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SPREAD-SPECTRUM RANDOM-ACCESS COMMUNICATIONS FOR HF CHANNELS

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SPREAD-SPECTRUM RANDOM-ACCESS COMMUNICATIONS FOR HF CHANNELS

for the period August 21, 1980 - August 21, 1981

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ABSTRACT

A brief summary of the results obtained in research sponsored by the Naval Research Laboratory under Contract N00014-80-C-0802 is presented. The research covered several problems in the area of spread-spectrum random-access communications for fading channels. The results are applicable to the Navy's intra-task-force communications network.

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SUMMARY OF RESEARCH RESULTS

Our work in spread-spectrum communications that was supported by this contract focused on the performance of slow-frequency-hopped (SFH) spread-spectrum communications. A variety of channel models were considered to reflect varying degrees of amplitude fading and selectivity. Of primary interest is the class of slow, nonselective, Rician fading channels in which there are two components of the received signal: a non-faded or direct-path component and a faded or scatter component. The scatter component is assumed to undergo slow, nonselective, Rayleigh fading in this model, so that the Rayleigh fading channel is obtained as a special case (no direct-path component). In addition to the received signal, additive white Gaussian noise is present at the front end of the receiver, so the non-faded additive white Gaussian noise channel is also obtained as a special case of the general model (no scatter component). The effects of selective fading were also considered, especially frequency-selectivity which produces intersymbol interference. Details of the various channel models are described in Appendix A.

Several bounds and approximations for the bit error probability in a SFH spread-spectrum multiple-access system are presented in Appendix A and in [1]-[3]. Both FSK and DPSK data modulation and selective and nonselective fading channels are considered. These results are very general in nature and can be adapted to a wide range of systems and channel models.

A specific problem that can arise in a system like the intra-task-force (ITF) communications network is due to the possibility of a specular multipath signal with a relative delay greater than the dwell time of the frequency hopper. This problem, although not addressed specifically in

Appendix A, can be analyzed by the results developed in our research. The results of Appendix A are applicable to a system in which there are K simultaneous SFH signals. If there are K' simultaneous transmitters and if each signal produces one nonselective Rician faded component plus one specular multipath component, then there are $K = 2K'$ interfering signals. If the relative delay of the specular multipath component exceeds the dwell time then the bounds and approximations given in Appendix A apply with $K = 2K'$. In such a system there are $K-1 = 2K'-1$ interfering signals for each receiver.

Application of the results of Appendix A to such a system is illustrated by the data of Table 1. The only reason for presenting numerical results for this special situation is that perhaps larger values of K are of interest than for a system without the specular multipath components. The results of Table 1 are for the same model as described in Appendix A, and the notation is exactly as used in Table 2 of Appendix A. The approximation P_G given in Table 1 is a new result (obtained after [4] was submitted for publication), which we believe to be slightly more accurate than the approximation $\bar{P}_A^{(i)}$ described in Appendix A. Both P_G and $\bar{P}_A^{(i)}$ are for channels with fading which is slow relative to the hopping rate (case (i) described on page 10 of Appendix A).

The methods and results developed in Appendix A can also be applied to determine the probability of error in a coded SFH spread-spectrum system. For a fully interleaved system these results can be applied directly. This is because the interleaving breaks up the error bursts due to the fading and multiple access interference, in which case the bit error probability is the performance measure of interest. Thus, for interleaved systems the performance of various codes can be determined from the results given in Appendix A and published data on the performance of the codes for the binary memoryless channel.

Table 1. Bit error probability for nonselective Rayleigh fading and specular multipath.

a) $K = 10$ ($K' = 5$), $q = 100$, and $N_b = 10$

$\bar{\mathcal{E}}/N_0$ (dB)	$\tilde{P}_L$	P_G	$\tilde{P}_A$	P_U
6	0.161	0.175	0.182	0.199
8	0.118	0.131	0.137	0.156
10	0.085	0.096	0.102	0.123
12	0.060	0.071	0.076	0.098
15	0.036	0.046	0.051	0.074
20	0.018	0.028	0.032	0.056
∞	0.009	0.019	0.023	0.047

b) $K = 20$ ($K' = 10$), $q = 100$, and $N_b = 10$

$\bar{\mathcal{E}}/N_0$ (dB)	$\tilde{P}_L$	P_G	$\tilde{P}_A$	P_U
6	0.153	0.187	0.193	0.230
8	0.115	0.145	0.151	0.192
10	0.085	0.113	0.118	0.162
12	0.063	0.089	0.093	0.140
15	0.042	0.066	0.069	0.119
20	0.025	0.049	0.052	0.103
∞	0.018	0.041	0.043	0.095

c) $K = 50$ ($K' = 25$), $q = 250$, and $N_b = 10$

$\bar{\mathcal{E}}/N_0$ (dB)	$\tilde{P}_L$	P_G	$\tilde{P}_A$	P_U
6	0.157	0.188	0.191	0.227
8	0.119	0.147	0.149	0.189
10	0.088	0.114	0.116	0.158
12	0.066	0.090	0.091	0.136
15	0.044	0.067	0.068	0.114
20	0.028	0.050	0.050	0.098
∞	0.020	0.043	0.041	0.090

Table 2. Bit error probabilities for uncoded and coded SFH systems with nonselective Rayleigh fading. ($K = 15$, $q = 1000$, and $N_b = 40$)

$\bar{\mathcal{E}}/N_0$ (dB)	$\tilde{P}_A$	P_b
14	4.0×10^{-2}	1.1×10^{-1}
16	2.7×10^{-2}	1.1×10^{-3}
18	1.9×10^{-2}	1.7×10^{-8}
20	1.3×10^{-2}	4.0×10^{-15}

For a system without full interleaving the bit errors are not independent and thus the bit error probability does not completely describe the channel performance. However, we have evaluated the performance of certain Reed-Solomon codes with partial interleaving, and typical results are given in Table 2. The approximation $\tilde{P}_A$ to the probability of error for an uncoded system is compared with the bit error probability P_b for a system which uses a (255,127) Reed-Solomon code with partial interleaving. Notice that for values of $\bar{\mathcal{E}}/N_o$ greater than 18 dB, the coded system gives several orders of magnitude improvement in the bit error rate. Further work on the performance of coded SFH spread-spectrum systems is in progress (primarily under other sponsorship).

The data in Table 2 gives a comparison between the performance of uncoded and Reed-Solomon coded systems. Another interesting comparison is the performance of a Reed-Solomon coded SFH system for two different sets of assumptions on the frequency hopping and interleaving: (i) no frequency hopping and no interleaving vs. (ii) frequency hopping with interleaving of the code symbols. The channel model that we consider for this comparison is the very slow, nonselective Rayleigh fading channel. In case (i) we assume that the instantaneous power in the received signal is constant for the duration of the code word, but in case (ii) the instantaneous power is constant for the duration of a code symbol but (because of hopping and interleaving) the power levels for different symbols in the same code word are independent. In Table 3 numerical values for the <u>block</u> error probability are presented for the (31,15) and (255,127) Reed-Solomon codes. The probabilities $P_E^{(i)}$ and $P_E^{(ii)}$ are the block error probabilities for cases (i) and (ii), respectively. The data is for a system with only one transmitter ($K = 1$) in order to isolate the effects of fading from the effects of multiple-access interference.

Table 3. Block error probabilities for a coded SFH system with nonselective Rayleigh fading.

a) (31,15) Reed Solomon code

$\bar{\mathcal{E}}/N_0$ (dB)	$P_E^{(i)}$	$P_E^{(ii)}$
20	8.29×10^{-2}	8.44×10^{-5}
22	5.31×10^{-2}	2.50×10^{-6}
24	3.39×10^{-2}	5.92×10^{-8}
26	2.15×10^{-2}	1.21×10^{-9}
30	8.63×10^{-3}	3.97×10^{-13}

b) (255,127) Reed Solomon code

$\bar{\mathcal{E}}/N_0$ (dB)	$P_E^{(i)}$	$P_E^{(ii)}$
15	2.85×10^{-1}	8.86×10^{-2}
16	2.34×10^{-1}	1.50×10^{-3}
17	1.90×10^{-1}	3.52×10^{-6}
18	1.55×10^{-1}	1.53×10^{-9}
19	1.25×10^{-1}	1.65×10^{-13}
20	1.01×10^{-1}	5.62×10^{-18}

A significant area of progress in the random-access area under this contract has been the design and analysis of retransmission control policies for a random-access broadcast channel [5]-[7], [9]. The policies can be implemented in a distributed fashion. Analysis of delay and throughput is provided in these papers using the concept of local Poisson approximation which is introduced in these papers.

Versions of the recursive retransmission control policies which are relatively insensitive to the traffic statistics, and modifications which reduce feedback information requirements are also reported in [5].

It is proven in [7] that the retransmission policies in [5] provide stable throughput at rates of up to e^{-1} packets per slot. Moreover, a general methodology for proving such stability results is provided in [7] and the methods are also applied in [7] to prove a strong stability property of G/G/1 queues which is of general interest for queueing network studies.

Even though the papers [5]-[7], [9] do not deal explicitly with a spread-spectrum system, they were developed for spread-spectrum applications because these papers assume that channel feedback information is very limited, which is characteristic of spread-spectrum systems. Indeed, in the appendices of this report the traffic intensity vs. packet error probability tradeoff (Appendix B) and a possible implementation of recursive retransmission procedures as in [5] (Appendix C) are each given in the context of a FH-system such as the Navy's intra-task-force communications network.

In Appendix D some results from [9] are summarized. In this paper the delay/throughput tradeoff of a random-access system is studied under the assumption of a very limited amount of feedback. It is found that there is a potential for instabilities if the feedback information is insufficient.

In [8] we developed a numerical method for finding the invariant distribution for a class of Markov processes. The method is useful for performance evaluation of certain random access strategies, as shown in [10]. In Appendix E the method of [8] is outlined and some of the results from [10] are summarized.

PUBLICATIONS ACKNOWLEDGING NRL CONTRACT

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[2] E. A. Geraniotis, "Error Probability for Coherent Hybrid Slow-Frequency-Hopped Direct-Sequence Spread-Spectrum Multiple-Access Communications," 1981 IEEE National Telecommunications Conference, Conference Record (Nov. 1981, to appear).

[3] E. A. Geraniotis and M. B. Pursley, "Error Probability Bounds for Slow-Frequency-Hopped Spread-Spectrum Multiple-Access Communications over Fading Channels," IEEE International Conference on Communications, Conference Record, vol. 4, pp. 76.3.1-7 (June 1981).

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APPENDIX A

ERROR PROBABILITIES FOR SLOW-FREQUENCY-HOPPED SPREAD-SPECTRUM MULTIPLE-ACCESS COMMUNICATIONS OVER FADING CHANNELS

(by E. A. Geraniotis and M. B. Pursley, accepted for publication in the IEEE Transactions on Communications)

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I. INTRODUCTION

Several communications systems currently being developed have the following common features. Frequency-hopped spread-spectrum modulation is employed with a hopping rate not greater than the data rate. Multiple-access capability is required, because with high probability two or more terminals will be transmitting simultaneously. During transmission the spread-spectrum signals encounter severe fading, which causes reduced signal strength and may produce intersymbol interference or other dispersive effects. These systems are described in current terminology as slow-frequency-hopped (SFH) spread-spectrum multiple-access (SSMA) communications systems with fading channels.

In this paper we present bounds and approximations for the average probability of error for SFH/SSMA communications over fading channels. Two important classes of fading models are considered: the class of nonselective Rician fading channels--which includes the additive white Gaussian noise channel and the nonselective Rayleigh fading channel as special cases--and the selective wide-sense-stationary uncorrelated-scattering fading channel. The data modulation is binary frequency-shift keying (FSK), but many of the results apply to differential phase-shift-keying (DPSK) as well. Noncoherent demodulation of the data is employed, partly because we do not require coherent frequency hopping and dehopping. The communications network is assumed to be asynchronous; that is, a given terminal makes no attempt to coordinate its transmissions with those of other terminals. This may be due to the lack of an accurate timing reference or because of the variation in propagation times among the different communication paths in the network. The point here is that even if the transmitters have a common clock they cannot adjust their transmission times to provide coordinated arrival times at all of the receivers in the network.

In the analysis of SFH/SSMA systems there are two approaches to the modeling of the frequency hopping patterns: general random-process models may be employed or specific (deterministic) sets of hopping patterns may be considered. The random-process models are often used in an attempt to match certain characteristics of extremely complex hopping patterns which have very long periods. Also random-process models serve as substitutes for deterministic models when the communications engineer is given little or no information about the structure of the hopping patterns to be used in the system. Both random patterns and a special class of deterministic patterns (based on Reed-Solomon codes) are considered in this paper.

The results obtained in this paper are bounds and approximations for the bit error probability. These results are useful for both uncoded FH/SSMA systems and fully-interleaved coded FH/SSMA systems. For coded systems which employ random-error-correcting codes, full interleaving is usually necessary for satisfactory performance. We have also obtained results (similar to those presented in Section III) on the probability of error for FH/SSMA systems which employ certain burst-error-correcting codes and "partial interleaving", but this topic is beyond the scope of the present paper.

A brief outline of the paper is as follows. The model for the SFH/SSMA system is presented in Section II where our models for the various subsystems and signals are described. The effect of nonselective fading on the probability of error in a SFH/SSMA system is considered in Section III. A more precise analysis is given in Section IV for the special case in which the channel exhibits nonselective Rayleigh fading. Finally, selective fading is considered in Section V.

II. SYSTEM MODEL

The transmitter for the slow-frequency-hopped spread-spectrum signal is shown in Figure 1. There are K such transmitters in the spread-spectrum multiple-access system. The k-th data signal $b_k(t)$ is a sequence of positive and negative rectangular pulses of duration T. The amplitude of the ℓ-th pulse for the k-th signal is denoted by $b_\ell^{(k)}$ (i.e., $b_k(t) = b_\ell^{(k)}$ for $\ell T \le t < (\ell+1)T$), and $b_\ell^{(k)}$ is either +1 or -1 for each k and ℓ. The data signal $b_k(t)$ is the input to an FSK modulator, and the corresponding output is

$$c_k(t) = \cos\{2\pi[f_c + b_k(t)\Delta]t + \theta_k(t)\} \tag{1}$$

where Δ is one-half the spacing between the two FSK tones. The signal $\theta_k(t)$ is the phase signal introduced by the FSK modulator; that is, if $b_\ell^{(k)} = m$ then $\theta_k(t) = \theta_{k,m}$ for $\ell T \le t < (\ell+1)T$ where $\theta_{k,m}$ is the phase of the tone at frequency $f_c + m\Delta$ for m = +1 or m = -1.

The FSK signal is then frequency-hopped according to the k-th hopping pattern $f_k(t)$ which is derived from a sequence $(f_j^{(k)}) = \ldots, f_{-1}^{(k)}, f_0^{(k)}, f_1^{(k)}, \ldots$ according to

$$f_k(t) = f_j^{(k)}, \quad jT_h \le t < (j+1)T_h. \tag{2}$$

The parameter T_h is the time between hops (also called the dwell time). For slow-frequency-hopping T_h is an integer multiple of T. The frequencies $f_j^{(k)}$ are all from the set $S = \{\nu_n : 1 \le n \le q\}$ which is ordered such that $\nu_n < \nu_{n+1}$ for each n. Let Δ' be the minimum spacing between the frequencies in the set S, and let $N_b \triangleq T_h/T$ be the number of data bits per hop.

The band-pass filter shown in Figure 1 removes unwanted frequency components present at the output of the multiplier. The signal at the

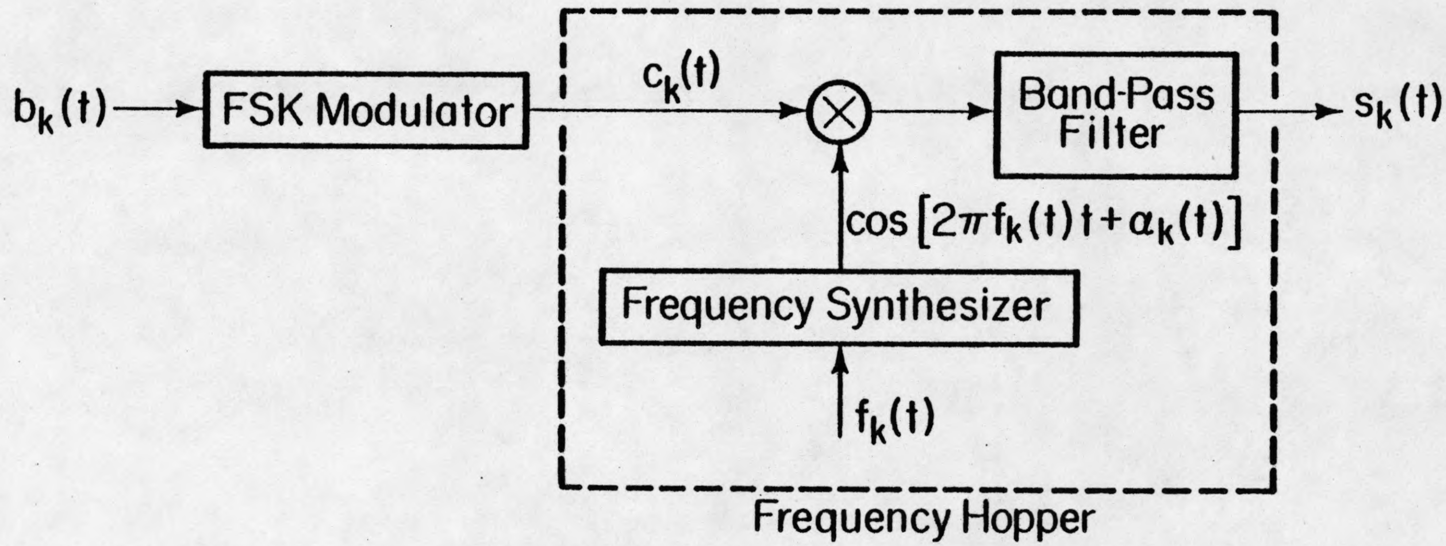

Figure 1. Transmitter Model.

output of the filter is

$$s_k(t) = \sqrt{2P}\cos[2\pi\tilde{f}_k(t)t + \tilde{\varphi}_k(t)], \tag{3}$$

where

$$\tilde{f}_k(t) = f_c + b_k(t)\Delta + f_k(t) \tag{4}$$

and

$$\tilde{\varphi}_k(t) = \theta_k(t) + \alpha_k(t). \tag{5}$$

The signal $\alpha_k(t)$ represents the phase shifts introduced by the frequency hopper as it switches from one frequency to another. Accordingly, $\alpha_k(t)$ is constant on the time intervals that $f_k(t)$ is constant. Let $\alpha_j^{(k)}$ denote the value of $\alpha_k(t)$ on the interval $[jT_h, (j+1)T_h)$.

