A MULTI-OBJECTIVE SEQUENTIAL STOCHASTIC ASSIGNMENT PROBLEM FOR EBOLA ENTRY SCREENING

BY

GE YU

THESIS

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Adviser:

Associate Professor Negar Kiyavash

ABSTRACT

The 2014 Ebola outbreak in West Africa prompted a need to assess how deplaning passengers from West Africa should be managed. A 21-day quarantine requirement for deplaning passengers, based on their risk factors, was implemented at five international airports in the United States in late 2014. This thesis formulates the multi-objective sequential stochastic assignment problem (MOSSAP) to improve the process for managing such quarantine assignments. In MOSSAP, each passenger is assessed with a two-dimensional risk vector, revealed upon entering the United States, which is used to make the quarantine assignment. The objective is to maximize the expected number of passengers assigned to the correct level of monitoring (quarantine, self-monitoring), subject to quarantine capacity constraint. The weighted sum method is used to generate Pareto optimal policies for MOSSAP. Statistics available from Ebola entry screening and related public health sources are used to illustrate how such a policy would operate in practice.

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CHAPTER 1

INTRODUCTION AND BACKGROUND

The 2014 Ebola Hemorrhagic Fever (or simply, Ebola) outbreak in West Africa created widespread concern of its possible spread to other countries. As of Dec 15, 2015, there have been 28,638 cases and 11,315 deaths [1]. Besides West African countries, two imported cases and two locally acquired cases have been reported in the United States [1]. These cases prompted the United States Government to take measures and monitor the flow of air traveling out of West Africa to the United States. Together with exit screening at airports in three West African countries (Guinea, Liberia, and Sierra Leone), *enhanced entry screening* was enforced from October 11, 2014, at New York JFK international airport and from October 16, 2014, at another four international airports (Washington-Dulles, Newark Liberty, Chicago-O'Hare, and Atlanta Hartsfield-Jackson) [2]. Under this enhanced screening, all passengers coming from or transferring through these three West African countries are required to be routed to one of these five airports to undergo *risk assessment*, and a possible 21-day quarantine requirement based on their risk factors is implemented [3]. Developing an effective screening assignment policy to prevent the spread of Ebola, as well as the outbreak of other epidemics in the future, is an essential component of contagious disease control.

The enhanced entry screening used by the United States Customs and Border Protection (CBP) checks each passenger's exposure history and symptoms [4] and assesses each passenger's risk with one of four risk levels (High risk, Some risk, Low (but not zero) risk, and No identifiable risk) [5]. Different monitoring levels are required for passengers assessed with different risk levels [6]. Among the monitoring levels, only *quarantine monitoring* (or simply, *quarantine*) requires hospital space and medical intervention while *direct active monitoring* and *active monitoring* do not. Moreover, quarantine costs are large compared to other monitoring levels. Therefore, we limit monitoring assignments for Ebola entry screening (or simply, passenger assignments) to two categories: *selectees*, denoting the category where passengers are quarantined, and *non-selectees*, denoting the category where passengers are not quarantined (direct active monitoring, active monitoring or other self-monitoring may be needed). Thus, this *binary assignment* (selectee or non-selectee) can be viewed as a first-step assignment for enhanced entry screening [6]; a more refined assignment can be made for non-selectees after the initial assignment.

The Ebola entry screening problem and the aviation security screening problem have similarities but differences. Both can be modeled as a sequential stochastic assignment problem with capacity constraints. Research on the aviation security screening problem has resulted in several discrete optimization models, which motivated the model for the Ebola entry screening problem in this work. [7] introduces the sequential stochastic passenger screening problem (SSPSP) and models it as a Markov decision process. [8] formulates a discrete-time model for real-time binary passenger assignments in airport screening. [9] discusses a sequential stochastic multi-level passenger screening problem. [10] introduces probability-based metrics to evaluate the performance of sequential passenger assignments with respect to the retrospective off-line optimal policy. [11] models the passenger screening problem with dynamic risk update as a multi-stage sequential assignment problem. For all these aviation security screening models, each passenger is assessed with a one-dimensional risk value.

Naively adopting the aviation security screening policy for Ebola entry screening is not appropriate. The main issue is the transmission potential of a sick passenger without protection measures. If a sick passenger is not quarantined, then his or her social contacts have the potential of being infected, which may lead to additional people infected. The *basic reproduction number* (defined as the average number of infected persons caused by a sick person without protection measures) of Ebola is around 2 for the three West African countries [12], which indicates the possibility of a nationwide outbreak if left unattended. In addition, although the CDC reports that Ebola has transmission potential only after symptoms appear [13], there is some controversy concerning this observation. For example, [14] claims that Ebola is contagious even when no symptoms are present. Moreover, Ebola symptoms are similar to those of influenza [15, 16], and hence, it is possible that passengers with Ebola may not be recognized at the onset of symptoms. To prevent the spread of Ebola, *contact tracing* may need to be implemented, which incurs additional costs. Contact tracing is defined as the identification and follow-up of persons who may have interacted with a sick person with no protection measures, as recommended for Ebola by the World Health Organization (WHO) [17] and the CDC [18]. For a suspected passenger with a large number of social contacts, if this person is not quarantined and becomes sick with Ebola, then contact tracing costs may exceed the quarantine cost. Therefore, a one-dimensional risk measure is insufficient to represent the transmission potential of a passenger. The trade-off between quarantine costs and passenger assignments efficiency is the fundamental objective for Ebola entry screening studied in this work.

The primary contribution of this work is a mathematical model for multi-objective sequential stochastic assignment problems (MOSSAP), which extends the classic single-objective sequential stochastic assignment problem proposed by [19]. This problem, motivated by the Ebola entry screening problem, requires passengers to be sequentially assigned to either the selectee or the non-selectee category. The model of MOSSAP can be used to improve the process for managing monitoring assignments for Ebola entry screening at airports, as well as to prevent the spread of other epidemics in the future. Pareto optimal policies for MOSSAP are generated by solving a sequence of weighted objective sequential stochastic assignment problems. Mixed policies for sequential stochastic assignment problems are defined formally and discussed in detail. Moreover, values of the multiple objective functions under Pareto optimal policies for MOSSAP are provided with recursive equations. These approaches can be applied directly to solve the general type of stochastic sequential assignment problems with multiple objectives, as long as the objective functions are in the form of accumulating a product-form reward for each assignment (referred to as *product-form MOSSAP*). Simulation results using public health data from Ebola entry screening are presented to illustrate operational implications of MOSSAP.

1.1 Organization

This thesis is organized as follows. Chapter 2 describes the mathematical model for MOSSAP admitting mixed policies. Chapter 3 presents the structural properties of MOSSAP, defines mixed policies for sequential stochastic assignment problems, and simplifies MOSSAP into a sequence of weighted objective sequential assignment problems (WOSA). Chapter 4 models WOSA as a Markov decision process and a sequential stochastic assignment problem, respectively, with optimal policies for WOSA proposed. Chapter 5 provides the values of the objective functions under different Pareto optimal policies for MOSSAP. Chapter 6 presents simulation results using Ebola entry screening data. Chapter 7 outlines operational implications and limitations of MOSSAP. Chapter 8 summarizes the work with directions for future research.

CHAPTER 2

MULTI-OBJECTIVE SEQUENTIAL STOCHASTIC ASSIGNMENT PROBLEM

This chapter describes the mathematical model for MOSSAP, which is motivated by the Ebola entry screening problem at airports in the United States. This model can be applied to the general type of product-form MOSSAP.

Passengers coming from or transferring through West African countries with an Ebola outbreak are required to provide their recent travel and contact histories before entering the United States [3]. This information will be used as input to a *prescreening system* to assess each passenger with a two-dimensional risk vector upon arrival, which consists of a *primary risk* measure, defined as the probability of a passenger being sick, and a *secondary risk* measure, defined as the number of social contacts that need to be covered by contact tracing if a passenger is not quarantined and becomes sick. Bringing in this secondary risk measure here is motivated by the implementation of contact tracing in epidemic prevention and control. Note that the *disease status* (i.e., whether a passenger is sick with Ebola) is not known at the time of assignment. There are two categories for assignments based on passengers' assessed risk vectors: selectee and non-selectee category. Passengers assigned to the selectee category are to be quarantined, while those assigned to the non-selectee category are not (though contact tracing may be needed later). Therefore, only the capacity of the selectee category is limited. A sequential policy that makes assignments upon passenger arrival is defined as the *on-line policy*.

Several assumptions need to be clarified before formulating the MOSSAP model for the Ebola entry screening problem. First, passenger assignments are irrevocable and made on-line. Since passengers may not display symptoms for Ebola until entry into the United States, risk assessment cannot be made until arrival. Moreover, passenger assignments must be made immediately upon arrival at airports. Second, we assume passengers are assigned during a fixed time period. Since the quarantine monitoring lasts for an incubation period (21 days for Ebola), the available quarantine space is reset every 21 days. Under these assumptions, MOSSAP is formulated as a sequential stochastic assignment problem that assigns arriving passengers during an 21-day period. The limitations are discussed in Chapter 7.

2.1 Mathematical Model for MOSSAP

Consider T passengers to be sequentially assigned to one of the two categories during the fixed time period. T is assumed to be known since the aircraft that arrive to an airport each day are typically pre-scheduled and the number of passenger enplanements is known from the check-in and boarding procedure prior to landing. For the t^{th} passenger, denote the primary and secondary risk values by the random variables $\mathcal{A}(t)$ and $\mathcal{B}(t)$. Both the primary and secondary risk measures are assumed independent and identically distributed (IID) with known probability mass functions (pmf), denoted by $p_{\mathcal{A}}(\alpha)$ and $p_{\mathcal{B}}(\beta)$, respectively. The realized primary and secondary risk values of the t^{th} passenger, denoted by (α_t, β_t), become known upon the passenger's arrival. Denote the selectee category capacity by $\eta_o \in \mathbb{Z}^+$, which guarantees quarantine space for up to η_o passengers.

A policy for MOSSAP defines a sequence of passenger assignments. Let the binary random variable $X_t^\Phi \in \{0,1\}$ denote the t^{th} passenger assignment under policy Φ: $X_t^\Phi = 1(0)$ denotes a selectee (non-selectee) assignment. Policy Φ may be *pure* or *mixed*. Pure policies have only one level of randomness, which is generated by the joint distribution of the sequence of passenger risk vectors. However, mixed policies have an additional level of randomness, which is referred to as the randomness of the policy assignments. We will discuss mixed policies in more detail in Chapter 3. The screening system outputs binary assignment variables $\{X_t^\Phi\}_{t=1}^T$ sequentially. If a policy is pure, then passenger assignments are *deterministic* given a sequence of passenger risk vectors.

The objective function components are the expected number of passengers assigned to the two categories and the expected number of social contacts to be covered by contact tracing. These expected numbers show the trade-off between quarantine costs (both the medical direct costs and societal indirect tracing costs) and passenger assignments efficiency. On the one hand, if a healthy passenger is quarantined, the incurred medical costs and indirect costs (such as disruption in personal life) are unnecessary. If a sick passenger is not quarantined, the contact tracing costs incurred by his transmission potential may exceed quarantine costs. On the other hand, passenger assignments will be accurate when healthy passengers are not quarantined but sick ones are. The costs incurred in these cases are of course warranted. Table 2.1 defines the five objective functions for MOSSAP, in the unit of number of passengers/people, which are our chosen performance metrics for evaluating a policy Φ. In Table 2.1, the expectations are taken with respect to the distributions of two passenger risk measures and the randomness of the policy assignments, when applicable, i.e., for mixed policies only.

Next, we formulate MOSSAP as a discrete sequential stochastic optimization problem as follows.

Table 2.1: Five objective functions of MOSSAP.

Objective function	Definition
$W_{ns}(\Phi)$	Expected number of *right-screened healthy passengers* (i.e., healthy passengers assigned to the non-selectee category).
$W_s(\Phi)$	Expected number of *right-screened sick passengers* (i.e., sick passengers assigned to the selectee category).
$V(\Phi)$	Expected number of *over-screened passengers* (i.e., healthy passengers assigned to the selectee category).
$U(\Phi)$	Expected number of *under-screened passengers* (i.e., sick passengers assigned to the non-selectee category).
$U_{st}(\Phi)$	Expected number of social contacts covered by contact tracing (only incurred by under-screened passengers).

Multi-objective Sequential Stochastic Assignment Problem (MOSSAP)

Instance:

T passengers;

$p_{\mathcal{A}}(\alpha)$, the probability mass function for passengers' primary risk value;

$p_{\mathcal{B}}(\beta)$, the probability mass function for passengers' secondary risk value;

$\eta_o \in \mathbb{Z}^+$, the capacity of the selectee category.

Random Variables:

$\mathcal{A}(t)$, the assessed primary risk value for the t^{th} passenger for $t = 1, 2, \ldots, T$, with $\mathcal{A}(t) \in \{A_1, A_2, \ldots, A_M\}$ and $0 < A_1 < A_2 < \ldots < A_M < 1$;

$\mathcal{B}(t)$, the assessed secondary risk value for the t^{th} passenger, with $\mathcal{B}(t) \in$

$\{B_1, B_2, \ldots, B_K\} \subset \mathbb{Z}^+$ and $B_1 < B_2 < \ldots < B_K$.
Assignment Variables:
X_t^Φ, the binary assignment of the t^{th} passenger under policy Φ: if $X_t^\Phi = 1(0)$, then the passenger is assigned to the selectee (non-selectee) category.
Objective: Find an admissible on-line policy Φ (may be a mixed policy) that assigns passengers to one of the two categories $\{X_t^\Phi \in \{0,1\}\}_{t=1}^T$, such that the expected number of passengers assigned to the correct category is maximized, the expected number of passengers assigned to the wrong category is minimized, and the expected number of social contacts covered by contact tracing is minimized. More precisely, the on-line policy Φ optimizes

$$\begin{cases} W_{ns}(\Phi) = \mathbb{E}[\sum_{t=1}^{T}(1 - X_t^\Phi)(1 - \mathcal{A}(t))] \quad (\max_{\Phi\in\Psi}), \\ W_s(\Phi) = \mathbb{E}[\sum_{t=1}^{T} X_t^\Phi \mathcal{A}(t)] \quad (\max_{\Phi\in\Psi}), \\ V(\Phi) = \mathbb{E}[\sum_{t=1}^{T} X_t^\Phi (1 - \mathcal{A}(t))] \quad (\min_{\Phi\in\Psi}), \\ U(\Phi) = \mathbb{E}[\sum_{t=1}^{T}(1 - X_t^\Phi)\mathcal{A}(t)] \quad (\min_{\Phi\in\Psi}), \\ U_{st}(\Phi) = \mathbb{E}[\sum_{t=1}^{T}(1 - X_t^\Phi)\mathcal{A}(t)\mathcal{B}(t)] \quad (\min_{\Phi\in\Psi}), \end{cases} \tag{2.1}$$

where $(\max_{\Phi\in\Psi})$ and $(\min_{\Phi\in\Psi})$ specify whether the optimization is a maximization or a minimization for each objective function. Ψ denotes the set of admissible policies, referred to as *feasible region*. Let $SL(\Phi) \triangleq \sum_{t=1}^{T} X_t^\Phi$ denote the total number of passengers assigned to the selectee category under policy Φ, then

$$\Psi \triangleq \{\Phi : SL(\Phi) \le \eta_o\} = \cup_{\eta=0}^{\eta_o} \Psi^\eta, \tag{2.2}$$

i.e., the feasible region consists of $\eta_o + 1$ mutually exclusive and exhaustive *sub-feasible regions*, defined as $\Psi^\eta \triangleq \{\Phi : SL(\Phi) = \eta\}$, indexed by $\eta = 0, 1, \ldots, \eta_o$.

MOSSAP has multiple objectives that typically do not admit the same optimal policy. Therefore, the feasible region of MOSSAP consists of *Pareto optimal* policies, as defined by Definition 1.

Definition 1. *A policy* $\Phi \in \Psi$ *is said to be Pareto optimal for MOSSAP if there does not exist another policy* $\Phi' \in \Psi$ *such that* $W_{ns}(\Phi) \leq W_{ns}(\Phi')$, $W_s(\Phi) \leq W_s(\Phi')$ *and* $V(\Phi) \geq V(\Phi')$, $U(\Phi) \geq U(\Phi')$, $U_{st}(\Phi) \geq U^s(\Phi')$, *with at least one strict inequality.*

For multi-objective optimization problems, Pareto optimal solutions are typically not unique. In Chapter 3, we provide an approach for generating the set of Pareto optimal policies for MOSSAP.

CHAPTER 3

STRUCTURAL PROPERTIES AND AN APPROACH FOR MOSSAP

This chapter presents the structural properties of MOSSAP, which will be used to derive Pareto optimal policies. First, we show that in each sub-feasible region Ψ^η, the five objective functions in (2.1) are reduced to two objective functions. Then, we prove that the set of Pareto optimal policies for MOSSAP can be obtained from Pareto optimal policies for a sequence of bi-objective sequential stochastic assignment problems (or simply, bi-objective optimization problems). Lastly, weighted sum method is used to generate Pareto optimal policies for each bi-objective optimization problem by solving a sequence of weighted objective sequential stochastic assignment problems.

3.1 MOSSAP and Bi-objective Optimization Problems

Theorem 1 states that the five objective functions of MOSSAP can be simplified in each sub-feasible region.

