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BASIC GAS DYNAMIC EQUATIONS IN A MOVING ORTHOGONAL COORDINATE SYSTEM

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by

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C. J. Woan and T. J. Akai

1. Introduction

We shall expand in this report the basic gas dynamic equations written by Walkden [1] in tensor form for an arbitrary moving coordinate system for the following specific systems:

(1) Orthogonal moving,

(2) Elliptic-cylindrical moving, and

(3) Orthogonal stationary.

2. Basic Equations in Tensor Form for an Arbitrary Moving Coordinate System

The basic equations of motion of a viscous, compressible gas referred to general coordinates $(\xi^0, \xi^1, \xi^2, \xi^3)$ are given by Walkden [1] as follows.

Continuity equation:

$$\frac{\partial}{\partial \xi^0}(J\rho) + \frac{\partial}{\partial \xi^\beta}(J\rho V^\beta) = 0 \qquad (1)$$

ξ^α-momentum equation:

$$\frac{1}{J}\left[\frac{\partial}{\partial \xi^0}(J\rho V_\alpha) + \frac{\partial}{\partial \xi^\beta}\left(J(\rho V_\alpha V^\beta + (p + \frac{2}{3}\mu\Delta)\delta_\alpha^\beta - \mu g^{\epsilon\beta}(V_{\alpha,\epsilon} + V_{\epsilon,\alpha}))\right)\right] - \frac{1}{2}\rho\frac{\partial g_{00}}{\partial \xi^\alpha} - \rho V^\beta \frac{\partial g_{0\beta}}{\partial \xi^\alpha}$$

$$- \frac{1}{2}\ \frac{\partial g_{\epsilon\beta}}{\partial \xi^\alpha}\left(\rho V^\epsilon V^\beta + (p + \frac{2}{3}\mu\Delta)g^{\epsilon\beta} - \mu g^{\omega\epsilon} g^{\gamma\beta}(V_{\gamma,\omega} + V_{\omega,\gamma})\right) = 0 \qquad (2)$$

Energy equation:

$$\rho C_p \frac{DT}{D\xi^0} = \Phi + \frac{Dp}{D\xi^0} + \frac{1}{J}\frac{\partial}{\partial \xi^\alpha}(J g^{\alpha\beta} k \frac{\partial T}{\partial \xi^\beta}) \qquad (3)$$

In the general coordinates, ξ^0 represents time and ξ^α represents the space variable. The general system is related to the cartesian system by

$$x^0 = \xi^0 \qquad (4)$$

and

$$x^\alpha = x^\alpha(\xi^0, \xi^1, \xi^2, \xi^3). \qquad (5)$$

The symbols used in equations (1), (2), and (3) and their relations are summarized as follows.

J = the Jacobian $\frac{\partial(x^1, x^2, x^3)}{\partial(\xi^1, \xi^2, \xi^3)}$,

δ_α^β = the Kronecker delta.

g_{ij} = elements of the metric tensor given by

$$g_{\alpha j} = g_{j\alpha} = \frac{\partial x^1}{\partial \xi^\alpha}\frac{\partial x^1}{\partial \xi^j} + \frac{\partial x^2}{\partial \xi^\alpha}\frac{\partial x^2}{\partial \xi^j} + \frac{\partial x^3}{\partial \xi^\alpha}\frac{\partial x^3}{\partial \xi^j}\text{, and}$$

$$g_{00} = -c^2\left(1-\left\{\left(\frac{\partial x^1}{\partial \xi^0}\right)^2 + \left(\frac{\partial x^2}{\partial \xi^0}\right)^2 + \left(\frac{\partial x^3}{\partial \xi^0}\right)^2\right\}\Big/c^2\right)$$

where c is the speed of light.

g^{ij} = elements of a tensor associated with the metric tensor and are given by $g_{ij}g^{jk} = \delta_i^k$.

