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RANDOM-ACCESS TECHNIQUES FOR COMMUNICATION NETWORKS WITH SPREAD-SPECTRUM SIGNALING

BRUCE HAJEK

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A brief summary of results obtained primarily during the second year (August 21, 1981 to August 20, 1982) under Contract N00014-80-C-0802 sponsored by the Naval Research Laboratory is presented. This report complements an earlier report which summarized the first year of research under this contract. The research covered several aspects of random access, routing and transmission scheduling and it also included some performance analysis of slow-frequency-hopped spread-spectrum multiple-access in a fading environment. The results obtained are applicable to the Navy's intra-task-force mobile radio network.

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Final Report for

RANDOM-ACCESS TECHNIQUES FOR COMMUNICATION NETWORKS WITH SPREAD-SPECTRUM SIGNALING

for the period August 21, 1981 - August 20, 1982

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ABSTRACT

A brief summary of results obtained primarily during the second year (August 21, 1981 to August 20, 1982) under Contract N00014-80-C-0802 sponsored by the Naval Research Laboratory is presented. This report complements an earlier report which summarized the first year of research under this contract. The research covered several aspects of random access, routing and transmission scheduling and it also included some performance analysis of slow-frequency-hopped spread-spectrum multiple-access in a fading environment. The results obtained are applicable to the Navy's intra-task-force mobile radio network.

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SUMMARY OF RESEARCH

This report briefly summarizes progress made during the past year. The emphasis is on results which are applicable to the Navy's proposed intra-task-force (ITF) communication network. One goal of our research was to study basic random access strategies. An important feature was to consider buffered stations and to allow the packet arrival rates at the different stations to independently fluctuate. The rate fluctuation leads to more "bursty" traffic than the usual Poisson or Bernoulli arrival models. Much to our surprise, even for rather bursty models, the time division multiple access (TDMA) strategy produces delays which are not much larger than the ideal (but unrealizable) perfect scheduling strategy. (See Appendix D for analysis and numerical results.)

As a result, we shifted more attention to considering how a network wide TDMA strategy might be implemented using spread-spectrum signaling. We sought a strategy which incorporates the multiple-access capability of spread-spectrum signaling. The result, reported in [11] (see Appendix A), is a procedure for controlling multiple access interference in a scheduled spread-spectrum network. The procedure is compatible with existing link activation scheduling algorithms which are not designed to control such interference.

Another research goal was to determine how several frequency bands with different attenuations should be used. This problem is addressed in [13] and in Appendix B. One of our main conclusions is that network throughput can be significantly increased if the transmission range used to transmit a packet depends on the packet's ultimate destination. The implication of this for the Navy's ITF network is that a single network operating on all frequency bands should be implemented so that a typical packet can be transmitted in different frequency bands at different stages along its route (see Appendix B).

Error probability bounds are given in [14] and [4] for a coded slow-frequency-hopped system subject to a Rayleigh fading channel. In Appendix C additional numerical results are given. These results are for a larger number of frequency slots which is of interest in the Navy's proposed ITF network. Also given are upper bounds on the error probability for Rician fading channels.

The need for effective routing procedures to avoid local regions of high traffic in a bandwidth limited radio network is indicated in Appendix B. In this direction, new efficient optimal dynamic routing procedures for communication networks are given in [12] (portions contained in Appendix E). These procedures appear to be the first optimal dynamic routing procedures which are computationally feasible for large networks. One of the algorithms is suitable for decentralized implementation.

PUBLICATIONS SUPPORTED BY NRL CONTRACT

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APPENDIX A August 1982

BALANCED SCHEDULING IN A PACKET SYNCHRONIZED SPREAD SPECTRUM NETWORK

by

Bruce Hajek

ABSTRACT

A simple method is presented for controlling secondary multiple access interference in a packet synchronized spread spectrum network with scheduled transmissions. The method can be incorporated into existing scheduling algorithms which have been designed to avoid primary multiple access interference. Certain transmission schedules called uniformly most balanced schedules are shown to exist which are optimal simultaneously under a variety of criteria. It is shown that such schedules can be found by an easily decentralized local improvement algorithm, and a preliminary performance analysis is given.

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I. INTRODUCTION

A viable access strategy for a mobile radio network is to attain network-wide synchronization with resolution small compared to packet transmission times and to govern packet transmission times with a schedule. A schedule designates which stations are to transmit packets and which stations are to receive the packets during each time slot. A schedule is periodic of period M if the same transmitter-receiver pairs are designated every M time slots. Such a schedule need only be specified for a frame which consists of M consecutive time slots.

We can associate a directed graph to the network at a given time instant. The nodes of the graph represent stations in the network and the links represent ordered pairs of stations. We say that link ℓ is activated in a slot if during the slot the first station in the pair of stations represented by ℓ transmits a packet intended for the second station in the pair. A periodic transmission schedule can be represented by a vector $\underline{f} = (f_\ell(k))$ where for each link ℓ and each slot k with $1 \le k \le M$, $f_\ell(k)$ is equal to one if link ℓ is activated during slot k of each frame and $f_\ell(k)$ is equal to zero if not.

For a narrowband channel it may be desirable to schedule transmissions so that during any given time slot each transmitted packet is the only packet which can be heard at the receiver intended for the packet, thus preventing collisions entirely. This has been called spatial TDMA [7].

For spread spectrum systems it is possible for multiple disjoint ordered pairs of stations all within communication range of each other to successfully complete packet transmissions simultaneously. To be definite, we assume that the spread spectrum codes or hopping pattern assignments are <u>receiver</u> oriented. For such systems we can identify two types of conflicts - primary and secondary.

A primary conflict occurs if two or more transmitters simultaneously send packets to the same receiver. If a primary conflict occurs then none (or perhaps one if the system has a capture capability) of the conflicting transmissions are successful and at least one packet does not get through. We also say a primary conflict occurs if a single transmitter is scheduled to simultaneously transmit two packets.

A secondary conflict occurs at a receiver if packet transmissions which are not intended for the receiver are heard at the receiver in addition to a transmission which is intended for the receiver. For typical spread spectrum systems it is not critical that the level of secondary conflicts be zero, although it should be kept to an acceptably low level.

In [1] a decentralized algorithm called the Link Activation Algorithm was described which generates a periodic transmission schedule that avoids primary conflicts. This algorithm can be generalized so that for a given vector $\underline{G} = (G_\ell)$ called a link demand vector the algorithm generates a periodic transmission schedule such that link ℓ is activated G_ℓ times during each frame. The Link Activation Algorithm produces a schedule to avoid primary conflicts but by itself does not attempt to choose a schedule which also yields an acceptable level of secondary conflicts. The main issue we address in this paper is how to improve scheduling algorithms to minimize the level of secondary conflicts as well as to completely avoid primary conflicts.

The solution we suggest is the following. First, replace the demand vector $\underline{G}$ by a larger demand vector $\underline{G}'$ (i.e. $G'_\ell \geq G_\ell$ for each link ℓ and we write $\underline{G}' \geq \underline{G}$) as the input to the Link Activation Algorithm. For example, let $G'_\ell = G_\ell + 1$ for ℓ with $G_\ell \neq 0$, or set $G'_\ell = 2G_\ell$ for all ℓ. The algorithm will then produce a schedule which we denote by $\underline{C} = (C_\ell(k))$ and which we call a skeleton schedule. Then $\underline{C}$, as well as any transmission schedule $\underline{f}$ with $\underline{f} \leq \underline{C}$, will avoid primary conflicts. The second part of our suggested solution is to choose a particular schedule $\underline{f}$, from among all schedules $\underline{f}$ with $\underline{f} \leq \underline{C}$ and which satisfy the link demand requirements $\underline{G}$, in order to minimize the adverse effects of secondary conflicts.

This procedure is motivated by the fact that once $\underline{C}$ has been selected, $\underline{f}$ can be readily varied while still preventing primary conflicts. For example, $\underline{f}$ can be chosen in the following way. For a given link ℓ there are G'_ℓ time slots per frame during which link ℓ could be activated. These slots are designated by the skeleton schedule $\underline{C}$. By monitoring the level of secondary conflicts at the receiver, the pair of stations corresponding to link ℓ can choose the best G_ℓ slots from among these G'_ℓ possibilities.

In a practical network we envision that a transmission schedule will be used which is only quasi-periodic (i.e., periodic over time intervals on the order of several frames in duration). This would be implemented by updating the link activation vector $\underline{f}$ in response to changing network topology and traffic demands. The schedule updating and the initialization of the schedule would all be done in a decentralized way. Each station in the network need only know a small part of the schedule. The overall transmission control scheme is pictured in Figure 1.

In this paper we investigate a criterion and a method for choosing $\underline{f}$ given a skeleton schedule $\underline{C}$ and link demand $\underline{G}$. We do not here consider details about how $\underline{G}$ and $\underline{C}$ are generated. In fact, our results in the next two sections do not depend on how $\underline{G}$ and $\underline{C}$ are generated and our performance analysis model given in Section 4 assumes that $\underline{C}$ is generated at random in a specified manner. Our results support the thesis that the use of a skeleton schedule can decouple the problem of secondary conflict control from the problems of primary conflict control, routing, etc.

We assume for analysis purposes that all stations are within range of each other. This assumption is discussed in the next section. We find in the next section that there exist schedules which are optimal simultaneously for a large number of different optimality criteria. We call such schedules "uniformly most balanced" and in Section 3 we show that such schedules can be readily found by an easily decentralized local improvement algorithm. An outline for performance analysis and some preliminary progress are presented in Section 4.

