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DESIGN CHARACTERISTICS OF COHERENT RADAR FOR SNORKEL DETECTION

Report R-39

November 1953

Prepared by:

C. W. Sherwin

CONTROL SYSTEMS LABORATORY
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Abstract

If the calculations in the body of the report are correct, snorkel targets in high seas should give signals which are only 1/10 to 1/100 the sea clutter power under typical search conditions. (Range 10 to 15 miles, altitude of 2,000 to 3,000 feet, X-band radar, 1.5 degree beam, and 0.5 microsecond pulse.) This low signal to clutter ratio is the crux of the snorkel detection problem. Nothing short of really drastic clutter rejection in the early stages of the data processing will really do any good. It is well known that for effective search, within a reasonable number of looks or scans (10 to 100) the signal to noise during each look must be considerably greater than one. As the subsequent calculations show, sea clutter must be rejected by the order of 20 db <u>before</u> scan to scan integration begins if effective detection is to occur in 10 to 20 scans. This is a stiff requirement indeed; however, under some conditions the velocity displacement of target with respect to clutter should produce the desired result. We have actually observed (in low sea states) that clutter is in fact rejected by as much as 30 to 40 db, so that we know that at least under some conditions coherent radar can produce adequate clutter rejection.

In the light of these facts, polarization techniques and rapid scan techniques look unpromising since there is no reason to hope for large clutter rejection effects. Also, targets without adequate velocity displacement must be considered undetectable, even with coherent doppler radar.

Now there are all sorts of subtle things about the way that sea clutter and target signals vary with range, depression angle, and sea conditions, and many of these things are poorly known, if at all. One must not take

our conclusions too seriously, since we still lack actual observations of snorkel targets in high sea states, under practical search conditions. One can still hope that a few db, cheaply found and judiciously exploited, will do the trick. To the extent that our observations and calculations are right, however, this hope is dim.

The coherent radar data upon which these conclusions rest are (a) 52 one second duration snorkel detections in low sea states (1 and 2) and (b) sea clutter observations in sea states 1 through 4.

We calculate that a target signal, displaced 4 knots from the center of the sea clutter spectrum and observed with a filter 25 cps wide, will compete with less than 1/100th of the clutter power.

We also find that the sea clutter spectrum is displaced downwind by about 3 knots. We calculate that 40% of all 5 knot targets (of random heading with respect to the wind, and searched for by an aircraft of random heading with respect to the wind) will have the signal displaced 4 knots or more from the center of the clutter spectrum, and thus be detectable. Operational techniques could in principle improve the probability of obtaining adequate velocity displacement.

I THE CSL EXPERIMENTS

During the past year, the Control Systems Laboratory has made some observations of snorkel signals and sea clutter with coherent airborne X-band radar. These observations were made with relatively simple equipment, with the purpose of exploring the potential value of using moving target detection techniques on the snorkel detection problem. Three reports describing these observations have been written.*

The problem of snorkel detection is very elaborate, if all its aspects are considered carefully. Furthermore, the basic signal and clutter information are too incomplete for an accurate analysis to be performed. Consequently, only rough order of magnitude calculations will be attempted, liberally sprinkled with the usual number of simplifying assumptions. We do have a new take-off point, however, namely, the detailed analysis of 52 actual snorkel detections obtained with our experimental coherent equipment. These detections were made in low sea states (1 and 2, as judged by observers from the Naval Air Station in Key West where the experiments were run), at close ranges (2,500 to 5,000 yards) for a limited range of depression angles (2° to 7°), and near the ground track of the aircraft.

Fortunately, one can make a reasonable extrapolation from the experimental data. None the less, one must regard our conclusions as tentative until more extensive experimental data taken under realistic search conditions, are available.

The depression angle with which one views the sea surface is an important parameter. At X-band there is a transition angle, around 1 to 2 degrees, depending upon sea conditions, where the clutter power begins to fall off very rapidly with range. In the "near zone" the received clutter power falls off as $1/R^3$, and in the far or "interference" zone the clutter falls off as $1/R^7$. The target echo is expected to fall off as $1/R^4$.

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* CSL Reports R-27--"Preliminary Report on the Observations of Snorkels and Sea Clutter Using Coherent Airborne Radar," (SECRET), November, 1952.
R-36--"Snorkel Detection Using Airborne Coherent Radar II," (SECRET), April, 1953.
R-37--"Sea Clutter Studies Using Airborne Coherent Radar II," (CONFIDENTIAL), June, 1953.

According to our understanding, practical snorkel detections using operational equipment usually occur on the "fringe" of the clutter which is either in the $1/R^7$ region, or in the transition between the $1/R^7$ region and the $1/R^3$ region.

Our experimental detection data were all obtained in the "near zone", however, since our calculations in this report assume that detections occur in the region where the received clutter power is proportional to $1/R^3$. It is possible that a more favorable ratio of signal to clutter can be found in the transition region or in the interference zone, and therefore our conclusions tend to be more pessimistic than necessary. For another thing, it has been observed that in low sea states at least, the clutter spectrum in the interference zone is much more narrow than in the near zone.

Before discussing the detection problem, we shall outline the significant characteristics of both sea clutter and snorkel signals as observed on coherent X-band radar.

II SUMMARY OF SEA CLUTTER CHARACTERISTICS

With coherent radar, one can distinguish an omnipresent "core" spectrum of gaussian shape and with a full width at half power of 2.8 knots. This spectrum appears to be unchanged in shape or half width over sea states ranging from 1 through 4. A target displaced 3.8 knots from the center of the spectrum is in a region where the clutter power is down 20 db from the power at the center.