$$V^i = \frac{D\xi^i}{D\xi^0}\ .$$

$$V_0 = -c^2.$$

$$V_\alpha = g_{\alpha m}V^m.$$

$$V_{\alpha,\epsilon} = \frac{\partial V_\alpha}{\partial \xi^\epsilon} - \frac{1}{2}V^\ell\left(\frac{\partial g_{\ell\alpha}}{\partial \xi^\epsilon} + \frac{\partial g_{\ell\epsilon}}{\partial \xi^\alpha} - \frac{\partial g_{\alpha\epsilon}}{\partial \xi^\ell}\right).$$

Φ = the dissipation function

$$= \frac{1}{2}\mu g^{\alpha\epsilon}g^{\beta\gamma}e_{\alpha\gamma}e_{\epsilon\beta} - \frac{2}{3}\mu\Delta^2.$$

$$\Delta = \frac{1}{J}\left(\frac{\partial J}{\partial \xi^0} + \frac{\partial}{\partial \xi^\beta}(JV^\beta)\right).$$

$$e_{\alpha\gamma} = V_{\alpha,\gamma} + V_{\gamma,\alpha}\ .$$

μ = coefficient of viscosity.

p = pressure.

T = temperature.

ρ = density.

C_p = specific heat at constant pressure.

k = thermal conductivity.

$$\frac{D}{D\xi^0} = \frac{\partial}{\partial\xi^0} + V^\beta \frac{\partial}{\partial\xi^\beta} .$$

To expand equations (1),(2), and (3), i.e., to carry out the summation, the letter indices are replaced by numerical indices. The quantities, J, δ^β_α, g_{ij}, g^{ij}, V^α, V_α, $V_{\alpha,\epsilon}$, Φ, Δ, and $e_{\alpha\beta}$ are then replaced by the relations given in the preceding summary.

It should be noted that the summation convention for Greek and Latin indices are different. Greek indices take on the values 1, 2, and 3 whereas Latin indices take on the values 0, 1, 2, and 3. For example,

$$V^jV^j = V^0V^0+V^1V^1+V^2V^2+V^3V^3$$

and

$$V^\alpha V^\alpha = V^1V^1+V^2V^2+V^3V^3.$$

3. Basic Equations for a Moving Orthogonal System

We consider an orthogonal system that is moving but is not distorted in time; i.e.; the curves of constant ξ^1, ξ^2, and ξ^3 remain fixed relative to each other. We wish to introduce the scale factors h_α, defined by

$$(h_\alpha)^2 = \left(\frac{\partial x^1}{\partial\xi^\alpha}\right)^2 + \left(\frac{\partial x^2}{\partial\xi^\alpha}\right)^2 + \left(\frac{\partial x^3}{\partial\xi^\alpha}\right)^2 . \qquad (6)$$

We then have, for this system,

$$J = h_1 h_2 h_3,$$

$$g_{\alpha\beta} = (h_\alpha)^2 \delta_\alpha^\beta \,, \qquad \text{(no sum intended)}$$

$$g^{\alpha\beta} = \delta_\alpha^\beta / (h_\alpha)^2, \qquad \text{(no sum intended)}$$

$$V^\beta = u^\beta / h_\beta, \qquad \text{(no sum intended)}$$

where u_β is the velocity component in the ξ^β-direction, and

$$V_\beta = h_\beta u_\beta + g_{\beta 0}. \qquad \text{(no sum intended)}$$

Since the coordinate system is not distorted, the quantities h_α, J, $g_{\alpha\beta}$ and $g^{\alpha\beta}$ are not functions of time.

Introducing the above expressions in equations (1), (2) and (3), we obtain the following equations of motion for a moving orthogonal system.

