STRATEGIC MANAGEMENT OF OPERATIONAL RESOURCES UNDER UNCERTAINTY

BY

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DISSERTATION

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ABSTRACT

This dissertation addresses and answers two questions: (1) What are the impacts of market uncertainty and technological uncertainty? (2) What is the best way for a firm to manage demand information and technological knowledge in the face of competition?

The first essay (Chapter 2) investigates a problem of competitive investment with payoff externalities and uncertain but partially observable profitability. This essay examines a duopoly game, which, under appropriate conditions, reduces to a war of attrition game in the sense that both firms have incentives to be the follower. We find that due to the strategic interactions, payoff externalities and learning opportunities have counterintuitive effects on investment strategies and on the time to the first investment. In particular, we find that an increase in the rate of learning, which usually benefits the follower, may hasten or delay the first investment depending on the rate of learning and the prior probability that the investment is profitable. Overall, the results of this chapter suggest that firms facing entry into an unproven market need to consider the strategic effects arising from learning and externalities.

The second essay (Chapter 3) investigates the strategy of investment in R&D projects when completion time of R&D is uncertain. By examining a game theoretic model of two firms competitively engaged in R&D projects, we find that the more innovative firm may or may not have an incentive to unilaterally share technological knowledge with its opponent; the result depends on the more innovative firm's tradeoff between reduction of competitive pressure and reduction of the competitor's imitation. A direct implication of this result is that a firm may achieve superior performance by strategically managing its technological knowledge without incurring cost.

The third essay (Chapter 4) investigates a problem of competitive investment in R&D projects to examine (1) the impacts of uncertainties and (2) the strategies of managing demand information and technological knowledge. We find that market uncertainty can improve or diminish a firm's payoff due to strategic interactions between firms and the interplay of learning effects and externalities. Our results also indicate that technological uncertainty can alter the relationship between the time to completion and the fierceness of competition. More specifically, we find that an increase in the time to completion may or may not increase the fierceness of the competition. Lastly, this essay compares the impact of disclosing demand information and that of disclosing technological knowledge. The results show that disclosing technological knowledge can only improve a firm's ex-ante payoff, whereas disclosing demand information can improve both the ex-ante and ex-post payoffs. Hence, our results indicate that the disclosed contents and the time to disclose are important when firms consider voluntary disclosure to reduce competition.

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CHAPTER 1

INTRODUCTION

In the context of investment decisions under uncertainty, making "good" decisions can be a challenging task because of the unknown impacts of uncertainties and the strategic interactions between firms. My dissertation mainly investigates two aspects of the decision-making process in two aspects: (1) understanding the impacts of uncertainties and (2) providing managerial insights for managing operational resources, such as information regarding market demand, intellectual properties, and research and development (R&D) resources. With these two broad themes, the dissertation consists of three essays (Chapter 2 - Chapter 4).

1. Impacts of Uncertainties

This dissertation focuses on two types of uncertainty: *market uncertainty* and *technological uncertainty.* chapter 2 and Chapter 4 consider the impact of market uncertainty, and Chapter 3 and Chapter 4 consider technological uncertainty.

When there is significant uncertainty associated with the market reception of new products or the performance of technologies, it can be beneficial to delay an investment if time will yield more demand information, from which a firm can learn about the investment's future prospects. Furthermore, firms' investments can entail externalities, which means a firm's returns from an investment can improve or diminish with an increase in the number of firms in the market. Because of these two features, understanding the impact of learning and externalities becomes a preliminary step toward investigating the impact of market uncertainty. In chapter 2, by examining a game theoretical duopoly investment model, we find the interplay between learning and externalities gives rise to counterintuitive effects on investment strategies and payoffs: Contrary to the conventional theory of war of attrition where an increase in follower's payoff generally delays the first move, an increased rate of learning that tends to benefit the follower may hasten or delay the first investment, depending on the rate of learning and the initial beliefs of firms (prior probability) that the investment is profitable. Utilizing the impact of learning and externalities found in chapter 2, Chapter 4 examines the impact of market uncertainty, and shows that market uncertainty has non-monotonic relationships with both investment time and payoffs. Unlike other works that explore the impact of uncertainty with decision theoretical models or empirical data, the results in Chapter 4 are driven by the strategic interactions between firms, and the interplay of learning and externalities.

Chapter 3 and Chapter 4 study two models related to technological uncertainty in the context of R&D investment. Although Chapter 3 focuses on firms' incentives to unilaterally increase technological knowledge spillover to competitors, our results indicate that technological uncertainty reduces such an incentive. Chapter 4 also investigates the impact of uncertain time-to-completion (the lag between a firm's time of investment and the time to complete the project) on the fierceness of competition. The results show that longer time-to-completion may or may not result in fiercer competition, depending on the firms' beliefs about market demand.

2. Managing Operational Resources Strategically

Two types of resources are discussed in this work: demand information and technological knowledge. Demand information is defined as the information that can help to resolve market uncertainty, such as forecasts about future demand, or sales data that imply current demand. Technological knowledge is the knowledge which helps to facilitate or accelerate the innovation of products or processes, such as the designs for new products or the expertise of skilled workers.

Chapter 3 focuses on managing technological knowledge resources. More specifically, it examines the impact of natural spillover on R&D investment strategies when the R&D completion times are uncertain and one firm can receive spillover from another. The analysis shows that natural spillover can improve the profit of an innovative firm. Furthermore, an innovative firm may even have an incentive to unilaterally share technological knowledge with its opponent. Chapter 4 focuses on managing demand information resources. It shows that firms can also have an incentive to freely disclose more demand information to competitors. Although both disclosing demand information and technological knowledge can be beneficial to firms, Chapter 4 points out the difference between these two types of disclosing: only disclosing demand information benefits firms' ex-post payoffs. Hence, our results indicate that the disclosed contents as well as timing are important when firms consider voluntary disclosure.

CHAPTER 2

IMPACT OF BAYESIAN LEARNING AND EXTERNALITIES ON STRATEGIC INVESTMENT

2.1 Introduction

Investment decisions in business such as the introduction of products in unproven markets or the adoption of unproven technologies often involve significant uncertainty associated with the market reception of new products or the performance of technologies. In competitive scenarios, returns on investments can also depend upon the timing of the investments. For example, it may be beneficial to delay an investment if time will bring more information from which a firm can learn about the future prospects of the investment, provided the opportunity to invest does not disappear (Carruth et al. 2000, Dixit 1992). Returns on investments can also depend upon the investment decisions made by other firms in the market. If there are positive externalities, a firm's returns can improve with an increase in the number of firms in the market; and if there are negative externalities, a firm's returns can diminish with an increase in the number of firms. The existing literature on investment under uncertainty has examined these two factors, i.e., learning effects and externalities, separately.

In this paper, we examine a duopoly game of investment with uncertain profitability where learning effects and externalities coexist. In our model, if one firm enters the market as a leader, then the other firm (the follower) has the opportunity to observe the leader's performance in the market and learn about the true profitability of the market. By observing the leader's performance, the follower can also make an investment to enter the market. If the two firms are in the market at the same time, their profit streams exhibit externalities. We study the following comparative statics in detail: (1) how the follower's time to invest changes with the rate of learning , (2) how the leader's payoff changes with the rate of learning and (3) how the time to first investment changes with the rate of learning. We find the presence of a ubiquitous *single-crossing property* (property of changing its sign at most once) of the derivatives of the payoff and the time to investment with respect to the rate of learning. This result is driven by the interplay between externalities and learning, which are the salient features of the model that we investigate.

Under appropriate conditions, our model reduces to a war of attrition, which is a game in which the leader's payoff is less than the follower's payoff. In a mixed strategy equilibrium in the conventional war of attrition, each player may delay their investments in an attempt to be the follower (Section 2.4.2). It is well known that in such an equilibrium, an increase in the follower's reward delays both players' decisions to move first (Hendricks et al. 1988). Hence, one would expect that an increased rate of learning, which enables the follower to learn faster about the profitability of the investment, would induce both firms to delay their investments. However, we find that the effect of an increased rate of learning is more nuanced.

In this study, there are three main findings. The first finding is that an increased rate of learning has two opposing effects on the follower's time of investment: (a) it can hasten the follower's investment because the follower can acquire more meaningful information within a shorter time, or (b) it can delay the follower's investment because of increased value of waiting (to collect more information on the profitability of the investment). Either effect can dominate; we find that the derivative of the follower's time to investment with respect to the rate of learning exhibits a single-crossing property. To understand the underlying mechanism of this result, we note that the follower's optimal policy is to invest when p (the probability that the market has a high profit) exceeds a threshold θ_F (Proposition 1). As is well-known in the literature (see, for example, Proposition 2 of Kwon and Lippman 2011), the threshold θ_F increases in the learning rate of the follower because a higher learning rate increases the value of waiting and learning, which delays the follower's investment. Thus, when p is sufficiently close to θ_F, the follower's time of investment is strongly influenced by the comparative statics of θ_F, and consequently, effect (b) dominates. In contrast, when p is sufficiently far away from θ_F (for sufficiently low values of p), the follower's time of investment is less influenced by the comparative statics of θ_F. In this case, due to the influence of effect (a), the follower's investment is hastened by an increased rate of learning. Thus, the comparative statics of the follower's time to investment changes as p increases: effect (a) dominates for low values of p, and effect (b) dominates for high values of p. Indeed, our analysis shows a single-crossing property of the comparative statics as shown in Theorem 1 and illustrated in Figure 2.3.

The second finding is that the interplay of externalities with learning drives the single-crossing property of the comparative statics of the leader's payoff. Under positive externalities, an earlier investment of the follower improves the leader's payoff because of the time value of money (i.e., discounting). Therefore, for p sufficiently close to θ_F when effect (b) dominates, the leader's payoff (mixed strategy equilibrium payoff) decreases with the rate of learning. On the other hand, for sufficiently low p when effect (a) dominates, the leader's payoff increases with the rate of learning. Thus, the comparative statics of the leader's payoff also exhibits a single-crossing property. Under negative externalities, the comparative statics are reversed since an early investment of the follower diminishes the leader's payoff. These results are reported as Theorem 2 and illustrated by Figure 2.4.

The third finding is that the comparative statics of the time to the first investment also has a single-crossing property. For example, in the case of positive externalities, if the leader's payoff increases (decreases) in the learning rate, then the time to the first investment tends to decrease (increase) in the learning rate. Thus, the time to the first investment tends to increase in the learning rate for sufficiently high values of p due to effect (b), and it tends to decrease in the learning rate for sufficiently low values of p due to effect (a). Thus, the derivative of the time to first investment with respect to the learning rate also has a single-crossing property. These results are reported as Theorem 3(i) and illustrated by Figure 2.5(a). We also report similar results for the case of a second-mover advantage in Theorem 4 and Figure 2.6(b).

Investment problems with both positive and negative externalities are often seen in real life. Positive externalities can arise from a number of factors, including network externalities, product complementarity, and economies of scale. An example of positive externalities through economies of scale is illustrated by the introduction of organic cotton garments in the mid-1990s. At the time, introducing organic cotton garments was risky, as organic cotton garments are indistinguishable by sight and touch from garments made of conventionally grown cotton (Casadesus-Masanell et al., 2009), and so it was not clear whether customers would be willing to pay more for such products. It would also cost more to procure organic cotton, as firms would need to invest in growers to support their adoption of specific practices for cultivating organic cotton. Despite the risk, firms like Patagonia entered the market as first movers, and second movers such as Gap were able to observe the performance of the first movers and learn how the garments were received by consumers. However, Patagonia and Gap were not in competition with each other since they targeted different markets: Patagonia made garments for mountaineering-related activities, while Gap made garments for casual wear. As more firms introduced organic cotton garments, the growers of organic cotton benefited from the additional investments made by the new entrants, which helped lower the procurement costs of organic cotton due to economies of scale. The lower procurement costs could translate to lower retail prices for organic cotton garments and thus increased the uptake of such products by consumers. Thus, the firms' investments had positive externalities.

Negative externalities can often be observed in the context of new product launches or adoption of new technologies. For instance, in the mid-1980s, the steel maker Nucor was pondering the difficult question of whether to adopt a new thin slab casting technology called compact strip production (CSP). There was a significant upside profitability potential if the technology was successful; however, there was also significant uncertainty about the viability of the technology (Ghemawat and Stander 1998). Moreover, even if Nucor's adoption of the new technology turned out to be successful, it was unclear how large the first-mover advantage would be. Other steelmakers were bound to notice the performance of CSP and would follow suit within a few years if Nucor successfully adopted the technology, which could drive down the profits due to competition. Here, the firms' investments had negative externalities that could potentially disrupt the leader's efforts to appropriate profits from its investments or deter the investments of followers. These examples illustrate that when learning effects and externalities combine, investment decisions under uncertainty become inherently complex.

The proofs of all mathematical statements in the paper are provided in Appendix A.2.

2.2 Related Literature

Our work builds on and contributes to several streams of literature: on Bayesian decision models in investment under uncertainty, learning effects in investment games, externalities and complementarities in investment games, and the war of attrition.

Jensen (1982) was one of the first to apply sequential Bayesian decision models to investment decisions under uncertainty when he examined technology adoption under uncertain profitability. McCardle (1985) and Ulu and Smith (2009) studied the problem of technology adoption coupled with exit decisions when it is costly to acquire information about the technology's profitability. Using the continuous-time model of Shiryaev (1967) for Bayesian sequential decisions, Ryan and Lippman (2003) investigated the exit decision of a firm operating a project with uncertain underlying profitability. Using a similar framework, Kwon and Lippman (2011) examined the problem of a firm facing a choice between exit and expansion of a pilot project with uncertain profitability. These papers examine a single decision maker's problems. In contrast, this paper investigates investment decisions in a duopoly under uncertainty.

Another strand of literature has examined the role of externalities in investment games involving competing firms. Dybvig and Spatt (1983) and Katz and Shapiro (1986) viewed technology adoptions as providing complementarities to other firms. Nielsen (2002) studied a duopoly stochastic entry game in which the return on an investment depends on the number of firms in the market, through positive or negative externality. Femminis and Martini (2011) studied a similar stochastic entry game where profit improvement spills over from the leader to the follower at a Poisson time. Mamer and McCardle (1987) studied a Bayesian technology adoption game with positive or negative externalities, but their model separates the technology adoption stage from the the product launch and competition stage. Weeds (2002) studied an extreme form of first mover advantage in a winner-takes-all game of R&D competition. In her model the economic value of the patent follows a stochastic process and technological success is random. She found that investments are delayed in a symmetric equilibrium than in a cooperative equilibrium because firms hold back on investing for fear of starting a patent race.

When multiple firms consider similar investment decisions under uncertainty, they can learn from the investment decisions of the competing firms. A follower can learn from the investment decisions the performance of a leader, which gives firms an incentive to delay their investments. Such behavior reflects a war of attrition, which was first introduced by Smith (1974) and has subsequently seen widespread application in economics and game theory, particularly in the context of investment games. Kapur (1995) studied how the adoption decisions of other firms facilitate learning in a game of technology adoption between multiple players whose private payoffs are independent of the technological progress of other firms. Hoppe (2000) also studied a duopoly game of new technology adoption and showed that when the probability of success is low it results in a war of attrition because information externalities delay adoption.

In contrast to most papers on investment games, Décamps and Mariotti (2004) incorporated both externalities and Bayesian learning in a single model. They consider the investment decisions of two firms with respect to a project with unknown profitability, where the firms have private information about the cost of investment. The follower learns about the profitability by observing the leader's performance, and the resulting game is a war of attrition, analogous to the one that we identify, for which they find a unique symmetric equilibrium. Although Décamps and Mariotti (2004) is closely related to our paper in that they also study the impact of learning on a game of investment with externality, there are some notable differences from our paper. Their focus is on the interplay of the information externality and private information on costs while we focus on the interplay of externality and learning. Another important difference is that their model assumes that the leader's payoff is independent of the follower's time of investment once the leader-follower relationship has been established, and consequently the leader's payoff is independent of the follower's learning rate. In contrast, in our model, the leader's payoff depends on the follower's time of investment, which drives our main results.

Another paper that incorporated both externalities and Bayesian learning is Thijssen et al. (2006), which studied a preemption or an attrition equilibrium in a game of competitive investment with Bayesian learning about the profitability of the project. Although their model is similar to ours, it assumes that the leader's investment immediately reveals the true profitability to the follower, and hence, the follower's learning process is not incorporated. Hence, their model is not designed to address the question that we investigate.

Choi (1997) also studied a technology adoption process where there is an interplay between informational externalities and payoff interdependency through network externalities. However, his study was focused on the description of a herd behavior through a model of sequential technology choice between two new technologies. Frisell (2003) developed a market entry model in which payoff externalities and informational externalities coexist, and he found that stronger payoff externalities weaken the second-mover advantage and reduce the delay to market. In his model, each firm receives a private signal regarding the market demand and enters the market only if the market demand is favorable. Due to the information asymmetry between the firms, one firm's entry is considered a favorable signal for the other firm as well, and hence it causes an information spillover. In contrast, the information externalities in our model arise from Bayesian learning based on observing the other firm's profit streams, and our focus is on examining the combined impact of learning and externalities on equilibrium strategies.

2.3 The Game of Externality and Bayesian Learning

We consider two firms indexed as $i \in \{1, 2\}$, and we use j as an index to denote the opponent of firm i. Each firm has a one-time irreversible option to make an irreversible investment to enter a new market with unknown demand. The investments made by the two firms have mutually positive or mutually negative externalities. The time-averaged market demand can be either high or low, but neither firm knows the true state of the demand. If one firm enters the market first, then the other firm can observe its performance and learn about the true state of the market demand.

To formulate the game, we specify the strategy space, the payoff function, and the objective of each firm. Let $T_i \in [0, \infty]$ denote firm i's time of investment. Then $(T_1, T_2) \in [0, \infty] \times [0, \infty]$ is the strategy profile of the game. In this section, we assume without loss of generality that firm 1 is the leader and firm 2 is the follower, so that $T_1 \leq T_2$. We let $V_i(p; T_1, T_2)$ denote the *payoff* (defined as the expected cumulative discounted profit) for firm i given a strategy profile (T_1, T_2) conditional on the prior probability p (the initial belief of the firms) that the market demand is *high*. We define $\tau_2 = T_2 - T_1$, which represents the elapsed time between the leader's investment time T_1 and the follower's investment time T_2. Let $X = \{X_t : t \in [T_1, T_2]\}$ denote the process of the leader's cumulative profit before the follower invests, and let $r > 0$ denote the discount rate for both firms. We model the process X as a Brownian motion that satisfies

$$dX_t = \mu dt + \sigma dW_t \quad \text{for} \quad t \in [T_1, T_2] \,,$$

where $\sigma > 0$ is the noise level of the leader's income stream before the follower invests, and the drift μ represents the time-averaged profit per unit time. The process W_t is a Wiener process that represents the white noise in the profit stream. The true value of μ is unknown, but it is publicly known to be either h, if the demand is high, or ℓ, if the demand is low. We assume that both firms share the same prior belief about μ.

We assume that each firm wants to maximize its expected cumulative discounted profit. Using the notation $E^p[\cdot]$ for the expectation conditional on the prior probability p, we express the objective function $V_i(p; T_1, T_2)$ for each firm i as follows:

$$\begin{aligned} V_1(p; T_1, T_2) &= e^{-rT_1} E^p \left[-k + \int_0^{\tau_2} e^{-rt} dX_t + e^{-r\tau_2} \hat{U}_L \right] \,, \qquad (2.1) \\ V_2(p; T_1, T_2) &= e^{-rT_1} E^p \left[e^{-r\tau_2} (\hat{U}_F - k) \right] \,, \end{aligned}$$

where k is the upfront cost of investment for each firm. The random variables $\hat{U}_L$ and $\hat{U}_F$, defined in (2.2) and (2.3) below, respectively denote the leader's and follower's expected cumulative discounted incomes after the follower invests, conditional on the value of μ. Each firm's objective is to maximize its objective function by choosing the optimal time T_i given its opponent's strategy T_j.

We first consider the case where $\tau_2 > 0$, or equivalently, $T_2 > T_1$. For notational convenience, we use an index $I \in \{L, F\}$ to denote the role of each firm; $I = L$ represents the leader, and $I = F$ the follower. In this case (i.e., $T_2 > T_1$), the random variable $\hat{U}_I$ is given as follows:

$$\hat{U}_I \equiv E\left[\int_0^\infty \mu(1+\alpha_I)e^{-rt}dt + \int_0^\infty e^{-rt}\sigma_I dW_t^I|\mu\right] = \frac{\mu}{r}(1+\alpha_I) \quad \text{if } T_1 < T_2\,. \tag{2.2}$$

Here $\hat{U}_I$ is essentially the present value of a perpetual income of $\mu(1+\alpha_I)$ per unit time with a discount rate r. After the follower invests at time T_2, the income for the role I during an infinitesimal time dt is given by $\mu(1+\alpha_I)dt + \sigma_I dW_t^I$. The processes W_t^I is a Wiener process that represents the white noise in the income streams. For instance, if $\alpha_I > 0$ ($\alpha_I < 0$) for $I \in \{L, F\}$, then positive (negative) externality exists between the investments of the two firms (as the externality from each firm's investment will have a similar directional impact for the other firm, we assume that the signs of α_L and α_F coincide). Mixed signs of the externalities such as $\alpha_L > 0 > \alpha_F$ or $\alpha_F > 0 > \alpha_L$ are also possible.

In the second case, we consider simultaneous investment where $T_1 = T_2$. We assume that each player has an equal (50%) chance of being the leader or the follower, and hence the degree of externality is effectively $\alpha_S \equiv (\alpha_L + \alpha_F)/2$. Thus, $\hat{U}_L$ and $\hat{U}_F$ in the case of a simultaneous investment case can be expressed as follows:

$$\begin{aligned} \hat{U}_L &= \hat{U}_F = \hat{U}_S \\ &\equiv E\left[\int_0^\infty \mu(1+\alpha_S)e^{-rt}dt + \int_0^\infty e^{-rt}\sigma_S d(W_t^L + W_t^F)/2|\mu\right] \\ &= \frac{\mu}{r}(1+\alpha_S) \quad \text{if } T_1 = T_2\,. \end{aligned} \tag{2.3}$$

Except for Section 2.5.4, we make the following assumption in the rest of the paper:

Assumption 1 *$\alpha_I \in (-1, \infty)$ for $I \in \{L, F\}$, $\alpha_L \geq \alpha_F$, $0 < \frac{\ell}{r} < k < \frac{h}{r}$, and $0 < \frac{(1+\alpha_I)\ell}{r} < k < \frac{(1+\alpha_I)h}{r}$ for each I.*

This assumption implies that investment without learning would be profitable when $\mu = h$ and unprofitable when $\mu = \ell$. If $\alpha_F \leq -1$, then there is no incentive for the follower to invest, and the problem becomes trivial. Hence, we assume $\alpha_I \in (-1, \infty)$ for $I \in \{L, F\}$. The condition $\alpha_L \geq \alpha_F$ is based on the assumption of the first-mover advantage commonly observed in competitive contexts. In Section 2.5.4, we consider the case of a second-mover advantage.

Next we construct the Bayesian updating process for the posterior probability of $\mu = h$ for time $t \in (T_1, T_2)$, when the follower observes the leader's profit stream and learns about the market demand. Assume that X_t and μ belong to the same probability space $(\Omega, \mathcal{F}, \mathcal{P})$. Let $\{\mathcal{F}_t^X : t \geq 0\}$ denote the natural filtration with respect to the observable cumulative profit process X. We assume that the two firms share common prior and posterior probabilities concerning the profitability. Let $P_t = \mathcal{P}^p(\mu = h \mid \mathcal{F}_t^X) = E^p[\mathbf{1}_{\{\mu=h\}} \mid X_t]$ denote the posterior probability of $\mu = h$ at time t, conditional on the initial prior probability p (here $\mathbf{1}_{\{.\}}$ is the indicator function). In particular, if $T_1 = 0$ and $X_0 = 0$, then P_t can be expressed in terms of X_t and t as follows:

$$\begin{aligned} P_t &= \frac{\mathcal{P}[\{\mu = h\} \cap \{X_t\} | P_0 = p]}{\mathcal{P}[\{\mu = h\} \cap \{X_t\} | P_0 = p] + \mathcal{P}[\{\mu = \ell\} \cap \{X_t\} | P_0 = p]} \\ &= \frac{p \exp\{-\frac{(X_t - ht)^2}{2\sigma^2 t}\}}{p \exp\{-\frac{(X_t - ht)^2}{2\sigma^2 t}\} + (1-p)\exp\{-\frac{(X_t - \ell t)^2}{2\sigma^2 t}\}} \\ &= \left[1 + \frac{1-p}{p} \exp\left\{-\frac{(h-\ell)}{\sigma^2}\left[X_t - \frac{h+\ell}{2}t\right]\right\}\right]^{-1} , \end{aligned}$$

which can be derived from Bayes' rule (Peskir and Shiryaev 2006, p. 288) and the fact that $X_t - \mu t = \sigma W_t$ is normally distributed with mean zero and variance $\sigma^2 t$. Furthermore, the process $P = \{P_t : t \geq 0\}$ can be shown (Peskir and Shiryaev 2006, pp. 288-289) to be the unique strong solution to the stochastic differential equation:

$$dP_t = P_t(1 - P_t)\frac{h - \ell}{\sigma} d\hat{W}_t \text{ , where } \quad \hat{W}_t = \frac{1}{\sigma}\left(X_t - \int_0^t E[\mu \mid \mathcal{F}_s^X] ds\right) .$$

Here $\hat{W}_t$ is an observable Wiener process constructed purely from the observable process X. Lastly, note that P_t is defined only within the interval $[T_1, T_2]$, i.e., before the follower invests. Once the follower invests, it has no incentive to learn about the true value of μ. Since σ is the amplitude of the noise, the follower learns more quickly if $1/\sigma$ is higher (Bergemann and Valimaki, 2000). Thus, for the remainder of the paper, we use $\beta \equiv 1/\sigma$ to represent the *rate of learning.*

We are now in a position to express the payoff functions in terms of P_t.

Before the leader invests, neither firm receives any profit stream and therefore neither firm receives any information with which to update the posterior probability. Thus the probability of $\{\mu = h\}$ coincides with p for all $t \in [0, T_1]$. Further, if $T_1 = \infty$ and $T_2 = \infty$, then we have $V_i(p; \infty, \infty) = 0$ for both $i = 1, 2$ because there is no profit when neither firm invests. Next, for notational convenience, we define

$$m(p) \equiv E^p[\mu] = hp + \ell(1-p) ,$$

so that $E^p[\mu|\mathcal{F}_t^X]$ can be expressed as $m(P_t)$. Then we obtain the expressions for $V_i(p; T_1, T_2)$ in terms of the process P when $\tau_2 = T_2 - T_1$ is a stopping time. For ease of presentation, we consider the cases $\tau_2 > 0$ and $\tau_2 = 0$ separately; in Proposition 1, we show that the follower's optimal policy reduces to one of these two cases.

First, let us consider the case where $\tau_2 > 0$, or equivalently, $T_2 > T_1$. The leader's payoff reduces to

$$\begin{aligned} V_1(p; T_1, T_2) &= e^{-rT_1} E^p \left[\left(\frac{\mu}{r} - k \right) + \alpha_L \frac{\mu}{r} e^{-r\tau_2} \right] \\ &= e^{-rT_1} \left\{ \frac{1}{r} m(p) - k + \frac{\alpha_L}{r} E^p[e^{-r\tau_2} m(P_{\tau_2})] \right\} , \end{aligned} \tag{2.4}$$

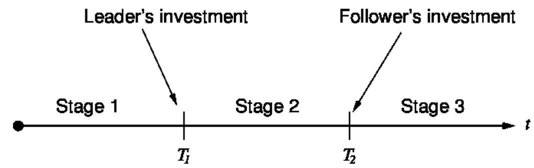
where we have used the equality $E^p[\mu e^{-r\tau_2}] = E^p[E^p[\mu e^{-r\tau_2}|\mathcal{F}_{\tau_2}^X]]$. Note that the term $m(p)/r$ is the expected value of μ/r, which is the cumulative discounted stream of time-averaged profit μ per unit time. The term $\alpha_L E^p[e^{-r\tau_2} m(P_{\tau_2})]/r$ is the expected value of the additional profit from the follower's investment at time $T_1 + \tau_2$. Similarly, the payoff to the follower is given by

$$\begin{aligned} V_2(p; T_1, T_2) &= e^{-rT_1} E^p \left[\left(\frac{\mu}{r}(1+\alpha_F) - k \right) e^{-r\tau_2} \right] \\ &= e^{-rT_1} E^p \left\{ e^{-r\tau_2} \left[(1+\alpha_F) \frac{m(P_{\tau_2})}{r} - k \right] \right\} . \end{aligned} \tag{2.5}$$

Here the term $\frac{\mu}{r}(1 + \alpha_F)$ is the cumulative discounted stream of the average profit $\mu(1 + \alpha_F)$ per unit time.

Second, let us consider the case of simultaneous investment, or $\tau_2 = 0$ (i.e.

Figure 2.1: The three-stage game.



$T_2 = T_1$), in which firms 1 and 2 obtain identical payoffs:

$$V_1(p; T_1, T_2) = V_2(p; T_1, T_2) = e^{-rT_1} E^p[\hat{U}_S - k] = e^{-rT_1} \left[(1 + \alpha_S) \frac{m(p)}{r} - k \right] ,$$

where $(1 + \alpha_S)m(p)/r$ is the expected value of the cumulative discounted stream of profit $(1+\alpha_S)\mu$ per unit time, which originates from the assumption that each firm has an equal chance of being a leader or a follower.

In summary, our model can be viewed as a three-stage game (see Figure 2.1 and Table 2.1). The first stage is the time period $t < T_1$, i.e., before the first investment. The firms are simply waiting for the first investment to happen, and neither firm earns any profit stream in this stage, so the probability of the event $\{\mu = h\}$ remains constant. The second stage is the time period $t \in [T_1, T_2)$ in the case where $T_2 > T_1$ (the second stage is absent if $T_1 = T_2$). In this stage, the leader (firm 1) earns a cumulative profit stream X and the follower (firm 2) updates the posterior process P based upon the observed process X. In our model, the processes X and P are terminated at the end of the second stage, i.e., as soon as the follower invests at time T_2. The third stage is the period after the follower's investment, i.e., $t \geq T_2$. In this stage, neither firm actively updates the probability of the event $\{\mu = h\}$ because neither firm has any investment decision to make, and both firms earn their final profit streams in perpetuity. The most frequently used notations and mathematical symbols are provided in Table 2.2.

2.4 Classification of Equilibria

In this section, we consider the cases of both positive externalities ($\alpha_I > 0$ for $I \in \{L, F\}$) and negative externalities ($-1 < \alpha_I < 0$) with the constraint that $\alpha_L \geq \alpha_F$, to obtain pure strategy (Section 2.4.1) and mixed strategy

Table 2.1: The three-stage game.

<table>
<tr><th colspan="2"></th><th>Stage 1</th><th>Stage 2</th><th>Stage 3</th></tr>
<tr><td colspan="2">Probability of $\{\mu = h\}$</td><td>p (No evolution over time)</td><td>P_t evolves through Bayesian learning</td><td>No Bayesian learning takes place</td></tr>
<tr><td rowspan="2">Time-averaged profit stream</td><td>Leader</td><td>No profit stream</td><td>μ per unit time</td><td>$\mu(1 + \alpha_L)$ per unit time</td></tr>
<tr><td>Follower</td><td>No profit stream</td><td>No profit stream</td><td>$\mu(1 + \alpha_F)$ per unit time</td></tr>
</table>

(Section 2.4.2) subgame perfect equilibria.

2.4.1 Pure Strategy Subgame Perfect Equilibria

In the spirit of backward induction, we first obtain the follower's optimal policy and its associated payoff. As in Section 2.3, we suppose that firm 1 is the leader in the sense that $T_2 \geq T_1$ (although we also allow for the possibility of simultaneous investment). Once the leader invests at time T_1, the objective of firm 2 (the follower) is to maximize its payoff (given by (2.5)) with respect to the stopping time $\tau_2 = T_2 - T_1$. We let $V_F(p) \equiv \sup_{\tau_2 \geq 0} V_2(p; 0, \tau_2)$ denote the optimal payoff for the follower for $T_1 = 0$ and $T_2 \geq 0$.

To obtain $V_F(p)$ and the optimal τ_2, we utilize the well-established verification theorem (see, for example, Theorem 3(A) of Alvarez (2001)) that stipulates a number of sufficient conditions an optimal value function must satisfy.

Proposition 1 *(i) At time T_1, the follower's optimal payoff is given by $V_F(p) = \max\{\Pi_F(p), \Pi_S(p)\}$, where*

$$\Pi_F(p) = \begin{cases} \frac{\psi(p)}{\psi(\theta_F)} \left[(1+\alpha_F)\frac{m(\theta_F)}{r} - k \right] & \text{for } p < \theta_F \\ \frac{1}{r}(1+\alpha_F)m(p) - k & \text{otherwise} \end{cases}, \tag{2.6}$$

$$\Pi_S(p) = \frac{1}{r}(1+\alpha_S)m(p) - k\,, \tag{2.7}$$

and θ_F and $\psi(x)$ are defined by (A.1) and (A.3) in Appendix A. Furthermore, the follower's optimal policy is to invest immediately at T_1 if $\Pi_S(p) \geq \Pi_F(p)$ and to wait and invest as soon as P_t hits the upper threshold θ_F if $\Pi_S(p) \leq$

Table 2.2: Frequently used notations

Notation	Definition
k	Cost of investment
$m(p)$	$m(p) \equiv E^p[\mu] = ph + (1-p)\ell$
P_t	The posterior probability of $\mu = h$ at time t
p	The initial belief of the firms that the market demand is high
r	Discount rate
T_i	The strategy of firm i (firm i's time of investment)
$\hat{T}_i$	Firm i's stage-1-strategy in the mixed strategy game
$V_i(p; T_1, T_2)$	Payoff to firm i with a prior p given a strategy profile (T_1, T_2)
$V_F(p)$	The follower's optimal payoff
$V_L(p)$	The leader's equilibrium payoff
$V_M(p)$	Symmetric mixed strategy equilibrium payoff
α_L, α_F, α	The degree of externality
β	The rate of learning ($\beta \equiv \sigma^{-1}$)
γ	$\gamma \equiv \sqrt{1 + 8r\sigma^2/(h-\ell)^2}$
θ_c	The boundary between the preemption and the war of attrition regime
θ_F	The follower's optimal threshold of investment in stage 2 when $T_2 > T_1$
θ_0	$\theta_0 \equiv \lim_{\beta \to 0} \theta_F$
θ_L	The leader's equilibrium threshold
θ_S	The lower boundary of the region of simultaneous investment
$\mu \in \{h, \ell\}$	The time-averaged profit per unit time
$\Pi_L(p)$	The leader's payoff from an immediate investment
$\Pi_S(p)$	The payoff from simultaneous investment
σ	Magnitude of the noise
$\bar{\tau}_M(p)$	Inverse of the arrival rate of $\hat{T}_i$ in the mixed strategy equilibrium
τ_2	$\tau_2 \equiv T_2 - T_1$
τ_F	The follower's optimal stopping time in stage 2 when $T_2 > T_1$

$\Pi_F(p)$.

(ii) There exists $\theta_S \leq \theta_F$ *such that* $\Pi_S(p) > \Pi_F(p)$ *if and only if* $p > \theta_S$.

Proposition 1 establishes that the follower's optimal strategy is to invest at time $T_2 = T_1 + \tau^*$, where

$$\tau^* = \begin{cases} \tau_F \equiv \inf\{t > 0 : P_t \geq \theta_F\} & \text{if} \quad \Pi_F(p) \geq \Pi_S(p)\ , \\ 0 & \text{if} \quad \Pi_F(p) < \Pi_S(p)\ . \end{cases} \tag{2.8}$$

Here $\Pi_F(p)$ represents the optimal value function for the follower under the constraint $T_2 > T_1$, and $\Pi_S(p)$ is the value function for $T_1 = T_2$, i.e., from simultaneous investment. The functional form of $\Pi_F(p)$ for $p < \theta_F$ gives the payoff for waiting until P_t hits the threshold θ_F, while $\Pi_F(p)$ for $p \geq \theta_F$ gives the payoff for immediate investment. Proposition 1 asserts that when $\Pi_F(p) \geq \Pi_S(p)$, the follower's optimal policy is to invest as soon as P_t hits the optimal upper threshold θ_F given by (A.1) in Appendix A. This optimal policy can be understood as the intuitive notion that a follower begins to learn about the market demand once the leader invests, and it invests only when its profit prospect (posterior probability P_t of a high profitability) hits a sufficiently high value θ_F.

Intuitively, the optimal threshold θ_F of the follower's investment can be obtained as follows. Let us define $\tau_\theta = \inf\{t > 0 : P_t \geq \theta\}$ as the hitting time for some threshold θ. By the theory of stopping (Chapter 9 of Oksendal 2003), it is known that the random discount factor $e^{-r\tau_\theta}$ has the expected value

$$E^p[\exp(-r\tau_\theta)] = \frac{\psi(p)}{\psi(\theta)}\ . \tag{2.9}$$

This leads to $V_2(p; 0, \tau_\theta) = [(1 + \alpha_F)m(\theta)/r - k]\psi(p)/\psi(\theta)$ because the follower's payoff from investment at time τ_θ is $(1 + \alpha_F)m(\theta)/r - k$. The optimal threshold θ_F can be obtained from the necessary first-order condition $dV_2(p; 0, \tau_\theta)/d\theta = 0$.

In the next proposition, we obtain the leader's (firm 1's) best response T_1 conditional on the follower's optimal stopping time τ^* given by (2.8). Similar to the follower's payoff, let $V_L(p) \equiv \sup_{T_1} V_1(p; T_1, T_1 + \tau^*)$ denote the optimal payoff for the leader. We establish that there exists a critical value θ_L such that the leader invests immediately if $p \geq \theta_L$ and never invests if $p < \theta_L$.

Proposition 2 *Given the follower's time of investment τ^*, the leader's optimal payoff is given by*

$$V_L(p) = \max\{\Pi_L(p), 0\}\ , \tag{2.10}$$

$$\begin{aligned} \textit{where} \qquad \Pi_L(p) &= \frac{m(p)}{r} - k + \frac{\alpha_L m(\theta_F)\psi(p)}{r\psi(\theta_F)} \ \textit{for}\, p < \theta_S\ , & (2.11) \\ &= \frac{1}{r}(1+\alpha_S)m(p) - k \qquad \textit{otherwise}\,. & (2.12) \end{aligned}$$

The leader's best response is to invest at $T_1 = 0$ if $p \geq \theta_L$ and at $T_1 = \infty$ if $p < \theta_L$, where $\theta_L \in (0, \theta_S]$ is defined by

$$\theta_L = \inf\{p : \Pi_L(p) > 0\}. \tag{2.13}$$

The function $\Pi_L(p)$ represents the leader's payoff from an immediate investment when the follower is expected to invest at time τ^*. Note that $\Pi_L(p)$ can be negative, while $V_L(p)$ is non-negative because the leader would not invest when $\Pi_L(p)$ is negative. The right-hand side of (2.11) is the leader's payoff from an immediate investment when the follower is expected to invest after time τ_F. On the other hand, equation (2.12) is the payoff from investment when the follower is expected to invest at the same time. The intuition behind Proposition 2 is that the leader immediately invests if and only if its net payoff from investment exceeds zero; otherwise the leader never invests.

Next, we obtain the strategies in the pure strategy subgame perfect Nash equilibria.

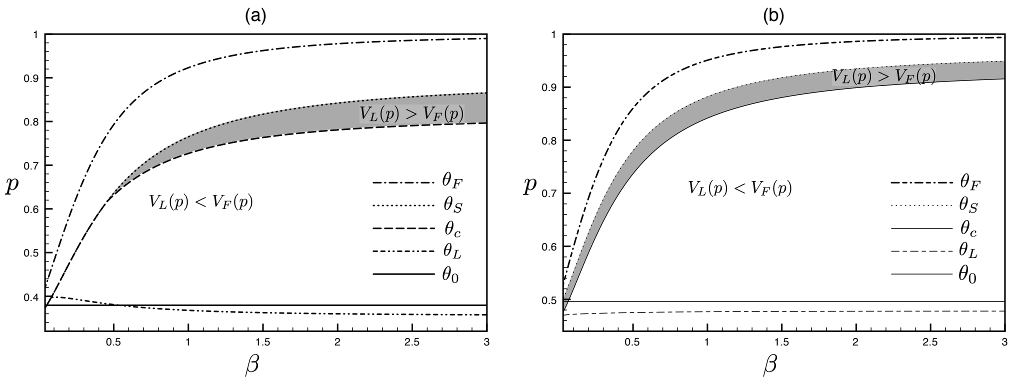
Proposition 3 *(i) If $p \in [0, \theta_L)$, neither player invests in the equilibrium.*

(ii) If $p \in [\theta_L, \theta_S)$, there are two pure strategy subgame perfect equilibria, each of which has a leader and a follower. The leader invests at $\tau_L = 0$, and the follower invests at a stopping time $\tau_F = \inf\{t > 0 : P_t \geq \theta_F\} > 0$, where θ_F is given by (A.1).

(iii) If $p \in [\theta_S, 1]$, then there exists a symmetric pure strategy Nash equilibrium in which players invest immediately at the same time at $t = 0$.

Under the pure strategy equilibria obtained in Proposition 3(ii) for $p \in [\theta_L, \theta_S)$, a leader and a follower exist. The role of the leader and the follower may be determined through the common expectation that both firms will

Figure 2.2: Values of θ_0, θ_L, θ_c, θ_S, and θ_F when $h = 2$, $\ell = 0.12$, $r = 0.1$, and $k = 10$. (a) shows the case $\alpha_L = 0.3$ and $\alpha_F = 0.2$, and (b) shows the case $\alpha_L = -0.02$ and $\alpha_F = -0.05$.



choose to play a predetermined role to attain a specific equilibrium (Fudenberg and Tirole 1991, p. 18). For example, the firm that is publicly known to be a more proactive investor will be the leader in this game. However, it is not essential for a natural leader and a follower to exist in our model, and the indeterminacy may result in a mixed strategy equilibrium. Under a mixed strategy equilibrium, each firm's strategy for its first investment is to invest at a *random* time with a probability distribution specified by its strategy. In this case, the leader is randomly determined, but the follower's best response reduces to that of the pure strategy equilibrium in Proposition 1. The details are discussed in Section 2.4.2.

Next, we report that both a war of attrition region ($V_F(p) > V_L(p)$) and a preemption region ($V_L(p) > V_F(p)$) can exist within the interval $[\theta_L, \theta_S)$.

