RESILIENCE ANALYSIS: A MATHEMATICAL FORMULATION TO MODEL RESILIENCE OF ENGINEERING SYSTEMS

BY

NEETESH SHARMA

THESIS

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Adviser:

Professor Paolo Gardoni

Abstract

Resilience of engineering systems is related to their ability of absorbing both gradual and abrupt changes under exposure conditions and rapidly recover from disruptions. In this thesis, we develop a general stochastic formulation to model the recovery process and quantify system's resilience. In particular, we develop models for time-dependent capacity of a system and the imposed demand, under joint effects of recovery and shock deterioration processes. Using the developed models, a recovery curve is formulated in terms of system's reliability, functionality and work progress. Furthermore, we propose a novel approach for resilience analysis by defining measures to capture characteristics of recovery curves. The proposed approach makes a distinction in resilience of systems with different recovery patterns. A numerical example is provided to illustrate the application of the model.

To Mommy, Papa & Didi

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Chapter 1

Introduction

Current design approaches of engineering systems are pushing towards options that are resilient under exposure conditions. The definition and quantification of the term resilience vary among different disciplines. One such definition given by the US Department of Homeland Security (Directive 2011) is "the ability to adapt to changing conditions and withstand and rapidly recover from disruption due to emergencies." The mathematical formulation of the recovery process and resilience analysis are key elements in assessing the performance of engineering systems and guiding the decision makers to optimally allocate limited resources. Resilience analysis gives rise to the requirement of developing tangible metrics that integrate major characteristics of a resilient system. The desirable characteristics of a resilient system are, 1) increased availability for operation, 2) enhanced reliability, and 3) reduced recovery time. Because resilience analysis involves the formulation of an uncertain recovery process that might also be subject to interrupting shocks, it is of utmost significance to appropriately treat various sources of the uncertainties.

The probabilistic modeling of the recovery process and resilience analysis of engineering systems have been the focus of much research over the past decade. The problem has been approached in a number of different ways. Chang and Shinozuka (2004) quantified resilience in terms of the probability that a system's performance loss, right after a disruption, and the corresponding recovery time would be less than predefined thresholds. Cimellaro et al. (2010) defined a resilience metric in terms of the area bounded between the recovery curve of a system and the time axis, over a given time horizon. Furthermore, they suggested different shapes of recovery curves that represent the level of preparedness of a system/society in responding to a disruption. Decò et al. (2013) used the same resilience metric as in Cimellaro et al. (2010) and suggested a parametric sinusoidal function for the recovery curve. Uncertainty in a recovery process is modeled by assigning probability distributions to the parameters of the recovery curves. A Monte Carlo simulation method is then used to obtain the probability distribution of the resilience metric. Iervolino and Giorgio (2015) proposed a non-homogeneous Markov process to model the recovery of a system, considering the possible effects of aftershock disruptions. A finite set of damage states are defined for a system and the possible transition from each damage state to others, in a unit time interval, is described by means of a time-dependent probability matrix, known as a transition matrix.

Though the mentioned works, among others, made significant progress over the years (see, for example, Hosseini et al. 2016 for a summary of the work in the literature), there remain challenges to be addressed. In particular, there are two major limitations with current approaches: 1) Predefined, monotonic continuous recovery curves do not replicate the reality of a recovery process. The progress in the recovery process does not lead to instantaneous changes in a system's functionality. For systems that work only after full restoration of functionality, the recovery curve should be modeled as a step function. Furthermore, for systems that work with partial functionality, the recovery curve should be modeled as a piecewise constant function, because recovery activities take time to make impacts on the functionality. On the other hand, in the stochastic modeling of recovery, using a non-homogenous Markov process, the key problem is the formulation of the time-dependent transition matrix. However, no practical solution has been proposed for this problem. 2) The current resilience metrics are insensitive to the shape of recovery curves, given that the bounded-area between a recovery curve and time axis is fixed.

This thesis develops a general stochastic formulation to model the recovery process and quantify systems' resilience. In particular, stochastic models are developed for time-dependent capacity of a system and the imposed demand, under joint effects of recovery process and shock deterioration. Using the developed capacity and demand models, equations are derived to compute the instantaneous reliability of a system, the probability distribution of 1) the number of recovery activities to restore a target functionality; 2) the number of shocks to recover, and 3) time to the end of recovery. Furthermore, a novel approach is proposed for resilience quantification. In addition, a theory of resilience analysis based on recovery curve characteristics is also proposed. Each segment of a recovery curve, between two possible successive shocks, is termed as a cumulative resilience distribution function (CRF). Partial descriptors of the CRFs can be aggregated to quantify the resilience of a system, which is analogous to probability analysis. In contrast to current approaches, the proposed resilience metrics (i.e., a set of partial descriptors) is sensitive to the shape of all recovery curves with the same bounded-area between the recovery curve and the time axis. An example is presented to explain the proposed formulation.

There are eight chapters in this thesis. Following this Introduction, Chapter 2 describes the resilience modeling of deteriorating systems. Chapter 3 discusses a general recovery process and the related stochastic modeling issues. Chapter 4 presents the proposed formulation to model the recovery process and quantify resilience. Chapter 5 presents solutions of quantities of interest. Chapter 6 provides details on estimation and calibration of parameters of the model. Chapter 7 presents a numerical example to illustrate the proposed formulation and Chapter 8 presents conclusions derived from this work.

Chapter 2

Proposed Mathematical Formulation for Resilience Analysis and Resilience Metrics

Assessing the resilience of engineering systems is crucial both for pre-disruption effective mitigation planning and post-disruption optimal resource allocation. There are many factors that influence resilience of engineering systems, including the design specifications, the availability of resources needed for the repairs (e.g., funding and materials), the accessibility of damaged components, preparedness of recovery plans, and environmental condition during the recovery. Developing a composite resilience metric that integrates all the influencing factors is critical for optimal decision-making.

In this chapter, we first review the current practices of quantifying resilience, the available metrics, and their limitations. Then, we propose a new mathematical formulation for resilience analysis that overcomes the current limitations.

Fig. 2.1.1 shows a typical recovery curve used in the literature to quantify resilience (Bocchini et al. 2012; Bonstrom and Corotis 2014; Cimellaro et al. 2010). An external shock (e.g., an earthquake) at time τ_I causes an instantaneous reduction in system's performance indicator, $\breve{Q}\left(\tau\right)$ (e.g., the system's functionality.) The residual performance of the system, Q_{res}, depends on the intensity of the shock, design specifications, and the reliability of the system. Subsequently, the system undergoes a recovery process to restore a desired performance level (e.g., the original functionality or a higher one, if desired.) After meeting the desired requirements, the recovery process terminates at time τ_L. The different factors affecting resilience, listed earlier, influence the shape of the recovery curve as well as the duration of the process, $T_R := \tau_L - \tau_I$. Resilience of the system is typically quantified as a function of the shaded area in Fig. 2.1.1. Mathematically, the typical resilience metric (see, for example,Bonstrom and Corotis 2014; Cimellaro et al. 2010; Decò et al. 2013) is defined as

$$R := \frac{\int_{\tau_I}^{\tau_L} \breve{Q}\left(\tau\right) d\tau}{T_R} = \frac{\int_0^{T_R} Q\left(t\right) dt}{T_R} \tag{2.1}$$

where, we used the change of variable $t = \tau - \tau_I$ to define $Q\left(t\right) := \breve{Q}\left(\tau + \tau_I\right)$. The limitation of the resilience metric, R, is that it gives the same value of resilience for different combinations of $Q\left(t\right)$ and T_R. We explain this limitation with the following example. Fig. 2.1.2 shows three possible recovery curves. Table 2.2.1

summarizes their functional forms along with the corresponding T_R's. The three different recovery curves are associated with different levels of resilience (e.g., the curve Linear 1 might be considered to be the most desirable recovery.) However, as shown in the last column of the Table 2.2.1, the calculated values of R for the three recovery curves are the same (i.e., equal to 0.75.)

To address this limitation, we need to develop a set of partial descriptors of $Q(t)$ that capture the differences in the shape of the recovery curves. Furthermore, it is desirable that the partial descriptors have simple and clear interpretations. This property can be helpful when comparing different $Q(t)$'s and prioritizing resource allocation for the recovery of damaged systems. We propose a new resilience analysis that describes the recovery process in terms of partial descriptors of $Q(t)$.

