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J. MEDANIC
CHENG-I CHEN

UNIVERSITY OF ILLINOIS - URBANA, ILLINOIS

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J. Medanic and Cheng-I Chen
Coordinated Science Laboratory
University of Illinois
Urbana, Illinois

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ABSTRACT

This paper develops the segment method for the problem of control with uncertainty. It is shown that due to disturbances a segment of values of the performance functional is associated with each admissible control. It is proposed that the optimal control for the problem with uncertainty be defined on the basis of an ordering of associated segments. The ordering is achieved by introducing a segment index. Two particular segments, the pessimistic and the optimistic segment, are shown to be important in the analysis of the problem while minimization of the introduced segment index is the basic synthesis problem. The approach is applied to a linear control problem where uncertainty is caused by external disturbances and specific results are presented. They show the optimal control to retain the advantages of the usual, minimax type designs and ensure better system performance for disturbances other than the worst.

INTRODUCTION

The control problem with uncertainty is commonly treated by stochastic methods where a distribution function is associated to the set of possible variations of plant parameters or initial conditions and where the optimal control is that which minimizes the expected value of the selected perform-

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[††]On leave from Mihailo Pupin Institute, Belgrade, Yugoslavia.

ance functional [1]. Unfortunately, it is difficult to find solutions in all but the simplest problems. Another commonly employed approach is based on the sensitivity of the system outputs and the performance functional to parameter variations where usually a correction δu to the nominal optimal control u^o is sought. For a discussion of such methods and some recent results see Perkins et al [2] and Kokotivić et al [3]. In many problems with uncertianty neither of these two approaches can be fully justified. It is possible that a distribution function does not even exist, but more commonly that it is unknown, while methods based on first order sensitivities are valid only for small parameter variation.

The problem with large parameter variations has not received as much attention. The only approach employed in practice in the minimax, or worst case design. Recently performance sensitivity-based approaches which consist in finding the minimax solution of a selected performance sensitivity index [4,5] have been proposed. In order to approach the problem where uncertainties due to large parameter variations or external disturbances are present, this paper introduces the notion of performance segments and develops the segment method for the design of controls for uncertain systems. It is shown that a segment of a real line (instead of a point as in the problem without uncertainty) is associated with each control. The optimal control for the problem with uncertainty is obtained by ordering the set of segments by employing a selected segment index.

In this paper the basic features of the segment approach are presented. Specifically, the set of segments is defined, the particular significance of the pessimistic and the optimistic segment in characterizing the nature of the problem with uncertainty is stressed and the segment index is introduced. The approach is then applied to a linear control problem with external disturbance inputs and results are presented.

SEGMENTS AND THE SEGMENT INDEX

Consider the system

$$\dot{x} = f(x,u,v),\ x(t_o) = x_o(v),\ u \in U,\ v \in V, \tag{1}$$

and an associated functional

$$J(u,v) = \int_{t_o}^{T} L(x,u,v)dt + g[x(T)] \tag{2}$$

where $x = [x_1(t),\ldots,x_n(t)]$ is the state and $u = [u_1(t),\ldots,u_m(t)]$ is the control. The perturbation vector $v = [v_1,\ldots,v_r]$ may be a parameter variation, or a variation in the initial conditions; it may also be an external disturbance input $v = [v_1(t),\ldots,v_r(t)]$. The only specification on the perturbation v is that it belongs to a given compact set V. It is assumed that $f(x,u,v)$, $L(x,u,v)$ and $g(x)$ are sufficiently smooth in all variables and that $J(u,v)$ is continuous on the compact sets U and V.

The problem of control under uncertainty is to design a system which will perform "satisfactorily" with respect to the functional (2) in the presence of any $v \in V$. The more specific problem is to define the best, in this paper also called the optimal control. To achieve this aim some preliminary concepts concerning the structure of the set $\{J(u,v) \mid u \in U, v \in V\}$ of all attainable values of the associated functional, are introduced.

Notice that for each given control $\tilde{u}$ and for all perturbations $v \in V$ equation (2) defines a set, denoted by $E(\tilde{u})$, of attainable values of the performance functional:

$$E(\tilde{u}) = \{J(\tilde{u},v) \mid v \in V\}. \qquad (3)$$

Because of continuity of J, the set $E(\tilde{u})$ is a segment determined by its upper and lower bounds:*

$$J_h(\tilde{u}) = \max_{v \in V} J(\tilde{u},v) \qquad (4)$$

$$J_l(\tilde{u}) = \min_{v \in V} J(\tilde{u},v). \qquad (5)$$

Hence, the set $\{J(u,v) \mid u \in U, v \in V\}$ may be considered to be composed of segments $E(u)$ corresponding to all controls $u \in U$. In this way, the set of controls U is mapped into a set of segments $I(U)$,

$$I(U) = \{E(u) \mid u \in U\}. \qquad (6)$$

The ordering of segments leads to an optimal segment and it is proposed that as optimal be selected the control corresponding to the optimal segment.