Lemma 1 *There exists $\theta_c \in (\theta_L, \theta_S]$ at which $V_L(p) > V_F(p)$ for $p \in (\theta_c, \theta_S)$ and $V_F(p) > V_L(p)$ for $p \in (\theta_L, \theta_c)$, with the understanding that (θ_c, θ_S) is empty whenever $\theta_c = \theta_S$.*

The relative values of $\theta_0 \equiv \lim_{\beta \to 0} \theta_F$, θ_L, θ_c, θ_S, and θ_F are illustrated in Figure 2.2.

For the remainder of the paper, we call the interval $[\theta_L, \theta_c)$ a *war of attrition* (WA) region, the interval (θ_c, θ_S) a *preemption* (PE) region, and the interval $[\theta_S, 1]$ a *simultaneous move* (SM) region.

2.4.2 Mixed Strategy Subgame Perfect Equilibrium in the War of Attrition Region

In this subsection, we obtain mixed strategy equilibria in the WA region $[\theta_L, \theta_c)$ by employing the results of Hendricks et al. (1988) pertaining to a war of attrition in continuous time. Unlike in the previous sections, we do not assume that any one firm is predetermined to take the leader's role.

In the WA region, a mixed strategy profile is completely characterized by (i) each firm's stopping time for investment as a follower in stage 2 in case the other firm invests first, and (ii) each firm i's probability distribution of the random time $\hat{T}_i$ of investment for stage 1. Note that the random strategy $\hat{T}_i$ is applicable only in the first stage of the game. For example, if $\hat{T}_i < \hat{T}_j$, then firm i becomes the leader at time $\hat{T}_i$, at which point stage 1 is terminated. In this case, firm j's initial strategy $\hat{T}_j$ is never realized because it becomes the follower in stage 2. As shown in Section 2.4.1, a subgame perfect equilibrium requires that the follower's best response should be to invest at time τ_F given by (2.8). Hence, we focus on specifying $G_p^{(i)}(\cdot) : \mathbb{R}_+ \to [0,1]$, which denotes firm i's cumulative probability distribution function for time $\hat{T}_i$ given the prior probability p. In what follows, to keep the notation brief, we let $G_p^{(i)}(\cdot)$ denote the strategy of firm i with the understanding that the follower's time of investment is τ_F. By our convention, the strategy profile is represented by $(G_p^{(1)}, G_p^{(2)})$.

Here we adopt the convention that $G_p^{(i)}(\cdot)$ is right-continuous with left limits. We let $q_p^{(i)}(t) = G_p^{(i)}(t) - \lim_{s\uparrow t} G_p^{(i)}(s)$ denote the discontinuity of $G_p^{(i)}$ at time t. Intuitively, $q_p^{(i)}(t)$ represents the probability that firm i will invest exactly at time t. Given a strategy profile $(G_p^{(1)}, G_p^{(2)})$, the payoff for firm i is given by

$$\begin{aligned} V_i(p; G_p^{(1)}, G_p^{(2)}) &= E[\mathbf{1}_{\{\hat{T}_i<\hat{T}_j\}} e^{-r\hat{T}_i} V_L(p) + \mathbf{1}_{\{\hat{T}_i>\hat{T}_j\}} e^{-r\hat{T}_j} V_F(p) \\ &\quad + \mathbf{1}_{\{\hat{T}_i=\hat{T}_j\}} e^{-r\hat{T}_i} \Pi_S(p) | (G_p^{(1)}, G_p^{(2)})] \qquad (2.14) \\ &= \int_0^\infty \left\{ e^{-rt}[1 - G_p^{(j)}(t)] V_L(p) + \left[\lim_{u\uparrow t} \int_0^u V_F(p) e^{-rs} dG_p^{(j)}(s)\right] \right. \\ &\quad \left. + e^{-rt} \Pi_S(p) q_p^{(j)}(t) \right\} dG_p^{(i)}(t) \,. \qquad (2.15) \end{aligned}$$

In (2.14), the term $\mathbf{1}_{\{\hat{T}_i<\hat{T}_j\}} e^{-r\hat{T}_i} V_L(p)$ represents the payoff for the event that firm i happens to invest before firm j, in which case firm i expects $V_L(p)$ at time $\hat{T}_i$. Note that firm j will not invest at time $\hat{T}_j$ if $\hat{T}_i < \hat{T}_j$, because $\hat{T}_j$ is j's investment time conditional on j being the first one to invest; if i happens to have invested first, then j will invest at time $\hat{T}_i + \tau_F$, which is j's best response. Analogously, $\mathbf{1}_{\{\hat{T}_i > \hat{T}_j\}} e^{-r\hat{T}_j} V_F(p)$ represents the payoff for the event that firm j invests before firm i, in which case firm i's expected payoff is $V_F(p)$ at time $\hat{T}_j$ because firm i will invest at time $\hat{T}_j + \tau_F$. Lastly, $\mathbf{1}_{\{\hat{T}_i = \hat{T}_j\}} e^{-r\hat{T}_i} \Pi_S(p)$ represents the payoff for the event of simultaneous investment in the case where $\hat{T}_i = \hat{T}_j$.

Note that there are no dynamics or updating of the probability p until the time $\min\{\hat{T}_i, \hat{T}_j\}$, because the time $t < \min\{\hat{T}_i, \hat{T}_j\}$ belongs to stage 1 of the game (see Figure 2.1 and Table 2.1). Therefore, at time $\min\{\hat{T}_i, \hat{T}_j\}$, the leader's payoff is $V_L(p)$ and the follower's payoff is $V_F(p)$ without any dependence on $\min\{\hat{T}_i, \hat{T}_j\}$. It follows that the mixed strategy game for $t \le \min\{\hat{T}_i, \hat{T}_j\}$ reduces to a *static* game of a war of attrition in the sense that the state variable p has no dynamics before the first move from either player.

Equation (2.15) is the integral representation of (2.14) with respect to the investment times of the two firms. Given firm i's investment time t, the probability that i will be the leader is $1 - G_p^{(j)}(t)$ and the probability that both firms invest at t is $q_p^{(j)}(t)$, which explains the terms $e^{-rt}[1 - G_p^{(j)}(t)]V_L(p)$ and $e^{-rt}\Pi_S(p) q_p^{(j)}(t)$ within the curly brackets in (2.15). The term $\lim_{u \uparrow t} \int_0^u V_F(p) e^{-rs} dG_p^{(j)}(s)$ is the integral over the payoff in the event that firm j invests before time t. Now we use the payoff function described above to characterize the mixed strategy equilibria through the following proposition:

Proposition 4 *(i) For $p \in [\theta_L, \theta_c)$, a strategy profile $(G_p^{(1)}, G_p^{(2)})$ with $q_p^{(1)}(0) < 1$ and $q_p^{(2)}(0) < 1$ is a subgame perfect mixed strategy equilibrium if and only if the following two conditions are satisfied:*

(a) $(q_p^{(1)}(0), q_p^{(2)}(0)) \in [0,1) \times [0,1)$ *and* $q_p^{(1)}(0) q_p^{(2)}(0) = 0$.

(b) For both $i = 1$ *and* 2,

$$G_p^{(i)}(t) = 1 - \left[1 - q_p^{(i)}(0)\right] \exp\left[-t/\bar{\tau}_M(p)\right] \tag{2.16}$$

$$\text{where} \quad \bar{\tau}_M(p) = \frac{V_F(p) - V_L(p)}{rV_L(p)} . \tag{2.17}$$

(ii) Under the subgame perfect mixed strategy equilibrium, the payoff for firm i is given by

$$V_i(p; G_p^{(1)}, G_p^{(2)}) = q_p^{(j)}(0)V_F(p) + [1 - q_p^{(j)}(0)]V_L(p) \, . \tag{2.18}$$

Furthermore, the expected time to the first investment is given by

$$E\left[\min\{\hat{T}_1, \hat{T}_2\}\right] = \left[1 - q_p^{(1)}(0) - q_p^{(2)}(0)\right] \frac{\bar{\tau}_M(p)}{2} \, . \tag{2.19}$$

Note that the equilibria are parameterized by the initial probabilities of the firms' entry, i.e., $q_p^{(1)}(0)$ and $q_p^{(2)}(0)$. Note also that $q_p^{(i)}(t) = 0$ for all $t > 0$. In other words, one of the firms may strategically allocate a positive probability of being the leader at time $t = 0$, but once the time has elapsed beyond $t = 0$, the two firms' strategies are characterized by a continuous probability distribution $G_p^{(i)}(\cdot)$.

The non-zero values of $q_p^{(i)}(t)$ are confined to $t = 0$ for the following reason: In a mixed strategy equilibrium for all $t > 0$, each player i must be indifferent regarding to the time $\hat{T}_i$ of investment; otherwise, the mixing of all strategies $\hat{T}_i > 0$ would not be feasible in a mixed-strategy equilibrium. It implies that the equilibrium strategy of distribution $G_p^{(i)}(t)$ must be time-invariant in the sense that at any time $t > 0$, the game must look exactly the same as at any other time $t' > 0$ for any $t' \neq t$. If, however, $q_p^{(i)}(\tau) > 0$ for some deterministic time $\tau > 0$, then the time-invariance is broken because the game before τ and after τ look different to player j; in this hypothetical case, player j would prefer to invest after time τ due to the advantage of being the follower. Therefore, $q_p^{(i)}(t) = 0$ must hold for all $t > 0$ for a mixed-strategy equilibrium. The time-invariance is not necessary at $t = 0$ because the players do not need to consider a strategy before $t = 0$, so either $q_p^{(1)}(0) > 0$ or $q_p^{(2)}(0) > 0$ is permissible even in a mixed strategy equilibrium.

Note also that at least one of $q_p^{(1)}(0)$ and $q_p^{(2)}(0)$ must be zero. If firm 1 chooses $q_p^{(1)}(0) > 0$, it is taking the role of the leader with a probability of $q_p^{(1)}(0)$ at time $t = 0$, so the equilibrium probabilistically takes a characteristic of a pure strategy equilibrium. In this case, because there is a non-zero probability that firm 1 will be a leader, firm 2 has no incentive to place any positive probability of investing at time $t = 0$ since being a follower is more profitable than being a leader; it would rather first wait to see if firm 1 does invest at time $t = 0$. Hence, when $q_p^{(1)} > 0$, firm 2's best response is to set $q_p^{(2)} = 0$. By symmetry of the game, it follows that there is no equilibrium in which $q_p^{(1)}(0) > 0$ and $q_p^{(2)}(0) > 0$ at the same time.

If we focus on a completely symmetric equilibrium between the two firms, we can set $q_p^{(1)}(0) = q_p^{(2)}(0) = 0$. Let $V_M(p)$ denote the mixed strategy equilibrium payoff for $q_p^{(1)}(0) = q_p^{(2)} = 0$. It is worth noting that by (2.18), $V_M(\cdot)$ coincides with $V_L(p)$ because the payoff for investment at any time for either firm is identically given when the opponent plays the equilibrium strategy given by (2.16). In this case, each firm's investment time $\hat{T}_i$ is exponentially distributed with a rate $1/\bar{\tau}_M(p)$. Thus, we can interpret the hazard rate $1/\bar{\tau}_M(p)$ as the rate of each firm's investment at any moment in time. From the property of exponential distributions, the expected time of the first investment from *any* firm is given by $E[\min\{\hat{T}_1, \hat{T}_2\}] = \bar{\tau}_M(p)/2$. Thus, $\bar{\tau}_M(p)/2$ characterizes how long it takes for the first investment to occur. In general, due to equation (2.19), $\bar{\tau}_M(p)$ is the single most important quantity that characterizes the expected time to the first investment, even if $q_p^{(1)}(0)$ or $q_p^{(2)}(0)$ is nonzero. We defer the investigation of the comparative statics of $\bar{\tau}_M(p)$ until Section 2.5.

Lastly, we briefly comment on the possibility of a mixed strategy equilibrium in the PE region. Just as in the case of the WA game, if there is no natural leader, then a mixed strategy equilibrium makes more sense in practice even in a PE game. We refer the reader to Thijssen et al. (2012) for details on mixed strategy equilibria when $V_L(p) > V_F(p) > \Pi_S(p)$. Nevertheless, it is worth noting that even in a mixed strategy equilibrium, the first investment occurs at time $t = 0^+$ (as soon as the game begins) in a preemption game. Since the focus of our paper is on a war of attrition, we forgo further discussion of the preemption equilibria.

2.5 Impact of Learning

In this section, we explore the impact of learning on the equilibrium strategies and show that there exists an interplay between learning and externalities due to strategic interactions between the firms. In Section 2.5.1, we first study a benchmark model in which externalities do not exist while Bayesian learning does, and we obtain a benchmark result regarding the impact of learning. Then we return to the model presented in Section 2.4 to obtain the comparative statics for the firms' equilibrium strategies and payoff with respect to learning. We study the comparative statics of the follower's payoff and strategy in Section 2.5.2. In Section 2.5.3, we study the impact of learning on $V_M(\cdot)$ and $\bar{\tau}_M(p)$. (As noted in Section 2.4.2, the comparative statics of $\bar{\tau}_M(p)$ coincide with that of the expected time to the first investment.) Lastly, in Section 2.5.4, we study the case of a second-mover advantage.

2.5.1 Benchmark Model

In our benchmark model, we show that the expected time to the first investment monotonically increases[1] with the rate of learning. This agrees with the intuition that an increase in benefits to the follower delays the first investment in a war of attrition.

Suppose that each firm can invest once, but there is no externality between the two investments. Then the mixed strategy equilibrium reduces to the one considered in Section 2.4 with $\alpha_L = \alpha_F = 0$. In this case, the leader's payoff is independent of the follower's action, so we have $V_L(p) = m(p)/r - k$ for $p \geq \theta_L$, which does not depend on β or the follower's strategy. The follower's optimal payoff is given by

$$V_F(p) = E^p\left[e^{-r\tau_F}\left(\frac{\mu}{r} - k\right)\right] = \left(\frac{m(\theta_F)}{r} - k\right)\frac{\psi(p)}{\psi(\theta_F)}.$$

By Proposition 2 of Kwon and Lippman (2011), $V_F(p)$ increases with β, which is consistent with the intuition that a higher rate of learning improves the follower's profit. Further, from (2.17), it follows that $\bar{\tau}_M(p)$ increases with β, which is consistent with the intuition that a player will delay investment if the followers payoff improves with a higher rate of learning.

2.5.2 The Follower's Payoff and Strategy

In this section, we study the comparative statics of $V_F(\cdot)$, θ_F, and $E^p[\tau_F \mid \tau_F < \infty]$ with respect to β for non-zero α_L and α_F. (We do not study the

[1] Throughout the paper, we make no distinction between increasing and non-decreasing functions; similarly, we do not distinguish between decreasing and non-increasing functions.

comparative statics of $E^p[\tau_F]$ because $E^p[\tau_F] = \infty$ for any $p < \theta_F$.) We will use these results to provide intuitive explanations for the main results in 2.5.3 and 2.5.3.

We first establish the comparative statics of $V_F(\cdot)$ and θ_F. By Proposition 2 of Kwon and Lippman (2011), the value function and the investment threshold are both non-increasing in σ, so $V_F(p)$ and θ_F are non-decreasing with β. $V_F(p)$ increases with β because a higher learning rate improves the follower's payoff. Due to the improved value of waiting and learning before investment, the follower delays its investment as the learning rate increases. This intuition explains why θ_F increases with β. This result is consistent with the conventional result that the signal-to-noise ratio $(h-\ell)/\sigma$ increases the value of waiting as well as the upper threshold of investment (Bergemann and Valimaki, 2000).

Next, we obtain the form of $E^p[\tau_F \mid \tau_F < \infty]$:

Lemma 2

$$E^p[\tau_F \mid \tau_F < \infty] = \log\left(\frac{\theta_F}{p}\frac{1-p}{1-\theta_F}\right)\frac{4\sigma^2}{(h-\ell)^2}. \tag{2.20}$$

For notational convenience, we define

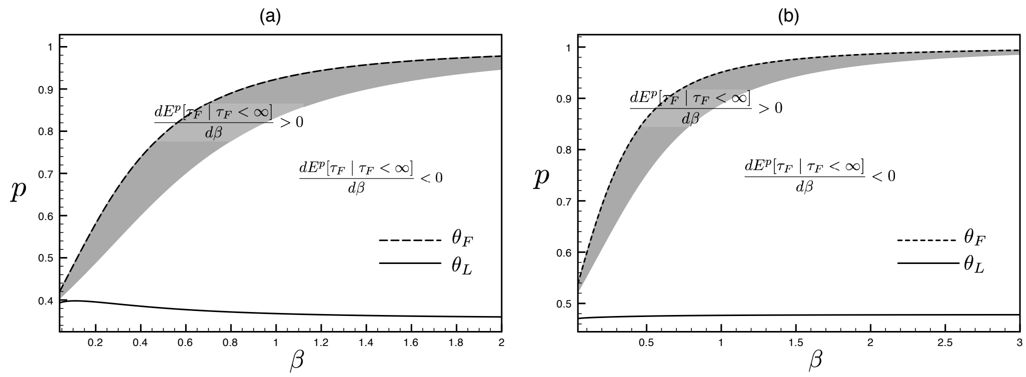
$$\theta_0 \equiv \lim_{\beta\to 0}\theta_F = \frac{kr-\ell(1+\alpha_F)}{(1+\alpha_F)(h-\ell)}\,. \tag{2.21}$$

By virtue of Proposition 2 of Kwon and Lippman (2011), $\theta_F > \theta_0$ for all values of β. Note also that θ_L may or may not be larger than θ_0, and there is no general ordering between θ_L and θ_0 as illustrated by Figure 2.2. Now we characterize the regions in which $E^p[\tau_F \mid \tau_F < \infty]$ increases or decreases with β.

Theorem 1 *For fixed $\beta \in (0,\infty)$, there exists $\hat{\theta}_F(\beta) \in (\theta_0, \theta_F)$ such that $E^p[\tau_F \mid \tau_F < \infty]$ decreases with β for $p \in (0, \hat{\theta}_F(\beta))$ and increases with β for $p \in (\hat{\theta}_F(\beta), \theta_F)$. For fixed $p \in (\theta_0, \theta_F)$, there exists $\hat{\beta}_F(p)$ such that $E^p[\tau_F \mid \tau_F < \infty]$ increases with β for $\beta \in (0, \hat{\beta}_F(p))$ and decreases with β for $\beta \in (\hat{\beta}_F(p), \infty)$. For fixed $p < \theta_0$, $E^p[\tau_F \mid \tau_F < \infty]$ decreases with β.*

A noteworthy feature of Theorem 1 is the single-crossing property: the sign change of $dE^p[\tau_F \mid \tau_F < \infty]/d\beta$ occurs at most once as β increases with

Figure 2.3: The impact of β on $E^p[\tau_F|\tau_F < \infty]$. The shaded (unshaded) area represents the region in which $E^p[\tau_F|\tau_F < \infty]$ increases (decreases) with β. (a) shows the case where $\alpha_F = 0.2$ and $\alpha_L = 0.3$, and (b) shows the case where $\alpha_F = -0.05$ and $\alpha_L = -0.02$. Here we set $h = 2$, $\ell = 0.12$, $r = 0.1$, and $k = 10$.



fixed p or as p increases with fixed β (see Figure 2.3 for a numerical illustration). Theorem 1 reflects the fact that an increase in the rate of learning has two countervailing effects on the follower's time to investment (Kwon and Lippman, 2011). On the one hand, such an increase may hasten the follower's investment because the follower acquires more meaningful information within a shorter time when the learning rate increases. On the other hand, an increase in the rate of learning may delay the follower's investment because of the increased value of waiting in order to collect more information. Either effect can be dominant, depending on the values of p and β.

When p is sufficiently close to θ_F, the comparative statics of $E^p[\tau_F|\tau_F < \infty]$ is strongly influenced by the comparative statics of θ_F. For instance, as the learning rate β increases, the investment threshold θ_F increases due to the increase in the value of waiting, so the follower's time to investment also increases if p is very close to θ_F. On the other hand, if p is sufficiently far away from θ_F, the comparative statics of θ_F has little effect on $E^p[\tau_F|\tau_F < \infty]$ since it takes a long time for P_t to reach the vicinity of θ_F. Furthermore, in this regime, the higher learning rate hastens the follower's investment because it takes less time to collect sufficient information to make a decision. For instance, if θ_F hypothetically did not have any dependence on β, then $E^p[\tau_\theta|\tau_\theta < \infty]$ can be shown to decrease in β. Thus, the comparative statics of the follower's time to investment changes as p increases from small values to the vicinity of θ_F.

The single-crossing property in p also explains the single-crossing property in β because θ_F strictly increases with β: For a fixed p, the threshold θ_F is sufficiently close to p for sufficiently low values of β. Thus, $E^p[\tau_F|\tau_F < \infty]$ increases with β for sufficiently low β. For sufficiently large values of β, θ_F takes a high value, so it is far from p. It follows that $E^p[\tau_F|\tau_F < \infty]$ decreases with β for large β. Thus, the comparative statics of $E^p[\tau_F|\tau_F < \infty]$ changes as β increases.

2.5.3 The Impact of Learning on the Mixed Strategy Equilibrium

In this section, we investigate the impact of learning on the equilibrium payoff $V_M(\cdot)$ and $\bar{\tau}_M(\cdot)$. We also supplement our analyses with selected numerical examples.

The dependencies of θ_F, θ_S, θ_L, and θ_c on β play an important role, so we will denote these dependencies as $\theta_F(\beta)$, $\theta_S(\beta)$, $\theta_L(\beta)$, and $\theta_c(\beta)$. We define the open set of pairs (p, β) in the WA region as

$$\mathcal{W} = \{(p, \beta) \in (0, 1) \times (0, \infty) : \theta_L(\beta) < p < \theta_c(\beta)\},$$

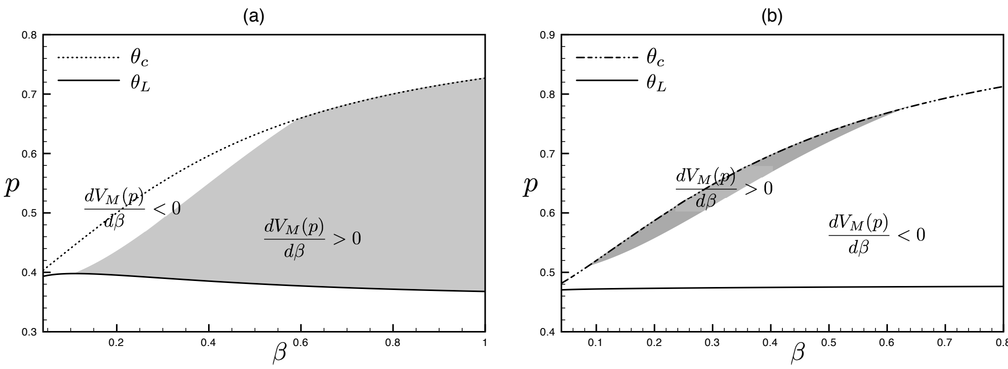
and we also define $\underline{\beta}_p \equiv \inf\{\beta : \theta_c(\beta) > p > \theta_L(\beta)\}$, which is the smallest value of β at which a given value of p belongs to the WA region. For a fixed value of p, if $\beta < \underline{\beta}_p$, then p may belong to the SM region or the PE region.

Comparative Statics of the Mixed Strategy Equilibrium Payoff

Now we obtain the comparative statics of $V_M(\cdot)$ and θ_L with respect to β under positive and negative externalities.

Theorem 2 *(i) Suppose $\alpha_I > 0$ for $I \in \{L, F\}$. For $p > \theta_0$, there exists $\hat{\beta}_M(p) \in [\underline{\beta}_p, \infty)$ such that $V_M(p)$ decreases with β for $\{\beta : (p, \beta) \in \mathcal{W}, \beta < \hat{\beta}_M(p)\}$ and increases with β for $\{\beta : (p, \beta) \in \mathcal{W}, \beta > \hat{\beta}_M(p)\}$. Furthermore, for a fixed β, there exists $\hat{\theta}_M(\beta) \in [\theta_L(\beta), \theta_c(\beta)]$ such that $V_M(p)$ increases with β if $\theta_L(\beta) < p < \hat{\theta}_M(\beta)$ and decreases with β if $\hat{\theta}_M(\beta) < p < \theta_c(\beta)$. If $\theta_L(\beta) < p < \theta_0$, then $V_M(p)$ increases with β for sufficiently large or sufficiently small values of β.*

Figure 2.4: The impact of β on $V_M(\cdot)$, θ_F, θ_L, and θ_c. The shaded (unshaded) area represents the region in which $dV_M(p)/d\beta > 0$ ($dV_M(p)/d\beta < 0$). (a) shows the case where $\alpha_L = 0.3$ and $\alpha_F = 0.2$, and (b) shows the case where $\alpha_L = -0.02$ and $\alpha_F = -0.05$. Here we set $h = 2$, $\ell = 0.12$, $r = 0.1$, and $k = 10$.



(ii) Suppose $\alpha_I < 0$ for $I \in \{L, F\}$. For $p > \theta_0$, there exists $\hat{\beta}_M(p) \in [\underline{\beta}_p, \infty)$ such that $V_M(p)$ increases with β for $\{\beta : (p, \beta) \in \mathcal{W}, \beta < \hat{\beta}_M(p)\}$ and decreases with β for $\{\beta : (p, \beta) \in \mathcal{W}, \beta > \hat{\beta}_M(p)\}$. Furthermore, for a fixed β, there exists $\hat{\theta}_M(\beta) \in [\theta_L(\beta), \theta_c(\beta)]$ such that $V_M(p)$ decreases with β if $\theta_L(\beta) < p < \hat{\theta}_M(\beta)$ and increases with β if $\hat{\theta}_M(\beta) < p < \theta_c(\beta)$. If $\theta_L(\beta) < p < \theta_0$, then $V_M(p)$ decreases with β for sufficiently large or sufficiently small values of β.

Figure 2.4 provides a numerical illustration of Theorem 2. The salient feature of this theorem is the single-crossing property of $dV_M(p)/d\beta$: the sign change of $dV_M(p)/d\beta$ occurs at most once, as p increases for fixed β or as β increases for fixed $p > \theta_0$. For fixed β, the sign change of $dV_M(p)/d\beta$ occurs when p crosses $\hat{\theta}_M(\beta)$, and for fixed $p > \theta_0$, the sign change of $dV_M(p)/d\beta$ occurs when β crosses $\hat{\beta}_M(p)$. We do not have analytical results for $p < \theta_0$, but numerical examples suggest that the sign of $dV_M(p)/d\beta$ does not change as β increases for fixed $p < \theta_0$.

Interestingly, the single-crossing property of the comparative statics of $E^p[\tau_F \mid \tau_F < \infty]$ (Theorem 1) provides an intuitive explanation for the single-crossing property of $dV_M(p)/d\beta$. First, we examine the case where $\alpha_I > 0$ for $I \in \{L, F\}$, as illustrated by Figure 2.4(a). For small values of β, Theorem 1 implies that an increase in β increases $E^p[\tau_F \mid \tau_F < \infty]$, which tends to decrease the leader's payoff $V_L(\cdot) = V_M(\cdot)$ because a delayed investment of the follower diminishes the leader's payoff due to the positive externality. This is reflected by Theorem 2(i) and Figure 2.4(a) for small values of β. For large values of β, Theorem 1 implies that $E^p[\tau_F \mid \tau_F < \infty]$ decreases (increases) with β for low (high) values of p. This tendency is exactly reflected in Theorem 2(i) and Figure 2.4(a) for large β: $V_L(\cdot) = V_M(\cdot)$ increases (decreases) with β for low (high) values of p. Then we examine the case where $\alpha_I < 0$ and find a similar result except that the sign of the comparative statics is opposite due to the opposite sign of α_L. This is illustrated in Figure 2.4(b).

Comparative Statics of $\bar{\tau}_M(p)$

Now we examine the impact of learning on $\bar{\tau}_M(p)$ and focus on the comparative statics of $\bar{\tau}_M(p)$ for large values of β. (The WA region shrinks to an almost null set in the limit $\beta \to 0$, so it is difficult to obtain meaningful analytical results in the small-β limit.)

Theorem 3 *(i) Suppose $\alpha_I > 0$ for $I \in \{L, F\}$.For sufficiently high values of p in the interval (θ_L, θ_c), $\bar{\tau}_M(p)$ increases with β. Furthermore, whenever $\beta > \beta_c$ for some $\beta_c > 0$, there exists $q(\beta) \in (\theta_L, \theta_c)$ such that $\bar{\tau}_M(p)$ decreases with β for $p \in (\theta_L, q(\beta))$ and increases with β for $p \in (q(\beta), \theta_c)$.*

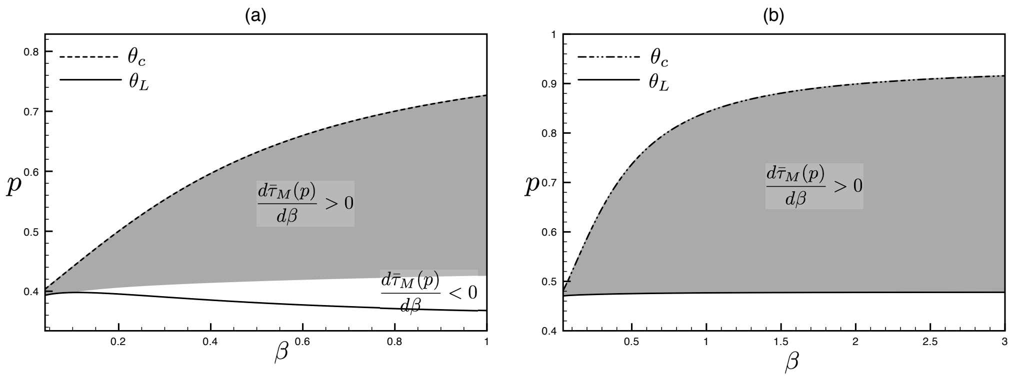
(ii) If $\alpha_I < 0$ for $I \in \{L, F\}$, then for each fixed value of p, there exists $\beta_c(p) > 0$ such that $\bar{\tau}_M(p)$ increases with β whenever $\beta > \beta_c(p)$.

Theorem 3 states that the comparative statics of $\bar{\tau}_M(p)$ also has a single-crossing property (changes the sign at most once). Most importantly, contrary to the naïve expectation that a higher rate of learning induces the firms to delay their first investment (based on the analysis of Section 2.5.1), Theorem 3 establishes that $\bar{\tau}_M(p)$ decreases with β under certain conditions.

Figure 2.5(a) numerically confirms Theorem 3. For positive externalities in Figure 2.5(a), $d\bar{\tau}_M(p)/d\beta$ changes its sign once as p increases. In contrast, even though there exists no general result for the intermediate values for negative externalities, Figure 2.5(b) suggests that $d\bar{\tau}_M/d\beta$ is always positive for p between θ_L and θ_c, which is consistent with our asymptotic results in Theorem 3.

In the presence of positive externalities, the parameter region (β, p) in which $E^p[\tau_F|\tau_F < \infty]$ decreases with β tends to coincide with the region in

Figure 2.5: The impact of β on $\bar{\tau}_M(\cdot)$. The shaded (unshaded) area represents the region in which $d\bar{\tau}_M(p)/d\beta > 0$ ($d\bar{\tau}_M(p)/d\beta < 0$). (a) shows the case where $\alpha_L = 0.3$ and $\alpha_F = 0.2$, and (b) is where $\alpha_L = -0.02$ and $\alpha_F = -0.05$. Here we set $h = 2$, $\ell = 0.12$, $r = 0.1$, and $k = 10$.



which $dV_M(p)/d\beta > 0$ and $d\bar{\tau}(p)/d\beta < 0$, i.e., for small p and large β. In the presence of negative externalities, we need to take into account another effect from the strategic behavior of the follower. On the one hand, an increase in β can increase (decrease) τ_F, which would increase (decrease) the value of the leader's investment as the follower delays (advances) the investment. On the other hand, an increase in β can also influence the follower's opportunistic behavior: If the follower learns faster, then it can selectively invest whenever the profit potential is high. Thus, the follower's selective action diminishes the leader's payoff when the profit prospect is good. As a result, a higher rate of learning can have a negative impact on the leader's payoff for high values of p. Hence, even though the leader's payoff can increase with β for higher values of p, the increase in $V_F(\cdot)$ tends to overshadow the increase in $V_L(\cdot)$, so we only observe $d\bar{\tau}_M(p)/d\beta > 0$. This phenomenon is illustrated in Figure 2.5(b).

Discussion on the Interplay of Externality and Learning

Overall, the impact of learning on the equilibrium payoffs and the time to the first investment is non-trivial. In the WA region, the follower has the opportunity to learn about the unknown profitability of the investment by observing the leader's performance. Therefore, it is reasonable to infer that firms will tend to delay their investments with an increased rate of learning because learning tends to benefit the follower. However, our results show that an increased rate of learning may improve the leader's payoff and hence hasten the firms' investments. In fact, we obtain a single-crossing property in the comparative statics of the payoff and the expected time to the first investment.

This finding is driven by the following two conditions: (a) *The value of the leader's investment decreases (increases) with the follower's time to investment under positive (negative) externalities.* (b) *The follower's time to investment depends on the learning rate β.* For example, in the case of positive externalities of our model, the follower's earlier investment improves the leader's payoff, so it satisfies condition (a). In addition, the follower's time to investment increases in β for high p and decreases in β for low p, which implies condition (b). The combined effect of these two conditions causes the leader's payoff to decrease with β for high p and increase with β for low p. Incidentally, condition (b) is satisfied by the benchmark model ($\alpha_L = \alpha_F = 0$) since Theorem 1 holds even when $\alpha_L = \alpha_F = 0$. However, as shown by Section 2.5.1, $\bar{\tau}_M(p)$ always increases with β because condition (a) is absent. We conclude that the combined effect of (a) and (b) leads to our results on the single-crossing property of the comparative statics of the payoff and the time to investment.

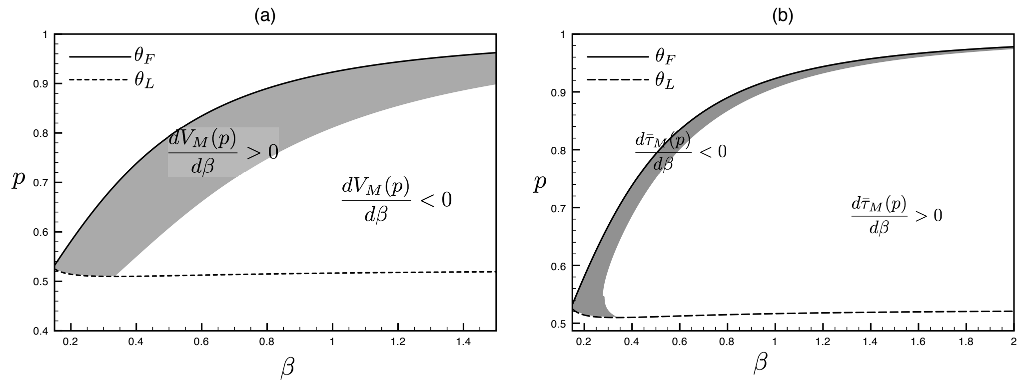
2.5.4 Case of Second-Mover Advantage

Next we consider an interesting case when $\alpha_F > 0 > \alpha_L$, which represents situations with second-mover advantage. For example, even though Apple was the first-mover in the smart-phone market, its position as the world's most profitable mobile phone maker was soon taken over by Samsung, which was the second-mover[2]. Furthermore, this was a situation with both learning and externalities. Samsung's smartphones benefited from Apple's pioneering efforts on the development of the smartphone and hence it enjoyed positive externalities from Apple's entry into the smart phone market. Moreover, by observing Apple's initial performance, Samsung was able to learn that there was very high demand in the smartphone market.

First, note that Theorem 1 always holds irrespective of the sign of α_F. Next, we establish the following:

[2]Samsung overtakes Apple as world's most profitable mobile phone maker, *The Guardian* (July 26, 2013)

Figure 2.6: The impact of β on $V_M(p)$ and $\bar{\tau}_M(p)$. Here we set $\alpha_L = -0.1$, $\alpha_F = 0.2$, $h = 2$, $\ell = 0.12$, $r = 0.1$, and $k = 10$.



Proposition 5 *If $\alpha_F > 0 > \alpha_L$, then $(\theta_L, 1)$ is the WA region.*

In other words, the regions of PE and SM do not exist because of the second-mover advantage. Furthermore, $V_M(\cdot)$ and $\bar{\tau}_M(\cdot)$ have no dependence on β for $p > \theta_F$ because $V_F(p) = \frac{1}{r}(1+\alpha_F)m(p) - k$ and $V_L(p) = \frac{1}{r}(1+\alpha_L)m(p) - k$ for $p > \theta_F$. Thus, we focus on the comparative statics within the interval $(0, \theta_F)$.

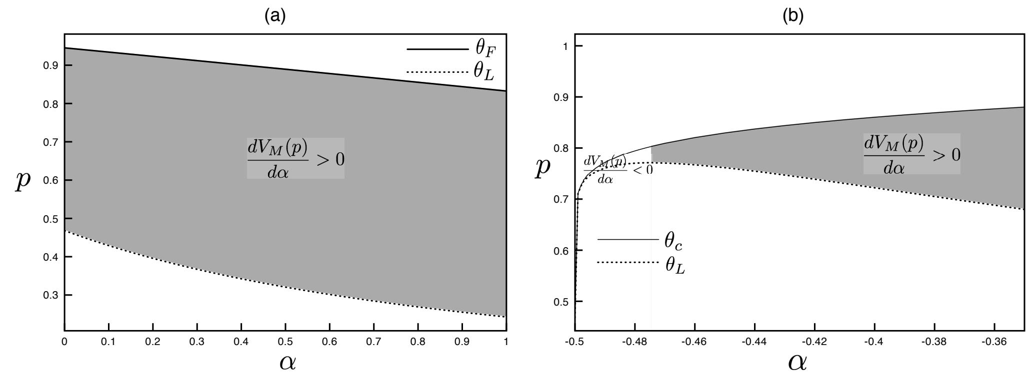
It is straightforward to prove that the statements of Theorem 2(ii) exactly apply for the case $\alpha_F > 0 > \alpha_L$. In other words, the sign of $dV_M(p)/d\beta$ changes at most once as p or β increases, and hence, the single-crossing property holds. This is illustrated in Figure 2.6(a). The proof is essentially identical to that for Theorem 2(ii), and hence omitted.

Lastly, we obtain the following comparative statics of $\bar{\tau}_M(p)$:

Theorem 4 *Suppose $\alpha_F > 0 > \alpha_L$. For sufficiently high values of p in the interval (θ_L, θ_F), $\bar{\tau}_M(p)$ decreases with β. Furthermore, there exist $\beta_c > 0$ and a function $p_c(\beta) \in (\theta_L, \theta_F)$ such that $\bar{\tau}_M(p)$ increases with β whenever $(\beta, p) \in (\beta_c, \infty) \times (\theta_L, p_c(\beta))$.*

This establishes that the comparative statics of $\bar{\tau}_M(p)$ changes in p at least once (an odd number of times) for large values of β. In fact, the numerical example in Figure 2.6(b) shows that $d\bar{\tau}_M(p)/d\beta$ changes its sign exactly once in p for sufficiently large β (for $\beta > 0.335$). For small values of β, the sign change of $d\bar{\tau}_M(p)/d\beta$ may not happen at all, as shown in Figure 2.6(b) for $\beta < 0.281$. For the intermediate values of β ($0.281 < \beta < 0.335$), Figure 2.6(b) shows that the sign change of $d\bar{\tau}_M(p)/d\beta$ happens twice as p increases. Even

Figure 2.7: The impact of α on $V_M(\cdot)$. The shaded (unshaded) area represents the region in which $dV_M(p)/d\alpha > 0$ ($dV_M(p)/d\alpha < 0$). Here we set $h = 2$, $\ell = 0.12$, $r = 0.1$, $k = 10$, $\beta = 1$.



if we account for regime of small to intermediate values of β, our numerical examples indicate that $d\bar{\tau}_M(p)/d\beta$ exhibits a single-crossing property as β changes from small to large values with a fixed value of p. Overall, Figure 2.6(b) demonstrates that the qualitative behavior of $d\bar{\tau}_M(p)/d\beta$ is the same as the case of the first-mover advantage in the sense that there exists at most one single boundary between the regions of $d\bar{\tau}_M(p)/d\beta > 0$ and $d\bar{\tau}_M(p)/d\beta < 0$.

In summary, the main effect of the second-mover advantage with different signs of α_F and α_L is that there is no PE region. This is because both the externalities and the opportunity of learning favor the follower's payoff. Nevertheless, the single-crossing property largely remains true for this case as well.

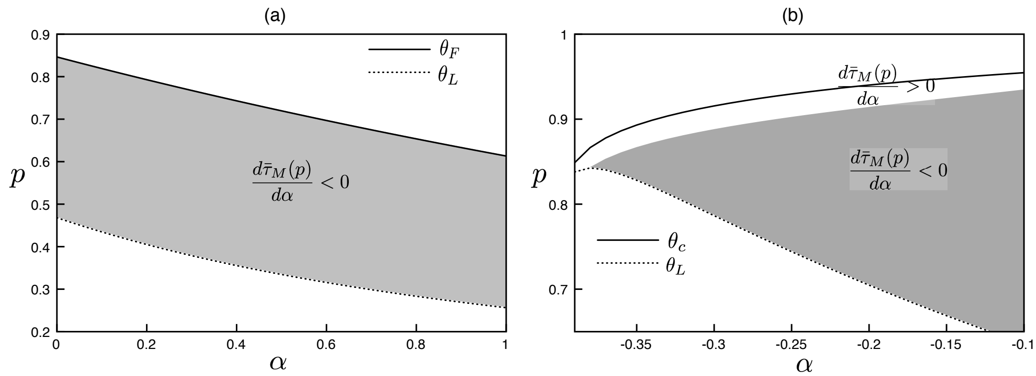
2.6 The Impact of Externality

In this section, we briefly discuss the impact of externality. For simplicity, we consider the case where $\alpha_L = \alpha_F = \alpha$ and illustrate examples of numerical comparative statics with respect to a single parameter α.[3] Here we remark that *an increase in the externality* means an increase in the value of α. This implies that when $\alpha < 0$, an increase in externality implies *a decrease in the magnitude of the externality* (a decrease in $|\alpha|$).

We first illustrate the comparative statics of $V_M(\cdot)$ in Figure 2.7. With

[3]The mathematical proofs of the main results illustrated in this section are available although they are not presented in this paper.

