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ABSTRACT

In this paper we show that it is NP-complete to determine whether an arbitrary GCRP can be laid out in the knock-knee mode using a specified number of tracks. Producing a layout of minimum separation (i.e., minimum number of tracks) is an NP-hard problem for which provably good approximation algorithms are known.

Keywords: General channel routing problem, knock-knee layout mode, computational complexity

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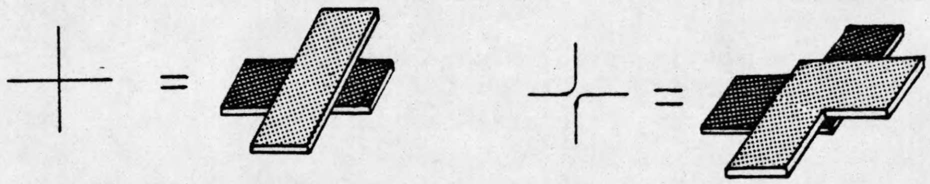
1. INTRODUCTION

One of the most crucial steps in the overall computer design process is that of routing (RP), the process of formally defining the precise conductor paths (wires) necessary to properly and efficiently interconnect the elements of a digital system. Due to the inherent complexity of the RP, it is necessary to partition the problem into simpler subproblems - each of which, because of comparative simplicity, can be studied and possibly solved with far better results than the routing problem as a whole.

We shall focus on the general channel routing problem (GCRP), which plays a central role in solving the routing problem (as discussed in [Sa]). As is customary, we view a channel of width t as being on a unit grid with grid points (x,y), where both x and y are integers with $0 \leq y \leq t+1$ and arbitrary x. The horizontal and vertical lines are called tracks and columns, respectively. A vertex (x,y) of this grid at either y = 0 or y = t+1 is a terminal; in particular, $(s_j,0)$ is a lower (or entry) terminal and $(t_i,t+1)$ is an upper (or exit) terminal. A wire is a subgraph of this grid whose edges are segments connecting adjacent vertices in the grid. A net N is an ordered pair of (not simultaneously empty) increasing integer sequences $((s_1,\ldots,s_k),(t_1,\ldots,t_h))$; thus, N contains lower terminals $s_1,\ldots,s_k$ and upper terminals $t_1,\ldots,t_h$. If $k+h > 2$, then we speak of a multiterminal net, as distinct from a two-terminal net. The reason for this distinction is that channel routing of two-terminal nets is much simpler and better understood than the corresponding multiterminal net problem.

A solution to a GCRP must have, for each net, a graph on the grid that contains a path between any two terminals of that net. Note also that no two nets may share the same terminal.

We must distinguish between the layout disciplines known as "Manhattan" and "knock-knee" [RBM,L,BB]. In the Manhattan mode two wires may share a grid point only by crossing at that point (crossing) as shown in Figure 1a. In the knock-knee mode two wires may cross at a vertex or may both bend at that vertex (knock-knee) as shown in Figure 1b. In both modes, no two wires are permitted to share an edge of the grid. In this paper, we shall adopt the "knock-knee" layout mode.



(a) Crossing (b) Knock-knee

Figure 1. Illustration of the basic construct in the knock-knee mode.

In the channel, there is a fixed number (two or more) of conducting layers, each of which is a graph isomorphic to the channel grid. These layers are ordered and placed one on top of another, and contacts between two distinct layers (vias) can be made only at grid points. If two layers are connected at a grid point, no layer in between can be used at that grid point.

We shall use the terms "layout" and "wiring" with the following distinct technical connotations (as in [PL]).

Definition 1. A wire layout (or simply layout) for a given GCRP is a subgraph of the layout grid, each of whose connected components corresponds to a distinct net of the GCRP, in the knock-knee mode.

Notice that we can, without loss of generality, restrict ourselves to connected subgraphs which are trees, which we shall call wire-trees. (Each non-tree graph can be replaced by one of its tree subgraphs on the same set of terminals.)

<u>Definition 2</u>. Given a wire layout consisting of wire-trees $w_1,\ldots,w_n$, a <u>wiring</u> is a mapping of each edge of wire-tree w_i (for $i = 1,2,\ldots,n$) to a conducting layer with vias established at layer changes.

<u>Example</u>. Consider a GCRP $\eta = \{N_1,N_2\}$ where $N_1 = ((2,4),(5))$ and $N_2 = ((3),(1,4))$ (see Figure 2a). Nets are laid out and wired as shown in Figures 2b and 2c.

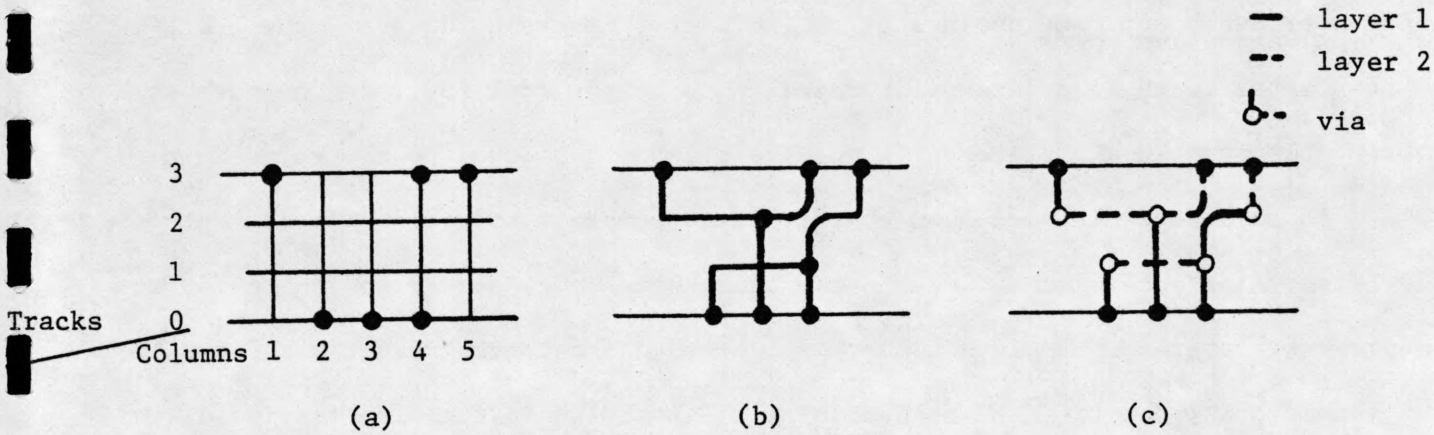


Figure 2. (a) Specification of the terminals of N_1 and N_2; (b) wire layout and (c) wiring of the nets N_1 and N_2.

An <u>optimal</u> layout of a given GCRP is a layout that uses the least possible number d of tracks. A simple-minded (and optimistic) lower bound to d can be readily established as follows.

