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NETWORK ANALYSIS OF CONJUNCTIVELY OPERATED GROUND WATER-SURFACE WATER SYSTEMS

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ABSTRACT

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The concept of network models is introduced and a general network model for a multiple purpose, multiple unit water resource system is developed. The "out-of-kilter" algorithm is then presented as a solution technique for network flow problems.

A model for preliminary screening of alternatives for water supply from conjunctive use of ground water and surface water in the Kaskaskia River Basin in Illinois is presented and analyzed. The results from the analysis demonstrate how the network analysis procedures can be used to determine optimum investment plans, to test policy constraints, and to develop policies for future development and future constraints.

Hamdan, Abdullatif S. and Dale D. Meredith
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PREFACE

This final report is for the OWRR Project A-059-ILL entitled "Management of Conjunctively Operated Ground Water-Surface Water Systems Using the Theory of Network Flows" and covers a study period of July 1972 to January 1974. The report is based in part upon the Ph.D. thesis "Network Approach to Management of Conjunctively Operated Ground Water-Surface Water Systems" prepared by A.S. Hamdan under the supervision of D.D. Meredith.

The theory of network flows was first applied to a certain family of linear programming problems known as the "transportation problem." A number of algorithms and theorems have been developed and the approach has been applied in the last two decades to successfully formulate and solve a large number of military and industrial management problems. These include problems of minimizing the cost of a given flow of a certain commodity in a network, maximizing flow of some commodity in a network, and coordination of activities in a network. Among the reasons why the theory of network flows has been so popular is the fact that it can solve problems with thousands of variables and hundreds of constraints. Because a ground water-surface water system may be thought of as a network through which water flows, it was believed that network flow theory might be useful in solving some problems in water resources.

The Kaskaskia River Basin in Illinois was used as a case study to demonstrate the modeling of ground water-surface water systems by networks. To indicate the potential applicability of network flow theory to water resources planning, the "out-of-kilter" algorithm was used to determine optimum investment plans under different policy constraints for the Kaskaskia River Basin network model.

The following thesis was completed:

Hamdan, A.S., "Network Approach to Management of Conjuntively Operated Ground Water-Surface Water Systems," Ph.D. thesis, University of Illinois at Urbana-Champaign, Urbana, Illinois, pages x + 187, 1973.

The help and patience of Louise Dumain in typing the report is appreciated.

Buffalo, N.Y. Dale D. Meredith

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1. INTRODUCTION

1.1 Background

Maximum water development can only be achieved through conjunctive (joint) operation of ground water and surface water reservoirs. In the 1950's the advantages of this approach were recognized and studies were qualitative in nature (Kazmann, 1951; Banks, 1953).

The quantitative aspects of the approach were emphasized in the 1960's. This appears in the works of Buras (1962), Chun et al (1964), Domenico et al (1966), Dracup (1966), Aron (1969) and Milligan (1970). Research was aimed at optimization of conjunctively operated water resource systems using linear and dynamic programming. Advantages and disadvantages of using linear and dynamic programming to optimize water resources systems have been reviewed (Chow and Meredith, 1969b; Dracup et al, 1972) and the literature on this subject has been documented (Chow and Meredith, 1969a; Gysi and Loucks, 1969; Kriss and Loucks, 1971; Loucks, 1972).

"Network flow theory" is a branch of linear programming theory that was first applied to a certain family of linear programming problems known as the"transportation problem." A number of algorithms and theorems have been developed and the approach has been applied in the last two decades to successfully formulate and solve a large number of military and industrial management problems. These include problems of minimizing cost of a given flow of a certain commodity in a network (Fulkerson, 1961), maximizing flow of some commodity in a network (Ford and Fulkerson, 1962), and other problems (Elmaghraby, 1970).

Among the reasons why the theory of network flows has been so successful is the fact that it can solve problems with thousands of variables and hundreds of constraints, which cannot be solved by other techniques (Ford and Fulkerson, 1962).

Using the Trans-Texas division of the planned Texas Water System as a case study, the Texas Water Development Board (1970) used the "out-of-kilter algorithm," of the theory of network flows in a 4-phase research project to develop and apply planning techniques to the planning of complex water resources systems. In the first phase the algorithm was used to estimate canal and reservoir sizes and to find reservoir operating rules. In the last phase the algorithm was used to evaluate and improve alternative development plans.

The Texas study was one of the first attempts to use network flow theory in the planning of water resources systems. The use of network flow theory concepts and analysis procedures in planning and management of water resources systems has not been fully explored.

1.2 Objective

The objective of this study is to demonstrate the application of network flow theory concepts and analysis procedures in water resources system planning and management. The development of a network model for a case study area and the application of network analysis procedures to provide answers to planning and management queries are demonstrated. The demonstration of the methodology of model development and network flow theory analysis procedures is felt to be more important than the model per se.

1.3 Scope

A demonstration of the application of network flow theory concepts and analysis procedures in water resource systems planning and management is presented in the remainder of this report.

In Chapter 2 the concept of network models is introduced and a general network model for a multiple purpose, multiple unit water resource system is developed. The "out-of-kilter" algorithm is then presented as a solution technique for network flow problems.

In Chapter 3 the Kaskaskia River Basin in Illinois is presented as a case study to demonstrate the modeling of ground water-surface water systems by network models. The model is for preliminary screening of alternatives for water supply from conjunctive use of ground water and surface water for a 50 year planning period.

The results from the analysis of the Kaskaskia River Basin case study are discussed in Chapter 4. The discussion demonstrates how the analysis procedures can be used to determine optimum investment plans, to test policy constraints, and to develop policies for future development and future constraints.

The summary and conclusions are presented in Chapter 5.

2. ANALYSIS PROCEDURE

2.1 Purpose

The analysis procedure provides the planner with a rigorous means for evaluation of alternative water resource systems. It includes the out-of-kilter algorithm which is a formal optimization technique.

The general purposes of the procedure are: (1) to evaluate alternative water resource development plans, (2) to find the sizes of system elements which correspond to superior alternatives, (3) to estimate timing for investment in water supply expansion, and (4) to indicate areas for nongrowth emphasis.

2.2 Concepts

The out-of-kilter algorithm is an optimization technique derived from network flow theory which has the capability to solve problems which can be stated in terms of flows and costs as

$$\text{Minimize } Z = \sum_{ij} c_{ij} q_{ij} \tag{2.1}$$

subject to:

$$\sum_{i=1}^{N} q_{ij} - \sum_{i=1}^{N} q_{ji} = 0 \qquad j = 1, \ldots M \tag{2.2a}$$

$$b_{ij} \leq q_{ij} \leq u_{ij} \quad \text{for all } ij \tag{2.2b}$$

in which c_{ij} is the cost of transferring one unit of flow from location i to location j; q_{ij} is the number of units of flow from location i to location j; q_{ji} is the number of units of flow from location j to location i; b_{ij} is the lower limit for q_{ij}; and u_{ij} is the upper limit for q_{ij}. The flow can represent any commodity, either real or conceptual. The locations are usually called nodes or junctions and represent points in time or space.

It often happens that the objectives of a water resource system are so well defined in terms of water demands that a minimization-of-cost criterion function (also sometimes referred to as an objective function) is appropriate. However, it may be more desirable to maximize the net returns. Although maximization of net returns and minimization of gross costs are two different forms of the objective function, it is possible to convert one form into the other by a simple mathematical manipulation. Constraints being the same, an alternative that maximizes the value of a criterion function Z(x,y) minimizes the value of another function Z'(x,y) defined as

$$Z'(x,y) = -Z(x,y) \tag{2.3a}$$

or

$$Z'(x,y) = L - Z(x,y) \tag{2.3b}$$

in which L is a constant larger than any value Z(x,y) may obtain.

Hence, the word "cost" may be interpreted to mean either the actual cost or some appropriately defined function of net returns synonymous with Z(x,y) above. In view of this interpretation one can use a minimization-of-"cost" criterion function in order to minimize actual costs or to maximize net returns.

A network structure, uniquely required for the analysis procedure, was devised to represent the space and time continuum for which the optimal solution was desired. This structure and the mathematical statements which characterize it are described below.

2.3 Network Structure

2.3.1 General

The elements of the water resource system is first represented as a network of arcs (branches, links) and nodes. Arcs fall into two groups: arcs which represent elements through which flow actually occurs and arcs which represent elements through which flow conceptually occurs. Flow actually occurs in elements such as river reaches or pipelines. Carry-over storage in a reservoir might be thought of as flow: the water remains in storage and no flow occurs but it can be presented as flowing from one time period to the next.

Nodes also fall into two groups, supply nodes and use nodes. A supply node represents a source where a supply of water can be obtained, i.e., a surface water reservoir, a ground water aquifer, an imported water source, etc. A use node represents a point where water is utilized to achieve some purpose, i.e., water supply, irrigation, hydroelectric power, artificial ground water recharge, etc.

Table 2.1 Symbols and Terms Used to Represent Water Resource Systems

symbol	system analog
$\xrightarrow{(b_{ij},\ u_{ij}, c_{ij})}$	arc
○	node
b_{ij}	lower bound for flow in arc between node i and node j
u_{ij}	upper bound for flow in arc between node i and node j
c_{ij}	cost to transfer one unit of flow from node i to node j

Table 2.1 contains the symbols used to illustrate a water resource system as a network of arcs and nodes.

Almost all water resource allocation systems may be rightly formulated, in more than one way, as networks amenable to network solution methods. The minimum-cost flow network model has been chosen here for two reasons: (1) the method is not restricted to capacitated flow networks, but it is also applicable to bounded flow networks which are the only ones that can represent real, complex water resource systems; and (2) to take advantage of the powerful out-of-kilter algorithm as a solution technique. Capacitated networks are those which have a lower bound of zero on all arcs and a finite maximum capacity for one or more arcs. Bounded flow networks are those which have one or more arcs with both an upper and lower bound different from zero.

Network structures for allocation of water from multiple sources to multiple use are presented for (1) a single time period and (2) for multiple time periods.

2.3.2 Spatial Representation

In this case water is to be allocated from N supplies, $N > 1$, each supply contains r_n units (e.g. r_n acre-feet) of water, $n \epsilon N$, to M uses, $M > 1$, each use demands d_m units of water, $m \epsilon M$, at a unit cost of c_{nm} from supply n to use m, $n \epsilon N$ and $m \epsilon M$. Where $n \epsilon N$ means for all values of n in the set N.

A network that can represent this system is shown in Fig. 2.1. The network consists of N + M nodes representing the supplies and uses plus two additional nodes which constitute a network source and a network sink. The source of the network is connected to each supply node, the sink is connected to each use node, and each supply node is connected to each use node by an arc with a source-sink orientation, i.e., the direction of flow in the arc is from the source toward the sink (flow-direction is indicated by arrow). The network source and the network sink are artificial nodes which have been added to transform the network into a flow network with a single source and a single sink in order to take advantage of the solution algorithm described below.

The upper and lower bounds and costs for a unit of flow in each arc must be assigned such that the network model represents the actual physical system. The minimum amount of water available at supply node 1 would be 0 units and the maximum amount available at supply node 1 is r_1 units. Therefore, flow in the arc from the network source to supply node 1 has a lower bound of 0 and an upper bound of r_1. There is no cost associated with the conceptual flow from the network source (or origin) to supply node 1, therefore c_{01} is equal to 0.

The flow in the arc from supply node 1 to use node m represents the amount of water allocated from supply 1 to use m. The amount of

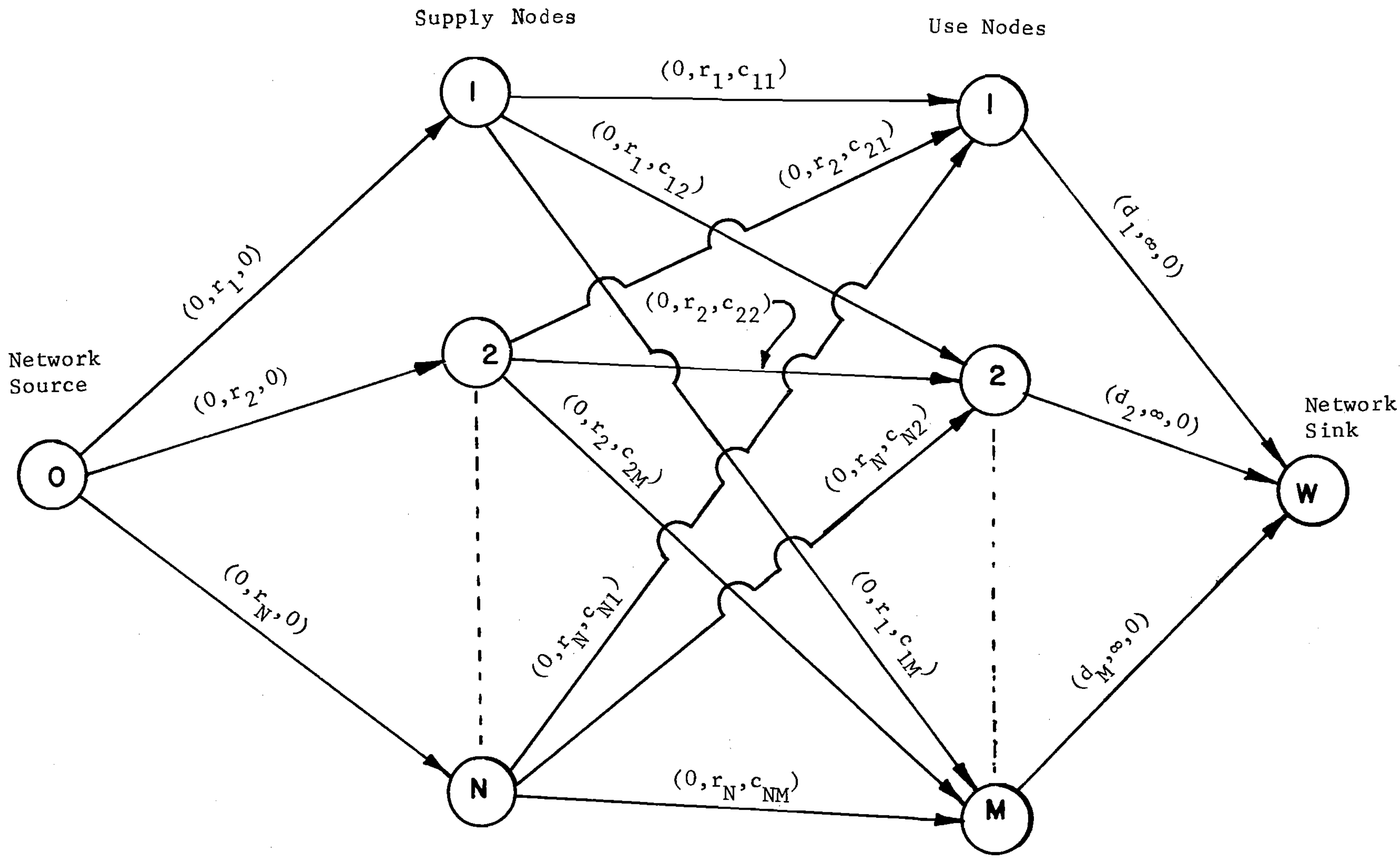

Fig. 2.1 Spatial Allocation

water that can be allocated from supply node 1 to any use node m, q_{1m}, must be greater than or equal to 0 and less than or equal to r_1. Therefore, the lower and upper bounds on the flow in this arc are 0 and r_1 respectively. The cost to allocate one unit of water from supply node 1 to use node m is c_{1m}.