The quantity P that appears in (3) is the power of the k-th signal at the receiver in the absence of fading. In order to account for fading, we will multiply P by a suitable factor to obtain the average power in the received signal. For simplicity we have assumed that the signals $s_k(t)$ all have the same power. However, as we will point out later, we obtain error probability bounds that are valid even if the power levels are not equal.

Since we are considering an asynchronous multiple-access system, we allow an arbitrary time delay τ_k for the k-th communication link ($1 \le k \le K$). Thus the received signals are $s_k(t-\tau_k)$, $1 \le k \le K$. For the random hopping patterns that will be considered in subsequent sections, it is sufficient to consider time delays modulo T_h. In order to allow for the possibility of deterministic periodic hopping patterns, we consider time delays modulo NT_h where N is the period of the patterns for deterministic hopping patterns or $N = 1$ for random hopping patterns. Thus we may restrict attention to time delays in the range $0 \le \tau_k < NT_h$. Similarly we are only concerned with phase angles modulo 2π, so we may restrict attention to phase angles in the interval $[0,2\pi]$.

The analysis presented in this paper does not account for adjacent channel interference in the frequency-hopping system or for interference between the two FSK tones of a given signal. Instead we are primarily concerned with multiple-access interference and the effects of fading such as intersymbol interference and reduced signal strength. In order to focus on multiple-access interference and fading, we made certain simplifying assumptions concerning the frequency spacings Δ and Δ'. It is enough for our purposes to have

$$\Delta' \gg \Delta + T^{-1} \tag{6a}$$

and

$$\Delta \gg T^{-1}. \tag{6b}$$

However, it is possible to relax these conditions somewhat, expecially for nonselective fading. For example if the fading is nonselective then it is sufficient to replace the constraint $\Delta \gg T^{-1}$ by the condition $\Delta = m/2T$ positive integer m (the case $m = 1$ is of greatest interest). In the absence of time-selective fading our results are valid if Δ' is about $3(\Delta + T^{-1})$ or larger, and they are likely to be fairly good approximations even if $\Delta' \approx 2(\Delta + T^{-1})$. However, frequency dispersion can expand the signal bandwidths so that $\Delta' \gg \Delta + T^{-1}$ is needed for time-selective fading.

Under our assumptions, the frequency band that contains the signals $s_k(t)$ is approximately the band from $f_c + \nu_1 - \Delta$ to $f_c + \nu_q + \Delta$. The center of this band is at frequency $f_c' = f_c + \frac{1}{2}(\nu_q - \nu_1)$. The (one-sided) bandwidth W is approximately $\nu_q - \nu_1 + 2\Delta$. Under our assumptions $W \approx \nu_q - \nu_1 \geq (q-1)\Delta'$.

In the absence of fading and noise the received signal is given by

$$s(t) = \sum_{k=1}^{K} s_k(t - \tau_k). \tag{7}$$

We focus our attention on the receiver for the i-th signal, and in doing so we may select the time reference such that $\tau_i = 0$. The variables τ_k are then delays (modulo NT_h) relative to this time reference.

The receiver for the i-th signal is shown in Figure 2. The received signal $\tilde{s}(t)$, which is a faded version of $s(t)$, is the input to the first band-pass filter. This filter has center frequency approximately f_c' and bandwidth approximately W so $\tilde{s}(t)$ is passed without distortion. This filter is followed by the i-th dehopper which is synchronized in frequency and time to the i-th frequency-hopping signal $f_i(t)$. The dehopper introduces a phase signal $\beta_i(t)$ which is analogous to the phase signal $\alpha_i(t)$ introduced by the frequency hopper. The phase signal $\beta_i(t)$ is constant during the time intervals between hops (i.e., when $f_i(t)$ is constant). The constant value of $\beta_i(t)$ for $jT_h \leq t < (j+1)T_h$ is denoted by $\beta_j^{(i)}$.

The time delays, phase angles, and data symbols are modeled as mutually independent random variables each of which is uniformly distributed on the appropriate set (cf. [4] or [6]). The random time delays are the random variables τ_k. The random phase angles that are of primary interest are $\theta_{k,m}$, $\alpha_j^{(k)}$, and $\beta_j^{(i)}$. An important feature of our model for asynchronous spread-spectrum multiple-access systems is that addition of phase angles is modulo-2π addition. This feature is critical to our assertions concerning

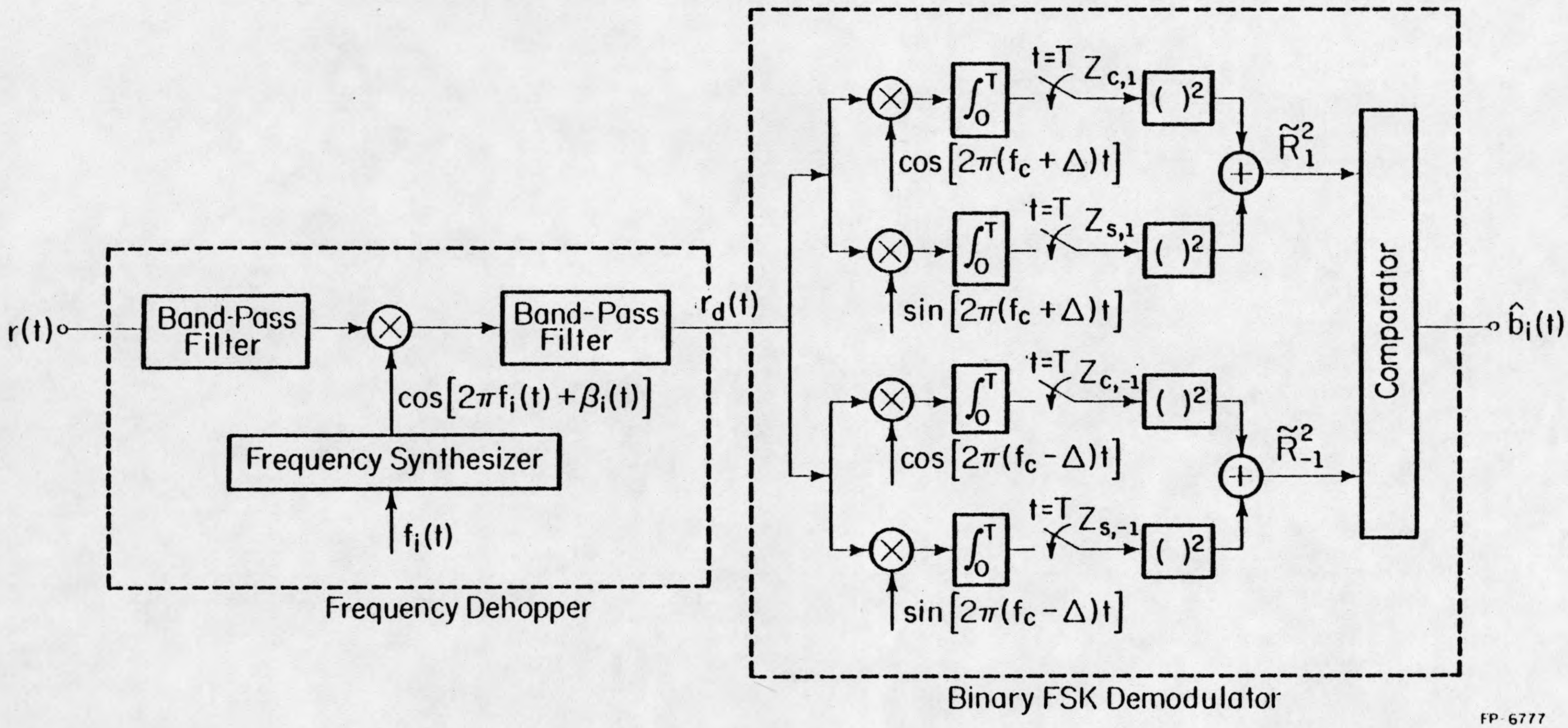

Figure 2. Receiver Model.

the distributions and the statistical independence of the phase angles (the basis for these assertions is given on pp. 159-160 of [6]).

The output of the dehopper is then passed through a band-pass filter which is designed to remove certain unwanted signals such as the double-frequency components of the i-th signal itself, the sum and difference frequency components due to the other K-1 signals (except, of course, those that happen to be at the same frequency as the i-th signal), and the thermal noise that is outside the frequency band occupied by the i-th signal. The bandwidth B of this band-pass filter is less than Δ' but usually larger than $2(\Delta + T^{-1})$. If $(\Delta + T^{-1}) \ll B < \Delta'$ then the thermal noise present at the output of the band-pass filter which follows the dehopper has a bandwidth larger than that of the FSK demodulator. This simplifies the analysis of the demodulator.

As shown in Figure 2 the FSK demodulator has two branches. Each branch forms a statistic $\tilde{R}_m^2$ where $m = 1$ corresponds to the upper branch and $m = -1$ corresponds to the lower branch. Each of these two branches has two components. In the in-phase component the signal is multiplied by $\cos[2\pi(f_c + m\Delta)t]$, and the quadrature component it is multiplied by $\sin[2\pi(f_c + m\Delta)t]$.

Consider the reception of the data bit $b_\ell^{(i)}$. The outputs of the in-phase components of the two branches are given by

$$Z_{c,m} = \int_{\ell T}^{(\ell+1)T} r_d(t)\cos[2\pi(f_c + m\Delta)t]dt \tag{8}$$

for $m = \pm 1$, where $r_d(t)$ is the output of the band-pass filter which follows the i-th dehopper (i.e., $r_d(t)$ is the input to the i-th FSK demodulator). Notice that in general $Z_{c,m}$ depends on both ℓ and i.

However, if the random hopping patterns are stationary and identically distributed and the fading process is stationary and not frequency selective, then the distribution of the random variable $Z_{c,m}$ will not depend on either ℓ or i. In case the hopping patterns are deterministic or the fading is frequency selective then we provide upper bounds on the probability of error which are independent of ℓ and i. The outputs of the quadrature components of the two branches are denoted by $Z_{s,m}$ for $m = \pm 1$. The random variables $Z_{s,m}$, which are defined by (8) with $\cos[\cdot]$ replaced by $\sin[\cdot]$, have the same properties as $Z_{c,m}$.

III. PERFORMANCE OF FH/SSMA SYSTEM WITH NONSELECTIVE FADING

The channels considered in this section are the nonselective slow-fading channels. For the frequency-hopped spread-spectrum system described in the previous section this means that the signal at the input to the first band-pass filter in the i-th receiver (see Figure 2) is

$$r(t) = n(t) + \sum_{k=1}^{K} y_k(t-\tau_k), \tag{9}$$

where for $\ell T \le t < (\ell+1)T$ the signal $y_k(t)$ is given by

$$y_k(t) = \sqrt{2P}\, A_\ell^{(k)} \cos[2\pi\tilde{f}_k(t)t + \tilde{\varphi}_k(t) + \Phi_\ell^{(k)}]. \tag{10}$$

The thermal noise $n(t)$ is white Gaussian noise with spectral density $\frac{1}{2}N_0$. Notice from comparisons of (9) and (10) with (7) and (3), respectively, that $y_k(t)$ is a faded version of $s_k(t)$ and, in the absence of noise, $r(t)$ is $\tilde{s}(t) = \Sigma_{k=1}^{K} y_k(t-\tau_k)$ which is a faded version of $s(t)$.

The amplitude of the fading signal $y_k(t)$ during the time interval $\ell T \le t < (\ell+1)T$ is represented by a nonnegative random variable $A_\ell^{(k)}$, and the phase shift due to the fading is denoted by $\Phi_\ell^{(k)}$. In this section the only assumption that we make concerning the signal amplitudes is that they are constant during the data bit interval. The sequence of amplitudes $(A_\ell^{(k)}) = \ldots, A_{-1}^{(k)}, A_0^{(k)}, A_1^{(k)}, \ldots$ may be any stationary random sequence. In particular we place <u>no</u> <u>restrictions</u> on the statistical dependence of amplitudes in different data bit intervals. Consider the set $\{A_\ell^{(k)} : jN_b \le \ell < (j+1)N_b\}$ of amplitudes for the data bits that are transmitted during the j-th hopping interval $[jT_h, (j+1)T_h)$. This interval contains the data bits $b_\ell^{(k)}$ for $jN_b \le \ell < (j+1)N_b$. Among the cases of interest are the two extreme cases (i) $A_{\ell}^{(k)} = A_{m}^{(k)}$ for all ℓ and m in the same hopping interval and (ii) $A_{\ell}^{(k)}$ and $A_{m}^{(k)}$ are independent if $\ell \neq m$ but ℓ and m are in the same hopping interval. Case (i) corresponds a system with no interleaving and a channel with slow fading relative to the hopping rate. An example of case (ii) arises in a system which is fully interleaved (e.g. if a random-error-correcting code is to be employed). Although these are the two specific cases of greatest interest, there is no need to restrict attention to such special cases in this section. Similarly, we impose no restrictions on the phase sequence $(\Phi_{\ell}^{(k)})$; all that is required is a constant value for the phase during the data bit intervals. Notice from (1)-(5) that for $\ell T \leq t < (\ell+1)T$ the phase of the signal in (10) is given by

$$\Theta_{\ell}^{(k)} = \theta_{k,m} + \alpha_{j}^{(k)} + \Phi_{\ell}^{(k)}, \tag{11}$$

where j is the integer part of ℓ/N_b. Under quite general conditions the phase $\Theta_{\ell}^{(k)}$ is uniformly distributed on $[0,2\pi]$ because the addition in (11) is modulo-2π. For example, it is enough to assume that one of the phase angles which appears on the right-hand side of (11) is uniformly distributed and that they are mutually independent (see pp. 159-160 of [6] for the rationale for this statement).

There are two different phenomena which contributed to errors in the system under consideration. First, even in the absence of noise and fading, errors may occur in a frequency-hopped spread-spectrum multiple-access system when a signal is hopped to a frequency slot that is occupied by another signal. Whenever two different signals simultaneously occupy one frequency slot we say a hit occurs. Second, even in the absence of hits, errors may occur due to the fading and additive noise. The first step in analyzing the overall probability of error is to evaluate the probability of a hit for various types of hopping patterns.

A. Probability of a Hit

Consider as before the i-th receiver during reception of the ℓ-th data bit. For a nonselective fading channel we say that a hit from the k-th signal occurs during the ℓ-th data bit if

$$f_k(t-\tau_k) = f_i(t) \tag{12}$$

for at least one value of t in the ℓ-th data bit interval $[\ell T,(\ell+1)T)$. As pointed out in Section II, we can let $N=1$ in considering stationary random hopping patterns. It follows that the probability $\mathcal{P}_\ell^{(k)}$ of a hit from the k-th signal during the ℓ-th data bit interval does not depend on ℓ for such patterns. If the K hopping patterns $\{(f_j^{(k)}) : 1 \le k \le K\}$ are also mutually independent and identically distributed then $\mathcal{P}_\ell^{(k)}$ does not depend on k either, and hence we denote it by $\mathcal{P}$ for such patterns. We first consider two different models for stationary random hopping patterns and give the value of $\mathcal{P}$ for each case.

Suppose the random process $(f_j^{(k)})$ is a stationary Markov process with transition probabilities given by

$$P(f_{j+1}^{(k)} = \nu_n \mid f_j^{(k)} = \nu_r) = (q-1)^{-1} \tag{13}$$

for $1 \le n \le q$, $1 \le r \le q$, and $n \ne r$. It follows that for these patterns

$$P(f_{j+1}^{(k)} = f_j^{(k)}) = 0 \tag{14}$$

and hence

$$\mathcal{P} = \frac{1}{q}(1 + \frac{1}{N_b}) \ . \tag{15}$$

Because of (13) the process $(f_j^{(k)})$ is a random process with first-order distribution given by

$$P(f_j^{(k)} = \nu_n) = q^{-1}, \quad 1 \leq n \leq q. \tag{16}$$

If instead of (13) we consider random hopping patterns for which $f_{j+1}^{(k)}$ is independent of $f_j^{(k)}$ and the distribution of $f_j^{(k)}$ is given by (16) for each j, then (14) should be replaced by

$$P(f_{j+1}^{(k)} = f_j^{(k)}) = q^{-2} \ , \tag{17}$$

and thus the probability of a hit is

$$\mathcal{P} = \frac{1}{q}[1 + \frac{1}{N_b}(1 - \frac{1}{q})] \ . \tag{18}$$

Notice that

$$\mathcal{P} \leq \frac{1}{q}(1 + \frac{1}{N_b}) \tag{19}$$

and if q is large then

$$\mathcal{P} \approx \frac{1}{q}(1 + \frac{1}{N_b}) \tag{20}$$

(cf. equation (15)). Thus for large q these <u>memoryless</u> hopping patterns give approximately the same probability of a hit as the first-order Markov patterns.

