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WATARU MAYEDA

UNIVERSITY OF ILLINOIS - URBANA, ILLINOIS

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Wataru Mayeda

Department of Electrical Engineering
University of Illinois, 1969

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Abstract

This paper gives a simple proof of the Tutte's realizability condition for a cut-set (circuit) matrix of a non-oriented graph [1,2]. First, a minimum non-realizable matris is defined as a matrix [N U] which satisfies (1) [N U] is not a cut-set (circuit) matrix, (2) [N U] does not satisfy the conditions in the Tutte's theorem, and (3) deleting any column other than that belongs to a unit matrix or any row of any normal form of [N U], the resultant matrix is realizable as a cut-set (circuit) matrix. A proof of the Tutte's theorem in this paper is accomplished by showing that minimum non-realizable matrices do not exist.

Introduction

The Tutte's realizability condition is stated as "a matrix F is a cut-set (circuit) matrix of a non-oriented graph if and only if it is regular and no normal form of F contains a circuit (cut-set) matrix of either of the two basic non-planar graphs of Kuratowski." A theorem for a matrix to be regular is given as "a matrix F is regular if and only if no normal form of F contains either N_o or N_o^t" where

$$N_o = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}.$$

The original proof for Tutte's condition is rather complicated. Here, we will use Whitney's geometric operations for 2-isomorphism and define a minimum non-realizable matrix to have a simple proof of the theorem.

Because Whitney's two geometric operations [5,6] are essential tools for the proof in this paper, we will give names to distinguish these operations. A "2-isomorphic operation of Type 1" is an operation to split a cut-vertex into two so that the number of maximal connected subgraphs will be increased by one. The reverse of the above operation is also called a "2-isomorphic operation of Type 1" which is to connect two maximal connected subgraphs g_r and g_s by coinciding a vertex in g_r and a vertex in g_s. A "2-isomorphic operation of Type 2" is the other Whitney's geometric operation which is to turn around one of two subgraphs, which are connected by two vertices, at these vertices.

Proof of Tutte's Theorem

It is obvious that a circuit (cut-set) matrix of either of basic non-planar graphs of Kuratowski cannot be a cut-set (circuit) matrix. Also, it is easily tested that neither matrix $[N_o U]$ nor matrix $[N_o{}^t \underline{U}]$ can be realized as a cut-set (circuit) matrix. Furthermore, if a matrix $[N\ U]$ contains a submatrix which is not a cut-set (circuit) matrix, matrix $[N\ U]$ cannot be a cut-set (circuit) matrix [7]. Thus the necessity part of the Tutte's theorem is obvious.

We will prove the sufficient part by contradiction. Suppose there exists a matrix $[N\ U]$ which is not a cut-set (circuit) matrix and does not satisfy the conditions in the Tutte's theorem. If there exists a submatrix $[\underline{N}\ \underline{U}]$ which is not a cut-set (circuit) matrix, we can consider $[\underline{N}\ \underline{U}]$ as a given matrix for the proof. Hence, without the loss of generality, we can assume that matrix $[N\ U]$, has the following four properties:

(1) No normal form of $[N\ U]$ and $[N^t\ \underline{U}]$ contains a circuit matrix of either of basic non-planar graphs of Kuratowski.

(2) No normal form of $[N\ U]$ contains either $[N_o U]$ or $[N_o{}^t \underline{U}]$ where

$$N_o = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} .$$

(3) Neither $[N\ U]$ nor $[N^t\ U]$ is a cut-set matrix.

(4) For any normal form $[N_1 U]$ of $[N\ U]$, if we delete a row p from $[N_1 U]$, the resultant matrix $[N_1 U]_{-p}$ is a cut-set matrix. Furtherform, if we delete a row q from $[N_1{}^t \underline{U}]$, the resultant matrix $[N_1{}^t \underline{U}]_{-q}$ is a cut-set matrix, where U and $\underline{U}$ are unit matrices.

For convenience, a matrix satisfying the above properties is called a minimum non-realizable matrix. Notice that the matrix [N U] is a normal form* itself. However, there may be many other normal forms of the matrix.

First, we will investigate some properties of minimum non-realizable matrices under the assumption that such a matrix exists.

<u>Theorem 1</u>: Let $[N\ U]_{-p}$ be a matrix obtained from a minimum non-realizable matrix [N U] by deleting a row p. Then $[N\ U]_{-p}$ is a cut-set matrix of a planar graph.

