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ON DECODING PROJECTIVE GEOMETRY CODES

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Abstract

In this report, it is shown that Projective Geometry codes are orthogonalizable in less than or equal to 3 steps. Thus, this class of codes is majority-logic decodable in no more than 3 steps.

The improved decoding algorithm reduces the decoding complexity of Projective Geometry codes enormously in most cases. This should make Projective Geometry codes very attractive for practical use on error-control systems.

1. Introduction

Projective Geometry (PG) codes[3,5,7,8] form a subclass of the class of cyclic codes that are majority-logic decodable.[4] Because these codes are majority-logic decodable, they can be very simply implemented. In addition, the decoding algorithm for them can automatically correct more error patterns than those guaranteed by the decoding algorithm itself without additional cost. Thus PG codes are attractive from a practical point of view.

The decoding complexity of PG codes grows exponentially with L, the number of levels (or steps) of majority logic required.[5,10] It is desirable, therefore, to decode these codes in as few steps as possible. Unfortunately, the Reed decoding algorithm[5,6] for this class of codes often require that L be large.

In this report, an improved decoding algorithm for PG codes is presented. It will be shown that PG codes can be orthogonalized[4,5] in no more than 3 steps. That is, these codes can be majority-logic decoded in less than or equal to 3 steps. The results reduce the decoding complexity of PG codes enormously in most cases. Thus, they should make this class of codes very attractive for practical use on error-control systems.

The basic concept behind the improved decoding algorithm is also applicable to the Euclidean Geometry (EG) codes[5,9] which also form an important subclass of the class of majority-logic decodable codes. The decoding of EG codes is presented in a separate report.[2]

In section 2 of this report, we shall review some of the properties of PG codes and discuss the existing decoding algorithms for these codes. In section 3, we shall propose an improved decoding algorithm for PG codes. We shall show that PG codes are majority-logic decodable in less than or equal to 3 steps.

In the following discussions the reader will be assumed to be familiar with the concept of orthogonality in majority-logic decoding[4,5] and the structure of finite geometries.[1,5,8] where possible the notation and conventions employed in Reference 5 will be used.

2. Projective Geometry Codes

It is well-known that the elements of a Galois field can be arranged in such a way that they form a concrete representation of a finite projective geometry.[1,5] Specifically, a projective geometry of dimension m over $GF(p^s)$, $PG(m,p^s)$, can be constructed as follows. A point (α) of the geometry is associated with the set of elements

$$(\alpha) = \alpha, \alpha\beta, \alpha\beta^2, \ldots, \alpha\beta^{p^s-2}$$

where α is a non-zero element of $GF(p^{(m+1)s})$ and β is a primitive element of $GF(p^s)$. The number of points in the geometry is equal to $\frac{p^{(m+1)s}-1}{p^s-1}$.

Let α be a primitive element of $GF(p^{(m+1)s})$. An r-flat consists of the points (α^j) such that

$$(\alpha^j) = \beta^{i_1}\alpha^{e_1} + \beta^{i_2}\alpha^{e_2} + \ldots + \beta^{i_{r+1}}\alpha^{e_{r+1}} \qquad (1)$$

where $\alpha^{e_1}, \alpha^{e_2}, \ldots, \alpha^{e_{r+1}}$ are linearly independent, and $\beta^{i_1}, \beta^{i_2}, \ldots, \beta^{i_{r+1}}$ take on all possible combinations of values in $GF(p^s)$ except that they cannot be simultaneously zero. Thus an r-flat contains $\frac{p^{(r+1)s}-1}{p^s-1}$ points.

The points of $PG(m,p^s)$ can be considered as the location number of a cyclic code of length $n = \frac{p^{(m+1)s}-1}{p^s-1}$ over $GF(p)$. An r-flat in $PG(m,p^s)$ can be associated with a polynomial in the algebra of polynomials modulo x^n-1. This polynomial is taken to have coefficient 1 in positions corresponding to the $\frac{p^{(r+1)s}-1}{p^s-1}$ points of the flat and 0 elsewhere.

