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METROPOLITAN WATER SUPPLY ALLOCATION AND OPERATION

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ABSTRACT

METROPOLITAN WATER SUPPLY ALLOCATION AND OPERATION

In metropolitan areas, water is supplied to consumers from one or more sources through separate but adjacent systems of facilities commonly owned and operated by municipal governments. Allocation of production and distribution is determined by the demand contained within municipal boundaries rather than on the basis of regional efficiency. Some systems may have more capacity than required to meet their needs, while others have insufficient capacity; and the excess capacity of one system could be used to augment the system that lacks capacity and thereby improve the overall efficiency of utilization. When viewed as a regional allocation problem, then, the challenge is to minimize the total cost of providing potable water with a given set of facilities (in the economic short-run sense). This can be accomplished by equating the marginal costs of production plus transportation among all interconnected systems of the region, while meeting, as constraints, water demands and capacity limitations. Production cost and transportation (distribution) cost functions were determined for selected water supply systems (or subsystems) in the Chicago area. Production cost and transportation cost functions were determined econometrically and, for transportation costs, technologically using a geometric programming procedure. The resulting cost functions were then used in an example problem to illustrate the utility of the proposed methodology for allocation and operation.

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I. INTRODUCTION

The management of metropolitan water resources is generally focused around two areas of primary responsibility: provision of water supplies for domestic, commercial and industrial use; and the collection, treatment and disposal of wastewater and storm runoff. A third area of responsibility, provision of flood protection and flood-water management, is often closely related to the second area; and its problems, beyond those of storm-water collection and disposal, are often less tractable to the development of general strategies for management, since the problems and, consequently, the solutions tend to be location - and site - specific.

Other problems of (or opportunities for) water resources management, dealing with aesthetics, recreation and transportation, are even more specific to the particular metropolitan area in question; still, the management strategies involved are intimately related to those of the primary mission areas of water supply and wastewater management.

The fundamental roles played by water supply and wastewater management in the development of methodologies for metropolitan water resources management led us to conclude that they provide the most promising entrée to the development of overall management strategies. The approach taken was to examine, by means of mathematical modeling, the concept and definition of "efficient size" as it relates to service districts for water supply and distribution and for wastewater collection and treatment. Once the characteristics of efficient service districts are delineated and procedures devised for identifying the limits or boundaries of such districts, a regional model can be constructed in which the many alternatives for metropolitan water resources management can be examined and the possible trade-offs between efficient operation of water supply systems, wastewater systems, and systems for flood control, water-related recreation, and other water resource management areas can be evaluated on a basis more firmly related to the concepts of efficiency.

In previously reported studies, we have examined the economics of wastewater collection networks (51), the performance of regionally-related wastewater treatment plants (52), and the economic and water quality aspects of wastewater treatment plant centralization and decentralization (54,55). This report addresses the problem of efficient service districts for water supply systems.

In most metropolitan areas, water is supplied to the general public, including commercial and industrial users, from one or more sources through facilities comprised of treatment plants, pumping stations, reservoirs, and distribution networks. These facilities are usually grouped into separate but adjacent supply systems that are owned and operated by municipal governments. The area served by each system tends to coincide with that of the municipality. A community that owns its own system ordinarily does not buy water from nor sell water to other communities that have their own systems. Allocation of production and transportation is determined by the demand contained within municipal boundaries and is thereby geopolitically dictated. When an individual supply system has more than one treatment plant or pumping station, production and transportation are allocated according to the design capacities of plants and pumps and the hydraulic characteristics of the distribution network.

If a regional allocative scheme based on municipal boundaries is economically efficient, this efficiency has been achieved largely by chance. Certainly not all systems in the metropolitan area are operated at their peak efficiency. At any given time, some systems will have more capacity than required to meet their needs while other systems will have insufficient capacity. The excess capacity of one system could be used to augment the system that lacks sufficient capacity and thereby improve the utilization of treatment, transportation, and storage facilities of both systems. Hydraulic considerations play an important role in determining the use of treatment plants, pumps, reservoirs, and distribution networks; however, in any complex supply system, there exist a great number of ways to combine these facilities in order to meet the demands for water. Hydraulic analysis does not identify the most economically efficient alternative. What is lacking in both hydraulic and geopolitical allocation schemes is consideration of the economic characteristics of the production and transportation facilities.

The feasible set of treatment and transportation facilities that can be employed to meet given water demands is defined by hydraulic and political considerations. The conditions of economic efficiency provide a rationale for selecting the optimal combination of treatment and transportation facilities from the feasible set. Economic efficiency is defined as that situation in which production and transportation are so allocated among the various facilities that any change in allocation will increase the cost of meeting demands and supplying water to the region. Of course, the various municipal or sub-systems must be physically interconnected in order to implement the optional strategy. Adjacent water supply systems are often interconnected for emergency purposes, although the links are seldom if ever used. However, if connections had to be constructed or enlarged, the cost would be insignificant compared to the cost of additional pump, tunnel or treatment capacity. Consequently, the small capital expenditures required to interconnect adjacent systems would not preclude the use of short-run economic analysis for evaluating alternative strategies.

The problem, then, is to minimize the total cost of providing potable water, given a multiple-plant, regional water supply system. In this paper, cost minimization will be dealt with only in terms of short-run economic considerations. Optimal operation or utilization of the regional supply system is the central issue and not the development or expansion of the system. Under the conditions of short-run analysis, the capacity of a system cannot be physically enlarged. However, any portion of the system's capacity can be left idle should it prove beneficial to do so.

Short-run economic analysis provides the means to evaluate alternative strategies for utilizing more efficiently that which already exists. As noted above, there is likely to be excess capacity in any given regional supply system. The inability to employ this capacity to meet critical demands is due, in large part, to the unfortunate distribution of the demands in time and space and to the inflexible disposition of water treatment, storage, and conveyance facilities. But before additional capacity is considered (which would require long-run economic analysis), the existing system should be scrutinized and an attempt made to fully employ the available facilities through operational controls.

In order to achieve the desired minimum cost, the capability or capacity of each system element (treatment plants, pumps, distribution networks) must be assessed; and, based on this knowledge, each element must be optimally employed. While most plants cannot be physically moved, their demand loads, or service areas, can be modified. Such changes may be required to more fully exploit the determined service and economic attributes of a particular element. Any changes that are made must preserve the integrity of the total system since optimality is based on the concerted operation of every element. As the demand for water must be satisfied, a reduction in the usage of one facility must be accompanied by the increased usage of at least one other element. By altering service areas, demand loads are moved to, and away from, particular facilities in the system, which in a sense is a reallocation of capacity. But regardless of whether demand is shifted or the capacity to supply demand is relocated, the solution to the problem follows from the optimal delineation of service areas.

In turn, determination of the minimizing market area configuration is based on equating, for all source areas, the sum of the marginal costs of production and transportation. For the i^{th} and j^{th} service areas, the marginal cost of producing an additional unit of water at the i^{th} plant and transporting it in the service area must be exactly equal to the marginal cost of producing an additional unit of water at the j^{th} plant plus the associated marginal cost of transportation to and within its service area. Each service area exerts a demand for water; and this demand dictates the quantity of water produced at a particular treatment plant and transported throughout the area. The quantities produced and transported define the marginal costs, and the marginal costs should be used to define the service area. In order to equate marginal costs between service areas, these areas must change in accordance with the economic characteristics of their treatment and transportation facilities. A statement by Moses (26), although taken somewhat out of context and of a more general application, explains the nature of the process: "...exogenous change in demand for a commodity in one region may affect... the spatial distribution of consumption, production, etc., of all commodities in all regions." As the service area boundaries change, the distribution of demand faced by each water treatment plant and pumping station changes; and, since supply must equal demand, the distribution of production and transportation changes.

The intention of this research effort, in part, is to establish a basis for effective planning and control of water distribution systems. As was noted by Pagnotto (38), Load Control Systems Engineer for the Philadelphia Water Department, this field of inquiry has been practically ignored and yet it is quite important "...considering that over fifty percent of plant investment is represented in the piping and the transmission facilities." Of a more general nature, Koenig indicated that as one of the purposes of his inquiry into the cost of water treatment (28) was to "...obtain at least some preliminary data bearing on the question: Is there some reasonable probability that water treatment costs could be reduced over present practice in design and operation by the application of modern process optimization techniques?" Although optimization was not attempted, he concluded that substantial savings could not be achieved through control techniques. However, the reduction in operating costs, whether treatment or distribution, is not the only issue in question. What is more important is how to operate a water supply system so as to avoid or defer further capital investment. To achieve this end, mean daily cost functions, critical event analysis and the underlying principles of marginal cost allocation can be used to evaluate feasible control strategies.

Before the relevant marginal costs can be established, quantitative relationships between the costs of both transportation and production and the quantity of water must be defined. Then, these functional relationships must be combined into an objective function that describes the variable cost of regional water supply. This research report presents a methodology for creating the objective function. This function incorporates production and transportation costs as well as the constraints of meeting water demands and capacity limitations. Using established techniques, the objective function is minimized, and marginal costs among service areas are equated.

For general perspective, a brief profile of the municipal water supply industry along with a description of physical facilities, treatment processes, and water supply problems existing in the test environment are given in this dissertation. Relevant economic theory, its development and application are presented. Actual cost functions for distribution facilities and treatment plants are given. Finally, an exemplary regional function is constructed, and its minimization is demonstrated.

II. MUNICIPAL WATER SUPPLY INDUSTRY

Output

The domestic water supply industry produces a "service-product" (44). The product output is potable water. The service output is providing a given quantity of water on demand at a prescribed pressure. The purity aspect of water is not dealt with in this dissertation in the sense that the obvious trade-offs between slightly less pure water and slightly reduced costs are not analyzed.

Treatment processes transform raw water input into a product that must meet specific standards of purity. Drinking water standards are promulgated by the United States Department of Health, and many states adopt these standards or some states enforce their own standards. Drinking water standards are designed to insure public health and safety. While the product must be safe for domestic consumption, it also should be palatable and free from objectionable odors. In addition, it must be delivered to the consumer with adequate pressure. The technological concepts of collection, production, and distribution are well established and present few problems to the industry.

Unlike the product output, the service output presents a formidable problem to the industry. The industry is constrained to meet demand. This means that water must be supplied more or less instantaneously and continuously. Consequently, the demand for water dictates both the magnitude and schedule of production. The water industry is responsible for supplying current demands as well as anticipating and meeting future demands. Physical plant must be operated, improved and expanded in such a manner as not to jeopardize the safety or the economic development of the dependent community.

The demand for water, as a function of time, does more to influence the output characteristics of a particular system than any other factor. Demand varies diurnally as well as from day to day, and from season to season. If demand were uniform throughout the day and year, production could be scheduled and cost and prices firmly established. However, this is not the case. In Figure II-1, supply is plotted against time for a typical summer day in 1968 for the Chicago water supply system. Since it is virtually impossible to measure demand, as such, it is considered to equal supply so long as no gross violations of the minimum pressure requirements of the system exist and few complaints have been reported by customers. At 4:00 in the morning the demand rate was about 900 million gallons per day (MGD), but by 4:00 in the afternoon the demand rate was approximately 2,200 MGD. For this same year and supply system, the pumpage for an annual average day was approximately 1,024 MGD, the minimum day 796 MGD, and the maximum day 1,666 MGD.

The factors affecting daily water demand are domestic and industrial consumption, air temperature, rainfall, etc. The influence of these factors varies both spatially and temporally. In the industrial areas of Chicago, the peak hourly pumpage occurs at about 8:00 a.m., remains constant for the next eight hours, and then rapidly tapers off. In suburban areas, demand starts about noon, peaks around 4:00 p.m. and again at 9:00 p.m., and then begins to decline. In order to satisfy the service constraint, the water supply system must have sufficient capacity to meet peak demands. Consequently, excess treatment and pumping or storage capacity must be maintained; and, since extreme events occur only one or two days each year (and then just for a few hours), a large part of the system remains idle for substantial periods of time.

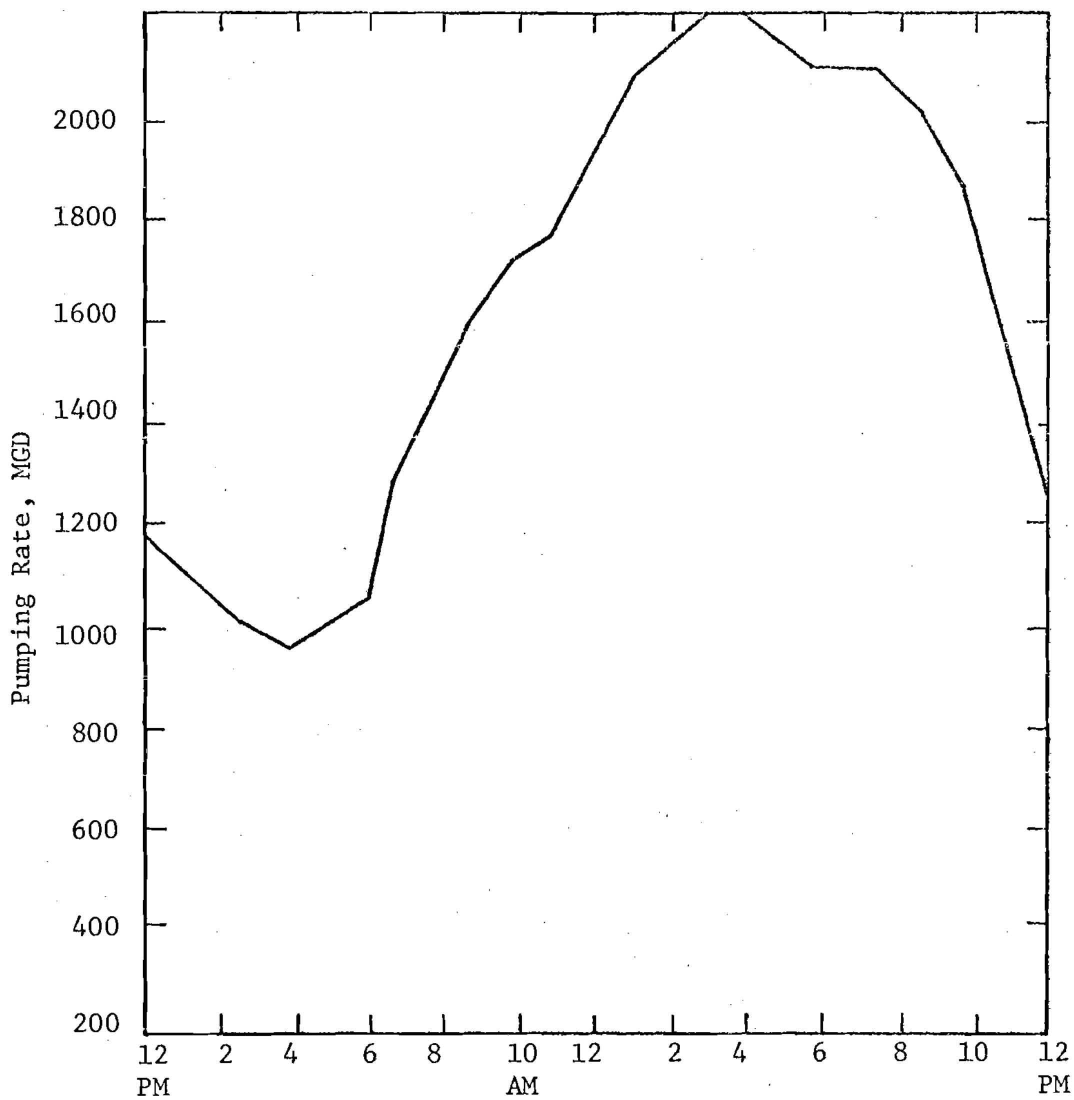


FIGURE II-1

TYPICAL PUMPAGE CURVE FOR THE CITY OF CHICAGO, 1968 (2)

Physical Plant

Along with demand, the quantity and quality of raw water input define the technology and physical plant of the water supply system. The municipal water supply system is physically characterized by its collection, treatment, and distribution facilities. Wells, rivers, natural lakes, reservoirs, and oceans are sources of raw water. Collection and diversion of these supplies to treatment works are accomplished by means of reservoirs and dams, aqueducts, tunnels, pumps, and other hydraulic structures. Treatment plants transform the raw water into potable output by one or more processes such as chlorination, settling, filtration, softening, and/or desalination. Treated water may be stored temporarily in ground surface or elevated reservoirs for later distribution or, immediately after treatment, it may be distributed by means of gravity or pumps through tunnels and pipe networks.