Figure 2.8: The impact of α on $\bar{\tau}_M(\cdot)$. The shaded (unshaded) area represents the region in which $d\bar{\tau}_M(p)/d\alpha < 0$ ($d\bar{\tau}_M(p)/d\alpha > 0$). Here we set $h = 2$, $\ell = 0.1$, $r = 0.1$, $k = 12$, and $\beta = 1$.



the positive externalities in Figure 2.7(a), $V_M(p)$ always increases with α. If $\alpha > 0$, then the equilibrium payoff $V_M(\cdot)$ increases with α because the leader's profit stream after the follower's investment is proportional to $(1 + \alpha)$, combined with the fact that the follower's expected time of investment $E^p[\tau_F \mid \tau_F < \infty]$ decreases with α. It follows that θ_L decreases with α. In contrast, if $\alpha < 0$, then an increase in α has two countervailing effects: On the one hand, an increase in α improves the leader's payoff after the follower invests. On the other hand, the strategic behavior of the follower diminishes the leader's payoff. More specifically, the follower tends to be discouraged from investing in the presence of negative externality; hence, if α increases, the follower is more encouraged to invest, and so the leader's payoff may decrease. Thus, if $\alpha < 0$, the comparative statics of θ_L or $V_M(\cdot)$ with respect to α are not clear a priori, based on intuitive reasoning alone. Figure 2.7(b) demonstrates the insight about the two countervailing effects of the negative externality. For the negative externalities in Figure 2.7(b), $V_M(p)$ is non-monotonic and may increase or decrease with α.

Next, Figure 2.8 provides a numerical illustration of the comparative statics for $\bar{\tau}_M(p)$. It illustrates that while an increase in positive externality encourages firms to invest earlier, the same is not necessarily true in the presence of negative externality. In the presence of the positive externality as in Figure 2.8(a), firms tend to invest earlier when α increases because greater externality definitely improves firms' payoff. In contrast, in the presence of the negative externality as in Figure 2.8(b), the time to the first investment does not necessarily decrease with α since $\bar{\tau}_M(p)$ increases with α near θ_c while it decreases with α near θ_L. For high values of p close to θ_c, due to the possibility of high profit, the follower has a greater incentive to invest as α increases, but the leader's payoff may not improve much as α increases because of the higher likelihood that the follower will invest. By the functional form of $\bar{\tau}_M(p)$ in (2.17), if the follower's payoff increases with α to a larger extent than does the leader's payoff, then it makes intuitive sense that $\bar{\tau}_M(p)$ increases with α. The intuition for p close to θ_L coincides with that for the case of positive externality.

Overall, the comparative statics results indicate that a higher degree of positive externality encourages firms to invest earlier. In contrast, the impact of negative externality is more nuanced.

2.7 Some Related Models

In this section, we briefly discuss two related models and check whether our main results hold.

2.7.1 Non-Zero Cost of Learning

We consider the case in which it is costly for the follower to collect information and learn the true market demand in stage 1. Let $V_i(p; T_1, T_2; k, c)$ denote firm i's payoff, where k is the cost of investment and c is the follower's per-unit-time cost of observing the leader's payoff in stage 1. For example, without loss of generality, suppose that firm 1 is the leader and firm 2 is the follower. Defining $k' \equiv k - c/r$, we obtain

$$\begin{aligned} V_1(p;0,\tau_2;k,c) &= E^p\left[-k + \int_0^{\tau_2} e^{-rt} dX_t + e^{-r\tau_2}\hat{U}_L\right] \\ &= -\frac{c}{r} + E^p\left[-k' + \int_0^{\tau_2} e^{-rt} dX_t + e^{-r\tau_2}\hat{U}_L\right] \\ &= V_1(p;0,\tau_2;k',0) - \frac{c}{r}\,, \\ V_2(p;0,\tau_2;k,c) &= E^p\left[\int_0^{\tau_2}(-c)e^{-rt}dt + e^{-r\tau_2}(\hat{U}_F - k)\right] \\ &= -\frac{c}{r} + E^p\left[e^{-r\tau_2}(\hat{U}_F - k')\right] \\ &= V_2(p;0,\tau_2;k',0) - \frac{c}{r}\,. \end{aligned}$$

Thus, the game with a non-zero c can be conveniently transformed into another game with $k' = k - c/r$ and its associated payoff reduced by c/r for each player. Thus, all the results of the previous sections continue to hold for this model, except that $\theta_L = \inf\{p : V_1(p;0,\tau_2;k,c) > 0\}$ takes a higher value than $\inf\{p : V_1(p;0,\tau_2;k',0) > 0\}$. We conclude that the follower's cost of collecting information does not alter the essential impact of learning and externalities.

2.7.2 Learning From a Public Signal

In some cases, the signal of the market demand is exogenous and public. Hence, it is useful to consider a model with learning from a public signal and to compare its results with our main findings. Let X denote the cumulative public signal that satisfies $dX_t = \mu dt + \sigma dW_t$ for $t \in [0,\infty)$ where $\mu \in \{h,\ell\}$ with $h > \ell > 0$, and μ is unknown. For example, X can be the cumulative profit stream from a closely related industry.

In stage 1, no one has invested, but the signal process X evolves, and the posterior probability P_t evolves as

$$P_t = \left[1 + \frac{1-p}{p}\exp\left\{-\frac{(h-\ell)}{\sigma^2}\left[X_t - \frac{h+\ell}{2}t\right]\right\}\right]^{-1}.$$

Suppose that firm 1 is the leader who invests at some time T_1. In stage 2, we assume that the leader's profit for the duration dt is simply ϕdX_t for some positive constant ϕ. This assumption ensures that the public signal is the only source of information regarding the quality of the market and that the leader's profit stream does not add any extra information.

Suppose that firm 2 invests at $T_2 \geq T_1$. Then in stage 3, we assume that both the leader and the follower earn $\phi(1+\alpha)dX_t$ for some α that represents the degree of externality.

Based on the model assumptions, the payoffs to the firms are given by the following:

$$\begin{aligned} V_1(p; T_1, T_2) &= E^p \left[\int_{T_1}^{T_2} e^{-rt} \phi dX_t + \int_{T_2}^{\infty} e^{-rt} \phi(1+\alpha) dX_t \right] , \\ V_2(p; T_1, T_2) &= E^p \left[\int_{T_2}^{\infty} e^{-rt} \phi(1+\alpha) dX_t \right] . \end{aligned}$$

We do not present a detailed analysis, but it can be shown that in this model of a purely public signal, a war of attrition never happens. This is because in such a model the signal is already being generated and neither player would need to wait for the other to invest first to learn about the market demand. In contrast, in the model of Section 2.3, the war of attrition occurs because each firm wants the other to invest first and start producing the signal, which generates a mixed strategy equilibrium in which both wait for the other to invest first. Since a war of attrition does not take place with purely public signals, we conclude that this model lies outside the scope of this paper.

2.8 Conclusions

Investments in new unproven projects in competitive situations are fraught with uncertainty. In general, returns from such investments are governed by positive or negative externalities from investments made by competing firms. Moreover, firms often have the opportunity to learn the potential value of investing in similar projects by observing the performance of their competitors' investments. In this paper, we investigate the impact of learning and externalities on equilibrium investment strategies. We find that due to the strategic interactions, externalities and learning opportunities have counterintuitive effects on investment strategies and on the time to the first investment. In particular, a single-crossing property in p and β is exhibited by the comparative statics of the payoff and the expected time to investment with respect to the rate of learning. Thus, depending on the values of p and β, a higher learning rate may hasten the first investment, which is in contrast to the conventional result from the benchmark model without externalities.

Overall, our results suggest that firms facing entry into an unproven market need to consider the strategic effects arising from the interplay between externalities and learning. In particular, the effect of externalities needs to be incorporated when modeling a competitive investment problem with learning opportunities as a war of attrition.

CHAPTER 3

R&D COMPETITION WITH SPILLOVERS AND UNCERTAIN COMPLETION TIMES

3.1 Introduction

Investment in research and development (R&D) projects, particularly in high-technology sector, entails many challenges. There is significant uncertainty in the completion times of R&D projects in high-tech industries (Lynn et al., 1996). Furthermore, in high-tech industries, there is significant spillover of knowledge to competitor firms, which will potentially diminish the innovator's payoff (Carlino and Carr, 2013). In the context of investments in R&D projects in the presence of knowledge spillovers and uncertain completion times, our paper addresses the following questions: (1) What is the impact of natural spillover upon innovative firms' payoffs? (2) Does an innovative firm have incentives to unilaterally increase the spillover to its competitor?

Throughout the paper, we make distinctions between "natural spillovers" and "controllable spillovers". The natural spillovers are the spillovers that naturally occur between firms, and they are often generated by some intrinsic characteristics of the resulting product or service from the innovation. For instance, when iPhone was launched in 2007, Google was able to assimilate similar features (look and feel) of iOS to develop Android OS by simply observing its appearance. To some extent, the degree of natural spillovers can be understood as a measure of intrinsic imitability of an innovation: high (low) degrees of natural spillovers occur because of high (low) degrees of imitability. Most often, the extent of these spillovers are symmetric between firms, and the direction of the spillovers is not pre-determined: regardless of the size or the capability, any firm can receive (send) spillovers from (to) other firms. Because imitability is an internal factor of innovations, a firm cannot change the degree of natural spillover unilaterally. In contrast, a firm can unilaterally change "controllable spillovers" from itself to other firms. For instance, firms can patent their innovations to decrease the spillover, or disclose some information about their innovations to other firms to increase spillover.

The resource-based theory suggests that imitability diminishes the durability of a firm's competitive advantage which is obtained by superior innovative resources (Barney, 1991). Hence, one might expect that natural spillovers tend to diminish an innovative firm's performance (profit) as well. Furthermore, conventional wisdom also suggests that spillovers should be undesirable from an innovator's standpoint (Dierickx and Cool, 1989). Hence, it behooves an innovative firm to minimize the controllable spillovers through such measures as patent protection and maintaining high wages to retain employees. However, this line of reasoning requires careful scrutiny. For instance, Pacheco-de Almeida and Zemsky (2012) provide a counterexample:

> "In the early 1980s, ..., AMD had access to Intel's 286 chip technology. Hence, AMD simply waited until Intel's release of 80286 chip, and developed its own chip AM286 based on Intel's product specifications in 1982. However, in 1984 Intel refused to share the design of the next generation "386" chip with AMD. Such a change in Intel's policy forced AMD .... adopted a strategy to design products in competition with Intel. ....AMD's change in its strategy diminished Intel's financial performance. "

In fact, as shown by Pacheco-de Almeida and Zemsky (2012) in their theoretical model, the impact of controllable spillovers (unilateral disclosure of knowledge) can be more complicated than what a naive intuition might suggest. Our paper extends this question and addresses the impact of natural spillover on an innovative firm's performance.

Natural spillovers can have two countervailing effects on an innovative firm's payoff: Increased spillovers allow competitors to be more competitive in the market by accelerating their paces of R&D, but increased spillovers also can reduce competition pressure by encouraging competitors to be followers who simply imitate the technology. Furthermore, if there is uncertainty in the completion times of R&D, then even the more innovative firm may end up being the imitator who is the beneficiary of natural spillover. Hence, the ultimate impact of spillovers is particularly difficult to discern in the presence of technological uncertainty.

To answer our research questions, we investigate a game theoretic duopoly model to examine the impact of natural spillovers on R&D investment strategies when the R&D completion times are random. Each firm can choose when to initiate the R&D project. Furthermore, each firm can control their effort levels on its R&D, which determine the expected completion times. Because of the randomness of completion times, either firm may randomly complete its R&D before the other. We define the firm that first completes R&D to be the *innovator* and the other firm to be the *imitator.* By this definition, even the more innovative firm may end up being the imitator rather than the innovator due to uncertain completion times. After the innovator's completion of R&D, knowledge spillovers take place, which boost the pace of the imitator's R&D.

3.2 Related Literature

Our work builds on and contributes to three streams of literature: resource-based view of firms, investment under uncertainty, and R&D investment with spillovers. Our paper contributes to the literature on the resource-based view of firms by examining the impact of imitability. The resource-based view in strategy suggests that imperfect imitability is one attribute of those resources that possess the potential for sustained competitive advantage (Barney, 1991), because imitability diminishes the durability of a firm's competitive advantage. Hence, it is intuitive to expect that low degrees of imitability is desirable for firms to achieve superior performance. This intuitive expectation is also corroborated by empirical evidence which shows that imitability has negative impact on firms' performance. (See, for example, De Carolis, 2003). In contrast, the empirical study by Autio et al. (2000) reveals positive effect of imitability on firms' international sales growth. Autio et al. (2000) suggest that this positive effect is because suppliers, customers, and exchange partners are more likely to accept an imitable technology that is easy to understand and learn. In our paper, we focus on the dynamics between two competing firms, and our work complements the resource-based theory by showing that imitability may improve performance of innovative firms under certain conditions.

In the industrial organization literature, there is an extensive body of work that examines investment strategies under uncertainties. There are mainly three sources of uncertainties that have often been addressed in the context of technology investment. The first uncertainty comes from the technological obsolescence, which is often considered in technology adoption problems, in which firms obtain new technologies from others (Balcer and Lippman, 1984; Hoppe, 2002). The second one is uncertain profitability. This uncertainty concerns the uncertain market demand once the resulting product of the projects are launched in the market in an investment problem, or it concerns the uncertain quality of a new technology in technology adoption problems. This uncertainty can be resolved gradually by observing signals, such as profit streams of other firms who have invested in the same or similar projects; see, for example, Miltersen and Schwartz (2004),Décamps and Mariotti (2004), Canan and Smith (2013), and Kwon et al. (2015). Although they do not specifically address the problems of R&D investment, both Décamps and Mariotti (2004) and Kwon et al. (2015) consider strategic investments under uncertain value of projects. Canan and Smith (2013) consider a technology adoption problem with an unknown value of the technology. The third one, which mainly exists in the research stage in R&D projects (Reinganum, 1985), is the technological uncertainty concerning the consequence of the effort exerted in R&D projects. For instance, we model this uncertainty by considering the uncertain completion time of R&D projects as other precedent work on R&D race models (see, for example, Weeds, 2002, Reinganum, 1985, and Reinganum, 1989)

In the literature on R&D investment, some papers address knowledge spillovers (for example, D'Aspremont and Jacquemin, 1988) whereas others do not (for example, Loury, 1979, and Lee and Wilde, 1980). In the well-explored theoretical work on R&D investment with spillovers, both Kort et al. (2007) and Leung and Kwok (2013) characterize various Nash equilibria. Kort et al. (2007) consider deterministic completion times by endogenizing the time-to-build of R&D projects, and find that the asymmetry between the firms' R&D efficiency determines whether the equilibrium is a preemption game or a war of attrition. Leung and Kwok (2013) incorporate uncertain completion times and show the existence of a sequential, preemptive and simultaneous equilibrium. The two major differences between Leung and Kwok (2013) and our work are the payoff structure and the timing of spillover. First, the payoff structure considered by Leung and Kwok (2013) is characterized by "winner-takes-all", whereas our model assumes that both firms can extract profit from their innovations. Second, Leung and Kwok (2013) assume that mutual spillover that occurs before the discovery of innovation (completion of R&D), whereas we examine spillovers that occur after the first completion of R&D. Furthermore, we assume that only one firm (imitator) can receive spillovers from the other firm (innovator). The structure of payoffs and the timing of spillover that our paper considers is similar to those considered by Pacheco-de Almeida and Zemsky (2012).

Although the majority of work on R&D spillover (for example, Harhoff, 1996) shows that spillovers are not desirable for innovators, Pacheco-de Almeida and Zemsky (2012) and Pacheco-de Almeida and Zemsky (2007) show that an increase in unilateral spillover may be beneficial for the innovator. Pacheco-de Almeida and Zemsky (2007) and Pacheco-de Almeida and Zemsky (2012) investigate a game of two competing firms who decide the timing of resource development in the presence of spillover. Instead of assuming the predetermined sequence of investments as done in Pacheco-de Almeida and Zemsky (2007), the model examined by Pacheco-de Almeida and Zemsky (2012) allows the possibility that the follower (less capable firm) develops concurrently with the leader (more capable firm). Pacheco-de Almeida and Zemsky (2012) find that there always exists a critical degree of unilateral spillover at which the leader has an incentive to freely reveal knowledge (increase the degree of unilateral spillover) to the follower to induce it to begin resource development after the leader's completion of innovation. Our work reexamines firms' incentives to voluntarily increase spillover when technological uncertainty exists. More importantly, we mainly focus on the impact of natural spillovers, which are not present in Pacheco-de Almeida and Zemsky (2012), to examine the impact of imitability on firms' performance. Driven by different research questions, our model is distinct from Pacheco-de Almeida and Zemsky (2012)'s model in three aspects: (a) Our work separates the natural spillovers from controllable spillovers; (b) Instead of assuming that the more capable firm never invests later than the weaker rival, our model endogenizes firms' decisions on timing of investment. (c) Pacheco-de Almeida and Zemsky (2012) investigates the development process of innovation, whereas we focus on the research process that entails uncertain completion times. Hence, our model incorporates random completion times that allow for the possibility that the less efficient firm wins the R&D race.

3.3 Model

In this section, we present the game theoretic model that we investigate. We also classify the model parameter regimes into four distinct types that correspond to four classes of equilibria.

We consider two firms indexed $i \in \{1, 2\}$. Each firm is about to initiate an R&D project and produce a new technology in order to enter the same market. The firm that completes its R&D project first (*innovator*) enjoys a monopoly profit until the other firm (*imitator*) completes its own R&D project. Let j denote the index of the opponent of firm i. Each firm can initiate an R&D project at any point in time. Let $T_i^0 \geq 0$ denote the time at which firm i initiates an R&D project. The initiation times T_1^0 and T_2^0 are strategic choices of the firms. If $T_1^0 < T_2^0$, we call firm 1 the *leader* and firm 2 the *follower*. We also define *time-to-completion* $T_i \geq 0$ as the duration of firm i's R&D project. It follows that firm i completes its R&D project at $T_i^0 + T_i$.

Let $\tau_I = \min_{i \in \{1,2\}}\{T_i^0 + T_i\}$ denote the time at which the innovator completes its R&D project and $\tau_M = \max_{i \in \{1,2\}}\{T_i^0 + T_i\}$ denote the time at which the imitator completes its R&D project. In the time period $t \in [\min\{T_1^0, T_2^0\}, \tau_I)$, the time-to-completion of firm i's R&D is an exponential random variable with a rate λ_i, where λ_i is firm i's choice of effort level. Suppose firm i completes its R&D first. In the time period $t \in [\tau_I, \tau_M)$, the imitator (firm j) gains additional efficiency of R&D as a result of inter-firm knowledge spillover. Hence, firm j's arrival rate of the completion of R&D is enhanced by a factor of $s_i \in [1, \infty)$, which is the degree of spillover from firm i to firm j. In this time period, firm j's (imitator's) choice of effort level is μ_j, which does not have to coincide with λ_j. Then firm j's arrival rate of R&D completion is $s_i \mu_j$. We also assume a symmetric mutual spillover between the two firms, i.e., $s_i = s_j = s$ except in Section 3.4.2.

We assume that each firm i's primary strategy is the R&D initiation time T_i^0. We also assume that the decisions on (λ_i, μ_i) are made after the *strategy profile* $(T_i^0, T_j^0) \in [0, \infty) \times [0, \infty)$ is determined, because the time to initiate R&D is a corporate level strategic decision, whereas the effort levels are largely operational decisions. If $T_1^0 = T_2^0$, then (T_1^0, T_2^0) is called a *concurrent strategy profile*. If $T_i^0 < T_j^0$, then (T_1^0, T_2^0) is called an *imitative strategy profile*, in which case firm i is the *leader* and firm j is the *follower*. The objective of each firm is to maximize its expected cumulative discounted profit given the opponent's strategy.

A firm earns no profit before its completion of R&D. After τ_I, the innovator earns π_{10} per unit time until τ_M, whereas the imitator earns zero profit stream until τ_M. After τ_M, each firm receives π_{11} per unit time. We assume that $\pi_{10} > \pi_{11} \geq 0$; π_{10} is the monopoly profit, whereas π_{11} is the duopoly profit, and hence it is natural to assume $\pi_{10} > \pi_{11}$. We call $\pi_{10} - \pi_{11}$ the *monopoly rent*. We assume two types of R&D expenses. One is the upfront setup cost $c \geq 0$ that occurs when a firm initiates its R&D. It includes, but is not limited to, expenditures on equipments and the costs of assembling a research team. The setup costs for the two firms are assumed to be identical. The other type of expenses is the variable cost $k_i \lambda_i^2$ per unit time, which is a convex function of λ_i (Kwon et al., 2010). Here, k_i is a measure of firm i's R&D cost efficiency.

In order to define the payoff as a function of the strategy profile, we need to specify how the strategy profile determines the equilibrium R&D effort levels. We let $\Pi_i(\Lambda_i, \Lambda_j; \sigma)$ denote firm i's expected cumulative discounted profit with a discount rate r when firms' effort levels are $\Lambda_i \equiv (\lambda_i, \mu_i)$ and $\Lambda_j \equiv (\lambda_j, \mu_j)$, and the strategy profile is given by $\sigma \equiv (T_i^0, T_j^0)$. Then we can define the *equilibrium effort levels* as $\Lambda_i^* \equiv (\lambda_i^*, \mu_i^*)$ and $\Lambda_j^* \equiv (\lambda_j^*, \mu_j^*)$ that satisfy the following:

$$\Pi_i(\Lambda_i^*, \Lambda_j^*; \sigma) = \max_{\Lambda_i} \Pi_i(\Lambda_i, \Lambda_j^*; \sigma), \Pi_j(\Lambda_i^*, \Lambda_j^*; \sigma) = \max_{\Lambda_j} \Pi_i(\Lambda_i^*, \Lambda_j; \sigma) \ . \quad (3.1)$$

We let $V_i(T_1^0, T_2^0; s_1, s_2) \equiv \Pi_i(\Lambda_i^*, \Lambda_j^*; \sigma)$ denote the expected cumulative discounted profit of firm i given a strategy profile (T_1^0, T_2^0) and the degrees of spillover (s_1, s_2).

In the spirit of examining the equilibria of the simplest structure, we limit our attention to the set of strategy profiles that respect the memoryless property of the exponential arrival of R&D completion. For example, given firm j's strategy T_j^0, firm i's best response is to initiate its R&D only when a significant event happens, i.e., at either T_j^0 or $T_j^0 + T_j$, because the arrival of the R&D completions is a memoryless exponential process. Let Σ denote the set of strategy profiles with either $T_i^0 = T_j^0$ or $T_i^0 = T_j^0 + T_j$, to which our analysis is confined throughout the paper. Focusing on the case in which the first initiation time $\min\{T_i^0, T_j^0\}$ is zero without loss of generality, we can classify $V_i(T_i^0, T_j^0; s_1, s_2)$ for each firm i into three distinct types as follows:

$$
\begin{aligned}
V_i^C(s_1, s_2) &\equiv V_i(0, 0; s_1, s_2) \\
V_i^{IL}(s_1, s_2) &\equiv V_i(0, T_i; s_1, s_2) \\
V_i^{IF}(s_1, s_2) &\equiv V_i(T_j, 0; s_1, s_2)\,.
\end{aligned}
$$

Here $V_i^C(s_1, s_2)$ is firm i's payoff from a concurrent strategy profile when $T_i^0 = T_j^0$. Furthermore, $V_i^{IL}(s_1, s_2)$ and $V_i^{IF}(s_1, s_2)$ are respectively firm i's payoffs as the leader (when $T_i^0 = 0$ and $T_j^0 = T_i$) and the follower (when $T_j^0 = 0$ and $T_i^0 = T_j$) from an imitative strategy profile. Here we can express

$$
\begin{aligned}
V_i^C(s_1, s_2) &= E\Bigg[\Bigg(-\int_0^{T_i} e^{-rt} k_i \lambda_i^2 dt + \int_{T_i}^{T_j} e^{-rt} \pi_{10} dt \qquad (3.2)\\
&\quad + \int_{T_j}^{\infty} e^{-rt} \pi_{11} dt\Bigg) 1_{\{T_i < T_j\}} + \Bigg(-\int_0^{T_j} e^{-rt} k_i \lambda_i^2 dt \\
&\quad - \int_{T_j}^{T_i} e^{-rt} k_i \mu_i^2 dt + \int_{T_i}^{\infty} e^{-rt} \pi_{11} dt\Bigg) 1_{\{T_i > T_j\}} - c\Bigg] \\
V_i^{IL}(s_1, s_2) &= E\Bigg[-\int_0^{T_i} e^{-rt} k_i \lambda_i^2 dt + \int_{T_i}^{T_j+T_i} e^{-rt} \pi_{10} dt \qquad (3.3)\\
&\quad -c + \int_{T_j+T_i}^{\infty} e^{-rt} \pi_{11} dt\Bigg] \qquad (3.4)\\
V_i^{IF}(s_1, s_2) &= E\Bigg[e^{-rT_j}\Bigg(-c - \int_0^{T_i} e^{-rt} k_i \mu_i^2 dt \qquad (3.5)\\
&\quad + \int_{T_i}^{\infty} \pi_{11} e^{-rt} dt\Bigg)\Bigg] \qquad (3.6)
\end{aligned}
$$

where λ_i, μ_i, λ_j, and μ_j are the equilibrium effort levels that satisfy (3.1).

We call (T_1^0, T_2^0) a *pure strategy profile* if $\min\{T_1^0, T_2^0\}$ is deterministic, i.e., when the first firm to initiate R&D does not randomize its initiation time. We say that (T_1^0, T_2^0) is a *mixed strategy profile* if both firms randomize their initiation times T_1^0 and T_2^0. The characteristics of the resulting Nash equilibrium depends on the model parameter of the game.

Now we assume $s_1 = s_2$ and characterize four distinct model parameter regimes depending on the relative sizes of $V_i^C(s,s)$, $V_i^{IF}(s,s)$ and $V_i^{IL}(s,s)$, which are the payoff functions when $\min\{T_1^0, T_2^0\} = 0$

(a) *Concurrent regime*: The game belongs to this regime if $V_i^C(s,s) \geq V_i^{IF}(s,s)$ for $i = 1, 2$. In a concurrent regime, being the follower is a dominated strategy for either firm because it gives the firm a lower payoff than does a concurrent strategy. Therefore, both firms have the incentives to initiate R&D at time 0, and the equilibrium is characterized by a concurrent strategy profile, i.e., $T_i^0 = T_j^0 = 0$. We call this equilibrium a *concurrent equilibrium*.

(b) *War of Attrition regime*: The game belongs to this regime if $V_i^{IF}(s,s) > V_i^{IL}(s,s)$ for $i = 1, 2$, and $V_i^{IF}(s,s) > V_i^C(s,s)$ for some $i \in \{1, 2\}$. In other words, there is at least one firm, say firm i, that satisfies $V_i^{IF}(s,s) > \max\{V_i^{IL}(s,s), V_i^C(s,s)\}$. Then each firm has a stronger incentive to be the follower than to be the leader. Hence, the equilibrium is characterized as a war of attrition, and there are exactly two pure strategy equilibria, each of which has a leader and a follower. In this regime mixed strategy equilibria also exist as discussed in Section 3.4. Note that once the leader-follower roles are already determined, the sequence of events under this equilibrium is identical to that of an *imitative equilibrium* defined below.

(c) *Preemption Regime*: The game belongs to this regime if $V_i^{IL}(s,s) > V_i^{IF}(s,s) > V_i^C(s,s)$ for $i = 1, 2$. Both firms have incentives to be the leader, so the equilibrium is characterized as a preemption game (Hendricks and Wilson, 1992).

(d) *Imitative Regime*: The game belongs to this regime if it belongs to none of the above regimes. In this regime, one of the two following conditions is satisfied: (1) For one firm i, $V_i^{IL}(s,s) > V_i^{IF}(s,s)$ holds while for the other firm j, $V_j^{IF}(s,s) > V_j^C(s,s)$ holds; in addition, $V_i^{IL}(s,s) > V_i^{IF}(s,s) > V_i^C(s,s)$ can hold for at most one firm i. (2) For one firm i, $V_i^{IL}(s,s) > V_i^{IF}(s,s) > V_i^C(s,s)$ holds while for the other firm j, $V_j^C(s,s) > V_j^F(s,s) > V_j^{IL}(s,s)$ holds. Under either conditions (1) and (2), the equilibrium is characterized as an imitative strategy profile in which firm i is the leader who initiates R&D at time zero and firm j is the follower who initiates it at time T_i. We call this equilibrium an *imitative equilibrium*.

3.4 Classification of Equilibria

In this section, we characterize Nash equilibria of the game. We first discuss a special case in which the firms are symmetric and the upfront setup cost c is sufficiently small in Section 3.4.1. Section 3.4.2 will examine the general case.

3.4.1 Case of Symmetric Firms in the Small c Limit

Consider the case of symmetric firms ($k_1 = k_2 = k$) with a very small setup cost c. It turns out that, in this case, the only possible regimes are the concurrent regime and the war of attrition regime.

In the spirit of backward induction, we first solve the optimal strategy for the imitator after τ_I. Assume that firm i is the innovator. Let $W_j(\mu_j; s_i)$ and $U_i(\mu_j; s_i)$ respectively be the firm j's and firm i's payoffs that are discounted to τ_I when firm j (the imitator) employs the equilibrium effort level μ_j after τ_I. Then $W_j(\mu_j; s_i)$ and $U_i(\mu_j; s_i)$ are given by

$$\begin{aligned} W_j(\mu_j; s_i) &= \max_{\mu_j} E\left[-\int_0^{T_j-\tau} e^{-rt} k_j \mu_j^2 dt + \int_{T_j-\tau}^{\infty} e^{-rt}\pi_{11} dt \mid \tau_I = \tau < T_j\right] \\ &= \frac{1}{r}\pi_{11}\frac{s_i\mu_j}{s_i\mu_j + r} - \frac{k_j\mu_j^2}{s_i\mu_j + r}, \\ U_i(\mu_j; s_i) &= E\left[\int_0^{T_j-\tau} \pi_{10} e^{-rt} dt + \int_{T_j-\tau}^{\infty} \pi_{11} e^{-rt} dt \mid \tau_I = \tau < T_j\right] \\ &= \frac{1}{r}\pi_{11}\frac{s_i\mu_j}{s\mu_j + r} + \frac{\pi_{10}}{s_i\mu_j + r}. \end{aligned}$$

From the first order condition $dW_j(\mu_j; s_i)/d\mu_j = 0$, the optimal μ_j^* that maximizes $W_j(\mu_i; s_i)$ is obtained as follows:

$$\mu_j^* = -\frac{r}{s} + \sqrt{(\frac{r}{s})^2 + \frac{\pi_{11}}{k_j}}. \tag{3.7}$$

Firm i's equilibrium payoff after τ_I is given by

$$W_j(s_i) \equiv W_j(\mu_j^*; s_i) = \frac{1}{r}\pi_{11}\frac{s_i\mu_j^*}{s_i\mu_j^* + r} - \frac{k_j\mu_j^{*2}}{s_i\mu_j^* + r}. \tag{3.8}$$

Firm j's equilibrium payoff after τ_I is given by

$$U_i(s_i) \equiv U_i(\mu_j^*; s_i) = \frac{1}{r}\pi_{11}\frac{s_i\mu_j^*}{s\mu_j^* + r} + \frac{\pi_{10}}{s_i\mu_j^* + r}. \quad (3.9)$$

Because we assume perfect symmetry between the two firms, the firm index i is dropped from the functions $W_i(\cdot)$ and $U_i(\cdot)$ for the remainder of this section.

Below we obtain the firms' strategies and payoffs of the two classes of equilibria. Throughout the paper, we assume $c < W_i(1)$, i.e., the upfront cost is sufficiently small so that both firms have incentives to initiate their R&D projects even without spillover.

Proposition 6 *(I) In an equilibrium with a concurrent strategy profile, firm i's strategy is given by $T_i^0 = 0$ for $i \in \{1, 2\}$. Then the effort levels μ_i^* for $t \in [\tau_I, \tau_M)$ are given by (3.7), and the effort levels λ_i^C for $t \in (0, \tau_I)$ are given by*

$$\lambda_i^C = \frac{\sqrt{(2kr - U(s) + W(s))^2 + 12krU(s)} - (2kr - U(s) + W(s))}{6k}. \quad (3.10)$$

The equilibrium payoffs are given by

$$V_i^C(s, s) = (U(s) + W(s))\frac{\lambda_i^C}{2\lambda_i^C + r} - \frac{k(\lambda_i^C)^2}{2\lambda_i^C + r} - c. \quad (3.11)$$

(II) In an equilibrium with an imitative strategy profile, where firm i is the leader, the strategy profile is given by $(0, T_i)$. Then firm i's effort level λ_i^{IL} for $t \in (0, T_i)$ is given by

$$\lambda_i^{IL} = -r + \sqrt{r^2 + \frac{rU(s)}{k}}, \quad (3.12)$$

and firm j's effort level μ_j^* for $t \in [T_i, \tau_M)$ is given by (3.7).

The equilibrium payoffs are given by

$$V_i^{IL}(s, s) = -\frac{k(\lambda_1^{IL})^2}{\lambda_1^{IL} + r} + U(s)\frac{\lambda_1^{IL}}{r + \lambda_1^{IL}} - c, \quad (3.13)$$

$$V_j^{IF}(s, s) = (W(s) - c)\frac{\lambda_1^{IL}}{r + \lambda_1^{IL}}. \quad (3.14)$$

Next, we establish some sufficient conditions for each parameter regime.

Proposition 7 *(I) If either of the following conditions is satisfied, then the game belongs to the concurrent strategy regime: (a)* $\pi_{11} \le 8r^2k$ *and* s *is sufficiently small or sufficiently large, or (b)* $\pi_{11} > 8r^2k$ *and* s *is sufficiently small.*

(II) If $\pi_{11} > 8rk^2$ *and* s *is sufficiently large, then the game belongs to the war of attrition regime.*

When the duopoly profit stream π_{11} is relatively small, a firm has a strong incentive to initiate its R&D earlier in order to attain the monopoly profit stream π_{10}. Thus, both firms initiate their R&D at time zero. When π_{11} is sufficiently large, delaying the initiation of an R&D project has two countervailing effects on a firm's payoff. On the one hand, a firm obtains higher cost efficiency after τ_I due to spillover, so delaying the R&D project and thereby being the imitator may increase the payoff. On the other hand, a delay in an R&D project may diminish the time value of the profit. When s is sufficiently small, the first effect is dominated by the second effect, because the increment in cost efficiency arising from delaying the R&D project is very small. Hence, both firms choose to initiate their R&D projects at time zero for small s.

If the spillover s and the duopoly profit π_{11} are sufficiently large, then it behooves a firm to be the imitator, because the first effect dominates the second effect. In this case, both firms prefer to be the follower since their incentives are identical if $k_1 = k_2$ and $s_1 = s_2$. Specifically, if $V_i^{IF}(s,s) > V_i^{C}(s,s)$ and $V_i^{IF}(s,s) > V_i^{IL}(s,s)$ are satisfied, then as discussed in Section 3.3, the game belongs to the war of attrition regime, and the equilibrium is characterized as a war of attrition (Tirole, 1988; Hendricks et al., 1988).

In a pure strategy equilibrium of a war of attrition, one firm takes the leader's role while the other takes the follower's role. The designation of the leader and the follower falls outside the scope of the mathematical specification of the game, but is rather determined by social expectation. If one of the firms is expected to be more proactive, possibly because of the past track records and public expectations, then it naturally takes the role of a leader.

If the social expectation cannot determine the leader and the follower roles, the game results in a mixed strategy equilibrium. Under a mixed strategy equilibrium, each firm initiates its R&D project at a random time. In this case, the firm that happens to initiate the R&D first becomes the leader. The best response of the other firm is to be the follower. A mixed strategy profile is characterized by each firm i's cumulative probability distribution for the random time T_i^0 of initiation of R&D. Let $F_i(t) : \mathcal{R}_+ \to [0, 1]$ denote firm i's cumulative probability distribution function (CDF) for time T_i^0. Then the mixed strategy profile is characterized by the pair of CDFs $(F_1(t), F_2(t))$. We let $q_i(t) = F_i(t) - \lim_{t_0 \uparrow t} F_i(t_0)$ represent the discontinuity of $F_i(t)$ at time t. Then by virtue of Hendricks et al. (1988), we obtain the following proposition:

Proposition 8 *In a war of attrition regime, a mixed strategy equilibrium characterized by a strategy profile* $(F_1(t), F_2(t))$ *with* $q_1(0) < 1$ *and* $q_2(0) < 1$ *exists, if and only if both of the following conditions are satisfied:*

(1) $(q_1(0), q_2(0)) \in [0, 1) \times [0, 1)$ *and* $q_1(0)q_2(0) = 0$.

(2) For both $i = 1$ *and* 2,

$$\begin{aligned} F_i(t) &= 1 - [1 - q_i(0)] \exp\left(-\frac{t}{\tau_i^M}\right), \\ \textit{where } \tau_i^M &= \frac{V_j^{IF}(s,s) - V_j^{IL}(s,s)}{rV_j^{IL}(s,s)}. \end{aligned} \tag{3.15}$$

The mixed strategy equilibrium payoff is given by

$$V_i^M = q_j(0)V_j^{IF}(s,s) + [1 - q_j(0)]V_j^{IL}(s,s).$$

Proposition 8 reveals that each firm's investment time T_i^0 is exponentially distributed with parameter $1/\tau_i^M$. The function $q_i(t)$ is the discontinuity in the CDF $F_i(t; s)$, and it is the probability that firm i initiates R&D at time t. The function $q_i(t)$ can never be positive when $t > 0$, but $q_i(0)$ can be positive.

The fact that $q_i(t) = 0$ for $t > 0$ can be understood as follows. Suppose that $q_i(t_0) > 0$ for some $t_0 > 0$, i.e., firm i initiates R&D at time t_0 with a positive probability. Then firm j has an incentive to initiate R&D after t_0 due to the higher payoff to the follower than that to the leader. However, this contradicts the fact that firm j should be indifferent among all $T_j^0 > 0$ in a mixed strategy equilibrium. Hence, $q_i(t)$ should be zero for all $t > 0$.

Moreover, Proposition 8 states that at most one of $q_1(0)$ and $q_2(0)$ is positive. Whenever $q_i(0) > 0$, i.e., firm i has non zero probability to initiate R&D at time 0, firm j's best response is to definitely delay R&D because it is more profitable to be the follower than to be the leader. Thus, $q_j(0) = 0$ whenever $q_i(0) > 0$.

In summary, in a war of attrition, even though the equilibrium has the characteristic of an imitative strategy profile, either firm can be the leader depending on the social expectation (in case of a pure strategy equilibrium) or by chance (in case of a mixed strategy equilibrium). As shown below, this feature of a war of attrition remains true even when the two firms are asymmetric.

3.4.2 Case of Asymmetric Firms

In this subsection, we generalize our model in Section 3.4.1 by assuming $c > 0$ and $k_1 \neq k_2$. Unless otherwise specified, we assume $k_1 \leq k_2$ in the remainder of the paper. For notational convenience, define $k \equiv k_2$ and $q \equiv k_1/k$ so that $q \in (0, 1]$. We call $1 - q \in [0, 1)$ *the degree of asymmetry.* Unlike the symmetric cases investigated in Section 3.4.1, we obtain the concurrent, imitative, and a war of attrition equilibria. In some limiting cases, we are able to obtain sufficient conditions for the three classes of equilibria.

We first obtain the strategies and payoffs for various equilibria.

Proposition 9 *(I) In a concurrent equilibrium, firm i's strategy is given by $T_i^0 = 0$ for $i \in \{1, 2\}$. Firm i's effort level μ_i^* for $t \in (\tau_I, \tau_M)$ is defined in (3.7), and firms' effort levels $(\lambda_1^C, \lambda_2^C)$ for $t \in (0, \tau_I)$ are the unique pair of solutions that satisfies*

$$r + \lambda_1^C + \lambda_2^C = \sqrt{(r + \lambda_1^C)^2 + (\lambda_1^C)\frac{U_2(s) - W_2(s)}{k_2} + \frac{r}{k_2}U_2(s)} \quad (3.16)$$

$$r + \lambda_1^C + \lambda_2^C = \sqrt{(r + \lambda_2^C)^2 + (\lambda_2^C)\frac{U_1(s) - W_1(s)}{k_1} + \frac{r}{k_1}U_1(s)} \quad (3.17)$$

$$\lambda_1^C \geq 0 \quad , \quad \lambda_2^C \geq 0 \quad (3.18)$$

where $U_i(s)$ and $W_i(s)$ are defined in (3.9) and (3.8).

(II) In an imitative or a war of attrition equilibrium where firm i is the leader, firm i's effort level λ_i^{IL} is given by

$$\lambda_i^{IL} = -r + \sqrt{r^2 + \frac{rU_i(s)}{k_i}}. \quad (3.19)$$

Although we cannot obtain the explicit expression of λ_i^C as in the symmetric case, Proposition 9 shows that there exists a unique pair of $(\lambda_1^C, \lambda_2^C)$ in a concurrent equilibrium. It is technically difficult to delineate the conditions for a concurrent equilibrium. However, progress can be made when we consider limiting cases of $q \to 1$ and $q \to 0$. Next, we establish some sufficient conditions for each parameter regime.

Proposition 10 *(I) If either of the following conditions is satisfied, the game belongs to the concurrent regime. (a) c and s are sufficiently small; (b) c is sufficiently small, q is sufficiently close to 1, $\pi_{11} < 8r^2k$, and s is sufficiently large.*

(II) If q is sufficiently close to 1, c is sufficiently small, $\pi_{11} > 8r^2k$, and s is sufficiently large, then the game belongs to the war of attrition regime.

(III) If q is sufficiently close to 0 and s is sufficiently large, then the game belongs to the imitative regime.

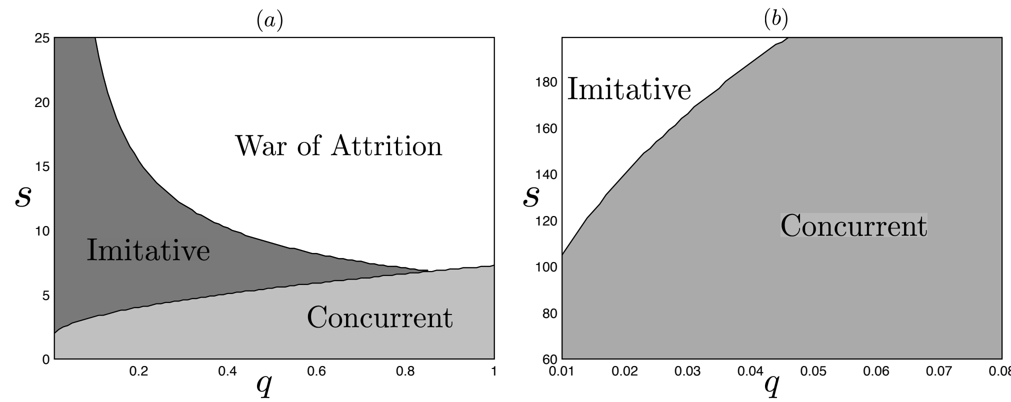
When the degree of spillover is sufficiently low and the upfront cost is sufficiently small, a firm does not have much incentive to wait for the other firm to complete its R&D. Hence, a concurrent equilibrium occurs as stated by Proposition 10 (I.a).