Theorem 1. *MOSSAP can be reduced to a bi-objective sequential stochastic assignment problem in each sub-feasible region $\Psi^\eta, \eta = 0, 1, \dots, \eta_o$. The two objective functions to be maximized are $R_s(\Phi) \triangleq \mathbb{E}[\sum_{t=1}^T X_t^\Phi \mathcal{A}(t)]$ and $R_d(\Phi) \triangleq \mathbb{E}[\sum_{t=1}^T X_t^\Phi \mathcal{A}(t)\mathcal{B}(t)]$.*

Proof. In each sub-feasible region Φ^η, the capacity constraint is *active* with η (i.e., $SL(\Phi) = \sum_{t=1}^T X_t^\Phi = \eta$), which implies

$$\max_{\Phi\in\Psi^\eta} W_{ns}(\Phi) \Leftrightarrow \min_{\Phi\in\Psi^\eta} V(\Phi) = \max_{\Phi\in\Psi^\eta} \mathbb{E}[\sum_{t=1}^T X_t^\Phi \mathcal{A}(t)] - \eta,$$

$$\max_{\Phi\in\Psi^\eta} W_s(\Phi) \Leftrightarrow \min_{\Phi\in\Psi^\eta} U(\Phi) = \max_{\Phi\in\Psi^\eta} \mathbb{E}[\sum_{t=1}^T X_t^\Phi \mathcal{A}(t)].$$

Rewrite the objective function $U_{st}(\Phi)$ defined by (2.1) as

$$\min_{\Phi\in\Psi^\eta} U_{st}(\Phi) = \min_{\Phi\in\Psi^\eta} \mathbb{E}[\sum_{t=1}^{T} \mathcal{A}(t)\mathcal{B}(t) - \sum_{t=1}^{T} X_t^\Phi \mathcal{A}(t)\mathcal{B}(t)] = \max_{\Phi\in\Psi^\eta} \mathbb{E}[\sum_{t=1}^{T} X_t^\Phi \mathcal{A}(t)\mathcal{B}(t)].$$

Define

$$R_s(\Phi) \triangleq \mathbb{E}[\sum_{t=1}^{T} X_t^\Phi \mathcal{A}(t)], \quad R_d(\Phi) \triangleq \mathbb{E}[\sum_{t=1}^{T} X_t^\Phi \mathcal{A}(t)\mathcal{B}(t)]. \tag{3.1}$$

Thus, the original MOSSAP is reduced to maximizing the two objective functions defined by (3.1) in each sub-feasible region. □

By Theorem 1, dividing the feasible region Ψ into mutually exclusive and exhaustive sub-feasible regions results in $(\eta_o + 1)$ bi-objective optimization problems

$$\max_{\Phi\in\Psi^\eta} R_s(\Phi), \quad \max_{\Phi\in\Psi^\eta} R_d(\Phi), \tag{3.2}$$

for $\eta = 0, 1, 2, \ldots, \eta_o$. Each of these problems is referred to as a *bi-objective optimization problem indexed by* η. Considering that $\max_{\Phi\in\Psi^\eta} R_s(\Phi)$ and $\max_{\Phi\in\Psi^\eta} R_d(\Phi)$ typically do not admit the same optimal policy, we seek *Pareto optimal* policies for each bi-objective optimization problem indexed by η, as defined by Definition 2.

Definition 2. *A policy $\Phi \in \Psi^\eta$ is said to be Pareto optimal for the bi-objective optimization problem indexed by η if there does not exist another policy $\Phi' \in \Psi^\eta$ such that $R_s(\Phi) \leq R_s(\Phi')$, $R_d(\Phi) \leq R_d(\Phi')$ with at least one strict inequality, for $\eta = 0, 1, 2, \ldots, \eta_o$.*

Theorem 2 shows the mapping relation between Pareto optimal policies for MOSSAP and Pareto optimal policies for the sequence of bi-objective optimization problems.

Theorem 2. *The set of Pareto optimal policies for MOSSAP is a subset of the set of Pareto optimal policies for the sequence of bi-objective optimization problems, indexed by $\eta = 0, 1, \ldots, \eta_o$.*

Proof. We prove this by contradiction. Let Φ be a Pareto optimal policy for MOSSAP and suppose that Φ is not Pareto optimal for any bi-objective optimization problem indexed by $\eta = 0, 1, \ldots, \eta_o$. From the exclusive and exhaustive divisions of the feasible region (2.2), there exists some

$\eta \in \{0, 1, \ldots, \eta_o\}$ such that $\Phi \in \Psi^\eta$. Then there exists $\Phi' \in \Psi^\eta$ such that $R_s(\Phi') \geq R_s(\Phi)$, $R_d(\Phi') \geq R_d(\Phi)$ with at least one strict inequality. Using these two inequalities in (2.1) leads to

$$W_{ns}(\Phi') \geq W_{ns}(\Phi),\ W_s(\Phi') \geq W_s(\Phi),$$
$$V(\Phi') \leq V(\Phi),\ U(\Phi') \leq U(\Phi),\ U_{st}(\Phi') \leq U_{st}(\Phi),$$

with at least one strict inequality, which is a contradiction. Therefore, Φ is Pareto optimal for the bi-objective optimization problem indexed by η. □

For the reverse direction, not every Pareto optimal policy for the sequence of bi-objective optimization problems is Pareto optimal for MOSSAP. However, if certain conditions are satisfied, a bijection between Pareto optimal policies for MOSSAP and Pareto optimal policies for the sequence of bi-objective optimization problems can be established. We provide such a condition (not necessarily tight) in Propositions 1 and 2. Proposition 1 shows that each bi-objective optimization problem generates at least one Pareto optimal policy for MOSSAP, and hence, the division of feasible region Ψ into $\eta_o + 1$ sub-feasible regions is not redundant. Moreover, Proposition 1 is a sufficient condition for determining whether a Pareto optimal policy for some bi-objective optimization problem is Pareto optimal for MOSSAP.

Proposition 1. *If a Pareto optimal policy $\Phi^*_\eta \in \Psi^\eta$ for the bi-objective optimization problem indexed by η satisfies $R_s(\Phi^*_\eta) = \max_{\Phi \in \Psi^\eta} R_s(\Phi)$ for any $\eta = 0, 1, \ldots, \eta_o$, then Φ^*_η is a Pareto optimal policy for MOSSAP.*

Proof. See Appendix A.1. □

Proposition 2 provides a sufficient condition under which all Pareto optimal policies for the sequence of bi-objective optimization problems satisfy the condition given in Proposition 1. Therefore, a bijection between these two sets of Pareto optimal policies can be established under this condition, and hence, MOSSAP may be transformed into a sequence of bi-objective optimization problems.

Proposition 2. *If realized values of $\mathcal{A}(t)$ and $\mathcal{B}(t)$ (i.e., $\{A_m\}_{m=1}^M$ and $\{B_k\}_{k=1}^K$) satisfy the following condition:*

$$A_m B_K \leq A_{m+1} B_1, \quad \text{for all } m = 1, 2, \ldots, M-1, \tag{3.3}$$

then every Pareto optimal policy for the bi-objective optimization problem indexed by η achieves $\max_{\Phi \in \Psi^\eta} R_s(\Phi)$, for all $\eta = 0, 1, \ldots, \eta_o$.

Proof. See Appendix A.2. □

Note that condition (3.3) is not a necessary condition. Theorem 3 formally states the bijection between Pareto optimal policies for MOSSAP and Pareto optimal policies for the sequence of bi-objective optimization problems under condition (3.3). Since bi-objective optimization problems are easier to solve and analyze compared to multi-objective optimization problems, we propose to generate the set of Pareto optimal policies for MOSSAP by solving the sequence of bi-objective optimization problems in (3.2).

Theorem 3. *If condition (3.3) is satisfied, then every Pareto optimal policy for the sequence of bi-objective optimization problems indexed by $\eta = 0, 1, \ldots, \eta_o$, is a Pareto optimal policy for MOSSAP.*

Proof. The result is immediate from Propositions 1 and 2. □

3.2 Bi-objective Optimization Problems and WOSA

We use the weighted sum method to generate Pareto optimal policies for each bi-objective optimization problem. The weighted sum method scales the two objective functions by a non-negative weight vector and sums them up into a weighted objective function.

Let $\mathbf{w} = (w_1, w_2)$ denote the *non-negative weight vector* for $R_s(\Phi)$ and $R_d(\Phi)$ of the bi-objective optimization problem indexed by η, with $w_1 \geq 0, w_2 \geq 0, w_1 + w_2 > 0$ (with abuse of notation, denote this by $\mathbf{w} \geq 0$). Using the weighted sum method, $R_s(\Phi)$ and $R_d(\Phi)$ are combined into a single weighted objective function. The maximization of this objective function over the sub-feasible region Ψ^η is referred to as the *weighted objective sequential assignment problem (WOSA) indexed by η and* $\mathbf{w}$:

$$\max_{\Phi \in \Psi^\eta} R_w(\Phi) = \max_{\Phi \in \Psi^\eta} (w_1 R_s(\Phi) + w_2 R_d(\Phi)) = \max_{\Phi \in \Psi^\eta} \mathbb{E}[\sum_{t=1}^{T} X_t^\Phi (w_1 \mathcal{A}(t) + w_2 \mathcal{A}(t)\mathcal{B}(t))], \tag{3.4}$$

for $\eta = 0, 1, \ldots, \eta_o$ and $\mathbf{w} \geq 0$. If one of the weight vector components is zero, then $R_w(\Phi)$ reduces to $R_s(\Phi)$ ($w_2 = 0$) or $R_d(\Phi)$ ($w_1 = 0$). The sequence of WOSA indexed by η and all $\mathbf{w} \geq 0$ is referred to as WOSA-η. A policy $\Phi \in \Psi^\eta$ that maximizes the objective function (3.4) for some $\mathbf{w} \geq 0$ is said to be *optimal* for WOSA-η.

Note that the weighted sum method in general does not guarantee a bijection between Pareto optimal policies for the bi-objective optimization problem indexed by η and optimal policies for WOSA-η. If the two objective functions are both affine and the feasible region is convex, then such a bijection holds [20]. Since neither redundant nor omitted policies are desired, if we are to benefit from the single objective optimization in WOSA-η, it is essential to establish a bijection. In the following, we use a straightforward *pruning* method to exclude redundant policies from the set of optimal policies for WOSA-η. Convexity of mixed policies in each sub-feasible region Ψ^η and affinity of $R_s(\Phi)$ and $R_d(\Phi)$ are provided to guarantee that no policies are omitted using the weighted sum method.

Definition 3 limits the set of optimal policies for WOSA-η to a subset, referred to as $\mathbb{M}$-*optimal* policies for WOSA-η.

Definition 3. *A policy $\Phi \in \Psi^\eta$ is said to be $\mathbb{M}$-optimal for WOSA-η if one of the following mutually exclusive and exhaustive conditions is satisfied, for $\eta = 0, 1, \ldots, \eta_o$:*

(a) policy Φ maximizes the objective function (3.4) for some $\mathbf{w} > 0$;

(b) policy Φ maximizes the objective function (3.4) for $\mathbf{w} = (w_1, 0), w_1 > 0$, and

$$R_d(\Phi) = \max_{\Phi' \in \Lambda_s^\eta} R_d(\Phi'), \ \textit{with } \Lambda_s^\eta \triangleq \{\Phi' \in \Psi^\eta : R_s(\Phi') = R_s(\Phi)\};$$

(c) policy Φ maximizes the objective function (3.4) for $\mathbf{w} = (0, w_2), w_2 > 0$, and

$$R_s(\Phi) = \max_{\Phi' \in \Lambda_d^\eta} R_s(\Phi'), \ \textit{with } \Lambda_d^\eta \triangleq \{\Phi' \in \Psi^\eta : R_d(\Phi') = R_d(\Phi)\}.$$

Moreover, a policy $\Phi \in \Psi$ is said to be $\mathbb{M}$-optimal for WOSA if there exists some $\eta \in \{0, 1, \ldots, \eta_o\}$ such that Φ is $\mathbb{M}$-optimal for WOSA-η.

3.2.1 Mixed Policies for Stochastic Sequential Assignment Problems

Since the formulation of MOSSAP defined in (2.1) is discrete and policy Φ consists of a sequence of binary passenger assignments, the convexity of the set of pure policies is not straightforward. Instead, we consider the set of mixed policies in each sub-feasible region, which will be shown to be a convex set (pure policies are considered as a special kind of mixed policies, and hence, included in the set of mixed policies). We will also show that $R_s(\Phi)$ and $R_d(\Phi)$ defined in (3.1) of the bi-objective optimization problem are both affine functions of Φ. Therefore, a bijection between Pareto optimal policies for the bi-objective optimization problem indexed by η and $\mathbb{M}$-optimal policies for WOSA-η can be established.

First, we extend the sub-feasible region Ψ^η of MOSSAP to $\Psi^{\eta+} \triangleq \{\Phi : \mathbb{E}[\sum_{t=1}^T X_t^\Phi] = \eta\}$ for $\eta = 0, 1, \ldots, \eta_o$, where the expectation is taken with respect to the randomness of the policy assignments X_t^Φ (i.e., $\mathbb{E}[\sum_{t=1}^T X_t^\Phi] = \eta$ holds for any sequence of passenger risk vectors). Note that for $\Phi \in \Psi^\eta$, $\sum_{t=1}^T X_t^\Phi = \eta$, and hence, $\Psi^\eta \subset \Psi^{\eta+}$. Moreover, if Φ is a pure policy, then $\mathbb{E}[\sum_{t=1}^T X_t^\Phi] = \eta$ and $\sum_{t=1}^T X_t^\Phi = \eta$ are equivalent, which means $\{\Phi_p : \Phi_p \text{ is pure and } \Phi_p \in \Psi^\eta\} = \{\Phi_p : \Phi_p \text{ is pure and } \Phi_p \in \Psi^{\eta+}\}$. In Chapter 4, we will show that optimal policies for WOSA are all pure policies, and hence, maximizing $R_s(\Phi)$ and $R_d(\Phi)$ over Ψ^η and $\Psi^{\eta+}$ are equivalent. Therefore, the extension of $\Psi^{\eta+}$ will not result in redundant policies. We only use $\Psi^{\eta+}$ given its convexity.

Next, we do a small example as a warm-up for the formal definition of mixed policies.

Example 1. *Suppose there are $T = 6$ passengers to be assigned with an active selectee capacity of $\eta = 2$. Consider a particular sequence of realized passenger risk vectors $\{(\mathcal{A}(t), \mathcal{B}(t))\}_{t=1}^6 = \{(\alpha_t, \beta_t)\}_{t=1}^6$. A mixed policy $\Phi \in \Psi^{2+}$ is determined by a sequence of probabilities, for example, $\boldsymbol{\chi}^\Phi = \{\chi_t^\Phi\}_{t=1}^6 = \{0, 0.7, 0, 0.5, 0.5, 0.3\}$ for the particular sequence of $\{(\alpha_t, \beta_t)\}_{t=1}^6$. Each component of $\boldsymbol{\chi}^\Phi$ is the probability of $X_t^\Phi = 1$. In this example, for instance, $\mathbb{P}\left(X_2^\Phi = 1 | \{(\mathcal{A}(t), \mathcal{B}(t))\}_{t=1}^2 = \{(\alpha_t, \beta_t)\}_{t=1}^2\right) = \chi_2 = 0.7$ (at time $t = 2$, only risk vectors of the first two passengers have been revealed). Since $\mathbb{E}[\sum_{t=1}^6 X_t^\Phi | \{(\mathcal{A}(t), \mathcal{B}(t))\}_{t=1}^6 = \{(\alpha_t, \beta_t)\}_{t=1}^6] = \sum_{t=1}^6 \mathbb{P}(X_t^\Phi = 1 | \{(\mathcal{A}(t'), \mathcal{B}(t'))\}_{t'=1}^t = \{(\alpha_{t'}, \beta_{t'})\}_{t'=1}^t) = 0.7 + 0.5 + 0.5 + 0.3 = 2$, then $\Phi \in \Psi^{2+}$.*

In the following, we will define mixed policies in $\Psi^{\eta+}$ using $\boldsymbol{\chi}^{\Phi}$. The *profile* of a mixed policy Φ is defined by a sequence of T random variables $\mathcal{X}_t^{\Phi} \in [0,1]$, and is denoted by $\mathcal{P}^{\Phi} = \{\mathcal{X}_t^{\Phi}\}_{t=1}^{T}$. Since the passenger assignment is sequential, at each time t, only the realized risk vectors of passengers that have arrived up to time t are known. Therefore,

$$\mathcal{X}_t^{\Phi} \triangleq \mathbb{P}(X_t^{\Phi} = 1 | \{(\mathcal{A}(t'), \mathcal{B}(t'))\}_{t'=1}^{t}), \text{ for } t = 1, 2, \ldots, T. \tag{3.5}$$

That is, $\mathcal{X}_t^{\Phi}$ is the probability of the t^{th} passenger to be assigned to the selectee category under policy Φ, given the sequence of passenger risk vectors till time t. The realized value of $\mathcal{P}^{\Phi}$ is a sequence of T probabilities, denoted by $\boldsymbol{\chi}^{\Phi} = \{\chi_t^{\Phi}\}_{t=1}^{T}$, where

$$\chi_t^{\Phi} \triangleq \mathbb{P}(X_t^{\Phi} = 1 | \{(\mathcal{A}(t'), \mathcal{B}(t'))\}_{t'=1}^{t} = \{(\alpha_{t'}, \beta_{t'})\}_{t'=1}^{t}), \text{ for } t = 1, 2, \ldots, T. \tag{3.6}$$

Clearly, $\chi_t^{\Phi} \in [0,1]$. The profile $\mathcal{P}^{\Phi} = \{\mathcal{X}_t^{\Phi}\}_{t=1}^{T}$ is dependent on the sequence of passenger risk vectors $\{(\mathcal{A}(t), \mathcal{B}(t))\}_{t=1}^{T}$. The realizations of $\{\mathcal{X}_t^{\Phi}\}_{t=1}^{T}$ for all sequences of realized passenger risk vectors determine a mixed policy Φ.

There are two levels of randomness in a mixed policy Φ, which are the randomness of the sequence of passenger risk vectors generated by the joint distribution of $\{(\mathcal{A}(t), \mathcal{B}(t))\}_{t=1}^{T}$ and the randomness of the policy assignments generated by $\boldsymbol{\chi}^{\Phi}$. Consider a sequence of realized passenger risk vectors $\{(\mathcal{A}(t), \mathcal{B}(t))\}_{t=1}^{T} = \{(\alpha_t, \beta_t)\}_{t=1}^{T}$. Let the corresponding realization of $\mathcal{P}^{\Phi}$ be $\boldsymbol{\chi}^{\Phi} = \{\chi_t^{\Phi}\}_{t=1}^{T}$. Then, there exists a corresponding pure policy Φ_p with deterministic assignments $X_t^{\Phi_p} = \mathbf{1}_{\chi_t^{\Phi} > 0}$ for $t = 1, 2, \ldots, T$, where $\mathbf{1}_{\chi_t^{\Phi} > 0}$ is the indicator function. Then, conditioning on $\{(\mathcal{A}(t), \mathcal{B}(t))\}_{t=1}^{T} = \{(\alpha_t, \beta_t)\}_{t=1}^{T}$,

$$\mathbb{P}\left(X_t^{\Phi} = 1\right) = \begin{cases} \chi_t^{\Phi} & \text{if } X_t^{\Phi_p} = 1, \\ 0, & \text{if } X_t^{\Phi_p} = 0, \end{cases} \tag{3.7}$$

and hence, the non-zero χ_t^{Φ} is the conditional probability of the $X_t^{\Phi} = 1$ given that $X_t^{\Phi_p} = 1$ (if $X_t^{\Phi} = 0$, then $X_t^{\Phi_p} = 0$ and the conditional probability is not well-defined) for $\{(\mathcal{A}(t), \mathcal{B}(t))\}_{t=1}^{T} = \{(\alpha_t, \beta_t)\}_{t=1}^{T}$. Therefore, $\boldsymbol{\chi}$ captures the randomness of the policy assignments. If $\chi_t^{\Phi} \in \{0, 1\}$, then the assignment X_t^{Φ} becomes deterministic. If $\chi_t^{\Phi} \in \{0, 1\}$ for all $t = 1, 2, \ldots, T$ and all $\{(\alpha_t, \beta_t)\}_{t=1}^{T}$, then policy Φ becomes pure.