<u>Continuity equation</u>:

$$\frac{\partial \rho}{\partial \xi^0} = - \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial \xi^1}(h_2 h_3 \rho u_1) + \frac{\partial}{\partial \xi^2}(h_1 h_3 \rho u_2) + \frac{\partial}{\partial \xi^3}(h_1 h_2 \rho u_3)\right]. \qquad (7)$$

Momentum equation, ξ^1-component:

$$\frac{\partial}{\partial \xi^0}\left[\rho u_1 + \rho \frac{g_{10}}{h_1}\right] + \mathrm{div}\left[\rho u_1 \vec{u} + \rho \frac{g_{10}}{h_1}\vec{u}\right]$$

$$+ \frac{\rho u_1}{h_1}\left\{\frac{u_1}{h_1}\frac{\partial h_1}{\partial \xi^1} + \frac{u_2}{h_2}\frac{\partial h_1}{\partial \xi^2} + \frac{u_3}{h_3}\frac{\partial h_1}{\partial \xi^3}\right\} - \frac{\rho}{h_1}\left\{\frac{u_1}{h_1}\left[\left(u_1 - \frac{g_{10}}{h_1}\right)\frac{\partial h_1}{\partial \xi^1} + \frac{\partial g_{01}}{\partial \xi^1}\right]\right.$$

$$+ \frac{u_2}{h_2}\left[u_2\frac{\partial h_2}{\partial \xi^1} - \frac{g_{10}}{h_1}\frac{\partial h_1}{\partial \xi^2} + \frac{\partial g_{02}}{\partial \xi^1}\right] + \frac{u_3}{h_3}\left[u_3\frac{\partial h_3}{\partial \xi^1} - \frac{g_{10}}{h_1}\frac{\partial h_1}{\partial \xi^3} + \frac{\partial g_{03}}{\partial \xi^1}\right]$$

$$\left. + \frac{1}{2}\frac{\partial g_{00}}{\partial \xi^1}\right\} = \frac{1}{h_1 h_2 h_3}\left\{\frac{\partial}{\partial \xi^1}\left[h_2 h_3(\tau_{11} - p)\right] + \frac{\partial}{\partial \xi^2}(h_1 h_3 \tau_{12}) + \frac{\partial}{\partial \xi^3}(h_1 h_2 \tau_{13})\right\}$$

$$+ \frac{\tau_{12}}{h_1 h_2}\frac{\partial h_1}{\partial \xi^2} + \frac{\tau_{13}}{h_1 h_3}\frac{\partial h_1}{\partial \xi^3} - \frac{(\tau_{22} - p)}{h_1 h_2}\frac{\partial h_2}{\partial \xi^1} - \frac{(\tau_{33} - p)}{h_1 h_3}\frac{\partial h_3}{\partial \xi^1} . \qquad (8)$$

The ξ^2- and ξ^3-components of the momentum equation can easily be obtained by rotating the indices 1, 2, and 3. For example, to obtain the ξ^2-component, replace 1 by 2, 2 by 3, and 3 by 1 in the ξ^1-equation.

Energy equation:

$$\rho\frac{\partial H}{\partial\xi^0}+\rho\left\{\frac{u_1}{h_1}\left[\frac{\partial H}{\partial\xi^1}-\left(\frac{u_1}{h_1}\frac{\partial g_{01}}{\partial\xi^1}+\frac{u_2}{h_2}\frac{\partial g_{02}}{\partial\xi^1}+\frac{u_3}{h_3}\frac{\partial g_{03}}{\partial\xi^1}+\frac{1}{2}\frac{\partial g_{00}}{\partial\xi^1}\right)\right]\right.$$

$$+\frac{u_2}{h_2}\left[\frac{\partial H}{\partial\xi^2}-\left(\frac{u_1}{h_1}\frac{\partial g_{01}}{\partial\xi^2}+\frac{u_2}{h_2}\frac{\partial g_{02}}{\partial\xi^2}+\frac{u_3}{h_3}\frac{\partial g_{03}}{\partial\xi^2}+\frac{1}{2}\frac{\partial g_{00}}{\partial\xi^2}\right)\right]$$

$$+\frac{u_3}{h_3}\left[\frac{\partial H}{\partial\xi^3}-\left(\frac{u_1}{h_1}\frac{\partial g_{01}}{\partial\xi^3}+\frac{u_2}{h_2}\frac{\partial g_{02}}{\partial\xi^3}+\frac{u_3}{h_3}\frac{\partial g_{03}}{\partial\xi^3}+\frac{1}{2}\frac{\partial g_{00}}{\partial\xi^3}\right)\right]$$