To explain the proposed resilience analysis, we first develop the tools for describing the recovery process and then derive the partial descriptors. The recovery curve $Q(t)$ that we term the *cumulative resilience distribution function* (CRF), represents the overall recovery progress by time t. The time derivative of the CRF represents the instantaneous recovery rate at any given time.

If the CRF is a continuous function of time, the instantaneous recovery rate is described by its *resilience density function* (RDF), defined as $q(t) := dQ/dt$. The RDF is undefined at a finite set of points where the CRF is not differentiable. We can obtain the recovery progress over an arbitrary interval $[t_u, t_v) \subseteq [0, T_R]$ as follows:

$$Q(t_u \leq t < t_v) = \int_{t_u}^{t_v} q(t)\, dt \tag{2.2}$$

If the CRF is a piecewise constant function, we can no longer use the RDF to describe the instantaneous recovery rate. In such cases, we define the *resilience mass function* (RMF) as $q(t) := \sum_{k=1}^{\infty} \Delta Q(t_k)\, \delta(t - t_k)$, where $\Delta Q(t_k) := Q(t_k) - Q\left(t_k^-\right)$ is the size of the jump in CRF at the discontinuity point $t = t_k$; t_k^- is the time instant immediately before t_k; and $\delta(\cdot)$ is the Dirac delta function. Similar to the continuous case, we can obtain the recovery progress over an arbitrary interval $[t_u, t_v) \subseteq [0, T_R]$ as follows:

$$Q(t_u \leq t < t_v) = \int_{t_u}^{t_v} \sum_{k=1}^{\infty} \Delta Q(t_k)\, \delta(t - t_k)\, dt = \sum_{k=1}^{\infty} \Delta Q(t_k)\, \mathbf{1}_{\{t_u \leq t_k < t_v\}} \tag{2.3}$$

where $\mathbf{1}_{\{t_u \leq t_k < t_v\}}$ is an indicator function such that $\mathbf{1}_{\{t_u \leq t_k < t_v\}} = 1$, when $t_k \in [t_u, t_v)$ and $\mathbf{1}_{\{t_u \leq t_k < t_v\}} = 0$, otherwise. To reflect that at the time of the shock ($t_1 = 0$) the CRF is equal to Q_{res} (which is typically$\neq 0$) we define $\Delta Q(0) := Q_{\text{res}}$.

In general, the CRF might be a combination of the previous two cases, where $Q(t)$ is a piecewise continuous function. In this case, we can write the time derivative of the CRF as

$$q(t) = \tilde{q}(t) + \sum_{k=1}^{\infty} \Delta\tilde{Q}(t_k)\,\delta(t - t_k) \tag{2.4}$$

where, $\tilde{q}(t)$ is the RDF, corresponding to the continuous part of the CRF and $\sum_{k=1}^{\infty} \Delta\tilde{Q}(t_k)\,\delta(t - t_k)$ is the RMF, accounting for the discontinuities of the CRF. Accordingly, we can write the recovery progress over $[t_u, t_v) \subseteq [0, T_R]$ as follows:

$$Q(t_u \leq t < t_v) = \int_{t_u}^{t_v} \tilde{q}(t)\,dt + \sum_{k=1}^{\infty} \Delta\tilde{Q}(t_k)\,\mathbf{1}_{\{t_u \leq t_k < t_v\}} \tag{2.5}$$

The CRFs or RDFs/RMFs of systems provide complete information about the recovery process and, thus, resilience. To help in the interpretation of the CRF, RDF and RMF, one can see the analogy between their definitions and those of the Cumulative Distribution Function (CDF), Probability Density Function (PDF) and Probability Mass Function (PMF) that are used to describe random variables in probability theory (see, for example, Ang and Tang 2007.)

To capture the degree of disparity between any pairs of recovery curves, we define the measure of *recovery disparity*, $\Delta(q_1, q_2)$, as follows:

$$\Delta(q_1, q_2) := \frac{\int_0^{T_R} q_1(t)\log_2[q_1(t)/\bar{q}(t)]\,dt + \int_0^{T_R} q_2(t)\log_2[q_2(t)/\bar{q}(t)]\,dt}{Q_1(T_{R_1}) + Q_2(T_{R_2})} \tag{2.6}$$

where, T_{R_1} and T_{R_2} are the recovery duration associated with q_1 and q_2; $T_R = \max(T_{R_1}, T_{R_2})$; and $\bar{q}(t) := [q_1(t) + q_2(t)]/2$. The recovery disparity is bounded between 0 and 1 such that $\Delta(q_1, q_2) = 0$, when $q_1(t) = q_2(t)$ for every $t \in [0, T_R]$ and $\Delta(q_1, q_2) = 1$, when the intersection of the supports of q_1 and q_2 in the interval $[0, T_R]$ is null. The proposed recovery disparity measure is analogous to divergence measures in probability theory. Specifically, if the CRFs are replaced with CDFs, $\Delta(q_1, q_2)$ corresponds to the Jensen-Shannon entropy (Lin 1991). In probability theory, this quantity is used as a measure of difference between two probability distributions. The applications can be found in approximation of probability distributions, in signal processing, and in pattern recognition.

Besides the recovery disparity that gives a general comparison of $q(t)$'s, partial descriptors can be defined to capture specific characteristics of the recovery process which are of importance in practice. First, we can define the central measures of the recovery process. We define the *center of recovery*, ρ, as

$$\rho := \frac{\int_0^{T_R} t q(t)\,dt}{\int_0^{T_R} q(t)\,dt} = \frac{\int_0^{T_R} t\tilde{q}(t)\,dt + \sum_{k=1}^{\infty} t_k \Delta\tilde{Q}(t_k)\,\mathbf{1}_{\{0 \leq t_k \leq T_R\}}}{Q(T_R)} \tag{2.7}$$

Two other central measures are the *median of recovery* and the *mode of recovery.* The median of recovery,

$\rho_{0.5}$, is the time instant at which the CRF is equal to $Q\left(T_R\right)/2$. The mode of recovery is the time instant corresponding to the maximum recovery rate. Mathematically, we can write it as $\rho_{\max} := \arg\max_{t\in[0,T_R]} q\left(t\right)$.

We extend our definitions of the partial descriptors and introduce measures of dispersion of the recovery process. We define the *recovery quantile*, ρ_w, which is the time instant corresponding to the w^{th} $(0 \le w \le 1)$ quantile of the CRF. Mathematically, we write the recovery quantile as $\rho_w := \min\left\{t \in [0, T_R] : w \le \left[Q\left(t\right)/Q\left(T_R\right)\right]\right\}$. The median of recovery, $\rho_{0.5}$, is a special case for which $w = 0.5$. Using ρ_w, we can define different measures of dispersion as the length of the intervals $\left[\rho_{w_i}, \rho_{w_j}\right]$, where $0 \le w_i < w_j \le 1$. We also define an alternative single measure to capture the dispersion that we call the *recovery bandwidth*, χ, and mathematically, we write it as

$$\chi^2 := \frac{\int_0^{T_R} \left(t-\rho\right)^2 q\left(t\right) dt}{\int_0^{T_R} q\left(t\right) dt} = \frac{\int_0^{T_R} \left(t-\rho\right)^2 \tilde{q}\left(t\right) dt + \sum_{k=1}^{\infty} \left(t_k-\rho\right)^2 \Delta\tilde{Q}\left(t_k\right) \mathbf{1}_{\{0\le t_k\le T_R\}}}{Q\left(T_R\right)} \tag{2.8}$$

The small values of χ represent a situation in which a large percentage of the recovery process is completed over a short period of time around ρ. In contrast, the large values of χ describe a situation in which the recovery process is spread over a long period of time. We can also define the *relative recovery bandwidth* as χ/T_R, which describes the spread of the recovery process with respect to the total recovery time, and the *bandwidth coefficient* as χ/ρ, which describes the spread of the recovery process with respect to the center of recovery.

