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ON DUAL OF A GRAPH ON A SURFACE OF GENUS 1

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Abstract

The dual of a graph on a surface of a certain genus is defined. Using the concept a necessary and sufficient condition for a graph to be drawn on a surface of genus 1 is derived.

ON DUAL OF A GRAPH ON A SURFACE OF GENUS 1

1. Introduction

It is well known that a linear graph can be drawn on a surface of sufficiently large genus without corssings of edges except at vertices. Hence a surface is decomposed into regions by a linear graph in exactly the same way as a plane is decomposed by a planar graph. And we can define the dual of a graph with respect to a surface of some genus.

In this note first we define the dual of a graph with respect to a surface of some genus. Next we will show that there is a relation between pseudo cuts and cutsets of a graph and circuits of its dual and that we can see if a graph can be drawn on a surface of some genus by testing the above-mentioned relation.

2. Definitions

In this section we define the dual of a graph which is the main subject of this note and we also define the concept of pseudo cuts on which some works have been done by Mayeda [1].

For a planar graph the dual is defined in two different but equivalent ways. One is by using relations between ranks and nullities and the other is by geometrical way. In the followings we define the dual of a linear graph on a surface of some genus geometrically.

Definition 1. Suppose a linear graph G is drawn on a surface of genus n without crossings of edges except at vertices. Then the surface is decomposed into regions by G. Let us denote these regions by Ri's i = 1,2,...,m, where m is the number of regions. Let us take a point Pi in each of Ri's. If two regions R_k and R_ℓ are adjacent, connect P_k and P_ℓ by edges P_kP_ℓ which cross the boundary edges common to R_k and R_ℓ only once. We will have a new graph G' with vertices $P_1, P_2, \ldots, P_m$. We call G' the dual of G on a surface of genus n. For example, the dual of $K_{3,3}$ on a surface of genus 1 is shown in Fig. 1.

Remark. By this definition a graph can have more than one dual according to how it is drawn on a surface of genus n.

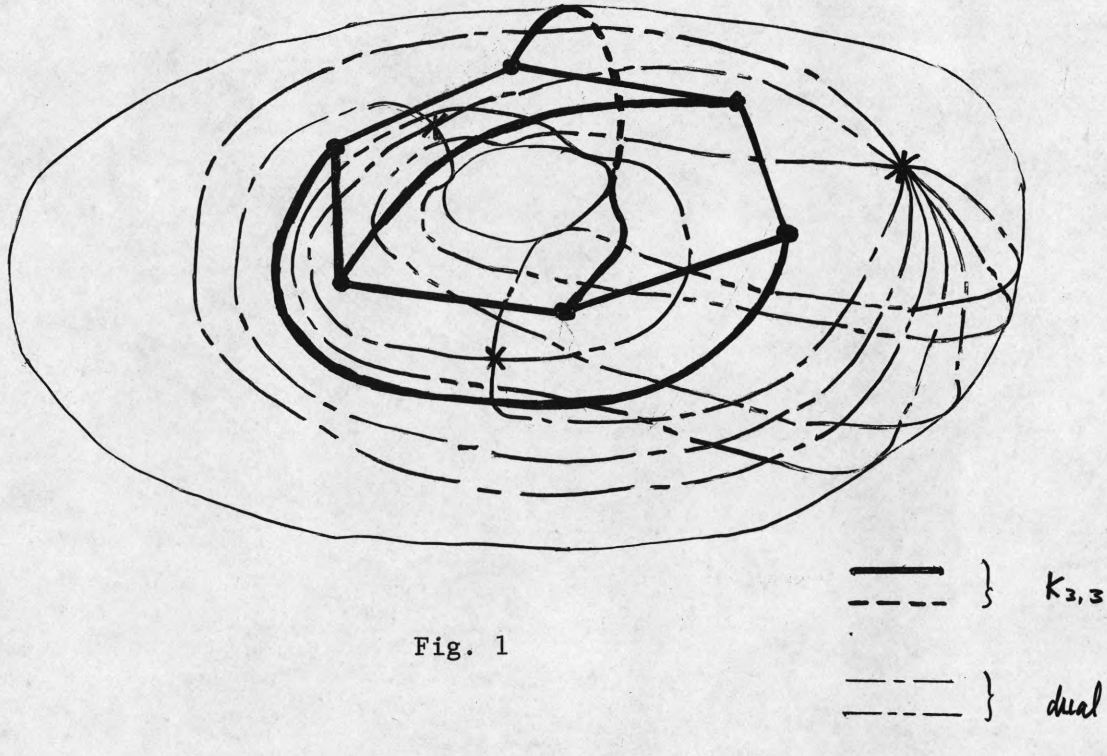

Fig. 1

Next we define pseudo cut. Before that we have to introduce the concept of Min. on a collection of sets.