Observations in one high sea state (4), show an unsymmetrical clutter spectrum. The asymmetry is due to a sea-clutter-like echo moving downwind at 3 or 4 knots, which is apparently due to the white caps, and is noticeable only on the crests of the swells. One edge of the unsymmetrical clutter spectrum (due to those scatterers that are moving upwind) appears normal. Viewed crosswind, both edges are normal (i.e. same as the core).

The whole clutter spectrum has an average motion of 2 to 4 knots downwind with respect to the mass of the water (as observed in one sea state, 3).

All of the above discussion applies to viewing sea clutter at depression angles of from 2 to 7 degrees. There is some preliminary data which suggest that the clutter spectrum is much more narrow (1-1/2 knots at half power) at smaller depression angles (below 2 degrees). If this is consistently true,* velocity discrimination of targets versus clutter should be considerably enhanced when detection occurs at small depression angles.

The increase of sea clutter power with increasing sea state is not too accurately known. Bartholomay** quotes an increase of 20 db in the echo power (per unit area of the sea) in going from 1-1/2 foot waves to 7 to 8 foot waves (horizontal polarization, X-band). Changes in the intensity of the "core" and "white cap echoes" with changing sea state have not to our knowledge been measured yet.

On the basis of what has been discussed so far, one can already draw some conclusions about using MTI to improve snorkel detectability. A really significant reduction in clutter power (20 db) is possible only when the target has a displacement of about 4 knots from the center of

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* Informal communication from T.R.E. on England's work on coherent X-band studies of clutter and snorkel indicate that in low sea states, in the interference zone, the clutter spectrum is in fact narrow, but in sea state 4, the clutter spectrum is 90 to 100 cps wide, even at very small depression angles.

** A.F. Bartholomay, "Sea Clutter Studies I," Project Lincoln, January 15, 1953, (SECRET).

the clutter if the spectrum has a normal or "core" shape.

In the higher sea states, a target proceeding downwind will be heavily masked by the white cap echoes. However, it should be very exposed when moving upwind since the upwind edge of the clutter is normal, and furthermore, the whole clutter spectrum is moving downwind at several knots. Finally, if velocity discrimination is effective in rejecting clutter, one should be able to search at depression angles of 5 to 10 degrees, and consequently, have an unshadowed view of the snorkel during its maximum exposure in the troughs of the swells. Unfortunately, we have no observations of targets in sea states 3 and 4 and the above speculations may not prove true.

In any case, it is clear that targets which are moving upwind and which are viewed from either the upwind or downwind direction are most likely to be detected by coherent MTI techniques.

To avoid excessive spreading of the clutter spectrum due to the geometrical beam width, one must use a narrow beam. At 150 knots ground speed, 45° off the ground track, a 1.5° beam has a spread of 2.8 knots, which is equal to the inherent spread due to the scatterers themselves. For this case, the clutter spectrum will be $\sqrt{2}$ times broader than on the ground track, namely, 4 knots between half power points.

III SUMMARY OF SNORKEL SIGNAL CHARACTERISTICS

For detectability calculations we must know not only the average target cross section, but also the nature of the fluctuations about the mean. To this end, we have measured the frequency spectra of 52 one-second observations on operating targets observed at Key West in low sea states (1 and 2). One can distinguish sea clutter, wake, and the direct snorkel echo. On amplitude versus frequency graphs, one can make quantitative comparison of the relative intensities of the three classes of echoes. The equipment lacked means of absolute intensity measurement, so that we are limited to measuring merely the ratios of snorkel and wake power to clutter power.

The one-second observations are spaced several minutes apart so that they are independent.

Figure 1 shows the frequency of occurrence of the average signal to clutter power for each of 52 one-second observations. For example, Figure 1 shows that there were 12 detections for which the snorkel signal to clutter ratio lay between 1 and 1.5. The wake echo from the overwater exhaust submarine is so infrequent, and weak, that it is plotted separately.

The wake echo is typically as broad as the clutter but has an average displacement (usually in the direction of target motion) of 2-1/2 to 3 knots.

The direct or snorkel echo (apparently less than 12 cps wide) is typically displaced from the clutter by an amount calculated from known target motion. There may be some exceptions to this rule, since targets approaching the aircraft seem to move too slowly and targets receding from the aircraft seem to move too fast. However, this effect is supported by limited observations, and has no reasonable explanation. Consequently, we regard this effect with suspicion for the present.

Due to the confusion caused by this target aspect effect, we have little information on the net motion of the sea clutter scatterers in sea states 1 and 2. (However, one target whose course was 90° with respect to the ground track was displaced 3 knots with respect to the center of the clutter.)

The direct snorkel echo fluctuates violently in intensity at an erratic rate in the neighborhood of 2 or 3 cycles per second. The data in Figure 1 have this rapid fluctuation averaged out.

In correcting the signal to clutter ratios to a standard range of 2,500 yards, the inverse first power range correction was used. All observations were made in the depression angle range of 2 to 7 degrees, where it is known that clutter power falls off as $1/R^3$, and we assume that target power falls off as $1/R^4$. In any case, the 1/R correction gives data that are internally consistent (e.g. signal to clutter ratios observed at 5,000 yards have the same average value when corrected, as ratios actually observed at 2,500 yards).

For all the data in Figure 1, about half of the target power is in the snorkel echo, and about half in the wake echo.

