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#### CHARACTERIZING VEGETATION STRUCTURE USING WAVEFORM LIDAR

BY

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#### THESIS

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#### Abstract

The structure of light penetration through the canopy plays an important role in water, carbon, and energy fluxes between the biosphere and the atmosphere. Total foliage and foliage distribution are major aspects of canopy structure that significantly influence light and vegetation interaction. Airborne full-waveform LiDAR (Light Detection and Ranging) data contains large amounts of vegetation structural information, and is a powerful tool for providing detailed physical information for large areas of vegetation.

In this thesis, we first provide a complete work flow that extracts and processes waveform LiDAR data for an area of interest. Then we test the feasibility of using waveform LiDAR data to estimate individual tree biomass with limited field samples. We use a voxelization method to generate pseudo-waveforms for individual trees and apply a stepwise regression to find the relationship between pseudo-waveform structural characteristics and biomass estimated by allometric equations using tree survey data. Next, we present a method for describing physical canopy clumping structure for individual trees that provides detailed spatial clumping variations. We utilize the K-means clustering algorithm to extract structure from the large amount of canopy architecture information provided by full-waveform LiDAR. Finally we use representative cluster traits to identify structurally significant clusters. This thesis demonstrates that large amount of canopy structural information can be extracted from waveform LiDAR data. The fine resolution canopy architecture for ecological models.

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# CHAPTER 1 INTRODUCTION

## 1.1 Light and Vegetation Interaction

Vegetation is a major component in all climate studies. It is a major source of evapotranspiration and respiration. The primary environmental factor that influences vegetation functioning is solar radiation. The amount of light striking the canopy strongly affects photosynthesis, which leads to gas exchanges. Therefore, understanding the interaction between radiation and vegetation is essential in quantifying water, carbon, and energy fluxes between the biosphere and the atmosphere [Kucharik et al., 1999].

There are several important factors that affect light penetration, and therefore its absorption, through the canopy. These include:

- Total foliage. The total amount of foliage can be estimated by leaf area index (LAI), which is one of the most important and thoroughly studies measures of canopy characteristics [Bonan, 1993]. It can also be described through vegetation biomass.
- Vertical foliage distribution. The vertical distribution of foliage can be described by leaf area density (LAD), generally presented as a probability density function ranging over the entire height of the canopy. Dutta et al. [submitted] presents a method of estimating tree-wise LAD using the same dataset as that of this thesis.
- Foliage clumping. The non-random distribution of leaves in the canopy leads to clumping. Many indices have been developed to describe the severity of foliage clumping. However, the majority of these are large scale estimates that assumes uniform amount of clumping through out the canopy.
- Leaf orientation. There are currently no accurate estimations of the orientation of each leaf in the canopy. A mathematical model is generally used as replacement in current models. Random orientation is generally chosen for simplicity.

• Leaf transmittance. The amount of light that can penetrate each leaf in the canopy depends on leaf's internal structure and chemistry. The LiDAR (Light Detection and Ranging) technology used in this thesis mainly provides structural information, and is unable to quantify this canopy characteristic.

LiDAR data can be used to estimate the total foliage, its distribution and clumping structure. In Dutta et al. [submitted], we used discrete LiDAR data, imaging spectrometer data, and field data to estimate LAD, thus giving us a tree-wise description of vertical leaf distribution. Tree-wise LAD provides a relatively fine scale canopy description. However, LAI and vegetation biomass, as an estimation of total vegetation, has traditionally been estimated on the plot level. Foliage clumping, generally described by clumping indices, are also coarse scale measurements.

As current ecological models increase in resolution, these large scale estimates of vegetation characteristics can no longer adequately describe variations in canopy structure. The goal of this thesis is to use waveform LiDAR data to estimate biomass and foliage clumping. Combined with previous works, we hope to provide fine scale estimates of above ground vegetation biomass, foliage distribution, and foliage clumping in order to inform fine resolution models of vegetation structure.

## 1.2 Light and Biomass

LAI is one of the most important measurements of total foliage, and it has been widely studied. Our estimation of LAD in Dutta et al. [submitted] can also be used to find the LAI of a tree. Above ground vegetation biomass information is also an important vegetation characteristic used in many ecological models that include terrestrial vegetation in their simulations. Biomass has strong influences on carbon, water, and nutrient cycles. Traditionally biomass estimation requires intensive, and often destructive, field measurements. This involves cutting down all sample trees, drying and then weighing each parts of the tree, such as trunk, branches, and leaves, separately [Bombelli et al., 2009]. With the increase in size and sophistication of todays models, acquiring enough biomass data with field sampling as input to such models has become increasingly difficult. Methods exist to estimate biomass from less demanding field measurements. The most common of such methods is using existing allometric equations that are based on the previously mentioned destructive sampling [Ter-Mikaelian and Korzukhin, 1997]. These equations generally require some information about the trees as input. The most common are diameter at breast height (DBH), i.e., the diameter of the tree measured at 1.3 m from the ground [He et al., 2013], and height of the tree. Measuring DBH and tree height in the field is certainly easier than cutting down and weighting the entire tree. However, with the increase in model range and resolution, acquiring enough data, even the simple measurements needed for allometric equation inputs, might present challenges. One such challenge might be that the areas covered by the model is just too large to traverse easily. Another reason might be that the data is needed on too fine resolution to measure. The most common reason, and also the challenge encountered at our test site, is accessibility. In a dense forest, there are many trees that cannot be measured simply because one cannot reach it due to either terrain, or, as is our case, dense understory. These difficulties impede many models from using rigorous biomass data as input and forces them to use large scale generalized or simulated results. With advances in technology, airborne LiDAR has the potential to become a convenient tool for acquiring such information on a large scale at fine resolution. The first goal of this thesis is to use LiDAR data to estimate species based tree-wise biomass.

## 1.3 Light and Foliage Clumping

Clumping of vegetation in forest canopy have important effects on light penetration in the canopy. It affects photosynthesis and many other land-atmosphere interactions such as carbon and water fluxes [Chen et al., 2003, Pisek et al., 2013]. Deriving LAI through optical methods also require knowledge about foliage clumping to obtain the actual LAI [Pisek et al., 2011, 2013]. In this case, foliage clumping accounts for overlaps in foliage that optical methods cannot account for. Due to its importance in a wide range of studies, foliage clumping has been estimated by various methods such as hemispherical photography, sun fleck analysis, and remote sensing [Chen and Black, 1992, Chen et al., 2005, 2003, Pisek et al., 2013, Walter et al., 2003. Apart from the various methods of obtaining clumping, there are also multiple ways of quantifying it. Currently, the most frequently used description of clumping comes from Chen and Cihlar [1995a,b] and Chen [1996], where the gap fraction and gap size information from hemispherical photos are used along with a ray tracing model to derive a clumping index. Many other indices have been developed, such as that of Pielou [1962], and are used today in conjunction with that of Chen and Cihlar. These indices may vary in the method of derivation, but they all describe the canopy on a fairly large scale. While these traditional clumping indices may be easily ingested by ecological models, they tend to obscure spatial variations of foliage clump characteristics within the canopy. The second goal of this theis is to provide a physical description of the density and spatial distribution of vegetation using LiDAR derived foliage clump characteristics.

## 1.4 Waveform LiDAR

With advances in technology, airborne LiDAR has the potential to become a convenient tool for acquiring vegetation structural information on a large scale at fine resolution. Airborne LiDAR is a growing technology where a laser pulse is shot toward the ground from a moving aircraft in a sideways back and forth sweeping motion. When the laser pulse encounters an obstacle, the pulse is scattered back toward the aircraft and the return wave is recorded. The planes location is recorded by a global positioning system (GPS) unit. The plane's orientation in space is recorded by the inertial measurement unit (IMU) on board [Fernandez-Diaz and Carter, 2013]. By measuring how long it took the laser pulse to return to the plane, the distance from the plane to the obstacle can be calculated. Knowing the angle at which the laser pulse is shot enables us to locate the obstacle relative to the plane. Then, with the plane's location and orientation in space, we can identify the geolocation of the obstacle. In other words, we know the longitude, latitude and elevation (with respect to any datum) of the obstacle that the laser pulse hit. As the aircraft fly multiple passes over the area of interest, overlapping swaths of land is scanned by thousands of laser pulses per second. This generates a high resolution 3D model of the land surface.

