1,2] In this note some more interesting properties of a Euler graph are listed.

To begin with we define some basic concepts which are closely related to a Euler graph. For the definition of other terms see [3].

Definition 1. An ordered sequence of edges in a linear graph is called an edge train if the following are satisfied:

(1) For any edge e other than the first edge and the last edge in the sequence, one endpoint of e is an endpoint of edge and the other endpoint of e is an endpoint of the succeeding edge.

(2) One endpoint of the first edge is an endpoint of the succeeding edge and the other endpoint of the first edge is the initial vertex.

(3) One endpoint of the last edge is an endpoint of the preceding edge and the other endpoint of the last edge is the final vertex.

(4) Every edge appears exactly once in the sequence.

Definition 2. An edge train is an open edge train if the initial vertex is different from the final vertex. Otherwise, it is a closed edge train.

It is well known that an open edge train has exactly two vertices of odd degree, namely the initial vertex and the final vertex. Hence by connecting the initial vertex of an open edge train to its final vertex by an edge we obtain a Euler graph. Conversely if we remove one edge from a

Euler graph we have an open edge train. Thus some of the properties of a Euler graph can be obtained from those of an open edge train. Therefore we first examine one of the properties of an open edge train.

Theorem 1. Let G be an open edge train with terminal vertices v_i and v_j. Then the number of distinct paths between v_i and v_j is odd.

Proof Claim. The number of all open edge trains between v_i and v_j is odd.

Proof of the Claim. To obtain an open edge train between v_i and v_j we proceed as follows:

(1) At the initial vertex v_i we choose an arbitrary edge e_1 incident at it and let v_1 be the other terminal vertex of e_1.

There are odd number of edges incident at v_i.

(2) At v_1 we choose an arbitrary edge e_2 incident at is and different from e_1 and let v_2 be the other terminal vertex of e_2.

There are even number of edges incident at v_1 including edge e_1 but since any edge appears only once in an edge train, edge e_1 must be excluded from the collection of possible succeeding edges of e_1. Hence the number N_1 of possible edges we can choose as the succeeding edges of e_1 is odd.

(3) At v_2 we choose an arbitrary edge e_3 incident at it and different from e_1 and e_2 and let v_3 be the other terminal vertex of e_3.

If $v_2 \neq v_i$ then by the same reasoning as the one in (2) we can say that the number N_2 of possible edges we can choose as the succeeding edge of e_2 is odd.

If $v_2 = v_i$ both e_1 and e_2 are incident at v_i and both of them are not eligible as the succeeding edge of e_2. Since the degree of v_i is odd N_2 is again odd.

(4) At v_3 we choose an arbitrary edge e_4 different from e_1, e_2, and e_3 and let v_4 be the other terminal vertex of e_4.

If $v_3 = v_i$ then by the same reasoning as the one for $v_2 = v_i$ in (3) the number N_3 of possible edges we can choose as the succeeding edge of e_3 is odd.

If $v_3 = v_1$ then all e_1, e_2, and e_3 are incident at v_1. Hence, any of them are not eligible as the succeeding edge of e_3. Since the degree of v_3 is even N_3 is odd.

If $v_3 \neq v_i$ then by the same reasoning as the one in (2) we can say that N_3 is odd.

(5) In general at vertex v_k we choose an edge e_{k+1} which is incident at it and which was not traversed before and let v_{k+1} be the other terminal vertex of e_{k+1}.

If $v_{k+1} = v_\ell$ for some vertex v_ℓ which we already traversed before, then by the same reasoning as the one for $v_3 = v_1$ or $v_3 = v_i$ in (4) we can say that the number N_k of possible edges we can choose as the succeeding edge of e_k is odd.

If $v_{k+1} \neq v_\ell$ then by the same reasoning as the one in (2) we can say that N_k is odd.

As we can see from the above (1) - (5) each time we arrive at a certain vertex trying to obtain an open edge train there are always odd number of edges from which we choose a succeeding edge. Hence the number of all possible open edge trains with the initial vertex v_i and the final vertex v_j is odd.