The optimal segment is dependent on the appropriate meaning of optimality when considering the set of associated segments. The nature of the problem with uncertainty and the role of the associated functional, (2), suggests that a reasonable way to define optimality is by associating the index

$$S(u) = \sqrt{J_h^2(u) + J_l^2(u)} \qquad (7)$$

to the segment $E(u)$; optimal is the segment minimizing this index. Intuition suggests that if an entire segment of values characterizes the performance of the system for a selected control $\tilde{u}$, the feature desired of the optimal

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*Only bounds, inf and sup, in the general case exist, but here it is assumed that all extremas are attained. Most important is that the assumption is satisfied in many problems of interest.

control is to displace and "squeeze" the associated segment of values downward. This is also the motivation behind minimax type designs; in such designs, however, what is achieved by selecting the control associated with the minimal upper bound is to squeeze down only the upper bound of the segment. In the segment method, on the other hand, the intention is to displace the complete segment of values downward and this is accomplished by selecting the control which minimizes the sum of squares of both bounds of the associated segment. The values of the two bounds of a segment are mutually dependent and this minimization must be joint.

To justify the segment approach and the employment of the introduced segment index, two particularly important segments, the pessimistic and the optimistic segment, are introduced in the following section and their influence on the solution of the problem of control with uncertainty is examined. The approach is then illustrated on a specific linear control problem where uncertainty is due to the presence of external disturbances. Some computer results are presented and compared with those attainable by other methods. Computation of the control that minimizes $S(u)$ is the basic synthesis problem in the segment approach. A gradient procedure is utilized in this paper to solve the linear control problem but it cannot be considered general.

THE PESSIMISTIC AND THE OPTIMISTIC SEGMENT

Recall that a segment $E(\tilde{u})$ is associated with a specific control $\tilde{u}$, and is defined by its upper and lower bounds (4) and (5). To define the pessimistic segment consider u^s and v^s satisfying

$$J(u^s,v^s) = \min_{u\in U} \max_{v\in V} J(u,v). \tag{8}$$

The segment associated with u^s is called the pessimistic segment, $E(u^s)$.

By (4) and (5), with $\tilde{u} = u^s$, its upper and lower bounds are

$$P_h = J_h(u^s) = \max_{v\in V} J(u^s,v) = \min_{u\in U} \max_{v\in V} J(u,v), \tag{9}$$

$$P_1 = J_1(u^s) = \min_{v\in V} J(u^s,v). \tag{10}$$

Analogously, to define the optimistic segment consider u^b and v^b satisfying

$$J(u^b,v^b) = \min_{u\in U} \min_{v\in V} J(u,v). \tag{11}$$

The segment associated with u^b is called the optimistic segment, $E(u^b)$, and from (4) and (5), with $\tilde{u} = u^b$, has bounds

$$0_h = J_h(u^b) = \max_{v\in V} J(u^b,v), \tag{12}$$

$$0_1 = J_1(u^b) = \min_{v\in V} J(u^b,v) = \min_{u\in U} \min_{v\in V} J(u,v). \tag{13}$$

A relationship between the two segments is given by

Theorem 1: The pessimistic segment is contained in the optimistic segment,

$$E(u^s) \subset E(u^b). \tag{14}$$

Proof: By definition $J(u^b,v^b) \equiv 0_1 \leq P_1$.

Moreover, $P_h \equiv J(u^s,v^s) \leq J_h(u)$ for all $u\in U$ and therefore for $u = u^b$.

Hence, $P_h \leq J_h(u^b) \equiv 0_h$.

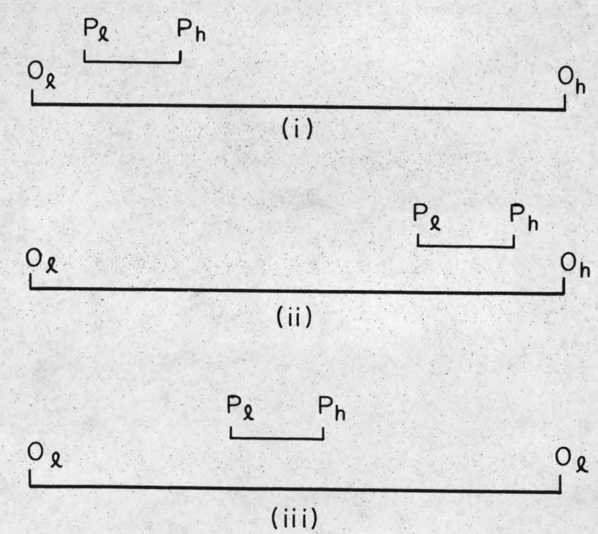

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Fig. 1a

The pessimistic segment and the optimistic segment are characteristics of the system, independent of the particular segment index used and out of influence of design. They characterize the nature of the problem of control with uncertainty. To further reveal their role consider, in view of the theorem, the three representative situations depicted in Fig. 1a, where: (I) $d_l \ll d_h$, $O_h - O_l \gg P_h - P_h$; (II) $d_l \gg d_h$, $O_h - O_l \gg P_h - P_l$; (III) $d_l \approx d_h$, $O_h - O_\ell \gg P_h - P_\ell$, with $d_l \equiv P_l - O_l$ and $d_h \equiv O_h - P_h$.