There are two common types of LiDAR data used today, the multiple return discrete LiDAR and the full waveform LiDAR. In this study, we will be using both forms of LiDAR data collected for the same area at the same time. The multiple return discrete LiDAR, or just discrete LiDAR, as shown by the black dots in Figure 1.1, is where a point is recorded each time the outgoing laser pulse hits an obstacle. In a forested area, the first return is generally the top of the canopy, the later returns are likely the understory, and the last return is generally assumed to be the ground. However, under dense canopies, it is possible that neither the understory nor the ground is captured. The reason for the multiple returns is that the laser pulse is not an infinitely thin beam, but spreads out in a narrow cone as it travels through the atmosphere. The size of the beam is called the footprint of the laser pulse. When the cone of light hits the top of the trees, only part of the energy within the footprint is returned, and the rest continues on to generate more returns [Fernandez-Diaz and Carter, 2013]. When the top of the canopy is dense enough to return all energy of the laser pulse, then only one return is generated. The discrete LiDAR is the most popular LiDAR data used today because it is relatively simple to work with. The discrete LiDAR data generally contains only up to four returns per laser pulse, the first three, and the last return. This limit on the amount of data leads to a compact dataset where each data point contains a lot of information. Therefore, the discrete LiDAR data, or point cloud data, is small and provides a good representation of the Earth's surface.

The full waveform LiDAR data is derived from the same physical laser return signal as the discrete LiDAR. However, where the discrete LiDAR only records a few points, the full waveform LiDAR digitizes the entire return wave and records the intensity of the return at 1GHz frequency. This translates to one record for about 0.3m travel for the laser. Recording everything at such high resolution generates a much larger dataset compared to the discrete LiDAR. Therefore, the waveform LiDAR data is much harder to work with. However, the full waveform data may be able to resolve finer details not captured in the discrete LiDAR. For instance, discrete LiDAR only captures the first three and the last return. In complex canopy structures, the discrete system will miss everything between the third and last return which the waveform LiDAR will not. Also, when the discrete LiDAR registers a return, there is a set time interval when it will not register another return. This prevents the discrete system from recording an obstacle twice. However it also prevents the discrete LiDAR from observing two closely spaced objects. These situations are resolvable in the waveform LiDAR. In addition, because the shape of each return wave depends on its interactions with the canopy and the ground, waveform LiDAR data can provide the most information for characterizing canopy structure.

Therefore working with LiDAR data presents a trade off. Discrete LiDAR data is easy to work with, but may miss finer details. Waveform LiDAR data is difficult to work with, but captures as much vertical structural information as possible. The key is to use a combination of the two forms of data. Use discrete data when doing larger scale studies, and use waveform data only when finer details are necessary.

## 1.5 Thesis Organization

Ecological models involving canopy processes have long since started migrating from the simpler big-leaf canopy models to more complex models where processes strongly depend on canopy structure [Chen et al., 2003]. At the same time, models have increasingly higher resolution as computational power grows. These two advancements in ecological modeling

lead to a strong demand for more detailed descriptions of canopy architecture.

In this thesis, we describes our work of using waveform LiDAR data to provide such detailed canopy structural information at fine resolution. Four main factors describe canopy structure as described above, total amount of foliage, foliage distribution, canopy clumping, and leaf orientation. In Dutta et al. [submitted], we estimate tree-wise LAD as a description of foliage distribution. In this thesis, we tackle two other factors that describe canopy structure, total amount of vegetation, and canopy clumping. We estimate total above ground biomass, as a description of total amount of vegetation, using stepwise multiple regression between each tree's pseudo-waveform characteristics and its biomass derived from field data. The pseudo-waveform is generated by voxelizing all waveform data of the tree. In this thesis, we provide new physical description of canopy clumping structure by applying cluster analysis to the waveform data of each tree. Then we classify the clumps found in waveform data to better understand the connection between clumps in data and actual clumps in the foliage. Unfortunately, airborne LiDAR cannot provide leaf level descriptions.

The thesis is organized as follows:

- Chapter 1 provides background information on the vegetation characteristics currently used in estimating fluxes between land and air, why finer resolution descriptions are needed, and an introduction to airborne LiDAR technology used in this thesis.
- Chapter 2 presents the data used in this thesis as well as the data processing procedure used to prepare raw waveform LiDAR data into usable form.
- Chapter 3 outlines the steps in using LiDAR data for estimating vegetation biomass as well as present the results
- Chapter 4 outlines the steps in using LiDAR data for characterizing foliage clumping and provides the associated results.
- Chapter 5 summarizes results from the two previous chapters.
- Chapter 6 provides discussion about the results as well as issues encountered in the study.
- Chapter 7 finishes with conclusions from this research and suggestions for future works.

# 1.6 Figures



Figure 1.1: LiDAR acquisition process and symbolic representation of discrete and full-waveform LiDAR [Fernandez-Diaz and Carter, 2013].

# CHAPTER 2

# DATA

### 2.1 Data Collection

The data used in this thesis is collected for sites located in the Upper Sangamon River Basin (USRB) as part of of Critical Zone Observatory for Intensively Managed Landscapes (IMLCZO). USRB is located in east-central Illinois, which has a humid continental climate with hot summers and cold winters. Vegetation mainly consists of row crops, corn and soybean, and some dense mixed forests. The methods in this thesis are mainly targeted toward application in these dense forests.

In the summer of 2014, both remote sensing data and field data were collected around the same time. Airborne remote sensing data includes imaging spectrometer data, also called hyperspectral, and LiDAR data. Lidar data set contains both discrete and waveform data. Flights to obtain the remote sensing data were conducted by the National Center for Airborne Laser Mapping (NCALM). Tree survey data was also collected by students and faculty at the University of Illinois. Only the LiDAR data and field data are used in this study.

#### 2.1.1 LiDAR Data

The LiDAR data was collected using the Optech Gemini Airborne Laser Terrain Mapper (ALTM) on the 4th of August 2014. An infrared laser with wavelength of 1064 nm is used. Nominal flight altitude is 600 m. The scan angle is  $\pm 18^{\circ}$ , resulting in a swath width of 367 m. Nominal swath overlap is 185 m. So on average, all areas are scanned twice by the plane to increase point density. With laser pulse repetition frequency (PRF) of 100 kHz and two passes, the resulting discrete point cloud has a density of 7.8 points/m<sup>2</sup>. Beam divergence is 0.8 mrad, which results in a 0.24 m radius footprint at ground level. Compared to others, this LiDAR system can be considered a small footprint LiDAR with medium to high density.

The discrete LiDAR data was processed by NCALM and is divided into 1 km square tiles. The waveform data was not processed and remains as separate binary files.

#### 2.1.2 Field Data

Tree survey data was collected from July 31<sup>st</sup> to August 4<sup>th</sup> 2014 in order to coincide with the airborne remote sensing data collection. Four sites within the USBR were sampled Allerton Park-1, Allerton Park-2, Home Forest Site, and Lake of the Woods. These four sites are located along the riparian forest corridor of the Sangamon River and are mainly dense mixed forests as previously mentioned.

For each site, tree characteristics data collected included stem location, species, crown width, tree height, height of the bottom of the crown, and diameter at breast height (DBH). Various tools used include tape measures, field GPS, total station, and clinometer. Plot level LAI was measured using LAI-2200C plant canopy analyzer. Tree species were identified by comparing leaf samples with published species identification guides [Mohlenbrock, 1973].

## 2.2 Data Processing

Out of the three datasets used in this thesis, tree survey data (in csv form) and discrete LiDAR data (as tiled point clouds) can be ingested relatively easily. However, our goal is to use waveform LiDAR data to quantify details that may not be captured in the discrete LiDAR data. The discrete LiDAR data was processed by NCALM after they collected the data, but the waveform data was not. They are given as a set of raw binary files. Figure 2.1 shows a summary of the important waveform data in each file and their relations to each other. We must first process the waveform data into usable form for our study. One challenge of working with waveform LiDAR data is the limited software support. So the waveform LiDAR data used in our study are completely processed by original code using Python starting with raw binary files using an object oriented approach. The processing steps are detailed below. A schematic summarizing processing steps is shown in Figure 2.2.

