CONTROL SYSTEMS LABORATORY

WIND WAVES ON THE WATER

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By

Bruce L. Hicks and Clive G. Whittenbury

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ABSTRACT

This paper reviews the origin and nature of wind waves on deep water. In order that the subject be treated properly it is necessary to include an extended discussion of the air boundary layer over water. This emphasis upon the aerodynamics of wind waves distinguishes the paper from earlier treatments of the subject. Although unusual emphasis is also placed upon the characteristics of small waves and of whitecaps, almost all important properties of wind waves receive some attention. Frequent references to the literature are used to avoid the necessity of giving all of the details of each topic embraced by the broad subject of wind waves.

By way of summary, we list a number of topics (including new results) that are to be found in the paper. Those dealing with hydrodynamics will be mentioned first. We find it to be useful to define primary and secondary waves as those of wave length greater and less than one foot, respectively. Secondaries are responsive to the local wind while primaries are not, and these characteristics have many consequences, which are explored in the paper. We extend the classical theory of viscous damping of waves on water to cover short-crested waves on a contaminated water surface. A partial explanation is found for the observed asymmetry of small secondary waves. The reality of so-called "nonlinear" motions is discussed briefly in an appendix, since nonlinearity of the wave motion is frequently assumed for the wind-wave interaction and the breaking of waves, or given as a reason for the inadequacy of the theory. We review various statistics of the sea surface and derive a few new and simple relations among them. The

origin and nature of instability leading to breaking of a wave is discussed at length. The fraction of the sea surface, described as a Gaussian process, which cannot possibly remain unbroken is calculated. This fraction is related to visual estimates of the "sea state." An alternative method of estimating this fraction is also outlined. Broken water is described qualitatively, and parameters for its quantitative description are suggested.

The difficult problem of wind-wave interaction is discussed on a simplified basis by relating the problem to three principal topics: the wave initiation, the aerodynamic forces on the water surface, and energy dissipation within the waves. The importance of boundary layer characteristics is stressed, in particular the history and depth of the air boundary layer, which is generally turbulent.

The so-called "critical wind speeds," quoted for example for the <u>initiation</u> of waves, are found to have little meaning unless accompanied by a statement of the air boundary layer characteristics. Through a discussion of the aerodynamic surface forces it is shown that the character of the boundary layer and of the wind-wave interaction is determined by the ratio of the thickness δ of the air boundary layer to the wave amplitude a, and that three cases should be considered separately: $\delta < a$, $\delta \sim a$, and $\delta \gg a$, the last including the very important case of the thick turbulent boundary layer over wind waves at sea. The aerodynamical interaction is then analyzed in terms of the tangential drag, which is important particularly at low wind speeds for all waves, but also for the largest waves at high wind speeds and generally when separation of the air boundary layer from the water surface is not present locally;

and in terms of the pressure drag which becomes important at high wind speeds and when separation is present from any of the waves.

The third case mentioned above, that of the thick turbulent boundary layer probably most commonly occurs in wind-wave generation and is discussed in the paper at length. In this case we assume (with explicit reservations) that the flow is similar to the flow of a turbulent boundary layer over a rough plate (flat or curved), a situation which has already been investigated in wind tunnels. In addition to the flow generally being turbulent over the water, we suggest that it is usually in the "fully rough" condition and that the roughness effects are caused mainly by secondary waves or by small parts of primary waves such as unstable or sharp crests. These roughness effects are conveniently discussed in terms of the "equivalent sand roughness" whose relation to the height of the relevant waves is an important but seldom measured sea state parameter. The functional relations among size of roughness elements, boundary layer profile and aerodynamic energy input to the waves are also of basic importance, both in transitional and in fully rough flow. In one particular case of air flow over the sea, the transition to fully rough flow was found by Hay to occur at a wind speed of 13 knots. About the same wind speed is generally quoted for the incipience of whitecaps, a significant coincidence.

Our discussion of drag for turbulent flow over a rough plate shows that usually the drag coefficient would be determined <u>entirely</u> by the thickness of the boundary layer and by the character of the roughness elements, especially by their height. Drag data from the literature, representing various methods and conditions of determination of drag, are summarized in the paper. Complete agreement with wind tunnel data is not to be expected because the aerodynamic situation in nature can be modified by the effects of thermal instability in the air boundary layer. A dimensional analysis of drag data predicts various dependencies of the drag coefficient on the fetch for the various flow regimes considered.

After a review of present methods of calculating wave dissipation, we conclude the paper with a preliminary account of the growth of complex systems of finite waves and with recommendations for future research.

SYMBOLS

a	wave amplitude
$\overline{A(\omega,\theta)^2}$	energy spectral density
c_f	skin friction coefficient ($2\tau_o/\rho u^2$)
C	wave velocity
C_g	group velocity
d_o	number of whitecaps per unit area of sea surface
$d\overline{F}$	force on a surface element
e	wave energy per unit area
E	"wave energy" (total energy/unit area).(2/ρg)
$\mathcal{E}(a_{max})$	expectation value of maximum amplitude among N waves
f_u	fraction of the sea surface that is unstable
fw	fraction of the sea surface that is covered by whitecaps
g	acceleration of gravity
$H = 2a$	wave height
$\overline{H}_\lambda$	average height of λ-highest waves
$\overline{i}, \overline{j}, \overline{k}$	unit vectors
k	height of roughness element
k_s	equivalent sand roughness
L	longitudinal wave length
L'	transverse wave length
$m = 2\pi/L$, $n = 2\pi/L'$	wave numbers
M_{ij}	spectral moments
N	number of waves
p	pressure
p_o	initial pressure distribution

r	$(m^2 + n^2)^{1/2}$
s	sheltering coefficient
t	time variable
t_1, t_2	decay times for wave damping
T	wave period
u, v, w	components of wind velocity
$u_1(x')$	air velocity above boundary layer
u_∞	wind speed
$u^* = (\tau_o/\rho)^{1/2}$	friction velocity
U, V, W	components of water velocity
x, y, z	coordinates relative to mean sea level (direction of wave propagation, x; along crests, y; vertical, z)
x', y', z'	coordinates relative to a plane tangent to the water surface locally
z_o	roughness parameter
α, α'	classes of water masses
β, β'	classes of air masses
γ	Hibbs' coefficient
δ	boundary layer thickness
ϵ	phase angle
η	local elevation of water surface
ϑ	parameter of smallness (Appendix B)
θ	azimuthal angle
λ	fraction of a set of N waves
μ	laminar viscosity coefficient (air)
μ_ϵ	eddy viscosity coefficient (water)
ν	kinematic viscosity coefficient (air)

ρ	air density
P	water density
σ^2_ζ	variance of the quantity ζ
Σ	surface tension (air-water interface)
$\Sigma' = \Sigma/P$	capillary constant
$\tau_o = \mu(\partial u/\partial z)_{z=0}$	shear stress
$\Upsilon(a_{max})$	modal value of maximum amplitude among N waves
$\omega = 2\pi/T$	radian frequency
$\xi(t)$	envelope of $\eta(t)$
$\phi = \partial\eta/\partial x$	x component of wave slope

TABLE OF CONSTANTS

1 knot = 51.48 cm/sec = 1.689 ft/sec

Kinematic viscosity* - cm^2/sec

	0°C	15°C
air	0.135	0.148
water	0.0179	0.0114

Capillary constant* Σ' - cm^3/sec^2

	0°C	15°C
air-water interface	75.6	73.4

Characteristic lengths and velocities of capillary-gravity water waves at 15°C (see App. A and B)

condition	wave-length-cm	wave velocity-cm/sec	group velocity cm/sec
minimum wave velocity	1.72	23.2	23.2
minimum group velocity	4.37	28.1	17.8

Modulus of decay t_1 (sec) = $1.11\ L_r^2$ ($L_r = (r/m)\ L$ in cm)

*Derived from data in Smithsonian Physical Tables, 9th Revised Ed. (1954)

LIST OF FIGURES

INTRODUCTION

Present understanding of the generation and decay of systems of wind waves is incomplete, being limited to those having narrow spectra. The surface of a body of water disturbed by the wind, seldom, however, exhibits the periodicity which is characteristic of a narrow spectrum. Instead, wind waves usually produce a random shape of the surface with a very broad spectrum, so that it is not surprising that a fairly complete connection between a system of wind waves and the hydrodynamic and aerodynamic forces controlling this system is yet to be formulated.

In searching for such a connection it is probably necessary to seek first an understanding of the dynamics of simpler and idealized but still relevant systems. In particular, we must seek to understand three fields: the hydrodynamics of simple wave systems including their damping and dispersion; the qualitative and statistical descriptions that have been developed of different properties of the sea surface; and the nature of the aerodynamic interactions between wind and waves. No very considerable part of this rather diversified body of information is available in any one place in the literature. We have, therefore, sought to bring together in this paper the important ideas in each of the three fields of information mentioned, emphasizing those subjects which have been especially neglected, namely, the characteristics of small waves, the nature of breakers and whitecaps, and the properties of the air boundary layer* over wind waves.

In collecting the information needed for this report we tried to keep in mind the various interests of oceanographers, many of whom are already specialists in the subject of wind waves, of aerodynamicists, more of whom we hope will become specialists in this subject, and of radar and sonar people, who study the scattering of energy by the sea surface and would like to be able to define its nature. In view of these very varied interests, we chose to organize the collected information rather rigidly into fourteen sections and several appendices to facilitate ready reference to any topic of interest. More often than not the longer sections refer to phenomena that are imperfectly understood, for we try to indicate (implicitly in the text and explicitly in the section, "Recommendations for Future Research") where new research is needed as well as the successes of past researches.

The first eight sections of the report present methods for describing and predicting wind waves, based upon hydrodynamic theory of capillary-gravity waves and semi-empirical oceanographic theory. The ninth section contains an account of breakers and whitecaps at sea, somewhat speculative in nature because of the small amount of theory and experimental data available. The last five sections are concerned with the interaction of wind and wave. Although much yet remains to be done in this area of research, it is possible, having made one important assumption, to outline the principle features of

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* In an earlier, introductory discussion of wind waves (Hic-1), one of the authors attempted to summarize the noteworthy characteristics of wind waves and their formation. Even though discussion there was based upon but part of the relevant oceanographic and aerodynamic literature, it was possible to see that only a part of the resources of aerodynamics had been exploited by those investigating what was manifestly an aerohydrodynamical problem.

of the interaction that manifest themselves in natural circumstances. The assumption is that of the approximate similarity of turbulent flow over wind waves to turbulent flow over a fixed rough plate. In Appendices A and B, formulae are collected for the frequencies, wave and particle velocities, and laminar viscous damping of monochromatic capillary-gravity waves for all crest and wave lengths. In Appendix C, the research of the Control Systems Laboratory on small wind waves is outlined. In Appendix D we try to clarify the significance of the term non-linear which is so freely applied in this and other papers to wind-wave phenomena that do not appear to follow the predictions of linear or small-displacement theory.

Because we discuss only waves caused by wind, we ignore several phenomena that affect the sea surface, such as surf beat, shelf noise and tides, that are characterized by frequencies less than 0.05 cps. We also ignore, for the most part, the surface effects of internal waves and the 0.07 cps band of frequencies arising from changes in the average meteorological conditions.

This summary of the contents of our paper illustrates its broad scope. We cannot hope to have treated adequately every relevant phenomenon and topic*. Many of these phenomena are not understood and are therefore still controversial, so that we have not been able to synthesize a compact and wholly self-consistent description of wind waves. Our critical examination of a wide range of oceanographic and aerodynamic material has, however, permitted us to formulate

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* As we have had little personal, first-hand experience with wind-waves at sea, we suggest to the reader that he should balance his diet by assimilating selected passages in the books by Cornish and by Bigelow and Edmundson. (See References at the end of the report.)

some new and useful concepts and a few small theories, which are listed in the Abstract to whet the appetite of the reader. Thus, in addition to reviewing the status of the subject of wind waves, we are able also to present new results and to make recommendations for future research.

Papers referred to in the body of the report are listed under "REFERENCES." For the convenience of the reader, additional papers are listed under "BIBLIOGRAPHY", but only a few of these have been used in preparing the report. With some exceptions our examination of the literature has not extended into 1956. Items marked with asterisks in REFERENCES and BIBLIOGRAPHY are relevant especially to the aerodynamic features of wind waves.

After the greater part of the present report was completed we received a copy of Sections I, II and III of the Manual on Amphibious Oceanography. (See Bibliography at the end of the report.) Several sections of the Manual should be mentioned. Section II contains a very complete account of the hydrodynamic theory of waves and gives useful graphs and tables of functions. Section I treats breakers and surf and wave statistics, and Section III discusses wave forecasting. The authors of the Manual sought to collect all information on these and other topics that was available in 1951. We shall give specific references in the text of our paper to only a few of the many relevant pages in these three Sections of the Manual, but we do recommend this valuable compendium to the reader who wishes to study further theoretical, experimental or historical details of the topics mentioned.

We should like to record here our indebtedness to a number of scientists whose comments helped us to prepare this paper.

Its defects are due not to them but to the authors. In particular, we should like to mention several long and profitable discussions with Dr. Walter Munk and Dr. Charles Cox of the Scripps Institution of Oceanography; with Dr. G. B. Schubauer, Dr. Garbis H. Keulegan and Dr. C. M. Tchen of the National Bureau of Standards; with Professor W. J. Pierson, Jr., and Professor G. Neumann of New York University; and with Dr. Jack Schule of the U. S. Navy Hydrographic Office. Dr. Isodore Katz and Dr. Charles I. Beard of the Applied Physics Laboratory, Dr. M. S. Longuet-Higgins of the National Institute of Oceanography, and Dr. Munk kindly sent us their written comments on our earlier paper on wind waves. Finally, the Allerton Conference on Sea State and Sea Clutter, held in April 1955 and sponsored by the Control Systems Laboratory and the Office of Naval Research, facilitated a stimulating exchange of ideas and served to define some of the chief problems of wind wave behavior that were unsolved at that time.

I. HYDRODYNAMICS OF PROGRESSIVE SURFACE WAVES ON WATER

1. Monochromatic Wave Systems

The simplest monochromatic progressive surface waves are sinusoidal and long-crested. Such simple waves are seldom found in nature but will be described first, as a basis for later discussion of the more complicated, naturally occurring wind waves.

The surface elevation η of these simple waves is representable by

$$\eta = a \cos \left[m(x \cos \theta + y \sin \theta) - \omega t + \epsilon \right] \tag{1-1}$$

Such a long-crested wave train is thus identified by the five parameters, amplitude* a, wave number m (or wave length $L = 2\pi/m$), radian frequency ω (or period $T = 2\pi/\omega$), phase angle ϵ, and angle θ between the positive x axis (lying in the plane of mean sea level) and the direction of propagation of the train. Fig. 1 is a section, perpendicular to the direction of propagation, of the instantaneous position of a long-crested wave for which $\theta = 0$. Waves of such a form can exist, theoretically, in a body of water bounded above by air if the water depth is greater than L/2**; if the amplitude a is much less than L; and if L is greater than a length, discussed later, for which viscous effects become significant.

Wave number and frequency of these waves are related by the condition

$$\omega^2 = \Sigma' m^3 + gm \tag{1-2}$$

where $\Sigma' = \Sigma/P$, Σ being the surface tension of an air-water

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* Wave height, H, the difference in height between a crest and an adjacent trough, is commonly used in oceanography rather than amplitude. For a sinusoidal wave H = 2a.

**We shall not discuss shallow water waves in this paper.

interface, ρ the density of the water and g the acceleration of gravity. The wave velocity is $C = \omega/m$. The waves are called gravity waves if L is large (say for $L > 5.1$ cm), and ripples or capillary waves if L is small (say for $L < 0.6$ cm), in recognition of the restoring force that is dominant in each of the two cases. Formulas for wave, particle, and drift velocities of long-crested waves are given in Appendix A. Comparison of measured velocities of waves with various theoretical calculations may be found in the literature. (See for example BE and Jo-1.)

If we remove the second restriction, that (am) be small, thereby admitting the possibility of non-linear effects, the wave shape is changed from sinusoidal to trochoidal, with correspondingly flatter troughs and steeper crests. A limiting shape, that is well approximated by pieces of cycloids (see Wi) forming sharp crests, is reached when $(am) = \pi/7$.* The crest angle is then 120°, and the wave velocity (crest velocity) is larger by ~ 12% than it would be for a sinusoidal wave of the same length. At these sharp crests the particle velocity is equal to the crest velocity, and no higher waves will be kinematically stable.**

Individual long-crested waves whose properties we have thus summarized correspond approximately to swell propagated to some area distant from the generating area and possibly also to wavelets generated in some early stage of wave growth under the action of light winds. Monochromatic trains of wind waves are almost non-existent.

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* The term "wave steepness" defined as $H/L = (am/\pi)$ is often found in the literature.

**In an earlier paper (Hic-1) we conjectured that long-crested waves may also be unstable with respect to break-up into short-crested waves. The rôle of wind forces in such a break-up might be an important one, as mentioned by Neumann (Ne-2, p. 43).

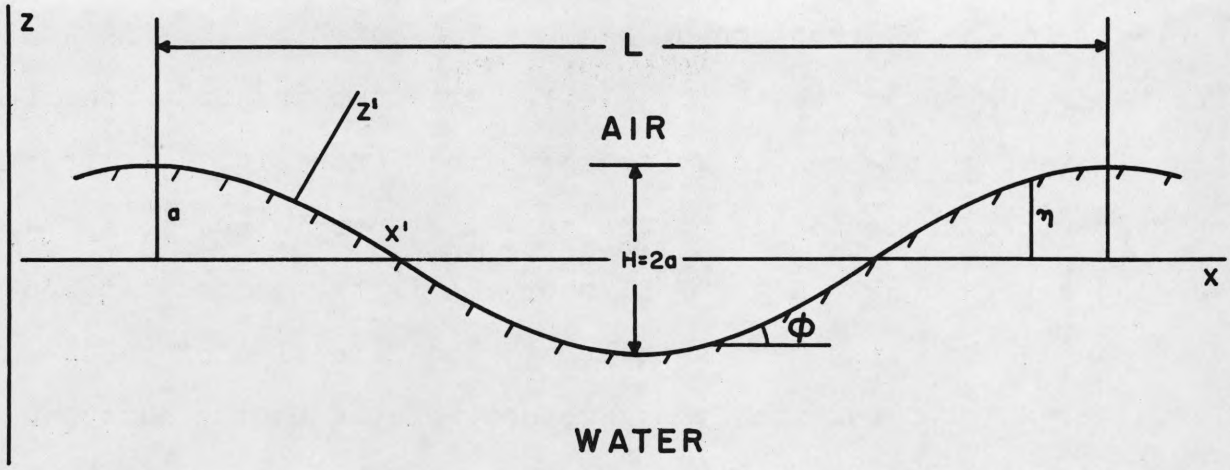

FIG. 1 PROFILE OF LONG-CRESTED WAVE

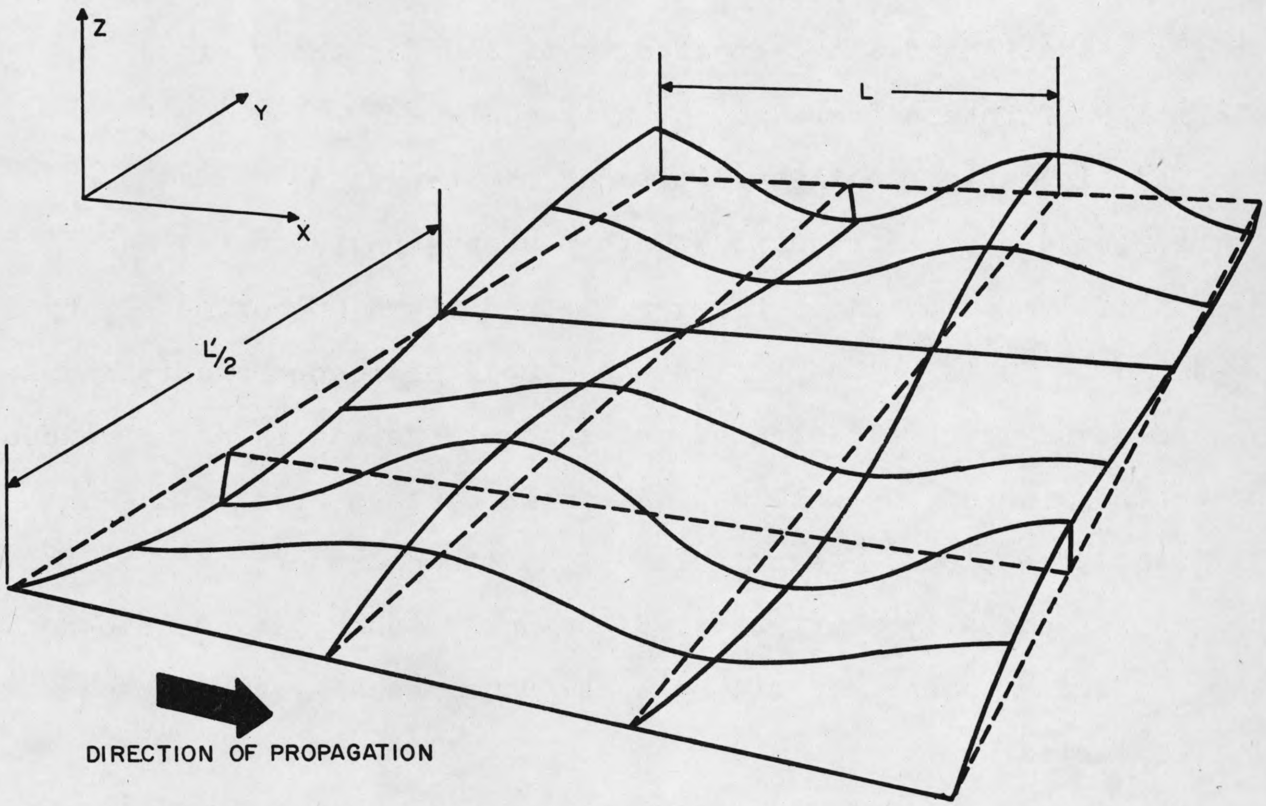

FIG. 2 IDEALIZED SHORT-CRESTED SEA.

Let us next consider the superposition of two wave trains which differ only in their direction of propagation, each satisfying the restrictions enumerated below Eq. (1-1). This superposition results in the simplest short-crested system of waves. (See Fig. 2.) It can be characterized conveniently by the same five parameters a, m, ω, ϵ, θ plus a wave number $n = 2\pi/L'$ where L' is the length between crests along a direction perpendicular to the direction of propagation. Wave number and frequency are now related by the equation

$$\omega^2 = \Sigma' r^3 + gr \qquad (1\text{-}3)$$

where $r^2 = m^2 + n^2$. The wave velocity as before is given by $C = \omega/m$. Noting that the condition $n > m$ is seldom observed, we can define gravity waves and capillary waves again as those for which, respectively, $L > 5.1$ cm or < 0.6 cm. The equations of the surface elevation of short-crested waves and for the various velocities of interest are given in Appendix A.

The kinematic stability limits of short-crested waves have not been studied.* The shape of steep short-crested waves and their drift velocity have, however, been derived approximately by Fuchs (Fu). It is probable that the simple or monochromatic short-crested waves we have described are seldom observed in nature except at the crossing of two well defined swell systems propagating in different directions. Nevertheless, the structure of the sea surface is generally short-crested, being composed, as described later, of wave trains covering a continuous, usually broad band of frequencies.

We are now ready to consider the generation or decay of simple

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* The extreme form of a radially symmetrical elevation has been found to be characterized by a crest angle of 130.5° (Sect. IIA 4a of the Manual on Amphibious Oceanography. Cf. Bibliography.).

progressive wave systems. The wind can in different circumstances act either to generate or to damp a wave system.* We summarize what is known about wind-wave interaction in Part III. We shall summarize now the damping effect, as predicted by rather idealized classical theory, of the water's viscosity when the motion of the water is laminar, deferring until Sect. 13 a short discussion of turbulent damping.

We shall find it convenient** to classify all waves as primary or secondary accordingly as their wave lengths are greater or less than one foot. A primary, monochromatic, long-crested structure may take many minutes or even hours to grow or die as the wind changes. Capillary waves, however, may follow the changes of the wind within a fraction of a second. This qualitative statement can be made reasonably quantitative, in a one-sided way, by referring to a classical formula (see App. B) which states that the time t_1 necessary for damping by a factor 1/e of the amplitude of a sinusoidal wave on clean water by the viscosity of the water is proportional to the square of the length of the wave. The proportionality constant is such that a long-crested wave of length 1.7 cm, which has the minimum velocity possible for a classical gravity-capillarity wave, will be damped by the viscosity of the water (at 15° C) to 1/e of its amplitude in 3.3 sec. The amplitude of a long-crested wave six meters in length will decrease, on the other hand, to 1/e th of its value in 115 hours. When there is a film of oil or of organisms on the water surface, the damping time

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* The damping of small water waves is increased by rain (cf. Sect. 13 and Sa), snow, and hail striking the water and by ice (see Sh) floating on the water surface.

**See Sect. 3.1 for a discussion of this definition.

t_2 is less than for a clean water surface and the percentage decrease is greater for the longer waves (see App. B). Thus the damping times quoted above for the long-crested waves 1.7 cm and 6 m long are reduced in theory by factors of 4.2 and 280, respectively, when there is a compact film on the surface.*

We have generalized theories of damping of waves on contaminated surfaces. The results of our generalization to short-crested waves on a contaminated water surface and of Jeffreys' theory for short-crested waves on clean water (Je-1) are given in Appendix B. These results show that the moduli of decay, t_1 and t_2, for clean and contaminated water surfaces respectively, are functions only of $r = (m^2 + n^2)^{1/2}$ and not of m and n separately. Accordingly, for a clean water surface, for example, a long-crested wave (1) of length L_1 will damp less rapidly by a factor of two than will a short-crested wave (2) of lengths $L_2 = L'_2 = L_1$. (For comparison we note that a change of temperature of the seawater from 15° C to 0° C will decrease t_1 and t_2 by factors of 1.60 and 1.26 respectively.)

Another way of illustrating the laminar viscous damping of waves is to state how many cycles of a wave of given length are executed before this damping has decreased its amplitude by the factor 1/e. This number is given by the equations in Appendix B. We find for example that the amplitude of a wave of length 0.6 cm on clean water will be damped in 20 cycles by the factor 1/e. The shape, as well as the amplitude, of waves much shorter than this will be appreciably perturbed by the viscosity of the water. When

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*The cut-off wavelength, below which a slick tends to damp out all waves, lies between one inch and one foot.

the surface is contaminated by a slick, all waves of length less than 5 cm will, by the same criterion, be appreciably perturbed. We have no quantitative information about the damping of waves so steep that they cannot be satisfactorily represented as sinusoids. It is probable, however, that in the presence of viscous damping, the "highest wave" (for which am = $\pi/7$) will never be exactly realized because the sharp crest characteristic of this wave requires for its existence the presence of very high frequency harmonics which are damped very much faster than the fundamental corresponding to the wave length.