When q is sufficiently close to 1, the game is very close to the symmetric case. Hence the intuition behind Proposition 10 (I.b) is exactly the same as that of Proposition 7 (I.a), and both firms choose to initiate R&D at time 0. When c is sufficiently small, the intuition behind Proposition 10 (II) is exactly the same as that of Proposition 7 (II), and both firms have incentives to be the follower, so the game becomes a war of attrition. Larger values of c only reinforce a firm's incentive to delay its R&D because of the additional upfront cost.

When q is sufficiently close to 0 and s is sufficiently large, the less efficient firm can benefit from significant cost reduction due to spillover, so it has an incentive to wait for the other firm's completion. In contrast, the more efficient firm has an incentive to initiate R&D immediately because waiting for the less efficient firm's completion is extremely time consuming due to the extreme asymmetry. Therefore, as stated by Proposition 10 (III), the game belongs to the imitative regime.

Figure 3.1 provides numerical illustrations of Proposition 7 and 10. When $\pi_{11} < 8kr^2$, the war of attrition regime does not exist for sufficiently large q,

Figure 3.1: Various regimes of the game. The parameters for (a) are: $k = 1, \pi_{10} = 200, \pi_{11} = 20, r = 0.1, c = 150$; the parameters for (b) are $k = 50, \pi_{10} = 200, \pi_{11} = 1, r = 0.1, c = 0.6$.



i.e., $q \to 1$, as illustrate by Figure 7 (b). Figure 7 (a) represents the case in which $\pi_{11} > 8kr^2$.

Although our analytical results of Proposition 7 and 10 only discuss the limiting cases of q and s, various model parameter regimes are numerically illustrated in Figure 3.1.

There are two noteworthy features of Figure 3.1 (a). First, if q is sufficiently close to 1, the imitative regime does not occur; if the two firms are nearly symmetric, then because their payoffs are similar, neither firm has strong incentive to be either the leader or the follower, and hence the imitative regime does not take place. Secondly, if the two firms are extremely asymmetric, there is no war of attrition regime even if s is very large. If the asymmetry is sufficiently high, the more efficient firm never prefers to be the follower because it takes the less efficient firm a long time to complete the R&D.

Another noteworthy feature of the model is that the war of attrition regime does not have to occur in the space of (s, q). Figure 3.1 (b) illustrates a model parameter regime ($k = 50$) where the war of attrition regime is absent even for large values of s or q.

Lastly, we characterize the mixed strategy equilibrium in a war of attrition regime for an asymmetric case. The following proposition is analogous to Proposition 8 for the symmetric case.

Proposition 11 *In a war of attrition regime, a mixed strategy equilibrium that is characterized by a strategy profile* $(F_1(t), F_2(t))$ *with* $q_1(0) < 1$ *and*

$q_2(0) < 1$ *exists if and only if both of the following conditions are satisfied:*

(1) $(q_1(0), q_2(0)) \in [0, 1) \times [0, 1)$ *and* $q_1(0)q_2(0) = 0$.

(2) For both $i = 1$ *and* 2,

$$\begin{aligned} F_i(t; s) &= 1 - [1 - q_i(0)] \exp[-\frac{t}{\tau_i^M}], \\ \textit{where } \tau_i^M &= \frac{V_j^{IF}(s, s) - V_j^{IL}(s, s)}{rV_j^{IL}(s, s)} \end{aligned} \tag{3.20}$$

The mixed strategy equilibrium payoff is given by

$$V_i^M = q_j(0)V_j^{IF}(s, s) + [1 - q_j(0)]V_j^{IL}(s, s).$$

One striking feature of the war of attrition is that the more efficient firm is not necessarily the first firm to initiate R&D in either the pure strategy equilibrium or the mixed strategy equilibrium.

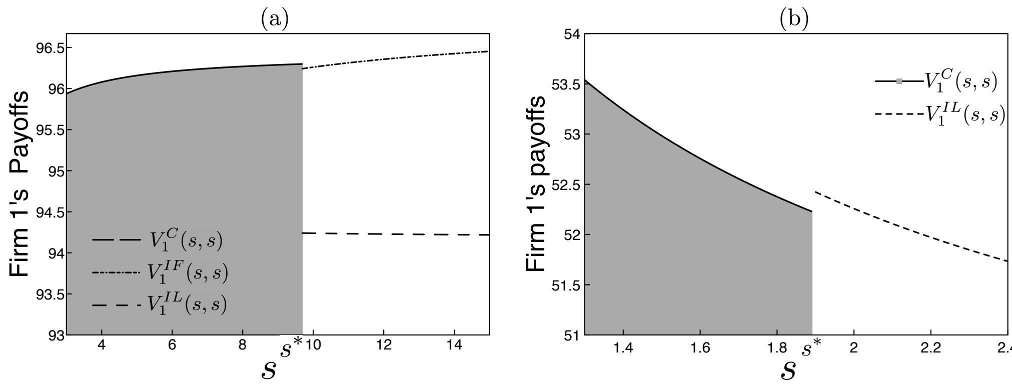
3.5 The Impact of Spillover

In this section, we examine the impact of natural spillover on the equilibrium payoffs in various regimes. Then we show that a firm may or may not have an incentive to unilaterally increase the spillover depending on the magnitude of the natural spillover and the degree of asymmetry. As in Section 3.4, we assume $k_1 \leq k_2$ so that firm 1 is the more efficient firm and firm 2 is the less efficient firm in the remainder of the paper.

Conventional wisdom suggests that an increased spillover should increase the imitator's payoff whereas it decreases the innovator's payoff. However, the following two examples demonstrate that this wisdom does not always hold.

Figure 3.2 (a) shows an example in which the game switches from the concurrent regime to the war of attrition regime as s increases. In this example where the degree of asymmetry is low ($q = 0.02$) and the monopoly rent $\pi_{10} - \pi_{11}$ is not very large ($\pi_{10} = 30, \pi_{11} = 10$), even the more efficient firm benefits from increased spillover when the game belongs to the concurrent regime because both firms have a chance to be the imitator due to random completion times. However, in the war of attrition regime, if firm 1 (the

Figure 3.2: The impact of spillovers upon equilibrium payoffs. In (a), the shaded (unshaded) area represents concurrent (war of attrition) regime; In (b), the shaded (unshaded) area represents concurrent (imitative) regime. Here, the parameters for (a) are
$k = 1, \pi_{10} = 30, \pi_{11} = 10, r = 0.1, c = 50, q = 0.02$; the parameters for (b) are $k = 1; \pi_{10} = 12, \pi_{11} = 10, r = 0.1, c = 0, q = 0.9$.



more efficient firm) is the leader then it cannot benefit from spillover, and its payoff decreases in s as a result of a shorter length of time over which it receives monopoly profit; if firm 1 is the follower, firm 1's payoff increases in s.

Figure 3.2 (b) shows an example in which the game switches from the concurrent regime to the imitative regime as s increases. In the imitative regime, firm 1 initiates R&D first. In the concurrent regime, because of the high degree of the asymmetry ($q = 0.9$), spillover tends to decrease firm 1's payoff. However, at the transition point s^*, firm 1's payoff increases discontinuously in s. The intuition behind this phenomenon is that, across the transition from the concurrent regime to the imitative regime, firm 2 switches from the concurrent strategy to the imitative strategy, which discontinuously increases firm 1's payoff because firm 2's imitative strategy lessens its competition against firm 1.

From these two examples, we see that an increased spillover does not always diminish the more efficient firm's payoff. Now we investigate how prevalent these results and associated insights are. We first investigate the comparative statics of $V_i^C(s,s)$,$V_i^{IL}(s,s)$ and $V_i^{IF}(s,s)$ with respect to s in various regimes to examine the impact of natural spillover upon firms' payoffs.

Theorem 5 *(I) For q sufficiently close to 1 and c sufficiently small, we obtain the following: (a) Suppose that the conditions of Proposition 10 (I)* *are satisfied so that the game belongs to the concurrent regime. Then* $V_i^C(s,s)$ *decreases in* s *if* $\pi_{10}-\pi_{11}$ *is sufficiently large, and it increases in* s *if* $\pi_{10}-\pi_{11}$ *is sufficiently small. (b) Suppose that the conditions of Proposition 10 (II) are satisfied so that the game belongs to the war of attrition regime. Then* $V_i^{IL}(s,s)$ *decreases in* s*. Furthermore* $V_i^{IF}(s,s)$ *decreases in* s *if* $\pi_{10}-\pi_{11}$ *is sufficiently large, whereas it increases in* s *if* $\pi_{10}-\pi_{11}$ *is sufficiently small.*

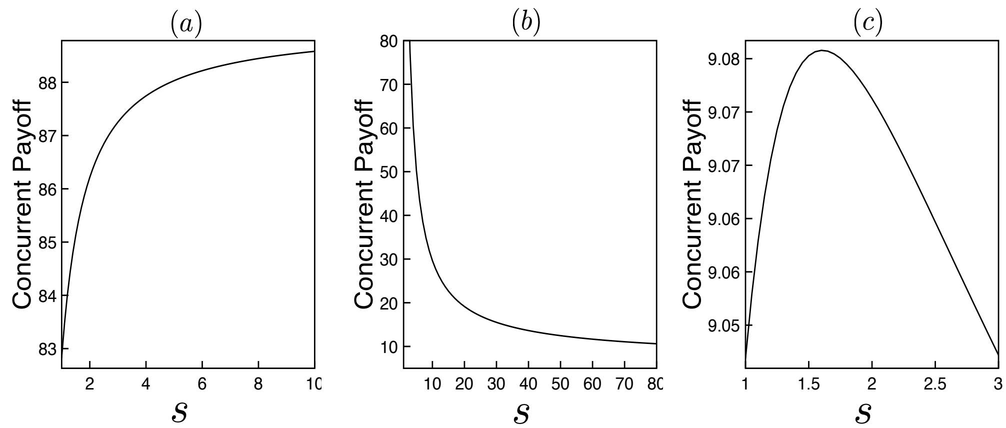
(II) For q *is sufficiently close to 0, we obtain the following: (a) Suppose that the conditions of Proposition 10 (I) are satisfied so that the game belongs to the concurrent regime. Then* $V_1^C(s,s)$ *decreases in* s *whereas* $V_2^C(s,s)$ *increases in* s*. (b) Suppose that the conditions of Proposition 10 (III) are satisfied so that the game belongs to the imitative regime. Then* $V_1^{IL}(s,s)$ *decreases in* s *whereas* $V_2^{IF}(s,s)$ *increases in* s*.*

Theorem 5 (II.a) can be understood as follows. In the concurrent regime, for high degrees of asymmetry between the two firms, i.e., if q is sufficiently close to 0, the more efficient firm is more likely to become the innovator. Hence, firm 1's payoff mainly depends on the payoff that it receives as an innovator, and firm 2's payoff mainly depends on the payoff that it receives as an imitator. Because an increase in s diminishes the innovator's payoff and increases the imitator's payoff by shortening the imitator's completion time, firm 1' payoff decreases in s whereas firm 2's payoff increases in s.

If the degree of asymmetry $1-q$ is sufficiently low, i.e., if q is sufficiently close to 1, the two firms are almost equally likely to be the innovator or the imitator. Hence, from the formula (3.11), a firm's payoff mainly depends on the sum of the payoffs it receives as an innovator and an imitator. Then Theorem 5 (I.a) can be understood as follows. If the monopoly rent $\pi_{10}-\pi_{11}$ is sufficiently small, the innovator's payoff is insensitive to the completion time of the imitator. It follows that the innovator's payoff is relatively insensitive to s, because the only effect of s is to decrease the completion time of the imitator. However, the imitator's payoff remains sensitive to s. Because the imitator's payoff increases in s, it follows that each firm's payoff increases in s. If the monopoly rent is sufficiently large, the innovator's payoff is diminished dramatically by the imitator's earlier completion time when s increases, so the impact of s upon the innovator's payoff dominates the impact of s upon the imitator's payoff. As a result, the firms' payoffs decrease in s.

Figure 3.3 illustrates the impact of the natural spillover s upon a firm's

Figure 3.3: The impact of the natural spillover s upon $V_i^C(s,s)$ under different values of the monopoly rent. (a) $k_1 = k_2 = 10, \pi_{10} = 12, \pi_{11} = 10, r = 0.1, c = 0$; (b) $k_1 = k_2 = 10, \pi_{10} = 200, \pi_{11} = 10, r = 0.1, c = 0$; (c) $k_1 = k_2 = 1$, $\pi_{10} = 3$, $\pi_{11} = 1$, $r = 0.1$, $c = 0$.



concurrent payoff under various values of the monopoly rent. Figure 3.3 (a) and (b) are consistent with Theorem 5 (I.a) when the monopoly rent is sufficiently small or sufficiently large. Figure 3.3 (c) illustrates the case where the value of monopoly rent is intermediate. For sufficiently small s, the change in the imitator's completion time is relatively insignificant, so the innovator's payoff does not decrease much in s. Thus the comparative statics of the concurrent payoff with respect to s are mainly affected by the increment in the imitator's payoff. In contrast, for sufficiently large s, the innovator's payoff is much more sensitive to large decrement in the imitator's completion time than the imitator's payoff. Hence, the concurrent payoff is mainly affected by the reduction in the innovator's payoff.

In the war of attrition regime, higher spillover diminishes $V_i^{IL}(s,s)$ because the leader must be the innovator. As stated in Theorem 5 (I.b), the follower's payoff $V_i^{IF}(s,s)$ does not always increase in spillover. Instead, the comparative statics of $V_i^{IF}(s,s)$ with respect to s depend on the magnitude of the spillover and the monopoly rent. This is because an increase in spillover has two opposing effects on the follower's payoff: On the one hand, increased spillover improves the follower's payoff by raising the follower's efficiency. On the other hand, increased spillover may decrease the follower's payoff because it may increase the leader's time-to-completion; the leader's time-to-completion may increase in s because the leader reduces the effort level when s increases. If the monopoly rent is small, the leader's effort level does not change significantly as s increases, so the first effect dominates and, as a result, $V_i^{IF}(s,s)$ increases in s. In contrast, if the monopoly rent is sufficiently large, then the leader's effort level is sufficiently sensitive to s. It follows that the second effect dominates, and $V_i^{IF}(s,s)$ decreases in s.

Now we examine the impact of increased spillover across the transition point s^*. In particular, by the definitions of various regimes in Section 3.3, we define s^* as the smallest degrees of natural spillover at which a transition from the concurrent regime to the other regime occurs.

Theorem 6 *(I) For q sufficiently close to 1, and $\pi_{11} > 8kr^2$, We obtain $V_i^C(s^*,s^*) \geq V_i^{IF}(s^*,s^*)$, and $V_i^C(s^*,s^*) > V_i^{IL}(s^*,s^*)$ for both $i \in \{1,2\}$.*

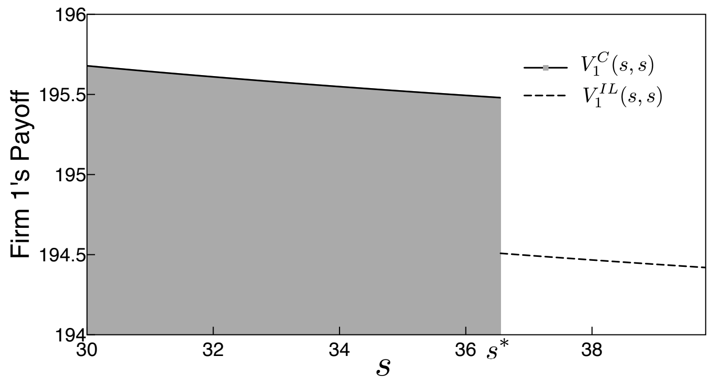
(II) For q sufficiently close to 0, Firm 1 is the leader, and we obtain $V_1^C(s^*,s^*) < V_1^{IL}(s^*,s^*)$ and $V_2^C(s^*,s^*) = V_2^{IF}(s^*;s^*)$.

As illustrated by Figure 3.2 (a), if the degree of asymmetry is sufficiently low, firm 1's payoff discontinuously decreases as s crosses s^* regardless of whether firm 1 is the leader or the follower in the war of attrition regime.

If the degree of asymmetry is high, an increase in spillover across s^* (at the transition from the concurrent to the imitative regime) has two countervailing effects on firm 1's payoff. (a) An increase in spillover increases firm 1's payoff, which has been demonstrated by Figure 3.2 (b) and explained by accompanying comments; (b) An increase in spillover across s^* induces firm 2 to switch to imitative strategy, and consequently firm 1 loses a chance to benefit from spillover. However, in the presence of high asymmetry in efficiency, firm 1's chance of receiving spillover is very small even in the concurrent regime. Hence, the first effect dominates the second effect, and firm 1's payoff increases in s across s^*.If q is not very small, Figure 3.4 shows an example in which the negative effect dominates the positive effect, and firm 1's profits decreases in s across s^* as the game crosses over from the concurrent to the imitative regime.

Theorems 5 and 6 examine the impact of natural spillover. Next we investigate the impact of unilateral increase in spillover. Specifically, we assume that firm 1 can change the spillover s_1 from firm 1 to firm 2 and examine its impact. (The spillover s_1 has been defined in Section 3.3). For instance, firm 1 (the more innovative firm) can achieve this by disclosing information about its innovation to its competitor after it completes the project. The

Figure 3.4: Firm 1's payoff decreases when the game crosses over from concurrent regime to imitative regime. The shaded (unshaded) area represents concurrent (imitative) regime. Here we set $k = 1; \pi_{11} = 20; \pi_{10} = 200; r = 0.1; c = 1; q = 0.4$.



following theorem establishes the impact of unilateral increase in spillover s_1 by an infinitesimal amount.

Theorem 7 *For sufficiently small increase $\epsilon > 0$ in the unilateral spillover from firm 1 to firm 2, we obtain the following:*

(I) For q sufficiently close to 1, (a) if the game belongs to the concurrent regime, then the equilibrium payoffs $V_1^C(s+\epsilon, s) < V_1^C(s,s)$ for both sufficiently large π_{10} and sufficiently small π_{10}; (b) if the game belongs to the war of attrition regime, then $V_1^{IL}(s+\epsilon, s) < V_1^{IL}(s,s)$ and $V_1^{IF}(s+\epsilon, s) = V_1^{IF}(s,s)$; (c) if $k < \pi_{11}/8r^2$ such that the game crosses over from the concurrent regime to the imitative regime at s^*, then $V_1^{IL}(s^*+\epsilon, s^*) < V_1^C(s^*, s^*)$.

(II) For q sufficiently close to 0, (a) if the game belongs to the concurrent regime, then $V_1^C(s+\epsilon, s) < V_1^C(s,s)$; (b) if the game belongs to the imitative regime, then $V_1^{IL}(s+\epsilon, s) < V_1^{IL}(s,s)$; (c) at the transition point s^*, we have $V_1^{IL}(s^*+\epsilon, s^*) > V_1^C(s^*, s^*)$.

Theorem 7 (I.a), (I.b), (II.a) and (II.b) state that, regardless of the value of s, the payoffs decrease in ϵ within the given regime. In another word, a firm has no incentive to unilaterally increase spillover as long as the increased spillover does not move the game to another equilibrium regime. However, at s^*, the unilaterally increased spillover may have different impacts upon firm 1's payoffs. If the game crosses over from concurrent to the war of attrition regime at s^*, the unilaterally increased spillover diminishes firm 1's payoff whether firm 1 is the leader or the follower. In contrast, Theorem 7 (II.c) states that, in case of high asymmetry, firm 1 can ease the competition by unilaterally increasing the spillover to induce firm 2 to delay the initiation of R&D.

Although Pacheco-de Almeida and Zemsky (2012) also addresses an innovative firm's incentives to unilaterally increase spillovers, our results are different from theirs due to the presence of technological uncertainty. Pacheco-de Almeida and Zemsky (2012) finds that the more efficient firm always has an incentive to unilaterally increase spillover beyond the critical value s^* if the completion times are deterministic. In contrast, our results suggest that the more efficient firm may not have the incentive to increase spillover beyond s^* if the completion times are random. (See Figure 3.4). This phenomenon sometimes happens because the more efficient firm can also benefit from the natural spillover in the concurrent regime due to substantial probability that the more efficient firm may end up being the imitator. In contrast, in the imitative regime, the more efficient firm cannot benefit from the natural spillover. Hence, the difference in our results is driven by the uncertainty in the completion times.

3.6 Sequence of Initiation of R&D

In this section, we investigate which firm initiates its R&D first in the war of attrition regime. In the concurrent regime, both firms initiate R&D at time zero. Hence, we only need to focus on the imitative regime and the war of attrition regime. In the imitative regime, the sequence of initiation of R&D is deterministic: the more efficient firm always initiates R&D earlier than the less efficient firm firm. In the war of attrition regime, because there are two pure strategy equilibria, either firm can be the leader or the follower depending on nom-mathematical factors such as social expectation or tacit agreement. Even in the mixed strategy equilibrium, if $q_i(0) > 0$, then by the meaning of $q_i(0)$ explained in Section 3.4, there is tacit expectation that firm i is more likely to be the leader to initiate R&D first. The interesting question is what happens when there is no such tacit propensity for any firm to be the leader, i.e., $q_i(0) = q_j(0) = 0$. Thus, we examine the probability that a given firm initiates R&D before its competitor in the completely mixed strategy equilibrium when $q_i(0) = q_j(0) = 0$. We say that firm i is more likely to initiate R&D earlier than firm j if $P(T_i^0 < T_j^0 \mid \min\{T_i^0, T_j^0\} > 0) > 1/2$. By virtue of Proposition 11,we obtain

$$P(T_i^0 < T_j^0 \mid \min\{T_i^0, T_j^0\} > 0) \;=\; \frac{\tau_j^M}{\tau_i^M + \tau_j^M}, \tag{3.21}$$

where τ_i^M is given in Proposition 11.

Theorem 8 *(I) Suppose that q is sufficiently close to 1, $\pi_{11} > 8r^2k$, and s is sufficiently large so that the game belongs to the war of attrition regime. In the mixed strategy equilibrium, we have $P(T_2^0 < T_1^0 \mid \min\{T_1^0, T_2^0\} > 0) > 1/2$.*

(II) Suppose that q is sufficiently close to 0 and s is sufficiently large so that the game belongs to the imitative regime. In the equilibrium, we have $T_1^0 < T_2^0$.

Theorem 8 reveals that the less efficient firm is more likely to initiate R&D first in the completely mixed strategy equilibrium of a war of attrition. In fact, extensive numerical studies indicate that this result is robust for the war of attrition regime even if the conditions of Theorem 8(I) are not satisfied. (The theorem is formally proved only for analytically tractable model parameter regime).

In a mixed strategy equilibrium, firm i chooses its strategy depending on firm j's payoff structure in such a way that firm j is indifferent among all initiations times $T_j^0 > 0$. Even though both firms have the incentive to be the follower, the less efficient firm has stronger incentive to wait than the more efficient firm because it takes longer for the less efficient firm to complete R&D. If the more efficient firm strategically places high likelihood to initiate R&D too early, then the less efficient firm would prefer to delaying its initiation, and firm j would not be indifferent among all $T_j^0 > 0$. Therefore, the more efficient firm needs to place significant amount of likelihood on later times of initiation. This gives the more efficient firm stronger incentive to place higher probability of initiation at later times. In general, a similar characteristics is shared by all mixed strategy equilibria of a war of attrition (Kornhauser et al. 1989, Myatt 1999).

3.7 Conclusions

Investments in R&D projects are often accompanied by knowledge spillovers and uncertainty in the projects' completion times. In this paper, we develop a game theoretic model to investigate the impact of natural spillover, which always takes place from the innovator to the imitator. We find the existence of three equilibrium regimes (concurrent, imitative, and war of attrition) depending on the degree of spillover and that of asymmetry between firms' cost efficiencies. Conventional wisdom implies that spillover decreases the more efficient firm's profit whereas it increases the less efficient firm's profit. In stark contrast, our analysis suggests that the impact of spillover does not always follow this conventional wisdom. In particular, an increase in natural spillover may benefit the more efficient firm (a) under low degrees of asymmetry,low degrees of spillover and small monopoly rent, or (b) under high degrees of asymmetry at the transition point between a concurrent regime and an imitative regime.

We also examine which firm is more likely to initiate R&D first. It is generally accepted that the more efficient firm initiates earlier than the less efficient rival. In the imitative regime, our result is consistent with this conventional wisdom. In contrast, in the war of attrition regime, we find the conventional wisdom does not hold anymore: in the pure strategy equilibrium, either firm initiates first depending on the social expectation; in the mixed strategy equilibrium, the less efficient firm is more likely to invest earlier.

We also investigate the incentive of a firm to increase the degree of R&D spillover to the competitor despite the possibility of imitation by the competitor. Extant literature of management suggests that knowledge spillover is thought to diminish an innovator's payoff, so one would expect that the more efficient firm should minimize the knowledge spillover. By allowing the more efficient firm to increase spillover, we find that the conventional expectation holds only under certain circumstances. More specifically, our analysis shows that under high degrees of asymmetry, the more efficient firm is better off by unilaterally increasing spillover to induce the competitor to adopt an imitative strategy. In contrast, under sufficiently low degrees of asymmetry, even if the competitor is induced to adopt an imitative strategy, the more efficient firm's payoff is not increased at the transition point. This result is in contrast to that of Pacheco-de Almeida and Zemsky (2012)

Overall, our paper suggests that firms facing investment decisions in R&D projects need to take the uncertain direction of the spillover into account. In particular, our analysis suggests that, in the presence of uncertainty and natural spillover, the asymmetry of firms is an important factor that impacts the R&D investment strategies. Lastly, we show that reduced imitability of new innovations does not always improve the performance of innovative firms.

CHAPTER 4

MANAGING R&D RESOURCES THROUGH REAL OPTIONS LENS

4.1 Introduction

Research and development (R&D) is essential for the growth of a firm. To obtain sustainable competitive advantages, firms invest significant resources and time in R&D activities. For instance, in 2015, Intel spent 20.6% of its revenue on R&D, Google spent 14.9%, and Pfizer spent 16.9% (Jaruzelski et al., 2015). Because of the importance of R&D, the valuation of R&D projects and R&D investment strategies are critical to investors' decisions. However, determining a R&D project's value and making investment decisions can be difficult tasks because of two challenges. The first challenge is the uncertainty. R&D projects are often fraught by two types of uncertainties. One is the *market uncertainty* regarding the market demand for the developed products or processes. Many external factors can cause the market uncertainty, such as the uncertainty in the customers' preferences or government policies. For example, the huge demand for smartphones was not anticipated before iPhone was launched. The other type of uncertainty is *technological uncertainty*, which is the uncertainty around the duration of R&D projects[1]. For instance, in pharmaceutical industry, the timeline of drug development and regulatory approval can be very uncertain. The second challenge is the presence of strategic interaction among firms. For example, high market demand for an innovator's product can provide favorable signals to potential competitors and encourage them to invest in developing similar products. Hence, a firm's revenue from R&D also depends on other firms' R&D investment strategies. Due to these two challenges, a general question is important for managers: How should firms manage their R&D

[1] Some literature (for example, Oriani and Sobrero, 2008) refers technological uncertainty as the uncertain life of a technology. For example, a technology may become obsolete because a new technology emerges.

resources under uncertainty? To cope with the uncertainties, firms prefer the flexibility in the timing of decisions. For instance, they can wait to invest in R&D projects until the market demand of R&D products turns out to be high, or they can abandon projects when the demand turns out to be low. The disadvantage of employing traditional net present value (NPV) approach to evaluate R&D projects is that the NPV approach neglects the economic value of flexibility. To capture the economic value of flexibility under uncertainty, real options approach has been widely used in theory (see Li et al., 2007 for a review). The existing literature on investment under uncertainty has explored the answers to this general question (for example, Dixit and Pindyck 1994). This chapter contributes to this strand of research by answering three research questions using a game theoretic model:

(1) What is the impact of market uncertainty on firms' investment strategies and payoffs?

Since McDonald and Siegel (1986) first studied value of waiting in an investment under uncertainty, a few work have examined the relationship between value of waiting and market uncertainty (see Carruth et al. 2000 and Li et al. 2007 for a review). The general implication of this strand of research is that the market uncertainty increases the value of waiting. Then one might expect that firms tend to postpone investments. However, this expectation might not be true because of the following reasons: Firstly, a higher value of waiting does not necessarily mean firms wait longer. The intuition behind can be seen from Chapter 2. Secondly, most of the previous studies focus on decision theoretical models. Hence, they leave the question whether this expectation still holds in the presence of competition, because the threat of preemption can encourage firms to hasten investment. Due to these two reasons, it is unclear that whether firms hasten or delay their investments under higher market uncertainty.

To re-examine the impact of market uncertainty on firms' investment decisions and payoffs, we first adopt one type of definition of market uncertainty in the literature. In this chapter, market uncertainty is defined as the difference between the best demand and worst demand. Moreover, we assume firms can learn about the true attractiveness of a market if the firms are uncertain about it. More specifically, we assume firms learn about market demand through two ways: (i) firms always observe some public signals that partially resolve the market demand uncertainty; (ii) if one firm (the innovator) completes its R&D earlier, then the other firm has the opportunity to observe the innovator's profit stream from R&D products and learn about the true demand. Learning is very common in practice. For instance, Dixit and Chintagunta (2007) gave an example that discount airlines use sales data to learn market demand and make exit decisions. Hitsch (2006) also provided a model where firm learn the true demand from observed sales and he used the data of U.S. ready-to-eat breakfast cereal industry to support the predictions of his model. Because of the definition of market uncertainty and this learning effect, market uncertainty improves the signal-to-noise ratio, which is a measurement of learning speed. In other words, higher market uncertainty can improve the quality of signals, which come from firms' profit streams.

Our analysis shows that firms may or may not delay their investment for higher market uncertainty. Moreover, we also show that there can be a non-monotonic relationship between market uncertainty and the equilibrium payoff of the firm that invests first. This result is similar to that in Oriani and Sobrero (2008), but we provide an alternative explanation. Oriani and Sobrero (2008) show that the non-monotonic relationship is driven by growth options, whereas our results indicate that, even without a growth option, such a relationship can still exist due to the strategic interactions between firms. Hence, we suggest that firms need to consider the strategic interactions between firms when they evaluate R&D projects.

Moreover, our results contribute to the empirical studies and practices on the valuation of R&D projects. We suggest that managers and researchers have to be careful about how to measure market uncertainty. There are at least two kinds of measurement in the literature. One is noise in profit streams. This measurement is used when the true state of demand is known. The other is the variation in the true state of demand. The assumption of this measurement is that the true demand is unknown. These two measurements have qualitative different impacts on firms' investment payoffs. Firstly, the net present value of projects is not affected by uncertainty under the first measurement whereas it is affected under the second measurement. Secondly, in the presence of learning effect, the impacts of market uncertainty on the rate of learning are different under two measurements.

(2) How should an innovator manage its demand information resources? A relevant question is that whether an innovator has an incentive to voluntarily disclose market demand information to its competitor.

A motivating example for this question comes from the competition between Intel and AMD. In Chapter 3, we have illustrated the example that Intel shared intellectual properties with AMD in the early 1980s. In the early 1990s, before AMD launched AM486 in 1993, Intel publicly indicated that the promising demand of its new microprocessor 80486, which had been launched in 1989. "The demand growth was continuing in its latest-generation 386 and 486 microprocessors, or computer 'brain chip', and that growth for those products during the third quarter is expected to offset declines in the older Intel 386 chips."(Carlton, 1991). However, not all firms are willing to disclose demand information or knowledge like Intel did. Hence, we explore the incentives of voluntary disclosure, and our results indicate that firms can delay their potential competitors' entry by strategically disclosing demand information.

This question is related to the results in Chapter 3. In chapter 3, we show that a firm can benefit from unilaterally sharing its technological knowledge with its competitors. However, such benefits are only ex-ante benefits. In other words, unilaterally disclosing technological knowledge to competitors is not a subgame perfect equilibrium strategy. In this chapter, we show that disclosing demand information can benefit both ex-ante and ex-post payoffs. Hence, unilaterally disclosing demand information can be a subgame perfect equilibrium strategy. This difference can help to explain the above example in which Intel still had an incentive to give the news press after its 80486 microprocessor had launched. Therefore, our results suggest that the disclosed contents (demand information or technological knowledge) and the time to disclose play important roles when firms consider voluntarily disclosure.

(3) In the presence of technological uncertainty, how does the technological uncertainty affect a firm's incentive to invest? Does the longer time-to-completion induce fiercer competition?

When a firm wants to invest in new R&D projects to enter into a new industry or field, managers concern about the industry's fierceness of competition, because the fiercer competition increases the difficulty to gain competitive advantages for making sustainable profits. Moreover, from a practical perspective, the duration of R&D projects can be long and highly uncertain. Hence, there is a lag between a firm's time of investment in a project and the time to complete the project. We call this lag *time-to-completion*. The presence of non-zero time-to-completion has countervailing effects on firms' incentive to invest. On the one hand, the lag discourages investments, because the time discounting reduces the value of the projects and thus firms increase their thresholds for investment. On the other hand, the lag encourages investment, because firms may compete to secure the position of an innovator. Longer lags can increase an innovator's profit because the innovator can obtain monopoly profits for a longer time. Besides, in the presence of learning effect, the lag reduces a firm's incentive to wait for better signals from the innovators, because now the signals are not available immediately after the innovator's investment. Hence, the impacts of time-to-completion on firms' incentives to invest and the fierceness of competition are unclear. Moreover, the impacts are even more ambiguous when the time-to-completion is uncertain. Our results show that the competition may or may not be fiercer for a longer time-to-completion.

4.2 Related Literature

This study builds on the literature on competitive investment with investment lags under uncertainty. Under the umbrella of this topic, two streams of research are closely related to our work. The first stream considers the impact of market uncertainty on the value of waiting. Market uncertainty can affect the value of waiting in two ways. First, it encourages firms to wait to collect more demand information to mitigate risks (see Li et al., 2007 for a review). Secondly, it affects how fast firms can learn about the true state of the market demand. Higher learning rate also increases the value of waiting. When discussing the impact of market uncertainty, the prior research only takes the first force into account (for example, McDonald and Siegel 1986 and Oriani and Sobrero 2008), whereas the second force receives less attention. Also, competition can affect the impact of market uncertainty. In the presence of competition, the fear of being preempted can encourage firms to hasten their investment (Grenadier, 2002). We re-examine the impact of market uncertainty in a real option game in which firms can learn about the demand.

The second stream of research considers the impact of investment lags. The investment lag is also called time-to-build if the lag is deterministic. Majd and Pindyck (1987) and Friedl (2002) studied sequential investment problems with the assumption of endogenous time-to-build. With the assumption of exogenous time-to-build, Pacheco-de Almeida and Zemsky (2003) considered a duopoly game of investment under market uncertainty and show that the industries with longer time-to-build are more competitive. Pacheco-De-Almeida et al. (2008) used the data from petrochemical industry to empirically support this theoretical result. By assuming the randomness investment lags and firms have prior belief about market demand, our study finds that a longer investment lag may or may not induce fiercer competition.

This chapter also contributes to the literature on incentives of voluntary disclosure. Sometimes, firms are willing to freely reveal more knowledge or information. The incentives of voluntary disclosure are different, depending on who to share with. Firms have an incentive to share financial information or product demand information with their investors to mitigate the agency problem or signal their investors (Healy and Palepu, 2001; Campbell et al., 2001). The vertical sharing of information regarding market demand or production cost along a supply chain aims to eliminate the information asymmetry among supply chain members and achieve supply chain coordination (For example, see Chen 1998; Gavirneni et al. 1999; Lee et al. 2000). Compare to the extensive literature on sharing information with investors or supply chain members, the investigation of incentives to share information among competitors receives much less attention. The prior research suggests that firms are willing to share demand information for Bertrand competition, whereas share production cost information for Cournot competition (Gal-Or, 1985; Gal-or, 1986; Vives, 1984). These studies (for example, Gal-Or 1985; Gal-or 1986; Dong and Orhun 2016; Guo et al. 2014; Vives 1984), however, are about bilateral information sharing between the competitors. Our study considers the incentive of unilateral disclose. From this perspective, our work is more related to the study of Pacheco-de Almeida and Zemsky (2012) and Chapter 3. They show that unilateral technological knowledge spillover can be beneficial for an innovator, because the technological knowledge spillover can decrease competition pressure. We examine the impact of unilateral demand information sharing and show that it can also alleviate the competition.

4.3 Base Model

We now consider two identical firms indexed as $i \in \{1, 2\}$. Denote firm j to be the opponent of firm i. Each firm has a one-time opportunity to make an R&D investment, with cost k to enter a new market. Neither firm knows the true market demand, and we assume the time averaged market demand can be either high (μ^H) or low (μ^L). Before either of them invests, the firms receive some public information that can partially reveal the market demand. For instance, such information can be from public marketing research reports or demand information of complementary products. We assume that a firm starts to obtain profits immediately after its investment. We relax this assumption in Section 4.6. Once one firm invests first, the other firm can use its performance as an additional signal to learn about the market. Although this additional signal is still imperfect, it provides better quality than the public signal.

We denote τ_i to be firm i's investment time. Without loss of generality, we assume firm 1 to be the leader and firm 2 to be the follower, i.e. $\tau_1 \leq \tau_2$. The strategy profile of the game is given by $(\tau_1, \tau_2) \in [0, \infty] \times [0, \infty]$. We denote processes X^0,X^{1L}, and X^{iF} as follows:

(1) $X^0 = \{X_t^0 : t \in [0, \tau_2]\}$: the public information process before the follower invests.

(2) $X^{1L} = \{X_t^{1L} : t \in [\tau_1, \tau_2]\}$: the leader's cumulative profit before the follower invests. The follower does not receive any profit before τ_2.

(3) $X^{iF} = \{X_t^{iF} : t \in [\tau_2, \infty]\}_{i \in \{1,2\}}$: firm i's cumulative profit after the follower's investment.

We model X^0, X^{1L}, and X^{iF} as Brownian motions that satisfy

$$
\begin{aligned}
dX_t^0 &= \mu dt + \sigma_0 dW_t^0 \qquad \text{for } t \in [0, \tau_2] \\
dX_t^{1L} &= \mu dt + \sigma dW_t^1 \qquad \text{for } t \in [\tau_1, \tau_2] \\
dX_t^{iF} &= s\mu dt + \sigma dW_t^i \qquad \text{for } t \in [\tau_2, \infty], i \in \{1, 2\},
\end{aligned}
$$

where $s \in (0, 1]$ evaluates the negative externality between the two firms' investments. The parameters σ_0 and σ are the noise levels in the public signal and profit stream of the first firm. The drift $\mu > 0$ represents the time-averaged profit per unit time. It can be either μ^H or μ^L, but neither firm knows the true state of μ. The process W_t^0, W_t^1, and W_t^2 are mutually independent Wiener processes that represent white noises. Denote $\{\mathcal{F}_t : t \geq 0\}$ to be the natural filtration with respect to the observable cumulative profit process of firm 1 and the pre-investment signal process $\{X^0, X^L : t \geq 0\}$. Let p be the prior probability that $\mu = \mu^H$. Let $V_i(p; \tau_1, \tau_2)$ denote firm i's expected cumulative payoff that is discounted to time 0, given a strategy profile (τ_1,τ_2) conditional on p. The objective of each firm is to choose optimal time τ_i to maximize $V_i(p; \tau_1, \tau_2)$, given its opponent's strategy τ_j. The objective functions are as follows:

$$\begin{aligned} V_1(p; \tau_1, \tau_2) &= E^p\left[-ke^{-r\tau_1} + \int_{\tau_1}^{\tau_2} e^{-rt} dX_t^{1L} + \int_{\tau_2}^{\infty} e^{-rt} dX_t^{1F}\right], \\ V_2(p; \tau_1, \tau_2) &= E^p\left[-ke^{-r\tau_2} + \int_{\tau_2}^{\infty} e^{-rt} dX_t^{2F}\right], \end{aligned} \tag{4.1}$$

where r is the discount rate for both firms.

Let $P_t = \mathcal{P}[\mu = \mu^H \mid \mathcal{F}_t]$ represent the posterior belief. We obtain the posterior probability process for the case where neither firm invests and the case where only the leader invests.

Following steps of Bayes rule (Peskir and Shiryaev, 2006), we have

$$\begin{aligned} P_t &= E^{p_0}[1_{\{\mu=\mu^H\}} \mid \mathcal{F}_t] \\ &= \begin{cases} E^{p_0}[1_{\{\mu=\mu^H\}} \mid X_t^0] & \text{for } t < \tau_1 \\ E^{p_0}[1_{\{\mu=\mu^H\}} \mid X_t^0, X_t^{1L}] & \text{for } t \in [\tau_1, \tau_2] \end{cases} \\ &= \begin{cases} \left[1 + \frac{1-p_0}{p_0} \exp\{\frac{(X_t^0-\mu^L t)^2}{2\sigma^2 t} - \frac{(X_t^0-\mu^H t)^2}{2\sigma^2 t}\right]^{-1} & \text{for } t \in [0, \tau_1) \\ \Big\{1 + \frac{1-p_{\tau_1}}{p_{\tau_1}} \exp\Big[\frac{[X_t^0-X_{\tau_1}^0-\mu^L(t-\tau_1)]^2}{2\sigma_0^2(t-\tau_1)} + \frac{[X_t^L-X_{\tau_1}^L-\mu^L(t-\tau_1)]^2}{2\sigma^2(t-\tau_1)} & \\ \frac{[X_t^0-X_{\tau_1}^0-\mu^H(t-\tau_1)]^2}{2\sigma_0^2 t} - \frac{[X_t^L-X_{\tau_1}^L-\mu^H(t-\tau_1)]^2}{2\sigma^2 t}^{-1}\Big]\Big\}^{-1} & \text{for } t \in [\tau_1, \tau_2]. \end{cases} \end{aligned}$$

We define the following in order to construct the posterior belief process:

$$\begin{aligned} \tilde{\sigma} &= \sqrt{\frac{\sigma_0 \sigma}{\sigma_0 + \sigma}} \\ \tilde{W}_t &= \frac{\sigma^2}{\sigma_0^2 + \sigma^2} W_t^0 + \frac{\sigma_0^2}{\sigma_0^2 + \sigma^2} W_t^1 \\ \tilde{X}_t &= \frac{\sigma_i^2}{\sigma_0^2 + \sigma^2} X_t^0 + \frac{\sigma_0^2}{\sigma_0^2 + \sigma^2} X_t^L. \end{aligned} \tag{4.2}$$

The posterior belief process is constructed as

$$P_t = \begin{cases} \left\{1 + \frac{1-p_0}{p_0} \exp\left[-\frac{\mu^H-\mu^L}{\sigma_0^2}\left(X_t^0 - \frac{\mu^H+\mu^L}{2}t\right)\right]\right\}^{-1} & \text{for } t \in [0, \tau_1) \\ \left\{1 + \frac{1-p_{\tau_1}}{p_{\tau_1}} \exp\left[-\frac{\mu^H-\mu^L}{\tilde{\sigma}^2}\left(\tilde{X}_t - \frac{\mu^H+\mu^L}{2}t\right)\right]\right\}^{-1} & \text{for } t \in [\tau_1, \tau_2]. \end{cases}$$

Furthermore, P_t is the unique strong solution of the SDE:

$$\begin{cases} dP_t = P_t(1-P_t)\frac{\mu^H-\mu^L}{\sigma_0}d\hat{W}_t^0 & \text{for } t \in [0, \tau_1) \\ dP_t = P_t(1-P_t)\frac{\mu^H-\mu^L}{\tilde{\sigma}}d\hat{W}_t & \text{for } t \in [\tau_1, \tau_2]. \end{cases} \tag{4.3}$$

4.3.1 Follower's Optimal Strategy and Leader's Best Response

To solve the game and obtain the equilibrium, we first derive the follower's optimal strategy. Suppose firm 1 has invested at τ_1, and the current posterior belief is $P_{\tau_1} = p_1$. Denote $\tau_F = \tau_2 - \tau_1$. Given τ_1 and p_1, Firm 2's maximization problem $\sup_{\tau_2} V_2(p_1; \tau_1, \tau_2)$ is equivalent to

$$\sup_{\tau_F \geq 0} E^{p_1}\left[\left(\frac{s}{r}E^{P_{\tau_F}}[\mu] - k\right)e^{-r\tau_F}\right].$$

Let $V_{\tau_1}^F(p_1)$ denote the optimal payoff that is discounted to time τ_1 for the follower. The following lemma establishes $V_{\tau_1}^F(p_1)$ and the follower's optimal strategy at time τ_1.