Next, we characterize the set of all admissible mixed policies in $\Psi^{\eta+}$ by giving constraints for $\mathcal{P}^{\Phi}$. For $\boldsymbol{\chi}^{\Phi}$, since $\Phi \in \Psi^{\eta+}$, then $\mathbb{E}[\sum_{t=1}^{T} X_t^{\Phi}] = \eta$. Therefore, from (3.6),

$$\begin{aligned}\sum_{t=1}^{T} \chi_t^{\Phi} =& \sum_{t=1}^{T} \mathbb{P}(X_t^{\Phi} = 1 | \{(\mathcal{A}(t'), \mathcal{B}(t'))\}_{t'=1}^{t} = \{(\alpha_{t'}, \beta_{t'})\}_{t'=1}^{t}) \\ =& \mathbb{E}[\sum_{t=1}^{T} X_t^{\Phi} | \{(\mathcal{A}(t), \mathcal{B}(t))\}_{t=1}^{T} = \{(\alpha_t, \beta_t)\}_{t=1}^{T}] = \eta,\end{aligned}$$

which holds for any sequence of realized passenger risk vectors $\{(\alpha_t, \beta_t)\}_{t=1}^{T}$. Therefore, profiles for all admissible mixed policies in $\Psi^{\eta+}$ form a convex set defined by

$$\Xi^{\eta} \triangleq \Big\{ \mathcal{P}^{\Phi} = \{\mathcal{X}_t^{\Phi}\}_{t=1}^{T} : \sum_{t=1}^{T} \mathcal{X}_t^{\Phi} = \eta, \text{ for } \mathcal{X}_t^{\Phi} \in [0, 1] \Big\}. \tag{3.8}$$

Let $\Phi_1 \in \Psi^{\eta+}$ and $\Phi_2 \in \Psi^{\eta+}$ denote two mixed policies. Then for any $0 < \lambda < 1$, $\Phi_\lambda = \lambda \Phi_1 + (1 - \lambda)\Phi_2$ is defined by the profile $\mathcal{P}^{\Phi_\lambda} = \{\mathcal{X}_t^{\Phi_\lambda}\}_{t=1}^{T}$,

$$\mathcal{P}^{\Phi_\lambda} \triangleq \lambda \mathcal{P}^{\Phi_1} + (1 - \lambda)\mathcal{P}^{\Phi_2}, \tag{3.9}$$

where $\mathcal{P}^{\Phi_1}$ and $\mathcal{P}^{\Phi_2}$ denote the profiles for Φ_1 and Φ_2, respectively. It is clear to see that $\mathcal{P}^{\Phi_\lambda} \in \Xi^{\eta}$ and $\Phi_\lambda \in \Psi^{\eta+}$. Therefore, $\Psi^{\eta+}$ is convex for all $\eta = 0, 1, \ldots, \eta_o$.

We need Proposition 3 for establishing the bijection between Pareto optimal policies for the bi-objective optimization problem indexed by η and $\mathbb{M}$-optimal policies for WOSA-η.

Proposition 3. *The objective functions $R_s(\Phi)$ and $R_d(\Phi)$ of the bi-objective optimization problem are both affine functions of $\Phi \in \Psi^{\eta+}$, for $\eta = 0, 1, \ldots, T$.*

Proof. See Appendix A.3. □

Theorem 4 formally states the bijection between Pareto optimal policies for the bi-objective optimization problem indexed by η and $\mathbb{M}$-optimal policies for WOSA-η.

Theorem 4. *There is a bijection between the set of Pareto optimal policies for the bi-objective optimization problem indexed by η and the set of $\mathbb{M}$-optimal policies for WOSA-η.*

Proof. See Appendix A.4. □

3.3 An Approach for MOSSAP

Given these two bijections in Theorems 3 and 4, MOSSAP may be solved as follows. First, MOSSAP is broken down into a sequence of bi-objective optimization problems, indexed by $\eta = 0, 1, \ldots, \eta_o$. Then, each bi-objective optimization problem indexed by η is solved by generating $\mathbb{M}$-optimal policies for WOSA-η. Theorem 5 guarantees this method will return Pareto optimal policies for MOSSAP by establishing a bijection between such policies and the set of $\mathbb{M}$-optimal policies for WOSA under condition (3.3).

Theorem 5. *If condition (3.3) is satisfied, then there is a bijection between the set of Pareto optimal policies for MOSSAP and the set of* $\mathbb{M}$*-optimal policies for WOSA.*

Proof. The result is immediate from Theorems 3 and 4. □

If condition (3.3) is not satisfied, $\mathbb{M}$-optimal policies for WOSA include the Pareto optimal policies for MOSSAP, as well as some redundant policies. In this case, an extra pruning step is needed to extract the Pareto optimal policies for MOSSAP from the set of $\mathbb{M}$-optimal policies for WOSA. The discussion of such a pruning algorithm is not covered in this thesis and is left for future work. Instead, we provide an optimal policy for WOSA indexed by η and $\mathbf{w}$. Additionally, values of the objective functions of MOSSAP under such policies are provided in Chapter 5. These results can be used by *brute-force* pruning. For brute-force pruning, values of multiple objective functions under all possible policies are enumerated for comparison.

CHAPTER 4

OPTIMAL POLICIES FOR WOSA

This chapter provides optimal policies for WOSA indexed by η and $\mathbf{w}$ by formulating WOSA as a Markov decision process (MDP) and a sequential stochastic assignment problem (SSAP), respectively. We prove that optimal policies for WOSA in $\Psi^{\eta+}$ are pure policies when T is fixed and finite. Therefore, they lie in Ψ^{η} for $\eta = 0, 1, \ldots, T$, when T is fixed and finite. By Definition 3, if the weight vector $\mathbf{w}$ is positive, then the optimal policy for WOSA indexed by η and $\mathbf{w}$ is $\mathbb{M}$-optimal for WOSA. If one of the weight vector components is zero, an extra pruning step is needed to select $\mathbb{M}$-optimal policies for WOSA.

4.1 Markov Decision Process Model for WOSA

WOSA indexed by η and $\mathbf{w}$ can be formulated as a $T + 1$ *stage* MDP, for which the *state* at each stage, denoted by the random variable $\mathcal{S}(t)$, is defined as the remaining selectee capacity before the t^{th} passenger assignment for $t = 1, 2, \ldots, T + 1$. By this definition, the realized state at each stage is $s(t) \in S = \{0, 1, 2, \ldots, \eta\}$, with $s(1) = \eta$. Stage $T + 1$ is the final stage after all passenger assignments.

For a mixed policy $\Phi \in \Psi^{\eta+}$ with profile $\mathcal{P}^{\Phi}$, passenger assignments are determined by $\mathcal{P}^{\Phi} = \{\mathcal{X}_t^{\Phi}\}_{t=1}^{T}$, which depends on the sequence of passenger risk vectors $\{(\mathcal{A}(t), \mathcal{B}(t))\}_{t=1}^{T}$. In the MDP formulation, $\mathcal{X}^{\Phi}$ only depends on the state at stage t and the risk vector of the t^{th} passenger. That is, $\mathcal{X}_t^{\Phi} = \mathbb{P}\left(X_t^{\Phi} = 1 | \mathcal{S}(t), (\mathcal{A}(t), \mathcal{B}(t))\right)$. Moreover, the *transition probability* of the Markov chain is given by

$$\mathbb{P}\left(\mathcal{S}(t+1)|\mathcal{S}(t)\right) = \begin{cases} \mathcal{X}_t^{\Phi}, & \text{if } \mathcal{S}(t+1) = \mathcal{S}(t) - 1, \\ 1 - \mathcal{X}_t^{\Phi}, & \text{if } \mathcal{S}(t+1) = \mathcal{S}(t), \\ 0, & \text{otherwise,} \end{cases}$$

for $t = 1, 2, \ldots, T$. Therefore, state transitions only occur if $X_t^{\Phi} = 1$, which happens with probability $\mathcal{X}^{\Phi}$, otherwise, states remain the same.

The objective function $R_w(\Phi)$ in (3.4) accumulates a *reward* at each stage over a finite horizon T. Define the *assignment reward* as

$$\kappa_t(\mathcal{S}(t), (\mathcal{A}(t), \mathcal{B}(t)), X_t^{\Phi}) \triangleq X_t^{\Phi}(w_1\mathcal{A}(t) + w_2\mathcal{A}(t)\mathcal{B}(t))\mathbf{1}_{\mathcal{S}(t)>0}, \quad t = 1, 2, \ldots, T, \tag{4.1}$$

where $\mathbf{1}_{\mathcal{S}(t)>0}$ is the indicator function. Thus, if a passenger is assigned to the non-selectee category or there is no selectee capacity, the reward is zero.

Define the *value function* $\Pi_t(s(t))$ at stage $t = 1, 2, \ldots, T$ as the optimal expected reward given the realized state $\mathcal{S}(t) = s(t)$, which is the maximum of the immediate expected reward for assignment X_t^{Φ} plus the optimal expected reward for assignments of the remaining $T - t$ passengers given by the recursive value functions,

$$\begin{aligned} \Pi_t(s(t)) &= \max_{\Phi \in \Psi^{\eta}} \mathbb{E}\left[\sum_{j=t}^{T} \kappa_j(\mathcal{S}(j), (\mathcal{A}(j), \mathcal{B}(j)), X_j^{\Phi}) | \mathcal{S}(t) = s(t)\right] \\ &= \max_{\Phi \in \Psi^{\eta}} \mathbb{E}\left[\kappa_t(s(t), (\mathcal{A}(t), \mathcal{B}(t)), X_t^{\Phi}) + \sum_{j=t+1}^{T} \kappa_j(\mathcal{S}(j), (\mathcal{A}(j), \mathcal{B}(j)), X_j^{\Phi}) | \mathcal{S}(t) = s(t)\right] \\ &= \max_{\mathcal{X}_t^{\Phi} \in [0,1]} \mathbb{E}\left[X_t^{\Phi}(w_1\mathcal{A}(t) + w_2\mathcal{A}(t)\mathcal{B}(t)) + \Pi_{t+1}(s(t) - X_t^{\Phi})\right]. \end{aligned} \tag{4.2}$$

Consider the t^{th} passenger with a realized risk vector as $(\mathcal{A}(t), \mathcal{B}(t)) = (\alpha_t, \beta_t)$ and denote the realized value of $\mathcal{X}_t^{\Phi}$ by χ_t^{Φ}. Then from (4.2),

$$\begin{aligned} \Pi_t\left(s(t)|(\mathcal{A}(t), \mathcal{B}(t)) = (\alpha_t, \beta_t)\right) &= \max_{\chi_t^{\Phi} \in [0,1]} \mathbb{E}\left[X_t^{\Phi}(w_1\alpha_t + w_2\alpha_t\beta_t) + \Pi_{t+1}(s(t) - X_t^{\Phi})\right] \\ &= \max_{\chi_t^{\Phi} \in [0,1]} \left(\chi_t^{\Phi}\left(w_1\alpha_t + w_2\alpha_t\beta_t + \Pi_{t+1}(s(t) - 1)\right) + (1 - \chi_t^{\Phi})\Pi_{t+1}(s(t))\right) \\ &= \max_{\chi_t^{\Phi} \in [0,1]} \left(\chi_t^{\Phi}\left(w_1\alpha_t + w_2\alpha_t\beta_t + \Pi_{t+1}(s(t) - 1) - \Pi_{t+1}(s(t))\right) + \Pi_{t+1}(s(t))\right). \end{aligned} \tag{4.3}$$

Note that if $\chi_t^\Phi = 0$ or 1, then $X_t^\Phi = 0$ or 1 is a deterministic passenger assignment. Value functions at each stage are computed using dynamic programming, with the optimal value for $R_w(\Phi)$ given by $\Pi_1(\eta)$, and boundary conditions given by

$$\begin{aligned} \Pi_{T+1}(s(T+1)) &= 0, \qquad \text{for all } s(T+1) \in S, \\ \Pi_t(0) &= 0, \qquad \text{for all } t = 1, 2, \dots, T+1. \end{aligned}$$

The value functions are zero at stage $T+1$ and realized state $s(t) = 0$. Taking the derivative with respect to χ_t^Φ on the right-hand side of (4.3) leads to the optimal value of χ_t^Φ at each stage

$$\chi_t^\Phi = \begin{cases} 1, & \text{if } w_1\alpha_t + w_2\alpha_t\beta_t > \Pi_{t+1}(s(t)) - \Pi_{t+1}(s(t)-1), \\ 0, & \text{if } w_1\alpha_t + w_2\alpha_t\beta_t \le \Pi_{t+1}(s(t)) - \Pi_{t+1}(s(t)-1), \end{cases} \tag{4.4}$$

which indicates the optimal mixed policy is pure. Therefore, the optimal passenger assignment at each stage is deterministic and binary, and hence, the optimal policy is in the sub-feasible region Ψ^η. From (3.6), the optimal passenger assignment is given by

$$X_t^\Phi = \begin{cases} 1, & \text{if } w_1\alpha_t + w_2\alpha_t\beta_t > \Pi_{t+1}(s(t)) - \Pi_{t+1}(s(t)-1) \text{ and } s(t) > 0, \\ 0, & \text{if } w_1\alpha_t + w_2\alpha_t\beta_t \le \Pi_{t+1}(s(t)) - \Pi_{t+1}(s(t)-1) \text{ or } s(t) = 0. \end{cases} \tag{4.5}$$

Denote the *combined risk* value for the t^{th} passenger by the random variable $\mathcal{G}(t)$, defined as

$$\mathcal{G}(t) \triangleq w_1\mathcal{A}(t) + w_2\mathcal{A}(t)\mathcal{B}(t). \tag{4.6}$$

Then, the realized combined risk value of the t^{th} passenger is given by $\gamma_t \triangleq w_1\alpha_t + w_2\alpha_t\beta_t$. Equation (4.5) reveals that the optimal policy is determined by thresholds for the passenger's combined risk at each stage, which are computed using the value functions (4.3) for a fixed and finite T. To see the computational effort based on this MDP for WOSA indexed by η and $\mathbf{w}$, note that both $\mathcal{A}(t)$ and $\mathcal{B}(t)$ are discrete random variables, and hence, the two-dimensional risk vector for each passenger can only take on a value from a finite set, $\{A_1, A_2, \dots, A_M\} \times \{B_1, B_2, \dots, B_K\}$, with cardinality bounded above by $M \times K$. Therefore, the time complexity for computing the expectations in value functions is $O(MK)$ (since the optimal policy is pure). Moreover, capacity η is bounded above by T. Therefore, the computational efforts for solving the MDP using dynamic programming will grow in $\mathbb{O}(T^2MK)$ time and $\mathbb{O}(T^2)$ space.

4.2 Sequential Stochastic Assignment Problem Model for WOSA

[19] introduces the sequential stochastic assignment problem, where T workers with known success rates $\tau_1 \leq \tau_2 \leq \ldots \leq \tau_T$ are to be assigned to T sequentially arriving tasks with values $\{\mathcal{C}_t\}_{t=1}^T$ (random variables) revealed upon arrival. The objective is to maximize the total expected reward $\mathbb{E}[\sum_{t=1}^T \tau_{j_t}\mathcal{C}_t]$, where j_t is the index of the worker assigned to perform the t^{th} task with value $\mathcal{C}_t$. The authors propose an optimal pure policy based on recursive equations to compute threshold values for each task assignment. This policy motivated the optimal policy proposed in this work, which considers a more general type of mixed policies.

To model WOSA admitting mixed policies as an SSAP, first we introduce the extension of SSAP with a *fixed success rate sum* to show that optimal mixed policies for WOSA are pure policies. For the SSAP with a fixed success rate sum, the sum of success rates of T workers is fixed as $\sum_{j=1}^T \tau_j = \Upsilon$ for $0 \leq \Upsilon \leq T$, while individual success rates can be reallocated upon every task arrival, with constraints $0 \leq \tau_j \leq 1,\ j = 1, 2, \ldots, T$. Moreover, there exists a corresponding SSAP with *almost-binary success rates* for each SSAP with a fixed success rate sum Υ, where $\lfloor \Upsilon \rfloor$ workers have a success rate one, one worker has a success rate $\Upsilon - \lfloor \Upsilon \rfloor$, and $T - 1 - \lfloor \Upsilon \rfloor$ workers have a success rate zero ($\lfloor \cdot \rfloor$ denoting the floor function, $\lfloor x \rfloor = \max_{n \in \mathbb{Z}} n \leq x$). Proposition 4 shows the relation between the optimal policy for the SSAP with a fixed success rate sum and the optimal policy for the corresponding SSAP with almost-binary success rates.

Proposition 4. *The optimal policy for the SSAP with a fixed success rate sum as $\sum_{j=1}^T \tau_j = \Upsilon$ for $0 \leq \Upsilon \leq T$ is the same as the optimal policy for the corresponding SSAP with almost-binary success rates.*

Proof. See Appendix A.5. □

WOSA indexed by η and $\mathbf{w}$ may be modeled as an SSAP as follows. The t^{th} passenger with the combined risk $\mathcal{G}(t)$ corresponds to an arriving task with value $\mathcal{G}(t)$. The objective of WOSA is to maximize the expected reward for the T passenger assignments, $\mathbb{E}[\sum_{t=1}^{T} X_t^{\Phi}\mathcal{G}(t)]$. For a mixed policy $\Phi \in \Psi^{\eta+}$ with profile $\mathcal{P}^{\Phi} = \{\mathcal{X}_t^{\Phi}\}_{t=1}^{T} \in \Xi^{\eta}$, from the definition of $R_w(\Phi)$ (3.4),

$$\begin{aligned}
R_w(\Phi) &= \mathbb{E}[\sum_{t=1}^{T} X_t^{\Phi}(w_1\mathcal{A}(t) + w_2\mathcal{A}(t)\mathcal{B}(t))] \\
&= \mathbb{E}\left[\mathbb{E}[\sum_{t=1}^{T} X_t^{\Phi}(w_1\mathcal{A}(t) + w_2\mathcal{A}(t)\mathcal{B}(t))|\{(\mathcal{A}(t),\mathcal{B}(t))\}_{t=1}^{T}]\right] \\
&= \mathbb{E}\left[\sum_{t=1}^{T} \mathbb{P}\left(X_t^{\Phi} = 1|\{(\mathcal{A}(t'),\mathcal{B}(t'))\}_{t'=1}^{t}\right)(w_1\mathcal{A}(t) + w_2\mathcal{A}(t)\mathcal{B}(t))\right] \\
&\overset{(3.5),(4.6)}{=} \mathbb{E}\left[\sum_{t=1}^{T} \mathcal{X}_t^{\Phi}\mathcal{G}(t)\right],
\end{aligned} \tag{4.7}$$

where for the second equality, the inner and outer expectations are taken with respect to $\{X_t^{\Phi}\}_{t=1}^{T}$ and the joint distribution of $\{(\mathcal{A}(t),\mathcal{B}(t))\}_{t=1}^{T}$, respectively. Let $\mathcal{X}_t^{\Phi}$ for $t = 1, 2, \ldots, T$ correspond to the success rates of T workers. Hence, from (4.7), assignment of the t^{th} passenger under policy Φ corresponds to the t^{th} task with value $\mathcal{G}(t)$ being assigned to a worker with the success rate $\mathcal{X}_t^{\Phi}$. Since $\sum_{t=1}^{T} \mathcal{X}_t^{\Phi} = \eta$, then the optimal expected reward given by (4.7) is the same as the optimal expected reward under the SSAP with a fixed success rate sum as η. From Proposition 4, the optimal policy is the same as the optimal policy for the corresponding SSAP with almost-binary success rates. Since $\eta \in \mathbb{Z}^+$, then the optimal policy for WOSA indexed by η and $\mathbf{w}$ is a pure policy with binary assignments, and hence lies in the sub-feasible region Ψ^{η}.