In (I) the position of $E(u^s)$ in $E(u^b)$ shows the pessimistic segment to be satisfactory: u^s achieves the minimal value of the upper bound of the associated segment while a comparatively low value of P_1, as compared to 0_1, indicates satisfactory system performance for all $v \in V$. In (II) the position of $E(u^s)$ in $E(u^b)$ shows the optimistic segment to be satisfactory: u^b secures the minimal value of the lower bound of the associated segment while a comparatively low value of 0_h, as compared to P_h, indicates satisfactory system performance even for worst perturbations in V.

While $E(u^s)$ in (I) or $E(u^b)$ in (II) are not the segments that strictly minimize (7) in the given situations, there is negligible gain in system performance to be expected by further sophistication of design. Design can, therefore, be terminated by selecting the pessimistic design in (I) and the optimistic design in (II).

It is because of the general situation in (III) that there is need of more sophisticated design. The pessimistic design displays a short-coming in the high value of P_1 while the optimistic design displays a short-coming in the high value of 0_h. While now the position of $E(u^s)$ within $E(u^b)$ does not indicate the optimal mode of system control. It is stressed, however, that even in this general case the pessimistic and the optimistic segment retain their significance due to the following result stated as a corollary to the above theorem:

<u>Corollary</u>: The segment $E(u^*)$ associated to the optimal control u^* minimizing $S(u)$ must satisfy the condition

$$E(u^s) \subset E(u^*) \subset E(u^b). \qquad (15)$$

Or, in expanded form,

$$0_l \le J_l(u^*) \le P_l < P_h \le J_h(u^*) \le 0_h. \qquad (16)$$

Proof is trivial: Suppose a control $\tilde{u}$ violates (16). This is possible in one of the following ways: (i) let $J_h(\tilde{u}) > 0_h$, $J_l(\tilde{u}) \le P_l$. Necessarily $J_l(\tilde{u}) \ge 0_l$ and therefore $S(\tilde{u}) > S(u^b)$ and $\tilde{u}$ cannot be optimal; (ii) $J_h(\tilde{u}) \le 0_h$, $J_l(\tilde{u}) > P_l$. Necessarily $J_h(\tilde{u}) \ge P_h$ and therefore $S(\tilde{u}) > S(u^s)$ and again $\tilde{u}$ cannot be optimal; (iii) the case $J_h(\tilde{u}) > 0_h$, $J_l(\tilde{u}) > P_l$ is evident from the first two.

Moreover, in view of theorem I, there are only four basically distinct positions a segment $E(\tilde{u})$ corresponding to a $\tilde{u} \in U$ can have with respect to the position of $E(u^s)$ and $E(u^b)$. The first is given in the statement of the corollary, the other three in its proof. When the four positions are displayed graphically, Fig. 1b, intuitive feeling agrees with the selection of a control satisfying (16).

In conclusion, the segments $E(u^s)$ and $E(u^b)$ characterize the nature of the problem with uncertainty and moreover confine a range of possible improvements in system design by confining the range within which the optimal segment must be located. In certain cases the pessimistic and optimistic segment directly furnish the solution making further design unnecessary.