The design of the water supply system is most commonly based on the criterion of maximum day demand. By not including storage for treated water in the design, substantial treatment and distribution capacity would be required and capital cost would be quite high. By judicious placement of storage in the distribution system, treatment plants, pumps, and pipe networks can be reduced in size which will reduce capital cost. Also, the use of storage permits more uniform production schedules and presumably improvement in water quality and better control over chemical costs. Storage is expensive, and in most cases the maintenance of sufficient storage to keep production at a constant level would be prohibitive. There is, of course, an economic balance between treatment and distribution capacity and storage capacity. There is evidence (4) that a six percent to 16 percent reduction in capital cost can be achieved by designing the water supply system for the average day demand while using storage to meet maximum day and maximum hour demand.

Economics

The cost of constructing, maintaining and operating the required physical plant for a water supply system is high relative to other industries. The capital turnover ratio (21), i.e., gross revenues to capital investment, is one to five for the water supply utilities as compared to three to five for gas utilities and two to one for private manufacturing (see Table II-1). The low return on capital investment is, according to Hirshleifer (23), an indication that there is overinvestment in the water supply industry. Hirshleifer maintains that there is typically premature and overly ambitious investment in the industry. A further explanation of the low return for capital invested is given by Hurter (23A): "...(the low return) may also be an indication that whoever controls the water supply system is using an exceptionally low rate of discount, either on the grounds of social or community benefit, or out of ignorance or error."

Table II-1
SELECTED CAPITAL TURNOVER RATIOS

INDUSTRY	C.T.R.
Electric utilities	0.30
Natural gas utilities	0.60
Natural gas pipelines	0.40
Bell Telephone System	0.40
Water utilities	0.20
Total manufacturing	2.00

Costs of the "service-product" result from energy, materials, wages, interest, maintenance, and depreciation. These costs can be grouped into three categories (44): fixed, fixed operating, and variable operating. Fixed costs are capital costs such as interest and depreciation: they are related to plant capacity but are not affected by the actual level of production. Fixed operating costs are maintenance, wages, and other overhead items such as plant lighting and heating. If a particular facility or system were to be totally shut down, fixed operating costs could be avoided whereas fixed costs could not. Variable operating costs are chemical and power costs and other costs which relate directly to the level of production. In 1970, Chicago's Bureau of Water reported that total costs (the sum of fixed, fixed operating and variable operating costs) were approximately $65,000,000 (7). Of that amount, 14 million dollars were fixed costs, 46 million dollars were fixed operating costs, and 5 million dollars were variable operating costs. This means that 92 percent of the total costs were fixed and only eight percent of the costs were variable and sensitive to production levels. In other utility industries, fixed costs accounted for 60 to 80 percent of the total costs (44). In a study done by Forn and Warford (17), variable operating costs for the Manchester Water Supply System were approximately four percent of total costs.

Given the dominance of fixed costs, the water supply industry has been characterized, not surprisingly, as one of decreasing average costs (8) (44) (45). Increases in production do little to affect total cost. Consequently, average total cost decreases as production levels increase. Considerable attention has been given to this aspect, particularly in regard to establishing the price of treated water (18) (19) (42). Factors contributing to declining average costs are overinvestment, investment in physical rather than operational alternatives, and, of course, the highly variable temporal distribution of demand. This latter factor, along with the constraint that demand must be met, affects total average costs because the treatment plant, pumps and storage capacities, required to meet peak demands, remain idle most of the time. During their idle period, they are not contributing to the making of revenue, yet interest payments must continue whether or not the facility is fully utilized.

Management

Public ownership in the water supply industry is so strongly entrenched that it is virtually taken for granted (8). The unusually high capital investment required to construct, operate, and maintain a water supply system requires considerable financial power. The low return on invested capital makes such a venture unattractive for private capital (18). The assessment of costs and value of services is particularly difficult to establish because of the joint relationship between water supply and public health, sanitation, community growth, and fire protection. While private industry would require compensation to cover total production and distribution costs, governmental ownership can insure production even at a loss, recognize external benefits, and subsidize the supply system if necessary. However, there are some successful, privately owned water utilities.

Governmental ownership usually takes the form of municipal ownership. Troxel (45) notes that a single city is usually the optimum market area for a supply system. This market definition probably is not correct; however, there are few examples upon which to base this opinion. A community will build a system to serve its own needs. Boundaries are established and changed not from a consideration of optimizing water supply service but rather from political considerations.

While the Chicago's water supply system serves the city of Chicago and 72 suburbs, it is not really a regional system. Operators of the Chicago system have no control over suburban systems which they serve. For example, storage capacity available in the suburban systems cannot be used by Chicago, nor are there effective controls placed on the withdrawals from the Chicago system by the suburban systems. Chicago is a semi-regional system: it serves a major portion of the metropolitan area; it has to deal with major problems of peak demand and fixed costs; yet, it cannot take advantage of excess capacity available in this complex system. A truly regional water supply system is one that serves a metropolitan area by employing one or more sources and treatment plants and has operational control over all production and distribution facilities.

Example Region and Problem

While the general methodology of regional cost minimization has universal application, development and testing was conducted using facilities in the Chicago Metropolitan Area (see Figure II-2). Existing in this area is a nearly complete set of source types, system configurations, institutional controls, constraints, and managerial problems. In fact, the imminent need for solutions to the problems of water management and allocation in northeastern Illinois was one of the compelling factors which led to this research effort.

In 1970, water production and distribution amounted to 1,304 MGD (1) for the metropolitan area. Of this total quantity, Lake Michigan supplied 1,104 MGD. The remaining 200 MGD were supplied from ground water sources. Approximately 46 percent of production is used for domestic purposes. Twenty-one percent is used by large industrial concerns, ten percent is used by large commercial interests, three percent is used for

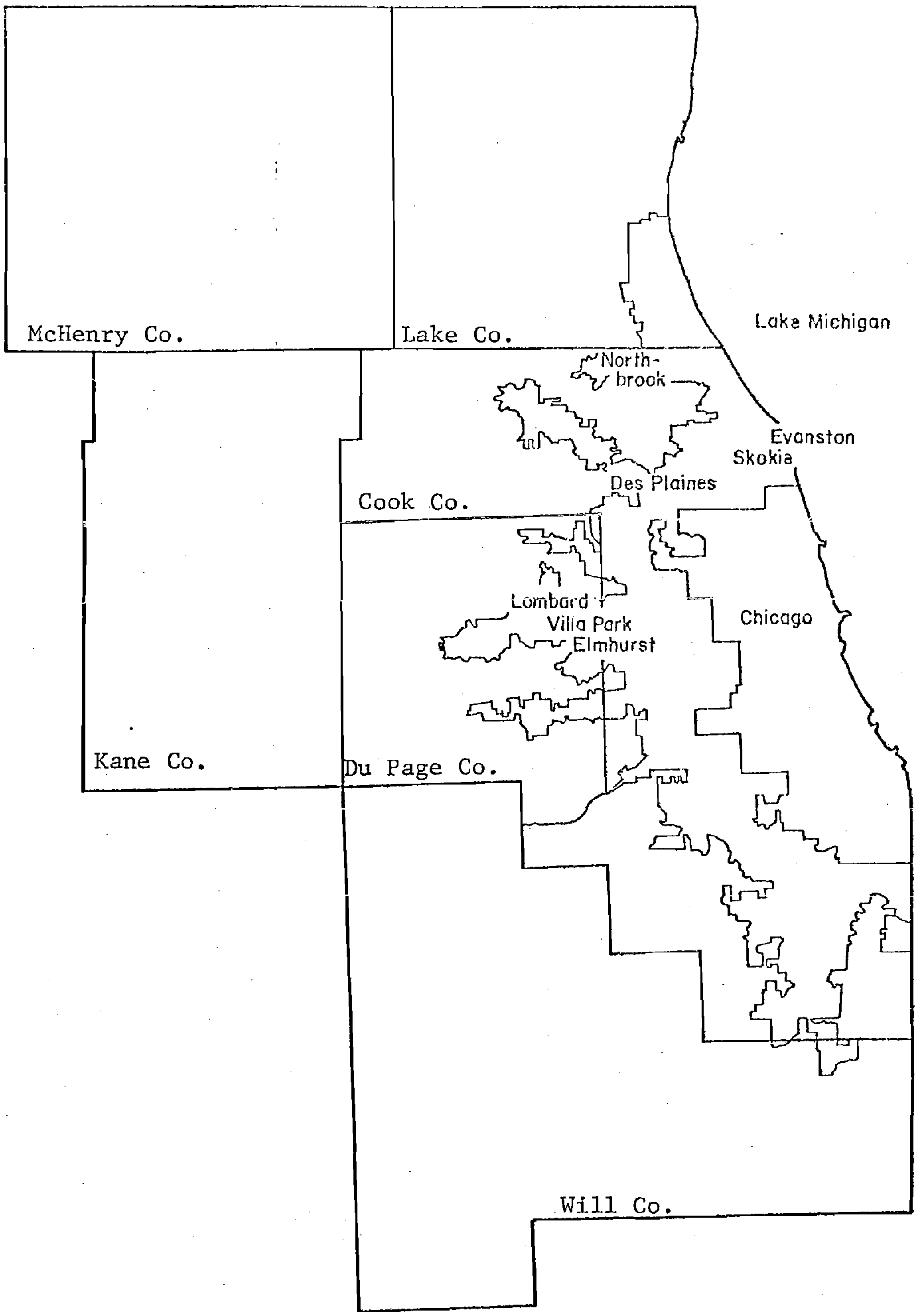


FIGURE II-2

NORTHEASTERN ILLINOIS METROPOLITAN AREA

irrigation (lawn sprinkling), and 20 percent is lost through leakage (37). At last count, approximately 680 governmental agencies in the six-county area had authority to make developmental and managerial decisions affecting the water resources of the region (37). In the municipal water supply sector alone there were 342 administrative and operating agencies. Approximately 85 of these agencies, including the City of Chicago, obtained their water from Lake Michigan. Of these 85, Chicago supplies itself and 72 other municipalities. The remaining 257 municipal systems obtained their water from ground water sources (37).

With one exception (Northbrook, Illinois), the communities west of Chicago and the other lake-front cities and towns either purchase water from the lake-front municipalities or they are dependent on groundwater. Groundwater is obtained from shallow wells in gravel or dolomite or from deeper wells in sandstone. Although the deeper wells are more expensive to construct, their expected yield, on the average, is much greater than that for shallow wells. The large suburbs have tended to draw their water from the sandstone formation; and, as demand has increased, more deep wells have been constructed and the existing wells have been more heavily pumped. Consequently, the piezometric level in the sandstone formation has fallen from well above ground level (artesian) to its present level of about two to three hundred feet below ground level. Lower water levels require more energy to deliver the water to the customer; and, therefore, the cost of producing water is higher. As the suburban population increases (one estimate is 50 percent (1) in the next three decades) and water levels decline, demands will become increasingly more difficult and expensive to meet. Already, the situation is critical for some suburbs, e.g., Elmhurst, Villapark, and Lombard, and the public is quite aware of the problem (10).

Although Lake Michigan represents a vast source of water, there is a Federal restriction on the withdrawal of water from the Lake by Illinois (47). The restriction was the result of a very lengthy litigation involving most of the Great Lake states and Canada. The U.S. Supreme Court ruled on March 1, 1970, that Illinois could withdraw only 2,080 MGD. Out of this allotment, the ruling set 969 MGD as the average withdrawal that the Metropolitan Sanitary District of Greater Chicago (MSDGC) could use for maintaining navigation and sanitary conditions on the Chicago Sanitary and Ship Canal. The difference between the total diversion and the MSDGC's diversion is available for municipal water supply. However, in the past, the City of Chicago along with the twelve other lake-oriented municipal systems consumed the remaining supply, leaving the western suburbs without access or right to this source.

In recent years, improvements in waste water treatment have reduced the quantities of water needed by the Metropolitan Sanitary District to maintain conditions on the Ship Canal. Reduction in withdrawals by the Metropolitan Sanitary District releases Lake Michigan water to be used elsewhere. Since there is an unsatisfied demand for water in the western suburbs, reallocation of production and transportation will be made to meet this need. In fact, the legal processes of permitting the western suburbs access to Lake Michigan have been initiated. The question under consideration is how this reallocation can be achieved so as to minimize total regional costs.

Example Facilities

Chicago is a city of superlatives, and its water supply system is no exception. The two filtration plants, providing the City of Chicago and 72 suburban communities with treated water, are two of the largest in the world. The Central Water Filtration Plant was built in 1964 and is located on the shores of Lake Michigan adjacent to the central business district. This plant services the central business district, the central and west central suburbs as well as the entire northern part of Chicago and the north and northwest suburbs (see Figure II-3). The South Water Filtration Plant, also located on the shore of Lake Michigan, was built in 1947 and expanded in 1966. It supplies water to the south part of Chicago and the south and southwest suburban communities. Distribution of treated water from these two plants is achieved in two steps: treated water is supplied to a series of pumping stations, 11 in all, by means of tunnels constructed in rock formations some 400 feet beneath the city. The city and suburbs are divided into three tunnel zones: north, central, and south. The central filtration plant provides treated water to the north and central tunnel zones, and the south filtration plant provides water to the south tunnel zone. Treated water is then pumped from the tunnels and into the distribution network. In 1970, the Chicago water supply system produced 1,035 MGD on the average (7). Average pumpage to the city alone was 867 MGD, and average pumpage to the suburban communities was 168 MGD (7). Total population served in that same year was 4,506,000 people, with 3,367,000 people living in the City of Chicago and 1,139,000 people in the suburban communities. The per capita consumption of water in the City of Chicago was 257 gallons per day. The per capita consumption for residents in the suburban communities was 147 gallons per day.