The flow in the arc from the use node m to the network sink represents the amount of water utilized by use m and, therefore, its lower bound is the water required for this use, d_m. If it is assumed that only d_m units of water can be utilized at use node m, the upper bound for the flow in the arc from use node m to the network sink is d_m. However, because the criterion function is minimize total cost for flow in the network, the solution will result in the minimum flow coming to use node m such that the demand d_m is satisfied. This allows the use of a large value for the upper bound on the flow in the arc from use node m to the sink without any effect on the solution. There is no cost associated with the flow in the arc and thus $c_{mw} = 0$.

The lower and upper bounds and the unit costs can be assigned for flow in the other arcs by following the above method. The network is a bounded flow network through which a minimum cost flow corresponding to the optimal allocation schedule can be found providing that the total supply available equals or exceeds the total demand, or

$$\sum_{n=1}^{N} r_n \geq \sum_{m=1}^{M} d_m \tag{2.4}$$

2.3.3 Spatial and Temporal Representation

Here the spatial allocation model is extended such that the water can be allocated over T time periods, T > 1. Carry-over storage from one time period to another constitutes an additional characteristic of the network model of the physical system. In this case water is to be allocated over T time periods, $T > 1$, from N supplies, $N > 1$, with r_{nt} units, to M uses, $M > 1$, with demands of d_{mt} units, at unit costs of c_{nmt}, $n \epsilon N$, $m \epsilon M$, $t \epsilon T$.

A network model of this system is shown in Fig. 2.2. The network consists of N x T supply nodes, M x T use nodes, and two artificial nodes representing the network source and sink. Each supply node is connected to the source node and each use node is connected to the sink node by a directed arc (flow in direction of arrow). The supply nodes are connected to the use nodes at each time period by directed arcs representing allocation alternatives. The nodes representing a particular supply at successive time periods are connected by directed arcs whose flows represent carry-over storage. All arcs have a source to sink orientation.

The flows in the arcs from the network source to supply nodes for $t = 1$ represent the initial conditions for the supplies. The upper bound is the initial storage conditions. This arc can be considered as a special type of carry-over storage. The flows in the arcs from the network source to the supply nodes for $t > 1$ represent the net input to that supply during that time period. The lower bound for these arcs with $t > 1$ is 0 and the upper bound is the maximum net input available during that period. The flows in the arcs from the use nodes to the network sink are defined and bounded as in the spatial allocation problem. The flows in the arcs connecting supply nodes in successive time periods represent carry-over storage and the lower bound is zero, or the minimum permissible water level for the storage facility, and the upper bound is the maximum storage capacity of the particular supply source. The

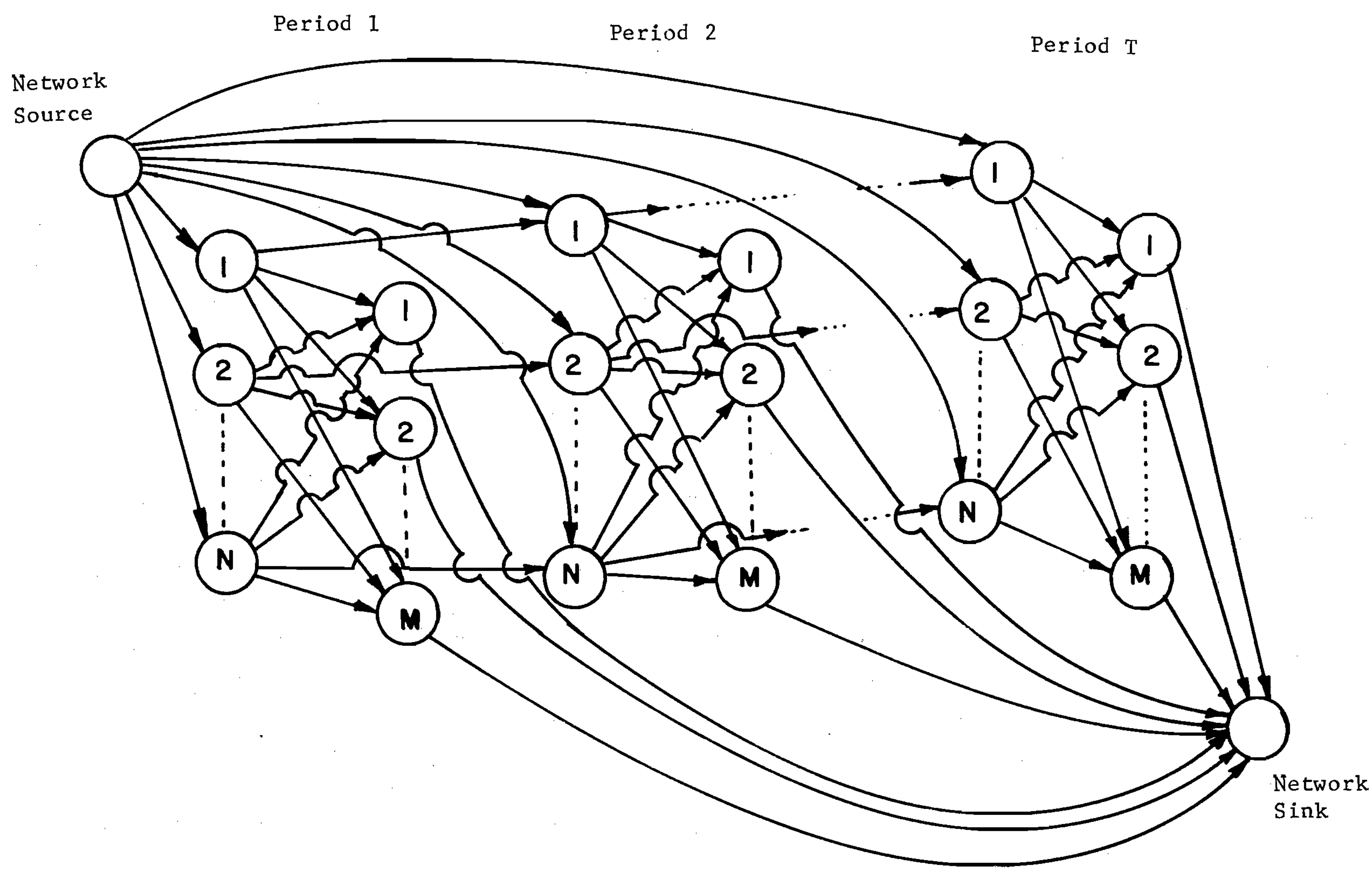

Fig. 2.2 Spatial and Temporal Allocation

unit costs for these flows are unit costs for storage.

The network in Fig. 2.2 is a bounded flow network through which a minimum cost flow corresponding to the optimal allocation schedule for the system can be found providing that

$$\sum_{n=1}^{N} r_{nt} \geq \sum_{m=1}^{M} d_{mt} \qquad \text{for } t = 1, 2, \ldots.. T \qquad (2.5)$$

in which r_{nt} is the total amount of water available at node n at time t, i.e., r_{nt} equals the carry over storage plus input.

The formulation above assumes that all of the available input during a period does not have to be accounted for. This means that if the input to a supply source plus the carry over storage from the previous time period minus the releases for demands from the various uses is greater than the storage capacity for carry over storage to the next time period the difference is not accounted for in the model,i.e., part of the input is dumped out of the basin in the case of streamflow into a reservoir or unused in the case of available water for import.

All of the input for a particular supply can be forced into the system by setting the lower bound equal to the upper bound on that input arc. Then in order to take care of any necessary spill from a supply source another use node can be added to represent a downstream channel to which spills are made. The water from this downstream channel can then flow out of the system as represented by flow in an arc from this use node to the network sink or it can flow into another supply source as input for the next time period as represented by flow in an arc from this use node to another supply source at the next time period. The lower and upper bounds for the flow in the arc from a supply node to this new use node would be the same as any other arc from the supply node. The unit cost might be zero or it could be positive if the spill resulted in flooding. The lower bound on the flow in the arc from this use node would be zero and the upper bound would be the same as the upper bound on the inflow to the node. The unit cost for flow in the arc from this use node would be zero unless the flow was pumped to a supply source with a higher elevation in the next time period in which case the unit cost would be positive.

A final storage content for a particular supply source can be specified by setting the lower bound on the flow in an arc from the supply node to the network sink at time $t = T$ equal to the desired final storage value.

Flows in rivers and pipe lines are normally permitted to vary between zero and a specific upper limit; however, lower limits can be raised, for example, to provide for quality control or to guarantee prior rights to appropriated water. Storage contents are likely to be constrained between zero and some design capacity which may be predetermined or, may itself, be a part of the problem solution. Upper limits may be stipulated as zero for elements not available for service and the limit raised to storage capacity when each element is added.

Initial reservoir storage contents, final reservoir storage contents, inputs, and demands can be forced into the model by setting upper and lower bounds identical. An assumption is that evaporation losses can be estimated a priori. Imported water is limited between zero and the maximum available. Flows in spill arcs are limited between zero and the maximum capacities of spillways or outlet works. Flow in a final storage arc is normally limited to between zero and the actual storage capacity.

Table 2.2 contains a summary of node types, arc types, flows, and upper and lower bounds and costs for flows.

Table 2.2 Node Types, Arc Types, Flows, Upper and Lower Bounds, and Costs for Flows

Nodes	Arc Types (Inflows/Outflows)		Flow Bounds		Costs
			Lower	Upper	
Supply Nodes:					
Surface Reservoir	Inflows	Natural streamflow	Streamflow during period	Streamflow during period	None
		Water transferred from another reservoir	0	Capacity of other reservoir plus inflow to other reservoir	Transmission & capital
		Initial conditions	Initial Conditions	Initial conditions	None
		Carry-over from previous period	0	Maximum reservoir capacity	Storage
	Outflows	Allocation to each use	0	Reservoir capacity plus inflow or maximum demand*	Transmission & treatment
		Allocation to aquifer (artificial recharge)	0	Reservoir capacity plus inflow or maximum recharge*	Transmission & artificial recharge operational cost & capital
		Water transferred to another reservoir	Low flow augmentation req.	Reservoir capacity plus inflow	Transmission & capital
		Carry over to next period	0	Maximum reservoir capacity	Storage
Ground Water Aquifer	Inflows	Carry over from previous period	0	Maximum aquifer capacity	None
	Outflows	Allocation to each use	0	Maximum aquifer capacity or maximum demand*	Transmission & treatment
		Water left in storage for carry over	0	maximum aquifer capacity	None
Use Nodes					
Use	Inflows	Allocation from reservoir	0	reservoir capacity plus inflow or maximum demand*	Transmission & treatment
		Allocation from aquifer	0	maximum aquifer capacity or maximum demand*	Transmission & treatment & capital
	Outflows	Allocation to the network sink	demand during period	demand during period	None
Ground Water Aquifer	Inflows	Water left in storage from previous periods	0	maximum aquifer capacity	None
		Natural recharge to aquifer	specified natural recharge	specified natural recharge	None
		Allocation from reservoir	0	maximum reservoir capacity & inflow or maximum recharge*	Transmission & artificial recharge operation cost
		Recycled water	0	specified portion of total water used during period	Treatment & transmission & artificial recharge operational cost
	Outflows	Carry over to the next period	0	maximum aquifer capacity	None
Artificial Nodes	Inflows	None			
Network Source	Outflows	Natural streamflow at reservoir	streamflow during period	streamflow during period	None
		Natural recharge to aquifer	natural recharge during period	natural recharge during period	None
		Initial conditions	initial conditions	initial conditions	None
Network Sink	Inflows	Downstream flow from reservoir	low flow augmentation require	maximum reservoir capacity & inflow	None or flood damanges
		Flow from demand region	region demand	region demand	None
	Outflows	None	0		None

*Take smaller value.

2.4 Mathematical Description

The mathematical description of the network flow problem consists of a criterion (objective) function and constraint equations. The criterion function is minimize the total cost of flow through the network and is given by

$$\text{Minimize } Z = \sum_{t=1}^{T} \sum_{n=1}^{N} c_{ont}\, q_{ont} + \sum_{t=1}^{T} \sum_{n=1}^{N} \sum_{m=1}^{M} c_{nmt} q_{nmt} + \sum_{t=1}^{T} \sum_{m=1}^{M} \sum_{n=1}^{N} c_{mnt} q_{mnt} + \sum_{t=1}^{T} \sum_{m=1}^{M} c_{mwt} q_{mwt} \qquad (2.6)$$

in which the first set of terms on the right hand side of the equation is the total cost of flow from the network source (origin) to the supply nodes, the second set of terms is the total cost of flow from the supply nodes to the use nodes, the third set of terms is the total cost of recycled flow from the use nodes back to supply nodes for use in the next period of time and the fourth set of terms is the cost of flow from the use nodes to the network sink.

One set of constraint equations requires that continuity be satisfied at the supply nodes. Hence,

$$\sum_{m=1}^{M} q_{mnt} - \sum_{m=1}^{M} q_{nmt} = 0 \qquad n = 1,\ldots,N \quad t = 1,\ldots,T \qquad (2.7)$$

in which the first summation yields the total flow into the supply node and the second summation yields the total flow out of the node. Continuity must also be satisfied at the use nodes. Thus,

$$\sum_{n=1}^{N} q_{nmt} - \sum_{n=1}^{N} q_{mnt} = 0 \qquad m = 1,\ldots,M \quad t = 1,\ldots,T \qquad (2.8)$$

The remaining constraint equations describe the upper and lower limits on flow in arcs of the network. Therefore

$$b_{ont} \leq q_{ont} \leq u_{ont} \qquad n = 1,\ldots,N \quad t = 1,\ldots,T \qquad (2.9a)$$

$$b_{nmt} \le q_{nmt} \le u_{nmt} \qquad \begin{array}{l} n=1,\ldots,N \\ m=1,\ldots,M \\ t=1,\ldots,T \end{array} \qquad (2.9b)$$

$$b_{mnt} \le q_{mnt} \le u_{mnt} \qquad \begin{array}{l} m=1,\ldots,M \\ n=1,\ldots,N \\ t=1,\ldots,T \end{array} \qquad (2.9c)$$

$$b_{mwt} \le q_{mwt} \le u_{mwt} \qquad \begin{array}{l} m=1,\ldots,M \\ t=1,\ldots,T \end{array} \qquad (2.9d)$$

Eqs. (2.9a) are the constraints for flow from the network source to the supply nodes. Constraints for flow from supply nodes to use nodes are given by equations (2.9b). Equations (2.9c) are the constraints for recycled flow from use nodes to source nodes. Constraints on flow from use nodes to the network sink are given by equations (2.9d).