A significant thing about the data in Figure 1 is the large fluctuation in the target echo, even after a one-second averaging process. It looks as if there are really two different distributions, a "weak" one where the average target echo is about equal to the clutter power, and a "strong" one where the average target echo is 6 or 7 times the clutter power.* How rapidly the target signal goes from the "weak" to the "strong" distribution is a subject of further study.

Practical detection will undoubtedly depend upon detecting the strong signals--say the upper 50 percent. The median points are indicated on the probability distributions in Figure 1, where it is shown that 50 percent of the snorkel signals and 50 percent of the wake signals are somewhat stronger than the clutter, and 50 percent of the total signal (snorkel plus wake) are greater than about twice the clutter power.

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* These "weak" and "strong" distributions do not appear to be correlated with any one target, or with any one sea state, or with target aspect.

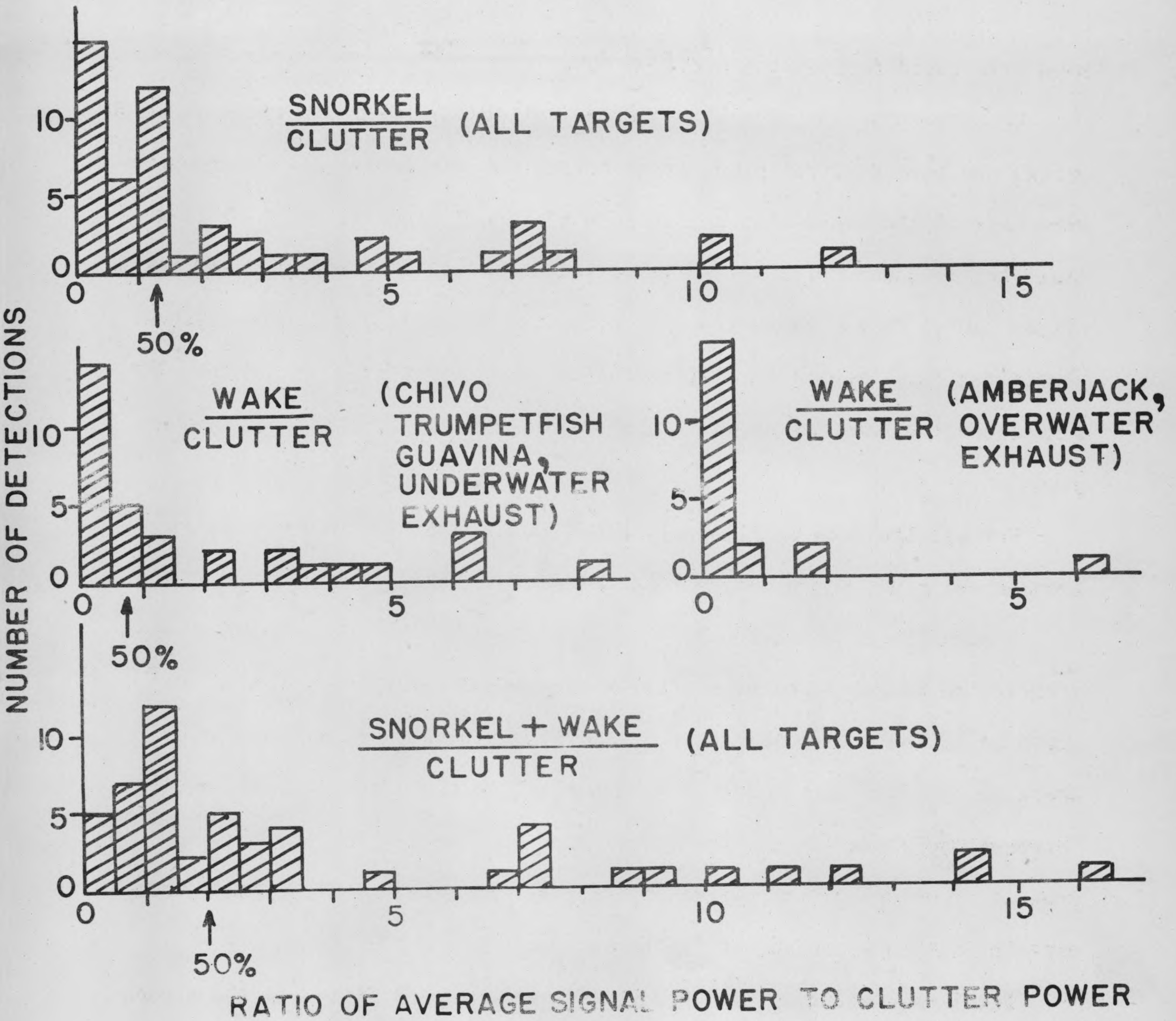

FIGURE 1

SNORKEL AND WAKE SIGNALS COMPARED TO SEA CLUTTER _ 52 ONE SECOND DETECTIONS.

ALL DETECTIONS NORMALIZED TO 2500 YARDS RANGE.

X-BAND, 4.1° BEAM, 0.5 μSEC. PULSE, P.R.F. = 2000 AIRCRAFT ALTITUDE 500 TO 1000 FEET, HORIZONTAL POLARIZATION, SEA STATE 1 AND 2.