<u>Proof</u>: By Property (4), $[N\ U]_{-p}$ is a cut-set matrix. Suppose $[N\ U]_{-p}$ is a cut-set matrix of a non-planar graph, then a normal form of $[N^t \underline{U}]_{-p}$ must contain a circuit matrix of either of basic non-planar graphs of Kuratowski which violates Property (1).

<u>Theorem 2</u>: Let $[N_1 U]$ be a normal form of a minimum non-realizable matrix. Also let $N_1(-q)$ be a matrix obtained from N_1 by deleting a column q. Then $[N_1(-q)U]$ is a cut-set matrix.

<u>Proof</u>: Consider $[N_1{}^t \underline{U}]$. By Property (4) and Theorem 1, $[N_1{}^t \underline{U}]_{-q}$ is a cut-set matrix of a planar graph. It is clear that if $[F\ \underline{U}]$ is a fundamental cut-set matrix of a planar graph, then $[F^t U]$ is also a fundamental cut-set matrix. Thus $[N_1(-q)U]$ is a fundamental cut-set matrix.

<u>Theorem 3</u>: Let [N U] be a minimum non-realizable matrix. Then any row and any column of $\underline{N}$ of any normal form $[\underline{N}\ \underline{U}]$ of [N U] have either two non-zero entries or three non-zero entries.

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*A normal form of a matrix is of the form [R U] where U is a unit matrix.

<u>Proof</u>: Let N(-c) be a matrix obtained from N by deleting a column c and N(-r) be a matrix obtained from N by deleting a row r. By Theorem 2, [N(-c) U] is a cut-set matrix. Let G(-c) be a linear graph whose cut-set matrix is [N(-c) U]. By Property (4), we can assume that G(-r) to be a linear graph whose cut-set matrix is [N(-r) $\underline{U}$].

Consider a matrix [N(-c-r) $\underline{U}$] which is obtained from [N U] by deleting row r and column c. Let G_o(-c-r) be a linear graph whose cut-set matrix is [N(-c-r) $\underline{U}$].

Let chord c be the edge corresponding to column c and branch r be the edge corresponding to the column in U which has 1 at row r. Then if we delete chord c in G(-r), we will have a linear graph G(-r-c) whose fundamental cut-set matrix is [N(-r-c) $\underline{U}$]. Thus linear graphs G_o(-c-r) and G(-r-c) are 2-isomorphic each other.

If we short branch r in G(-c) (i.e. coincide the endpoints of branch r and delete branch r), we will have a linear graph G(-c-r) whose fundamental cut-set matrix is again the same as that of G_o(-c-r). Thus G_o(-c-r) and G(-c-r) are 2-isomorphic. Hence G(-c-r) and G(-r-c) are 2-isomorphic each other.

Let $r_1, r_2, \ldots,$ and r_k be the rows in [N U] which have 1 at column c. Also let branch r_p be the edge corresponding to column U which has 1 at row r_p for $p = 1,2,\ldots,k$. In G($-r_1$), branches $r_2, r_3, \ldots, r_k$ and chord c must form a circuit in order that [N($-r_1$) $\underline{U}$] is a fundamental cut-set matrix of G($-r_1$). Hence branches $r_2, r_3, \ldots,$ and r_k form a path is G($-r_1$). On the other hand, branches $r_1, r_2, r_3, \ldots,$ and r_k should not form a path, not only in G(-c) but also in all linear graphs obtained from G(-c) by 2-isomorphic operations. Because if these branches form a path in a linear graph $G'(-c)$ which is obtained from $G(-c)$ by 2-isomorphic operation, we can obtain a linear graph G' by inserting a chord c to $G'(-c)$ so that chart c branches $r_1, r_2, \ldots,$ and r_k form a circuit in G'. Then a fundamental cut-set matrix of G' will be $[N\ U]$ because a fundamental cut-set matrix of $G'(-c)$ is $[N(-c)\underline{U}]$ and chord c and branches $r_1, r_2, \ldots,$ and r_k in G' form a fundamental circuit. However $[N\ U]$ is a minimum non-realizable matrix. Thus branches $r_1, r_2, \ldots,$ and r_k do not form a path in any linear graph which is obtained from $G(-c)$ by 2-isomorphic operations. However, branches $r_1, r_2, \ldots,$ and r_k in $G(-c)$ must be located so that when we short branch r_1 to obtain $G(-c-r)$, the remaining branches $r_2, r_3, \ldots,$ and r_k will be a path in a linear graph obtained from $G(-c-r)$ by 2-isomorphic operations. Instead of shorting r_1, we can short r_p $(1 < p \leq k)$ to have the same result. Hence $G(-c)$ must have the following properties:

(I) branches $r_1, r_2, \ldots, r_k$ can neither be a path nor become a path by any 2-isomorphic operations, and

(II) when any one of $r_1, r_2, \ldots,$ and r_k is shorted, the remaining branches will either be a path or become a path by 2-isomorphic operations.