Lemma 3 *At time τ_1, the follower's optimal strategy is to invest at time $\tau_2 = \tau_1 + \tau_F^*$, where $\tau_F^* = \inf\{t > 0 : P_t \geq \theta_F\}$. Moreover, its optimal payoff $V_{\tau_1}^F(p_1)$ is given by*

$$V_{\tau_1}^F(p_1) = \begin{cases} \frac{\psi(p_1,\gamma_1)}{\psi(\theta_F,\gamma_1)}(\frac{s}{r}E^{\theta_F}[\mu] - k); & \text{if } p_1 < \theta_F \\ \frac{s}{r}E^{p_1}[\mu] - k; & \text{if } p_1 \geq \theta_F \end{cases}, \tag{4.4}$$

where

$$\begin{aligned} \theta_F &= (1 + \frac{\gamma_1 - 1}{\gamma_1 + 1}\frac{s\mu^H - kr}{kr - s\mu^L})^{-1}, \\ \psi(x, \gamma) &= x^{\frac{\gamma+1}{2}}(1-x)^{-\frac{\gamma-1}{2}}, \\ \gamma_1 &= \sqrt{1 + \frac{8r\tilde{\sigma}^2}{(\mu^H - \mu^L)^2}}. \end{aligned} \tag{4.5}$$

Now we derive the leader's best response to the follower's optimal strategy. Denote $V^L_{\tau_1}(p)$ to be the leader's payoff from an immediate investment when the follower is expected to invest at time at τ^*_F and the posterior belief at τ_1 is p_1. It is given by

$$\begin{aligned} V^L_{\tau_1}(p_1) &= E^p[\int_0^{\tau^*_F} e^{-rt}dX^{1L}_t + \int_{\tau^*_F}^{\infty} e^{-rt}dX^{1F}_t - k] \\ &= \begin{cases} \frac{1}{r}E^{p_1}(\mu) - \frac{1-s}{r}E^{\theta_F}(\mu)\frac{\psi(p,\gamma_1)}{\psi(\theta_F,\gamma_1)} - k & \text{if } p < \theta_F \\ \frac{s}{r}E^p(\mu) - k & \text{if } p \geq \theta_F, \end{cases} \end{aligned} \tag{4.6}$$

where θ_F, γ_1, and $\psi(p,\gamma)$ are defined in Lemma 3. Then, given the initial belief at time zero p, firm 1's objective function $V_1(p;\tau_1,\tau_2)$ in (4.1) can be expressed as

$$V_1(p;\tau_1,\tau_1+\tau^*_F) = E^p\left[e^{-r\tau_1}(V^L_{\tau_1}(P_{\tau_1}) - k)\right].$$

Let

$$V^L_0(p) = \sup_{\tau_1 \geq 0} V_1(p;\tau_1,\tau_1+\tau^*_F) \tag{4.7}$$

be the optimal payoff that is discounted to time zero for the leader. Lemma 4 shows the leader's best response.

Lemma 4 *Given the follower's time of investment $\tau_2 = \tau_1 + \tau^*_F$, the leader's optimal strategy is to invest at $\tau^*_1 = \inf\{t \geq 0 : P_t \geq \theta_L\}$, where $\theta_L \in (0,\theta_F)$ is the unique root to the following equation*

$$\frac{1+\gamma_0-2\theta_L}{2\theta_L(1-\theta_L)}(\frac{1}{r}E^{\theta_L}(\mu)-k) = \frac{1}{r}(\mu^H-\mu^L) - \frac{1-s}{r}E^{\theta_F}(\mu)\frac{\psi(\theta_L,\gamma_1)}{\psi(\theta_F,\gamma_1)}\frac{\gamma_0-\gamma_1}{2\theta_L(1-\theta_L)}. \tag{4.8}$$

Here, $\gamma_0 = \sqrt{1+\frac{8r\sigma_0^2}{(\mu^H-\mu^L)^2}}$.

Moreover, the leader's optimal payoff $V^L_0(p)$ is given by

$$V^L_0(p) = \begin{cases} \frac{\psi(p,\gamma_0)}{\psi(\theta_L,\gamma_0)}[V^L_{\tau_1}(\theta_L) - k] & \textit{if } p \in [0,\theta_L), \\ V^L_{\tau_1}(p) - k & \textit{if } p_0 \in [\theta_L,\theta_F), \\ \frac{s}{r}E^p(\mu) - k & \textit{if } p_0 \in [\theta_F,1]. \end{cases}$$

Given the strategy profile $(\tau_1^*, \tau_1^* + \tau_F^*)$, we consequently obtain the follower's expected payoff $V_0^F(p)$ that is discounted to time 0:

$$V_0^F(p) = \begin{cases} \frac{\psi(p,\gamma_0)}{\psi(\theta_L,\gamma_0)}[\frac{\psi(\theta_L,\gamma_1)}{\psi(\theta_F,\gamma_1)}(\frac{s}{r}E^{\theta_F}(\mu) - k)] & \text{if } p_0 \in [0, \theta_L), \\ \frac{\psi(p,\gamma_1)}{\psi(\theta_F,\gamma_1)}(\frac{s}{r}E^{\theta_F}(\mu) - k) & \text{if } p_0 \in (\theta_L, \theta_F), \\ \frac{s}{r}E^p(\mu) - k & \text{if } p_0 \in [\theta_F, 1]. \end{cases}$$

4.3.2 Characterization of the Equilibrium

In this section, we obtain the pure strategy Markov perfect equilibrium. Firstly, we compare $V_{\tau_L}^L(p)$ and $V_{\tau_L}^F(p)$ to determine whether the equilibrium is a preemption equilibrium or a war of attrition equilibrium. If $V_{\tau_L}^L(p) > V_{\tau_L}^F(p)$, then both firms would have an incentive to preempt their opponents to receive higher payoff. In contrast, if $V_{\tau_L}^L(p) < V_{\tau_L}^F(p)$, then both firms have an incentive to wait to be the follower. Lemma 5 shows that both of these two types of equilibrium can exist.

Lemma 5 *(1) There exists a unique $\theta_C \in (0, \theta_F)$ at which $V_{\tau_L}^L(p) < V_{\tau_L}^F(p)$ for $p \in [0, \theta_C)$ and $V_{\tau_L}^L(p) > V_{\tau_L}^F(p)$ for $p \in (\theta_C, \theta_F)$.*

(2) The inequality $\theta_L < \theta_C$ holds if only if

$$\frac{1}{r}(\mu^H - \mu^L) - \frac{\psi_x'(\theta_1, \gamma_1)}{\psi(\theta_F, \gamma_1)}\left[\frac{1}{r}E^{\theta_F}(\mu) - k\right] < 0. \tag{4.9}$$

Moreover, the inequality (4.9) holds for sufficiently small σ_1.

Lemma 5 reports the sufficient and necessary condition under which both types of equilibrium can arise for $p \in (\theta_L, \theta_F)$. Furthermore, Lemma 5 (2) indicates that, if the second mover's signal quality is significantly improved after the first mover's investment (sufficiently small $\tilde{\sigma}$), then a war of attrition equilibrium can arise for $p \in (\theta_L, \theta_F)$ due to the second-mover advantage. For the remainder of the paper, we call the interval $[0, \theta_C)$ to be the war of attrition (WA) region, the interval $[\theta_C, \theta_F)$ to be the preemption (PE) region.

Now we are in the position to determine pure strategy Markov perfect equilibrium (MPE). In this paper, we adopt the definitions of preemption policy in Agrawal et al. (2016) as follows: For $p < \theta_F$, if both firms have an incentive to preempt their opponent, to avoid the situation that firms invest concurrently, we assume that once one firm successfully invests earlier than its opponent with 50% chance, the other firm who loses the game immediately switches to a follower's optimal policy that is defined in Lemma 3. Using this definition of preemption policy and Theorem 1 in Agrawal et al. (2016), we obtain pure strategy MPE for our game.

Proposition 12 *(1) If $\theta_L \geq \theta_C$, there is a pure strategy equilibrium. In the equilibrium, both firms wait for $P_t \in [0, \theta_C]$; both firms take the preemption policy for $P_t \in [\theta_C, \theta_F)$, and for $P_t \in [\theta_F, 1]$, both firms invest immediately.*

(2) If $\theta_L < \theta_C$, there are two pure strategy equilibria. In the equilibrium, for $P_t \in [0, \theta_C]$, one firm invests at time $\tau_1^* = \inf\{t \geq 0, P_t \geq \theta_L\}$, and the other firm invests at τ_2^*. For $P_t \in [\theta_C, \theta_F]$, both firms take the preemption policy, and for $P_t \in [\theta_F, 1]$, both firms invest immediately. Here $\tau_2^* = \tau_1^* + \tau_F^*$, and $\tau_F^* = \inf\{t \geq 0, P_t \geq \theta_F\}$.

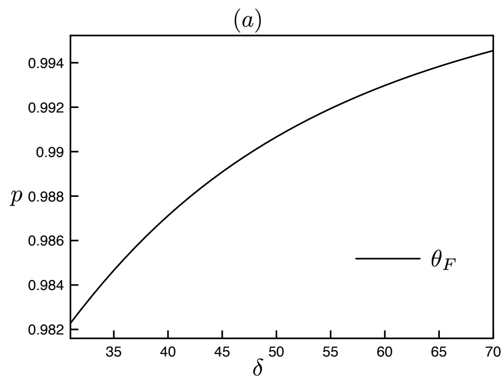
Proposition 12 shows that the characterization of equilibria depends on the relative magnitudes of θ_L and θ_C. If $\theta_L > \theta_C$, the first-mover advantage of monopoly profits always dominates the second-mover advantage of learning. Then both firms execute the preemption policy when P_t reaches θ_C. On the other hand, if $\theta_L < \theta_C$, the second-mover advantage dominates for $P_t \in (\theta_L, \theta_C)$ and the first-mover advantage dominates for $P_t \in (\theta_C, \theta_F)$. Hence, the game for $P_t \in (\theta_C, \theta_F)$ is exactly the same as in the case of $\theta_L \geq \theta_C$. For $P_t \in (\theta_L, \theta_C)$, the game becomes a war of attrition game and there are two pure strategy equilibria: either firm 1 invests as the leader or firms 2 does. Next we discuss the impact of uncertainty and incentives of voluntary disclosure based on the equilibrium strategies that are described in Proposition 12.

4.4 Impact of Market Uncertainty

In this section, we examine the impact of market uncertainty on the leader's investment strategy and equilibrium payoff. Before proceeding, we first define market uncertainty. We measure market uncertainty by demand variation. More specifically, we define $\delta \in (0, 1)$ as market uncertainty, where

$$\delta = \frac{\mu^H - \mu^L}{\mu^H + \mu^L}$$

Figure 4.1: The impact of δ on θ_F. Here we set $\bar{\mu} = 35$, $k = 200$, $r = 0.1$,$\sigma_0 = 50$,$\sigma_1 = 1$,$s = 0.5$.



so that μ^H and μ^L can be expressed as $\mu^H = \frac{\mu^H+\mu^L}{2}(1+\delta)$ and $\mu^L = \frac{\mu^H+\mu^L}{2}(1-\delta)$. Then for a fixed average $\bar{\mu} = (\mu^H + \mu^L)/2$, larger δ represents higher market uncertainty. We also assume $\bar{\mu} \geq kr/s$. Hence, if there is no market uncertainty, both firms have incentives to invest.

For the remainder of this paper, we focus on the leader's case, because the follower's investment strategy has been discussed in the literature (eg. Kwon and Lippman, 2011) that consider decision theoretical models. From Proposition 12, the equilibrium strategies are characterized by thresholds θ_L, θ_C and θ_F, so we first examine the comparative statics of the thresholds with respect to the market uncertainty.

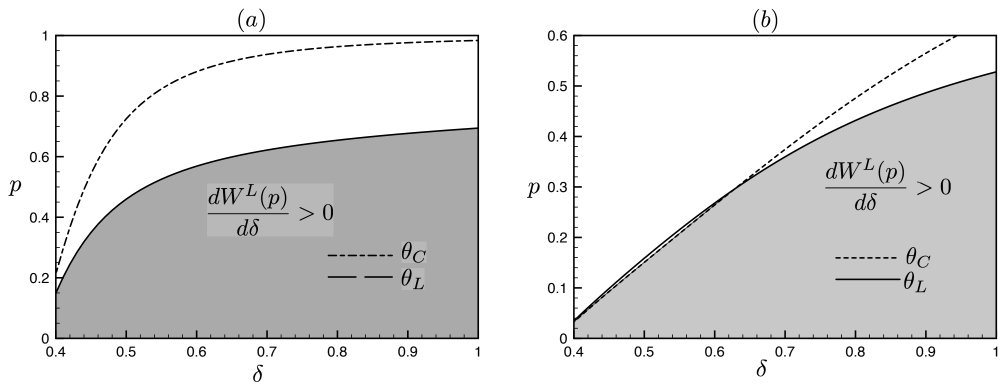
Lemma 6 *(1) θ_F increases in δ.*

(2) For sufficiently small σ_0, θ_L increases in δ.

In a single decision maker's case, higher makes uncertainty can affect a firm's investment threshold through two forces. Firstly, it can change the net present value (NPV) of the project. More specifically, given the belief p about the market demand, for a single decision maker, the NPV of the project is $E^p[\mu] = \mu^H p + \mu^L(1-p) = \bar{\mu}[1 + (2p-1)\delta]$. Hence, the NPV can increase or decrease in market uncertainty depending on the value of p. Secondly, according to the definition in Chapter 1, the rate of learning is defined as $(\mu^H - \mu^L)/\sigma$, which is rewritten as $2\bar{\mu}\delta/\sigma$. The results in Chapter 1 show that the single decision maker's investment threshold increases as the rate of learning increases.[2] Lemma 6 (1) indicates the net effect of these two

[2]In Chapter 2, we derive the follower's investment strategy given the leader has invested.

Figure 4.2: Impact of δ on the Leader's Value of Waiting: The parameters for (a): $\bar{\mu} = 4$; $k = 25, r = 0.1, s = 0.5, \sigma_1 = 1, \sigma_0 = 50$. The parameters for (b): $\bar{\mu} = 4$; $k = 25, r = 0.1, s = 0.5, \sigma_1 = 80, \sigma_0 = 100$

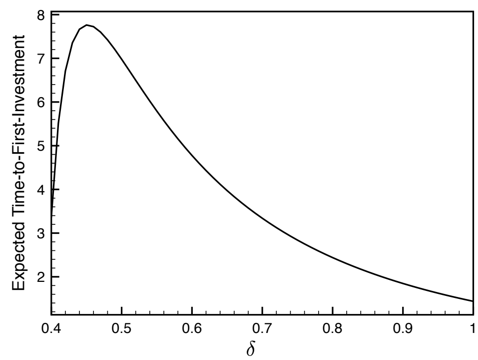


forces, and shows that higher market uncertainty always increases the single decision maker's threshold (θ_F). Figure 4.1 supplements this analysis with a numerical example.

From Proposition 12, the leader invests either at θ_L or θ_C, depending on the relative magnitudes of the thresholds. For notational simplicity, let $\tilde{\theta} = \min\{\theta_L, \theta_C\}$ denote the leader's investment threshold. Lemma 6 indicates that, without the threat of preemption (in the absence of θ_C), higher market uncertainty increases the leader's investment threshold $\tilde{\theta} = \theta_L$. Due to technical complexity, we could not obtain the general analytical results for the comparative statics of θ_L or θ_C with respect to δ. Hence, we provide numerical examples in Figure 4.2 to get some insights. Figure 4.2 (a) gives an example in which θ_L is always higher than θ_C, and Figure 4.2 (b) gives an example in which θ_C is lower than θ_L for low market uncertainty. Both Figure 4.2 (a) and (b) show that θ_L and θ_C always increase in market uncertainty. The intuition behind the growing pattern of the threshold is as follows. Firstly, θ_L increases in δ because higher uncertainty increases the leader's incentive to wait. Secondly, the threshold θ_C measures the competition in a preemption game where both firms prefer being the leader,and the competition for preemption is not fierce if θ_C is high. When δ increases, the improved learning rate benefits the follower more than the leader, then the competition pressure for preemption is reduced. Consequently, the threshold

This is the same as a single decision maker's case. Chapter 2 shows that θ_F increases in the rate of learning β.

Figure 4.3: Impact of δ on expected time-to-first-investment. Here we set $\bar{\mu} = 4$, $k = 25$, $r = 0.1$,$s = 0.5$,$\sigma_0 = 50$,$\sigma_1 = 1$, and $p = 0.1$.



θ_C decreases.

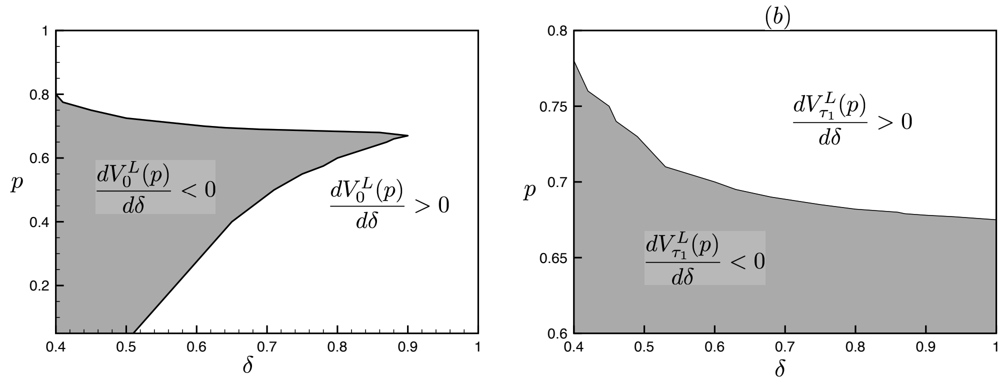
Knowing the comparative statics of $\tilde{\theta}$, we now examine the impact of market uncertainty on the leader's equilibrium strategy. From Proposition 12, the leader's strategy is to invests at $\tau_L^* = \inf\{t \geq 0 : P_t \geq \tilde{\theta}\}$. More specifically, we study the comparative statics of $E^p[\tau_L^* \mid \tau_L^* < \infty]$. Similar to the analysis in Chapter 2, we obtain

$$\begin{aligned} E^p[\tau_L^* \mid \tau_L^* < \infty] &= \frac{4\sigma_0^2}{(\mu^H - \mu^L)^2} \log\left(\frac{p}{\tilde{\theta}}\frac{1-\tilde{\theta}}{p}\right) \\ &= \frac{\sigma_0^2}{\bar{\mu}^2\delta^2} \log\left(\frac{p}{\tilde{\theta}}\frac{1-\tilde{\theta}}{p}\right), \end{aligned}$$

where p is the prior belief about the market demand. An increase in δ increases both $\tilde{\theta}$ and the rate of learning, so $E^p[\tau_L^* \mid \tau_L^* < \infty]$ can either increase because the leader needs more time to reach a higher threshold, or decrease because the leader needs less time to acquire the same amount of information. Figure 4.3 provides an numerical example to show the net effect of these two forces. From Figure 4.3, we observe that the expected time-to-first-investment first increases then decreases in market uncertainty.

Next we investigate the leader's equilibrium payoff (ex-ante) $V_0^L(p)$. Figure 4.4 (a) shows that $V_0^L(p)$ can either increase or decrease in market uncertainty, depending on the prior belief p. More specifically, for sufficiently small p, $V_0^L(p)$ first decreases then increases in market uncertainty. For sufficiently

Figure 4.4: Impact of δ on Leader's Payoff $V_0^L(p)$. Here we set $\bar{\mu} = 4$; $k = 25, r = 0.1, s = 0.5, \sigma_1 = 1, \sigma_0 = 50$.



large p, market uncertainty always increases $V_0^L(p)$. This non-monotonic relationship is also seen in Oriani and Sobrero (2008). They find the same relationship between market uncertainty and R&D project's valuation. Their explanation is that an embedded growth option drives this non-monotonic relationship. However, there is no growth option in our model, then why does this U-shape relationship still hold in some situations? To derive the insights, we decompose $V_0^L(p)$ into two parts. One is the leader's NPV of the project The NPV at time zero coincides with the leader's ex-post payoff $V_{\tau_1}^L(p_1)$ in equation (4.6) with $p_1 = p$. The other is the leader's value of waiting. We express the value of waiting $W^L(p)$ as follows.

$$
\begin{aligned}
W^L(p) &= V_0^L(p) - V_{\tau_1}^L(p) \\
&= \begin{cases} \frac{\psi(p,\gamma_0)}{\psi(\tilde{\theta},\gamma_0)}[\frac{1}{r}E^{\tilde{\theta}}(\mu) - \frac{1-s}{r}E^{\theta_F}(\mu)\frac{\psi(\tilde{\theta},\gamma_1)}{\psi(\theta_F,\gamma_1)} - k] & \\ -\left[\frac{1}{r}E^{p}(\mu) - \frac{1-s}{r}E^{\theta_F}(\mu))\frac{\psi(p,\gamma_1)}{\psi(\theta_F,\gamma_1)} - k\right] & \text{if } p < \tilde{\theta} \\ 0 & \text{if } p \geq \tilde{\theta} \end{cases}
\end{aligned}
\quad (4.10)
$$

We first examine the comparative statics of $W^L(p)$ with respect to δ. Both Figure 4.2 (a) and (b) shows that the value of waiting increases in the degree of market uncertainty. This result supports the conventional view (McDonald and Siegel 1986; Li et al. 2007; Oriani and Sobrero 2008; Carruth et al. 2000), and it indicates that the impact of market uncertainty on value of waiting still holds even in the presence of threats of preemption.

Next, we examine the comparative statics of NPV with respect to δ. Figure 4.4 (b) exhibits similar pattern in Figure 4.4 (a): For a fixed p, market uncertainty can first decrease then increase NPV. This result is from the strategic interactions between the two firms: Higher market uncertainty improves the rate of learning, and consequently affects the follower's time-to-investment. As we discussed in Chapter 2 and the case for the leader, follower's time-to-investment can either increase or decrease in the rate of learning. Due to the negative externality between the two firms' investments, the follower's time-to-investment ultimately affects the leader's NPV of the project, i.e., earlier investment of the follower decreases the leader's NPV.

By examining the comparative statics of NPV and the value of waiting with respect to market uncertainty, we provides a different explanation from that of Oriani and Sobrero (2008): in the absence of growth options, a firm's ex-ante payoff can still have a non-monotonic relationship with market uncertainty due to the strategic interactions between firms. An implication of this result suggests that managers need to take the strategic interactions into account when evaluating R&D projects.

Another implication of the results is about the measurement of market uncertainty. In the literature, we find that people measure market uncertainty differently. Some work (for example, Avner Bar-Ilan, 1996;Weeds, 1999,Oriani and Sobrero 2008) use the noise in profit streams to measure the uncertainty, and some use the difference in payoffs between the best scenario and the worst scenario (for example, Pacheco-de Almeida and Zemsky 2003). One difference between these two measurements is that the first one does not affect a firm's NPV, but the second one does. Moreover, when firms can use profit streams to learn about a new market's demand, these two measurements can affect the rate of learning oppositely. Hence, our results suggest that managers and researchers need to be careful when they select the measurement of market uncertainty.

4.5 Incentive of Voluntary Disclosure

In this section, we explore the leader's incentive to voluntarily disclose market demand information to its competitor. We show that, because the follower has an option to wait after the leader's investment, the leader can be better off through strategically sharing market demand information with the follower. Furthermore, we show the difference between demand information spillover and technological knowledge spillover by comparing the results in this section with those of Chapter 3.

To examine the impact of voluntary disclosure, we assume that the leader can change the noise σ_1 in its profit stream $\{X_t^{1L}\}$. For instance, in the example of Intel and AMD in Section 4.1, Intel reduced the noise σ by publicly revealing more information about its sales of 80486 microprocessor. Next, we establish the comparative statics of leader's ex-post payoff $V_{\tau_1}^{p}(p_1)$ (the payoff after the investment) with respect to σ.

Proposition 13 *(1) For fixed $p_1 < \theta_F$, $V_{\tau_1}^{L}(p_1)$ decreases in σ for sufficiently small σ.*

(2) For sufficiently large p_1 that is close to θ_F , $V_{\tau_1}^{L}(p_1)$ increases in σ for sufficiently large σ.

Proposition 13 asserts the fact that reducing σ can increase the leader's ex-post payoff. The intuition is as follows: In our model, the signals are imperfect, and the follower has an option to wait after the leader's investment. Hence, the leader's ex-post payoff depends on when the follower invests: the follower's early investment is detrimental to the leader's payoff because of the negative externality between firms' investments. The follower's time-to-investment depends on its rate of learning $(\mu^H - \mu^L)/\tilde{\sigma}$ [3] and value of p_1. As shown in Chapter 2 and Kwon and Lippman (2011), higher learning rate can either increase or decrease the firm's time-to-investment due to the trade-off between a higher value of waiting and less time needed to gather enough information. Hence, the leader can strategically disclosing information (decrease σ) to increase the follower's rate of learning, and furthermore induce the follower to delay the investment.

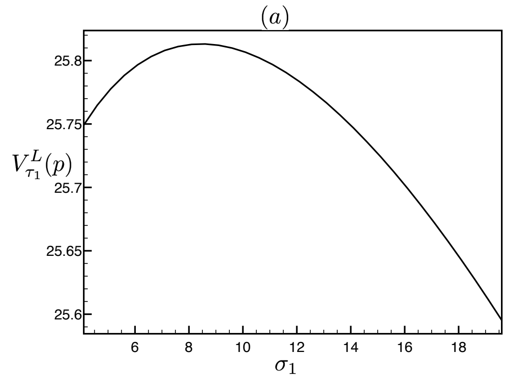
Now we examine the leader's ex-ante payoff $V_0^L(p)$ (the payoff before the investment). Because the leader's objective function at time 0 is to maximize $E^p[e^{-r\tau_1}V_{\tau_1}^L(p)]$, we have the following corollary that shows the impact of reducing σ_1 on $V_0^L(p)$.

Corollary 1 *(1) For fixed $p < \theta_F$, $V_0^L(p)$ decreases in σ_1 for sufficiently small σ_1.*

(2) For sufficiently large p that is close to θ_F , $V_0^L(p)$ increases in σ_1 for sufficiently large σ_1

[3]From the expression of $\tilde{\sigma}$ in formula (4.2), $\tilde{\sigma}$ increases in σ.

Figure 4.5: Impact of σ_1 on Leader's ex-post payoffs



Now we compare these results with those of Chapter 3. In Chapter 3, we find that an innovative firm can have an incentive to commit to sharing technological knowledge with its competitor unilaterally. The intuition is that, by committing sharing technological knowledge before R&D investments, the innovative firm can induce the follower to delay its investment, and thus reduce the competition pressure in R&D stage. However, once the innovative firm completes its R&D project, it tries to reduce the technological knowledge spillover as much as possible, because the competitor can utilize the technological knowledge as a substitute for its effort to reduce the R&D completion time, and consequently compete with the innovative firm in the product market sooner. In other words, sharing technological knowledge unilaterally can only improve an innovative firm's ex-ante payoff, but always diminishes the firm's ex-post payoff. Hence, unilaterally increasing technological knowledge spillover to competitors is not a subgame perfect equilibrium strategy. Likewise, this result that increasing technological knowledge spillover only works ex-ante is found in Pacheco-de Almeida and Zemsky (2012), which only discussed ex-ante payoffs. In contrast, Proposition 13 and Corollary 1 show that disclosing the demand information can benefit both the leader's ex-ante and ex-post payoffs. This is because the follower may delay its investment even after the leader's completion. Hence, increasing demand information spillover can be a subgame perfect equilibrium strategy. This difference between demand information spillover and technological knowledge spillover can help to explain the Intel-AMD examples in Section 1 of this Chapter and in Chapter 3: After its completion of 80486 microprocessor in 1989, Intel still voluntarily released market demand information of 80486 in 1991 by news press.

However, in the 1980s, Intel provided AMD with the access to its intellectual properties about Intel's successfully developed microprocessors (8086, 80186, 80286) only because it was required by the technology exchange agreement between the two firms in 1982. Due to the different mechanisms of technological knowledge spillover and demand information spillover, our results suggest that firms should be careful about the disclosed contents (demand information or technological knowledge) and the time to disclose when they consider voluntary disclosure to reduce competition.

4.6 Extended Model: Non-zero Time-to-Completion

In the previous section, we assume that a firm obtains profits immediately after its investment. However, from a practical perspective, the duration time of R&D projects can be long and highly uncertain. For example, the R&D time of a new drug can vary from 10 to 20 years (Dickson and Gagnon, 2009). To incorporate this characteristic, we define time-to-completion $T_i > 0$ as the duration of firm i's R&D project. It follows that firm i completes its R&D project at $T_i + \tau_i$ if it invests at τ_i. In line with the assumption in Section 4.3, we still assume firm 1 to be the leader who invests in R&D first. Because of the uncertain time-to-completion, a firm invests late can complete its R&D earlier depending on its opponent's time of investment and time-to-completion. Hence, in this section, the pre-investment signal and firms' profit streams become

$$\begin{aligned} dX_t^0 &= \mu dt + \sigma_0 dW_t^0 \qquad \text{for } t \in [0, \max_{i\in\{1,2\}}(\tau_i + T_i)), \\ dX_t^{iL} &= \mu dt + \sigma dW_t^i \qquad \text{for } t \in [\max_{i\in\{1,2\}}(\tau_i + T_i), \min_{i\in\{1,2\}}(\tau_i + T_i)), \\ dX_t^{iF} &= s\mu dt + \sigma dW_t^i \qquad \text{for } t \in [\min_{i\in\{1,2\}}(\tau_i + T_i), \infty), i \in \{1, 2\}. \end{aligned}$$

We assume that T_i is an exponential random variable with a rate λ_i, where λ_i represents firm i's R&D capability that is exogenously given. Higher λ_i indicates that firm i is expected to need less time to complete its R&D. In this section, we focus on the case of symmetric firms by assuming $\lambda_1 = \lambda_2 = \lambda$. There are two types of R&D expenses. One is the upfront setup cost $k \geq 0$ that occurs when a firm initiates its R&D. The other is the variable cost $c \geq 0$

per unit time[4]. We assume that these costs are identical for both firms. We investigate this extended model and examine the following question:what is the impact of non-zero time-to-completion on firms' investment strategies? By investigating this question, we ultimately find out the relationship between the length of time-to-completion and fierceness of competition.

4.6.1 Follower's Optimal Policy

In the spirit of backward induction, we first obtain the follower's optimal policy at time t after the leader's completion. Given that $t \geq \tau_1 + T_1$, the follower's payoff from an immediate investment is given by

$$\begin{aligned}\bar{R}_F(p) &= E^p[-\int_0^{T_2} ce^{-rt}dt + \int_{T_2}^{\infty} e^{-rt}dX_t^{2F}] \\ &= -c/(r+\lambda) + [s\lambda/r(r+\lambda)]E^p[\mu].\end{aligned} \tag{4.11}$$

For notational simplicity, let $\hat{k} = -k - c/(r+\lambda)$, $\bar{s} = s\lambda/(r+\lambda)$. Moreover, the quality of firm 2's signals (i.e., rate of learning) are improved after $T_1+\tau_1$, at which time the leader begins to obtain profits. Hence, firm 2's objective is to choose τ_2 to maximize

$$\begin{aligned}&E^p\left[e^{-r\tau_2}\left(\bar{R}_F(P_{\tau_2}) - k\right) \mid \mathcal{F}_{\tau_1+T_1}\right] \\ =\ &E^p\left[e^{-r\tau_2}\left(\tfrac{\bar{s}}{r}E^{P_{\tau_2}}[\mu] - \bar{k}\right) \mid \mathcal{F}_{\tau_1+T_1}\right].\end{aligned}$$

Solving this maximization problem yields that, for $t \geq T_1 + \tau_1$, the follower's optimal policy is to invest at $\tau_2 = \tau_1 + T_1 + \hat{\tau}_F^0$, where $\hat{\tau}_F^0 = \inf\{t \geq \tau_1 + T_1 : P_t \geq \bar{\theta}_F\}$, and

$$\bar{\theta}_F = \left(1 + \frac{\gamma_1 - 1}{\gamma_1 + 1}\frac{\bar{s}\mu^H - \hat{k}r}{\hat{k}r - \bar{s}\mu^L}\right)^{-1}. \tag{4.12}$$

[4]Here we assume the variable cost has no dependence on λ, because we will study the firms' incentive to increasing λ by excluding the impact of costs in the future plan.

Let $\hat{V}^F_{\tau_1+T_1}(p) = \sup_{\tau_2 \geq \tau_1+T_1} E^p \left[e^{-r\tau_2} \left(\bar{R}_F(P_{\tau_2}) - k \right) \mid \mathcal{F}_{\tau_1+T_1} \right]$ be the optimal payoff discounted to time $\tau_1 + T_1$ for the follower. It is given by

$$\hat{V}^F_{\tau_1+T_1}(p) = \begin{cases} \frac{\psi(p,\gamma_1)}{\psi(\bar{\theta}_F,\gamma_1)} \left(\bar{R}_F(\bar{\theta}_F) - k \right) & \text{if } p < \bar{\theta}_F, \\ \bar{R}_F(p) - k & \text{if } p \geq \bar{\theta}_F. \end{cases}$$

Secondly, we derive the follower's policy if the current time $t \in [\tau_1, \tau_1 + T_1)$, i.e., when the leader is still active in R&D. At time $t \in [\tau_1, \tau_1 + T_1)$, the follower's objective is to choose τ_2 to maximize its expected payoff $\hat{V}(p, t; \tau_1, \tau_2)$ and obtain the optimal expected payoff $\bar{V}^F_t(p)$. Mathematically, $\bar{V}^F_t(p)$ is expressed as the following:

$$\begin{aligned} \bar{V}^F_t(p) &= \sup_{\tau_2} \hat{V}(p, t; \tau_1, \tau_2) \\ &= \sup_{\tau_2} E^p \Big[1_{\{\tau_2 < T_1 + \tau_1\}} e^{-r\tau_2} \left(\tilde{R}_F(P_{\tau_2}) - k \right) \qquad (4.13) \\ &\quad + 1_{\{\tau_2 \geq T_1 + \tau_1\}} e^{-r\tau_2} \left(\bar{R}_F(P_{\tau_2}) - k \right) \mid \mathcal{F}_t \Big] . \qquad (4.14) \end{aligned}$$

Here $\bar{R}_F(p)$ is defined in (4.11), and $\tilde{R}_F(p)$ is the follower's payoff from an immediate investment before the leader's completion. It is given by

$$\begin{aligned} \tilde{R}_F(p) &= E^p \left[1_{\{T_1 < T_2\}} \int_{T_2}^{\infty} e^{-rt} dX^{2F}_t dt + 1_{\{T_1 \geq T_2\}} \left(\int_{T_2}^{T_1} e^{-rt} dX^{2L}_t dt \right. \right. \\ &\quad \left. \left. \int_{T_2}^{\infty} e^{-rt} dX^{2F}_t dt \right) \right] - \frac{c}{r + \lambda} \\ &= \frac{\tilde{s}}{r} E^p[\mu] - \frac{c}{r + \lambda}, \end{aligned}$$

where $\tilde{s} = [\lambda/(r + 2\lambda)][s\lambda/(r + \lambda) + (r + s\lambda)/(r + \lambda)]$.

We obtain the follower's optimal policy by solving the optimal stopping problem in (4.13). It is summarized in the following lemma.

Lemma 7 *Define*

$$\hat{\pi}^F_t(p) = \left[\frac{\lambda}{\tilde{r} + \lambda} \right] \frac{\psi(p, \gamma_1)}{\psi(\bar{\theta}_F, \gamma_1)} (\bar{R}_F(\bar{\theta}_F) - k), \qquad (4.15)$$

where $\tilde{r} = r(1 - \tilde{\sigma}^2/\sigma_0^2)$. For $t \in [\tau_1, \tau_1 + T_1)$, there exists $\hat{p} \in (0, \bar{\theta}_F)$ such that the follower's optimal policy is to invest at time $\bar{\tau}_2 = \inf\{\tau_1 \leq t < \tau_1 + T_1 : P_t \geq \hat{p}\}$, where $\hat{p}$ satisfies $\hat{\pi}^F_t(\hat{p}) = \tilde{R}_F(\hat{p}) - k$. Moreover, at time t

, *the follower's optimal payoff* $\bar{V}_t^F(p)$ *is given by*

$$\bar{V}_t^F(p) = \begin{cases} \hat{\pi}_t^F(p) & \text{if } p < \hat{p}, \\ \tilde{R}_F(p) - k & \text{if } p \geq \hat{p}. \end{cases} \tag{4.16}$$

Lemma 7 implies the impact of technological uncertainty on the follower's investment strategy. In the presence of random time-to-completion, waiting has two countervailing effects on the follower's payoffs. On the one hand, waiting benefits the follower because the follower can collect more information about the market demand. On the other hand, waiting can be detrimental because the leader is more likely to complete its R&D first. Due to the second effect, the investment threshold $\hat{p}$ is smaller than $\bar{\theta}_F$, which is the follower's investment threshold in the absence of technological uncertainty (i.e., the follower definitely completes its R&D late.).

4.6.2 Characterization of Equilibrium

In this section, we obtain the leader's best response given the follower's optimal policy that is described in Lemma 7, and characterize the pure strategy equilibrium. As a preliminary step, we compute the leader's expected payoff from an immediate investment.

Given the current posterior belief p, denote the leader's payoff from an immediate investment to be $\bar{R}^L(p)$. It satisfies the following differential equation:

$$\begin{aligned} (1 + rdt)\bar{R}^L(P_t) &= \lambda dt \left[\int_0^{T_2+\tau_2} e^{-rs} dX_s^{1L} + \int_{T_2+\tau_2}^{\infty} e^{-rs} dX_s^{1F} \right] \\ &\quad + (1 - \lambda dt)\bar{R}^L(P_{t+dt}) - cdt\,. \end{aligned}$$

Solving the above differential equation yields that

$$\bar{R}^L(p) = \frac{\lambda}{r+\lambda}\frac{1}{r}E^p[\mu] - \frac{1-s}{r}\frac{\lambda}{\tilde{r}+\lambda}\frac{\lambda}{r+\lambda}E^{\bar{\theta}_F}[\mu]\frac{\psi(p,\gamma_1)}{\psi(\bar{\theta}_F,\gamma_1)} - \frac{c}{r+\lambda},$$

where $\tilde{r}$ is the same as in equation (4.15). Then the leader's optimal payoff at time 0 is given by $\hat{V}_0^L(p) = \sup_{\tau_1} E^p[e^{-r\tau_1}(\bar{R}^L(P_{\tau_1}) - k)]$. Similar to the analysis in Section 4.3, we have the following lemma to characterize the leader's best response if the follower's optimal strategy is $\bar{\tau}_2$ that is defined in Lemma 7.

Lemma 8 *The leader's best response to firm 2's optimal strategy $\bar{\tau}_2$ is to invest at $\bar{\tau}_1 = \inf\{t \geq 0 : P_t \geq \bar{\theta}_L\}$, where $\bar{\theta}_L \in (0, \hat{p})$ is the unique root to the equation*

$$\frac{1+\gamma_0-2\bar{\theta}_L}{2\bar{\theta}_L(1-\bar{\theta}_L)}\left(\bar{R}^L(\bar{\theta}_L)-k\right) = \frac{\lambda}{r+\lambda}\frac{1}{r}(\mu^H-\mu^L) - \frac{1-s}{r}\frac{\lambda}{\tilde{r}+\lambda}\frac{\lambda}{r+\lambda} E^{\bar{\theta}_F}[\mu]\frac{\psi(\bar{\theta}_L,\gamma_1)}{\psi(\bar{\theta}_F,\gamma_1)}\frac{1+\gamma_1-2\bar{\theta}_L}{2\bar{\theta}_L(1-\bar{\theta}_L)}. \tag{4.17}$$

Given the follower's optimal policy that is to invest once the posterior belief reaches $\hat{p}$, we obtain the leader's payoff $\bar{V}^L_{\tau_1}(p)$ at the time of its investment as follows:

$$\bar{V}^L_{\tau_1}(p) = \begin{cases} \bar{R}^L(p) - k & \text{if } p < \hat{p}, \\ \bar{V}^F_{\tau_1}(p) & \text{if } p \geq \hat{p}. \end{cases}$$

For $p > \hat{p}$, $\bar{V}^L_{\tau_1}(p)$ is identical to $\bar{V}^F_{\tau_1}(p)$ that is defined in (4.16), because two identical firms invest simultaneously. Before deriving the equilibrium, we compare the leader's payoff $\bar{V}^L_{\tau_1}(p)$ and the follower's payoff $\bar{V}^F_{\tau_1}(p)$ to determine the war of attrition region and the preemption region.

Lemma 9 *For $p \in [0, \hat{p})$, there exists $\hat{\theta}_C \in (0, \hat{p})$ such that $\bar{V}^F_{\tau_1}(p) > \bar{V}^L_{\tau_1}(p)$ for $p \in (0, \hat{\theta}_C)$ and $\bar{V}^L_{\tau_1}(p) > \bar{V}^L_{\tau_1}(p)$ for $p \in (\hat{\theta}_C, \hat{p})$.*

Similar to Lemma 5, Lemma 9 shows the existence of both war of attrition and preemption region. Also, there is a simultaneous-move region, which is the interval $[\hat{p}, 1]$. Now we are in the position to characterize the pure strategy equilibrium.