The Central Water Filtration Plant has a designed capacity of 960 MGD; however, it has operated at well over 1,000 million gallons per day. Water is obtained from Lake Michigan through an intake crib located 2-1/2

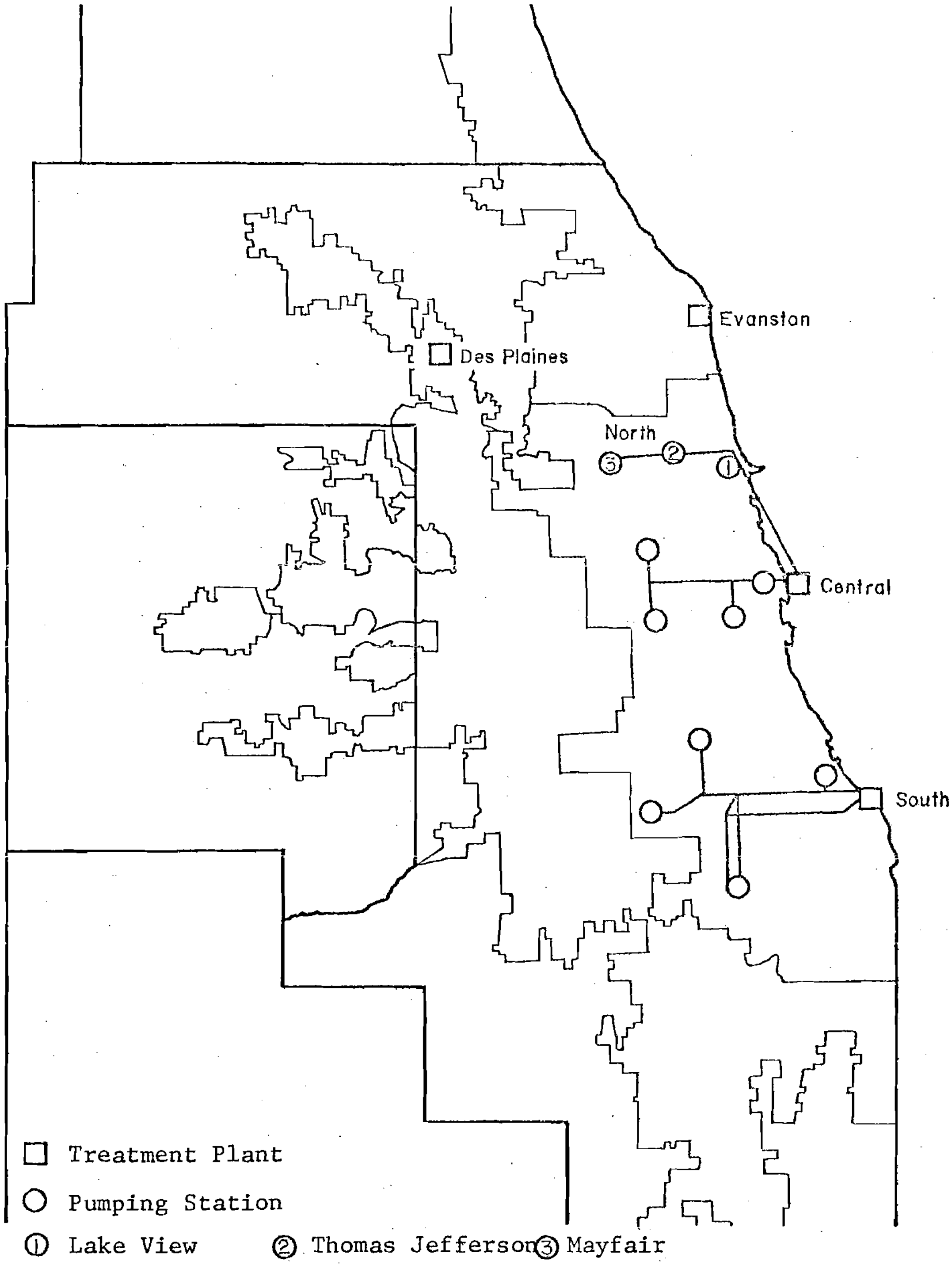


FIGURE II-3

EXAMPLE TREATMENT AND PUMPING FACILITIES miles offshore or through shore intake gates located on the north side of the treatment plant. The raw water passes through screens which remove floating debris, aquatic weeds and fish. Eight low lift pumps with an aggregate capacity of 1,900 MGD lift the water to permit gravity flow through the plant. Next, the water flows through chemical application chambers where chlorine, anhydrous ammonia, aluminum sulfate or oxidized ferrous sulfate, and activated carbon and fluoride are added. Chlorine is used to kill bacteria and the anhydrous ammonia to stabilize the chlorine residual. Aluminum sulfate or the oxidized ferrous sulfate is used as a coagulant to facilitate removal of suspended material by sedimentation and filtration. Activated carbon is used to absorb objectionable taste and odors and fluoride to reduce caries in teeth. The chemicals then are mixed by means of slow moving paddles to promote the growth of floc particles. In the settling basin which follows, the floc, heavy with adsorbed material, settles to the bottom of the chamber. Nearly 85 to 90 percent of the floc and suspended material is removed at this point in the treatment process. The final step is rapid sand filtration. There are 96 filter units in this plant. The filters are 48 inches deep and composed of graded sand and gravel. Water is placed on top of the filter beds and drained down through the material, removing nearly all remaining suspended matter. Prior to distribution of the treated water, more chlorine and, if used, ammonia are applied to the water for further disinfection. Lime or caustic may be applied for pH control. In 1970, the Central Water Filtration Plant processed some 246,080 million gallons.

There are three pumping stations in the north tunnel zone: Lake View, Thomas Jefferson, and Mayfair. Lake View has a rated capacity of 70 MGD, Thomas Jefferson a capacity of 120 MGD, and Mayfair a capacity of 320 MGD. Due to large peak demands in the north tunnel zone, the

Mayfair pumping station is operated at nearly its maximum hydraulic capacity. There are four pumping stations in the central tunnel zone—Chicago Avenue, Cermak, Springfield, and Central Park. The rated capacity for each is 210 MGD, 250 MGD, 280 MGD, and 280 MGD, respectively. In the south tunnel zone there are four pumping stations—68th St., Western Ave., Roseland, and South West. Their rated capacities are 200 MGD, 235 MGD, 230 MGD, and 125 MGD, respectively. The 11 pumping stations are operated at a pressure of between 130 ft. and 230 ft. of water. They maintain pressure and flow in a distribution network which contains over 4,100 miles of water mains.

Storage in Chicago's water supply system is extremely small. The central water filtration plant maintains a reservoir of 111 million gallons, the south water filtration plant a reservoir of 47 million gallons, and there is additional storage in the south tunnel zone of 30 million gallons. Total storage is approximately 188 million gallons. Due to the small amount of storage in the system, the treatment plants must respond almost directly to variations in demand.

While the Chicago water system serves itself and 72 suburbs, it is not really a regional system. The operators of the Chicago system have no control over the suburban components beyond certain contractual agreements regulating price and total quantities of water supplied. In the city-suburban system, there is approximately 140,000 MG of storage which, if properly utilized, could dampen the effects of peak demands and relieve overextended elements in the system. Yet, Chicago controls only 0.2 percent of the total storage. Further, there are no controls over when the suburbs draw water from the system. Consequently, water can be taken at peak periods, thereby aggravating a bad situation. If treated water could be stored in the suburbs during off peak periods and then returned to the city during peak periods, considerable savings might be achieved.

Like Chicago, Evanston takes its water from Lake Michigan. The water is treated at its lakeside plant and distributed to its customers through 300 miles of water mains. Water is drawn from Lake Michigan through three intake pipes. At the end of the intake pipes, activated carbon is added to the water to remove objectionable tastes and odors. The water then passes through coarse screens to remove fish, leaves, and large floating debris, and is pumped, by low lift pumps, to provide gravity flow through the rest of the treatment processes. The rated capacity of the low lift pumps is 100 MGD. Liquid aluminum sulfate and chlorine are now added to the water. Chlorine is used to disinfect the water and oxidize organic materials, while the aluminum sulfate is added as coagulant to facilitate removal of suspended material in the water.

After application of the chemicals, the water enters a flash mixer which facilitates chemical reaction. Leaving the flash mixer, the water enters the slow mix basin containing horizontal, slowly moving paddles. In this basin the floc is increased to sufficient size for settling. In the settling basin, sufficient time (four to eight hours) is allowed for a settling of the floc particles. Over 90 percent of the suspended impurities are removed at this step in the process. The final treatment step is sand filtration. The Evanston plant has 24 filters which have an aggregate capacity of 72 MGD. The filters are constructed as follows: graded gravel provides the base upon which 28 inches of coarse sand is placed. One-half of the filters have a five-inch layer of anthracite coal on top of the sand which tends to lengthen the filter operation.

Following filtration and before entering the storage reservoirs, chlorine, ammonia, and fluoride are added.

Approximately 21.9 million gallons of storage exist in the system. Nine and a half million gallons of storage is in the treatment process and in the filtered or treated water reservoir at the plant. Twelve and four tenths MG are available in the distribution system itself. Of that storage, 7.4 MG is available in elevated tanks or standpipes floating on the system, and 5 MG of storage is at ground level and can be input into the system by means of a booster pumping station. This storage is approximately 30 percent of the rated output for the treatment plant. With the average daily pumpage of 25 MGD for Evanston, the storage capacity in the system represents approximately 88 percent of the average daily total. This is in sharp contrast to storage found in the Chicago system, and it undoubtedly leads to cost savings for the treatment and distribution system.

Evanston produces and, in terms of short-run economic analyses, distributes water not only for its own residents but for those of Skokie as well. In fact, the Village of Skokie purchases over 50 percent of Evanston's output. A total population of 154,000 people is supplied—83,000 in Evanston and 71,000 in Skokie. Distribution to meet these demands is accomplished by seven electric pumps and two gasoline electric pumps with a total rated capacity of 124 MGD. Again, distribution is aided under peak conditions by elevated storage floating on line located at critical points in the system.

An example of a deep well, water supply system can be found in Des Plaines, Illinois. However, the Des Plaines water supply system is unique in that it obtains its water from two sources. It obtains part of its water from seven deep wells and another portion from Lake Michigan via the Chicago water supply system. Under terms of a ten-year contract with the City of Chicago, Des Plaines must purchase 3.5 MGD; however, it can purchase up to 7 MGD at any one time. The remaining demand is met by the deep wells and treatment plant owned by the City of Des Plaines.

The rated capacity of the treatment plant of Des Plaines is 6 MGD. The treatment process is mainly that of softening. At the beginning of the treatment process, lime, sodium phosphate and coagulant aid are added to the well water. These chemicals are then mixed in a basin to form a precipitate much as in the processes described for Evanston and Chicago. The mixed chemicals and water are slowly passed through a sedimentation basin where the floc is allowed to settle out, carrying with it bacteria and suspended matter. Before the settled water is filtered, carbon dioxide is added to the water by means of submerged burners to reduce pH. The water is then filtered through rapid sand filters. There are five filter beds each 51 inches deep. Following filtration, chlorine is added as disinfectant and to oxidize any remaining organic material.

There are three million gallons of storage at the softening plant and three and fifteen hundredths million gallons of elevated storage in the distribution system. In addition to this, ten million gallons of storage exist at ground level at the pumping station which supplies water from Chicago. In 1970, an average of about 6 MGD was pumped in the City of Des Plaines. The available storage is 275 percent of the daily pumpage. As mentioned above, this storage can be used to control production levels within Des Plaines or the City of Chicago.

The City of Des Plaines water distribution system contains approximately 140 miles of water main. Part of the energy to drive the water through the network is supplied by four pumps at the softening plant.

The aggregate capacity of these pumps is 20 MGD. Water from Chicago is supplied by means of pumps at the Maple Street Station having an aggregate capacity of three MGD. Total pumping capacity for distribution in the City of Des Plaines is six MGD.

The water supply system of Des Plaines services approximately 59,000 people, as well as industrial users. In 1970, the softening plant treated approximately 876 MG of water, while the Maple Street Pumping Station (supplying water from Chicago) provided 1,825 MG. Consequently, water supplied from wells amounted to 32 percent of the total demand, while water supplied from Lake Michigan via the Chicago system provided for 68 percent of the demand.

III. MULTIPLE-PLANT FIRM

Regional System Concept

Given a metropolitan area with multiple municipal water supply systems, how might these systems be jointly operated so as to minimize regional cost of production and service and avoid further capital costs? Recognizing the major role of fixed and fixed operating costs in the water supply industry, an immediate answer to this question would be to reduce these costs. However, fixed costs (interest and debt retirement) cannot be eliminated or reduced once the capital has been committed. Even if the plant is shut down, the operating agency still is responsible for fixed costs. Fixed operating costs can be manipulated, but control of these costs, in large part, is a matter of bureaucratic, managerial, political, and social concern. For example, at Chicago's Central Water Filtration Plant, salaries and wages for 1970 were approximately $2,890,000 (7) (the largest single expenditure), while approximately $1,370,000 were expended on chemical material and supplies (the second largest expenditure). Over 50 percent of the money spent for power went for lighting (fixed variable cost) and not pumping (variable cost). Lighting, of course, is necessary for operation, safety, and security of the plant, but a considerable amount is used for display of the building and grounds. Similar comparisons are available for distribution costs.

The employment policy of the operating agency and the community's desire to have ample treatment and distribution capacity are major factors affecting fixed operating costs. Although fertile ground for inquiry, neither managerial schemes nor community goals and objectives are at issue here. Presumably, social values have been assessed, capital has been committed as treatment plants and distribution systems have been constructed, and operating personnel are on the job. Consequently, what is of concern is the efficient utilization of the existing investment—plant, labor, materials, etc. Optimal allocation of the available resources will minimize variable operating costs while reducing future capital investment and future increases in fixed operating costs.

Now, how can the variable costs (power and chemicals) of production and distribution be minimized on a regional basis? The problem of cost minimization is not unique to the urban water supply system. In fact, power utilities use computational algorithms to minimize the costs of operating their generating and distribution systems (31). However, the problem and solution was first formulated as a cost minimization for the multiple-plant firm. In this formulation and in subsequent extensions, the answer to minimizing regional costs for water supply can be found.

Development, Theory, and Conditions

In 1947, Patinkin authored a paper (39) and comments (40) dealing with the comparative economic advantages of monopolies, oligopolies, cartels, and perfect competition. In these works, he explored the optimal size of the associated firms while supporting his conclusions with arguments based on a cost minimization model for the multiple-plant firm. Although most of his mathematical presentation dealt with the two-identical-plant firm, he did acknoweldge the feasibility of analyzing the n-plant firm with each plant having a different cost function. Patinkin's short-run n-plant model has the mathematical form:

$$C_i = f(Q_i) \qquad i = 1, 2, \ldots, n,$$

$$\text{minimize } C = \sum_{i=1}^{n} C_i \; ,$$

$$\text{subject to } \sum_{i=1}^{n} Q_i = \text{total demand}$$

where C_i is the variable operating cost of the i^{th} plant, and Q_i is the output of the i^{th} plant.

The monopolistic multiple-plant firm was the first case considered by Patinkin. He assumed a linear demand function, completely unspecialized factors of production which precluded any form of monopsony, a single product output, and no interplant economies of scale. Also, he assumed that the firm started off with 100 plants, that each plant was at its long-run optimal size and that all plants were identical. First analyzing the short-run behavior of the firm, Patinkin found that the firm, in maximizing profit, does not minimize cost and would make no effort to do so; however, in the long run, the monopolist would adjust his firm's position by adding or liquidating plants so as to minimize long-run cost. He demonstrated that, after adjustment, the short-run minimum cost point of production coincides with the point of production which maximizes long-run profits. As long as demand remains constant, the monopolist will act to minimize his cost while maximizing his profit. The behavior of the oligopolist or those engaged in perfect competition would hardly be the same, as Patinkin points out. Nevertheless, each wishes to minimize production costs. Why the position of minimum cost is pursued, whether by a desire to maximize profits or by the need to maintain a competitive position, is not important. What is important are the conditions by which the firm achieves the state of minimum cost.

Patinkin arrived at two conclusions concerning the n-plant case by use of a Lagrangian function. He concluded that (a) if two or more plants are operated simultaneously, the rates of output in all plants must be such as to equate their marginal costs; and (b) no plant should be kept idle if its marginal costs at zero output are lower than the marginal costs of any other plant at its actual rate of operation. He went on to state that these conditions do not permit the formulation of general rules to determine how many plants should be used to produce a given output. The selection of a feasible, let alone optimal, set of plants remains an arbitrary matter--an interesting dilemma and one that has yet to be resolved.

Leontief (32), commenting on Patinkin's works, contributed two additional conditions to insure that the operating, or allocating, scheme would lead to minimum costs. These conditions were (c) no more than one plant should be operated at decreasing marginal costs; and (d) if one plant is actually operated with falling marginal costs, the reciprocal of its rate of decrease must not be smaller in its absolute magnitude than the reciprocals of the rates of increasing marginal costs of all other operating plants put together.

Two more conditions resulted from Cohen's (9) review of Leontief's and Patinkin's conclusions. In effect, they are extensions of Leontief's condition (c) as given above. They are (e) if no plant is operating at its point of minimum marginal costs, i.e.,

$$\frac{\partial MC_i}{\partial Q_i} \neq 0 \qquad i = 1, 2, \ldots, n$$

where MC = marginal cost, then at most one of the plants can be operated on the decreasing portion of its marginal cost curve: and (f) if one or more plants are operating at their minimal marginal costs, i.e.,

$$\frac{\partial MC_i}{\partial Q_i} = 0 \qquad i = 1, 2, \ldots, n ,$$

then no plant in operation can be operating on the decreasing portion of its marginal cost curve.

These five conditions define the minimum cost solution for the multiple-plant firm. Reiterating these conditions in a numerical order they are:

a. If two or more plants are operated simultaneously, the rates of output in all these plants must be such as to equate their marginal costs;

b. no plant should be kept idle if its marginal costs at zero output are lower than the marginal costs of any other plant at its actual rate of production;

c. if no plant is operating at its point of minimum marginal costs, then at most one of these plants can be operated on the decreasing portion of its marginal cost curve;

d. if one or more plants are operated at their minimum marginal costs, then no plant in operation can be operating on the decreasing portion of its marginal cost curve;

e. if one plant is actually operating at falling marginal costs, the reciprocal of its negative rate of decrease must not be smaller in its absolute magnitude than the reciprocals of the rates of increases of marginal costs of all other operating plants added together.

The definitions and theorems necessary to prove the five conditions of cost minimization can be found in the works of Fiacco and McCormick (16) and in the work of Cohen (9).