Notice that branches $r_1, r_2, \ldots,$ and r_k are edges in a tree consisting of edges corresponding to columns of U of $[N(-c)U]$. Also notice that for $k \leq 2$, any structure which satisfies Property I will satisfy Property II. Hence we will consider all possible structures of $G(-c)$ with $k \geq 3$ which have the above properties.

There are five structures in which

(I') Branches $r_1, r_2, \ldots,$ and r_k can neither be a path nor become a path by 2-isomorphic operations, and

(II') By shorting branch r_1, the remaining branches $r_2, r_3, \ldots,$ and r_k will either be a path or become a path by 2-isomorphic operations. These are shown in Fig. 1. We will investigate these linear graphs one by one to see whether Properties I and II hold or not.

Consider linear graph G_a in Fig. 1(a). If $p < 2$, that is, all $r_2, r_3, \ldots,$ and r_k are in subgraph H_2, G_a is the same structure as G_d. So we assume that $p \geq 2$. Notice that when branch r_1 is shorted, vertices v_1 and v_2 become one vertex. This new vertex and vertex v_3 become a pair of vertices of a 2-isomorphic operation of Type 2 by which $r_2 \ldots$ and r_p and $r_{p+1}, \ldots,$ and r_k become one path.

Instead of shorting r_1, suppose we short r_2. If a 2-isomorphic operation which becomes possible by shorting r_2 is that of Type 1, the original graph G_a must be one shown in Fig. 2. Hence, it is clear that the remaining branches $r_1, r_3, \ldots,$ and r_k cannot be a path by 2-isomorphic operations.

If a 2-isomorphic operation which becomes possible by shorting r_2 is that of Type 2, then the use of this operation will not change the structure of neither H_2 nor the location of r_1. Thus in order to make $r_1, r_3, \ldots,$ and r_k a path, some of branches $r_3, \ldots,$ and r_p must be shifted to form a path between v_2 and v_3 which is impossible unless the path was there to begin with. Since the path cannot be there by Property I, we can conclude that G_a is not $G(-c)$.

Consider G_b in Fig. 1(b). If $p_1 < 2$, it became G_a so $p_1 \geq 2$. If $p_2 = p_1$ which means none of $r_1, r_2, \ldots,$ and r_k is in H_1 and $r_1, r_2,$ and r_k form a path, p_2 must be larger than p_1. Furthermore it is clear that $k \geq p_2 + 1$. Suppose $p_1 > 2$, then shorting r_2, we have the resultant graph which is identical to G_b except that the number of series edges which form a path between v_2 and v_3 is reduced by 1. Thus $r_1, r_3, \ldots,$ and r_k cannot become a path by 2-isomorphic operations. This means that $p_1 = 2$.

With $p_1 = 2$, suppose $p_2 > 3$. Then, there exist at least two branches r_3 and r_4 in H_1 for this case. Now shorting r_2 will give the same results as for G_a so we can conclude that G_b is not G(-c). This is also true when $k > p_2 + 1$. Thus only the case which is left to be considered is when $p_2 = 3$ and $k = 4$ which is given in Fig. 3. In order that this structure is valid, there must be paths P_1, P_2, P_3, and P_4 as shown in the figure. This linear graph obviously has a subgraph which is homorphic to linear graph G_3 in Fig. 4 [7,8]. Hence, there is a fundamental cut-set matrix Q_f of G_b which contains a fundamental cut-set matrix $\underline{Q}_f$ of G_3 as its submatrix. Consider a fundamental cut-set matrix $\underline{Q}_f$ of G_3 where

$$\underline{Q}_f = \begin{array}{c} \\ \\ \\ \\ \end{array}\overset{\begin{array}{cccccccc} 1 & 2 & 3 & 4 & r'_1 & r'_2 & r'_3 & r'_k \end{array}}{\left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right]}.$$

If G_b is G(-c), then a normal form of [N U] must contain the following matrix $[\underline{N}\ \underline{U}]$:

$$[\underline{N}\ \underline{U}] = \left[\begin{array}{cc} 1 & \\ 1 & \underline{Q}_f \\ 1 & \\ 1 & \end{array}\right] = \overset{\begin{array}{ccccccccc} c & 1 & 2 & 3 & 4 & r'_1 & r'_2 & r'_3 & r'_4 \end{array}}{\left[\begin{array}{ccccc|cccc} 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right]}.$$

It is easily seen that $[\underline{N}\ \underline{U}]$ is not a cut-set matrix. Hence [N U] is not a minimum non-realizable matrix because of Property (4) unless $[N\ U] = [\underline{N}\ \underline{U}]$. This means that G_b must be G_3.