<u>Definition 2</u>. Let A be a collection of sets. Then Min. A is a sub-collection of A such that any set a in A is in Min.A if every set b in A satisfies $b \not\subset a$ as long as b is not the empty set.

For example suppose A = {(a,b),(abc),(cd)}, then Min A = {(ab),(cd)}.

According to Mayeda [1] pseudo cut is defined as follows.

<u>Definition 3</u>. Let {S} be the collection of all possible cutsets, edge disjoint unions of cutsets of a graph G and the empty set. Let u_1 be a proper subset of a cutset in {S}.

Let $\{u\}_{u_1} = \mathrm{Min}\{u_1 \oplus S,\ s\epsilon\{S\}$, where $\oplus$ is ring sum operation. Then each set in $\{u\}_{u_1}$ is called a pseudo-cut with respect to u_1 and $\{u\}_{u_1}$ is called the set of pseudo-cuts with respect to u_1.

For example in Fig. 2 let S_1 = (a,b,e), S_2 = (c,d,e), and u_1 = (a,e).

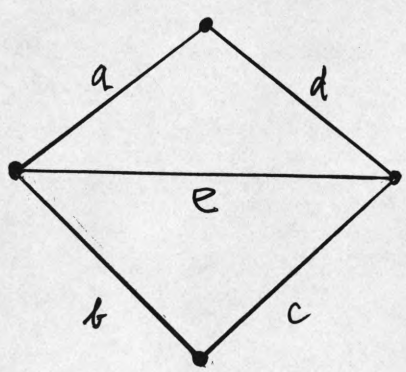

Fig. 2

Then u_1 = (ae,) and u_1 + s_1 = (b) are a pseudo cut with respect to u_1.

Mayeda also defines pseudo cut in more general form. We give the other definition too.

Definition 4. Let E be a set of edges of a graph G. Let $\{u\}_E = \text{Min}\{E \oplus S,\ s\epsilon\{S\}\}$. Then each set in $\{u\}_E$ is called a pseudo-cut with respect to E. $\{u\}_E$ is called the set of pseudo cut with respect to E . Here E can be any set of edges (not necessarily a proper subset of a cut set).

Definition 5. Let $\{S_1,S_2,...,S_n\}$ be a collection of sets. Then a set expressed by the ring sum of some of $\{S_1,S_2,...,S_n\}$ is called a set generated by $\{S_1,S_2,...,S_n\}$.

3. Relation Between Pseudo-Cuts and Circuits

For a planar graph there is a one to one correspondence between all of its circuits and all the cutsets of its dual. Similarly it is expected that there is some relation between pseudo-cuts of a graph and circuits of its dual on a surface of some genus. In this section we will establish that relation.

In the followings we assume that we can draw a graph G on a surface of genus 1 without crossings of edges except at vertices and we let G' be the dual of G on the surface. Then we have the following lemma on the number of independent circuits of G and that of vertices of G'.

Lemma 1. The number of independent circuits of G is equal to the number of vertices of G' plus 1.

Proof. Let N_v and N_e be the number of vertices and edges of G respectively. Let N'_v and N'_e be the number of vertices and edge of G' respectively. Then by the definition of G' we have $N'_e = N_e$

and $$N'_v = (N_e - N_v + 1) - 1 = N_e - N_v.$$

Hence $N_e - N_v + 1 = N'_v + 1$ QED

Now we have a theorem on a relation between pseudo-cuts of G and circuits of G'. Before the theorem we make some assumption on G.

Assumption 1. We assume that G has no self-loops. Note that we do not lose generality by this assumption because we can always substitute circuits with two edges for self-loops.

Theorem 2. There exist two pseudo-cuts u_1 and u_2 of G which satisfy the followings.