IV EXTRAPOLATION OF SNORKEL ECHO CHARACTERISTICS TO LONGER RANGES

A practical search radar should reliably detect snorkel targets at ranges of 10 miles or more. We shall assume that the radar will search at the same depression angles (2° to 7°) as used for the experimental results discussed above. Thus, the target-to-clutter power ratios will be 10 times smaller if we consider a range of 25,000 yards rather than 2,500 yards, assuming the same pulse length ($0.5\mu s$) and the same beam width (4.1°). If the beam width is reduced to 1.5°, the signal to clutter ratios at 25,000 yards should be only 3.7 times smaller. Thus, we extrapolate the data in Figure 1 as follows:

For 1.5° beam; $0.5\mu s$ pulse; 25,000 yard range, sea states 1 and 2, depression angles--2 to 10 degrees.

(a) 50 percent of the snorkel echoes will have an average power of 0.3 times the clutter. These echoes will be less than 10 cps wide, but will fluctuate strongly in intensity several times per second.

(b) 50 percent of the wake echoes will be greater than 0.25 times the clutter (assuming underwater exhaust).

(c) 50 percent of the snorkel plus wake echoes will be greater than 0.5 times the clutter.

If the sea state is higher, one can expect that the clutter will increase, say 10 times. Thus, the total signal (snorkel + wake) is expected to exceed 1/20th the clutter power 50 percent of the time, assuming that the snorkel exposure and wake echoes remain the same in the high sea states. Such a small signal must be well displaced in velocity to be reliably detected. The wake echo displacement is probably too small really to be useful. Thus, we confine our attention to the direct echo only.

Actually, one would expect that, if viewed at an adequate depression angle (say 5 to 10 degrees) the snorkel exposure, between the swells in high sea states should be fairly large giving a signal which, though increasing less rapidly than the clutter, could still make the gap between target power and clutter power smaller than we have assumed in the above discussion.

Thus, using the available information on both target and clutter, we conclude that detection using MTI methods will probably still be difficult in high sea states. None the less, with an adequate number of looks (say 20) at a target displaced at say 4 knots with respect to the clutter, it appears as if detection would be possible*--particularly if the display were such that the operator loss factor was samll.

V COHERENT RADAR CHARACTERISTICS

(a) General Requirements

We assume that the radar has adequate power to detect the target in thermal noise at a useful search range. The basic detection problem, therefore, involves clutter rejection, particularly in high sea states.

We also assume that the false target problem is important, and we require, therefore, that after initial detection, the radar be capable of giving a fairly reliable indication that the target is in fact a snorkeling submarine and not floating debris, a school of porpoises,** a small boat, etc. The constant frequency, velocity displaced snorkel signal with the

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* See later examples.

** The unidentified moving target discussed on page 11 of CSL Report R-36 turned out, after investigation of the submarine's log, to be a school of porpoises.

associated wake echo is quite distinctive if studied for several seconds with adequate doppler frequency resolution. Thus, the radar equipment should have the capability of making a more detailed examination of the target after initial detection than is necessary in the search phase.

The search display should be either very simple to view, or give audio alarms so that the operator's tedious job is made much easier.

We also assume that sweep widths of at least 10 to 20 miles are essential if the equipment is to be of general use.

The problem of enemy passive ECM will be bypassed since the smallness of the target appears to make radar detection, before enemy listening contact, hopeless at least with a scanning radar. The low side-lobe, side-looking radar is a possible solution to this problem, but this has its own practical difficulties, particularly of antenna design. Should such antennas be available, however, the coherent MTI approach should be useful.

(b) Radar Beam Width

The spread in radial velocity, Δv, of fixed targets spaced Δ radians apart in azimuth is

$$\Delta v = V \sin \gamma \Delta \gamma \qquad (1)$$

where V is the aircraft ground speed and γ is the angle between the aircraft flight vector and the targets.

Assume that the antenna has a gaussian pattern whose full width at half power is $\Delta\gamma_A$. Thus, for off-the-ground track viewing, even fixed ground targets would have a gaussian velocity spread Δv_A which can be calculated from equation (1) when $\Delta\gamma = \Delta\gamma_A$.

If the targets (e.g. sea reflectors) have their own velocity dispersion with full width at half power Δv_C, then the resultant clutter spectrum will have a full width at half power of

$$\Delta v = \sqrt{(\Delta v_A)^2 + (\Delta v_C)^2} \quad (2)$$

If V = 150 knots, $\gamma = 45^\circ$ and $\Delta\gamma_A = 1.5^\circ$ (60" dish at X-band), then Δv_A is 2.8 knots which is also equal to the spread in the clutter. Thus, the observed clutter spectrum will appear to be 40 percent broader than along the ground track. Actually, the difference between expected target velocities and the spread in the clutter itself is already marginal for reliable detection. We conclude, therefore, that for effective off-the-gound track viewing (which is essential if a significant sweep width is to be attained), the radar beam should be narrower than 1.5 degrees. This applies, of course, to a 150 knot aircraft. For a 50 knot airship, a 1.5 degree beam would widen the clutter spectrum by only 10 percent even at 90° with respect to the ground track.

It is interesting to note that the requirement of not appreciably broadening the clutter spectrum sets our upper limit on actual geometrical width of the radar beam, independent of radar wave length. Thus, it appears that X and K-band are the most logical wavelengths for doppler ASW, at least in aircraft, if large antennas are to be avoided.

(c) Radar P.R.F. (f_r)

Since the range of unambiguous frequencies in pulsed doppler is only 1/2 f_r, this number should be great enough to cover the expected range of target velocities--say ± 6 knots. Therefore, at X-band f_r should be at least 800 pulses per second. The maximum f_r is, of course, set by the expected maximum detection range--say about 30 miles ($f_r \approx$ 3KC), independent of wavelength.