Denote the cumulative distribution function (cdf) for $\mathcal{G}(t)$ by $F_{\mathcal{G}}(\gamma)$. Since $\mathcal{A}(t)$ and $\mathcal{B}(t)$ are discrete, $\mathcal{G}(t)$ is also discrete. Denote these discrete values by $0 < G_1 < G_2 < \ldots < G_L$ ($G_0 = 0$ and $F_{\mathcal{G}}(G_L) = 1$). Applying the law of total probability, pmf $p_{\mathcal{G}}(\gamma)$ for $\mathcal{G}(t)$ is

$$\begin{aligned}
p_{\mathcal{G}}(\gamma) &= \sum_{\alpha} \mathbb{P}(\mathcal{A}(t) = \alpha)\mathbb{P}(\mathcal{G}(t) = \gamma|\mathcal{A}(t) = \alpha) \\
&= \sum_{\alpha} p_{\mathcal{A}}(\alpha)p_{\mathcal{B}}(\frac{1}{w_2}(\frac{\gamma}{\alpha} - w_1)), \text{ for } w_2 > 0.
\end{aligned} \tag{4.8}$$

Let $\eta(t)$ denote the *remaining selectee capacity* before the t^{th} passenger assignment for $t = 1, 2, \ldots, T$, with $\eta(1) = \eta$. Now we can apply the optimal policy given by [19] to obtain the optimal policy for WOSA. We state the optimal policy in a way that is trimmed particularly for the SSAP with discrete distributions. For the t^{th} passenger assignment, there exist threshold values

$$-\infty = a_{0,t} \leq a_{1,t} \leq \ldots \leq a_{T-t+1,t} = +\infty, \tag{4.9}$$

obtained using the recursive equations

$$a_{i,t} = \left(\sum_{\gamma=g^u_{i-1,t+1}}^{g^l_{i,t+1}} \gamma p_{\mathcal{G}}(\gamma)\right) + a_{i-1,t+1} F_{\mathcal{G}}(a_{i-1,t+1}) + a_{i,t+1}(1 - F_{\mathcal{G}}(a_{i,t+1})), \tag{4.10}$$

where

$$g^u_{i-1,t+1} \triangleq \min_{l' \in \{1,2,\ldots,L\}} G_{l'} > a_{i-1,t+1}, \qquad g^l_{i,t+1} \triangleq \max_{l' \in \{1,2,\ldots,L\}} G_{l'} \leq a_{i,t+1}, \tag{4.11}$$

for $i = 1, 2, \ldots, T - t$ and $t = 0, 1, 2, \ldots, T$ ($t = 0$ has no passenger arrival and describes the *initial stage* with T passengers to be assigned). If $\mathcal{G}(t) \in (a_{i-1,t}, a_{i,t}]$ for some $i \geq T - t - \eta(t) + 2$, then the t^{th} passenger is assigned to the selectee category. Specifically, this policy denoted by $\Phi 1$ is given by

$$X_t^{\Phi 1} = \begin{cases} 1, & \text{if } \mathcal{G}(t) > a_{T-t-\eta(t)+1,t}, \\ 0 & \text{otherwise,} \end{cases} \tag{$\Phi 1$}$$
$$\eta(t+1) = \eta(t) - X_t^{\Phi 1}, \qquad t = 1, 2, \ldots, T.$$

Theorem 6 states that policy ($\Phi 1$) is optimal for WOSA indexed by η and $\mathbf{w}$.

Theorem 6. *Policy* ($\Phi 1$) *with threshold values defined by (4.10) maximizes the objective function of WOSA indexed by* η *and* $\mathbf{w}$ *defined in (3.4), when* T *passengers are to be assigned. Moreover, the threshold values in the initial stage,* $\{a_{i,0}\}_{i=1}^T$*, are the expected combined risk values for the* T *passengers.*

Proof. See Appendix A.6. □

Policy ($\Phi 1$), referred to as the *SSAP optimal policy*, reveals that the optimal policy for WOSA is determined by thresholds for each passenger's combined risk value, which is consistent with the optimal policy from the MDP model (4.5). Corollary 1 provides the optimal value of the weighted objective function of WOSA indexed by η and $\mathbf{w}$.

Corollary 1. *The threshold values in the initial stage, $\{a_{i,0}\}_{i=T-\eta+1}^{T}$ (4.10), are the expected combined risk values for the η passengers assigned to the selectee category, when T passengers are to be assigned. Therefore,*

$$\max_{\Phi \in \Psi^\eta} R_w(\Phi) = \max_{\Phi \in \Psi^\eta} \mathbb{E}[\sum_{t=1}^{T} X_t^\Phi \mathcal{G}(t)] = \sum_{i=T-\eta+1}^{T} a_{i,0}. \tag{4.12}$$

Proof. The result follows from Theorem 6, where passengers with the η largest combined risk values are expected to be assigned to the selectee category under policy (Φ1). □

Since each passenger's combined risk value is discrete and can only assume one of the L values $\{G_1, G_2, \ldots, G_L\}$ ($L \leq MK$), the time complexity to compute each threshold value in (4.10) for the SSAP optimal policy is $O(L)$. For the t^{th} passenger out of T passengers, $T - t$ threshold values are required. Therefore, the total time complexity is $O(T^2 L)$ and the space requirement is $O(T^2)$.

CHAPTER 5

MOSSAP UNDER THE SSAP OPTIMAL POLICY

This chapter provides values of the five objective functions of MOSSAP under the SSAP optimal policy ($\Phi 1$). We compute values of the two objective functions of the bi-objective optimization problems under policy ($\Phi 1$) with different η and $\mathbf{w}$, which will be used to compute values of the five objective functions of MOSSAP. These values of the objective functions capture the trade-off between Pareto optimal policies for MOSSAP.

5.1 Bi-objective Optimization Problems under Policy ($\Phi 1$)

Denote the i^{th} smallest combined risk value of T passengers to be assigned by the random variable $\hat{\mathcal{G}}_T^{(i)}$. Then from Theorem 6, $\mathbb{E}[\hat{\mathcal{G}}_T^{(i)}] = a_{i,0}$, for $i = 1, 2, \ldots, T$ with $\{a_{i,0}\}_{i=1}^{T}$ defined by (4.10). Denote the primary risk value resulting in $\hat{\mathcal{G}}_T^{(i)}$ by the random variable $\hat{\mathcal{A}}_T^{(i)}$, with the subscript indicating the number of passengers to be assigned. Define $b_{i,t} \triangleq \mathbb{E}[\hat{\mathcal{A}}_{T-t}^{(i)}]$, for $i = 1, 2, \ldots, T-t$ and $t = 0, 1, \ldots, T-1$. Therefore,

$$b_{i,0} = \mathbb{E}[\hat{\mathcal{A}}_T^{(i)}] = \mathbb{E}\left[\mathbb{E}[\hat{\mathcal{A}}_T^{(i)} | \hat{\mathcal{G}}_T^{(i)}]\right], \tag{5.1}$$

and $\{b_{i,0}\}_{i=1}^{T}$ are the expected primary risk values of T passengers to be assigned.

Proposition 5 gives the recursive equations for computing $\{b_{i,t}\}$.

Proposition 5. *The expected value of $\hat{\mathcal{A}}_{T-t}^{(i)}$, denoted by $b_{i,t}$, is given by the* *recursive equations*

$$b_{i,t} = (\sum_{\gamma'=g^u_{i-1,t+1}}^{g^l_{i,t+1}} \mathbb{E}[\mathcal{A}(t)|\mathcal{G}(t)=\gamma']p_{\mathcal{G}}(\gamma')) + b_{i-1,t+1}F_{\mathcal{G}}(a_{i-1,t+1}) + b_{i,t+1}(1-F_{\mathcal{G}}(a_{i,t+1})), \tag{5.2}$$

with $b_{0,t}=0$ *and* $b_{T-t+1,t}=1$ *for* $i=1,2,\ldots,T-t$ *and* $t=0,1,2,\ldots,T-1$. *Here,* $\{a_{i,t}\}$ *are defined by (4.10) and* $g^u_{i-1,t+1}$, $g^l_{i,t+1}$ *are defined by (4.11), respectively.*

Proof. See Appendix A.7. □

Recall that Corollary 1 provides the optimal value of the weighted objective function $R_w(\Phi)$ for WOSA indexed by η and $\mathbf{w}$. However, policy (Φ1) does not necessarily maximize $R_s(\Phi)$ and $R_d(\Phi)$ at the same time. Since MOSSAP has multiple objectives, the value of $R_w(\Phi 1)$ alone is not enough to capture the performance of policy (Φ1). Corollary 2 provides the values of $R_s(\Phi 1)$ (3.1) under the SSAP optimal policy for WOSA indexed by η and $\mathbf{w}$.

Corollary 2.

$$R_s(\Phi 1) = \mathbb{E}[\sum_{t=1}^{T} X_t^{\Phi 1}\mathcal{A}(t)] = \sum_{i=T-\eta+1}^{T} b_{i,0}. \tag{5.3}$$

Proof. The result follows from Theorem 6 and the definition of $\{b_{i,t}\}$. □

Similarly, for the objective function $R_d(\Phi)$ (3.1), the same arguments can be applied. Denote the expected value of $\mathcal{A}(t)\mathcal{B}(t)$ in $\hat{\mathcal{G}}^{(i)}_{T-t}$ of $T-t$ passengers to be assigned by $c_{i,t}$, for $i=1,2,\ldots,T-t$ and $t=0,1,\ldots,T-1$. We state the following Proposition without proof, since the results follow from the same arguments as Proposition 5 and Corollary 2 with $\mathcal{A}(t)$ substituted by $\mathcal{A}(t)\mathcal{B}(t)$.

Proposition 6. $\{c_{i,t}\}$ *are given by the recursive equations*

$$c_{i,t} = (\sum_{\gamma'=g^u_{i-1,t+1}}^{g^l_{i,t+1}} \mathbb{E}[\mathcal{A}(t)\mathcal{B}(t)|\mathcal{G}(t)=\gamma']p_{\mathcal{G}}(\gamma')) + c_{i-1,t+1}F_{\mathcal{G}}(a_{i-1,t+1}) + c_{i,t+1}(1-F_{\mathcal{G}}(a_{i,t+1})), \tag{5.4}$$

with $c_{0,t} = 0$ *and* $c_{T-t+1,t} = B_K$. *Here,* $\{a_{i,t}\}$ *are defined by (4.10) and* $g^u_{i-1,t+1}$, $g^l_{i,t+1}$ *are defined by (4.11), respectively. Moreover,*

$$R_d(\Phi 1) = \mathbb{E}[\sum_{t=1}^{T} X_t^{\Phi 1}\mathcal{A}(t)\mathcal{B}(t)] = \sum_{i=T-\eta+1}^{T} c_{i,0}. \tag{5.5}$$

5.2 MOSSAP under the SSAP Optimal Policy

We consider the *expected rewards per passenger* for the five objective functions of MOSSAP under the SSAP optimal policy ($\Phi 1$). The expected reward per passenger for an objective function is defined as the value of the objective function normalized by the total number of passengers, which is consistent with the classic asymptotic settings. Denote the expected rewards per passenger for $R_s(\Phi 1)$ and $R_d(\Phi 1)$ for WOSA indexed by η and $\mathbf{w}$ by $r_s^\eta(\mathbf{w})$ and $r_d^\eta(\mathbf{w})$, respectively. Then

$$r_s^\eta(\mathbf{w}) \triangleq \frac{1}{T} R_s(\Phi 1) = \frac{1}{T}\sum_{i=T-\eta+1}^{T} b_{i,0} \text{ and } r_d^\eta(\mathbf{w}) \triangleq \frac{1}{T} R_d(\Phi 1) = \frac{1}{T}\sum_{i=T-\eta+1}^{T} c_{i,0}. \tag{5.6}$$

From (2.1),

$$\begin{aligned} \frac{1}{T}W_{ns}(\Phi 1) &= r_s^\eta(\mathbf{w}) - \frac{\eta}{T} + 1 - \mathbb{E}[\mathcal{A}(t)], & \frac{1}{T}W_s(\Phi 1) &= r_s(\mathbf{w}), \\ \frac{1}{T}V(\Phi 1) &= -r_s(\mathbf{w}) + 1 - \frac{\eta}{T}, & \frac{1}{T}U(\Phi 1) &= -r_s(\mathbf{w}) + \mathbb{E}[\mathcal{A}(t)], \end{aligned} \tag{5.7}$$

$$\frac{1}{T}U_{st}(\Phi 1) = -r_d(\mathbf{w}) + \mathbb{E}[\mathcal{A}(t)\mathcal{B}(t)],$$

with $r_s^\eta(\mathbf{w})$ and $r_d^\eta(\mathbf{w})$ given by (5.6). The expressions in (5.7) show that the expected rewards per passenger for MOSSAP are functions of the active capacity constraint η of the sub-feasible region Ψ^η and the weight vector $\mathbf{w}$. Brute-force pruning is feasible using expressions in (5.7). Therefore, even if condition (3.3) is not satisfied, Pareto optimal policies for MOSSAP may be extracted by enumerating values of the five objective functions of MOSSAP under all optimal policies for WOSA.

5.3 Trade-off between Pareto Optimal Policies for MOSSAP

The trade-off between Pareto optimal policies for MOSSAP is captured by (5.7). For SSAP optimal policies in the same sub-feasible region (η is constant, $\mathbf{w}$ is different), the five objective functions of MOSSAP can be reduced to $R_s(\Phi)$ and $R_d(\Phi)$ as shown in Theorem 1. This also follows from (5.7). Therefore, we focus on $r_s^\eta(\mathbf{w})$ and $r_d^\eta(\mathbf{w})$ when comparing performance of SSAP optimal policies for WOSA indexed by the same η and different $\mathbf{w}$. Note that from the definition of $R_w(\Phi)$ (3.4), if $\mathbf{w} = (1, 0)$, then $R_w(\Phi) = R_s(\Phi)$ and the resulting $R_w(\Phi 1)$ given by Corollary 1 is the optimal value of $R_s(\Phi)$ in the sub-feasible region Ψ^η. Similarly, if $\mathbf{w} = (0, 1)$, then $R_w(\Phi) = R_d(\Phi)$ and the resulting $R_w(\Phi 1)$ is the optimal value of $R_d(\Phi)$ in Ψ^η. Define the *achievement ratios* for $R_s(\Phi)$ and $R_d(\Phi)$ under the SSAP optimal policy for WOSA indexed by η and $\mathbf{w}$ in Ψ^η as

$$\delta_s^\eta(\mathbf{w}) \triangleq \frac{R_s(\Phi 1)}{\max_{\Phi\in\Psi^\eta} R_s(\Phi)} = \frac{r_s^\eta(\mathbf{w})}{r_w^\eta((1,0))} \text{ and } \delta_d^\eta(\mathbf{w}) \triangleq \frac{R_d(\Phi 1)}{\max_{\Phi\in\Psi^\eta} R_d(\Phi)} = \frac{r_d^\eta(\mathbf{w})}{r_w^\eta((0,1))},$$

where $r_w^\eta(\mathbf{w}) \triangleq R_w(\Phi 1)/T = (\sum_{i=T-\eta+1}^{T} a_{i,0})/T$ from Corollary 1. That is, for an SSAP optimal policy for WOSA indexed by η and $\mathbf{w}$, $\delta_s^\eta(\mathbf{w})$ is the ratio of the value of $R_s(\Phi)$ under this policy to the optimal value of $R_s(\Phi)$ in Ψ^η and $\delta_d^\eta(\mathbf{w})$ is the ratio of the value of $R_d(\Phi)$ under this policy to the optimal value of $R_d(\Phi)$ in Ψ^η. In general, the magnitude of $\delta_s^\eta(\mathbf{w})$ and $\delta_d^\eta(\mathbf{w})$ measures the closeness of $R_s(\Phi 1)$ and $R_d(\Phi 1)$ under an SSAP optimal policy $\Phi 1$ to their optima (the larger the closer), respectively, with $0 \le \delta_s^\eta(\mathbf{w}) \le 1$ and $0 \le \delta_d^\eta(\mathbf{w}) \le 1$. It can be proved that if $w_1 = 1$ is fixed, then $\delta_s^\eta(\mathbf{w})$ decreases with w_2 while $\delta_d^\eta(\mathbf{w})$ increases with w_2.

For SSAP optimal policies in different sub-feasible regions (η and $\mathbf{w}$ are both different), from Corollary 2 and Proposition 6, $r_s^\eta(\mathbf{w})$ and $r_d^\eta(\mathbf{w})$ both increase with the active capacity constraint η when $\mathbf{w}$ is fixed. However, from (5.7), the trends of the expected rewards per passenger for the five objective functions of MOSSAP with η cannot be concluded theoretically. We show the trade-off between different SSAP optimal policies by presenting simulation results in Chapter 6.

CHAPTER 6

SIMULATION STUDY

This chapter illustrates how the SSAP optimal policy ($\Phi 1$) would operate in practice using available data from Ebola entry screening at airports and public health sources. The trade-off between Pareto optimal policies for MOSSAP is shown by providing expected rewards per passenger for the bi-objective optimization problem $r_s^\eta(\mathbf{w})$ and $r_d^\eta(\mathbf{w})$, and the expected rewards per passenger for the five objectives of MOSSAP. Sensitivity analysis is provided to identify how variations in the weight vector $\mathbf{w}$ and the distributions of the two risk measures influence the performance of the SSAP optimal policy. No extra pruning step is implemented in these simulation experiments to select $\mathbb{M}$-optimal policies for WOSA or Pareto optimal policies for MOSSAP.