Equations (2.6) through (2.9) are the common form required by the out-of-kilter algorithm.

2.5 Solution Procedure

2.5.1 General

The out-of-kilter algorithm will not be proved here. It will be described in general and then outlined in step form following Ford and Fulkerson (1962). In this method minimization of cost is made for a circulating flow, rather than a one-way flow in a network. When flow circulates in a network, the network becomes source and sink free and it is a circulation network. A single source and single sink network can easily be transformed into a circulation network by adding an artificial arc directed from the sink to the source and having lower and upper bounds and a unit cost for flow such that no additional flow constraints are imposed on the flow in the original network.

The out-of-kilter algorithm has the following important advantages. First, it can accommodate lower bounds as well as upper bounds for each arc flow. Second, it can be initiated with any circulation flow, feasible or not. Also, the status of no arc of the network is worsened at any step of the computation.

In order to avoid the cumberson notation of equations (2.6) through (2.9), number the nodes of the network consecutively from 1 to V with the network source as node 1 and the network sink as node V. Once the upper and lower bounds and the unit costs for flows have been assigned to each arc, the algorithm does not differentiate between supply nodes and use nodes. Then to put the network into circulation form, add an arc from node V to node 1 with a lower bound of 0, an upper bound of ∞, and a unit cost of 0. The flow in the arc from node V to node 1, q_{V1}, is called the circulation of the network. Hence, equations (2.6) through (2.9) become

$$\text{Minimize } Z = \sum_{i=1}^{V} \sum_{j=1}^{V} c_{ij} q_{ij} \tag{2.10}$$

subject to

$$\sum_{i=1}^{V} q_{ij} - \sum_{i=1}^{V} q_{ji} = 0 \qquad j = 1,\ldots.V \tag{2.11}$$

$$0 \leq b_{ij} \leq q_{ij} \leq u_{ij} \qquad \begin{matrix} i = 1,\ldots.V \\ j = 1,\ldots.V \end{matrix} \tag{2.12}$$

All variables will be restricted to integer values. This is necessary to guarantee convergence of the algorithm. However, this is not a restrictive assumption in water resources because in the case where fractions of units are involved, a small enough unit can always be found so that only integer values are considered.

The out-of-kilter algorithm is based on duality theory. The solution is initiated with any positive integral flow and any set of node numbers, π_i, i = 1,....V, one assigned to each node. The node numbers are analogous to dual variables in duality theory. A test is made to determine whether the arc is in-kilter or out-of-kilter. The algorithm concentrates on the out-of-kilter arcs, one at a time, increasing or decreasing flow, using a labeling procedure until all arcs in the network are in-kilter, at which time the flow in the network is optimal. If at least one arc of the network can not be put in-kilter, no feasible circulation can possibly take place in the network and the algorithm terminates.

One feature contributing to the efficiency of this algorithm is that the status of no arc in the network is worsened as computation progresses: an in-kilter arc stays in-kilter, whereas an out-of-kilter arc either stays the same or becomes less out-of-kilter.

For given node numbers π_i, i = 1,....V, compute

$$c'_{ij} = c_{ij} + \pi_i - \pi_j \tag{2.13}$$

Then, for the given π_i, i = 1,....V, and circulation q_{V1}, an arc a_{ij} is in just one of the following states:

$$c'_{ij} > 0,\ q_{ij} = b_{ij} \tag{2.14}$$

$$c'_{ij} = 0,\ b_{ij} \leq q_{ij} \leq u_{ij} \tag{2.15}$$

$$c'_{ij} < 0,\ q_{ij} = u_{ij} \tag{2.16}$$

$$c'_{ij} > 0,\ q_{ij} < b_{ij} \tag{2.17}$$

$$c'_{ij} = 0,\ q_{ij} < b_{ij} \tag{2.18}$$

$$c'_{ij} < 0,\ q_{ij} < u_{ij} \tag{2.19}$$

$$c'_{ij} > 0,\ q_{ij} > b_{ij} \tag{2.20}$$

$$c'_{ij} = 0,\ q_{ij} > u_{ij} \tag{2.21}$$

$$c'_{ij} < 0,\ q_{ij} > u_{ij} \tag{2.22}$$

An arc is said to be <u>in kilter</u> if it is one of the states given by equations (2.14), (2.15) or (2.16); otherwise, the arc is <u>out-of-kilter</u>. Therefore, to solve the problem it suffices to get all arcs in kilter, because from duality the optimality properties are

$$\text{if } c'_{ij} < 0;\ \text{then } q_{ij} = u_{ij} \tag{2.23}$$

$$\text{if } c'_{ij} > 0;\ \text{then } q_{ij} = b_{ij} \tag{2.24}$$

A non-negative number called the kilter number is associated with each arc in the given state. An in kilter arc has a kilter number of 0. The arc kilter numbers, K_{ij}, corresponding to each out-of-kilter state are:

for equation (2.17) or (2.18) $K_{ij} = b_{ij} - q_{ij}$ (2.25)

for equation (2.19) $K_{ij} = c'_{ij}(q_{ij} - u_{ij})$ (2.26)

for equation (2.20) $K_{ij} = c'_{ij}(q_{ij} - b_{ij})$ (2.27)

for equation (2.21) or (2.22) $K_{ij} = q_{ij} - u_{ij}$ (2.28)

Thus, out-of-kilter arcs have positive kilter numbers. The kilter numbers for states defined by equations (2.17), (2.18), (2.21) and (2.22) measure infeasibility for the arc flow q_{ij}, while the states defined by equations (2.19) and (2.20) are a measure of the degree to which optimality properties, equations (2.23) and (2.24), fail to be satisfied.

The algorithm concentrates on a particular out-of-kilter arc and attempts to put it in-kilter. It does this in such a way that all in-kilter arcs stay in kilter, whereas the kilter number for any out-of-kilter arc either decreases or stays the same. Thus, all arc kilter numbers are monotone non-increasing throughout the computation.

2.5.2 The Out-of-Kilter Algorithm

The algorithm presented here is due to Ford and Fulkerson (1962).

Assign any integer circulation q_{V1} and any set of node integers π_i, i = 1,....V. One can begin with $\pi_i = 0$, i = 1,....V, and $q_{ij} = 0$ for all i and j. Next locate an out-of-kilter arc a_{st} and go to the appropriate case below.

Equation (2.17) is satisfied such that $c'_{st} > 0$, $q_{st} < b_{st}$. Start a labeling process at t, trying to reach s, first assigning to t the label $[s^+, x_t = b_{st} - q_{st}]$. The labeling rules are:

If i is labeled $[z^{\pm}, x_i]$, j is unlabeled, and if a_{ij} is an arc such that either

$$c'_{ij} > 0, \ q_{ij} < b_{ij} \tag{2.29}$$

$$c'_{ij} \leq 0, \ q_{ij} < u_{ij} \tag{2.30}$$

then j receives the label $[i^+, x_j]$ in which

$$x_j = \min [x_i, b_{ij} - q_{ij}] \tag{2.31}$$

if arc a_{ij} satisfies equation (2.29) or

$$x_j = \min [x_i, u_{ij} - q_{ij}] \tag{2.32}$$

if arc a_{ij} satisfies equation (2.30). If i is labeled $[z^{\pm}, x_i]$, j is unlabeled, and if arc a_{ji} is an arc such that either

$$c'_{ji} > 0, \ q_{ji} > b_{ji} \tag{2.33}$$

$$c'_{ji} < 0, \ q_{ji} > u_{ji} \tag{2.34}$$

then j receives the label $[i^-, x_j]$ where

$$x_j = \min [x_i, q_{ji} - b_{ji}] \tag{2.35}$$

if arc a_{ji} satisfies equation (2.33) or

$$x_j = \min [x_i, q_{ji} - u_{ji}] \tag{2.36}$$

if arc a_{ji} satisfies equation (2.34). If a breakthrough occurs, i.e., s receives a label so that a path from t to s has been found, change the circulation by adding x_s to the flow in the forward arcs of this path, substracting x_s from the flow in reverse arcs, and finally adding x_s to q_{st}. If non-breakthrough, i.e., s cannot be labeled, then let X denote the set of nodes with labels, and $\bar{X}$ denote the set of nodes without labels. Now define two subsets of arcs:

$$A_1 = \{a_{ij} \mid i \epsilon X, j \epsilon \bar{X}, c'_{ij} > 0, q_{ij} \leq u_{ij}\} \tag{2.37}$$

$$A_2 = \{a_{ji} \mid i \epsilon X, j \epsilon \bar{X}, c'_{ji} < 0, q_{ji} > b_{ji}\} \tag{2.38}$$

such that A_1 is the set of all arcs a_{ij} with i being a labeled node and j being an unlabeled node which have $c'_{ij} > 0$, and $q_{ij} \leq u_{ij}$. Then let

$$\delta_1 = \min_{A_1} [c'_{ij}] \quad (2.39)$$

$$\delta_2 = \min_{A_2} [-c'_{ji}] \quad (2.40)$$

and

$$\delta = \min [\delta_1, \delta_2] \quad (2.41)$$

Here δ_1 is a positive integer or ∞ according as to whether A_1 is non-empty or empty. Change the node integers by adding δ to all π_j, for $j \epsilon \bar{X}$.

Equation (2.18) or (2.19) is satisfied such that $c'_{st} = 0$, $q_{st} < b_{st}$ or $c'_{st} < 0$, $q_{st} < u_{st}$. The procedure is the same as if equation (2.17) is satisfied except $x_t = u_{st} - q_{st}$.

Equation (2.20) or (2.21) is satisfied such that $c'_{st} > 0$, $q_{st} > b_{st}$ or $c'_{st} = 0$, $q_{st} > u_{st}$. Here the labeling process starts at s in an attempt to reach t. Node s is assigned the label $[t^-, x_s = q_{st} - b_{st}]$. The labeling rules are again given by equations (2.29) through (2.36). If breakthrough, change the circulation by adding and subtracting x_t to arc flows along the path from s to t; then subtract x_t from q_{st}. If non-breakthrough, change the node numbers as above.

Equation(2.22) is satisfied such that $c'_{st} < 0$, $q_{st} > u_{st}$. Here the process is the same as for Equation (2.20) and (2.21) except $x_s = q_{st} - u_{st}$.

The labeling process is repeated for the arc a_{st} until either a_{st} is in kilter, or until a non-breakthrough occurs for which $\delta = \infty$. In the latter case, stop because there is no feasible circulation. In the former case, locate another out-of-kilter arc and continue until all arcs are in kilter or non-breakthrough occurs for which $\delta = \infty$.

2.6 Input Data Requirements

The input data requirements for the algorithm are: the direction of flow in each arc given by the order of the subscripts, the upper and lower bounds and the unit cost for flow in each arc, the initial flow in each arc, and the initial node numbers. The initial flow values and initial node numbers provide values with which to begin the algorithm and may be chosen arbitrarily. In many cases it is convenient to begin with the initial flow values and initial node numbers set equal to zero.

2.7 Capabilities

The results obtained from applying the algorithm to the water resources network model will indicate the amount of water allocated to each use from each supply during each time period. This will indicate sizes of facilities required, times to expand supply facilities, and by repeated application alternative systems can be evaluated. In addition, the final node numbers represent shadow pieces which indicates the value of additional units of water and thus indicates those areas which are costly for expansion and hence areas where development should not be encouraged.

3. CASE STUDY-PLANNING FOR WATER SUPPLY IN THE KASKASKIA RIVER BASIN

3.1 General Description

Much of the preliminary description, initial conditions, and projected basin needs are adopted from an Illinois State Water Survey study of "Plans for Meeting Water Requirements in the Kaskaskia River Basin, 1970-2020" (Singh et al, 1972).

The Kaskaskia River Basin in south-central Illinois (Fig. 3.1) covers an area of 5,840 square miles. The river originates west of the city of Champaign in Champaign County and flows southwesterly in a meandering course for about 150 miles before it enters the Mississippi River, 8 miles upstream of Chester in Randolph County.

The basin has 115 small-to-medium size towns belonging to Bond, Champaign, Clinton, Coles, Douglas, Effingham, Fayette, Madison, Marion, Monroe, Montgomery, Moultrie, Piatt, Randolph, St. Clair, Shelby, and

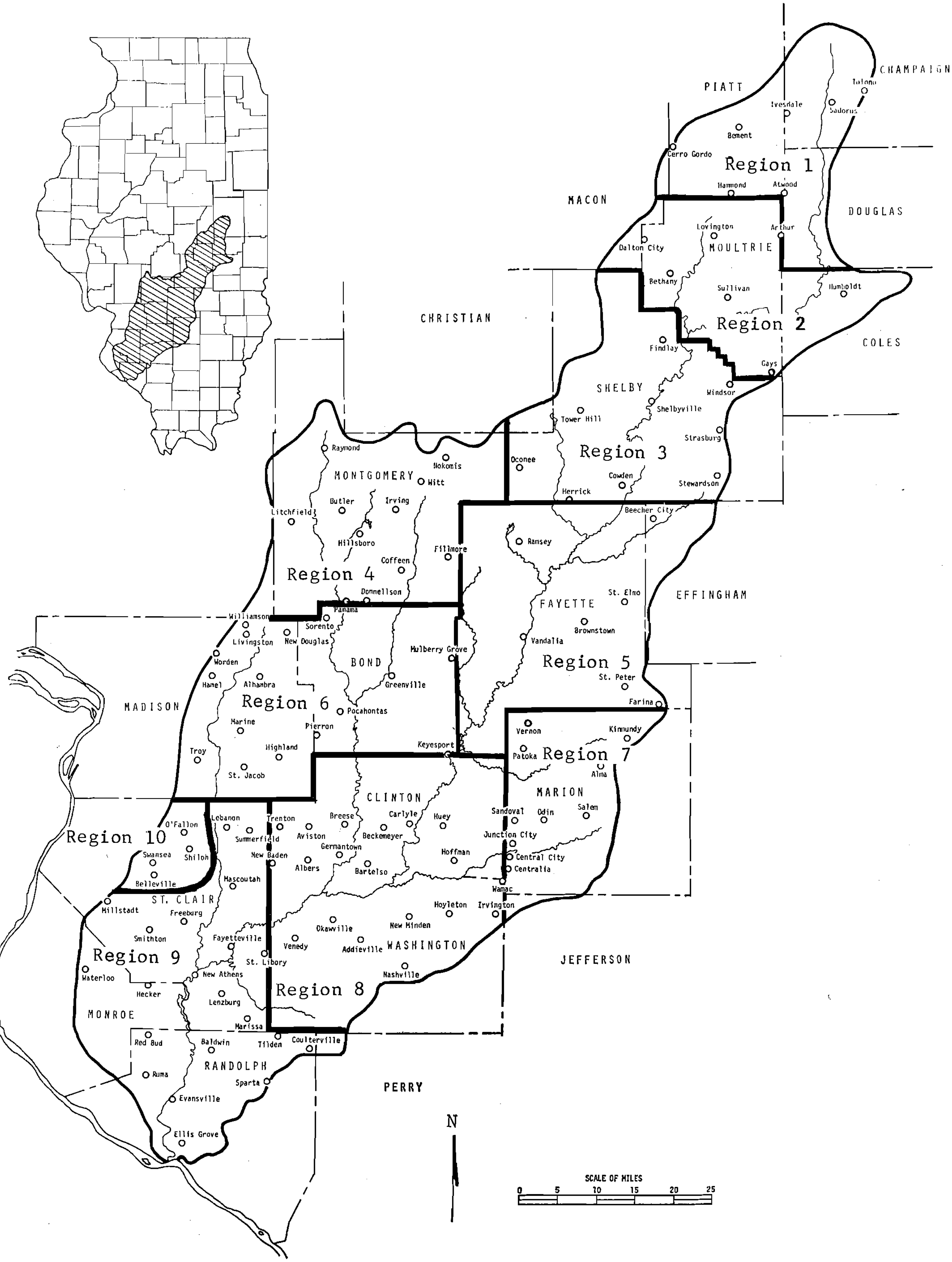

Fig. 3.1 Regions of Kaskaskia River Basin
(modified from Singh et al, 1972)

Washington Counties. At present, water supplies for these towns are furnished from ground and surface waters within the basin and from waters imported from the Mississippi and Embarrass River Basins.