Consider a GCRP $\eta = \{N_1,\ldots,N_n\}$, where $N_i = ((s_1^i,\ldots,s_{k_i}^i),\ (t_1^i,\ldots,t_{h_i}^i))$, and let $\ell_i = \min(s_1^i,t_1^i)$ and $r_i = \max(s_{k_i}^i,t_{h_i}^i)$. The interval $[\ell_i,r_i]$ represents an obvious lower bound to the horizontal track demand raised by N_i, since a terminal in column ℓ_i must be connected to a terminal in column r_i. In other words N_i is replaced by a fictitious two-terminal net N_i^* (whose two terminals may belong to the same track). We now consider the channel routing problem $\eta^* = \{N_1^*,\ldots,N_n^*\}$, and use standard methods to obtain its density δ (i.e., the maximum number of two-terminal nets which must cross any vertical section of the channel). It is clear that δ is a lower bound for the minimum number of horizontal tracks, and we call δ the essential density of the GCRP..

The method of Preparata-Lipski [PL], devised for the two-terminal net CRPs, produces optimal routings (i.e., it uses δ tracks). However, the best known upper bound on t for multiterminal nets is $t \leq 2\delta-1$ [SP], and no one has been able to make any statement about the tightness of this bound.

In this paper we will show that it is NP-complete to decide whether an arbitrary instance of the GCRP, in the knock-knee mode, can be routed (using any number of layers) in a given number of tracks (an analogous result was shown by Szymanski [Sz] for the Manhattan mode). The most frustrating implication of this result is that, unless p = NP, no efficient algorithm can find the least possible number, d, of tracks required to route a given GCRP (using three or more conducting layers).

2. GENERAL CONSTRUCTION

In this section we shall prove the NP-completeness of the GCRP by verifying that a known NP-complete problem is (polynomial-time) transformable to the GCRP (details about the theory of NP-completeness can be found in [GJ]. The NP-complete problem that we shall use is 3-satisfiability (3SAT):

Instance: A collection $C = \{c_1, c_2, \ldots, c_m\}$ of clauses on a finite set U of variables such that $|C_i| = 3$ for $1 \leq i \leq m$, and $|U| = n$.

Problem: Find a truth assignment for U that satisfies all the clauses, that is, in each clause, at least one literal is true.

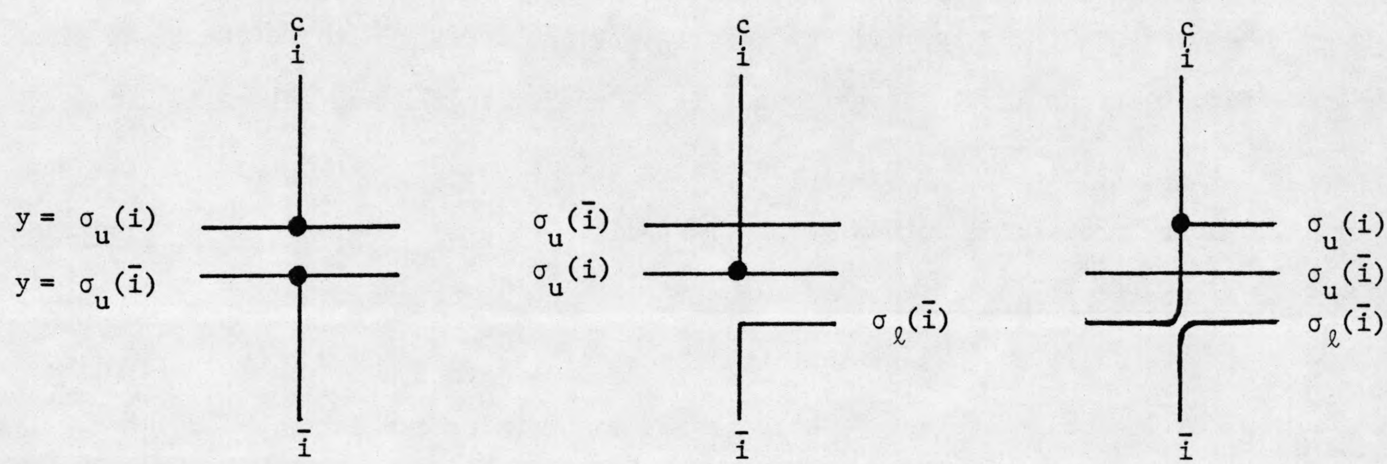
Following the definition in [SP], for any integer c, the interval (c,c+1) is called a vertical section (the vertical strip comprised between two columns). We say that a net $N = ((s_1, \ldots, s_k), (t_1, \ldots, t_h))$ is upper-active in (c,c+1) if $t_1 \leq c < t_h$ and lower-active in (c,c+1) if $s_1 \leq c < s_k$; N is active in (c,c+1) if it is both upper-active and lower-active in (c,c+1). Any vertical line $x = x_0$, $x_0 \in (c,c+1)$ for $\min(s_1,t_1) \leq c \leq \max(s_k,t_h)-1$ cuts N in at least one point; each intersection of N with $x = x_0$ identifies a strand of N at x_0. For a column c, let $d_-(c)$ and $d_+(c)$ be the local densities of the problem in the vertical sections (c-1,c) and (c,c+1), respectively. With reference to a left-to-right scan, we say that c is a density increasing column (d.i.c.) if $d_-(c) < d_+(c)$, is a density decreasing column (d.d.c.) if $d_-(c) > d_+(c)$, and a density preserving column (d.p.c.) if $d_-(c) = d_+(c)$. Hereafter, a terminal will be labeled with the index of the net to which it belongs.

We shall now assume that the layout of the given GCRP η uses tracks $1,2,\ldots,t$. We assume that each net N_m occupies at most two tracks in (c,c+1). We call the upper track $\sigma_u(m)$ and the lower track $\sigma_\ell(m)$. Note that $\sigma_u(m) \geq \sigma_\ell(m)$ and $\sigma_u(m) = \sigma_\ell(m)$ if and only if net N_m occupies only one track in (c,c+1)

2.1 Construction

We shall construct an instance of the GCRP corresponding to an instance of 3SAT. We partition the channel into blocks of contiguous columns where each block either corresponds to a clause of 3SAT - a clause block - or ensures a fixed orderingof nets - an enforcer or an end block. A similar idea has been exploited by Szymanski [Sz], but due to the inherent difference between the knock-knee and the Manhattan modes, the two constructions are correspondingly different.

Now we introduce the GCRP constructs corresponding to 3SAT variables. A pair of nets $(N_i, N_{\bar{i}})$ will correspond to a variable $\nu_i (\nu_i \in U)$. Consider a column c with upper-terminal i and lower-terminal $\bar{i}$. If $\sigma_u(i) > \sigma_u(\bar{i})$ (at the vertical section (c-1,c)) the layout of column c is straightforward; indeed, we connect the upper-terminal to $\sigma(i)$ and lower-terminal to $\sigma(\bar{i})$, as shown in Figure 3a. Instead, if $\sigma_u(i) < \sigma_u(\bar{i})$ in (c-1,c) we must have two strands of N_i or $N_{\bar{i}}$ at the vertical section (c,c+1), as shown in Figure 3b. The convention is that the variable ν_i, corresponding to the pair $(N_i, N_{\bar{i}})$, is true if



(a) $\sigma_u(i) > \sigma_u(\bar{i})$: ν_i true (b) $\sigma_u(i) < \sigma_u(\bar{i})$: ν_i false

Figure 3. A pair of nets correspond to a boolean variable.