Another useful measure is the skewness of the recovery. Mathematically, we can write the *recovery skewness*, ψ, as

$$\psi := \frac{\int_0^{T_R} \left(t-\rho\right)^3 q\left(t\right) dt}{\int_0^{T_R} q\left(t\right) dt} = \frac{\int_0^{T_R} \left(t-\rho\right)^3 \tilde{q}\left(t\right) dt + \sum_{k=1}^{\infty} \left(t_k-\rho\right)^3 \Delta\tilde{Q}\left(t_k\right) \mathbf{1}_{\{0\le t_k\le T_R\}}}{Q\left(T_R\right)} \tag{2.9}$$

The magnitude of the recovery skewness determines the degree of asymmetry of the recovery with respect to ρ. Its algebraic sign defines the direction of the skewness. From Eq. (2.9), we can see that $\psi = 0$, when the RDF and RMF are symmetric with respect to ρ. Furthermore, $\psi > 0$, when the RDF and RMF have longer tails to the right of ρ; and $\psi < 0$, when the left tails of the RDF and RMF are longer. We can interpret $\psi = 0$ as the condition in which the progress in a recovery process has the same pace before and after ρ. When $\psi < 0$, the process is slow during the interval $[0, \rho)$ and then it becomes faster over the next period, $[\rho, T_R]$. This is the most typical case for recovery processes that include a lengthy planning phase in the post-disruption period. If planning is done ahead of the disruptive event, as pre-disruption planning and preparation, then we can have $\psi > 0$ in which the progress picks up quickly and relatively the most time-consuming portion is the actual repair/reconstruction (faster in interval $[0, \rho)$, and it slows down over the next interval, $[\rho, T_R]$.) We can also define the *relative recovery skewness* as ψ/T_R^3, which describes the skewness of the recovery process with respect to the total recovery time, and the *recovery skewness coefficient* as ψ/χ^3, which describes the skewness of the recovery process with respect to the recovery bandwidth.

As a generalization, we can define the n^{th} recovery moment as follows:

$$\rho^{(n)} := \frac{\int_0^{T_R} t^n q\left(t\right) dt}{\int_0^{T_R} q\left(t\right) dt} = \frac{\int_0^{T_R} t^n \tilde{q}\left(t\right) dt + \sum_{k=1}^{\infty} t_k^n \Delta \tilde{Q}\left(t_k\right) \mathbf{1}_{\{0 \le t_k \le T_R\}}}{Q\left(T_R\right)} \tag{2.10}$$

Any recovery process can be completely defined in terms of all its $\rho^{(n)}$'s. However, in practice, typically ρ and χ are sufficient to characterize a recovery process. We can write ρ and χ as functions of the first two recovery moments as $\rho = \rho^{(1)}$ and $\chi = \sqrt{\rho^{(2)} - \rho^2}$.

It is useful to note the analogy between the proposed recovery measures and equivalent ones in probability theory and mechanics. In particular, ρ, χ, and ψ correspond to the mean, standard deviation, and skewness of a random variable when the CRF is replaced with a CDF (see, for example, Ang and Tang 2007.) Similarly, ρ and χ are analogous to the centroid and the radius of gyration of the area under $q\left(t\right)$ (see, for example, Gere and Timoshenko 2001.)

A closer look at the definitions of R and ρ shows that there are similarities in the way the two metrics quantify the recovery process and, thus, resilience. In particular, the following equation shows that ρ is an affine function of R:

$$\rho = \frac{Q\left(T_R\right) - R}{Q\left(T_R\right)} \times T_R \tag{2.11}$$

Hence, for a given recovery process (i.e., given $Q\left(T_R\right)$ and T_R,) the information that R provides about resilience is equivalently captured by ρ. However, ρ can differentiate between recovery curves with the same $Q\left(t\right)$ but different T_R. In addition, R lacks the extra information that higher order recovery moments like χ and ψ provide to characterize different $Q\left(t\right)$'s and quantify resilience. To illustrate this point, we calculate (ρ, χ) for three $Q\left(t\right)$'s in Fig. 2.1.2. The results in Table 2.2 show that, in contrast to R, the pair (ρ, χ) can characterize the three different $Q\left(t\right)$'s. Furthermore, a decision-maker can use the ρ_w's and/or combine ρ and χ (and, if desired, the higher order moments) to create composite resilience metrics (e.g., $\rho \pm \chi$, when the two values are in the range $[0, T_R]$.)

In practice, the recovery process of engineering systems might be interrupted by external shocks at different time instants (see Fig. 2.1.3). Each shock might cause a sudden reduction in $Q\left(t\right)$. The formulation of the recovery moments in Eq. (2.10) accounts for this situation by letting $\Delta \tilde{Q}\left(t_k\right) < 0$, when a shock occurs at $t = t_k < T_R$. Alternatively, we can rearrange the terms in Eq. (2.10) to write the overall recovery moments in terms of the recovery moments $\left(\rho^{(1)}, \ldots, \rho^{(n)}\right)_j$ derived for each segment j of $Q\left(t\right)$ (see Fig. 2.1.3), that follows the j^{th} shock and before the $(j+1)^{\text{th}}$ shock. Using Eqs. (2.7)-(2.10), we can write the overall recovery moments as follows:

$$\rho_{\text{tot}}^{(n)} = \sum_{j=1}^{m} \rho_j^{(n)} \times \frac{Q\left(t_{j+1}\right)}{Q\left(T_R\right)} + \sum_{j=1}^{m} t_j^n \times \frac{Q\left(t_{j+1}^-\right) - Q\left(t_j^-\right)}{Q\left(T_R\right)} \tag{2.12}$$

where m is the total number of shocks and by convention $t_{m+1} := T_R$.

2.1 Figures

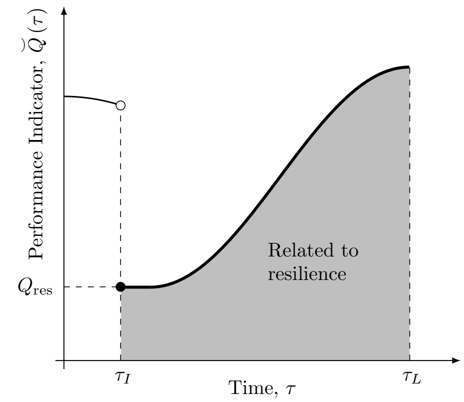

Figure 2.1: A typical recovery curve used in the literature to quantify systems' resilience

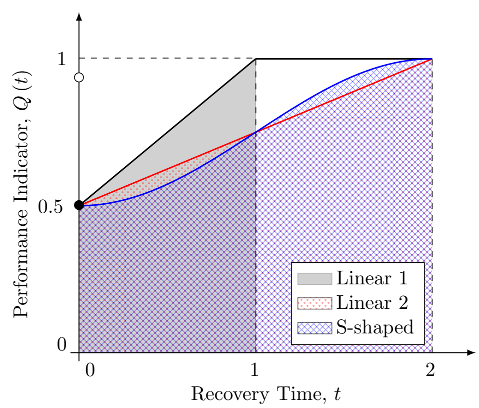

Figure 2.2: The current resilience metric cannot differentiate among different recovery curves in quantifying resilience

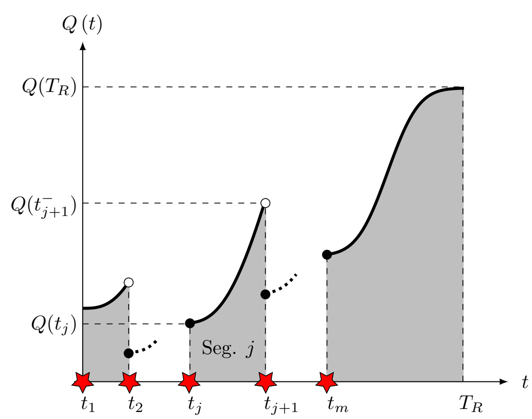

Figure 2.3: The recovery process might be repeatedly interrupted by external shocks

2.2 Tables

Table 2.1: Functional form of the recovery curves in Fig. 2.1.2 and the associated resilience

Description	T_R	Recovery function	R
Linear 1	1	$0.5 + 0.5t$	0.75
Linear 2	2	$0.5 + 0.25t$	0.75
S-shaped	3	$0.75 - 0.25\cos(\pi t/2)$	0.75

Table 2.2: Partial descriptors of resilience for recovery curves in Fig. 2.1.2

Description	ρ	χ
Linear 1	0.25	0.32
Linear 2	0.25	0.65
S-shaped	0.50	0.59

Chapter 3

Phases of the Recovery Process and Their Role in Resilience Quantification

In this chapter, we argue that different activities in the recovery of engineering systems can generally be grouped into three phases, which we call 1) recovery planning, 2) recovery execution, and 3) recovery closure. Next, we explain how the proposed time-dependent performance indicators (e.g., reliability and functionality) are affected by the recovery activities and the proposed phases.