Although some wells in the Kaskaskia River Basin derive their water from sandstones and limestones of the Pennsylvanian and Mississippian bedrock formations, the majority of wells in the basin are furnished in the overlying sand and gravel of the unconsolidated glacial drift deposits. The sand and gravel deposits underlie a good portion of the basin's area (Fig. 3.2), and they constitute the major source of ground water in the basin.

Sites of four existing and four potential surface water reservoirs in the basin are shown in Fig. 3.3. Carlyle, Shoal Creek and Silver Lake Reservoirs are currently in existence. Shelbyville Reservoir is scheduled for completion in 1974.

The four towns of Belleville, O'Fallon, Shiloh and Swansea in St. Clair County are being supplied by Mississippi River water through the East St. Louis and Interurban Company. The fifth town in the basin which is supplied by an outside source is Humboldt in Coles County. Humboldt receives its water from the town of Matton which is two miles south in the Embarrass River Basin.

Project demands over the next 50 years indicate a continuous increase over time such that neither local ground water nor existing surface water reservoirs within the basin can alone meet the basin's demands.

Therefore, an integrated plan that incorporates ground water and surface water within the basin and possibly imported water must be established if future water demands are to be met at reasonable costs. This plan should include (1) the amount of water to be allocated from

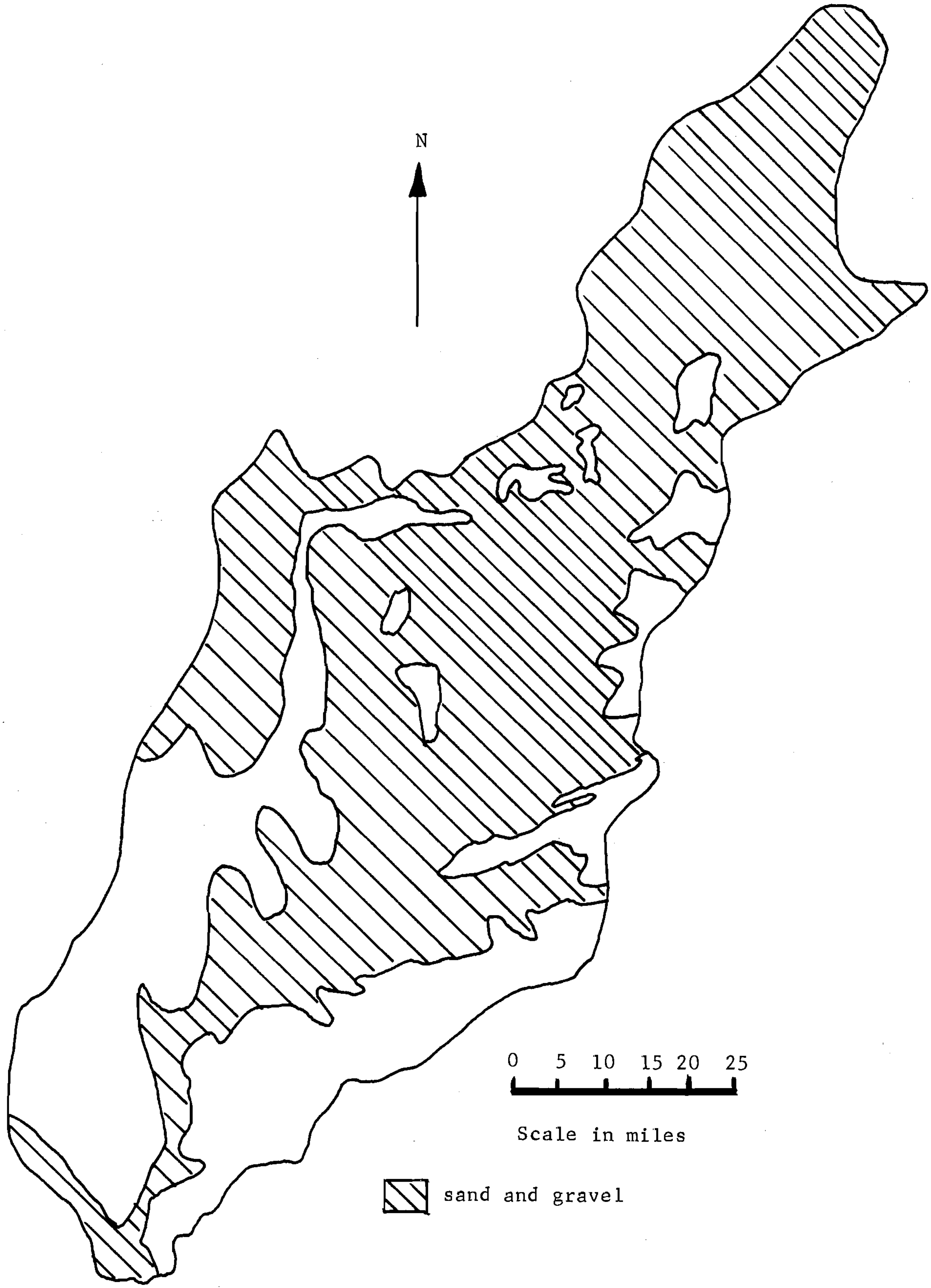

Fig. 3.2 Distribution of Sand and Gravel (composit from Pryor (1956, Selkregg et al (1957), and Selkregg and Kempton (1958))

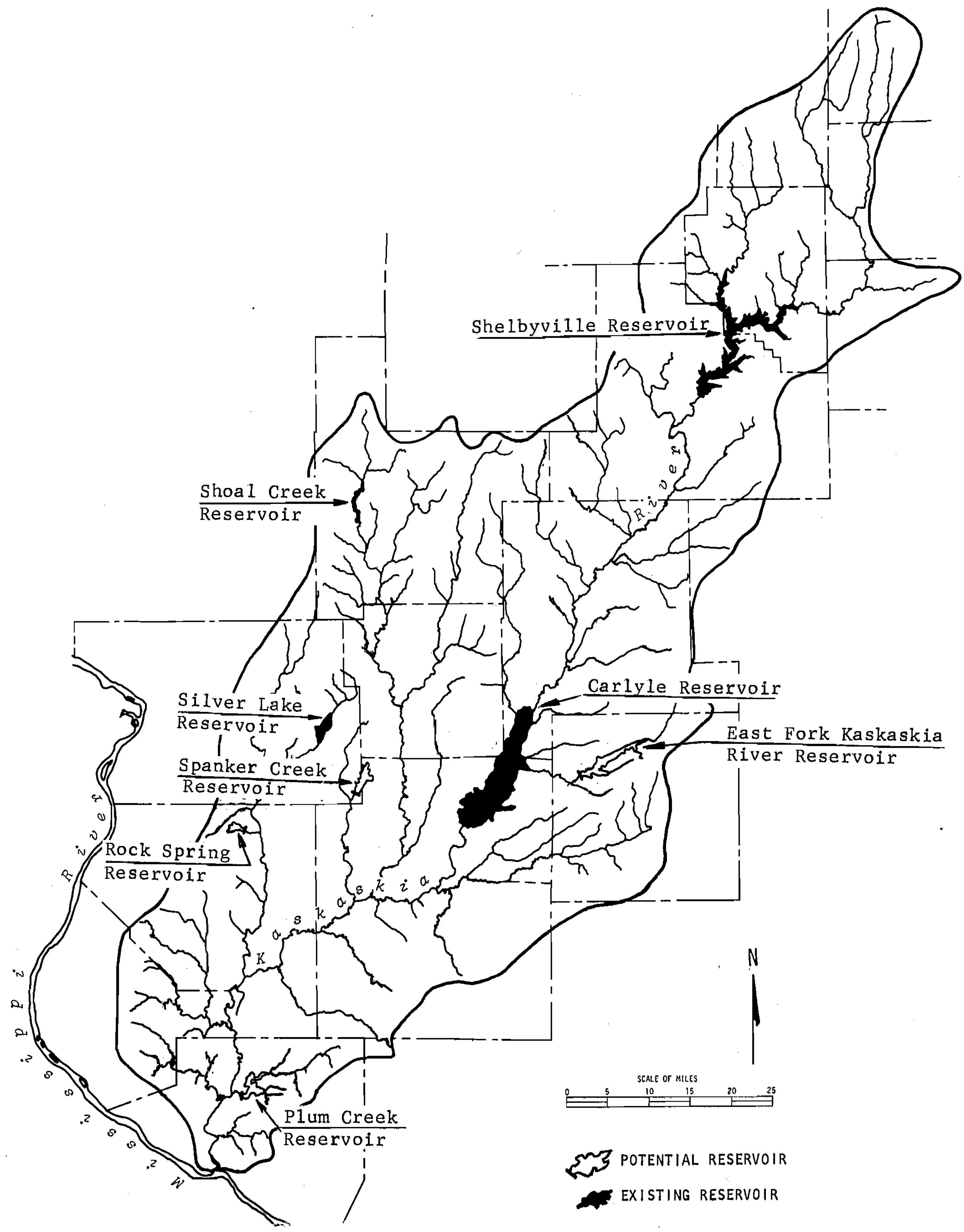

Fig. 3.3 Reservoirs Analyzed for Surface Water Supply (modified from Singh et al, 1972)

each source to various demand regions at all times during the planning period (operating rules); and (2) the time and magnitude of development of each of the water sources (design rules).

3.2 Network Model

In order to reduce the basin's demand centers into a manageable number, the basin's 115 towns were grouped into ten regions as shown in Fig. 3.1. The division attempted to follow county lines except for Region 10 which includes the four towns that are being supplied by the East St. Louis and Interurban Company from Mississippi River water.

The allocation problem has been formulated as a network with 175 nodes and 805 arcs (Fig. 3.4). At each time period the eight surface reservoirs, the imported water supply, and the ground water supplies in Regions 1,2,3,4,6,7,9, and 10 are represented by 17 supply nodes; the ground water in Regions 5 and 8 is represented by a single supply node and a single use node reflecting the continuity of the aquifer and the feasibility of ground water artificial recharge in these two regions; and the demand regions are represented by 10 use nodes. Finally, an artificial source node, an artificial sink node and an arc from sink to source are added to the network. The definitions of flows, flow bounds, and flow costs are summarized in Table 3.1.

The following assumptions were incorporated into the network in Fig. 3.4: (1) water from the surface reservoirs can be made available for use in any demand region, (2)ground water can only be used in the region in which it is pumped, (3) water may be imported into region 10, and (4) only the water allocations for the years 1970, 1980, 1990, 2000, 2010, and 2020 need to be computed to obtain an optimal solution. The total cost for the system can then be computed using linearly interpolated

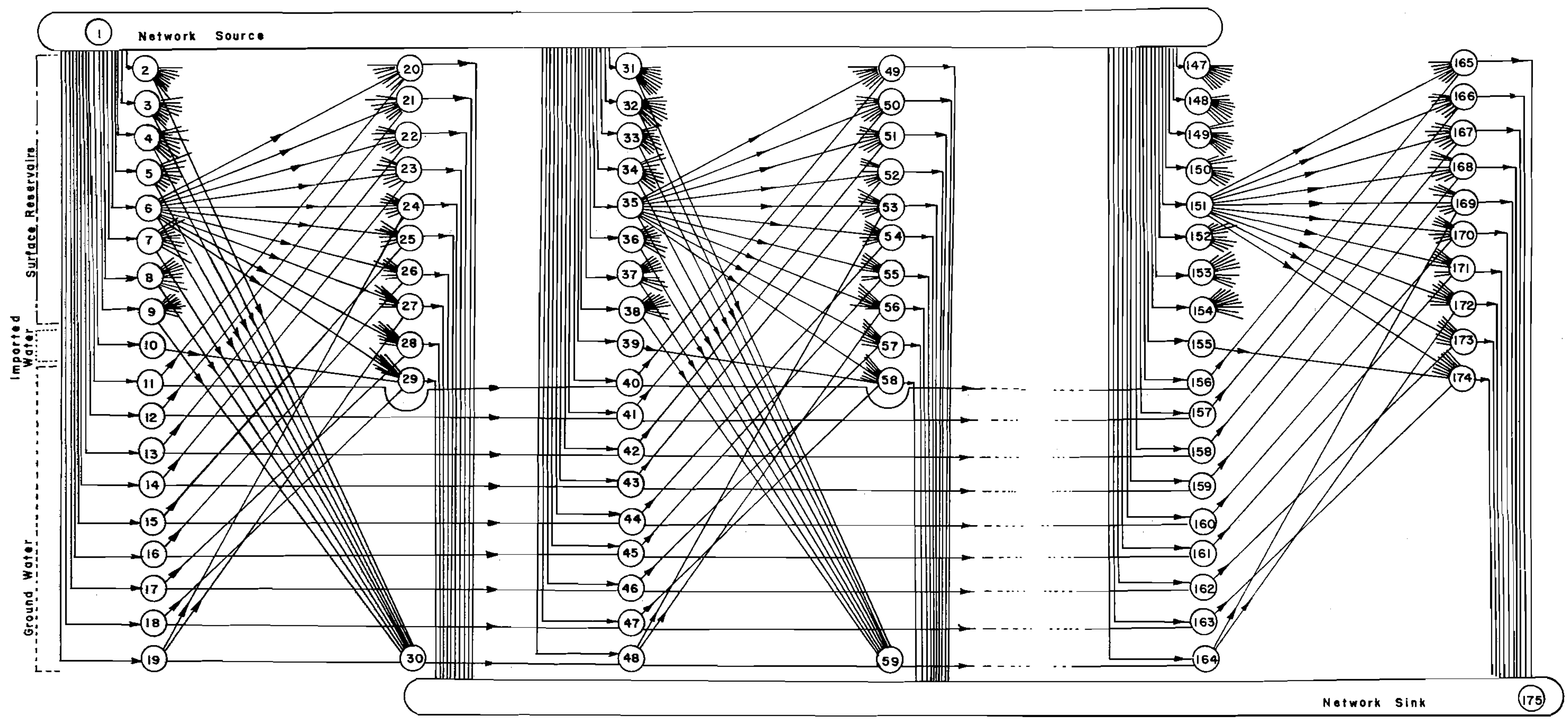

Fig. 3.4 Network Model for Kaskaskia River Basin Water Supply

Table 3.1 Inflows into and Outflows from Various Nodes, and Related Data for the Network in Fig. 3.4