Application

While the mathematical proofs demonstrate the validity of the stated conditions, they do not provide an efficient computational scheme for the problem of the urban water system. And, of course, they do not transcend the limitation and the underlying assumptions of Patinkin's model. The general multiple-plant model discussed so far implicitly precludes the need to consider transportation costs. In fact, the cost of transporting the factors of production to the processing centers and the output to the consumer were not given consideration in Patinkin's model. Uniformly distributed demand and uniformly distributed, nonspecific factors of production argue for uniformly distributed production--a conclusion that finds support in work done by Lösch (33). Lösch found, under similar assumptions, that optimal market areas, for identical plants and uniformly distributed demand, are regular hexagons all having the same area. Therefore, transportation cost can be uniformly and linearly distributed to each plant or production center without affecting the minimizing conditions. Consequently, neglecting transportation cost is justified in the case of Patinkin's model.

In the case of the multiple-plant water supply system, or most similar systems, transportation cost must be considered if a realistic allocation scheme is to be created. Water varies in quality and quantity from location to location. The characteristics of a supply source, its location, and its reference to demand define an important part of the cost function. The remaining definition is provided by the size and technical characteristics of the treatment and transportation facilities as indicated earlier. Various size treatment plants exist in most urban areas. This is due to the nature of the supply sources and to the demand, the design criteria, and the technology that existed when the facilities were built. In that the existing plants represent a certain capital investment, and that they are still operationally sound, their inclusion in the regional supply system would appear to be justified. The existing plants can serve as the initial set of production centers, thus avoiding the problem, as noted by Patinkin, of selecting the set of plants to produce the specified output. Given the varying size, nature, and location of water treatment plants, even if demand is assumed to be totally inelastic and uniformly distributed within a market area, the associated transportation cost would apply to each plant nonuniformly and nonlinearly; and, therefore, they should be considered in the minimization process.

As reported by Westfield (49) the power industry employs an economic model which contains a transportation cost function. Power loss, in part, is a function of distance of transmission; and, therefore, it represents a transportation cost. The transportation function is included in the model as part of the production, demand, equality constraint. There are several problems with transferring this experience to the urban water supply system. If the distance over which the commodity is transported is known before the allocation of production is made, demand must be discrete. In fact, the power economic model described by Westfield deals only with discrete demand and fixed transportation networks. To employ such a scheme to represent an urban water supply system would mean a considerable loss of definition, or it would result in a computationally infeasible problem. Further, water (unlike power) is not consumed (barring leakage) in the process of transportation, so the cost of transporting water must be incorporated into the model by means other than the production-demand constraint.

Certain assumptions about the nature of demand for water, the operation of the treatment plants, and the capacity of the transportation system have been made in order to facilitate the computational solution. Hopefully, these assumptions are sufficiently realistic so as to lead to a solution having practical value. The demand for water is assumed to be totally inelastic. This assumption permits each plant to function as an individual contributor toward meeting the total demand. While a change of production at one plant will affect production levels at other plants, it will not affect product prices or cause a change in total demand. A further economic assumption is that the physical system is fixed, i.e., the solution will be limited to short-run consideration. No new plants can be brought into the system, nor can the existing plants be expanded. The only possible choice would be between operating or not operating a particular facility. Short-run economic analysis provides a means to evaluate alternative strategies for utilizing more efficiently that which already exists. Insofar as the technical operation of the water supply system is concerned, each element, whether transportation or production, is assumed to be operated with technical efficiency and to be consistent with its allocated task. The assigned task may or may not be the optimal for each individual element; however, the objective is to optimize the system as a whole in order to minimize regional costs.

Minimization

One of the most useful techniques, given relatively simple mathematical functions, for determining and examining constrained minimum solutions employs the Lagrangian function (31). The function is

$$L(q, \lambda) = f(q) - \sum_{i=1}^{m} \lambda_i g^i(q)$$

where the λ_i, i=1, ..., m are arbitrary variables. The proof and conditions for q^* and λ^* to be a minimum point of $L(q, \lambda)$ are given in the works of Lancaster (31) and Wagner (48) among others. Their works also demonstrate that if q^* and λ^* is a minimum point of $L(q, \lambda)$ then q^* is a minimum point of $f(q)$. For convenience, the assumptions of minimization are given below.

Assumptions (48):

1. Each constraint is uniquely defined, finite and convex for all values of $q_1, q_2, \ldots, q_n$.
2. Each $\partial g^i(q)/\partial q_j$ is continuous for all q satisfying the constraints.
3. The objective function, $f(q)$, is single-valued and finite for each q satisfying the constraints.
4. Every $\partial f(q)/\partial q_j$ is single-valued, finite and continuous at each q satisfying the constraints.

Conditions:

1. $g^i(q^*) \leq 0$ $\quad i = 1, \quad , m$
2. There exist $\lambda_i^* \geq 0$ $\quad i = 1, \ldots, m$
   such that $g^i(q^*) \cdot \lambda_i^* = 0$ $\quad i = 1, \ldots, m$
3. $\partial f(q^*)/\partial q_j - (\sum_{i=1}^{m} \partial g^i(q^*)/\partial q_j)\lambda_i^* = 0$ $\quad j = 1, 2, \ldots, n$

An important aspect of the Lagrangian function is the economic interpretation of the λ's. λ_i is the marginal value (or marginal cost) of relaxing the i^{th} constraint. Consider, for example, an objective function $f(q)$ which is subject to a single constraint $g(q) = b$ where q is a vector $q_1, q_2, \ldots, q_n$. By the chain rule

$$\partial f(q^*)/\partial b = \sum_{j}^{n} (\partial f(q^*)\partial q_j)\partial q_j/\partial b$$

The partial derivative of the constraint is

$$(\partial g(q^*)/\partial q_j)\partial q_j/\partial b = 1 \quad .$$

Multiplying the above equation by λ^* results in

$$(\partial g(q^*)/\partial q_j)(\partial q_j/\partial b)\lambda^* = \lambda^*$$

and subtracting this equation from the partial derivative of the objective function results in

$$(\partial f(q^*)/\partial b) - \lambda^* = \sum_{j=1}^{n} (\partial f(q^*)/\partial q_j)(\partial q_j/\partial b) - (\partial g(q^*)/\partial q_j)(\partial q_j/\partial b)\lambda^*$$

Rearranging this equation

$$(\partial f(q^*)/\partial b) = \lambda^* + [\Sigma\ \partial f(q^*)/\partial q_j - (\partial g(q^*)/\partial q_j)\lambda^*]\partial q_j/\partial b \quad .$$

The terms inside the bracket must equal zero under the conditions of minimization (see the third condition given above), therefore

$$\partial f(q^*)/\partial b = \lambda^*$$

For a small increase or decrease in b, λ^* indicates the marginal change in the objective function.

The practical application of the Lagrange function is limited by the nature of the minimizing equation set generated by the third condition. If the set of equations is not linear or if substitution or elimination techniques cannot be applied, q^* and λ^* cannot be easily determined. However, another optimizing technique, geometric programming, can deal with such nonlinear conditions (11)(12)(25).

The advantages of employing geometric programming are best stated in the words of Duffin, Peterson and Zener: "We foresee three benefits from using geometric programming. The first benefit is an overall picture of the relative importance of the various design parameters. The second benefit is that geometric programming (particularly when constraints are involved) is more amenable to digital computers than are standard approaches. The third benefit is the engineering discipline that geometric programming imposes on its user. In particular, equality constraints must be replaced by inequality constraints, and such conversion forces the user at the very beginning to analyze the way in which the various design parameters interact with one another."

The solution technique can be applied to any problem that can be expressed in terms of posynomials, i.e.,

$$g = u_1 + u_2 + \ldots + u_n$$

where $u_i = c_i t_1^{a_{i1}} t_2^{a_{i2}} \ldots t_m^{a_{im}}$

and $c_i > 0$, $t_j > 0$ and a_{ij} are real numbers.

The primal problem, termed Program A, is converted to the dual problem, termed Program B, which can be more easily solved. The programs are as follows:

Primal Program A

$$\min q_o(t)$$

subject to

$$t_j > 0 \qquad j = 1, \ldots, m \text{ (natural constraints)}$$

$$q_i(t) \leq 1 \qquad i = 1, \ldots, p \text{ (forced constraints)}$$

where

$$q_k(t) = \sum_{i\varepsilon J[k]} c_i t_i^{a_{i1}} t_2^{a_{i2}} \ldots t_m^{a_{im}} \qquad k = 0, 1, \ldots, p$$

$$J[k] = \{m_k, m_k+1, m_k+2 \ldots n_k\} \qquad k = 0, 1, \ldots, p$$

and

$$m_o = 1 \quad m_1 = n_o + 1 \quad m_2 = n_1 + 1 \ldots,$$

$$m_p = n_{p-1} + 1 \quad n_p = n .$$

Dual Program B

$$\max v(\delta) = \left[\prod_{i=1}^{n} (c_i/\delta_i)^{\delta_i}\right] \prod_{k=1}^{p} \lambda_k(\delta)^{\lambda_k(\delta)}$$

where

$$\lambda_k(\delta) = \sum_{i \varepsilon J[k]} \delta_i \qquad k = 1, 2, \ldots, p$$

subject to

$$\delta_i \geq 0 \qquad i = 1, \ldots, n \text{ (positivity condition)}$$

$$\Sigma\delta_{i\varepsilon J[o]}\ {}_i = 1 \qquad \text{(normality condition)}$$

$$\Sigma a_{ij}\delta_i = 0 \qquad j = 1, 2, \ldots, m \text{ (orthogonality condition)}$$

Again for convenience, the first theorem of geometric programming is (12):

Suppose that primal program A is superconsistent and that the primal function $g_o(t)$ attains its constrained minimum value at a point that satisfies the primal constraints. Then

(i) The corresponding dual program B is consistent and the dual function $v(\delta)$ attains its constrained maximum value at a point which satisfies the dual constraints.

(ii) The constrained maximum value of the dual function is equal to the constrained minimum value of the primal function.

(iii) If t^* is a minimizing point for primal program A, there are non-negative Lagrange multipliers μ_k^*, $k = 1, 2, \ldots, p$, such that the Lagrange function

$$L(t, \mu) = g_o(t) + \sum_{k=1}^{p} \mu_k [g_k(t) - 1]$$

has the property

$$L(t^*, \mu) \leq g_o(t^*) = L(t^*, \mu^*) \leq L(t, \mu^*)$$

for arbitrary $t_j > 0$ and arbitrary $\mu_k \geq 0$. Moreover, there is a maximizing vector δ^* for dual program B whose components are

$$\delta_i^* = \begin{cases} \dfrac{c_1 t_1^{a_{11}} \cdots t_m^{a_{tm}}}{g_o(t)}, & i \varepsilon J[0], \\[2ex] \dfrac{\mu_k c_i t_1^{a_{11}} \cdots t_m^{a_{tm}}}{g_o(t)}, & i \varepsilon J[k], \quad k = 1, \ldots, p \end{cases}$$

where $t = t^*$ and $\mu = \mu^*$. Furthermore,

$$\lambda_k(\delta^*) = \frac{\mu_k^*}{g_o(t^*)}, \qquad k = 1, 2, \ldots, p.$$

(iv) If δ^* is a maximizing point for dual program B, each minimizing point t^* for primal program A satisfies the system of equations

$$c_i t_1^{a_{11}} \ldots t_m^{a_{tm}} = \begin{cases} \delta_i^* \, v(\delta^*), & i \varepsilon J[0] \, , \\ \dfrac{\delta_i^*}{\lambda_k(\delta^*)} \, , & i \varepsilon J[k] \, , \end{cases}$$

where k ranges over all positive integers for which $\lambda_k(\delta^*) > 0$.

Theorem 1 establishes the relationship between the dual and primal variables (δ, t). It provides a computation means for determining the optimal values of t^* from the optimal dual variables, δ^*. Further, it relates the Lagrange multiplier, μ_k^* to the dual variables by

$$\lambda_k(\delta^*) = \mu_k^*/g_o(t^*) \; .$$

The μ_k^* in the case of geometric programming has the same economic interpretation as λ^* in the above discussion of the Lagrangian function.

The water supply problem, as dealt with in this dissertation, is formulated as both a Lagrangian function and as a geometric programming problem. The cost functions describing production and transportation of water were found to be convex with continuous derivatives which permitted the Lagrangian formulation (see Chapter VI). Geometric programming requires that the problem be formulated as a posynomial in the primal program. The primal program may or may not be convex, but this program can be transformed to one that is convex (12). The dual of this transformed primal program is linear in the logs of the variables and, as such, it can be easily solved using linear programming techniques. Geometric programming has been applied to a wide class of engineering problems (e.g., transportation planning, transformer design, and chemical equilibrium). Certainly, the water supply problem, as formulated, is not unique in the application of geometric programming, but the problem seems well suited to this solution technique. Additional discussion on this topic is given in Chapter IV, where both techniques are applied to the problem of minimizing the cost of water distribution.

IV. TRANSPORTATION COSTS

Short-run transportation costs result from the consumption of energy to move water from the treatment plant to the point of demand. In a water system where gravity is the sole source of energy, variable transportation costs are zero. However, very few cities are fortunate enough to be located so that mechanical means of distribution are not necessary. In the case of Chicago, distribution is achieved entirely by hydraulic machinery. Gravity is used to move water through delivery tunnels, but the loss in elevation has to be overcome by pumping.

Typically, pumps are driven by electrical motors, gas or steam turbines, or diesel engines. To simplify the investigation of transportation costs, only electrical motor driven pumps were considered and unit energy costs were assumed to be constant. Nevertheless, the general form of the analysis is transferable to other pumping systems.

Pump performance is characterized by three relationships (3):

1. Head-Capacity
2. Efficiency-Capacity
3. Horsepower-Capacity

The distribution network and delivery system (i.e., pipes or tunnels connecting the pumps to their supply source) are characterized by the system-capacity relationship. Pump characteristics specify the total dynamic head and power costs at which a given quantity of water will be delivered. The system-capacity relationship specifies the energy necessary to deliver a given quantity of water at the desired service pressure.

The distinction between these two relationships is further explained later in this chapter. Also, a simplified transportation cost function, based on mean daily output, is proposed and substantiated as a means for analyzing the operation of pumping and distribution systems under daily demand conditions. Geometric programming is used to solve the complex set of equations describing the hydraulic behavior of the transportation system given peak demands.

Pumping

A variety of pump types (e.g., reciprocating, centrifugal and turbine) are used in the water supply industry. However, centrifugal pumps are most commonly used because of their simplicity, low cost, and ability to operate under a wide range of conditions (29). All of the pumps that currently are being used in Chicago and Evanston, for example, are centrifugal type. Although the following analysis is based on the characteristics of the centrifugal pump, again, any pumping system can be similarly analyzed given the appropriate characteristics.

As indicated above, the performance of a single pump or multiple pumps is defined by the associated head-capacity, efficiency-capacity, and horsepower-capacity relationships. A graphical representation of these relationships is given in Figure IV-1. The shape of these curves is typical for centrifugal pumps; however, the curves are based on actual measurements taken during a test of the No. 2 pump at Lake View Pumping Station in Chicago. Similar results were obtained for the other two pumps at Lake View and for the four pumps at the Thomas Jefferson Pumping Station (See reference 50, Appendix B).