6.1 An Illustrative Example with Application in Ebola

We use the Ebola entry screening as an illustrative application example for MOSSAP. The incubation period for Ebola is 21 days [5], and hence, T is set as the total number of passengers entering the United States coming from epidemic West African countries over a 21-day period. The average daily number of passengers arriving to the five international airports originating in the three West African countries (Guinea, Liberia and Sierra Leone) was approximately 150 [21]. Therefore, $T = 150 \times 21 = 3150$. Nine selected medical facilities in the United States have a capacity of at least 10 patients for Ebola quarantine monitoring [22], and hence, $\eta_o = 9 \times 10 = 90$. For $p_{\mathcal{A}}(\alpha)$, a four-level primary risk measure was used in the simulations, which is consistent with the current risk levels used by the CDC [5]. $p_{\mathcal{A}}(\alpha)$ was estimated using statistics from [23]. There were a total of 10,344 passengers monitored for a 21-day period from November 3, 2014, to March 8, 2015 (125 days), with the average daily number of passengers monitored estimated as

$\lfloor 10,344/125 \rfloor = 82$. None of these monitored passengers became sick with Ebola. Since passengers assessed with No identifiable primary risk were not monitored, then the average daily number of passengers assessed with No identifiable primary risk was estimated as $150 - 82 = 68$. Among those passengers monitored, the average daily number of passengers assessed with Low (but not zero) primary risk was estimated as $\lfloor 82 \times 0.97 \rfloor = 79$; the average daily number of passengers assessed with Some primary risk was estimated as $82 - 79 = 3$; we assumed that 1 out of these 3 passengers was assessed with High primary risk.

There was no publicly available data to estimate the realized values of $\mathcal{A}(t)$ (i.e., $\{A_m\}_{m=1}^4$). Similarly to the aviation security screening problem [10], a truncated exponential distribution with parameter $\lambda > 10$ was used to approximate the primary risk distribution, where $1/\lambda$ represented the average primary risk value (and hence, a larger λ indicated a lower average primary risk value). In these simulations, we assume passengers assessed with different primary risk levels have a probability of being sick no greater than 0.1, and they differ among each other by at least one order of magnitude. Therefore, three points were selected between $(0, 0.1)$, and these three points together with 0.1 defined the values $\{A_m\}_{m=1}^4$. Thus, they divided the primary risk value range $(0, 0.1)$ into 4 disjoint intervals. The probability mass for each A_m was estimated as the probability of each interval $(A_{m-1}, A_m]$, for $m = 1, 2, 3, 4$ (with $A_0 = 0$) following the truncated exponential distribution. In this way, each A_m represented the maximal probability of a passenger being sick with Ebola if the passenger was assessed with primary risk level i. Then to approximate the monitoring statistics reported by the [23], we chose $\lambda = 100$ and three points $\{10^{-5}, 10^{-4}, 10^{-2}\}$ (with $A_4 = 0.1$). Table 6.1 lists the primary risk distribution $p_{\mathcal{A}}(\alpha)$ used in the simulations.

For $p_{\mathcal{B}}(\beta)$, there was no available data from public health sources; the only available case for reference was the first confirmed Ebola case in the United States, in which approximately 100 social contacts had been covered by contact tracing [24]. Since Ebola can only be transmitted through direct contact rather than airborne [5], vulnerable contacts of a sick person are limited to family members (2 to 20), friends (5 to 40), co-workers (5 to 40) and other close contacts (5 to 100), with a variation caused by occupation and personal lifestyle. A three-level secondary risk measure with $\{B_k\}_{k=1}^3 = \{20, 100, 200\}$ was used in the simulations and Table 6.2 lists the estimated secondary risk distribution $p_{\mathcal{B}}(\beta)$.

Table 6.3 provides the expected rewards per passenger, $r_s^\eta(\mathbf{w})$ and $r_d^\eta(\mathbf{w})$,

Table 6.1: $p_{\mathcal{A}}(\alpha)$, pmf for the primary risk random variable $\mathcal{A}(t)$ $(\times 10^{-2})$.

α	$A_1(10^{-5})$	$A_2(10^{-4})$	$A_3(10^{-2})$	$A_4(10^{-1})$
$p_{\mathcal{A}}(\alpha)$	45.3	52.7	1.3	0.7

Table 6.2: $p_{\mathcal{B}}(\beta)$, pmf for the secondary risk random variable $\mathcal{B}(t)$ $(\times 10^{-2})$.

β	$B_1(20)$	$B_2(100)$	$B_3(200)$
$p_{\mathcal{B}}(\beta)$	5	85	10

under the SSAP optimal policy for WOSA indexed by $\eta = 30, 60, 90$ and $\mathbf{w} = (1,0), (0,1), (1,1), (1,1000), (1000,1)$. These results were computed from Corollary 2 and Proposition 6 following the two risk distributions of $p_{\mathcal{A}}(\alpha)$ and $p_{\mathcal{B}}(\beta)$. $r_s^\eta(\mathbf{w})$'s in the $(1,0)$ column are the optimal expected rewards per passenger for $R_s(\Phi)$ and $r_d^\eta(\mathbf{w})$'s in the $(0,1)$ column are the optimal expected rewards per passenger for $R_d(\Phi)$ in the corresponding Ψ^η. Both $r_s^\eta(\mathbf{w})$ and $r_d^\eta(\mathbf{w})$ increase as η increases. However, $r_s^\eta(\mathbf{w})$ and $r_d^\eta(\mathbf{w})$ remained unchanged with variations in the positive weight vector when η is fixed since $p_{\mathcal{A}}(\alpha)$ and $p_{\mathcal{B}}(\beta)$ in Tables 6.1 and 6.2 satisfy condition (3.3).

Table 6.3 also shows that an extra pruning step is necessary to obtain Pareto optimal policies for MOSSAP. To see this, consider the SSAP optimal policy with $\eta = 30,\ \mathbf{w} = (1,0)$ in Table 6.3. This policy is not $\mathbb{M}$-optimal for WOSA by Definition 3 (by comparing with the SSAP optimal policy with $\eta = 30,\ \mathbf{w} = (1,1)$). However, every SSAP optimal policy with a positive weight vector is $\mathbb{M}$-optimal for WOSA by Definition 3. Moreover, since condition (3.3) is satisfied, then every $\mathbb{M}$-optimal policy for WOSA is Pareto optimal for MOSSAP from Theorem 5.

Table 6.4 provides estimates for the expectations and standard deviations of $r_s^\eta(\mathbf{w})$ and $r_d^\eta(\mathbf{w})$ and Table 6.5 is for the five objective functions of MOSSAP, under the SSAP optimal policy ($\Phi 1$). These estimates were computed by averaging the results of 10,000 independently seeded replications for each $\eta = 30, 60, 90$ and $\mathbf{w} = (1,0), (0,1), (1,1), (1,1000), (1000,1)$ ($\mathbf{w} = (1,0), (0,1), (1,1)$ for MOSSAP). In each replication, a sequence of passenger risk vectors following distributions $p_{\mathcal{A}}(\alpha)$ and $p_{\mathcal{B}}(\beta)$ was simulated,

Table 6.3: Expected rewards per passenger $r_s^\eta(\mathbf{w})$ and $r_d^\eta(\mathbf{w})$ ($\times 10^{-2}$).

$\mathbf{w} = (w_1, w_2)$	$(1,0)$	$(0,1)$	$(1,1)$	$(1000,1)$	$(1,1000)$
$r_s^{30}(\mathbf{w})$	0.070	0.070	0.070	0.070	0.070
$r_d^{30}(\mathbf{w})$	7.4	7.5	7.5	7.5	7.5
$r_s^{60}(\mathbf{w})$	0.079	0.079	0.079	0.079	0.079
$r_d^{60}(\mathbf{w})$	8.5	8.5	8.5	8.5	8.5
$r_s^{90}(\mathbf{w})$	0.080	0.080	0.080	0.080	0.080
$r_d^{90}(\mathbf{w})$	8.5	8.5	8.5	8.5	8.5

and the SSAP optimal policy was applied to the sequence of passenger risk vectors. The threshold values for each passenger assignment for the SSAP optimal policy were computed using MATLAB R2011a on an Intel i7-2600 3.4 GHz processor with 8 GB RAM, which took 41.2-second CPU time for $T = 3150$. These threshold values were computed in advance and used in all replications. $r_s^\eta(\mathbf{w})$ and $r_d^\eta(\mathbf{w})$ for the SSAP optimal policy in Table 6.4 are close to the corresponding values in Table 6.3, which indicates the effectiveness of Corollary 2 and Proposition 6. Moreover, the results in Table 6.5 are consistent with those computed from (5.7) using values provided in Table 6.4.

Table 6.4: Expectations (standard deviations) of $r_s^\eta(\mathbf{w})$ and $r_d^\eta(\mathbf{w})$ under the SSAP optimal policy ($\times 10^{-2}$).

$\mathbf{w} = (w_1, w_2)$	$(1,0)$	$(0,1)$	$(1,1)$	$(1000,1)$	$(1,1000)$
$r_s^{30}(\mathbf{w})(std)$	0.069(0.013)	0.068(0.012)	0.069(0.012)	0.069(0.013)	0.069(0.012)
$r_d^{30}(\mathbf{w})(std)$	7.3(1.4)	7.4(1.4)	7.4(1.4)	7.4(1.4)	7.4(1.4)
$r_s^{60}(\mathbf{w})(std)$	0.078(0.013)	0.078(0.014)	0.078(0.014)	0.078(0.013)	0.078(0.014)
$r_d^{60}(\mathbf{w})(std)$	8.3(1.5)	8.3(1.5)	8.3(1.5)	8.3(1.5)	8.3(1.5)
$r_s^{90}(\mathbf{w})(std)$	0.080(0.015)	0.080(0.015)	0.080(0.015)	0.080(0.015)	0.080(0.015)
$r_d^{90}(\mathbf{w})(std)$	8.5(1.6)	8.5(1.6)	8.5(1.6)	8.5(1.6)	8.5(1.6)

Table 6.5: Expectations (standard deviations) of rewards per passenger for MOSSAP ($\times 10^{-2}$).

	$\eta = 30$			$\eta = 60$			$\eta = 90$		
$\mathbf{w} = (w_1, w_2)$	$(1,0)$	$(0,1)$	$(1,1)$	$(1,0)$	$(0,1)$	$(1,1)$	$(1,0)$	$(0,1)$	$(1,1)$
$W_{ns}^\eta(std)$	99(0)	99(0)	99(0)	98(0)	98(0)	98(0)	97(0)	97(0)	97(0)
$W_s^\eta(std)$	0.069(0.013)	0.068(0.012)	0.069(0.012)	0.078(0.013)	0.078(0.013)	0.078(0.013)	0.080(0.015)	0.080(0.014)	0.080(0.015)
$V^\eta(std)$	0.88(0.01)	0.88(0.01)	0.88(0.01)	1.83(0.01)	1.83(0.01)	1.83(0.01)	2.78(0.01)	2.78(0.01)	2.78(0.01)
$U^\eta(std)$	0.017(0.003)	0.018(0.004)	0.018(0.004)	0.008(0.002)	0.008(0.002)	0.008(0.002)	0.006(0)	0.006(0)	0.006(0)
$U_{st}^\eta(std)$	1.8(0.3)	1.7(0.4)	1.7(0.4)	0.81(0.24)	0.77(0.20)	0.77(0.20)	0.60(0.01)	0.60(0.01)	0.60(0.01)

6.2 Sensitivity Analysis for the SSAP Optimal Policy

This section presents a sensitivity analysis for the SSAP optimal policy with respect to variations in the distributions of the two risk measures. First, we show how the weight vector influences the performance of the SSAP optimal policy when condition (3.3) is not satisfied. Then we show variations in the values of objective functions with variations in the distributions of the two risk measures. Since the expected rewards per passenger for the five objective functions of MOSSAP can be easily derived from (5.7) using $r_s^\eta(\mathbf{w})$ and $r_d^\eta(\mathbf{w})$, we present the values of $r_s^\eta(\mathbf{w})$ and $r_d^\eta(\mathbf{w})$ for comparing the performance of SSAP optimal policies for simplicity.

Tables 6.6 and 6.7 give two risk distributions, $p_{\mathcal{A}}(\alpha)_1$ and $p_{\mathcal{B}}(\beta)_1$, with $\{A_m\}_{m=1}^4$ and $\{B_k\}_{k=1}^3$ not satisfying condition (3.3) (not necessarily realistic). Table 6.8 provides estimates for the expectations and standard deviations of $r_s^\eta(\mathbf{w})$ and $r_d^\eta(\mathbf{w})$, under the SSAP optimal policy for WOSA indexed by $\eta = 30, 60, 90$ and $\mathbf{w} = (1,0), (0,1), (1,1), (10,1), (20,1), (1,10)$. The estimates were computed by averaging the results of 10,000 independently seeded replications for each η and $\mathbf{w}$. In each replication, a sequence of passenger risk vectors following distributions $p_{\mathcal{A}}(\alpha)_1$ and $p_{\mathcal{B}}(\beta)_1$ was simulated and the corresponding SSAP optimal policy was applied. When $w_1 = 1,\ w_2 > 0$, $r_s^\eta(\mathbf{w})$ and $r_d^\eta(\mathbf{w})$ do not increase with w_2. However, when $w_1 > 0,\ w_2 = 1$, $r_s^\eta(\mathbf{w})$ increases and $r_d^\eta(\mathbf{w})$ decreases as w_1 increases. Note that w_1 is the scaling weight for $\mathcal{A}(t)$ and w_2/w_1 is the scaling weight for $\mathcal{B}(t)$, and the realized values of $\mathcal{B}(t)$ are much larger than those of $\mathcal{A}(t)$. Therefore, only by scaling the values of $\mathcal{A}(t)$ to be comparable or even larger than those of $\mathcal{B}(t)$ will there be differences in $r_s^\eta(\mathbf{w})$ and $r_d^\eta(\mathbf{w})$. Moreover, when η is fixed, the variations in $r_s^\eta(\mathbf{w})$ and $r_d^\eta(\mathbf{w})$ are not proportional to the variations in w_1 or w_2. This is consistent with the well-known drawbacks of the weighted sum method for multi-objective optimization problems, where evenly distributed weight vectors cannot generate evenly distributed Pareto optimal solutions in the objective value space [25].

Next, we show the sensitivity of the SSAP optimal policy with respect to the distributions of the two risk measures. Table 6.9 lists five primary risk distributions $p_{\mathcal{A}}(\alpha)_\lambda$, indexed by $\lambda = 40, 100, 200, 300, 400$, which were estimated using truncated exponential distributions with parameter λ. Ta-

Table 6.6: $p_{\mathcal{A}}(\alpha)_1$, pmf for the primary risk random variable $\mathcal{A}(t)$ ($\times 10^{-2}$).

α	$A_1(0.005)$	$A_2(0.01)$	$A_3(0.05)$	$A_4(0.1)$
$p_{\mathcal{A}}(\alpha)_1$	45.3	52.7	1.3	0.7

Table 6.7: $p_{\mathcal{B}}(\beta)_1$, pmf for the secondary risk random variable $\mathcal{B}(t)$ ($\times 10^{-2}$).

β	$B_1(1)$	$B_2(10)$	$B_3(100)$
$p_{\mathcal{B}}(\beta)_1$	5	85	10

Table 6.8: Expectations (standard deviations) of $r_s^\eta(\mathbf{w})$ and $r_d^\eta(\mathbf{w})$ under the optimal SSAP policy ($\times 10^{-2}$).

$\mathbf{w} = (w_1, w_2)$	$(1,0)$	$(0,1)$	$(1,1)$	$(10,1)$	$(20,1)$	$(1,10)$
$r_s^{30}(\mathbf{w})$	0.081(0.010)	0.027(0.006)	0.070(0.011)	0.071(0.011)	0.078(0.007)	0.069(0.010)
$r_d^{30}(\mathbf{w})$	1.5(0.4)	2.1(0.5)	2.1(0.5)	2.1(0.5)	2.0(0.5)	2.1(0.5)
$r_s^{60}(\mathbf{w})$	0.13(0.01)	0.045(0.008)	0.081(0.013)	0.082(0.013)	0.12(0.01)	0.080(0.013)
$r_d^{60}(\mathbf{w})$	2.3(0.5)	3.0(0.5)	3.0(0.5)	3.0(0.5)	2.5(0.5)	3.0(0.5)
$r_s^{90}(\mathbf{w})$	0.14(0.01)	0.063(0.009)	0.090(0.013)	0.092(0.013)	0.14(0.02)	0.090(0.013)
$r_d^{90}(\mathbf{w})$	2.6(0.6)	4.0(0.5)	4.0(0.5)	4.0(0.5)	3.4(0.5)	4.0(0.5)

ble 6.10 lists five secondary risk distributions $p_{\mathcal{B}}(\beta)_h$, indexed by $h = 50, 70^-, 70, 70^+, 90$, where $h(\times 10^{-2})$ captures the fraction of passengers assessed with a medium secondary risk level (the majority fraction). Note that $p_{\mathcal{A}}(\alpha)_\lambda$ and $p_{\mathcal{B}}(\beta)_h$ do not satisfy condition (3.3), which captures a more general case. In this sensitivity analysis, $T = 5000$, $\eta = 50$ and $\mathbf{w} = (100, 1)$. The SSAP optimal policy Φ^* was computed following the *estimated distributions* (i.e., the primary and secondary risk distributions as $p_{\mathcal{A}}(\alpha)_{200}$ and $p_{\mathcal{B}}(\beta)_{70}$) and used in the simulations for all pairs of *realized distributions* (i.e., the primary and secondary risk distributions as $\{(p_{\mathcal{A}}(\alpha)_\lambda, p_{\mathcal{B}}(\beta)_{70})\}$, $\lambda = 40, 100, 200, 300, 400$ and $\{(p_{\mathcal{A}}(\alpha)_{200}, p_{\mathcal{B}}(\beta)_h)\}$, $h = 50, 70^-, 70, 70^+, 90$).