Nodes	Inflows/Outflows		Arc Bounds Lower	Arc Bounds Upper	Costs
Supply Nodes					
Surface water reservoirs (8 nodes)	Inflows	Natural streamflow	0	Maximum net yield designated for the water supply use	None
	Outflows	Allocations to Regions 1 through 10	0	Reservoir yield or maximum demand*	Costs of raw water treatment and transmission
		Allocation to artificial recharge use node	0	Reservoir yield or maximum recharge*	Costs of raw water, transmission and artificial recharge operation
Imported water (1 node)	Inflows	Water import	0	Maximum demand	None
	Outflows	Allocation to Region 10	0	Maximum demand	Costs of raw water, transmission and treatment
Ground water in Region 1, 2,3,4,6,7,9, and 10 (8 nodes)	Inflows	Natural ground water recharge.	0	Safe yield	None
		Carry over from previous period	0	Aquifer yield	None
	Outflows	Allocations to appropriate regions	0	Aquifer yield or maximum demand*	Costs of raw water, transmission and treatment
		Ground water left in storage to following period	0	Aquifer yield	None
Ground water in regions 5 and 8 (1 node)	Inflows	Natural ground water recharge.	0	Safe yield	None
		Carry over from artificial recharge use node of previous period	0	Aquifer yield	None
	Outflows	Allocations to Regions 5 and 8		Aquifer yields or maximum demand*	Costs of raw water, transmission and treatment
		Ground water left in storage to following period	0	Aquifer yield	None
Use Nodes					
Demand Regions (10 nodes)	Inflows	Allocations from reservoirs	0	Reservoir yield or maximum demand*	Costs of raw water, transmission and treatment
		Ground water pumpage in region	0	Aquifer yield or maximum demand*	Costs of raw water, transmission and treatment
	Outflows	Allocations to meet region's demand	Demand during period	Demand during period	None
Artificial recharge use node (1 node)	Inflows	Allocations from reservoirs	0	Reservoir yield or maximum recharge*	Costs of raw water, transmission, and artificial recharge operation
		Ground water left in storate in regions 5 & 8 in previous period	0	Aquifer yield	None
	Outflows	Carry over to ground water node of Regions 5 and 8 of following period	0	Aquifer yield	None
Source and sink nodes:					
Source node (1 node)	Inflows	Flow from sink	0	∞	None
	Outflows	Natural ground water recharge	0	Safe yield	None
		Imported water flow	0	Maximum demand	None
		Natural streamflow to surface reservoirs	0	Maximum reservoir net yield designated for the water supply use	None
Sink node (1 node)	Inflows	Water demands of regions 1 through 10	Demand during period	Demand during period	None
	Outflows	Flow to source	0	∞	None

*Take smaller value. Reservoir yield may be taken as large as demand in order to let algorithm determine optimal size. Aquifer yield may be safe or mining yield depending on assumptions made.

yearly costs for the years between the times for which yearly costs were computed.

The equations (2.10), (2.11) and (2.12) can be applied to the network in Fig. 3.4 to obtain an optimal solution providing the values for the flow bounds and flow costs are available. The procedures for finding these values are described in the remainder of this chapter.

3.3 Flow Bounds and Flow Costs

Mathematically, the equations necessary to describe the system are of the simplest algebraic type. The necessary equations include those for projected demands, for projected supplies and for costs of supply elements.

3.3.1 Water Demands

Water requirements are computed by

$$Q = 11.0 \cdot P^{1.252} \tag{3.1}$$

where Q is the average yearly consumption in gallons per day and P is the population. This equation was developed by Singh et al (1972) for towns in the Kaskaskia River Basin. The estimated water requirements for each town in the basin are reported in Table 1 of Singh et al (1972). These values are used to determine the flow bounds for flow in the arcs from the use nodes.

3.3.2 Water Supplies

Singh et al (1972) provide estimates of ground water safe yield at each of the basin's 115 towns. Pumping ground water in excess of the safe yield is referred to as mining. The mining yield was determined assuming that the aquifer is 10 feet thick, has a storage coefficient of 0.10, that a town may pump ground water from an area of 9 square miles, and that ground water in a region may be completely mined over a period of 10 years.

Artificial recharge in the basin would increase the ground water yields. According to Smith (1967) artificial recharge by the pit method to the unconsolidated sand and gravel deposits is practicable over an area approximately 100 miles long and 5 miles wide along the Kaskaskia River.

Providing that enough recharge water is available the rate of artificial recharge in a certain region is limited by the operational recharge rate that can actually be achieved and the rate that would not, at any time during the planning period, result in water accumulation in the region in excess of available ground water storage. Up to 33.5 million gallons per day (mgd) have been artificially recharged through a gravel packed pit at Peoria, Illinois. A recharge pit usually occupies an area of less than one acre. Therefore, the operational recharge rate is not likely to be the limiting factor on the maximum rate of recharge in the Kaskaskia Basin. The available storage space in the ground water reservoir is the limiting factor and it is taken into consideration in order to prevent any water logging in the basin during the planning period.

The State of Illinois has a reserve net water supply of 22.3 million gallon per day (mgd) and 29.5 mgd in Shelbyville and Carlyle Reservoirs, respectively (personal communication, U.S. Army Corps of Engineers, St. Louis District, 1973).

The maximum amounts of water available from the other existing and potential reservoirs must be computed as a function of topography, capacity losses because of sedimentation and evaporation losses.

Data from Stall (1964) was fitted by Singh et al (1972) to a single equation such that the annual reservoir capacity loss because of sedimentation in the Kaskaskia Basin is computed by

$$\text{capacity loss} = 0.0191 \cdot A^{-0.1473 \cdot (\log_{10} A)^{0.64}} \qquad (3.2)$$

in which capacity loss is in inches per year and A is the drainage area in square miles. The net reservoir storage is determined by substracting the capacity loss over the period of time from the reservoir capacity.

The maximum amount of water that may be made available for water supply use from each of the reservoirs is assumed to be 50% of the reservoir net yield. The net yield from a reservoir is the difference between the gross reservoir yield during a period of critical drawdown and the net evaporation loss during this period. Singh et al used data for percent draft rate or reservoir yield as percent of mean flow, p, pertaining to a 40-year recurrence interval (Stall, 1964); the net reservoir storage as percent of mean flow, S; and the drainage area, A, to derive the general equation

$$S = C_p A^{n_p} \qquad (3.4)$$

for reservoirs in the Kaskaskia Basin. In equation (3.4), C and n are a coefficient and exponent respectively and the subscript p refers to the percent draft rate.

The evaporation loss during the critical drought is determined from the critical drawdown duration, T_c, in months for a given value of p and the net evaporation data at Springfield and Carbondale. Evaporation loss from the reservoir surface minus precipitation falling directly on the lake surface during the critical drawdown period yields the net evaporation. The effective surface area for evaporation is $0.65 \cdot A_s$ where A_s represents the pool area in acres. Values for C, n, T_c, net evaporation at Springfield and Carbondale, and the relative weights for computing weighted evaporation from the data for the two towns are given in Tables 3, 4,and 5 of Singh et al (1972).

The Mississippi River water imported by the East St. Louis and Interurban Water Company constitute the only source of imported water considered in this study. At present the Mississippi River water is being used to furnish supplies to region 10. It is assumed that adequate water may be imported to meet the demands of this region over the entire planning period.

The maximum amount of water available from each supply (Tables 3.2 and 3.3) was used to determine the bounds for flow in the arcs to and from each supply node.

3.3.3 Costs

The cost of water supply includes the cost of raw water, the cost of treatment, and the cost of transmission.

Cost of Raw Water

Groundwater

Based on the work of Gibb and Sanderson (1969), Singh et al (1972) determined the total annual cost of untreated ground water, TCG, as

$$TCG = C_w(CRF)_{20} + C_{pm}(CRF)_{12.5} + C_{op} + C_e \qquad (3.5)$$

in which the subscripts 20 and 12.5 refer to the useful life in years for wells and pumps, respectively, for calculating capital recovery factors, (CRF), for various rates of interest. C_w is the cost of wells and is given by

$$C_w = 850 \cdot d_w^{0.299} N_{wt}/\alpha \qquad \text{tabular} \qquad (3.6a)$$

$$C_w = 680 \cdot d_w^{0.408} N_{wt}/\alpha \qquad \text{gravel packed} \qquad (3.6b)$$

in which d_w is the well depth in feet, α is the factor to convert 1966 dollars to dollars in year of interest, and the number of wellsto meet the water requirement plus standby wells, N_{wt}, is given by

Table 3.2 Ground Water Yields in 1000 gpd (approximated to the next lower thousand)

Region	Safe Yield[a]	Mining Yield[b]
1	3,983	4,113
2	2,518	3,085
3	3,900	4,627
4	1,074	6,684
5	1,110	3,599
6	3,494	8,740
7	707	5,655
8	2,720	9,254
9	2,548	11,311
10	204	2,056
Totals	22,258	58,124

[a]Source: Table 2 of Singh et al (1972)

[b]Based on a 10-year mining period and assumptions of Section 3.3.2.

$$N_{wt} = N_w + 1 \qquad \text{if } N_w \leq 3 \tag{3.7a}$$

$$N_{wt} = N_w + 2 \qquad \text{if } N_w > 3 \tag{3.7b}$$

The number of wells to meet the water requirement, N_w, is given by

$$N_w = \frac{\text{water requirement in gpd}}{1440 \cdot q_w} \times 1.5 \tag{3.8}$$

rounded to next higher integer. Here q_w in gallon per minute (gpm) is the average long-term well yield. The multiplier 1.5 allows 50% excess requirement over the mean. The maximum value which N_w can assume is the maximum number of wells that an aquifer can sustain, N_m, and is given by

$$N_m = Q_y/(1440 \cdot q_w) \tag{3.9}$$

rounded to next lower integer. Q_y is the aquifer potential yield in gallon per day (gpd). In equation (3.5) C_{pm} is the cost of submersible

Table 3.3 Maximum Water Available for Water Supply (1000 gpd) from Surface Water Reservoirs

1.	East Fork Kaskaskia River Res.	11,465
2.	Plum Creek Res.	5,873
3.	Rock Spring Branch Res.	847
4.	Shoal Creek Res.[a]	4,726
5.	Silver Lake Res.[a]	2,854
6.	Spanker Creek Res.	1,470
7.	Shelbyville Res.[b]	22,300
8.	Carlyle Res.[a]	29,500

[a]completed

[b]to be completed in 1974

turbine and vertical turbine pumps and motors and is given by

$$C_{pm} = 5.629 \cdot q_w^{0.541} H_d^{0.658} N_{wt}/\alpha \qquad q_w \leq 100 \qquad (3.10a)$$

$$C_{pm} = (800 + 7.309 \cdot q_w^{0.453} H_d^{0.642} N_{wt}/\alpha \qquad q_w > 100 \qquad (3.10b)$$

in which H_d is the design head for the pump and equals the depth of the well plus 25 feet to allow for pumping to treatment plant. The \$800 in equation 3.10b is for the motor housing for vertical turbine pumps. Wells are assumed to have a useful life of 20 years. The useful life of a pump is assumed to be 12.5 years. C_{op} is the annual cost of operation, maintenance, and repair on wells and pumps and is given by

$$C_{op} = 100 + 75 \cdot N_{wt} \qquad (3.11)$$

C_e is the annual electrical charges for pumping and is computed by using the prevalent electric rate schedule and the total for kilowatt hours per year, kwh, as calculated by

$$kwh = 0.0011476 \cdot Q \cdot H_d/E_g \qquad (3.12)$$

in which E_g is the average overall efficiency during the year for pumping ground water.

Surface Water

Based on work by Dawes and Wathne (1968), Singh et al (1972) determined the annual cost of raw water from reservoirs, RWCR, in dollars as

$$RWCR = IWS \times CRF_{40} + OMRWS \tag{3.13}$$

in which CRF_{40} represents the capital recovery factor for a 40-year period. IWS is the investment cost chargeable to water supply and is given by

$$IWS = 0.5(RC + LC + RLC) + ITC \tag{3.14}$$

in which the reservoir cost, RC, is given by

$$RC = 6250 \cdot S_g^{0.87} \tag{3.15}$$

in which S_g is the gross storage in acre-feet for a given percent draft rate. The land cost, LC, is given by

$$LC = 1.5 \times 260 \cdot K \cdot S_g^{0.87} \tag{3.16}$$

in which the average cost of land is \$260 per acre, the factor 1.5 indicates that the land required is 1.5 times the reservoir surface area, and K defines the relationship between reservoir storage and surface pool area in acres. The relocation cost, RLC, is given by

$$RLC = 80{,}000 \cdot L_a + 200{,}000 \cdot (L_r + L_h) + 60{,}000 \cdot L_{og} \tag{3.17}$$

in which L_a is the length in miles of new access roads and L_r, L_n, and L_{og} are the lengths in miles for relocating railroads, highways, and oil and gas lines respectively. ITC is the cost in dollars of the intake tower and is given by

$$ITC = 30{,}000 + 3000 \cdot x \tag{3.18}$$

in which x is the water supply in mgd.