In general for a set of deterministic hopping patterns the probability $\mathcal{P}_{\ell}^{(k)}$ depends on both k and ℓ. One set of deterministic hopping patterns that has very good properties is derived from a Reed-Solomon code, so we refer to it as a set of Reed-Solomon (RS) patterns [7]. Given a prime number q of frequency slots, the particular set of RS patterns of interest here consists of N = q-1 sequences of period N (of course we can always choose a subset if fewer patterns are needed). Each sequence is nonrepeating; that is, for each sequence (f_j), $\delta(f_j, f_n) = 0$ for $n \neq j$ and $0 \leq n \leq N-1$, where

$$\delta(u,v) = \begin{cases} 1 & u = v \\ 0 & u \neq v \end{cases} \quad . \tag{21}$$

The property of RS patterns that is of primary importance here is that for any two patterns $(f_j^{(k)})$ and $(f_j^{(i)})$,

$$\sum_{n=0}^{N-1} \delta(f_n^{(k)}, f_j^{(i)}) \leq 1 \tag{22}$$

for each j. Property (22) is actually valid for any set of nonrepeating patterns.

Since τ_k is uniform on $[0, NT_h]$ for $k \neq i$, then it follows from (22) that

$$\mathcal{P}_{\ell}^{(k)} \leq \mathcal{P} \triangleq \frac{T_h + T}{NT_h} = \frac{1}{N}\left(1 + \frac{1}{N_b}\right) = \frac{1}{q-1}\left(1 + \frac{1}{N_b}\right) . \tag{23}$$

Actually (22) implies the stronger statement that either $\mathcal{P}_{\ell}^{(k)} = 0$ or $\mathcal{P}_{\ell}^{(k)} = \mathcal{P}$. Since the number of frequency slots q is larger than the period N = q-1, then it is possible to choose the hopping pattern $(f_j^{(k)})$ such that $\mathcal{P}_{\ell}^{(k)} = 0$ for N_b different values of ℓ in the range $0 \leq \ell \leq NN_b$.

Notice that for large q

$$\mathcal{P} \approx \frac{1}{q}\,(1+\frac{1}{N_b}) \tag{24}$$

is a good approximation for the upper bound (cf. (15) and (20)).

Of primary interest for our subsequent analysis is the probability $\hat{\mathcal{P}}_\ell$ of one or more hits from the K-1 signals (corresponding to $k \neq i$) during the ℓ-th data bit interval. For stationary <u>random</u> patterns $\hat{\mathcal{P}}_\ell$ does not depend on ℓ so we denote it by $\hat{\mathcal{P}}$. If the patterns are also mutually independent and identically distributed then

$$\hat{\mathcal{P}} = 1 - (1-\mathcal{P})^{K-1}\ , \tag{25}$$

where $\mathcal{P}$ is the probability of a hit from a given signal. For the first-order Markov patterns (25) and (15) imply

$$\hat{\mathcal{P}} = 1 - \{1 - \frac{1}{q}\,(1+\frac{1}{N_b})\}^{K-1}\ . \tag{26}$$

If the patterns are sequences of independent random variables (i.e. memoryless patterns) satisfying (16) then

$$\hat{\mathcal{P}} = 1 - \{1 - \frac{1}{q}[1+\frac{1}{N_b}(1-\frac{1}{q})]\}^{K-1}\ . \tag{27}$$

Next we consider the probability $\hat{\mathcal{P}}_\ell$ of one or more hits in the ℓ-th data bit interval for <u>deterministic</u> patterns. Since the random variables τ_k, $k \neq i$, are mutually independent, then for any deterministic hopping patterns

$$\hat{\mathcal{P}}_\ell = 1 - \left\{\prod_{\substack{k=1\\k\neq i}}^{K} [1 - \mathcal{P}_\ell^{(k)}]\right\}\ . \tag{28}$$

For RS patterns (23) implies

$$\hat{\mathcal{P}}_\ell \leq \hat{\mathcal{P}} \triangleq 1 - [1 - \mathcal{P}]^{K-1} = 1 - \{1 - \frac{1}{q-1}(1+\frac{1}{N_b})\}^{K-1} , \quad (29)$$

where the symbol $\hat{\mathcal{P}}$ is used to denote an upper bound. Notice from (27) and (29) that for large q

$$\hat{\mathcal{P}} \approx 1 - \{1 - \frac{1}{q}(1+\frac{1}{N_b})\}^{K-1} \quad (30)$$

for the sequences of independent random elements and the RS sequences. Notice from (26) that the expression given in (30) is the exact value of $\hat{\mathcal{P}}$ for the first-order Markov patterns.

B. Bounds and Approximations for the Probability of Error

For a nonselective fading channel the bit error probability $P_{e,\ell}$ in a slow-frequency-hopped spread-spectrum multiple-access communications system can be written as

$$P_{e,\ell} = P_0(1 - \hat{\mathcal{P}}_\ell) + P_{1,\ell}\,\hat{\mathcal{P}}_\ell , \quad (31)$$

where P_0 is the conditional probability of error for the ℓ-th bit given that there are no hits and $P_{1,\ell}$ is the conditional probability of error for the ℓ-th data bit given there is at least one hit. Notice that P_0 does not depend on ℓ. In general $P_{1,\ell}$ depends on ℓ but, as will be seen from the numerical results, it is sufficient for many purposes to use the bounds $0 \leq P_{1,\ell} \leq \frac{1}{2}$.

Recall that for stationary random hopping patterns $\hat{\theta}_\ell$ does not depend on ℓ (and hence it is denoted by $\hat{\theta}$). For RS patterns $\hat{\theta}_\ell$ depends on ℓ but its upper bound $\hat{\theta}$ given by (29) does not. Hence for all of these patterns we have the lower bound

$$P_{e,\ell} \geq P_L \triangleq P_0(1-\hat{\theta}) \tag{32}$$

and the upper bound

$$P_{e,\ell} \leq P_U \triangleq P_L + \tfrac{1}{2}\hat{\theta} = P_0 + (\tfrac{1}{2} - P_0)\hat{\theta} \tag{33}$$

where $\hat{\theta}$ is given by (26), (27), or (29), depending on which type of hopping patterns are employed. The lower bound is the same as we previously presented in [5], but the upper bound of (33) is a slight improvement of the upper bound presented in [5].

It is tempting to use $P_{e,\ell} \geq P_0$ in place of (32), and we certainly believe this tighter lower bound to be valid for independent time delays, data streams, and hopping patterns. Under these conditions it is intuitively clear that multiple-access interference cannot decrease the average probability of error. However, the lower bound of (32) has the advantage that it holds under more general conditions (such as for dependent time delays, data streams, and hopping patterns).

The bounds given in (32) and (33) are valid even if the power levels are not the same for the various signals or the hopping patterns are statistically dependent. As might be expected, the imposition of additional restrictions on the system leads to more precise results. In Section IV we present such results for a more restrictive channel model. However, even with the full generality of the nonselective fading channel model considered in this section, we can improve the lower bound and obtain a useful approximation if we consider equal power signals and add certain constraints on the hopping patterns and the binary data streams. The hopping patterns are assumed to be stationary, mutually independent, identically distributed random patterns, and the data sequences are stationary, memoryless, independent random sequences with distribution given by $P(b_n^{(k)} = m) = \frac{1}{2}$ for $m = +1$ and $m = -1$.

The lower bound can be improved for such systems by providing a nonzero lower bound for the term $P_{1,\ell}\hat{\mathscr{P}}_\ell$ of (31). One such bound is obtained as follows. Consider the conditional probability of error in the ℓ-th data bit given a "full" hit from the k-th signal (i.e., given that (12) holds for <u>all</u> t in $[\ell T, (\ell+1)T))$ and given the k-th signal transmits $-b_\ell^{(i)}$ for the <u>two</u> consecutive bit intervals of interest. This conditional probability of error is equal to $\frac{1}{2}$. The conditional probability of a "full" hit (given a hit has occurred) is not smaller than $(N_b-1)/(N_b+1)$, and the probability of two consecutive transmissions of a particular tone is $\frac{1}{4}$. Finally, we use the fact that (25) implies

$$\hat{\mathscr{P}}_\ell \geq (K-1)\mathscr{P}(1-\mathscr{P})^{K-2},$$

which is just the statement that the probability of one or more hits is not less than the probability of exactly one hit. From the above we conclude that

$$P_{1,\ell}\hat{\mathscr{P}}_\ell \geq \frac{(N_b-1)}{8(N_b+1)}(K-1)\mathscr{P}(1-\mathscr{P})^{K-2},$$

so that the improved lower bound is

$$P_{e,\ell} \geq \tilde{P}_L \triangleq P_L + \frac{(N_b-1)}{8(N_b+1)}(K-1)\mathscr{P}(1-\mathscr{P})^{K-2}. \qquad (34)$$

We use tilde ($\tilde{}$) to denote bounds and approximations which are valid for the restricted class of systems only (i.e., equal power signals, memoryless independent data sequences, independent hopping patterns).

An approximation which is valid under the same conditions is

$$P_{e,\ell} \approx \tilde{P}_A \triangleq P_L + \tfrac{1}{2}\,(\tfrac{1}{2}+P_0)(K-1)\mathcal{P}(1-\mathcal{P})^{K-2} \quad . \tag{35}$$

This approximation is very accurate whenever q/K is large because it is based on the assumption that the probability of a multiple hit (i.e. hits from two or more signals in a given data bit interval) is negligibly small in comparison to the probability of a hit from only one signal.

Comparisons of the bounds and the approximation are given in Table 1 for various values of P_0, K, q, and N_b. The hopping patterns are the first-order Markov patterns for the data in Table 1, but in view of (30) the results would not be significantly different for the other patterns described above.

C. The Nonselective Rician Fading Channel

The bounds and approximation given in (32)-(35) can be applied to any particular nonselective fading channel by substituting the appropriate expression for P_0 in these results. In this section we consider the Rician nonselective fading model in which each transmitted signal results in a received signal that is the sum of a nonfaded version of the transmitted signal and a (nonselective) Rayleigh faded version of the transmitted signal. The difference in the propagation times for these two components is sufficiently small compared with the data bit duration T that the overall channel is nonselective. This model is discussed in [9] where the nonfaded component is called the fixed or specular component and the Rayleigh-faded component is called the random or scatter component. In some applications the nonfaded component arises from a direct path between the transmitter and the faded component arises from a reflection.

Table 1. Lower bounds, approximation, and upper bound on the probability of error for a FH/SSMA system.

a) $K = 15$, $q = 1000$, and $N_b = 10$

P_O	P_L	$\tilde{P}_L$	$\tilde{P}_A$	P_U
0.100	0.098	0.100	0.103	0.106
0.050	0.049	0.051	0.053	0.057
0.030	0.030	0.031	0.034	0.037
0.020	0.020	0.021	0.024	0.027
0.010	0.010	0.011	0.014	0.017
0.005	0.005	0.006	0.009	0.013

b) $K = 15$, $q = 100$, and $N_b = 5$

P_O	P_L	$\tilde{P}_L$	$\tilde{P}_A$	P_U
0.100	0.084	0.096	0.128	0.162
0.050	0.042	0.054	0.081	0.120
0.030	0.025	0.037	0.063	0.103
0.020	0.017	0.029	0.054	0.095
0.010	0.008	0.020	0.045	0.086
0.005	0.004	0.016	0.041	0.082

c) $K = 25$, $q = 250$, and $N_b = 20$

P_O	P_L	$\tilde{P}_L$	$\tilde{P}_A$	P_A
0.100	0.090	0.100	0.118	0.138
0.050	0.045	0.056	0.070	0.093
0.030	0.027	0.037	0.051	0.075
0.020	0.018	0.028	0.042	0.066
0.010	0.009	0.019	0.032	0.057
0.005	0.005	0.015	0.027	0.053

The amplitude S of the sum of the two components of the received signal is a random variable with a Rician distribution (see [8] or [9]). Since we are interested in the conditional probability of error given there are no hits, we can assume in all that follows that only the components of the i-th signal are present at the i-th receiver (during the data bit interval under consideration). Let ρ be the normalized bit energy to noise density ratio, so that $S^2\rho$ is the actual received energy to noise density ratio. Hence for noncoherent FSK the probability of error given $S = a$ is $\frac{1}{2}\exp(-\frac{1}{2}a^2\rho)$. For the Rician channel the density function f_S for the amplitude S is

$$f_S(a) = (a/\sigma^2)\exp\{-\tfrac{1}{2}(a^2+\alpha^2)/\sigma^2\}I_0(\alpha a/\sigma^2) \tag{36}$$

for $a > 0$, where α^2 represents the strength of the nonfaded component, $2\sigma^2$ is the expected value of the strength of the faded component, and I_0 is the zero-th order modified Bessel function. The average probability of error for noncoherent FSK* is [9]

$$P_0 = \int_0^\infty \tfrac{1}{2}\exp(-\tfrac{1}{2}a^2\rho)\, f_S(a)\, da$$

$$= \frac{\exp\{-\frac{1}{2}\alpha^2\rho/(\sigma^2\rho+1)\}}{2(\sigma^2\rho+1)} \quad . \tag{37}$$

If $\overline{\mathcal{E}}$ denotes the average energy per bit in the received signal then

$$\Lambda \triangleq \overline{\mathcal{E}}/N_0 = (\alpha^2 + 2\sigma^2)\rho \quad . \tag{38}$$

Let γ^2 denote the ratio of the power in the faded component to the power in the unfaded component; that is, $\gamma^2 = 2\sigma^2/\alpha^2$

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*Corresponding results for binary DPSK are obtained by replacing ρ by 2ρ in (37).

Then we can write

$$P_0 = \frac{(\gamma^2+1)}{\gamma^2\Lambda + 2(\gamma^2+1)} \exp\{-\Lambda/[\gamma^2\Lambda + 2(\gamma^2+1)]\} \quad . \tag{39}$$

Two limiting cases of interest are $\sigma^2 = 0$ and $\alpha^2 = 0$. If $\sigma^2 = 0$ ($\gamma^2 = 0$) then there is no faded component, and the channel is just an additive white Gaussian noise channel. In this case $\Lambda = \alpha^2\rho$, and the probability of error reduces to

$$P_0 = \tfrac{1}{2} \exp\{-\tfrac{1}{2}\Lambda\} \quad . \tag{40}$$

If $\alpha^2 = 0$ the channel is a nonselective Rayleigh fading channel, $\Lambda = 2\sigma^2\rho$, and the probability of error is

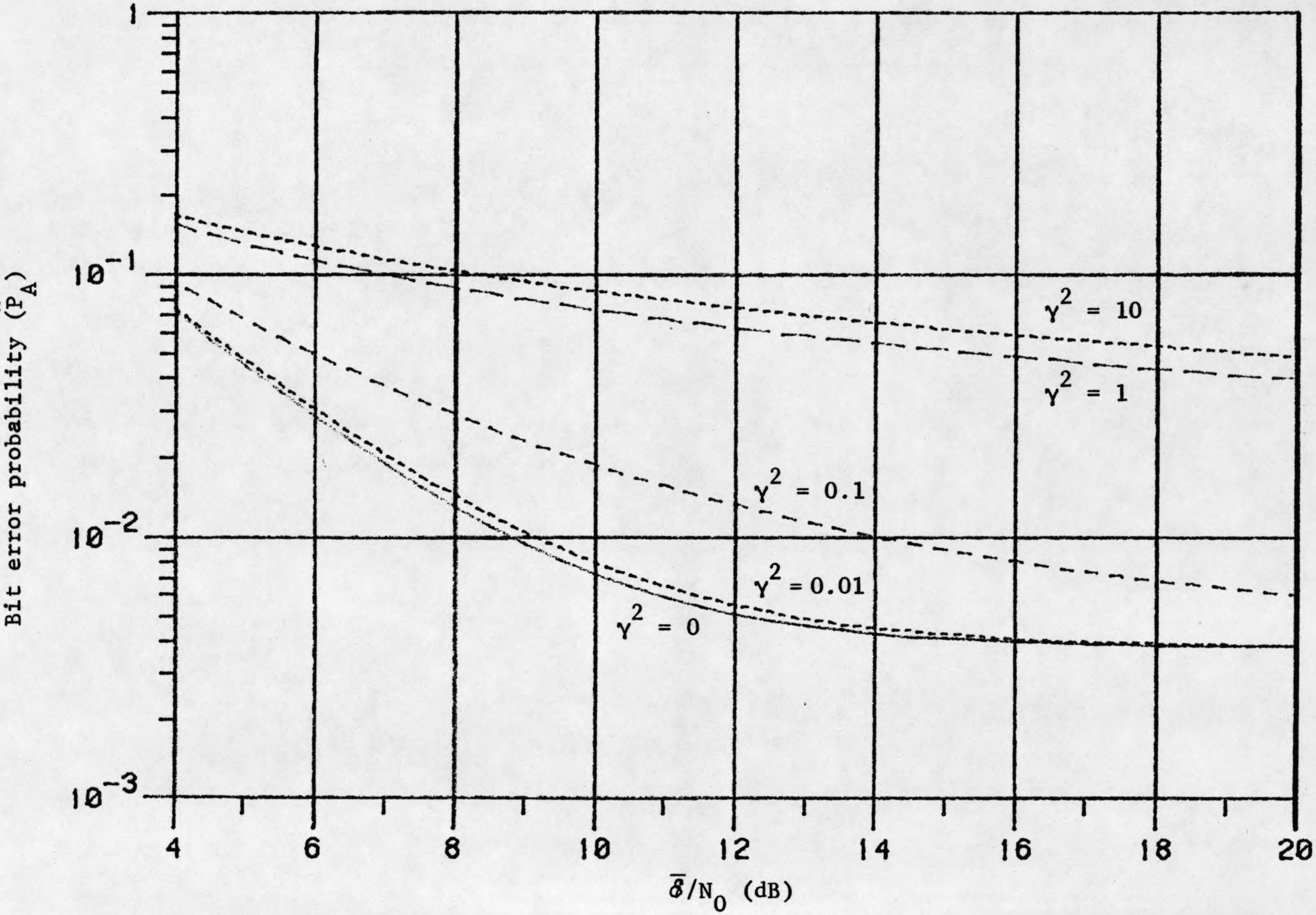
$$P_0 = \frac{1}{\Lambda + 2} \quad . \tag{41}$$

An examination of (39) as a function of γ^2 shows that for $\gamma^2 = 10$ the probability of error for Rician fading is nearly the same as for Rayleigh fading. For example, if Λ is 12 dB then P_0 is 1.81×10^{-2} for $\gamma^2 = 0$, 4.41×10^{-2} for $\gamma^2 = 0.1$, 4.53×10^{-2} for $\gamma^2 = 1.0$, and 5.58×10^{-2} for $\gamma^2 = 10.0$. The value of P_0 for Rayleigh fading ($\gamma^2 = \infty$) is 5.60×10^{-2}.