$\sigma_u(i) > \sigma_u(\bar{i})$ and false if $\sigma_u(i) < \sigma_u(\bar{i})$. Obviously, the layout of a column with upper-terminal i and lower-terminal $\bar{i}$ is simpler (requires only two tracks in (c,c+1)) if the corresponding variable ν_i is true.

Now we can construct the clause blocks $(B_1,\ldots,B_m)$, each of which corresponds to a clause of 3SAT (B_i corresponds to c_i, for $1 \leq i \leq m$). We will insert an enforcer block E_i $(0 \leq i \leq m)$ to the right of the clause block B_i, and to the left of the clause block B_{i+1}. The main purpose of the enforcers is to guarantee that both strands of nets N_i and $N_{\bar{i}}$ are ordered with respect to strands of nets N_j and $N_{\bar{j}}$, that is, $\sigma_u(j),\sigma_u(\bar{j}) < \sigma_u(i),\sigma_u(\bar{i})$ for $1 \leq j < i \leq n$. We also introduce two end blocks, F_0 and F_1, to the left of E_o and to the right of E_m, respectively, to ensure that (some) nets traverse the entire length of E_0 and E_m. An overview of the channel is shown in Figure 4.

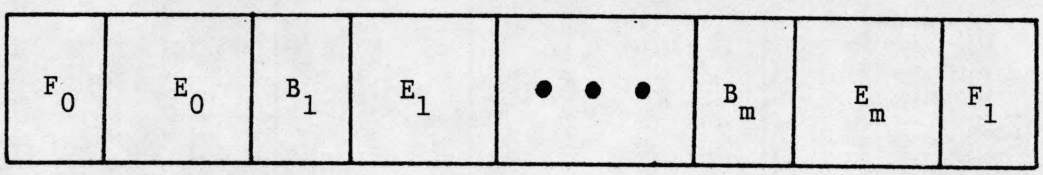


Figure 4. Relative positions of the enforcer, clause, and end blocks.

The essential density of our proposed problem is 2n+4:2n nets (N_i,$N_{\bar{i}}$ for $1 \leq i \leq n$) for n variables, 2 long nets N_e and N_f (which traverse the entire length of the channel), and pairs of nets N_{e_i} and N_{f_i} that begin in B_{i-1} and terminate in B_i. Thus there are a total of 2(n+m+1) nonoverlapping nets. Next we give a more detailed description of the individual blocks.

We begin by describing the clause blocks. Each clause block consists of seven columns; B_i contains columns $c_i, c_i+1, \ldots, c_i+6$. Columns c_i and c_i+1 are d.d. columns at which N_{e_i} and N_{f_i} terminate, respectively. Columns c_i+2, c_i+3, and c_i+4 are d.p. columns. Finally, nets $N_{e_{i+1}}$ and $N_{f_{i+1}}$ begin at column c_i+5 and c_i+6 respectively. The position of terminals in B_i is shown in Figure 5, where Λ is the index of the empty net. For convenience, we have used a, b, and c for the indices of nets corresponding to the variables (of the blocks) with the convention that $\sigma_u(a), \sigma_u(\bar{a}) < \sigma_u(b), \sigma_u(\bar{b}) < \sigma_u(c), \sigma_u(\bar{c})$. As we shall verify later, this ordering of the nets at the vertical sections (c_i-1, c_i) and (c_i+6, c_i+7) is imposed by E_{i-1} and E_i, respectively.

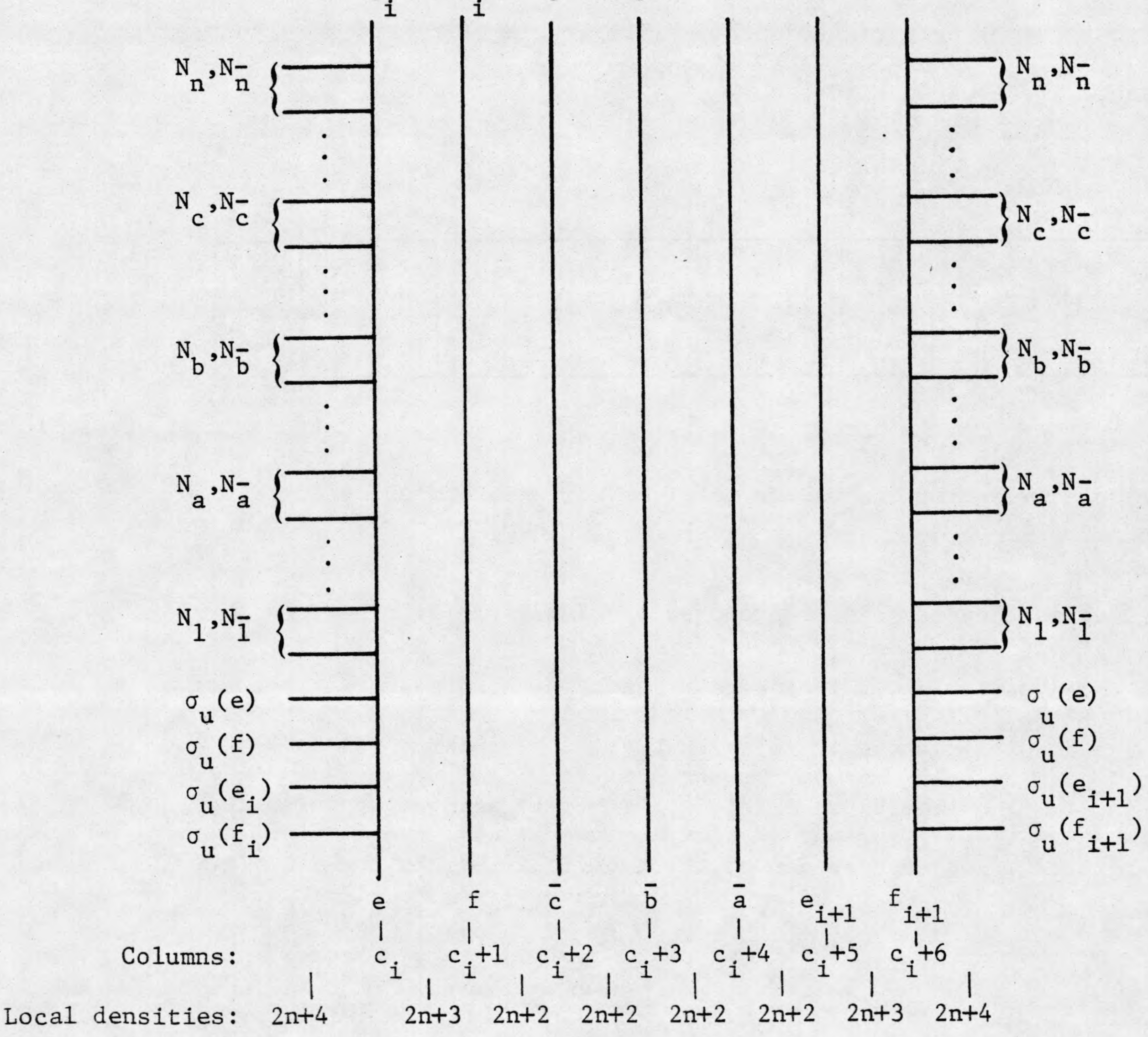


Figure 5. Clause block B_i.