The operation, maintenance, and repair costs chargeable to water supply, OMRWS, are given by

$$\text{OMRWS} = 5000 \qquad \text{IWS} \leq 10^5 \qquad (3.19a)$$

$$\text{OMRWS} = 0.05 \cdot \text{IWS} - 0.025 \cdot (\text{IWS} - 10^5) \qquad 10^5 \leq \text{IWS} \leq 10^6 \qquad (3.19b)$$

$$\text{OMRWS} = 0.05 \cdot \text{IWS} - 0.025 \cdot (\text{IWS} - 10^5) - 0.01 \cdot (\text{IWS} - 10^6) \quad \text{IWS} > 10^6 \qquad (3.19c)$$

Equation (3.13) assumes that 1/2 of the reservoir, land, relocations, and operation, maintenance, and repair costs are charged to water supply. Cost of the intake tower and its OMR are fully charged to water supply.

The annual cost of raw water obtained directly from the Kaskaskia River, RWCKR, is given by

$$\text{RWCKR} = \text{ITC}(\text{CRF}_{40} + 0.05) \qquad (3.20)$$

in which ITC is the cost of the intake tower and the OMR is taken as 5% of ITC. The capital recovery factor, CRF_{40}, is for a 40-year period.

The cost of raw water from the Carlyle and Shelbyville reservoirs can be assumed to be a constant 6¢/1000 gallons (Illinois Department of Transportation, personal communication).

Cost of Treatment of Raw Water

Ground Water

The ground water treatment includes iron removal softening, and chlorination. Based on work by Illinois State Water Survey (1968) and Koenig (1967), Singh et al (1972) determined the total annual cost of ground water treatment, TCTPG,by

$$\text{TCTPG} = \text{ICTPG} \times \text{CRF}_{25} + \text{OMRTPG} \qquad (3.21)$$

in which CRF_{25} denotes the capital recovery factor for a useful plant life of 25 years. ICTPG is the investment cost of a treatment plant and is given by

$$\text{ICTPG} = 115{,}000 \cdot Q_d^{0.63} \qquad (3.22)$$

in which Q_d is the design plant capacity in mgd. Q_d is the water requirement for any given year or the amount of water available, whichever is the smaller. OMRTPG is the annual operation, maintenance, and repair cost and is given by

$$OMRTGG = 0.05783 \cdot ICTPG \qquad (3.23)$$

For utilization factor (ratio of mean daily pumpage to design plant capacity) of one, equation (3.23) is modified to

$$OMRTPG = 0.08069 \cdot ICTPG \cdot (Q_d^{-0.02074}) \qquad (3.24)$$

Surface Water

Surface water treatment includes chemical coagulation, sedimentation, rapid sand filtration, and chlorination. The annual cost of surface water treatment, TCTPR, is given by

$$TCTPR = ICTPR \times CRF_{30} + OMRTPR \qquad (3.25)$$

in which CRF_{30} denotes the capital recovery factor. ICTPR is the investment cost of a treatment plant and is given by

$$ICTPR = 267{,}900 \cdot Q_d^{0.65} \qquad (3.26)$$

OMRTPR is the operation, maintenance, and repair costs and is given by

$$OMRTPR = 0.08069 \cdot ICTPR\ (Q_d^{-0.02074}) \qquad (3.27)$$

<u>Cost of Transmission of Water</u>

Ground Water

The total annual cost of transmission of ground water TCT, is given by

$$TCT = (C_1 + C_3) \cdot CRF_{50} + (C_4) \cdot CRF_{25} + C_2 + C_5 + C_6 \qquad (3.28)$$

in which subscripts 50 and 25 refer to the amortization period in years for the pipeline and pumping station, respectively. C_1 is the pipeline construction cost and is given by

$$C_1 = 2160 \cdot D^{1.2} L \qquad (3.29)$$

in which D is the inside diameter of pipe in inches, and L is the length of the pipe in miles. In equation (3.28), C_2 is the annual cost for repairing leaks and breaks in the pipeline and is given by

$$C_2 = 10 \cdot D \cdot L \tag{3.30}$$

C_3 is the easement cost and is given by

$$C_3 = 1700 \cdot L \tag{3.31}$$

C_4 is the pumping station cost and is given by

$$C_4 = 17{,}000 \cdot (H/300) + 135 \cdot [nP_s^{1.01} - (P_i - nP_s)^{1.01}] \tag{3.32}$$

in which n equals the integer part of the ratio H/300 and P_s refers to the installed horsepower when H is 300 feet. H is the total head which is equal to the static head, H_s, plus the friction head, H_f. The pipeline is designed to carry a maximum of 1.5 times the average water requirement, Q. Thus

$$H = 2.25 \cdot H_o + H_s \tag{3.33}$$

in which H_o is 1.05 times the frictional head loss (based on Colebrook and White equation). The multiplier 1.05 allows for losses in bends, etc. P_i is the installed housepower at the pumping station and is given by

$$P_i = 0.2634 \cdot Q \cdot H \cdot J/E \tag{3.34}$$

in which E is the overall efficiency at peak load, Q is the flow in gallons per day, and J is the firming or standby factor. J is given by

$$J = 2.08 - 0.18 \cdot x \qquad J \leq 2.0 \qquad x \leq 2.0 \tag{3.35a}$$

$$J = 1.9666 - 0.1233 \cdot x \qquad 2.0 \leq x \leq 5.0 \tag{3.35b}$$

$$J = 1.42 - 0.014 \cdot x \qquad 5.0 \leq x \leq 10.0 \tag{3.35c}$$

$$J = 1.30 - 0.002 \cdot x \qquad 10.0 \leq x \leq 20.0 \tag{3.35d}$$

in which x equals flow in mgd. It is assumed that a pumping station can produce a maximum of 300 feet of head. If H is greater than 300 feet then two or more pumping stations are necessary.

C_5 is the pumping cost and is given by the product of cost per killowatt hour (kwh) and total kwh per year used. The total energy consumption is given by

$$\text{Total kwh per year} = kQ(p_t H_o + p_s H_s) \tag{3.36}$$

in which k is a conversion factor and is given by

$$k = (0.1337 \times 365.24 \times 62.4 \times 0.7457)/(550 \times 3600 \times E_a) \tag{3.37}$$

in which E_a is the overall efficiency during the pumping period. p_f and p_s are the ratios of energy consumption for varying flow to that for constant flow in respect to frictional and static heads, respectively. These energy ratios are evaluated by integration over the pumping period for the varying flow rate.

C_6 is the operation, maintenance, and repair cost for the pumping station and is given by

$$C_6 = 850 \cdot P_i' + 8[nP_s'^{1.05} + (P_i' - nP_s')^{1.05}]\, \Delta t \tag{3.38}$$

in which

$$P_i' = 0.85 \cdot P_i / J \tag{3.39}$$

$$P_s' = 0.85 \cdot P_s / J \tag{3.40}$$

and Δt is the pumping period as a fraction of the year. The multiplier 0.85 converts the installed horsepower to firm wire horsepower.

The value of the inside pipe diameter, D, is determined by solving equation (3.28) such that a minimum cost is determined for the flow requirement Q.

Surface Water

The total cost for transmission of surface water, TCTSW, is given by

$$TCTSW = (C_1 + C_3)CRF_{50} + (C_7)CRF_{25} + C_2 + C_8 + C_9 \tag{3.41}$$

in which C_1, C_2, and C_3 are given by equations (3.29), (3.30) and (3.31) respectively. C_7 is the pumping station cost (Hazen and Sawyer, 1971) and is given by

$$C_7 = 1390 \cdot Q_d^{0.34} H^{0.65} + 1700 \cdot Q^{0.34} \qquad Q_d \leq 6 \text{ mgd} \qquad (3.42a)$$

$$C_7 = 573 \cdot Q_d^{0.83} H^{0.65} + 7100 \cdot Q^{0.83} \qquad Q_d > 6 \text{ mgd} \qquad (3.42b)$$

in which Q_d is the design flow in mgd and Q is the flow in gpm.

C_8 is the pumping station operation, maintenance and repair cost (Hazen and Sawyer, 1971) and is given by

$$C_8 = \frac{1150 \cdot K \cdot Q_d H}{E_a} + 0.01 \cdot C_7 \qquad (3.43)$$

in which K is the power cost in dollars per kwh.

C_9 is the pumping cost (Illinois State Water Survey, 1968) and is given by

$$C_9 = 1.88 \times 10^{-4} \cdot K \cdot Q \cdot H \cdot T / E_a \qquad (3.44)$$

in which T is the pumping time in hours.

The optimal transmission pipe diameter (Linaweaver and Clark, 1964) in inches is given by

$$D = 8.55 \cdot Q_d^{0.463} \qquad (3.45)$$

rounded to the next higher multiple of 6 inches. Q_d is taken as the smaller of the water requirement and the maximum amount of water that can be made available from the surface water source.

The pumping heads (Higgins and Okum, 1972) are given by

$$H = \frac{10.1 \times 10^9 \times (Q/c)^{1.85} L_w}{D^{4.67}} + H_s \qquad (3.46)$$

in which c is the Hazen and Williams pipe coefficient, L_w is the transmission distance in miles, and H_s is zero if the demand area elevation is below the elevation of the reservoir outlet.

Cost of artificial ground water recharge

The costs of artificial ground water recharge include the cost of transmission, and the cost of operation. In computing the artificial recharge costs, the raw water costs and the transmission costs from the various reservoirs to Regions 5 or 8, whichever is less, were used.

According to Suter and Harmeson (1960), the operational cost of artificial recharge using the pit method near Peoria, Illinois, ranged between 1.6 to 2.9¢/1000 gallons of recharge water. Adjusting the higher of these two values to 1970 values, a constant cost of 4¢/1000 gallons was used in the present study.

The costs of raw water, transmission, and operation were added to compute the total artificial recharge costs from all 8 reservoirs. The easement cost of recharge pits was found to be less than 0.01¢/1000 gallons and for this study it was ruled insignificant.

Cost of imported water

The July 1972 rates of water imported to Region 10 from the Mississippi River by the East St. Louis and Interurban Company were used to compute costs of water at 10 year time intervals over the planning period.

Cost Summary

All costs are present value costs for unit flow in each arc. The values used in this study are presented in Appendix A.

4. CASE STUDY RESULTS AND DISCUSSION

4.1 Case Study Results

4.1.1 Alternatives Studied

The out-of-kilter algorithm was used to determine a minimal cost flow through the network of Fig. 3.4. The algorithm was written in Fortran IV and the IBM-360/75 at the University of Illinois at Urbana-Champaign was used to obtain the results. The computer output gave the optimal circulating flow through the network which represents the minimal-cost allocation schedule over the planning period and the shadow prices of supplies and demands. The shadow prices indicate the marginal savings in total cost for a unit decrease in demand or a unit increase in supply.

The case study problem was solved for the following three conditions of supply availability:

Alternative I. Water supplies include imported water, local ground water, and the existing surface reservoir of Shoal Creek, Silver Lake and Carlyle which have already been completed. It was assumed that Shelbyville Reservoir, which is scheduled for completion in 1974, would yield water for water supply use in the year 1975 and beyond. These conditions were imposed by setting upper bounds of zero on each arc exiting from the network source and entering the supply node representing a nonexisting reservoir. The solution under this condition determines if existing facilities can meet the water demands over the planning period, and if so, determines a minimal cost plan.

Alternative II. Water supplies include local ground water and existing and potential surface reservoirs but water cannot be imported at any time during the planning period. This condition is imposed by setting upper bounds of zero on each of the imported water arcs exiting from the network source. The solution under this condition determines if the local supplies of the basin are adequate to meet the demands in the basin over the planning period, and if so, determines a minimal-cost plan.

Alternative III. Water supplies include imported water, local ground water and existing and potential surface water reservoirs. The solution under this condition determines a minimal cost plan providing that all sources can be developed as soon as needed.

Because ground water mining is basic to the practice of conjunctive use of ground water and surface water, and because the ground water costs in Chapter 3 are based on the safe yield criterion, the network was initially solved on the assumption that ground water supplies are operated on a safe yield basis. This solution indicated that under all three alternatives, ground water pumpage was less than the safe yield in Regions 1,2,3, and 8. Increasing the ground water costs in the remaining regions by 10% whenever pumping reached the safe yield, and thereafter, the network was then solved assuming that ground water may be mined at the increased costs. All results presented in this report are for the conditions under which ground water can be mined.

4.1.2 Results for Alternatives Studied

The results for the three cases studied are presented in Tables 4.1 through 4.6. The only elements of the systems listed in these tables for each case are those from which allocations were greater than zero at some time in the planning period.

Alternative I. The existing sources of water supply including imported water, ground water, and completed and scheduled for completion surface reservoirs within the basin are adequate to meet the basin demands over the entire planning period. None of the remaining potential surface reservoir sites need to be developed as far as water supply is concerned. Furthermore, the Silver Lake Reservoir, already completed, need not be used for water supply use.

The ground water supplies are to be developed on a safe-yield basis in Regions 1,2, and 3 and beyond safe yield in Regions 4,5,6,7,9 and 10. Ground water is to be completely mined in Region 7 during the first two decades of the planning period and in Region 10 during the last decade of the planning period.

The overall cost of this plan over the entire planning period is

Table 4.1 Optimal Allocation Plans for Alternative I (Allocations in 1,000 gpd. Only allocations greater than zero are shown.)

Supply	Use Region	1970	1980	1990	2000	2010	2020
Shoal Creek	4	-	1895	2304	2735	3190	3660
Reservoir	5	-	-	-	-	-	1066
Shelbyville Reservoir	3	-	1095	1413	1749	2074	-
Carlyle	5	-	-	-	2333	2745	2105
	7	-	194	3379	4801	5510	6238
	8	1643	2040	2592	3169	3755	4359
	9	-	-	-	5131	5987	6849
Imported Water	10	7938	9496	11242	13045	14726	14808
Ground Water	1	799	1065	1277	1496	1719	1948
	2	527	534	596	725	837	951
	3	901	-	-	-	-	2409
	4	1815	-	-	-	-	-
	5	1125	1462	1887	-	-	-
	6	1904	2350	2813	3297	3838	4396
	7	3109	3253	707	-	-	-
	9	2868	3622	4356	-	-	-
	10	-	-	-	-	204	2056
Cost ($/day)		7614	8668	9748	10127	10416	10821

Total Plan Cost ($) 175,844,225

Table 4.2 Optimal Allocation Plans for Alternative II. (Allocations in 1,000 gpd. Only allocations greater than zero are shown.)