A few miscellaneous notes concerning slicks are worth recording. It is generally known that oil calms a rough sea. It is said, for example, that (see Big, p. 149) "Oil extinguishes the smaller wavelets and prevents the waves from breaking." The second effect has not been examined scientifically. The few experiments in which a detailed examination of the first effect has been made are reviewed in Sect. 13. There is no known effect of films upon the shape of waves (Munk*) although the skewness of the slope distribution is removed by a film (Cox). The orientation of natural slicks, for wind speeds less than 6.6 knots, is governed by internal waves (Ew-1, p. 162), and bears no relation to the direction of the wind; indeed two independent trains of slicks may cross (Ew-1, p. 183).

2. Polychromatic Wave Systems

As has already been noted, a system of wind waves is seldom even approximately monochromatic. It is therefore worthwhile

*Private communication.

mentioning some of the characteristics of polychromatic systems, i.e., those possessing neither narrow spectra nor the special line spectra corresponding to permanent waves of finite amplitude.

We note first that the crest angle of the steepest stable crest formed by a two-dimensional polychromatic wave system containing waves with various propagation velocities is 120°, just as in the case of the classical steepest "permanent" wave with a single propagation velocity. This generalization of Stokes' classical result has recently been obtained by Hibbs (Hib, Sect. V c). The stability of waves is of course related to their shape (cf. Sect. 9).

Since the wave velocity of water waves is dependent upon their wave length, a polychromatic system of such waves is dispersive. In a dispersive medium, the group velocity at any one frequency is defined as

$$C_g = d\omega/dm \qquad (2\text{-}1)$$

In many simple cases and in particular for the asymptotic motion away from a disturbance it has been proved that the energy is propagated with this velocity. For capillary waves, the group velocity is larger than the wave velocity, $C_g = 3C/2$; for gravity waves, the opposite is true, $C_g = C/2$. The two velocities are equal when the wave velocity is a minimum, i.e., for $C = C_g = 23.2$ cm/sec. The minimum group velocity occurs at a wavelength of 4.37 cm where $C_g = 17.8$ cm/sec and $C = 28$ cm/sec. A packet of waves of lengths near 4.37 cm will exhibit much slower spreading than packets centered on other frequencies. (See Je-3, Sec. 17.09 for discussion of further effects of a minimum C_g.) For a packet with an initially Gaussian wave number distribution (Cou, p. 138) the rate of spreading, to a first order of approximation, will be proportional to $d^2\omega/dm^2$ and to the square of the half-width* of the wave number distribution. The spreading is of course accompanied by a corresponding decrease of amplitude. The asymptotic properties of the propagation of other groups of capillary-gravity waves is discussed in more detail elsewhere (Je-3, Sect. 17.09).

Many different groups of (inviscid) water waves have been studied mathematically** (Lam, Sects. 238-244, 248, 255, 256, 269-272; Fu; Pi-2, 3). The groups differ from one another in the initial conditions assumed. Thus in problems of the classical Cauchy-Poisson type, a disturbance of a limited part of the surface is assumed at time $\underline{t = 0}$ (Lam, loc. cit.). Pierson (Pi-2, Sect. 4.4) has analyzed the motion of a group of sinusoidal waves of large but finite crest length which are observed to pass the line $\underline{x = 0}$ during a finite time interval. Except for the theory given in Lamb (Lam, Sects. 269-272), such calculations have been completed only for the case of gravity waves even though their extension to short-crested, capillary and capillary-gravity waves would be useful in understanding the behavior of the secondary structure of water surfaces.

The changes of amplitude, of width, and of velocity of a packet as it propagates will all be changed slightly when viscous damping of clean water is considered, and much more, at a given wave length, when the water surface is contaminated or when there

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* This half-width is inversely proportional to the half-width of the packet's initial extension in space.

**Unfortunately, recent studies of individual problems seldom include discussions of <u>both</u> the small-time and the large-time behavior. In fact one <u>rarely</u> finds a statement of what is meant by small- or large-time in terms of the parameters of a problem. Lamb (Lam, Sects. 241, 269 and elsewhere) does discuss this question for many of the classical problems of wave motion.

is turbulent rather than laminar viscous dissipation on the water. (Strong viscous effects may, of course, preclude description of the surface in terms of sinusoidal waves.) It is possible that the nature of slicks on the surface or of turbulent dissipation just beneath the surface could be studied by measuring the propagation characteristics of simple wave packets.

The damping of a polychromatic system of water waves is usually calculated in the linear approximation which assumes that each component frequency damps independently of the others. In at least one simple and important situation this approximation may be inadequate, namely when an underlying primary crest (trough) steepens (flattens) the secondary riding on it (cf. Un and Hib). The damping rate as well as the stability of the secondary may then be changed.

II. CHARACTERIZATION OF WIND WAVES AND WHITECAPS

3. Qualitative Characterization

3.1. Some Useful Concepts. The sea surface appears to the eye to consist of a more or less ramdom collection of peaks and hollows which retain their approximate form and identity for a matter of seconds. Certain concepts are useful in characterizing qualitatively this random collection, as viewed at any one instant--for example, the height and steepness of waves that have already been defined in Sect. 1. Quantitative characterization will be deferred until Sect. 4.

The first additional concepts to be distinguished are "sea" and "swell". Their individuality is recognized in setting up different scales for sea and swell conditions (for Tables see PNJ; HO-1; and Locke). Sea and swell may occur separately or may exist together in any combination. In a given area of the ocean surface, the "sea" refers to the large-scale condition of the surface brought about primarily by winds within, say, 1-10 miles. We shall find later that the small-scale surface structure is determined by the existence and character of winds much closer to the point of observation. "Swell" refers to the large scale surface condition brought about, independently of the local wind, by waves that have propagated into the given area from remote storm or generating areas and are thus in the process of decaying.

We have found that a second pair of concepts, namely primary and secondary structure of the sea surface, are useful in discussing wind waves. As mentioned previously, we define primary waves to be those of wave length greater than one foot and secondary waves to be those of wave length less than one foot.* This definition requires some elaboration since the actual sea surface is random (see Sects. 3.2, 4, 6). The lengths of the "waves" appearing on the sea surface have no simple relationship to the lengths of the individual waves (spectral components) of different frequencies whose superposition yields the actual sea surface. It is, however, often assumed, as we shall also assume here, that actual waves of a given length are composed mostly of spectral components of very roughly that same length. The "primary structure" is thus largely the result of superposing individual waves of length greater than one foot. Our somewhat artificial, but nonetheless useful, distinction between primary and secondary waves may also be expressed in another way. Suppose we decide to ignore all waves, whether those actually observed or the spectral components, whose lengths lie between 0.8 and 1.5 feet. Then very few of the component waves of length less than 0.8 feet can make a contribution to the height of the resultant (primary) waves longer than 1.5 feet, and very few of the component waves, of length greater than 1.5 feet, can make contribution to the height, relative to the underlying primaries, of the resultant (secondary) waves shorter than 0.8 feet.

Different wind wave shapes may also be distinguished, as were the monochromatic waves of Sect. 1, according to the distribution in plan of their troughs and crests, as being long-crested ($L \gg L'$) or short-crested** ($L \sim L'$) where L and L' are the average crest lengths measured parallel and perpendicular, respectively, to the

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* Neumann (Ne-2, Ch. II, Sect. 2) uses the term "general profile" in place of our term "primary wave."

**Another measure of the short-crestedness is the beam width which measures the range in azimuth of the different directions of propagation of the component waves making up a sea (see Sect. 5).

direction of propagation of the waves. If waves from a distant storm are propagated into a region where the wind is not blowing vigorously, if there are no other wave trains crossing the region, and if there are no currents present*, then the swells that will be observed are long-crested or two-dimensional (see Big, pp. 47-51). The swell, which is part of the primary structure, is the swiftest wave in a storm (Je-1). Two-dimensional waves also are believed to occur in the secondary structure. Thus those wavelets first formed by the gentlest wind that can ruffle the water at all are two-dimensional, according to Jeffreys' theory (see Sect. 11 and Je-1). Both the fastest and slowest waves, relative to the water, that can be generated by a given (greater) wind are two-dimensional; the three-dimensional or short-crested waves that are formed by this greater wind will then be travelling with intermediate velocities.

If the waves are observed, however, in a region where the wind has been blowing vigorously for some time, as in a storm area (Fu, p. 194), the sea will contain waves that are short-crested. Johnson and Rice (Jo-1) state that it is probably short-crested waves that are observed in naturally occurring bodies of water. Pierson (Pi-2, Sect. 11) implies that the surface structure generated by the wind at sea is always short-crested. In windy weather L' is seldom more than a few times as large as L (Big, p. 66), (we are referring now to primary structure), and in stormy weather (L'/L) ranges from one (Je-1) to five (Big, p. 46). A

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*Even small currents may affect significantly the height, length velocity and slope distribution of waves on the sea (Big, p. 53). Ocean currents (Big, p. 56) do not "quiet" wave trains crossing them as do tidal currents.

sea with (L'/L) = 1 is also formed, in first approximation, by two wave trains of the same amplitude and wave length that cross at an angle of nearly 90°. The smaller the angle is between the directions of propagation of the two trains, the longer are the crests. The crossing of two trains of storm waves, can generate a "confused" sea containing dangerously high pyramidal waves.

On the basis of this evidence from the literature we shall assume that it is rarely that pure, two-dimensional or long-crested waves are found alone, but that it is common though not universal for long-crested swells to make an easily visible contribution to the structure of the sea surface.

Little can be said about the profile of individual waves at sea, although it is generally believed that the down-wind faces of a medium length primary is steeper than its up-wind face.

3.2 Variability and Irregularity. The sea surface generally is random, complex and variable rather than appearing to consist of regular trains of the monochromatic waves described in Sect. 1. Randomness, like short-crestedness, is characteristic of wind-generated waves. The finer properties even of well-defined swells also show large fluctuations. It is clear that such random configurations must be described statistically, except for some of the smallest waves, which show some regularity (Sect. 8 and Mu-1, Hic-2). The statistics required for description of the primary structure can in many instances be taken to be quasi-steady and controlled by the mean wind. This has been shown by observation and can also be made plausible by the following argument concerning the factors that can conceivably influence the primaries. The drag of the air is insufficient to change rapidly the motion of the primary waves; likewise its variability affects only smaller waves. Furthermore, the damping by the viscous action of the water is small. Thus the rate of variation of the time average properties owing to these factors is small, which is equivalent to a quasi-steady statistical situation.

The idea that the primary structure is a realization of a quasi-steady random process has been extensively exploited, in modern oceanographic theory as we shall outline in the next section. The theory assumes steady wind and makes use of linear superposition of sinusoidal gravity waves. In applying this theory it is important to keep in mind the question: what part of the irregular surface structure and its time variation is not representable as a quasi-steady random process? As background to this question, remarks on two topics are pertinent. The first topic concerns the variability with time and position, of the wind. For example, a freshening wind is always gusty, even in the trade wind belts where the wind is most constant (Big, p. 51).* The subsidiary question which was dodged in the plausibility argument given above thus arises as to how rapidly does the surface structure of the sea adjust itself to the variable wind. It is only the secondary structure that generally is immediately sensitive to departures in magnitude and direction of the wind from its mean values. However, in extreme instances, the primary structure can be seen to reflect the variability of the wind (cf. discussion of wind variability _vs._ wave height in Sect. 6). Thus a severe squall lasting for a few minutes may be accompanied by much higher waves than are observed immediately before or after the squall

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* A recent paper (Hay-1) quotes measurements of the lateral and vertical components of gustiness at a site on the water 800 m. downwind from land.

(Cor, pp. 8-11). Although this group of high waves passing an observer appear to be a transitory phenomenon, the waves have presumably been built up rather slowly by the movement of the squall over a considerable distance. A second extreme case is that of a "confused sea" where the variability of wind direction can be the direct cause of the high pyramidal peaks observed*. This appears to be a second instance in which it is not possible to describe the situation by a mean wind.

The second topic concerns the (Eulerian) linear superposition (cf. App. D) usually employed in the theory. The shape (at a given instant) of <u>any</u> rough surface can be represented by such a superposition. If, however, there are non-linear interactions affecting the wave system, the change of its shape with time cannot, at present, be derived from the usual linear superposition nor from the Eulerian dynamical equations. Non-linear effects may include, the interconversion of long-and short-crested waves**, rotationality, turbulent dissipation, and the breaking of the waves. The mechanism of breaking is fundamental in all but the lowest sea states, for it limits the height which the secondary structure can attain. In heavy cross chop or in very heavy seas, breaking also limits the height of the primary waves. (See Sect. 9 for further discussion of breakers at sea.)

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* It has been said that the typical storm condition in the North Atlantic is one of constantly changing wind force and direction (St.D, p. 337).

** <u>Apparent</u> transformation of short- into long-crested waves of a group spreading from a source can occur even in the absence of wind and of non-linear effects, if the spectrum of the source has well-defined maxima in two or more directions (Fu, Sect. 7).

4. Outline of Statistical Description

Observed properties of the sea surface can usually be described in terms of multivariate Gaussian probability distribution functions (Munk)*. It is therefore natural, at least in hindsight, to try to represent the random surface elevation as a Gaussian stochastic process in three dimensions, the two space coordinates x and y and the time coordinate t. Pierson (see Pi-2, p. 136 for a review of the history of these developments) achieved such a representation by summing elementary long-crested gravity waves, each of which is characterized by the five parameters displayed in Eq. (1-1). The phases $\epsilon(\omega,\theta)$ of the component waves are assumed to be random with a rectangular probability distribution. In the limit, this sum converges to the integral representation** of an ensemble of surfaces,

$$\eta(x,y,t) = \int_0^\infty \int_{-\pi}^{\pi} \cos\left[(\omega^2/g)(x\cos\theta + y\sin\theta) - \omega t + \epsilon(\omega,\theta)\right] \cdot \sqrt{\overline{A(\omega,\theta)^2}\, d\omega\, d\theta} \qquad (4\text{-}1)$$

in which $\overline{A(\omega,\theta)^2}$ is the energy spectrum (see Sect. 5). The expression under the radical in Eq. (4-1) is the energy*** in the

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* Private communication. The calculation of shadowing by the rough sea surface probably requires more detailed statistical information however (Hic-3).

** For an explanation of this convenient but rather unusual representation the reader is referred to Pierson (Pi-2, Sects. 5 and 6) and Tukey (St.D, p. 350). A more conventional Fourier representation of a random process is used by Cox and Munk (Cox) and was examined earlier by Birkhoff and Kotik (Bir).

*** The total wave energy is usually represented by

$$E = \int_{-\pi}^{\pi} \int_0^\infty \overline{A(\omega,\theta)^2}\, d\omega\, d\theta \qquad (4\text{-}2)$$

which has the dimensions of area. The total wave energy, kinetic and potential is equal to ($\rho g E/2$) per unit area of surface for gravity waves.

band $d\omega d\theta$ of frequency and azimuthal angle. For a given continuous distribution of phases $\epsilon(\omega, \theta)$ over the (ω, θ) space, $\eta(x,y,t)$ is one possible realization of the random process representing the ocean surface. Pierson shows that the ensemble of these functions η represents a stationary, multivariate Gaussian process.

There is considerable indirect evidence that Eq. (4-1) is a very good first approximation to the nature of the sea surface for steady (or properly averaged) winds. In fact no observations of gravity waves have given results in serious disagreement with the representation, except where non-linear effects were thought to be present. Many reliable statistics of the random process can therefore be computed, some of which we shall list later.

The method does have its limitations however. It applies only when the sea surface is in a quasi-steady statistical state, a requirement that is difficult to formulate explicitly although it is possibly very often satisfied. Even when the surface is quasi-steady, the method gives a complete statistical description of the sea surface only when that surface is a realization of a multivariate Gaussian process, a surface characteristic which has not yet been verified directly. When the surface is not a realization of a Gaussian process, which has been known to occur*, then all of the statistical properties of the surface cannot be derived from the spectrum. (See Law, Ch. 3 for a brief discussion of this question.) The representation given by Eq. (4-1) also is linear in the component waves and cannot therefore predict changes of wave shape** or exchange

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* Probability distribution functions for non-Gaussian or otherwise "irregular" waves have been derived from experiment (Pu-1, 2).

**Tukey feels (St.D, p. 351) that representation of (Eulerian) non-linear random processes will in some useful sense become possible in the future. One possibility is that a non-Gaussian wave system will be described as the output of a non-linear mechanism whose input is a Gaussian process. If such a description is appropriate, identification of a non-linear mechanism with a real physical mechanism is clearly an important problem.

of energy by various mechanisms among component waves of different frequency and direction, owing to non-linear effects. Since the representation reproduces at best only the kinematics of the sea surface and the underlying water, it gives no physical picture of the exchange of energy among component waves of different frequencies, of the changes of wave energy accompanying viscous dissipation in the water, nor of the nature of the aerodynamic interaction between wind and water. These dynamical factors must however be of such a nature that they result in the observed energy spectrum.

The secondary structure is believed to play a role in aerodynamic interaction, but the nature of this role, for many reasons, cannot be derived from Eq. (4-1). (See Sect. 12.4.) In the first place the component waves in the representation by Eq. (4-1) are gravity waves whose phase velocity is always greater than their group velocity. The shorter secondary waves are, however, capillary-gravity waves whose phase velocity may be either less than or greater than their group velocity, depending on the frequency in question. Also, since the secondary structure can show regularities* (see Sect. 8), which are never exhibited by the primary structure, there is accordingly some doubt whether any purely statistical representation of the secondary structure can be adequate.

In the second place, there is little reason to believe that the spectrum used in the stochastic representation of the primary waves should, when extrapolated to high frequencies, give the spectrum of the secondary waves. One partial test of the validity

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* Note that the regularities of secondary structure occur for the same wave lengths for which capillarity strongly affects the water waves.

of this extrapolation is the comparison by Cox and Munk (Cox) of mean square slopes computed from the various wave spectra that have been proposed with those measured in glitter experiments. The Neumann spectrum (see Sect. 5) gives better agreement than other suggested spectra but only for waves damped by slicks. A second test of the extrapolation is provided by attempting to calculate the average distance between maxima on a Gaussian sea surface described by the Neumann spectrum.* As noted by Pierson (Pi-2, p. 152) this computed length is equal to zero, indicating that the secondaries are not properly represented by the Neumann spectrum. Finally, the secondary waves are attenuated so rapidly compared to the primaries that it seems inadvisable to represent them in terms of undamped waves until such representation is shown to be feasible by detailed consideration of the generation and decay of the secondaries in a steady state situation. Owing to the rapid generation and decay of secondary waves and to unsteadiness of the wind, they can probably be considered to be in a steady state statistically only for short intervals of time compared to the duration of steadiness for primary waves.

5. Energy Spectra

A representation such as that of Eq. (4-1) gives us a convenient model of the primary structure of a random ocean surface from which we may derive many different properties of that model surface for comparison with experimental observations or for other purposes.

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* A formula has been derived (Ste) for the variance of the number of zeros of a random process. This formula has apparently not been applied to a Gaussian process having a Neumann spectrum.

The spectrum $\overline{A(\omega,\theta)^2}$ characterizes the model completely since the process is assumed to be Gaussian. It also characterizes the sea surface insofar as the model is valid. Some properties of the model depend only upon the moments of the spectrum, as we shall see in Sect. 6. Other properties require for their computation more complicated integrals over the spectral function.

Neumann derived (Ne-3) a spectrum for the primary waves from his stop-watch observations of waves at sea. Many additional observations seem to check his spectrum, although none of the checks are as direct as one could wish. Neumann's spectrum as modified by Pierson (Pi-2, Sect. 11) is of the following form:

$$\overline{A(\omega,\theta)^2} = \left(K'\omega^{-6}\right)\cos^2\theta \, \exp\left(-2C^2/u_\infty^2\right) \qquad |\theta| \leq \pi/2 \qquad (5\text{-}1)$$

$$= 0 \qquad |\theta| > \pi/2 \qquad (5\text{-}2)$$

in which the wave velocity $C = g/\omega$, u_∞ is the wind speed and K' is a constant whose experimental value is $3.05 \ m^2 sec^{-5}$. The frequency dependent part of $\overline{A(\omega,\theta)^2}$ is proportional to the total energy in each frequency band and is plotted in Fig. 3 for several wind speeds. The area under any one curve is the wave energy E for that wind speed, as given by Eq. (6-9).

Pierson's modification amounted to suggesting the azimuthal dependence given in Eq. (5-1) corresponding to a 180° beam width. Glitter experiments (Cox) suggest a smaller beam width than 180° in the absence of slicks. On the whole it is too early to say with certainty just what the azimuthal variation should be in any given situation or even if this variation should be independent of the frequency. It is sure however that the beam width cannot be greater than the angle subtended by the storm area at the point of observation (Al). Marks (see Al; Mar; and Pi-2, p. 151) has made

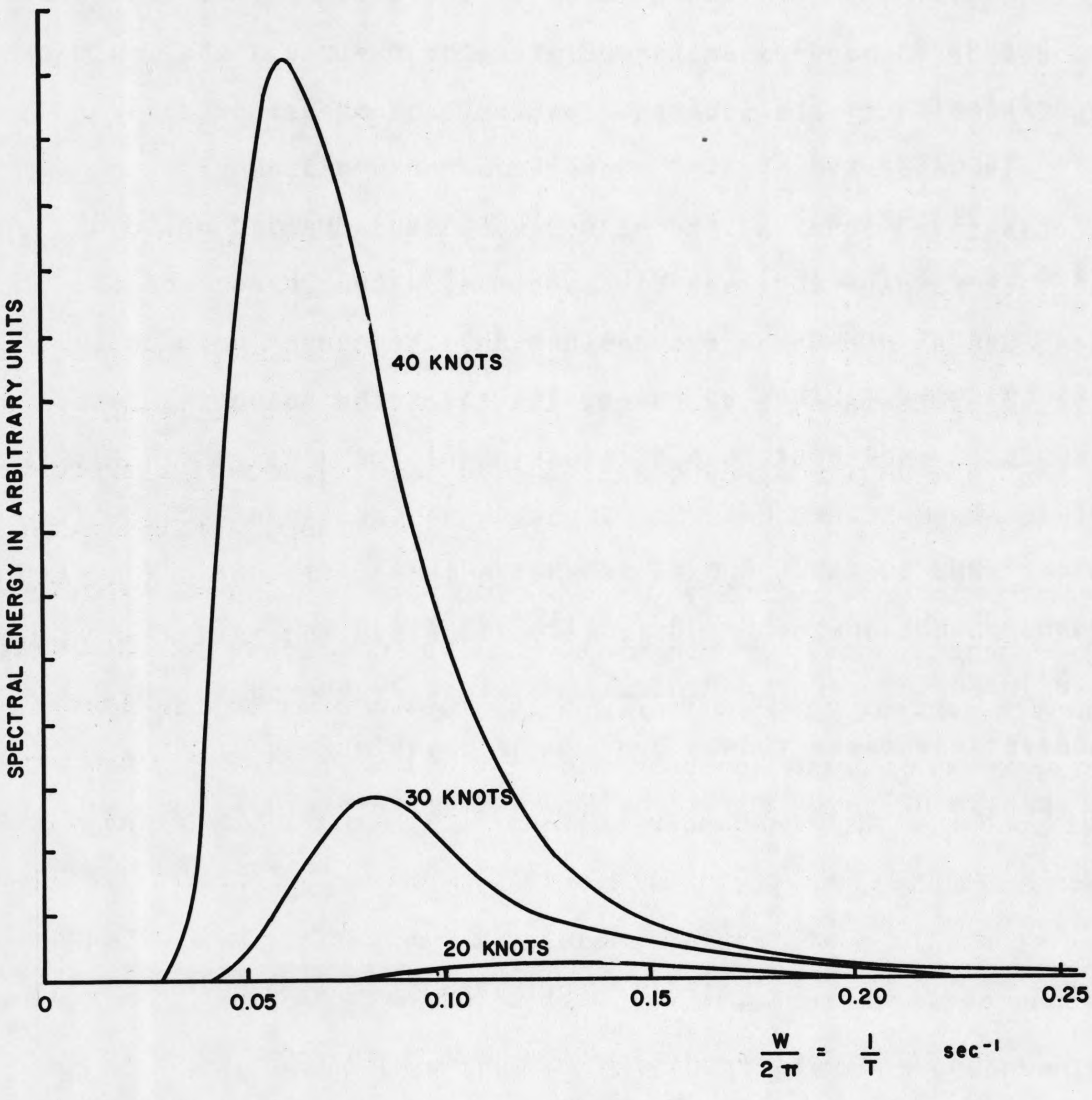

FIG. 3 ENERGY SPECTRA OF FULLY DEVELOPED SEAS

photogrammetric measurements from which the topography of the sea surface can be derived at contour intervals of three feet. Analysis of this data will yield the two-dimensional spectrum of the primaries, for the sea conditions observed, but will not give the sense of the wave propagation vector, which must be derived from other information. We note that several oceanographic laboratories have recently built electronic analyzers which can extract quickly the maximum amount of reliable spectral information from a wave record of finite length. (See Pi-2, Sect. 13.)

Only part of this primary spectrum is developed, for a given wind speed, if either the fetch or the duration is too small. (See Sect. 7.) The spectrum probably builds up from the high frequency end, that is, the secondaries are developed first. Unfortunately no detailed energy spectra of secondary waves have been reported in the literature.* We have noted in the previous section that extrapolation of primary spectra to give secondary spectra is inadequate for several reasons. In general, the Neumann spectrum should not be trusted for wave lengths less than about several meters (A1). However it is possible to discuss some aspects of secondary structure (see Sects. 8, 9 and Part III).

6. Some Calculable Statistics of Waves

Specific statistics of the waves can be calculated by two complementary methods. In the first method, calculations are based on the probability distribution function (p.d.) for the envelope of a stochastic process $\eta(t)$, and the details of the spectrum need not be known. The process must however satisfy two conditions. In the

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* Appendix C describes current CSL work in this area.

second method, all calculations are based ultimately on the spectrum function for $\eta(x, y, t)$, but the function is required to satisfy only general, non-restrictive conditions.

The first method applies with accuracy when 1) the spectrum A^2 is narrow and 2) the component waves making up the process are randomly phased. In this situation, the envelope $\xi(t)$ of the process $\eta(t)$ has the probability distribution

$$F(\xi) = (2\xi/E)\exp(-\xi^2/E) \quad (6\text{-}1)$$

Because A^2 is narrow, ξ varies slowly and may be taken to represent the amplitude A of successive waves. Then $F(\xi)$ is also the probability distribution function of the wave amplitudes. From this p.d. Longuet-Higgins derived a number of useful statistics (Lon-1). He considered for example a large number N of waves and a group of these, λN in number, each of which were higher than any of the remaining $(1-\lambda)N$ waves. He derived then the average amplitude $a^{(\lambda)}$ of the λ-highest waves and related $a^{(\lambda)}$ to the energy E of the waves.