(1) $u_1 \subsetneq S$, $u_2 \subsetneq S'$ and $u_1 \neq u_2$ where S and S' are a cutset of G.

(2) There is no cutset S'' such that $u_1 + u_2 \supseteq S''$.

(3) There is a one to one correspondence between circuits and edge disjoint unions of circuits of G' and pseudo-cuts, cutsets and edge disjoint unions of cutsets of G generated by $\{u_1, u_2, S_1, \ldots, S_{n_v - 1}\}$ where $S_1, \ldots, S_{n_v - 1}$ are independent cutsets of G.

Proof. Since the number of independent circuits of G' is $N'_e - N'_v + 1$ which is equal to $N_v + 1$ by lemma 1, all we have to do is show that $\{u_1, u_2, S_1, \ldots, S_{n_v - 1}\}$ corresponds to a set of independent circuits of G'.

It is well known from combinatorial topology that G can be expressed as in Fig. 3. See for example [2].

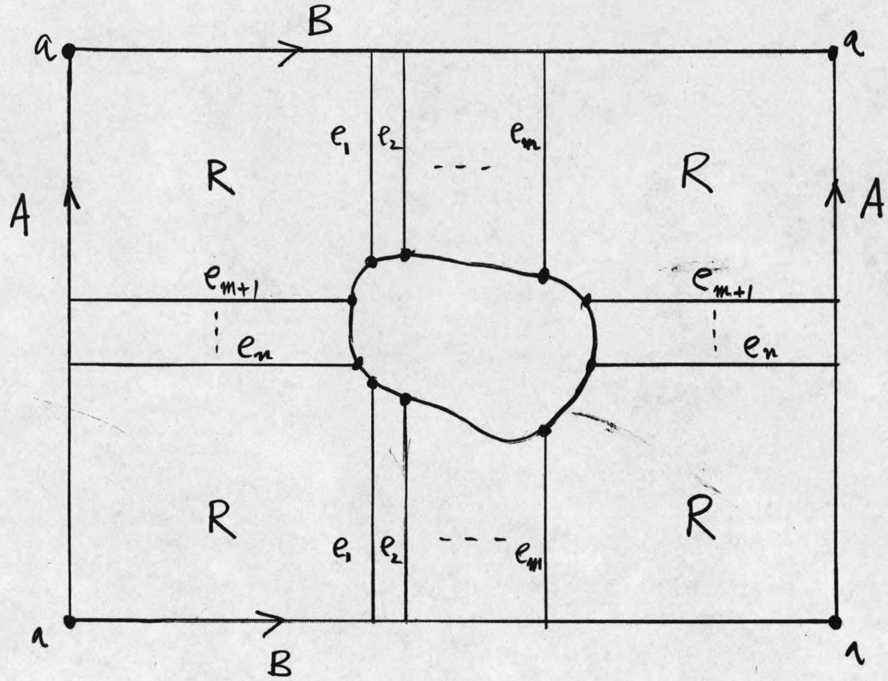

Fig. 3

Let $u_1 = \{e_1, e_2, \ldots, e_m\}$

and $u_2 = \{e_{m+1}, \ldots, e_n\}$.

then clearly u_1 and u_2 are two distinct pseudo-cuts and it is not difficult to see that there are cutsets S and S' such that $u_1 \subsetneqq S$, $u_2 \subsetneqq S'$. Hence (1).

Now suppose there is a cutset S" such that $u_1 + u_2 \supseteq S''$. Then $u_1 \cup u_2 \supseteq S''$, which means that $G-(u_1 \cup u_2)$ is disconnected. Hence either u_1 or u_2 is not a part of a circuit which is linked with the hole. This is a contradiction. Hence (2).