II. UNIFORMLY MOST BALANCED TRANSMISSION SCHEDULES

To simplify our analysis of secondary conflicts we specialize to the situation when all stations are within range of each other. Then our analysis can be viewed as a worst case analysis or as an approximate analysis of a local region in a network. Then, roughly speaking, the problem of choosing a transmission schedule $\underline{f}$ given link demand requirements $\underline{G}$ and a skeleton schedule $\underline{C}$ becomes:

"Optimize" $X = (X_1, X_2, \ldots, X_M)$

over $(\underline{f}, \underline{X})$

subject to
$$\sum_{\ell} f_{\ell}(k) = X_k \qquad (2.1)$$

$$\sum_{k=1}^{M} f_{\ell}(k) = G_{\ell} \qquad (2.2)$$

$$f_\ell(k) \in \{0,1\} \qquad (2.3)$$

$$f_\ell(k) \le C_\ell(k) \qquad (2.4)$$

Note that X_k is the number of transmissions in a slot. An interpretation of equations (2.1) through (2.4) which will be quite useful later is as flows on a certain single-source single-destination graph as pictured in Figure 2.1. The node set of the graph consists of a source node s, a destination node, d, nodes $\{a_\ell: 1 \le \ell \le L\}$ and nodes $\{b_k: 1 \le k \le M\}$. There is a link leading from S to each node a_ℓ, a link leading from each node a_ℓ to each node b_k, and a link from each node b_k to the destination node d. Then $f_\ell(k)$ and $C_\ell(k)$ represent the flow and capacity of link (a_ℓ, b_k) for each ℓ and k, G_ℓ represents the flow on link (s, a_ℓ), and X_k represents the flow on link (b_k, d). The choice of a schedule $\underline{f}$ given a link demand $\underline{G}$ and skeleton schedule $\underline{C}$ is thus a special case of a flow assignment (i.e., routing) problem.

In a real system the performance of a schedule $\underline{f}$ depends in a complex way on packet error probabilities as a function of the level of secondary conflicts, the routing strategy, delay-throughput requirements, etc. Therefore an important problem is to find a tractable yet meaningful interpretation for the word "optimize" in the above problem formulation.

For example, let Φ be a convex function on the nonnegative integers. It may be desirable for $\underline{f}$ to be a solution to the minimization problem $\underline{P}$ defined by

$$(P) \qquad \min \sum_{k=1}^{M} \Phi(X_k)$$

over $\underline{f}$, $\underline{X}$ subject to Eqs. (2.1) - (2.4).

The mean number of transmissions per slot m(X) is constrained by equation (2.1) to be

$$m(\underline{X}) = \frac{1}{M} \sum_{\ell} G_{\ell}$$

and does not depend of $\underline{f}$. The sample variance of the number of transmissions per slot is

$$V(X) = \frac{1}{M} \sum_{k=1}^{M} X_k^2 - m(X)^2$$

and the mean number of transmissions which occur during the same slot as a transmission chosen at random is

$$m_1(X) = m(X) + V(X)/m(X)$$

Thus, both V(X) and $m_1(X)$ are minimized when $\underline{f}$ is a solution to problem P for $\Phi(x) = x^2$.

In another example, if $\Phi(x) = 1$ for $x = 0$ and $\Phi(x) = 0$ for $x \geq 1$ then the solution $\underline{f}$ to problem P_o minimizes the number of slots with no transmissions.

It is not clear what the best choice for Φ would be, but fortunately, it doesn't matter as we show next.

Proposition 2.1. There exists a (not necessarily unique) pair $(\underline{f}^*, \underline{X}^*)$ which solves problem P simultaneously for all convex functions Φ.

A schedule $\underline{f}$ satisfying the conditions of Proposition 2.1 will be called uniformly most balanced (for the link demand $\underline{G}$ and skeleton schedule $\underline{C}$). Although uniformly most balanced schedules may often be desired in practice, that is not always the case. For example, suppose that $m(X) = 2$ but that the channel is narrow band. Then for a perfectly balanced schedule with two transmissions per slot, no transmission gets through. However, for the highly unbalanced schedule with one transmission in each of the first M-1 slots and the rest of the transmissions in the Mth slot, a packet is successfully transmitted during all but one slot.

Generally speaking, achieving a balanced schedule is probably appropriate whenever the <u>mean</u> level of secondary interference is acceptable. If the mean level is unacceptably high then an unbalanced schedule, for which occasionally the secondary interference is lower than the mean, may be acceptable. Accordingly, balanced schedules should be quite generally desirable if the <u>mean</u> level of secondary interference is appropriately controlled. These considerations lead us to the following modification of the concept of most balanced schedule.

Given a convex function Φ, consider the minimization problem P_s defined by

$$(P_s) \qquad \min \sum_{k=1}^{M} \Phi(X_k)$$

$$\text{over } \underline{f}, \underline{X} \text{ subject to } \quad \sum_{\ell} f_\ell(k) = X_k$$

$$\sum_{k=1}^{M} f_\ell(k) \le G_\ell \tag{2.5}$$

$$f_\ell(k) \in \{0,1\}$$

$$f_\ell(k) \le C_\ell(k)$$

Problem P_s is equivalent to problem P except there is an inequality in the second constraint in (2.5) instead of an equality. Thus, the resulting solution schedule $\underline{f}$ may not satisfy the given link demand $\underline{G}$.

Proposition 2.2. For each integer $b \geq 0$ there exists a (not necessarily unique) pair $(\underline{f}^b, \underline{X}^b)$ which solves problem P_s simultaneously for all convex functions Φ such that $\Phi(x)$ is minimized at $x = b$. For each k it holds that $X_k^b \leq b$.

A schedule satisfying the conditions of Proposition 2.2 for a fixed integer b will be called a uniformly most balanced schedule about traffic level b. If we choose $\Phi(x) = \max(b - x, 0)$, then we see that such a schedule maximizes

$$\sum_{k=1}^{M} X_k I\{X_k \leq b\}$$

which is the throughput for a channel that can support up to b transmissions per slot and for which all packets are lost when more than b transmissions occur in a slot.

III. A LOCAL IMPROVEMENT ALGORITHM FOR ACHIEVING UNIFORMLY MOST BALANCED SCHEDULES

Given a link demand vector $\underline{G}$ and skeleton schedule $\underline{C}$, let F denote the set of transmission schedules $\underline{f}$ which satisfy the constraints (4.2) - (4.4). In the previous section we showed that F contains schedules which are uniformly most balanced. We show in this section that such schedules can be arrived at in a simple decentralized way. The procedure is simply the heuristic schedule selection method suggested in Section 1 applied under our simplifying assumption that all stations are within range of each other and under the assumption that stations can detect which of two given slots have more transmissions.

Given schedules f and f' in F we say that f is at least as balanced as f' if for any convex function Φ

$$\sum_{k=1}^{M} \Phi(X_k) \leq \sum_{k=1}^{M} \Phi(X_k') \tag{3.1}$$

where X and X' correspond to f and f' by equation (2.1). We say that f is strictly more balanced than f' if strict inequality holds in relation (3.1) whenever Φ is a strictly convex function.

A simple way to modify a schedule f in F to obtain another schedule g in F is to change one slot assignment for one link. More precisely, if for some link ℓ_o and some slots k_o and j_o it holds that

$$f_{\ell_o}(k_o) = 1, \quad f_{\ell_o}(j_o) = 0 \quad \text{and} \quad C_{\ell_o}(j_o) = 1$$

then we can define a new schedule g in F by

$$g_\ell(k) = \begin{cases} 1 - f_\ell(k) & \text{if } \ell = \ell_o \text{ and } (k = k_o \text{ or } k = j_o) \\ f_\ell(k) & \text{otherwise.} \end{cases}$$

We say that g is obtained from f by an elementary transition. By Jensen's inequality the new schedule g is as balanced (respectively, strictly more balanced) than f if and only if $X_{k_o} \geq X_{j_o} + 1$ (respectively, $X_{k_o} \geq X_{j_o} + 2$) where X is the vector associated with f by equation (2.1).

An easily decentralized way to search for a uniformly most balanced schedule is to obtain a sequence of schedules so that each successive schedule is more balanced and is obtained from the previous schedule by an elementary transition. The question arises as to whether or not a most balanced schedule is always eventually obtained. As we show the answer is yes if one does not insist that each successive schedule in the sequence be strictly more balanced.

Consider Fig. 3.1 for example. If the link using slot 2 shifted to use slot 1 (this preserves the distribution of $\underline{X}$) and then if the other link which could use slot 2 shifted to slot 2 from slot 3 then a more balanced schedule is achieved. However, no single elementary transition leads to a more balanced schedule.

In general, unless $\underline{f}$ is uniformly most balanced, we can show that $\underline{f}$ can be modified by a sequence of elementary transitions so that each successive schedule produced is as balanced as the previous one and eventually a strictly more balanced schedule is obtained. We express a probabilistic statement of this fact as a proposition.

Proposition 3.1. Consider the Markov process with finite state space F with generator matrix Q defined by

$$Q(\underline{f}, \underline{g}) = \begin{cases} 1 & \text{if } \underline{g} \text{ is in F, is obtained from } \underline{f} \text{ by an} \\ & \text{elementary transition, and is at least as} \\ & \text{balanced as } \underline{f} \\ 0 & \text{for other } \underline{g} \text{ not equal to } \underline{f} \end{cases}$$

Then the balance of the state is nondecreasing along each sample path with probability one and the set of ergodic states of the chain consists of the set of uniformly most balanced schedules.

IV. PERFORMANCE ANALYSIS OR "HOW BALANCED ARE UNIFORMLY MOST BALANCED SCHEDULES?"

A given schedule defines a probability distribution $\underline{p}$ which is the distribution of the number of transmissions in a typical slot. It is defined by

$$p_n = (\#\text{ of slots } k \text{ with } X_k = n)/M$$

where $\underline{X}$ is defined by equation (2.1). Now for any link demand vector $\underline{G}$ and skeleton schedule $\underline{C}$ there exist uniformly most balanced schedules $\underline{f}$ and they each give rise to the same distribution $\underline{p}$. Thus, $\underline{G}$ and $\underline{C}$ together determine $\underline{p}$. Broadly speaking, we want to analyze $\underline{p}$ when $\underline{G}$ and $\underline{C}$ are in some sense "typical", and we do this by letting them be random.

To simplify our analysis, we suppose that each link is to be activated exactly once per frame (i.e., $G_\ell = 1$ for all ℓ). We also suppose that $\underline{C}$ assigns two distinct slots to each link. Thus, if L denotes the number of links, there are

$$\binom{M}{2}^L,$$

such possible skeleton schedules $\underline{C}$. We assume that $\underline{C}$ is random and equal to each of these possibilities with equal probability. So far, exact analysis of $\underline{p}$ for M and L large appears intractable, so our aim so far has been to obtain exact asymptotic results as M and L tend to infinity with $L = \alpha M$ for some fixed constant α which denotes the mean number of transmissions per slot. Some preliminary results are presented in Figs. 4.1 through 4.3. The asymptotic analysis results rely on a branching process argument. Details will be presented elsewhere.