The head-capacity curve defines the head (measured in feet of water) at which a specified quantity of water can be delivered. When the elevation of water on the suction side of the pump varies and when

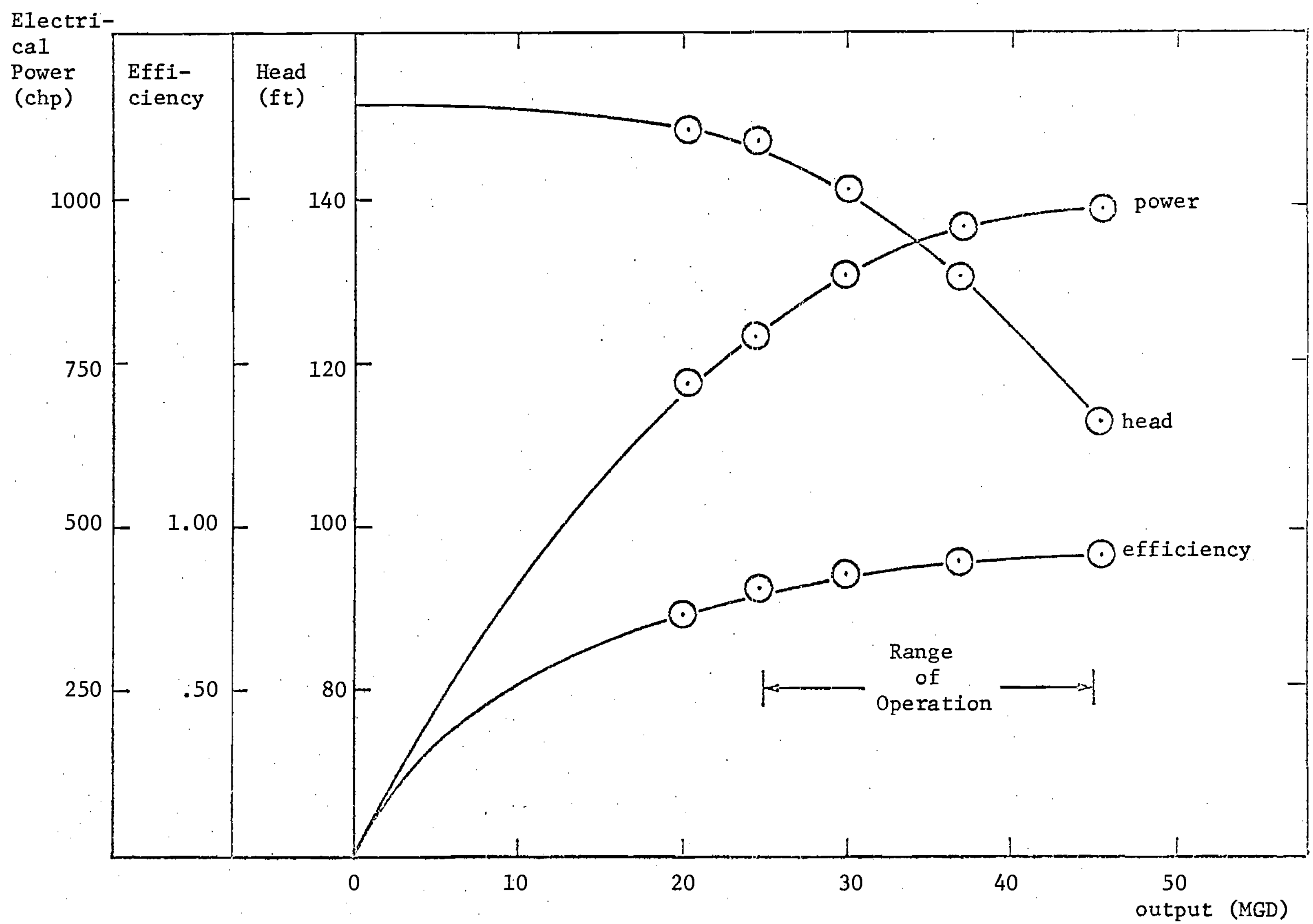


FIGURE IV-1

PUMP CHARACTERISTICS

the elevation to which the pump must deliver water varies, a better term for head is total dynamic head. The components of the dynamic head are suction lift, velocity conversion head, velocity head, and pressure head. Figure IV-2 schematically distinguishes between the suction and pressure sides of a pump. Bernoulli's equation can be used to mathematically define the components of total dynamic head and the relationship between energy and output (see Figure IV-2).

$$E_p = Z_1 - Z_o + \frac{P_1}{\gamma} - \frac{P_o}{\gamma} + \frac{V_1^2}{2g} - \frac{V_o^2}{2g} + H_{L_{0-1}}$$

Here E_p is the total dynamic head (or energy expressed in foot-pounds/pounds of mass), $Z_1 - Z_o$ is the difference in elevation of the measuring points, P_o/γ is the suction pressure head or suction lift (P expressed in pounds/ft^2 and γ is the specific weight of water which is 62.4 lbs/ft^3), P_1/γ is the pressure head, $V_1^2/2g - V_o^2/2g$ is the velocity conversion factor (V is expressed in ft/sec and g is the gravity constant, 32.2 ft/sec^2), and $H_{L_{0-1}}$ is the headloss due to friction between point 0 and 1. At a known flow, q (ft^3/sec), velocity is computed using the equation of continuity, V = q/A where A is the cross sectional area of the pipe. In the above example, $V_o = q/.25\pi D_o^2$ and $V_1 = q/.25\pi D_1^2$ where D_o and D_1 are the diameters of the pipes at point 0 and 1. The pressure heads are measured at their respective gages. The headloss, due to friction, takes the form $H_{L_{0-1}} = f \frac{L}{D} \frac{V^2}{2g}$, where L is the length of pipe and f is the friction factor. Since L is very short, headloss is quite small and usually this term is neglected.

Measurements are taken for several different flow rates. The total

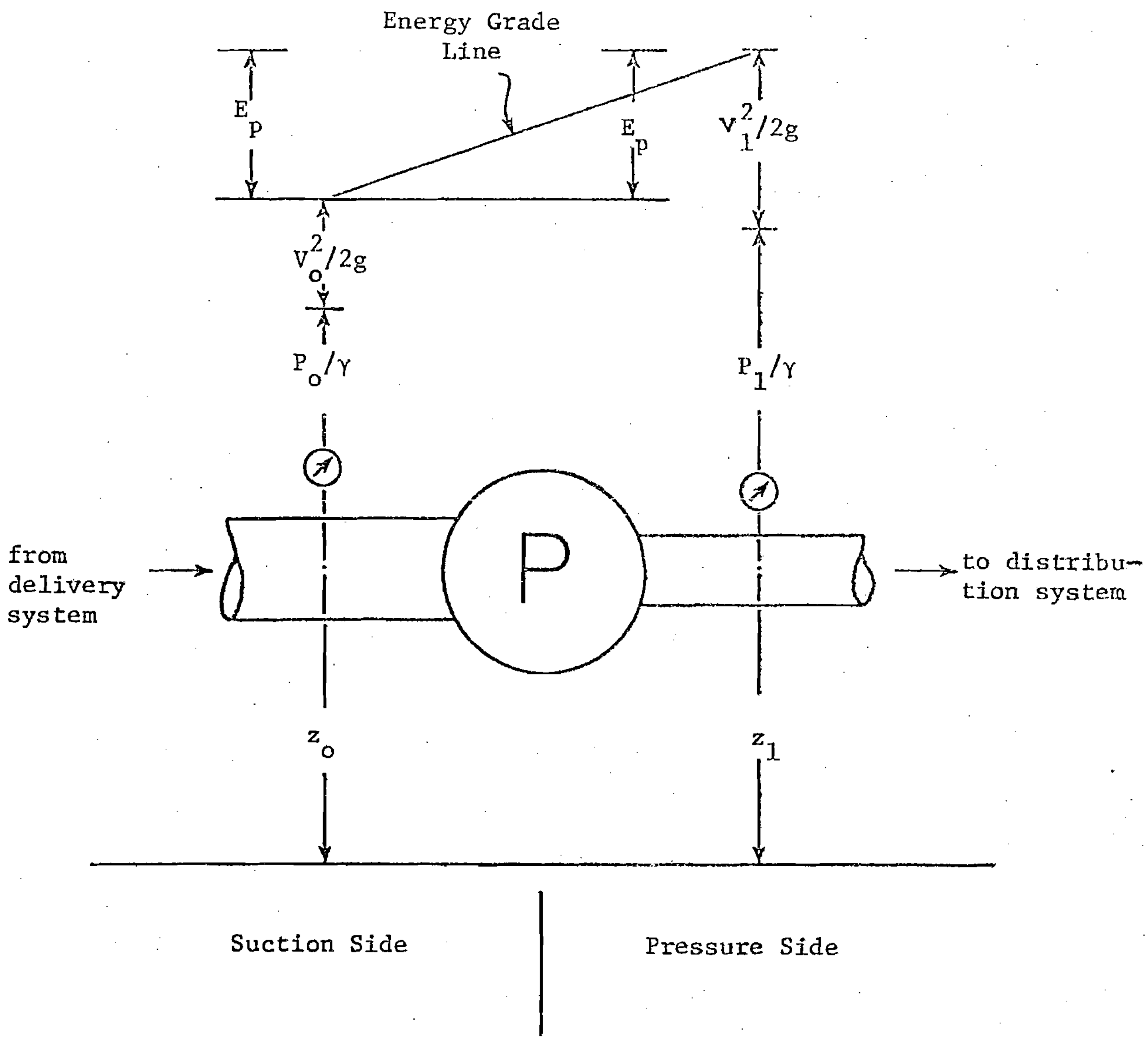


FIGURE IV-2

DYNAMIC HEAD

dynamic head is computed using Bernoulli's equation as given above. Plotting these values against the associated flow rate yields the head-capacity curve. Water horsepower (whp) is computed by the relationship

$$whp = .176\ q\ h$$

where q is expressed in million gallons per day (MGD), h is the total dynamic head, and .176 is a factor that converts the units to horsepower. One other measurement is made during a pump test and that is the electrical horsepower (ehp) necessary to operate the pump at the various flow rates. This information along with the water horsepower is used to determine the efficiency-capacity curve. For,

$$e = whp/ehp$$

where e is efficiency. Brake horsepower (bhp) and thereby the horsepower-capacity curve, is defined by

$$bhp = .176\ qh/e\ .$$

Not all pumps have different diameter suction and pressure pipes. For example, all three pumps at the Lake View Pumping Station have thirty inch intake pipes and twenty-four inch outlet pipes, while at the Thomas Jefferson Pumping Station all four pumps have the same diameter intake and outlet pipes. One reason for the intakes being larger is to avoid cavitation. The velocity conversion factor used to compute total dynamic head during the August 1971 pump test (see Appendix B) at Lake View was $.00221\ q^2$. The basis of this relationship is

$$\text{velocity conversion head} = \frac{V_1^2}{2g} \quad \frac{V_o^2}{2g} ,$$

but

$$V_i = q/A_i = q/.25\pi D_i^2 .$$

Therefore,

$$\text{velocity conversion head} = q^2 \quad \frac{1}{2g(.25\pi D_1^2)^2} \quad \frac{1}{2g(.25\pi D_o^2)^2}$$

where the terms inside the brackets are constant.

In Chapter II, the service-product of the water industry was defined as providing potable water on demand at a prescribed pressure. Maintenance of adequate pressure is necessary for fire protection and the delivery of water to elevated points throughout the service area. As demand increases (i.e., q increases), the head-capacity curve dictates that the total dynamic head must decrease which, in turn, means that the pressure head, or system pressure, must decrease. In order to maintain service pressure at or near the desired level, additional pumps must be brought into use. Generally, pumps are added in parallel. (An exception to this would be a situation where extremely high pressures are required. Then, pumps would be added in series.) When pumps are operated in parallel a new set of pumping characteristics is generated every time a pump is brought into or taken out of service. The capacity curves are aggregated by adding the output from each pump at a given head, brake horsepower, or efficiency. Example aggregate head-capacity relationships are given in Figure IV-3. The solid line is the head-capacity curve for the No. 2 pump at Lake View. For the purposes

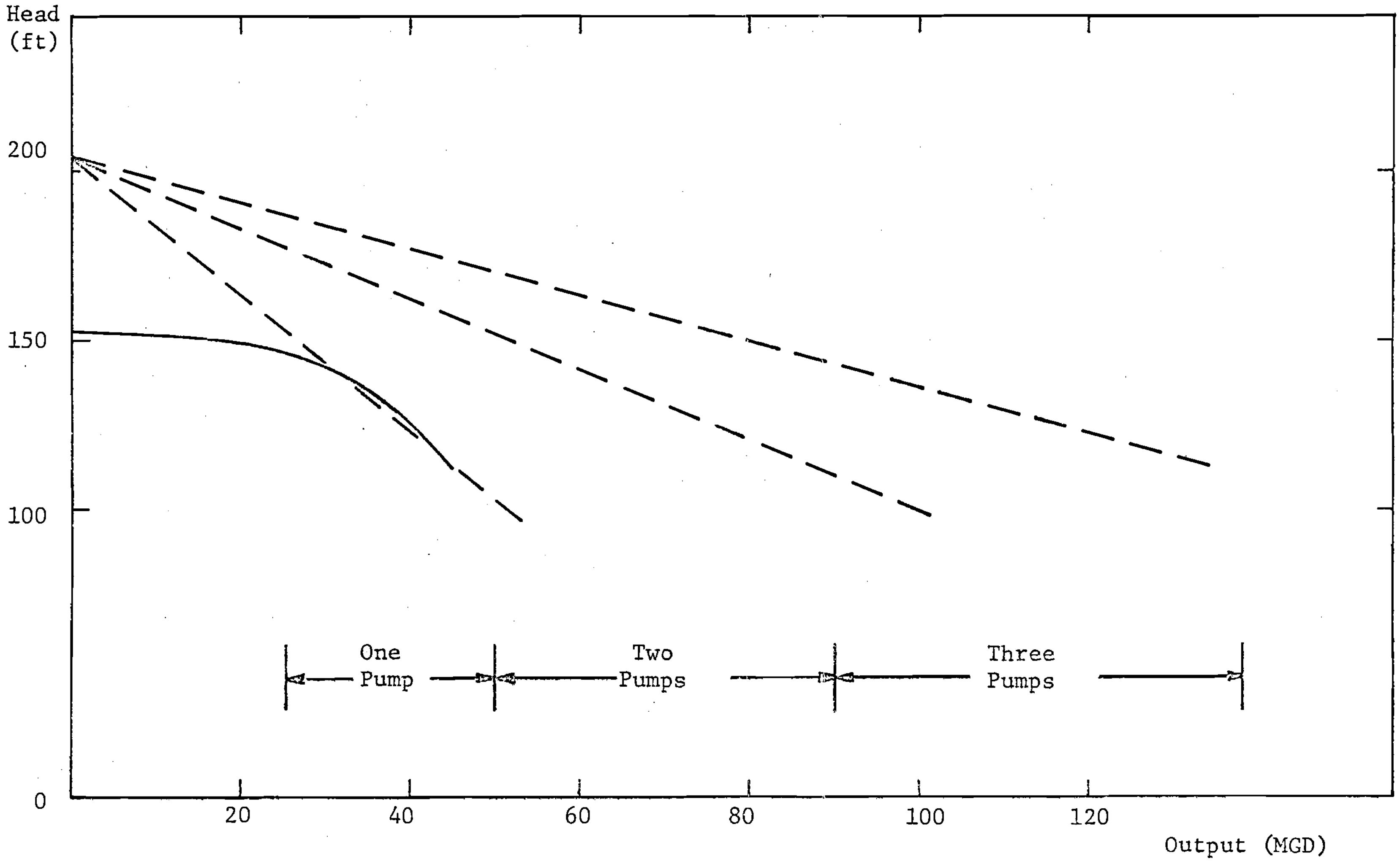


FIGURE IV-3

AGGREGATED PUMP CHARACTERISTICS

of this study, this curve was approximated, over the normal range of operation, by the tangent, dashed line. Similar approximations of the head-capacity curves for pumps No. 1 and No. 3 were made and the aggregate relationships developed. The linear equations representing the approximation for each pump are:

$$h = 207 - 2.12\ q_1$$

$$h = 195 - 1.76\ q_2$$

$$h = 206 - 2.19\ q_3\ .$$

To facilitate the following presentation, the general form of these equations will be used, i.e., $h = a_i - b_i q_i$. Since only q is added in developing the aggregate head-capacity relationship, the following transposition is necessary:

$$q_i = \frac{a_i}{b_i} - \frac{1}{b_i}\ h\ .$$

When pumps 1 and 2 are operating in parallel (as they are designed to do), the head-capacity relationship at the Lake View Pumping Station is

$$h_{1,2} = \frac{a_1 b_2 + a_2 b_1}{b_1 + b_2} - \frac{b_1 b_2}{b_1 + b_2}\ (q_1 + q_2)$$

or

$$h_{1,2} = 202 - .981\ (q_1 + q_2)\ .$$

Similarly,

$$h_{1,2,3} = \frac{a_1 b_2 b_3 + a_2 b_1 b_3 + a_3 b_1 b_2}{b_1 b_2 + b_2 b_3 + b_1 b_3} - \frac{b_1 b_2 b_3}{b_1 b_2 + b_2 b_3 + b_1 b_3}\ (q_1 + q_2 + q_3)$$

or

$$h_{1,2,3} = 203 - .665\ (q_1 + q_2 + q_3)\ .$$

As pumps are added in parallel, the dynamic head at zero output (i.e., the intercept or the head axis) tends to be a constant. This is due to the fact that each pump has nearly the same intercept value. For example, the values of a_1 and a_2 are within six percent of each other. The aggregate intercept is very nearly the average of a_1 and a_2 and this average closely approximates a constant since a_1 and a_2 are approximately the same.

$$\frac{a_1 + a_2}{2} = 201$$

$$a_{1,2} = \frac{a_1 b_2 + a_2 b_1}{b_1 + b_2} = 202\ .$$

At a given pumping station, the head intercept for all pumps are designed to be approximately the same because, as a pump is brought into service, it must be operating at the same head or pressure as the other pump or pumps in operation; otherwise flow through the pump would be affected. Since the a_i's can be assumed constant, the parameters which control output are the b_i's.