Proposition 14 *(1) If $\bar{\theta}_L \geq \hat{\theta}_C$, there is a pure strategy equilibrium. In the equilibrium, both firms wait for $P_t \in [0, \hat{\theta}_C]$; both firms take the preemption policy for $P_t \in [\hat{\theta}_C, \hat{p})$, and when $P_t \in [\hat{p}, 1]$, both firms invest immediately.*

(2) If $\bar{\theta}_L < \hat{\theta}_C$, there are two pure strategy equilibria. In the equilibrium, for $P_t < \hat{\theta}_C$, one firm invests at time $\bar{\tau}_1$, and the other firm invests at $\bar{\tau}_2$. For $P_t \in [\hat{\theta}_C, \hat{p}]$, both firms take the preemption policy, and when $P_t \in [\hat{p}, 1]$, both firms invest immediately.

At each point of time, whether a firm waits or invests depends on benefit and cost of waiting. Here, the benefit of postponing an investment is that firms can collect more information about market uncertainty. The opportunity cost of waiting can be in different forms. Firstly, waiting reduces the value of future payoffs due to time discounting. Secondly, waiting of the follower reduces its likelihood of leapfrogging the leader to complete R&D earlier than the leader. Thirdly, if there are no firms on the market, waiting can cost a firm to lose the chance to preempt. The incentive of preemption can be either to discourage the opponent's investment, or to obtain a high monopoly profit. The trade-off between the benefit and cost of waiting determines the investment thresholds $\hat{p}$,$\bar{\theta}_L$, and $\hat{\theta}_C$. In the next section, we investigate how the length of time-to-completion affects this trade-off, and further affects firms' investment strategies and the fierceness of competition.

4.6.3 Impact of Time-to-Completion

When a firm wants to enter into a new industry, managers concern about the industry's fierceness of competition, because the fiercer competition increases the difficulty to gain competitive advantages to make sustainable profits. In this section, we investigate the impact of time-to-completion on the fierceness of competition.

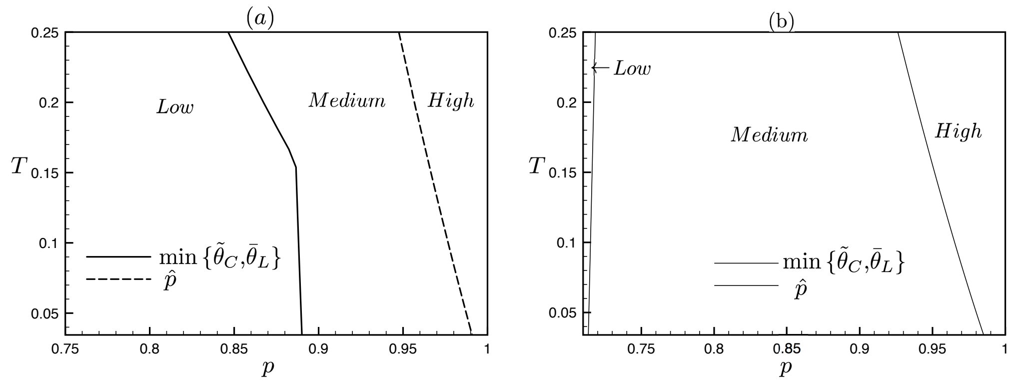
According to the property of Exponential distribution, let $T^* = 1/\lambda$ be the expected time-to-completion. In our paper, we measure the fierceness of competition by the number of firms that invest in R&D at time 0. From Proposition 14, both firms wait for the initial belief $p \in [0, \min\{\bar{\theta}_L, \hat{\theta}_C\})$, firms invest sequentially for $p \in [\min\{\bar{\theta}_L, \hat{\theta}_C\}, \hat{p})$, and both firm invest immediately at time 0 if $p \in [\hat{p}, 1]$. Hence, we call the degree of competition is low for $p \in [0, \min\{\bar{\theta}_L, \hat{\theta}_C\})$, medium for $p \in [\min\{\bar{\theta}_L, \hat{\theta}_C\}, \hat{p})$, and high for $p \in [\hat{p}, 1]$.

Because we do not have the closed form solution for neither $\min\{\bar{\theta}_L, \hat{\theta}_C\}$ or $\hat{p}$, we cannot obtain the analytical results for the impact of time-to-completion on the thresholds. Hence, we illustrate the results through numerical examples in Figure 4.6 to gain some insights. Figure 4.6 (a) shows the impact of expected time-to-completion upon the degree of competition for sufficiently small duopoly share s, and Figure 4.6 (b) shows the case for sufficiently large s.

Firstly, we observe that when the initial belief p is sufficiently large, higher T^* increases the degree of competition, regardless of the size of s. This observation indicates that if the market demand is promising (p is high), the competition is fiercer for a longer time-to-completion. The intuition behind it is as follows: When the new market is promising, at least one firm invests immediately. Whether other firms follow simultaneously or wait for the resolution of the market uncertainty depends on the value of waiting. Higher T^* reduces the follower's value of waiting, because the follower has to wait longer for better signals and the better signals become available only after the leader's completion. Hence, the followers are encouraged to invest earlier to gain profits sooner. Consequently, the degree of the competition is high.

For sufficiently small p, We observe that a longer time-to-completion can have different impacts on the degree of competition depending on the size of s. If the new market is not promising (p is low), once one firm invests, the other firm tends to wait for the leader's completion and invests until its belief about the market demand is high enough. Then a firm has the incentive to invest as the leader only when either it is confident that the market demand is good enough ($P_t \geq \bar{\theta}_L$), or it is better to be the leader than to be the follower. For sufficiently small s, the follower's payoff is so small that both firms have the incentive to preempt each other. Higher T^* even strengthens such incentives of preemption because now the follower needs more time to finish its R&D and consequently the leader enjoys the monopoly profit for a longer period. Therefore, as shown in Figure 4.6 (a), longer time-to-completion increases the degree of competition. This result is consistent with Pacheco-De-Almeida et al. (2008). In contrast, different from Pacheco-De-Almeida et al. (2008), for sufficiently large s, the opportunity cost of waiting is small, because the time average profits (the drifts in the profit streams) are similar for two firms. Hence, the firms are encouraged to wait until their belief of the demand is sufficiently high. Ceteris paribus, a firm's reward from an immediate investment decreases in T^* due to the time discounting. Hence, firms increase their investment threshold $\bar{\theta}_L$ for higher T^*. Consequently, as suggested in Figure 4.6 (b), longer time-to-completion reduces the degree of competition.

Figure 4.6: Impact of T on $\hat{\theta}$: The parameters are $\mu^H = 60$, $\mu^L = 0$, $\sigma_1 = 1;\sigma_0 = 50;k = 110,c = 1$, $r = 0.1$; $s = 0.3$ for (a) and $s = 0.5$ for (b).



4.7 Conclusions

The valuation of R&D projects and deciding R&D investment strategies can be challenging due to the uncertainties and the strategic interactions between firms. In this chapter, we develop a game theoretical model by using real options approach. We mainly provide insights to the managers of firms that want to invest in R&D on understanding the impact of uncertainty and managing demand information resources strategically. Our results have four implications. Firstly, our results suggest practitioners and researchers need to choose proper measurement of market uncertainty because different measurements of market uncertainty can have different impacts on firms' investment strategies and payoffs. Secondly, the results suggest that strategic interaction also needs to be considered when evaluating R&D projects, because the interactions can affect the net present value of a project. Thirdly, we provide insights for firms that want to invest in R&D projects by showing the impact of time-to-completion on the fierceness of competition. Lastly, this chapter points out the difference between disclosing technological knowledge and demand information, and our results indicate that the disclosed contents (demand information or technological knowledge) and the time to disclose are important when firms consider voluntary disclosure.

In the future, some extensions of the current work can be undertaken. One possible extension is to examine how firms manage their R&D resources. A relevant research question can be, if firms control their efforts in R&D (such as the capital or human resources allocated to R&D), does a firm always want to put more efforts (even the cost of inserting extra efforts are small)? The answer to this question may provide an alternative explanation to the phenomenon that R&D spending does not necessarily increase profits. For example, according to the data from Bloomberg, in 2015, Apple Inc. spent only 3.5% of its revenue, which number is relatively low compared to other large technology companies, such as Facebook, Qualcomm, and Alphabet that all spent much more than 10% of their revenues. Another extension is to continue the study of understanding the impact of market uncertainty. The current work only discusses the impact of market uncertainty in the absence of technological uncertainty. In the future, I plan to study how the presence of technological uncertainty affects the impact of market uncertainty on firms' investment strategies.

CHAPTER 5

CONCLUSION

In this dissertation, I study firms' decision-making regarding the management of operational resources under uncertainty by allowing the endogenization of the time-dimension of firms' decisions. I study two types of problems: (1) the impacts of uncertainty and (2) how to strategically manage operational resources.

The impacts of uncertainty have been widely studied in the literature. My dissertation contributes to this strand of research by examining these impacts in the context of real options games. Real options game theoretical models allow my work to investigate the impacts of uncertainties under strategic interactions and the flexibility of delaying actions. There are two types of uncertainties discussed in this work: market uncertainty and technological uncertainty.

Market uncertainty gives firms who want to enter a new market an incentive to collect signals to learn about the uncertainty. Such signals can come from various sources, like market research reports, news press, market performance from early investors, or sales data of complementary products. In the presence of learning effects, Chapter 4 shows that market uncertainty has non-monotonic relationships with both a firm's investment time and payoffs. With higher market uncertainty, firms' rates of learning increase, so the impact of market uncertainty affects the impact of learning on firms' strategies. The relationships between firms' learning and time-to-investment, as well as between their learning and payoffs, are illustrated in Chapter 2. Due to strategic interactions and externalities, in a war of attrition game, an increased rate of learning that tends to benefit the follower may hasten or delay the first investment (and increase or decrease the leader's payoffs), depending upon the rate of learning and the firms' initial beliefs that the investment is profitable.

Chapter 3 and Chapter 4 study R&D investment models under technological uncertainty. In the presence of technological uncertainty, a firm that invests first is not necessarily the firm that completes R&D first. Chapter 3 shows that the presence of technological uncertainty reduces an innovative firm's incentive to unilaterally share technological knowledge with competitors, due to the fear of losing the R&D race. Similarly, Chapter 4 shows longer time-to-completion may or may not result in fiercer competition, depending on the firms' beliefs about market demand.

Conventional wisdom suggests that possession of resources is critical to attaining competitive advantages and sustainable profits. However, in addition to resources, when time is another dimension of firms' strategies, providing resources to competitors can be also beneficial. Chapter 3 and Chapter 4 support this view by investigating how to manage demand information resources and technological knowledge resources. More specifically, these two chapters examine the impacts of technological knowledge and demand information spillovers on firms' payoffs and find that disclosing a certain amount of demand information or technological knowledge to competitors can reduce competition pressure by inducing competitors to delay their investment in R&D projects. Moreover, Chapter 4 points out the difference between these two spillovers: only demand information spillover can benefit firms' ex-post payoffs. Hence, the results imply that the timing and contents of disclosure are also important.

APPENDIX A

PROOFS FOR CHAPTER 2

A.1 Some Preliminaries for Proposition 1

Here we provide the expressions for the notations used in Proposition 1:

$$\theta_F \equiv \left[1+\frac{\gamma-1}{\gamma+1}\cdot\frac{h(1+\alpha_F)-kr}{kr-\ell(1+\alpha_F)}\right]^{-1}, \tag{A.1}$$

$$\gamma \equiv \sqrt{1+8r\frac{\sigma^2}{(h-\ell)^2}} = \sqrt{1+\frac{8r}{\beta^2(h-\ell)^2}}, \tag{A.2}$$

$$\psi(x) \equiv x^{(\gamma+1)/2}(1-x)^{(1-\gamma)/2}. \tag{A.3}$$

In particular, the function $\psi(x)$ is an increasing fundamental solution to the differential equation $\mathcal{A}\psi(x) = 0$ (see, for example, Kwon and Lippman 2011), where

$$\mathcal{A} \equiv \frac{1}{2}\left(\frac{h-\ell}{\sigma}\right)^2 p^2(1-p)^2\partial_p^2 - r \tag{A.4}$$

is the r-excessive *characteristic operator* (Alvarez, 2003) for the process P.

A.2 Proofs of Mathematical Statements

Proof of Proposition 1

(i) We first consider the case $\tau_2 > 0$ to obtain $\sup_{\tau_2>0} V_2(p;T_1,T_1+\tau_2)$, i.e., the optimal policy under the condition that firm 1 already invested. We employ Theorem 3(A) of Alvarez (2001) to prove this proposition. For convenience, we follow the convention of Alvarez (2001) and define $g(p) = (1+\alpha_F)m(p)/r - k$, which is the follower's value of investing immediately. We also define a function $\Pi_1(p) = g(p)/\psi(p)$, where $\psi(p)$ is defined in (A.3).

Our goal is to prove that $\tau_F = \inf\{t > 0 : P_t \geq \theta_F\}$ is the optimal stopping time and that the function $\Pi_1(\theta_F)\psi(p) = \Pi_F(p)$ in (2.6) is the optimal value function. In order to prove it, by virtue of Theorem 3(A) of Alvarez (2001), we only need to prove that $g(\cdot)$ is non-decreasing, that $\Pi_1(\cdot)$ attains a unique global interior maximum at θ_F, and that $\Pi_1(\cdot)$ is non-increasing for $p > \theta_F$.

The derivative of $\Pi_1(\cdot)$ is given as follows:

$$\Pi_1'(p) = \frac{\psi'(p)\{(1-\gamma)[h(1+\alpha_F) - kr] + (1+\gamma)[\ell(1+\alpha_F) - kr]\}}{r\psi^2(p)(\gamma+1-2p)}(p - \theta_F)\,.$$

Note that $\gamma > 1$, $h(1+\alpha_F) > kr > \ell(1+\alpha_F)$, and $\gamma+1 > 2p$ for all $p \in (0,1)$. Hence, $\Pi_1'(p) > 0$ for $p < \theta_F$ and $\Pi_1'(p) < 0$ for $p > \theta_F$, and if follows that $\Pi_1(p)$ attains its global maximum at θ_F and that $\Pi_1(p)$ is decreasing for $p > \theta_F$. We conclude that $\Pi_F(p) = \sup_{\tau_2 > 0} V_2(p; 0, \tau_2) = V_2(p; 0, \tau_F)$.

If $\Pi_S(p) > \Pi_F(p)$, however, then the follower's optimal policy is to invest immediately at T_1 when the leader invests. For $p \geq \theta_F$, the inequality $\Pi_S(p) > \Pi_F(p)$ is always satisfied because $\alpha_S > \alpha_F$. Also, note that $\tau_F > 0$ a.s. if $p < \theta_F$. Thus, the optimal value function is given by $V_F(p) = \sup_{\tau_2 \geq 0} V_2(p; 0, \tau_2) = \max\{\Pi_F(p), \Pi_S(p)\}$, and the optimal stopping time is τ_F if $\Pi_F(p) \geq \Pi_S(p)$ and 0 if $\Pi_S(p) > \Pi_F(p)$.

(ii) Note that $\Pi_S(\theta_F) > \Pi_F(\theta_F)$ because $\alpha_S > \alpha_F$. Furthermore, $\Pi_S(0) < \Pi_F(0)$ because $\Pi_S(0) = \ell(1+\alpha_S)/r - k < 0$ by Assumption 1 while $\Pi_F(p) > 0$ for all $p \in (0,1)$. We also note that $\Pi_S(p)$ is linear while $\Pi_F(p)$ is convex. We conclude that $\Pi_S(p)$ and $\Pi_F(p)$ can cross only once in the interval $(0, \theta_F)$. The statement of the proposition follows.

■

Proof of Proposition 2

First, we consider $p < \theta_S$, in which case the follower's strategy is to invest at τ_F with the threshold θ_F. Consider $\tau_\theta = \inf\{t > 0 : P_t \geq \theta\}$ for some $\theta \in (0,1)$ and the quantity $f(p) = E^p[e^{-r\tau_\theta} m(P_{\tau_\theta})]$. From the well-known theory of stopping values (see, for example, Chapter 9 of Oksendal, 2003) $f(\cdot)$ satisfies $\mathcal{A}f(p) = 0$ for $p < \theta$ with the boundary condition $f(\theta) = m(\theta)$. It follows that $E^p[e^{-r\tau_\theta} m(P_{\tau_\theta})] = \psi(p)m(\theta)/\psi(\theta)$ for any $p \leq \theta$. Hence, from (2.4), we obtain (2.11) for $p < \theta_S$. Second, if $p \geq \theta_S$, then the follower will also immediately invest at the same time, so both the leader and the follower expect a payoff given by $\Pi_S(p)$ in (2.12).

Given the follower's strategy of investment at τ^* given by (2.8), the leader's optimal policy is then to invest immediately if $V_1(p;0,\tau^*) \geq 0$ and never invest if $V_1(p;0,\tau^*) < 0$. Hence, the leader must compare $V_1(p;0,\tau^*)$ given by (2.11) and (2.12) with $V_1(p;\infty,\tau^*) = 0$ and choose the maximum of the two. It follows that (2.10) holds.

Note that $\Pi_L(p)$ increases in p because both $m(\cdot)$ and $\psi(\cdot)$ are increasing functions. Also, because $\Pi_L(0) = \ell/r - k < 0$ and $\Pi_L(\theta_S) = V_F(\theta_S) > 0$, θ_L defined by (2.13) must satisfy $\theta_L \in (0, \theta_S]$. Thus, $V_L(p) > 0$ if and only if $p > \theta_L$.

■

Proof of Proposition 3

This proposition follows from Propositions 1 and 2 which detail the best responses of the leader and the follower.

(i) In $[0, \theta_L)$, the firm that invests first (the leader) expects a negative payoff, so none of the players invest.

(ii) In the region $[\theta_L, \theta_S)$, consider the strategy profile (T_1, T_2) in which firm 1 takes the leader's role with an investment threshold θ_L and firm 2 takes the follower's role with a threshold θ_F. We now prove that this strategy profile is a Nash equilibrium and that it is subgame perfect.

To show that it is a Nash equilibrium, we only need to show that each firm's strategy is the best response given the other firm's strategy. We first consider firm 1's best response. Since it is already known that firm 2 will wait until the probability P_t reaches θ_F before investment, firm 1's best response is to invest with the upper threshold of θ_L as was established in Proposition 2.

Now we suppose that firm 1's strategy is to invest immediately. Then firm 2's optimal policy (best response) is to wait until P_t reaches the threshold θ_F by virtue of Proposition 1. It follows that (T_1, T_2) is a Nash equilibrium.

Now we prove that (T_1, T_2) is a subgame perfect equilibrium. After firm 1 invests, firm 2's optimal policy is a stationary Markov policy, and hence, (T_1, T_2) is still a Nash equilibrium. Before firm 1 invests, even if firm 1 (the leader) waits for any length of time, because the prior probability never changes, (T_1, T_2) still remains a Nash equilibrium. Therefore, the Nash equilibrium (T_1, T_2) is subgame perfect because it remains a Nash equilibrium at any point in time.

Finally, because firms 1 and 2 are symmetric, there exists another subgame perfect equilibrium (T_2, T_1) in which firm 2's threshold is θ_L while firm 1's threshold is θ_F.

(iii) In $[\theta_S, 1]$, because $\Pi_S(p) \geq \Pi_F(p) > 0$, both firms invest immediately.

■

Proof of Lemma 1

Define the following function

$$f(p) = \frac{m(p)}{r} - k + \frac{\psi(p)}{\psi(\theta_F)}\left[(\alpha_L - 1 - \alpha_F)\frac{m(\theta_F)}{r} + k\right],$$

so that $f(p) = \Pi_L(p) - \Pi_F(p)$ for $p < \theta_S$. (The function $f(\cdot)$ is defined above for any value of p, however.) Note that $f(0) = \ell/r - k < 0$, $f(\theta_L) < 0$ (because $V_L(\theta_L) = 0$ and $V_F(p) > 0$ for all p), but $\lim_{p\to\theta_S} f(p)$ may or may not be positive.

If $(\alpha_L - 1 - \alpha_F)\frac{m(\theta_F)}{r} + k \geq 0$, then $f(\cdot)$ is a strictly increasing convex function. Hence, even if $\lim_{p\to\theta_S} f(p) > 0$, there exist $\theta_c \in (\theta_L, \theta_S)$ such that $f(p) > 0$ if and only if $p \in (\theta_c, \theta_S)$.

If $(\alpha_L - 1 - \alpha_F)\frac{m(\theta_F)}{r} + k < 0$, then $f(\cdot)$ is a concave function. Then, using the fact that $V'(\theta_F) = (\alpha_F + 1)m'(\theta_F)/r = \frac{\psi'(\theta_F)}{\psi(\theta_F)}\left[(1 + \alpha_F)\frac{m(\theta_F)}{r} - k\right]$, we obtain

$$\begin{aligned} f'(\theta_F) &= \frac{m'(\theta_F)}{r} + \frac{\psi'(\theta_F)}{\psi(\theta_F)}\left[(\alpha_L - 1 - \alpha_F)\frac{m(\theta_F)}{r} + k\right] \\ &= \frac{m'(\theta_F)}{r}\frac{(\alpha_L - \alpha_F)(1 + \alpha_F) + k\alpha_F}{(1 + \alpha_F)m(\theta_F)/r - k}. \end{aligned}$$

Suppose that $f'(\theta_F) \geq 0$. From the concavity of $f(\cdot)$, we deduce that $f'(p) > 0$ for all $p \in (0, \theta_S)$. Thus, even if $\lim_{p\to\theta_S} f(p) > 0$, there exist $\theta_c \in (\theta_L, \theta_S)$ such that $f(p) > 0$ if and only if $p \in (\theta_c, \theta_S)$. Now suppose that $f'(\theta_F) < 0$. We also know that $f'(0) > 0$, $f(\theta_L) < 0$, and $f(\theta_F) = (\alpha_L - \alpha_F)m(\theta_F)/r \geq 0$, so $f(p)$ crosses zero (from negative to positive) only once in the interval (θ_L, θ_F), and at most once in the interval (θ_L, θ_S) because $\theta_S \leq \theta_F$. Thus, there exists $\theta_c \in (\theta_L, \theta_S]$ such that $f(p) > 0$ if and only if $p \in (\theta_c, \theta_S)$.

■

Proof of Proposition 4

(i) The proof is based on the analytical results from Hendricks et al. (1988). In order to utilize Hendricks et al.'s results on the war of attrition for our problem, it is necessary to make a change of variable $z = t/(t+1)$ so that

$z \in [0,1]$ where $z = 1$ is understood as the limit $t \to \infty$. We also define the following: $L(z) \equiv \Pi_L(p) \exp\left(-r\frac{z}{1-z}\right)$, $F(z) \equiv V_F(p) \exp\left(-r\frac{z}{1-z}\right)$, $S(z) \equiv \Pi_S(p) \exp\left(-r\frac{z}{1-z}\right)$, and $I(z_1, z_2) \equiv \exp\int_{z_1}^{z_2} \frac{dL(z)}{F(z)-L(z)}$. Here $L(\cdot)$ and $F(\cdot)$ are the discounted payoffs to the leader and the follower respectively, and $S(\cdot)$ is the discounted payoff of simultaneous entry.

First, we note that $L(\cdot)$, $F(\cdot)$, and $S(\cdot)$ are continuous on $[0,1]$. Second, by definition of WA region, $V_F(p) > \Pi_S(p)$ and $V_F(p) > V_L(p)$ for p within WA. We also note that $L(z)$ is strictly decreasing because $\Pi_L(p) > 0$ for $p > \theta_L$ and $\exp[-rz/(1-z)]$ is strictly decreasing in z. Therefore, all the assumptions made by Hendricks et al. (1988) are satisfied here.

Now we check the conditions for a Nash equilibrium. Theorem 2 of Hendricks et al. (1988) stipulates the necessary and sufficient condition for the existence of equilibrium with $q_p^{(i)}(0) < 1$. From the definition of $I(z_1, z_2)$ given by Hendricks et al. (1988), we have $I(z_1, z_2) = \exp[-\int_{z_1}^{z_2}(1-z)^{-2}/\bar{\tau}_M(p)dz]$ where $\bar{\tau}_M(p)$ is given in (2.17). We note that $I(0,0) = 1$ and that $I(z,1) = 0$ so that $I(1,1) \equiv \lim_{z\uparrow 1} I(z,1) = 0$. Thus, our model satisfies the conditions of Theorem 2 of Hendricks et al. (1988), and it allows for an equilibrium.

Next, Theorem 3 of Hendricks et al. (1988) provides the necessary and sufficient conditions for a strategy profile to be an equilibrium. The theorem essentially states that, if $L(z) > S(1) = 0$ for any $z < 0$, which is satisfied by our model, then the only possible form of equilibrium strategy profile is one in which $(q_p^{(1)}(0), q_p^{(2)}(0)) \in [0,1) \times [0,1)$ and $q_p^{(1)}(0)q_p^{(2)}(0) = 0$ and $G_p^{(i)}$ is exactly given by (2.16).

Then we show that the Nash equilibrium we obtained is subgame perfect. We only need to prove that at any time $s > 0$, the conditional probability distributions (conditional on the fact that neither firm has invested yet by time s) constitute a Nash equilibrium. Let $G_p^{(i)}(t|s)$ be the conditional distribution for time $t > s$. Then

$$G_p^{(i)}(t|s) = 1 - \frac{1 - G_p^{(i)}(t)}{1 - G_p^{(i)}(s)} = 1 - \exp\left[-\frac{t-s}{\bar{\tau}_M(p)}\right] ,$$

which reduces to $G_p^{(i)}(t-s)$ when $q_p^{(i)}(0) = 0$. Therefore, the strategy profile $(G_p^{(1)}(t|s), G_p^{(2)}(t|s))$ is a Nash equilibrium. We conclude that the equilibria we obtained in the proposition are subgame perfect.

(ii) First, we obtain (2.15) from when $q^{(i)} = 0$ and $q^{(j)}(0) > 0$ where

$G_p^{(i)}$ and $G_p^{(j)}$ are given by (2.16). We also obtain (2.15) when $q^{(i)} > 0$ and $q^{(j)}(0) = 0$. (Note that $\Pi_L(p) = V_L(p)$ for $p \geq \theta_L$.)

Second, suppose that $q_p^{(i)}(0) = \mathbb{P}(\{\hat{T}_i = 0\}) \geq 0$ and $q_p^{(j)}(0) = 0$. Then

$$E[\min\{\hat{T}_i, \hat{T}_j\}] = E[\min\{\hat{T}_i, \hat{T}_j\}|\hat{T}_i > 0]\mathbb{P}(\{\hat{T}_i > 0\}) = \frac{\bar{\tau}_M(p)}{2}[1 - q_p^{(i)}(0)] ,$$

because $E[\min\{\hat{T}_i, \hat{T}_j\}|\hat{T}_i > 0] = E[\min\{\hat{T}_i, \hat{T}_j\}] = \bar{\tau}_M(p)/2$ from the fact that $\hat{T}_i$ and $\hat{T}_j$ are exponentially distributed. In general, because $q_p^{(i)}(0)q_p^{(j)}(0) = 0$, equation (2.19) holds.

■

Proof of Lemma 2

As a preliminary step, we study $E^p[\tau_\theta \mid \tau_\theta < \infty]$ where $\tau_\theta = \inf\{t > 0 : P_t \geq \theta\}$ for some fixed value of θ. Note that for any $r > 0$, we can express $\mathbf{1}_{\{\tau_\theta < \infty\}} = \mathbf{1}_{\{\tau_\theta < \infty\}} + e^{-r\tau_\theta}\mathbf{1}_{\{\tau_\theta = \infty\}}$. Hence, we can express

$$\begin{aligned} \mathcal{P}(\tau_\theta < \infty) &= E^p(\mathbf{1}_{\{\tau_\theta < \infty\}}) = E^p[\mathbf{1}_{\{\tau_\theta < \infty\}} + e^{-r\tau_\theta}\mathbf{1}_{\{\tau_\theta = \infty\}}] \\ &= E^p(\lim_{r \to 0} e^{-r\tau_\theta}) = \lim_{r \to 0} E^p(e^{-r\tau_\theta}) , \end{aligned}$$

where the last equality is due to the bounded convergence theorem. From (2.9), we have $\mathcal{P}(\tau_\theta < \infty) = \lim_{r\to 0} \psi(p)/\psi(\theta) = p/\theta$. Similarly, we obtain

$$\begin{aligned} E^p[\tau_\theta 1_{\{\tau_\theta < \infty\}}] &= E^p \left\{ \lim_{r \to 0} [\tau_\theta e^{-r\tau_\theta}\mathbf{1}_{\{\tau_\theta < \infty\}} + \tau_\theta e^{-r\tau_\theta}\mathbf{1}_{\{\tau_\theta = \infty\}}] \right\} , \\ &= E^p \left[-\lim_{r \to 0} \frac{d(e^{-r\tau_\theta})}{dr} \right] \\ &= \lim_{r \to 0} E^p \left[\frac{d(e^{-r\tau_\theta})}{dr} \right] \end{aligned}$$

where the last inequality is due to the bounded convergence theorem. Interpreting $d(e^{-rt})/dr = \lim_{r'\to r}(e^{-r't} - e^{-rt})/(r' - r)$, we can express $E^p \left[\frac{d(e^{-r\tau_\theta})}{dr}\right] = dE^p[e^{-r\tau_\theta}]/dr$ from the bounded convergence theorem, which allows us to exchange the order of the limit $r' \to r$ and the expectation $E^p[\cdot]$. Thus,

$$\begin{aligned} E^p[\tau_\theta 1_{\{\tau_\theta < \infty\}}] &= -\lim_{r \to 0} \frac{dE^p[e^{-r\tau_\theta}]}{dr} = -\lim_{r \to 0} \frac{d[\psi(p)/\psi(\theta)]}{dr} \\ &= \frac{p}{\theta} \log \left(\frac{\theta}{p} \frac{1-p}{1-\theta} \right) \frac{4\sigma^2}{(h-\ell)^2} . \end{aligned}$$

From the Bayes' rule, we finally obtain

$$E^p[\tau_\theta \mid \tau_\theta < \infty] = \frac{E^p[\tau_\theta \mathbf{1}_{\{\tau_\theta<\infty\}}]}{\mathcal{P}(\tau_\theta < \infty)} = \log\left(\frac{\theta}{p}\frac{1-p}{1-\theta}\right)\frac{4\sigma^2}{(h-\ell)^2}. \tag{A.5}$$

It follows that $E^p[\tau_F|\tau_F < \infty]$ is given by the right-hand-side of (A.5) with θ replaced by θ_F.

■

Proof of Theorem 1

From the equality $\sigma^2/(h-\ell)^2 = (\gamma^2-1)/(8r)$, we can express $E^p[\tau_F \mid \tau_F < \infty]$ as a function of γ as follows:

$$f(\gamma, p) = \log\left(\frac{\theta_F}{1-\theta_F}\cdot\frac{1-p}{p}\right)\frac{\gamma^2-1}{2r},$$

where θ_F has γ dependence as in (A.1). From the expression of θ_F in (A.1), the partial derivative of $f(\gamma, p)$ with respect to γ can be expressed as follows:

$$\begin{aligned} f_1(\gamma,p) &\equiv \frac{\partial f(\gamma,p)}{\partial\gamma} \\ &= \frac{\gamma^2-1}{2r\theta_F(1-\theta_F)}\frac{d\theta_F}{d\gamma} + \frac{\gamma}{r}\log\left(\frac{\theta_F}{1-\theta_F}\cdot\frac{1-p}{p}\right) \\ &= -\frac{1}{r} + \frac{\gamma}{r}\log\left(\frac{\theta_F}{1-\theta_F}\cdot\frac{1-p}{p}\right), \end{aligned} \tag{A.6}$$

where $d\theta_F/d\gamma$ is given by

$$\frac{d}{d\gamma}\theta_F = \frac{-2\theta_F^2[(1+\alpha_F)h - kr]}{(\gamma+1)^2[kr-(1+\alpha_F)\ell]} < 0\,.$$

To prove the theorem, we need to determine the sign of $f_1(\gamma, p)$. Because γ decreases in β, if $f_1(\gamma, p) > 0$, then $E^p[\tau_F \mid \tau_F < \infty]$ decreases in β, and vice versa.

We consider the following:

$$\frac{\partial(r\gamma^{-1}f_1(\gamma,p))}{\partial\gamma} = \frac{1}{\gamma^2} + \left(\frac{1}{\theta_F} + \frac{1}{1-\theta_F}\right)\frac{d\theta_F}{d\gamma} = -\frac{\gamma^2+1}{(\gamma^2-1)\gamma^2} < 0\,.$$

Furthermore, for a given $p \in (0, 1)$, $\lim_{\gamma\to1} f_1(\gamma, p) > 1$ because $\lim_{\gamma\to1}\theta_F = 1$. Hence, $f_1(\gamma, p)$ crosses zero (from positive to negative) at most once as γ

increases. If $p > \lim_{\gamma\to\infty}\theta_F = \theta_0$, then θ_F eventually coincides with p at a sufficiently large value of γ, at which point $f_1(\gamma,p) = -r^{-1} < 0$. Therefore, if $p > \lim_{\gamma\to\infty}\theta_F$, then $f_1(\gamma,p)$ crosses zero exactly once as γ increases from 1 to ∞.

If, on the other hand, $p < \theta_0$, then $\lim_{\gamma\to\infty} f_1(\gamma,p) > 0$ because the logarithmic term is positive, so $f_1(\gamma,p)$ never crosses zero as γ increases. It follows that $f_1(\gamma,p) > 0$ for all $p < \theta_0$.

Lastly, note that $f_1(\gamma,p)$ is strictly decreasing in p and that $\lim_{p\to 0} f_1(\gamma,p) > 0$ and $f_1(\gamma,\theta_F) < 0$. Thus, $f_1(\gamma,p)$ crosses zero exactly once as p increases from 0 to θ_F.

■

Proof of Theorem 2

As a preliminary step, we obtain the comparative statics of $\Pi_L(p)$ with respect to β when $(p,\beta)\in\mathcal{W}$. After some algebra, $d\Pi_L(p)/d\beta$ can be expressed as follows :

$$\frac{d\Pi_L(p)}{d\beta} = \frac{d\gamma}{d\beta}\cdot\frac{d\Pi_L(p)}{d\gamma} = \frac{d\gamma}{d\beta}\cdot\alpha_L\frac{m(\theta_F(\beta))\psi(p)}{r\psi(\theta_F(\beta))}\cdot g(\theta_F(\beta)), \quad \text{(A.7)}$$

$$\text{where} \quad g(x) \equiv \frac{m(\theta_0)}{2\theta_0(1-\theta_0)}\cdot\frac{x-\theta_0}{m(x)} - \frac{1}{2}\log\frac{x(1-p)}{(1-x)p}.$$

Here we used the fact that $\gamma = 1 + 2\theta_0(1-\theta_F)/(\theta_F-\theta_0)$ and expressed all the β-dependencies in terms of $\theta_F(\beta)$. Because it is already established that $d\gamma/d\beta < 0$ and $m(\theta_F) > 0$, the sign of $d\Pi_L(p)/d\beta$ depends on the signs of $g(\theta_F(\beta))$ and α_L. Note that $\theta_F(\beta)$ is strictly increasing in β by Proposition 2 of Kwon and Lippman (2011) and that $\lim_{\beta\to 0}\theta_F(\beta) = \theta_0$ and $\lim_{\beta\to\infty}\theta_F(\beta) = 1$.

As a preliminary step, we consider $p > \theta_0$, in which case the possible value of β is within $(\underline{\beta}_p, \infty)$ for some $\underline{\beta}_p > 0$, and the possible values of $\theta_F(\beta)$ are within the interval $(\theta_F(\underline{\beta}_p), 1)$ where $\theta_F(\underline{\beta}_p) \geq \theta_c(\underline{\beta}_p) = p$. For now, we extend the domain of the function $g(\cdot)$ to the interval $[p,1)$ and establish that this extended function $g(\cdot)$ crosses zero exactly once as x increases within the interval $[p,1)$. $g(\cdot)$ does cross zero because $\lim_{x\to 1} g(x) < 0$ and $g(p) > 0$ because $p > \theta_0$. We note that $dg(x)/dx = g_1(x)/[2m(x)^2\theta_0(1-\theta_0)x(1-x)]$,where $g_1(x) = m(\theta_0)^2x(1-x) - m(x)^2\theta_0(1-\theta_0)$. Note that $g_1(\cdot)$ is strictly concave quadratic function, and $g_1(x) = 0$ yields two roots: $x_1 = \theta_0$ and $x_2 = (1-\theta_0)\ell^2/[\theta_0h^2 + (1-\theta_0)\ell^2] < 1$. The first root x_1 is outside the interval $[p, 1)$ because $p > \theta_0$, so we focus on the location of the second root x_2.

Suppose that $x_2 \le p$. Then $g_1(x) < 0$ for $x \in [p, 1)$, and hence, $g(\cdot)$ strictly decreases in the interval $[p, 1)$. It follows that $g(\cdot)$ changes sign only once.

Suppose that $p < x_2$. Then $g_1(x) > 0$ for $x < x_2$ and $g_1(x) < 0$ for $x > x_2$, so $g(\cdot)$ is strictly increasing in the interval $[p, x_2)$ and strictly decreasing in $(x_2, 1)$. From the observation $g(p) > 0$ and $\lim_{x\to 1} g(x) < 0$, it follows that $g(x)$ changes sign only once in the interval $[p, 1)$ as x increases.

Now we restrict the domain of $g(\cdot)$ to $(\theta_F(\underline{\beta}_p), 1)$. By virtue of the analysis above, there is $\hat{\theta} \in [\theta_F(\underline{\beta}_p), 1)$ such that $g(x)$ changes sign at $\hat{\theta}$ as x increases within the domain $(\theta_F(\underline{\beta}_p), 1)$. (If $g(\cdot)$ does not change sign anywhere within $(\theta_F(\underline{\beta}_p), 1)$, then $\hat{\theta} = \theta_F(\underline{\beta}_p)$.) Thus, $d\Pi_L(p)/d\beta$ changes sign at some $\hat{\beta}_M(p) \in [\underline{\beta}_p, \infty)$ as β increases within the domain $\{\beta : (p, \beta) \in \mathcal{W}\}$. Note also that $V_M(p) = \Pi_L(p)$ within the WA region.

Next, we note that the sign of $\lim_{\beta\to\infty} d\Pi_L(p)/d\beta$ coincides with the sign of α_L because $d\gamma/d\beta < 0$ and $\lim_{x\to 1} g(x) < 0$. Thus, the statements of the theorem follow regarding the comparative statics of $V_M(p)$ with respect to β for $p > \theta_0$.

We also note that $g(x)$ is strictly increasing in p, so the sign of $g(\cdot)$ can change at most once as p increases from $\theta_L(\beta)$ to $\theta_c(\beta)$ for a fixed β. If the sign change happens within $(\theta_L(\beta), \theta_c(\beta))$, then $d\Pi_L(p)/d\beta$ in (A.7) changes from positive to negative as p increases if $\alpha_L > 0$, and from negative to positive as p increases if $\alpha_L < 0$.

Lastly, consider $p < \theta_0$. In the limit $x \downarrow \theta_0$ (or in the small-β limit) and in the limit $x \uparrow 1$ (or in the large-β limit), $g(x)$ is negative. Thus, $V_M(p)$ increases (decreases) in β for small or large values of β if α_L is positive (negative).

■

Proof of Theorem 3

As the first step, we obtain an analytical expression for the determinant of the sign of $d\bar{\tau}_M(p)/d\beta$. From (2.17), we obtain

$$\frac{d\bar{\tau}_M(p)}{d\gamma} = \frac{1}{r\Pi_L^2(p)} \left[\Pi_L(p) \frac{d}{d\gamma} V_F(p) - V_F(p) \frac{d}{d\gamma} \Pi_L(p) \right] .$$

Here $dV_F(p)/d\gamma$ is given by

$$\frac{dV_F(p)}{d\gamma} = \left[\frac{1}{r}(1+\alpha_F)m(\theta_F) - k\right] \frac{\psi(p)}{\psi(\theta_F)} \frac{1}{2} \ln\left(\frac{p}{1-p}\frac{1-\theta_F}{\theta_F}\right) < 0 \,,$$

so we only need to compute $d\Pi_L(p)/d\gamma$. From the expression (2.11) and the equality

$$\left[\frac{1}{r}(1+\alpha_F)m(\theta_F) - k\right] \frac{\psi'(\theta_F)}{\psi(\theta_F)} = \frac{1}{r}(1+\alpha_F)m'(\theta_F)\,, \tag{A.8}$$

which is derived from the continuous differentiability of $V_F(p)$ at $p = \theta_F$, we obtain

$$\begin{aligned} \frac{d}{d\gamma}\Pi_L(p) \quad = \quad & \frac{\alpha_L}{r}\frac{\psi(p)}{\psi(\theta_F)}\left[-\frac{d\theta_F}{d\gamma}\cdot\frac{\psi'(\theta_F)kr}{\psi(\theta_F)(1+\alpha_F)}\right. \\ & \left. + \frac{m(\theta_F)}{2}\log\left(\frac{p}{1-p}\cdot\frac{1-\theta_F}{\theta_F}\right)\right]. \end{aligned}$$

for $p < \theta_S$. Then it follows that, for $p < \theta_S$,

$$\begin{aligned} & \Pi_L(p)\frac{d}{d\gamma}V_F(p) - V_F(p)\frac{d}{d\gamma}\Pi_L(p) \\ = \quad & \frac{V_F(p)}{r}\left[\frac{1}{2}\left(m(p)-kr\right)\log\left(\frac{p}{1-p}\cdot\frac{1-\theta_F}{\theta_F}\right) - \right. \\ & \left. \frac{2\alpha_L kr\theta_F^2\theta_0}{(1+\alpha_F)(\gamma+1)^2}\cdot\frac{\psi'(\theta_F)\psi(p)}{\psi^2(\theta_F)}\right]. \end{aligned}$$

Once we substitute $\gamma = 1 + 2\theta_0(1-\theta_F)/(\theta_F - \theta_0)$, which is an equality that can be verified from the definition of θ_F and θ_0, we conclude that $\bar{\tau}_M(p)$ increases (decreases) in β if and only if

$$\frac{1}{2}\left(m(p)-kr\right)\log\left(\frac{p}{1-p}\cdot\frac{1-\theta_F}{\theta_F}\right) - \frac{\alpha_L kr\theta_0(\theta_F-\theta_0)^2}{2(1+\alpha_F)(1-\theta_0)^2}\cdot\frac{\psi'(\theta_F)\psi(p)}{\psi^2(\theta_F)}\,, \tag{A.9}$$

is negative (positive).

Let us write (A.9) as $A(p) + B(p)$ where

$$
\begin{aligned}
A(p) &= \frac{1}{2}\,(m(p) - kr)\log\left(\frac{p}{1-p}\cdot\frac{1-\theta_F}{\theta_F}\right)\,, \\
B(p) &= -\frac{\alpha_L kr\theta_0(\theta_F-\theta_0)^2}{2(1+\alpha_F)(1-\theta_0)^2}\cdot\frac{\psi'(\theta_F)\psi(p)}{\psi^2(\theta_F)}\,.
\end{aligned}
$$

(i) Suppose $\alpha_L > 0$. Then $B(p) < 0$ for all p, but the sign of $A(p)$ depends on the sign of $m(p) - kr$. (The logarithmic term is always negative because $p < \theta_F$.) Specifically, for sufficiently high p, either $m(p) > kr$ or $p = \theta_F$ is satisfied, so $A(p) \le 0$. Thus, for sufficiently high p, (A.9) is negative.