If the radar is to scan in azimuth, f_r should be as large as possible since this makes the fraction of the time that the clutter is folded over on itself* a minimum.

(d) Necessary Doppler Frequency Resolution

Since the basic detection problem is to reject clutter, it is clear that the frequency resolution of the radar system must be at least as narrow as the clutter. A rejection filter about 90 cps wide (for X-band) always centered on the clutter would reject clutter fairly well. However, the asymmetry in the clutter spectrum when looking upwind or downwind in high sea states would cause trouble. Such a system would merely report the presence of signal power outside the clutter spectrum but give no indication of direction or magnitude of target motion.

For recognition purposes, one needs to identify the low acceleration of the direct snorkel echo, and look for the characteristic wake echo. Filters as narrow as 5 or 10 cps would be profitable here. Our experiments show that floating objects and buoys shows characteristic accelerations.

In any case, when filters of band width Δf are being used, the antenna must take at least $1/\Delta f$ seconds to scan past the target. This allows the filters to build up properly and make full use of their resolution.

A flexible detection system would have a bank of filters covering a $\pm$ 6 knot range. These filters (each narrower than the clutter spectrum) would show the shape of the clutter spectrum, including any asymmetry. The alarm thresholds on these filters can then be set in a variety of ways, giving considerable freedom in the manner of clutter rejection. Filters, each up to 25 cps wide, could fairly accurately measure the shape of the clutter spectrum at X-band.

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* whenever the doppler frequency is near a multiple of 1/2 p.r.f.

(e) Necessary Extent of Clutter Rejection

If the estimated snorkel signal to clutter ratios for 25,000 yard range (see above section) are even approximately correct, there is little hope of detecting targets which are not velocity displaced. For example, we expect the total target signal to be in the range of 1/10th to 1/100th the clutter power in high sea states. For this case, even tens or hundreds of independent looks at the target are not adequate for reasonable detection (say 50 percent signal detection and 10^{-3} flase alarm probability, after all the data has been included). In practice, unless the signal to noise ratio for each independent input sample to the scan-to-scan integrator starts off at better than 1:1, there is no hope of building up a reliable detection state in a reasonable number of looks--say 10 to 100. We conclude, therefore, that the entire region of the clutter spectrum between the 20 db points or at least the 10 db points is a "hopeless" area to look for targets. Therefore, it should be eliminated. The remaining clutter power which extends beyond these limits should be appreciably less than the expected signal power. In lower sea states, the "blanked out" clutter band can be somewhat narrower.

Detectability can be improved, however, by going to lower velocity platforms (airships) and narrower radar beams (e.g. 0.5 degrees). This will improve the target to clutter ratio, increase the time of observation, and reduce the clutter broadening due to the geometrical beam width (but also reduce the searched area due to the lower velocity). The longer time of observation may be particularly important however if slow scintillations of target signal occur.

Automatic clutter rejection, which takes account of the clutter asymmetry can in principle be done, using banks of filters at each range