Consider $[\underline{N}^t\underline{U}']$ where

$$[\underline{N}^t\underline{U}'] = \left[\begin{array}{cccc|ccccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \end{array}\right].$$

It can easily show that this matrix is a cut-set matrix of one of two basic non-planar graph of Kuratowski. Thus by Property 1, $[\underline{N}\ \underline{U}]$ is not a minimum non-realizable matrix. Hence G_b is not G(-c).

Consider G_c in Fig. 1(c). If p = 2 (i.e., no branches of $r_2, \ldots,$ and r_k are in H_1), G_c becomes G_d. Hence we assume that $p > 2$. By shorting r_2, we can easily see that the remaining branches $r_1, r_3, \ldots, r_k$ cannot become a path by 2-isomorphic operations. Hence G_c is not G(-c).

Consider G_d in Fig. 1(d). Instead of shorting r_1, if we consider to short r_2, the structure of G_d will be either G_a or G_c. Thus G_d is not G(-c).

Consider G_e in Fig. 1(e). If by considering of shorting any branches other than r_1, the structure of G_e becomes one of the others, G_e will not be G(-c). Thus in order that G_e is G(-c), k must be 3 and G_e must be one shown in Fig. 5. Thus we can conclude that $k \leq 3$. Notice that k is the number of non-zeros in column c of [N U]. Since this must be true for taking any column of any normal form of [N U], we can say that any column in $\underline{N}$ has either two, or three non-zeros where $[\underline{N}\ U]$ is any normal form of [N U]. Also by Property 4 of a minimum non-realizable matrix, any column in $[N^tU']$ has the same property. Thus Theorem 3 is true.

<u>Theorem 4</u>: There are no minimum non-realizable matrices.

<u>Proof</u>: In order that a matrix is not a cut-set matrix, there must be at least one column having at least three 1's. Since by Theorem 3, any column has either two or three 1's, we can assume that there is at least one column which has exactly three 1's. Thus N of a minimum non-realizable matrix [N U] must have the following configuration.

$$N = \begin{bmatrix} 1 & a_{12} & a_{13} & a_{14} \\ 1 & a_{22} & a_{23} & a_{24} \\ 1 & a_{32} & a_{33} & a_{34} \\ 0 & a_{42} & a_{43} & a_{44} \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

This is also true for N^t. Suppose we assume that $a_{12} = a_{13} = 1$. Then [N U] becomes

$$[N\ U] = \left[\begin{array}{cccccc|cccccc} 1 & 1 & 1 & 0 & \dots & 0 & 1 & 0 & 0 & & & \\ 1 & a_{22} & a_{23} & a_{24} & \dots & & 0 & 1 & 0 & & & 0 \\ 1 & a_{32} & a_{33} & a_{34} & \dots & & 0 & 0 & 1 & & 1 & \\ 0 & a_{42} & a_{43} & a_{44} & \dots & & 0 & 0 & 0 & & \ddots & \\ \vdots & & & & & & \vdots & \vdots & \vdots & 0 & & 1 \\ 0 & & & & & & 0 & 0 & 0 & & & \end{array}\right] \qquad (1)$$

If $a_{22} = a_{32}$, then $a_{22} = a_{32} = 1$ makes the 1st column and the 2nd column of N identical because of at most three 1's in every row of N. Thus [N U] is not a minimum non-realizable matrix. When $a_{22} = a_{32} = 0$, we have

$$
[N\ U] = \left[\begin{array}{cccccc|cccccc}
1 & 1 & 1 & 0 & \cdots & & 1' & 2' & 3' & \cdots & & \\
1 & 0 & a_{23} & a_{24} & \cdots & 0 & 0 & 1 & 0 & & & \\
1 & 0 & a_{33} & a_{34} & \cdots & & 0 & 0 & 1 & & 0 & \\
0 & a_{42} & a_{43} & a_{44} & & & 0 & 0 & 0 & 1 & & \\
\vdots & \vdots & \vdots & \vdots & & & \vdots & \vdots & \vdots & & \ddots & \\
0 & & & & & & 0 & 0 & 0 & 0 & & 1
\end{array}\right].
$$