Now consider the limit $\beta \to \infty$. From the expression of (A.1) and (A.3), $\psi'(\theta_F)\psi(p)/\psi^2(\theta_F)$ converges to a finite value in the limit $\gamma \to 1$ ($\beta \to \infty$), so $\lim_{\beta\to\infty}|B(p)| < \infty$. On the other hand, $\lim_{\beta\to\infty}\ln[(1-\theta_F)/\theta_F] = -\infty$ because $\lim_{\beta\to\infty}\theta_F = 1$. Thus, $\lim_{\beta\to\infty}A(p) + B(p) > 0$ if $m(p) - kr < 0$ and $\lim_{\beta\to\infty}A(p) + B(p) < 0$ if $m(p) - kr > 0$. From (2.11), we find that $m(\theta_L) - kr = -\frac{\alpha_L m(\theta_F)\psi(\theta_L)}{r\psi(\theta_F)} < 0$. Because $m(\cdot)$ is strictly increasing, we conclude that $m(p) - kr < 0$ for sufficiently low p that satisfies $p > \theta_L$.

(ii) Suppose $\alpha_L < 0$. From (2.11), we find that $m(\theta_L) - kr = -\frac{\alpha_L m(\theta_F)\psi(\theta_L)}{r\psi(\theta_F)}$ is positive, so $m(p) - kr > 0$ for all $p > \theta_L$. By an analogous argument above, we conclude that $A(p) + B(p) < 0$ for sufficiently large β.

■

Proof of Proposition 5

We consider the intervals $I_1 = (\theta_L, \theta_F)$ and $I_2 = [\theta_F, 1)$ separately below. (We define $I_1 = \emptyset$ in case $\theta_L > \theta_F$.

(i) We first study the interval I_1. First, we prove that $\Pi_F(p) > \Pi_S(p)$ for all $p \le \theta_F$. By 2.6 and 2.7 and the definition $\alpha_S = (\alpha_L + \alpha_F)/2 \le \alpha_F$, we can derive

$$
\Pi_S(p) = \frac{1}{r}(1+\alpha_S)m(p) - k < \frac{1}{r}(1+\alpha_F)m(p) - k \le \Pi_F(p)
$$

for all $p \le \theta_F$. Here the second inequality holds because $\Pi_F(\cdot)$ is convex, $m(\cdot)$ is linear, and $\Pi_F'(\theta_F) = (1+\alpha_F)m(\theta_F)/r$. It follows that the SM region does not exist within $(0, \theta_F)$.

Second, we prove that $V_F(p) \ge V_L(p)$ for all $p \le \theta_F$. Because $\alpha_L < 0$, we have $\Pi_L(p) \le m(p)/r - k \le (1+\alpha_F)m(p)/r - k \le V_F(p)$ for all $p \le \theta_F$. Since $V_F(p) > 0$ for all p, it follows that $V_F(p) \ge \max\{\Pi_L(p), 0\} = V_L(p)$.

(ii) Next, we consider the interval $I_2 = [\theta_F, 1)$. In this interval, if the leader invests first, then the follower can choose to invest an infinitesimal time later to be the follower. Thus, $V_F(p) = \frac{1}{r}(1+\alpha_F)m(p) - k$ and $V_L(p) = \frac{1}{r}(1+\alpha_L)m(p) - k$. Because $\alpha_L < \alpha_F$, we have $V_F(p) > V_L(p)$.

■

Proof of Theorem 4

We use the same notation employed in the proof of Theorem 3. Since $\bar{\tau}_M(p)$ has no dependence on β for $p \in (\theta_F, 1)$, we focus on the interval (θ_L, θ_F) in case $\theta_L < \theta_F$.

For p sufficiently close to θ_F and for any fixed value of β, $A(p)$ is negligible compared to $B(p)$. Since $B(p) > 0$ for $\alpha_L < 0$, we have $A(p) + B(p) > 0$ in the limit $p \to \theta_F$, which implies that $\bar{\tau}_M(p)$ decreases with β for p sufficiently close to θ_F.

Let us now consider a fixed $p < \theta_F$ and for sufficiently large values of β. By the same argument used in the proof of Theorem 3(ii), $m(p) - kr > 0$ for all $p > \theta_L$, so we have $A(p) < 0$. Furthermore, in the limit $\beta \to \infty$, $|A(p)| > |B(p)|$ by the argument used in the proof of Theorem 3(i). It follows that $A(p) + B(p) < 0$ for a fixed $p < \theta_F$ and for sufficiently large values of β. Thus, we conclude that $\bar{\tau}_M(p)$ increases with β when p is not too close to θ_F and for sufficiently large values of β.

■

APPENDIX B

PROOFS FOR CHAPTER 3

Proof of Proposition 6

Because we only consider pure strategy equilibrium, we set $\min\{T_1^0, T_2^0\} = 0$. By the distributions of completion times $T_1$and T_2 before and after τ_I, and by (3.2), (3.3) and (3.5), we obtain the expressions for payoffs as functions of (λ_i,λ_j), where $W_i(s)$, $W_j(s)$, $U_i(s)$ and $U_j(s)$ are given by (3.8) and (3.9):

$$\begin{aligned} V_i^C(\lambda_i, \lambda_j; s; s) &= U_i(s)\frac{\lambda_i}{\lambda_i + \lambda_j + r} + W_i(s)\frac{\lambda_j}{\lambda_i + \lambda_j + r} \quad \text{(B.1)} \\ &\quad -\frac{k_i(\lambda_i)^2}{\lambda_i + \lambda_j + r} - c; \\ V_i^{IL}(\lambda_i, \lambda_j; s; s) &= -\frac{k_i(\lambda_i)^2}{\lambda_i + r} + U_i(s)\frac{\lambda_i}{r + \lambda_i} - c\,; \quad \text{(B.2)} \\ V_j^{IF}(\lambda_i, \lambda_j; s; s) &= (W_j(s) - c)\frac{\lambda_i}{r + \lambda_i}\,. \quad \text{(B.3)} \end{aligned}$$

(i) In a concurrent pure strategy equilibrium, we obtain the expression of λ_i^C as in (3.10) by solving the first order condition $dV_i^C(\lambda_i, \lambda_j; s, s)/d\lambda_i = 0$ for $i = 1, 2$.

(ii) In an imitative pure strategy equilibrium in which $T_i^0 < T_j^0$, firm j invests at time T_i. Firm i's best response can be obtained by solving the first order condition $dV_i^{IL}(\lambda_i, \lambda_j; s; s)/d\lambda_i = 0$. Then we obtain λ_i^{IL} as (3.12) that maximizes $V_i^{IL}(\lambda_i, \lambda_j; s, s)$.

■

Proof of Propositions 7

Note that firms are symmetric, we only need obtain one firm's best response given the other firm's strategy. First as a preliminary result, we need to prove that $2\lambda_i^C < \lambda_i^{IL}$, $\forall s \in [1, \infty)$.

$$\begin{aligned}
2\lambda_i^C &= -\frac{1}{3k}(2kr - U(s) + W(s)) + \sqrt{\frac{(2kr - U(s) + W(s))^2}{9k^2} + \frac{4rU(s)}{3k}} \\
&> -\frac{1}{3k}(2kr - U(s) + W(s)) + \sqrt{\frac{(2kr - U(s) + W(s))^2}{9k^2} + \frac{rU(s)}{k}} \\
&> -r + \sqrt{r^2 + \frac{rU(s)}{k}} = \lambda_i^{IL},
\end{aligned}$$

where we use (3.10). The last inequality is from the fact that

$$(2kr - U(s) + W(s))/3k < r,$$

which is due to $U(s) - W(s) > 0$, and the fact that the function $-x + \sqrt{x^2 + y}$ is a decreasing function of x whenever y is positive. The fact that $U(s) > W(s)$ is apparent from the expressions of (3.8) and (3.9).

Given that firm j initiates at time 0, firm i receives $V_i^C(s,s)$ if it chooses a concurrent strategy, and $V_i^{IF}(s,s)$ if it chooses to initiate its R&D at time τ_I. Hence we only need to compare $V_i^C(s,s)$ and $V_i^{IF}(s,s)$ to obtain firm i's best response. In the limit $c \to 0$, we have

$$\begin{aligned}
& V_i^C(s,s) - V_i^{IF}(s,s) \qquad \text{(B.4)} \\
&= \frac{\lambda_i^C}{2\lambda_i^C + r}[U(s) + W(s) - k\lambda_i^C] - \frac{\lambda_j^{IL}}{\lambda_j^{IL} + r}W(s) \\
&> \frac{\lambda_i^C}{2\lambda_i^C + r}[U(s) - W(s) - k\lambda^C]
\end{aligned}$$

Now we investigate the inequality (B.4) in the limits of very small s and very large s. In the limit $s \to 1$, $\lambda_i^C/(2\lambda_i^C + r)$ converges to a fixed positive number from (3.10), and we obtain

$$\begin{aligned}
\lim_{s\to 1} U(s) - W(s) - k\lambda_i^C &= \frac{1}{3}kr + \frac{5}{6}(U(1) - W(1)) \qquad \text{(B.5)} \\
&\quad - \left\{ \left[\frac{1}{3}kr + \frac{5}{6}(U(1) - W(1))\right]^2 + f_1 \right\}^{\frac{1}{2}}
\end{aligned}$$

where

$$f_1 = \frac{rkU(1)}{3} + \left[\frac{1}{3}kr - \frac{U(1)-W(1)}{6}\right]^2 - \left[\frac{1}{3}kr + \frac{5}{6}\left(U(1)-W(1)\right)\right]^2.$$

From (B.5), if $f_1 < 0$, then we obtain $\lim_{s\to 1} U(s) - W(s) - k\lambda_i^C > 0$. From equations (3.7), (3.8), and (3.9), and the fact that $\pi_{10} > \pi_{11}$, we obtain $f_1 < -2k^2\mu_j^2(r^2 + r\mu_j + \mu_j^2) - k(r^2 + 3\mu_j^2)\pi_{10} - 2\pi_{10}^2 < 0$. The inequality is from the fact that μ_j,k,r,k and π_{10} are non-negative. Therefore $\lim_{s\to 1} U(s) - W(s) - k\lambda_i^C > 0$ and $\lim_{s\to 1} V_i^C(s,s) - V_i^{IF}(s,s) > 0$.

Next we investigate the other limit. From (3.11) and (3.14), we obtain

$$\lim_{s\to\infty} V_i^C(s,s) - V_i^{IF}(s,s) = \frac{\pi_{11}(2\lambda_i^C - \lambda_{j\infty}^{IL}) - k(\lambda_{i\infty}^C)^2(\lambda_{j\infty}^{IL} + r)}{(2\lambda_{i\infty}^C + r)(\lambda_{j\infty}^{IL} + r)},$$

where $\lambda_{i\infty}^C = \lim_{s\to\infty} \lambda_i^C$ and $\lambda_{i\infty}^{IL} = \lim_{s\to\infty} \lambda_i^{IL}$. Because both $\lambda_{i\infty}^C$ and $\lambda_{j\infty}^{IL}$ are non-negative, the sign of $\lim_{s\to\infty} V_i^C(s,s) - V_i^{IF}(s,s)$ coincides with the sign of the following expression

$$\pi_{11}(2\lambda_{i\infty}^C - \lambda_{j\infty}^{IL}) - k(\lambda_{i\infty}^C)^2(\lambda_{j\infty}^{IL} + r). \tag{B.6}$$

Below we investigate the sign of (B.6). First we prove that (B.6) decreases in $\lambda_{i\infty}^C$ and $\lambda_{j\infty}^{IL}$. Then the sign of (B.6) is determined by the relative sizes of k and $\pi_{11}/8r^2$.

First we establish $\lambda_{i\infty}^C \geq \pi_{11}/[k(\lambda_{j\infty}^{IL}+r)]$. Suppose $\lambda_{i\infty}^C < \pi_{11}/[k(\lambda_{j\infty}^{IL}+r)]$. Because $\lambda_i^C > \lambda_j^{IL}/2, \forall s$, which we have shown at the beginning of this proof, we have $\lambda_{j\infty}^{IL}/2 \leq \pi_{11}/[k(\lambda_{j\infty}^{IL} + r)]$, which can be reexpressed as

$$\lambda_{j\infty}^{IL} \geq -\frac{r}{2} + \sqrt{\frac{r^2}{4} + \frac{2\pi_{11}}{k}} > -r + \sqrt{r^2 + \frac{\pi_{11}}{k}}\,.$$

The second strict inequality is because $-x + \sqrt{x^2 + y}$ decreases in x and $2\pi_{11}/k > \pi_{11}/k$. The inequality $\lambda_{j\infty}^{IL} > -r + \sqrt{r^2 + \pi_{11}/k}$ contradicts the fact that $\lambda_{j\infty}^{IL} = -r + \sqrt{r^2 + \pi_{11}/k}$ from the expression of (3.12). Therefore, we obtain $\lambda_{i\infty}^C \geq \pi_{11}/[k(\lambda_{j\infty}^{IL} + r)]$.

Then we prove that (B.6) decreases in $\lambda_{i\infty}^C$ and $\lambda_{j\infty}^{IL}$. Define $f_2(x) = \pi_{11}(2x - \lambda_{j\infty}^{IL}) - kx^2(\lambda_{j\infty}^{IL} + r)$. The function $f_2(x)$ decreases in x if $x > \pi_{11}/[k(\lambda_{j\infty}^{IL}+r)]$. Because of the inequality $\lambda_{i\infty}^C \geq \pi_{11}/[k(\lambda_{j\infty}^{IL}+r)]$, $\pi_{11}(2\lambda_{i\infty}^C -$

$\lambda_{j\infty}^{IL}) - k(\lambda_{i\infty}^{C})^2(\lambda_{j\infty}^{IL} + r)$ decreases in $\lambda_{i\infty}^{C}$. Also, $\pi_{11}(2\lambda_{i\infty}^{C} - \lambda_{j\infty}^{IL}) - k(\lambda_{i\infty}^{C})^2(\lambda_{j\infty}^{IL} + r)$ decreases in $\lambda_{j\infty}^{IL}$.

Now we consider the two regimes $k \geq \pi_{11}/8r^2$ and $k < \pi_{11}/8r^2$. By (3.10) and (3.12), if $k \geq \pi_{11}/8r^2$, we obtain $\lambda_{i\infty}^{C} \leq 4r/3$, $\lambda_{j}^{IL} \leq 2r$, and $\pi_{11}(2\lambda_{i}^{C} - \lambda_{j\infty}^{IL}) - k(\lambda_{i\infty}^{C})^2(\lambda_{j\infty}^{IL} + r) \geq 0$, which proves Proposition 7. Similarly, if $k < \pi_{11}/8r^2$, we obtain $\lambda_{i\infty}^{C} > 4r/3$ and $\lambda_{j}^{IL} > 2r$. As a result, $\pi_{11}(2\lambda_{i}^{C} - \lambda_{j\infty}^{IL}) - k(\lambda_{i\infty}^{C})^2(\lambda_{j\infty}^{IL} + r) < 0$.

It remains to prove $V_i^{IL}(s,s) < V_i^{IF}(s,s)$ whenever $s \in \{s : V_i^{C}(s,s) < V_i^{IF}(s,s)\}$ in order to complete the proof of Proposition 7. Note that

$$V_i^{IL}(s,s) - V_i^{IF}(s,s) = (U(s) - W(s) - k\lambda_i^{IL})\frac{\lambda_i^{IL}}{\lambda_i^{IL} + r} .$$

Hence it suffices to prove that $U(s) - W(s) - k\lambda_i^{IL} < 0$ whenever $V_i^{C}(s,s) < V_i^{IF}(s,s)$.

Claim: $\lambda_i^{IL} > \lambda_i^{C}$ when $s \in \{s : V_i^{C}(s,s) < V_i^{IF}(s,s)\}$.

Proof of the claim: Assume $\lambda_i^{IL} < \lambda_i^{C}$. Then $\forall s \in [1, \infty)$,

$$\begin{aligned} V_i^{C}(s,s) - V_i^{IF}(s,s) &= \frac{\lambda_i^{C}}{2\lambda_i^{C} + r}[U(s) + W(s) - k\lambda^{C}] - \frac{\lambda_i^{IL}}{\lambda_i^{IL} + r}W(s) \\ &> \frac{\lambda_i^{C}}{2\lambda_i^{C} + r}[U(s) + W(s) - k\lambda^{C}] - \frac{\lambda_i^{C}}{\lambda_i^{C} + r}W(s) \\ &= \frac{\lambda_i^{C}}{(2\lambda_i^{C} + r)(\lambda_i^{C} + r)}g(\lambda_i^{C}), \end{aligned}$$

where $g(x) = -kx^2 + x(U(s) - W(s) - kr) + rU(s)$. The concave quadratic equation $g(x) = 0$ has a positive root

$$x_0 = -\frac{1}{2}r + \frac{U(s) - W(s)}{2k}) + \sqrt{\left(\frac{1}{2}r - \frac{U(s) - W(s)}{2k}\right)^2 + \frac{rU(s)}{k}} .$$

It follows that $g(x) > 0$ for $x \in [0, x_0)$. After some algebra, it is straight forward to verify the following:

$$\begin{aligned} \lambda_i^{C} &= -\frac{1}{3}r + \frac{U(s) - W(s)}{6k} + \sqrt{\left(\frac{1}{3}r - \frac{U(s) - W(s)}{6k}\right)^2 + \frac{rU(s)}{3k}} \\ &< -(\frac{1}{2}r - \frac{U(s) - W(s)}{2k}) + \sqrt{(\frac{1}{2}r - \frac{U(s) - W(s)}{2k})^2 + \frac{rU(s)}{k}} = x_0. \end{aligned}$$

Therefore, $g(\lambda_i^C) > 0$ and $V_i^C(s,s) - V_i^{IF}(s,s) > 0$, which contradicts the fact that $s \in \{s : V_i^C(s,s) < V_i^{IF}(s,s)\}$, and hence we conclude that $\lambda_i^{IL} > \lambda_i^C$.

From the inequalities $\lambda_i^{IL} > \lambda_i^C$, $\lambda_i^{IL} < 2\lambda_i^C$ and $V_i^C(s,s) < V_i^F(s,s)$, we obtain

$$\frac{\lambda_i^C}{2\lambda_i^C + r}[U(s) + W(s) - k\lambda_i^C] < \frac{\lambda_i^{IL}}{\lambda_i^{IL} + r}W(s) < \frac{2\lambda_i^C}{2\lambda_i^C + r}W(s),$$

from which we obtain $U(s) - W(s) < k\lambda_i^C < k\lambda_i^{IL}$. Hence we prove Proposition 7.

■

Proof of Proposition 8

The proofs of part (1) and (2) are a straightforward modification of the proof of Proposition 4 in Kwon et al. (2015).

■

Proof of Proposition 9

Case (II) is identical to the symmetric case, so the proof of 7 directly applies here. Hence we only need to prove case (I).

The payoff functions are given by the same expressions as in (B.1), (B.2), and (B.3). In a concurrent pure strategy equilibrium, we obtain equations (3.16)-(3.18) from the first order conditions $dV_i^C(\lambda_i, \lambda_j; s, s)/d\lambda_i = 0$.

Below we prove that there exists a unique pair of (λ_1, λ_2) that satisfies (3.16)-(3.18). For notational simplicity, we define

$$q(x) = \sqrt{(r+x)^2 + x[U_2(s) - W_2(s)]/k_2 + rU(s)/k_2}\,.$$

From (3.16)-(3.18), we find that λ_1^C is given as the unique nonnegative solution to $f(x) = 0$, where

$$f(x) = x^2 - 2xq(x) + (\frac{U_1(s) - W_1(s)}{k_1})(q(x) - r - x) + \frac{r}{k_1}U_1(s) \qquad \text{(B.7)}$$

and λ_2^C is given by

$$\lambda_2^C = -r - \lambda_1^C + \sqrt{(r+\lambda_1^C)^2 + (\lambda_1^C)\frac{U_2(s) - W_2(s)}{k_2} + \frac{r}{k_2}U_2(s)}.$$

Notice that $f(0) = [(U_1(s) - W_1(s))/k_1](\sqrt{r^2 + rU_2(s)/k_2} - r) + rU_1(s)/k_1 > 0$. In the large x limit, $f(x) = -\infty$. By the continuity of $f(x)$, there must exist at least one positive root.

Also, we obtain $f"(x) < 0$. Therefore $f(x)$ crosses 0 only once, and there is a unique pair of $(\lambda_1^C,\ \lambda_2^C)$ that satisfies the simultaneous equations (3.16)-(3.18).

■

Proof of Proposition 10

(I)(a) From (3.16)-(3.18),

$$\begin{aligned}
\lambda_i^C + \lambda_j^C &= -r + \sqrt{(r + \lambda_j^C)^2 + \lambda_j^C \frac{U_i(s) - W_i(s)}{k_i} + \frac{rU_i(s)}{k_i}} \\
&> -r + \sqrt{r^2 + \frac{rU_i(s)}{k_i}} = \lambda_i^{IL},
\end{aligned}$$

where the inequality is because of $\lambda_j^C \geq 0$ and $U_i(s) > W_i(s)$. Hence, by (3.2) and (3.5), and from the inequality $\lambda_i^C + \lambda_j^C > \lambda_i^{IL}$, we obtain the inequality $V_i^C(s,s) - V_i^{IF}(s,s) > [\lambda_i^C/(\lambda_i^C + \lambda_j^C + r)](U_i(s) - W_i(s) - k_i\lambda_i^C)$ in the limit $c \to 0$. Then it suffices to show that $U_i(s) - W_i(s) - k_i\lambda_i^C$ is nonnegative in the limit $s \to 1$. Note that if $\lim_{s\to 1} \lambda_i^C < (U_i(1) - W_i(1))/2k_i$, we have $\lim_{s\to 1} U_i(s) - W_i(s) - k_i\lambda_i^C > (U_i(1) - W_i(1))/2 > 0$. Then it remains to prove that $\lim_{s\to 1} \lambda_i^C < (U_i(1) - W_i(1))/2k_i$ always holds true.

As a preliminary step, we show that $rW_i(s)/k_i - (U_i(s) - W_i(s))^2/4k_i^2 < 0$ for $i = 1, 2$, when $s \to 1$. By (3.8) and (3.9), we obtain

$$\begin{aligned}
\frac{rW_i(s)}{k_i} - \frac{(U_i(s) - W_i(s))^2}{4k_i^2} &= \frac{(\pi_{10} - \pi_{11})}{4k_i^2(k_j r^2 + \pi_{11})} . [-k_j(\pi_{10} - \pi_{11}) \\
&\quad + 4\sqrt{k_j\left(k_j r^2 + \pi_{11}\right)} \left(k_i r - \sqrt{k_i\left(k_i r^2 + \pi_{11}\right)}\right)\Big] .
\end{aligned}$$

The right hand side of the above equation is negative because $\pi_{10} > \pi_{11}$ and $k_i r < \sqrt{k_i\left(k_i r^2 + \pi_{11}\right)}$.

From (3.16)-(3.18), we obtain

$$\lambda_i^C + \lambda_j^C + r = \sqrt{\left(r + \lambda_j^C + \frac{U_i(s) - W_i(s)}{2k_i}\right)^2 + \frac{rW_i(s)}{k_i} - \frac{(U_i(s) - W_i(s))^2}{4k_i^2}}.$$

Because of the inequality $rW_i(s)/k_i - (U_i(s) - W_i(s))^2/4k_i^2 < 0$, we obtain $\lim_{s\to 1} \lambda_i^C < (U_i(1) - W_i(1))/2k_i$.

(I)(b) When $q \to 1$, we obtain

$$\begin{aligned}
\lambda_1^C &= \lambda^C + \mathcal{O}(q-1)\,, \\
\lambda_2^C &= \lambda^C + \mathcal{O}(q-1)\,, \\
\mu_2 &= -r + \sqrt{r^2 + \frac{U(s)}{k}} + \mathcal{O}(q-1)\,,
\end{aligned} \tag{B.8}$$

where λ^C is identical to the right hand side of (3.10). Because the game reduces to the symmetric case in the limit $q \to 1$, the proof is complete by virtue of Proposition 7.

(II) In the simultaneous limit $c \to 0$ and $q \to 1$, Proposition 10 applies. It follows that $V_i^{IF}(s,s) > V_i^{IL}(s,s)$ and $V_i^{IF}(s,s) > V_i^C(s,s)$ in this limit.

(III) Below we prove that $V_1^{IL}(s,s) > V_1^{IF}(s,s)$ and

$$V_2^{IF}(s,s) \;>\; \max\{V_2^{IL}(s,s), V_2^C(s,s)\}$$

are satisfied, thereby showing that the game belongs to the imitative regime.

In the limit $q \to 0$, we solve the system of equations (3.16)-(3.18), and obtain

$$\lambda_1^C = \frac{A_2}{\sqrt{q}} + A_0 + \mathcal{O}(\sqrt{q}) \quad ; \quad \lambda_2^C = B_0 + B_1\sqrt{q} + \mathcal{O}(q)$$

where

$$\begin{aligned}
\tilde{u}_2 &= \left[(\pi_{10} - \pi_{11})\sqrt{\frac{k}{\pi_{11}}}\right]/s \\
\tilde{w}_1 &= \frac{2}{s}\sqrt{k\pi_{11}} \\
B_0 &= \frac{1}{2k}[\frac{\pi_{11}}{r} - W(s)] \qquad\qquad (\text{B.9}) \\
A_2 &= \sqrt{\frac{B_0}{k}[U(s) - \frac{\pi_{11}}{r}] + r\frac{U(s)}{k}} \\
B_1 &= \frac{1}{2A_2}(\frac{A_2\tilde{u}_2}{k} + \frac{\pi_{11}}{r} - 2rB_0 - B_0^2) \qquad\qquad (\text{B.10}) \\
A_0 &= -B_0 - r + \frac{1}{2A_2k}[B_0\tilde{w}_1 + B_1(U(s) - \frac{\pi_{11}}{r})]
\end{aligned}$$

Then we obtain

$$
\begin{aligned}
V_1^C(s,s) &= UC + \left(\frac{B_0\pi_{11}}{A_2 r} - \frac{U(s)(B_0+r)}{A_2} - A_2 k\right)\sqrt{q} + \mathcal{O}(q) &\text{(B.11)}\\
V_2^C(s,s) &= WC + [-B_0^2 k + B_0\pi_{11}/r - W(s)(B_0+r)] &\text{(B.12)}\\
&\sqrt{q}A_2^{-1} + \mathcal{O}(q) &\text{(B.13)}\\
V_1^{IL}(s,s) &= UC - 2\sqrt{krU(s)}\sqrt{q} + 2krq + \mathcal{O}(q^{\frac{3}{2}}) &\text{(B.14)}\\
V_2^{IF}(s,s) &= WC + \sqrt{\frac{kr}{U(s)}}\left[\frac{2kr}{s^2}\left(\frac{\pi_{10}/r - U(s)}{U(s)-\pi_{11}/r}\right)\right. &\text{(B.15)}\\
&\left.+c - \frac{\pi_{11}}{r}\right]\sqrt{q} + \mathcal{O}(q^{\frac{3}{2}})\\
V_1^{IF}(s,s) &= (\frac{\pi_{11}}{r} - c)\left(1 - \frac{kr}{\sqrt{k(kr^2+\pi_{11})}}\right) + \mathcal{O}(q^{\frac{1}{2}}) &\text{(B.16)}\\
V_2^{IL}(s,s) &= \left(1 - \frac{kr}{\sqrt{k(kr^2+\pi_{11})}}\right)\left[\frac{\pi_{11}}{r} + kr\right. &\text{(B.17)}\\
&\left. - \sqrt{k(kr^2+\pi_{11})}\right] - c + \mathcal{O}(q^{\frac{1}{2}}).
\end{aligned}
$$

where $UC = U(s) - c$ and $WC = W(s) - c$.

Hence, we prove $V_2^{IF}(s,s) > V_2^C(s,s)$, $V_2^{IF}(s,s) > V_2^{IL}(s,s)$, and $V_1^{IL}(s,s) > V_1^{IF}(s,s)$ in the large s limit.

(i) We obtain $V_2^C(s,s) - V_2^{IF}(s,s) = g_1(s)\sqrt{q} + \mathcal{O}(q)$, where

$$
g_1(s) = -\frac{B_0^2 k}{A_2} + W(s)\left(\sqrt{\frac{rk}{U(s)}} - \frac{B_0+r}{A_2}\right) + \frac{B_0\pi_{11}}{A_2 r} - c\sqrt{\frac{rk}{U(s)}}\,.
$$

Taking the limit $s \to \infty$ yields $\lim_{s\to\infty} g_1(s) = -cr\sqrt{k/\pi_{11}} < 0$. As a result, $V_2^{IF}(s,s) > V_2^C(s,s)$ in the large s limit.

(ii) We obtain

$$
V_2^{IL}(s,s) - V_2^{IF}(s,s) = 2kr(s^2-1)/s^2 - 2m + \sqrt{k(kr^2+\pi_{11})/s^2} + \mathcal{O}(\sqrt{q}).
$$

In the large s limit, the leading order term is $2kr - 2\sqrt{k^2r^2 + k\pi_{11}} < 0$, so $V_2^{IF}(s,s) > V_2^{IL}(s,s)$.

(iii) We obtain

$$
V_1^{IF}(s,s) - V_1^{IL}(s,s) = \pi_{11}/r - U(s) - kr(\pi_{11}/r - c)/\sqrt{k^2r^2 + k\pi_{11}} + \mathcal{O}(\sqrt{q}).
$$

In the large s limit, the leading order term is negative because $c < \pi_{11}/r$ that is from $c < W_i(1)$, and $\lim_{s\to\infty} U(s) = \pi_{11}/r$. Therefore $V_1^{IL}(s,s) > V_1^{IF}(s,s)$.

■

Proof of Proposition 11

The proof is a straightforward modification of the proof of Proposition 4 in Kwon et al. (2015).

■

Proof of Theorem 5

(I)(a) If q is sufficiently close to 1, from the expansion of effort level λ_i^C in (B.8), firm i's payoff in the concurrent regime in the asymmetric case is exactly the same as in the symmetric case except for the sub-leading order terms $\mathcal{O}(1-q)$. Therefore, we only need to obtain the comparative statics of $V_i^C(s,s)$ with respect to s to prove the theorem.

In the large s limit, the derivative $dV_i^C(s,s)/ds$ is given by

$$\frac{d}{ds}V_i^C(s,s) = \frac{v_a}{v_b}\left(\frac{1}{s}\right)^2 + \mathcal{O}(s^{-3}),$$

where $v_a = 2k^{3/2}(\pi_{10} - 2\pi_{11})g_2$, $g_2 = k^{3/2}r^3 - (kr^2 + 12\pi_{11})\sqrt{kr^2 + 3\pi_{11}}$, and $v_b = 3\sqrt{\pi_{11}(kr^2 + 3\pi_{11})}[kr + 2\sqrt{k(kr^2 + 3\pi_{11})}]^2$. Here, it is straightforward to confirm that $g_2 < 0$ and $v_b > 0$. If π_{10} is sufficiently large, we have $v_a > 0$. Then $V_i^C(s,s)$ decreases in s. In contrast, when π_{10} is sufficiently small, $\pi_{10} < 2\pi_{11}$, then $v_a < 0$ and $V_i^C(s,s)$ increases in s.

Now we examine the sign of $dV_i^C(s,s)/ds$ in the small s limit. We note that

$$\begin{aligned}
&\frac{d}{ds}V_i^C(s,s) \\
= \ & \lambda^C(2\lambda^C + r)\frac{d}{ds}(U(s) + W(s)) + [k(\lambda^C)^2 - \lambda^C(U(s) - W(s)) \\
&+ rW(s)]\frac{d\lambda^C}{ds}.
\end{aligned}$$

For notational simplicity, we denote $f_1 = d(U(s) + W(s))/ds$, $f_2 = d\lambda^C/ds$, $f_3 = k(\lambda^C)^2 - \lambda^C(U(s) - W(s)) + rW(s)$, and $f_4 = \lambda^C(2\lambda^C + r)$. Then the derivative $dV_i^C(s,s)/ds$ can be expressed as $f_1 f_4 + f_2 f_3$. Next, we expand f_i for $i = 1, 2, 3, 4$ with respect to s in the small s limit. In the limit $\pi_{10} \to \pi_{11}$, we have $f_4 > 0$. We also obtain $\lim_{\pi_{10}\to\pi_{11}} f_1 = 2\sqrt{k/(kr^2+\pi_{11})}(\sqrt{kr^2+\pi_{11}} - \sqrt{kr^2})^2 + \mathcal{O}(s-1)$, $\lim_{\pi_{10}\to\pi_{11}} f_3 = \mathcal{O}(s-1)$, and

$$\lim_{\pi_{10}\to\pi_{11}} f_2 = \frac{\sqrt{k}(4kr^3+3\pi_{11}r)+\sqrt{kr^2+\pi_{11}}(\pi_{11}+4kr^2)}{\sqrt{kr^2+\pi_{11}}\left(2\sqrt{kr^2+\pi_{11}}-\sqrt{kr^2}\right)} + \mathcal{O}(s-1)\,.$$

Hence, $f_1 f_4 + f_2 f_3 > 0$, i.e., $dV_i^C(s,s)/ds > 0$ for π_{10} sufficiently close to π_{11}. For sufficiently large π_{10}, we obtain $\lim_{\pi_{10}\to\infty} dV_i^C(s,s)/ds = -\infty + \mathcal{O}(s-1)$, i.e., $dV_i^C(s,s)/ds < 0$ for sufficiently large π_{10}.

(I)(b) If $q = 1$, $V_i^{IF}(s,s)$ is given by (B.3). In the large s limit, the derivative of the leading order term in the expansion of $V_i^{IF}(s,s)$ with respect to $1-q$ is given by

$$\begin{aligned}
\frac{d[\lambda_j^{IL}(W(s)-c)/(\lambda_j^{IL}+r)]}{ds} &= \eta_1\frac{1}{s^2} + \mathcal{O}(s^{-3}), \\
\text{where } \eta_1 &= kr\frac{cr(\pi_{10}-\pi_{11})-\pi_{11}4kr^2+(\pi_{10}+3\pi_{11})}{2\sqrt{\pi_{11}}\,(kr^2+\pi_{11})^{3/2}} \\
&\quad -2\sqrt{k\pi_{11}}.
\end{aligned}$$

If π_{10} is sufficiently large, then $\eta_1 < 0$ because of $c - \pi_{11}/r < 0$, which is from the fact that $c < W_i(1) < \pi_{11}/r$. Hence, $V_i^{IF}(s,s)$ decreases in s. If π_{10} is sufficiently close to π_{11}, then $\eta_1 > 0$, and therefore, $V_i^{IF}(s,s)$ increases in s.

For s sufficiently close to 1, we obtain

$$\frac{d[\lambda_j^{IL}(W(s)-c)/(\lambda_j^{IL}+r)]}{ds} = \eta_2 + \mathcal{O}(s-1),$$

where

$$\begin{aligned}
\eta_2 &= \frac{-r+\sqrt{r^2+rU(1)/k}}{\sqrt{r^2+rU(1)/k}}\left[-4kr+\sqrt{\frac{k}{kr^2+\pi_{11}}}(2\pi_{11}+4kr^2)\right] \\
&\quad + (W(1)-c)\left[\frac{kr^{1/2}\pi_{11}(-\pi_{10}+\pi_{11})}{2[(kr+U(1))(kr^2+\pi_{11})]^{3/2}}\right].
\end{aligned}$$

In the limit $\pi_{10}\to\pi_{11}$, we obtain

$$\eta_2 \to [-4k^2r^3 - 3kr\pi_{11} + (4kr^2+\pi_{11})\sqrt{k(kr^2+\pi_{11})}]/(kr^2+\pi_{11}) > 0.$$

In the limit $\pi_{10} \to \infty$, we obtain

$$\eta_2 \to [-4k^2r^3 - 4kr\pi_{11} + (4kr^2 + 2\pi_{11})\sqrt{k(kr^2+\pi_{11})}]/(kr^2+\pi_{11}) > 0$$

. Hence, $dV_i^{IF}(s,s)/ds > 0$ in the small s limit when π_{10} is either sufficiently large or sufficiently close to π_{11}.

From the expression of $V_i^{IL}(s,s)$ given by (B.2), we obtain the comparative statics of $V_i^{IL}(s,s)$ with respect to s.

$$\begin{aligned}
&\frac{d}{ds}\left(-\frac{k(\lambda_i^{IL})^2}{\lambda_i^{IL}+r} + U_i(s)\frac{\lambda_i^{IL}}{r+\lambda_i^{IL}} - c\right) \\
=\ &\frac{\partial}{\partial\lambda_i^{IL}}\left(-\frac{k(\lambda_i^{IL})^2}{\lambda_i^{IL}+r} + U_i(s)\frac{\lambda_i^{IL}}{r+\lambda_i^{IL}} - c\right)\frac{d\lambda_i^{IL}}{ds} \\
&+\frac{\partial}{\partial U_i(s)}\left(-\frac{k(\lambda_i^{IL})^2}{\lambda_i^{IL}+r} + U_i(s)\frac{\lambda_i^{IL}}{r+\lambda_i^{IL}} - c\right)\frac{dU_i(s)}{ds} \\
=\ &\frac{\lambda_i^{IL}}{r+\lambda_i^{IL}}\frac{dU_i(s)}{ds}.
\end{aligned} \tag{B.18}$$

The second equality holds due to the envelope theorem because $V_i^{IL}(s,s)$ is an optimal function with respect to λ_i^{IL}. From the fact that $U_i(s)$ decreases in s, which can be checked from the expression of $U_i(s)$ in (3.9), we find that (B.18) is negative.

(II.a) and (II.b) From the expressions in (B.11)-(B.17), Both $V_1^C(s,s)$ and $V_1^{IL}(s,s)$ decrease in s because $U(s)$ decreases in s, and, both $V_2^C(s,s)$ and $V_2^{IF}(s,s)$ increase in s because $W(s)$ increases in s.

■

Proof of Theorem 6

(I) First, we prove $V_i^{IF}(s^*,s^*) \leq V_i^C(s^*,s^*)$. By the definition of s^*, $V_j^{IF}(s^*,s^*) = V_j^C(s^*,s^*)$ for at least one of the firms labeled as $j \in \{1,2\}$. For the other firm i, if $q=1$, $V_i^{IF}(s^*,s^*) = V_i^C(s^*,s^*)$. If $q<1$, suppose $V_1^{IF}(s^*,s^*) > V_1^C(s^*,s^*)$. Then because of the continuity of $V_1^{IF}(s,s)$ and $V_1^C(s,s)$ with respect to s, we obtain $V_1^{IF}(s^*-\triangle s, s^*-\triangle s) > V_1^C(s^*-\triangle s, s^*-\triangle s)$ for sufficiently small $\triangle s > 0$. This contradicts the fact that the game belongs to the concurrent regime when $s < s^*$. Therefore $V_1^{IF}(s^*,s^*) \leq V_1^C(s^*,s^*)$.

If q is sufficiently close to 1, it suffices to compare the leading order terms in the expansions of $V_i^C(s^*,s^*)$ and $V_i^{IL}(s^*,s^*)$ with respect to $1-q$. Or equivalently, we compare $V_i^C(s^*, s^*)$ given by 3.11 and $V_i^{IL}(s^*, s^*)$ given by (3.13) in symmetric cases.

By the definition of s^*, we have $V_j^{IF}(s^*, s^*) = V_j^C(s^*, s^*)$. Also, due to the sufficiently low degree of asymmetry (i.e., $q \to 1$), $V_j^C(s^*, s^*) = V_i^C(s^*, s^*)$. Hence, it remains to show that $V_j^{IF}(s^*, s^*) > V_i^{IL}(s^*, s^*)$, i.e.,

$$(U(s^*) - W(s^*))\frac{\lambda_i^{IL}}{r + \lambda_i^{IL}} \quad < \quad \frac{k(\lambda_i^{IL})^2}{r + \lambda_i^{IL}} + \frac{cr}{\lambda_i^{IL} + r}. \tag{B.19}$$

We have proved that $U(s^*) - W(s^*) < k\lambda_i^{IL}$ in the proof of Proposition 7. Therefore, the inequality (B.19) holds.

(II) To prove Theorem 6 (II) we only need to prove $V_1^{IL}(s^*, s^*) > V_1^C(s^*, s^*)$ whenever $V_2^{IF}(s^*, s^*) = V_2^C(s^*, s^*)$. In order for the equality $V_2^{IF}(s^*, s^*) = V_2^C(s^*, s^*)$ to hold, the following condition has to be satisfied by virtue of (B.11)-(B.17) when q is sufficiently small.

$$\frac{-B_0^2k + B_0\pi_{11}/r - W(s)(B_0 + r)}{A_2} = \sqrt{\frac{kr}{U(s)}}\left[\frac{2kr}{s^2}\left(\frac{\pi_{10}/r - U(s)}{U(s) - \pi_{11}/r}\right) + c - \frac{\pi_{11}}{r}\right], \tag{B.20}$$

where B_0, A_2 are defined in (B.9). Now we obtain the sign of $V_1^{IL}(s^*, s^*) - V_1^C(s^*, s^*)$. From the expressions in (B.11)-(B.17), we obtain

$$\begin{aligned} V_1^{IL}(s^*, s^*) - V_1^C(s^*, s^*) &= \sqrt{q}\left[-2\sqrt{krU(s^*)} + A_2k + \frac{U(s^*)}{A_2}(B_0 + r)\right. \\ &\quad \left. - \frac{B_0\pi_{11}}{A_2 r}\right] + \mathcal{O}(q) \\ &= \left\{-2\sqrt{krU(s^*)} - \frac{B_0^2k}{A_2} + A_2k + \sqrt{\frac{kr}{U(s^*)}}\right. \\ &\quad \left.\left[\frac{2kr}{(s^*)^2}\left(\frac{\pi_{10}/r - U(s^*)}{U(s^*) - \pi_{11}/r}\right) + c - \frac{\pi_{11}}{r}\right]\right\}\sqrt{q} + \mathcal{O}(q). \end{aligned}$$

The second equality is from (B.20). Hence, in order to prove $V_1^{IL}(\delta_1^{IL}, \delta_2^{IF}; s^*, s^*)$ $-V_1^C(\delta_1^C, \delta_2^C; s^*, s^*)$ is positive, it suffices to prove that the coefficient of $\sqrt{q}$

is positive. We note that

$$
\begin{aligned}
& -2\sqrt{krU(s^*)} - A_2^{-1}B_0^2 k + A_2 k + \sqrt{\frac{kr}{U(s^*)}}\left[\frac{2kr}{(s^*)^2}\right. \\
& \left.\left(\frac{\pi_{10}/r - U(s^*)}{U(s^*) - \pi_{11}/r}\right) + c - \frac{\pi_{11}}{r}\right] \\
\geq \quad & -2\sqrt{krU(s^*)} - \frac{B_0^2 k}{A_2} + A_2 k + \sqrt{krU(s^*)}\left[\frac{2kr}{s^2 U(s^*)}\right. \\
& \left.\left(\frac{\pi_{10}/r - U(s^*)}{U(s^*) - \pi_{11}/r}\right) - \frac{\pi_{11}}{rU(s^*)}\right] \\
= \quad & \frac{f_1^{LC}(s^*)}{A_2}\,.
\end{aligned}
$$

Here $f_1^{LC}(s) = f_A(s) + A_2\sqrt{krU(s)}f_B(s)$ where$f_A(s) = (A_0^2 - B_0^2)k$ and

$$
f_B(s) = 2kr\frac{\pi_{10}/r - U(s)}{(U(s) - \pi_{11}/r)s^2 U(s)} - \frac{\pi_{11}}{rU(s)} - 2\,.
$$

Because of $\lim_{s\to\infty} f_1^{LC}(s) = 0$, if we prove that $f_1^{LC}(s)$ decreases in s, then we prove $f_1^{LC}(s^*) > 0$.