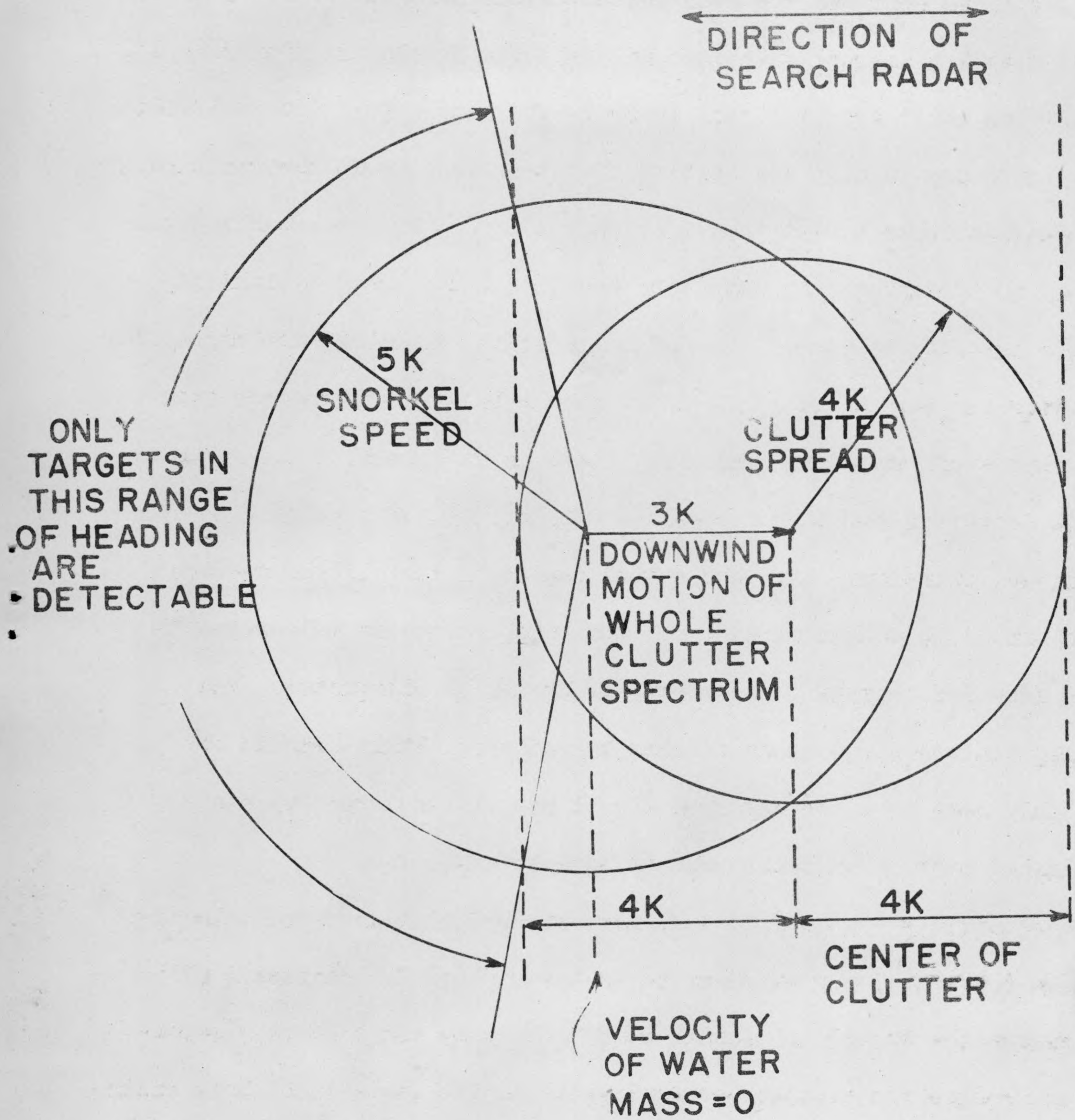

FIGURE 2. GEOMETRICAL METHOD OF DETERMINING THE HEADING OF 5 KNOT TARGETS WHICH ARE DISPLACED MORE THAN 4 KNOTS WITH RESPECT TO THE CENTER OF THE CLUTTER WHEN THE RADAR IS SEARCHING UPWIND OR DOWNWIND.

bin. One merely averages the power level of the same frequency filter from N range bins to determine the correct threshold.

VI DETECTABILITY AS A FUNCTION OF TARGET VELOCITY, WIND, AND SEARCH AIRCRAFT HEADING

We make the following simplifying assumptions:

(a) the scatterers causing the sea clutter are moving downwind with an average speed of 3 knots, relative to the mass of the water.

(b) a target to be detectable must have a velocity whose component in the direction of the aircraft exceeds 4 knots.

Figure 2 shows that under these assumptions, a 5 knot target, proceeding within $\pm$ 78° of upwind will be detectable if the search aircraft is flying upwind or downwind. Targets moving downwind at less than 7 knots are masked by the core of the clutter. Consequently the downwind motion of the white caps, in practice, will not add to the masking.

Similarly for an aircraft flying crosswind one can determine that a target whose heading is within $\pm$ 37° of crosswind will be detectable.

Extending the same method to other snorkel speeds, we have the following table. In calculating the fraction of targets that are "detectable", we have assumed that target headings are uniformly distributed with respect to the wind direction.

Snorkel Speed	Aircraft Searching Upwind or Downwind		Aircraft Searching Crosswind	
	True Heading of Detectable Target	Percent of all Targets	True Heading of Detectable Target	Percent of all Targets
3K	$\pm$71° of upwind	39%	not detectable	
4K	$\pm$76° of upwind	42%	not detectable	
5K	$\pm$78° of upwind	43%	$\pm$37° of crosswind	41%
6K	$\pm$80° of upwind	44%	$\pm$48° of crosswind	53%

Thus, if we take 5 knots as the typical snorkel speed and assume no knowledge of probable snorkel course, we estimate that roughly 40 percent of the targets would be in the "detectable" class, for upwind-downwind, or crosswind search. Similar numbers are obtained for other directions of aircraft search.

Of course, operational methods can in principle improve the probability of viewing the target from a favorable direction. However, it seems likely that only targets proceeding generally crosswind or upwind are potentially detectable--at least in high sea states.

VII FIRST EXAMPLE: DETECTION OF A 5 KNOT TARGET IN A HIGH SEA STATE--20 LOOKS

Such a target would have about a 40 percent chance of being in the "detectable" category, i.e. of having a speed displacement in excess of 4 knots with respect to the clutter in the direction of the search aircraft.

Suppose the search radar takes 1/20th second looks (i.e., it takes 1/20th second to scan one beam width) spaced an average of 3 seconds apart (e.g. a $\pm 45^{\circ}$ scan with a 1.5° beam with a period of 6 seconds for a complete cycle).

Let the range be 25,000 yards where we have estimated (Section IV) that 50 percent of the direct snorkel echoes (excluding wake echoes, and averaged over one second) exceed about 1/30th the clutter power. (Section IV, (a)) in a 1/20th second look, we assume that the target will either be "on" (have $>$ 1/15th the clutter power) or "off" (zero power). Thus, each scan or look has about one chance in four of obtaining a signal which is greater than 1/15th the clutter power.

Assume that the filters are effective in blanking out the center of the clutter spectrum leaving only about 1/100th of the clutter power to spill into the filters where the target is expected (i.e. 20 db clutter rejection).*

Thus, in one out of four looks (scans), an echo from the snorkel will exceed by 7 times the clutter power in a filter displaced 4 knots (or more) from the center of the clutter spectrum. Such a signal should give a clear alarm when a signal occurs. The false alarm probability for this threshold** (set at $\sqrt{7}$ times the r.m.s. noise voltage) is 10^{-3}, for a single "look" at one filter.