#### 2.2.1 Read Files

The raw waveform LiDAR data is recorded in three separate binary file with different formats DF2, IX2, and CSD. In Figure 2.1, files are enclosed in rectangles, and data are enclosed in ellipses. To construct each waveform, pieces of data must be extracted from all three files. The DF2 file, shown as light red in Figure 2.1, contains the actual digitized intensities of the outgoing and return signals as well as time interval information between when the laser pulse left the digitizer and when it returned. The time interval is the number of cycles of the digitizer's processor. A PCount factor is needed to convert the time interval information in the DF2 file to seconds. This factor is found in the IX2 file, shown as dark red in Figure 2.1, which is an indexing file that corresponds to the DF2 file. Together, the time interval information and the PCount factor gives the exact linear distance between the start of the outgoing signal (assumed to be the plane location) and that of the return signal.

To fully geolocate the waveform, plane location and orientation as well as scan angle of the laser are needed. These location information is recorded in the CSD file, labeled *CSD Record* in Figure 2.1, as each laser fires, and each record contains an associated GPS time. The IX2 file can also be used to retrieve GPS time for each waveform. Waveforms can then be matched to its corresponding location information using the two GPS times. To retrieve waveform GPS time from the IX2 file, the following equation is used:

$$T(W_{f,r}) = T_f + r(\frac{T_{f+1} - T_f}{R_f})$$
(2.1)

Where T refers to GPS time, and  $W_{f,r}$  is the  $r^{th}$  waveform in frame number f.  $T_f$  and  $T_{f+1}$  refers to GPS time of frame f and that of the subsequent frame.  $R_f$  is the total number of waveform in frame f. Here a frame is just an organizational unit used in the IX2 file can contains around 13000 waveforms. Since the IX2 file only contains GPS time record for the start of each frame, interpolation is used to assign GPS time location to each waveform. This process of combining pieces of information from all raw binary files is labeled as *Read from files* in Figure 2.2. The result is a Python object that we defined called Wave. It contains all intensity records of a single wave as well as geolocation information.

#### 2.2.2 Process Waveform

After all information for one waveform is collected, we then geolocate the wave by assigning location to each digitized intensity record in the waveform. To do this, the range  $D_i$ , linear distance of intensity record i in the return wave from the plane, is calculated. Then rotational matrices are used to apply scan angle and plane orientation to the range. Calculations are shown in the following equation:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{W_i} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{pl} + R^{RPY} \left( R_x(a) \begin{bmatrix} 0 \\ 0 \\ D_i \end{bmatrix} - \begin{bmatrix} l_x \\ l_y \\ l_z \end{bmatrix} \right)$$
(2.2)

In a simpler representation, equation 2.2 can be written as follows:

$$L_{W_i} = L_{pl} + R^{RPY} \left( R_x(a) \mathbf{D}_i - e \right)$$
(2.3)

Where  $R^{RPY} = R_z(Y)R_y(P)R_x(R)$ .  $R_x(a)$  represents a 3D rotation matrix that applies rotation of angle *a* around the x-axis to some vector in Euclidean space. Similarly,  $R_y(P)$ means rotation of angle *P* around the y-axis. *a* is the scan angle of the laser, *R*, *P*, *Y* are the *roll*, *pitch*, *yaw* angles of the plane.

 $L_W$  is the location of one digitized record in the waveform.  $L_{pl}$  is the location of the plane. **D**<sub>i</sub> is the vector representation of  $D_i$ , and e is an error term that account for misalignment and IMU offset. Error values can be found in the header of the CSD file labeled as *CSD Header* in Figure 2.1.

Once the wave is geolocated, baseline intensity registered by the sensor is removed from the wave by subtracting a constant representative value to maintain comparability between waveforms. This step of geolocating and processing individual waveforms is labeled as *Process Wave* in Figure 2.2.

#### 2.2.3 Correct Errors

At this point, the geolocation of each wave is not exactly correct. Empirical offsets in the data exists due to offset in sensors and datum conversion. The resulting waveform data must be compares to the discrete LiDAR data produced by NCALM using commercial software for corrections. These offsets include offset in range, heading, as well as an offset in GPS time used to match waveform data to plane location and orientation. In order to compare the waveform to the discrete point cloud data, the record with the max intensity in each wave is extracted and combined into a set of point cloud data.

Range offset can be found automatically by comparing corresponding data points in each dataset. Heading offset can be found by comparing the angle of one scan line of data.

However, GPS time offset must be corrected by guess and check. A GPS time offset results in one waveform using the geolocation of an earlier (or later) waveform. This error is most noticeable between two adjacent scan lines in sloped surfaces such as angled roofs. If the offset exists, then the two adjacent scan lines of the same roof will display a horizontal offset. So the offset must be corrected by adding a constant to the GPS time of each waveform. Different constants are tried and checked visually until adjacent scan lines in roofs match. This process of finding offsets in the waveform data is labeled as *Correct Errors* in Figure 2.2. Once offset values are found, either by automatic or visual comparison, we must restart the process at the beginning by reading from raw data and geolocate waveforms while including the calculated offsets.

#### 2.2.4 Extract Waveforms

The previous sections describe how data is extracted from raw binary files and combined into a list of waveforms (Wave List). However, waveform LiDAR data is large and difficult to work with, and generally data for the entire site is not needed. Therefore, we develop an algorithm to extract a section of interest in the waveform data based on discrete LiDAR point clouds. Using the point cloud of an area of interest, we find all GPS time intervals the data spans. These intervals will not be continuous in time as the point cloud can contain data from multiple flight lines and the laser can pass in and out of the area of interest as it scan along the flight line. By applying the processes described in previous sections to only waveforms with in the GPS time intervals, we can extract needed waveform information with minimum computation. Figure 2.3 shows the Allerton Park-1 site, and 2.4 shows the sample tree used to test the foliage cluster analysis method described in Chapter 4. In order to display the waveform data for an extracted region, all records in each waveform is converted into a geolocated point and shown as a point cloud by the free CloudCompare software.

#### 2.2.5 Process Wave List

Once a Wave List extracted for a region of interest, further processing might be needed to prepare waveform data for our models. This step is labeled as *Process Wave List* in Figure 2.2. However, different procedures are required for different models. Therefore, this step will be described in the following Chapter along with studies that use the data produced by the procedures described in this Chapter.

# 2.3 Figures



Figure 2.1: Summary of relations between raw binary files and the pieces of waveform data each file contains.







Figure 2.3: Waveform data extracted for a region where each record in the waveform is converted into a point.



Figure 2.4: Waveform data of tree displayed as points by giving geolocating each record.

# CHAPTER 3

# BIOMASS ESTIMATION

In this chapter, we describe how we used waveform data extracted for individual trees along with field data in an attempt to estimate above ground biomass for each tree. Traditional methods for calculating biomass involves destructive sampling of trees. More recent methods such as using allometric equations also require some field measurements such as DBH and tree height.

Using airborne LiDAR data may be the only way to find fine scale biomass data for large areas. LiDAR data gives a detailed description of tree size and structure, with directly influences biomass. So in this chapter, we attempt to relate a tree's waveform characteristics to it biomass calculated from field data.

First we describe the methods used in the analysis, then we present our results. Conclusions will be presented in Chapter 6 and 7.

## 3.1 Methods

In this section, we first describe how we extract waveform characteristics for individual trees, then we present our regression analysis that tries to relate the waveform characteristics of each tree to biomass information derived from field data.

#### 3.1.1 Process Wave List

To find individual trees, the canopy height model is used as input to Envi-Lidar. This program assumes circular tree crowns and delineates individual trees with location and radius as output. Next, using the delineated point cloud information as input to the data processing procedure described in Chapter 2, the list of waveforms, or Wave List, for each tree can be extracted.

Using the tree delineated point cloud data, we find trees that match those that were sampled in the field. After extracting all waveforms for each sampled tree, we combine all waveforms associated with each tree using a voxelization method which allows up to capture vertical structure of biomass distribution [Hosoi and Omasa, 2006]. In this method, each tree is approximated as a cylinder. The height of the cylinder is the vertical range of the all waveforms in each tree, and the radius is the output from Envi-Lidar when using the discrete data as input for tree delineation. By dividing the cylinder into 1 cm layers, disk shaped voxels along the height of the tree are generated. Then by totaling all records that fall within each voxel, pseudo-waveforms, as shown in Figure 3.1, are generated for each tree. In order for each waveform to be comparable, each wave record is normalized with respect to the max intensity in the wave. The equation below demonstrates this process.

$$V_h = \sum \frac{w_i}{w_{max}} \tag{3.1}$$

 $V_h$  is a voxel at a certain height,  $w_i$  is a wave record, and  $w_{max}$  is the max intensity of the wave that contains  $w_i$ .