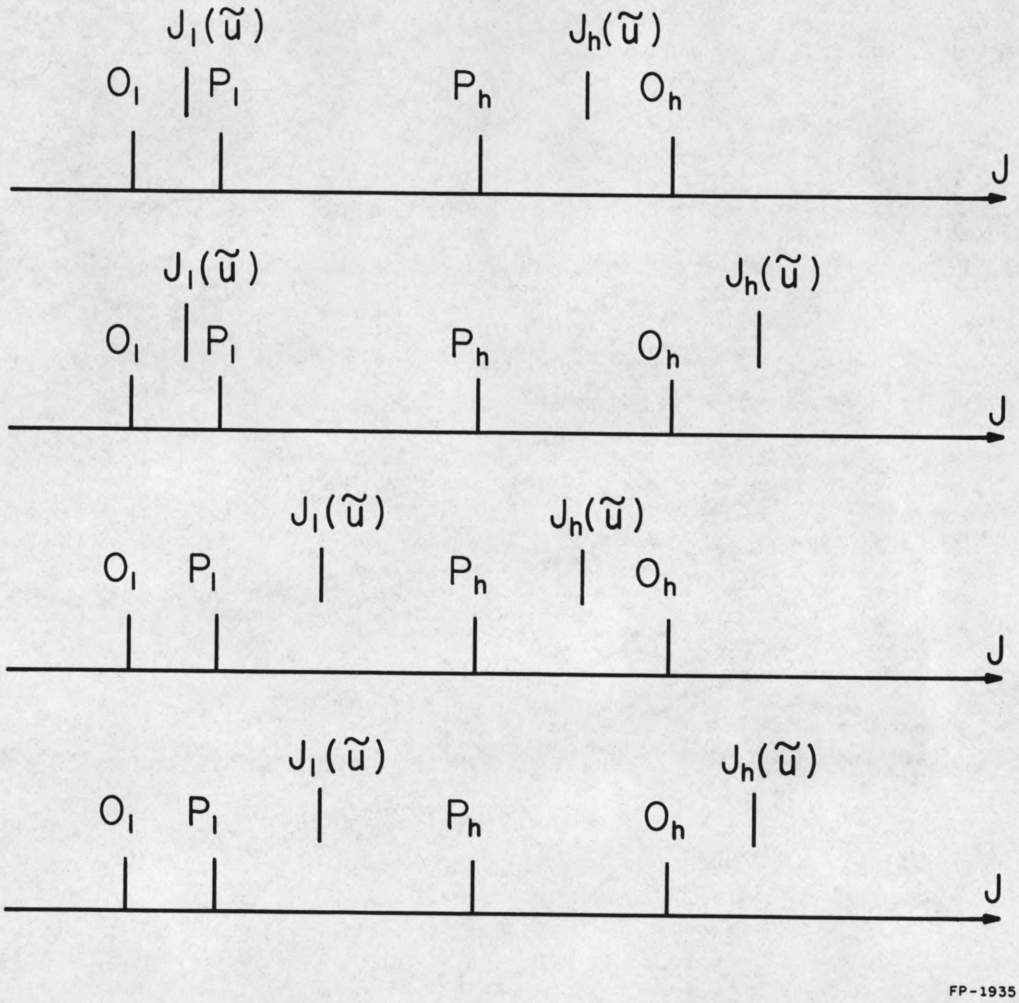

Fig. 1b

APPLICATION TO THE DESIGN OF LINEAR CONTROL SYSTEMS

Consider now the application of the segment approach to the problem of designing controls for a linear control system in which uncertainty exists and is caused by external disturbance inputs. The associated functional is assumed to be quadratic in the state and control variables.

Hence, (1) and (2) take the form

$$\dot{x} = A_o(t)x + B(t)u + v, \; x(t_o) = x_o \tag{17}$$

$$J(u,v) = \tfrac{1}{2}\int_{t_o}^{T} (x^*Qx + u^*Ru)dt + \tfrac{1}{2}x(T)^*Fx(T). \tag{18}$$

The corresponding problem without uncertainty ($v \equiv 0$) is the linear regulator problem whose solution is well known and the details are therefore omitted [7,8], with the remark that the relevant conditions are imposed on $A_o(t)$, $B(t)$, Q, R, and F. The optimal control is

$$u^o = -R^{-1}B^*K_o x \tag{19}$$

where the optimal gain matrix, $K_o(t)$, is the solution of the Riccati equation

$$\dot{K}_o + A_o^*K_o + K_oA_o - K_oS_oK_o + Q = 0, \; K_o(T) = F, \tag{20}$$

$$S_o = BR^{-1}B^*, \tag{21}$$

and the minimal value of the performance index is

$$J(u^o) = \tfrac{1}{2}x_o^*K_o(t_o)x_o. \tag{22}$$

When disturbances from a specified set V act on the system it may be necessary to modify the optimal control. The first step is to analyze the problem by obtaining the pessimistic and the optimistic segment as well as to obtain the position of the segment $E(u^o)$ corresponding to the optimal control u^o, (19). If necessary, the optimal control for the problem with uncertainty is then modified and obtained by minimizing (7).

The nature of the problem with uncertainty depends on the set of disturbances considered as admissible. Here and in the following section admissible disturbances are implicitly defined as those satisfying

$$\int_{t_o}^{T} v^*v\,dt \leq \gamma \int_{t_o}^{T} x^*x\,dt, \tag{23}$$

where γ is a specified constant. Such a characterization of the set V indicates a relative dependence of disturbances on the system trajectory.

Consider now the problem of finding the bounds of the pessimistic segment. P_h the minimax of $J(u,v)$ is simply found since it is at the same time a saddle point [9,10]. In view of (23) the Hamiltonian for the saddle point problem becomes

$$H = \tfrac{1}{2}[x^*(Q + \gamma m_1 I)x + u^*Ru - m_1 v^*v] + p^*(A_o x + Bu + v). \tag{24}$$

From the necessary conditions for a saddle point, the minimizing control and the maximizing disturbance may be written in the form†

$$u^s = -R^{-1}B^*K_s x, \quad v^s = \frac{1}{m_1} K_s x, \tag{25}$$

where $K_s(t)$ is the solution of the Riccati equation

$$\dot{K}_s + A_o^*K_s + K_s A_o - K_s S_s K_s + Q + \gamma m_1 I = 0, \quad K_s(T) = F \tag{26}$$

with

$$S_s = BR^{-1}B^* - \frac{1}{m_1}I = S_o - \frac{1}{m_1}I. \tag{27}$$

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†It is not implied by (25) that the disturbances appear in the system in some feedback realization but that the worst disturbance $v(t)$, given u^s is expressible in this way.