Table 6.11 provides estimates for the expectations and standard deviations of $r_s^{50}(\mathbf{w})$ and $r_d^{50}(\mathbf{w})$ under policy Φ^* with realized distributions $\{(p_{\mathcal{A}}(\alpha)_\lambda, p_{\mathcal{B}}(\beta)_{70})\}$, $\lambda = 40, 100, 200, 300, 400$ and Table 6.12 with realized distributions $\{(p_{\mathcal{A}}(\alpha)_{200}, p_{\mathcal{B}}(\beta)_h)\}$, $h = 50, 70^-, 70, 70^+, 90$. For each pair of realized distributions $(p_{\mathcal{A}}(\alpha)_\lambda, p_{\mathcal{B}}(\beta)_h)$, the estimated values were computed by averaging the results of 10,000 independently seeded replications, where a sequence of passenger risk vectors following the primary and secondary risk distributions $p_{\mathcal{A}}(\alpha)_\lambda$ and $p_{\mathcal{B}}(\beta)_h$ was simulated and policy Φ^* was applied in each replication. The corresponding *optima* (i.e., $r_s^{50}(\mathbf{w})_{opt}$ and $r_d^{50}(\mathbf{w})_{opt}$) for

Table 6.9: $p_{\mathcal{A}}(\alpha)_\lambda, \lambda = 40, 100, 200, 300, 400$, pmf's for the primary risk random variable $\mathcal{A}(t)$ ($\times 10^{-2}$).

α	$A_1(0.005)$	$A_2(0.01)$	$A_3(0.05)$	$A_4(0.1)$
$p_{\mathcal{A}}(\alpha)_{40}$	18.2	14.8	65.2	1.8
$p_{\mathcal{A}}(\alpha)_{100}$	39.4	23.9	36.0	0.7
$p_{\mathcal{A}}(\alpha)_{200}$	63.2	23.3	13.0	0.5
$p_{\mathcal{A}}(\alpha)_{300}$	77.7	17.3	4.8	0.2
$p_{\mathcal{A}}(\alpha)_{400}$	86.5	11.7	1.8	0

Table 6.10: $p_{\mathcal{B}}(\beta)_h, h = 50, 70^-, 70, 70^+, 90$, pmf's for the secondary risk random variable $\mathcal{B}(t)$ ($\times 10^{-2}$).

β	$B_1(1)$	$B_2(10)$	$B_3(100)$
$p_{\mathcal{B}}(\beta)_{50}$	25	50	25
$p_{\mathcal{B}}(\beta)_{70^-}$	20	70	10
$p_{\mathcal{B}}(\beta)_{70}$	15	70	15
$p_{\mathcal{B}}(\beta)_{70^+}$	10	70	20
$p_{\mathcal{B}}(\beta)_{90}$	5	90	5

each pair of realized distributions $(p_{\mathcal{A}}(\alpha)_\lambda, p_{\mathcal{B}}(\beta)_h)$ are also provided in Tables 12 and 13, which were computed from Corollary 2 and Proposition 6 following the realized distributions $p_{\mathcal{A}}(\alpha)_\lambda$ and $p_{\mathcal{B}}(\beta)_h$. When the estimated distributions for the two risk measures are accurate, $r_s^{50}(\mathbf{w})$ and $r_d^{50}(\mathbf{w})$ achieve their corresponding optima, respectively (see the $p_{\mathcal{A}}(\alpha)_{200}$ column in Table 6.11 and the $p_{\mathcal{B}}(\beta)_{70}$ column in Table 6.12). When the estimated primary risk distribution has a lower average risk value than the realized value, then $r_s^{50}(\mathbf{w})$ and $r_d^{50}(\mathbf{w})$ are close to their corresponding optima. In this case, passengers have higher realized risk values than the estimated risk values, and hence, the threshold values for policy Φ^* are relatively low and passengers assessed with high risk values were assigned to the selectee category. However, when the estimated primary risk distribution has a higher average risk value than the realized value, then $r_s^{50}(\mathbf{w})$ and $r_d^{50}(\mathbf{w})$ deviate from their corresponding optima ($r_s^{50}(\mathbf{w})$ and $r_d^{50}(\mathbf{w})$ achieve 0.93 and 0.83 of their corresponding optima in the $p_{\mathcal{A}}(\alpha)_{300}$ column, and $r_s^{50}(\mathbf{w})$ and $r_d^{50}(\mathbf{w})$ achieve 0.50 and 0.82 of their corresponding optima in the $p_{\mathcal{A}}(\alpha)_{400}$ column in Table 6.11). In this case, passengers have lower realized risk values than the estimated risk values, and hence, the threshold values for policy Φ^* are relatively high and passengers were not assigned to the selectee category until the latter part of passenger arrivals. The larger the error in the estimation for the primary risk distribution, the larger the deviation in $r_s^{50}(\mathbf{w})$ and $r_d^{50}(\mathbf{w})$ from their corresponding optima. However, differences between the estimated and realized secondary risk distributions result in small deviations in both $r_s^{50}(\mathbf{w})$ and $r_s^{50}(\mathbf{w})$ from their corresponding optima ($r_s^{50}(\mathbf{w})$ and $r_d^{50}(\mathbf{w})$ achieve 0.99 and 1.00 of their corresponding optima in the $p_{\mathcal{B}}(\beta)_{50}$ column, and $r_s^{50}(\mathbf{w})$ and $r_d^{50}(\mathbf{w})$ achieve 0.99 and 0.87 of their corresponding optima in the $p_{\mathcal{B}}(\beta)_{90}$ column in Table 6.12). This indicates that the SSAP optimal policy is less sensitive to variations in the secondary risk distribution than variations in the primary risk distribution.

Table 6.11: Expectations (standard deviations) of $r_s^{50}(\mathbf{w})$ and $r_d^{50}(\mathbf{w})$ under the SSAP optimal policy Φ^* ($\times 10^{-2}$), with the realized distributions $\{(p_{\mathcal{A}}(\alpha)_\lambda, p_{\mathcal{B}}(\beta)_{70})\}$, $\lambda = 40, 100, 200, 300, 400$.

	$p_{\mathcal{A}}(\alpha)_{40}$	$p_{\mathcal{A}}(\alpha)_{100}$	$p_{\mathcal{A}}(\alpha)_{200}$	$p_{\mathcal{A}}(\alpha)_{300}$	$p_{\mathcal{A}}(\alpha)_{400}$
$r_s^{50}(\mathbf{w})(std)$	0.10(0.00)	0.082(0.006)	0.076(0.005)	0.056(0.005)	0.025(0.003)
$r_s^{50}(\mathbf{w})_{opt}$	0.10	0.084	0.077	0.060	0.050
$r_d^{50}(\mathbf{w})(std)$	3.0(0.5)	3.2(0.5)	3.6(0.4)	3.3(0.5)	1.4(0.3)
$r_d^{50}(\mathbf{w})_{opt}$	3.5	3.1	3.5	4.0	1.7

Table 6.12: Expectations (standard deviations) of $r_s^{50}(\mathbf{w})$ and $r_d^{50}(\mathbf{w})$ under the SSAP optimal policy Φ^* ($\times 10^{-2}$), with the realized distributions $\{(p_{\mathcal{A}}(\alpha)_{200}, p_{\mathcal{B}}(\beta)_h)\}$, $h = 50, 70^-, 70, 70^+, 90$.

	$p_{\mathcal{B}}(\beta)_{50}$	$p_{\mathcal{B}}(\beta)_{70^-}$	$p_{\mathcal{B}}(\beta)_{70}$	$p_{\mathcal{B}}(\beta)_{70^+}$	$p_{\mathcal{B}}(\beta)_{90}$
$r_s^{50}(\mathbf{w})(std)$	0.076(0.005)	0.076(0.005)	0.076(0.005)	0.076(0.005)	0.076(0.005)
$r_s^{50}(\mathbf{w})_{opt}$	0.077	0.077	0.077	0.077	0.077
$r_d^{50}(\mathbf{w})(std)$	4.0(0.5)	3.3(0.4)	3.6(0.4)	3.8(0.4)	2.7(0.4)
$r_d^{50}(\mathbf{w})_{opt}$	4.0	3.3	3.5	3.8	3.1

CHAPTER 7

OPERATIONAL IMPLICATIONS AND LIMITATIONS

The simulation results indicate that assessing each passenger with a two-dimensional risk vector has advantages over a one-dimensional risk value. If passengers are assessed with a one-dimensional risk value, then $R_s(\Phi)$ becomes the only objective function, as in the classic SSAP setting studied by [19]. In this case, consider $R_s(\Phi)$ as the single objective function and $R_d(\Phi)$ as an additional performance evaluation metric (the larger the better). With some proper positive weight vector for some η, the SSAP optimal policy for WOSA (passenger assignments with two-dimensional risk vectors, see the $\mathbf{w} = (1000, 1)$ and $\eta = 30$ cell in Table 6.4, and the $\mathbf{w} = (20, 1)$ and $\eta = 90$ cell in Table 6.8) can achieve the optimum for $R_s(\Phi)$ and a larger value for $R_d(\Phi)$, compared with those achieved under the SSAP optimal policy for the single objective function $R_s(\Phi)$ (passenger assignments with one-dimensional risk values, see the $\mathbf{w} = (1, 0)$ and $\eta = 30$ cell in Table 6.4, and the $\mathbf{w} = (1, 0)$ and $\eta = 90$ cell in Table 6.8). Therefore, assessing each passenger with an extra dimension of risk measure may improve the process for managing quarantine assignments for Ebola entry screening. A comparison between policies using the one-dimensional risk values and the two-dimensional risk vectors in different scenarios is provided by [26].

MOSSAP for the Ebola entry screening problem has several limitations. We assume passenger assignments to be irrevocable in MOSSAP. Therefore, if a passenger with high primary and secondary risk values arrives after the quarantine capacity has been reached, quarantine space will not be available. If this passenger becomes sick with Ebola, high contact tracing cost will be incurred. However, if passenger assignments are revocable, we can replace one passenger being quarantined that has lower primary and secondary risk values with this passenger. The resulting model is currently under investigation.

CHAPTER 8

CONCLUSIONS

This thesis formulates a mathematical model for multi-objective sequential stochastic assignment problem (MOSSAP), motivated by Ebola entry screening and quarantine assignments at airports in the United States. Each passenger is assessed with a two-dimensional risk vector, with Pareto optimal policies determining on-line binary passenger assignments (selectee or non-selectee) to maximize the expected number of passengers assigned to the correct category, subject to the selectee capacity constraint. When a certain condition is satisfied, the set of Pareto optimal policies for MOSSAP can be generated by solving a sequence of bi-objective optimization problems. Each of these bi-objective optimization problems can then be solved using the weighted sum method, with SSAP optimal policies provided. Moreover, values of the multiple objective functions under the SSAP optimal policy for discrete risk distributions are provided with recursive equations. Our proposed MOSSAP generates the classic single-objective SSAP, and the analysis techniques in this work can be directly applied to the general type of product-form MOSSAP. Simulation results using publicly available Ebola data are provided to illustrate practical implications of the proposed policies.

There are several directions to extend this work. First, this assignment problem can be generalized to include more than two assignment categories. In this case, the results in this work still apply with minor changes on decision thresholds, while recursive equations for the thresholds and values of objective functions remain unchanged. Another direction to be considered is MOSSAP with revocable passenger assignments. Moreover, an efficient pruning algorithm rather than the brute-force enumerating needs to be considered to select Pareto optimal policies for MOSSAP. Last but not least, although the weighted sum method works well for MOSSAP, it may not be the only approach to multi-objective on-line optimization problems. Developing other exact and approximation algorithms to solve multi-objective on-line optimization problems is yet another direction for future consideration.

APPENDIX A

PROOFS OF THEOREMS AND PROPOSITIONS

A.1 Proof of Proposition 1

We prove this by contradiction. Suppose that policy Φ^*_η is not Pareto optimal for MOSSAP. Then there exists another Pareto optimal policy for MOSSAP $\Phi' \in \Psi^{\eta'}$, where $\eta' \in \{0, 1, \ldots, \eta_o\}$ and

$$\begin{aligned} W_{ns}(\Phi') \geq W_{ns}(\Phi^*_\eta),\ W_s(\Phi') \geq W_s(\Phi^*_\eta), \\ V(\Phi') \leq V(\Phi^*_\eta),\ U(\Phi') \leq U(\Phi^*_\eta),\ U_{st}(\Phi') \leq U_{st}(\Phi^*_\eta), \end{aligned} \tag{A.1}$$

with at least one strict inequality. Then from Theorem 2, policy Φ' is Pareto optimal for the bi-objective optimization problem indexed by η'.

If $\eta' = \eta$, then from Theorem 1 and (A.1), $R_s(\Phi') \geq R_s(\Phi^*_\eta),\ R_d(\Phi') \geq R_d(\Phi^*_\eta)$ with at least one strict inequality. This is contradictory to Pareto optimality of Φ^*_η by Definition 2.

If $\eta' > \eta$, then since $0 < \mathcal{A}(t) < 1$, from (2.1)

$$\begin{aligned} \mathbb{E}[\sum_{t=1}^{T} X_t^{\Phi'} \mathcal{A}(t) | \sum_{t=1}^{T} X_t^{\Phi'} = \eta'] &< \max_{\Phi \in \Psi^\eta} \mathbb{E}[\sum_{t=1}^{T} X_t^{\Phi} \mathcal{A}(t) | \sum_{t=1}^{T} X_t^{\Phi} = \eta] + \eta' - \eta \\ &= \mathbb{E}[\sum_{t=1}^{T} X_t^{\Phi^*_\eta} \mathcal{A}(t) | \sum_{t=1}^{T} X_t^{\Phi^*_\eta} = \eta] + \eta' - \eta \end{aligned}$$

$$\Rightarrow W_s(\Phi') < W_s(\Phi^*_\eta) + \eta' - \eta, \quad V(\Phi^*_\eta) - V(\Phi') = (\eta - W_s(\Phi^*_\eta)) - (\eta' - W_s(\Phi')) < 0,$$

which is a contradictory to (A.1).

If $\eta' < \eta$, then since $0 < \mathcal{A}(t) < 1$, (2.1) leads to

$$\begin{aligned} W_s(\Phi') &= \mathbb{E}[\sum_{t=1}^{T} X_t^{\Phi'} \mathcal{A}(t) | \sum_{t=1}^{T} X_t^{\Phi'} = \eta'] \\ &< \max_{\Phi \in \Psi^\eta} \mathbb{E}[\sum_{t=1}^{T} X_t^{\Phi} \mathcal{A}(t) | \sum_{t=1}^{T} X_t^{\Phi} = \eta] = W_s(\Phi_\eta^*), \end{aligned}$$

which is contradictory to (A.1).

Therefore, Φ_η^* is Pareto optimal for MOSSAP.

A.2 Proof of Proposition 2

We prove this by contradiction. Let Φ_η denote the Pareto optimal policy for the bi-objective optimization problem indexed by η with $R_s(\Phi_\eta) = \max_{\Phi \in \Psi^\eta} R_s(\Phi) = \mathbb{E}[\sum_{t=1}^{T} X_t^{\Phi_\eta} \mathcal{A}(t)]$. Suppose $\Phi' \in \Psi^\eta$ is another Pareto optimal policy for the bi-objective optimization problem but $R_s(\Phi') < R_s(\Phi_\eta)$. Since condition (3.3) is satisfied, then it further implies that if $m < m' \in \{1, 2, \ldots, M\}$, inequality

$$A_m B_k \leq A_{m'} B_{k'} \tag{A.2}$$

holds for all $k,\ k' \in \{1, 2, \ldots, K\}$. Therefore,

$$\begin{aligned} &R_s(\Phi') = \mathbb{E}[\sum_{t=1}^{T} X_t^{\Phi'} \mathcal{A}(t)] < \mathbb{E}[\sum_{t=1}^{T} X_t^{\Phi_\eta} \mathcal{A}(t)] = R_s(\Phi_\eta) \\ \Rightarrow &R_d(\Phi') = \mathbb{E}[\sum_{t=1}^{T} X_t^{\Phi'} \mathcal{A}(t)\mathcal{B}(t)] \leq \mathbb{E}[\sum_{t=1}^{T} X_t^{\Phi_\eta} \mathcal{A}(t)\mathcal{B}(t)] = R_d(\Phi_\eta), \end{aligned}$$

which is contradictory to the Pareto optimality for Φ'. Therefore, $R_s(\Phi') = R_s(\Phi_\eta)$.

A.3 Proof of Proposition 3

We show that for any $0 < \lambda < 1$,

$$R_s(\Phi_\lambda) = \lambda R_s(\Phi_1) + (1-\lambda) R_s(\Phi_2) \text{ and } R_d(\Phi_\lambda) = \lambda R_d(\Phi_1) + (1-\lambda) R_d(\Phi_2), \tag{A.3}$$

where $\Phi_\lambda = \lambda\Phi_1 + (1-\lambda)\Phi_2$. Thus, $R_s(\Phi)$ and $R_d(\Phi)$ are both convex and concave functions of Φ. Therefore, $R_s(\Phi)$ and $R_d(\Phi)$ are both affine functions of Φ. In the following, we consider $R_s(\Phi_\lambda)$ first. The derivation for $R_d(\Phi_\lambda)$ follows the same arguments, with $\mathcal{A}(t)$ substituted by $\mathcal{A}(t)\mathcal{B}(t)$.

Rewrite the objective function $R_s(\Phi_\lambda)$ by conditioning on the sequence of $\{(\mathcal{A}(t), \mathcal{B}(t))\}_{t=1}^T$ as

$$R_s(\Phi_\lambda) = \mathbb{E}[\sum_{t=1}^T X_t^{\Phi_\lambda} \mathcal{A}(t)] = \mathbb{E}\left[\mathbb{E}[\sum_{t=1}^T X_t^{\Phi_\lambda} \mathcal{A}(t) | \{(\mathcal{A}(t), \mathcal{B}(t))\}_{t=1}^T]\right], \tag{A.4}$$

where the outer expectation is taken with respect to the joint distribution of $\{(\mathcal{A}(t), \mathcal{B}(t))\}_{t=1}^T$ and the inner expectation is taken with respect to $\{X_t^{\Phi_\lambda}\}_{t=1}^T$.

Consider $\mathbb{E}[\sum_{t=1}^T X_t^{\Phi_\lambda} \mathcal{A}(t) | \{(\mathcal{A}(t), \mathcal{B}(t))\}_{t=1}^T = \{(\alpha_t, \beta_t)\}_{t=1}^T]$. Then,

$$\begin{aligned}
&\mathbb{E}[\sum_{t=1}^T X_t^{\Phi_\lambda} \mathcal{A}(t) | \{(\mathcal{A}(t), \mathcal{B}(t))\}_{t=1}^T = \{(\alpha_t, \beta_t)\}_{t=1}^T] \\
=& \sum_{t=1}^T \mathbb{E}[X_t^{\Phi_\lambda} | \{(\mathcal{A}(t), \mathcal{B}(t))\}_{t=1}^T = \{(\alpha_t, \beta_t)\}_{t=1}^T] \alpha_t \\
=& \sum_{t=1}^T \mathbb{P}(X_t^{\Phi_\lambda} = 1 | \{(\mathcal{A}(t'), \mathcal{B}(t'))\}_{t'=1}^t = \{(\alpha_{t'}, \beta_{t'})\}_{t'=1}^t) \alpha_t,
\end{aligned} \tag{A.5}$$

where (A.5) follows from $X_t^{\Phi_\lambda}$ being a sequential passenger assignment with information of passenger risk vectors till time t. From (3.6) and (3.9),

$$\mathbb{P}(X_t^{\Phi_\lambda} = 1 | \{(\mathcal{A}(t'), \mathcal{B}(t'))\}_{t'=1}^t = \{(\alpha_{t'}, \beta_{t'})\}_{t'=1}^t) = \chi_t^{\Phi_\lambda} = \lambda \chi_t^{\Phi_1} + (1-\lambda)\chi_t^{\Phi_1},$$

for $t = 1, 2, \ldots, T$, where $\{\chi_t^{\Phi_1}\}_{t=1}^T$, $\{\chi_t^{\Phi_2}\}_{t=1}^T$ and $\{\chi_t^{\Phi_\lambda}\}_{t=1}^T$ denote the realized profiles of policy Φ_1, Φ_2 and Φ_λ with respect to $\{(\alpha_t, \beta_t)\}_{t=1}^T$, respectively.