#### 3.1.2 Waveform Characteristics

To describe the pseudo-waveform for each tree, we chose several waveform structural characteristics listed below:

- Total energy (tE). Total energy is the total area under the pseudo-wave of each tree. Found by integrating the wave with respect to elevation.
- Total Height (tH). Total height is the vertical range of the pseudo-wave. The peak that likely represents ground return is not removed. Our reasoning is that the ground return characteristics below a tree is also an expression of tree structure. Dense trees are likely to have less significant ground return compared to foliage return, and the reverse is true for sparse trees.
- Max energy (maxE). The maximum energy of the wave is the peak intensity value of the wave. In the case of the pseudo-waveform, this is the x-axis value (pseudo-intensity) shown in Figure 3.1. Due to normalizing the waveform, this value does not have a physical meaning, but is an indirect expression of the intensity of all waveforms for a tree.

- Median energy (midE). The median energy is the median value of all pseudo-intensity values of the pseudo-waveform for each tree.
- Relative height at 25% energy (RH25). RH25 is the relative height below which lies 25% of the pseudo-wave's area. Integrating the pseudo-wave from minimum elevation to elevation at 25% results in 25% of total energy. Relative height entails a distance measurement that it is the elevation at 25% energy subtracted by the minimum pseudo-wave elevation.
- Relative height at 50% energy (RH50). Similar to RH25, this is the height, relative to the minimum elevation, below which lied 50% of the pseudo-wave's energy.
- Relative height at 75% energy (RH75). Similar to RH25 and RH50, this is the height, relative to the minimum elevation, below which lied 75% of the pseudo-wave's energy.

All wave characteristics above are calculated for the pseudo-waveform of each tree. Some are energy characters that describes the waveform intensity of each tree, and some are height characteristics that describes the pseudo-waveform structure and the distribution of energy. We believe this to be a thorough description of the different characteristics of the pseudowaveform.

#### 3.1.3 Field Biomass

In order to relate the pseudo-waveform characteristics described in the previous section to each tree's biomass, we must have actual biomass values for some trees for guidance. We use field tree survey data and existing allometric equations to estimate biomass. There are many published allometric equations in existing literature. The National Biomass Estimator Library (NBEL) compiled many of them, and is a great tool for exploring tree biomass [Wang, 2014]. For each tree, the NBEL requires USDA forest service region code (09 for Illinois), DBH, tree height, and tree species as input. As a result, it returns biomass in kg for the tree calculated using all known allometric equations for the region. We then choose a reasonable number based on how suitable the original study is to the current situation. If a species of tree does not have an equation in this region, that of a similar tree, such as another type of maple for an unknown maple, is chosen.

Currently, since cutting down trees to measure biomass is no longer prevalent, biomass derived from allometric equations is generally accepted as valid biomass measurements. Therefore, even though these are estimations, we assume that the biomass results estimated from field data are correct biomass values. Using the field survey data for the four sites mentioned in Chapter 2 Section 2.1.2, 33 usable trees are identified and above ground biomass, the mass of all vegetation above the ground, is calculated for each tree.

#### 3.1.4 Regression

After estimating above ground biomass in the previous section for each surveyed tree, we can now relate each tree's pseudo-waveform structural characteristics to its biomass through linear regression.

The simplest method would be to use multiple linear regression with each tree's biomass as the dependent variable and the pseudo-waveform characteristics as explanatory, or independent, variables. However, when we chose pseudo-waveform characteristics in Section 3.1.2, many of them describe similar traits. This leads to high cross correlation between explanatory variables, rendering all variables insignificant.

Therefore, we perform stepwise multiple linear regression between biomass estimated from field measurements and pseudo waveform structural characteristics using Matlabs Linear-Model. Stepwise regression builds a linear model, but repeatedly add or removed explanatory variables based on a given criterion. Criterion used in this thesis is  $R^2$ . Therefore, by using a stepwise regression we can filter out the least significant explanatory variables that may negatively influence the model fit. In order to check the validity of results, we only use 80% of trees, training data, as input to the stepwise regression model. The regression generates a model similar in the form to equation 3.2 shown below:

$$y \sim 1 + x1 + x2 + x1 * x2 \tag{3.2}$$

Here y is biomass, the dependent variable, and x1andx2 represent pseudo-waveform characteristics, the explanatory variables. To test the validity of the resulting model, the remaining 20% of tree data, test data, is used as input and the resulting biomass value is compared with the value derived from allometric equations.

### 3.2 Results

Since we choose a random 80% of data as regression input, the results of each repeated regression analysis are different. Figure 3.2 and 3.3 shows regression results for different

runs of the stepwise multiple linear regression. One outlier in the data had to be physically removed for reasonable results. In the model shown in Figure 3.2, the regression results are fairly good. The  $R^2$  value is not high by general standards, but is considered pretty good for this type of study. However, the  $R^2$  for the model shown in Figure 3.3 is extremely low. This unstable performance by the stepwise regression method is also evident in that each run results in a model based on different explanatory variables. In other words, we are not able to determine the most important pseudo-waveform structural characteristics that affects above ground biomass of each tree.

The variation in results from the regression analysis is likely due to the limited number of data points we have. Unfortunately, the inaccessibility of trees to measure due to dense understory is inevitable in dense forest as we have at USRB. Many other situations can also lead to limited amount of data. Since limited data is a prevalent problem, we then hope to test the effectiveness of statistical methods in finding significant explanatory variables. In order to determine the most significant pseudo-waveform structural characteristics that indicate biomass, we use a bootstrapping method and run the stepwise regression 500 times. Each time, the regression is applied to a randomly chosen 80% of the data as training data, and the remaining 20% is test data used as input to the resulting model which gives a predicted biomass value.

Figure 3.4 shows the results from the 500 runs of stepwise regression. The y-axis is the biomass of surveyed trees based on allometric equations and field data. The x-axis indicates biomass predicted by the model. Each blue point is a test data point, part of the 20% not used in building the model. Because the training data is chosen randomly, each surveyed tree has served as test data multiple times to different models. This leads to a horizontal spread of predicted biomass for each field measured biomass as shown in Figure 3.4. The red circles indicate the median value of all predicted biomass for each surveyed tree.

The model for each stepwise regression is also recorded and processed. In order to find the pseudo-waveform structural characteristics that has the most influence on biomass, we want to find the model that occurs most frequently from randomly chosen training data. Out of 500 runs, there are 149 unique models. By our definition, two models are the same if they used the exact same explanatory variable and operations between variables. For example,  $y \sim x1 + x2$  is the same as  $y \sim x2 + x1$ . Their coefficients may differ. The model that occurred most frequently is:

$$Biomass \sim 1 + tE + RH50 + tE * RH50 \tag{3.3}$$

which occurred 33 time out of 500. This model is relatively simple and only uses two terms, tE and RH50. Figure 3.5 shows biomass prediction results of using only tE and RH50 as explanatory variables. In this case,  $R^2$  value is based on all data instead of just the test data.

Closely following the model shown in Equation 3.3 in terms of frequency of occurrence are:

$$Biomass \sim 1 + midE + tE + RH50 + tE * RH50 \tag{3.4}$$

$$Biomass \sim 1 + tE + tH + tE * tH \tag{3.5}$$

which occurred 32 and 31 times respectively. The model shown in Equation 3.4 is very similar to that of Equation 3.3. The only difference is the addition of the linear midE term. The model in Equation 3.5 is similar in form to that of Equation 3.3. The difference here is the used of tH instead of RH50, which are both height terms. tE occurred in all three models. RH50 occurred in two. midE and tH each occurred once.

# 3.3 Figures



Figure 3.1: Examples of pseudo-waveforms for individual trees in Allterton Park-1 site.



Figure 3.2: Example of relatively well fitted model. Model based on randomly chosen 80% of data. Blue points are data used in the regression. Red points are the remaining 20% test data.  $R^2$  value is based on test data only.



Figure 3.3: Example of poorly fitted model. Model based on randomly chosen 80% of data. Blue points are data used in the regression. Red points are the remaining 20% test data.  $R^2$  value is based on test data only.



Figure 3.4: Biomass results from stepwise regression combined with bootstrapping. All test data are shown in blue. Red circles show the median of all predicted biomass for each tree.



Figure 3.5: Biomass prediction results using the most prevalent model from 500 bootstrap runs shown in Equation 3.3.