3.1 Phases of the recovery process

The scope of a recovery process is defined by the magnitude and nature of the damage sustained by the system. The corresponding recovery process can typically be decomposed into three phases (shown in Fig. 3.3) that we call 1) recovery planning, 2) recovery execution, and 3) recovery closure. These phases can be sequential or overlapping. The recovery planning phase includes the following activities: identification of the objectives of the recovery (i.e., where we want to be at the end of the process), development of the recovery strategies (i.e., definition of the activities that will need to take place in order to reach the desired objectives), and securing the required resources for the recovery. The recovery execution phase includes activities where physical progress is made towards achieving the objectives developed in the recovery planning phase. This phase consumes the majority of the resources (i.e., time, material, and labor.) The recovery closure phase involves quality control activities to ensure that the recovery completion criteria are met and the system is ready to be put back into operation.

In the specific case of recovery of a civil engineering structure (e.g., building or bridge) or system/network (e.g., transportation, water or power network), the three phases of the recovery process correspond to what is known in construction management as 1) pre-construction, 2) construction, and 3) post-construction (Klinger and Susong 2006). The pre-construction phase includes planning, designing, deciding on repair strategies, budgeting financial and other resources, and obtaining work permits from relevant authorities. The construction phase involves physical onsite activities required to build new or repair damaged components. Inspection, quality assurance, safety management, cost and schedule control, and field engineering functions (e.g., onsite decisions) are also activities in this phase. The post-construction phase involves closing activities, like final inspection, handing over, and certification. In mechanical engineering applications such as installation of a new equipment (e.g., a boiler, compressor, or pump), the above three phases correspond to 1) design and planning 2) mechanical erection, and 3) commissioning (Wirtz 1987). Likewise, when the equipment undergoes a reactive maintenance, the three phases correspond to 1) fault detection, 2) system repairs and 3) recommissioning.

The duration of each recovery phase depends on the level of damage, the preparedness of the recovery plan, the repairability of the system and the accessibility of the damaged components. Estimation of absolute and relative duration of each recovery phase can guide how to expedite the recovery process and improve resilience. For instance, a disaster response plan, prepared before disrupting event strikes, can improve resilience by saving valuable time in the recovery planning phase. There might be updates in the planning as new information becomes available during the post-disruption period; however, a general planning can be developed a head of time. Similarly, a well-designed system could favor repairability over constructability to save time in the recovery execution phase.

3.2 Tracking performance indicators during recovery

The phases of the recovery process can be divided into a hierarchy of activities. A work breakdown structure can be designed where activities are further divided up to a required level of detail, based on functional requirement or available data. The lowest level of activities is where standardized crews, equipment, means, and methods are defined and relevant data are readily available (see, for example, RS Means database (Means 1996).) Activities in a recovery process have precedence, constraints, and tentative duration associated with them which give rise to a network of activities (see, Fig. 5). In a typical construction project, the number of such activities in the network can be as high as several thousands.

In a recovery process, work on individual activities might continuously progress over time; however, contributions to system performance (e.g., functionality or reliability) occur at discrete time instants when the activities are completed. The magnitude of each contribution and the completion time of each activity depend on the activity network, the type of activities, and the metric used by a decision-maker to measure work progress. For example, the progress might be measured in terms of the expenditure incurred in completing each activity with respect to the total project expenditure or in terms or the duration of each activity.

Fig. 3.3 shows a comparison between the actual work progress in a recovery process and the changes in the reliability and the functionality of the system. The figure illustrates that the work progress might be near-continuous; however, it is only the completion of a group of activities that contributes to increments in the reliability and in the functionality of the system. The completion of the recovery activities in a group changes both the capacity and the demand on the system (as described in Chapter 4) and, hence, the reliability (as described in Chapter 5.) Functionality typically has discrete increments when the work completion and the associated reliability reach specific target values.

The time duration of individual activities and the contribution to the overall work progress can be modeled as random variables. The initial estimates of the activity duration (derived from available databases and past engineering experience) can be updated to include the effects of environmental factors, system characteristics, and resource availability, among other factors. For example, Moselhi et al. (1997) developed a model to derive the distributions of the activities' duration, considering the weather impact. Gardoni et al. (2007) developed a Bayesian formulation to update the work progress estimates as a function of the work completed up to date. Thus, given a well-defined initial state, a stochastic network of activities can be generated. Furthermore, the time duration of individual activities and milestones corresponding to change in the reliability and functionality can be identified. An instance of the stochastic activity network maps to a corresponding instance of recovery process with respect to progression of work, reliability, and functionality.

3.3 Figures

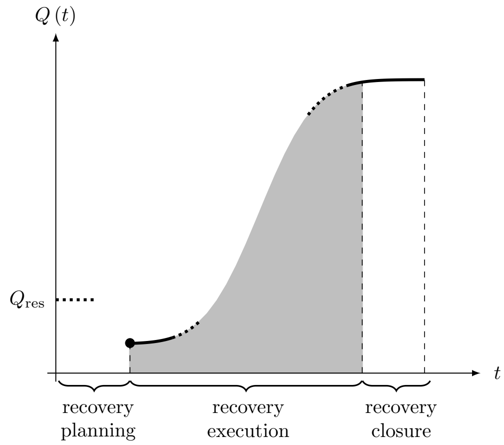

Figure 3.1: The three phases of the recovery process in the aftermath of a disruption are 1) recovery planning, 2) recovery execution, and 3) recovery closure

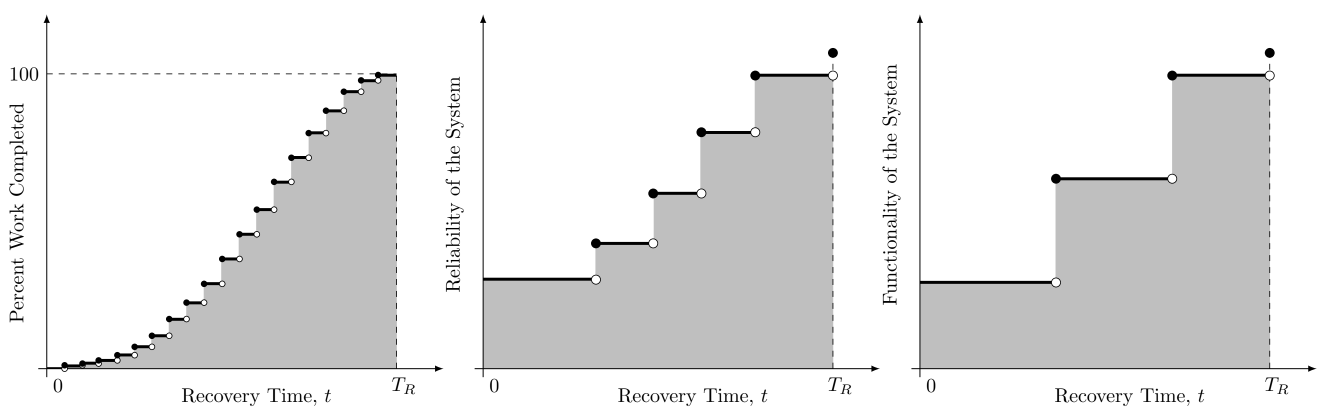

Figure 3.2: The recovery process affects different system performance indicators differently

Chapter 4

Proposed Stochastic Formulation of the Recovery Process

The scope of the present formulation is to model the recovery process of engineering systems that also might be interrupted by potential external shocks (e.g., aftershocks in case of seismic damage.) System capacity and demand are the quantities we use to describe the system states and calculate the reliability and functionality of the system.

Fig. 4.3 shows the sample paths of the time-dependent capacity and demand of a damaged system over the course of its recovery. The figure shows how the recovery process and the interrupting shocks can affect the capacity and the imposed demand on the system. We model the recovery process as a series of actions such that after the completion of each recovery activity group i, the capacity of the system instantaneously increases by ΔC_i^r and the demand on the system reduces by ΔD_i^r. The changes in the capacity and demand due to the recovery activities are occurring at time instances $\{t_i^r\}_{i\in\mathbb{N}}$. Furthermore, the system is subject to a sequence of potential interrupting shocks at time instances $\left\{t_j^s\right\}_{j\in\mathbb{N}}$. Shock j, occurs at time t_j^s, and changes the capacity of the system by ΔC_j^s and the demand by ΔD_j^s.