In order to apply (36)-(41) to the slow-frequency-hopped spread-spectrum multiple-access system, consider first the expressions (9) and (10) for the received signal. The amplitudes $A_\ell^{(k)}$ are random variables with a density function of the form given in (36). In general the parameters α and σ may depend on i, in which case Λ and γ also depend on i. The probability P_0 then depends on i and is given by (37) with α and σ replaced by α_i and σ_i or by (39) with γ and Λ replaced by γ_i and Λ_i. It follows from (9) and (10) that the parameter ρ is given by $\rho = PT/N_0$.

The next step is to substitute for P_0 in (32)-(35) using the expressions (37) or (39). If P_0 depends on i the bounds of (32) and (33) are valid, but of course they will also depend on i. Notice that if α and σ depend on i, then the average power in the received signal also depends on i. That is, the signals are not required to have equal power. The approximation given in (35) is also valid even if P_0 depends on i, provided that $\alpha_k \approx \alpha_i$ and $\sigma_k \approx \sigma_i$ for all k.

In Figure 3 the approximation $\tilde{P}_A$, which is given by (35) with P_0 replaced by the expression in (39), is shown as a function of $\Lambda = \bar{\mathcal{E}}/N_0$ for various values of γ^2. For the data presented in Figure 3, the values of α and σ (and hence γ and Λ) do not depend on i. Additional numerical data can be obtained from Table 1 by evaluating P_0 from (37) or (39). Notice that for Rayleigh fading with $\bar{\mathcal{E}}/N_0$ less than 20 dB the value of P_0 is less than 0.01. From Table 1 we see that for $P_0 \leq 0.01$, the value of P_U is always less than $2\tilde{P}_A$ and the value of $\tilde{P}_A$ is always less than $2\tilde{P}_L$ for the values of K, q, and N_b considered in Table 1. For K = 15, q = 1000, and N_b = 10 we see that for $P_0 \leq 0.01$, we always have $\tilde{P}_A \leq 1.2\ \tilde{P}_L$ and $P_U \leq 1.25\ \tilde{P}_A$. Thus, for Rayleigh fading or Rician fading with $\gamma^2 \geq 1$, the bounds and approximations given in this section are sufficiently accurate for the design of slow-frequency-hopped spread-spectrum multiple-access systems. Further evidence of this is given in the next section.



<u>Figure 3</u>. Bit error probability ($\tilde{P}_A$) for FH/SSMA system with Rician fading ($K = 15$, $q = 1000$, and $N_b = 10$).

IV. NONSELECTIVE RAYLEIGH FADING

In this section we present a more exact analysis of the effects of multiple-access interference and nonselective fading for the special case in which the fading is Rayleigh. This analysis provides a more accurate approximation and a tighter upper bound for the probability of error than is obtained by specializing the results of Section III to Rayleigh fading. The system and channel models are as presented in Section III, and the received signal is as given in (9) and (10).

Since we are considering only Rayleigh fading in the present section, the random amplitudes $A_\ell^{(k)}$ have a Rayleigh distribution. The density function for $A_\ell^{(k)}$ is given by (36) with $\alpha = 0$ and $\sigma = \sigma_k$. In general the second moments $\mu_k \triangleq 2\sigma_k^2$ are different for different signals. For the analysis presented in this section we assume that the fading for different signals is statistically independent. Stated precisely, the requirement is that $A_{\ell_1}^{(1)}, A_{\ell_2}^{(2)}, \ldots, A_{\ell_K}^{(K)}$ are mutually independent for any choice of $\ell_1, \ell_2, \ldots, \ell_K$.

The starting point for the analysis of the receiver is (8). Since in practice $f_c \gg T^{-1}$ for a spread-spectrum system, the high frequency terms in the integrand of (8) may be ignored. The output of the integration at the sampling instant is then given by

$$Z_{c,m} = D_{c,m} + (P/8)^{\frac{1}{2}} T \sum_{k \neq i} I_{c,m}^{(k,i)} + N_{c,m} . \tag{42}$$

The first term $D_{c,m}$ is the component due to the signal $s_i(t)$. If the transmitted data bit is $b_\lambda^{(i)}$ for $\lambda = jN_b + p$ then

$$D_{c,m} = (P/8)^{\frac{1}{2}} T A_\lambda^{(i)} \delta(b_\lambda^{(i)}, m) \cos[\theta_{i,m} + \alpha_j^{(i)} - \beta_j^{(i)} + \Phi_\lambda^{(i)}] . \tag{43}$$

Since the component is the output of the integrator in the absence of multiple-access and channel noise, it is called the desired signal component.

The multiple-access interference $I_{c,m}^{(k,i)}$ from the k-th signal depends upon the delay τ_k. For convenience let $\ell_k = \lfloor \tau_k/T_h \rfloor$ and $n_k = \lfloor (\tau_k - \ell_k T_h)/T \rfloor$, where $\lfloor u \rfloor$ denotes the integer part of the real number u. Then $I_{c,m}^{(k,i)}$ can be expressed as

$$I_{c,m}^{(k,i)} = d(\ell_k)[A_{L(n_k+1)}^{(k)} e_1(\ell_k,n_k)\cos\psi'(\ell_k,n_k) + A_{L(n_k)}^{(k)} e_2(\ell_k,n_k)\cos\psi''(\ell_k,n_k)] \tag{44}$$

for $0 \le n_k < p$. The following expressions define the various terms in (44). First we have

$$d(\ell) = \delta(f_{j-\ell}^{(k)}, f_j^{(i)}) \tag{45}$$

for $0 \le \ell < N$. Second, if $L(n) = (j-\ell_k)N_b + p - n$ then

$$e_1(\ell,n) = \delta(b_{L(n+1)}^{(k)}, m)[\tau_k - \ell T_h - nT]/T. \tag{46}$$

and

$$e_2(\ell,n) = \delta(b_{L(n)}^{(k)}, m)[(n+1)T - \tau_k + \ell T_h]/T . \tag{47}$$

Finally, if $b_{L(n+1)}^{(k)} = m'$ and $b_{L(n)}^{(k)} = m''$ then

$$\psi'(\ell,n) = \theta_{k,m'} + \alpha_{j-\ell}^{(k)} - \beta_j^{(i)} - 2\pi[f_c + m'\Delta + f_{j-\ell}^{(k)}]\tau_k + \Phi_{L(n+1)}^{(k)} \tag{48}$$

and $\psi''(\ell,n)$ is given by (48), with m' replaced by m'' and L(n+1) replaced by L(n). For $p < n_k < N_b$ equation (44) is replaced by

$$I_{c,m}^{(k,i)} = d(\ell_k+1)[A_{L(n_k+1)}^{(k)} e_1(\ell_k,n_k)\cos\psi'(\ell_k+1,n_k) + A_{L(n_k)}^{(k)} e_2(\ell_k,n_k)\cos\psi''(\ell_k+1,n_k)] . \tag{49}$$

The only remaining case is $n_k = p$ for which we have

$$I_{c,m}^{(k,i)} = d(\ell_k+1)A_{L(p+1)}^{(k)} e_1(\ell_k,p)\cos\psi'(\ell_k+1,p) + d(\ell_k)A_{L(p)}^{(k)} e_2(\ell_k,p)\cos\psi''(\ell_k,p) . \tag{50}$$

Notice that if we set $A_{L(n)}^{(k)} = 1$ and $\Phi_{L(n)}^{(k)} = 0$ in the above expressions, then we obtain expressions for the in-phase components of the desired signal and the multiple-access interference for a system with an additive white Gaussian noise channel.

The remaining component of $Z_{c,m}$ is the component $N_{c,m}$ which is due to the channel noise process n(t). It is easy to show that $N_{c,m}$ is a zero-mean Gaussian random variable with variance $N_0T/16$.

The quadrature components are defined by expressions which are analogous to (42)-(44). In fact $Z_{s,m}$ and $N_{s,m}$ are defined in the same way as above, and the only change that must be made in the definitions of $D_{s,m}$ and $I_{s,m}^{(k,i)}$ is that cos(•) should be replaced by -sin(•) in (43), (44), (49) and (50).

We next consider the average probability of error where the average is computed with respect to the phase angles, time delays, and data symbols. We start by assuming that the transmitted data bit is $b_{\lambda}^{(i)} = +1$ where $\lambda = jN_b+p$ as before. Also the probabilities and expectations below are all conditioned upon the data sequences $(b_{\ell}^{(k)})$ and time delays τ_k. For m = +1 or -1, let $\sigma_{c,m}^2$ and $\sigma_{s,m}^2$ be the variances of the in-phase and quadrature components $Z_{c,m}$ and $Z_{s,m}$ respectively. Since, as we discuss below, these components are Gaussian random variables with equal variances, the probability of error is given [8, p. 587] by

$$P_e = \sigma_{c,-1}^2(\sigma_{c,1}^2+\sigma_{c,-1}^2)^{-1} \tag{51}$$

for slow nonselective Rayleigh fading and noncoherent FSK detection. Under the assumptions about the fading model that were made above, the desired signal component $D_{c,1}$ is a zero-mean Gaussian random variable with variance $(PT^2/16)\mu_i$. Also notice that $D_{c,-1} = 0$.

In order to proceed further in the analysis of the multiple-access interference, we need to consider the nature of the statistical dependence between $A_\ell^{(k)}$ and $A_{\ell+1}^{(k)}$ and between $\Phi_\ell^{(k)}$ and $\Phi_{\ell+1}^{(k)}$ for ℓ and $\ell+1$ in the same hopping interval. These are the random variables which describe the fading during adjacent data bits. We consider the two extreme cases described in Section III: (i) the fading is constant in the sense that $A_\ell^{(k)} = A_{\ell+1}^{(k)}$ and $\Phi_\ell^{(k)} = \Phi_{\ell+1}^{(k)}$ whenever ℓ and $\ell+1$ are in the same hopping interval and (ii) the fading is independent for adjacent data bits in the same hopping interval.

Under our assumptions, the multiple-access interference component $I_{c,m}^{(k,i)}$ is a zero-mean Gaussian random variable with variance $\frac{1}{2}\mu_k\sigma_m^2(k,i)$. For constant fading, as described by case (i) above, we have

$$\sigma_m^2(k,i) = d(\ell_k)[e_1(\ell_k,n_k) + e_2(\ell_k,n_k)]^2 \tag{52}$$

for $0 \leq n_k < p$,

$$\sigma_m^2(k,i) = d(\ell_k+1)[e_1(\ell_k,n_k)]^2 + d(\ell_k)[e_2(\ell_k,n_k)]^2 \tag{53}$$

for $n_k = p$, and

$$\sigma_m^2(k,i) = d(\ell_k+1)[e_1(\ell_k,n_k) + e_2(\ell_k,n_k)]^2 \tag{54}$$

for $p < n_k < N_b$. For independent fading, as described by case (ii) above, we have

$$\sigma_m^2(k,i) = d(\ell_k)\{[e_1(\ell_k,n_k)]^2 + [e_2(\ell_k,n_k)]^2\} \tag{55}$$

for $0 \leq n_k < p$. If $n_k = p$, $\sigma_m^2(k,i)$ is given by (53), and for $p < n_k < N_b$

$$\sigma_m^2(k,i) = d(\ell_k+1)\{[e_1(\ell_k,n_k)]^2 + [e_2(\ell_k,n_k)]^2\} \quad . \tag{56}$$

Because of the independence of the fading for different signals, the random variables $I_{c,m}^{(k,i)}$ are independent when conditioned on the data bits and time delays. As a result

$$\sigma_{c,m}^2 = (PT^2/16)\{\delta(1,m)\mu_i + \sum_{k\neq i} \mu_k \sigma_m^2(k,i)\} + N_0T/16 \quad . \tag{57}$$

By symmetry we see that $\sigma_{s,m} = \sigma_{c,m}$. Thus (51) can be written as

$$P_e = \frac{(\bar{\mathcal{E}}/N_0)^{-1} + \sum_{k\neq i} \mu_k\mu_i^{-1}\,\sigma_{-1}^2(k,i)}{1+2(\bar{\mathcal{E}}/N_0)^{-1} + \sum_{k\neq i} \mu_k\mu_i^{-1}[\sigma_{-1}^2(k,i)+\sigma_1^2(k,i)]} \tag{58}$$

where $\bar{\mathcal{E}} = \mu_i PT$ is the energy per bit for the received signal (in the absence of multiple-access interference).

In order to evaluate the average probability of error $\bar{P}_e$, we must average the expression in (58) with respect to the time delays and data symbols. This is of course a difficult computation since it involves the evaluation of K-1 dimensional integrals. However we can obtain an approximation $\bar{P}_A$ and an upper bound $\bar{P}_U$ which are relatively easy to compute. This is accomplished by observing that P_e depends on τ_k only through t_k, ℓ_k and n_k where $t_k = (\tau_k - \ell_k T_h - n_k T)/T$. We can thus obtain a discrete <u>approximation</u> to the integral with respect to t_k by approximating the uniform distribution on [0,1] by the discrete distribution with probability mass J^{-1} at points $J^{-1}, 2J^{-1}, \ldots, (J-1)J^{-1}$ and probability mass $(2J)^{-1}$ at the end points 0 and 1. We find that for the first-order Markov patterns and constant fading (case (i))

$$P\{\sigma_m(k,i) = jJ^{-1}\} = p_j, \qquad 0 \le j \le J, \tag{59}$$

where the quantities p_j are defined completely by the fact that their sum is 1 and

$$p_j = \begin{cases} (2Jq)^{-1}(1+N_b^{-1}), & j = 1,2,\ldots,J-1, \\ (4Jq)^{-1}(1+N_b^{-1}) + (4q)^{-1}(1-N_b^{-1}), & j = J\ . \end{cases} \tag{60}$$

For independent fading (case (ii))

$$P\{\sigma_m(k,i) = jJ^{-1}\} = p_j\ , \quad 0 \le j \le J, \tag{61a}$$

$$P\{\sigma_m(k,i) = [j^2 + (J-j)^2]^{\frac{1}{2}}J^{-1}\} = q_j\ , \quad 0 \le j \le \tfrac{1}{2}J, \tag{61b}$$

where p_j and q_j are defined by

$$p_j = \begin{cases} (2Jq)^{-1}(1+N_b^{-1}), & j = 1,2,\ldots,J-1\ , \\ (2Jq)^{-1}\ , & j = J, \end{cases} \tag{62a}$$

$$q_j = \begin{cases} (2Jq)^{-1}(1-N_b^{-1}), & j = 1,2,\ldots,J/2-1\ , \\ (4Jq)^{-1}(1-N_b^{-1}), & j = J/2\ , \end{cases} \tag{62b}$$

and

$$p_0 = 1 - \sum_{j=1}^{J} p_j - \sum_{j=1}^{J/2} q_j\ . \tag{62c}$$

In (61) and (62) we assume J is an even integer.

An upper bound can be obtained as follows. The conditional probability of error P_e given by (58) is not convex in t_k $(1 \le k \le K,\ k \ne i)$. However if we upper bound the sum of squares of (53) for case (i) or of (55), (53), and (56) for case (ii) by the square of the sum, the upper bound on P_e

becomes a convex function of the t_k's. We then obtain a discrete approximation to the integral with respect to t_k as for $\overline{P}_A$. The upper bound $\overline{P}_U$ is the same for cases (i) and (ii) and the distribution of $\sigma_m(k,i)$ is given by (59), where the p_j are defined by

$$p_j = \begin{cases} (4J_q)^{-1}(1+N_b^{-1}) , & j = 1,2,\ldots,J-1 , \\ (1+J^{-1})(4q)^{-1}(1+N_b^{-1}), & j = J \quad . \end{cases} \tag{63}$$

A similar approximation and upper bound can be obtained for the sequences of independent random elements.

Finally we note that in order that the approximation and bound presented in this section be tight and computationally efficient we need to assume that $\mu_k = \mu_i$ for all $k \neq i$. If this is not the case, we can still work with $\mu_k' = \max_k\{\mu_k\}$, but the approximation and the upper bound obtained above are not expected to be very tight, so that it might be preferable to work with the bounds suggested in Section III which are not affected by the different power levels.