# CHAPTER 4 FOLIAGE CLUMPING

In this Chapter we propose an easily scalable method to estimate physical canopy clumping structure for individual trees using airborne full-waveform LiDAR data. Canopy clumping is hard to quantify, since what entails a clump can be subjective. Traditionally clumping indices are used to describe the amount of clumping on a large scale. These indices are generally derived using optical tools such as hemispherical photos.

Using airborne LiDAR data will not only provide wide coverage, but can potentially give a fine scale description of the canopy clumping characteristics that clumping indices cannot provide. In this Chapter, we attempt a new method that describes clumping structure in waveform LiDAR data, which should give an indication of actual canopy clumping.

We first describe our method used for clustering the waveform data, then we present the clustering results. Interpretations of the clustering results and conclusions will be described in Chapter 6 and 7.

## 4.1 Methods

In this section we describe why we choose to use K-means clustering, and how we use simulated LiDAR data test the most applicable method of finding the K in K-means. Then we present how we cluster the records (each recorded intensity) in the waveform LiDAR data and our clustering results for a sample tree selected from USRB.

#### 4.1.1 Cluster by K-means

Waveform LiDAR data can provide detailed canopy structural information. However, the relation between individual waveforms and records in each waveform is unknown. In order to find patterns in the LiDAR data, clustering analysis can be extremely useful [Jain, 2010].

Clustering is a form of unsupervised pattern recognition involving only unlabeled data, such as our waveform LiDAR records [Duda et al., 2012]. The purpose of data clustering is to find the natural groupings of a set of data, or points, in order to gain insight into the underlying structure of the data set. Clustering characteristics in LiDAR data can help us gain insight into canopy foliage clumping.

Cluster analysis is an extremely useful but also difficult process due to the subjective nature of the problem. This challenge has led to an increasing number clustering algorithms [Jain, 2010]. Two main categories of clustering algorithms are *hierarchical* and *partitional*. Hierarchical clustering recursively combines or divides clusters while partitional clustering divides the data into a set number of clusters simultaneously. The more resource intensive hierarchical clustering is unsuitable for the large amount of waveform LiDAR data. Therefore, K-means, a widely used partitional clustering algorithm, is chosen.

K-means is one of the most popular clustering algorithms due to its simplicity, efficiency, and empirical success [Jain, 2010]. The algorithm seeks to minimize the sum of the squared errors (SSE) of all clusters shown in Equation below [Jain, 2010]:

$$J(C) = \sum_{k=1}^{K} \sum_{x \in c_k} ||x - \mu_k||^2$$
(4.1)

Here, K is the total number of cluster,  $c_k$  represents one cluster, and  $\mu_k$  is its center.

Given K, a distance metric (usually Euclidean distance), and a cluster initialization method, the K-means algorithm first partitions the data into K clusters according to the given initialization method. Next, it re-clusters by assigning each data point to its closest cluster center based on the given distance metric. Using cluster centers from new clusters, re-clustering is repeated until clusters are stable [Jain and Dubes, 1988].

In this thesis, four features for each data point are selected as input to the K-means clustering algorithm, xyz location based on UTM N16 and record intensity. Point location serves to group points of close spatial proximity. Intensity is used to group points of similar return characteristics. In order to only use the most informative data, all records with intensities below half of the max intensity of their respective peaks are removed. Intensities are then linearly scaled so that the max value is the range of elevation. Scaling is necessary for comparability between variables. We use Euclidean distance as the distance metric and use randomly selected data points as initial cluster centers.

#### 4.1.2 Cluster Evaluation

The challenge in using K-means clustering is choosing the correct K, the number of clusters, since what constitutes the correct number of clusters may be subjective [Han et al., 2011]. Often K is determined through trial and error and there may be no justification for selecting a particular value [Pham et al., 2005]. However many methods exist for selecting the K in K-means. In this thesis, we consider many representative methods for choosing K. These include:

• Nominal K [Kodinariya and Makwana, 2013]. The nominal K is a rule of thumb for selecting K given by Equation 4.2 below:

$$K \approx \sqrt{n/2} \tag{4.2}$$

It is only based on the total number of data points being clustered by the K-means algorithm. Therefore it may not be applicable in all situations. However, it does give a large enough K estimate in our case to ensure we are not over simplifying data structure.

- Elbow Method [Ng, 2012]. The elbow method is a traditional method for estimating K in K-means. In this method, clustering results are evaluated by the SSE shown in Equation 4.1. K-means is run for a range of K values, and the SSE is calculated for each clustering result. The calculated SSE is then plotted against their respective K values. In an ideal case, the resulting plot shows a curve with a sharper bend in the middle, looking like a bent arm. The K value where the curve bends the sharpest, the elbow, is the correct K to use for K-means.
- Calinski-Harabasz or Pseudo-F Index [Caliński and Harabasz, 1974]. Similar to the elbow method, finding the correct number of clusters, *K*, to use using the Pseudo-F index also requires running the K-means algorithm for a range of *K* values. In this case, the Pseudo-F Index, shown below is used to evaluate cluster results instead of the SSE.

$$PseudoF = \frac{BGSS/(K-1)}{WGSS/(N-K)}$$
(4.3)

Here BGSS is the between group sum of squares, and WGSS is the within group sum of squares. BGSS is calculated as the squared error of all cluster centroids, and WGSS is the SSE of the clustering result. N is the total number of data points, and K is the number of clusters. This index is named Pseudo-F index because it is analogous to the F-statistic used by Edwards and Cavalli-Sforza [1965] in cluster analysis. Larger values indicate tighter and more separated clusters. Ideally a good K value is indicated by peaks in the plot of this index with respect to the number of clusters [Wilkinson et al., 2012].

• Silhouette Score [Kaufman and Rousseeuw, 2009]. The Silhouette Score is the mean Silhouette Coefficient or all clusters. This index should also be calculated for clustering results from a range of K values. The equation for calculating Silhouette Coefficient for each cluster is shown below:

$$Silhouette = \frac{b-a}{\max(a,b)}$$
(4.4)

Here a is the mean within cluster distance, or the mean distance between all pairs of data points in a cluster. b is the mean nearest-cluster distance, or the mean distance from all points in the cluster to the nearest point that is not in the cluster. This coefficient measures how cohesive each cluster is and how separate it is from neighboring clusters. As we can tell from Equation 4.4, the coefficient ranges from -1 to 1. Larger value indicate more distinct clusters. A suitable value of K can be chosen using the plot of the Silhouette Score, mean Silhouette Coefficient for all cluster, with respect to K, the number of clusters.

Bayesian Information Criterion (BIC) [Pelleg et al., 2000, Schwarz et al., 1978]. The BIC used in Pelleg et al. [2000] as the stopping criteria for the x-means algorithm, which is simply an accelerated K-means clustering that chooses K automatically based on BIC. The Bayesian Information Criterion, also called the Schwarz Criterion, is first used by Schwarz et al. [1978]. Since then, many form of the criterion has developed. The following equation comes from Wit et al. [2012].

$$BIC = -2l(\theta) + p\log(N) \tag{4.5}$$

Here,  $l(\hat{\theta})$  is the maximum log likelihood of all clusters, p is the number of parameters in the clustering data, four in our case. N is the total number of data points. The BIC measures the posterior probability as an evaluation of a clustering result. Using Equation 4.5, minimizing BIC gives the maximum posterior probability. By evaluating clustering results for a range of K values using BIC, the correct K should ideally occur at the minimum BIC value.

• Dunn's Index [Dunn, 1973]. This is another cluster evaluation index that measures the compactness within clusters and separation between clusters. It is defined as the minimum Euclidean distance between any two points in the data set that belongs to different clusters.

$$DI_K = \frac{\min_{1 \le i < j \le K} \delta(c_i, c_j)}{\max_{1 \le h \le K} \Delta_h}$$
(4.6)

where the numerator is the minimum Euclidean distance between any pair of cluster centers, and the denominator is the maximum diameter of any cluster. Here  $\Delta_h$ , the diameter of a cluster, is defined as the maximum distance between any two points in the cluster. It is a measure of the spread of data in a cluster. It can also be define as the mean distance between all data pairs in a cluster (same as *a* in Equation 4.4) or the mean distance between each point and its respective cluster center. Larger Dunn's index indicates more tightly grouped clusters.