In order to develop the mathematical formulation of the capacity and demand on the system, we make following assumptions:

1. During the recovery process, the effect of gradual deterioration is minimal and can be neglected. This assumption is justified because the rate of gradual deterioration is typically low with respect to the recovery rate (i.e., the change in the capacity and demand due to the gradual degradation is negligible given the general short duration of the recovery phase.) However, we consider the effect of possible interrupting shocks.

2. The distribution of the inter-arrival times of the recovery activity groups (i.e., $t_i^r - t_{i-1}^r$) is independent of the interrupting shocks. The implication of this assumption is that the occurrence of the interrupting shocks does not change the recovery rate. In practice, this assumption might be justified noting that typically the recovery process does not start immediately after the main shock (e.g., main seismic shock). This time lag in case of seismic events means decreased rate of aftershocks that can significantly affect the recovery rate. Furthermore, in case of general disruptive events, it might be justifiable to

assume that the recovery duration is generally short with respect to the inter-arrival times of the main shocks that can significantly affect the recovery rate.

3. The effects of the recovery process and the interrupting shocks on the capacity and demand are additive (i.e., ΔC_i^r and ΔC_j^s can be added, and similarly for ΔD_i^r and ΔD_j^s .)

4.1 Formulation of capacity and demand models under joint effects of recovery and interrupting shocks

With reference to Fig. 4.3, the general analytical expression for time-dependent capacity of a system is written as

$$C\left(t^{-}\right) = C_{\text{res}} + \sum_{i=1}^{N^r\left(t^{-}\right)} \Delta C_i^r + \sum_{j=1}^{N^s\left(t^{-}\right)} \Delta C_j^s \tag{4.1}$$

where $C\left(t^{-}\right)$ is the capacity of the system immediately before time t; C_{res} is the residual capacity of the system at the beginning of the recovery process; $N^r\left(t^{-}\right)$ is the number of completed recovery activities by time t^{-}; ΔC_i^r is the capacity improvement due to the completion of the recovery activity group i; $N^s\left(t^{-}\right)$ is the number of shocks by time t^{-}; and ΔC_j^s is the loss of capacity caused by the shock j. Similarly, the time-dependent demand model, with a sample path shown in Fig. 4.3, is written as

$$D\left(t\right) = D_{\text{res}} + Y_j \times \mathbf{1}_{\left\{t=t_j^s\right\}} \sum_{i=1}^{N^r\left(t^{-}\right)} \Delta D_i^r + \sum_{j=1}^{N^s\left(t^{-}\right)} \Delta D_j^s \tag{4.2}$$

where, $D\left(t\right)$ is the imposed demand on the system at time t; D_{res} is the residual demand at the beginning of the recovery process; Y_j is the demand that would have imposed by shock j on the intact system; $\left\{Y_j\right\}_{j\in\mathbb{N}}$ is modeled as a sequence of independent and identically distributed (i.i.d.) random variables; ΔD_i^r is the effect of the recovery activity group i on $D\left(t\right)$; and ΔD_j^s is the effect of shock j on $D\left(t\right)$.

4.2 Termination of the recovery process

Construction projects typically end when all the prescribed activities in the work plan are completed. This condition is the typical case of the post-event recovery assessment where the level of damage and, thus, the scope of the work is known with high certainty. Subsequently, the work breakdown structure is designed and the type and the number of the required recovery activities are determined. Mathematically, this termination condition corresponds to set $N\left(T_R\right) = n$ in Eqs. (4.1)-(4.2), where n is the number of determined recovery activities. However, in a pre-event recovery assessment (e.g., for the life-cycle analysis) both the level and the type of damage to the system are unknown. Accordingly, the scope of work and the type and the number of recovery activities are unknown and can only be described in a probabilistic sense. Thus, we cannot use the same termination criterion as in the post-event assessment. Instead, we can frame the problem in a slightly different way. In the post-event assessment, the completion of the recovery process corresponds to achieving a certain level of reliability. Equivalently, we can say that the recovery process ends when the event $\{C(t^-) - D(t) > 0\}$ occurs with a prescribed probability level. We can write this event as

$$C\left(t^-\right) - D(t) = C_{\text{res}} - D_{\text{res}} - Y_j \times \mathbf{1}_{\left\{t=t_j^s\right\}} + \sum_{i=1}^{N^r\left(t^-\right)} \left(\Delta C_i^r - \Delta D_i^r\right) + \sum_{j=1}^{N^s\left(t^-\right)} \left(\Delta C_j^s - \Delta D_j^s\right) > 0 \tag{4.3}$$

Defining $Z_{\text{res}} := C_{\text{res}} - D_{\text{res}}$, $\Delta Z_i^r := \Delta C_i^r - \Delta D_i^r$, and $\Delta Z_j^s := \Delta C_j^s - \Delta D_j^s$, we rewrite Eq. (4.3) as

$$C\left(t^-\right) - D(t) = Z_{\text{res}} - Y_j \times \mathbf{1}_{\left\{t=t_j^s\right\}} + \sum_{i=1}^{N^r\left(t^-\right)} \Delta Z_i^r + \sum_{j=1}^{N^s\left(t^-\right)} \Delta Z_j^s > 0 \tag{4.4}$$

where, Y_j and ΔZ_j^s are statistically dependent, because both are the results of shock j.

4.3 Figures

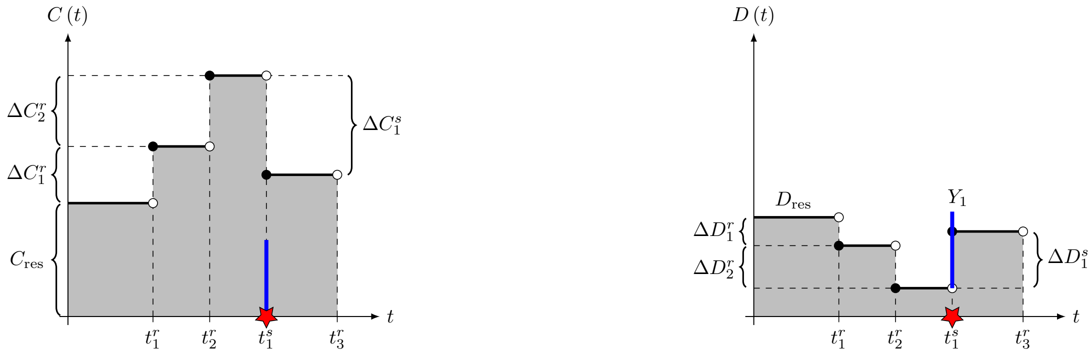

Figure 4.1: Sample paths of the time-dependent capacity (left) and demand (right) models under joint effects of the recovery and interrupting shocks

Chapter 5

Estimation of Recovery Quantifiers

Various features of the recovery process can be quantified and used, for example, in system life cycle analysis. These quantifiers can also serve as the basis to predict and compare system performance for different designs and operation strategies. Some of the useful features that can be quantified from the proposed model are the amount of the required work in a recovery process (in terms of the number of recovery activities), amount of risk involved (in terms of the number of shocks during the recovery process), system down/partial functionality time (in terms of required recovery time) and resilience of the system (in terms of the partial descriptors of resilience.)