The multiplier $m_1 > 0$ is such that equality in (23) is satisfied. The value of the upper bound of the pessimistic segment is

$$P_h = \tfrac{1}{2}x_o^* K_s(t_o)x_o. \tag{28}$$

The lower bound P_1 is obtained by closing the loop with the feedback control u^s and finding the most favorable disturbance in the resulting system. The system equation becomes

$$\dot{x} = (A_o - S_o K_s)x + v,\ x(t_o) = x_o. \tag{29}$$

with

$$J(u^s, v) = \tfrac{1}{2}\int_{t_o}^{T} x^*(Q + K_s S_o K_s)x\,dt + \tfrac{1}{2}\, x(T)^* F x(T). \tag{30}$$

The most favorable, minimizing, disturbance is

$$v^1 = -\frac{1}{m_2} N_1 x, \tag{31}$$

where $N_1(t)$ is the solution of the Riccati equation

$$\dot{N}_1 + (A_o - S_o K_s)^* N_1 + N_1(A_o - S_o K_s) - \frac{1}{m_2} N_1^2 + Q + K_s S_o K_s - \gamma m_2 I = 0, N_1(T) = F, \tag{32}$$

where m_2 is a positive multiplier such that equality in (23) is satisfied. The value of the lower bound of the pessimistic segment is

$$P_1 = \tfrac{1}{2}\, x_o^* N_1(t_o)x_o. \tag{33}$$

A similar set of equations can be shown to define the bounds of the optimistic segment:

$$u^b = -R^{-1}B^*K_b x, \tag{34}$$

$$v^b = \frac{1}{m_3} K_b x, \tag{35}$$

$$0_1 = \tfrac{1}{2}\, x_o^* K_b(t_o)x_o, \tag{36}$$

$$v^h = -\frac{1}{m_4} N_h x, \tag{37}$$

$$O_h = \tfrac{1}{2} x_o^* N_h(t_o) x_o . \tag{38}$$

$K_b(t)$ and $N_h(t)$ are solutions of (26) and (32) respectively, where m_3, m_4 are now, respectively, negative multipliers. The values of the miltipliers m_1, m_2, m_3, and m_4 are initially unknown and are found by the iterative solution of the corresponding Riccati equation together with the system equation and the constraint condition (23) until equality in (23), in each case, becomes satisfied.

Two examples are presented for a discussion of the results. Both examples are of second order with the same associated functional characterized by

$$Q = \begin{Vmatrix} 1 & 0 \\ 0 & 1 \end{Vmatrix}, \quad R = 1,$$

with $T = \infty$. In the first example, denoted by S, the system is characterized by

$$A_o = \begin{Vmatrix} -1 & 1 \\ 0 & -1 \end{Vmatrix}, \qquad B = \begin{Vmatrix} 0 \\ 2 \end{Vmatrix}.$$

In the second example, denoted by C, the system is characterized by

$$A_o = \begin{Vmatrix} 0 & 1 \\ 0 & -1 \end{Vmatrix}, \qquad B = \begin{Vmatrix} 1 \\ 1 \end{Vmatrix}.$$

The results were obtained on a digital computer and have been plotted in function of the parameter γ. Hence, they represent an analysis of a family of problems and show how the systems react under increasing level of external disturbances. As $\gamma \to 0$, both the pessimistic and the optimistic segment degenerate to the point $J(u^o)$ while the controls u^s, u^b converge to the optimal control u^o. Both examples, Fig. 2, show the typical position of $E(u^s)$ in $E(u^b)$ characteristic to this problem. The supremacy of the pessimistic over the optimistic segment, particularly for greater values of γ, is evident. As expected, 0_1 decreases with the increase of γ indicating the presence of disturbances not adverse to system performance. On the other side, $0_h \to \infty$ as $\gamma \to \gamma_{crit}$, where γ_{crit} is smaller for less stable plants. Concerning the pessimistic segment, it is noted that although the sufficiency condition for the existence of a saddle point (positive semi-definiteness of S_s) is violated even for arbitrary small m_1, a saddle point with a finite value of P_h exists in some systems for relatively large γ. In other examples, particularly those where the plant is less susceptible to control action, P_h tends to infinity as $\gamma \to \gamma_{max}$† (e.g., in system C but with $B^* = ||0 \quad 1||^*$, $P_h \to \infty$ at about $\gamma \approx 1$). The dependence of P_1 on γ is also characteristic. As expected, its value decreases as γ increases but only until a minimal value of P_1 is reached for some γ_{min}. Increasing γ further causes an increase in the value of P_1. This manifests the necessity of employing a rigid control to give the system relatively good performance characteristics even for worst disturbances.

SYNTHESIS OF THE OPTIMAL CONTROL

The examples demonstrate that the pessimistic design might be a satisfactory choice in this problem, particularly for large γ. For smaller γ the supremacy of the pessimistic over the optimistic design is not as

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†The limiting values γ_{crit} and γ_{max} depend on the system, were obtained experimentally and have not been investigated at the present time.

pronounced and it is meaningful to undertake further design.