Since regions marked by R in Fig. 3 actually form one region on a surface of genus 1 to each of u_1 and u_2 there corresponds a circuit of G' composed of edges corresponding to edges of u_1 and u_2 respectively. To each incidence set of G there corresponds a circuit of G' as in the case of planar graphs. Therefore we have N_v-1 independent cutsets $S_1, S_2, \ldots, S_{n_v-1}$ of G to each of which there corresponds a circuit of G'. Now since $u_1 \subset S$ for some cutset S we know that $u_1, S_1, \ldots, S_{n_v-1}$ are linearly independent with respect to ring sum operation. Similarly $u_2, S_1, \ldots, S_{n_v-1}$ are also linearly independent. Now if $u_1, u_2, S_1, \ldots, S_{n_v-1}$ are not linearly independent we have $u_2 = u_1 \oplus X$ where X is a cutset or an edge disjoint union of cutsets. Hence $u_1 \oplus u_2 = X$. Hence $u_1 \oplus u_2 \supset S''$. So by the same reason as for (1) $u_1, u_2, S_1, \ldots, S_{n_v-1}$ must be linearly independent with respect to ring sum operation. QED

For a surface of genus greater than one we can obtain similar results by induction on genus.

Corollary 3. Let G be a graph which can be drawn on a surface of genus n without crossings of edges except at vertices. Let G' be the dual of G on the surface. Then there exist 2n pseudo-cuts $u_1, u_2, \ldots, u_{2n}$ of G which satisfy the followings.

(1) $u_1, u_2, \ldots, u_{2n}$ are a proper subset of a cutset of G.

(2) There is no cutset S such that $u_1 \oplus u_2 \oplus \ldots \oplus u_{2n} \supseteq S$.

(3) There is a one to one correspondence between circuits and edge disjoint unions of circuits of G' and pseudo-cuts, cutsets and edge disjoint unions of cutsets of G generated by $\{u_1, u_2, \ldots, u_{2n}, S_1, S_2, \ldots, S_{n_{v-1}}\}$ where S_1, $S_2, \ldots, S_{n_v-1}$ are independent cutsets of G.

The proof of this corollary is easy. We do not do that here.

The next theorem is the converse of theorem 2.

Theorem 4. Let G be a graph. Let u_1 and u_2 be a pseudo-cut of definition 3 of G and let $\{S_1, S_2, \ldots, S_{n_v-1}\}$ be a set of independent cutsets of G such that $\{u_1, u_2, S_1, \ldots, S_{n_v-1}\}$ are linearly independent with respect to ring sum operation. If $\{u_1, u_2, S_1, \ldots, S_{n_v-1}\}$ is realizable as a set of independent circuits of some graph, then G can be drawn on a surface of genus 1 without crossings of edges except at vertices.

Proof. It is well known (see [3]) that any graph can be drawn on a surface without crossings of edges except at vertices if the surface is of suitable genus. Hence we can draw a graph H which has $\{u_1, u_2, S_1, \ldots, S_{n_v-1}\}$ as a set of independent circuits on a surface of certain genus. Without loss of generality we can assume that the surface is of genus 2. Then the dual H' of H on the surface has $\{u_1, u_2, S_1, \ldots, S_{n_v-1}\}$ as a set of independent pseudo-cuts and cutsets. However by corollary 3 four of them have to be a pseudo-cut. Suppose u_1, u_2, S_1 and S_2 are a pseudo-cut of H'. Then there are cutset S and S' of H', (hence cutset of G) such that $S_1 \subsetneq S$ and $S_2 \subsetneq S'$. This is a contradiction since S_1 and S_2 are also a cutset of G. Hence H can be drawn on a surface of genus 1. Hence G can be drawn on a surface of genus 1. QED

4. Conclusion and Further Problems

Now we know that a graph G can be drawn on the surface of genus 1 without crossings of edges except at vertices if and only if there are independent pseudo-cuts u_1 and u_2 and cutsets $S_1, S_2, \ldots, S_{n_v-1}$ such that $\{u_1, u_2, S_1, \ldots, S_{n_v-1}\}$ can be realized as a set of independent circuits of some graph. Since there are $N_e - N_v + \rho$ pseudo-cuts u_i's which give linearly independent sets of the form of $\{u\}_{u_i} \cup \{S\}$, we do not have to test all possible pseudo-cuts to see if a graph can be drawn on a surface of genus 1. As a future problem there is a problem of reducing the number of pseudo-cuts to be tested. Another problem is to extend theorem 4 to general case, which is not so difficult. The third problem is to define dual by using ranks and nullities.

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13. ABSTRACT

The dual of a graph on a surface of a certain genus is defined. Using the concept a necessary and sufficient condition for a graph to be drawn on a surface of genus 1 is derived.

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