A projective Geometry (PG) code of order r and length $n = \frac{p^{(m+1)s}-1}{p^s-1}$ with symbols from GF(p) has the property that the polynomials corresponding to all r-flats of $PG(m,p^s)$ are in its null space. Because of this property, the Reed algorithm[5,6] can be used to decode a PG code. The key point of the Reed algorithm for PG codes is that the check sums corresponding to the r-flats intersecting a particular (4-1)-flat are orthogonal on the check sum corresponding to the (r-1)-flat. The number J of r-flats intersecting a particular (r-1)-flat can be shown[3,5,8] to be equal to

$$J = \frac{p^{(m-r+1)s}-1}{p^s-1} \qquad (2)$$

Now, the Reed decoding algorithm for an r-th order PG code can be described as follows. At the first step of decoding, the check sum corresponding to an (r-1)-flat is determined by the value assumed by a majority of the check sums corresponding to the J (r+1)-flats that intersect on the given (r-1)-flat. The check sums corresponding to all (r-1)-flats can be correctly determined in this way provided that $[\frac{J}{2}]$* or fewer errors occurred. In the next step of decoding, each of the check sums corresponding to all (r-2)-flats is determined by a majority decision on a set of check sums corresponding to some (r-1)-flats that intersect on the given (r-2)-flat. It is easy to see from Eq.(2) that the number of (r-1)-flats that intersect on a given (r-2)-flat is

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*[x] is equal to the largest integer less than or equal to x.

at least equal to J. Therefore, each of the check sums corresponding to all (r-2)-flats can be determined provided that $[\frac{J}{2}]$ or fewer errors occurred. This decoding process can be repeated until the error digits corresponding to all 0-flats are determined. It requires r steps of majority logic elements for this decoding algorithm.

The decoding complexity of the Reed algorithm for PG codes grows exponentially with the number of decoding steps employed. It is important, therefore, to try to cut down the number of decoding steps. In this regard, Weldon proposed a modified decoding algorithm for PG codes.[5,10] The modified decoding algorithm can correct as many guaranteed error patterns as the original Reed algorithm.

The modified decoding algorithm by Weldon requires only two steps of majority logic elements. At the first step of decoding, the check sums corresponding to all (r-1)-flats are determined from the r-flats that intersect on the given (r-1)-flats in exactly the same way as in the original Reed algorithm. At the second step, the error digits corresponding to all 0-flats are determined from the (r-1)-flats using the idea of non-orthogonal check sums due to Rudolph.[7] Though this modified algorithm reduces the number of decoding steps to two, the decoder may not cost less than the decoder using the original algorithm. The reason is that a single majority gate with a very large number of inputs has to be used in the second step of the decoding.

In the next section, we shall introduce an improved decoding algorithm for PG codes. This improved algorithm can correct as many guaranteed error patterns as the original algorithm. In addition, the new algorithm reduces the number of decoding steps without increasing the number of inputs to the majority gates in every step of the decoding. Applying the idea behind the new algorithm, we shall show that PG codes are orthogonalizable in less than or equal to 3 steps. Thus, PG codes are majority-logic decoding in no more than 3 steps.

3. Improved Decoding Algorithm

The Reed algorithm for PG codes is a step-by-step decoding algorithm. It determines the next lower dimension of the associated projective geometry from the given dimension of the associated projective geometry. Actually, it is possible to speed up the decoding process by jumping from the flats of a given dimension to the flats of several dimensions lower. Based upon this fact, we have a new improved decoding algorithm.

In the first step of the improved decoding algorithm, (r-1)-flats are determined from the sets of r-flats that intersect on the given (r-1)-flats in the same way as in the Reed algorithm. Now, suppose that k is the smallest number such that a set of J (r-1)-flats that intersect on the given k-flat can be constructed. Obviously k is less than or equal to (r-2). Then, in the second step of decoding, each of the k-flats is determined from a set of J (r-1)-flats that are orthogonal on the given k-flat. This process can be repeated again and again until all error digits or 0-flats are determined.

In general, the problem of finding the smallest k such that a set of at least J (r-1)-flats orthogonal on a given k-flat can be constructed has not been solved. Thus, the number of decoding steps that can be reduced by the improved decoding algorithm can not be expressed explicitly. However, using the basic concept of the improved algorithm, we shall show that PG codes are orthogonalizable in less than or equal to 3 steps. Thus, the number of decoding steps is reduced to no more than 3 for any r-th order codes.

We have demonstrated in the last section that $PG(m,p^s)$ can be constructed from $GF(p^{(m+1)s})$. By this construction, the elements α^j, $\beta\alpha^j$, $\beta^2\alpha^j$, ..., $\beta^{p^s-2}\alpha^j$ of $GF(p^{(m+1)s})$ represent the same point of $PG(m,p^s)$, where α and β are primitive elements of $GF(p^{(m+1)s})$ and $GF(p^s)$, respectively. Thus, there are (p^s-1) replications of each of the points of the r-flat in $GF(p^{(m+1)s})$.