Now to collect all paths between v_i and v_j from the collection E of all open edge trains between v_i and v_j we must remove those edge trains from E which contain circuits. If an edge train contains a circuit consisting of edges $e_k, e_{k+1}, \ldots, e_{k+n}$ in this order as in Fig. 1 then an edge train $e_{k-1}\ e_{k+n}\ e_{k+n-1} \ldots e_{k+1}\ e_k\ e_{k+n+1} \ldots$ also contains a circuit and it must also be removed from E. This means that open edge trains which are not a path always appear as pairs. Hence the number of edge trains to be removed from E is even. Hence the number of distinct paths between v_i and v_j is odd. Q.E.D.

From theorem 1 we can obtain the next theorem which gives one of the properties of a Euler graph.

<u>Theorem 2</u>. The number of distinct circuits containing as edge in a Euler graph is odd.

<u>Proof</u>. Let G be a Euler graph. Let e be an edge in G. Remove e from G and let $G' = G-\{e\}$. Then G' is an open edge train with terminal vertices v_i and v_j where v_i and v_j are the terminal vertices of e in G. Then from Theorem 1 the number of distinct paths between v_i and v_j in G' is odd. Each of those paths forms a circuit together with e in G. Hence the theorem. Also from Theorem 1 we can obtain the next theorem which also gives one of the properties of a Euler graph.

<u>Theorem 3</u>. The number of paths between any two vertices in a Euler graph is even.

<u>Proof</u>. Let G be a Euler graph. Let v_i and v_j be two vertices of G. If v_i and v_j are connected by an edge e then remove e from G and let $G' = G-\{e\}$. G' is an open edge train between v_i and v_j. Hence by Theorem 1 the number of paths between v_i and v_j in G' is odd. Since e is a path between v_i and v_j in G and $G = G' \cup \{e\}$ the number of distinct paths between v_i and v_j is even. If v_i and v_j are not connected by an edge then we insert an edge e between v_i and v_j. Let $G' = G \cup \{e\}$. Then G' is an open edge train between v_i and v_j. Hence by Theorem 1 the number of paths between v_i and v_j is odd. e is included as a path in G'. Hence in G which is obtained from G' by removing e the number of distinct paths between v_i and v_j is even. Q.E.D.

Now the next theorem, which is also listed as an exercise in [2], is the last property of a Euler graph in this note. Here we list it without proof.

<u>Theorem 4</u>. The number of edges in a cut-set in a Euler graph is even.

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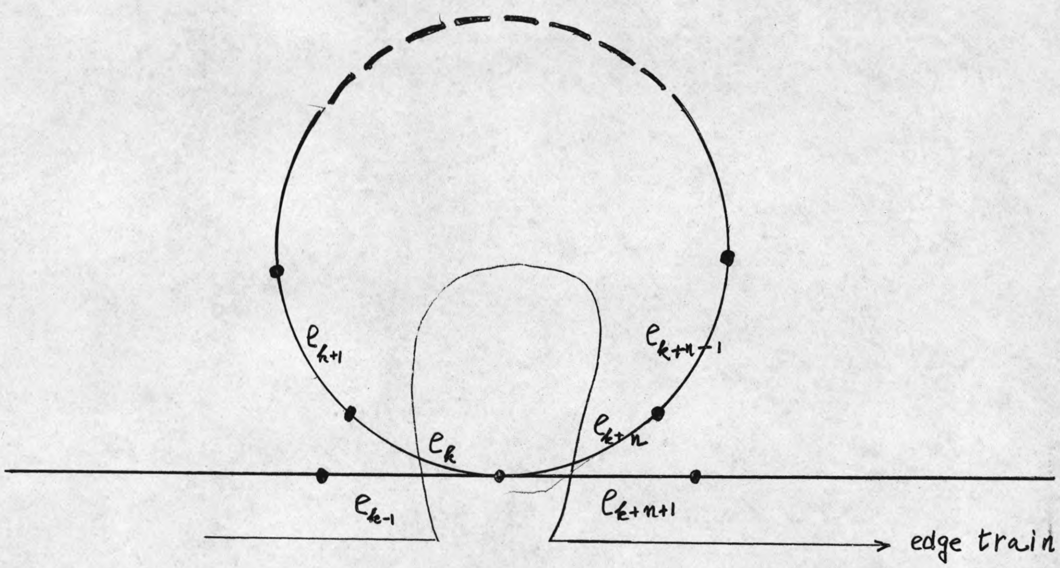

fig. 1

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