$$+u_1\frac{\partial}{\partial\xi^0}\left(\rho\frac{g_{10}}{h_1}\right)+u_2\frac{\partial}{\partial\xi^0}\left(\rho\frac{g_{20}}{h_2}\right)+u_3\frac{\partial}{\partial\xi^0}\left(\rho\frac{g_{30}}{h_3}\right)$$

$$+\frac{1}{h_1h_2h_3}\left\{\frac{u_1}{h_1}\left[\frac{\partial}{\partial\xi^1}\left(h_2h_3\rho u_1g_{10}\right)+\frac{\partial}{\partial\xi^2}\left(h_1h_3\rho u_2g_{10}\right)+\frac{\partial}{\partial\xi^3}\left(h_1h_2\rho u_3g_{10}\right)\right]\right.$$

$$+\frac{u_2}{h_2}\left[\frac{\partial}{\partial\xi^1}\left(h_2h_3\rho u_1g_{20}\right)+\frac{\partial}{\partial\xi^2}\left(h_1h_3\rho u_2g_{20}\right)+\frac{\partial}{\partial\xi^3}\left(h_1h_2\rho u_3g_{20}\right)\right]$$

$$\left.+\frac{u_3}{h_3}\left[\frac{\partial}{\partial\xi^1}\left(h_2h_3\rho u_1g_{30}\right)+\frac{\partial}{\partial\xi^2}\left(h_1h_3\rho u_2g_{30}\right)+\frac{\partial}{\partial\xi^3}\left(h_1h_2\rho u_3g_{30}\right)\right]\right\}$$

$$=\frac{\partial p}{\partial\xi^0}+\frac{1}{h_1h_2h_3}\left\{-\frac{\partial}{\partial\xi^1}\left(h_2h_3q_1\right)-\frac{\partial}{\partial\xi^2}\left(h_1h_3q_2\right)-\frac{\partial}{\partial\xi^3}\left(h_1h_2q_3\right)\right\}$$

$$+\frac{1}{h_1h_2h_3}\left\{\frac{\partial}{\partial\xi^1}\left(h_2h_3\left[u_1\tau_{11}+u_2\tau_{12}+u_3\tau_{13}\right]\right)\right.$$

$$+\frac{\partial}{\partial\xi^2}\left(h_1h_3\left[u_1\tau_{21}+u_2\tau_{22}+u_3\tau_{23}\right]\right)$$

$$+\frac{\partial}{\partial\xi^3}\left(h_1h_2\left[u_1\tau_{31}+u_2\tau_{32}+u_3\tau_{33}\right]\right)\ . \qquad (9)$$

In these equations,

$$H = C_p T + \frac{1}{2} u_\beta u_\beta ,$$

$$q_\alpha = \frac{-k}{h_\alpha} \frac{\partial T}{\partial \xi^\alpha} ,$$

$$\tau_{11} = 2\mu \left\{ \frac{1}{h_1} \frac{\partial u_1}{\partial \xi^1} + \frac{u_2}{h_1 h_2} \frac{\partial h_1}{\partial \xi^2} + \frac{u_3}{h_1 h_3} \frac{\partial h_1}{\partial \xi^3} \right\} + C ,$$

$$\tau_{22} = 2\mu \left\{ \frac{1}{h_2} \frac{\partial u_2}{\partial \xi^2} + \frac{u_3}{h_2 h_3} \frac{\partial h_2}{\partial \xi^3} + \frac{u_1}{h_2 h_1} \frac{\partial h_2}{\partial \xi^1} \right\} + C ,$$

$$\tau_{33} = 2\mu \left\{ \frac{1}{h_3} \frac{\partial u_3}{\partial \xi^3} + \frac{u_1}{h_3 h_1} \frac{\partial h_3}{\partial \xi^1} + \frac{u_2}{h_3 h_2} \frac{\partial h_3}{\partial \xi^2} \right\} + C ,$$

