CONTROL SYSTEMS LABORATORY

EMPIRICAL FLUCTUATIONS IN INFORMATION MEASURES

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Prepared by:

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In the earlier applications of information theory to telecommunication the problem of sample size did not arise, since virtually infinite sample sizes could be obtained in a msec. In the application of information theory to other fields, e.g., psychology, very large samples are not obtainable. Accordingly, the measures obtained are bound to be affected by sampling fluctuations. It is imperative that the type and amount of these fluctuations be known, otherwise information measures based on small samples would be meaningless. Thus, we have endeavored to obtain some knowledge of the small sample statistics of the information measures most commonly used.

A sampling distribution depends upon the sampling scheme used. Of the large number of sampling schemes which might conceivably serve as models of human behavior, a very simple one is random multinomial sampling. The mathematics of it are discussed in the preceding report (R-76). In this report we will investigate empirical sampling fluctuations and compare them with the results obtained by Monte Carlo techniques employing multinomial sampling.

For this purpose we used data obtained in the flash recognition of playing cards. A detailed description of the experimental procedure has been given elsewhere* and only a brief description will be presented here. Displays containing ordinary playing cards (only aces and numbers)

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* Control Systems Laboratory Report R-69 (1956).

were shown to the subjects for 100 msec. Two seconds later they were asked to identify one of the cards which had been selected randomly and was unknown to the subject before the display was shown. The information transmitted by suits and numbers was treated separately: suits transmission was evaluated by a matrix method and numbers transmission by the method of error magnitude (R-69). The transmission values were calculated for each display position based upon twenty responses for that position. The transmission values for the individual positions were then summed to give a total transmission, T, value for that type of display.

We obtained four estimates of information transmitted, T, from three-card displays and three from six-card displays for each of two subjects. For one of the subjects we also had data observed three months earlier. The transmission values found in each run and the standard deviation between runs are shown in table I.

TABLE I

Subject

	W	K	K(earlier data)
Transmission for 3-card displays	11.6 (bits)	10.7	9.9
	13.2	10.5	10.8
	12.0	10.8	10.5
	12.3	12.0	
Average	12.3	11.0	10.4
Empirical σ	0.68	0.68	0.46
Monte Carlo σ	0.69	0.59	
Transmission for 6-card displays	17.8	16.1	14.6
	17.2	15.3	14.1
	16.4	16.0	
Average	17.1	15.8	14.4
Empirical σ	0.71	0.44	0.36
Monte Carlo σ	0.86	0.86	

The results obtained with our two subjects were compared with results obtained by Monte Carlo techniques. The input-output frequencies for each display position from all experiments with a given subject and display type were pooled and normalized. These probability sets were then used to generate samples of twenty from which transmission values were computed. Two hundred such samples were generated and the standard deviation between these samples computed. The standard deviations for the individual display positions were then pooled to form a standard deviation for each display type. These results are also entered in table I.

It appears that there are many occasions where the results of one act of transmitting information affects subsequent acts. There are instances of positive feedback: in this case, errors in preceding acts of information transmission cause the subject to become "rattled" and increase his probability of making additional errors. On the other hand, there exists negative feedback: previous errors tend to make the subject more careful and thereby reduce his probability of making errors. Both feedbacks have the effect of reducing the effective number of independent trials. Negative feedback would tend to make T more stable than it would be with independent successive random samplings, and thus reduce the variance between runs. In the case of positive feedback, the effective reduction in the number of independent trials is not opposed by a T-stabilizing effect and therefore there should be an increased variance between runs. Table I shows that empirical standard deviations are in general slightly smaller than those obtained from the Monte Carlo method. This indicates that there is a small negative feedback; however, non-coherent multinomial sampling is seen to be a good approximation to the behaviour of our human subjects in the experiment discussed.