Like the head-capacity curve, the efficiency-capacity curve is aggregated as additional pumps are brought into service. For a given efficiency, output is equal to $\sum_{i=1}^{n} q_i$ where n is the number of pumps in operation.

Inspection of the efficiency-capacity curve shown in Figure IV-1 reveals a flat slope in the normal range of operation. As the efficiency relationship is modified to reflect the operation of an increasing number of pumps, the slope approaches zero. This is further demonstrated by the following linear approximations:

$$e = c_i + d_i q_i$$

then

$$e_{1,2} = \frac{c_1 d_2 + c_2 d_1}{d_1 + d_2} + \frac{d_1 d_2}{d_1 + d_2} (q_1 + q_2)$$

and

$$e_{1,2,3} = \frac{c_1 d_2 d_3 + c_2 d_1 d_3 + c_3 d_1 d_2}{d_1 d_2 + d_2 d_3 + d_1 d_3} + \frac{d_1 d_2 d_3}{d_1 d_2 + d_2 d_3 + d_1 d_3} (q_1 + q_2 + q_3) \ .$$

Since $d_i < 1$, $i = 1,2,3$, the slope parameter (i.e., the coefficient of the q_i's) becomes smaller as more pumps are added. This implies that the efficiency relationship approaches a constant. In fact, efficiency is usually assumed to be a constant (3). For purposes of this study and in the examples, the efficiency is assumed to be a constant (e.g., efficiency at Lake View is estimated to be .78 and that at Thomas Jefferson to be .76).

Having characterized the physical processes of pumping, the next step is to develop the cost function for the head-capacity relationship. However, before doing this, the system-capacity function needs to be described so that a comparison can be made between pump and system characteristics in determining the transportation cost function.

Distribution

The distribution network is characterized by the length, diameter, and roughness of the pipe elements; by the way the pipe elements are interconnected; by the rate and spatial distribution of flow (or demand); and by the service pressure maintained in the network. Again, Bernoulli's equation describes the controlling energy relationships.

$$E_p = Z_1 - Z_o + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + H_{L_{1-2}} + Z_2 - Z_o$$

where P_2/γ is the service head and P_2 is the service pressure expressed in pounds per square feet. Energy supplied through the pumping station, E_p, is used to lift the water from Z_o to Z_2 and to supply the desired pressure and quantity of water at the point of demand. Figure IV-4 is a schematic representation of the energy and hydraulic grade lines. Generally, the service pressure in a municipal water supply system is between 3600 psf and 5040 psf.

For the very simple system represented in Figure IV-4, the headloss term for a given flow, q, can be directly estimated by use of the Hazen-Williams formula (2T), $H_{L_{1-2}} = Kq^{1.85}$ where K is a constant for a given pipe length, diameter and roughness. In a more complex pipe network, flow occurs so as to balance the headlosses in each loop of the network, but in order to determine the headloss across each pipe element the flow in each element must be known. The best known solution technique to this problem is the Hardy Cross Method. It is essentially a method of trial and error and it requires considerable expense for the solution of complex systems.

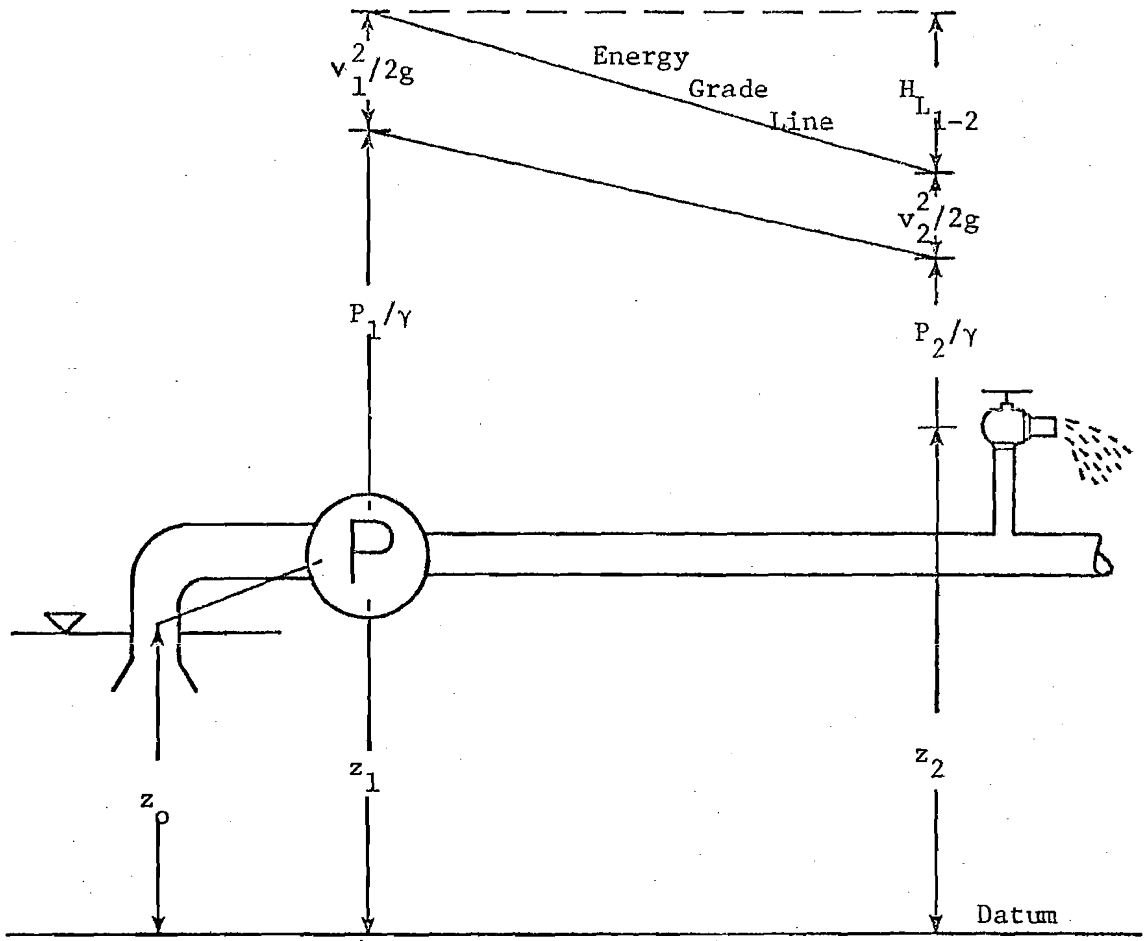


FIGURE IV-4

ENERGY AND HYDRAULIC GRADE LINES

However, the determination of headlosses between the pumping station and every point in the distribution system is not necessary for the development of the system-capacity function. There is usually one critical point in the system that dictates the energy head at the pumping station if service pressure is to be maintained at that point. In the case of the Lake View Pumping Station, the critical point is represented by Node 58 in Figure IV-5 while Node 80 is critical for the Thomas Jefferson Pumping Station (as determined by Hardy Cross analyses of the system: see reference 50). These nodes or junctions are critical points because of their elevation and distance from the pumping station and because of the connecting pipe network and the associated water demands. Field test and Hardy Cross analysis are used to identify such points. Although only one point controls, the headloss to this point from the pumping station is a function of the total flow in the system and the flow along each pipe element, i.e., a Hardy Cross type analysis still is required. However, resarch done by McPherson (34) showed that, once the headloss between two points in a pipe network is established, headloss for any flow conditions could be represented by $H_{L_{i-j}} = K_{i-j}q^m$ where K_{i-j} is a constant between point i and j, m varies between 1.86 and 2.00 (m is usually taken to be 1.85), and q is the total flow in the system.

In the case where water is supplied to the network from more than one source (e.g., two or more pumping stations), $H_{L_{i-j}} = K_{i-j}(q_i/q_d)^n q_d^m$ where K_{i-j} and m are as before, n is a constant based on known headlosses, q_i is the input from the i^{th} pumping station and q_d is the total flow in the network. The assumption that makes these equations reasonable approximations of headloss between two points is that demand at

Node 0 = Lakeview Pumping Station

Node 42 = Thomas Jefferson Pumping Station

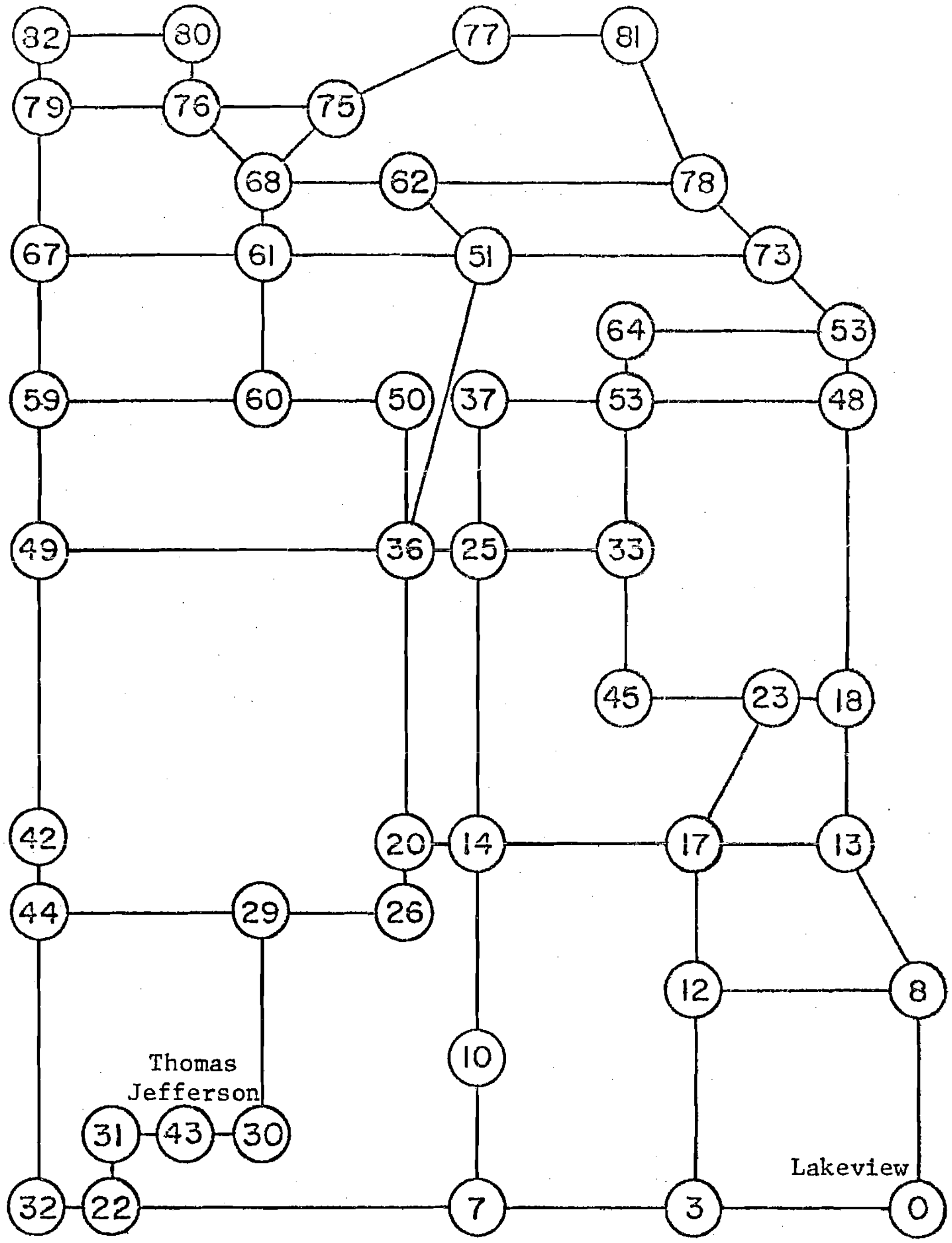


FIGURE IV-5

LAKEVIEW/THOMAS JEFFERSON NETWORK

each junction in the representative network fluctuates in direct proportion to the total system demand; i.e., $q_i \propto q_d$. McPherson indicated that violation of this assumption was not serious in systems largely serving residential areas, but he cautioned against the unrestricted use of the generalized headloss relationships in highly industrialized systems.

One important factor which McPherson did not deal with is the relationship between the energy boundary, or no-flow line, and headloss. There is a boundary, defined by equal energy, which separates the service areas of each pumping station (see Figure IV-6). Since there is no flow across this boundary, the value of the headloss function for a point outside a service area has no meaning. Headlosses can be computed only between points within the same service area.

In Bernoulli's equation defining the energy for distribution, the suction lift was considered to be static (i.e., $Z_1 - Z_o$). This is not always the case. Pumping stations in Chicago are supplied by tunnels and the hydraulic grade line fluctuates at each station which means that the suction lift fluctuates. The elevation of the water on the suction side of the pump varies according to the elevation of the supply reservoir at the Central Water Filtration Plant and the headloss through the tunnel. Consequently the suction head is

$$sh_{1v} = Z_{1v} - (Z_R - H_{L_{R-1v}})$$

where Z_R is the elevation of the reservoir, Z_{1v} is the elevation of the pump at Lake View, and $H_{L_{R-1}}$ is the headloss in the connecting tunnel. Again, the headloss term is approximated by Kq^m. The actual headloss function used by the City of Chicago for computing headlosses between the

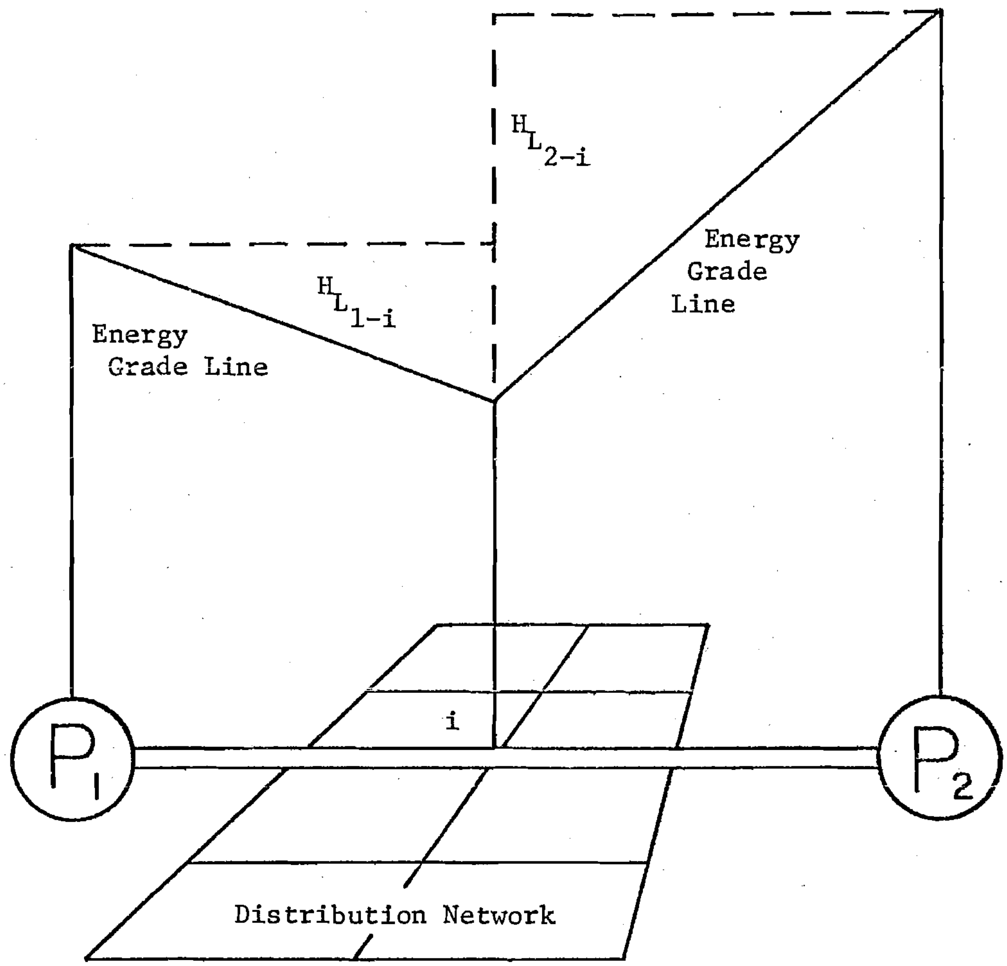


FIGURE IV-6

ENERGY BOUNDARY

Central Water Filtration Plant and the Lake View Pumping Station is $.0000292q^2$, where q is expressed in MGD.