We quote the results for $\lambda = 1$, giving the average wave height and for $\lambda = 1/3$, giving the "significant wave height".

$$\overline{H} = 2a^{(1)} = 1.77E^{1/2} \quad (6\text{-}2)$$

$$\overline{H}_{1/3} = 2a^{(1/3)} = 2.83E^{1/2} \quad (6\text{-}3)$$

He also was able to calculate the maximum wave amplitude $\mathcal{E}(a_{max})$ to be expected, on the average, for N waves where now N need not be large. The approximate equations are:

$$(1/\overline{a})\,\mathcal{E}(a_{max}) = \begin{cases} (\pi^{1/2}/2)N\left[1-2^{-3/2}(N-1)+\ldots\right], \text{ N small or moderate} & (6\text{-}4) \\ (\log N)^{1/2} + O(\log N)^{-1/2}, \text{ N very large} & (6\text{-}5) \end{cases}$$

Similarly, the most probable maximum (modal) wave amplitude $\Upsilon(a_{max})$

in N waves was found to be

$$(1/\bar{a})\Upsilon(a_{max}) = (\log N)^{1/2} + O(\log N)^{-3/2} (N \text{ large}) \qquad (6\text{-}6)$$

The assumption of a narrow band spectrum does not apply where there are two distinct frequency bands in the spectrum. It should apply with some accuracy though to (primary) storm waves at such a distance from a single generating area that the filtering action of dispersion has narrowed the spectrum compared to that which existed within the generating area. In the case of the secondary structure, the waves may never propagate far enough from the generating area without attenuation for this filtering action to narrow the spectrum and permit application of the above results. Experiments are needed to resolve this question.

Longuet-Higgins concludes from comparisons of his theoretical results with observations of primary waves, that the theoretical relations may, for all practical purposes, be assumed to be accurate. Perhaps this is not surprising if one remembers that the p.d. of the envelope of any reasonable process, even though its spectrum is not narrow, must have the general shape of the Rayleigh distribution $F(\xi)$. Longuet-Higgins also infers from the extreme improbability of waves very high compared to $\Upsilon(a_{max})$ that variation of the wind is more important in producing an observed <u>wide</u> variability of wave height than is statistical variation of the wave amplitude a.

In the second method of calculating specific statistics of the waves, use is made of the fact that differentiation or integration of $\eta(x, y, t)$ yields other stochastic processes whose spectra are simply related to that of η. Since these derived processes are likewise Gaussian we can characterize them by their variance

σ_{ζ}^2, where ζ stands for the variable of interest, such as slope, particle velocity etc. We shall now state the relations between these variances and the moments of the spectrum $\overline{A(\omega,\theta)^2}$.

Let us define the moment M_{ij} as

$$M_{ij} = 1/2 \int_{-\pi}^{\pi} \int_{0}^{\infty} \omega^{2i} \sin^{2j}\theta \, \overline{A(\omega,\theta)^2} \, d\omega \, d\theta \quad (6\text{-}7)$$

Then the total wave energy E is given by

$$M_{o,o} = E/2 = \sigma_{\eta}^2 \quad (6\text{-}8)$$

For the Neumann spectrum

$$E = 3(2^{-6})(\pi^3/2)^{1/2} K'(u_{\infty}/g)^5 \quad (6\text{-}9)$$

where (Pi-2, Sect. 10.3)

$$K' = 3.05 \times 10^4 \text{ cm}^2/\text{sec}^5 \quad (6\text{-}10)$$

From Eq. (6-8) then, a wave height statistic such as $\overline{H}_{1/3}$ or $\overline{H}_{1/10}$ will vary as the 5/2 power of the wind speed u_{∞}, a variation which agrees reasonably well with experiment.

Let U and V be the horizontal components of particle velocity parallel and perpendicular to the direction of wave propagation, and let η_x and η_y be the components of wave slope* in these two directions. Let W be the vertical component of particle velocity. Then it may be shown that

$$\sigma_U^2 = M_{1,0} - M_{1,1} \quad , \quad \sigma_V^2 = M_{1,1} \quad , \quad \sigma_W^2 = M_{1,0} \quad (6\text{-}11)$$

$$g^2\sigma_{\eta x}^2 = M_{2,0} - M_{2,1} \, , \quad g^2\sigma_{\eta y}^2 = M_{2,1} \quad (6\text{-}12)$$

If the Neumann spectrum (Sect. 5) is used, one finds (Cox; Pi-2) that $\sigma_{\eta x}^2$ and $\sigma_{\eta y}^2$ increase linearly with wind speed, as is also found to be the case by experiment (Cox, Scho-1; also see Sect.

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* The spectra of slopes and of particle accelerations are essentially identical for gravity waves (Cox). This relationship is made use of in Sect. 9.2.

9.2). The decrease predicted in $(\sigma_{\eta x}^2 + \sigma_{\eta y}^2) = \sigma^2$ because of the presence of a slick on the water surface is also in approximate agreement with experiment, although there is now added uncertainty in the theory because the mechanism of the effect of a slick is not clearly understood (Sect. 13 and App. B). Detailed quantitative agreement between experiment and theory based on the Neumann spectrum would not, in any case, be expected because this spectrum was derived for waves longer than 10 m, and shorter waves make considerable contribution to the slope spectrum.

The curvature distribution of wind generated waves has been measured in the laboratory (Scho-2) and could also be calculated from a spectrum that was known to be reliable at short wavelengths. Since these experimental results have not been verified and because there is no such reliable spectrum at present known, we shall not discuss this comparison in detail. We note however, that from the experiments there appeared to be a critical wind speed of about seven knots. For lower wind speeds, water bumps having appreciable curvature were not observed. Schooley has also made a preliminary investigation of the separation of reflecting "facets" on a wind-disturbed water surface (Scho-3).

Other statistics have been derived from the spectrum using zero-crossing theory developed by various workers (see references in Pi-2). For example, the average "period" of the waves* (the average time interval between zero up-crossings) is

$$\tilde{\bar{T}} = 2\pi (E/2 \cdot M_{1,0})^{1/2} \tag{6-13}$$

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* Pierson has also shown (Pi-2, Eq. 11.3) that the average wavelength $\tilde{\bar{L}}$ of a fully developed sea is given by

$$\tilde{\bar{L}} = (2/3)\, g\, \tilde{\bar{T}}^2 / 2\pi$$

and the average time between maxima is

$$\tilde{\bar{T}}_{max} = 2\pi(M_{1,0}/M_{2,0})^{1/2} \qquad (6\text{-}14)$$

For the Neumann spectrum these formulae give 73% more maxima than zero up-crossings.

If we define angular frequencies Ω and Ω_{max} by the equations

$$\Omega\ \tilde{\bar{T}} = \Omega_{max}\ \tilde{\bar{T}}_{max} = 2\pi \qquad (6\text{-}15)$$

then

$$\sigma_W^{\ 2} = (E\Omega^2)/2 \qquad (6\text{-}16)$$

and

$$g^2(\sigma_{\eta x}^{\ \ 2} + \sigma_{\eta y}^{\ \ 2}) = \sigma_W^{\ 2}\Omega_{max}^{\ \ 2} = (E\Omega^2\ \Omega_{max}^{\ \ 2})/2 \qquad (6\text{-}17)$$

There are thus seen to be connections between the statistics of, on the one hand, particle velocities and frequency of zero crossing and, on the other hand, particle accelerations, particle velocities and the frequency of maxima.

The spectrum $A^*(\gamma_o, \theta_D)$ of the waves, considered at a fixed time and as a function of distance along a direction at an angle θ_D to the wind, has been derived by Pierson (Pi-2, Sect. 6.4). The variable γ_o is related to ω, θ and θ_D, and the new spectrum, $A^*(\gamma_o, \theta_D)$ can be calculated from $A(\omega, \theta)$. It is possible to compute the frequency of maxima and of zero crossings in the direction θ_D.

Higher statistics that have been derived by Longuet-Higgins (Al; Lon-2) in terms of the moments of a two-dimensional wave-number spectrum include: the mathematical conditions for a spectrum to be narrow in angle, for two swells to cross, and for almost monochromatic waves; the frequency of zeros, crests, inflection points, humps, hollows, saddle points, and twinkles; and the velocities of zeros, crests, contours, and specular points.

Wooding and Barber (see Pi-2, Sect. 9.2) have shown how to express the p.d. of the amplitudes of the horizontal tangents to a wave record in terms of certain integrals of the spectrum. Their p.d. reduces, as it should, to Eq. 6-1 (reflected about the origin) for narrow spectra and to a normal distribution with variance E/2 when the spectrum contains important high frequency components.

7. Prediction of Primary Structure

A successful method of predicting sea and swell must describe the overall effect* of several important dynamical factors--the wind-wave interaction, the dispersion of water waves, and their dissipation by viscous action of the water--each of which affects differently the component waves in different parts of the spectrum. All three of these factors introduce considerable complexity into prediction methods, and understanding of the first and third is far from complete. (See Part III.) In spite of these handicaps, oceanographers have been able to evolve practical prediction methods for primary waves.

In outline this is their approach. Sea and swell are considered separately, their effects being added later as necessary. A storm area generating a "sea" is characterized at any point by an average wind speed u_{∞}, a fetch F (distance over which the average wind has blown), and a duration D (time during which the average wind has blown). These three parameters must generally be derived from meteorological predictions or observation.

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* The dynamical details of wind-wave interaction are not described by models used in present prediction methods which must thus be regarded as essentially empirical.

In the pioneer work of Sverdrup and Munk (Sv) the state of the sea is represented by a single statistic, the height $\overline{H}_{1/3}$ of the significant waves. In more recent work (PNJ) the sea state is described by the energy spectrum (Sect. 5). In general, the larger the speed, fetch, or the duration of the wind, the heavier the sea state and the larger is its total energy. Either limited fetch or limited duration may limit the development of the sea and this limitation is reflected in the value of $\overline{H}_{1/3}$ or in the nature of the PNJ spectrum. The "fully developed sea" characteristic of a given wind speed is produced for finite fetch and duration, and possesses a wave energy E given by Eqs. (4-2) and(6-9).

At some distance from the generating area swells appear, that is, waves having a relatively narrow spectrum because of their selective propagation from the storm area. Sverdrup and Munk describe the swell at a given time and position again in terms of significant waves and their characteristic period and group velocity. The more recent approach of PNJ is to "filter" the spectrum of waves in the storm area so that only those waves with proper group velocity can arrive from the generating area at the point of observation and at the time of observation. Pierson notes (Pi-2, Sect. 10.4) that "no appreciable amount of energy associated with components with group velocities greater than or equal to the wind velocity is present in a fully developed sea."

The longer swells tend to predominate as the distance from the storm area increases. This so-called "period increase of ocean swell" (cf. Pi-2, Sect. 16.5, for example) has been attributed both to the higher group velocity of the longer waves and also to their lower rate of energy dissipation, relative to the shorter waves, by backwards drag in windless areas and by viscous dissipation in the water. The relative importance of these possible causes of "period increase" and, in fact, the reality of this phenomenon (see Ec, p. 1494), cannot be determined until accurate measurements have been made of the spectra in storm areas, of the swells they produce, and possibly also not before we have reached a much better understanding of the mechanisms of dissipation of wave energy by wind and water.

The accuracy of either of the two prediction* methods depends a great deal upon the quality and quantity of meteorological data available, upon the availability of a few check points for comparing with the predictions, and upon the skill and experience of the man making the prediction. It also depends upon the fluctuation of the weather and sea away from the idealized conditions assumed in prediction, and possibly also upon unrealized defects of the prediction methods which cause errors even though the idealized conditions assumed by the models are satisfied. It is thus not often practicable to estimate the uncertainty of a given prediction, nor is it yet possible to say which method of prediction is more reliable. It appears however that the PNJ method, though more complex, is capable of providing more information than is the Sverdrup-Munk method.

8. Description of Secondary Structure

Except for Roll's informative study (Rol-2) which has been discussed by Ursell (Ur-2), few experimental studies of secondaries have been made. Thus accurate wave records of the secondary

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* A prediction may be used to give either forecasts or hindcasts, each being useful in appropriate circumstances.

structure, even in the absence of primary structure, have not yet been reported in the literature.* However, Neumann (Ne-2, Ch. II, Sect. 1) has given a useful discussion of qualitative observations of secondaries. The glitter experiments of Cox and Munk (Cox) show that the slope distribution for water surfaces uncontaminated by slicks is skewed. The skewing may be interpreted as the effect of some unknown nonlinear action that is dependent upon wind speed. The disappearance of the skewness when slicks are present is due to the elimination of the secondaries by the slicks but does not necessarily imply that the slope distribution of the primaries on clean water is not skewed, because the action of the wind in steepening primaries may be affected by the presence or absence of the secondaries.

There is a corresponding lack of quantitative theoretical information. Thus there is no theoretical description of the secondaries themselves, such as is provided for primaries by the Pierson-Neumann statistical description as represented by the equations in Sections 4 and 5. We have already mentioned (Sect. 4) why Eq. (4-1) cannot generally be extrapolated to the high frequencies corresponding to secondaries, and indeed why even its form may not be adequate to describe secondaries. There also is no complete theoretical description of the secondaries' important role in wind-wave interaction.**

We can say something about the kinematics of skewed and steepened capillary--gravity waves, although the physical origin of the steepening will remain obscure until the aerodynamic forces

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* See App. C for a description of current CSL work.

** See Part III, however, for a treatment of this subject.

can be specified. A small amplitude surface wave described by the equation

$$\eta = a \cos m(x - Ct) + b \sin 2m(x - Ct) \qquad (8\text{-}1)$$

is steepened on the downwind faces, downwind implying in the direction of propagation. Such a wave propagates without change of form (in the absence of viscous damping) if $m = m_1 = 2^{-1/2} m_m$ or $L = 2^{1/2} L_m = 2.4$ cm; for the waves with $m = m_1$ and $m = 2m_1$ propagate with the same velocity, $C_1 = 1.03\, C_m = 24$ cm/sec. (See App. B.) The waves travelling with the velocity C_1 can be more asymmetric than any other waves, ignoring nonlinear effects, the largest negative slope being greater by a factor of $(1 + 2\, b/a)$ than for a wave described by the first term alone. Fig. 4 showed a comparison between such an asymmetric wave with $b/a = 1/8$ and a symmetric wave of equal length and amplitude. The asymmetry produced by this value of b/a is rather small. For larger values of b/a the asymmetry is increased and new inflection points appear on the wave. No new extrema appear until $b/a > 1/2$. Since the group velocity exhibits a minimum, a group of capillary-gravity waves can likewise exhibit asymmetries. (See Je-3, Sect. 17.09; Lam, Sects. 269-272.)

Even though we cannot yet describe the detailed behavior of secondaries it is certain that this behavior is quantitatively different from that of the primaries. Secondaries do at times exhibit regularity of structure, at least for wave lengths less than 10 cm, (see Fig. 5) whereas primaries do not exhibit regularity of this kind. (Hic-2; Mu-1.) Less regular capillary patterns are often observed at higher wind speeds (Fig. 6). The development of secondaries by the wind is usually limited by their breaking*;

* We may postulate that secondaries in the absence of primaries show no white water when they break (cf. Sect. 9).

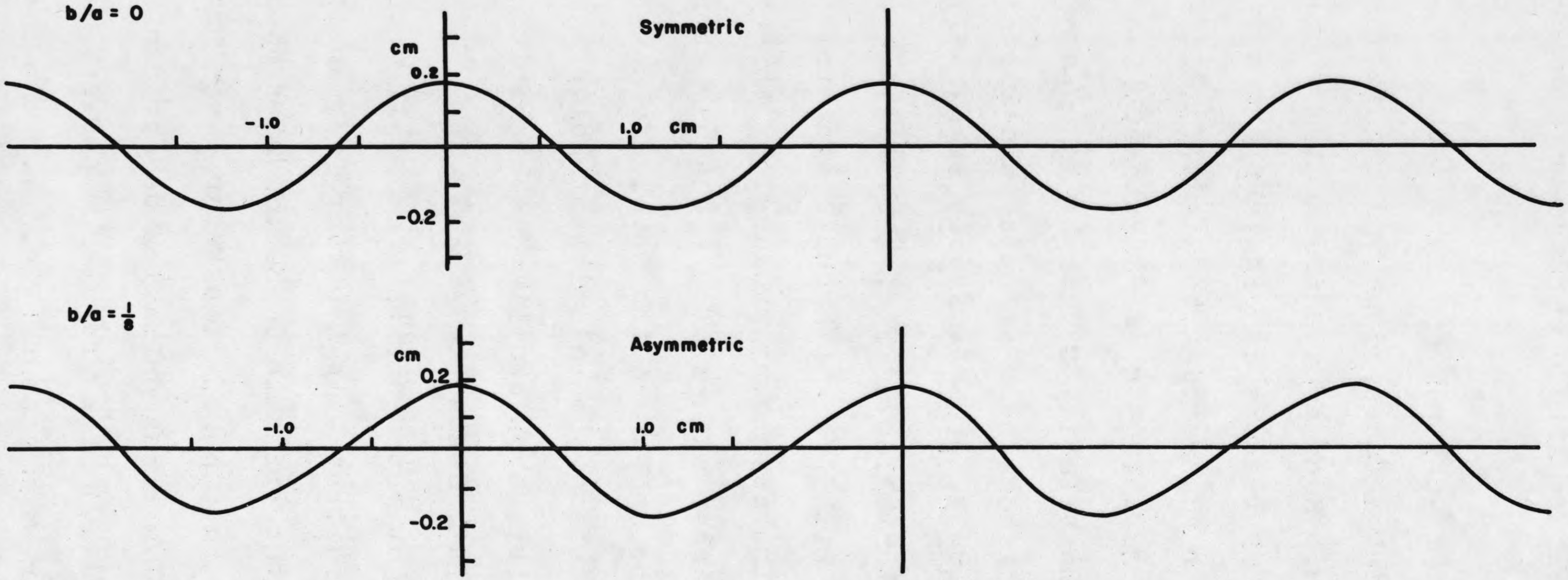

FIG. 4 ASYMMETRIC WAVE OF PERMANENT FORM AND SYMMETRIC WAVE OF SAME LENGTH AND HEIGHT.

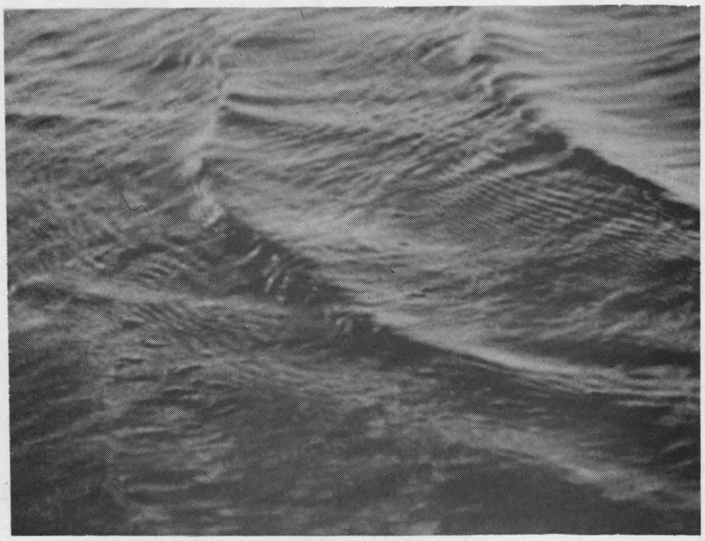

FIG. 5a LONG-CRESTED CAPILLARY-GRAVITY WAVES

FIG.5b SHORT-CRESTED CAPILLARY-GRAVITY WAVES

the development of primaries is limited usually by fetch or duration. Secondaries are so responsive to the local wind that one can say as a good first approximation that they are in equilibrium with it.* Primaries are responsive to the wind (averaged in suitable fashion) acting only over considerable distances and lengths of time and even then may not be in equilibrium with it. In particular, the mean propagation direction of secondaries follows the mean direction of the local wind, averaged say for one minute, whereas correlation of primaries with the direction of the local wind is wholly accidental. (See Big, p. 41.)

The implicit idea that the secondary structure depends only upon the mean local wind speed and not upon fetch or duration has perhaps been carried too far in the literature in the use of so-called critical wind speeds which are supposed to be associated with more or less well-defined changes in the nature of the secondary structure. For example, critical wind speeds have been given for the first "darkening" of a previously unruffled surface, for the appearance of the first appreciable waves of given length, and for change of the water surface from aerodynamically smooth to aerodynamically rough. A number of such critical speeds were summarized in an earlier report (Hic-1). A more careful analysis (Sect. 11) can now be made on the basis of relevant aerodynamic factors.

There is no method at present available for predicting the secondary structure because, as we have seen, there are no proven methods for describing it, and, furthermore, there is no information concerning the microstructure of the wind over the water. Even

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* It has been estimated that a fetch of about 100 yards is needed to produce secondaries one or two inches in length in equilibrium with the wind (A1).

FIG. 6 IRREGULAR SURFACE STRUCTURE PRODUCED BY GUSTS

over land the microstructure of the wind has seldom been measured and cannot now be predicted. One major difficulty is that gusts over land can cause as much as 100% variation of wind speed, even though the rms fluctuations are much smaller than this (Su, pp. 249-252). Such gusts can also occur over water (Hay-1) and will cause larger than average growth rates of secondary waves during the lifetime of the gusts, as well as great variation over the surface of its roughness (see Fig. 6). More generally, the secondary structure tends to follow any fluctuations of the wind, whether owing to large scale turbulence or gusts. The shorter secondary waves reproduce the fluctuations of the wind more faithfully than do the longer ones, for these shorter waves build up and decay more rapidly. In consequence there must be a kind of filtering action in the response of the secondaries to the wind forces.

9. Breakers and Whitecaps

9.1 Introduction. The energy of a breaking wave is transformed into energy of turbulent motion of the water and probably also into energy of waves of length different from that which broke. The energy of the primary structure for example is probably partly degraded to energy of short wave length components by energy exchange promoted by the breaking. The growth of wave amplitude for the small waves is certainly limited by breaking. Breaking waves thus play a definite role in the dynamics of wind waves and deserve some space in our consideration of this subject. We have accordingly tried to collect here the information about breakers at sea that appears to be potentially useful even though partly speculative. Because there has been no quantitative investigation in this area of oceanography, the literature yields only qualitative information that specifically concerns breakers at sea together with some quantitative information on related phenomena.

The breaking of waves on a sloping beach is one related phenomenon that is similar in some respects to the breaking of waves at sea, although there are substantial differences in detail. Any wave will break when it has assumed a state, described by its shape and kinematics, such that it is unstable. Its history during the process of breaking and at points not too far beneath the surface may be qualitatively the same no matter what has produced the unstable state. With implicit reservations we shall therefore quote in this section some of the characteristics of breakers on beaches, derived from experiment or theory, that may have counterparts in breakers at sea.

We would naturally like to appeal also to the classical (i.e., the purely hydrodynamical and non-statistical) theory of instability of permanent waves (see Sect. 1), but we shall see that a much more general theoretical formulation than this is needed. Before abandoning the classical picture altogether we remark that it can be used to suggest three informative determinations (either calculations or measurements). The first of these is that of the minimum length of permanent surface wave in a viscous medium which could break. Because the high frequency components which contribute the unstable peak to a non-sinusoidal permanent wave are damped so much faster than the fundamental, it is possible that, for a given fetch, duration and speed of wind that the steepness of a wave of too short a length cannot be built up enough to cause it to break. This minimum wave-length would also depend upon other factors such as turbulence and the presence of slicks on the surface, both of which increase the damping rate. We do not attempt to calculate this minimum length here even though the basis for it might be entirely hydrodynamical. The second informative determination is that of the maximum length wave that could break, for given fetch, duration and speed of the wind. Such a maximum length should exist because the steepness of the longer waves build up slowly, and also perhaps their high frequency components are dissipated directly by the breaking of short wave length waves. There would be little basis for an attempted calculation of this length at the present time. The third determination is that of the fraction of the sea surface, according to one model, that is unstable in a classical sense at any one time. This calculation will be given later in this section. Each of these three classical calculations would give an incomplete physical picture because wind forces are ignored.

9.2 Origin and Nature of Unstable Waves. Let us first consider the influences which may bring a region of the water surface to an unstable state in which it will break. If we observed, at a fixed point, the irregular water surface created by winds, we should see pass in succession a random arrangement of surface elevations, of slopes, of curvatures and of other geometrical characteristics of the surface. Even in the absence of local wind, the surface under observation would then exhibit breakers caused not directly by wind but merely by the occasional kinematic instability of the local surface underlying the random geometrical characteristics. A specific example of this behavior is that suggested by Unna (Un) and examined mathematically by Hibbs (Hib) in which the smaller waves (not necessarily part of the secondary structure) are destabilized near the crest of the larger waves on which they ride.

The presence of wind can conceivably be the direct cause of breakers at sea according to two mechanisms. The first mechanism probably predominates for high wind speeds and the second mechanism for low wind speeds. In the first mechanism, the wind exerts a shear on the tops of high waves. (This shear can be so large in heavy seas that the tops of the waves are blown off, causing large masses of water to be carried through the air (Big, p. 46).) Secondly, wind flowing in the direction of propagation of a wave acts to produce a breaker by progressively steepening the front face of the wave. More specifically, the pressure of the wind forces the backs of the crests ahead faster than the leeward sides are advancing until the crests come to overhang the troughs (Big, p. 110). A somewhat contradictory (?) view has been expressed implicitly (Ma-M-1) to the effect that wind opposing propagation of waves also tends to steepen them.

Although it is apparent that our understanding of the effect of wind upon wave shape is rather incomplete, it is useful to mention one "critical" wind speed, 13 knots (~ 6m/sec), with the further reservation that of course a value of the local wind speed will not alone fully determine the structure of the sea surface. At this wind speed whitecaps owing to breaking waves generally make their first appearance (Mu-3). Observation indicates that this threshold is rather sharp. These whitecaps must be associated in some way with the secondary structure* because a slick on the surface tends to inhibit whitecap formation and to destroy secondary structure while having no direct effect (and only indirectly, through the secondaries) upon the primary structure.

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*See Sect. 14 for discussion of the aerodynamic situation.

Currents, especially those running contrary to the direction of wave propagation can steepen waves and therefore increase their tendency to break*. For example, a current of speed 2.2 knots would cause waves five feet high, 100 feet long, and travelling with a speed of 13.4 knots to start breaking (Big, p. 54). The backwash in surf is an example of an opposing current in very shallow water. The intersection of a sea with swell from a distant storm may also cause breakers at sea (Cor, p. 42, footnote).

Generally, any influence which tends to retard the forward face of a wave relative to the back face is conducive to breaking. An obvious example is the breaking caused by a shoaling bottom, such as a beach. We remark that it is possible to derive theoretically the shapes of waves breaking on a beach of simple shape (cf. Bie). The characteristics of these theoretical breakers appear to be reasonable although they were not compared with experiment. It may be that similar theory could be developed for the upper parts of breaking waves at sea.

Until there is new experiment or theory it is not profitable to speculate further about the causes of unstable wave shapes at sea. In particular it may be difficult to distinguish for any real sea surface between the effects of the various direct and indirect causes of breakers that we have enumerated. Both observation and theory, however, can give us information about the nature of waves that are about to break or have broken. In the classical picture (cf. for example Lam, Art. 250; Dav-1), a "permanent" wave, that is, one which progresses without change of form, is unstable if the

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* A current probably must be present before wind waves can form on an initially calm surface (Sect. 11).

forward particle velocity at its crest becomes larger than the propagation velocity of the wave. Data on waves breaking on a sloping beach give qualitative support to this view. Thus in model experiments, Iversen (Iv) found that the ratio of the crest particle velocity at breaking to the velocity $V_d = (gd)^{1/2}$ ranged from 0.4 to 1.0. The latter velocity, with d equal to the height of the crest above the beach, is of course the wave velocity for a wave in shallow water of this depth.