$$C = \frac{-2}{3}\mu \frac{1}{h_1 h_2 h_3} \left\{ \frac{\partial}{\partial \xi^1}\left(h_2 h_3 u_1\right) + \frac{\partial}{\partial \xi^2}\left(h_3 h_1 u_2\right) + \frac{\partial}{\partial \xi^3}\left(h_1 h_2 u_3\right) \right\} ,$$

$$\tau_{12} = \tau_{21} = \mu \left\{ \frac{h_2}{h_1} \frac{\partial}{\partial \xi^1}\left(\frac{u_2}{h_2}\right) + \frac{h_1}{h_2} \frac{\partial}{\partial \xi^2}\left(\frac{u_1}{h_1}\right) \right\} ,$$

$$\tau_{13} = \tau_{31} = \mu \left\{ \frac{h_3}{h_1} \frac{\partial}{\partial \xi^1}\left(\frac{u_3}{h_3}\right) + \frac{h_1}{h_3} \frac{\partial}{\partial \xi^3}\left(\frac{u_1}{h_1}\right) \right\} ,$$

$$\tau_{23} = \tau_{32} = \mu \left\{ \frac{h_3}{h_2} \frac{\partial}{\partial \xi^2}\left(\frac{u_3}{h_3}\right) + \frac{h_2}{h_3} \frac{\partial}{\partial \xi^3}\left(\frac{u_2}{h_2}\right) \right\} ,$$

$$\operatorname{div}(F\vec{u}) = \frac{1}{h_1 h_2 h_3} \left\{ \frac{\partial}{\partial \xi^1}\left(h_2 h_3 F u_1\right) + \frac{\partial}{\partial \xi^2}\left(h_1 h_3 F u_2\right) + \frac{\partial}{\partial \xi^3}\left(h_1 h_2 F u_3\right) \right\}$$

4. Basic Equations for a Moving, Elliptic-Cylindrical System

We shall now apply equations (7), (8), and (9) to an accelerating elliptic system:

$$x^0 = \xi^0, \tag{10}$$

$$x^1 = a \cosh \xi^1 \cos \xi^2 + \varphi(\xi^0), \tag{11}$$

$$x^2 = a \sinh \xi^1 \sin \xi^2 + \psi(\xi^0). \tag{12}$$

For this system, $h_1 = h_2 = a(\sinh^2\xi^1 + \sin^2\xi^2)^{\frac{1}{2}}$

$$g_{01} = g_{10} = a\left(\frac{\partial\varphi}{\partial\xi^0} \sinh\xi^1\cos\xi^2 + \frac{\partial\psi}{\partial\xi^0}\cosh\xi^1\sin\xi^2\right)$$

and $$g_{02} = g_{20} = a\left(\frac{-\partial\varphi}{\partial\xi^0} \cosh\xi^1\sin\xi^2 + \frac{\partial\psi}{\partial\xi^0}\sinh\xi^1\cos\xi^2\right).$$

It is also noted that g_{00} is independent of ξ^α.

The basic equations for this system are:

Continuity equation:

$$\frac{\partial\rho}{\partial\xi^0} = \frac{-1}{h_1h_2}\left[\frac{\partial}{\partial\xi^1}(h_2\rho u_1) + \frac{\partial}{\partial\xi^2}(h_1\rho u_2)\right]. \tag{13}$$

Momentum equation, ξ^1-component:

$$\frac{\partial}{\partial \xi^0}\left[\rho u_1 + \frac{\rho g_{10}}{h_1}\right] + \mathrm{div}\left(\rho u_1 \vec{u} + \rho \frac{g_{10}}{h_1} \vec{u}\right)$$

$$+ \frac{\rho u_1}{h_1}\left\{\frac{u_1}{h_1}\frac{\partial h_1}{\partial \xi^1} + \frac{u_2}{h_2}\frac{\partial h_1}{\partial \xi^2}\right\} - \frac{\rho}{h_1}\left\{\frac{u_1}{h_1}\left[\left(u_1 - \frac{g_{10}}{h_1}\right)\frac{\partial h_1}{\partial \xi^1} + \frac{\partial g_{01}}{\partial \xi^1}\right]\right.$$