5.1 Instantaneous reliability

Instantaneous reliability during the recovery process is given by the probability of the system capacity being greater than the system demand. This can be obtained by calculating the probability of the event represented in Eq (4.4). At a given time we define the system performance terms of system reliability as $Q_{rel}(t)$. Which can be written as

$$Q_{rel}(t) = P\left(C\left(t^-\right) - D\left(t\right)\right) = P\left(Z_{\text{res}} - Y_j \times \mathbf{1}_{\left\{t=t_j^s\right\}} + \sum_{i=1}^{N^r\left(t^-\right)} \Delta Z_i^r + \sum_{j=1}^{N^s\left(t^-\right)} \Delta Z_j^s > 0\right) \tag{5.1}$$

5.2 Number of recovery steps to completion

The number of recovery steps required to fully restore the system performance quantifies the amount of work required in the recovery process. The probability that the recovery process has not been completed by the n^{th} recovery activity group can be written as

$$\mathbf{P}\left(N_R > n\right) = \mathbf{P}\left(Z_{\text{res}} - Y_m \times \mathbf{1}_{\{t_m^s = t_n^r\}} + \sum_{i=1}^{n} \Delta Z_i^r + \sum_{j=1}^{m} \Delta Z_j^s < 0\right) \tag{5.2}$$

where, $m := \max\left\{j : t_j^s \leq t_n^r\right\}$. Because $\{t_m^s = t_n^r\}$ is a zero probability event, we can simplify Eq. (5.2) as

$$\mathbf{P}\left(N_R > n\right) = \mathbf{P}\left(Z_{\text{res}} + \sum_{i=1}^{n} \Delta Z_i^r + \sum_{j=1}^{m} \Delta Z_j^s < 0\right) \tag{5.3}$$

Defining $W_m^s := \sum_{j=1}^{m} \Delta Z_j^s$ and conditioning on $\{m, W_m^s, Z_{\text{res}}\}$, we obtain

$$\mathbf{P}\left(N_R > n \mid m, W_m^s, Z_{\text{res}}\right) = F_{W_n^r|W_m^s, Z_{\text{res}}}\left(-w - z \mid W_m^s = w, Z_{\text{res}} = z\right) \tag{5.4}$$

where, $F_{W_n^r|W_m^s, Z_{\text{res}}}(\cdot)$ is the conditional CDF of the random variable $W_n^r := \sum_{i=1}^{n} \Delta Z_i^r$. The individual recovery steps are assumed to be i.i.d.; We can obtain the conditional PDF of W_n^r, $f_{W_n^r|W_m^s, Z_{\text{res}}}$, by evaluating the n-fold convolution of the conditional PDF of ΔZ_i^r, $f_{\Delta Z_i^r|W_m^s, Z_{\text{res}}}$, with itself. Accordingly, we can obtain the unconditional distribution of N_R as follows:

$$\mathbf{P}\left(N_R > n\right) = \mathbf{E}_{\{m, W_m^s, Z_{\text{res}}\}}\left[F_{W_n^r|W_m^s, Z_{\text{res}}}\left(-w - z \mid W_m^s = w, Z_{\text{res}} = z\right)\right] \tag{5.5}$$

where $\mathbf{E}\left[\cdot\right]$ is the expected value operator.

5.3 Number of interruptions to completion

A recovery process might be interrupted by the external shocks (e.g., earthquake aftershocks.) These events can extend the amount of work in the initial plan and elongate the recovery time. The probability that the recovery process has not completed by the time of occurrence of the m^{th} shock can be written as

$$\mathbf{P}\left(N_S > m\right) = \mathbf{P}\left(Z_{\text{res}} - Y_m + \sum_{i=1}^{n} \Delta Z_i^r + \sum_{j=1}^{m} \Delta Z_j^s < 0\right) \tag{5.6}$$

Defining $n := \max\{i : t_i^r \leq t_m^s\}$ and conditioning on $\{n, W_n^r, Z_{\text{res}}, Y_m\}$, we can write

$$\mathbf{P}\left(N_S > m \mid n, W_n^r, Z_{\text{res}}, Y_m\right) = F_{W_m^s|W_n^r, Z_{\text{res}}, Y_m}\left(y - w - z \mid W_m^s = w, Z_{\text{res}} = z, Y_m = y\right) \tag{5.7}$$

Assuming individual shocks are i.i.d., the conditional PDF of W_j^s, $f_{W_m^s|W_n^r, Z_{\text{res}}, Y_m}$, can be obtained by evaluating an m-fold convolution of the conditional PDF of ΔZ_j^s, $f_{\Delta Z_j^s|W_m^s, Z_{\text{res}}}$, with itself. Accordingly, we can obtain the unconditional distribution of N_S as follows:

$$\mathbf{P}\left(N_S > m\right) = \mathbf{E}_{\{n, W_n^r, Z_{\text{res}}, Y_m\}}\left[F_{W_m^s|W_n^r, Z_{\text{res}}, Y_m}\left(y - w - z \mid W_m^s = w, Z_{\text{res}} = z, Y_m = y\right)\right] \tag{5.8}$$

5.4 Time to completion

Time is typically one of the most important variables because of its direct impact on opportunity costs and performance metrics. If T_R is the random variable representing the time to completion of the recovery process, then the probability that the recovery process has not been completed by time t can be written as

$$\mathbf{P}\left(T_R > t\right) = \mathbf{P}\left(Z_{\text{res}} - Y_j \times \mathbf{1}_{\{t_m^s = t\}} + \sum_{i=1}^{n} \Delta Z_i^r + \sum_{j=1}^{m} \Delta Z_j^s < 0\right) \tag{5.9}$$

where $n := \max\{i : t_i^r \leq t\}$ and $m := \max\left\{j : t_j^s \leq t\right\}$. Conditioning on $\{n, m, Z_{\text{res}}\}$ and also removing the zero-probability event $\{t_m^s = t\}$, we can rewrite Eq. (5.9) as

$$\mathbf{P}\left(T_R > t \mid n, m, Z_{\text{res}}\right) = F_{W|Z_{\text{res}}}\left(-z \mid Z_{\text{res}} = z\right)$$

where $F_{W|Z_{\text{res}}}(\cdot)$ is the conditional CDF of the random variable $W := W_n^r + W_m^s$. Assuming W_n^r and W_m^s are statistically independent, we can find their conditional CDF as their convolution.

5.5 Resilience Metrics

Using the results in Sections 5.1-5.4, we can estimate the resilience partial descriptors defined in Chapter 2. Because the recovery process is stochastic, the resilience partial descriptors would be stochastic as well. A predictive estimate of such partial descriptors can be obtained by taking the expected values of the partial descriptors. For example, the expected value of the center of recovery can be written as

$$\mathbf{E}\left[\rho\right] = \mathbf{E}_{\{T_R, Q(t)\}}\left[\frac{\sum_{k=1}^{\infty} t_k \Delta\tilde{Q}\left(t_k\right) \mathbf{1}_{\{0 \leq t \leq T_R\}}}{Q\left(T_R\right)}\right] \tag{5.10}$$

where, the expected value of ρ can be calculated considering the uncertainty in T_R and the possible sample paths of $Q(t)$. It should be noted, that due to the form of the recovery curves (piecewise constant), Eq. (5.10) does not have the continuous part.

Chapter 6

Calibration of Capacity and Demand Models

The unknown model parameters to be estimated include the parameters of the distributions of random variables: $N^r(t)$, $N^s(t)$, ΔC_i^r, ΔC_j^s, ΔD_i^r, ΔD_j^s, and Y_j. We cluster the random variables into two groups: 1) The first group which corresponds to the recovery-related variables includes $\{N^r(t), \Delta C_i^r, \Delta D_i^r\}$. 2) The variables in the second group, which are related to the interrupting shocks, include $\left\{N^s(t), \Delta C_j^s, \Delta D_j^s, Y_j\right\}$.

The unknown parameters of the variables in the first group can be estimated considering past data and engineering judgment. As explained in Section 3.2, corresponding to each value of C_{res} and D_{res}, a set of recovery activities can be identified. Kumar et al. (2015) have developed models which can estimate C_{res} and D_{res}, distributions for such a system considering shock and gradual deterioration. A work breakdown structure can be prepared and contributions of individual activity completion can be calculated. Thus, to estimate ΔC_i^r, and ΔD_i^r, only descriptions and number of activities involved in recovery process are required, which is only dependent on the initial state (C_{res} and D_{res}) and system design. To estimate the rate of re-occurrence of recovery steps, the random activity network (as shown in Fig 6.1) can be simulated corresponding to a given value of system and environmental factors. A regression model can then be employed to capture the dependence of the recovery process on system and environmental characteristics. The regression model is then used to obtain a time dependent rate function which can be used in the recovery process model.

Parameters of the shock process are dependent on the characteristics of the hazard under consideration. For example, in the case of a civil engineering structure under seismic hazard, rate of recurrence of the shocks may correspond to the recurrence rate of the aftershocks experienced post-earthquake. Similarly, ΔC_j^s and ΔD_j^s are dependent on the fragilities of the structure. Thus, these parameters can be estimated using data from previous events and simulations using a finite element model of the structure.