Most waves of interest do not propagate without change of form and a criterion of kinematic instability is needed that is more general than the classical one. Perhaps the needed criterion can be found by comparing the particle velocity at a given point of the surface with the propagation velocity of a geometrical characteristic of the surface at that point, such as elevation, slope, or curvature. It is clear that any one such characteristic may have different propagation velocities for different values of that characteristic and at different points of the surface, and also that different characteristics need not have the same propagation velocity. The situation on the real sea surface is thus more complex than in the case of the single, progressive, permanent wave for which the propagation velocities of all such characteristics at all points are equal.

In discussing this proposed criterion further, let us assume that we are considering one specific geometrical characteristic, say the slope, or a kinematic characteristic such as a particle velocity component. (We do not know now which characteristic or combination of characteristics is relevant to breaking.) The theory of Longuet-Higgins (Al; Lon-2) would permit the calculation of the average propagation velocity of the chosen characteristic from knowledge of the spectrum of the waves. Even though his theory assumes (Eulerian) linear superposition of the spectral components, it should give approximately the range of propagation velocities to be expected in the presence of some non-linearity of superposition. In particular, his theory could be extended to give the probability distribution of propagation velocities for points having a given horizontal component of particle velocity. From this probability distribution one could then determine the fraction of the sea surface area for which locally the particle velocity exceeds this propagation velocity. We note that the drag of the air directly influences the particle velocities of the water, an influence not yet treated theoretically.

The above considerations have to some extent been prompted by the experiments of Mason (Ma-M-1) who concluded that, for a wave breaking on a sloping beach, the breaking can occur when a vertical slope of the water surface has been produced, and by a similar observation which indicates that discontinuities of slope initiate breaking (cf. Sect. 9.3). While this first criterion may be a satisfactory upper stability limit, our preliminary observations would indicate that waves in deep water may break before a vertical slope is reached, or at least that a rolling break may occur that produces no white water. (Also see Sect. 14.)

We conclude this subsection with a calculation of a lower limit to the fraction of the surface covered by whitecaps. The basis for this calculation is simpler than that of the more general theory outlined above. Let us assume that any part of the surface is unstable, in the sense that it cannot remain unbroken*, if its downward vertical acceleration is greater than g. The fraction f_u of the surface with downward vertical acceleration greater than g is equal to the probability that this acceleration exceeds g or to the probability that the local slope η_s of the surface, in the direction of minimum slope, is less than -1, or that

$$\eta_s = -(\eta_x^2 + \eta_y^2)^{1/2} < -1 \qquad (9\text{-}1)$$

For simplicity we assume that the probability distribution of this slope is Gaussian with variance σ^2. This is known to be a good first approximation. (See Sect. 6 and Cox.) Then

$$f_u = (1/2)\ \mathrm{erfc}\ (2^{-1/2}\ \sigma^{-1}) \qquad (9\text{-}2)$$

$$= \left[\sigma/(2\pi)^{1/2}\right] \exp\ (-1/2\ \sigma^2) \qquad (9\text{-}3)$$

since $\sigma \ll 1$.

The variance σ^2 is a linear function of wind speed u_∞. According to the Neumann spectrum, the relation is (Cox)

$$\sigma^2 = 1.59\mathrm{x}10^{-3} u_{\infty 1} \qquad (9\text{-}4)$$

From observation (Cox) the relation is

$$\sigma^2 = 1.56\mathrm{x}10^{-3} u_{\infty 2} + 0.008 \qquad (9\text{-}5)$$

when a slick is present and

$$\sigma^2 = 5.12\mathrm{x}10^{-3} u_{\infty 3} + 0.003 \qquad (9\text{-}6)$$

when no slick is present. The wind speed here is in m/sec and the subscripts 1, 2, 3 on u_∞ refer to the three linear equations.

The fraction f_u is plotted in Fig. 7 as a function of σ and wind speed for the three cases. Several remarks may be made about these curves. The fraction should be interpreted as the fraction of

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* A different kind of stability criterion refers to the fraction of the surface whose slope is less than the classical limit of -tg 30° (see Sects. 1,2) and which cannot therefore be moving as part of an irrotational flow. This fraction is very much larger than the one we compute.

the surface that is unstable in the sense that it cannot remain unbroken. This fraction f_u is generally very small, especially in the presence of slicks which, in eliminating the secondaries, appear to reduce f_u by a factor of 10^4 or so. Once breaking has occurred, the unstable regions are presumably transformed into much larger regions of white water. It is clear that the ratio of whitecap to unstable area should be studied experimentally as a function of wind speed, sea state and the occurrence of slicks. The number $f_u^{1/2}$, which is of course much larger than f_u, can be interpreted as the average ratio of the diameter of the unstable areas to their separation.

We note that this calculation applies to seas of any beam width but obviously does not take into account many of the factors affecting breaking that we have discussed previously. It should be useful however as a first estimate* of a lower limit to the fraction of the sea surface that is unstable. The calculation has not given us a critical wind speed such as is observed at about 6 m/sec. corresponding to the first formation of whitecaps. Consideration of the contribution of the secondaries to instability suggest two mechanisms, however, that might lead to a critical wind speed. Thus we may postulate that for low wind speeds the longest secondaries that can break (cf. Sect. 9.1) rarely produce white water when they break because the two mechanisms of viscosity and surface tension both tend to blunt the crests of secondaries**. For a high enough wind speed, such as 6 m/sec. for example, the contribution to sharp and steep

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* A specific indication of the uncertain basis of this calculation is the fact that we assume that the slope has Gaussian distribution in a range of slopes <u>much</u> larger in absolute value than those measured experimentally.

**Note our comment elsewhere (Sect. 14) that small wind waves observed visually appear to break by a rolling action that does not produce white water.

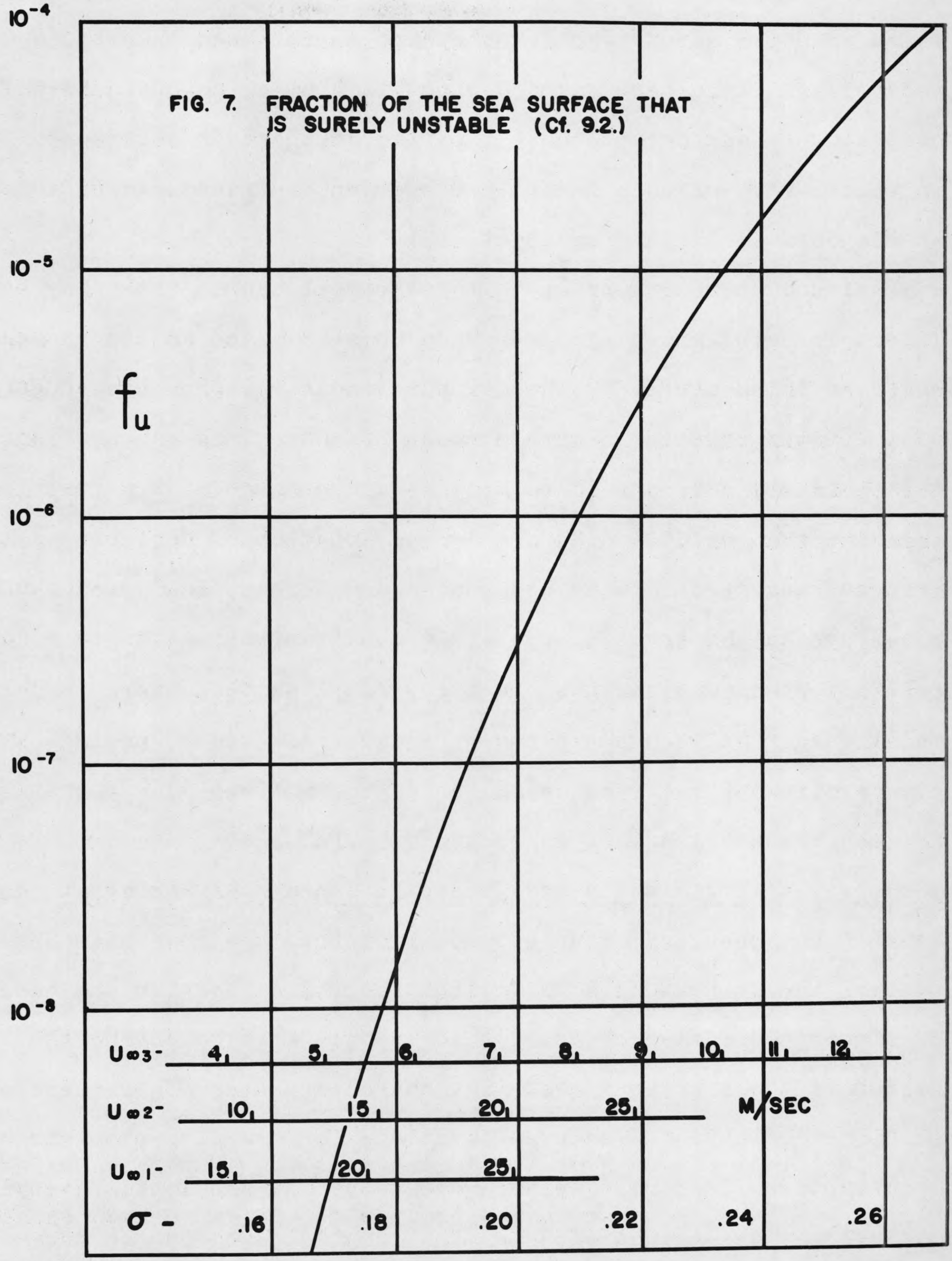

FIG. 7. FRACTION OF THE SEA SURFACE THAT IS SURELY UNSTABLE (Cf. 9.2.)

waves by wave crests by lower frequency components has perhaps increased enough so that these longer components produce white water when the waves of which they are a part become unstable and break. It is perhaps more probable that an aerodynamic effect, namely the shearing action of the wind on the crests suddenly increases at 6 m/sec owing to the emergence of secondaries through the laminar sub-layer (see Sect. 14).

Finally a remark relevant to practical oceanography may be made. For wind speeds greater than about 6 m/sec an experienced observer in an aircraft can estimate wind speed from the appearance of the water surface to within about 20-30%. This range of uncertainty in u_{∞} corresponds to a much larger range of the fraction f_u, namely about 500%. Such a large variation of unstable or whitecap areas should be noticeable and perhaps indicates why purely visual observation can yield qualitatively accurate wind speeds. We note also that the theory implies that there is a linear relationship between the two wind speeds required to produce any given value of the fraction f_u (or of the whitecap concentration d_o) in the presence and in the absence of slicks.

9.3 Breaking Waves and Their Products. Having spent some time on why breakers occur and how incipient breakers might be recognized, we next turn to qualitative classification and description of the breakers themselves. It is useful first to define the different types of breakers. The characteristics of breakers used in these definitions, such as duration, for example, probably depend upon factors like the complete wave spectrum, the local wave kinematics, and the amount of water that participates in the breaking action.

Quoting Iversen's description (Iv) for beach breakers, we may say that for "spilling" breakers "the crest became unstable in a mild fashion with the appearance of 'white-water' at the crest, which expanded down the front face of the breaker". In this case the appearance of the white water signaled the beginning of the wave's breaking. For "plunging" breakers, "the crest overshot the body of the wave to project ahead of the wave face".* In this second case, the appearance of a vertical slope signaled the commencement of breaking. Finally for "surging" breakers, "the front face of the wave became unstable over a major portion of the face in a large-scale turbulent fashion", and the appearance of turbulence signaled the beginning of breaking. Mason (Ma-M-1) considers also a general category that is intermediate between the cases of spilling and plunging and which would apparently include the surging case.

Among the various types of breakers formed on beaches, Mason (Ma-M-1) observed that it was always at the point of most abrupt change of slope of the surface that broken water first appeared at the beginning of the breaking process. We quote Mason's graphic description of the subsequent history of a breaker. "High-speed photography showed the broken water to be small masses or slugs or water probably mixed with entrapped air giving then a characteristic foamy appearance, that detach from the advancing face but are not projected or do not travel far from the face. In fact, they appear simply to be detached and roll over the advancing face. In a spilling breaker the separation of these masses appears to spread

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* This appears to be the type of breaker visualized in one discussion of their causes (Big, p. 110). Sometimes the front face of a wave before it breaks becomes very smooth except for small scale striations. We suspect that only plunging or spilling breakers may exhibit this behavior.

like a grass fire, over the advancing forward face without penetrating into the wave but rather forming a bulbous protuberance of the advancing face. In a plunging breaker the initial separation is of very short duration, being succeeded by the forward and downward projection of a tongue or jet of water from the crest of the forward face, the jet curling down much in the form of a free discharge jet to entrap the initial broken water and plunge into the backwash of the preceding wave."

It is profitable next to classify in detail the various products of breaking because of their varied behavior and importance in different connections. The classification can be made in terms of connectivity. (See Fig. 8.) Let any water mass that is connected with the main body of water be in class α and a mass that is surrounded by air and is not so connected be in class α'. The two classes are separated in space, for any given sea structure, by a more or less smooth imaginary surface S coinciding with what the eye would normally call the sea surface. Class α water is of course the relatively stable variety. Class α' water contains the water in the air, which we may briefly designate as spray. The spray falls to S from its maximum height h above S in a time not less than $(2h/g)^{1/2}$. We remark that spray can be blown off the crests of breaking waves, can be ejected from them, or can be caused by splash of other spray that falls back into the main body of water. Spray is of course readily observable visually as a product of waves breaking on a beach and is likewise noticeable in pictures of heavy breakers at sea.

We can also distinguish two classes of air masses. An air mass is in class β if it is surrounded by water and in class β' if not. Class α' water and class β' air generally lie above the surface S except when breaking of a bubble at the surface produces an air cavity and subsequent water jet (cf. Ma-B, Ki) or during the plunging break of a small wave. The inhomogeneity of the material below S might be described by giving the distribution of bubble sizes in it, that is, of regions of class β air. The small bubbles tend to last longer than the larger ones. Many bubbles will persist longer than does that part of the class α' water that was originally formed by the breaker. However, subsequent small subsidiary breakers and breaking bubbles, as well as splash from falling spray continue to produce class α' water for some time after the initiation of the breaker. The foam, which contains the smallest bubbles, has the largest lifetime and is among the products of breaking that are most easy to see or photograph. (Presumably the easily observable "whitecaps" or "white horses" consist of foam and spray.) The persistence of both bubbles and foam depend upon contamination of the water by organic impurities whose identity is not known (Bik, p. 179), upon the wind speed, and perhaps upon the degree of turbulence of the water.*

We are now in a position to make some guesses about the connection between these various breaker products and the dynamics of wind waves. The boundary layer of the air above the water surface forms breakers directly through its drag and indirectly through its effect upon wave shape.** (See Sect. 9.2.) The character of the air boundary layer also determines the distance that the spray

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* The subject of foams has been studied extensively because of its industrial importance. See for example the book by Bikerman (Bik).

**Normally we would not expect the air boundary layer to be affected by the presence of breakers because they cover but a small fraction of the sea surface.

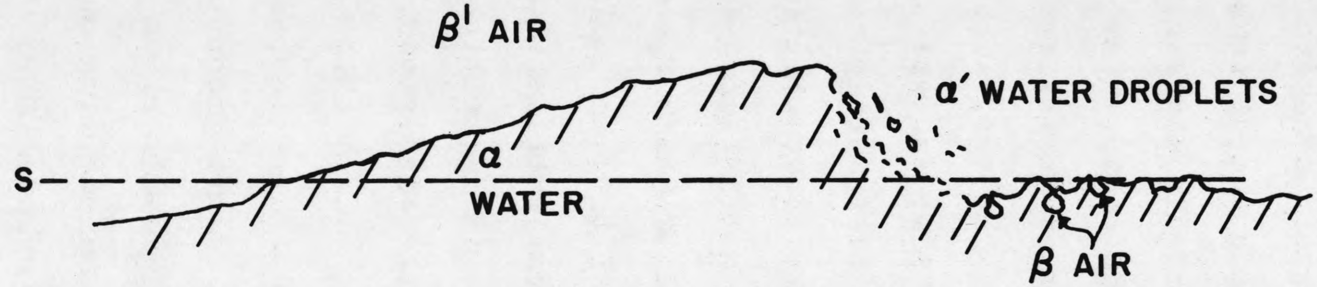

FIG. 8 CLASSIFICATION OF WHITECAP CONSTITUENTS

is transported through the air. (See Sect. 12.4) Variation of direction as well as speed of the wind should change the amount and transport of the spray. It is known (HO-2, p. 28) that more spray is formed by a wind opposing breakers on a beach than by a following wind. The transport of foam along the surface likewise is determined by the boundary layer conditions. In very stormy weather so much foam is blown along that the sea appears to consist of sheets of foam (Big, p. 46) but the long-lived white patches observed in calmer seas are "mere films" that are not deformed by the wind (Cor, p. 39).

The products of breaking waves affect the water boundary layer more than that of the air. A breaker projects water of high momentum forward over the water surface in the form of water rolling over the surface and of spray. The kinetic energy of more or less ordered motion resident in the wave before breaking is thus converted into considerable turbulent energy. The rotational motion that should arise from the early stages of the interaction of the broken water with the underlying water is probably rapidly dissipated into turbulent motion of large intensity and scale, if we assume that the results of Mason (Ma-M-1, p. 219) for breakers on a beach carry over in some measure to breakers at sea. The momentum of the wave that broke is also communicated to the underlying water together with whatever momentum is imparted to the spray in its formation and during its acceleration by the drag of the wind.

So far we have devoted attention to individual breakers. This procedure is appropriate because breaking is believed to be primarily a local phenomenon involving only limited areas of water (cf. Ma-M-1, p. 220), and the breakers may thus be considered to be independent of one another. If we wish now to describe breaker characteristics averaged over a large area of ocean surface, what are the most important parameters that we should specify?

Since the areas of breaking or broken water (whitecaps for short) are usually isolated from one another, an obviously important parameter is the number d_o of such whitecaps per unit area of sea surface. This parameter, like the other ones to be mentioned, should be studied as a function of wind speed. A second parameter is the fraction f_w of the sea surface covered by whitecaps. At the time of the Allerton Conference (A1), no one had measured this fraction but we have given a provisional theoretical estimate, in sub-section 9.2, of a related quantity, the fraction f_u of the surface that cannot be stable. To understand the origin of the breakers we should also know a factor $\psi(\omega)$ with which to correct the spectrum of particle acceleration predicted from linear theory. To understand the influence of the breakers on energy transformation we should know a factor $\Psi(\omega)d\omega$ which gives the rate of increase of energy for a narrow frequency band $\omega \rightarrow \omega + d\omega$ owing to transfer of energy to or from other frequency bands through the action of the breakers. For the low frequencies of primary waves, ψ would approach unity, corresponding to no distortion of the linear prediction of particle acceleration, and Ψ would approach zero corresponding to non-participation of the primaries in breaking processes. For frequencies larger than some intermediate frequency ω' dependent upon wind speed, non-linearities should cause $(\psi - 1)$ and Ψ to become appreciably different from zero.

Finally there should be specified the mass and velocity distribution of spray components. These distributions would be expected to be related to wind speed and wave spectra.

III. THE DYNAMICS OF WIND WAVES

10. Problems to be Discussed

The production and maintenance of wind waves involve hydrodynamic and aerodynamic phenomena characteristic of two unsteady, coupled boundary layers between two fluids of semi-infinite extent. The problem of constructing a general theory of these phenomena proves to be, at the present, insuperably difficult: even under simplified conditions retaining some degree of physical reality, analysis is complex, and its results may not possess a clear interpretive meaning. One may mention difficulties encountered in finding solutions to the Navier Stokes' equations which become very much more complicated on considering turbulent and separated flow at the boundary of the fluids; indeed the unsteady shape of the boundary itself, which is what one seeks in the theory, at once removes any hope for a general meaningful analysis. Despite the aforementioned considerable analytical difficulties, the theory of wind wave dynamics may be divided logically into three main topics, which yield to comparatively simple qualitative discussions and manageable, but simplified theoretical treatments.

To introduce this classified division, let us formulate the problem of wind wave generation as follows: given a plane undisturbed interface between air and water, what is the nature and history of the <u>initial</u> disturbances generated by a wind over this interface; what factors contribute to the generation and decay of monochromatic <u>finite</u> amplitude waves; further, what theoretical aspects of the hydrodynamics and aerodynamics of the wind and <u>complex</u> (random, polychromatic) wave structure (see Figs. 5a, 5b, 6 and 9) may be discussed? In attempting to answer these questions we shall deal with three main topics in the three sections of the report: the initiation of the disturbances (Sect. 11), the generation of finite amplitude waves (Sect. 12), and the growth and development of a complex wave structure on the interface (Sect. 14). A subordinate but relevant topic, the dissipation of wave energy, is discussed in Sect. 13.

In discussing the aerodynamic forces on the disturbed surface in Sect. 12, it is necessary to recognize an important aspect of the problem: the characteristics of the air boundary layer in relation to the wave shape and amplitude. As preparation for later discussions of the air boundary layer, we shall now discuss briefly some boundary layer concepts.

In the neighborhood of the surface of discontinuity of any two fluid media which are in bulk relative motion, there exist layers of retarded fluid which are the result of the viscous effects and velocity continuity across such a boundary (see Fig. 10). The aggregate of these retarded layers in either fluid is called the boundary layer, and its thickness is a measure of the region in which vorticity due to the shear between the retarded layers becomes appreciable (Schl, Chapter II). If no large scale turbulence is present in either fluid, these boundary layers are fairly thin: for example, a laminar air boundary layer on an aircraft wing moving at 600 m.p.h. is about one hundredth of an inch thick. However, in a turbulent boundary layer, the strong mixing action due to turbulence thickens the boundary layer considerably, and in the case of the wind blowing over a well-developed sea the thickness of the air boundary layer is many meters. The thickness δ of laminar and turbulent boundary layers may be expressed in terms of the velocity u_1 outside the boundary layer, the fetch x (see Sect. 7 for definition),

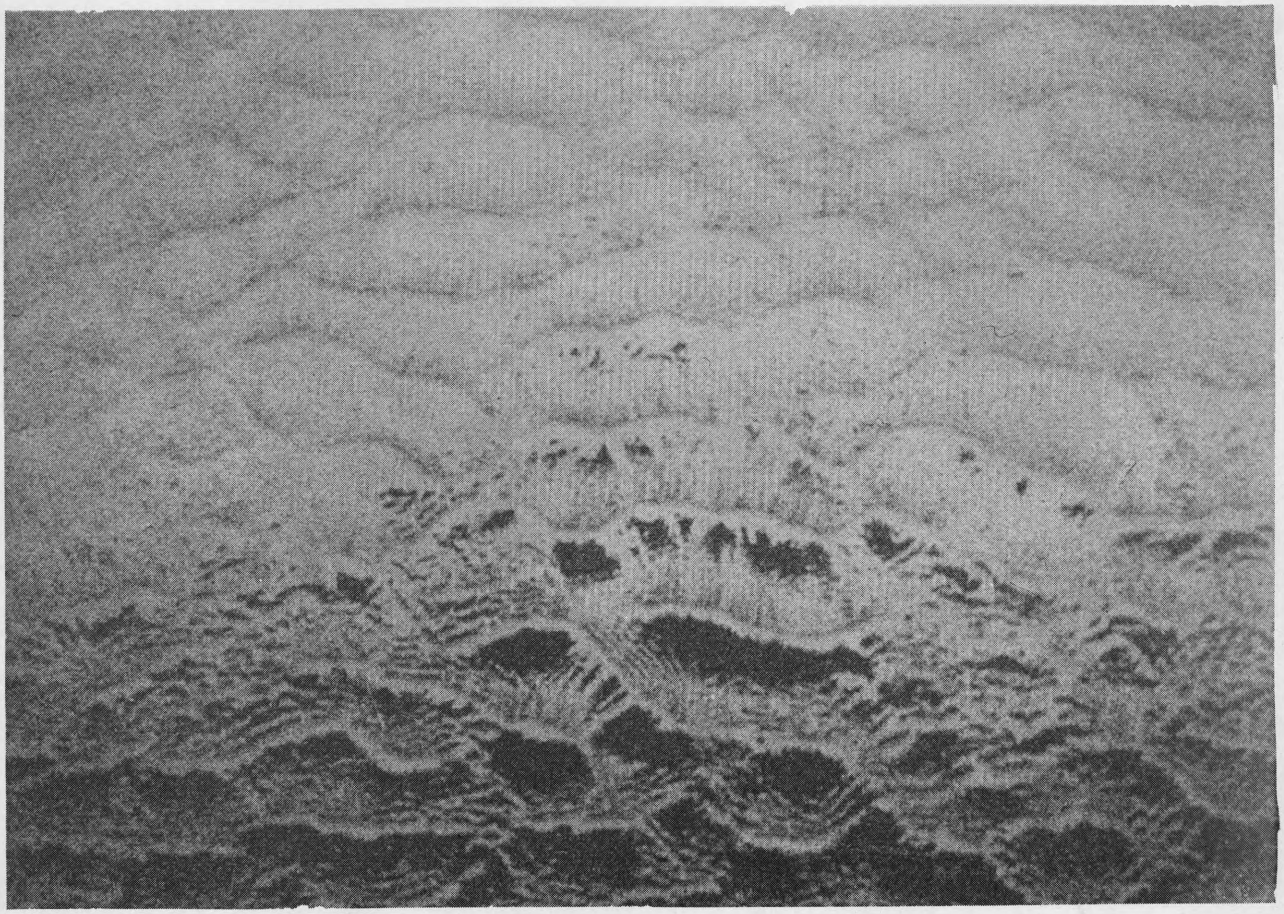

FIG.9 PHOTOGRAPH OF WATER NEAR BEGINNING OF FETCH

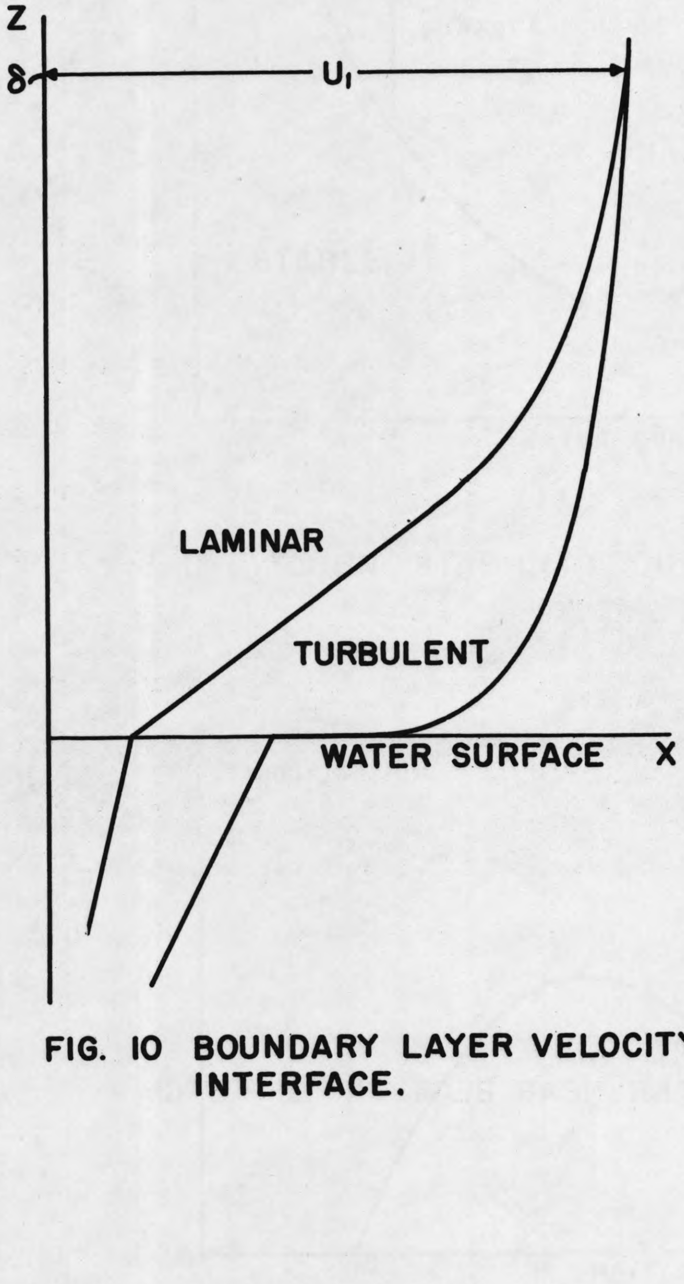

FIG. 10 BOUNDARY LAYER VELOCITY PROFILES NEAR AIR-WATER INTERFACE.

the kinematic viscosity ν, as follows (Schl, Chapter II):

$$\delta_{\text{laminar}} = 5\,(x\,\nu/u_1)^{1/2}$$

$$\delta_{\text{turbulent}} = 0.37\,(\nu/u_1)^{1/5}\,x^{4/5}$$ (hydraulically smooth flow, Sect. 12.4)

It is a sufficient approximation to assume potential flow outside the air boundary layer, but, in calculating forces on a body, the usefulness of such an assumption clearly depends upon the relative size of the body and the boundary layer thickness. As we shall see below, the wave height in predominantly all cases of natural wind wave generation is less than the boundary layer thickness; a knowledge of the boundary layer characteristics is therefore essential for a correct theoretical understanding of the wind wave interaction, and for a useful interpretation of experimental data relating wave statistics to wind velocities above the surface.