In Table 2 the approximation obtained in this section is compared with the improved lower bound, the approximation, and the upper bound of Section III.B for the first-order Markov hopping patterns and $\mu_k = \mu_i$ for all k. The approximation $\overline{P}_A$ (for both cases (i) and (ii) and the bound $\overline{P}_U$ are evaluated for J = 4. It turns out that they are rather insensitive to increases in J as long as $J \geq 4$. Values for $\overline{P}_A$ are given in Table 2(a) for (i) constant fading and (ii) independent fading. The notations $\overline{P}_A^{(i)}$ and $\overline{P}_A^{(ii)}$, respectively, are used for these two cases. Independent-fading turns out to be the most favorable case although the difference is less than ten percent. Also notice

Table 2. Bit error probability for nonselective Rayleigh fading.

a) $K = 5$, $q = 100$, and $N_b = 10$

$\bar{\mathcal{E}}/N_0$ (dB)	$\tilde{P}_L$	$\bar{P}_A^{(ii)}$	$\bar{P}_A^{(i)}$	$\bar{P}_U$	$\tilde{P}_A$	P_U	
6	1.64	1.71	1.72	1.72	1.74	1.82	$(\times 10^{-1})$
8	1.19	1.25	1.26	1.27	1.28	1.37	$(\times 10^{-1})$
10	8.41	8.95	9.02	9.09	9.21	10.14	$(\times 10^{-2})$
12	5.80	6.29	6.36	6.44	6.54	7.52	$(\times 10^{-2})$
15	3.28	3.73	3.81	3.90	3.97	5.01	$(\times 10^{-2})$
20	1.37	1.78	1.87	1.97	2.02	3.10	$(\times 10^{-2})$
∞	0.44	0.83	0.92	1.02	1.06	2.16	$(\times 10^{-2})$

b) $K = 10$, $q = 1000$, and $N_b = 10$

$\bar{\mathcal{E}}/N_0$ (dB)	$\tilde{P}_L$	$\bar{P}_U$	$\tilde{P}_A$	P_U	
6	1.67	1.68	1.69	1.70	$(\times 10^{-1})$
8	1.20	1.22	1.22	1.24	$(\times 10^{-1})$
10	8.35	8.51	8.54	8.74	$(\times 10^{-2})$
12	5.65	5.79	5.82	6.04	$(\times 10^{-2})$
15	3.05	3.18	3.20	3.44	$(\times 10^{-2})$
20	1.07	1.20	1.22	1.46	$(\times 10^{-2})$
∞	0.10	0.23	0.24	0.49	$(\times 10^{-2})$

c) $K = 15$, $q = 1000$, and $N_b = 10$

$\bar{\mathcal{E}}/N_0$ (dB)	$\tilde{P}_L$	$\bar{P}_U$	$\tilde{P}_A$	P_U	
6	1.66	1.69	1.70	1.72	$(\times 10^{-1})$
8	1.20	1.23	1.23	1.26	$(\times 10^{-1})$
10	8.36	8.60	8.65	8.97	$(\times 10^{-2})$
12	5.67	5.90	5.94	6.28	$(\times 10^{-2})$
15	3.08	3.30	3.33	3.69	$(\times 10^{-2})$
20	1.12	1.33	1.35	1.73	$(\times 10^{-2})$
∞	0.16	0.36	0.38	0.76	$(\times 10^{-2})$

that the bound $\overline{P}_U$ (common for both cases) differs from $\overline{P}_A$ of (i) or (ii) by at most twenty percent; therefore, in Tables 2(b) and 2(c) we present data on $\overline{P}_U$ only (not on $\overline{P}_A$). The purpose is comparison with $\tilde{P}_L$, $\tilde{P}_A$, and P_U. In comparing $\overline{P}_U$ and $\tilde{P}_A$ we note that $\tilde{P}_A$ appears to be an upper bound for the nonselective Rayleigh case. Also the results of Table 2(c) show that for $q = 1000$, $K = 15$, $N_b = 10$, and $\overline{\mathcal{E}}/N_0 \leq 20$ dB the results of the Table show that $\overline{P}_U \leq 1.17\, \tilde{P}_L$, $\tilde{P}_A \leq 1.3\, \overline{P}_U$ and $P_U \leq 1.52\, \overline{P}_U$. Similar observations can be made for the data provided in Tables 2(a) and 2(b). As a final comment we point out that since the approximations $\overline{P}_A$ and the bound $\overline{P}_U$ are expected to be very close to the true probability of error, their favorable comparison with the simpler bounds $\tilde{P}_L$ and P_U and the approximation $\tilde{P}_A$ strongly suggests the use of the latter for the design of SFH/SSMA systems.

V. SELECTIVE FADING

In this section we consider a general wide-sense stationary uncorrelated-scattering (WSSUS) fading channel. This model is described in detail in [1] and [8, Ch. 9] and is employed in the analysis of direct-sequence SSMA communications over fading channels in [4]. We assume that $f_c \gg q\Delta'$, so that narrow band signal models can be employed. The input to the k-th channel is $s_k(t-\tau_k)$ where

$$s_k(t) = \mathrm{Re}\{x_k(t)\exp(j\,2\pi f_c t)\} \tag{64}$$

and

$$x_k(t) = \sqrt{2P}\,\exp\{j(2\pi[b_k(t)\Delta + f_k(t)]t + \theta_k(t) + \alpha_k(t))\}. \tag{65}$$

The corresponding output is $y_k(t-\tau_k)$ where

$$y_k(t) = \gamma_0 s_k(t) + \mathrm{Re}\{u_k(t)\exp(j2\pi f_c t)\} \tag{66}$$

and

$$u_k(t) = \gamma_k \int_{-\infty}^{\infty} h_k(t,\tau)x_k(t-\tau)d\tau\ , \tag{67}$$

so that the received signal for this channel is given by (9).

If $\gamma_0 = 1$ then there is a (non-faded) specular component present in the output of the channel, and the channel is a Rician fading channel (as in [4]). In this case γ_k^2 plays the same role as the parameter γ^2 of Section III. If $\gamma_0 = 0$ there is no specular component, and the channel is a Rayleigh fading channel. In this case γ_k^2 plays the same role as the parameter μ_k of Section IV.

The fading process $h_k(t,\tau)$ (which can be thought of as the time-varying impulse response of a lowpass filter) is a zero-mean complex Gaussian random process with autocovariance

$$E\{h_k(t,\tau)h_k^*(s,\sigma)\} = \rho_k(t-s,\tau)\delta(\tau-\sigma) , \tag{68}$$

where $\delta(\cdot)$ is the Dirac delta function and

$$\int_{-\infty}^{\infty} \rho_k(0,\tau)d\tau = 1 .$$

Two special cases of the model considered in [1] and [4] are the purely time-selective and purely frequency-selective WSSUS fading channels (see also [2] and [3]).

In the present paper we consider a somewhat more general model which is both time and frequency selective. This is a special doubly-dispersive model that is characterized by

$$\rho_k(t-s,\tau) = r_k(t-s)g_k(\tau) . \tag{69}$$

If $g_k(\tau) \equiv \delta(\tau)$ the channel is not frequency selective. If $r_k(\xi) \equiv 1$ it is not time selective.

As usual ([1]-[4]), some limitations are imposed on the selectivity of the channel. First it is assumed that

$$g_k(\tau) \approx 0 \text{ for } |\tau| > T, \tag{70}$$

which is a constraint on the frequency selectivity of the channel that allows us to restrict attention to the intersymbol interference from the two adjacent data bits. This assumption can be relaxed, but the error probability computations become more difficult. The second assumption is that two signals which are transmitted at different frequencies have non-overlapping spectra at the receiver. This is primarily a limitation on the time selectivity of the channel, but it also is related to the spacing Δ.

The analysis of the receiver follows that of Section IV, so many of the details are omitted. The output of the in-phase component of each of the two branches of the i-th receiver is

$$Z_{c,m} = \gamma_0(D_{c,m} + I_{c,m}) + (P/8)^{\frac{1}{2}}T(\gamma_i F_{c,m} + \sum_{k\neq i} \gamma_k \hat{I}_{c,m}^{(k,i)}) + N_{c,m} . \quad (71)$$

The terms $D_{c,m}$, $I_{c,m}$, and $N_{c,m}$ are as in Section IV if we replace $A_\ell^{(i)}$ by 1 and $\Phi_\ell^{(i)}$ by 0. The terms $F_{c,m}$ and $\hat{I}_{c,m}^{(k,i)}$ are (normalized) faded versions of the desired signal and the multiple-access interference due to the k-th signal. These terms are defined for the λ-th decision bit ($\lambda = jN_b + p$) by

$$F_{c,m} = \mathrm{Re}(\tilde{F}_m) \quad (72)$$

and

$$\hat{I}_{c,m}^{(k,i)} = \mathrm{Re}(\tilde{I}_m^{(k,i)}) , \quad (73)$$

where

$$\tilde{F}_m = T^{-1} \int_{\lambda T}^{(\lambda+1)T} \int_{-\infty}^{\infty} h_i(t,\tau)\eta_{i,m}(t,\tau)\exp[j\psi_i(t,\tau)]d\tau dt \quad (74)$$

and

$$\tilde{I}_m^{(k,i)} = T^{-1} \int_{\lambda T}^{(\lambda+1)T} \int_{-\infty}^{\infty} h_i(t-\tau_k,\tau)\eta_{k,i,m}(t,\tau_k+\tau)\exp[j\psi_{k,i}(t,\tau_k+\tau)]d\tau dt. \quad (75)$$

In (74) and (75) the functions $h_{k,i,m}$ and $\psi_{k,i}$ are given by

$$\eta_{k,i,m}(t,\tau) = \delta[f_k(t-\tau),f_i(t)]\delta[b_k(t-\tau),m] \tag{76}$$

and

$$\psi_{k,i}(t,\tau) = -2\pi[f_c + b_k(t-\tau)\Delta + f_k(t-\tau)]\tau + \theta_k(t-\tau) + \alpha_k(t-\tau) - \beta_i(t). \tag{77}$$

The functions $\eta_{i,i,m}$ and $\psi_{i,i}$ are denoted by $\eta_{i,m}$ and ψ_i, respectively.

Notice that $\tilde{F}_m$ is nonzero if and only if both $f_i(t-\tau) = f_i(t)$ and $b_i(t-\tau) = m$ for some t and τ (similarly for $\tilde{I}_m^{(k,i)}$). This is a result of our assumption for the time-selectivity of the channel and the size of Δ.

In the analysis below, the expectations and probabilities are conditioned on $b_\ell^{(k)}$ and τ_k for $1 \le k \le K$. However, the error probabilities that are obtained do not depend upon τ_k or $b_\ell^{(k)}$ for $k \neq i$. So in the last step we only have to average over $b_\ell^{(i)}$.

A. WSSUS Rayleigh Fading Model ($\gamma_0 = 0$)

The bounds of (32)-(34) and the approximation of (35) are employed except that fading must be accounted for in P_0 and $\mathcal{P}$. For $\gamma_0 = 0$ and $K = 1$, $Z_{c,m}$ is the sum of two random variables ($(P/8)^{\frac{1}{2}}T\ F_{c,m}$ and $N_{c,m}$) of which the first is conditionally Gaussian and the second is Gaussian. Furthermore, it is not hard to see that $Z_{c,1}$ and $Z_{c,-1}$ are conditionally independent, and so are $Z_{s,1}$ and $Z_{s,-1}$. Since $\sigma_{c,m} = \sigma_{s,m}$ then [8, p. 587]

$$P_{e,0} = \sigma_{c,-1}^2(\sigma_{c,1}^2 + \sigma_{c,-1}^2)^{-1} \tag{78}$$

is the conditional probability of error given there are no hits where

$$\sigma_{c,m}^2 = (PT^2/8)\gamma_i^2\ \mathrm{Var}\{F_{c,m}\} + N_0T/16\quad . \tag{79}$$

It is convenient to normalize $\sigma^2_{c,m}$ and write (78) as

$$P_{e,0} = v_{-1}(v_1 + v_{-1})^{-1} \tag{80}$$

where v_m is given in terms of $\bar{\mathcal{E}} = \gamma_i^2\,PT$ by

$$v_m = 2\,\mathrm{Var}\{F_{c,m}\} + (\bar{\mathcal{E}}/N_0)^{-1} \quad . \tag{81}$$

The expression for $\mathrm{Var}\{F_{c,m}\}$ depends on the position of the data bit within the interval $[jT_h, (j+1)T_h)$. For the p-th bit of the j-th hop ($\lambda = jN_b + p$) de define $\delta_m = \delta(1,m)$, $\delta'_m = \delta(b^{(i)}_{\lambda-1}, m)$, and $\delta''_m = \delta(b^{(i)}_{\lambda+1}, m)$. Let

$$H_i(v) = 2T^{-2}\int_0^v (v-u)\,r_i(u)\,du, \tag{82}$$

and define

$$F_i = \int_0^T g_i(\tau)H_i(\tau)\,d\tau, \tag{83a}$$

$$\hat{F}_i = \int_0^T g_i(\tau)H_i(T-\tau)\,d\tau, \tag{83b}$$

and

$$G_i = T^{-2}\int_0^T g_i(\tau)\int_0^{\tau}\int_0^{T-\tau} r_i(t-s)\,dt\,ds \quad . \tag{83c}$$

The following expressions for $\mathrm{Var}\{F_{c,m}\}$ are derived in the Appendix. First for $p = 0$ we find

$$\mathrm{Var}\{F_{c,m}\} = \tfrac{1}{2}[(\delta''_m + \delta_q)F_i + 2\delta_m\hat{F}_i + 2\delta_m(\delta''_m + \delta_q)G_i] \quad . \tag{84}$$

For $p = N_b - 1$ (84) is valid provided we replace δ''_m by δ'_m. Finally, for $0 < p < N_b - 1$, the expression is

$$\mathrm{Var}\{F_{c,m}\} = \tfrac{1}{2}[(\delta'_m + \delta''_m)F_i + 2\delta_m\hat{F}_i + 2\delta_m(\delta'_m + \delta''_m)G_i] \quad . \tag{85}$$

For the first-order Markov hopping patters and the RS hopping patterns the quantity δ_q that appears in (84) is identically 0. For the sequences of independent random elements δ_q is a random variable with $P\{\delta_q = 1\} = q^{-1}$ and $P\{\delta_q = 0\} = 1 - q^{-1}$.

Notice that for $0 < p < N_b - 1$ (i.e. for the internal bits of each dwell interval), $\mathrm{Var}\{F_{c,m}\}$ does not depend on the hopping pattern. It turns out that the average probability of error for these bits ($0 < p < N_b - 1$) is larger than that of the first and last bits ($p = 0$ and $p = N_b - 1$). Thus we use (85), and not (84), in order to obtain an upper bound on P_0 which applies for all values of p. As a consequence of using (85), we obtain a bound on P_0 which does not depend on the hopping pattern.

In order to obtain the limiting error probability (as the channel becomes nonselective) it suffices to let $g_i(\tau) = \delta(\tau)$ and $r_i(u) = 1$. We then have $F_i = G_i = 0$ and $\hat{F}_i = \frac{1}{2}$ so that P_0 is given by (41) with $\Lambda = \bar{\mathcal{E}}/N_0 = \gamma_i^2 \rho$. Similarly, to obtain the irreducible error probability (as $\rho \to \infty$) we simply disregard the second term in the right-hand side of (81).

For the WSSUS Rayleigh fading model we say that a <u>hit</u> occurs from the k-th signal whenever τ_k, $b_k(t)$, and $f_k(t)$ are such that $\mathrm{Var}\{\hat{I}_{c,m}^{(k,i)}\} \neq 0$. The probability $\mathcal{P}$ of such a hit depends upon N_b and q. For the first-order Markov hopping patterns we have

$$\mathcal{P} \leq \mathcal{P}_u \triangleq \frac{1}{q}\,(1 + \frac{3}{N_b}) \quad . \tag{86}$$

In deriving (86) we used the fact that for the selective fading model used in this section, as many as 4 adjacent bits from the k-th signal may interfere with each bit of the i-th signal. The expressions (25) and (32)-(35) apply with $\mathcal{P}$ replaced by $\mathcal{P}_u$ and P_0 evaluated as explained above. For memoryless hopping patterns the corresponding result is

$$\mathcal{P} \le \mathcal{P}_u \triangleq \frac{1}{q}\left[1 + \frac{3}{N_b}\left(1 - \frac{1}{q}\right)\right] . \tag{87}$$

Both bounds in (86) and (87) are tight for $N_b \ge 3$. For the RS hopping patterns the corresponding result is

$$\mathcal{P} \le \mathcal{P}_u = \frac{1}{q-1}\left(1 + \frac{1}{N_b}\right) . \tag{88}$$

Notice that the bound in (88) is the same as in (23) which was obtained under nonselective fading conditions. This is due to the fact that the RS hopping patterns do not repeat within a period.

B. <u>WSSUS Rician Fading Model ($\gamma_0 = 1$)</u>

In this case the conditional error probability given there are no hits is [8, p. 587]

$$P_{e,0} = \sigma^2_{c,-1}(\sigma^2_{c,1} + \sigma^2_{c,-1})^{-1}\exp[-\tfrac{1}{2}(D^2_{c,1} + D^2_{s,1})(\sigma^2_{c,1} + \sigma^2_{c,-1})^{-1}] . \tag{89}$$

Upon normalization, (89) reduces to

$$P_{e,0} = v_{-1}(v_1 + v_{-1})^{-1} \exp\{-[\gamma_i^2(v_1 + v_{-1})]^{-1}\} \tag{90}$$

where v_m is defined by

$$v_m = 2\,\mathrm{Var}\{F_{c,m}\} + (1 + \gamma_i^2)(\gamma_i^2\,\overline{\mathcal{E}}/N_0)^{-1}, \tag{91}$$

$\overline{\mathcal{E}}/N_0 = (1 + \gamma_i^2)\rho$, and $\mathrm{Var}\{F_{c,m}\}$ is given by (82)-(85). Finally in order to obtain P_0 we have to average $P_{e,0}$ with respect to the data bits $(b^{(i)}_{\lambda+1}, b^{(i)}_{\lambda-1})$.