The last row of Fig. 4.2 has the following interpretation. If M links are each independently assigned two out of M slots by a skeleton schedule and if M is very large, then under a most balanced transmission schedule about 16% of the slots will have no transmissions, 68% will have one, 16% will have none, and perhaps surprisingly, a negligible fraction of slots will have three or more transmissions. From Figure 4.3 we observe that 0.5 is a critical value of α since if and only if α exceeds 0.5 will a nontrivial proportion (as $M \to \infty$) of slots be used by two or more links for activation.

For values of α larger than one we have only been able to compute p_o (as $M \to \infty$). We conjecture that (as $M \to \infty$) $p_n = 0$ for n larger than $[\alpha] + 3$. This is because we believe (and this belief is supported by Figures 4.1 through 4.3) that uniformly most balanced schedules quite effectively reduce peak (over a frame) interference levels to be near the mean interference level. These issues pose interesting problems for further investigation.

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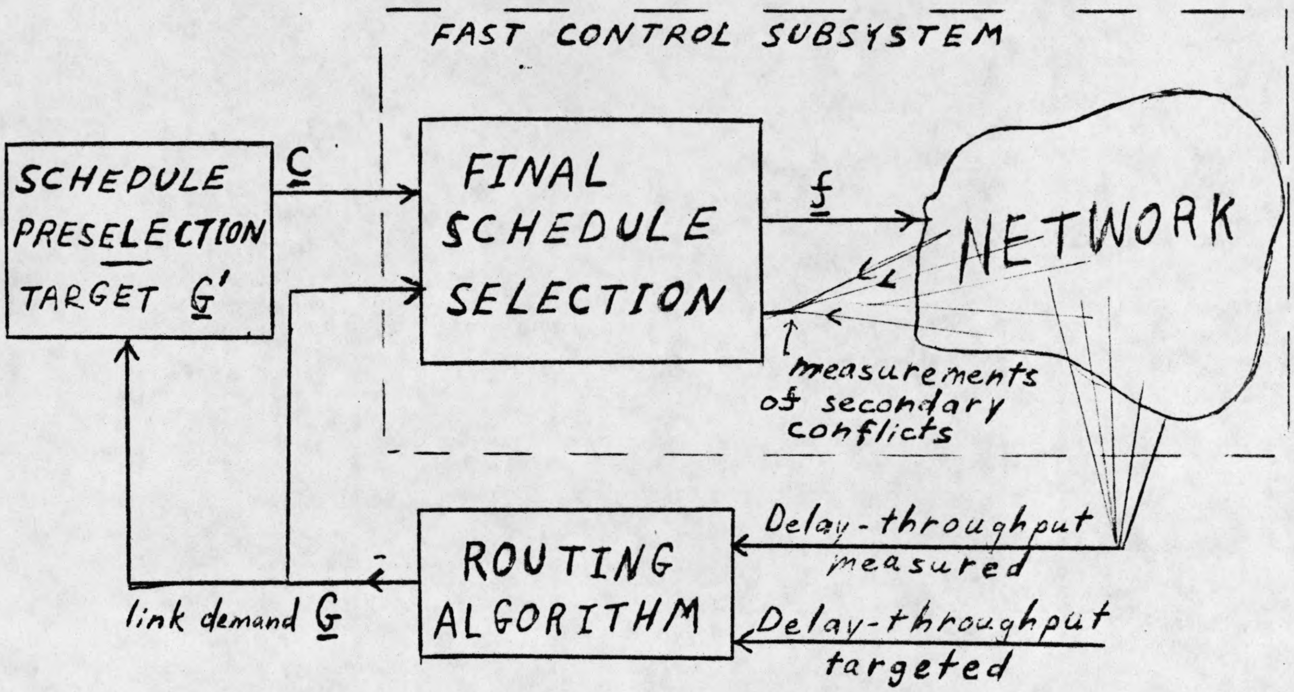

Fig. 1.1. Routing and link activation using a skeleton schedule

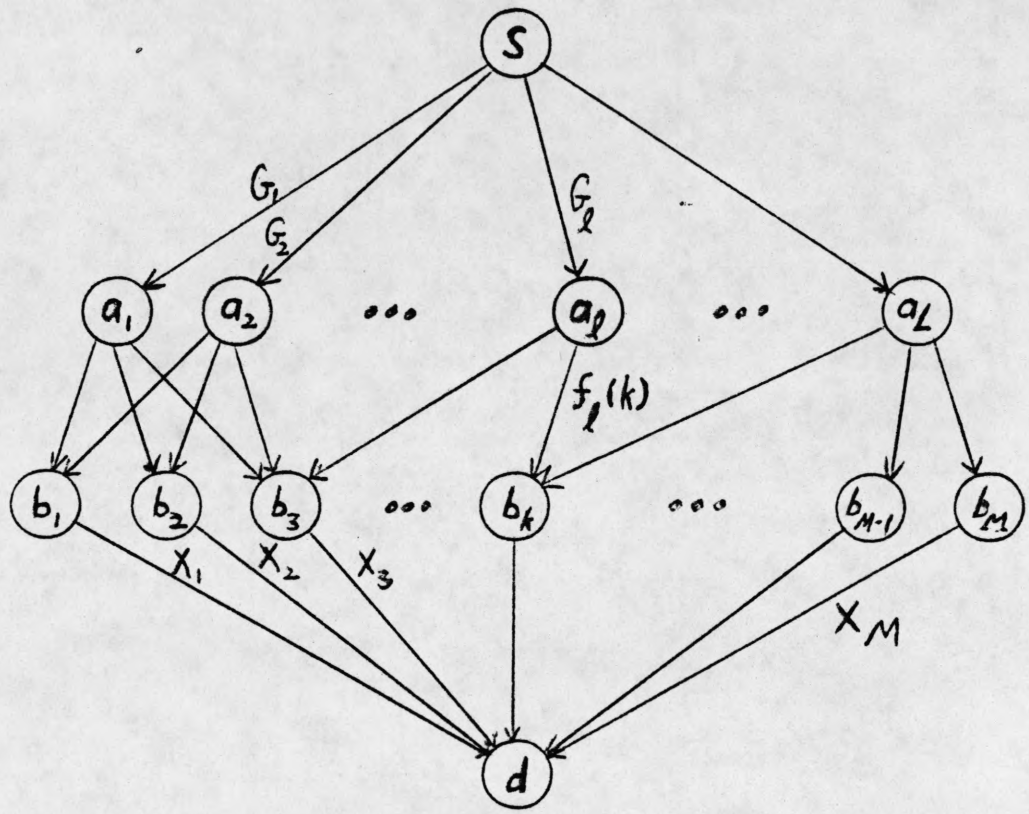

Fig. 2.1. Interpretation of a link activation schedule as a directed flow

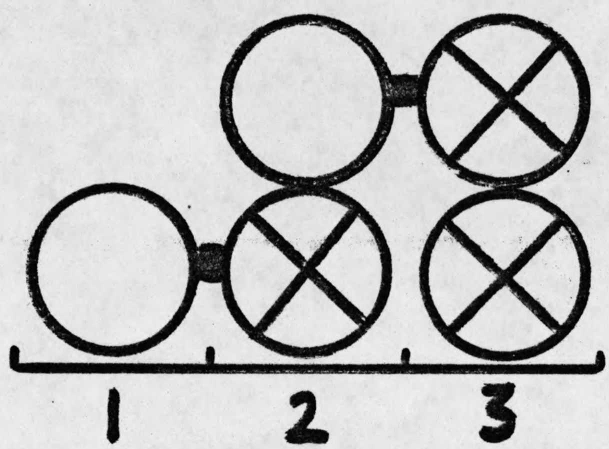

Fig. 3.1. Link schedule for which two but not one elementary transition suffice to produce a more balanced schedule

M	p_o	p_1	p_2	p_3
2	.5	.5	0	0
4	.5	.5	0	0
6	.50074	.49852	.00074	0
8	.50124	.49752	.00124	0
10	.50081	.49838	.00081	2.43×10^{-7}
$+\infty$	.5	.5	0	0

Fig. 4.1. Distribution $\underline{p}$ induced by uniformly most balanced schedules for random skeleton schedule with two slot choices per link and $M = 2L$ ($\alpha = .5$).

M	p_o	p_1	p_2	p_3
2	0	1	0	0
3	.0370	.9259	.0370	0
4	.0682	.8636	.0682	0
5				
$+\infty$	.1619	.6762	.1619	0

Fig. 4.2. Distribution $\underline{p}$ induced by uniformly most balanced schedules for random skeleton schedule with two slot choices per link and $M = L$ ($\alpha = 1.0$).

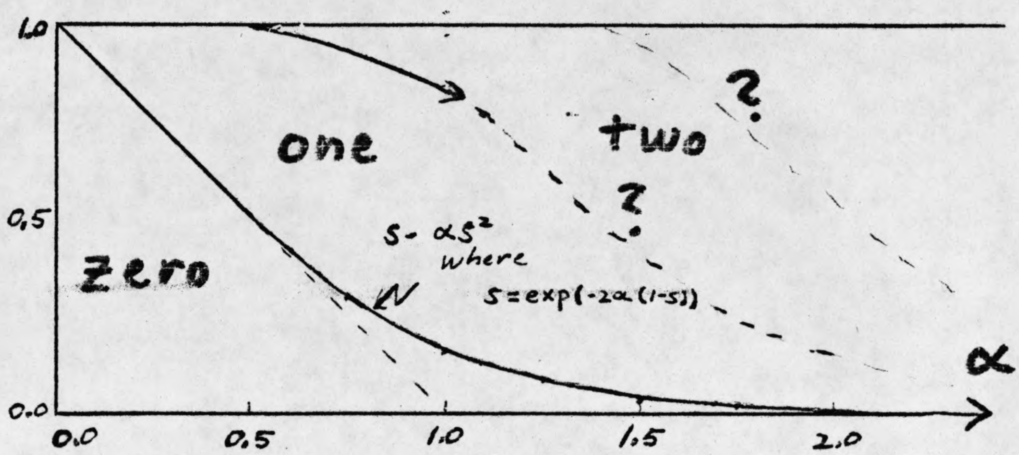

Fig. 4.3. Distribution p vs α in the limit as M tends to infinity for random skeleton schedule with two slot choices per link and L = mα.