The effect of thermal instability on the air boundary layer is very important but is far from being understood and will therefore be discussed rather briefly in Sect. 12.4 and ignored elsewhere. The water boundary layer will not be discussed explicitly (in Lock's stability analysis, Sect. 11, it is important) since its effect on wave motion is not understood, and the use of the term boundary layer will thus in general refer to the air boundary layer only. The water boundary layer may have a significant effect on the shape and breaking of waves (cf. Sect. 9.2), particularly those of amplitudes of the same order as the water boundary layer thickness.

11. Initiation of Disturbances on a Calm Air-Water Interface

When the wind blows over a limited stretch of water, for example a pond or small lake, it is well known from observation that the very beginning of the fetch is characterized by a ruffled appearance in which it is not generally possible to distinguish regularities by the eye. A few meters downwind of this area, regular wave systems emerge which develop into the secondary* wave structure. The ruffled appearance marks the initiation of unstable small disturbances of the water surface whose nature and history determine the early characteristics of the secondary waves. Not only is it important to investigate these instabilities in order to understand the early development of the secondary structure from a calm surface, but also to understand the rapid generation of capillary-gravity waves in a well developed sea by strong gusts of wind (cf. Sect. 8, Sect. 12.2 and see Fig. 6). These patches may have a significant effect on the aerodynamics and hydrodynamics associated with the larger underriding waves. The stability analysis is also fundamental to a discussion of the so-called "critical wind speeds" required for the initial formation of water waves (Hic-1).

Comprehensive theoretical work is limited concerning the initiation and early development of the surface wave structure. Quantitative experimental measurements appear to be confined to laboratory investigations** (Fr-1, Keu-1,-2, Jo-1), and Roll's study in the open (Rol-2). In this section we shall outline what theory there is of unstable small disturbances of the water surface caused by wind, and make some brief comments on its relation to observations.

The initiation of waves on a calm air-water interface is fundamentally a problem in the stability of two coupled boundary layers. If a small disturbance of the interface from its equilibrium position

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* For the definition of secondary waves, see Sect. 3.1.

** An experimental investigation on a small lake is being conducted by the authors. See Appendix C for details.

is resisted, then no waves form. If the amplitude of the disturbance increases with time, i.e., the motion is unstable, then waves will develop on the surface.

In the neighborhood of a calm water surface the air usually possesses a turbulent boundary layer due to its passage over land before reaching the water, or, more generally, due to its earlier passage over a disturbed area of water; the characteristics of this boundary layer naturally depend upon its earlier history and its thickness may vary from a few centimeters to several meters (cf. Sect. 12.4). It is reasonable, for a calm water surface, to assume that there is a laminar boundary layer in the water, but in the case of ripple generation in well-developed seas by wind gusts, the layer will be turbulent. If one assumes that the Navier-Stokes' equations can describe turbulence, then the most complete formal approach requires a stability analysis of the solution of these equations with the appropriate boundary conditions. The nature of the equations does not now permit this. Furthermore, the requisite empirical guidance for this particular turbulent boundary layer problem is not available.

Under simplifying assumptions, however, the stability of the coupled air and water boundary layers has been investigated by Lock (Lock). The analysis is restricted to two-dimensional laminar flow, and the solution is not valid for small values of $2\pi x/L = 2\pi\delta R/L$, and large values of $2\pi\delta/L$, where R is the boundary layer Reynolds number i.e., $u_1\delta/\nu = (u_1 x/\nu)^{1/2}$, L is the wavelength of the disturbance, and x is the fetch. The analysis is based upon successful methods developed by several authors, and summarized by Lin's papers (Li), for investigating the stability of boundary layers in confined air flows in pipes and along plates.

Three-dimensional disturbances have been shown by Squire (Sq) to cause instability at higher Reynolds numbers than for two-dimensional flow, so that Lock's dimensional restriction is not serious.

Lock's results indicate that the neutral stability curve in the wavelength--fetch plane consists of a single closed loop for low wind speeds and at higher speeds ($>$ 250 cm. sec.$^{-1}$) this splits into two portions, each increasing in size with wind speed. Lock did not compute a critical wind velocity, (i.e., a threshold below which no instabilities occur) but his curves indicate that it is less than 100 cm. sec.$^{-1}$. At a wind speed of 100 cm. sec.$^{-1}$ the 10 cm. waves amplify most rapidly and at 200 cm. sec.$^{-1}$, the 15 cm. waves. The applicability of the results is limited by assumption of laminar flow conditions, and interpretation for wind wave generation does not seem to be straightforward. The dependence on fetch is also not clear although the results do indicate those wavelengths subject to maximum amplification for a given wind speed.

Tchen (Tc) has noted that previous stability analyses have not employed those physical arguments which would simplify the theory. Since the inflexion point in the velocity profile at the surface may be regarded as the "seat" of the instability, Tchen has reduced the problem, with physical simplification as the prime objective, to that of Helmholtz stability in which the two fluid basic currents possess uniform velocities (Fig. 11). Viscosity is therefore not admitted in the basic flow, but is introduced into the boundary condition for stress continuity for the perturbations at the surface. Solutions were found for small and large wave numbers and an approximate analytic solution found between these extremes by interpolation. A neutral stability curve may be plotted in the (wave number) - (water current) plane (Fig. 12). There does

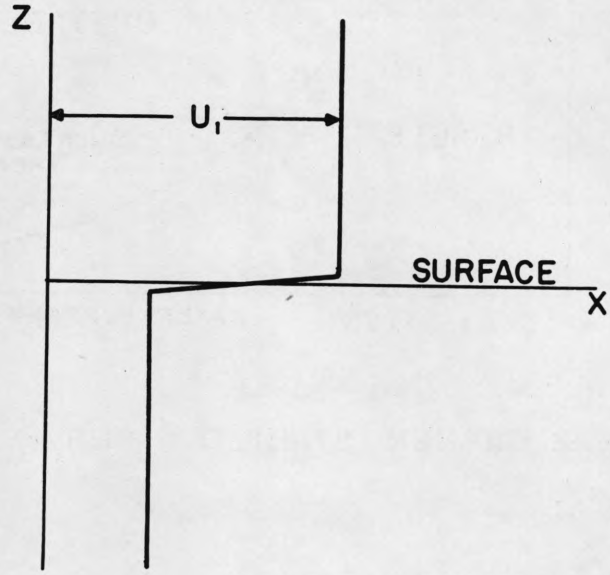

FIG. 11 BOUNDARY LAYER VELOCITY PROFILE (TCHEN)

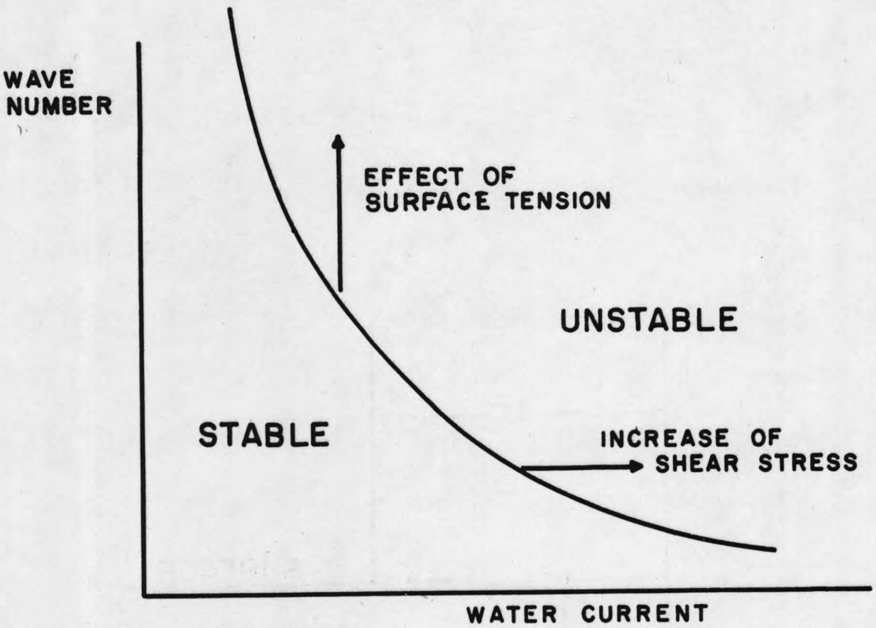

FIG. 12. TCHEN STABILITY CURVE

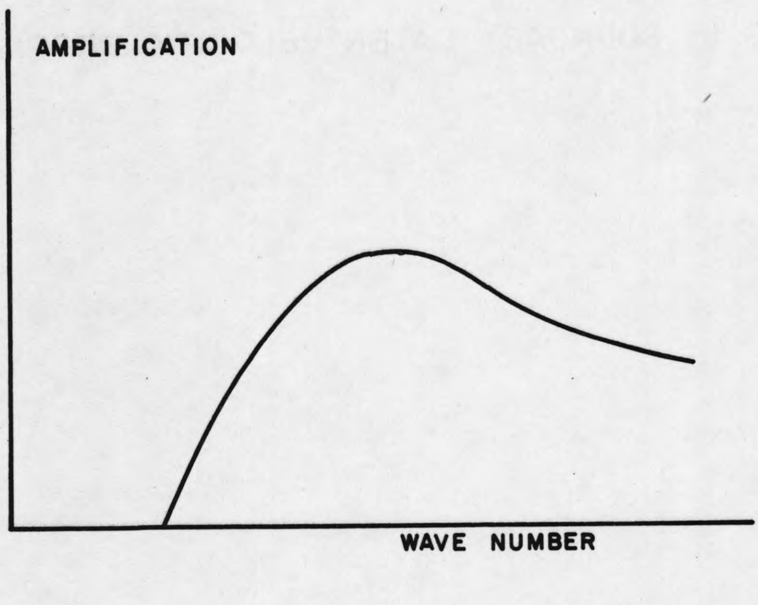

FIG. 13. TCHEN AMPLIFICATION CURVE

not appear to be any stability loop: i.e., although there is the possibility of a critical water current below which no waves are unstable, all waves beyond this with k larger than the neutral stability value are unstable, but receive varying amplification. For any given water current, the amplification rises from zero for the neutral stability wave number to a maximum and then falls slightly to an asymptote as k increases (Fig. 13). The effect of increased surface tension stabilizes the perturbations up to higher k, while an increase in kinematic viscosity (eddy viscosity for turbulent air flow) stabilizes the perturbations up to higher surface water currents. Tchen's results promise very useful physical application.

The most drastic simplification in the stability problem is to neglect viscous effects altogether, but this yields results misleading beyond any defense of the parent approximation. This case was first solved by Lord Kelvin (Kel) who calculated that the first instabilities occurred for a wind speed of 660 cm. sec.$^{-1}$ and were of wavelength 1.8 cm. Jeffreys (Je-1) recognized that the inviscid restriction was not valid, and assumed that separation of the boundary layer, leaving a "dead air" region between the surface and boundary layer, takes place on the lee side of the crests. The separation phenomenon is represented in his mathematical formulation by an empirical constant, the sheltering coefficient, s, defined in the expression for the effective pressure on the wave surface at any point, $s\rho u_{\infty}^2\phi$, where ϕ is the slope at that point. The condition for a wind to raise a wave at all was found to be given by

$$u_{\infty} \geqslant 73s^{-1/2} \text{ cm/sec,} \qquad (11\text{-}1)$$

where u_{∞} is the wind speed, and s is less than unity. The corresponding wave number m for the first waves formed, was found to be

$$m^3 = 5s^2. \qquad (11\text{-}2)$$

For u_{∞} = 100 cm/sec, a wavelength of 8 cm. for the first waves was computed from (11-1), (11-2) which was in general agreement with Jeffreys' visual observations. Since he states that this theory described only the initial stages of wave formation in which the wave steepness is still small it does seem implausible to assume separation particularly if the boundary layer is turbulent since turbulence delays separation. Lack of separation would nullify his theory and imply that the shear stress is the most important contribution in the initiation (see discussion of equation (12-6), Sect. 12.1).

The above theoretical predictions of critical wind speeds may now be summarized and compared with observation. The inviscid case of Kelvin yields a value for the critical wind speed which is much too high. Jeffreys' results, which cannot apply to the initial formation but to a slightly later stage, do give values in fair agreement with observation. Lock's results do not give a critical wind speed, but show that this must be lower than 100 cm/sec. and that the corresponding critical wavelengths must be approximately 10 cm. Tchen's analysis bears unpicked fruit, but it indicates that a current must be present before instabilities may be generated, and that there is no lower limit to their wavelength. All the stability analyses so far have shown two dimensional disturbances to be the most unstable.

Our visual and photographic observations indicate that instabilities occur at the low wind speeds, about 100 cm. $sec.^{-1}$, but that they are of small wavelength, 2-3 cm., are short-crested, and possess profiles considerably distorted from a sinusoid.* The waves rapidly assume asymmetric profiles, the crests collapsing in a series of small capillaries on their lee side (see Fig. 9). The short-crested nature of the initial disturbances is very evident in Keulegan's (Keu-1) water tank tunnel, and it is his opinion that they cannot form unless a water current is present. Waves generated in a very shallow water-wind tunnel (Han) indicate that two dimensional disturbances do initially form, but that they rapidly break up into 'pebbly' disturbances which grow in a manner similar to turbulent spots in laminar-turbulent transitions in boundary layer flow (Schu-1). Tchen is investigating the possibility that this short-crestedness may be due to turbulence in the water acting through the non-linear coupling terms in the Navier-Stokes' equations.

A random and dense collection of short-crested waves of small amplitude can presumably be produced just by the turbulence of the wind even for very low wind speeds. This mechanism has been investigated by Eckart for large (gravity) waves. With viscous damping neglected, his results (Ec, Sect. 9) indicated that the height of small waves generated in this way is proportional to the rms turbulent pressure fluctuation and to the fourth root of the number of (micro-) gusts in the "storm" area. The critical quantity for first initiation may then be the turbulence level rather than the wind speed, and, as Ursell has concluded earlier (Ur-2, Sect. 6), the critical speed for first initiation of waves may be essentially zero.

The concept of critical wind speeds for initial formation of

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* The laboratory observation by Schooley (Scho-2) of a critical wind speed of seven knots for first formation of high curvatures on the water surface has not been noticed yet for secondaries produced on lakes.

waves is clearly meaningful, but it is necessary to proceed with caution in their interpretation and use (cf. Sect. 8). The character of both the air and water boundary layers, must at least be qualitatively known before a citation of the corresponding critical speed is justified. To the present, good quantitative experimental data necessary to establish the validity of any of the above theory is non-existent; furthermore, the difficulty of matching the amenable theoretical stability analyses with conditions encountered in practice make it necessary and important to classify critical wind speeds in relation to prevailing air-water boundary layer characteristics by careful experimentation.

12. Aerodynamic Forces on a Disturbed Water Surface

12.1 Classification

The growth of wind waves depends on the energy input of the aerodynamic forces in a manner discussed in Sect. 14. In this section these forces will be treated for a disturbed water surface, i.e., one supporting finite amplitude waves (as distinct from the infinitesimal waves treated in the previous section). The treatment of the general problem may be simplified by the following classification, which distinguishes two important cases: (i) the air boundary layer thickness, δ, is less than the wave amplitude, a; (ii) the air boundary layer thickness is of the same order as, or appreciably greater than, the wave amplitude. It must be emphasized that in natural cases of wave generation the boundary layer thickness is usually several times the wave amplitude, but in laboratory investigations this may not always be so. The two cases are discussed in Sect. 12.2 and 12.3 and the relation of these cases to circumstances met in nature is discussed in Sect. 12.4. The effects of thermal instability also are outlined in Sect. 12.4.

Before analysing the two cases, we first introduce expressions for the aerodynamic surface forces. Consider a coordinate system with x in the direction of the wind and of the propagation of a two dimensional wave, z perpendicular to the horizontal plane containing x. The aerodynamic forces in the x and z directions on an element of surface, calculated from the stress tensor, are

$$d\overline{F}_x = \overline{i} \left[(-p + 2\mu \frac{\partial u}{\partial x}) \, dz + \mu (\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}) \, dx \right] , \qquad (12\text{-}1)$$

$$d\overline{F}_z = \overline{k} \left[(-p + 2\mu \frac{\partial w}{\partial z}) \, dx + \mu (\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}) \, dz \right] , \qquad (12\text{-}2)$$

for unit distance in the y direction and where u, v, w, are the wind velocity components.

In case (i) it is convenient to make a transformation of axes into directions tangential x', and normal, z', to the wave surface to facilitate a discussion of the boundary layer velocity derivatives. Designating the wave slope at any point by the angle ϕ, the expressions (12-1, 2) become, neglecting certain very small terms,

$$d\overline{F}_x = \overline{i} \left[-p\phi + 2\mu\phi \frac{\partial w'}{\partial z'} + \mu(\phi^2 + 1) \frac{\partial u'}{\partial z'} + \mu \frac{\partial w'}{\partial x'} \right] dx, \qquad (12\text{-}3)$$

$$d\overline{F}_z = \overline{k} \left[-p + 2\mu \frac{\partial w'}{\partial z'} - \mu\phi \frac{\partial u'}{\partial z'} \right] dx . \qquad (12\text{-}4)$$

These expressions may be simplified by retaining only the largest terms. The relative orders of the velocity derivatives are found to be (Schl, p. 95).

$$\left(\frac{\partial u'}{\partial z'}\right) : \left(\frac{\partial w'}{\partial z'}\right) : \left(\frac{\partial w'}{\partial x'}\right) :: \delta^{-1} : 1 : \delta , \qquad (12\text{-}5)$$

and if ϕ is small one has

$$d\overline{F} = d\overline{F}_x + d\overline{F}_z = \left[\overline{i} \mu \frac{\partial u'}{\partial z'} - \overline{k} p \right] dx. \qquad (12\text{-}6)$$

The velocity derivative in (12-6) is evaluated at the wave surface : its value depends on whether a laminar or turbulent boundary layer is present, being higher in the latter case.

12.2 Case (i) : $\delta < a$

This condition enables one to use potential flow above the wave surface, thereby permitting calculation of pressures at the wave surface. The force $d\overline{F}$ varies over the water surface due to both the presence of the waves, and the nonuniformity and variability of the wind. In calculating for case (i) the average aerodynamic force, energy input and resulting growth for a single wave (see Sect. 14, Eq. 14-5), locally uniform and steady wind may be assumed, but over larger regions* of the water surface, the nonuniform distribution of surface forces due to wind variability alone can be responsible for the generation of waves. The formation of storm waves in the latter case has been treated by Eckart (Ec), but by itself his theory predicts a wave growth rate well below that observed. For a complex sea, in addition to the effect of unsteady wind conditions, one must take into account the variation of drag contribution with wavelength due to different characteristics of the waves through the spectrum. The dependence of the drag on the wave spectrum has received attention from Munk (Mu-2). His results indicate that the form (pressure) drag is proportional to u_∞^2 multiplied by a statistical parameter related to the mean square slope. In his paper mention is made of the wind to wave momentum transfer by rain, and similarly by spray, which is a significant contribution as was found in Van Dorn's experiments (Va).

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* For example, regions comparable to the extent of a wind gust.

The pressure distribution $p = p(x)$ in (12-6) will depend upon the wave profile, and for small but finite amplitudes will give no contribution to wave growth. For steeper crested waves these pressure forces assume importance due to the thickening and resultant separation of the boundary layer on the lee side of the crest. The shear stress $\mu(\frac{\partial u'}{\partial z'})$ may be large if any wavelet superstructure is present on the wave which gives rise to rough turbulent flow. (For the relevant conditions see (12-20) and discussion thereon later in this section.) For the largest superstructure one may expect the form drag, owing to the pressure term, to be significantly changed, whereas for the capillary superstructure one would probably find the viscous stresses increase to those for the fully rough condition, (cf. (12-20)). Strong gusts spread these capillaries over the surface wherever they travel, but since one usually has waves of moderately large dimensions present for higher wind velocities, the effects of increase of drag due to capillary superstructure on the larger waves may be small since the capillaries are only short lived (see Fig. 6). For waves of long length, the boundary layer flow becomes similar to that over a curved plate.

The case of separation or boundary layer asymmetry has been considered by Jeffreys (Je-1) and recently by Hibbs (Hib). Jeffreys allowed for these effects by means of the sheltering coefficient already discussed in Sect. 11. The pressure distribution for potential flow over a symmetrical wave profile is out of phase with the particle velocity, and does not result in an energy transfer, but any boundary layer asymmetries yield an in-phase component.*

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* Wind waves themselves are also generally asymmetric, but we are not considering here the effect of this asymmetry upon energy transfer.

By representing the stream line limiting the potential flow below as a Fourier series, Hibbs has calculated this in-phase component from a model which limits the region of potential flow between crests to be that above a straight line of slope γ from one crest to a point below and windward of the succeeding crest. It is this in-phase component that may be associated with boundary layer asymmetry and separation. Tiza's results (A1) for a single wave show no asymmetries in the air boundary layer profile, but calculations by Schroeder (Schr) indicate that for laminar boundary layer flow over a regularly wavy surface asymmetries are quite noticeable after a few wavelengths only, and separation is induced fairly rapidly.

In one experimental study (Schu-3), separation of a turbulent boundary layer from a wing occurred when $(-d \log u_1/d \log x) \sim 1.4$. For sinusoidal long-and short-crested waves, the largest values of this ratio are about 0.7 and 0.5 indicating that separation of the turbulent layer may occur only over waves appreciably distorted from sinusoidal shape.

The form drag of a complex wind wave system depends upon the relative velocity of wind and wave, and in calculations the representative velocity requires careful choice. Whether it should be the particle, group, or individual wave velocity is undecided. Since the form of the waves is important here it would appear to be the group velocity.

12.3 Case (ii): $\delta \sim a, \delta >> a$.

For case (ii) potential flow does not exist near the wave surface, and the internal characteristics of the boundary layer become important. The condition $\delta \sim a$ is not clear cut and is therefore very difficult to treat. This is unfortunate, for this condition can also occur and forms the bridge (cf. Fig. 14) between case (i) and the case for thick turbulent boundary layers given below (Sect. 12.4). However, the velocity derivatives $\frac{\partial w'}{\partial z'}$ and $\frac{\partial w'}{\partial x'}$ may no longer be insignificant, and no data is available for their discussion.

One may immediately distinguish between two extreme subcases fulfilling the condition $\delta \gg a$. First one has the problem of initial instability, which has been treated in Sect. 11, and, secondly, the problem of the turbulent boundary layer flowing over a wave structure whose height is an order of magnitude less than δ. There is good evidence that this latter subcase is met with commonly: generally at sea, and on inland waters where the boundary layer has thickened due to its earlier passage over land. This subcase is therefore extremely important in our understanding of natural wind wave generation, and will be dealt with rather fully in the next sub-section.

12.4 The Thick Turbulent Boundary Layer over Water

Even in the idealized case (i) where a thin boundary layer was considered, separation of the boundary layer from the wave crests due to the steepening of the waves establishes strong turbulent mixing and rapid thickening of the boundary layer. A pictorial summary of the idealized transition from case (i) to case (ii) is given in Fig. 14. When the boundary layer is so thick that $\delta \gg a$, which, as we have mentioned above, is already present in most natural wind flows, the flow becomes similar in many but not all respects to that over a roughened plate in wind tunnel experiments. If the wave structure possesses a random character the roughness elements in the analogous wind tunnel flow are three dimensional

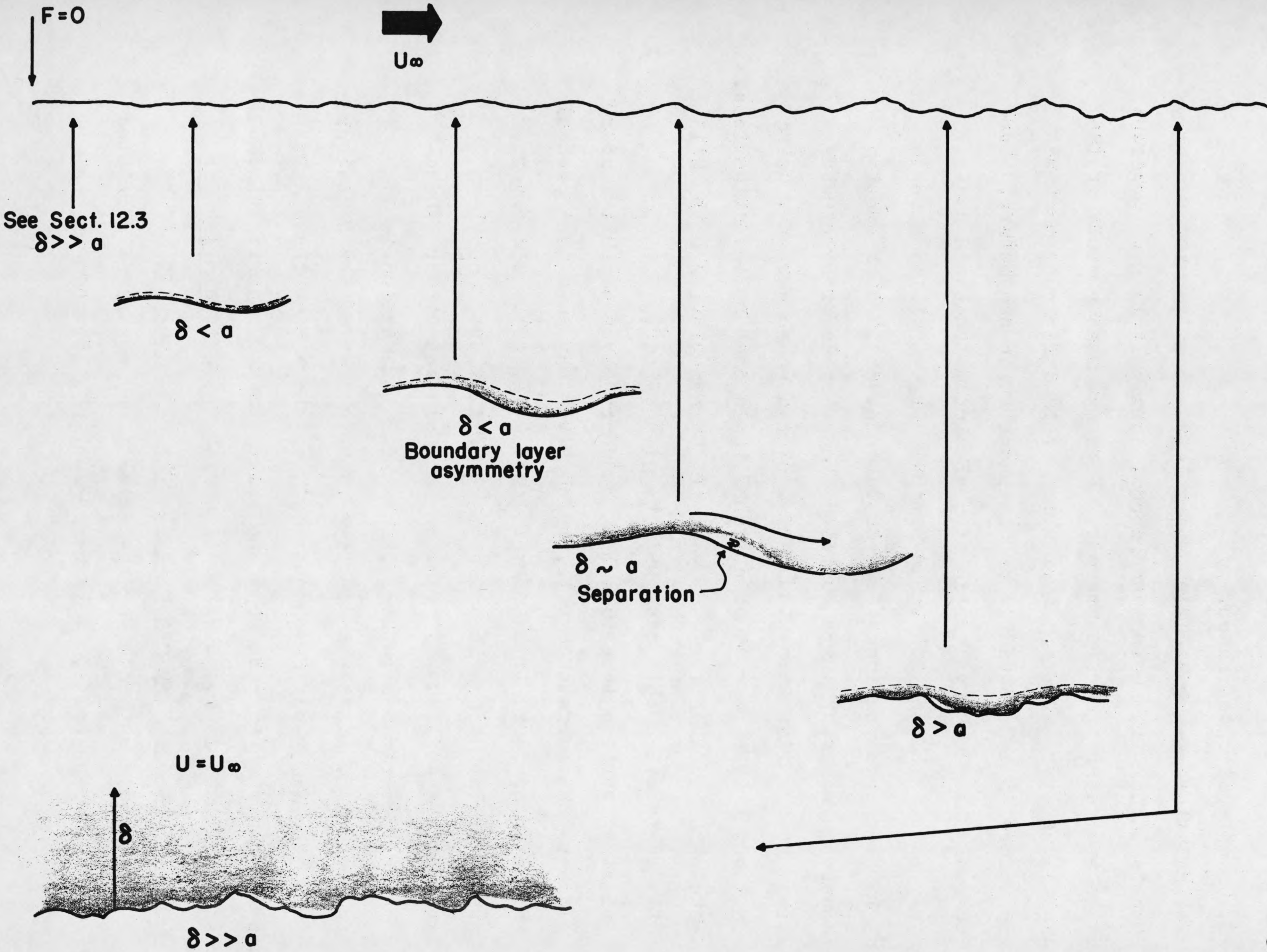

FIG. 14 CLASSIFICATION OF BOUNDARY LAYER CONDITIONS

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and of variable size and shape.