By interchanging columns 1 and 1', we have

$$
\begin{array}{c}
1 \\ 2 \\ 3 \\ 4 \\ \vdots \\ 0
\end{array}
\left[\begin{array}{cccccc|cccccc}
1 & 1 & 1 & 0 & \cdots & 0 & 1 & 0 & 0 & & & \\
0 & 0 & a_{23} & a_{24} & \cdots & & 1 & 1 & 0 & & & \\
0 & 0 & a_{33} & a_{34} & \cdots & & 1 & 0 & 1 & & 0 & \\
 & a_{42} & a_{43} & a_{44} & \cdots & & 0 & 0 & 0 & 1 & & \\
\vdots & \vdots & \vdots & \vdots & & & \vdots & \vdots & \vdots & & \ddots & \\
 & & & & & & 0 & 0 & 0 & 0 & & 1
\end{array}\right]
$$

In order to make this a normal form, we must add (modular 2) the 1st row to the 2nd and to the 3rd row which gives

$$
\begin{array}{c}
\\ 1 \\ 2 \\ 3 \\ \\ \\ \\
\end{array}
\begin{array}{cccccc|cccccc}
1' & 2 & 3 & 4 & & & 1 & 2' & 3' & & & \\
\hline
1 & 1 & 1 & 0 & \cdots & 0 & 1 & 0 & 0 & & & \\
1 & 1 & 1+a_{23} & a_{24} & \cdots & & 0 & 1 & 0 & & & \\
1 & 1 & 1+a_{33} & a_{34} & \cdots & & 0 & 0 & 1 & & 0 & \\
0 & a_{42} & a_{43} & a_{44} & \cdots & & 0 & 0 & 0 & 1 & & \\
\vdots & \vdots & \vdots & \vdots & & & \vdots & \vdots & \vdots & & \ddots & \\
0 & & & & & & 0 & 0 & 0 & 0 & & 1
\end{array}.
$$

By Theorem 3, column 2 must be identical to column 1', thus [N U] is not a minimum non-realizable matrix. Thus $a_{22} \neq a_{32}$. Without the loss of generality, let $a_{22} = 1$ and $a_{32} = 0$ as

$$[N\ U] = \left[\begin{array}{cccccc|ccccc}
1 & 1 & 1 & 0 & \cdots & 0 & 1 & 0 & 0 & & \\
1 & 1 & a_{23} & a_{24} & \cdots & & 0 & 1 & 0 & & 0 \\
1 & 0 & a_{33} & a_{34} & \cdots & & 0 & 0 & 1 & & \\
0 & a_{42} & a_{43} & a_{44} & \cdots & & 0 & 0 & 0 & 1 & \\
\vdots & \vdots & \vdots & \vdots & & & \vdots & \vdots & \vdots & & \ddots \\
 & & & & & & & & & 0 & 1 \\
0 & 0 & 0 & & & & 0 & 0 & 0 & &
\end{array}\right].$$

Similarly, $a_{23} \neq a_{32}$. It is also true for $[N^t U']$. Hence $a_{22} \neq a_{23}$ and $a_{32} \neq a_{33}$. Thus

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ \vdots \\ \\ \end{array}
\begin{array}{cccccc|ccccc}
1 & 2 & 3 & 4 & & & 1' & 2' & 3' & & \\
1 & 1 & 1 & 0 & \cdots & 0 & 1 & 0 & 0 & & \\
1 & 1 & 0 & a_{24} & \cdots & & 0 & 1 & 0 & & 0 \\
1 & 0 & 1 & a_{34} & \cdots & & 0 & 0 & 1 & & \\
0 & a_{42} & a_{43} & a_{44} & \cdots & & 0 & 0 & 0 & 1 & \\
\vdots & \vdots & \vdots & \vdots & & & \vdots & \vdots & \vdots & 0 & \ddots \quad 1 \\
0 & & & & & & 0 & 0 & 0 & &
\end{array}$$

Suppose $a_{42} \neq a_{43}$. Without the loss of generality, let $a_{42} = 1$ and $a_{43} = 0$. Then by interchanging columns 2 and $1'$ and make the resultant matrix a normal form by adding row 1 to rows 2, 4, etc. we will produce at least four 1's in column 3. Thus by Theorem 3, $[N\ U]$ is not a minimum non-realizable matrix. Thus $a_{42} = a_{43}$.