ADAPTIVE TRANSMISSION STRATEGIES AND ROUTING IN MOBILE RADIO NETWORKS

by Bruce Hajek

Local throughput in a mobile radio network is roughly defined as the rate at which packets are propagated in specified directions in local network regions. A key factor determining local throughput in an ALOHA or spatial TDMA network with randomly spaced stations is the transmission radius used by the stations. We demonstrate that allowing the transmission radius to depend on the desired direction of propagation can significantly increase local throughput. Practical implications are discussed for the Navy's ITF Network.

The local throughput capabilities of a radio network can be effectively used only if adequate routing strategies are employed. This is illustrated by an example based on a symmetric demand assumption for stations uniformly distributed over a disc.

I. OPTIMAL FIXED RADIUS SELECTION

Suppose that the population of n stations is uniformly distributed within a circle of radius R. If n is large then in small regions the stations are distributed like a Poisson point process with intensity λ satisfying $n = \pi R^2 \lambda$. Assuming that the traffic demand is symmetric (i.e., uniformly distributed over all n(n-1) directed pairs of distinct stations) the mean distance between the source and destination of a packet is (see [2], or see [1] which is a chapter from [2])

$$\frac{128}{45\pi}\left(\frac{n}{\lambda\pi}\right)^{\frac{1}{2}}$$

The network throughput in packet-hops for the ALOHA random access protocol can be approximated by

$$\frac{n}{\overline{A}_c \lambda e}$$

where A_c is the area covered by a transmission and $\overline{A}_c$ denotes the mean of A_c. (If the transmission radius is always a constant r then A_c is not random and is equal to πr^2.) Thus, if $\overline{L}$ denotes the mean forward progress per successful transmission, the network end-to-end throughput in packets per time slot is

$$\gamma = \overline{L}\left(\frac{n}{\overline{A}_c \lambda e}\right) \bigg/ \left(\frac{128}{45\pi}\left(\frac{n}{\lambda\pi}\right)^{\frac{1}{2}}\right)$$

$$= (\eta/\sqrt{\lambda})(.72017)\ \sqrt{n} \qquad (1.1)$$

where

$$\eta = \overline{L}/\overline{A}_c \qquad (1.2)$$

Since by equation (1.1) the end-to-end throughput is proportional to η we call η the <u>efficiency</u> of the transmission radius policy. The constant

η is perhaps more meaningful than γ since it does not depend on the network's global geometry and is thus a "local" measure.

In [2] it was assumed that the transmission radius r is the same for all transmissions. The fixed radius r was chosen to maximize the efficiency η over all positive values. It was found that the optimum value of r is such that $\lambda A_c \approx 5.89$ - that is, the optimum fixed transmission radius is such that the mean number of stations within range of a given station is about six. This choice of transmission radius leads to efficiency

$$\eta_{opt,fixed\ r} \cong .135\ \lambda^{\frac{1}{2}} \qquad (1.3)$$

Using (1.1), this leads to the optimal network throughput $.0976\ \sqrt{n}$ reported in [2].

We are quick to remark that much of this analysis is heavily laden with approximations. For example, there is a problem in defining mean forward progress when no station is within range of the transmitter. (We avoid that particular problem in the next section.) See [2] for further discussion.

II. OPTIMAL ADAPTIVE TRANSMISSION RADIUS

Suppose that transmitters can vary their transmission radius with time (rather than using a fixed - although optimized - radius as in [2]), possibly as a function of the location of the other stations. How might the efficiency be improved?

First, we note that once a transmitter has identified an intended receiver for a packet transmission, it should use a transmission radius just large enough to reach that station. It remains to see which of the other stations should be the intended receiver.

Suppose a transmitter located at (0,0) must transmit a packet which is ultimately destined for a station with coordinates (z,0) where z is large. If a transmitter at (x,y) is chosen to receive the packet (and to then relay it on) the efficiency for that transmission would be

$$\eta(x,y) \triangleq \frac{x}{\pi(x^2 + y^2)} .$$

Clearly the adaptive transmission radius rule which maximizes η (defined in (1.2)) is to use the radius just large enough to reach the station whose coordinates (x,y) maximize $\eta(x,y)$ over all the stations. This rule is illustrated in Figures 1 and 2. Geometrically, one "scans" the region within a circle centered on the positive x axis which passes through (0,0). The diameter of the circle continuously increases until some station is contained in the region. That station becomes the intended receiver and the transmission radius used is the distance to that receiver. We will now compute the efficiency of this rule. All our analysis is under the assumption that n is so large that the distribution of stations located near the fixed station can be assumed to be Poisson with intensity λ per unit area.

Let A_s be the area of the region which is scanned. Then

$$\begin{aligned} P[A_s \geq c] &= P[\text{no stations are in a given circle which has area } c] \\ &= e^{-\lambda c} \end{aligned} \tag{2.1}$$

Hence, A_s is exponentially distributed and $\overline{A}_s = \lambda^{-1}$. Since the diameter L of the region scanned is equal to $2(A_s/\pi)^{\frac{1}{2}}$ we have that

$$f_L(\ell) = \frac{\pi\lambda}{2} \exp(-\lambda\pi\ell^2/4) \text{ for } \ell \geq 0$$

and

$$\overline{L} = \lambda^{-\frac{1}{2}}. \tag{2.2}$$

Given that $L = \ell$, the coordinates (X,Y) of the designated receiver is a solution to

$$x^2 + y^2 = \ell x \tag{2.3}$$

or

$$(x - \frac{\ell}{2})^2 + y^2 = \frac{\ell^2}{4}$$

For fixed x we compute from equation (2.3) that

$$\frac{dy}{dx} = \frac{x}{2y} = \frac{x}{2(\ell x - x^2)^{\frac{1}{2}}}$$

Now given that a region contains exactly one point of a Poisson point process with constant intensity, the point is uniformly distributed over the region. Applying this fact to the shaded region in Figure 3 yields that

$$f_{X|L}(x|\ell) = c_\ell \frac{dy}{d\ell}$$

where c_ℓ is chosen so that the conditional density integrates to one for ℓ fixed. To find c_ℓ we note that

$$\int_0^\ell \frac{dy}{d\ell}(x)\ dx = \frac{1}{2}\frac{dA_s}{d\ell} = \frac{1}{2}\frac{d\,\frac{\pi\ell^2}{4}}{d\ell} = \frac{\pi\ell}{4}$$

Therefore

$$f_{X|L}(x|\ell) = \frac{2x}{\pi\ell((\ell-x)x)^{\frac{1}{2}}} \quad \text{for } 0 \leqslant x \leqslant \ell \tag{2.4}$$

From this we can compute that

$$E[X|L = \ell] = \int_0^\ell \frac{2x^2}{\pi\ell((\ell-x)x)^{\frac{1}{2}}}\ dx = \frac{3}{4}\ell \tag{2.5}$$

Now using (2.2) and (2.5) we compute that

$$\overline{X} = E[E[X|L]] = \frac{3}{4}\,\overline{L} = \frac{3}{4}\,\lambda^{-\frac{1}{2}}. \tag{2.6}$$

Furthermore, the area of the region within transmission range is

$$A_c = \pi\,(X^2 + Y^2) = \pi LX$$

so that

$$\begin{aligned} \overline{A}_c &= \pi\, E[LX] \\ &= \pi\, E[LE[X|L]] \\ &= \frac{3\pi}{4}\, E[L^2] = 3\, E[\frac{\pi L^2}{4}] = 3\overline{A}_s = 3\lambda^{-1}. \end{aligned} \tag{2.7}$$

From (2.6) and (2.7) we compute that the efficiency of the rule is

$$\eta_{opt} = \frac{\frac{3}{4}\,\lambda^{-\frac{1}{2}}}{3\lambda^{-1}} = .25\,\lambda^{\frac{1}{2}}$$

Comparing with equation (1.3) we see that by optimally adapting the transmission radius as a function of ultimate packet destination and other station locations, the efficiency can be increased by about 85%.

Equation (2.7) suggests that the optimal adaptive transmission radius policy we have chosen is such that on average, three stations will be in transmission range. (In analogy to [2], one might say that 3 is a magic number.) However, since the transmission range is random and is dependent on the station locations, the distribution of stations within transmission range is <u>not</u> conditionally Poisson. In fact, with probability one there is exactly one station within (actually, on the boundary of) the region scanned. The conditional distribution of stations in the unscanned region is Poisson, however.

Using (2.4) and the fact that $X = L\cos^2\Theta$ we easily derive that L and Θ are independent and

$$f_\Theta(\theta) = \frac{2\cos^2(\theta)}{\pi} \qquad -\frac{\pi}{2} < \theta < \frac{\pi}{2} \tag{2.8}$$

Now for fixed θ the area of the region which is scanned but which is <u>not</u> within range of the transmission can be shown to equal

$$\Delta = \ell^2(\sin(2\theta) - 2\theta\cos(2\theta))/4$$

and using (2.8) the mean of this area is found (after elementary computations) to be

$$\overline{\Delta} = \left(\overline{L^2}\right)\left(\frac{17-\pi^2}{16\pi}\right) = \frac{17-\pi^2}{4\pi^2}\ \lambda^{-1}$$

Now the mean area of the region which is scanned <u>and</u> within the transmission range is $\lambda^{-1} - \overline{\Delta}$, and so the mean area of the unscanned portion of the region within transmission range is $2\lambda^{-1} + \Delta$ (see Fig. 4). Thus, the mean number of stations within transmission range, including the designated receiver, is

$$1 + (2 + \overline{\Delta}\lambda) \approx 3.18$$

(One might say that 3.18 is a magic number.) It would be interesting (and it appears difficult) to find a dynamic transmission radius rule which minimizes the mean forward progress divided by the mean number of stations in transmission range, other than the intended receiver.

III. PRACTICAL CONCLUSIONS

It may not be feasible for stations to continuously vary their range from transmission to transmission. However, even if each station has only two available ranges, the same principles apply and thus using a destination dependent transmission range can increase throughput. Moreover, our study was motivated by the fact that in some situations the transmissions from a single station might be forced to have variable ranges. This situation is expected for the Navy's ITF Network due to the large variation in propagation characteristics over distinct frequency bands. It is important to emphasize that the variable ranges imposed by the environment can only be effectively exploited if packets can be transmitted at different frequencies for different hops along its path. That is, our results argue against the concept of independently running several networks, each in a different frequency band - rather they argue for a single network using all frequency bands simultaneously.