Consider therefore the synthesis problem of finding the linear feedback control that minimizes S(u). The gradient technique developed below restricts further discussion to time-invariant linear systems and an infinite integration interval, as in the examples. The synthesis problem reduces to finding the optimal feedback control u* characterized by the optimal gain matrix K*. The procedure consisted in the following. Let the feedback control

$$\tilde{u} = -R^{-1}B^*\tilde{K}x \qquad (39)$$

be implemented where $\tilde{K}$ is an initial guess for K*. Elements of $\tilde{K}$ are adjusted in accordance with the usual algorithm,

$$\delta k_{ij} = -\frac{1}{\rho}\frac{\partial S(\tilde{u})}{\partial k_{ij}} \qquad (40)$$

where

$$\frac{\partial S}{\partial k_{ij}} = \frac{1}{S}\left[J_h \frac{\partial J_h}{\partial k_{ij}} + J_l \frac{\partial J_l}{\partial k_{ij}}\right]$$

Implementation of the algorithm demands that for each $\tilde{u}$, the values of $J_h(\tilde{u})$, $J_l(\tilde{u})$, $S(\tilde{u})$, $\frac{\partial J_h(\tilde{u})}{\partial k_{ij}}$, $\frac{\partial J_l(\tilde{u})}{\partial k_{ij}}$ be computed. Bounds, $J_h(\tilde{u})$, $J_l(\tilde{u})$, of the segment associated with $\tilde{u}$ are found by solving the Riccati equation (32) (to find the steady state solution); the lower bounds with a negative multiplier m_l, the upper bound with a positive multiplier m_h. $S(\tilde{u})$ is then found from (7). The gradients are found from

$$\frac{\partial J_h}{\partial k_{ij}} = \tfrac{1}{2}x_o^* \frac{\partial N_h}{\partial k_{ij}} x_o, \qquad (42)$$

$$\frac{\partial J_1}{\partial k_{ij}} = \tfrac{1}{2} x_o^* \frac{\partial N_\ell}{\partial k_{ij}} x_o . \tag{43}$$

where the sensitivity coefficients $\frac{\partial N_h}{\partial k_{ij}}$, $\frac{\partial N_1}{\partial k_{ij}}$ are the (steady state) solutions of the sensitivity equation of the Ricatti equation (32):

$$\frac{\partial N_s}{\partial k_{ij}} + (A_o - S_o K - \frac{1}{m_s} N_s)^* \frac{\partial N_s}{\partial k_{ij}} + \frac{\partial N_s}{\partial k_{ij}} (A_o - S_o K - \frac{1}{m_s} N_s)$$

$$+ \frac{\partial K}{\partial k_{ij}} S_o (K - N_s) + (K - N_s) S_o \frac{\partial K}{\partial k_{ij}} = 0,$$

$$\frac{\partial N_s}{\partial k_{ij}} (T) = 0, \; s = 1,h. \tag{44}$$

The pairs (N_h, m_h), (N_1, m_1) are known and have been obtained as solutions of (32) in the course of determining $J_h(\tilde{u})$, $J_1(\tilde{u})$. The gradient procedure is easily implemented on a digital computer and has been employed to determine the optimal control gains for the examples considered earlier.

The results illustrate the type of results expected and obtained from the segment approach. They have again been plotted for the whole family of problems when γ is allowed to lie in the range $0 \leq \gamma \leq 1$. For example, in system C, for $\gamma = .9$ the optimal control gain matrix K_o, for the problem without disturbances, and the pessimistic gain matrix K_s, serving as the initial guess for K^* were found in the analysis to be

$$K_o = \begin{Vmatrix} .813 & .174 \\ .174 & 1.157 \end{Vmatrix}, \quad \tilde{K} = K_s = \begin{Vmatrix} 1.109 & .274 \\ .274 & .561 \end{Vmatrix}$$

After the iterative procedure was judged to have approached the minimum within prescribed bounds, the result was

$$K^* = \begin{Vmatrix} 1.108 & .237 \\ .237 & .508 \end{Vmatrix} .$$

It is most appropriate to compare these results with corresponding results in minimax (wrost case) designs. Such designs minimize the upper bound of segments, correspond, therefore, to the pessimistic design and result in the selection of the pessimistic control. The optimal control from the segment approach as demonstrated, Fig. 3, 4, retains the advantages of the worst case designs while improving the system performance for perturbations other than the worst. With the optimal control from the segment approach the upper bound of the associated segment remains virtually unchanged (an increase of .8% from P_h) while the lower found undergoes a substantial decrease (about 10% decrease from the value of P_1 or about a 20% decrease of the range $d_1 = P_1 - 0_1$). The implementation of the optimal control must be made at the risk of greater expenditure of control energy. It should, however, be noted that with minimax type designs generally even more energy expenditure must be allowed.

The problem with uncertainty considered above is of course specific, its nature depending on the set V of admissible disturbances as defined by (23). It is, however, emphasized that this study does characterize the reaction of linear system to increasing level of external disturbances. Moreover, with simple modifications it is possible to consider the following frequently appearing problems with uncertainty: (i) a more general statement of the above problem is obtained if in (17) the term Cv is put in place of v. Moreover, the approach is not restricted to problems with complete state feedback; (ii) energy constrained external disturbances may be considered by assuming V is given by

$$\int_{t_o}^{T} v^* v dt \leq \gamma \tag{45}$$

With slight modifications in the computational algorithm it is possible to apply the gradient procedure to obtain the optimal feedback control. The authors have considered such problems for time invariant systems and infinite integration interval and have obtained results corresponding to those presented here for the constraint set (23); (iii) a frequently discussed problem with uncertainty occurs when amplitude bounded external disturbances act on the system. The set V is then assumed to consist of piecewise continuous time functions such that for all $t \in [0,T]$ $|v_i| \leq 1$, $i = 1,\ldots,r$ and the computational requirements become more complex. The authors do not have sufficient experience with such problems at the present time; (iv) Another common problem with uncertainty occurs in linear systems when some plant parameters are known and the set V consists of a closed and bounded domain in the plant parameter space. For any $v \in V$ there exists an optimal linear feedback controller and it is again advantageous to look for the optimal linear feedback control when applying the segment approach; (v) Finally, one of the most common problems, due to variations in the initial conditions is solved trivially since the optimal control (19) is independent of the initial conditions and modifications of the optimal control is unnecessary.