- Minimum Between-cluster Distance (minBdist). The minimum between cluster distance is an additional index used in this thesis to test for the correct K value in K-means clustering. The reasoning behind using this measure to evaluate clustering results is that as K value increases, larger clusters are, ideally, being divided into more tightly grouped smaller ones. Therefore, when K is small, the minBdist should be large, and as K increase, minBdist should become gradually smaller until it reaches the minimum distance between any two data points. Ideally the correct K value should be when the minBdist starts to decrease more slowly, indicating that any more increase in K will result in dividing more tightly grouped clusters.
- Minimum Center Distance (minCdist). The minimum center distance is also another index developed in this thesis to estimate the correct K value when K-means is applied to LiDAR data. This index measures the minimum distance between any two cluster centers. The reasoning and expected behavior of minCdist is similar to that of minBdist. When K is small, there should be few clusters where cluster centers are far apart. As K increase, minCdist should also become gradually smaller since there

are more and more small clusters. Ideally the correct K value should be when the minCdist starts to decrease more slowly. This should be an indication that additional increase in K will result in dividing tightly grouped small clusters.

#### 4.1.3 Point Cloud Simulation

We test the applicability of each method of choosing K mentioned above using simulated LiDAR point clouds with given degree of foliage aggregation and known number of clusters. Similar simulation procedures has been used by [de Castro and Fetcher, 1999], then [Walter et al., 2003]. In these previous works, LAI is needed as input to simulate foliage canopy. In this thesis, the number of points in the discrete LiDAR point cloud of an area of interest  $(N_p)$  is used instead of LAI to represent the amount of foliage in the canopy. Additional parameters needed as input include the number of clusters  $(N_c)$ , and the cluster percentage (fraction clumping,  $F_c$ ).  $F_c$  is a fraction between 0 and 1 used to scale the distance between each point and its closest cluster center. To generate a point cloud with given degree of foliage aggregation, the first step is to generate  $N_p$  points randomly distributed within the same volume as that of the original point cloud. Then  $N_c$  cluster centers are also randomly located within the volume. Each of the  $N_p$  points in the point cloud is displaced toward its closest center according to

$$d' = (1 - F_c)d (4.7)$$

The displacement of each point occurs along the vector between the point and its closest center. d is the original length of the vector, and d' is the distance after displacement. As should be clear from Equation 4.7 above,  $F_c = 0$  corresponds to a completely random canopy, and  $F_c = 1$  represents a completely compacted one.

#### 4.1.4 K for Simulated Point Clouds

Once a point cloud with a given number of clusters is simulated, we test the applicability of each cluster evaluation index by repeatedly clustering the point cloud using increasing number of clusters (K). Each cluster result is tested with the cluster evaluation methods mentioned in Section 4.1.2, and an index is generated for each given K. This testing process is run multiple times, and an average index from all runs for each K is used to judge the performance of the cluster evaluation method.

Most cluster evaluation methods mentioned in Section 4.1.2 are unable to correctly identify the number of clusters in the simulated point clouds. Figure 4.2 and 4.3 shows examples of failed cluster evaluation methods. Figure 4.2 shows the results from using BIC as cluster evaluation index on a simulated point cloud with 60 clusters and  $F_c$  of 0.6. A range of Kvalues are used to cluster the point cloud and BIC is calculated for each cluster result. In the resulting line plot, the BIC decreases with increased K, indicating better clustering, however, it shows no sign that K = 60 is any better than other K values in that range. Therefore, we cannot use BIC as a cluster evaluation for waveform data clustering when looking correct K. Another example of a failed cluster evaluation index is minimum between cluster distance. This index is applied to a simulated point cloud with similar parameters as that of the BIC test, 60 clusters and  $F_c$  of 0.6. Again a range of K values are used for clustering. The plot of minBdist with respect to K is shown in Figure 4.3. Similar to BIC, this evaluation index give no indication that the clustering result of K = 60 is any different from others in that range. Based on the results, we would most likely choose K = 45, which does not agree with the actual number of clusters.

Only two of the cluster evaluation methods mentioned in Section 4.1.2 yields promising results. One is the Dunn's index described by Equation 4.6. The Dunn's index is also used for evaluating clustering for a simulated point cloud with 60 clusters and  $F_c$  of 0.6. The results are shown in Figure 4.4. In theory, larger Dunn's index indicates tighter clustering. However, the index gradually decreases for increasing number of clusters until the K grows past the correct cluster number. This method may not be suitable for evaluating cluster validity given the index value at the correct number of clusters, it does indicate when the correct number is reached by comparing multiple runs of K-means clustering.

Another cluster evaluation measure that yields good results is the minimum center distance described in Section 4.1.2. This measure is simply the numerator of Dunn's index, and its behavior is similar as well. However, it is significantly smoother compared to the Dunn's index as shown in Figure 4.5. From the Figure, it is easy to see that the correct cluster number occurs after the bend in the graph. The measure generally remains constant with increasing K after that.

#### 4.1.5 K for Waveform Data

The cluster evaluation methods that performed well with the simulated point clouds, Dunn's index and minCdist, is then applied to the waveform data of the tree of interest. Results are shown in Figure 4.6. Cluster evaluation results for the waveform data do not show as strong indication of the correct cluster number as previous simulations. We believe this might be due to higher point densities in the waveform data as well as less distinct clusters. However, the results still shows similarities. In the results for Dunn's index, shown in Figure 4.6a, the index decreases at first, but seems to reach a plateau after K = 130. There seems to be another plateau after K = 210. However, the large variations in the index value after this point prevents any conclusive decisions.

The results from the minimum cluster center distance measure, shown in Figure 4.6b are also inconclusive. With this measure, we encounter the same problem which plagues the oldest method for determining K in K-means, the elbow method [Ng, 2012]. Similar to the ideal elbow method result, the result from the minimum cluster center distance measure has a distinct bend, or elbow, at the correct number of clusters. However, often times there is no distinct bend in the result from the elbow method, as is the case with our measure and the waveform data. In this situation, we can only specify a range that the correct K is likely to be in. From Figure 4.6b, the range 100 < K < 250 seems to be the center section of the elbow.

In such ambiguous situations, several K values can be used [Pham et al., 2005]. In this study, we choose to use K = 130 and K = 220. These numbers are close to the likely K values from the evaluation using Dunn's index, they also represent the range of K identified in the minimum center distance measure. Also, K = 220 is chosen because it also corroborates with the rule of thumb for selecting K defined as follows where n is the number of data points.

### 4.2 Results

Waveform data for an individual tree is extracted, and all records in each waveform are converted to points in space. The K-means clustering algorithm is then applied to the waveform data using K = 130,220. The tree used and the cluster results are shown in Figure 4.7.

Next we calculate the following four traits for each cluster for further analysis.

- Average Cluster Intensity. The average intensity of all records in each cluster. Results are shown in Figure 4.8. Clustering results for both K value are very similar to the unclustered intensity data shown in Figure 4.7a.
- Cluster Count. The number of records in each cluster. Results are shown in Figure 4.9. There are slight differences for the cluster results from different K values. Clusters for K = 220 have less variation in the number of records per cluster.
- Cluster Volume. The volume of each cluster calculated by fitting a convex hull around all points. Results are shown in Figure 4.10. The most notable result is that data for the tree canopy form significantly larger clusters than data representing the ground.
- Cluster Diameter. The maximum distance between any two points in the cluster. Results are shown in Figure 4.11. The results for K = 220 have many blue, or small diameter, clusters. This indicates that there are many small clusters and a few large clusters. Results for K = 130 seems more evenly distributed with small, medium and large clusters.

Cluster volume and cluster diameter are both measures of the spread of a cluster. Clusters with larger diameter should theoretically have larger volume. As expected, we find that they exhibit a strong positive correlation shown in Figure 4.12. Therefore, cluster volume is not used for further analysis. Figure 4.13 shows the distribution of values for the three remaining cluster traits. We can see that their distribution are very similar. This leads us to believe the K values we chose are in the correct range for describing the data structure.