Another practical consideration is that it would most likely not be efficient (if even possible) for stations to execute the scanning procedure we suggested in order to determine the preferred receiver for a given ultimate destination. However, if a (yet to be developed) routing strategy is used which effectively incorporates station interdistance measurements, then we believe that routes will automatically be chosen which are roughly consistent with those determined by our rule. In this light, our calculations suggest that under effective routing strategies, the multiple access interference will be less (or else the throughput larger) than that predicted by [2].

IV. GLOBAL NETWORK PERFORMANCE

The efficiency η we defined in Section I can be thought of as a measure of local throughput capacity. The formula (1.1) relates it to a global performance measure (end-to-end throughput) for a specific network topology and traffic demand. As noted in [2], in simulations, the end-to-end throughput was only a small fraction of that predicted by (1.1). In this section we indicate why indeed one should expect that the actual end-to-end throughput should be only one third that predicted by equation (1.1) as long as a minimum hop routing rule is used.

Consider a continuum of stations uniformly distributed over a disc of radius one. Suppose that the traffic demand is uniform so that the amount of traffic originating within one region and destined for another is proportional to the product of the areas of the regions. We wish to compute the spatial traffic distribution, assuming that line-of-sight routing is used.

Refer to Figure 5 and fix a point at distance r from the origin, and let θ denote the angle between the radial line and another line through the point. Now the amount of traffic passing through a small neighborhood of the point and traveling at an angle between θ and $\theta + d\theta$ is proportional to the product of the areas of regions a and b indicated. Therefore the traffic density profile is

$$\rho(r) = (\text{const.}) \int_0^{2\pi} (\frac{1}{2} x^2)(\frac{1}{2} y^2)\, d\theta$$

But for each θ, $xy = 1 - r^2$ so that

$$\rho(r) = \frac{2}{\pi} (1 - r^2) \qquad (4.1)$$

where the constant $2/\pi$ was chosen so that

$$\int_0^{2\pi}\int_0^1 \rho(r)\; r\; dr\; d\theta = 1\ .$$

From (4.1) we compute that

$$\frac{\text{Peak traffic density (set } r = 0)}{\text{Spatial average of traffic density}} = \frac{2/\pi}{1/\pi} = 2$$

Since maximum throughput is limited by the peak traffic density and since formula (1.1) is based on mean traffic density, we thus see that if line-of-sight routing (which is essentially dictated if n is large and minimum hop routing is used for the network considered in earlier sections) is used then the network throughput will be one third that predicted by (1.1). The high peak to average density ratio indicates the need for effective routing strategies which are not restricted to minimum hop routes.

REFERENCES

[1] L. Kleinrock and J. Silvester, "Optimum transmission radii for packet radio networks or Why six is a magic number," Proceedings of the National Telecommunications Conference, pp. 4.3.1-5, 1978.

[2] J. Silvester, "On the spatial capacity of packet radio networks," Ph.D. Dissertation, Computer Science Department, University of California, Los Angeles, March 1980.

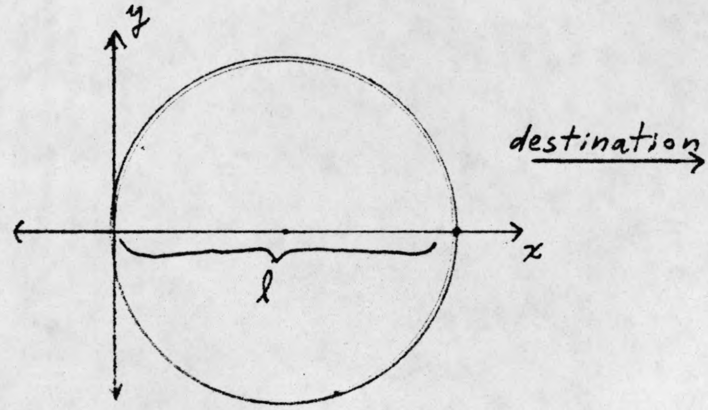

Figure 1. Locus of $\{(x,y) : \frac{x}{x^2 + y^2} = \frac{1}{\ell}\}$ for ℓ fixed.

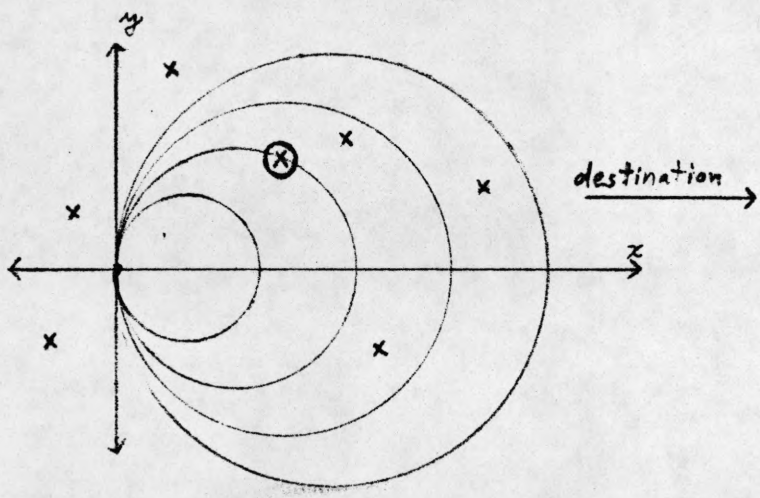

Figure 2. Illustration of selection rule - the preferred next receiver is circled.

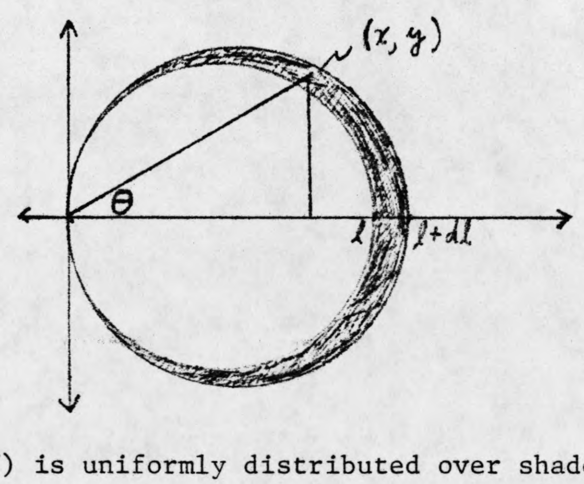

Figure 3. (X,Y) is uniformly distributed over shaded region given that $\ell \leqslant L \leqslant \ell + d\ell$.

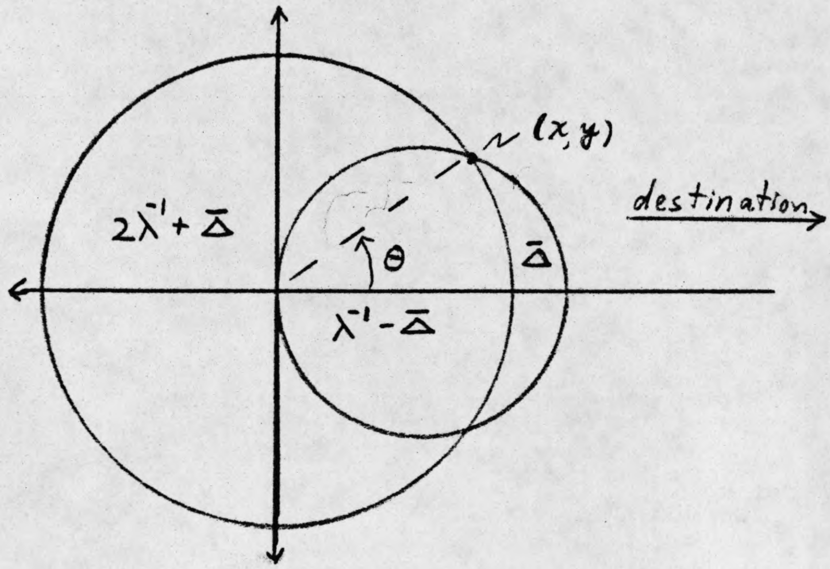

Figure 4. From left to right, the region unscanned and in transmission range, the region scanned and within range, and the region scanned and out of range are pictured and are labeled with their mean areas.

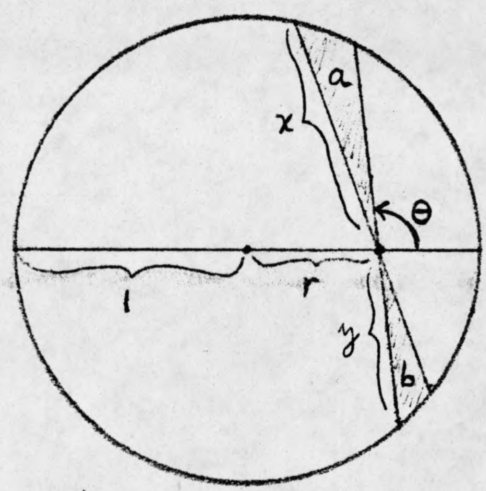

Figure 5. Traffic from region a passing to region b passes through a neighborhood of the point at distance r from the center of the disc. The angle of travel is roughly θ.

APPENDIX C by M. B. Pursley

PERFORMANCE OF A CODED SLOW-FREQUENCY-HOPPED SYSTEM WITH FADING

In Table 2 of [1] some preliminary results are given on the performance of Reed-Solomon (RS) coded slow-frequency-hopped (SFH) spread-spectrum multiple-access communications in a nonselective Rayleigh fading environment. The parameter values employed in that table were chosen for illustrative purposes only. In Table 1 we present numerical results for parameter values of interest in the Navy's proposed intra-task-force communications network. The key parameters are the number q of frequency slots, the number K of active transmitters, and the number N_b of bits per dwell interval. All of the data reported here is for q = 100, and the modulation is binary noncoherent FSK. The values of K are 1, 5, and 10. Although we evaluated error rates for K = 20, the error rates are not given here because they are too large to be of interest (in excess of 0.15 for all values of the bit-energy-to-noise density ratio).