An enforcer block E_{i-1} consists of 4n+1 columns: (i) n-1 subblocks each of which contains four columns; the j-th block ensures that the strands of nets $N_{j+1}, N_{\overline{j+1}}$ lie above the strands of nets $N_j, N_{\bar{j}}$, that is, $\sigma_u(j+1), \sigma_u(\overline{j+1}) > \sigma_u(j), \sigma_u(\bar{j})$ in (c_i-1, c_i), (ii) five columns that guarantee $\sigma_u(1), \sigma_u(\bar{1}) > \sigma_u(e) > \sigma_u(f) > \sigma_u(e_i) > \sigma_u(f_i)$ in (c_i-1, c_i). The enforcer block E_{i-1} is shown in Figure 6.

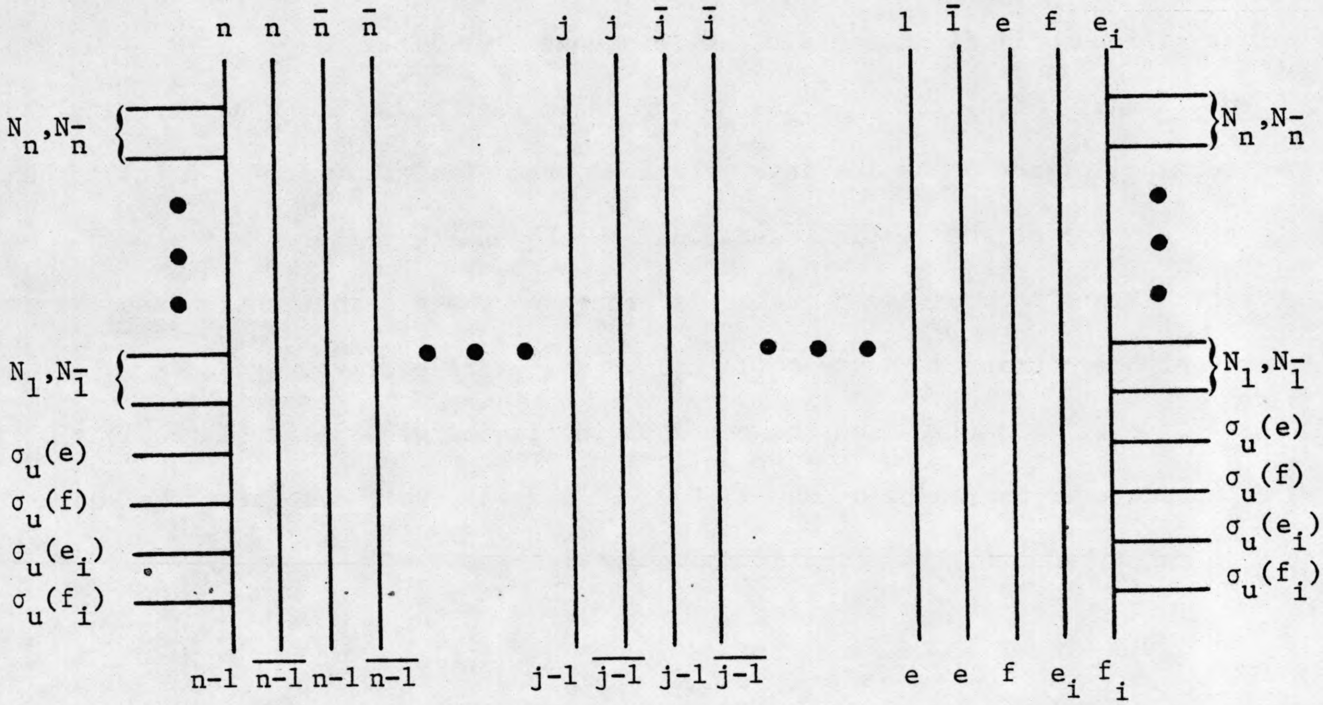


Figure 6. Enforcer block E_{i-1}.

Finally, each of the end blocks F_0 and F_1 contain 2n+4 columns (actually n+2 columns could suffice): 2n+2 columns for $N_i, N_{\bar{i}}$ $(1 \le i \le n)$, N_e, and N_f, and two columns for N_{e_1}, N_{f_1} in F_0, and $N_{e_{m+1}}, N_{f_{m+1}}$ in F_1.

2.2 Proof of the Reduction

First, we must show that all of the nets must have a fixed ordering as they enter (in a left-to-right scan of the channel) a clause block (see Figures 5-6). We will also show that a clause block can be laid out using $\delta = 2n+4$ tracks if and only if the corresponding 3SAT-clause is satisfiable (i.e. if and only if it contains at least one true literal).

Lemma 1: If a d.p. column c has a terminal of net N_i as its upper terminal and a terminal of net N_j as its lower terminal, and if $t = d_+(c)$ $(= d_-(c))$, then $\sigma_u(i) > \sigma_u(j)$ at the vertical sections (c-1,c) and (c,c+1).

Proof: Since each of the t tracks is occupied by a distinct net, N_i and N_j can only be connected to $y = \sigma_u(i)$ and $y = \sigma_u(j)$, respectively.

If $\sigma_u(j) > \sigma_u(i)$, then the vertical section between $y = \sigma_u(i)$ and $y = \sigma_u(j)$ at column c is shared by N_i and N_j (see Figure 7a), which violates the knock-knee mode rules, so $\sigma_u(i) > \sigma_u(j)$, as shown in Figure 7b. □

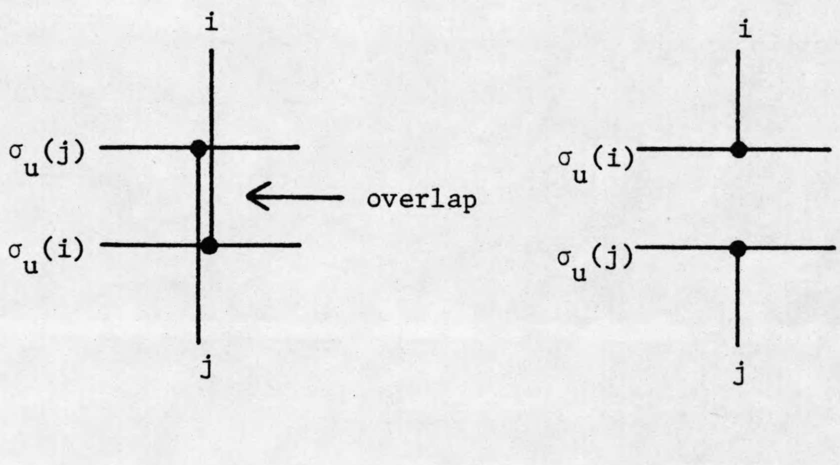


Figure 7. Layout of column c when $t = d_-(c) = d_+(c)$.