Supply	Use Region	1970	1980	1990	2000	2010	2020
East Fork	5	-	-	1887	2333	2745	3171
Kaskaskia River	7	-	3447	4086	4801	5510	6238
Reservoir	9	-	-	-	-	-	976
	10	-	-	-	-	-	1080
Plum Creek	9	-	-	4356	5131	5873	5073
Reservoir	10	2181	5873	1517	742	-	-
Rock Spring Branch Reservoir	10	847	847	-	-	-	-
Shoal Creek	4	-	1895	2304	2735	3190	3660
Reservoir	10	-	-	-	-	-	1066
Silver Lake Reservoir	10	2854	2572	2854	-	-	-
Shelbyville Reservoir	3	901	1095	1413	1749	2074	-
Carlyle	8	1643	2040	2592	3169	3755	4359
Reservoir	9	-	-	-	-	114	-
	10	-	-	6667	12099	14726	14514
Ground Water	1	799	1065	1277	1496	1719	1948
	2	527	534	596	725	837	951
	3	-	-	-	-	-	2409
	4	1815	-	-	-	-	-
	5	1125	1462	-	-	-	-
	6	1904	2350	2813	3297	3838	4396
	7	3109	-	-	-	-	-
	9	2868	3622	-	-	-	-
	10	2056	204	204	204	204	204
Cost ($/day)		12,532	12,878	11,523	10,405	10,106	10,059
Total plan costs ($)		205,157,375					

Table 4.3 Optimal Allocation Plans for Alternative III. (Allocations in 1000 gpd. Only allocations greater than zero are shown.)

Supply	Use Region	1970	1980	1990	2000	2010	2020
East Fork	5	-	-	1887	2333	2745	3171
Kaskaskia River	7	-	3447	4086	4801	5510	6238
Reservoir	8	-	-	-	-	-	1080
	9	-	-	-	-	-	976
Plum Creek Reservoir	9	-	3622	4356	5131	5873	5873
Shoal Creek Reservoir	4	-	1895	2304	2735	3190	3660
Shelbyville Reservoir	3	901	1095	1413	1749	2074	-
Carlyle	8	1643	2040	2592	3169	3755	3279
Reservoir	9	-	-	-	-	114	-
Imported Water	10	7938	9496	11242	13045	14726	14808
Ground Water	1	799	1065	1277	1496	1719	1948
	2	527	534	596	725	837	951
	3	-	-	-	-	-	2409
	4	1815	-	-	-	-	-
	5	1125	1462	-	-	-	-
	6	1904	2350	2813	3297	3838	4396
	7	3109	-	-	-	-	-
	9	2868	-	-	-	-	-
	10	-	-	-	-	204	2056
Cost ($/day)		7532	8435	8291	8519	9118	9689
Total Plan Cost ($) 146,853,275							

Table 4.4 Shadow Prices for Alternative I. (¢/1000 gallons. Only values greater than zero are shown.)

Supply or Demand	1970	1980	1990	2000	2010	2020
Supply						
East Fork Kaskaskia River Reservoir	96.09	96.09	96.09	96.09	96.09	96.09
Plum Creek Reservoir	96.09	96.09	96.09	96.09	96.09	96.09
Rock Spring Branch Reservoir	96.09	96.09	96.09	96.09	96.09	96.09
Shoal Creek Reservoir	-	-	-	-	-	0.17
Spanker Creek Reservoir	96.09	96.09	96.09	96.09	96.09	96.09
Shelbyville Reservoir	94.84	-	-	-	-	-
Ground water:						
Region 7	27.96	27.96	5.21	-	-	-
Region 10	-	-	-	-	0.49	1.79
Demand						
Region 1	17.40	16.66	15.83	14.95	13.99	13.19
Region 2	30.43	30.31	29.41	24.89	23.05	21.29
Region 3	32.09	21.39	20.30	19.47	18.88	17.95
Region 4	52.02	43.14	31.31	25.02	21.45	19.32
Region 5	60.47	57.76	48.64	37.92	28.86	23.97
Region 6	33.39	29.02	25.89	23.98	24.77	22.45
Region 7	68.79	68.79	44.41	32.21	25.88	22.31
Region 8	22.74	21.29	20.26	19.45	18.87	18.33
Region 9	47.87	49.03	45.09	39.54	30.05	24.67
Region 10	22.04	21.89	21.79	21.70	21.64	21.58

Table 4.5 Shadow Prices for Alternative II. (¢/1000 gallons. Only values greater than zero are shown.)

Supply or Demand	1970	1980	1990	2000	2010	2020
<u>Supply</u>						
East Fork Kaskaskia River Reservoir	-	-	-	-	-	2.02
Plum Creek Reservoir	22.91	2.20	5.91	3.25	8.40	5.81
Rock Spring Branch Reservoir	-	5.19	-	-	-	-
Shoal Creek Reservoir	1.46	-	-	-	-	0.28
Silver Lake Reservoir	-	0.90	1.68	-	-	-
Imported Water	251.78	251.78	251.78	251.78	251.78	251.78
Ground Water: Region 10	80.70	44.84	27.85	14.18	7.20	3.95
<u>Demand</u>						
Region 1	17.40	16.66	15.83	14.95	13.99	13.19
Region 2	30.43	30.31	29.41	24.89	23.05	21.21
Region 3	22.95	21.39	20.30	19.47	18.88	17.95
Region 4	52.02	43.14	31.31	25.02	21.45	19.43
Region 5	60.47	57.76	37.32	25.22	19.22	17.89
Region 6	33.39	29.02	25.89	23.98	24.77	22.45
Region 7	40.83	36.57	25.28	19.35	16.09	16.13
Region 8	22.74	21.29	20.26	19.45	18.87	18.33
Region 9	47.87	49.03	39.55	29.26	30.05	24.64
Region 10	108.90	71.13	52.27	36.57	28.35	23.74

Table 4.6 Shadow Prices for Alternative III (¢/1000 gallons. Only values greater than zero are shown.)

Supply or Demand	1970	1980	1990	2000	2010	2020
Supply						
East Fork Kaskaskia River Reservoir	-	-	-	-	-	0.20
Plum Creek Reservoir	-	-	-	-	8.40	3.99
Ground Water: Region 10	-	-	-	-	0.49	1.79
Demand						
Region 1	17.40	16.66	15.83	14.95	13.99	13.19
Region 2	30.43	30.31	29.41	24.89	23.05	21.29
Region 3	22.95	21.39	20.30	19.47	18.88	17.95
Region 4	52.02	43.14	31.31	25.02	21.45	19.15
Region 5	60.47	57.76	37.32	25.22	19.22	16.07
Region 6	33.39	29.02	25.89	23.98	24.77	22.45
Region 7	40.83	36.57	25.28	19.35	16.09	14.31
Region 8	22.74	21.29	20.26	19.45	18.87	18.33
Region 9	47.87	48.16	33.64	26.01	30.05	22.82
Region 10	22.04	21.89	21.79	21.70	21.64	21.58

$175,844,225 which is $4,121,768 less than the cost of the same plan developed on a safe yield basis. Developed on a safe yield basis this plan would require water from Silver Lake Reservoir to provide water to Region 9 during the first two decades and to Region 6 during the last three decades.

Alternative II. The local ground water and surface water supplies are adequate to meet the basin demands over the entire planning period. However, all reservoirs except Spanker Creek Reservoir would have to be developed at the start of the planning period.

The ground water supplies are to be developed on a safe-yield basis in Regions 1,2, and 3 and beyond safe-yield in Regions 4,5,6,7,9 and 10 with the ground water being completely mined in Region 10 during the first decade of the planning period.

The overall total cost of this plan is $205,175,375 which is $2,685,402 less than the cost of the same plan developed on a safe yield basis.

Alternative III. The local ground water, imported water, three existing reservoirs, and two potential reservoirs are used in this plan to meet the basin demands.

The ground water development for Case III is similar to that for Case II except the development in Region 10 would be delayed until the last two decades of the planning period.

The overall total cost for this plan is $146,853,275 which is $9,603,353 less than the cost of the same plan developed on a safe yield basis. Developed on a safe yield basis this plan would require water from Silver Lake Reservoir to provide water to Region 6 during the last three decades.

4.2 Discussion of Results

Network analysis was used in this case study as a preliminary screening tool to identify potentially optional alternative plans. The procedure permits the planner to evaluate several alternative plans to determine size of structures and the point in time at which they will be needed and to determine information helpful in setting policies for future developments.

4.2.1 Evaluation of Alternatives

The analysis procedure allows the planner first to determine if the alternative under consideration is capable of meeting the basin demands over the planning period.

If the alternative system under consideration can meet the basin demands then the least cost plan to meet the demands is identified. The cost as determined here is the present value of the costs attributed to providing water supply in the basin over the planning period. The total cost for each element has been converted to a present value per unit of water provided by that element in each specific period of time. These present value unit costs are obtained by assuming an amount to be provided by each element in each time period. If the assumed allocation values for each element are approximately equal to the values obtained by the analysis procedure or if the cost functions for the elements are in all ways linear, then the resulting total cost represents the minimal cost plan. If the cost functions are not in all ways linear and the assumed allocation values for each element are not approximately equal to the values obtained by the analysis procedure, then the present value of the unit costs will have to be revised for a second analysis in light of the results from the first analysis similar to what was done above regarding ground water mining.

Once the basic network model and the supporting data have been developed the procedure can readily be used to evaluate several alternatives as illustrated above by simply changing flow bounds or costs. These alternative systems can then be ranked as to least cost such that the analysis can be refined for a second or more iterations on those alternative systems which are most likely to result in the optimal plan.

The analysis procedure also allows the planner to evaluate policy decisions easily. In the case study the policy to allow or disallow the importation of water into the basin can be evaluated by simply changing the flow bounds on a few arcs. The results above illustrate that the importation of water is a basic element of any minimal cost plan for water supply in the Kaskaskia River Basin. Results such as these then inform the planner which policies are of primary importance and which policies might be compromised when making political trade offs with other planning agencies.

4.2.2 Size and Time of Element Development

The results from the analysis procedure can be used to determine the size required for each element and the time at which it should be ready for use. For example, examine the profile of allocations from ground water developments in Region 10 for Alternative I as shown in Table 4.1 and Fig. 4.1. The well field for this alternative in this region doesn't need to be developed before the year 2000 and then can be developed in stages until it is fully developed by the end of the planning period. The same profile of allocations results from Alternative III as shown in Table 4.3. However, Alternative II, from Table 4.2 would require that the well field in this region be fully developed at the beginning of the period and then used only a small amount after the first decade.

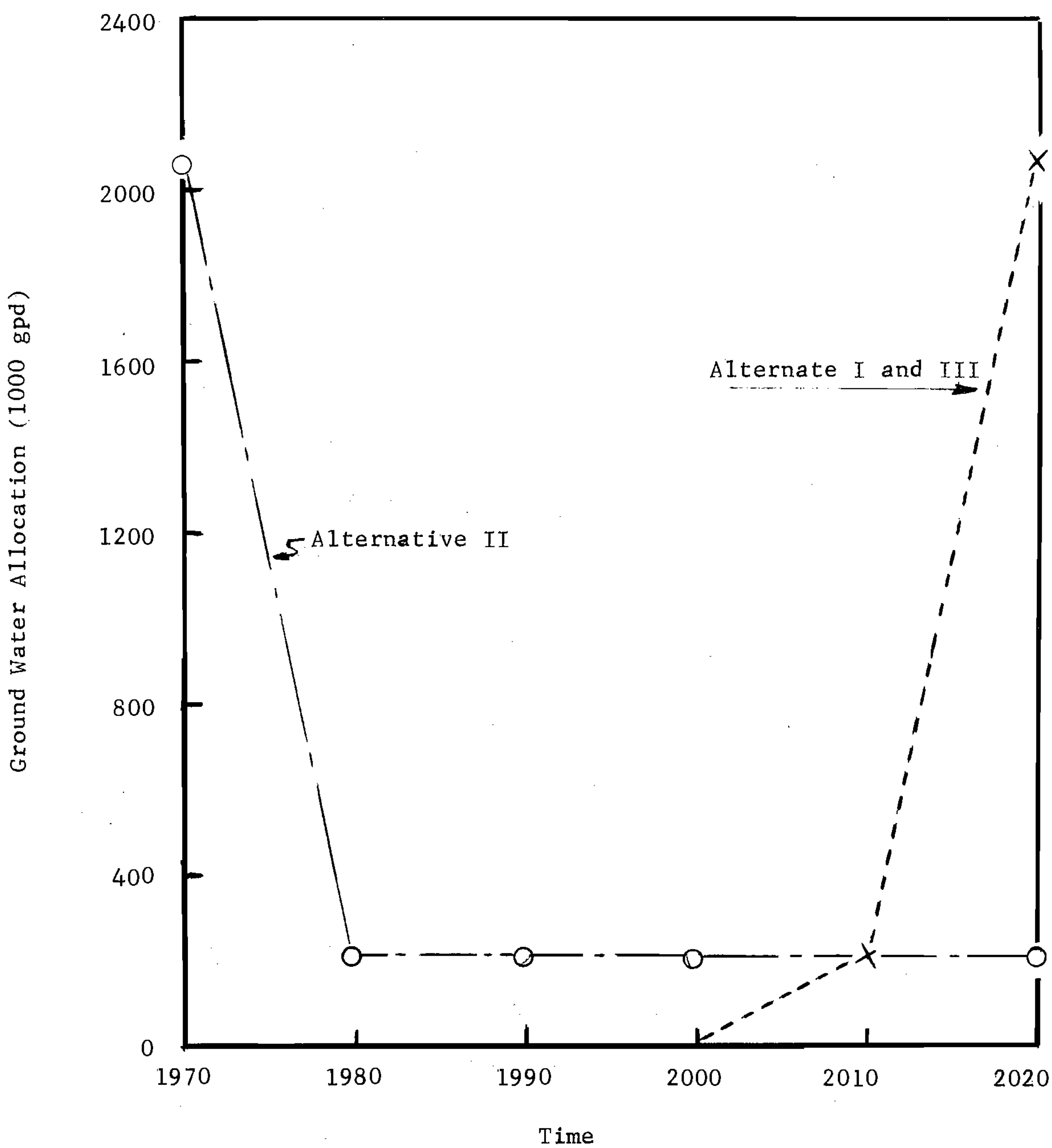

Fig. 4.1 Profile of Allocation from Ground Water in Region 10

The information about the sizes of the elements and the time at which they are required can be used by the planner in the selection of the final alternative. Factors other than costs must be considered in the selection of the final alternative. It might be poor policy to choose the least cost alternative which would require the development of a major element which would be used for only a brief period and then discarded when for only a slightly higher cost all of the elements of the system would be more fully used. The analysis procedure here allows the planner to look at these alternatives and evaluate the policies that might be made.

4.2.3 Future Development Potential

The value of the dual variables (the final node numbers for the out-of-kilter algorithm) of a programming problem can be interpreted as shadow prices. Shadow prices represent marginal values. In the context of determining the least cost of meeting demands in a supply-demand allocation problem,the shadow prices represent the marginal values of various supplies and demands. The shadow prices for supplies and demands for the three alternatives for the case study are presented in Tables 4.4, 4.5 and 4.6. The numbers in these tables represent the cost reductions in cents that may be obtained at a particular time period if a supply is increased by 1000 gallons during that time period and the cost reduction in cents that may be obtained at a particular time period of a demand is decreased by 1000 gallons during that time period.