The behavior of turbulent boundary layers even in wind tunnel experiments is far from being understood, particularly when pressure gradients and separation are present. Very few measurements of the boundary layer over water are available: this deficiency must be made up before an understanding of wave generation can mature. The character of the boundary layer in natural circumstances is considerably complicated by its three dimensionality (giving rise to cross flows) and unsteadiness. We shall endeavor in this section to discuss the natural, thick turbulent boundary layer over water in analogy to the flow in a wind tunnel over a rough surface, using those notions that have been established in laboratory investigations. A discussion of turbulent boundary layer characteristics which will be useful in treating this case will be given first.

Velocity profiles in a turbulent boundary layer exhibit a sharp rise close to the surface or wall, which gives way to a more gradual rise in the outer four fifths of the layer (Fig. 10). The outer flow resembles turbulence in a free wake, whilst the inner is similar to the so-called wall flow in channels and pipes. These flows may be described separately, but their interaction is a difficult problem. In the inner layer, where most of the turbulent energy is produced, the flow depends only on the shear stress, the viscosity, and the distance from the wall. In the majority of the boundary layer a velocity distribution plot which is <u>similar for all turbulent boundary layers without pressure gradients</u> may be obtained by nondimensionalizing the velocity by a parameter related to the shear stress (Cla). This parameter, u^*, is known as the friction velocity and is defined by

$$u^* = \sqrt{\frac{\tau_o}{\rho}} \quad , \quad \tau_o = \mu \left(\frac{\partial u}{\partial z}\right)\Bigg|_{z = o} = \frac{c_f}{2} \rho u_1^2, \qquad (12\text{-}7)$$

where c_f is the local skin friction coefficient. The form of this distribution is

$$u = u_1 + u^* f_1 \left(\frac{z}{\delta}\right) , \qquad (12\text{-}8)$$

where the function f_1 does not have a simple analytic form. In the inner layer the velocity distribution is of the form

$$u = u^* f_2\left(\frac{u^* z}{\nu}\right), \qquad (12\text{-}9)$$

while very close to the wall the form is

$$u = u^* \left(\frac{u^* z}{\nu}\right). \qquad (12\text{-}10)$$

This form, (12-10), exists in the region where the viscous stresses are greater than the turbulent stresses. The region is distinguished as the viscous layer or the laminar sublayer. The depth of this layer relative to the size of any protuberances on the surface is very important in determining the flow characteristics near the surface, as we shall see below. The two velocity distributions, (12-8) and (12-9), overlap near to the surface (and always outside the laminar sublayer), and their consistency requires (Cla) that

$$f_1 = A \log \frac{z}{\delta} + B', \qquad (12\text{-}11)$$

and

$$f_2 = A \log \frac{zu^*}{\nu} + B, \qquad (12\text{-}12)$$

respectively. The quantity B' depends on the type of flow and on the definition of δ. B' may be chosen as zero so that z/δ is unity when the free stream is reached by the extrapolated logarithmic distribution. The boundary layer thickness then, if defined in this way, is given by

$$\delta = \frac{\nu}{u^*} \exp 2.3 \left[\frac{1}{A}\left(\frac{u_1}{u^*} - B\right)\right] \qquad (12\text{-}13)$$

The logarithmic form of the velocity distribution in the inner layer appears to hold closely for the inner 10% of the boundary layer and to a sufficient approximation, for the inner 20%. It does not however hold further out than this (that is, in the outer layer), and any calculations based on this assumption are untrustworthy. Calculations of the very important quantity, u^*, for instance, should be based on velocity measurements in the inner 20% of the layer. It may be noted that only two such measurements are necessary to determine u^*, since if u_i and u_{ii} are measured at z_i and z_{ii}, (12-8) and (12-11) give

$$\frac{u_{ii} - u_i}{u^*} = A \log \frac{z_{ii}}{z_i} . \qquad (12\text{-}14)$$

The values of the constants A, B, and B', and the range of validity of the logarithmic distributions have been by no means sharply defined by experiment. We shall quote, however, some results: in the case of pipe flow the law appears to hold for $1.5 < \log \frac{zu^*}{\nu} < 4.5$, with A = 5.75, B = 5.5, (Schl, p. 405, 407). It appears that for flow along a curved or flat wall the law holds for $1.5 < \log \frac{zu^*}{\nu} < 2.5$ (Cla). Clauser claims A = 5.6, B = 4.9, while Schlichting quotes A = 5.85, B = 5.56. The inner limit to the validity is, of course, set by the increasing importance of the viscous forces on approaching the wall or surface. The inner limit quoted above then implies that the viscous stresses have become negligible at $\frac{zu^*}{\nu} = 30$. The exact figure still does not seem to be certain, and figures as high as 60 have been quoted for the attainment of the logarithmic velocity distribution (Dr). The value of B' for which the "true" velocity distribution reaches the free stream velocity, at $\frac{z}{\delta}$ equal to unity, is -2.5 (Cla). The "extrapolated logarithmic" definition of the boundary layer thickness given by Eq. (12-13) is about twice the true thickness.

The outer edge of the laminar sublayer occurs (Cla) at approximately $z = 12\,\nu/u^*$, and any roughness elements whose disturbance projects noticeably beyond this render the above distributions invalid. The flow is no longer aerodynamically smooth at this point and the laminar sublayer becomes disturbed by the roughness elements. The momentum transfer in smooth wall flow takes place due only to the tangential stresses at the surface and the first term in (12-6) is sufficient to determine the aerodynamic force on the wave surface. However, when artificial mixers, or vortex generators (the roughness elements) are present at the wall or surface the eddy-stabilizer effect of the viscous layer is decreased, and finally nullified, at which point the flow is said to become fully rough, and fully turbulent flow exists clear to the wall. In rough flow, the separation at the roughness elements and consequent strong mixing at the surface causes an additional momentum transfer to the wall or surface due to the forces acting normal to the wall. The aerodynamic forces on the waves then must be calculated from the full expression (12-6), although now it must be remembered that the waves are well inside the boundary layer. The apparent shear stress at the wall is then representative of the normal forces due to separation. The shear stress may still be calculated because the effect of rough flow on the logarithmic distributions quoted above is known. Thus Clauser (Cla) has shown, that the function f_2 must be modified to include the roughness Reynolds number, ku^*/ν, where k is the height of the roughness elements*. The effect of this new dependence is to displace

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* For waves k is some appropriate average wave height (see our later discussion of k in this sub-section).

the velocity distribution of (12-12) by $\Delta u/u^*$,

$$u/u^* = A \log zu^*/\nu + B - \Delta u/u^*, \qquad (12\text{-}15)$$

where

$$\Delta u/u^* = f_3 \, (ku^*/\nu). \qquad (12\text{-}16)$$

The form of this function, f_3, depends on the geometry of the roughness elements as well as their height, and must be determined empirically. This important function has not yet been determined for the sea surface. Experiments at the National Bureau of Standards by G. Keulegan, and at the University of Illinois by T. Hanratty, are being made to determine wind velocity distributions above the surface which should indicate the behavior of this function under a limited range of circumstances. Francis' tunnel experiments (Fr-1) should also prove useful in its evaluation.

When the flow becomes fully rough, f_3 becomes a linear function of $\log ku^*/\nu$

$$f_3 \, (ku^*/\nu) = A \log ku^*/\nu + D, \qquad (12\text{-}17)$$

where D is a constant depending on the type of roughness (Ham). Combining this equation with (12-12), one has

$$u/u^* = A \log z/k + (B - D), \qquad (12\text{-}18)$$

which may be expressed in the alternative form

$$u/u^* = A \log z/z_o \qquad (12\text{-}19)$$

where z_o is the so-called roughness parameter. This quantity may be written in terms of the constants A, B, D and k. A and B are known, and if D is determined by experiment for say a random wave structure, measurements of z_o from wind velocity profiles in the inner boundary layer give the k characteristic of the wave roughness elements. The k thus found will indicate indirectly what wave amplitudes produce the roughness effect under different circumstances. The motion of the roughness elements will be considered later.

In the absence of information concerning the expression (12-17) for a wave structure, we may still discuss the roughness effects in terms of the concept of "equivalent sand roughness." All types of roughness can be scaled down to a standard so that the various forms of f_3 become exactly equal (i.e., D becomes a Universal constant). In the past, this standard has been chosen as a representative of sand grains for which the constant (B-D) is 8.5. The corresponding k in the equation is then known as the equivalent sand roughness, denoted by k_s. Combining (12-18) and (12-19) for this value of (B-D), one then has $k_s = 30z_o$. The relation between k for any type of roughness and the standard k_s may be found by comparing (B-D) found experimentally for the former with the constant 8.5 for the latter. The ratio k_s/k varies considerably with the roughness character: pipe flow data suggests that for surface ridges running transverse to the flow (and in this respect similar to long-crested waves) the ratio may be as much as 30 (Schl, p. 246). The qualitative similarity in appearance between a short-crested wave structure and a collection of sand grains suggests that in this case the ratio is probably near unity. We return to these figures below.

The type of flow in the boundary layer may be conveniently characterized by the following criteria involving the quantity k_s (Schl, p. 418)

a) hydraulically smooth $u^* k_s/\nu < 5$

b) transition $5 < u^* k_s/\nu < 70$ (12-20)

c) completely rough $u^* k_s/\nu > 70$

The criteria were qualitatively discussed in the introduction to (12-15). Measurements of velocity profiles at sea yield values of z_o and u^*, so that it is more convenient to express the above flow characterization by Nikuradse's conditions, using the relation between

z_o and k_s given above (Schl, p. 420)

a) $z_o u^*/\nu < 0.17$

b) $0.17 < z_o u^*/\nu < 2.3$ (12-21)

c) $z_o u^*/\nu > 2.3$

In relation to the above discussion, we shall now consider some velocity profiles measured at sea by Hay (Hay-1). These measurements were made 800 meters from the land for off-shore winds. Calculations of z_o indicated that it is a linear function of the wind speed, as measured at two meters, with values of 0.03 cm. and 0.25 cm. at 500 cm/sec and 900 cm/sec, respectively. Transition to fully rough flow, according to the conditions (12-21), takes place at a wind speed (at a height of two meters) of about 530 cm/sec. Two typical velocity profile plots are shown in Fig. 15, with one corresponding to the latter wind speed (profile 1). The free stream velocity is in the neighborhood of 650 cm/sec. An estimate of the thickness of the boundary layer in this case may be made from (12-13) in which, since we now have rough flow, the constant B must be replaced by B- $\Delta u/u^*$, where $\Delta u/u^*$ is seen to be approximately 7 from the logarithmic plot of the velocity profiles in Fig. 16. The thickness of the extrapolated logarithmic plot is about 30 meters, meaning that the true thickness is about 15 meters. This figure agrees well with the data of Fig. 15. According to the figures for z_o quoted above, the equivalent sand roughness, $k_s = 30\ z_o$, varies linearly with wind speed (at two meters) from about 1 cm. at 500 cm/sec. to 7.5 cm. at 900 cm/sec. The height of the 'significant waves' calculated from Sverdrup and Munk's data (Sv-1, Fig. 6) should be 16 cm. and 18 cm. respectively. (These predicted heights are about one-half as large as those listed by Hay.) If $k \sim k_s$, then these

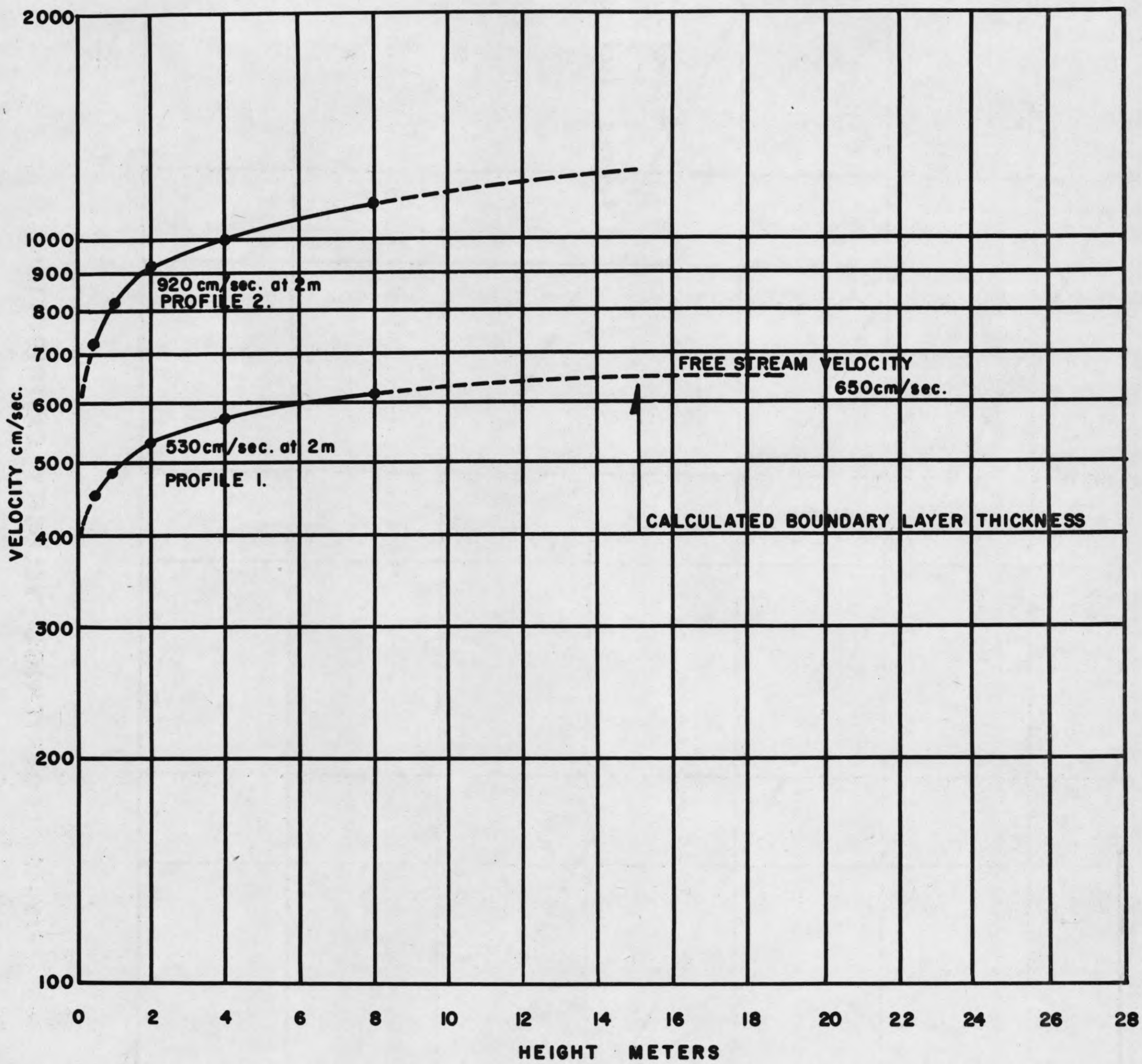

FIG. 15 VELOCITY PROFILES OVER THE SEA SURFACE (HAY).

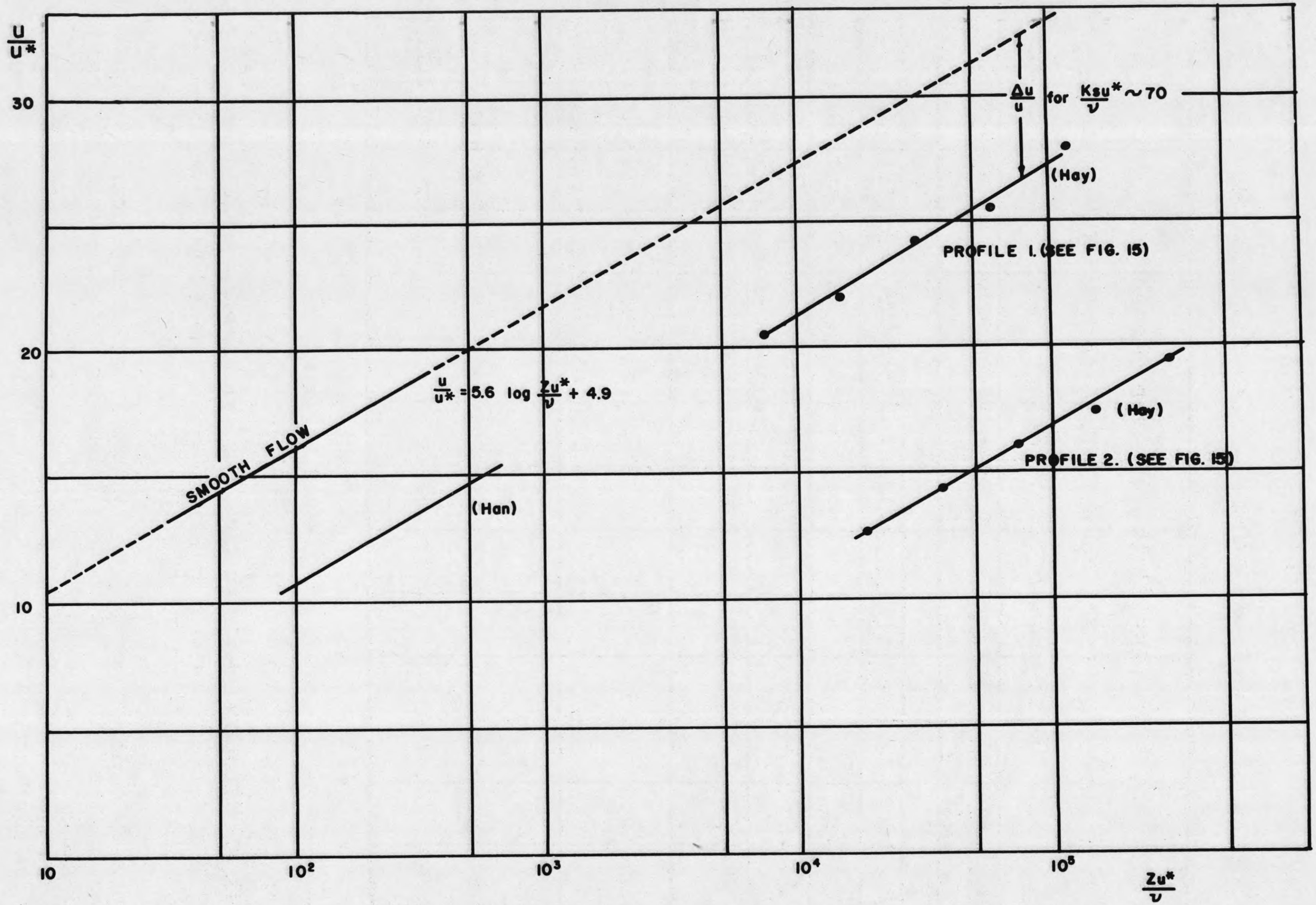

FIG. 16 LOGARITHMIC VELOCITY DISTRIBUTIONS.

results would imply that the roughness effects are due to the smaller short-crested secondary waves riding on the larger primaries.*

It should be noted that the data in Fig. 16 fall at high values of log $z u^*/\nu$ (of the order 4) compared with the range of validity of the logarithmic law given above. The consistency of the data with the form (12-12) is, however reassuring. This does not mean that care should not be exercised in the extrapolation of the wall law (12-12) from wind tunnel data to measurements in the open sea. Included in Fig. 16 are some very recent measurements in a turbulent boundary layer flowing over a thin film of water (Han). The flow is seen to be in the fully rough condition even though the fetch was a few meters instead of 800 meters as in Hay's experiments.** The equivalent sand roughness was found to be 0.03 inches. Photographs of the water surface showed a well developed pebbled appearance, and the height of these pebbles, which constituted the roughness elements, was observed to be close to 0.03 inches. The equality of k_s and k is evidently justified under such conditions.

We have noted above that the roughness effects of a wave structure may possibly be identified with the secondary structure. A very important question then is, "what is the relation between k

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* The earlier calculation (Hic-1) of critical wind speeds, corresponding to the Reynolds numbers of 5 and 70 (see (12-20)), assumed that the roughness elements were the larger waves and is therefore probably in error. These critical wind speeds were derived only for monochromatic wave trains.

** In Hanratty's experiments the boundary layer was thin at the beginning of the fetch, which was probably not the case for Hay's experiments. This illustrates the point emphasized by Clauser (Cla) that the thickness is a more important parameter than the fetch in characterizing the boundary layer.

and k_s for a given shape of secondary structure?". As we have mentioned above, pipe flow data suggests that k_s/k for surface ridges running transverse to the flow may be as large as 30 (Schl, p. 246). For short-crested waves this ratio is probably not far from unity, a supposition supported by Hanratty's measurements quoted above. (A "pebbled" wave surface is, of course, of a short-crested nature.) Thus it is possible, for a given wind speed and fetch, that long-crested capillaries 0.1 cm high would produce fully rough flow and short-crested gravity waves 1 cm high might still allow aerodynamically smooth flow, with a corresponding considerable difference in drag coefficient.

Description of sea surface roughness by equivalent sand roughness may not appear entirely satisfactory since as Munk has pointed out (private communication) there is no geometrical similarity between the waves and a collection of sand particles, particularly in view of the smoothing effect of surface tension. One must keep in mind, however, that the disruption of the laminar sublayer by flow separation over the roughness elements is caused by adverse pressure gradients, and there is reason to suppose that severe pressure gradients exist both over the sea surface with its moderately "sharp" crests and over sand grains. This argument may justify the use of the concept of equivalent sand roughness, although the spacing of the roughness at sea is much larger than on roughened places. We note further that for fully rough flow (i.e., for the most common case for wind waves) only one parameter such as k_s is needed to describe the effect of roughness (Cla) whereas for the intermediate and less common case the <u>shape</u> and <u>distribution</u> of the roughness must also be somehow specified.

It is perhaps not the shape of the roughness elements which may preclude the use of k_s or z_o in describing turbulent air flow over water, but rather the motion of the water carrying the roughness. The velocity of a roughness element is about equal to the particle velocity of the underlying primary wave, and may be appreciable compared to the velocity in the boundary layer at the top of the roughness element. Thus for the two air velocity profiles shown in Fig. 15 (from Hay) the velocity at the top of the roughness elements* (for $zu^*/\nu = k_s u^*/\nu$) is about 200 cm/sec and 500 cm/sec respectively, and the particle velocity of the primaries is at most about 60 cm/sec. For larger primaries the particle velocity may be as large as the air velocity at the top of the roughness elements. Furthermore, the variation of particle velocity of a large primary wave along the surface of the wave may then be so large as to cause corresponding variation of the growth and separation of the turbulent air boundary layer. We have no data to support a more definitive discussion of this aspect of the wind-wave interaction problem or to assess its importance.

Unfortunately there are natural situations which require extension of the discussions of cases (i) and (ii) above. The nature of turbulent flow of atmospheric air has been reviewed recently by Ellison (El). Temperature gradients nearly always exist in the air boundary layer, giving rise to significant effects on wind-wave dynamics. The effects depend on the atmospheric stability, which is defined in the following way. The atmosphere is stable, neutral or unstable accordingly as the rate of change of temperature is less than, equal to, or greater than the adiabatic lapse rate (Su, p. 10).

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* In estimating this velocity we must make the somewhat contradictory assumption that the roughness elements are fixed.

Although the effect of atmospheric stability on the air boundary layer is not well understood, it is known to be significant in the wind-wave interaction (Fle). Unstable conditions give rise to a momentum transfer in addition to that associated with the turbulent transfer found in a neutrally stable boundary layer.* This additional transfer has two important consequences: the drag on the water surface increases, and the products of breaking waves is carried higher into the boundary layer and transported further over the sea surface than for neutrally stable boundary layers. Darbyshire, designating conditions as stable, unstable and neutral for temperature differences between the air and water of +2°F, -2°F and 0°F respectively, has investigated the effect of stability on the aerodynamic drag over a large area of water (Dar-1). He found that, relative to the neutral condition, the drag increased by fifty percent for unstable conditions, and decreased by twenty-five percent for stable conditions. Some indication was found that the wave heights, for a given wind speed, were higher in unstable as compared to neutrally stable conditions. This indication has been confirmed by Fleagle (Fle) who found that wave heights increase by about ten percent per degree centigrade difference under unstable conditions.

We have already indicated the relationship of the cases (i) and (ii) to natural conditions of wave generation. Our detailed discussion of case (ii) has shown that the flow of the thick turbulent boundary layer over water under rough conditions is the most relevant in a majority of circumstances. Case (i) may certainly

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* Inversion, in which the temperature increases from the surface, (a stable condition), suppresses turbulent transfer.

be important for laboratory wind wave tunnels, however. The aerodynamic forces on the waves, in summary, are characterized by the following: the history of the boundary layer (cf. Cla); the free stream velocity; the shear stress, and the relation between the secondary structure and the equivalent sand roughness (the form of the function $f_3\,(ku^*/\nu)$ in (12-16) is extremely important for the transitional region (12-20)). Calculations of the energy input to the waves is directly related to u^*, which gives the total aerodynamic drag on the water surface locally; however in a complex surface motion as one finds at sea, the waves receive individual aerodynamic attention due to differing phase velocity, and varying form, and the energy input to individual waves, or different parts of the wave spectrum, will be difficult to calculate. Resort will have to be made, then, to a combination of individual wave element theory, associated wave statistics, and the boundary layer characteristics. In the case of separation within the turbulent boundary layer (rough flow conditions) data will probably be required on velocities very close to the wave surface. Simultaneous measurements of wind velocity profiles and wave statistics are very important. Although no figures are available on the ratio of form or tangential drag to the total drag on the water surface associated with various wave structures, a great deal of experimental data has been collected on the total drag of a wave supporting surface, both locally and over a large area of water surface. In the section below, we briefly discuss this drag.