Now consider the enforcer block E_i (see Figure 6). Since there are exactly $t = 2n+4$ tracks available and $t = d_-(c) = d_+(c)$, where c is any column of E_i, we can apply Lemma 1 to each column. The application of Lemma 1 to the first column of E_i dictates that the strand of N_n lie above the strand of N_{n-1}. After applying this lemma to every column of E_i, we obtain the following result.

<u>Lemma 2</u>: In the vertical section $(c_i'+k-1, c_i'+k)$ for $0 \leq k \leq 4n+1$, when c_i' is the leftmost column of the enforcer block E_i, the following invariants are maintained: (i) if $i > j$ then $\sigma_u(i), \sigma_u(\bar{i}) > \sigma_u(j), \sigma_u(\bar{j})$; (ii) nets N_e, N_f, N_{e_i}, and N_{f_i} occupy tracks 4, 3, 2, and 1, respectively.

Next, we consider the layout of a clause block. We must show that a clause block can be laid out if and only if its corresponding clause in 3SAT is satisfiable, that is, (with reference to Figure 5) $\sigma_u(a) > \sigma_u(\bar{a})$, or $\sigma_u(b) > \sigma_u(\bar{b})$, or $\sigma_u(c) > \sigma_u(\bar{c})$ in (c_i-1, c_i). We must also guarantee that the ordering of pairs of nets corresponding to a variable remains fixed as we lay out a block.

The upper terminal of column c_i (the leftmost column of B_i) is e_i and the lower terminal is e (see Figure 5). Since $\sigma_u(e) > \sigma_u(e_i)$, either N_e or N_{e_i} must have two strands in the section (c_i, c_i+1) (this is a trivial variation of Lemma 1). Since N_{e_i} and N_e can occupy at most two tracks in the section (c_i, c_i+1), and N_e is forced to have at least one strand in this section (since N_e is a continuing net), N_e must be the net with two strands at this vertical section. Therefore, column c_i can be <u>uniquely</u> laid out; we connect the upper-terminal to $\sigma_u(e_i)$, thereby terminating N_{e_i}, and we connect the lower-terminal to $\sigma_u(e_i)$, as shown in Figure 8a. Using a similar argument and considering that the two strands of N_e cannot be spliced at column c_i+1, we can verify the existence of a unique layout for this column as shown in Figure 8b.

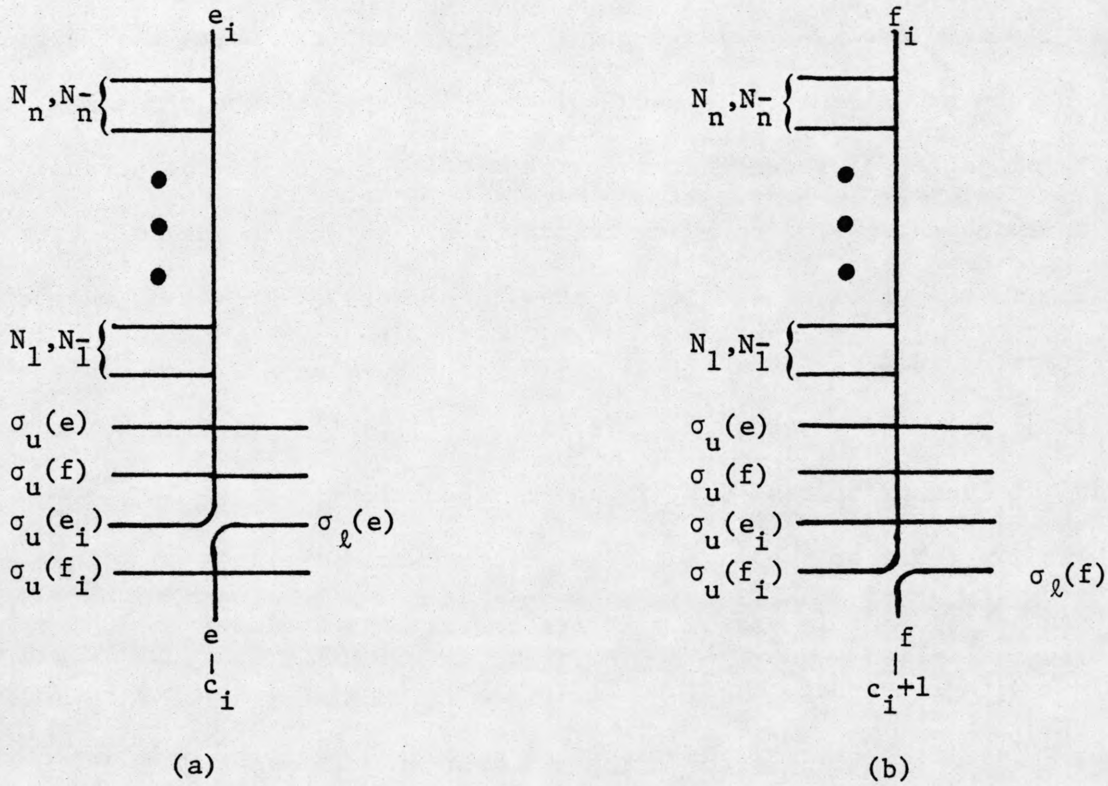
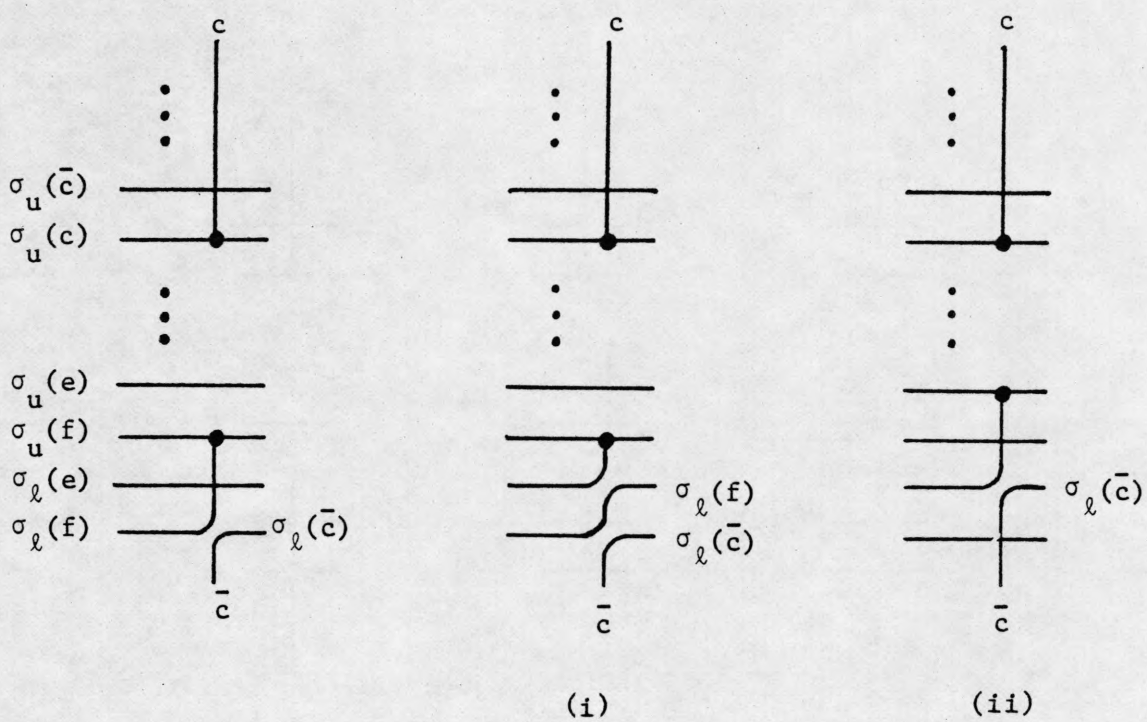