We assume there are ten 25 cps wide filters where, due to snorkel velocity limitations, a target is expected to occur. Let each filter have the same noise power. Then the total false alarm probability per scan for all ten filters in any one range-angle cell (i.e. one pulse length x one beam width) is about 1/100. Unfortunately, even a "detectable" target has only a probability of 1/4th of alarming on any one scan. Consequently, we need several scans over the target, perhaps as many as 10 or 20 to establish its presence with any reliability.

To attain scan-to-scan memory for 20 scans or so requires some storage device--such as a drum or CRT--in which the aircraft ground speed has been taken out so that a target will continue to alarm the same range-angle cell.

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* For a simple tuned circuit of 25 cps full width at half power centered 130 cps (4 knots at X-band) away from a gaussian clutter spectrum (of 90 cps full width at half power), numerical integration shows that 1/160th of the clutter power leaves the filter (-22 db).

** Assume a Raleigh Distribution for the filter output and see for example, Control Systems Laboratory report R-42, "Signal Detection" (UNCLASSIFIED). Actually, this false alarm probability may be slightly pessimistic since a 25 cps wide filter has about 3 "independent samples" in 1/20th second, and some integration of the filter output is possible before the threshold judgment is made.

Let us assume that a binary storage system exists, and has a capacity of 20 scans. Let each scan have 60 azimuth bins and 50 range bins* or 3,000 range-angle cells. The total capacity of the system is, therefore, 60,000 bits, and it has a new load of information every minute. A cathode ray tube could perform this function fairly well.

If the detection probability per scan is 1/4th, and the false alarm probability in each range-angle cell during one scan is 10^{-2}, then using the Bernoulli distribution, we calculate the following results:**

(1) If the alarm threshold requires 4 or more hits out of 20 scans:

Target Detection Probability = 75 percent.

False Alarm Probability (for all 3,000 range-angle cells) after 20 scans = 1/8 (one in 8 minutes since 20 scans require one minute).

(2) If the alarm threshold requires 5 or more hits, out of 20 scans, in the same range-angle cell:

Target Detection Probability = 55 percent.

False Alarm Probability (for all 3,000 cells) after 20 scans = 1/250 (one in 250 minutes).

Thus, _if_ adequate record is kept of the output alarms of all 50 range elements at each of 60 azimuth positions, for a time of one minute (20 scans), one can expect fairly staisfactory target detection--even in high sea states-- for the "detectable" targets (4 knots displacement from the clutter).

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* In 20 scans at 3 seconds/scan, a 150 knot aircraft will fly about 2-1/2 miles, which is covered by 50, 1/2 μs range gates. Thus 50 range bins are necessary for 20 looks at the target.

** See Appendix A for detailed calculation.

VIII SECOND EXAMPLE: DETECTION WITH ONLY FOUR LOOKS AT THE TARGET

Suppose that there were a limited number of range gates (say 10) for which frequency analysis were available, and the aircraft speed and azimuth scan rates were such that one had just four looks at the target while it traversed the detection range band. All other conditions are the same as in the above section.

What then are the consequences of cutting down the number of observations by a factor of 5?*

We assume as before that for high sea states and for 20 db clutter rejection, we have a 1/4th chance per look (i.e., scan) of getting an alarm on a "detectable" target, and a 10^{-2} chance per range-angle cell, of getting a false alarm in one scan past the cell in question.

Again, using the Bernoulli distribution, and requiring one hit or more out of 4 scans for an alarm, we have:

70 percent signal detection after 4 scans

24 false alarms after 4 scans for all 600 range-angle cells

(or 120 false alarms per minute).

This is not a practical detection condition. However, it is not far from being good enough, and should the target-to-clutter ratio be slightly more favorable, even 3 db, effective detection would be possible. In fact if we repeat the above calculation with the initial signal to noise ratio 3 db better, we get 70 percent signal detections and one false alarm in 10 minutes, which is substantially the same state as in Section VII where we had 5 times as many observations.

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* In this range of input signal to noise ratios ($\sim$10:1) the power level of a detectable signal is proportional approximately to $\frac{1}{\sqrt{n}}$ where n is the number of observations. Thus decreasing n by a factor of 5 should require a signal about 3 db stronger.

IX THIRD EXAMPLE: LONGER RANGE GATES

If we are still limited to 10 range gates, we can increase the time of observation by lengthening each one--from 1/2 microsecond to 2-1/2 microseconds--a factor of 5.

Now we have 5 times as many looks at a target, but each look has a signal-to-noise ratio which is 5 times worse.* If we keep the threshold set for the upper 50 percent of the distribution in Figure 1, it turns out that the clutter noise, being 5 times greater than before, makes detection impossible--even in 20 scans. However, if we use the upper 20 percent of the targets, the threshold can be set much higher (nearly a factor of 4 in power level). This is possible because of the peculiar shape of the signal/clutter distribution (there is an excessive number of large values). If in a 1/20th second look the target is either off or on, we have a probability of 1/10 of getting an alarm from a target in the "detectable" class (i.e. from the standpoint of velocity discrimination).

For this same threshold, the false alarm probability per look in one filter is 4×10^{-3}, or 4×10^{-2} in 10 filters.

Thus, for one look in one range-angle cell, the signal detection probability is 10^{-1} and the false alarm probability is 4×10^{-2}.

Using the Bernoulli distribution, the probability of getting 2 or more hits on a target out of 20 looks is 67 percent.

The probability that any one range-azimuth cell should give 2 or more false alarms out of 20 looks is 0.13. Unfortunately, there are 600 range-angle cells in each radar scan; thus there will be an average of 80 false alarms in each integration operation of 20 scans, or 80 false alarms per minute.