Moreover, substituting Φ_λ with Φ_1 and Φ_2 in (A.5) respectively, we have

$$\begin{aligned}
&\mathbb{E}[\sum_{t=1}^{T} X_t^{\Phi_1}\mathcal{A}(t)|\{(\mathcal{A}(t),\mathcal{B}(t))\}_{t=1}^{T}=\{(\alpha_t,\beta_t)\}_{t=1}^{T}] \\
&=\sum_{t=1}^{T}\mathbb{P}(X_t^{\Phi_1}=1|\{(\mathcal{A}(t'),\mathcal{B}(t'))\}_{t'=1}^{t}=\{(\alpha_{t'},\beta_{t'})\}_{t'=1}^{t})\alpha_t=\sum_{t=1}^{T}\chi_t^{\Phi_1}\alpha_t, \\
&\mathbb{E}[\sum_{t=1}^{T} X_t^{\Phi_2}\mathcal{A}(t)|\{(\mathcal{A}(t),\mathcal{B}(t))\}_{t=1}^{T}=\{(\alpha_t,\beta_t)\}_{t=1}^{T}]=\sum_{t=1}^{T}\chi_t^{\Phi_2}\alpha_t.
\end{aligned}$$

Therefore,

$$\begin{aligned}
&\mathbb{E}[\sum_{t=1}^{T} X_t^{\Phi_\lambda}\mathcal{A}(t)|\{(\mathcal{A}(t),\mathcal{B}(t))\}_{t=1}^{T}=\{(\alpha_t,\beta_t)\}_{t=1}^{T}]=\sum_{t=1}^{T}\chi_t^{\Phi_\lambda}\alpha_t \\
=&\sum_{t=1}^{T}\left(\lambda\chi_t^{\Phi_1}+(1-\lambda)\chi_t^{\Phi_1}\right)\alpha_t \\
=&\lambda\mathbb{E}[\sum_{t=1}^{T} X_t^{\Phi_1}\mathcal{A}(t)|\{(\mathcal{A}(t),\mathcal{B}(t))\}_{t=1}^{T}=\{(\alpha_t,\beta_t)\}_{t=1}^{T}] \\
&+(1-\lambda)\mathbb{E}[\sum_{t=1}^{T} X_t^{\Phi_2}\mathcal{A}(t)|\{(\mathcal{A}(t),\mathcal{B}(t))\}_{t=1}^{T}=\{(\alpha_t,\beta_t)\}_{t=1}^{T}], \qquad \text{(A.6)}
\end{aligned}$$

which holds for any sequence of realized passenger risk vectors $\{(\alpha_t,\beta_t)\}_{t=1}^{T}$. Substituting (A.6) into (A.4) leads to

$$\begin{aligned}
R_s(\Phi_\lambda)=&\lambda\mathbb{E}\left[\mathbb{E}[\sum_{t=1}^{T} X_t^{\Phi_1}\mathcal{A}(t)|\{(\mathcal{A}(t),\mathcal{B}(t))\}_{t=1}^{T}=\{(\alpha_t,\beta_t)\}_{t=1}^{T}]\right] \\
&+(1-\lambda)\mathbb{E}\left[\mathbb{E}[\sum_{t=1}^{T} X_t^{\Phi_2}\mathcal{A}(t)|\{(\mathcal{A}(t),\mathcal{B}(t))\}_{t=1}^{T}=\{(\alpha_t,\beta_t)\}_{t=1}^{T}]\right] \\
=&\lambda R_s(\Phi_1)+(1-\lambda)R_s(\Phi_2),
\end{aligned}$$

where the last equality follows from substituting Φ_λ with Φ_1 and Φ_2 in (A.4), respectively. Therefore, $R_s(\Phi)$ is an affine function of Φ.

Following the same arguments with $\mathcal{A}(t)$ substituted by $\mathcal{A}(t)\mathcal{B}(t)$, we have $R_d(\Phi)$ is an affine function of Φ, which finishes the proof.

A.4 Proof of Theorem 4

First, we show every $\mathbb{M}$-optimal policy for WOSA-η is a Pareto optimal policy for the bi-objective optimization problem indexed by η, for $\eta = 0, 1, \ldots, \eta_o$. Let Φ_1 be an $\mathbb{M}$-optimal policy for WOSA-η, then Φ_1 must satisfy one of the three conditions in Definition 3. If Φ_1 satisfies Definition 3(a), then $\mathbf{w} > 0$, and hence, Φ_1 is a Pareto optimal policy for the bi-objective optimization problem by *Theorem 3.1.2* [20, p. 78].

If Φ_1 satisfies Definition 3(b), then $\mathbf{w} = (w_1, 0)$ with $w_1 > 0$ and $\max_{\Phi \in \Psi^\eta} R_w(\Phi) \Leftrightarrow \max_{\Phi \in \Psi^\eta} R_s(\Phi) = R_s(\Phi_1)$. If Φ_1 is the unique policy that maximizes $R_s(\Phi)$ in Ψ^η, then Φ_1 is a Pareto optimal policy for the bi-objective optimization problem by *Theorem 3.1.3* [20, p. 79]. Otherwise, Φ_1 is not unique and we prove Φ_1 is Pareto optimal for the bi-objective optimization problem by contradiction. Suppose Φ_1 is not Pareto optimal, then there exists another policy $\Phi' \in \Psi^\eta$ such that $R_s(\Phi') \geq R_s(\Phi), R_d(\Phi') \geq R_d(\Phi)$ with at least one strict inequality. Since $R_s(\Phi_1) = \max_{\Phi \in \Psi^\eta} R_s(\Phi)$, then $R_s(\Phi') \leq R_s(\Phi_1)$, and hence, $R_s(\Phi') = R_s(\Phi_1), R_d(\Phi') > R_d(\Phi_1)$. However, this is contradictory to $\mathbb{M}$-optimality of Φ_1. Therefore, Φ_1 is a Pareto optimal policy for the bi-objective optimization problem.

If Φ_1 satisfies Definition 3(c), similar arguments can be applied.

For the reverse direction, we show every Pareto optimal policy for the bi-objective optimization problem indexed by η is $\mathbb{M}$-optimal for WOSA-η, for $\eta = 0, 1, \ldots, \eta_o$. Let $\Phi_2 \in \Psi^{\eta+}$ be a mixed policy, which is Pareto optimal for the bi-objective optimization problem indexed by η. Then, by Proposition 3, $\Psi^{\eta+}$ is convex and the objective functions $R_s(\Phi)$ and $R_d(\Phi)$ are affine functions of Φ. Then from *Theorem 3.1.4* [20, p. 79], there exists a non-negative weight vector $\mathbf{w}$ such that Φ_2 maximizes $R_w(\Phi)$, which is the objective function of WOSA indexed by η and $\mathbf{w}$ defined in (3.4). From § 4, optimal policies for WOSA are all pure policies, and hence, $\Phi_2 \in \Psi^\eta$. We are left to show Φ_2 satisfies one of the three conditions in Definition 3 to prove Φ_2 is $\mathbb{M}$-optimal for WOSA-η.

If $\mathbf{w} > 0$, then Definition 3(a) is satisfied and Φ is $\mathbb{M}$-optimal for WOSA-η.

If $\mathbf{w} = (w_1, 0)$ with $w_1 > 0$, then $R_s(\Phi_2) = \max_{\Phi \in \Psi^\eta} R_s(\Phi)$. We prove $R_d(\Phi_2) = \max_{\Phi \in \Lambda_s^\eta} R_d(\Phi)$ with Λ_s^η defined by Definition 3(b) by contradiction. Suppose $R_d(\Phi_2) \neq \max_{\Phi \in \Lambda_s^\eta} R_d(\Phi)$, then there exists $\Phi' \in \Lambda_s^\eta$ such that

$R_d(\Phi') = \max_{\Phi \in \Lambda_s^\eta} R_d(\Phi)$. Therefore, $R_s(\Phi') = R_s(\Phi_2)$, $R_d(\Phi') > R_d(\Phi_2)$, which is a contradictory to the Pareto optimality of Φ_2. Therefore, Φ_2 is $\mathbb{M}$-optimal for WOSA-η.

If $\mathbf{w} = (0, w_2)$ with $w_2 > 0$, similar arguments can be applied, which finishes the proof.

A.5 Proof of Proposition 4

The proof is based on induction on T. First, we consider the corresponding SSAP with almost-binary success rates for T tasks. In this case, the SSAP optimal policy, denoted by Φ_B, is a direct application of *Theorem 1* given by [19].

Theorem 7 (*Theorem 1*, [19])**.** *For the t^{th} task arrival with the task value $\mathcal{C}_t$, there are $T-t+1$ workers available for $t = 1, 2, \ldots, T$. The thresholds for $\mathcal{C}_t$ are given by $-\infty = a_{0,t} \leq a_{1,t} \leq \ldots \leq a_{T-t+1,t} = +\infty$, obtained based on the recursive equations*

$$a_{i,t} = \int_{a_{i-1,t+1}}^{a_{i,t+1}} x dF_{\mathcal{C}}(x) + a_{i-1,t+1} F_{\mathcal{C}}(a_{i-1,t+1}) + a_{i,t+1}(1 - F_{\mathcal{C}}(a_{i,t+1})), \quad \text{(A.7)}$$

for $i = 1, 2, \ldots, T-t$. If the t^{th} task value $\mathcal{C}_t \in (a_{i-1,t}, a_{i,t}]$, then the worker with the i^{th} smallest success rate among the $T-t+1$ available workers will be assigned to the t^{th} task under the optimal policy. Moreover, $a_{i,t}$ is the expected task value that will be assigned to the worker with i^{th} smallest success rate among the $T-t$ available workers for $i = 1, 2, \ldots, T-t$.

Let $a_{i,t}$ denote the threshold values defined by (A.7) for T, with $i = 1, 2, \ldots, T-t$ and $t = 0, 1, \ldots, T-1$. Then, the first task assignment under policy Φ_B is

$$\tau_{j_1}^{\Phi_B} = \begin{cases} 1, & \text{if} \quad \mathcal{C}_1 > a_{T-\lfloor \Upsilon \rfloor, 1}, \\ \Upsilon - \lfloor \Upsilon \rfloor, & \text{if } a_{T-\lfloor \Upsilon \rfloor - 1, 1} < \mathcal{C}_1 \leq a_{T-\lfloor \Upsilon \rfloor, 1}, \\ 0, & \text{if} \quad \mathcal{C}_1 \leq a_{T-\lfloor \Upsilon \rfloor - 1, 1}. \end{cases} \quad \text{(A.8)}$$

Moreover, the optimal expected assignment reward is achieved under policy Φ_B as $\mathbb{E}[\sum_{t=2}^{T} \tau_{j_t}^{\Phi_B} \mathcal{C}_t] = \sum_{i=T-\lfloor \Upsilon \rfloor + 1}^{T} a_{i,0} + a_{T-\lfloor \Upsilon \rfloor, 0}(\Upsilon - \lfloor \Upsilon \rfloor)$.

When $T = 1$, there is only one task to be assigned to one worker, and hence, Φ_B is trivially optimal for the SSAP with the fixed success rate sum.

Suppose Proposition 4 holds for $T' \leq T-1$. When there are T tasks to be assigned, we need to show $\tau_{j_1}^{\Phi_B}$ is optimal for the first task assignment in the fixed success rate sum scenario, and the remaining $T-1$ task assignments are optimal under policy Φ_B by the induction assumption. Let $a'_{i,t}$ denote the threshold values defined by (A.7) for $T' = T-1$, with $i = 1, 2, \ldots, T-t$ and $t = 0, 1, \ldots, T-2$. We compute the optimal conditional expected reward for T tasks assignments given the first task value,

$$\begin{aligned}
&\max_{\{\tau_{j_t}\}_{t=1}^T} \mathbb{E}[\sum_{t=1}^T \tau_{j_t}\mathcal{C}_t | \sum_{t=1}^T \tau_{j_t} = \Upsilon, \mathcal{C}_1 = x_1] \\
&= \max_{\{\tau_{j_t}\}_{t=1}^T} \mathbb{E}[\tau_{j_1}x_1 + \sum_{t=2}^T \tau_{j_t}\mathcal{C}_t | \sum_{t=1}^T \tau_{j_t} = \Upsilon, \mathcal{C}_1 = x_1] \\
&= \max_{0\leq\tau_{j_1}\leq 1}(\tau_{j_1}x_1 + \max_{\{\tau_{j_t}\}_{t=2}^T} \mathbb{E}[\sum_{t=2}^T \tau_{j_t}\mathcal{C}_t | \sum_{t=2}^T \tau_{j_t} = \Upsilon - \tau_{j_1}]),
\end{aligned} \tag{A.9}$$

where x_1 denotes the realized value of the first task.

The second term on the right-hand side of (A.9) is the optimal expected reward for $T-1$ task assignments with the success rate sum of $T-1$ workers as $\Upsilon - \tau_{j_1}$, and

$$\lfloor \Upsilon - \tau_{j_1} \rfloor = \begin{cases} \lfloor \Upsilon \rfloor - 1, & \text{if } \tau_{j_1} > \Upsilon - \lfloor \Upsilon \rfloor, \\ \lfloor \Upsilon \rfloor, & \text{if } \tau_{j_1} \leq \Upsilon - \lfloor \Upsilon \rfloor. \end{cases} \tag{A.10}$$

Then by the induction assumption, the maximum of the second term on the right-hand side of (A.9) is achieved under the SSAP optimal policy for the almost-binary success rate scenario with $T-1$ workers. Therefore,

$$\begin{aligned}
&\max_{\{\tau_{j_t}\}_{t=2}^{T}} \mathbb{E}[\sum_{t=2}^{T} \tau_{j_t}\mathcal{C}_t | \sum_{t=2}^{T} \tau_{j_t} = \Upsilon - \tau_{j_1}] \\
&= \sum_{i=T-\lfloor \Upsilon-\tau_{j_1} \rfloor}^{T-1} a'_{i,0} + a'_{T-\lfloor \Upsilon-\tau_{j_1}\rfloor-1,0}(\Upsilon - \tau_{j_1} - \lfloor \Upsilon - \tau_{j_1} \rfloor) \\
&= \begin{cases} \sum_{i=T-\lfloor\Upsilon\rfloor+1}^{T-1} a_{i,1} + a_{T-\lfloor\Upsilon\rfloor,1}(\Upsilon - \tau_{j_1} - \lfloor\Upsilon\rfloor + 1), & \text{if } \tau_{j_1} > \Upsilon - \lfloor\Upsilon\rfloor, \\ \sum_{i=T-\lfloor\Upsilon\rfloor}^{T-1} a_{i,1} + a_{T-\lfloor\Upsilon\rfloor-1,1}(\Upsilon - \tau_{j_1} - \lfloor\Upsilon\rfloor), & \text{if } \tau_{j_1} \le \Upsilon - \lfloor\Upsilon\rfloor, \end{cases}
\end{aligned} \tag{A.11}$$

where the second equality follows from (A.10) and the recursive definitions of threshold values (i.e., $a'_{i,0} = a_{i,1}$ for $i = 1, 2, \ldots, T-1$). Then, we substitute (A.11) into (A.9) and compute the optimal expected conditional reward (A.9) for two cases: (a) $\tau_{j_1} > \Upsilon - \lfloor\Upsilon\rfloor$ and (b) $\tau_{j_1} \le \Upsilon - \lfloor\Upsilon\rfloor$, respectively, and then combine them together to obtain the optimal assignment for the first task. Note that (A.9) is an affine function of τ_{j_1} in both cases, and hence, the maximum of (A.9) is achieved at the boundary point of τ_{j_1}. Therefore, in each case, we take derivative with respect to τ_{j_1} and if the derivative is positive, the maximum (supremum) of (A.9) is achieved when τ_{j_1} takes the maximum (supremum) value. Otherwise, the maximum (supremum) of (A.9) is achieved when τ_{j_1} takes the minimum (infimum) value. Therefore, for case (a), $\tau_{j_1} > \Upsilon - \lfloor\Upsilon\rfloor$,

$$\begin{aligned}
&\max_{\{\tau_{j_t}\}_{t=1}^{T}} \mathbb{E}[\sum_{t=1}^{T} \tau_{j_t}\mathcal{C}_t | \sum_{t=1}^{T} \tau_{j_t} = \Upsilon, \mathcal{C}_1 = x_1] \\
&= \begin{cases} \sum_{i=T-\lfloor\Upsilon\rfloor+1}^{T-1} a_{i,1} + x_1 + a_{T-\lfloor\Upsilon\rfloor,1}(\Upsilon - \lfloor\Upsilon\rfloor), & \text{if } x_1 > a_{T-\lfloor\Upsilon\rfloor,1}, \\ \sum_{i=T-\lfloor\Upsilon\rfloor+1}^{T-1} a_{i,1} + x_1(\Upsilon - \lfloor\Upsilon\rfloor) + a_{T-\lfloor\Upsilon\rfloor,1}, & \text{if } x_1 \le a_{T-\lfloor\Upsilon\rfloor,1}, \end{cases}
\end{aligned} \tag{A.12}$$

where the second line is the supremum of the left-hand side and

$$\tau_{j_1} = \begin{cases} 1, & \text{if } x_1 > a_{T-\lfloor\Upsilon\rfloor,1}, \\ \Upsilon - \lfloor\Upsilon\rfloor, & \text{if } x_1 \le a_{T-\lfloor\Upsilon\rfloor,1}, \end{cases}$$

with the second line being the infimum of the left-hand side.