The method that we have employed to generate the data given in Table 1 can be generalized to permit the evaluation of the bit error probability for RS-coded SFH systems with Rician fading channels. In Table 2 we present a comparison of bit error rates for three values of γ^2, the Rician channel parameter defined in [3]. The case $\gamma^2 = 0$ is just the additive white Gaussian noise channel, and $\gamma^2 \to \infty$ gives the Rayleigh fading channel. A comparison of error rates for Rician fading channels with $\gamma^2 = 0$, 0.1, and ∞ is given in Table 2.

The probabilities that are listed in Tables 1 and 2 are upper bounds on the bit error probability for the RS coded SFH system. This upper bound is based on the method reported in Appendix A of [3] (see especially pp. 15-17). The bound is very robust in the sense that it is valid for arbitrary relative power levels and time delays. It turns out the bound is also fairly tight in many cases as can be seen from Table 3 where we present results on an approximation to the bit error probability. The approximation method is an extension of a method for uncoded systems reported in [3].

In the process of obtaining numerical results for the RS-coded SFH spread spectrum systems, we discovered that for low bit energy to noise density the bit error probabilities for noncoherent FSK are larger for the AWGN channel than for the Rayleigh fading channel. This phenomenon shows up in Table 2 for $\overline{\mathcal{E}}_b/N_0 = 4$ dB. A further investigation revealed that for the (255,127) RS code, the Rayleigh channel gives better error rates than the AWGN channel for values of $\overline{\mathcal{E}}_b/N_0$ less than 6.81 dB. For values of $\overline{\mathcal{E}}_b/N_0$ greater than 6.82 dB, the AWGN channel gives superior performance as expected. This phenomenon is apparently due to the tail of the Rayleigh density function: large values of the Rayleigh distributed amplitude give a better byte error probability than the constant amplitude of the AWGN. At low bit energy to noise density ratios, these large values of the amplitude are required to obtain a low error rate. At present it appears that this phenomenon is of little practical interest, because we have observed it at high error rates only (in excess of 0.3).

Table 1. Upper bounds on the bit error probability for RS-coded SFH systems with nonselective Rayleigh fading (q = 100).

(a) (31,15) RS code with $N_b = 5$.

$\bar{\mathcal{E}}_b/N_0$ (d)	K = 1	K = 5	K = 10
4	0.41E + 00	0.41E + 00	0.42E + 00
10	0.22E + 00	0.24E + 00	0.27E + 00
16	0.47E - 02	0.33E - 01	0.10E + 00
22	0.38E - 02	0.10E - 02	0.25E - 01
28	0.34E - 11	0.15E - 03	0.13E - 01

(b) (255,127) RS code with $N_b = 8$.

$\bar{\mathcal{E}}_b/N_0$ (d)	K = 1	K = 5	K = 10
4	0.44E + 00	0.45E + 00	0.45E + 00
10	0.26E + 00	0.27E + 00	0.30E + 00
16	0.20E - 03	0.47E - 01	0.16E + 00
22	0.62E - 28	0.10E - 08	0.52E - 02
28	0.00E + 00	0.97E - 15	0.15E - 03

Table 2. Upper bounds on the bit error probability for RS coded SFH systems with nonselective Rician Fading (q = 100, K = 5, (255,127) RS code, $N_b = 8$).

$\bar{\mathcal{E}}_b/N_0$ (d)	$\gamma^2 = 0$	$\gamma^2 = 0.1$	$\gamma^2 = \infty$
4	0.46E + 00	0.46E + 00	0.44E + 00
10	0.17E + 00	0.20E + 00	0.27E + 00
16	0.12E - 17	0.33E - 14	0.47E - 01
22	0.11E - 17	0.13E - 17	0.10E - 08
28	0.11E - 17	0.11E - 17	0.97E - 15

Table 3. Bit error probabilities for RS-coded SFH systems with nonselective Rayleigh fading ($q = 100$, $K = 5$).

(a) (31,15) RS code with $N_b = 5$

$\overline{\mathcal{E}}_b/N_0$ (d)	Approximation	Upper Bound	
4	0.41	0.42	(E + 00)
10	0.24	0.24	(E + 00)
16	0.30	0.33	(E - 01)
22	0.77	1.02	(E - 03)
28	0.10	0.15	(E - 03)

(b) (255,127) RS code with $N_b = 8$

$\overline{\mathcal{E}}_b/N_0$ (d)	Approximation	Upper Bound	
4	0.44	0.45	(E + 00)
10	0.27	0.27	(E + 00)
16	0.42	0.47	(E - 01)
22	0.44	1.04	(E - 09)
28	0.24	0.97	(E - 15)

Reference

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APPENDIX D

by B. Hajek

WAITING TIME FOR PERFECT SCHEDULING WITH INDEPENDENTLY FLUCTUATING ARRIVAL RATES, IN DISCRETE TIME

A topic of investigation during the past year has been the investigation of random access procedures for a group of stations with on-going access needs, rather than with one-time access needs. We have developed several principles which are still under investigation. It has become apparent that the proper selection of a random access policy is critically dependent on the statistics of the user demands. Following is a report on our investigation of models for user demands.

In general, random access techniques are needed when several stations attempt to use a common channel but have little knowledge about the instantaneous availability of the channel and the identity of the other stations. If the group of active stations remains fixed over a sufficiently long period, then even with small feedback rates the stations can eventually learn to effectively cooperate with each other. In that case, the random access aspect of the problem is only relevant for a transient "locking period."

It is thus apparent that random access capability becomes a critical factor only when the population of active stations is changing relatively fast compared to the rate at which stations can learn the system state.

For example, the usual infinite population model is appropriate for studying random-access techniques since each station transmits only one packet and then remains silent thereafter (thus by the time other stations learn of its identity, the information is useless). In an effort to obtain more realistic models, some authors consider a finite number of buffered stations with Poisson arrivals. However, the schemes evaluated to date for such models outperform TDMA only for extremely low traffic rates, and often do significantly worse. The reason for this, we propose, is that a Poisson arrival process does not reflect a bursty source of packets at a station. This has led us to use a variation of Poisson arrival processes in which the arrival rate is time varying. As reported previously [2], we have analyzed the TDMA policy for such arrival models. Below we outline an evaluation method for perfect scheduling. The method is based on non-linear matrix iteration. The analysis of these two basic schemes provides a benchmark for evaluating other proposed random access strategies.

The Single Station Model

Each station generates packets to be transmitted and stores the packets in a buffer of unlimited capacity until transmission. Each station (more precisely, the packet-generating mechanism at each station) is assumed to be in state zero or state one. A station in state i (where i is zero or one) at the beginning of a time slot generates a packet during the slot with probability σ_i. Typically $\sigma_1 > \sigma_0$ so that state one represents an active state.

The state of each station is modeled by a two-state Markov chain with transition probability matrix

$$\tilde{P} = \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix} .$$

It is assumed that the event of a state transition during a slot and the event of a packet generation during a slot are conditionally independent given the state at the beginning of the slot. The invariant distribution for $\tilde{P}$ is $(a + b)^{-1}$ (b,a) so that the mean arrival rate is

$$\bar{\sigma} = (a + b)^{-1} (b\sigma_0 + a\sigma_1)$$

For comparison of numerical results, it is desirable to reparameterize the matrix $\tilde{P}$ so that one parameter represents the rate at which the generation probability fluctuates. To this end, we let

$$\tilde{P} = \exp(\nu\tilde{Q})$$

where $\tilde{Q}$ is the transition rate matrix (or generator matrix)

$$\tilde{Q} = \begin{bmatrix} -\alpha & \alpha \\ \beta & -\beta \end{bmatrix} .$$

Then the discrete time chain with transition matrix $\tilde{P}$ can be viewed as a continuous time chain with generator $\tilde{Q}$ which is sampled every ν time units. An alternative equivalent expression for $\tilde{P}$ is $\tilde{P} = I + \gamma\tilde{Q}$ where γ and ν are related by

$$\gamma = (\alpha + \beta)^{-1} \{1-\exp (-\nu/(\alpha + \beta))\}.$$

As ν ranges over $\mathbb{R}_+$, γ ranges over the interval $[0,(\alpha + \beta)^{-1}]$. Note that $\nu(1/\alpha + 1/\beta)^{-1}$ represents the rate at which the continuous time process completes its two step cycles.

Other reparameterizations of $\tilde{P}$ are also possible. It might be noted that no such reparameterization will make the discrete-time model for packet generation completely equivalent to a continuous time model with a modulated Poission arrival process since we allow only one packet arrival per slot per station.

Perfect Scheduling with m Stations.

Suppose that each of m stations independently generates packets according to independent discrete-time doubly-stochastic point processes modulated by two-state Markov processes, with common parameters a, b, σ_0 and σ_1, as described above. A perfect scheduling multiple-access policy is a scheme whereby exactly one of the stations successfully transmits a packet during each slot for which at least one station has a packet to transmit at the beginning of the slot. Of course, this does not completely characterize the policy. For an example, the stations might be assigned a priority and then the highest priority non-empty station would always transmit. In general, a complete Markov (or state space) description of the system would include a list of the states of each of the m stations, a list of the number of packets at each station, and possibly some state information involving the policy.

However, the average queueing delay can be computed using a simpler model as follows. Let θ_t denote the number of stations in state one at time t, and let L_t denote the number of packets in the system, summed over all m stations. Then (θ_t, L_t) is a discrete time Markov chain on $\{(i,n): 0 \le i \le m, n \ge 0\}$ whose transition probability matrix can be written in the block form (only non-zero blocks are indicated):

$$P = \begin{bmatrix} P(0) & P(1) & P(2) & \cdots & P(m) & & & \\ P(0) & P(1) & P(2) & \cdots & P(m) & & & \\ & P(0) & P(1) & P(2) & \cdots & P(m) & & \\ & & P(0) & P(1) & P(2) & \cdots & P(m) & \\ & & & & \ddots & & & \end{bmatrix}$$

where the (m+1) x (m+1) submatrices are defined by

$$P(k)_{ij} = P\{k \text{ arrivals in slot } [t,t+1), \theta_{t+1} = j \mid \theta_t = i]$$

If $\overline{L}$ denotes the expected value of L_t under the invariant distribution for P, then $\overline{N}$, the average number of packets at each station (averaged over all m stations) is given by

$$\overline{N} = \frac{1}{m} \overline{L} .$$

The matrix P is a block matrix of M/G/1 type (for discrete time). By specializing and adapting the techniques summarized in NEUTS (1980) we can give a numerically tractable way to compute $\overline{L}$.