To update all the distributions estimated by the process described above a bayesian updating framework can be used. Gardoni et al. (2007) provided such a framework which uses the approach given by Box and Tiao (1992). Using which the updated or posterior distribution of a set of parameters can be obtained considering a recorded set of observation as

$$p(\Theta \mid W) = \kappa L(\Theta \mid W)\, p(\Theta) \tag{6.1}$$

where, $p(\Theta \mid W)$ is the posterior distribution representing our updated status of knowledge about Θ and, in a sense, tells us what is known about Θ given knowledge from the data, $L(\Theta \mid W)$ is the likelihood function representing the objective information on Θ contained in a set of observations, $p(\Theta)$ is the prior distribution reflecting our state of knowledge about Θ prior to obtaining the observations $W = (W_1, \ldots, W_T)$, and $\kappa = \left[\int L(\Theta \mid W)\, p(\Theta)\, d\Theta\right]^{-1}$ is a normalizing factor.

It should be noted that if the marginal distributions of recovery process parameters are used, the results correspond to the overall resilience of the system. However, during a post disruptive shock scenario, the distributions defining the initial state can be updated by incorporating data obtained from inspections. Also updated conditional distributions of ΔC^r and ΔD^r can be used to get the results for the specific cases. System performance results from these specific cases can be used to compare repair strategies and optimize recovery efforts.

Algebraic operations on random variables is a fairly involved process. While solving for the various recovery quantifiers as explained in Chapter 5, summations of different random variables is encountered. These summations can be computed by obtaining convolutions of corresponding probability distributions if the involved variables are statistically independent. The probability distribution of summation of two statistically independent random variables A and B can be obtained by

$$f_{A+B}(x) = \int_{-\infty}^{\infty} f_A(a)\, f_B(x-a)\, dx = (f_A * f_B)(x) \tag{6.2}$$

Similarly, for the case of finding the n^{th} scalar integer multiple of A, this is written as

$$f_{\sum_{i=1}^{n} A}(x) = (f_{A_1} * \cdots * f_{A_n})(x) = f_A^{(n)}(x) \tag{6.3}$$

So, the distribution $f_{nA}(x)$ can be obtained by calculating a n fold convolution of the distribution of A with itself. Properties of Fourier transformations $\mathcal{F}\{f\}$ can be used to calculate this efficiently. Some of them are

$$\mathcal{F}\{f_A * f_B\} = \mathcal{F}\{f_A\}\,\mathcal{F}\{f_B\}$$

$$f_A * f_B = \mathcal{F}^{-1}\{\mathcal{F}\{f_A\}\,\mathcal{F}\{f_B\}\}$$

Fast Fourier transformation algorithms are conveniently available in common computational packages, which makes the implementation of the proposed model efficient.

6.1 Figures

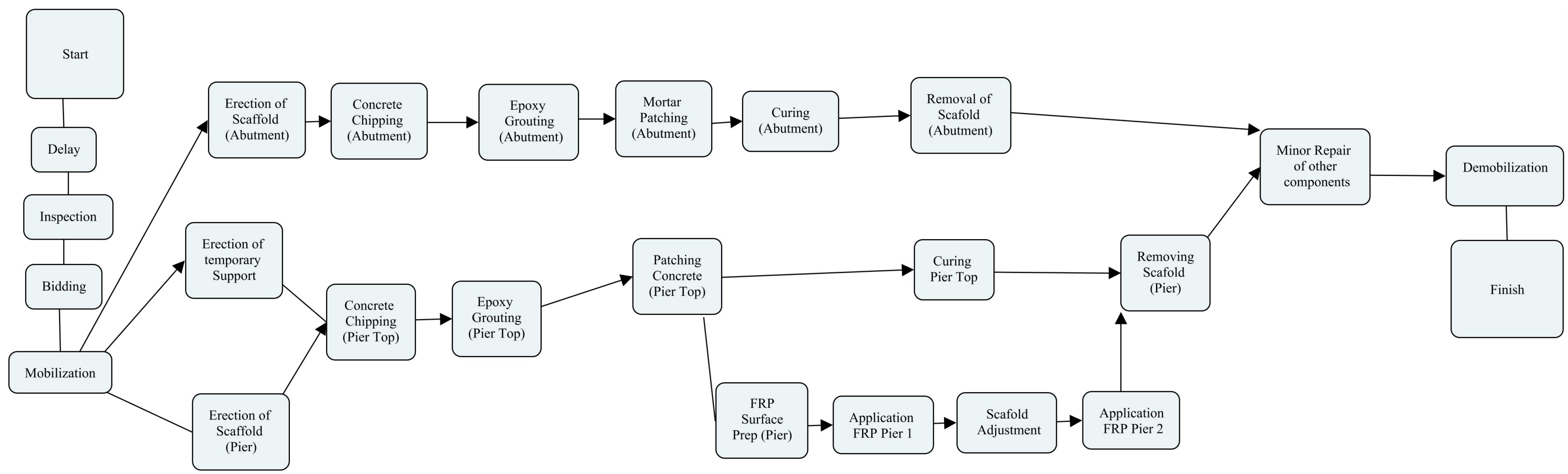

Figure 6.1: Activity network for a sample recovery process (bridge repair project)

6.2 Tables

Table 6.1: Durations and Precedence for activities in a sample recovery process for network in Fig. 6.1

S No.	Activity	Optimistic	Most Likely	Pessimistic	Predecessor
1	Delay	30	35	60	
2	Inspection	2	3	5	1
3	Contracting	15	20	30	2
4	Mobilization	5	7	15	3
5	Erection of Scaffold Abutment	1	2	3	4
6	Erection of temporary Support	1	2	3	4
7	Erection of Scaffold Pier	1	2	3	4
8	Concrete Chipping Abutment	1	2	3	5
9	Epoxy Grouting Abutment	1	2	3	8
10	Mortar Patching Abutment	1	2	3	9
11	Curing Abutment	7	10	15	10
12	Removal of Scaffold Abutment	1	2	3	11
13	Concrete Chipping Pier Top	1	2	3	5,6
14	Epoxy Grouting Pier Top	2	3	3	13
15	Patching Concrete Pier Top	1	2	3	14
16	Curing Pier Top	7	10	15	15
17	FRP Surface Prep Pier	0.5	1	2	16
18	Application FRP Pier 1	0.5	1	2	17
19	Scaffold Adjustment	0.5	1	2	18
20	Application FRP Pier 2	0.5	1	2	19
21	Removal of Scaffold Pier	0.5	1	2	15,20
22	Minor Repair of other components	3	4	5	11,21
23	Demobilization	2	3	4	22

Chapter 7

Illustrative Example

Parameters for various input variables can be estimated using the process described in Chapter 6. For an illustration, we assume the input parameters as listed in Table 7.1. Residual Capacity and demand are assumed to be lognormal because they are assumed to be non negative. Assuming exponential distributions for ΔZ_r and ΔZ_s may simplify the calculation process because n-fold convolutions of i.i.d exponential distribution can be easily calculated to be corresponding gamma distributions. Since exponential distribution has same expressions for mean and standard distribution, it limits the representation of uncertainty in relevant variables. Hence lognormal distribution is used to model step sizes. For simplicity, we use homogeneous poisson processes to model number of recovery steps and interrupting shocks.

We measure system performance in terms of reliability. For the purpose of calculating system resilience we assume $Q_{rel}(t)$ to be equal to system functionality $Q(t)$. Results for the estimation of recovery quantifiers is shown in Figs. 7.1 to 7.3 and Table 7.2.