For Rician fading the hits from the k-th signal may occur from either the direct-path component or the faded component. The probability of a hit from the k-th signal due to the direct-path component is the same as for nonselective Rayleigh fading (this was evaluated in Section III). The probability of a hit due to the faded component is evaluated above (for Rayleigh fading). The union bound provides simple upper bound on the probability of a hit. This is given by

$$\wp \leq \wp'_u = \frac{2}{q}\left(1 + \frac{2}{N_b}\right) \tag{92}$$

for first-order Markov hopping patterns and

$$\wp \leq \wp'_u = \frac{2}{q}\left[1 + \frac{2}{N_b}\left(1 - \frac{1}{q}\right)\right] \tag{93}$$

for memoryless random hopping patterns. For RS hopping patterns $\hat{\wp}$ is still bounded as in (88); that is,

$$\wp \leq \wp'_u = \wp_u \quad . \tag{94}$$

By substituting for P_0 in (32)-(35) and replacing $\wp$ by $\wp'_u$ in (25) we have lower bounds, an approximation and an upper bound on the average probability of error.

In Tables 3 and 4 the approximation $\tilde{P}_A$ given in (35) is obtained for purely frequency-selective Rayleigh and Rician fading channels, respectively. The system parameters are $K = 15$, $q = 1000$, and $N_b = 10$. First-order Markov hopping patterns are employed. The covariance function of the frequency-selective channel is triangular, so that the rms multimath spread σ defined by $\sigma^2 = \int_{-\infty}^{\infty} \tau^2 \, g(\tau)d\tau$ is related to the parameter d of [3] by $d = 2.22\,\sigma/T$. We let $\gamma_k = \gamma$ for all k. Then in Table 3, $\tilde{P}_A$ is given as a function of

Table 3. Bit error probability for Rayleigh frequency-selective fading ($K = 15$, $q = 1000$, and $N_b = 10$).

$\bar{\mathcal{E}}/N_0$ (dB)	σ=0.05T	σ=0.1T	σ=0.15T	σ=0.2T	
6	1.75	1.82	1.91	2.01	($\times 10^{-1}$)
8	1.28	1.35	1.44	1.54	($\times 10^{-1}$)
10	0.91	0.97	1.06	1.17	($\times 10^{-1}$)
12	6.31	6.88	7.71	8.84	($\times 10^{-2}$)
15	3.63	4.13	4.94	6.08	($\times 10^{-2}$)
20	1.59	2.04	2.82	3.95	($\times 10^{-2}$)
∞	0.58	1.00	1.76	2.89	($\times 10^{-2}$)

Table 4. Bit error probability for Rician frequency-selective fading ($K = 15$, $q = 1000$, $N_b = 10$, and $\sigma = 0.05T$).

$\bar{\mathcal{E}}/N_0$ (dB)	γ^2=.1	γ^2=.5	γ^2=1	γ^2=10	γ^2=1000	
6	0.98	1.42	1.60	1.77	1.78	($\times 10^{-1}$)
8	0.49	0.94	1.13	1.30	1.31	($\times 10^{-1}$)
10	0.23	0.61	0.78	0.93	0.94	($\times 10^{-1}$)
12	1.23	3.96	5.34	6.58	6.63	($\times 10^{-2}$)
15	0.86	2.26	3.14	3.94	3.97	($\times 10^{-2}$)
20	0.82	1.24	1.58	1.92	1.94	($\times 10^{-2}$)
∞	0.81	0.83	0.86	0.93	0.94	($\times 10^{-2}$)

$\overline{\mathcal{E}}/N_0 = (1+\gamma^2)\rho$ for $\sigma = 0.05\ T$ and for five different values of γ^2. Notice that as $\gamma^2 \to \infty$ the probability $\tilde{P}_A$ is <u>not</u> the same as the second column of Table 3. Although P_0 is the same in this limiting case, the fact that $\mathcal{P}_u < \mathcal{P}'_u$ (compare (86) to (92)) implies that the two cases give different values of the bit error probability.

Finally we compare $\tilde{P}_A$ for nonselective and frequency-selective Rayleigh fading for $K = 15$, $q = 1000$, and $N_b = 10$, (first-order Markov hopping patterns are employed). From Tables 2(c) and 3 we see that the probability of error for the frequency-selective case is, for $\overline{\mathcal{E}}/N_0 = 12$ dB and $\sigma = 0.05$, 1.1 times that for nonselective fading, and it becomes 1.5 times the corresponding probability for nonselective fading as σ increases to 0.2 T. Similarly for $\overline{\mathcal{E}}/N_0 = 20$ dB the ratio of the two probabilities ranges from 1.2 for $\sigma = 0.05$ T to 2.9 for $\sigma = 0.2$ T.

APPENDIX

In this appendix we develop the expressions for $\mathrm{Var}\{F_{c,m}\}$. As in [4] we can write $\mathrm{Var}\{F_{c,m}\}$ as

$$\mathrm{Var}\{F_{c,m}\} = E[\mathrm{Re}\{\tilde{F}_m\}]^2 = \tfrac{1}{2}\, E[\tilde{F}_m \tilde{F}_m^*] , \tag{A-1}$$

where we used the fact that [1] $E\{h_i(t,\tau)h_i(s,\sigma)\} = 0$. Upon substitution for (74), (68) and (69) in (A-1) we find

$$\mathrm{Var}\{F_{c,m}\} = T^{-2} \int_{-\infty}^{\infty} g_i(\tau) \int_{\lambda T}^{(\lambda+1)} \int_{\lambda T}^{(\lambda+1)T} r_i(t-s)\eta_{i,m}(t,\tau)\eta_{i,m}(s,\tau)$$

$$\exp\{j[\psi_i(t,\tau) - \psi_i(s,\tau)]\} dt\, ds\, d\tau \; . \tag{A-2}$$

Notice that $\eta_{i,m}(t,\tau)\eta_{i,m}(s,\tau) \neq 0$ only for those t, s, and τ for which the following three conditions hold: $f_i(t-\tau) = f_i(t)$, $f_i(s-\tau) = f_i(s)$, and $b_i(t-\tau) = b_i(s-\tau) = m$. But these three conditions imply $\alpha_i(t-\tau) = \alpha_i(t)$, $\alpha_i(s-\tau) = \alpha_i(s)$, $\theta_i(t-\tau) = \theta_i(s-\tau)$, respectively. Also $\alpha_i(t) = \alpha_i(s) = \alpha_j^{(i)}$ and $\beta_i(t) = \beta_i(s) = \beta_j^{(i)}$ for t and s in $[\lambda T, (\lambda+1)T)$. Consequently, $\psi_i(t,\tau) = \psi_i(s,\tau)$ for these values of t, s, and τ. As a result we may let

$$\exp\{j[\psi_i(t,\tau) - \psi_i(s,\tau)]\} = 1$$

in equation (A-2).

The next step is to write (A-2) as

$$\mathrm{Var}\{F_{c,m}\} = \sum_{\ell=-\infty}^{\infty} [d(\ell) \sum_{n=0}^{p-1} A_m(\ell,n) + A_m(\ell,p) + d(\ell+1) \sum_{n=p+1}^{N_b-1} A_m(\ell,n)] , \tag{A-3}$$

where for $n \neq p$

$$A_m(\ell,n) = \int_0^T g_i(\tau+\ell T_h+nT)[\,\Delta_m(\ell,n+1)F(\tau) + \Delta_m(\ell,n)\hat{F}(\tau) + 2\Delta_m(\ell,n+1)\Delta_m(\ell,n)G(\tau)]\,d\tau, \tag{A-4a}$$

and for $n = p$

$$A_m(\ell,p) = \int_0^T g_i(\tau+\ell T_h+pT)[\,d(\ell+1)\Delta_m(\ell,p+1)F(\tau) + d(\ell)\Delta_m(\ell,p)\hat{F}(\tau) + 2d(\ell+1)d(\ell)\Delta_m(\ell,p+1)\Delta_m(\ell,p)G(\tau)]\,d\tau. \tag{A-4b}$$

In (A-3) - (A-4) we also need the definitions

$$d(\ell) = \delta(f^{(i)}_{j-\ell},\ f^{(i)}_{j}), \tag{A-5}$$

$$\Delta_m(\ell,n) = \delta(b^{(i)}_{\lambda-\ell N_b-n},\ m), \tag{A-6}$$

and (cf. (82) - (83))

$$F(\tau) = T^{-2}\int_{\lambda T}^{\lambda T+\tau}\int_{\lambda T}^{\lambda T+\tau} r_i(t-s)\,dtds = H_i(\tau), \tag{A-7a}$$

$$\hat{F}(\tau) = T^{-2}\int_{\lambda T+\tau}^{(\lambda+1)T}\int_{\lambda T+\tau}^{(\lambda+1)T} r_i(t-s)\,dtds = H_i(T-\tau) \tag{A-7b}$$

$$G(\tau) = T^{-2}\int_{\lambda T}^{\lambda T+\tau}\int_{\lambda T+\tau}^{(\lambda+1)T} r_i(t-s)\,dtds = \int_o^{\tau}\int_{\tau}^{T} r_i(t-s)\,dtds. \tag{A-7c}$$

Notice that the result of (A-3) is quite general and it accounts for the intersymbol interference due to many data bits. However, because of the assumption (70) only the terms $\ell=0$, $n=0$ and $\ell=-1$, $n=N_b-1$ of (A-3) give nonzero contributions, and thus (A-3) reduces to (84) - (85).

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APPENDIX B

ANALYSIS OF A SLOW FREQUENCY-HOPPED SYSTEM WITH POISSON TRAFFIC

In this section the packet error probability and the throughput rate are determined for a particular frequency-hopped system when the number of packet transmissions in a slot is given by a Poisson random variable. The packet error rate and throughput under the Poisson traffic assumption are significant in view of the local Poisson approximation and recursive retransmission control strategies discussed in the next section.

The system of interest is assumed to be packet-synchronized. A sufficient time-guard-band must be maintained between packet slots to maintain sychronization in the face of differential delays due to the spatial distribution of the stations. Synchronization at the level of bits or bytes is not assumed.

Each packet transmission is declared successful or not according to some criteria (a specific choice is given below). The following definitions will be used

$$r(m|k) = P[m \text{ successful} | k \text{ packets transmitted in slot}]$$

$$\bar{r}(k) = \sum_{m=0}^{k} m\, r(m|k)$$

and

$$P(k) = 1 - \bar{r}(k)/k.$$

Thus $r(\cdot|k)$ is the distribution of the number of successes, $\bar{r}(k)$ is the mean number of successes, and $P(k)$ is the average probability of failure for a typical packet, all given that k packets are transmitted in the slot.

Similarly, define

$$r_p(m|G) = E[r(m|K)]$$

$$\bar{r}_P(G) = E[r(K)]$$

and

$$P_P(G) = 1 - \bar{r}_P(G)/G$$

where K is a Poisson random variable with mean G. Thus $r_P(\cdot|G)$ and $\bar{r}_P(G)$ are the distribution and mean of the number of successes and $P_P(G)$ is the probability of failure of a typical packet, all given that the number of transmissions in the slot is a Poisson random variable with mean G.

The specific FH system will now be described. The frequency spectrum is divided into q frequency slots and the packets are divided into n bytes each. Each byte is transmitted at a frequency chosen from the q frequencies with equal probability, independently of the frequencies chosen for other bytes. It is then appropriate to use a burst-error correcting code -- we will assume that a Reed-Solomon code is used. We will also assume that a packet consists of exactly one codeword from a RS code for which up to t byte errors can be corrected. This provides us with a natural definition of a successfully transmitted packet. A packet is declared successfully transmitted if at most t byte errors occur. Both the (31,15)-code (with $n = 31$ bytes, five bits per byte, 15 information bytes and $t = 8$) and the (255,127)-code (with $n = 255$ bytes, eight bits per byte, 127 information bytes and $t = 64$) will be considered.

In the following let $\lambda = G/q$, so that λ is the traffic intensity <u>per frequency slot</u>. Also, $P(k,q)$ and $P_P(\lambda,q)$ will be written in place of $P(k)$ and $P_P(G)$ in order to make the dependence on q explicit. Finally, let $\eta(\lambda,q) = \lambda(1-P_P(\lambda,q))$. Thus η denotes the average throughput per frequency slot.

Assume now that byte errors are independent in the absence of multi-access interference. This independence assumption is true for an additive white Gaussian noise (AWGN) channel. The assumption is also approximately true for an AWGN channel with fading if q is so large that very few frequency slots are hit by more than one byte for any packet, or if the fading process of the channel model has a short correlation time compared to the typical elapsed time between visits to a given frequency. Also, by interleaving codewords, it is possible to approximately achieve the situation with independent byte errors even for relatively slowly fading channels.

Now let p_1 be the byte error probability in the absence of multi-access interference. Then the byte error probability <u>given</u> that k packets are transmitted in a slot is

$$p_k = 1 - \left(1 - \left(\frac{2}{q} - \frac{1}{q^2}\right)\right)^{k-1}(1 - p_1) \qquad (k \geqslant 1)$$

If byte synchronization were possible, the term $\frac{2}{q} - \frac{1}{q^2}$ in this expression could be replaced by $1/q$. By the assumed independence of byte errors in the absence of multi-access interference and the memorilessness of the random hopping pattern, the byte errors (including multi-access interference) are conditionally independent given k. Thus the packet error probability is

$$\mathcal{P}(k,q) = \sum_{i=t+1}^{n} \binom{n}{i} p_k^i (1 - p_k)^{n-i}$$

which can be used to compute

$$\mathcal{P}_p(\lambda,q) = \sum_{k=1}^{\infty} \frac{e^{-\lambda q} (\lambda q)^k}{k!} k\, \mathcal{P}(k,q)/\lambda q$$

and

$$\eta(\lambda,q) = \lambda(1-P_p(\lambda,q))$$

which are the desired packet error probability and throughput per frequency slot for Poisson traffic.

The computation of $P_p(\lambda,q)$ (and hence also $\eta(\lambda,q)$) simplifies in two special cases: First, when $q=1$,

$$P_p(\lambda,1) = 1 - e^{-\lambda}\ (1 - P(1,1))$$

since $P(k,1) = 1$ for $k \geqslant 2$. This implies that

$$\eta(\lambda,1) = \lambda e^{-\lambda}\ (1 - P(1,1)).$$

That is, when $q=1$, the throughput is $\lambda e^{-\lambda}$ (which is the throughput for a noiseless slotted-ALOHA channel) times the success rate in the absence of multiple-access interference.

The second special case is obtained by letting q and G tend to infinity with $\lambda = G/q$ fixed. The limiting packet error probability is then

$$P_p(\lambda,+\infty) = \lim_{q\to\infty} P_p(\lambda,q) = \lim_{\substack{q\to\infty \\ k\to\infty \\ \lambda=k/q}} P(k,q)$$

$$= \lambda \sum_{i=t+1}^{\infty} \binom{n}{i} p_\infty^i (1 - p_\infty)^{n-i}$$

where

$$p_\infty = \lim_{\substack{q\to\infty \\ k\to\infty \\ \lambda=k/q}} p_k = 1 - e^{-2\lambda}\ (1 - p_1).$$

Numerical results are given in Figs. 1-4 and in Tables 1-3. We see in Fig. 1 that for no channel noise and using the (255,127) Reed-Solomon code, a smaller packet error probability is achieved by $q = +\infty$ than by $q = 1$ if and only if λ is smaller than about 0.13. For an intuitive understanding of this it is important to keep in mind the following two facts. First, at the level of byte errors, the essential effect of varying the parameter q is that as q decreases, the occurrences of byte errors within a single packet become more positively correlated. Secondly, since λ is the traffic normalized per frequency slot, for fixed λ the byte error probability and therefore also the <u>mean</u> number of byte errors per packet does not strongly depend on q. Summarizing these two facts, for larger q the distribution of the number of byte errors tends to be more tightly concentrated near the (almost q-independent) mean number of byte errors.

Thus, whether or not the packet error probability is smaller for large q than for small q is determined by whether or not the error correcting capability of the code can accomodate any number of byte errors "near" the mean number of byte errors. Since the mean number of byte errors increases with λ, it follows that for small enough λ the packet error probability is smaller for large q, and conversely for large λ the packet error probability is smaller for small q.

When the byte error probability in the absence of multi-access interference p_1 is increased from zero to 0.1, the packet error probability does not significantly increase for $q = 1$ while it does for larger values of q. (Compare Figs. 1 and 2.) As a result, the value of λ at which the packet error probability for $q = \infty$ surpasses the packet error probability decreases to $\lambda = 0.078$. (See Fig. 2.) Thus in the presence of channel noise, the crossover value of λ can become quite small.

Turning to Figs. 3 and 4 we observe that the maximum throughput (over all λ) is much greater for $q = 1$ than for $q = \infty$. However, the maximum throughput for $q = 1$ can only be achieved by maintaining a mean traffic intensity $\lambda = 1$ which causes the packet error probability to exceed .63. Hence, although $q = 1$ offers greatly increased maximum throughput, the increase comes at the expense of either many retransmissions (which, if possible at all, generally increase delays) or a high packet loss rate.

Discussion of Method

The method of using the local Poisson approximation as discussed here and in the next section is admittedly only an approximation. It is important to emphasize however that, as shown in [7], the method does lead to channel stability (even taking approximations into account).