Figure 8. Layout of column c_i and c_i+1.

Next we show that if all the variables of a clause in 3SAT are false then the corresponding clause block cannot be laid out (in 2n+4 tracks). With reference to Figure 5, assume that ν_c (the variable associated with $(N_c,N_{\bar{c}})$) is false, that is, $\sigma_u(\bar{c}) > \sigma_u(c)$. We must then have a free track in the vertical section (c_i+2,c_i+3) to be able to run the second strand of net $N_{\bar{c}}$ (or N_c). We can have this free track if we splice the two strands of N_e or N_f at column c_i+2 (note that we cannot splice the strands of both nets). If we splice the strands of N_f then there is a unique way to layout column c_i+2; we connect the upper-terminal to $y = \sigma_u(c)$, the lower-terminal to $y = \sigma_\ell(f)$, and $y = \sigma_u(f)$ to $y = \sigma_\ell(f)$ (see Figure 9a). Instead, if we splice the strands of N_e then there are two ways to layout this column, as shown in Figure 9b.

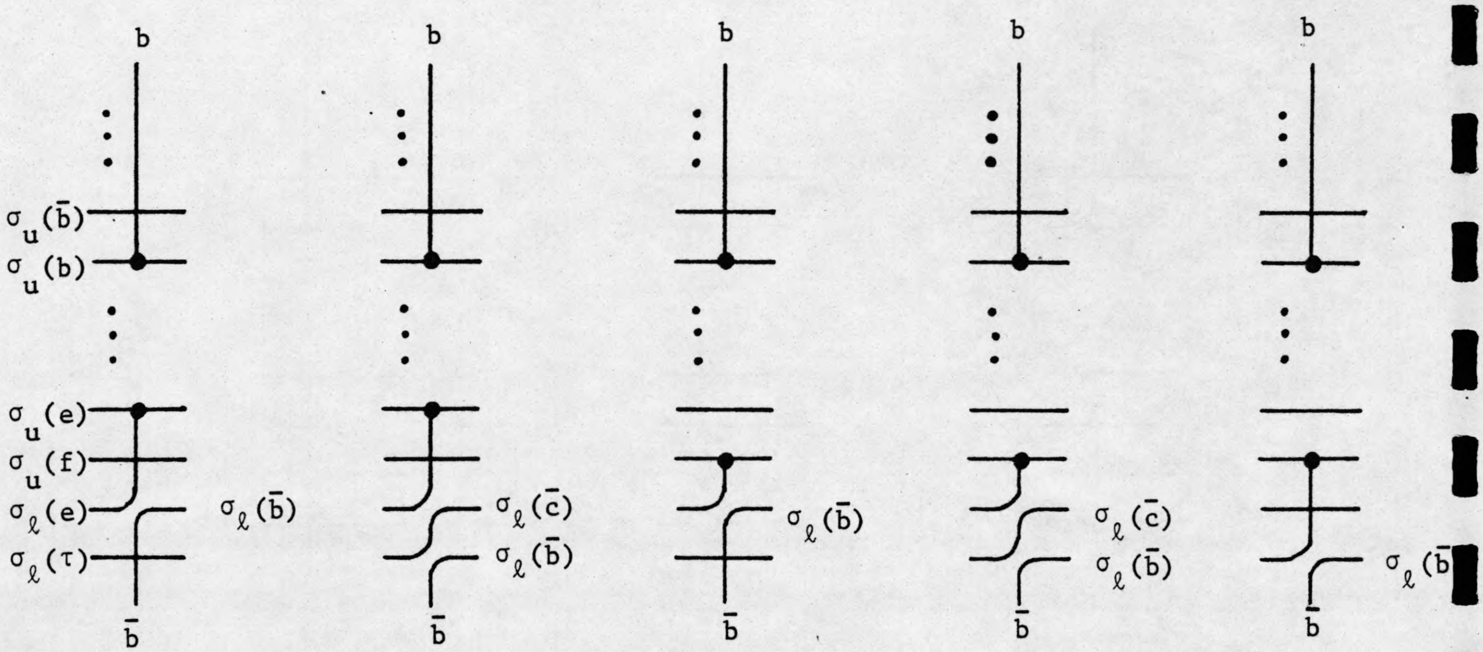


(a) Strands of N_f are spliced (b) Strands of N_ℓ are spliced

Figure 9. Layout of column c_i+2.

If ν_b is false, that is, $\sigma_u(\bar{b}) > \sigma_u(b)$, then we can apply the preceding argument to show that net N_e or N_f (depending on which one was spliced at c_i+2) must be spliced at column c_i+3. It is crucial to observe that $N_{\bar{c}}$ cannot be spliced at this column; if it is, then we are unable to connect the upper- or the lower-terminal to any tracks. All possible ways to layout column c_i+3 are shown in Figure 10, which provides sufficient explanation.

A closer look at column c_i+4 reveals that neither the strands of $N_{\bar{c}}$ nor the strands of $N_{\bar{b}}$ can be spliced at this column, since $\sigma_u(c),\sigma_u(b) > \sigma_u(a),\sigma_u(\bar{a}) > \sigma_\ell(c),\sigma_\ell(b)$. We conclude that if ν_a is false -- that is, $\sigma_u(\bar{a}) > \sigma_u(a)$ --



(a) If c_i+2 is laid out as shown in Figure 9a.

(b) If c_i+2 is laid out as shown in Figure 9b(i).

(c) If c_i+2 is laid out as shown in Figure 9b(ii).

Figure 10. Layout of column c_i+3.

then B_i cannot be laid out in 2n+4 tracks. In fact, to lay out column c_i+4 we must add one more track. The preceding discussion gives us the following result.