Before formalizing the system-capacity function, one further aspect should be noted. The velocity head given in Bernoulli's equation can be neglected. Its value relative to the other terms is quite small. In most supply systems, velocity is maintained at eight feet per second or less which means that $V^2/2g$ is approximately equal to one foot. The pressure head, at 3600 psf, is 57.7 feet, differences in elevation may be as much as 40 to 50 feet, and headlosses may be 60 to 70 feet. In all, the velocity head is only about one percent of the total system head. Consequently, it will be neglected.

The system-capacity function for a single pumping station is composed of the following significant terms:

$$\text{suction head (sh)} = Z_1 - (Z_R - H_{L_{R-1}})$$
$$\text{velocity conversion head (vh)} = Cq^2$$
$$\text{delivery head (dh)} = Z_2 - Z_1$$
$$\text{service pressure head (ph)} = P_2/\gamma$$
$$\text{headloss } (H_L) = K\, q^{1.85}$$

where q is expressed in MGD. The energy (in foot-pounds/pound mass) required to move a quantity of water through the delivery and distribution system while meeting service conditions at the critical point is

$$E_s = Z_1 - (Z_R - H_{L_{R-1}}) + Cq^2 + Z_2 - Z_1 + P_2/\gamma + Kq^{1.85} .$$

The requisite brake horsepower is

$$\text{bhp} = .00182\, q\, (E_s) .$$

Now, the quantity of water that can be delivered for a given amount of energy is specified by the head-capacity function. Conversely, the amount of energy necessary to deliver a given quantity of water is specified by the system-capacity function. In the next section both of these relationships will be converted into cost functions.

Variable Operating Costs

The variable operating costs for distributing water are a function of consumed energy and the unit costs of energy. Energy consumed in the distribution process is a function of pump and system characteristics and operating policies. The variable cost of transportation or power cost is

$$PC = .0346\ q\ h$$

where PC is expressed in dollars per day, q in MGD, and h in feet of head (or foot-pounds per pound mass). The constant, .0346 is the product of the factor for converting foot-pounds to kilowatt hours (3.766×10^{-7}), the unit weight of water per million gallons ($8.346 \times 10^{+6}$), and the unit cost of power (.011 dollars per kilowatt hour). The actual, variable cost function is based on h from the head-capacity function. As long as a fixed number of pumps remain in operation, the variable cost function has the form

$$PC = .0346\ q\ (a - bq)\ .$$

If pumps are brought into and taken out of operation in an attempt to meet demands and maintain service pressure, the cost function is based on h from the system-capacity function. The variable cost relationship is then

$$PC = .0346\ q\ [Z_1 - (Z_R - K_{R-1} q^{m_{R-1}}) + Cq + Z_2 - Z_1 + P_2/\gamma + K_{1-2} q^{1.85}]\ .$$

In Figure IV-7, example head-capacity and system-capacity cost functions are presented. The head-capacity cost functions A, B and C are for the Lake View Pumping Station with one, two and three pumps in operation The exemplary system-capacity function reflects a fixed suction lift, service pressure of 30 psi or 69.2 feet of water, velocity conversion head equal to 0.774q, and a headloss function of $.012q^{1.85}$ (the coefficient K was based on the results of a network analysis).

Assuming a pumping station efficiency of 0.78, the variable cost functions for one, two, and three pumps are

$$PC_1 = 0.442\ q\ (207 - 2.12q)$$

$$PC_{1,2} = 0.442\ q\ (202 - .981q)$$

$$PC_{1,2,3} = 0.442\ q\ (203 - .665q)\ .$$

If service conditions are to be satisfied, pumping operations must be conducted so that the system-capacity function, explained above, is not violated. Variable costs resulting from this function are

$$PC = .0442q\ (9.00 + 69.2 + .0774q + .012q^{1.85})\ .$$

For the example system, when demand is in the neighborhood of 30 MGD and only one pump is in operation, the variable cost of transportation will be greater than that indicated by the system-capacity cost function (see Figure IV-7). When demand is 100 MGD with three pumps in operation, service conditions are violated and the cost indicated by the head-

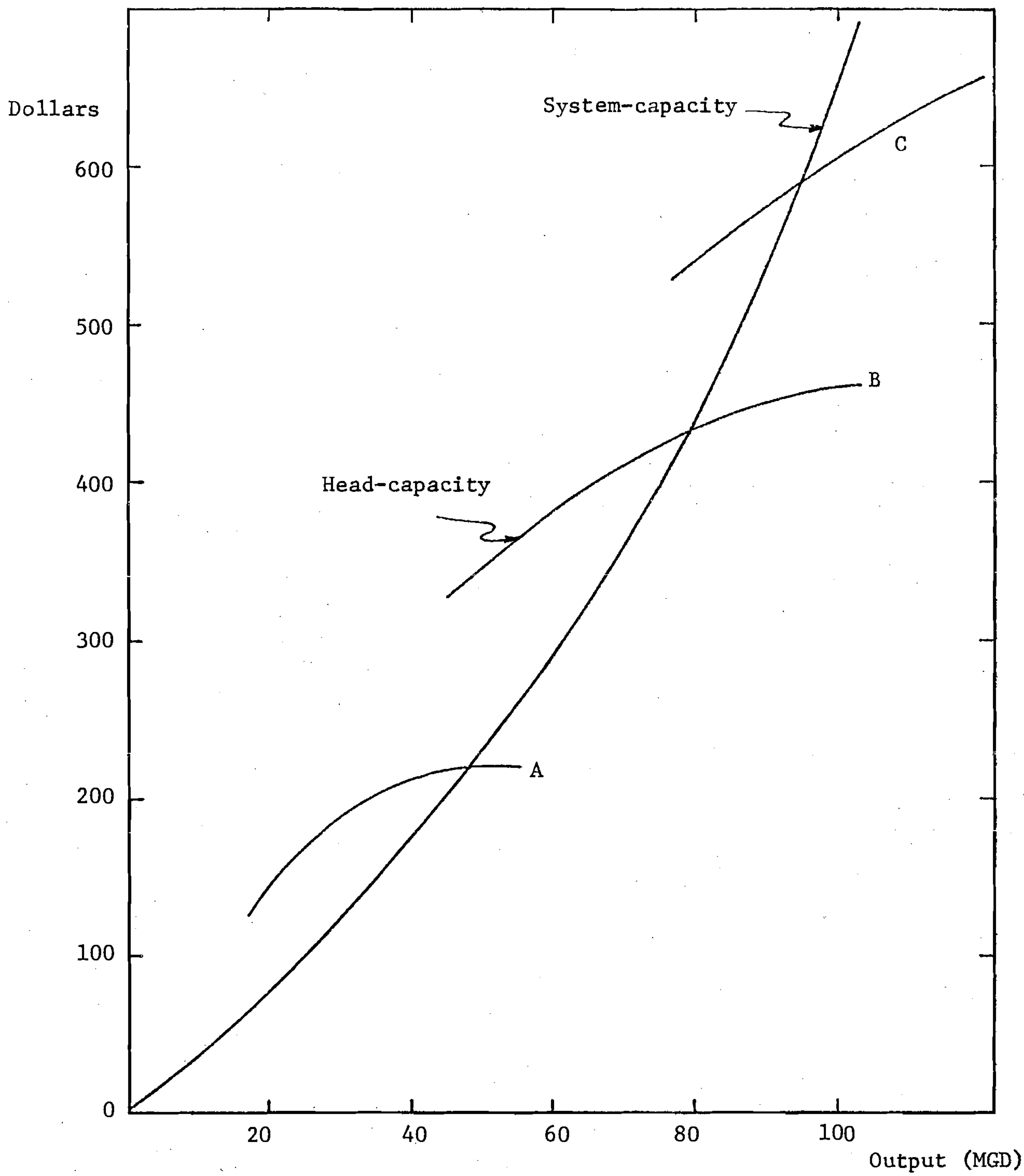


FIGURE IV-7

TRANSPORTATION COST FUNCTIONS

capacity function is less than that indicated by the system-capacity function. In both cases, the cost of transportation is dictated by the head-capacity relationship. Further, at any point in time, the head-capacity function determines the cost. However, over a period of time such as a day, pumps are brought into operation and taken out of operation in order to meet variations in demand and pressure conditions. Through control policy, pumping stations are operated so as to approximate the system-capacity function. As can be seen from Figure IV-7, the approximation of the system-capacity function results in the least costs while satisfying the demand and pressure constraints.

Daily Cost Model

For mean daily output, the variable cost of transportation is defined by the system-capacity function. A practical demonstration of this is provided by daily operating data from the Lake View Pumping Station. Observed daily power costs, in dollars, were regressed with power costs computed from observed daily output using the system-capacity function:

$$CPC_{lv} = .0442\ Q_{lv}\ (\Delta Z_{lv} + P_{lv}/\gamma + .077Q_{lv} + .012Q_{lv}^{1.85})$$

where $\Delta Z_{lv} = 9.0$, $P_2/\gamma = 69.2$, and Q_{lv} is mean daily flow in MGD. The regression model was

$$PC_{lv_i} = \alpha + \beta CPC_{lv_i} + u_i$$

where PC and CPC are observed and computed power costs, α and β are the estimated regression coefficients, and u_i is the difference (also termed error or residual) between the observed and computed costs for the i^{th} observation. In order for α and β to be unbiased estimators of their true values, the u_i's must be normally distributed, they must have a mean of zero, they must be independent of CPC_{1v} and u_i and u_{i+s} must be independent. Stated more formally (19),

$$E(u_i) = 0 \text{ for all } i$$

$$E(u_i u_j) = \begin{cases} 0 \text{ for } i \neq j,\ i,j, = 1,2,\ldots,n \\ \sigma_u^2 \text{ for } i=j,\ i,j = 1,2,\ldots,n\ . \end{cases}$$

Least-squares linear regression procedures result in $E(u_i) = 0$. A scattergram plot of the residuals against computed power costs (see Figure IV-8) indicates an independent random pattern. The linear correlation coefficient is .00041. To insure that the other conditions were met, certain statistical tests were made on the residuals. The Durban-Watson statistics were used to test for serial correlation (u_i and u_{i+s} not independent); and, the Kolmogorov-Smirnov statistic was used to test for normality.

The hypothesis to be tested by the Kolmogorov-Smirnov statistic, D, is that the u_i's are normally distributed. For the known sampling distribution of D, the computed statistic D, for the residuals from the above regression model, was not significant at the five percent level. Consequently, the hypothesis is not rejected, i.e., the residuals can be taken to be normally distributed.

However, the Durban-Watson statistic, d, was significant at the five percent level which indicated serial correlation. The computed statistic is equal to 1.54 which is less than the lower boundary, $d_L = 1.65$,

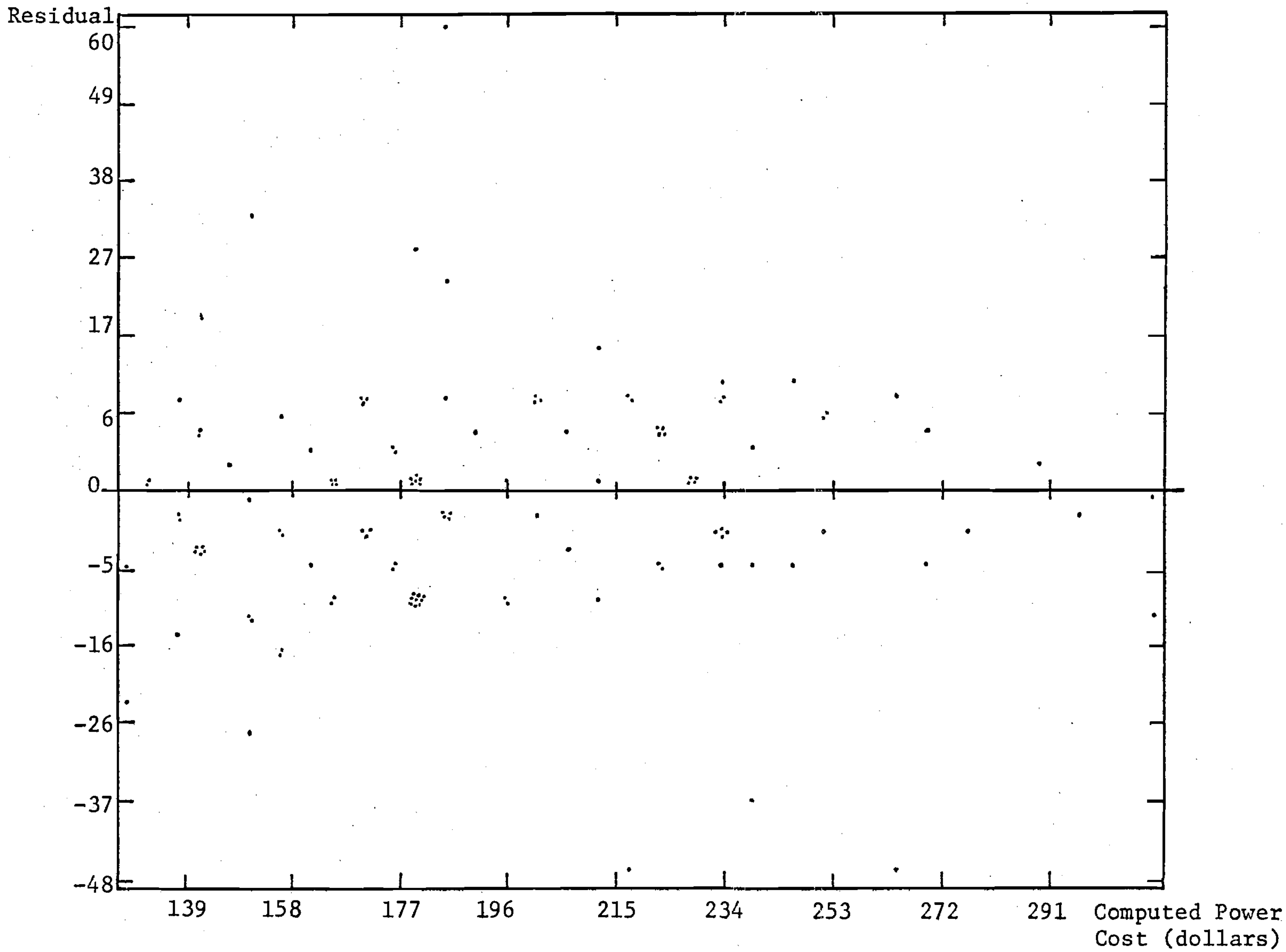


FIGURE IV-8

COMPUTED POWER RESIDUALS

for one independent variable and the number of observations, n = 120, greater than one hundred. Since the computed d value is less than d_L there is significant positive serial correlation (13). That the residuals are autocorrelated is not surprising since the observed variables, Q_{1v}'s, are time-series type data and they are serially correlated. One procedure for removing autocorrelation is a first-order autoregressive scheme operating on the residuals (a more complete discussion of the autoregressive model is given in Chapter V).

$$u_i = ru_{i-1} + e_i$$

then

$$r = \frac{\sum_{i=2}^{n} u_i u_{i-1}}{\sum_{i=2}^{n} u_{i-1}^2} .$$

The new variables become

$$PC'_{1v_i} = PC_{1v_i} - rPC_{1v_{i-1}}$$

$$CPC'_{1v_i} = CPC_{1v_i} - rCPC_{1v_{i-1}} .$$

Least squares regression is applied to the transformed variables which results in new estimates α' and β'. Since α' is an estimate of $\alpha(1 - r)$, $\alpha = \alpha'/(1 - r)$.