Another approach to the analysis of delay in a random-access system would be to use a more detailed model of the transmitters -- allowing them to obtain multiple packets and then buffer delay could be discussed. For such analysis so far in the literature, the total system is usually described as a (many state) Markov chain. For such analysis, the main obstacle has been the large size of the state-space. Here we wish to point out another difficulty which arises for such detailed analysis when one considers spread-spectrum systems. The problem is that a detailed exact analysis would require knowledge of the conditional distribution $r(\cdot|k)$ of the number of successes given k transmissions (whereas our analysis only required use of the mean number of successes). Some authors propose (implicitly) that the distribution of the number of successes is binomial under the assumption that the outcomes of transmissions of distinct packets form independent events. It is clear, due to the mutually destructive effect of collisions that this assumption is not true.

In summary -- before more detailed models can be effectively used, the distribution $r(\cdot|k)$ must be better characterized. Nevertheless, we have found retransmission control schemes which insure stable throughput, even without knowledge of this distribution (see next section).

Table 1. Packet error probability and throughput vs. traffic-intensity-per-frequency-slot λ for $q = 1$ (no hopping during packet transmission).

$q = 1$	$p_1 = 0.0$ Either Code		$p_1 = 0.1$ RS-(31,15)		RS-(255,127)	
λ	$P = 1-e^{-\lambda}$	η	$P = 1-e^{-\lambda}(.9974)$	η	$P = 1-e^{-\lambda}(1-1.2\times10^{-12})$	η
0.00	0	0	.0026	0	1.2×10^{-12}	0
0.02	.0198	.0196	.0223	.0196	Same as columns for $p_1 = 0.0$	
0.04	.0392	.0384	.0417	.0383		
0.06	.0582	.0565	.0607	.0564		
0.08	.0768	.0738	.0793	.0737		
0.10	.0951	.0905	.0975	.0902		
0.12	.1131	.1064	.1154	.1062		
0.14	.1306	.1217	.1329	.1214		
0.16	.1479	.1363	.1501	.1360		
0.18	.1647	.1503	.1669	.1500		
0.20	.1812	.1637	.1834	.1633		
0.25	.2212	.1947	.2232	.1942		
0.30	.2592	.2222	.2611	.2216		
0.40	.3300	.2681	.3314	.2674		
0.50	.3934	.3032	.3950	.3025		
0.60	.4512	.3293	.4526	.3284		
0.70	.5034	.3476	.5047	.3467		
0.80	.5501	.3595	.5518	.3585		
0.90	.5934	.3659	.5945	.3650		
1.0	.6321	.3679	.6331	.3669		
1.5	.7769	.3347	.7774	.3338		
2.0	.8646	.2707	.8650	.2700		
2.5	.9180	.2052	.9181	.2047		

Table 2. Packet error probability and throughput vs. traffic-intensity-per-frequency-slot λ for $q = 10$ frequency slots.

q=10	$p_1 = 0.0$ RS-(31,15)		$p_1 = 0.0$ RS-(255,127)		$p_1 = 0.1$ RS-(31,15)		$p_1 = 0.1$ RS-(255,127)	
λ	P	η	P	η	P	η	P	η
0.00	0	0	0	0	.0026	0	1.24×10^{-12}	0
0.02	.0335	.0193	.0185	.0196	.0958	.0180	.1386	.0172
0.04	.0820	.0367	.0632	.0375	.1864	.0325	.2599	.0296
0.06	.1400	.0516	.1240	.0525	.2725	.0436	.3655	.0381
0.08	.2031	.0637	.1934	.0645	.3531	.0517	.4571	.0434
0.10	.2684	.0731	.2664	.0733	.4274	.0572	.5364	.0464
0.12	.3335	.0800	.3395	.0792	.4953	.0606	.6047	.0474
0.14	.3968	.0844	.4102	.0825	.5568	.0620	.6635	.0471
0.16	.4572	.0868	.4769	.0837	.6121	.0620	.7140	.0457
0.18	.5140	.0874	.5388	.0830	.6615	.0610	.7572	.0437
0.20	.5667	.0866	.5955	.0809	.0753	.0589	.7942	.0411
0.25	.6801	.0800	.7138	.0715	.7937	.0515	.8645	.0339
0.30	.7983	.0695	.8010	.0595	.8574	.0428	.9113	.0266
0.40	.8835	.0465	.9080	.0365	.9337	.0265	.9626	.0150
0.50	.9438	.0280	.9597	.0201	.9700	.0150	.9845	.0078
0.60	.9737	.0157	.9827	.0104	.9860	.0079	.9936	.0038

Table 3. Byte error probability, packet error probability and throughput vs. traffic-intensity-per-frequency-slot for $q = \infty$.

$q = \infty$	$p_1 = 0.0$					$p_1 = 0.1$				
		RS-(31,15)		RS-(255,127)			RS-(31,15)		RS-(255,127)	
λ	$p = 1-e^{-2\lambda}$	P	η	P	η	$p = 1-.9e^{-2\lambda}$	P	η	P	η
0.00	0	0	0	0	0	.100	.00260	0	1.24×10^{-12}	0
0.02	.0392	2.00×10^{-6}	.020	8.72×10^{-33}	.020	.135	.0183	.020	2.41×10^{-7}	.020
0.04	.0768	3.93×10^{-4}	.040	4.96×10^{-18}	.040	.169	.0658	.037	3.37×10^{-4}	.040
0.06	.113	5.91×10^{-3}	.060	2.36×10^{-10}	.060	.202	.158	.051	.0236	.059
0.08	.148	3.15×10^{-2}	.077	5.84×10^{-6}	.080	.233	.285	.057	.224	.062
0.10	.181	9.36×10^{-2}	.091	2.06×10^{-3}	.100	.263	.431	.057	.639	.036
0.12	.213	.199	.096	6.19×10^{-2}	.113	.292	.576	.051	.961	.010
0.14	.244	.337	.093	.366	.089	.319	.697	.042	.989	.0015
0.16	.274	.490	.082	.773	.036	.349	.798	.032	.999	.0001
0.18	.302	.623	.068	.958	.008	.372	.871	.023		
0.20	.330	.741	.052	.996	.001	.397				
0.22	.356	.829	.036							
0.30	.451	.978	.007							

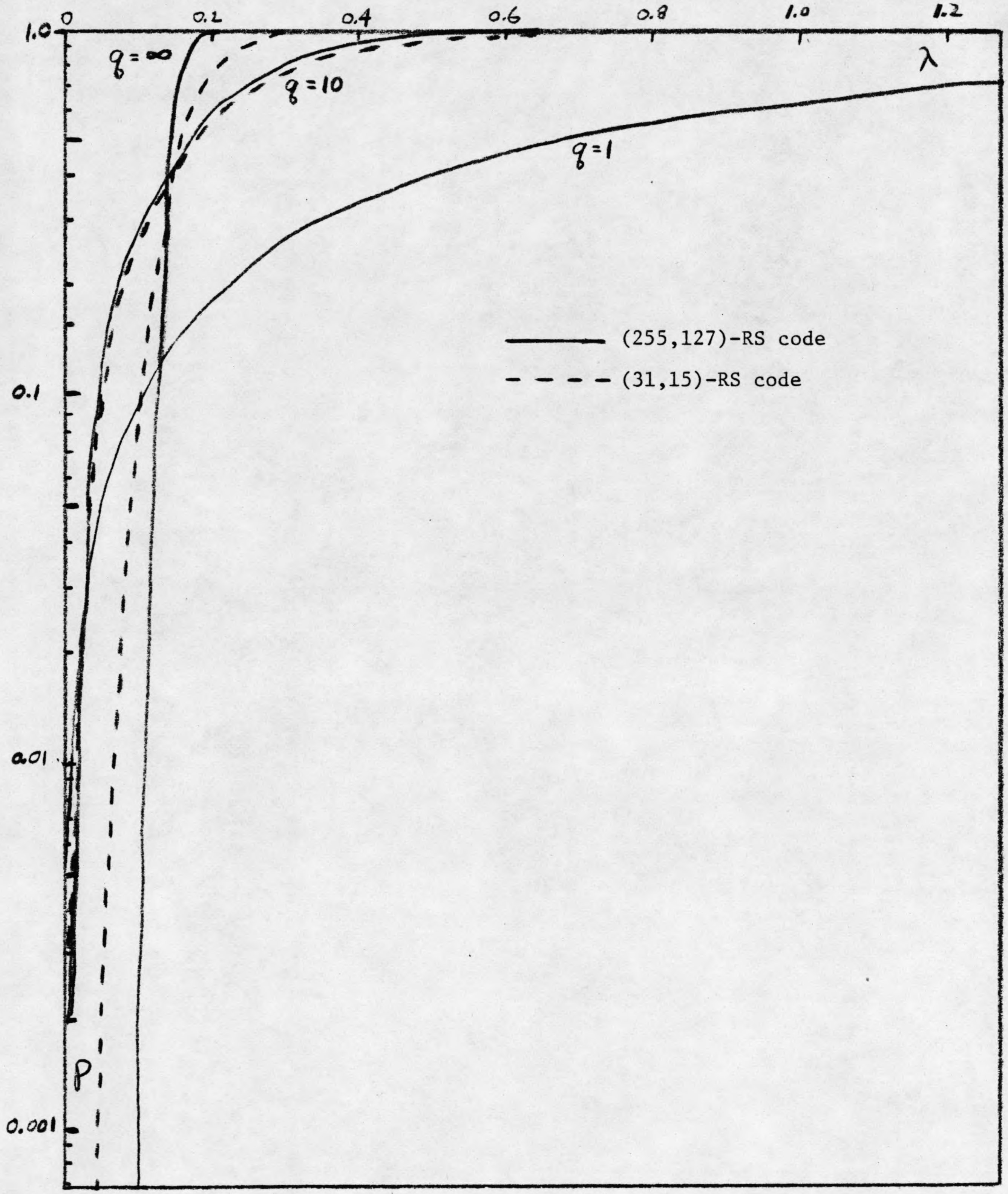

Fig. 1. Block error probability vs. traffic-intensity-per-frequency-slot λ. No channel noise ($p_1 = 0$). For q = 1, curves coincide.

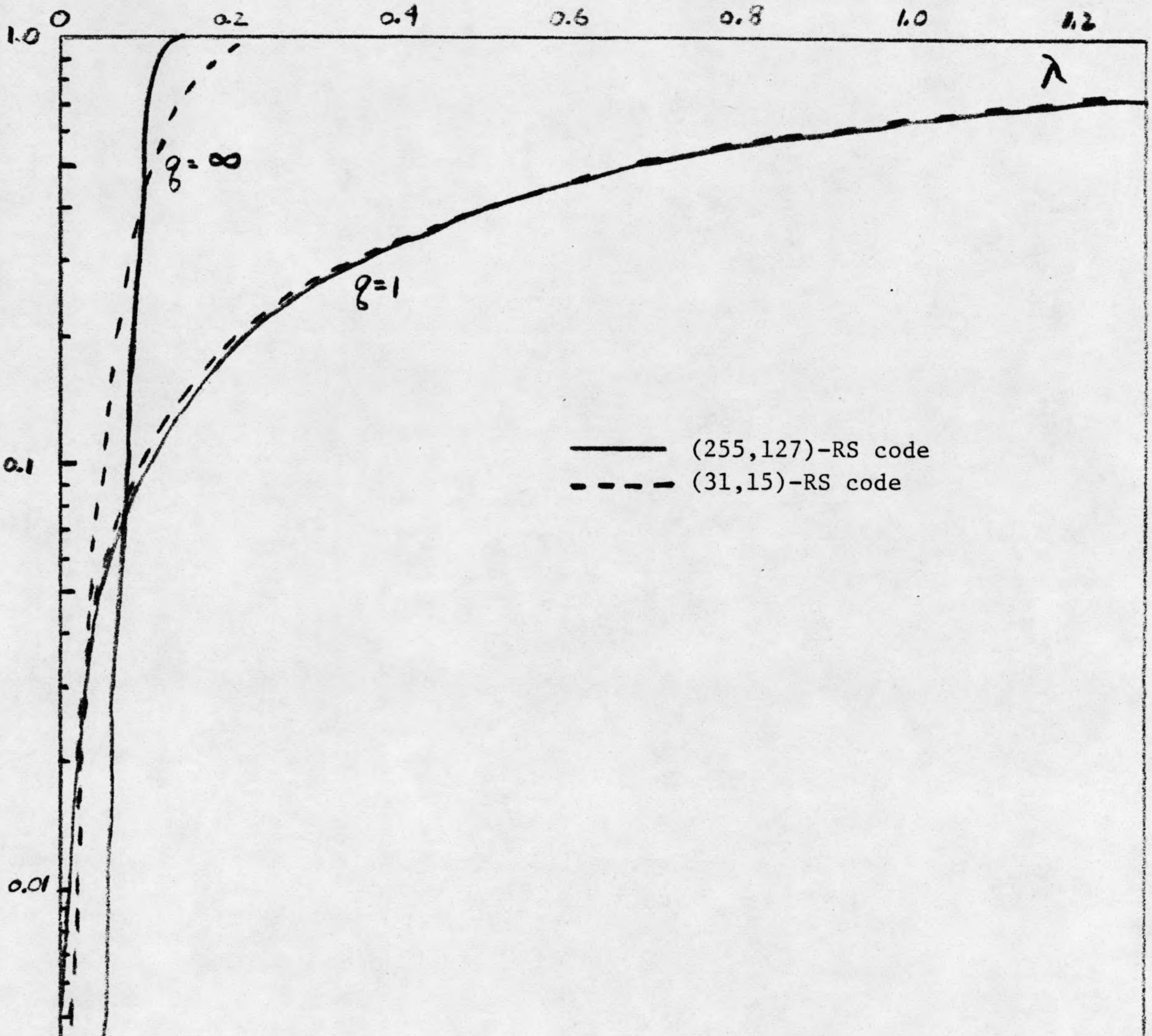

Fig. 2. Block error probability vs. traffic-intensity-per-frequency-slot λ.
Independent byte errors -- error probability $p_1 = 0.1$.

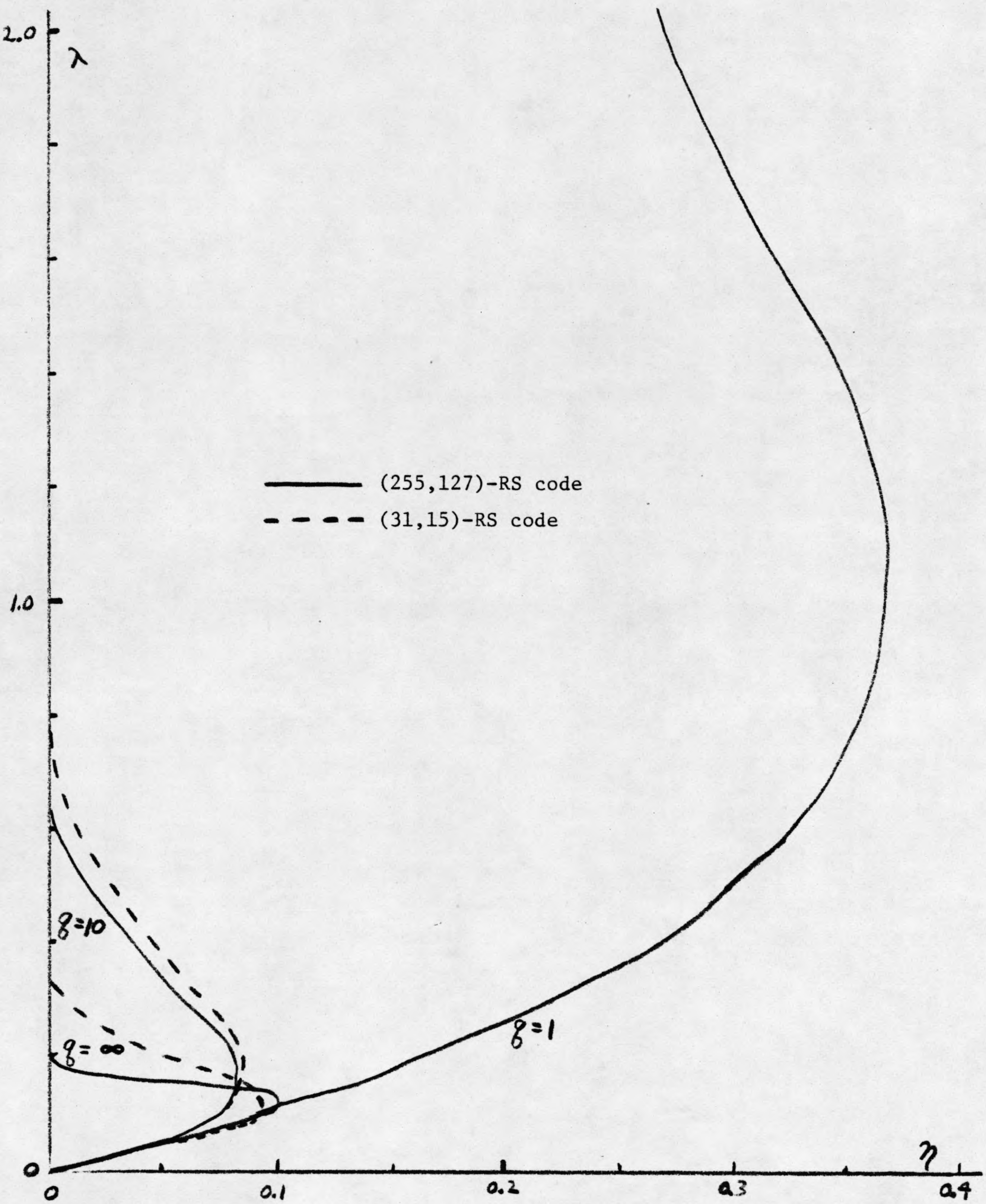

Fig. 3. Throughput vs. traffic-intensity-per-frequency-slot λ. No channel noise ($p_1 = 0$). For $q = 1$, curves coincide.