It is well-known that an m-dimensional Euclidean geometry $EG(m+1,p^s)$ can also be constructed from $GF(p^{(m+1)s})$. Every element in $GF(p^{(m+1)s})$ is taken to be a point of $EG(m+1,p^s)$. An (r+1)-flat that passes through the origin consists of the points α^j such that

$$\alpha^j = \beta^{i_1}\alpha^{e_1} + \beta^{i_2}\alpha^{e_2}, \ldots + \beta^{i_{r+1}}\alpha^{e_{r+1}} \qquad (3)$$

where $\alpha^{e_1}, \alpha^{e_2}, \ldots, \alpha^{e_{r+1}}$ are linearly independent, and $\beta^{i_1}, \beta^{i_2}, \ldots, \beta^{i_{r+1}}$ take on all possible combinations of values in $GF(p^s)$.

Comparing Eq.(1) and Eq.(3), we have the following result.

<u>Lemma 1</u> The points in the (p^s-1) replications of an r-flat of $PG(m,p^s)$ in $GF(p^{(m+1)s})$ plus the origin form an (r+1)-flat in $EG(m+1,p^s)$ that passes through the origin.

The number M of r-flats in $PG(m,p^s)$ can be shown to be equal to[5,10]

$$M = \frac{(p^{s(m+1)}-1)(p^{sm}-1)\ldots(p^{s(m-r+1)}-1)}{(p^{s(r+1)}-1)(p^{sr}-1)\ldots(p^s-1)} \qquad (4)$$

It is easy to show that the number of (r+1)-flats in $EG(m+1,p^s)$ that passes through the origin is also equal to M. (See Appendix.) Thus, the converse of Lemma 1 is true.

Lemma 2 Every (r+1)-flat in EG$(m+1,p^s)$ that passes through the origin consists of (p^s-1) replications of an r-flat of PG(m,p^s).

Lemma 1 and Lemma 2 set up a unique correspondence between an r-flat of PG(m,p^s) and an (r+1)-flat of EG$(m+1,p^s)$. From this correspondence, any two (r+1)-flats of EG$(m+1,p^s)$ that intersect on a given (k+1)-flat passing through the origin correspond to two r-flats of PG(m,p^s) that intersect on a k-flat. Therefore, we have

Lemma 3 If it is possible to construct I (r+1)-flats that are orthogonal on a given (k+1)-flat passing through the origin in EG$(m+1,p^s)$, then it is possible to construct I r-flats that are orthogonal on a given k-flat in PG(m,p^s).

In Reference 2, it has been shown that EG codes can be orthogonalized in less than or equal to 3 steps. By Lemma 3, we can show that PG codes are also orthogonalizable in no more than 3 steps. To prove this, we shall first list some of the results in Reference 2 as the following three lemmas.

Lemma 4 If it is possible to construct I r-flats that are orthogonal on a given k-flat in EG(m,p^s), then it is also possible to construct I (r+1)-flats that are orthogonal on a given (k+1)-flat in EG$(m+1,p^s)$, and vice versa.

Lemma 5 If $r \leq \frac{m}{2}$, it is possible to construct $I = \frac{p^{(m-r)s}-1}{p^s-1}$ r-flats that are orthogonal on the origin in EG(m,p^s).

Lemma 6 If $r > \frac{m}{2}$, then the r-th order EG code associated with EG(m,p^s) can be orthogonalized in 3 steps.

Lemma 7 If $r \leq \frac{m}{2} + 1$, the r-th order PG code associated with PG(m,p^s) can be orthogonalized in 1 or 2 steps.

Proof: If $r = 1$, then the first order PG code is 1 step orthogonalizable. For the case $r > 1$, we shall show that a 0-flat can be determined from a set of at least $J = \frac{p^{(m-r+1)s}-1}{p^s-1}$ $(r-1)$-flats that are orthogonal on the given 0-flat.

By Lemma 5, J $(r-1)$-flats that are orthogonal on the origin in $EG(m,p^s)$ can be constructed. In addition, from Lemma 4, it is possible to construct J r-flats that are orthogonal on a 1-flat passing through the origin in $EG(m+1,p^s)$. Finally, from Lemma 3, it is possible to construct J $(r-1)$-flats that are orthogonal on a given 0-flat in $PG(m,p^s)$. Therefore, if $1 < r \leq \frac{m}{2} + 1$, the r-th order PG code can be 2-step orthogonalized.

Q.E.D.