<u>Outline of Computation of $\overline{L}$</u>.

<u>Step 1</u>. Compute and store P(k) for $0 \leq k \leq m$.

<u>Step 2</u>. Find the transition probability matrix G satisfying

$$G = \sum_{k=0}^{m} P(k)G^k.$$

This can be done by matrix iteration where successive iterates are obtained by substituting the previous interate into the right hand side, using initial value 0. (For each iteration, the right side can be computed efficiently by computing G^k in the loop used to compute the sum.)

<u>Step 3</u>. Determine the probability vector r with rG = r. (By iteration.)

<u>Step 4</u>. Compute

$$\overline{L} = \tfrac{1}{2}(1-\rho)^{-1} \{\eta''(1) + 2\underline{y}_0 Z \underline{\mu}_1\}$$

where

$p = m\overline{\sigma}$ is the system utilization.

$y_0 = (1-\rho)\ \underline{r}$ is the invariant distribution of (L_t, θ_t) restricted to the states where $\theta_t = 0$.

π = the invariant distribution of P* (defined below).

$$Z = [I - P^* + \Pi]^{-1} \text{ where } \Pi = \begin{bmatrix} \pi \\ --- \\ \pi \\ --- \\ \vdots \\ --- \\ \pi \end{bmatrix}$$

$$\mu_1 = \begin{bmatrix} m\sigma_0 \\ (m-1)\sigma_0 + \sigma_1 \\ (m-2)\sigma_0 + 2\sigma_1 \\ \vdots \\ m\sigma_1 \end{bmatrix}$$ is the mean arrival vector given θ_t.

$$\Pi''(1) = \pi\underline{\mu}_2 - 2\rho^2 + 2\pi\,\Delta(\underline{\mu}_1)P^* Z\Delta(\underline{\mu}_1)\underline{e} \qquad \underline{e} = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} \in \mathbb{R}^{m+1}$$

and

$$\underline{\mu}_2 = \begin{bmatrix} \underline{\mu}_2(0) \\ \vdots \\ \underline{\mu}_2(m) \end{bmatrix}$$ where $\mu_2(i) = i\sigma_1(1-\sigma_1)+(m-i)\sigma_0(1-\sigma_0)+ (i\sigma_1 + (m-i)\sigma_0)(1-(i\sigma_1 + (m-i)\sigma_0))$

Let us elaborate on Step 1. By the statistical assumptions, θ_{t+1} and N_{t+1} are conditionally independent given θ_t, so that

$$P(k) = \Delta(\underline{s}(k))P^*$$

where

$$P^*_{ij} = P[\theta_{t+1} = j \mid \theta_t = i]$$

and $\Delta(\underline{s}(k))$ denotes the diagonal matrix whose diagonal entries are given by the column vector $\underline{s}(k)$ which is defined by

$$s_i(k) = P[k \text{ arrivals in } (t,t+1) \mid \theta_t = i].$$

Computation of $P^*_{ij} = P[\theta_{t+1} = j \mid \theta_t = i] \quad 0 \le i, i \le m$

Write $P^*_{i\cdot}$ $0 \le i \le m$ for the m+1 rows of the $(m+1)\times(m+1)$ matrix $\underline{P}^*$.

Then

$$P^*_{i\cdot} = (\underline{g}_0)^{(m-i)*} * (\underline{g}_1)^{i*}$$

where

$$\underline{g}_0 = (1-a, a, 0, 0 \ldots 0) \in \mathbb{R}^{m+1}$$

$$\underline{g}_1 = (b, 1-b, 0, 0, \ldots 0) \in \mathbb{R}^{m+1}$$

and the following notation is used: For two vectors $\underline{f}$ and $\underline{g}$ in $\mathbb{R}^{m+1}$, $\underline{f}*\underline{g}$ denotes the m+1 row vector $\underline{h}$ defined by

$$h_k = \sum_{i=0}^{k} f_i \, g_{k-i} \text{ for } 0 \le k \le m,$$

and $\underline{g}^{i*}$ is obtained by the i-fold product $\underline{g}*\underline{g}* \ldots *\underline{g}$. (Organization: First, for u=0 and for u=1, compute $(\underline{g}_u)^{k*}$ for $0 \le k \le m$ recursively by

$$(\underline{g}_u)^{(k+1)*} = (\underline{g}_u^{k*})*\underline{g}_u \quad k \ge 0; \; \underline{g}^{0*} = \delta_0 \stackrel{\Delta}{=} (1, 0, \ldots 0) \in \mathbb{R}^{m+1}$$

Computation of $s_i(k) = P\{k \text{ new packets arrive} \mid \theta_t = i\}$

Write $\underline{s}_i = (s_i(0), \ldots, s_i(m)) \in \mathbb{R}^{m+1}$. Then

$$\underline{s}_i = \underline{f}_0^{(m-i)*} * \underline{f}_1^{i*}$$

where

$$\underline{f}_0 = (1-\sigma_0, \sigma_0, 0, \ldots, 0) \in \mathbb{R}^{m+1}$$

$$\underline{f}_1 = (1-\sigma_1, \sigma_1, 0, \ldots, 0) \in \mathbb{R}^{m+1}$$

(Storage: Let $S = \begin{bmatrix} \underline{s}_0 \\ \underline{s}_1 \\ \cdot \\ \cdot \\ \cdot \\ \underline{s}_m \end{bmatrix} \in \mathbb{R}^{(m+1)\times(m+1)}$

Note that the computation of S and P* is identical.)

Numerical Results

The method for analyzing perfect scheduling outlined above was implemented on a DEC-10 computer. The most time consuming step was to solve the nonlinear equation for G by iteration. For M = 4 stations very little computation was required whereas for M = 10 the required computation time became substantial.

In Figure 1 we have graphed $\overline{N}$ for both the perfect scheduling and TDMA protocols as a function of γ for M = 4. The method reported in [2] was used to obtain the curve for TDMA. We choose parameter values $\sigma_o = .2$ and $\sigma_1 = .75$ and the relative rates of transition were chosen so that $\rho = .96$. Note that during periods when one or more stations are in the active state the mean packet arrival rate exceeds one.

Perhaps the most striking feature of Figure 1 is that as γ decreases to zero (indicating that stations switch between active and inactive states more slowly) the difference in $\overline{N}$ for TDMA compared to perfect scheduling remains almost constant (it actually slightly increases). Contrary to what we had originally anticipated, TDMA performs quite well (when compared to the best possible - perfect scheduling) even for our quite bursty traffic model.

Of course the delay for TDMA does become large as γ tends to zero - our observation is merely that the delay for perfect scheduling becomes nearly as large. It does not appear, therefore, that significant improvements in delay can be achieved over TDMA by other implementable access schemes. Rather, the approach of scheduling transmissions in a "spatial TDMA" fashion may be quite effective even for bursty traffic.

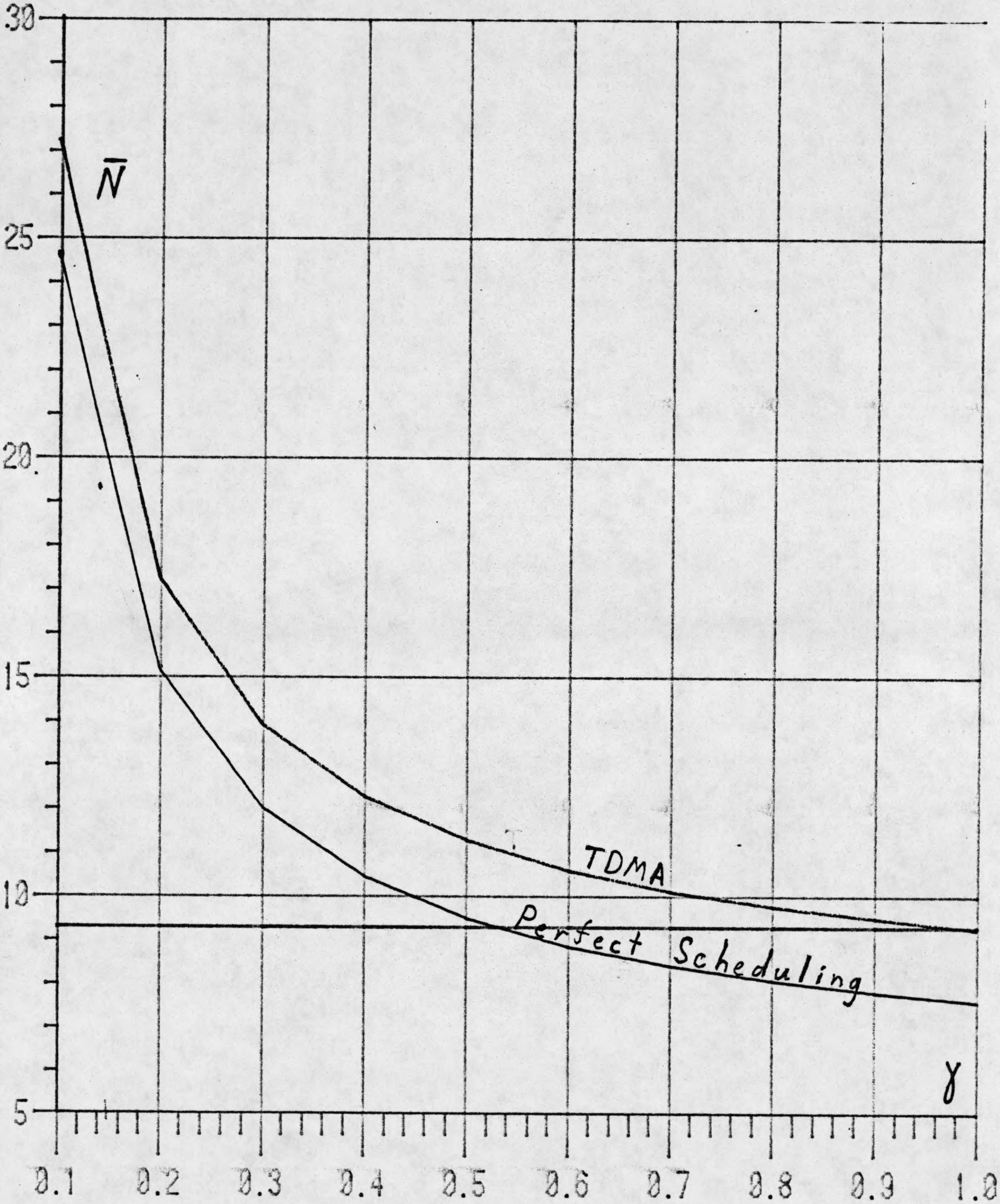

Figure 1. Mean number $\overline{N}$ in system per station vs. switching rate γ for perfect scheduling and for TDMA multiple access. ($M = 4$ stations, $\sigma_o = 0.2$, $\sigma_1 = 0.75$ and $\rho = .96$).