CONCLUSIONS

In conclusion, it is hoped that the results presented in this paper will contribute to better understanding of control problems in the presence of uncertainty. The intention of this paper was to introduce the basic notions and justify the approach by presenting specific results for a particular problem. Hence, a large portion of the paper was devoted to a specific linear control problem although the method can, with slight modifications, be used to solve other commonly discussed problems with uncertainty in linear control systems with a quadratic associated functional. If the obtained results are representative of the general situation and acceptable from the design point of view the basic problem is the synthesis problem consisting in the determination of the optimal control u^* which minimizes $S(u)$. Methods used to minimize $S(u)$ depend on the problem, in particular on the nature of the sets U and V. Gradient and other local procedures may be used when the solution is local [5]. It is however, possible that the solution is not local in which case global algorithms such as presently being developed for the minimax problem must be employed [5,13,14]. A complete discussion of all computational aspects even for this type of problems is out of the scope of this paper.

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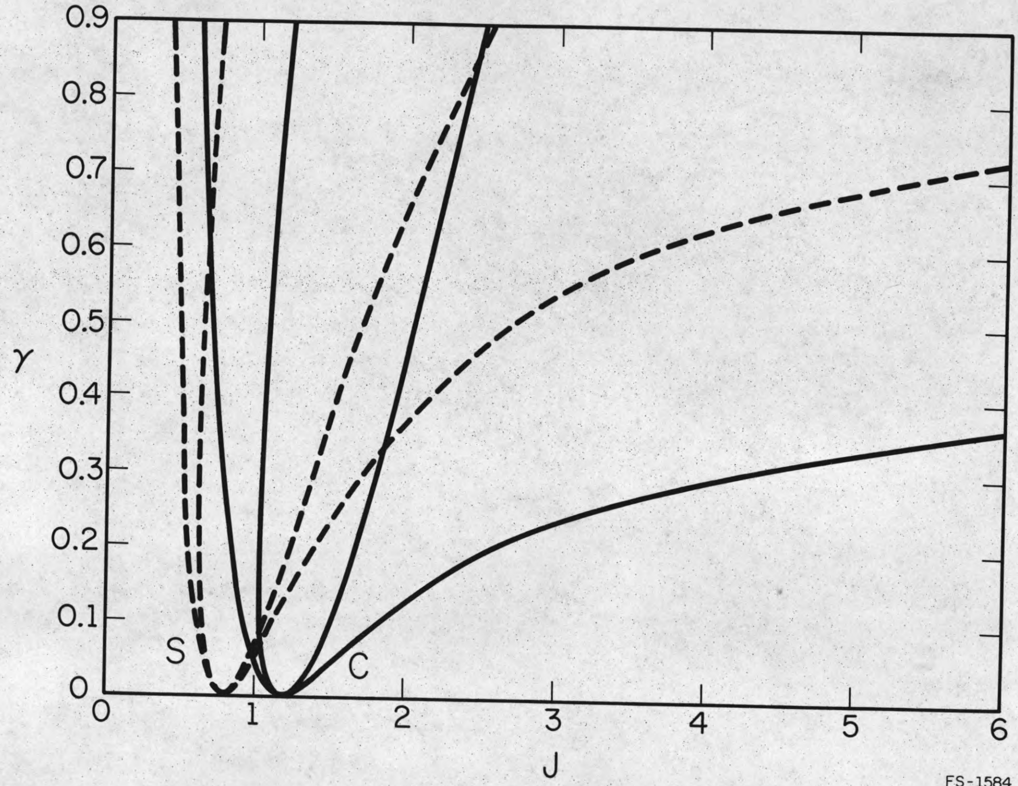

Fig. 2. The dependence of bounds O_l, P_l, and O_h on the amount of external disturbances, i.e. on the parameter γ. Full lines belong to system C, dotted lines to system S.