$$\left. + \frac{u_2}{h_2}\left[u_2 \frac{\partial h_2}{\partial \xi^1} - \frac{g_{10}}{h_1}\frac{\partial h_1}{\partial \xi^2} + \frac{\partial g_{02}}{\partial \xi^1}\right]\right\} = \frac{1}{h_1 h_2}\left\{\frac{\partial}{\partial \xi^1}\left[h_2(\tau_{11} - p)\right] + \frac{\partial}{\partial \xi^2}(h_1 \tau_{12})\right\}$$

$$+ \frac{\tau_{12}}{h_1 h_2}\frac{\partial h_1}{\partial \xi^2} - \frac{(\tau_{22} - p)}{h_1 h_2}\frac{\partial h_2}{\partial \xi^1} \, . \tag{14}$$

Energy equation:

$$\rho \frac{\partial H}{\partial \xi^0} + \rho\left\{\frac{u_1}{h_1}\left[\frac{\partial H}{\partial \xi^1} - \left(\frac{u_1}{h_1}\frac{\partial g_{01}}{\partial \xi^1} + \frac{u_2}{h_2}\frac{\partial g_{02}}{\partial \xi^1}\right)\right]\right.$$

$$\left. + \frac{u_2}{h_2}\left[\frac{\partial H}{\partial \xi^2} - \left(\frac{u_1}{h_1}\frac{\partial g_{01}}{\partial \xi^2} + \frac{u_2}{h_2}\frac{\partial g_{02}}{\partial \xi^2}\right)\right]\right\}$$

$$+ u_1 \frac{\partial}{\partial \xi^0}\left(\frac{\rho g_{10}}{h_1}\right) + u_2 \frac{\partial}{\partial \xi^0}\left(\frac{\rho g_{20}}{h_2}\right)$$

$$+ \frac{1}{h_1 h_2}\left\{\frac{u_1}{h_1}\left[\frac{\partial}{\partial \xi^1}(h_2 \rho u_1 g_{10}) + \frac{\partial}{\partial \xi^2}(h_1 \rho u_2 g_{10})\right]\right.$$

$$\left. + \frac{u_2}{h_2}\left[\frac{\partial}{\partial \xi^1}(h_2 \rho u_1 g_{20}) + \frac{\partial}{\partial \xi^2}(h_1 \rho u_2 g_{20})\right]\right\}$$

$$= \frac{\partial p}{\partial \xi^0} + \frac{1}{h_1 h_2}\left\{-\frac{\partial}{\partial \xi^1}(h_2 q_1) - \frac{\partial}{\partial \xi^2}(h_1 q_2)\right\}$$

$$+ \frac{1}{h_1 h_2}\left\{\frac{\partial}{\partial \xi^1}\left[h_2(u_1 \tau_{11} + u_2 \tau_{12})\right] + \frac{\partial}{\partial \xi^2}\left[h_1(u_1 \tau_{21} + u_2 \tau_{22})\right]\right\} \, . \tag{15}$$

5. Basic Equations for a Stationary Orthogonal System

Finally we consider the stationary orthogonal system

$$X^0 = \xi^0 \tag{16}$$

$$X^\alpha = X^\alpha(\xi^1, \xi^2, \xi^3) \tag{17}$$

We note that $g_{\alpha 0} = g_{0\alpha} = 0$ and that g_{00} is not a function of ξ^α. Thus, we set $g_{\alpha 0} = g_{0\alpha} = 0$ and $\frac{\partial g_{00}}{\partial \xi^\alpha} = 0$ in equations (7), (8), and (9) and arrive at the following results:

Continuity equation:

$$\frac{\partial \rho}{\partial \xi^0} = - \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial \xi^1}(h_2 h_3 \rho u_1) + \frac{\partial}{\partial \xi^2}(h_1 h_3 \rho u_2) + \frac{\partial}{\partial \xi^3}(h_1 h_2 \rho u_3)\right] \tag{18}$$