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APPENDIX E (June 1982)

OPTIMAL DYNAMIC ROUTING IN COMMUNICATION NETWORKS WITH CONTINUOUS TRAFFIC*

by

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ABSTRACT

New characterizations of optimal state-dependent routing strategies are obtained for the continuous traffic network model proposed by A. Segall for linear cost with unity weighting at each node and for constant inputs. The concept of <u>flow relaxation</u> is introduced and is used to transform the optimal routing problem into an <u>initial flow</u> optimization problem with convex cost and linear constraints.

Three algorithms are given for open-loop computation of the optimal initial flow. The first is a simple iterative algorithm based on gradient descent with bending and it is well suited for <u>decentralized</u> computation. The second algorithm reduces the problem to a series of max-flow problems and it computes the exact optimal initial flow in $O(|N|^4)$ computations where $|N|$ is the number of nodes in the network. The third algorithm is based on a search for successive <u>bottlenecks</u> in the network.

<u>Key words</u>. Dynamic routing, state-constrained optimal control, minimal cost flow, communication network, traffic network, decentralized algorithms.

*Only the first two sections of this paper and references are given in this report.

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1. INTRODUCTION

An optimal dynamic routing strategy for a model which includes random queueing delays and the discrete nature of packets would certainly be too complex to implement for reasonably large systems. However, if the traffic to be distributed is measured in continuous rather than discrete variables, and if a deterministic pure flow model is used to describe the links, then there is some hope of computing optimal dynamic routing strategies. Such a model was introduced by Segall [16] and should be accurate when the number of packets in the network is large compared to the number of nodes.

This paper is organized as follows. The dynamic routing problem is formulated in Section 2 following Segall [16]. In Section 3 the concept of flow relaxation is introduced and an algorithm for its efficient computation is described. In Section 4 the problem of finding an optimal time-dependent control is transformed via flow relaxation into a vector optimization problem with convex cost and simple linear equality and inequality constraints. As a corollary, new characterizations are obtained for arbitrary optimal controls.

Three algorithms are provided in Sections 5-7, respectively, for solving the vector optimization problem. The algorithm in Section 5 is a decentralized iterative algorithm, while the algorithm in Section 6, called OPTFLO, is a "combinatorial" algorithm. OPTFLO reduces the problem to a series of max-flow problems and it computes the exact optimal vector in at most $O(|N|^4)$ computations. Often even fewer computations are required.

The algorithms given in Sections 5 and 6 are quite different from each other although each appears to be numerically efficient and able to handle larger networks than previously known algorithms. The third algorithm given in Section 7 reduces the vector optimization problem to a search for "bottlenecks" in the network. This third algorithm is not yet computationally competitive with the other two, but it is included for conceptual purposes.

The optimal routing problem we consider in this paper is dynamic in the sense that the optimal control depends on the state of the network [16]. Since the state evolution (given the control) is deterministic, optimal controls can be implemented in closed-loop or in open-loop form.

Closed-loop implementation requires that an optimal control value be found for each state (i.e., a feedback control policy is needed). This problem is extensively studied in the remarkable series of papers [4],[5],[10]-[12],[16] of A. Segall and his co-workers. Their main approach is to apply the theory of necessary conditions for state-constrained optimal control problems. The resulting methods are based on the backwards evolution of state and dual variables. (The connection between this approach and ours is summarized in a remark at the end of Section 4.) Through ingenious use of max-flow algorithms, Jodorkovsky and Segall [4],[5] are able to compute closed-loop control laws for small to moderate size networks (with about four to seven nodes). However, it appears that both the computational requirement and the buffer requirement for storage of the feedback policy itself grow exponentially with the size of the network, and therefore the method is impractical for large networks. (Since computation of closed-loop policies can be made off-line, the buffer requirement is the most critical limitation.)

Thus, we are led in this paper to seek open-loop solutions whereby the optimal control is computed (usually in real time) as a function of the given initial state. Since the optimal control is sought for only a single initial state it is hoped that the computation time (which is a critical parameter for open-loop implementation) will be much smaller than that required for closed-loop solutions. We believe that the algorithms presented in this paper bear this out.

An interesting algorithm for solving the open-loop problem is given in Shats and Segall [18]. Our algorithm OPTFLO is more efficient primarily for two reasons. First, although both the algorithm of Shats and Segall and OPTFLO are recursive, only OPTFLO divides the original problem into independent (i.e., decoupled) subproblems during each iteration. Secondly, OPTFLO takes advantage of existing special-purpose algorithms for solving max-flow problems whereas the algorithm given by Shats and Segall requires the solutions of linear programming problems which are solved by (less efficient) general linear-programming methods.

Yet another approach to the open-loop dynamic routing problem is to begin by considering a discrete-time formulation. Then if the network is time-expanded [2, pp. 145-146] into a new network which contains a duplicate of the original network for each unit of time, then (the discrete-time version of) the problem P we consider in this paper is equivalent to a certain static weighted minimal cost flow problem (which is a special type of linear programming problem). However, the well-known technique of "building up" optimal solutions for such static problems [2, Sect. III.3] is much less efficient for the particular minimal cost problem than is the discrete-time version of our algorithm OPTFLO. Indeed, the technique of [2, Sect. III.3] requires the solution of at least one max-flow problem per time step whereas OPTFLO often requires many fewer since it exploits the fact that an optimal control will often be constant over many consecutive time intervals.

An important feature of any closed-loop or open-loop solution to the dynamic routing problem is its degree of suitability for decentralized implementation. The algorithm given in Section 4 appears to be the first optimal dynamic routing algorithm which is readily implemented in a decentralized way.

2. PROBLEM FORMULATION

A single destination network $\mathcal{N}$ is a 4-tuple $(N,L,\underline{C},d)$ where (N,L) is a finite directed graph with a set N of nodes and a set L of links where $L \subset N \times N$, d is a distinguished node in N called the destination, and $\underline{C} = (C_\ell: \ell \in L)$ is a capacity assignment vector so that $C_\ell \geq 0$ for each link ℓ. The notation $|A|$ will be used to denote the number of elements in a set A. A link leading from node i to node j will often be denoted by (i,j). Define $E(i)$ to be the collection of nodes k such that (i,k) is a link and $I(i)$ to be the collection of nodes k such that (k,i) is a link. For each node i in N, $x_i(t)$ is a real value which denotes the amount of traffic at node i at time t (measured in bits, packets, vehicles or messages, for example).

A demand for the network is a pair $(\underline{x}(0),\underline{r})$ where $x_i(0)$ for i in N denotes the (nonnegative) initial amount of traffic at node i and r_i for i in N denotes the (nonnegative) rate at which traffic enters node i from outside the network. By convention, $x_d(t) = r_d = 0$ for all $t \geq 0$. Given a control $\underline{u} = (u_\ell(t): \ell \in L,\ t \geq 0)$, where $u_\ell(t)$ denotes the instantaneous flow on link ℓ at time t, the state equation of the network is

$$\dot{x}_i(t) = r_i + \sum_{k \in I(i)} u_{ki}(t) - \sum_{j \in E(i)} u_{ij}(t) \qquad \text{for } i \neq d .$$

This state equation can be written in vector notation as

$$\dot{\underline{x}}(t) = \underline{r} + B\,\underline{u} \qquad (2.1)$$

where B is the $|N| \times |L|$ matrix such that the column corresponding to link (i,j) consists of a -1 at row i (if $i \neq d$) and a $+1$ at row j (if $j \neq d$) and zeros elsewhere. It is convenient to define $C_{ij} = u_{ij} = 0$ for pairs of nodes i,j such that a link (i,j) does not exist.

Throughout this paper when sets of nodes are used as subscripts the summation convention is implied. Thus, by definition,

$$x_A(t) = \sum_{i \in A} x_i(t)$$

$$r_A = \sum_{i \in A} r_i$$

and

$$C_{AB} = \sum_{i \in A} \sum_{j \in B} C_{ij} ,$$

and the state equations can be written as

$$\dot{x}_A(t) = r_A + u_{N-A,A}(t) - u_{A,N-A}(t) \text{ for } A \subset N-d .$$

Note that in this last equation we have abused notation by writing d when we really mean the set {d} with the single element d.

A state trajectory $(\underline{x}(t): t \geq 0)$ is well-defined by (the integrated version of) the state equation (2.1) for any demand $(\underline{x}(0),\underline{r})$ and any control $\underline{u}$ in the set

$$\mathcal{U} = \{\underline{u}: \underline{u} \text{ is a measurable function on } \mathbb{R}_+ \text{ and } \underline{0} \leq \underline{u} \leq \underline{C}\}$$

where $\underline{0}$ is the vector of all zeros and vector inequalities such as $\underline{0} \leq \underline{u} \leq \underline{C}$ are to be interpreted coordinate-wise. A control $\underline{u}$ in $\mathcal{U}$ is termed <u>admissible</u> for the demand $(\underline{x}(0),\underline{r})$ if the corresponding state trajectory statisfies the constraint $\underline{x}(t) \geq \underline{0}$ for all $t \geq 0$. For such a control $\underline{u}$ the number

$$D(\underline{u}) = \int_0^\infty x_N(t)dt$$

is the total waiting time in the network incurred by all traffic. Given a network $\mathcal{N}$ and demand $(\underline{x}(0),\underline{r})$, the problem we consider is to find an admissible control $\underline{u}^*$ which minimizes $D(\underline{u})$ over all admissible controls $\underline{u}$. This will be called problem P and we write

$$\text{(P)} \qquad \min\{D(\underline{u}): \underline{u} \in \mathcal{U}, \ \underline{u} \text{ is admissible}\}$$

and we let D^* denote the minimum delay. Since the input rates r_i are not time varying, if the total delay is finite for some control then it is possible to empty each of the nodes in finite time. Thus, the problem we consider might be termed an "optimal evacuation" problem.

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