Now we prove that both $f_A(s)$ and $A_2\sqrt{krU(s)}f_B(s)$ decrease in s. By the definition of A_2 in (B.9) we have $f_A(s) = -B_0^2 + B_0(U(s) - \pi_{11}/r)/k + rU(s)/k$. Define $\tilde{f}_A(b) = -b^2 + b(U(s) - \pi_{11}/r)/k + rU(s)/k$ in such a way that $\tilde{f}_A(B_0) = f_A(s)$. The function $\tilde{f}_A(b)$ increases in b if $b < (U(s) - \pi_{11}/r)/2k$. It can be verified that $B_0 = (\pi_{11}/r - W(s))/2k < (U(s) - \pi_{11}/r)/2k$ from the definition of B_0 in (B.17) and definitions of $U(s)$ and $W(s)$ in (3.8) and (3.9). Therefore, $\partial\tilde{f}_A(b)/\partial b > 0$ at $x = B_0$. Moreover, we define $\hat{f}_A(u) = -B_0^2 + B_0(u - \pi_{11}/r)/k + ru/k$ such that $\hat{f}_A(U(s)) = f_A(s)$. The derivative $\partial\hat{f}_a(u)/\partial u$ is positive because B_0 is positive. Furthermore, B_0 decreases in s because $W(s)$ increases in s, and $U(s)$ decreases in s. Hence, we have $df_A(s)/ds = (\partial\tilde{f}_A(b)/\partial b)\mid_{b=B_0}(dB_0/ds) + (\partial\hat{f}_a(u)/\partial u)\mid_{u=U(s)}(dU(s)/ds) < 0$.

Next we prove that $A_2\sqrt{krU(s)}f_B(s)$ decreases in s. We have shown that $U(s)$ decreases in s, and $dA_2/ds = (\partial A_2/\partial B_0)(dB_0/ds) < 0$ due to the facts that $\partial A_2/\partial B_0 > 0$ and $dB_0/ds < 0$. Therefore, we only need to prove that $f_B(s)$ decreases in s. Taking the derivative of this expression with respect to

s yields that $df_B(s)/ds$ is given by

$$\frac{-kr^2 f_g}{s^3\sqrt{k(kr^2+s^2\pi_{11})}[kr(\pi_{10}-\pi_{11})+\pi_{11}\sqrt{k(kr^2+s^2\pi_{11})})]^2}, \tag{B.21}$$

where

$$\begin{aligned} f_g \quad &\equiv \quad \pi_{10}\left[2r\sqrt{k(kr^2+s^2\pi_{11})}-(2kr^2+s^2\pi_{11})^2\right] \\ &\qquad +\pi_{11}\left[(s^2\pi_{11})^2-2(\sqrt{kr^2(kr^2+s^2\pi_{11})}-kr^2)^2\right] . \end{aligned}$$

After some algebra, we can show the following:

$$(s^2\pi_{11})^2-2(\sqrt{kr^2(kr^2+s^2\pi_{11})}-kr^2)^2 > \frac{1}{4}(s^2\pi_{11})^2 > 0\,,$$

so we conclude that $f_g > 0$. Therefore, the derivate in (B.21) is negative, and $f_B(s)$ decreases in s.

■

Proof for Theorem 7

(I) If q sufficiently is close to 1, from the expressions for effort level λ_i^C in (B.8), the payoff is given by the payoff in the symmetric case ($q = 1$) plus a sub-leading order term $\mathcal{O}(1-q)$.

(a) We only need to compare $V_i^C(s,s)$ in (3.11) and $V_i^C(s+\epsilon,s)$, where $V_i^C(s+\epsilon,s)$ can be obtained from (3.2):

$$\begin{aligned} V_1^C(s+\epsilon,s) \quad &= \quad \frac{\hat{\lambda}_1^C}{\hat{\lambda}_1^C+\hat{\lambda}_2^C+r}U(s+\epsilon)+\frac{\hat{\lambda}_2^C}{\hat{\lambda}_1^C+\hat{\lambda}_2^C+r}W(s) \\ &\qquad -\frac{k(\hat{\lambda}_1^C)^2}{\hat{\lambda}_1^C+\hat{\lambda}_2^C+r}; \end{aligned} \tag{B.22}$$

$$\begin{aligned} V_2^C(s+\epsilon,s) \quad &= \quad \frac{\hat{\lambda}_2^C}{\hat{\lambda}_1^C+\hat{\lambda}_2^C+r}U(s)+\frac{\hat{\lambda}_1^C}{\hat{\lambda}_1^C+\hat{\lambda}_2^C+r}W(s+\epsilon) \\ &\qquad -\frac{k(\hat{\lambda}_2^C)^2}{\hat{\lambda}_1^C+\hat{\lambda}_2^C+r}. \end{aligned} \tag{B.23}$$

Here, in equilibrium, $\hat{\lambda}_i^C$ is chosen by firm i to maximize $V_i^C(s+\epsilon,s)$ given firm j's effort level $\hat{\lambda}_j$. Following the proof of Proposition 9, the pair of effort levels $(\hat{\lambda}_1^C, \hat{\lambda}_2^C)$ is the unique solution of the following system of equations:

$$\begin{aligned}
\hat{\lambda}_1^C + \hat{\lambda}_2^C + r &= \sqrt{(\hat{\lambda}_2^C + r)^2 + (\hat{\lambda}_2^C)\frac{U(s+\epsilon) - W(s)}{k} + \frac{r}{k}U(s+\epsilon)} \\
\hat{\lambda}_1^C + \hat{\lambda}_2^C + r &= \sqrt{(r + \hat{\lambda}_1^C)^2 + (\hat{\lambda}_1^C)\frac{U(s) - W(s+\epsilon)}{k} + \frac{r}{k}U(s)} \\
\hat{\lambda}_1^C \geq 0 \quad &, \quad \hat{\lambda}_1^C \geq 0\,.
\end{aligned}$$

Now we expand $\hat{\lambda}_i^C$ and $V_i^C(s+\epsilon, s)$ in (B.22) with respect to ϵ:

$$\begin{aligned}
\hat{\lambda}_1^C &= \hat{\lambda} + v_1\epsilon + \mathcal{O}(\epsilon^2) \\
\hat{\lambda}_2^C &= \hat{\lambda} + v_2\epsilon + \mathcal{O}(\epsilon^2) \\
V_1^C(s+\epsilon, s) &= V_1^C(s,s) - \frac{1}{(2\hat{\lambda}+r)^2}\frac{g_A}{g_B}\epsilon + \mathcal{O}(\epsilon^2).
\end{aligned}$$

Here, $\hat{\lambda}$ is exactly the same as λ_i^C in (3.10), and

$$\begin{aligned}
v_1 &= \frac{2k\tilde{u}_1(r+\hat{\lambda})(r+2\hat{\lambda}) - \tilde{w}_2\hat{\lambda}(2k\hat{\lambda} - U(s) + W(s))}{4k^2(\hat{\lambda}+r)(3\hat{\lambda}+r) + 4k\hat{\lambda}(U(s)-W(s)) - (U(s)-W(s))^2} \\
v_2 &= \frac{\tilde{u}_1(r+\hat{\lambda})(2k\hat{\lambda} - U(s) + W(s)) - 2\tilde{w}_2 k\hat{\lambda}(r+2\hat{\lambda})}{4k^2(\hat{\lambda}+r)(3\hat{\lambda}+r) + 4k\hat{\lambda}(U(s)-W(s)) - (U(s)-W(s))^2} \\
g_B &= [2k(\hat{\lambda}+r) + U(s) - W(s)][2k(3\hat{\lambda}+r) - (U(s)-W(s))], \\
g_A &= -g_B\Big\{\hat{\lambda}\Big[(v_1 - v_2)(U(s) - W(s) - k\hat{\lambda}) + (\tilde{w}_2 - \tilde{u}_1)(2\hat{\lambda}+r) \\
&\quad -2kv_1(r+\hat{\lambda})\Big] + r(v_1U(s) + v_2W(s))\Big\},
\end{aligned}$$

where

$$\tilde{u}_1 = [s\pi_{11}(\pi_{10} - \pi_{11})/(kr^2 + s^2\pi_{11})]\sqrt{k/(kr^2 + s^2\pi_{11})}$$

,

$$\tilde{w}_2 = (2rW(s)/s)\sqrt{k/(kr^2 + s^2\pi_{11})}.$$

In order to prove the theorem, it suffices to prove that $g_A/g_B > 0$. In the limit $\pi_{10} \to \infty$, we have $g_A/g_B \to \infty$.

Now we examine the sign of g_A/g_B in the limit $\pi_{10} \to \pi_{11}$. In this limit, we find that g_B converges to a positive finite number because of the inequality $2k(3\hat{\lambda}+r) > (U(s) - W(s))$ which follows from the definition of $\hat{\lambda}$ in (3.10). Note that $\lim_{\pi_{10}\to\pi_{11}} \tilde{u}_1 \to 0$, and $\hat{\lambda}$, $U(s)$, $W(s)$ and $\tilde{w}_2$ all converge to positive finite numbers. Therefore, g_A reduces to $g_A = \tilde{w}_2\hat{\lambda}\tilde{g}_A$ in the limit

$\pi_{10} \to \pi_{11}$, where

$$\begin{aligned}\tilde{g}_A &= -(r+\hat{\lambda})\left[(U(s)-W(s))^2 + 2krW(s)\right] + (U(s)-W(s)) \\ &\quad (6kr\hat{\lambda} - rW(s) + 9k\hat{\lambda}^2) - k\hat{\lambda}^2(6r+10\hat{\lambda})\,.\end{aligned}$$

Now we will show $g_A > 0$ by proving $\tilde{g}_A > 0$, because $\tilde{w}_2$ and λ^* are positive by their definitions. We reexpress $\tilde{g}_A$ as $\tilde{g}_A(U,W)$ by replacing $U(s)$ by U and $W(s)$ by W. We obtain the sufficient condition for $\tilde{g}_A(U,W) > 0$ is $k > (U-W)^2/4rW$. By definitions of $U(s)$ in (3.9)and $W(s)$ in (3.8), we obtain $k > (U(s)-W(s))^2/4rW(s)\ \forall s$. The sufficient condition for $\tilde{g}_A(U,W) > 0$ is automatically satisfied when $U = U(s)$ and $W = W(s)$. Hence, we obtain $g_A > 0$ and $g_A/g_B > 0$ in the limit $\pi_{10} \to \pi_{11}$.

(b) If firm 1 is the leader, let $(\hat{\lambda}_1^{IL}, \hat{\lambda}_2^{IF})$ be the equilibrium effort level before the first innovation Then by definition of $V_i^{IL}(s,s)$ in (3.13) and λ_1^{IL} in (3.19), we have

$$\begin{aligned}V_i^{IL}(s,s) &= U(s)\frac{\lambda_1^{IL}}{\lambda_1^{IL}+r} - \frac{k(\lambda_1^{IL})^2}{\lambda_1^{IL}+r} - c \geq U(s)\frac{\hat{\lambda}_1^{IL}}{\hat{\lambda}_1^{IL}+r} - \frac{k(\hat{\lambda}_1^{IL})^2}{\hat{\lambda}_1^{IL}+r} - c \\ &> U(s+\epsilon)\frac{\hat{\lambda}_1^{IL}}{\hat{\lambda}_1^{IL}+r} - \frac{k(\hat{\lambda}_1^{IL})^2}{\hat{\lambda}_1^{IL}+r} - c = V_i^{IL}(s+\epsilon, s).\end{aligned}$$

Here, the first inequality is because of the optimality of λ_1^{IL}, and the second inequality is from the fact that $U(s)$ decreases in s.

If firm 1 is the follower, the spillover from firm 1 to firm 2 never occurs, so the value functions has no dependence on ϵ. Hence, $V_1^{IF}(s+\epsilon, s)$ is the same as $V_1^{IF}(s,s)$.

(c) In the proof of part (a), we have obtained $V_i^C(s+\epsilon, s) = V_i^C(s,s) + \mathcal{O}(\epsilon)$. We also obtain $V_i^{IL}(s+\epsilon, s) = V_i^{IL}(s,s) + \mathcal{O}(\epsilon)$ by expanding $V_i^{IL}(s+\epsilon, s)$ with respect to ϵ. By Theorem 6, we know that $V_i^C(s^*, s^*) > V_i^{IL}(s^*, s^*)$. Therefore, $V_i^C(s^*+\epsilon, s^*) > V_i^{IL}(s^*, s^*)$.

(II) In the small q limit, we obtain the expansions of $V_i^C(s+\epsilon, s)$, $V_i^{IL}(s+\epsilon, s)$ and $V_i^{IF}(s+\epsilon, s)$ with respect to q as follows:

$$
\begin{aligned}
V_1^C(s+\epsilon,s) &= U(s+\epsilon)-c+\left(\frac{\hat{B}_0\pi_{11}}{\hat{A}_2 r}-\frac{U(s+\epsilon)(\hat{B}_0+r)}{\hat{A}_2}-\hat{A}_2 k\right)\sqrt{q} \\
&\quad +\mathcal{O}(q) \\
V_2^C(s+\epsilon,s) &= W(s+\epsilon)-c+\sqrt{q}\hat{A}_2^{-1}[-\hat{B}_0^2 k+\hat{B}_0\pi_{11}/r \\
&\quad -W(s+\epsilon)(\hat{B}_0+r)]+\mathcal{O}(q) \\
V_1^{IL}(s+\epsilon,s) &= U(s+\epsilon)-c-2\sqrt{krU(s+\epsilon)}\sqrt{q}+2krq+\mathcal{O}(q^{\frac{3}{2}}) \\
V_2^{IF}(s+\epsilon,s) &= W(s+\epsilon)-c+\sqrt{\frac{kr}{U(s+\epsilon)}}\sqrt{q}\left[\frac{2kr}{(s+\epsilon)^2}\right. \\
&\quad \left.\left(\frac{\pi_{10}/r-U(s+\epsilon)}{U(s+\epsilon)-\pi_{11}/r}\right)+c-\frac{\pi_{11}}{r}\right]+\mathcal{O}(q^{\frac{3}{2}}) \\
V_1^{IF}(s+\epsilon,s) &= (\frac{\pi_{11}}{r}-c)\left(1-\frac{kr}{\sqrt{k(kr^2+\pi_{11})}}\right)+\mathcal{O}(q^{\frac{1}{2}}) \\
V_2^{IL}(s+\epsilon,s) &= \left(1-\frac{kr}{\sqrt{k(kr^2+\pi_{11})}}\right)\left[\frac{\pi_{11}}{r}+kr-\sqrt{k(kr^2+\pi_{11})}\right] \\
&\quad -c+\mathcal{O}(q^{\frac{1}{2}}),
\end{aligned}
$$

where

$$
\begin{aligned}
\hat{B}_0 &= (\pi_{11}/r-W(s+\epsilon))/2k \\
\hat{A}_2 &= \sqrt{\hat{B}_0(U(s+\epsilon)-\pi_{11}/r)/k+rU(s+\epsilon)/k} \\
\hat{u}_2 &= \left[(\pi_{10}-\pi_{11})\sqrt{k/\pi_{11}}\right]/(s+\epsilon) \\
\hat{w}_1 &= 2\sqrt{k\pi_{11}}/(s+\epsilon).
\end{aligned}
$$

Therefore, for sufficiently small ϵ, the proofs of Theorem 5 (II) and Theorem 6 (II) carries over into the proof of Theorem 7 (II).

■

Proof of Theorem 8

(i) By virtue of Proposition 10, the game belongs to the war of attrition regime. From the definition of $P(T_2^0 < T_1^0 \mid \min\{T_1^0, T_2^0\} > 0)$, it suffices to prove $\tau_2^M < \tau_1^M$. First, we obtain the following:

$$\begin{aligned}
V_1^{IL}(s,s) &= -c - \frac{k(\lambda_0)^2}{r+\lambda_0} + \frac{\lambda_0 U(s)}{r+\lambda_0} + p_1(1-q) + \mathcal{O}((1-q)^2) \\
V_2^{IL}(s,s) &= -c - \frac{k(\lambda_0)^2}{r+\lambda_0} + \frac{\lambda_0 U(s)}{r+\lambda_0} + p_2(1-q) + \mathcal{O}((1-q)^2) \\
V_1^{IF}(s,s) &= \frac{\lambda_0}{r+\lambda_0}(W(s)-c) + q_1(1-q) + \mathcal{O}((1-q)^2) \\
V_2^{IF}(s,s) &= \frac{\lambda_0}{r+\lambda_0}(W(s)-c) + q_2(1-q) + \mathcal{O}((1-q)^2),
\end{aligned}$$

where $\lambda_0 = -r + \sqrt{r^2 + rU(s)/k}$, and

$$\begin{aligned}
p_1 &= \frac{k\lambda_0^2(r+\lambda_0) + l_1(-k\lambda_0^2 + rU(s) - 2k\lambda_0)}{(r+\lambda_0)^2} \\
p_2 &= \frac{\lambda_0(k\lambda_0 + \tilde{u}_2)(r+\lambda_0) + l_2(-k\lambda_0^2 + rU(s) - 2k\lambda_0)}{(r+\lambda_0)^2} \\
q_1 &= \frac{l_2 r(W(s)-c) + \tilde{w}_1\lambda_0(r+\lambda_0)}{(r+\lambda_0)^2} \\
q_2 &= \frac{l_1 r(W(s)-c)}{(r+\lambda_0)^2} \\
l_1 &= \frac{rU(s)}{2\sqrt{k}(r+\lambda_0)} \\
l_2 &= \frac{r\tilde{u}_2}{2\sqrt{k}(r+\lambda_0)} \\
\tilde{u_2} &= -\frac{1}{2}\frac{s^2\pi_{11}}{k}\frac{(U(s)-\pi_{11}/r)^3}{(\pi_{10}-\pi_{11})^2} \\
\tilde{w}_1 &= -\frac{2kr}{s^2} + \frac{(2kr^2 + s^2\pi_{11})(U(s)-\pi_{11}/r)}{s^2(\pi_{10}-\pi_{11})}.
\end{aligned}$$

Here $U(s)$,$W(s)$ are given in (3.9) and (3.8).From the expressions in (3.20), in order to prove $\tau_1^M > \tau_2^M$, it suffices to prove

$$V_2^{IF}(s,s)/V_2^{IL}(s,s) > V_1^{IF}(s,s)/V_1^{IL}(s,s),$$

or equivalently,

$$\left[\frac{\lambda_0}{r+\lambda_0}(W(s)-c)\right](p_1-p_2) - \left[-c - \frac{k(\lambda_0)^2}{r+\lambda_0} + \frac{\lambda_0 U(s)}{r+\lambda_0}\right](q_1-q_2) > 0$$

Taking the limit of the left hand side of the above inequality gives

$$\begin{aligned} & \lim_{s\to\infty}\left[\frac{\lambda_0}{r+\lambda_0}(W(s)-c)\right](p_1-p_2)-(q_1-q_2) \\ & \left[-c-\frac{k(\lambda_0)^2}{r+\lambda_0}+\frac{\lambda_0 U(s)}{r+\lambda_0}\right] \\ = \; & \sqrt{k}\pi_{11}(cr-\pi_{11})\gamma/[2r(kr^2+\pi_{11})^2], \end{aligned} \tag{B.24}$$

where $\gamma = \sqrt{k(k+\pi_{11})}(cr - 2kr^2 - \pi_{11}) + 2k^2r^3 + 2kr\pi_{11}$. The inequality $c < W(1)$ gives

$$\gamma < \sqrt{k(k+\pi_{11})}(W(1)r - 2kr^2 - \pi_{11}) + 2k^2r^3 + 2kr\pi_{11} = 0\,.$$

Moreover, the inequality $c < \pi_{11}/r$ holds because of the fact $c < W(1) < \lim_{s\to\infty} W(s) = \pi_{11}/r$. Hence the expression (B.24) is positive, which proves $\tau_2^M < \tau_1^M$ and furthermore proves Theorem (8) (I).

(ii) By virtue of Proposition 10, the game belongs to the imitative regime, in which firm 1 is the leader and firm 2 is the follower. Hence, $T_2^0 > T_1^0 = 0$.

■

APPENDIX C

PROOFS FOR CHAPTER 4

Proof of Lemma 4

Following the standard steps of solving optimal stopping problem, we derive that θ_L is the solution to (4.8) by applying smooth pasting condition and value matching condition. Now we only need to prove the equation (4.8) only has a unique solution in $(0, \theta_F)$.

First of all, we prove that the leader never chooses $\theta_L \in [\theta_F, 1]$. Suppose the leader chooses $\theta_L \in [\theta_F, 1]$, then $R_1(p) = sE^p(\mu)/r$ from expression in (4.6). We solve the optimal stopping problem and obtain a candidate of $V_0^L(p)$ as $[sE^p(\mu)/r - k][\psi(p, \gamma_0)/\psi(\theta_L, \gamma_0)]$ by applying value matching condition. Taking the first derivative of this candidate of $V_0^L(p)$ yields that

$$R_1'(\theta_L) - \frac{\psi'(\theta_L, \gamma_0)}{\psi(\theta_L, \gamma_0)}(R_1(p) - k) = \frac{s}{r}(\mu^H - \mu^L) - \frac{1 + \gamma_0 - 2\theta_L}{2\theta_L(1 - \theta_L)} (\frac{s}{r}E^{\theta_L}(\mu) - k) \quad \text{(C.1)}$$

When $\theta_L \to 1$, the expression (C.1) goes to $-\infty$. When $\theta_L \to \theta_F$, due to $\gamma_0 > \gamma_1$, we have $R_1'(\theta_L) - [R_1(\theta_L) - k]\psi_x'(\theta_L, \gamma_0)/\psi(\theta_L, \gamma_0) < s(\mu^H - \mu^L)/r - [sE^{\theta_F}(\mu)/r - k][(1 + \gamma_0 - 2\theta_F)/2\theta_F(1 - \theta_F)] < s(\mu^H - \mu^L)/r - [sE^{\theta_F}(\mu)/r - k][(1 + \gamma_1 - 2\theta_F)/2\theta_F(1 - \theta_F)] = 0$. Moreover, the expression $s(\mu^H - \mu^L)/r - [(1 + \gamma_0 - 2x)/2x(1 - x)][E^x(\mu) - k]$ decreases in x. Hence, $R_1'(x) - (R_1(x) - k_1)\psi_x'(x, \gamma_0)[\psi(x, \gamma_0)]^{-1} < 0$, $\forall x \in [\theta_F, 1]$. We conclude that the optimal θ_L cannot lie in $[\theta_F, 1]$.

Secondly we prove that there exist a unique $\theta_L \in (0, \theta_F)$ satisfies (4.8). Given $\theta_L \in [0, \theta_F)$, $R_1(x) = E^x(\mu)/r - [(1 - s)/r]E^{\theta_F}(\mu)\psi(x, \gamma_1)/\psi(\theta_F, \gamma_1)$ from (4.6). Solving the optimal stopping problem and applying value matching condition yields the candidate of $V_0^L(p)$ to be

$$\{E^p(\mu)/r - [(1 - s)/r]E^{\theta_F}(\mu)\psi(x, \gamma_1)/\psi(\theta_F, \gamma_1) - k\}\psi(x, \gamma_0)/\psi(\theta_L, \gamma_0)\,.$$

Define

$$
\begin{aligned}
f(x) &= \frac{\psi'(x,\gamma_0)}{\psi(x,\gamma_0)}(R_1(x)-k) - R_1'(x) \\
&= -\frac{1}{r}(\mu^H-\mu^L) + \frac{1-s}{r}E^{\theta_F}(\mu)\frac{\psi(x,\gamma_1)}{\psi(\theta_F,\gamma_1)}\frac{1+\gamma_1-2x}{2x(1-x)} \\
&\quad + \frac{1+\gamma_0-2x}{2x(1-x)}\left(\frac{1}{r}E^x(\mu) - \frac{1-s}{r}E^{\theta_F}(\mu)\frac{\psi(x,\gamma_1)}{\psi(\theta_F,\gamma_1)} - k\right).
\end{aligned}
$$

When $x \to 0$, the expression $f(0) = \lim_{x\to 0}(E^x(\mu)/r - k)(1+\gamma_0-2x)/[2x(1-x)] + [(1-s)/r]E^{\theta_F}(\mu)[\psi(x,\gamma_1)/\psi(\theta_F,\gamma_1)](\gamma_1-\gamma_0)/[2x(1-x)] - (\mu^H-\mu^L)/r < 0$, due to the fact that $\mu^L/r < k$ and $\lim_{x\to 0}\psi(x,\gamma_1)/2x(1-x) \to 0$. When $x \to \theta_F$, the expression

$$
\begin{aligned}
f(\theta_F) &= \left(\frac{s}{r}E^{\theta_F}(\mu) - k\right)\frac{1+\gamma_0-2x}{2x(1-x)} - \frac{1}{r}(\mu^H-\mu^L) + \frac{1-s}{r}E^{\theta_F}(\mu) \\
&\quad \frac{1+\gamma_1-2x}{2x(1-x)} \\
&> \left(\frac{s}{r}E^{\theta_F}(\mu) - k\right)\frac{1+\gamma_1-2x}{2x(1-x)} - \frac{1}{r}(\mu^H-\mu^L) + \frac{1-s}{r}E^{\theta_F}(\mu) \\
&\quad \frac{1+\gamma_1-2x}{2x(1-x)} \\
&= 0\,.
\end{aligned}
$$

By some algebra, we find that the sign of $f(x)$ is the same as the sign of $\tilde{f}(x)$, where

$$
\begin{aligned}
\tilde{f}(x) &= (1+\gamma_0)(\frac{1}{r}\mu^L - k) + x[(1+\gamma_0)\frac{1}{r}(\mu^H-\mu^L) - 2(\frac{1}{r}\mu^H - k)] \\
&\quad -\psi(x,\gamma_1)(\gamma_1-\gamma_0)\frac{1-s}{r}E^{\theta_F}(\mu)\frac{1}{\psi(\theta_F,\gamma_1)}.
\end{aligned}
$$

The function $\tilde{f}(x)$ is concave in x because $\tilde{f}"(x) < 0$. Furthermore, we have $\tilde{f}(0) < 0$, and $\tilde{f}(\theta_F) > 0$, because $f(0) > 0$ and $f(\theta_F) > 0$. Therefore, $\tilde{f}(x)$ only crosses 0 once, and consequently $f(x)$ only crosses 0 once. Thus, θ_L is the only root for $x \in (0, \theta_F)$.

■

Proof of Lemma 5

From expression of $V^L_{\tau_L}(p)$ in (4.6) and $V^F_{\tau_L}(p)$ in (4.4) for $p < \theta_F$, we define

$$\begin{aligned}\nu(x) &= V^L_{\tau_L}(x) - V^F_{\tau_L}(x) \\ &= \frac{1}{r}E^x(\mu) - [\psi(p,\gamma_1)/\psi(\theta_F,\gamma_1)]E^{\theta_F}(\mu)\frac{1}{r} - k(1-\psi(p,\gamma_1)/\psi(\theta_F,\gamma_1))\,.\end{aligned}$$

The second derivative $\nu"(x) = \psi_x"(x,\gamma_1)/\psi(\theta_F,\gamma_1)(-E^{\theta_F}(\mu)/r + k) < 0$ due to the fact that $E^{\theta_F}(\mu)/r > k$ and $\psi(x,\gamma_1)$ is a convex function of x. Hence, $\nu(x)$ is a concave function. Moreover, because $v(0) = \frac{1}{r}\mu^L - k < 0$ and $\nu(\theta_F) = 0$, it is sufficient to show that $\nu(x)$ first increases then decreases in x in order to prove that $\nu(x)$ crosses 0 only once. Hence next we prove that $v'(x)$ first is positive then is negative for x varies from 0 to θ_F. The derivative $v'(0) = \frac{1}{r}(\mu^H - \mu^L) > 0$, and

$$\begin{aligned}\nu'(\theta_F) &= (\mu^H - \mu^L)/r + \psi'_x(\theta_F,\gamma_1)/\psi(\theta_F,\gamma_1)[k - E^{\theta_F}(\mu)/r] \\ &< \frac{1}{r}(\mu^H - \mu)^L + [\psi'_x(\theta_F,\gamma_1)/\psi(\theta_F,\gamma_1)][k/s - E^{\theta_F}(\mu)/r] \\ &= 0\,.\end{aligned}$$

Furthermore, because $v''(x) < 0$, the derivative $v'(x)$ first is positive then is negative. Therefore, $\nu(x)$ only crosses 0 once. In other words, there exists a unique θ_C such that $V^F_{\tau_L}(p) > V^L_{\tau_L}(p)$ for $p \in (0,\theta_C)$ and $V^F_{\tau_L}(p) < V^L_{\tau_L}(p)$ for $p \in (\theta_C,\theta_F)$.

Secondly, we prove the condition for $\theta_L < \theta_C$. From the definition of θ_C, the inequality $V^F_{\tau_L}(\theta_L) > V^L_{\tau_L}(\theta_L)$ holds if only if $\theta_L < \theta_C$. Consider $\nu(\theta_L) = V^L_{\tau_L}(\theta_L) - V^F_{\tau_L}(\theta_L) = E^{\theta_L}(\mu)/r - [(1-s)/r]E^{\theta_F}(\mu)\psi(\theta_L,\gamma_1)/\psi(\theta_F,\gamma_1) - k - [\psi(\theta_L,\gamma_1)/\psi(\theta_F,\gamma_1)][sE^{\theta_F}(\mu)/r - k]$. Using the expression of θ_L in (4.8), we rewrite $\nu(\theta_L)$ to be $[\psi(\theta_L,\gamma_1)/\psi'(\theta_L,\gamma_1)][(\mu^H - \mu^L)/r - (E^{\theta_F}(\mu)/r - k)$ $\psi'(\theta_L,\gamma_1)/\psi(\theta_F,\gamma_1)]$. Because $\psi(\theta_L,\gamma_1)/\psi'(\theta_L,\gamma_1)$ is positive, the inequality $V^L_{\tau_L}(\theta_L) - V^F_{\tau_L}(\theta_L) < 0$ holds if (4.9) holds.

Lastly we prove the case of sufficiently small σ_1. For $\sigma_1 \to 1$, the left hand side of (4.9) goes to $-\infty$, because $\psi(\theta_F,\gamma_1) \to \infty$, $\psi'(\theta_L,\gamma_1)$ is positive and finite, and $(E^{\theta_F}(\mu)/r - k) > 0$.

■

Proof of Lemma 6

Define $f_2(\delta) = \theta_F^{-1} - 1 = [(\gamma_1 - 1)/(\gamma_1 + 1)][(s\mu^H - kr)/(kr - \mu^L)]$, so that the sign of $d\theta_F/d\delta$ is opposite to the sign of $df_2(\delta)/d\delta$, where

$$\begin{aligned} df_2(\delta)/d\delta &= 2f_2(\delta)[\frac{1}{\gamma_1^2-1}\frac{d\gamma_1}{d\delta}+\frac{(\bar{\mu}-kr/s)}{(\bar{\mu}-kr/s)^2-\delta^2}] \\ &= 2f_2(\delta)\left[-\frac{1}{\gamma_1\delta}+\frac{(\bar{\mu}-kr/s)}{(\bar{\mu}-kr/s)^2-\delta^2}\right]. \end{aligned}$$

Here $(\bar{\mu}-kr/s)^2-\delta^2=(\bar{\mu}-kr/s+\delta)(\bar{\mu}-kr-\delta)=(\mu^H-kr/s)(\mu^L-kr/s)<0$. Because $\bar{\mu}-kr/s\geq 0$, we have $df_2(\delta)/d\delta<0$ and $d\theta_F/d\delta>0$.

■

Proof of Proposition 13

No matter $\theta_L<\theta_C$ or not, the leader's ex-post payoff is always given by

$$V^L_{\tau_1}(p)=E^p(\mu)/r-[(1-s)/r]E^{\theta_F}(\mu)[\psi(p,\gamma_1)/\psi(\theta_F,\gamma_1)]-k$$

for $p<\theta_F$, and $V^L_{\tau_1}(p)=sE^p(\mu)/r$ when $p>\theta_F$.

We first discuss the case when $p<\theta_F$. From the expression of $V^L_{\tau_1}(p)$, only the term $[(1-s)/r]E^{\theta_F}(\mu)[\psi(p,\gamma_1)/\psi(\theta_F,\gamma_1)]$ has σ_1-dependence. Let $f_2(\gamma_1)=[(1-s)/r]E^{\theta_F}(\mu)[\psi(p,\gamma_1)/\psi(\theta_F,\gamma_1)]$. Due to the fact that $d\gamma_1/d\sigma_1>0$, the sign of $dV^L_{\tau_1}(p)/d\sigma_1$ is opposite of the sign of $df(\gamma_1)/d\gamma_1$. The derivative $df_2(\gamma_1)/d\gamma_1$ is given by

$$\begin{aligned} \frac{\partial f_2(\gamma_1)}{\partial\gamma_1}+\frac{\partial f_2(\gamma_1)}{\partial\theta_F}\frac{d\theta_F}{d\gamma_1} &= \frac{\psi(p,\gamma_1)}{\psi(\theta_F,\gamma_1)}\left\{\frac{2\theta_F^2}{(1+\gamma_1)^2}\frac{s\mu^H-kr}{kr-s\mu^L}-\Big[(\mu^H-\mu^L)\right. \\ &\quad +E^{\theta_F}(\mu)\frac{1+\gamma_1-2\theta_F}{2\theta_F(1-\theta_F)}\Big]+\frac{1}{2}E^{\theta_F}(\mu) \\ &\quad \left.\ln(\frac{p}{1-p}\frac{1-\theta_F}{\theta_F})\right\} \\ &= \left\{\frac{\theta_F}{1-\theta_F}\frac{s\mu^H-kr}{kr-s\mu^L}\frac{(1+\gamma_1)E^{\theta_F}(\mu)-2\theta_F\mu^H}{(1+\gamma_1)^2}\right. \\ &\quad \left.+\frac{1}{2}\ln(\frac{p}{1-p}\frac{1-\theta_F}{\theta_F})E^{\theta_F}(\mu)\right\}\frac{\psi(p,\gamma_1)}{\psi(\theta_F,\gamma_1)}. \end{aligned}$$

Because $\psi(p,\gamma_1)>0,\forall p\in(0,1)$, the sign of $df_2(\gamma_1)/d\gamma_1$ depends on the sign of the sum in the curly brackets. (1) For any fixed p, if $\sigma_1\to 0$, then $\gamma_1\to 1$, $\theta_F\to 1$, and $\ln\theta_F/(1-\theta_F)\sim o((\gamma_1-1)^{-1})$. Hence the sum in the curly brackets goes to ∞, the derivative $df_2(\gamma_1)/d\gamma_1>0$, and $dV^L_{\tau_1}(p)/d\sigma_1<0$. (2) If $p\to\theta_F$ and $\gamma_1\to\infty$, $E^{\theta_F}(\mu)/(\gamma_1-1)-2\theta_F\mu^H/[(\gamma_1+1)(\gamma_1-1)]\to 0$, while

$\ln[p(1-\theta_F)/(1-p)\theta_F]E^{\theta_F}(\mu)/2<0$. Hence the sum in the curly brackets is negative, and thus $df_2(\gamma_1)/d\gamma_1<0$ and $dV^L_{\tau_1}(p)/d\sigma_1>0$.

Then we discuss the case in which $p\to\theta_F$ so that an infinitesimal increase in γ_1 let the follower changes from waiting to immediate investment. Suppose $\gamma_1^{'}<\gamma_1^{''}$, denote $\theta_F^{\prime}$ and $\theta_F^{''}$ as the corresponding follower's investment thresholds. From the expression of θ_F, $\theta_F^{\prime}>\theta_F^{''}$. Consider the posterior belief at τ_1 satisfies $\theta_F^{''}<p<\theta_F^{\prime}$, then the difference in the leader's payoffs due to the increase in γ_1 is given by $sE^p(\mu)/r-k-\{E^p(\mu)/r-[(1-s)/r]E^{\theta_F}(\mu)[\psi(p,\gamma_1)/\psi(\theta_F,\gamma_1)]-k\}<sE^p(\mu)/r-k-(sE^p(\mu)/r-k)=0$, because of the optimality of $\theta_F^{'}$ at $\gamma_1=\gamma_1^{'}$. Hence, the leader's payoff decreases in this case.

Lastly, when $p>\theta_F$, $V^L_{\tau_1}(p)$ is a constant in σ_1.

■

Proof of Lemma 7

Given the current posterior belief $p<\bar{\theta}_F$, let $\hat{\pi}^F_t(p)$ be the follower's payoff if it chooses waiting, then $\hat{\pi}^F_t(p)$ satisfies the following ODE:

$$(1+rdt)\hat{\pi}^F_t(p)=\lambda dt\left[\frac{\psi(p,\gamma_1)}{\psi(\bar{\theta}_F,\gamma_1)}\left(\bar{R}_F(\bar{\theta}_F)-k\right)\right]+(1-\lambda dt)\hat{\pi}^F_{t+dt}(p).$$

Solving this ODE yields that $\hat{\pi}^F_t(p)=[\lambda/(\tilde{r}+\lambda)]\,[\psi(p,\gamma_1)/\psi(\bar{\theta}_F,\gamma_1)](\bar{R}_F(\bar{\theta}_F)-k)$. If the follower chooses to invest immediately, it receives payoff $\tilde{R}_F(p)-k$. Consider

$$\begin{aligned}g(p)&=\tilde{R}_F(p)-k-\hat{\pi}^F_t(p)\\&=(\frac{\tilde{s}}{r}E^p(\mu)-\hat{k})-\frac{\lambda}{\tilde{r}+\lambda}\frac{\psi(p,\gamma_1)}{\psi(\bar{\theta}_F,\gamma_1)}(\frac{\bar{s}}{r}E^{\bar{\theta}_F}(\mu)-\hat{k}).\end{aligned}$$

Firstly, when $p\to0$, $g(p)\sim-[\lambda/(\tilde{r}+\lambda)]\psi^{-1}(\bar{\theta}_F,\gamma_1)(\bar{s}E^{\bar{\theta}_F}(\mu)/r-\hat{k})p^{(\gamma_1+1)/2}+o(p^{(\gamma_1+1)/2})$. Hence $g(p)<0$ for sufficiently small p. When $p\to\bar{\theta}_F$,$g(p)\to\tilde{s}E^{\bar{\theta}_F}(\mu)/r-\hat{k}-[\lambda/(\tilde{r}+\lambda)](\bar{s}E^{\bar{\theta}_F}(\mu)/r-\hat{k})>0$, because $\tilde{s}-\bar{s}=(1-s)r\lambda]/[(r+\lambda)(r+2\lambda)]>0$ and $\lambda/(\tilde{r}+\lambda)<1$. Therefore, there exist at least one $\hat{p}$ such that $g(\hat{p})=0$.

Secondly, we prove the uniqueness of $\hat{p}$. The function $g(p)$ is a concave function because $\tilde{R}_F(p)$ is linear and $\hat{\pi}^F_t(p)$ is convex in p. Hence $g'(p)$ is a decreasing function, and $g'(p)>g'(\bar{\theta}_F)$ for all $p\in(0,\bar{\theta}_F)$. $g'(\bar{\theta}_F)=\tilde{s}(\mu^H-\mu^L)/r-[\lambda/(\tilde{r}+\lambda)]\,[\psi_p'(\bar{\theta}_F,\gamma_1)/\psi(\bar{\theta}_F,\gamma_1)](\bar{R}_F(\bar{\theta}_F)-k)=\tilde{s}(\mu^H-\mu^L)/r-$

$[\lambda/(\tilde{r}+\lambda)]\,\bar{s}(\mu^H-\mu^L)/r > 0$. The second equality is from the optimality condition of $\bar{\theta}_F$. Therefore, $g'(p) > g'(\bar{\theta}_F) > 0, \forall p \in [0,\bar{\theta}_F)$, i.e., $g(p)$ always increases in p. Consequently, there is a unique $\hat{p} \in (0,\bar{\theta}_F)$ such that $\tilde{R}_F(p) - k < \hat{\pi}_t^F(p)$ if only if $p < \hat{p}$.

■

Proof of Lemma 9

For $x \in [0,\hat{p})$, define

$$\begin{aligned} g_3(x) &= \bar{V}_{\tau_1}^F(x) - \bar{V}_{\tau_1}^L(x) = \hat{\pi}_t^F(p) - \bar{R}^L(p) - k \\ &= \left[\frac{\lambda}{\tilde{r}+\lambda}\right]\left[\frac{\psi(p,\gamma_1)}{\psi(\bar{\theta}_F,\gamma_1)}\right](\bar{R}_F(\bar{\theta}_F)-k) - \left\{\frac{\lambda}{r(r+\lambda)}E^p[\mu]\right. \\ &\quad \left. - \left(\frac{1-s}{r}\right)\frac{\lambda}{(r+\lambda)}E^{\bar{\theta}_F}[\mu]\frac{\lambda}{\tilde{r}+\lambda}\frac{\psi(p,\gamma_1)}{\psi(\bar{\theta}_F,\gamma_1)} - \frac{c}{r+\lambda}\right\}. \end{aligned}$$

When $x \to 0$, $\lim_{x\to 0}\hat{V}_{\tau_1}^F(x) > 0$ and $\lim_{x\to 0}\hat{V}_{\tau_1}^L(x) < 0$. Hence $\lim_{x\to 0} g_3(x) > 0$. When$x \to \hat{p}$, $\bar{R}^L(\hat{p}) - k > \bar{V}_{\tau_1}^F(p) = \hat{\pi}_t^F(\hat{p})$. Hence $\lim_{x\to\hat{p}} g_3(x) < 0$.

■

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