Lemma 8 If $r > \frac{m}{2} + 1$ the r-th order PG code associated with $PG(m,p^s)$ can be orthogonalized in 3 steps.

Proof: Lemma 4 and Lemma 6 imply that all 1-flats passing through the origin in $EG(m+1,p^s)$ can be orthogonalized in 3 steps from all of the $(r+1)$-flats in $EG(m+1,p^s)$ with $J = \frac{p^{(m-r+1)s}-1}{p^s-1}$. Thus, the r-th order PG code associated with $PG(m,p^s)$ is 3-step orthogonalizable by Lemma 3.

Q.E.D.

Directly from Lemma 7 and Lemma 8, we have our main theorem on decoding PG codes.

Theorem A PG code can be orthogonalized in less than or equal to 3 steps.

The new decoding procedure for the r-th order PG code associated with $PG(m,p^s)$ can be described as follows. At the first step, $(r-1)$-flats are determined from all r-flats. If $r = 1$, this is the end of the decoding. At the second step, there are two cases to consider. If $(r-1) \leq \frac{m}{2}$, all 0-flats are determined from the $(r-1)$-flats. This finishes the decoding. If $(r-1) > \frac{m}{2}$,

$(r-1-[\frac{m}{2}])$-flats are determined from the $(r-1)$-flats. It requires another step to finish up the decoding in this case. At the third step, all 0-flats are determined from the $(r-1-[\frac{m}{2}])$-flats. The decoding procedure is depicted as follows

a. $r = 1$ $\quad 1 \to 0$ $\quad$ (1 step)

b. $1 < r \leq \frac{m}{2} + 1$ $\quad r \to (r-1) \to 0$ $\quad$ (2 steps)

c. $r > \frac{m}{2} + 1$ $\quad r \to (r-1) \to r-1-[\frac{m}{2}] \to 0$ $\quad$ (3 steps)

In conclusion, the new improved decoding algorithm successfully reduces the number of majority-logic decoding steps from r to no more than 3 for any value of r. Thus it greatly reduces the decoding complexity of PG codes in most cases. These results together with the results on EG codes in Reference 2 should make the finite geometry codes very attractive for practical use on error-control systems.

Appendix

From Mann or CarMichael,[1,11] the number N of r-flats in EG(m,p^s) is equal to

$$N = \frac{(p^{ms}-1)(p^{(m-1)s}-1)\ldots(p^{(m-p+1)s}-1)p^{(m-r)s}}{(p^{rs}-1)(p^{(r-1)s}-1)\ldots(p^{s}-1)} \qquad (a)$$

Let A be the number of k-flats that are contained in an r-flat of EG(m,p^s). From Eq.(a)

$$A = \frac{(p^{rs}-1)(p^{(r-1)s}-1)\ldots(p^{(r-k+1)s}-1)p^{(r-k)s}}{(p^{ks}-1)(p^{(k-1)s}-1)\ldots(p^{s}-1)} \qquad (b)$$

Let B be the number of k-flats in EG(m,p^s). By Eq.(a) B is equal to

$$B = \frac{(p^{ms}-1)(p^{(m-1)s}-1)\ldots(p^{(m-k+1)s}-1)p^{(m-k)s}}{(p^{ks}-1)(p^{(k-1)s}-1)\ldots(p^{s}-1)} \qquad (c)$$

Now let M be the number of r-flats that contain a given k-flat in EG(m,p^s). Then

$$M = \frac{N \cdot A}{B}$$

$$= \frac{(p^{(m-k)s}-1)(p^{(m-k-1)s}-1)\ldots(p^{(m-r+1)s}-1)}{(p^{(r-k)s}-1)(p^{(r-k-1)s}-1)\ldots(p^{s}-1)} \qquad (d)$$

If $k = 0$, then the number of $(r+1)$-flats that pass through a given point of EG$(m+1,p^s)$ is equal to

$$\frac{(p^{(m+1)s}-1)(p^{ms}-1)\ldots(p^{(m-r+1)s}-1)}{(p^{(r+1)s}-1)(p^{rs}-1)\ldots(p^{s}-1)}$$

which is the same as the expression in Eq.(4).

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13. ABSTRACT

In this report, it is shown that Projective Geometry codes are orthogonalizable in less than or equal to 3 steps. Thus, this class of codes is majority-logic decodable in no more than 3 steps.

The improved decoding algorithm reduces the decoding complexity of Projective Geometry codes enormously in most cases. This should make Projective Geometry codes very attractive for practical use on error-control systems.

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