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* The target signal must compete with 5 times as many sea scatterers.

This situation is an improvement over the long pulse with the lower threshold, and about equivalent to the short pulse case discussed in the above section. Thus, the advantages of the long "tail" on the signal plus noise distribution plus the increased time of observation just overcome the handicap of lowered signal-to-noise ratio caused by the longer pulses. It may be that the longer observation time is actually necessary if the large target signal fluctuations are to be observed with any reliability. The average time between large target signals is an important item which is yet to be determined.

X NON-COHERENT MTI

Non-coherent radar can not distinguish the asymmetry of the sea clutter (due to white caps). Also, it can make only an inferior distinction between clutter and moving target signals even with symmetrical clutter spectrum. It is likely, therefore, that non-coherent MTI will be of little value, since it is distinctly inferior to the already marginal coherent MTI.

This conclusion may have to be modified, however, if the clutter spectrum in the interference zone (less than about 2° depression angle, on X-band) turns out to be consistantly much narrower than normal clutter.

XI NON-COHERENT RAPID SCAN

An ordinary non-coherent radar takes redundant data on both clutter and target plus clutter. If the signals are a gaussian spectrum 90 cps wide, the return pulses from one patch are correlated over a time of about 1/150 second. Thus, a radar with p.r.f. of 2,000 pulses/sec will give trains of echoes which are correlated over trains of about 14 pulses.

Rapid scan techniques* can reduce these trains to two pulses each** (since a minimum of 2 pulses per beamwidth is necessary, otherwise the antenna will have turned too far to receive the longer range echoes). Thus, there is a factor of 7 increase in the number of independent samples. The ability to detect weaker signals is improved by about the $\sqrt{7}$, thus rapid scan techniques offer an improvement of about 4 db over conventional radar.***

If our assumptions are correct, however, one needs about 20 db clutter rejection before conventional scan-to-scan integration begins if effective detection in high sea states is feasible. Thus, the theoretical 3 or 4 db offered by rapid scanning is inadequate. Furthermore, in high sea states, the clutter is broader and rapid scanning offers relatively less improvement over conventional radar.

XII CONCLUSIONS

Within the strength of the above assumptions, we conclude that targets moving generally upwind or crosswind can be detected at depression angles of 2 to 10 degrees with a scanning radar at ranges of 10-12 miles, using coherent MTI, even in high sea states (4). To attain this result, we were forced to require about 20 db clutter rejection and also scan-to-scan integration of 4 to 20 scans. Targets without velocity displacement should be quite undetectable in high sea states since coherence, and narrow filters, produce little improvement unless the signal is actually displaced outside of the interfering noise band.

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* Such as those developed at the Naval Electronics Laboratory.

** Control Systems Laboratory report, "Rapid Scan versus Coherent Doppler Radar for ASW." J. Ruina and C. W. Sherwin (to be published).

*** For off the ground track search using 150 knot aircraft and 3 to 5 degree beam widths, the clutter spectrum is appreciably broadened making the advantage of rapid scan over slow scan even less.

In any case, since clutter power in high sea states is 10, perhaps 20 db higher than in lower sea states, only a system offering really large clutter rejection will be effective.

Further improvement over our calculated detectabilities can be had by using radars with narrower beams, shorter pulses and slower platforms. Such systems will require more frequency analysis equipment, and if slower platforms are used, the area searched per unit time is reduced.

APPENDIX A

Example of the Calculation of Target Detection Using the Bernoulli Distribution

We take 20 looks at each of 3,000 range-angle cells. Target detection probability (if a target is present in one cell), $p_T = 1/4$. False alarm probability, $p_{F.A.} = 10^{-2}$. The Bernoulli distribution:

$$P(n) = \frac{N!}{n!(N-n)!} p^n (1-p)^{N-n}$$

gives the probability of observing n successes out of N trials where p is the probability of a success in one trial. It has been shown by Golay and also by Harrington* that binary integration (i.e. merely storing a "yes" or a "no" from each look at each cell is at most about 1.9 db worse than linear integration in a signal detection operation. Thus we use the Bernoulli distribution which is concerned only with success (i.e. exceed threshold) or failure, and calculate quite accurately the probability of signal detection.

We require that 4 or more hits out of 20 looks at a single cell be called an alarm.

If a target is actually present in a particular cell ($p = 1/4$, $N = 20$),

$P(0) = .004$
$P(1) = .025$
$P(2) = .074$
$P(3) = .150$

$.25$ = probability that less than 4 hits occur in 20 looks.

Thus the probability of a target causing 4 or more hits out of 20 is 0.75 for the particular cell where the target is located.

* J. V. Harrington, "An Analysis of the Detection of Repeated Signals in Noise by Binary Integration" August 1952, Tech. Report No. 13, Lincoln Laboratory, MIT.

If no target is present, $p = 10^{-2}$, and $N = 20$. Thus for any <u>one</u> cell,

$P(4) = 4 \times 10^{-5}$
$P(5) = 1 \times 10^{-6}$
$P(6) \cong 10^{-7}$ etc.

Thus the probability of a false alarm in any one cell, after 20 looks, is $\sim 4 \times 10^{-5}$.

Since there are 3,000 cells, the probability of one of them giving a false alarm is $3 \times 10^3 \times 4 \times 10^{-5} = .12 \cong 1/8$, after 20 looks, (spaced 3 seconds apart).

Since 20 looks requires one minute, there is one false alarm every 8 minutes on the average.