For example, during 1970 for Alternative I from Table 4.4, the shadow price for ground water supply in Region 7 is 27.96¢/1000 gallons and the shadow price for water demand in Region 7 is 68.79¢/1000 gallons. This means that if another 1000 gallons were available in the ground water supply in Region 7 such that it could be used in 1970 there would be a reduction of 27.96¢ in the total cost. Similarly if there were a reduction in the demand in Region 7 for 1970 of 1000 gallons there would be a reduction of 68.79¢ in the total cost.

Interesting information on the economic feasibility of certain practices in various regions may be obtained by analzying the shadow prices. For example, there would be a reduction of 27.96¢/1000 gallon in the total cost if the ground water supply in Region 7 in 1970 for Alternative I were increased. This indicates that it is economic to practice ground water artificial recharge in this region to increase ground water available in storage for 1970 as long as the total costs of recharging 1000 gallons are less than 27.96¢.

It is interesting to note that the shadow prices of demands at future time periods may be used to determine the growth potential of various regions. Limiting growth in a region with high shadow prices of future demands would result in greater savings than by limiting growth in a region with low shadow prices. It is obvious, for example, that reducing demands in Region 9 in future years for Alternative I would result in greater savings in water supply costs than reducing demands in Region 1. In other words, from a water supply cost viewpoint Region 1 has a greater growth potential than Region 9. There would be a savings of 24.67 - 13.19¢ = 11.48¢/1000 gallon for demand that can be transferred from Region 9 to Region 1. Therefore, it would be more economical if the projected growth in Region 9 could be encouraged to occur in Region 1 rather than Region 9.

5. SUMMARY AND CONCLUSIONS

5.1 Summary

The network approach to water resources systems optimization includes modeling the problem as a network and solving it with a network-based solution algorithm. The network model of a water resources system consists of nodes and directed arcs. The nodes represent supplies where water is originated and uses where water is "consumed." The "supply" nodes are connected to a common source node, the "use" nodes are connected to a common sink node, and the "supply" and "use" nodes are connected among themselves by directed arcs representing various allocation alternatives. Available supplies are imposed as integer-value upper bounds on arcs exiting from the network source and demands are imposed as integer-valued lower bounds on arcs entering the network sink. Linear unit "costs" are imposed on appropriate arcs. Any additional constraints on the amount of water to be allocated from a particular supply to a particular use may also be imposed on appropriate network arcs. Once modeled as a network, a water resources allocation problem may be solved by a network-based solution algorithm such as the out-of-kilter algorithm.

Water resources allocation problems with several supplies, including surface and ground water sources, and several uses, including ground water artificial recharge may be modeled as a network and solved by network-based solution methods,if the water "costs" can be made linear. The solution from the out-of-kilter algorithm results in an optimal plan for allocation of water from sources to uses if there is sufficient water to meet all demands. In addition the results indicate the optimal size of water supply facilities and the time at which they will be needed. The results also include the shadow prices which can be used to guide promotion of future economic and population growth in the basin. The network approach can also be used to evaluate various policies such as the policy to restrict water importation or to restrict the construction of any more reservoirs in the basin.

A network representation of the water resources system seems to reduce the problem to its bare essentials which helps to bring out the fundamental concepts necessary for its solution. This is due to the "special characteristics" inherent in the network structure. These characteristics are exemplified by the binary relationships between the nodes and arcs and the preservation of flow through all network nodes and in the network as a whole. Exploiting these "special characteristics" is what makes the network solution methods more plausible and efficient than the more common linear programming Simplex Method and its modifications.

5.2 Conclusions

1. The network representation of a system provides the planning agency with a physical picture revealing various components of the system structure and allocation alternatives.

2. The out-of-kilter algorithm is an efficient solution technique. It requires about 1/15 the time for the Simplex Method. Furthermore, the out-of-kilter algorithm points out the infeasibility of a problem if a feasible solution does not exist.

3. Large allocation problems consisting of thousands of variables and constraints may be solved at a reasonable computer cost. The computer time required to solve the case study which consisted of 175 nodes and 805 arcs was less than two minutes on an IBM 360/75.

4. The time variable can be incorporated into the network structure and therefore converted into a space variable, thus reducing the dimensionality of the problem.

5. The network-based solution solution techniques provide a ready means to evaluate alternative policies and alternative system designs.

6. The results from the out-of-kilter algorithm solution indicates

whether there is a feasible solution and if so the optional allocation plan, optimal sizes for each element,and time each element will be needed. The shadow prices can be used to evaluate possibility of trade offs or they can be used to guide the setting of policy toward future economic and population growth in the basin.

7. The linearity of the objective function is basic to models to be analyzed by the network approach. Because of the cost and benefit functions of water resources systems are hardly linear, linearization of these coefficients must precede any optimization process using this approach.

8. The fact that flow needs to be preserved at every node of a network necessitates that infiltration and evaporation losses be estimated independently prior to any network analysis.

9. Network modeling and the out-of-kilter algorithm provide an efficient tool for preliminary screening of alternative water resource systems.

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APPENDIX A PRESENT VALUE COSTS

Table A.1 Ground Water Costs (¢/1000 gallons)*

Region	1970	1980	1990	2000	2010	2020
1	17.40	16.66	15.83	14.95	13.99	13.19
2	30.43	30.31	29.41	24.89	23.05	21.29
3	32.09	29.66	26.59	23.21	21.23	17.95
4	47.29	47.04	45.18	41.03	37.70	34.36
5	54.97	52.51	44.22	38.55	34.58	31.59
6	33.39	29.02	25.89	23.98	22.52	20.41
7	37.12	37.12	35.64	34.64	31.93	29.37
8	42.83	37.46	32.56	29.00	26.51	25.44
9	47.87	44.57	40.99	37.91	35.33	33.11
10	25.64	23.90	22.20	20.35	19.23	17.99

*Compiled from Singh et al (1972)

Table A.2 Costs of Raw Surface Water (¢/1000 gallons)

Source	1970	1980	1990	2000	2010	2020
East Fork Kaskaskia River Res.	8.49	7.66	6.69	6.00	5.64	5.11
Plum Creek Res.	11.02	9.40	8.31	7.43	6.70	6.08
Rock Spring Branch Res.	33.74	28.55	25.81	23.33	21.34	19.50
Shoal Creek Res.	11.39	10.97	9.76	8.78	8.01	7.71
Silver Lake Res.	16.05	14.08	12.78	11.66	10.74	9.94
Spanker Creek Res.	30.14	25.54	21.89	19.12	17.11	15.32
Shelbyville Res.	6.00	6.00	6.00	6.00	6.00	6.00
Carlyle Res.	6.00	6.00	6.00	6.00	6.00	6.00

Table A. 3 Costs of Surface Water Treatment (¢/1000 gallons)

Source	1970	1980	1990	2000	2010	2020
East Fork Kaskaskia River Res.	9.20	8.97	8.46	8.01	7.52	7.34
Plum Creek Res.	13.96	12.67	11.88	11.28	10.83	10.45
Rock Spring Res.	24.53	21.91	20.90	19.91	19.20	18.50
Shoal Creek Res.	12.90	12.97	12.04	11.28	10.74	9.91
Silver Lake Res.	17.19	15.62	14.67	13.86	13.22	12.66
Spanker Creek Res.	20.10	18.43	17.09	16.14	15.60	15.08
Shelbyville Res.	16.31	15.10	14.17	13.41	12.85	12.32
Carlyle Res.	16.31	15.10	14.17	13.41	12.85	12.32

Table A.4 Water Transmission Cost from East Fork Kaskaskia River Reservoir (¢/1000 gallons)

To region	1970	1980	1990	2000	2010	2020
1	1176.54	532.85	270.86	144.29	79.99	45.78
2	275.57	152.49	78.86	39.48	20.98	11.52
3	221.94	104.63	48.21	23.87	12.60	6.96
4	218.84	118.58	58.93	30.82	16.86	9.59
5	106.11	47.44	22.17	11.21	6.06	3.42
6	310.47	149.18	75.63	40.12	22.17	12.68
7	38.33	19.94	10.13	5.34	2.93	1.66
8	157.38	73.81	35.28	18.14	9.91	5.68
9	260.74	122.40	61.54	32.42	17.82	10.17
10	173.10	93.17	52.94	29.73	16.60	9.27

Table A.5 Water Transmission Costs from Plum Creek Reservoir (¢/1000 gallons)

To region	1970	1980	1990	2000	2010	2020
1	1843.67	835.35	424.82	226.42	125.59	71.92
2	520.50	288.03	149.03	74.71	39.76	21.85
3	463.91	218.77	100.88	49.99	26.43	14.62
4	417.03	225.99	112.39	58.82	32.20	18.33
5	441.66	197.76	92.64	46.96	25.43	14.38
6	442.89	212.63	107.66	57.02	31.44	18.00
7	229.69	120.15	61.71	33.01	18.39	10.38
8	404.18	189.69	90.78	46.74	25.57	14.65
9	54.41	26.09	13.45	7.30	4.12	2.30
10	83.92	46.86	26.17	14.61	8.16	4.56

Table A.6 Water Transmission Costs from Rock Spring Branch Reservoir (¢/1000 gallons)

To region	1970	1980	1990	2000	2010	2020
1	355.22	196.51	109.73	61.27	34.22	19.11
2	353.10	195.50	101.70	51.66	27.84	15.52
3	239.04	133.48	74.53	41.62	23.24	12.98
4	133.28	74.72	41.56	23.20	12.96	7.24
5	144.75	80.83	45.13	25.20	14.07	7.86
6	68.83	38.43	21.46	11.98	6.69	3.74
7	148.89	83.14	46.42	25.92	14.48	8.08
8	80.60	45.01	25.13	14.03	7.84	4.38
9	57.15	31.91	17.82	9.95	5.56	3.10
10	27.72	15.48	8.64	4.83	2.70	1.51

Table A.7 Water Transmission Costs from Shoal Creek Reservoir (¢/1000 gallons)

To region	1970	1980	1990	2000	2010	2020
1	1034.23	468.31	238.04	126.81	70.30	40.24
2	259.51	143.60	74.26	37.18	19.77	10.86
3	195.84	92.26	42.49	21.03	11.11	6.15
4	35.46	19.20	9.51	4.96	2.70	1.53
5	190.80	85.35	39.94	20.22	10.95	6.18
6	183.22	87.89	44.48	23.55	12.98	7.43
7	122.76	65.37	34.89	19.48	10.88	6.08
8	456.74	212.70	100.34	50.72	27.19	15.59
9	150.45	75.21	40.67	22.82	12.75	7.12
10	107.64	60.10	33.56	18.74	10.47	5.84

Table A.8 Water Transmission Costs from Silver Lake Reservoir (¢/1000 gallons)

To region	1970	1980	1990	2000	2010	2020
1	1339.93	606.65	308.43	164.33	91.12	52.17
2	353.49	195.61	101.18	50.69	26.96	14.81
3	281.41	132.66	61.14	30.28	16.00	8.84
4	101.17	55.24	28.69	15.78	8.86	4.94
5	168.11	75.94	36.16	18.73	10.37	5.82
6	30.18	15.56	8.56	4.78	2.67	1.49
7	133.08	74.31	41.49	23.17	12.94	7.22
8	74.70	37.17	19.68	11.06	6.18	3.45
9	104.07	58.11	32.45	18.12	10.12	5.65
10	74.20	41.43	23.14	12.92	7.21	4.03

Table A.9 Water Transmission Costs from Spanker Creek Reservoir (¢/1000 gallons)

To region	1970	1980	1990	2000	2010	2020
1	740.36	357.23	192.94	109.33	61.05	34.09
2	358.51	198.39	102.62	51.41	27.34	15.02
3	424.53	218.09	119.40	67.19	37.52	20.95
4	202.23	112.92	63.06	35.21	19.66	10.98
5	204.01	112.78	63.06	35.21	19.66	10.98
6	97.19	54.27	30.31	16.92	9.45	5.28
7	212.19	118.49	66.16	36.94	20.63	11.52
8	83.35	46.54	25.99	14.51	8.10	4.53
9	164.40	91.80	51.26	28.62	15.98	8.93
10	131.69	73.54	41.06	22.93	12.80	7.15

Table A.10 Water Transmission Costs from Shelbyville Reservoir (¢/1000 gallons)

To region	1970	1980	1990	2000	2010	2020
1	531.00	240.26	122.01	64.92	35.95	20.55
2	72.74	40.24	20.78	10.36	5.49	3.00
3	.64	.29	.13	.06	.03	.01
4	231.66	125.52	62.40	32.66	17.89	10.19
5	208.87	93.44	43.73	22.15	11.99	6.77
6	517.41	248.40	125.83	66.70	36.82	21.11
7	213.57	111.18	56.54	29.88	16.45	9.36
8	572.23	268.57	128.57	66.24	36.27	20.81
9	491.22	231.27	116.33	61.32	33.72	19.26
10	324.62	160.77	82.56	44.21	24.54	14.04

Table A.11 Water Transmission Costs from Carlyle Reservoir (¢/1000 gallons)

To region	1970	1980	1990	2000	2010	2020
1	1316.06	596.11	303.05	161.46	89.52	51.24
2	334.50	185.10	95.74	47.95	25.50	14.00
3	279.09	131.59	60.65	30.04	15.86	8.76
4	220.39	119.42	59.35	31.03	16.97	9.64
5	174.69	78.17	36.57	18.51	10.01	5.65
6	236.58	113.53	57.45	30.40	16.74	9.58
7	91.60	47.69	24.24	12.80	7.03	3.99
8	.43	.19	.09	.04	.02	.01
9	157.58	74.51	37.80	20.13	11.20	6.35
10	126.45	62.59	32.10	17.16	9.50	5.42

Table A.12 Costs of Artificial Recharge

From Reser.	1970	1980	1990	2000	2010	2020
East Fork	118.60	59.10	32.86	21.21	15.70	12.53
Plum Creek	419.20	203.09	103.09	58.17	36.13	24.46
Rock Spring	118.34	77.56	54.94	41.36	33.18	27.88
Shoal Creek	206.19	100.32	53.70	33.00	22.96	17.89
Silver Lake	94.75	55.25	36.46	26.72	20.92	17.39
Spanker Crk.	117.49	76.08	51.88	37.63	29.21	23.85
Shelbyville	218.87	103.44	53.73	32.15	21.99	16.77
Carlyle	10.43	10.19	10.09	10.04	10.02	10.01

Table A.13 Costs of Imported Water (¢/1000 gallons)

Year	Cost
1970	22.04
1980	21.89
1990	21.79
2000	21.70
2010	21.64
2020	21.58