12.5 Drag Data

Much experimental work has been devoted to the determination of the drag coefficient for a developing wave structure. The drag coefficient as determined in these experiments depends upon the type of measurement. Calculation of u^* and the drag coefficient from wind velocity profiles (Eqs. (12-7,14)) yields the local drag, and since the complex conditions in the atmospheric boundary layer (in particular due to temperature effects) and deviation from logarithmic form of the outer part of the profile have not always been taken into account, these results have not always proved to be very reliable and considerable scatter has been a consequence (see Fig. 17). Many useful results have been obtained from observations on wind tides which give the average aerodynamic drag over a large area of water surface. Some data have also been obtained in laboratory experiments. The drag on the surface is usually expressed in terms of the coefficient $\tau_o/\rho u_\infty^2$. For drag coefficients based on area of water surface this is then equal to half the local skin friction coefficient, $c_f/2$.

In Fig. 17 drag data are plotted as a function of wind speed (these speeds have not been measured at similar heights, which introduces a further complication), representing results from several sources and indicating the wide range of values obtained. The measurements giving the average drag $\overline{c}_f/2$ and local drag $c_f/2$, which of course are not generally equal, are indicated in the key. Before a useful comparison of such data can be made it is important that additional data concerning wave statistics and boundary layer history should be given. Thus one or more parameters must be given in addition to the local wind speed (the free stream value of which should be estimated) in order to reduce them to a single consistent family. The increase in drag coefficient with decreasing wind speed at low wind speed observed in Fig. 17 is believed to be untrue, the erroneous data being attributed to inversion in the boundary layer and its effect on the interpretation of velocity profiles (Ch-1). There is however a distinct possibility that part of this increase might be real under certain conditions, for the skin friction increases with decreasing Reynolds number for flow over a flat plate. If the waves do not grow fast enough to induce rough flow in the turbulent boundary layer, then we may expect a similar increase for the water surface (see Fig. 17, curve xii). In general we should expect such increases (Cla) when the effect of unfavorable pressure gradients is decreasing and still causes only a small part of the drag. Neumann has also discussed the apparent increase of drag with decreasing wind speed (Ne-6).

If the turbulent flow over the water is similar to that over a rough plate, then dimensional arguments impose some restrictions upon the manner in which c_f and k depend upon u_∞ and F. As an example we consider roughness elements that are not affected by surface tension but only by the gravitational acceleration g. We assume that the viscosity of the water is not a significant parameter but that dissipation there is controlled by turbulence which in turn is controlled by the size of k as a measure of wave height and by $(gk)^{1/2}$ as a measure of water particle velocity. From the two constants g and ν (kinematic viscosity of air) and the two variables, wind speed u_∞ and fetch F, we can form two independent dimensionless ratios, $(u_\infty{}^3/g\nu)$ and (gF^3/ν^2). The skin friction coefficient c_f and the quantity k/F must be functions of these ratios only, if the stated assumptions are true and if the boundary layer and surface structure have built up together from a common origin for the fetch.

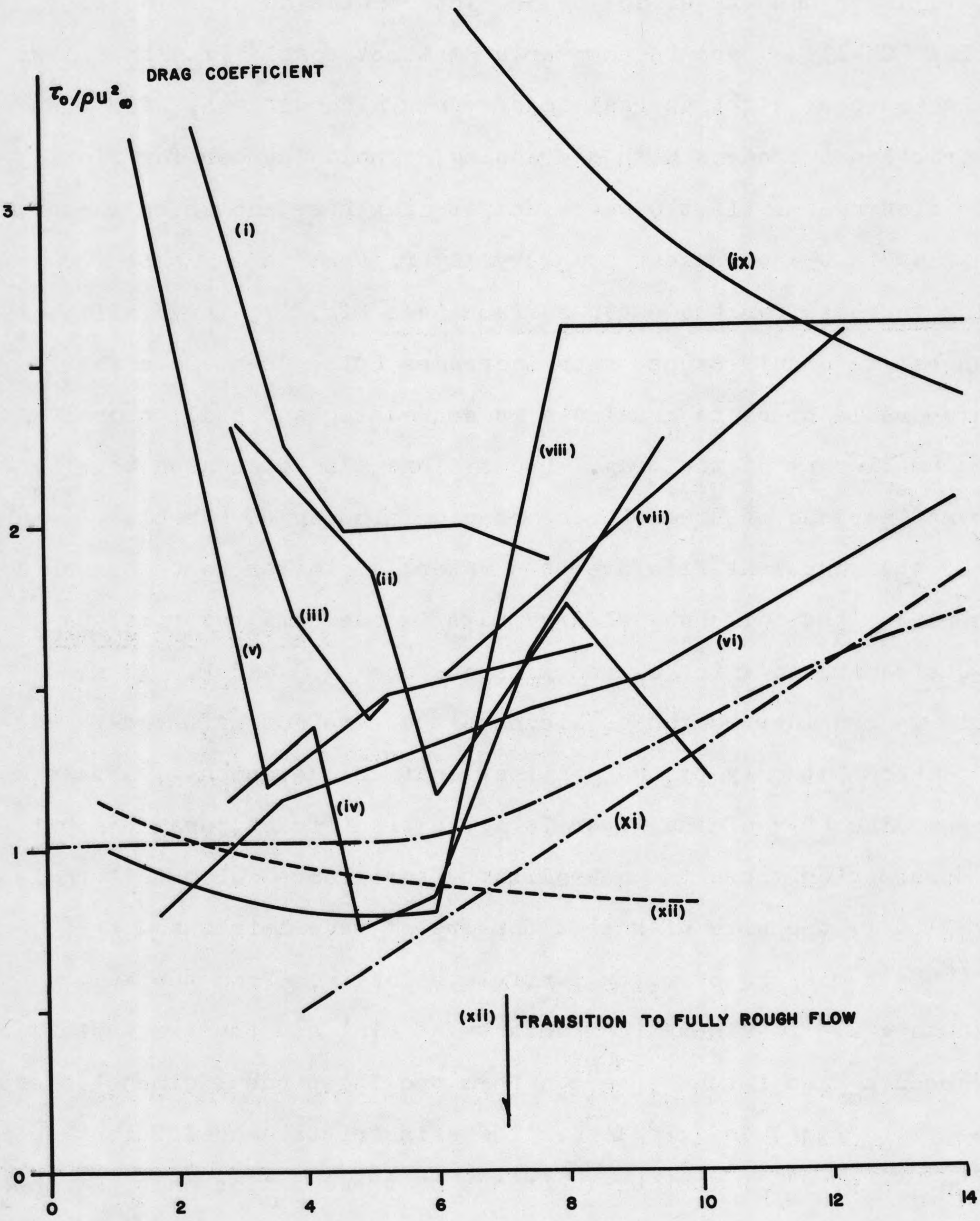
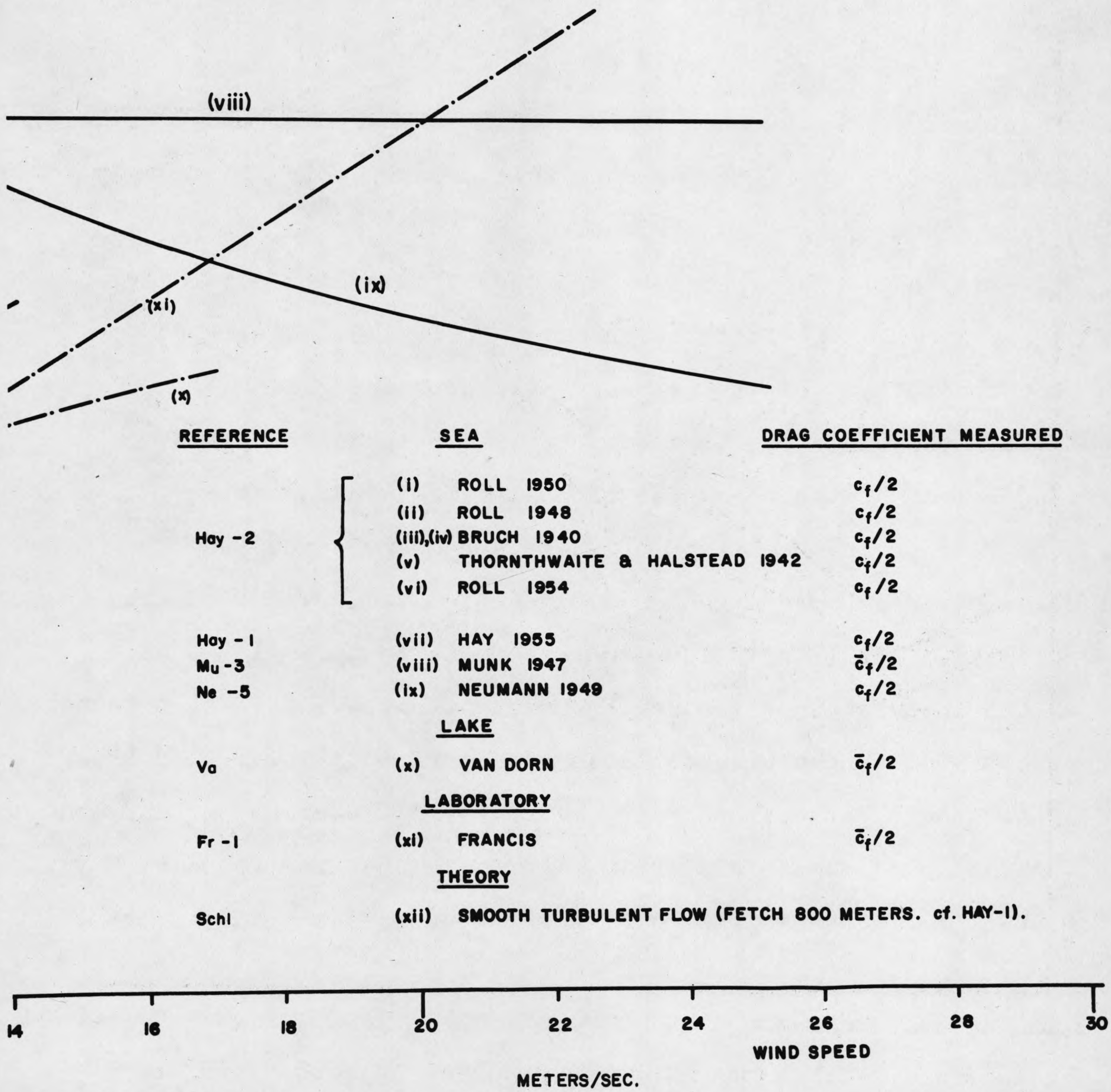


FIG. 17 DRAG DATA


(viii)
(ix)
(xi)
(x)
REFERENCE
SEA
DRAG COEFFICIENT MEASURED
Hay -2
(i) ROLL 1950 $c_f/2$
(ii) ROLL 1948 $c_f/2$
(iii),(iv) BRUCH 1940 $c_f/2$
(v) THORNTHWAITE & HALSTEAD 1942 $c_f/2$
(vi) ROLL 1954 $c_f/2$
Hay - I (vii) HAY 1955 $c_f/2$
Mu -3 (viii) MUNK 1947 $\bar{c}_f/2$
Ne -5 (ix) NEUMANN 1949 $c_f/2$
LAKE
Va (x) VAN DORN $\bar{c}_f/2$
LABORATORY
Fr - I (xi) FRANCIS $\bar{c}_f/2$
THEORY
Schl (xii) SMOOTH TURBULENT FLOW (FETCH 800 METERS. cf. HAY-I).
14
16
18
20
22
24
26
28
30
WIND SPEED
METERS/SEC.

Then

$$c_f = f_1 \left[(u_\infty{}^3/g\nu), (gF^3/\nu^2)\right] \quad (12\text{-}22)$$

$$k/F = f_2 \left[(u_\infty{}^3/g\nu), (gF^3/\nu^2)\right] \quad (12\text{-}23)$$

For a limited range of the variables, we can choose exponents q_1, ---q_4 such that the relations

$$c_f \propto (u_\infty{}^3/g\nu)^{q_1} (gF^3/\nu^2)^{q_2} \quad (12\text{-}24)$$

$$k/F \propto (u_\infty{}^3/g\nu)^{q_3} (gF^3/\nu^2)^{q_4} \quad (12\text{-}25)$$

agree closely with the more general expressions, Eqs. (12-22, 12-23).

If the flow over the water is fully rough, which is usually the case, then c_f is a function of (k/F) only, and $q_1/q_3 = q_2/q_4 = q_5$, say. Hay's results are described approximately by $k \propto u_\infty$ and $c_f \propto u_\infty$. In this case* $q_1 = q_3 = 1/3$. Now if we neglect the effect of the primary waves upon the growth of the roughness elements, it seems reasonable to suppose that k, the height of the roughness elements, is independent of the fetch, for the height of these secondary waves is limited by breaking and come rapidly into equilibrium with the wind speed. But if k is independent of F then $q_4 = -1/3 = q_2$. With the uncertainty attending our assumptions, we have thus found that $c_f \propto F^{-1}$, a result susceptible of experimental test. If the flow is smooth rather than rough, then c_f is a decreasing function of the Reynolds number (Fu_∞/ν). Therefore the flow found by Hay for which c_f increases with u_∞ cannot be aerodynamically smooth,

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* When $k \propto u_\infty$, then k cannot be independent of both F and ν. If Hay's measurements are fitted (about as well) to $k \propto u_\infty{}^2$ (as Ellison (El) prefers), then k can be independent both of F and ν. We are assuming implicitly that k_s/k is constant.

a conclusion reached by other arguments in Sect. 12.4.*

The case of capillary roughness elements controlled by the capillary constant Σ' rather than by g can be given similar dimensional discussion. In general, measurement of c_f, z_o and k for different fetches as well as different wind speeds should clarify, with the help of dimensional analysis, the effect of the physical parameters ν, g, and Σ' upon boundary layer growth over wind waves. Measurement of the rate of dissipation of the secondaries' energy owing to turbulence of the water would also be helpful. Unfortunately it is not convenient, for various reasons, to study on natural bodies of water the effect on c_f, k and z_o of varying ν, g and Σ'. In a wind-wave tunnel, it might be possible to vary Σ' and ν without introducing confusing side effects. The influence of artificial or natural slicks (Sect. 13), which destroy most of the secondaries and therefore reduce the drag considerably, should also be examined experimentally, especially if some appropriate means can be found for characterizing the slicks.

13. The Dissipation of Energy in the Wave Structure

In the case of a simple monochromatic train of waves, the dissipation of energy may be attributed directly to three phenomena in the main: the aerodynamic drag, of opposite sign but similar to the force analyzed in the preceding section, for those waves travelling with a wave velocity greater than the wind velocity; the

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* For transitional flow it was assumed by the authors in a recently submitted paper (1957 Annual Meeting of the American Physical Society) that $c_f \propto (ku_\infty/\nu)^{1/2}$ and $k/F \propto u_\infty (gF)^{-1/2}$, leading to the conclusion that $c_f \propto F^{+1/4}$. This conclusion must be regarded with suspicion because data for flow over a rough flat plate (Schl, p. 449) indicates that nowhere in the transition region is c_f dependent just on k and not also on F.

kinematic viscous dissipation (see Sect. 1, and Appendix B); and finally the dissipation due to turbulent processes caused by the wave motion. Unless one is dealing with fast waves of large wavelength travelling away from and outside a storm area, the aerodynamic drag need not be considered. It seems evident from observations of small suspended particles in water which is disturbed by the wind that turbulent processes are important even for comparatively small values of a/L and that they are induced primarily by the interaction of the wave motion and current. The kinematic viscosity has negligibly small effects compared with the turbulent dissipation. It is clear, therefore, that the latter is the only factor to be considered.*

In the complex sea there are further phenomena contributing to energy dissipation: the waves break as they approach limiting steepness, and energy may be transferred across the energy spectrum due to non-linear effects in the wave motion (cf. Sect. 4,9.1 and Ne-2, Ch. II, Sect. 6). To be associated with these effects is the behavior of the secondary superstructure on the larger underriding waves. The products of breaking mainly contribute to the generation of turbulence in the upper layers, although due to momentum conservation new waves will also be formed. The latter would contribute in part to the transfer from one wavelength to another. None of these phenomena associated with breaking have been examined quantitatively in the literature.

We have seen that turbulent dissipation is the most important energy sink that must be considered. The theory of turbulence at

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*The effectiveness of turbulent dissipation is vividly demonstrated by the suppression of wind waves in the wake of a ship where the turbulence generated by the ship's motion and propeller is considerable. Neumann has also discussed the role of turbulent dissipation in wind wave growth (Ne-2, Ch. II, Sect. 1).

present is not capable of dealing with problems without empirical guidance and its simplest concept, that of eddy viscosity, requires experimental determination of the eddy viscosity coefficient. The eddy viscosity, μ_ϵ, enters into the equations of motion in a manner similar to the kinematic viscosity, and the dissipation of energy per unit area of sea surface which is monochromatic and simply sinusoidal, may be found from the stress tensor as

$$\frac{de}{dt} = 2\mu_\epsilon m^3 C^2 a^2. \qquad (13\text{-}1)$$

It is thus necessary to determine the quantity μ_ϵ. The generation of the turbulence by the surface waves implies that μ_ϵ will be a function of the surface structure. Neumann (Ne-2) has assumed that μ_ϵ will depend only upon the wind velocity for a fully developed primary surface structure, and for a less complete stage of wave development, will lie between the value found for the fully developed case and the kinematic viscosity, Groen and Dorrenstein (Gro-1) have proposed a dependence of μ on $L^{4/3}$, and Bowden (Bo-1) suggests a proportionality to aC. Darbyshire (Da-2) has proposed the relation

$$\mu_\epsilon \propto a P u_\infty \, f(C^2/u_\infty^2) \qquad (13\text{-}2)$$

where the function f is constructed so as to tend to infinity for C becoming greater than u_∞, the wind velocity.

Dissipation of wave energy has up to the present only been discussed in the literature in terms of this useful but inadequate concept of eddy viscosity, but there clearly is need for a more complete understanding of the wave decay process. It is possible that the engineering concepts of the turbulent boundary layer, accompanied by plausible assumptions concerning the behavior of certain parameters (Em-H), may also prove useful for a clearer understanding of the turbulent dissipation in the water near the water surface and in the presence of water waves.

There are other factors affecting the dissipation of wave energy that may be briefly noted. When organic matter is distributed on the water surface it has been found that small wave formation by aerodynamic means is prohibited over that region which is contaminated. These calm patches are known as slicks, (see discussion also in Sect. 1) and have been attributed to the wave damping characteristics of a film of organic matter in compression (Ew-1), i.e., to increase viscous damping owing to an altered boundary condition. Surface active components consisting of long-chain molecules cause hysteresis effects in compression and extension of some organic films (Whi-1). If contamination on natural bodies of water possess components of this type the hysteresis effect, which is observed in the neighborhood of the sudden drop in surface tension due to the contamination, may have a considerable effect on wave dissipation.

An important effect of a slick is to reduce the mean square slope of the waves by a considerable factor, essentially removing the high frequency components in the wave spectrum and also reducing the aerodynamic drag on the surface considerably. Keulegan (Keu-1) found that with detergent on the surface of the water in his tank experiments no wind waves at all were formed. Van Dorn's (Va) work in California on a lake surface contaminated with detergent indicated a reduction of drag, as measured by set up, relative to that of a surface without waves. Early qualitative experiments on the effect of an oil slick were also made by Gibson (Gi).

The effect of rain in damping wave motion has been discussed by Sainsbury and Cheeseman (Sa) on the basis of the transport of momentum away from the surface by vortex rings generated by the rain drops. In view of the confused state of the upper layers, one may simplify the explanation by attributing it to a generation of turbulence which increases the rate of energy dissipation of the waves.

14. The Growth and Development of the Surface Wave Structure

Two of the three topics mentioned in Section 10 have been discussed in the preceding sections, and we have attempted therein to provide a basis for discussing the third topic, the growth and development of a complex surface wave structure. In spite of our increased use of the results of recent aerodynamic theory and experiment, our discussion of the growth of complex wave structure will still be relatively incomplete because of the difficulty of the subject and the lack of appropriate experimental data.

The importance of viscous effects in the air flow near the water surface has been emphasized in Section 12, and it is clear therefrom that several factors must be considered to gain a correct understanding of the aerodynamic forces on a wave. If the viscous effects are confined to regions in the air of depth very small compared with the wave heights (case (i) of Sect. 12.2) then the viscous drag may be neglected and the growth of finite amplitude waves may be calculated by solving the equations of motion of an inviscid fluid. This case has already been solved (Tek). His work includes the inviscid stability case previously treated by Lord Kelvin, and his computations give 693 cm/sec and 1.7 cm. for the critical wind speed and generated wavelength for wave initiation. Included in his calculations are plots for the growth rate of a wave against its wavelength for various wind velocities. The highest growth rates were found to be for waves between about 0.9 and 2 cm. As the wind velocity increases, the theoretical wave phase velocity decreases for a given wavelength, departing more and more from the well known curve exhibiting the minimum velocity for a wavelength of 1.7 cm. There is experimental evidence, however, that the wave phase velocity actually increases (Fr-1). This increase perhaps is associated with the current in the water at the surface.

The potential flow approximation is invalid for the real case of wind blowing over a developing wave structure; because the boundary layer is seldom thin one must look for a theoretical method which on the one hand avoids the considerable difficulties of solving the equations of motion for a viscid fluid (particularly when separation and turbulence must be included) but on the other hand is not limited by too severe approximations. As we have mentioned in Section 12, there are two methods which may be used in calculating the growth of finite amplitude waves and which have proved useful, but require further refinement in view of the discussion in Sections 12 and 13. We shall now discuss these two methods briefly.

The first method is due to Cauchy and Poisson: here one may start with an initial distribution of aerodynamic surface forces and calculate the subsequent response of the surface. This is essentially Hibbs' approach (Hib). Damping may be introduced into the resulting growth equation. Hibbs calculated the growth rate of waves due to a travelling periodic disturbance limited in space to the region over which the wind is blowing. In the response of the surface to the applied pressure distribution, Hibbs found that effects due to a variation in the latter travelled across the system at group velocity $C_g = C/2$, and that the magnitude of the effect measured by the change in amplitude Δa was $\omega p_o \Delta t/2 \rho g$ where p_o is the pressure at the point of origin. One therefore has for the resulting amplitude

$$a(x, t) = \frac{\omega}{2\rho g} \int_o^t p_o(x - C_g\tau, t - \tau)d\tau \qquad (14\text{-}1)$$

From this expression one may find the growth equation by differentiation,

$$\left(\frac{\partial a}{\partial t}\right)_{(x - C_g t)} = C_g \left(\frac{\partial a}{\partial x}\right)_t + \left(\frac{\partial a}{\partial t}\right)_x = \frac{\omega}{2\rho g} \; p_o(x,t) \qquad (14\text{-}2)$$

Hibbs calculated the surface force due to separation*, p_o, and shear, s_o, and found,

$$p_o(x, t) + s_o(x, t) = \rho \left[\underbrace{2\gamma(u_\infty - C)^2 \left(1 - \frac{\gamma L}{2a}\right)}_{\text{separation}} + \underbrace{c_f \frac{u_\infty^2}{2\pi}}_{\text{shear}} \sqrt{\frac{2\gamma L}{a}} \right] \qquad (14\text{-}3)$$

The inclusion of the shear force in the equation in the same manner as the normal force is justified by Hibbs. Solutions to this growth equation were found for the two cases of fetch limited and duration limited growth. With an appropriate value for γ (see Sect. 12.2), the results compared favorably with Bretschneider's (Br-1) data for the amplitude of the limiting waves ($a/L = 1/7$) at various fetches for the first case ($\partial a/\partial t = o$). Similar agreement was found with the data of Sverdrup and Munk (Sv-1) for the second case($\partial a/\partial x = o$). In neither case was there agreement with the data for the wavelength as a function of fetch or duration.

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* See Sect. 12.2, case (i).

The second method, based on energy considerations, is formally simple and provides a convenient explicit representation of the problem. The rate of change of energy in the wave structure is expressed as the difference of two terms: the rate of energy input to the waves and the rate of energy dissipation in the upper layers of the water. The energy input is due to aerodynamic forces and uncertain hydrodynamic means. In the latter category one may include energy introduced into a given wavelength range by its removal from another part of the spectrum due to non-linear effects in the wave motion and breaking. A corresponding term accounting for energy lost in this manner must then be introduced into the energy dissipation term for this wavelength range.

The energy equation may thus be written as follows,

$$\underset{\text{Wave System}}{\left(\frac{de}{dt}\right)} = \underset{\text{Energy Input}}{\left(\frac{de}{dt}\right)_I} - \underset{\text{Energy Dissipation}}{\left(\frac{de}{dt}\right)_L} . \qquad (14\text{-}4)$$

The term $(de/dt)_I$ may be split into the average rate of energy input per unit area over a single wave due to tangential stress, $(\mu \, \partial u'/\partial z')_{z'=0}$ and normal pressure forces, p. If U and W are the particle velocities in the wave

$$\left(\frac{de}{dt}\right)_I = \frac{1}{L} \int_0^L \left(\mu \frac{\partial u'}{\partial z'}\right)_{z'=0} U dx - \frac{1}{L} \int_0^L pW dx + \left(\frac{de}{dt}\right)_{NI}, \qquad (14\text{-}5)$$

where $(de/dt)_{NI}$ represents the input due to the non-linear effects mentioned above. The main contribution to the term $(de/dt)_L$ was found in Section 13 to be turbulent dissipation, but we must include any contribution due to the transfer of energy across the wave spectrum. A simple calculation of the aerodynamic input assuming separation and/or tangential shear shows that $(de/dt)_I$ due to aerodynamic forces is proportional to the term $(u_\infty - C)^2 \, C$. A plot of this dependency is shown in Fig. 18, and gives an indication of those waves receiving most favorable amplification.

Neumann has used the energy method to calculate the initial growth of finite amplitude waves (Ne-5) and the build up of the primary structure (Ne-2). In Neumann's first paper (Ne-5) laminar damping* was considered to be the source of energy dissipation and the energy input was calculated from a drag coefficient obtained from "wind turning" data over the open sea. The second paper uses the same data for the drag but allows for turbulent damping of the waves. The energy of a simple wave per unit area of surface, (Lam Art. 230),

$$\frac{1}{2} P \quad mC^2 a^2 \tag{14-6}$$

is used in the energy term so that all variables in the equation may be expressed in terms of the "wave age" C/u_{∞} through empirical relationships. Solutions are then found in terms of this quantity, and are found to have good correspondence with the fairly limited observational data presented.