By plotting the three remaining traits, intensity, count, and diameter, using a 3D scatter plot, we see distinct groups form in the traits data. To classify these groups, we also use the K-means algorithm (the term 'groups' is used to describe to cluster traits to distinguish from 'clusters' of the waveform data). First, the three sets of trait data for all clusters are normalized so that the maximum of each set is one. Then each cluster, each with three traits, is used as a data point as input to the K-means algorithm. This time, to find the optimal number of groups we apply the Dunn's index to evaluate and compare K-means grouping results as shown in Figure 4.14. From observation the we believe the traits data best fit into three groups. However, according to Dunn's index, dividing the data into 2 or 4 groups result in the best groups. Because 2 groups might not adequately describe the structure in the traits data, K-means is performed using 4 groups. Results are shown in Figure 4.15. By observation, we can tell that the groups colored blue and black in Figure 4.15 can be considered one group. Because the K-means algorithm is more suitable for spherical clusters, this elongated cluster in the data was split in two.

Judging from only three groups, where the blue and black groups are grouped into one, there are several noticeable trends in the traits data. Most noticeably, is that there are very few differences between results using K = 130 and K = 220. This can be an indication that the Ks are chosen in the correct range. In terms of the traits data, the elongated group (colored blue and black) is noticeably separate from the rest of the data. This group have very low intensity and low diameter. Their count is relatively high but varies. This means that these low intensity clusters are small and dense. There are also more data points in this group when compared with the others. The remaining traits data is split into two groups. In these data, cluster average intensity is positively correlated with cluster diameter, but negatively correlated with the cluster count, the number of data points in the cluster. This indicates that data clusters with high intensity, the red group in Figure 4.15, are large but with sparse data points. There are also the fewest clusters in this group. Compared to the red group, green group contains clusters with lower intensity, smaller diameter and higher count. These clusters, similar to those in the elongated group, are also small and dense, but have significantly higher intensity.

In summary, foliage clumping can be physically described by clustering waveform LiDAR data. Each cluster is described by three traits: average cluster intensity, number of data points, and cluster diameter. By using these traits, clumps in vegetation can be grouped into three main groups

- Group 1. Low intensity group that are small and dense. A majority of clusters fall in this group
- Group 2. Medium intensity group that are also small and dense. There are relatively less clusters in this group
- Group 3. High intensity group that are large but sparse. This groups contains the fewest number of clusters

Of the three groups, group 1, with low intensity, contains the least amount of clumping information. Waveform data clusters in this group might have resulted from few scattered leaves or ascending and descending edges of the return laser waveform due to a foliage clump. Group 2, with clusters similar in size and density to those of group 1, have significantly higher intensity. They likely represent slightly denser foliage or small foliage clumps. Group 3 provides the most information on the structure of the canopy due to foliage clumping. Clusters in this group have high intensity, indicating strong returns, likely from dense foliage clump with few to no gaps. Also, since here should be fewer returns in dense areas, the sparse data of clusters in group 3 is also indicative of dense vegetation.

# 4.3 Figures



Figure 4.1: Simulated point clouds with 30 cluster centers.



Figure 4.2: K-means cluster evaluation by BIC using a simulated point cloud with 60 clusters. The solid line represents the average of all repeated runs which are shown as dashed lines.



Figure 4.3: K-means cluster evaluation by minBdist using a simulated point cloud with 60 clusters. The solid line represents the average of all repeated runs which are shown as dashed lines.



Figure 4.4: K-means cluster evaluation by Dunn's index using a simulated point cloud with 60 clusters. The black line represents the average of all repeated runs, shown as colored lines.



Figure 4.5: K-means cluster evaluation by minimum cluster center distance using a simulated point cloud with 60 clusters. The black line represents the average of all repeated runs, shown as colored lines.



Figure 4.6: K-means cluster evaluation by a. Dunn's index and b. minimum cluster center distance using the waveform LiDAR data of a tree. The black line represents the average of all repeated runs, shown as colored lines.



Figure 4.7: a. Waveform data of the tree where high intensity shows as green and low as blue; b,c. Cluster results for K = 130 and K = 220, each color represents individual clusters.



Figure 4.8: Cluster results for K = 130 and K = 220 colored by cluster intensity. Blue is less intense, and red the most intense.



Figure 4.9: Cluster results for K = 130 and K = 220 colored by cluster count. Blue means less records, and red is more records per cluster.



Figure 4.10: Cluster volume for K = 130 and K = 220. Blue represents smaller clusters, and red represents larger clusters.







Figure 4.12: Relationship between cluster diameter and cluster volume. Each point represents a cluster found by K-means.



Figure 4.13: Comparison of the distribution of different cluster traits: average intensity, number of records, and cluster diameter.



Figure 4.14: Evaluation of K-means with different number of group (G) using Dunn's index.



Figure 4.15: Cluster grouping results by K-means for a. K = 130, and b. K = 220. Different groups are represented by different colors.

# CHAPTER 5 SUMMARY OF RESULTS

In this chapter, we summarize the important results from this thesis. There are three major parts to this work. The first part is processing raw waveform LiDAR data into a list of usable Python objects. The next is using a combination of discrete LiDAR data, waveform data and field data to build a model that can estimate tree biomass based on each tree's waveform characteristics. The last part focuses on finding structure in waveform data in order to quantify canopy clumping structure.

## 5.1 Data Processing

Data processing is the most difficult and time consuming part of this work. In this part of the work, we are able to produce a complete work flow that lets users extract geolocated waveform LiDAR data based on the corresponding discrete LiDAR point cloud. This process only takes a point cloud LiDAR file in .las format of the area of interest. Discrete LiDAR data can be easily viewed and extracted by many open source or free software unlike waveform LiDAR data. The major steps in this work flow are outlined below.

- 1. Find all GPS time ranges where data exists in the discrete point cloud.
- 2. Read from all LiDAR binary files and locate all waveform records that fall in any of the GPS time ranges in the discrete data.
- 3. Combine raw data for each waveform into a geolocated waveform object based on Equation 2.2 and 2.3.
- 4. Extract record of peak intensity of each waveform and convert into LiDAR point cloud.
- 5. Compare waveform data derived point cloud with discrete LiDAR point cloud. Based on the comparison correct geolocation errors due to heading, range and GPS time offsets.

The work flow is written in Python and has been optimized using shared memory parallelization. This software allows users to extract geolocated waveform data for only areas of interest. It not only prevent the user from having to work with raw binary files, it also limits the amount of data the users have to process to only those relevant to their work.

### 5.2 Biomass

Our goal for this part of the work is to develop a model that can estimate tree biomass based on the tree's waveform LiDAR data characteristics. Toward this aim, we first find biomass using allometric equations found in the National Biomass Estimator Library using tree survey data for four sites in USBR, Allerton Park-1, Allerton Park-2, Home Forest Site, and Lake of the Woods. Then we calculate a pseudo-waveform for each surveyed tree using a cylindrical voxelization method. Next we use stepwise multiple linear regression and try to relate each tree's pseudo-waveform structural characteristics to its biomass.

Since the regression is based on only 80% of the data with the remaining 20% serving as test data, results between regressions are extremely variable. The  $R^2$  values for fitting the test data ranges from 0.3 to 0.7. The resulting models are also very variable. In order to find the major pseudo-waveform characteristics that affect biomass, we then run the regression 500 times each time using a randomly chosen 80% of the data.

The three models that occurs the most frequently are shown in Equations 3.3, 3.4, and 3.5. Only a few pseudo-waveform characteristics reoccur in these models. The most frequently used is tE, total pseudo-waveform energy, which occurs in all three models. RH50, relative height at 50% energy, occurs twice. midE, median energy, and tH, total pseudo-waveform height, each occur once. The resulting biomass predictions for the 500 regressions are plotted against biomass calculated based on field survey data in Figure 3.4. In this figure, the red circles shows the median value for all predicted biomass from 500 different models. The median and the actual biomass does exhibit a positive trend. However, the trend is not one to one. The regression models tends to under predict for trees with high biomass values and over predict for those with low biomass values.

## 5.3 Foliage Clumping

In this part of the thesis, we use K-means clustering algorithm to explore the structure in waveform LiDAR data of individual trees. We first test multiple published methods of finding the best number of clusters to use based on simulated LiDAR data. Dunn's index and minimum cluster center distance are found to be the most suitable. When applied to the waveform LiDAR data of the sample tree shown in Figure 2.4, K = 130 and K = 220 are chosen. Using cluster results from the K-means clustering algorithm on waveform LiDAR data, we find four traits for each cluster, cluster intensity, count, volume and diameter. Then using cluster intensity, count and diameter, we further group all clusters into three groups.