Lemma 3. If $\sigma_u(\bar{a}) > \sigma_u(a)$, $\sigma_u(\bar{b}) > \sigma_u(b)$ and $\sigma_u(\bar{c}) > \sigma_u(c)$ in B_i, that is, $\nu_a + \nu_b + \nu_c = 0$, then B_i cannot be laid out in 2n+4 (=δ) tracks.

Assume that ν_c and ν_b are false, but ν_a is true. Column c_i+2 and c_i+3 are laid out as shown in Figures 8 and 9. Column c_i+4 can be laid out by connecting the upper-terminal to $y = \sigma_u(a)$ and lower-terminal to $y = \sigma_u(\bar{a})$. At column c_i+5 and c_i+6, we will splice the strands of $N_{\bar{c}}$ and $N_{\bar{b}}$ and begin $N_{e_{i+1}}$ and $N_{f_{i+1}}$, respectively. Note that there are only two possible ordering of nets in the section (c_i+3, c_i+4); we can lay out columns c_i+4, c_i+5, and c_i+6 correspondingly in two different ways (see Figure 11). We also have a choice of splicing the strands of $N_{\bar{c}}$ and $N_{\bar{b}}$ at c_i+5 or c_i+6, that is, we can trade the knock-knees at $(x,y) = (\sigma_\ell(\bar{c}), c_i+5)$ and $(\sigma_\ell(\bar{b}), c_i+6)$ with the one at $(\sigma_\ell(\bar{b}), c_i+5)$ in Figure 11a. A similar tradeoff exists for the case illustrated in Figure 11b.

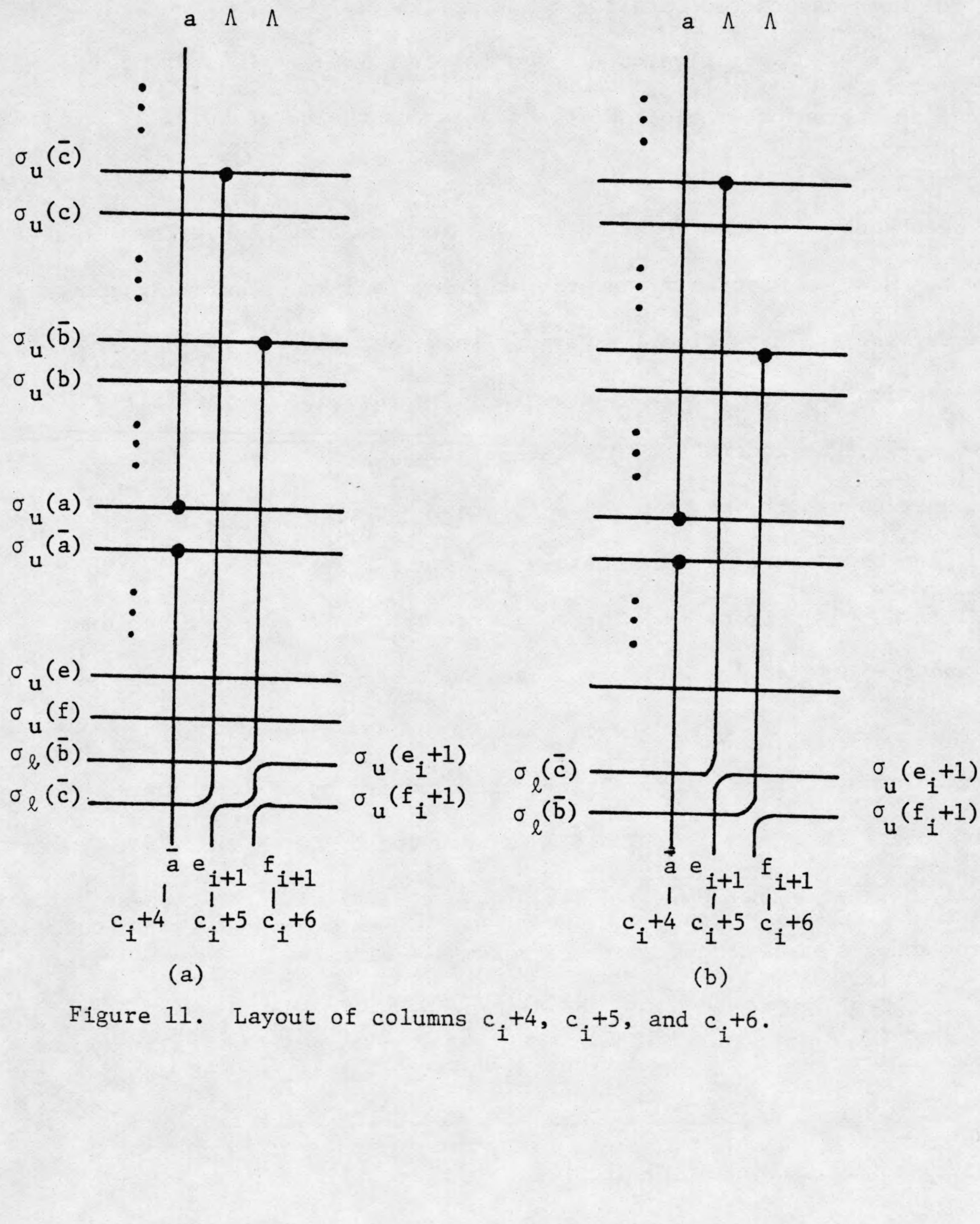


Figure 11. Layout of columns c_i+4, c_i+5, and c_i+6.

We conclude that if $(\nu_a,\nu_b,\nu_c) = (0,1,1)$, then we can lay out the clause block E_i (in 2n+4 tracks). Since only the pair of nets corresponding to false variables (ν_a in the preceding discussion) require an extra track, and there are at most two free tracks , we conclude that the same result holds if at least one of the variables is true which means that the two tracks that are free will be used for the nets associated with the false variables.

Lemma 4. If $\nu_a \vee \nu_b \vee \nu_c = 1$ (i.e. $\sigma_u(a) > \sigma_u(\bar{a})$ or $\sigma_u(b) > \sigma_u(\bar{b})$, or $\sigma_u(c) > \sigma_u(\bar{c})$) in the enforcer block B_i, we can layout the block using $2n+4$ $(=\delta)$ tracks.

It has been shown that the ordering of nets remains fixed as we lay out an enforcer block. Now, we must show that the ordering is also preserved when we layout a clause block. In particular, we must show that if $\sigma_u(N_i) \gtrless \sigma_u(N_{\bar{i}})$ at the vertical section (c_i-1,c_i) of B_i then $\sigma_u(N_i) \gtrless \sigma_u(N_{\bar{i}})$ at the vertical section (c_i+6,c_i+7). We accomplish this task by showing that net N_j for $1 \leq j \leq n$, cannot change tracks from $y = \sigma_u(j)$ to $y = \sigma_u(\bar{j})$.