7.1 Figures

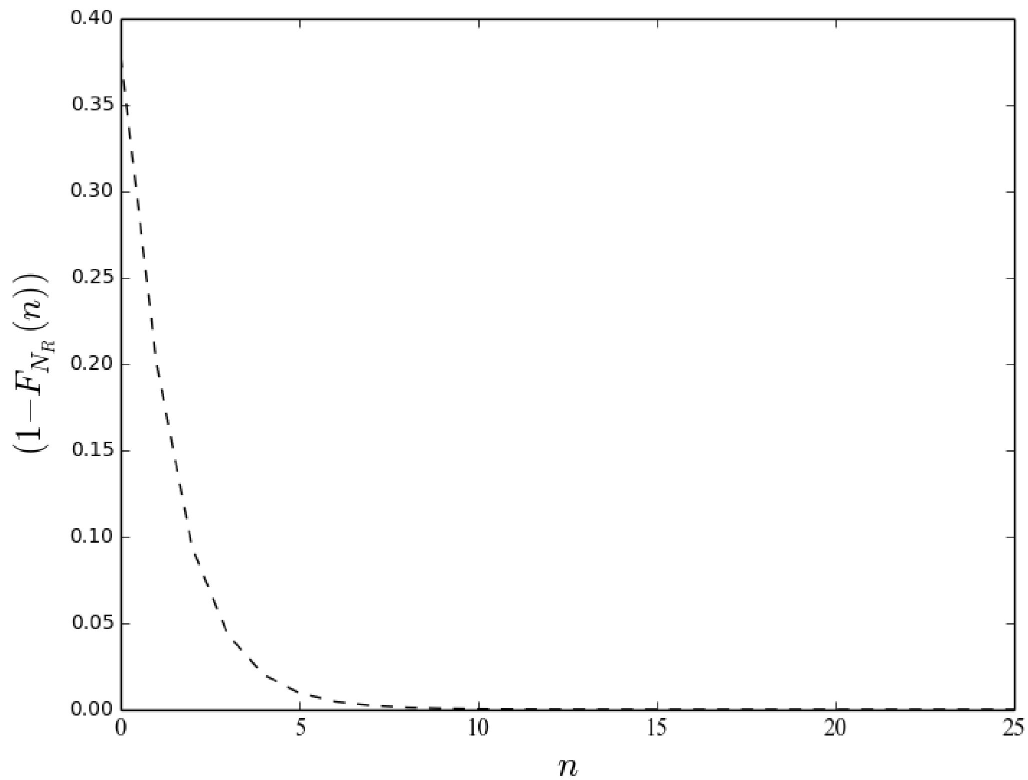

Figure 7.1: Complementary CDF of N_R, $P(N_R > n)$

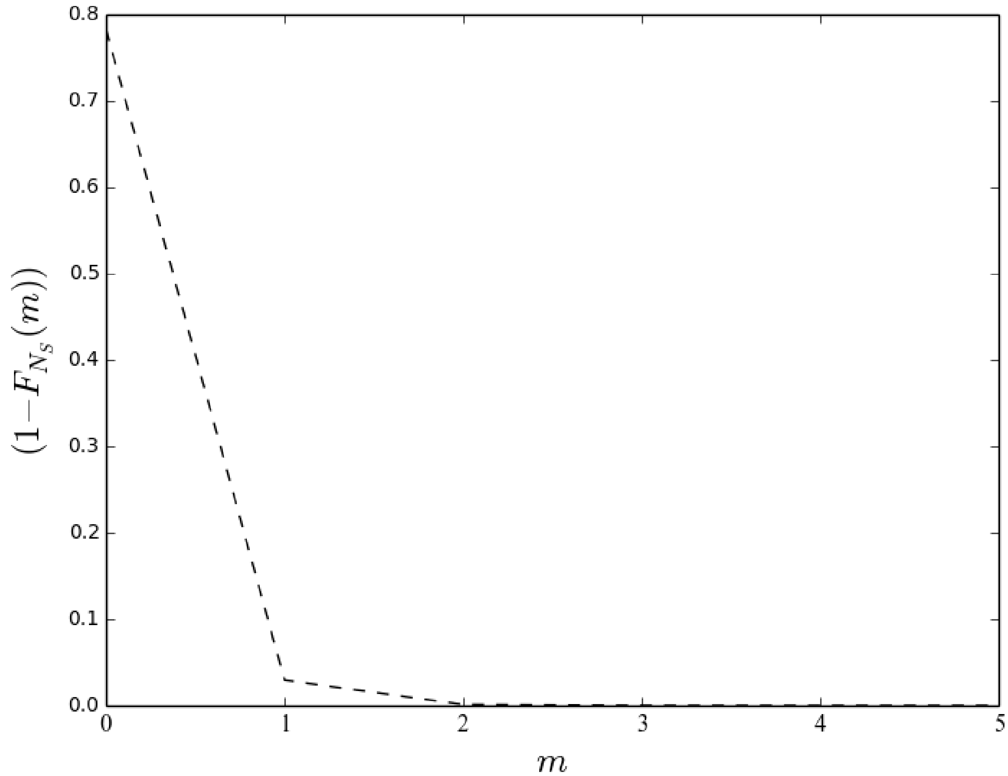

Figure 7.2: Complementary CDF of N_S, $P(N_S > m)$

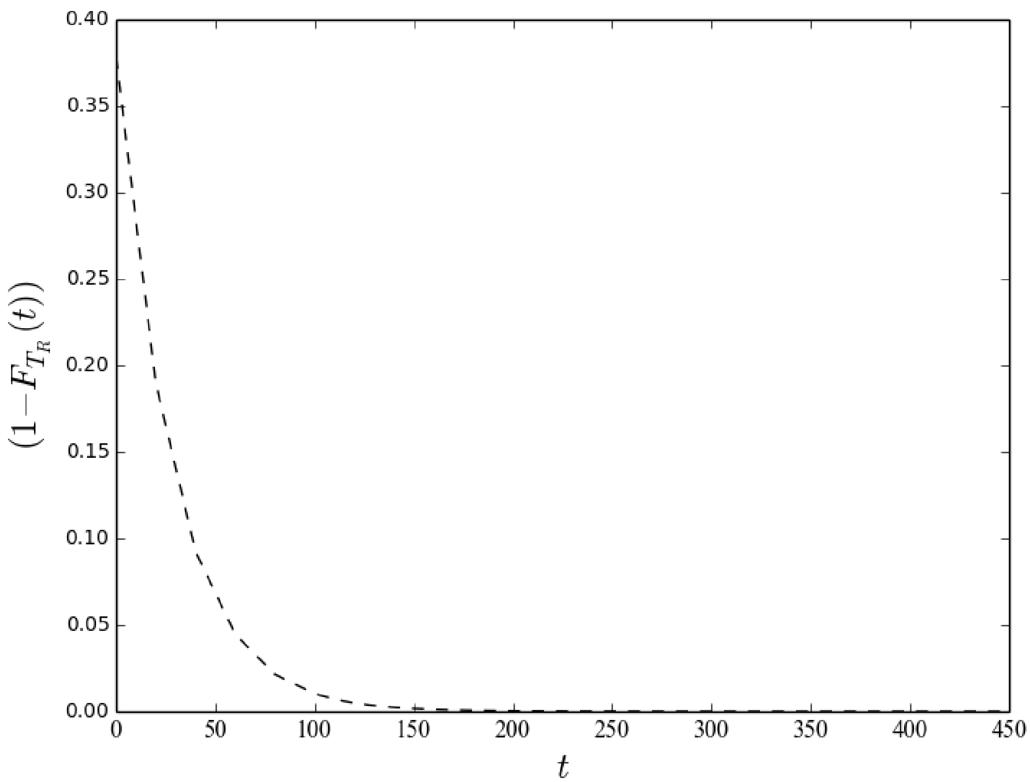

Figure 7.3: Complementary CDF of T_R, $P(T_R > t)$

7.2 Tables

Table 7.1: Assumed distributions for the model illustration

Variable/Function	Description	Distribution	Parameter	
			Symbol	Value
C_{res}	-	Lognormal	λ_C	-3.3769
			ζ_C	0.3246
D_{res}	-	Lognormal	λ_D	-3.5536
			ζ_D	0.4724
ΔZ^r	-	Lognormal	$\lambda^r_{\Delta Z}$	-4.4755
			$\zeta^r_{\Delta Z}$	0.3246
ΔZ^s	-	Lognormal	$\lambda^s_{\Delta Z}$	-4.1290
			$\zeta^s_{\Delta Z}$	0.4724
Y_j	-	Shifted exponential	y_0	0.005
			μ_Y	0.02
$N^r(t)$	Poisson process	-	λ^r	0.5/week
$N^s(t)$	Poisson process	-	λ^s	0.2/year

Table 7.2: Resilience metrics obtained for input parameters in Table 7.1

Resilience metric	Description	Mean	Covarience
ρ	Center of recovery	11.0908	0.023
χ^2	Recovery bandwidth	471.14	0.0449
ψ	Recovery skewness	33975	0.0.816

Chapter 8

Conclusions

This study has introduced a novel approach to quantify resilience of engineering systems. A mathematical formulation for resilience analysis has been proposed wherein the relevance of various partial descriptors to the corresponding characteristics of the recovery process has been explained. Resilience metrics have been improved to provide representative information of recovery characteristics important for decision making.

A stochastic framework to model the recovery process has been created which can both measure and predict resilience of deteriorating systems. Illustration of the model is provided with detailed instructions and suggestions on model parameter estimation and calibration. Future work may aim to make the stochastic recovery model more general by relaxing the assumptions regarding the statistical independence of recovery and shock interruption processes.

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