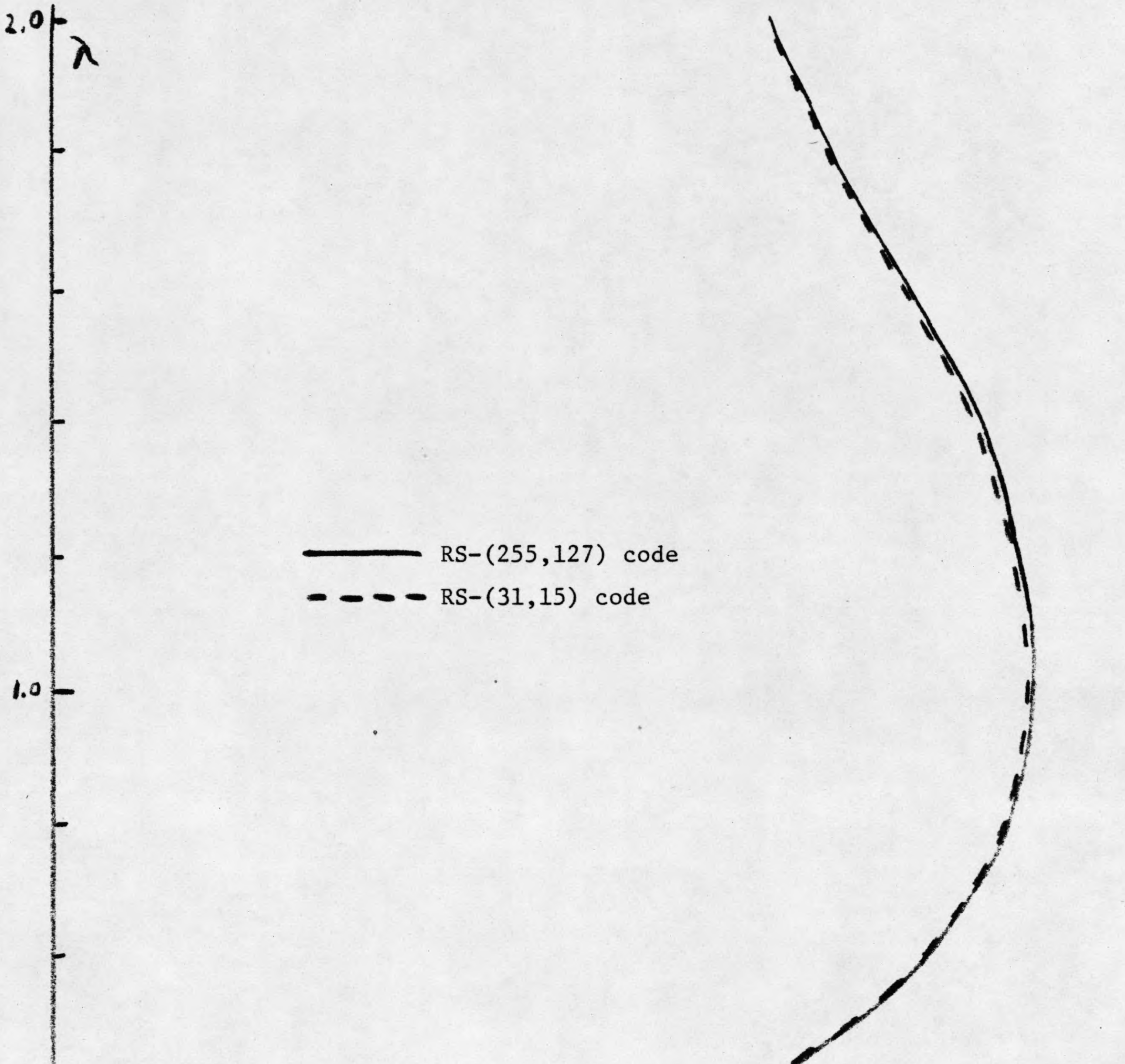

Fig. 4. Throughput vs. traffic-intensity-per-frequency-slot λ. Independent byte errors -- error probability $p_1 = 0.1$.

APPENDIX C

RECURSIVE RETRANSMISSION CONTROL -- APPLICATION TO FREQUENCY-HOPPED (FH) SPREAD-SPECTRUM SYSTEMS

Although the models of the user population and the feedback information are quite simple, the concepts of [5]-[7] readily extend to more complex and realistic settings. In order to illustrate this point, we shall briefly describe how the decentralized dynamic control procedure in [5] can be adapted to the frequency-hopping system described in the previous appendix. For definiteness, suppose that the Poisson model in [5] is used to describe how the population of stations acquires packets to be transmitted.

Our research has shown that it is desirable for the users to have some feedback information in order to suitably control the traffic level. We shall now describe a method for the users to obtain such information which is appropriate for use in the Navy's ITF network. During each slot, user α uses a random hopping pattern to hop a receiver among the q frequencies. The pattern has the same distribution as patterns used to transmit packets. For each dwell time the user decides (by a simple threshold test) whether or not the channel was free during that dwell time in the frequency monitored. The user then simply counts the number of dwell times in the slot for which it was decided that the channel frequency was not free. Let Y_t^α denote the count of user α for slot t. The variables Y_t^α comprise the feedback information upon which the retransmission control strategy described next is based.

Following [5], we suggest that user α recursively computes the sequence f_t^α via the multiplicative rule

$$f_{t+1}^\alpha = \min(f_t^\alpha a(Y_t^\alpha), 1) \tag{1}$$

Then if user α has a packet to transmit, it transmits it in slot t with probability f_t^α. A possible choice for the function a is

$$a(y) = 1 - (\frac{y}{q} - p^*)\gamma \qquad (2)$$

where

$$p^* = 1 - \exp(-2\lambda^*)$$

and λ^* is a desired value of the traffic intensity per frequency slot.

To understand this choice of retransmission policy, note first that if the number of packet transmissions in a slot is Poisson with mean $G = q\lambda$, then the probability that a frequency slot is used during any portion of a given one-byte dwell time is

$$p = 1 - \exp(-2\lambda)$$

When $\lambda = \lambda^*$, $p = p^*$. That is, p^* is the probability that a frequency is occupied during a given one-byte dwell time when the number of packet transmissions is Poisson with the desired mean-per-frequency-slot λ^*.

Now given the current values of the retransmission probabilities f_t^α for all α and given the set A_t of users which have a packet to transmit, the conditional distribution of the number of packet transmissions in slot t is approximately Poisson with mean

$$G_t = \sum_{\alpha \in A_t} f_t^\alpha + \mu \qquad (4)$$

(where μ is the rate at which new packets are transmitted) by the local Poisson approximation described in [5] and [9]. Hence,

$$E[Y_t^\alpha | f_t^\alpha, \text{ all } \alpha] = qp_t \qquad (5)$$

where

$$p_t = 1 - \exp(-2G_t/q)$$

Thus by (1) and (4),

$$E\,[\,G_{t+1}\,|\,f_t^{\alpha},\ \text{all}\ \alpha\,] = E\,[\sum_{\alpha\in A_{t+1}} f_t^{\alpha}a(Y_t^{\alpha}) + \mu\,|\,f_t^{\alpha},\ \text{all}\ \alpha\,]$$

$$= \mu + \sum_{\alpha\in A_{t+1}} f_t^{\alpha}E[a(Y_t^{\alpha})\,|\,f_t^{\alpha},\ \text{all}\ \alpha\,]$$

and since (by (2) and (5))

$$E\,[\,a(Y_t^{\alpha})\,|\,f_t^{\alpha},\ \text{all}\ \alpha\,] = E\,[\,1-\gamma(\frac{Y_t^{\alpha}}{q} - p^*)\,|\,f_t^{\alpha},\ \text{all}\ \alpha\,]$$

$$= 1-\gamma(p_t - p^*)$$

$$= 1+\gamma(\exp(-2G_t/q) - \exp(-2G^*/q))$$

we have

$$E\,[\,G_{t+1}\,|\,f_t^{\alpha},\ \text{all}\ \alpha\,] = \{1+\gamma(\exp(-2G_t/q) - \exp(-2G^*/q))\}G_t$$

Hence, G_{t+1} will tend to be larger than G_t (resp. smaller than G_t) if G_t is smaller (resp. larger) than the desired traffic level G^*. That is, G_t will tend to drift toward the desired traffic level G^*.

Using the methods of [7] it can be shown that with this transmission policy the channel is stable whenever the input rate μ is smaller than the average throughput rate $q\eta(\lambda^*,q)$ corresponding to the desired traffic intensity per frequency slot λ^*.

APPENDIX D

ACKNOWLEDGEMENT BASED RETRANSMISSION CONTROL

In [9] we define and analyze the class of Acknowledgement Based Retransmission Control (ABRC) schemes for random access. Such schemes require no more feedback information than the original ALOHA scheme in that each user need only learn whether or not its own transmission is successful. Such small feedback requirements are desirable in a spread spectrum environment when channel monitoring is difficult. On the basis of approximations developed in [5] under this contract and an equilibrium analysis, we have found that such schemes can provide satisfactory performance for both an infinite and finite number of users, as long as the retransmission probabilities are properly chosen.

We also introduced the possibility that after a packet has collided a certain number of times then it is rejected and no longer retransmitted. It appears that the probability that a given packet must be rejected can be kept to a satisfactorily low level, while allowing rejections improves stability considerably. In Fig. 1 the average probability of success and the probability of ultimate rejection P_R are given as a function of the allowed number of retransmissions k for the infinite user model with input rate $\lambda = 0.3$. For $k = 12$ the probability of rejection is about one in ten thousand. However if k is chosen to be too large then undesirable bistable behavior appears.

k	β		$P_R = (1-\beta)^k$	
1	.741		.741	
2	.662		.438	
3	.637		.258	
4	.622		.023	
5	.618		.009	
6	.615		.0035	
7	.614		.0013	
8	.614		5×10^{-4}	
9	.614		2×10^{-4}	
10	.613		8×10^{-5}	
11	.613		3×10^{-5}	
12	.613		1×10^{-5}	
13	.613		4×10^{-6}	
14	.613	.033	2×10^{-6}	.625
15	"	.0196	$<10^{-6}$	.743
16	"	.0125	"	.818
17	"	.0085	"	.865
18	"	.0059	"	.899
19	"	.0041	"	.925
$+\infty$	.6130	0.0	0.0	1.0

Fig. 1. Equilibrium values of success probability β and ultimate probability of rejection for $\lambda = 0.3$ packets/slot in infinite user model.

APPENDIX E

THE METHOD OF MARKOV PROCESSES WITH PHASES

Another area of progress under this contract has been the development of a numerical technique, the "method of phases", which we have discovered is suitable for evaluating certain random access algorithms in the presence of fluctuating traffic rates. Our main motivation is that the usual Bernoulli or Poisson models of arrival processes are not "bursty" enough to realistically model traffic which random access schemes are likely to face in practice. So far the method has been successful for evaluation of TDMA with buffered users and varying arrival rates. The method will now be briefly described, following [8].

Consider a TDMA system with m users and arrival rate σ packets/slot/user. Let N_t denote the number of packets in the first user's buffer and let $\theta_t \in \{1,\ldots,m\}$ denote which user is transmitting during slot t. (θ is the "phase" of the system relative to the first user.) Then (N_t, θ_t) can be modelled as a discrete-time Markov chain on $\mathbb{Z}_+ \times \{1,\ldots,m\}$ with transition matrix (in the following matrices, only non-zero entries are indicated):

$$P^* = \begin{vmatrix} A_{00} & A_{01} & & & \\ A_{10} & A_1 & A_2 & & \\ & A_0 & A_1 & A_2 & \\ & & A_0 & A_1 & A_2 \\ & & \ddots & \ddots & \ddots \end{vmatrix}$$

where the blocks of P^* are the mXm matrices

$$A_0 = \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \\ 1-\sigma & & & \end{bmatrix} \quad A_1 = \begin{bmatrix} 0 & 1-\sigma & & & \\ & & 1-\sigma & & \\ & & & \ddots & \\ & & & & 1-\sigma \\ \sigma & & & & 0 \end{bmatrix} \quad A_2 = \begin{bmatrix} 0 & \sigma & & & \\ & & \sigma & & \\ & & & \ddots & \\ & & & & \sigma \\ & & & & 0 \end{bmatrix}$$

$$A_{10} = \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \\ 1-\sigma & & & \end{bmatrix} \quad A_{00} = \begin{bmatrix} 0 & 1-\sigma & & & \\ & & 1-\sigma & & \\ & & & \ddots & \\ & & & & 1-\sigma \\ 1-\sigma & & & & 0 \end{bmatrix} \quad A_{01} = \begin{bmatrix} 0 & \sigma & & & \\ & & \sigma & & \\ & & & \ddots & \\ & & & & \sigma \\ \sigma & & & & 0 \end{bmatrix}$$

In [8] a general method is presented for obtaining the invariant distribution of such chains. The general procedure is as follows [8]:

<u>Step 1</u>: Compute B, where B is the minimum non-negative solution to the equation

$$B = BA_2BA_0(I-A_1)^{-1} + (I-A_1)^{-1}$$

Successive substitutions starting with $B^{(0)} = 0$ yields an increasing sequence converging to B.

<u>Step 2</u>: Find π_0, the invariant distribution for the m×m transition matrix $P_0^* = A_{00} + A_{01}BA_{10}$.

<u>Step 3</u>: Compute the constant

$$c = \pi_0(I + A_{01}B(I-R)^{-1})e$$

where $e = (1,1,\ldots,1)^T$ and $R = A_2B$.

Then the invariant distribution for P^* is $x = (x_0, x_1, \ldots)$ where

$$x_0 = \pi_0/c$$

and

$$x_k = x_0A_{01}BR^{k-1} \qquad k \geq 1.$$

In [8] analogous results are also derived when P^* is truncated to a finite number of levels and boundary states are added. This provides a computationally tractable method to analyze queues with finite buffers.

Using this approach, the invariant distribution for the TDMA example has been found analytically and numerically [10]. Our results, such as expressions for the average backlog, agree with those obtained by other methods. The advantage of this approach is that it readily extends to the case when the arrival rate fluctuates according to an underlying Markov process, for then the system still has the same form as above but for different choices of the A_i's. This extension has been carried out and is presented in [10].

An example of our numerical results are presented in Fig 2. In this example the number of users M was taken to be 4 or 10. In each case the mean arrival rate was ρ = 96% of the channel capacity, and the actual arrival rate fluctuated between two different arrival rates, where the switching was governed by a two state Markov chain. The dashed lines correspond to an example when both of the input rates were chosen below the system capacity while the solid lines correspond to an example when one of the two rates is above the system capacity. The curves give the average backlog $\overline{N}$ for a given user versus γ, where γ is a parameter which indicates how fast the rate is switched. For small γ the switching processes is slow so that a large backlog results when one of the two rates is above the system capacity (see solid lines).

Our reason for studying arrival processes with varying rates is that we feel it provides a more realistic model of bursty traffic than does the usual Poisson arrival model. We are now in the process of analyzing other random access disciplines in the presence of varying traffic rates. We suspect that many random access schemes will perform more favorably relative to the performance of TDMA when the traffic arrival rates vary dynamically.

In addition, we have found the method of phases developed in [8] under this contract to be useful in the analysis of certain routing schemes in a packet switched network [10]. (This portion of [10] was supported by a JSEP contract.) An important product has been an increase in our understanding of the advantages and limitations in the use of Markov processes with phases. We feel that it is an important and useful technique which will find many applications both within and beyond the multiple access area.

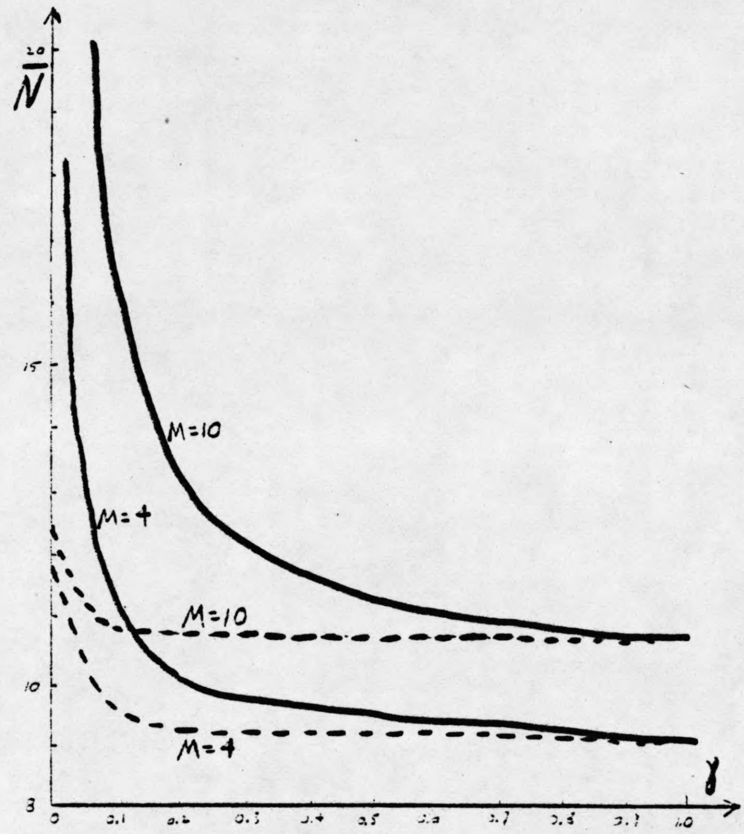

Fig. 1. Average backlog of a user in TDMA system as a function of the rate of traffic variation.