If $a_{42} = a_{43} = 1$, we have

$$[N\ U] = \left[\begin{array}{cccccccccc}
1 & 1 & 1 & 0 & \cdots & 0 & 1 & 0 & 0 & & \\
1 & 1 & 0 & a_{24} & \cdots & & 0 & 1 & 0 & & 0 \\
1 & 0 & 1 & a_{14} & \cdots & & 0 & 0 & 1 & 1 & \\
0 & 1 & 1 & a_{44} & \cdots & & 0 & 0 & 0 & & \ddots \\
\vdots & \vdots & \vdots & \vdots & & & \vdots & \vdots & \vdots & 0 & 1 \\
0 & & & & & & 0 & 0 & 0 & &
\end{array}\right]$$

in which $[N_o{}^t U]$ is a submatrix. Thus $[N\ U]$ is not a minimum non-realizable matrix. Thus $a_{42} = a_{43} = 0$, if any (r^2) entry for $r > 4$ is 1, we can interchange row r and row 4 then interchange columns belonging to U to make the resultant matrix a normal form. Thus $a_{42} = a_{43} = 0$ means $a_{r2} = a_{r3} = 0$ for $r = 4,5,\ldots$. This is also true for a_{24} and a_{34} by considering $[N^t U']$. Thus we have

$$[N\ U] = \left[\begin{array}{cccccc|ccccc}
1 & 1 & 1 & 0 & \ldots & 0 & 1 & 0 & 0 & & \\
1 & 1 & 0 & 0 & \ldots & 0 & 0 & 1 & 0 & & \\
1 & 0 & 1 & 0 & \ldots & 0 & 0 & 0 & 1 & & 0 \\
0 & 0 & 0 & & & & 0 & 0 & 0 & 1 & \\
\vdots & \vdots & \vdots & & \boxed{N'} & & \vdots & \vdots & \vdots & \ddots & \\
0 & 0 & 0 & & & & 0 & 0 & 0 & 0 & 1
\end{array}\right]$$

In order that $[N\ U]$ is not a cut-set matrix, we can see that $[N'U]$ cannot be a cut-set matrix. Thus $[N\ U]$ is not a minimum non-realizable matrix.

We have assumed that $a_{12} = a_{13} = 1$ for the above discussion. Suppose this is not the case. Without the loss of generality, let $a_{12} = 1$ and $a_{13} = 0$. Then $[N\ U]$ becomes

$$[N\ U] = \left[\begin{array}{cccccc|c}
1 & 1 & 0 & 0 & \ldots & 0 & \\
1 & a_{22} & a_{23} & a_{24} & & & \\
1 & a_{32} & a_{33} & a_{34} & & & U \\
0 & & & & & & \\
\vdots & & & & & & \\
0 & & & & & &
\end{array}\right]$$

$a_{22} = a_{32}$ cannot make $[N\ U]$ a minimum non-realizable matrix. Thus we take $a_{22} = 1$ and $a_{32} = 0$, or

$$[N\ U] = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ \vdots \\ \vdots \end{array} \left[\begin{array}{cccccc|c} 1 & 1 & 0 & 0 & \cdots & 0 & \\ 1 & 1 & a_{23} & & & & \\ 1 & 0 & & & & & U \\ 0 & \vdots & & & & & \\ \vdots & \vdots & & & & & \\ 0 & & & & & & \end{array}\right].$$

If $a_{23} = 1$, we have [N U] which is the same form as in Eq. (1). If any entry (2p) for $p > 3$ in N is 1, we can shift it to the (2,3) position. Thus $a_{23} = a_{24} = \ldots = 0$ in N. But this situation makes row 1 and row 2 identical. Thus [N U] is not a minimum non-realizable matrix. Hence as long as $a_{12} = 1$, we can change [N U] to the form in Eq. (1).

If there is an entry (1p) in N for $p > 2$ which has 1, we can interchange columns to make $a_{12} = 1$. Thus in order that $a_{12} = 0$, the 1st row of N must be [10 ... 0]. This means that in $[N^t U']$, there are two columns which are $[10 \ldots 0]^t$. Thus [N U] is not a minimum non-realizable matrix. This covers all possible cases of [N U]. Thus we can conclude that there are no minimum non-realizable matrix which proves Theorem 4.