We have seen that there is a unique way to layout column c_i and c_i+1 in the clause block B_i (see Figure 8). The only available free tracks at columns c_i+2, c_i+3, and c_i+6 are at $y = 1$ or $y = 2$, and they become available when we splice the two strands of N_e and N_f (each in a different column). This means that at these columns, tracks 3 through 2n+4 are occupied by distinct nets, and the vertical section $(y_1,y_2) = (\sigma(e_i),\sigma(f))$ is occupied by net N_e, N_f, $N_{\bar{c}}$, $N_{\bar{b}}$, or $N_{\bar{a}}$. Therefore, neither N_a nor N_b, nor N_c can have a vertical wire at these columns, that is, they cannot switch tracks at these columns. Finally, c_i+7 contain a segment $N_{e_{i+1}}$ and $N_{f_{i+1}}$ and also nets N_e, N_f, $N_{\bar{a}}$, $N_{\bar{b}}$, or $N_{\bar{c}}$ can be spliced at this column (exactly one net per column). Trivially, none of N_a, N_b, or N_c can have a vertical segment at column c_i+5 or c_i+6. The preceding discussion gives us the following result.

Lemma 5. The ordering of nets that corresponds to the truth assignment of the variables is an invariant from one block to the other. We now integrate the previous results to establish the main theorem of this section.

Theorem 1. Our proposed instance of the GCRP with $\delta = 2n+4$ can be laid out in 2n+4 tracks if and only if the corresponding 3SAT problem is satisfiable.

Proof: (if) If the corresponding 3SAT is satisfiable, then it means that $\nu_a^i + \nu_b^i + \nu_c^i = 1$ for all i, $1 \leq i \leq m$. Thus, in each clause block $\sigma_u(a) > \sigma_u(\bar{a})$, $\sigma_u(b) > \sigma_u(\bar{b})$, or $\sigma_u(c) > \sigma_u(\bar{c})$, so by Lemma 4, every one of the clause blocks can be laid out using 2n+4 tracks.

(only if) If the corresponding 3SAT is not satisfiable, that is, $\nu a^i + \nu_b^i + \nu_c^i = 0$ for some i, $1 \leq i \leq m$, then in the corresponding clause block $\sigma_u(\bar{a}_i) > \sigma_u(a_i)$ and $\sigma_u(\bar{b}_i) > \sigma_u(b_i)$, and $\sigma_u(\bar{c}_i) > \sigma_u(c_i)$. Lemma 3 implies that the corresponding clause block cannot be laid out in 2n+4 tracks; indeed, we must use more than 2n+4 tracks (i.e., 2n+5 tracks) to layout that clause block. Thus, the entire problem requires more than 2n+4 tracks. These arguments hold provided that the truth assignment is common to all bolcks and this was established by Lemma 5.

Theorem 2. It is NP-complete to decide whether an arbitrary instance of a GCRP with essential density δ can be laid out using a specified number of tracks.

Proof. In Theorem 1 we have shown that a known NP-complete problem is reducible to the GCRP in polynomial time. This implies that the GCRP is NP-complete. □

Immediate consequences of the previous theorem are:

<u>Corollary 1</u>: The following problems are NP-hard (or NP-complete) in knock-knee mode:

(i) To determine the minimum number of tracks needed to lay out an arbitrary instance of the GCRP.

(ii) To decide whether a given layout for an instance of the GCRP is optimal.

(iii) To obtain an optimal layout, that is, a layout that uses the least possible number of tracks.

3. NETS OF BOUNDED DEGREE

In this section, we shall extend the previous result to nets of bounded degree k. We will show that for $k \geq 5$ it is NP-complete to decide whether an arbitrary k-terminal net GCRP (where each net has at most k terminals) can be laid out with a specified number of tracks.

In order to establish this result, we need to modify the construction of Section 2 in such a way that each net has at most k terminals. Since each net has at most one terminal in a clause block (see Figure 5), we begin by cutting a k-terminal net N_i into s pieces, (each an individual net), called $N_i^1, N_i^2, \ldots, N_i^s$.

Each column c of an enforcer block with upper terminal i and lower-terminal j will be replaced by five columns. At the first two columns we terminate nets N_i^m and N_j^m, and, at the next two columns, we begin N_i^{m+1} and N_j^{m+1}. Finally, at the fifth column we enforce $\sigma_u(i) > \sigma_u(j)$ which is exactly what column c was originally meant to accomplish (see Figure 12).

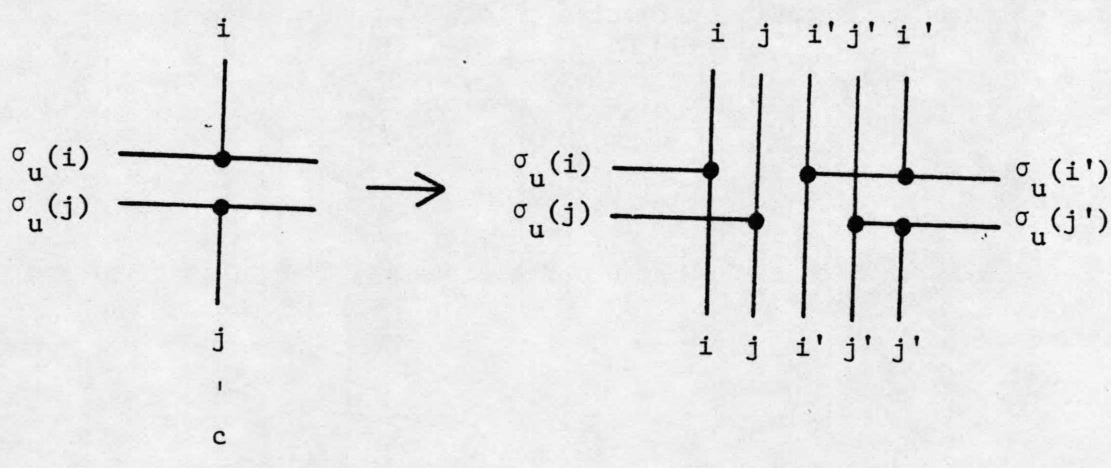
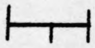
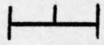
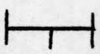
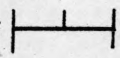
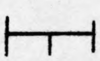
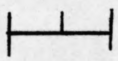


Figure 12. Transformation of a column in an enforcer block.

After this transformation, every net has exactly five terminals (with the exception of the nets that have a terminal in an end block). In fact, all the nets have either  or  form. Obviously, the ordering of the nets (see Lemma 2) is preserved and we conclude:

Theorem 3: It is NP-complete to decide whether an arbitrary instance of the k-terminal net CRP, for $k \geq 5$, can be laid out in a specified number of tracks. Obviously, Corollary 1 also holds for this problem (k-terminal net CRP).

As we have mentioned, all the nets in our final construction are of the form  or  . Preparata and Sarrafzadeh [PS] have shown that a 3-terminal net CRP with essential density δ can be laid out (and wired) using $3\delta/2$ tracks. An immediate consequence of this result is that a problem with nets of the form  or , and essential density δ, can also be laid out in $3\delta/2$ tracks, Theorem 3 shows that it is NP-hard to find an optimal layout (or routing) for this problem.

It should be noted that Theorem 3 holds for other routing models like routing through a rectangle or routing through a generalized switchbox.

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