It must be emphasized that all the a priori theoretical methods so far used in calculating wave growth have essentially referred to simple wave systems, as approximations to a real sea, in which only a narrow spectrum of waves can be discussed.** Real seas possess a broader spectrum, as discussed in Part II of the

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* Munk (see Ew, page 174) calculated the effect of a contaminating surface film on the initial generation of finite amplitude waves. His results indicate that the minimum wind speed for growth of waves is increased by approximately a factor of two, and the corresponding wavelength is reduced but still lies in the classification of gravity waves (cf. discussion of Jeffreys' theory, Section 11).

** Empirical theories (Sect. 7) have of course been used to predict many characteristics of the growth of systems of primary waves.

report, and the analyses of the growth equation require extension. No theoretical attempts have yet been made, however, to describe the growth of a multiple frequency system before the limit of breaking is reached: in this connection one may support the energy method as promising a relatively simple and useful analysis of the problem which should provide information on the behavior of the energy spectrum (e as a function of L) under various conditions of wind speed, duration, and the fetch.

At present, however, one may give only qualitative consideration to the growth and development of a many-component wave system. The details of the growth of a wave structure depend largely on the conditions of the air boundary layer, and it is therefore important to know the latter's characteristics for various situations encountered in nature. For instance, for a stretch of inland water the passage of the wind over the land will be largely responsible in determining the thickness of the turbulent boundary layer and the resulting characteristics of the waves first formed on the water. At sea a wind may commence by blowing over a calm area, in which case laminar conditions might initially be present. Even if transition is assumed to occur at a Reynolds number, $u_1 x/\nu$, of 10^7 then with a wind of 150 cm. sec.$^{-1}$ the maximum fetch for laminar boundary layer flow would be 100 m. The flow will often be fully rough within 1 km. (cf. Sect. 12.4) even though transition to this condition is delayed by the existence early in the fetch of a thick boundary layer. Probably a different initial wave structure will form for the different types of boundary layers. In either case, small waves will form initially, and will reflect even rapid variations in the wind velocity. Their wavelengths may vary between about two and fifteen centimeters,

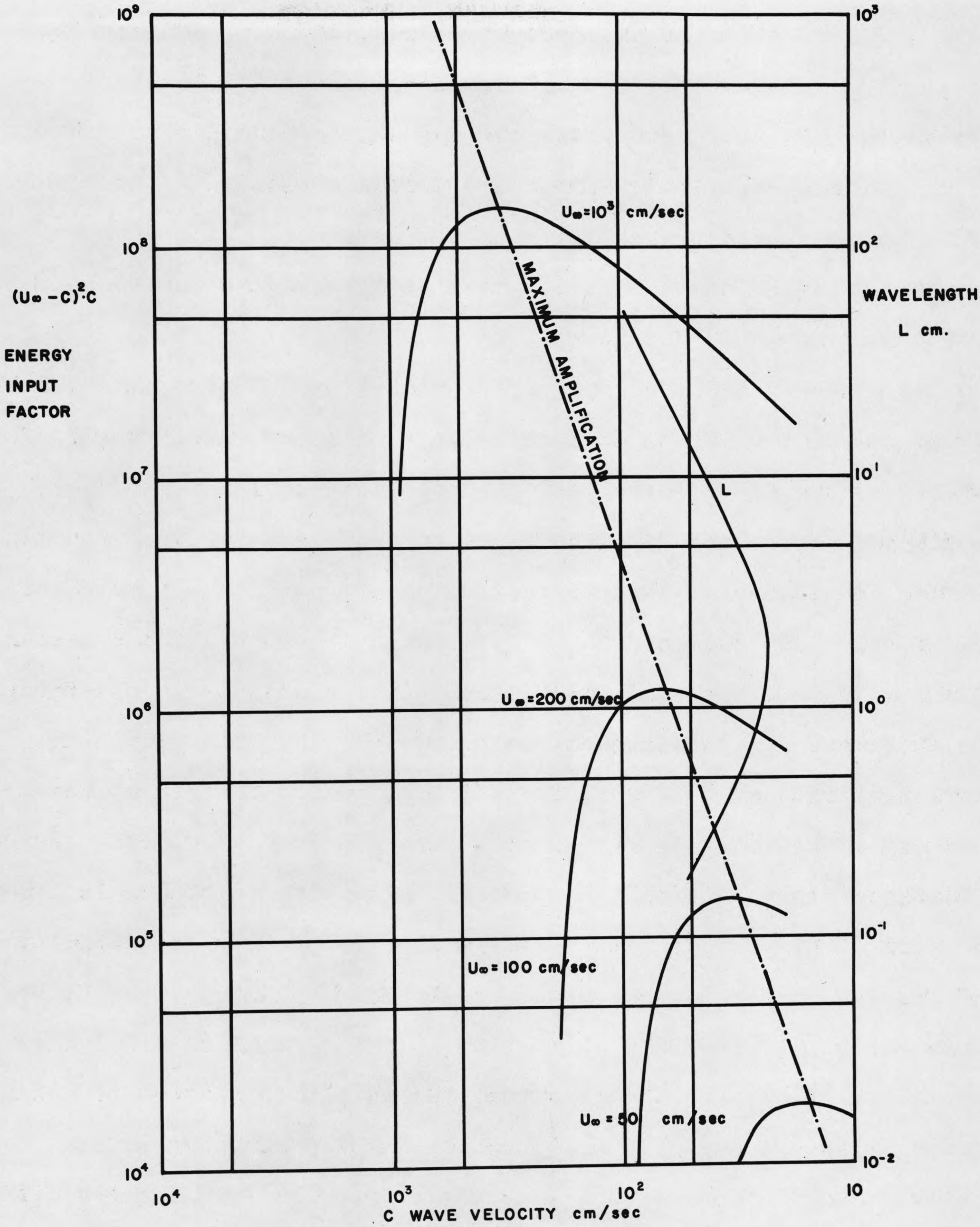

FIG. 18 AERODYNAMIC ENERGY INPUT FACTOR

and their growth be determined largely by the tangential or shear forces at the surface. Due to the action of the tangential stresses dominant in these early stages of wave growth, shear-induced asymmetries in the small waves may be responsible for the slopping over of the crests, without formation of white water but with the generation of lee side capillaries (see Fig. 9). Although waves of about fifteen centimeters length will characterize the early part of the fetch, following wave initiation, as they continue to develop, their wavelength increases as does their height. It is difficult to discuss the history of the short-crestedness of the waves but observation indicates that this short-crestedness is characteristic of even the earlier formed waves. The papers by Eckart (Ec) and Roll (Rol-2) are relevent here. In the early fetch the crest length L' (see Sect. 3.1) is usually several times the wavelength L, and may retain these values up to moderate values of the wavelength on small inland lakes, sometimes including some of the primary waves.

Not far beyond the point of initial formation of the waves, separation of the boundary layer may be expected, and the dominant role in energy input will be taken over by normal forces*. A further effect of separation may be in the "sheltering" of the surface over the region of separated flow, inhibiting the formation of very short waves, but this problem has received only very qualitative consideration.

The viscous and pressure stresses develop asymmetries in the

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* Since the onset and extent of boundary layer separation is determined by the adverse pressure gradients over a wave, the 'relieved' pressure distribution over the lee face of a short-crested wave will be less favorable to separation than the pressure distribution over a long-crested wave, (cf. discussion of roughness effects in Sect. 12.4).

wave, which further affect the boundary layer and its separation, thereby establishing a type of 'positive feedback process.' A rapid acceleration of the wave breaking process may then be expected when separation becomes predominant. In a well developed sea state one may expect a similar process to occur when the peaked crests of larger waves disrupt the laminar sublayer and separation takes place on their lee sides. Since most of the turbulent energy is produced in the region just outside the laminar sublayer, and the turbulent velocities are highest in this region, production of white water may well be expected. Referring to Hay's measurements, the appearance of whitecaps may then occur at a free stream velocity of about 6.5 m/sec or about 13 knots. The close agreement of this figure and that quoted in Sect. 9 for the incipience of whitecaps is significant. When smooth flow is present the roughness elements are not subject to appreciable turbulent stresses, and the viscous shear stresses may only induce rolling breakers, which are observed on small lakes. It will be remembered that there may be other reasons associated with viscosity and surface tension (see Sect. 9.2) that inhibit production of white water by the roughness elements.

IV. RECOMMENDATIONS FOR FUTURE RESEARCH

Further theoretical and experimental research is needed on many problems of wind waves. In this section we outline these problems and in some cases specify possible theoretical or experimental methods for their solution.

One of the few means available for affecting the nature of wind waves is by means of artificial slicks. A special effort should therefore be made to find the real nature of the effects of both natural and artificial slicks upon damping of water waves and upon the drag of the wind on the waves, and also to describe the composition, origin and variability of natural slicks. Slicks can apparently be used to eliminate the high frequency components of the wave spectrum (which account for most of the roughness effects) and can therefore be used as a tool to study the specific relation of secondary structure to aerodynamic drag.

There is no theory of the generation and structure of wind waves which starts from the hydrodynamical equations and includes the <u>randomness</u> of the surface. Even though natural winds are variable it is not known just how important this <u>variability</u> is in the accurate description of wind waves, and of the secondaries in particular whose scales of length and time are so much less than those of the primaries. The accuracy of the Neumann spectrum for primary waves and of the predictions based upon it is being actively studied. The azimuthal part of this spectrum remains uncertain as does the reality of the so-called period increase of ocean swell. Less effort is going into studying departures of the sea surface from an assumed stationary Gaussian process. We have suggested that non-Gaussian character of the sea surface can be described as the result of a nonlinear filtering of an essentially Gaussian process. If this suggestion is to have physical as well as mathematical validity, it must be possible to identify experimentally the physical processes which perform the nonlinear filtering and which constitute the original Gaussian process.

There has been a rather inadequate theoretical basis for predicting the instability of a randomly rough sea surface with respect either to breaking or to conversion of long into short-crested waves. The second stability problem has, indeed, not been treated. In the methods we suggest for treating the first stability problem, we assume that the breakdown, in one respect or another, of the linear description of a Gaussian sea surface corresponds to instability of the water. This assumption has not been tested directly or indirectly by experiment. Certain aspects of stability and of shadowing theory apparently require higher statistical information than either theory or experiment will be able to supply for some time to come.

Although whitecapping following breaking is clearly a complicated phenomena to describe theoretically, certain experimental measurements should be easy as well as informative. Thus it should be possible to measure photographically the fraction of the sea surface covered by whitecaps and their number density, and to study these parameters as a function of local wind speed, air boundary layer structure, primary wave spectrum and slick concentration. With qualitative studies of the types of breakers that can occur at sea as further background, this quantitative measurement should give several types of information: it should test our aerodynamic explanation of the observed critical wind speed for whitecapping; should give a better understanding of the rôle of slicks in controlling the production and dissipation of whitecaps and foam (and of the remarkable calming effect of oil on storm waters!); and should also provide a partial basis for estimating momentum transfer and energy exchange owing to the breakers.

We have stressed the importance of characterizing the air boundary layer. Successful characterization of the boundary layer formed by the wind over water demands moderately complicated experiments and more emphasis upon aerodynamic measurement than has often been given in the past. We may safely predict that there will not soon be a surplus of comprehensive studies in which both the wind and waves are adequately characterized under a wide variety of conditions. Minimum requirements for a study of wind-wave interaction (for a stable atmosphere) include determination of wind speed profiles and of wave statistics, and at least a qualitative specification of the history of the air boundary layer. Then from plots of the "universal" velocity distribution it should be possible to decide whether the flow experiences any appreciable pressure gradient, on the average. In analyzing the velocity profile it must be remembered that only the inner 20% of the air boundary layer may be expected to exhibit logarithmic behavior. On the basis of such data a systematic classification of drag measurements could be attempted.

More refined experiments will be required if a deeper understanding of wind-wave interaction is to be gained: for example how the energy input from the air is shared among waves of different length and the relation of this sharing to local separation of the flow and to the distribution of roughness over the sea surface; and what the behavior is of the function f_3 for transitional flow over various types of wave structures.

A very profitable area of future study would be that of the initiation, growth and establishment of secondary wave structure on limited fetches, a study foreshadowed by Roll's experiments. The relative importance in wave initiation of the Eckart mechanism and of shear induced asymmetries of the waves is not known at present, and there is little quantitative data on the shape and instability of secondaries. Experiments like those with the CSL wave probe should yield reliable quantitative data on such characteristics of the secondaries for any reasonable wind condition. Ideally, measurement of wave spectra and statistics should be accompanied by measurement of the boundary layer profile of the wind and by photographs of the surface in order that the great variety of wave patterns and shapes that accompany typical unstable behavior of secondaries can at least qualitatively be characterized. Particular attention should be given to the high frequency end of the secondary spectrum in order that it may be discovered which waves and what part of the wave profiles are responsible for roughness effects on the air flow. Several methods may be available for influencing the <u>history</u> of the air boundary layer before it encounters the patch of water being studied. The characteristics of the turbulence in the air can be controlled to some extent by use of vertical wire meshes or screens. The character of the boundary layer can also be altered by replacing the natural land or water surface over which the wind is blowing by a smooth flat plate or by one of known roughness characteristics. The effect on the water that is downwind of two such strips lying side by side and parallel to the wind could be easily judged qualitatively through photographs with an exposure time of less than say 1/200 of a second.

Presumably a final description of the production of secondaries will not be achieved unless allowance is made for the microstructure of the wind. The creation of both primaries and secondaries is also affected by the degree of thermal stability or instability of the air boundary layer. It is conceivable that both of these factors could be studied in specially equipped wind-water tunnels in which air motions and temperatures can be produced that are known to be similar to those found over natural bodies of water.

There is almost no theoretical knowledge of the rotationality of wind waves, caused either by viscosity or by wind drag; for classical theory confines itself, with few exceptions, to irrotational motions. It would be useful to learn how much the viscosity of the water also affects the crest length and the statistics of the capillary-gravity waves, which damp so much more rapidly than primary waves and whose statistics also differ. The rôles of the water boundary layer and the aerodynamic interaction in determining the wave profile and type of breaking of a wave has not yet been assessed. Empirical turbulence theory that can be applied to turbulence within a water wave is in a rudimentary state, but recent advances in the theory of turbulent shear flow should stimulate a more careful analysis of the problem of wave dissipation caused by the turbulence of the water. The turbulence of the water boundary layer should be measured (techniques for this measurement are available) as a function of fetch and wind speed and also in the absence of wind in order to ascertain what turbulence if any is associated with the free wave motion of a complex wave system on the water surface. In lakes or in water wind tunnels it may be possible to construct band pass or rejection filters in the water, either of which would aid in identifying the rôle of a given band of frequencies of waves in the processes of wave growth and of energy exchange across the spectrum. This process of energy exchange should also of course be studied theoretically if possible. If turbulence in the water could be removed by vertical or horizontal grids (as it can in a wind tunnel) or, more specifically, if the scale of the turbulence present could be reduced to a dimension much less than the dominant wave length of the water waves, then the growth and decay of small wind waves could be more easily compared with theoretical predictions. (More flexible dynamical scaling could be achieved by varying the liquid medium in order to permit various combinations of the effects of viscosity and surface tension.) The solution of these problems might also, as suggested in the paper, be aided by studying the profiles of secondary waves carefully and by observing the change in form of wave packets during their propagation.

APPENDIX A

Formulae for Sinusoidal Deep Water Waves

Let us visualize a right-handed system of coordinates with the y axis vertical upward and the x and z axes lying, respectively, parallel and perpendicular to the direction of propagation of the waves and in the plane of the undisturbed water (Fig. 2). Let U, W be the components of particle velocity and m, n the wave numbers, corresponding to the wavelengths of the disturbance, along the x and z axes, and let V be the vertical component of velocity. We use subscripts o and T for long-crested waves and for (translational) drift velocity respectively. The phase velocity is C_o and the amplitude is a.* The angular frequency is denoted by $\omega = mC_o$. The density and surface tension of the water are P and $\Sigma = P\Sigma'$, and g is the acceleration of gravity.

The motion of long-crested waves (n = 0) is parallel to the x-y plane; thus W = 0. Their phase is independent of z. The particle velocity U_o varies sinusoidally at any instant as a function of x whereas the drift velocity U_{To} and, of course, the phase or wave velocity C_o are independent of x at every instant. The following equations, which are special cases** of the equations given later for short-crested waves, give the dependence upon the wave number $m = 2\pi/L$ for long-crested waves of C_o, U_o, and other quantities of interest.

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* See first footnote in Sect. 1.

** Equations (A-1 to 5) for capillary-gravity waves are given essentially by Lamb (Lam, p. 459, 626). He derives Equation A-7 (p. 419) for pure gravity waves only ($\Sigma = 0$).

$$mL = 2\pi \qquad \text{(A-1)}$$

$$\omega_o^2 = \Sigma' m^3 + gm = (mC_o)^2 \qquad \text{(A-2)}$$

$$\eta = a \sin mx \qquad \text{(A-3)}$$

$$U_o = (am) C_o e^{my} \sin mx \qquad \text{(A-4)}$$

$$V_o = -(am) C_o e^{my} \cos mx \qquad \text{(A-5)}$$

$$W_o = 0 \qquad \text{(A-6)}$$

$$U_{To} = (am)^2 C_o \quad (\Sigma' = 0) \qquad \text{(A-7)}$$

The smallest possible phase velocity of a surface wave in deep water is that of a long-crested, capillary-gravity wave, for which (Lam, p. 459)

$$m = (g/\Sigma')^{1/2} = 2\pi/L_m, \quad L_m = 1.72 \text{ cm.*} \qquad \text{(A-8)}$$

This smallest velocity C_m is given by

$$C_m^2 = 2(g\Sigma')^{1/2}, \quad C_m = 23.2 \text{ cm/sec.*} \qquad \text{(A-9)}$$

For short-crested waves $n \neq o$, and particle motions no longer are confined to planes parallel to the (x-y) plane. The phase velocity C is still a constant, but the components of particle velocity at any instant now vary sinusoidally with both x and z, and the drift velocity U_T, which is still in the x direction, varies with z. The following equations** give the dependence upon wave numbers m, n of ω, C, and of U, V, W, and U_T.

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* See Table of Constants.

**Jeffreys (Je-1, 2) derived equations for short-crested, damped, gravity-capillary waves of small amplitude. Fuchs (Fu)gave higher order theory of undamped, short-crested gravity waves. We have extended Jeffreys' results slightly to cover first-order phase and particle velocities of short-crested capillary-gravity waves in an infinite depth of water (Equations A-10 to 16). Equation A-17 for drift velocity was derived only for pure gravity waves (see Fu, Eq. 15). Equations (A-7, 17) should be used with some discretion since they make no allowance for shear flow owing to the wind drag which can produce unpredictable drift velocities as large as U_T.

$$mL = nL' = 2\pi \qquad \text{(A-10)}$$

$$\omega^2 = \Sigma' r^3 + gr = (mC)^2 \qquad \text{(A-11)}$$

$$r^2 = m^2 + n^2 \qquad \text{(A-12)}$$

$$\eta = a \sin mx \cos nz \qquad \text{(A-13)}$$

$$U = C(am)(m/r)\, e^{ry} \sin mx \cos nz \qquad \text{(A-14)}$$

$$V = -C(am)\, e^{ry} \cos mx \cos nz \qquad \text{(A-15)}$$

$$W = C(am)(n/r)\, e^{ry} \cos mx \sin nz \qquad \text{(A-16)}$$

$$U_T = (1/2)\, C\, (am)^2 \left[1 + (m^2/r^2) \cos 2nz\right] \quad (\Sigma' = 0) \qquad \text{(A-17)}$$

We note that for given values of m, a:

$$\omega \geqslant \omega_o$$

$$C \geqslant C_o$$

APPENDIX B

Damping of Surface Waves by the Viscosity of the Water

Except for very short capillary waves, the viscous damping alters but very little the form of the progressive surface waves described by the Equations in Appendix A. The parameter

$$\vartheta = r^2 \; \nu/\omega, \tag{B-1}$$

which depends on r alone and not on m and n separately is a convenient parameter of smallness to use where ν is here the kinematic viscosity of the water and r, ω, have been defined previously. For a 1 mm wave, $\vartheta \sim 7 \times 10^{-3}$. The thickness, measured in wavelengths, of the "boundary layer" within which viscous effects are appreciable is $\sim\vartheta^{1/2}$. For an uncontaminated surface, the correction at the surface to the particle velocities U, W, for a sinusoidal wave of given amplitude and frequency and owing to the viscous damping, is of order $\vartheta^{1/2}$, and the correction to the particle velocity V and to the elevation η is of order ϑ. For the case of a slick on the surface, the perturbations owing to viscous damping also are small.

We might expect that ϑ would be a function of some Reynolds number R_r. This is indeed the case, for let us put

$$r = 2\pi/L_r = \omega/C_r, \quad L_r = rL/m \tag{B-2}$$

where L_r and C_r are a "wavelength" and "propagation velocity" characteristic of the wave number r rather than of m. Then the corresponding Reynolds number is

$$R_r = C_r L_r/\nu \tag{B-3}$$

and

$$\vartheta = 2\pi/R_r \tag{B-4}$$

Again, on dimensional grounds, we might expect that the modulus of decay t_i of the surface waves would be given by

$$t_i = K_i/\nu r^2 \qquad (B-5)$$

or

$$\omega t_i = K_i/\vartheta \qquad (B-6)$$

where K_1 (no film on surface) and K_2 (with film) may depend upon ϑ. Equation (B-6) shows that $(K_i/2\pi\vartheta)$ is the number of wave periods for the amplitude to decay by a factor of $1/e$, and therefore, since ϑ is small, that this number is large.

Jeffreys (Je-1, 2) treated damping of short-crested capillary-gravity waves for a clean surface, finding that $K_1 = 1/2$. We have generalized to the case of short-crested waves Lamb's treatment (La, pp. 625-632) of the damping of long-crested, capillary-gravity waves when a compact film is on the water surface* and find that

$$K_2 = (8\vartheta)^{1/2} \qquad (B-7)$$

Our calculation, like Lamb's, was made for sinusoidal waves.

Certain statements are true whether or not there is a film on the surface. Since ω depends upon $r = (m^2 + n^2)^{1/2}$ alone we see that for any (m/n), K_i and t_i are functions of r alone. Furthermore, since ϑ or ω increase with increase of r, t_i decreases as r increases. For fixed m, t_i decreases as n increases, that is,

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* It is assumed that an oil film prevents extension of the water surface in this condition so that U = W = 0, and the shear stresses in the immediate neighborhood of the surface are increased over their values for the free surface (clean water) condition. Ewing (Ew-1) discusses the apparent reasons why films of oil or organic material form slicks on the surface that produce increased damping of the waves, but he concludes that at present "the precise relation between film compaction and ripple damping cannot be calculated". (See also Sect. 13.) The inadequacy of this classical theory has been discussed by Cox and Munk (Cox).

short-crested waves damp more rapidly for given wavelength L in the direction of propagation than do long-crested ones.

APPENDIX C

CSL Research on Small Wind Waves

The authors have begun experimental investigations of secondary wave systems which will provide data to supplement the knowledge of primary waves. Of several techniques that have been discussed an electronic method has been chosen and developed for a first series of quantitative experiments. The other techniques, which are not at present being used, would provide mainly qualitative, but important, information concerning wind wave dynamics. We may list a few of those which we feel might considerably improve our understanding of the process of wind wave generation: investigation of instability patterns on an initially calm water surface by means of observations of the reflections of a wire grid above the surface; smoke pattern studies of the air boundary layer just above the water surface, particularly in the presence of steep waves, to investigate separation; observation of talcum powder dusted over the surface to detect separated air flow regions; observation of small particles just below the water surface to detect turbulence in the initial stages of wave generation; other flow visualization methods to investigate the character of the air boundary layer, and careful measurements of its velocity profiles in conjunction with wave measurements. These are but a few of many informative experiments which have yet to be performed by some laboratory.

A discussion is given now of the electronic method, which analyzes a wind wave system at a point in conjunction with the local wind speed.* The method is based upon the use of a capacity probe, a water height sensing device suggested by the National Institute of Oceanography (England).

A dielectric-coated metal rod when placed in water acts as a variable capacity to the water, the capacity being linearly dependent on the depth of immersion providing the coating is uniform. By generating a voltage output proportional to the capacity of the rod a signal in produced which is therefore proportional to the water height. This has been achieved by employing the capacity in series with a resistor to serve as the time constant determining the repetition interval of a pulse oscillator. The pulse oscillator drives a Miller run-down, the depth of whose run-down is proportional to the repetition interval and thereby the water height. Determination of the run-down depth by a bi-directional diode switch then provides the desired signal. The signal may then be fed into a band pass filter to select only the desired frequency band from the water waveform and subsequently recorded. The local wind speed is recorded simultaneously, using a Hastings thermocouple wind gauge. The output of the capacity probe wave gauge presently being used is about 8 volts/cm, and in conjunction with a Brush recorder, displacements of 0.006 cm can be detected. The usefulness of this sensitivity depends upon the uniformity of coating, and the effect of meniscus lag in dynamic operation.

A later report will contain results of the experiments presently being conducted on a small lake.

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* The reader is referred to Control Systems Laboratory Report R-84 (Whi-2) for a full description of the instrument and, for a brief account, to a paper at the 1957 Annual Meeting of the American Physical Society.

APPENDIX D

The Meaning of Non-linearity

Wave motion with a free boundary is most often described mathematically by the Eulerian equations, from which one seeks to find, for specified boundary and initial conditions, the pressure p and the Eulerian velocity $\vec{V}$ as functions of position and time. For rotational motions the continuity equation is linear, but both the equation of motion and the condition at the free boundary are non-linear in $\vec{V}$ and its derivatives. For irrotational motion, for which we seek to find a velocity potential ϕ, one boundary condition is non-linear in the derivatives of ϕ but the equation of motion can be integrated in all cases, and the non-linearity of this equation is no longer relevant. Since, then, at least one of the exact Eulerian equations of a motion with a free boundary is non-linear, it is customary to regard surface waves as a non-linear phenomenon, except in the limit of small amplitudes where the Eulerian equations can be linearized.

We wish to call attention to a second mathematical description of surface waves which appears at first sight to be linear for any irrotational wave motion (John). In the Lagrangian description of fluid motion, the most general, time dependent, two-dimensional free surface can be described by two functions which satisfy a pair of linear partial differential equations. John shows that, with reasonable restrictions on the functions involved, to any such general motion of the free surface there corresponds an irrotational wave motion of the entire fluid.* This result possibly holds also

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* Singularities within the fluid are not excluded.

for the three-dimensional case. Thus from the Lagrangian viewpoint almost every irrotational wave can be described linearly and exactly.

The difficulty in making use of this linear description lies in the fact that the condition at the bottom cannot be specified initially but is calculated from the velocity potential after the linear partial differential equation for the surface motion has been solved. A similar situation exists in the theory of compressible flow of a gas where a hodograph transformation linearizes the partial differential equation but loses sight of the boundary conditions in the physical plane.

In spite of the fact that there are no general solutions yet of the linear Lagrangian equations which are relevant to wind waves, we propose that non-linearity should only be ascribed in the future to the effects of breaking waves, of viscosity, and of rotationality and that the irrotational surface wave motion of an inviscid fluid should be regarded as a linear phenomenon. In view of this recommendation, we have used the prefix "Eulerian" before the word non-linear at many points in the text to remind the reader that the Lagrangian description of the motion may be linear.

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