By Theorem 4, any matrix which is regular and satisfies that no normal form contains a circuit(cut-set) matrix of either of basic non-planar graphs of Kuratowski is a cut-set (circuit) matrix. This proves the sufficient part of the Tutte's theorem.

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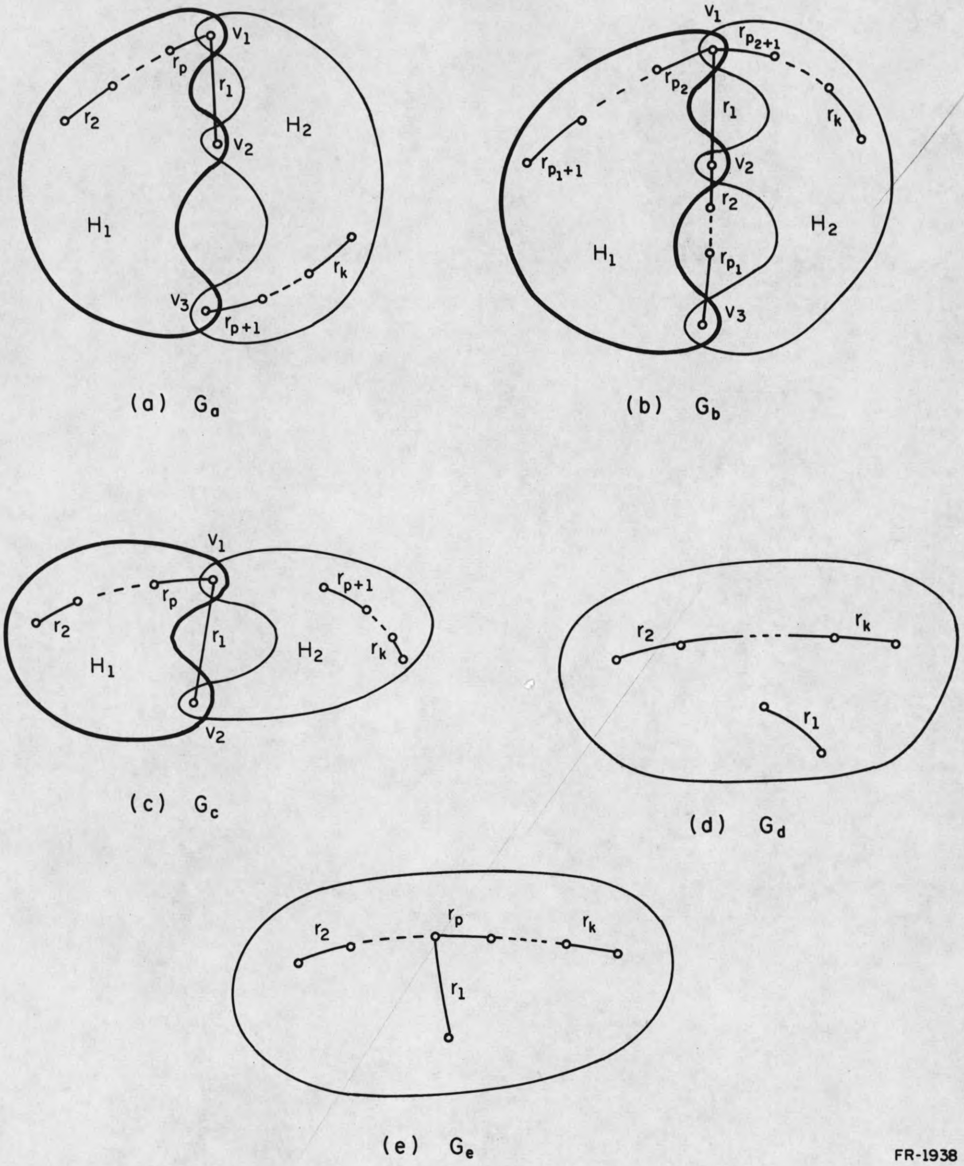
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rp
r1
r2
H2
v2
H1
v3
rk
rp+1
(a) Ga
v1
rp2+1
rp2
r1
rk
rp1+1
v2
r2
H1
rp1
H2
v3
(b) Gb
v1
rp
rp+1
r2
r1
H1
H2
rk
v2
(c) Gc
r2
rk
r1
(d) Gd
r2
rp
rk
r1
(e) Ge
FR-1938