For case (b), $\tau_{j_1} \leq \Upsilon - \lfloor \Upsilon \rfloor$,

$$\max_{\{\tau_{j_t}\}_{t=1}^{T}} \mathbb{E}[\sum_{t=1}^{T} \tau_{j_t} \mathcal{C}_t | \sum_{t=1}^{T} \tau_{j_t} = \Upsilon, \mathcal{C}_1 = x_1]$$
$$= \begin{cases} \sum_{i=T-\lfloor \Upsilon \rfloor}^{T-1} a_{i,1} + x_1(\Upsilon - \lfloor \Upsilon \rfloor), & \text{if } x_1 > a_{T-\lfloor \Upsilon \rfloor -1,1}, \\ \sum_{i=T-\lfloor \Upsilon \rfloor}^{T-1} a_{i,1} + a_{T-\lfloor \Upsilon \rfloor -1,1}(\Upsilon - \lfloor \Upsilon \rfloor), & \text{if } x_1 \leq a_{T-\lfloor \Upsilon \rfloor -1,1}, \end{cases} \tag{A.13}$$

where

$$\tau_{j_1} = \begin{cases} \Upsilon - \lfloor \Upsilon \rfloor, & \text{if } x_1 > a_{T-\lfloor \Upsilon \rfloor -1,1}, \\ 0, & \text{if } x_1 \leq a_{T-\lfloor \Upsilon \rfloor -1,1}. \end{cases}$$

Combining cases (a) and (b) together by comparing (A.13) and (A.12) leads to the optimal first task assignment as

$$\tau_{j_1} = \begin{cases} 1, & \text{if } x_1 > a_{T-\lfloor \Upsilon \rfloor,1}, \\ \Upsilon - \lfloor \Upsilon \rfloor, & \text{if } a_{T-\lfloor \Upsilon \rfloor -1,1} < x_1 \leq a_{T-\lfloor \Upsilon \rfloor,1}, \\ 0, & \text{if } x_1 \leq a_{T-\lfloor \Upsilon \rfloor -1,1}, \end{cases}$$

which is the same as the SSAP optimal policy assignment $\tau_{j_1}^{\Phi_B}$ in the almost-binary success rate scenario given by (A.8). By the induction assumption, the optimal expected reward for assigning the remaining $T-1$ tasks to $T-1$ workers with a fixed success rate sum $\Upsilon - \tau_{j_1}$ is achieved under the SSAP optimal policy Φ_B. This completes the proof.

A.6 Proof of Theorem 6

The proof is by induction on T, using similar techniques as in [19] but specifically trimmed for WOSA with discrete risk distributions.

Let $f(\eta, T)$ denote the optimal expected reward for T passenger assignments with an active selectee capacity η. Further, let $f(\eta, T|\gamma_1)$ denote the optimal conditional expected reward for T passenger assignments with an active selectee capacity η given the first passenger combined risk $\mathcal{G}(1) = \gamma_1$, then

$$f(\eta, T) = \max_{\Phi \in \Psi^\eta} \mathbb{E}[\sum_{t=1}^{T} X_t^\Phi \mathcal{G}(t)],$$
$$f(\eta, T|\gamma_1) = \max_{\Phi \in \Psi^\eta} \mathbb{E}[\sum_{t=1}^{T} X_t^\Phi \mathcal{G}(t)|\mathcal{G}(1) = \gamma_1].$$

When $T = 1$, there is only one passenger to be assigned and $a_{0,1} = -\infty$, $a_{1,1} = +\infty$. Then, under policy (Φ1), this passenger will be assigned to the selectee category if $\eta = 1$ and to the non-selectee category if $\eta = 0$. Therefore, policy (Φ1) is trivially optimal for $T = 1$. Moreover, the expected combined risk value of this passenger is the expected value of $\mathcal{G}(1)$, and from (4.10),

$$a_{i,t} = \sum_{\gamma=G_1}^{G_L} \gamma p_{\mathcal{G}}(\gamma) = \mathbb{E}[\mathcal{G}(1)].$$

Therefore, Theorem 6 holds for $T = 1$.

Suppose Theorem 6 holds for $T' \leq T-1$. Then, policy (Φ1) with threshold values defined by (4.10) maximizes the objective function $R_w(\Phi)$ (3.4) for $T' = T-1$ passengers. Let $\{a'_{i,0}\}_{i=1}^{T'}$ denote the threshold values in the initial stage for T' passengers, which are the expected combined risk values for T' passengers to be assigned by the induction assumption. Let $\{a_{i,t}\}$ denote the threshold values defined by (4.10) for T passengers, for $i = 1, 2, \ldots, T-t$ and $t = 0, 1, \ldots, T-1$. We show the first passenger assignment under policy (Φ1) is optimal for T passengers, and optimal assignments for the remaining $T-1$ passengers follow from the induction assumption. When there are T passengers to be assigned, conditional on the combined risk value of the first passenger,

$$f(\eta, T|\gamma_1) = \max_{X_1^\Phi \in \{0,1\}} (\gamma_1 X_1^\Phi + f(\eta - X_1^\Phi, T-1)). \tag{A.14}$$

Note that $f(\eta - X_1^\Phi, T-1)$ is the optimal expected reward for $T-1$ passenger assignments with an active selectee capacity $\eta - X_1^\Phi$. Then, by the induction assumption, the optimal expected reward for $T-1$ passenger assignments is achieved under policy (Φ1). Since $\{a'_{i,0}\}_{i=1}^{T'}$ are monotonically increasing by

(4.9), and hence,

$$f(\eta - X_1^{\Phi}, T-1) = \sum_{i=(T-1)-(\eta-X_1^{\Phi})+1}^{T-1} a'_{i,0}, \tag{A.15}$$

where the $(\eta - X_1)$ largest expected passenger combined risk values are assigned to the selectee category to maximize (3.4). Substitute (A.15) into (A.14); the optimal policy assigns $X_1 = 1$ if

$$\gamma_1 + \sum_{i=(T-1)-(\eta-1)+1}^{T-1} a'_{i,0} > \sum_{i=(T-1)-\eta+1}^{T-1} a'_{i,0},$$

or equivalently,

$$\gamma_1 > a'_{(T-1)-\eta+1,0} = a'_{T-\eta,0} = a_{T-\eta,1}, \tag{A.16}$$

where the last equality follows from the recursive definitions of threshold values (4.10), $a_{i,1} = a'_{i,0}$ for $i = 1, 2, \ldots, T-1$ (the first stage for T passengers $\{a_{i,1}\}_{i=1}^{T-1}$ are equal to the threshold values in the initial stage for $T-1$ passengers). Therefore, the optimal first passenger assignment given by (A.16) is the same as that given by policy (Φ1). By the induction assumption, the remaining $T-1$ passengers can be assigned under policy (Φ1) to maximize the objective function $R_w(\Phi)$ (3.4). Therefore, policy (Φ1) is optimal for T passengers.

Next, we compute the expected combined risk values for the T passengers to be assigned. By the monotonicity of the threshold values and the induction assumption, $a_{i,1} = a'_{i,0}, i = 1, 2, \ldots, T-1$ is the i^{th} smallest expected combined risk value for the $T-1$ passengers to be assigned. Let $\hat{\mathcal{G}}_T^{(i)}$ (random variable) denote the i^{th} smallest combined risk value for T passengers,

$i = 1, 2, \ldots, T$. Conditioning on the value of $\mathcal{G}(1)$ leads to

$$
\begin{aligned}
&\mathbb{E}[\hat{\mathcal{G}}_T^{(i)}] = \mathbb{E}[\mathbb{E}[\hat{\mathcal{G}}_T^{(i)}|\mathcal{G}(1)]] \\
=&\mathbb{E}[\mathcal{G}(1)|a_{i-1,1} < \mathcal{G}(1) \leq a_{i,1}]\mathbb{P}(a_{i-1,1} < \mathcal{G}(1) \leq a_{i,1}) + \ldots \\
&+ \mathbb{E}[\hat{\mathcal{G}}_{T-1}^{(i-1)}|\mathcal{G}(1) \leq a_{i-1,1}]\mathbb{P}(\mathcal{G}(1) \leq a_{i-1,1}) + \mathbb{E}[\hat{\mathcal{G}}_{T-1}^{(i)}|\mathcal{G}(1) > a_{i,1}]\mathbb{P}(\mathcal{G}(1) > a_{i,1}) \\
=&\left(\sum_{\gamma'=g_{i-1,1}^u}^{g_{i,1}^l} \gamma' p_{\mathcal{G}}(\gamma')\right) + a_{i-1,1}F_{\mathcal{G}}(a_{i-1,1}) + a_{i,1}(1 - F_{\mathcal{G}}(a_{i,1})) \\
=&a_{i,0},
\end{aligned}
$$

with $g_{i,1}^l$ and $g_{i-1,1}^u$ given by (4.11). Therefore, the threshold values in the initial stage $\{a_{i,0}\}_{i=1}^T$ are the expected combined risk values for the T passengers to be assigned. This completes the proof.

A.7 Proof of Proposition 5

The proof is based on induction on T. When $T = 1$, there is only one passenger to be assigned, and hence, $i = 1,\ t = 0$. The expected value of $\hat{\mathcal{A}}_1^{(1)}$ is just the expectation of $\mathcal{A}(t)$. From Proposition 5, only $b_{1,0}$ is defined by (5.2) when $T = 1$, which is given by

$$b_{1,0} = \sum_{\gamma'=G_1}^{G_L} \mathbb{E}[\mathcal{A}(t)|\mathcal{G}(t) = \gamma']p_{\mathcal{G}}(\gamma') = \mathbb{E}[\mathcal{A}(t)]. \tag{A.17}$$

Therefore, Proposition 5 holds for $T = 1$.

Suppose Proposition 5 holds for $T' \leq T - 1$ and $\{b'_{i,t}\}$ are defined by (5.2) for $T-1$, for $i = 1, 2, \ldots, T-1-t$ and $t = 0, 1, \ldots, T-2$. Then we show that (5.2) holds for T with $\{b_{i,t}\}$, for $i = 1, 2, \ldots, T - t$ and $t = 0, 1, \ldots, T - 1$. From the recursive definitions of $\{a_{i,t}\}$ and (5.2), $\{b_{i,t}\}$ are the same as $\{b'_{i,t-1}\}$ for $t = 1, 2, \ldots, T - 1$. We are left with $\{b_{i,0}\},\ i = 1, 2, \ldots, T$, which need to be verified as the expected value of $\hat{\mathcal{A}}_T^{(i)}$ for T passengers. Since the threshold values $\{a_{i,t}\}$ are monotonically increasing with respect to i, then conditioning on the combined risk value of the first passenger $\mathcal{G}(1)$ leads to

$$\begin{aligned}
&\mathbb{E}[\hat{\mathcal{A}}_T^{(i)}] = \mathbb{E}[\mathbb{E}[\hat{\mathcal{A}}_T^{(i)}|\mathcal{G}(1)]] \\
=&\mathbb{E}[\hat{\mathcal{A}}_T^{(i)}|a_{i-1,1} < \mathcal{G}(1) \leq a_{i,1}]\mathbb{P}(a_{i-1,1} < \mathcal{G}(1) \leq a_{i,1}) + \dots \\
&+ \mathbb{E}[\hat{\mathcal{A}}_T^{(i)}|\mathcal{G}(1) \leq a_{i-1,1}]\mathbb{P}(\mathcal{G}(1) \leq a_{i-1,1}) + \mathbb{E}[\hat{\mathcal{A}}_T^{(i)}|\mathcal{G}(1) > a_{i,1}]\mathbb{P}(\mathcal{G}(1) > a_{i,1}) \\
\overset{(a)}{=}&\mathbb{E}\left[\mathbb{E}[\mathcal{A}(1)|\mathcal{G}(1)]|a_{i-1,1} < \mathcal{G}(1) \leq a_{i,1}\right]\mathbb{P}(a_{i-1,1} < \mathcal{G}(1) \leq a_{i,1}) + \dots \\
&+ b_{i-1,1}\mathbb{P}(\mathcal{G}(1) \leq a_{i-1,1}) + b_{i,1}\mathbb{P}(\mathcal{G}(1) > a_{i,1}) \\
\overset{(b)}{=}&\left(\sum_{\gamma'=g_{i-1,1}^u}^{g_{i,1}^l} \mathbb{E}[\mathcal{A}(t)|\mathcal{G}(t) = \gamma']\right) + b_{i-1,1}F_{\mathcal{G}}(a_{i-1,1}) + b_{i,1}(1 - F_{\mathcal{G}}(a_{i,1})) \\
=&b_{i,0},
\end{aligned}$$

where: equality (a) follows from the induction assumption; and equality (b) follows from the fact that $\mathcal{A}(t)$ and $\mathcal{G}(t)$ are both IID. Therefore, $b_{i,0}$ defined by (5.2) is the expected value of $\hat{\mathcal{A}}_T^{(i)}$ for $i = 1, 2, \dots, T$, which completes the proof.

REFERENCES

[1] Centers for Disease Control and Prevention, "2014 Ebola Outbreak in West Africa," http://www.cdc.gov/vhf/ebola/outbreaks/2014-west-africa/, 2015, retrieved Dec. 17, 2015.

[2] U.S. Department of Homeland Security, "DHSs Coordinated Response to Ebola," http://www.dhs.gov/ebola-response, Aug 2015, retrieved Aug. 27, 2015.

[3] Centers for Disease Control and Prevention, "Fact Sheet: Screening and Monitoring Travelers to Prevent the Spread of Ebola," http://www.cdc.gov/vhf/ebola/travelers/ebola-screening-factsheet.html, Jun 2015, retrieved Aug. 27, 2015.

[4] Centers for Disease Control and Prevention, "Emergency Department Evaluation and Management of Patients under Investigation for Ebola Virus Disease," http://www.cdc.gov/vhf/ebola/pdf/ed-algorithm-management-patients-possible-ebola.pdf, May 2015, retrieved Aug. 27, 2015.

[5] Centers for Disease Control and Prevention, "Epidemiologic Risk Factors to Consider when Evaluating a Person for Exposure to Ebola Virus," http://www.cdc.gov/vhf/ebola/exposure/risk-factors-when-evaluating-person-for-exposure.html, May 2015, retrieved Aug. 27, 2015.

[6] Centers for Disease Control and Prevention, "Interim U.S. Guidance for Monitoring and Movement of Persons with Potential Ebola Virus Exposure," http://www.cdc.gov/vhf/ebola/exposure/monitoring-and-movement-of-persons-with-exposure.html, Jun 2015, retrieved Aug. 27, 2015.

[7] L. McLay, S. Jacobson, and A. Nikolaev, "A sequential stochastic passenger screening problem for aviation security," *IIE Transactions*, vol. 41, no. 6, pp. 575–591, 2009.

[8] A. Lee, L. McLay, and S. Jacobson, "Designing aviation security passenger screening systems using nonlinear control," *SIAM Journal on Control and Optimization*, vol. 48, no. 4, pp. 2085–2105, 2009.

[9] L. McLay, A. Lee, and S. Jacobson, "Risk-based policies for airport security checkpoint screening," *Transportation Science*, vol. 44, no. 3, pp. 333–349, 2010.

[10] A. Lee and S. Jacobson, "Evaluating the effectiveness of sequential aviation security screening policies," *IIE Transactions*, vol. 43, no. 8, pp. 547–565, 2011.

[11] A. Nikolaev, A. Lee, and S. Jacobson, "Optimal aviation security screening strategies with dynamic passenger risk updates," *Intelligent Transportation Systems, IEEE Transactions on*, vol. 13, no. 1, pp. 203–212, 2012.

[12] C. Althaus, "Estimating the reproduction number of Ebola virus (EBOV) during the 2014 outbreak in West Africa," *PLoS currents*, vol. 6, 2014, published online Sep. 2, 2014.

[13] Centers for Disease Control and Prevention, "Q and As on Transmission," http://www.cdc.gov/vhf/ebola/transmission/qas.html, Jun 2015, retrieved Aug. 27, 2015.

[14] J. Corsi, "Ebola victims without symptoms can still be contagious," Oct. 8, 2014. http://www.wnd.com/2014/10/ebola-victims-without-symptoms-could-still-be-contagious/, 2014, retrieved Jul. 14, 2015.

[15] Centers for Disease Control and Prevention, "Signs and Symptoms," http://www.cdc.gov/vhf/ebola/symptoms/, Nov 2014, retrieved Aug. 27, 2015.

[16] Centers for Disease Control and Prevention, "Flu Symptoms & Severity," http://www.cdc.gov/flu/about/disease/symptoms.htm, Aug 2015, retrieved Aug. 27, 2015.

[17] WHO Regional Office for Africa, "Contact tracing during an outbreak of Ebola virus disease," http://www.who.int/csr/resources/publications/ebola/contact-tracing/en/, Sep 2014, retrieved Aug. 27, 2015.

[18] Centers for Disease Control and Prevention, "Contact tracing can stop Ebola in its tracks," http://www.cdc.gov/vhf/ebola/outbreaks/what-is-contact-tracing.html, Dec 2014, retrieved Aug. 27, 2015.

[19] C. Derman, G. Lieberman, and S. Ross, "A sequential stochastic assignment problem," *Management Science*, vol. 18, no. 7, pp. 349–355, 1972.

[20] K. Miettinen, *Nonlinear Multiobjective Optimization*. New York, the United States: Springer Science & Business Media, 1999, vol. 12.

[21] S. Jones, "CDC: 150 People Enter U.S. Per Day from Ebola-Stricken Countries–or 4,500 Per Month," http://cnsnews.com/news/article/susan-jones/cdc-150-people-enter-us-day-ebola-stricken-countries-or-4500-month, Oct 2014, retrieved Sep. 10, 2015.

[22] U.S. Department of Health and Human Services, "HHS selects nine regional Ebola and other special pathogen treatment centers," http://www.hhs.gov/news/press/2015pres/06/20150612b.html?utm_src=email, Jun 2015, retrieved Aug. 27, 2015.

[23] Centers for Disease Control and Prevention, "Monitoring of Persons with Risk for Exposure to Ebola Virus Disease United States, November 3, 2014March 8, 2015," http://www.cdc.gov/mmwr/preview/mmwrhtml/mm6425a1.htm?s_cid=mm6425a1_e, Jul 2015, retrieved Aug. 27, 2015.

[24] R. Daigrepont and C. Ladd, "First Imported Case of Ebola Confirmed in the United States: Are Hospitals Prepared?" Oct. 2, 2014. http://www.wittobriens.com/go/doc/2000/2370506/First-Imported-Case-of-Ebola-Confirmed-in-the-United-States-Are-Hospitals-Prepared-, 2014, retrieved Aug. 27, 2015.

[25] I. Das and J. Dennis, "A closer look at drawbacks of minimizing weighted sums of objectives for Pareto set generation in multicriteria optimization problems," *Structural optimization*, vol. 14, no. 1, pp. 63–69, 1997.

[26] S. Jacobson, G. Yu, and J. Jokela, "A Double-risk Monitoring and Quarantine Policy for Ebola Entry Screening at Airports in the United States," 2015, technical report.