Group 1 are small and dense clusters with low intensity. Group 2 are clusters similar in size to that of group 1, but have significantly higher intensity. Group 3 contains large and sparse clusters with the highest intensity. From these results, clusters in both group 2 and 3 are highly indicative of non-randomness in the foliage. Therefore, the location and traits of these clusters from waveform LiDAR data can serve as physical representation of foliage clumping.

# CHAPTER 6 DISCUSSION

In this chapter, we present the conclusions we can draw from the results of two sections of our work. The first section is our work in trying to estimate tree biomass based on pseudowaveform characteristics of each tree. The second part of our work involves describing canopy clumping structure using waveform LiDAR data.

In our work with using stepwise regression to estimate biomass, each regression returns varying results. The results differ because a randomly chosen portion of the data is used. This instability likely means that the data we use in the regression cannot represent the full dataset, and that results from individual regressions may not reflect real physical processes.

The limited amount of data available was a concern since the beginning of this study. However, since researchers are likely to run into data limitations frequently, we hope to find a way to overcome this problem by using a bootstrapping method of running the regression many times. The results from this method show that the majority of models tends to over predict biomass for trees with low biomass values, and under predict for trees with large biomass values. However, the most frequently occurring model does give relatively high  $R^2$ value. Because there are two other models that came close to the best one in performance, the best model may not be a reliable estimator of biomass in application. However, these models are able to inform us of the pseudo-waveform characteristics that correlates most strongly with biomass, the total pseudo-wave energy and relative height at 50% energy. Knowing these characteristics can enable better understanding of how waveform data interacts with forest canopy.

In the second part of our work, we use the K-means clustering algorithm to find structure in waveform LiDAR data of individual trees in order to better describe canopy clumping. From the clustering results, we are able to identify three groups of clusters. Group 1 contains small, dense clusters with low intensity. The density of the waveform records indicates that the area is on the edge of a foliage clump, or is an area with sparse or scattered leaves. The low cluster intensity indicates that the former is more likely, and they may be edge affects produced in the waveform data by a denser patch of foliage. Therefore, we believe that these clusters contains the least amount of foliage, and thus the least amount of structural information. Group 2 contains clumps of similar size and density to that of group one, but they have significantly higher intensity. The higher intensity differentiates group 2 clusters from those in group 1. We can see from Figure 4.15 that groups 2 and 3 are very well separated from group 1, and not as much from each other. Due to the higher intensity, this group likely represents areas on the canopy with sparse or scattered foliage that results in small peaks in the return waveform, or the less dense foliage surrounding dense clump that are captured in the rising and falling legs of each return waveform. Group 3 contains large clusters with sparse data. The high intensity immediately indicates a strong laser return due to dense foliage. The sparseness may also be an indication of foliage density since there are not as many returns from the area. Therefore, we conclude that group 3 indicates dense clumps in the canopy, and combined with group 2 clusters, they can serve as indication of canopy clumping structure.

One difficulty we encounter in this study is choosing K for clustering the waveform LiDAR data. The difficulty in this choice is likely due to the high density of the waveform LiDAR data, and that the spatial location of LiDAR data is generally uniform. Clusters are results of differences in record intensity and vertical distribution. To overcome this issue two different numbers are chosen based on estimates from two cluster evaluation measures. By comparing the results of both choices of K shown in Figure 4.13, we notice that the shapes of the distributions of cluster traits are extremely similar. From Figure 4.15, we can see that the trends in the traits data are also similar. The most significant difference is the increased number of clusters in groups 1 and 2 when K is larger. By increasing K, the K-means algorithm seems to further divide the less intense clusters, but have little effect on the more intense ones.

Given our conclusion that only the more intense clusters in groups 2 and 3 should serve as indications of clumping, we believe that reasonable variations in the number of clusters will not affect the results of the clustering for this application. Therefore, when using this method on a larger number of trees, we recommend apply the cluster evaluation process to one representative tree, or simply use the rule of thumb (Eq.4.2) for finding K in the K-means algorithm.

In summary, for our work in estimating tree biomass using waveform LiDAR data, we are unable to overcome data limitations. However, using statistical techniques, we find that the total energy and relative height at 50% energy of pseudo-waveforms of each tree provides the most information about tree biomass. In our work with characterizing canopy structure, we classify the waveform data into three groups, and concludes that groups 2 and 3, those containing clusters with higher average intensity can serve as an estimate of canopy clumping structure.

# CHAPTER 7 CONCLUSION

Understanding the interactions between light and the canopy is very important in describing gas and energy fluxes between land and air. Both canopy structure and leaf chemistry affect light penetration. In this thesis, we use waveform LiDAR data to inform two important factors that affect canopy structure, total foliage, and foliage clumping, in order to better understand light and vegetation interactions.

Because of the importance of these canopy characteristics, many previous studies have been published. However, most of them, especially those about canopy clumping, are large scale while assuming uniform vertical canopy properties. In this thesis we describe our methods of finding biomass and canopy clumping for individual trees using raw waveform LiDAR data in order to satisfy high resolution models that are being developed with increasing frequency.

## 7.1 Summary of Methods

Waveform LiDAR data, unlike its discrete counterpart, has very limited software support. Therefore, in order to work with raw waveform LiDAR data, we first developed a complete data processing workflow extracts all waveform data for an area of interest by using the discrete LiDAR data of the same area as input. All records in each waveform are geolocated and corrected for errors by comparing with the processed discrete LiDAR point cloud.

Using the extracted waveform data and tree survey data from four field sites in USBR, we first try to estimate biomass of individual trees using stepwise linear regression. Biomass values for each surveyed tree are calculated using allometric equations from the National Biomass Estimator Library. These values are used as dependent variables in the regression. A pseudo-waveform for each tree is calculated by a voxelization method where all waveform record are averaged in thin cylindrical voxels over the height of the tree. Each cylinder has the same radius as that of the tree, and 0.2 m in height. Then structural characteristics of each pseudo-waveform are calculated, including tE, tH, maxE, midE, RH25, RH50, and

RH75. These characteristics serve as explanatory variable in the regression.

Individual regressions based on 80% training data and 20% test data return variable results likely due to the limited amount of data. So we apply a bootstrapping method and run 500 regressions based on 80% randomly chosen data. Results are summarized in the section below.

In the next part of our work, we use the K-means clustering algorithm to find clumping structure in waveform LiDAR data of individual trees. We first use simulated point cloud data to test a multitude of cluster evaluation criteria for finding the most suitable K for the K-means algorithm. From our test, Dunn's Index and minimum cluster center distance could best indicate the correct number of clusters specified in the simulated LiDAR data. However, when these evaluation criteria are applied to the waveform data, the best choice for K is not apparent. Therefore, two K values are chosen, K = 130, and K = 220. Using these K values, the K-means clustering algorithm is then used to cluster the waveform LiDAR data for a sample tree. For each resulting cluster, four traits are found, and three of them, cluster intensity, count and diameter, are used to classify all clusters by K-means. Results are presented in the following section.

### 7.2 Results and Conclusions

In our aim to describe vegetation structure, we used waveform data to tackle two aspects of canopy structural characteristics, total foliage through biomass, and foliage distribution through canopy clumping.

For our biomass estimation, from the results of 500 regressions using bootstrapping, we are not confident in the ability of the best performing model in predicting biomass results. However, we find that total energy and relative height at 50% energy of each tree's pseudo-waveform are the most correlated with tree biomass. This information will be extremely useful in future studies of biomass estimations, as well as providing insight into the relation between waveform LiDAR data and total foliage.

In our work with canopy clumping, we classify the cluster results into three groups. Groups 2 and 3, containing relatively high intensity clusters, represents most of the dense foliage in the canopy and thus provides important information about canopy structure. Using waveform LiDAR data, we are able to construct an unprecedentedly detailed replication of physical canopy architecture. In the process we are also able to develop a complete raw waveform LiDAR processing work flow as well as easily scalable method for describing canopy

structure.

In summary, we are not able to provide biomass estimations for individual trees based on our field data alone. However, we believe our method in sound and the results from our study can provide good insight for future studies. We are able to characterize detailed physical clumping structure in the canopy using LiDAR data. Our new physical description is unlike all previous works that seek to generalize the canopy using a single index. This description of canopy structure may be more difficult to ingest by current models than the traditional indices, and more work is needed to relate cluster traits to physical cluster characteristics. However it provides sub-meter characterizations of the canopy that can be extremely useful in fine resolution models such as those involving ray tracing. With more work, we believe description of foliage clumping provided in this study can have wide applications in the future.

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