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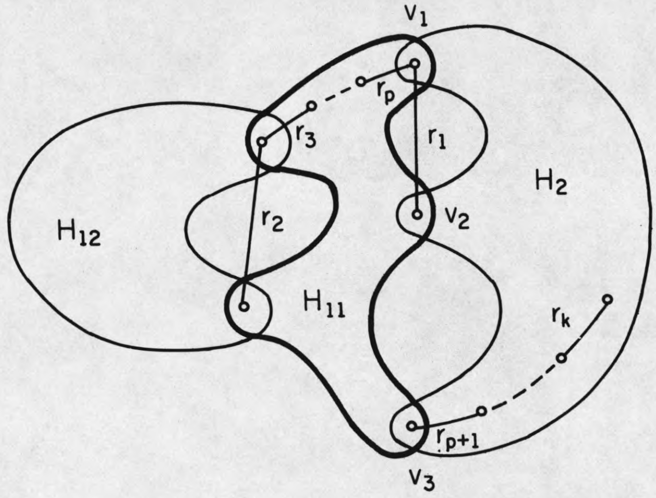
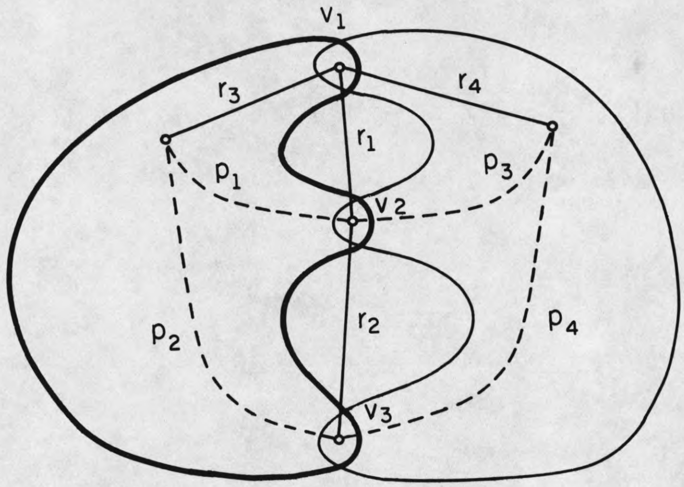


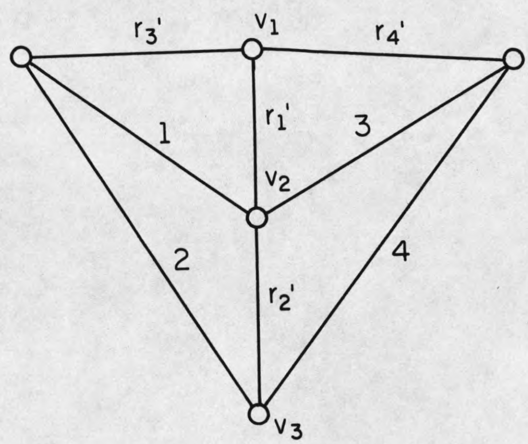
G_a with H_{12} and H_{22} FR-1940

Figure 2



Paths P_1, P_2, P_3 and P_4

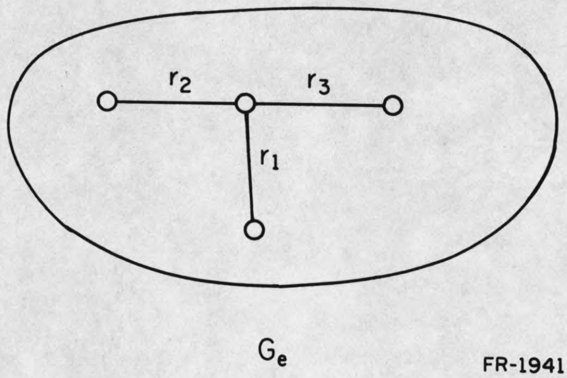
Figure 3



G_3

FR-1939

Figure 4



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13. ABSTRACT

This paper gives a simple proof of the Tutte's realizability condition for a cut-set (circuit) matrix of a non-oriented graph [1,2]. First, a minimum non-realizable matris is defined as a matrix [N U] which satisfies (1) [N U] is not a cut-set (circuit) matrix, (2) [N U] does not satisfy the conditions in the Tutte's theorem, and (3) deleting any column other than that belongs to a unit matrix or any row of any normal form of [N U] , the resultant matrix is realizable as a cut-set (circuit) matrix. A proof of the Tutte's theorem in this paper is accomplished by showing that minimum non-realizable matrices do no exist.

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