Momentum equation ξ^1-component:

$$\frac{\partial}{\partial \xi^0}(\rho u_1) + \mathrm{div}(\rho u_1 \vec{u}) + \frac{\rho u_1}{h_1}\left\{\frac{u_1}{h_1}\frac{\partial h_1}{\partial \xi^1} + \frac{u_2}{h_2}\frac{\partial h_1}{\partial \xi^2} + \frac{u_3}{h_3}\frac{\partial h_1}{\partial \xi^3}\right\}$$

$$- \frac{\rho}{h_1}\left\{\frac{u_1^2}{h_1}\frac{\partial h_1}{\partial \xi^1} + \frac{u_2^2}{h_2}\frac{\partial h_2}{\partial \xi^1} + \frac{u_3^2}{h_3}\frac{\partial h_3}{\partial \xi^1}\right\}$$

$$= \frac{1}{h_1 h_2 h_3}\left\{\frac{\partial}{\partial \xi^1}\left[h_2 h_3(\tau_{11} - p)\right] + \frac{\partial}{\partial \xi^2}\left[h_1 h_3 \tau_{12}\right] + \frac{\partial}{\partial \xi^3}\left[h_1 h_2 \tau_{13}\right]\right\}$$

$$+ \frac{\tau_{12}}{h_1 h_2}\frac{\partial h_1}{\partial \xi^2} + \frac{\tau_{13}}{h_1 h_3}\frac{\partial h_1}{\partial \xi^3} - \frac{(\tau_{22} - p)}{h_1 h_2}\frac{\partial h_2}{\partial \xi^1} - \frac{(\tau_{33} - p)}{h_1 h_3}\frac{\partial h_3}{\partial \xi^1} \tag{19}$$

Energy equation:

$$\rho \frac{\partial H}{\partial \xi^0} + \rho \left\{ \frac{u_1}{h_1} \frac{\partial H}{\partial \xi^1} + \frac{u_2}{h_2} \frac{\partial H}{\partial \xi^2} + \frac{u_3}{h_3} \frac{\partial H}{\partial \xi^3} \right\}$$

$$= \frac{\partial p}{\partial \xi^0} + \frac{1}{h_1 h_2 h_3} \left\{ - \frac{\partial}{\partial \xi^1}(h_2 h_3 q_1) - \frac{\partial}{\partial \xi^2}(h_1 h_3 q_2) - \frac{\partial}{\partial \xi^3}(h_1 h_2 q_3) \right\}$$

$$+ \frac{1}{h_1 h_2 h_3} \left\{ \frac{\partial}{\partial \xi^1} \left[h_2 h_3 (u_1 \tau_{11} + u_2 \tau_{12} + u_3 \tau_{13}) \right] \right.$$

$$+ \frac{\partial}{\partial \xi^2} \left[h_1 h_3 (u_1 \tau_{21} + u_2 \tau_{22} + u_3 \tau_{23}) \right]$$

$$\left. + \frac{\partial}{\partial \xi^3} \left[h_1 h_2 (u_1 \tau_{31} + u_2 \tau_{32} + u_3 \tau_{33}) \right] \right\} \qquad (20)$$

The energy equation given in reference 2 appears to be incorrect in that the last 3 terms on the right hand side of the equation have the factors $\frac{h_2 h_3}{h_1}$, $\frac{h_1 h_3}{h_2}$, and $\frac{h_1 h_2}{h_3}$ instead of $h_2 h_3$, $h_1 h_3$, and $h_1 h_2$ respectively.

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13. ABSTRACT

We shall expand in this report the basic gas dynamic equations written by Walkden in tensor form for an arbitrary moving coordinate system for the following specific systems:

(1) Orthogonal moving,

(2) Elliptic-cylindrical moving, and

(3) Orthogonal stationary.

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14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Gas Dynamic Equations						
Coordinate System						

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