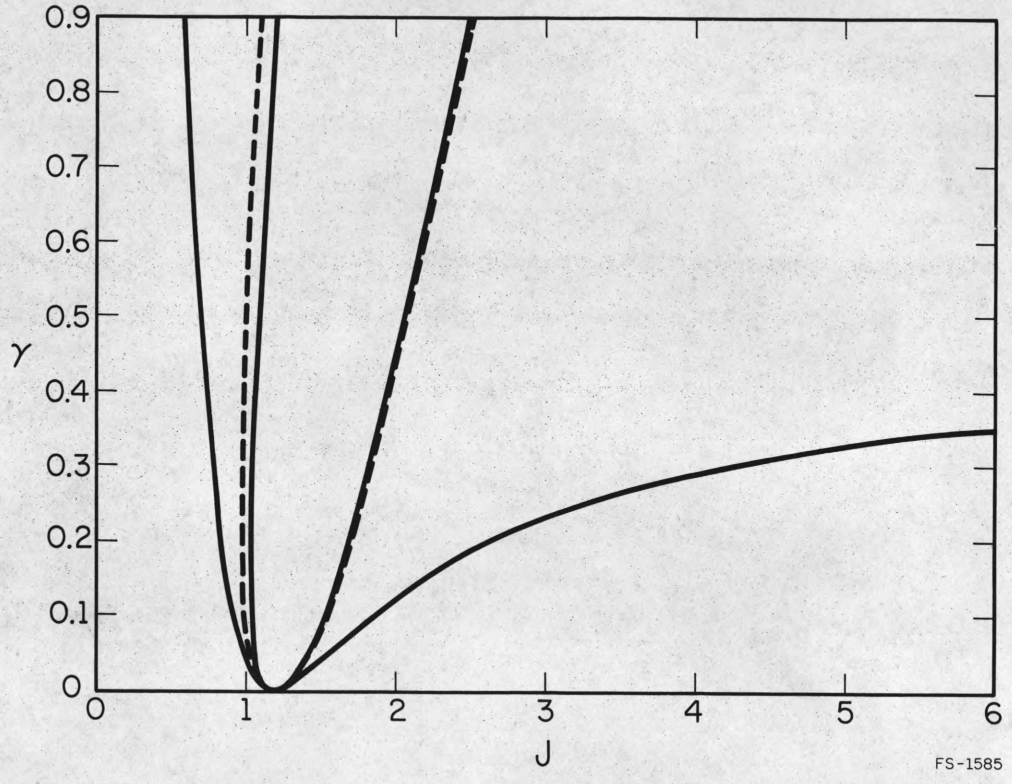

Fig. 3. Dotted lines indicate the location of the optimal segment, corresponding to the optimal control u*, for system C.

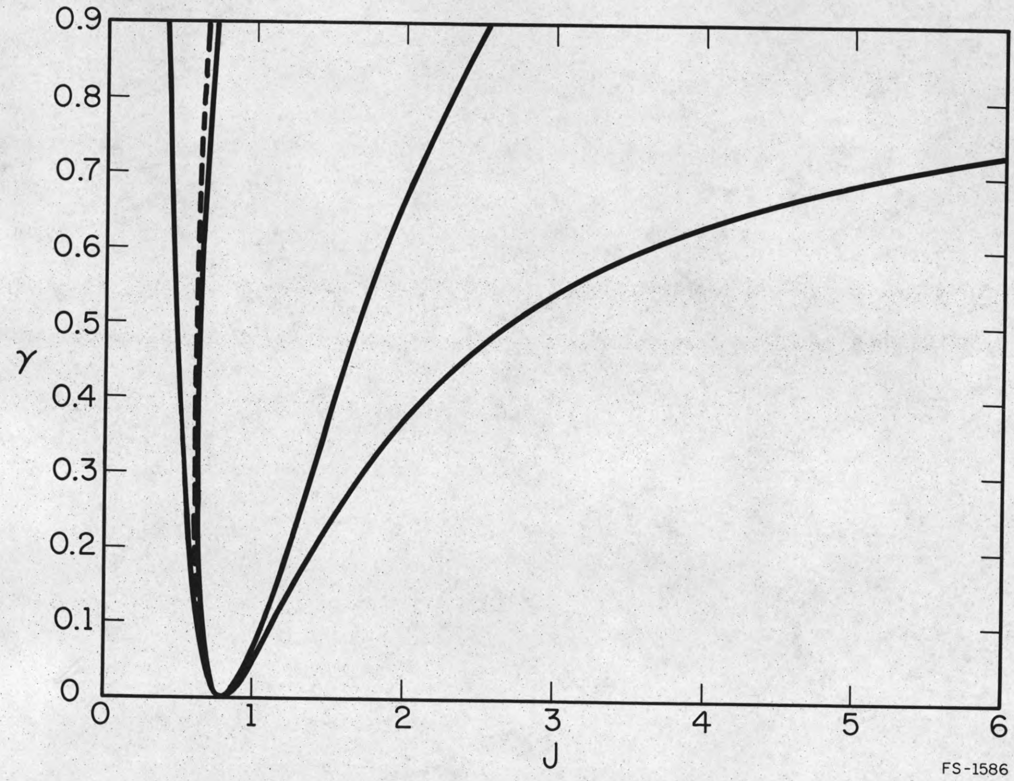

Fig. 4. Dotted lines indicate the location of the optimal segment for system S. The upper bound virtually coincides with P_h.

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This paper develops the segment method for the problem of control with uncertainty. It is shown that due to disturbances a segment of values of the performance functional is associated with each admissible control. It is proposed that the optimal control for the problem with uncertainty be defined on the basis of an ordering of associated segments. The ordering is achieved by introducing a segment index. Two particular segments, the pessimistic and the optimistic segment, are shown to be important in the analysis of the problem while minimization of the introduced segment index is the basic synthesis problem. The approach is applied to a linear control problem where uncertainty is caused by external distrubances and specific results are presented. They show the optimal control to retain the advantages of the usual, minimax type designs and ensure better system performance for disturbances other than the worst.

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