In the example using Lake View operating data, the initial estimates of α and β were 37.1 and .922. After applying the first-order autoregressive scheme (r = .229), α = 39.9 and β = .907. The F statistic, testing the joint hypothesis $\alpha = \alpha_o$ and $\beta = \beta_o$, was not significant at the five percent level for the above example or for any of the other examples given. Consequently, the hypothesis $\alpha = \alpha_o$ and $\beta = \beta_o$ is rejected in all cases.

If the suggested cost model, $.0442Q_{lv}(\Delta Z_{lv} + P_{lv}/\gamma + .077Q_{lv} + .012Q_{lv}^{1.85})$, incorporated the actual system-capacity function which was being approximated by pumping operations during the period of observation, β should have been unity and α should have been zero. The deviation from these values appears not to be serious and, in fact, it can be explained. The estimated efficiency for Lake View, $e_{lv} = .78$, may have been in error and the true value may be closer to $e_{lv} = .78/.907 = .860$. An explanation for α being greater than zero is that the approximation of the system-capacity curve is more difficult in the lower range of output since the minimum level of operation is one pump (see Figure IV-7). Still, the functional form of the system-capacity cost relationship is supported by the example cost data (the coefficient of correlation is .94). The assertion that the system-capacity function, on a daily basis, defined variable costs is thereby demonstrated.

However development of this type of cost model requires explicit knowledge of the distribution system. For the above example, a Hardy Cross analysis of the Lake View-Thomas Jefferson distribution network was necessary for the estimation of network characteristics. Hardy Cross type analyses are quite expensive and to implement such an analysis on a metropolitan basis would be impractical (the distribution network for the City of Chicago has yet to be analyzed as a single system). A greatly simplified cost function and a reasonable approximation of the system-capacity cost function is

$$PC = A + BQ^2$$

where A and B are estimated by linear regression from the available operating data. Again using Lake View as an example,

$$PC_{1v} = 99.2 + .0551Q_{1v}^2 \ .$$

All of the assumptions of linear regression are satisfied and the estimates of α and β reflect non-autocorrelated residuals. In Figure IV-9, the theoretical system-capacity cost function, the regression system-capacity function and the quadratic cost function are graphically represented. Over the range of observed data, there is very little difference between the latter two models. The advantages of the quadratic form are that it can be developed quite easily and it is a convenient form for marginal cost analysis. The slope of the quadratic function (the marginal cost) at $Q_{1v} = 40$ is 4.41 dollars per MGD and, at $Q_{1v} = 60$, it is 6.61. Slope values for the system-capacity regression cost function at the same outputs are 4.64 and 6.18 respectively.

However, care should be taken to apply the quadratic model only to the upper ranges of operation. As output approaches zero, the two marginal cost functions diverge (see Figure IV-10). In the lower ranges of operation, the system-capacity cost function more closely approximates the variable operating costs and marginal costs than does the quadratic function since it reflects the cost of maintaining service pressure.

The coefficient of correlation for the quadratic function was .939 and that for the system-capacity model was .941. The explained variance

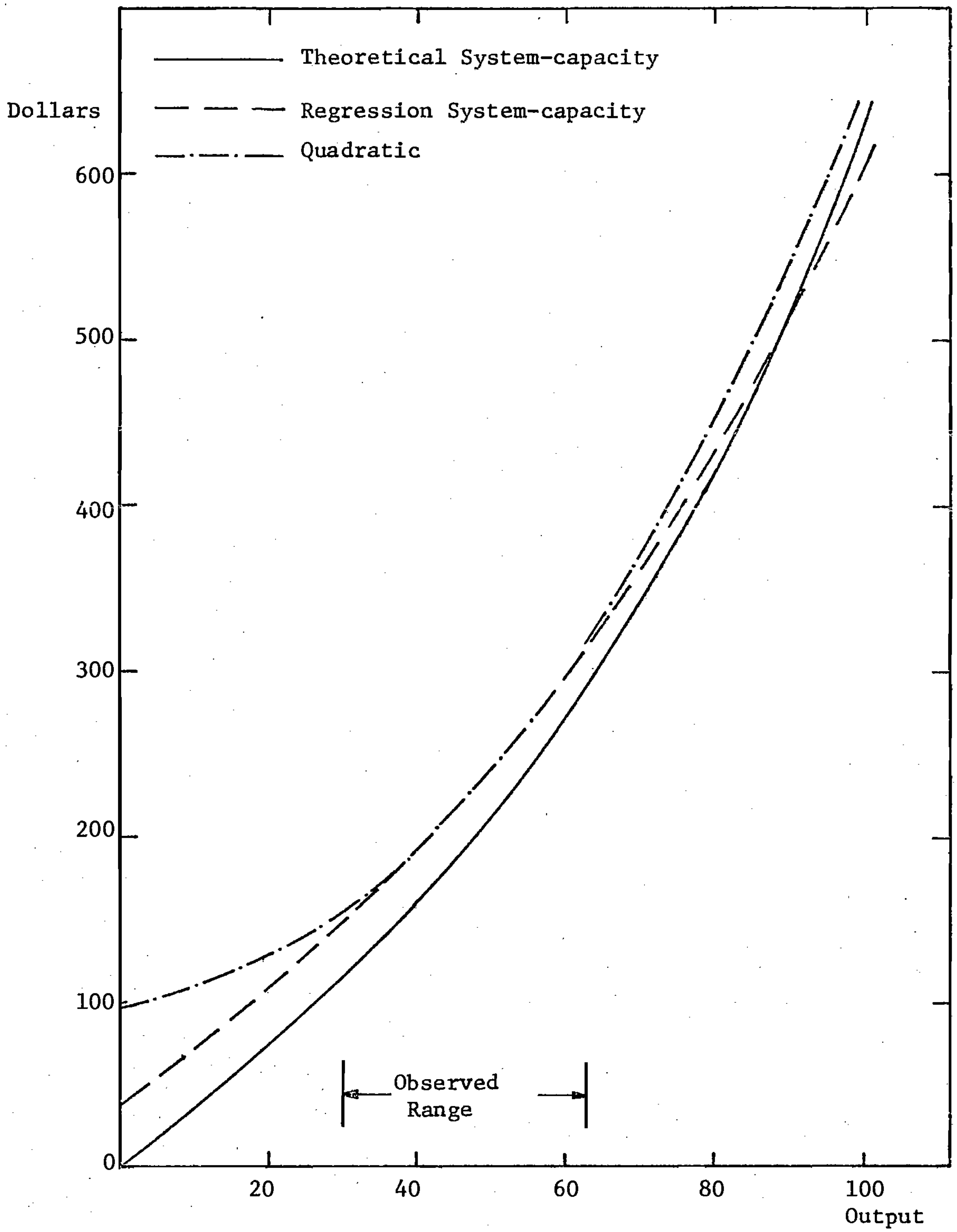


FIGURE IV-9

ALTERNATIVE TRANSPORTATION COST FUNCTIONS

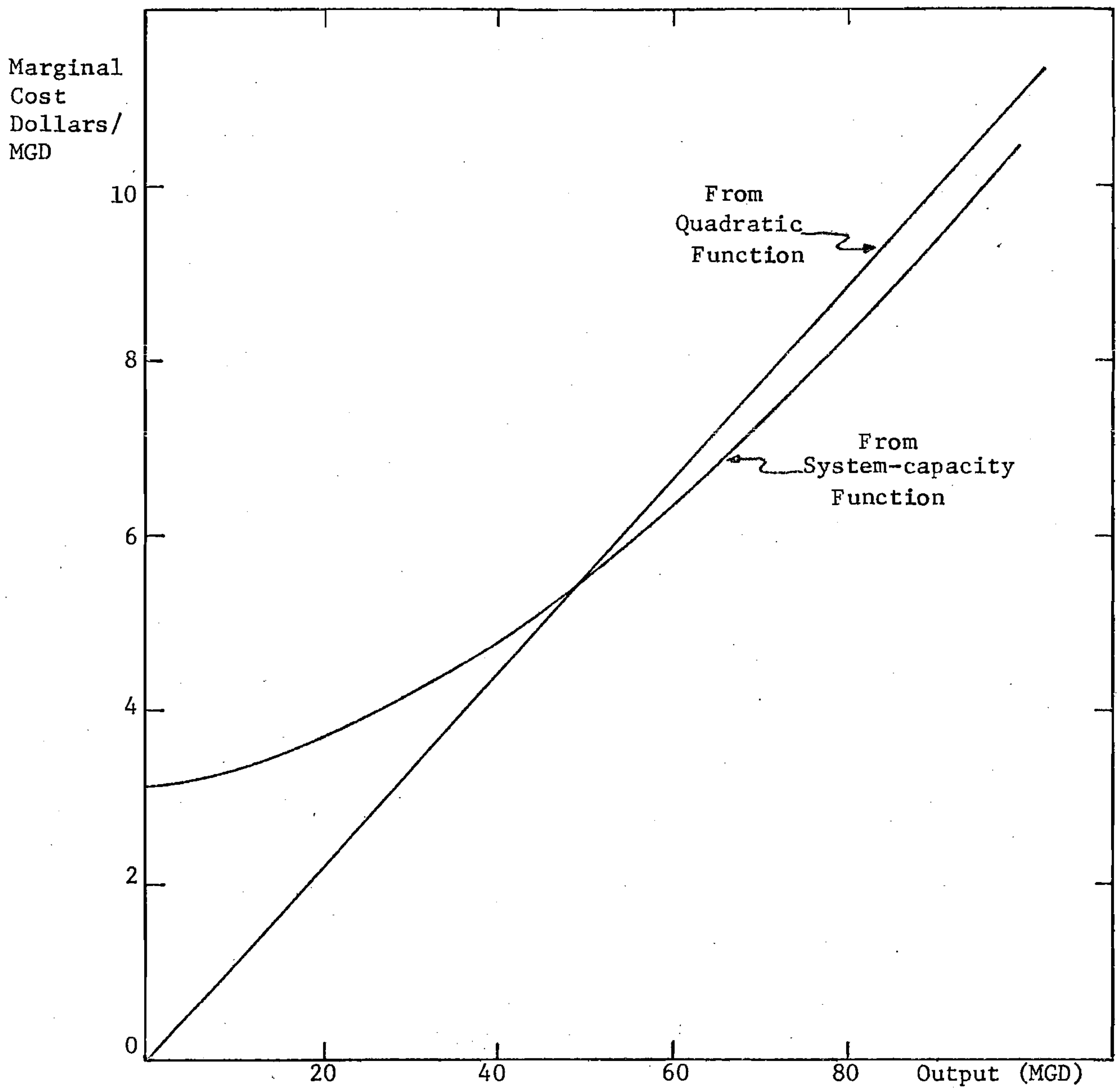


FIGURE IV-10

ALTERNATIVE MARGINAL COST FUNCTIONS

was .882 and .885 respectively. The unexplained variance in both models undoubtedly was due to the exclusion of the following factors:

1. The desired service pressure in the distribution system was not met, on the average, during the given time period.
2. Diurnal variations in demand.
3. Disproportional fluctuations in demand at take-off points with respect to changes in total system demand.
4. The influence of inter-connected pumping stations.

The first three factors cannot be dealt with in that there is insufficient data to quantify their effect. The effect of the fourth factor will be discussed in the next paragraph. However, the aggregate effect of these factors appears to be minor. Both functions give a reasonable estimate of transportation costs when compared with the theoretical function.

When more than one pumping station (or input source such as an elevated water tank) is in operation, headloss, in theory, is a function of all input sources. Such a headloss function is

$$.00368Q_{1v}^{.133}\ (Q_{1v} + Q_{tj})^{1.72}$$

where Q_{tj} is the quantity of water being pumped at Thomas Jefferson Station. The sensitivity of headloss to perturbations in Q_{tj} is

$$\frac{\dfrac{\partial H_{L_{iv}}}{H_{1_{1v}}}}{\dfrac{\partial Q_{tj}}{Q_{tj}}} = \frac{.00633Q_{1v}^{.133}\ (Q_{1v} + Q_{tj})^{.72}Q_{tj}}{.00368Q_{1v}^{.133}\ (Q_{1v} + Q_{tj})^{1.72}} .$$

Representing the left-hand side of the equation by el $\dfrac{H_{L_{1v}}}{Q_{tj}}$ and reducing the right-hand side

$$el\ \frac{H_L}{Q_{tj}} = \frac{1.72\ Q_{tj}}{Q_{1v} + Q_{tj}}\ .$$

Given a fixed output at Lake View, the greater the proportion of total demand supplied by Thomas Jefferson the more sensitive is headloss in the Lake View service area to fluctuations in Q_{tj}. Consequently, fluctuations in Q_{tj} can affect power consumption and costs at Lake View. Substituting the above headloss term into the system-capacity function, the sensitivity of power costs at Lake View relative to Q_{tj} is

$$el\ \frac{PC_{1v}}{Q_{tj}} = \frac{.00633Q_{1v}^{.133}\ (Q_{1v} + Q_{tj})^{.72}Q_{tj}}{9.0 + 69.2 + .0774Q_{1v} + .00368Q_{1v}^{.133}\ (Q_{1v} + Q_{tj})^{1.72}}\ .$$

In Table IV-1, a representative listing is given of observed output at Thomas Jefferson for a constant output at Lake View. Also listed in the table are total output $(Q_{1v} + Q_{tj})$, power costs at Lake View, and the ratio of output at Thomas Jefferson to total output. At the extremes,

Table IV-1

ACTUAL OPERATING DATA

Q_T(MGD)	Q_{tj}(MGD)	Q_{1v}(MGD)	PC_{1v}($)	$\frac{Q_{tj}}{Q_{1v} + Q_{tj}}$
84	42	42	184	.500
85	43	42	184	.506
85	43	42	184	.506
87	45	42	184	.517
88	46	42	184	.523
118	76	42	184	.644
121	79	42	200	.653
122	80	42	184	.656
114	72	42	231	.632
122	80	42	200	.656
117	75	42	200	.641
121	79	42	184	.653
122	80	42	184	.656
115	73	42	199	.633
118	76	42	200	.644

the sensitivity of $H_{L_{lv}}$ varies from .880 to 1.13 and the sensitivity of power costs at Lake View varies from .113 to .252. While the headloss at Lake View is quite sensitive to fluctuations in Q_{tj}, the power costs are not very sensitive. Further, the power cost data in Table IV-1 (as well as the other observed cost data) do not support the theoretical interrelationship. The other three factors mentioned above apparently mask this aspect of water distribution. Consequently, if the allocation scheme, e.g., equal marginal costs, does not greatly alter the output ratios, the hydraulic interrelationship between Lake View and Thomas Jefferson (and probably for most other pumping systems) can be neglected as far as daily allocation is concerned.

The preceding arguments have been directed toward establishing a transportation cost function to serve as a basis for least cost allocation of output from multiple pumping stations. The quadratic function appears to be an adequate representation of the variable costs of transportation. An example of how such functions might be used is one dealing with the Lake View and Thomas Jefferson Pumping Stations. Their cost functions are

$$PC_{lv} = 99.2 + .0551Q_{lv}^2$$

$$PC_{tj} = 156 + .0408Q_{tj}^2 \ .$$

The marginal cost functions are

$$MC_{lv} = .110Q_{lv}$$

$$MC_{tj} = .0816Q_{tj} \ .$$

(See Chapters III and VII for proof and conditions of minimization.)

Both marginal cost functions are graphically represented in Figure IV-11. The aggregate function for marginal cost is given in Figure IV-12. When $Q_T = 122$, at equal marginal costs of 5.65 dollars per MGD (see Figure IV-12), $Q_{lv} = 53$ and $Q_{tj} = 69$ (see Figure IV-11). Inspection of the actual operating data in Table IV-1 reveals that Lake View is underutilized. In fact, reallocation based on this marginal cost scheme, of the total daily demand for the 120 days of observed data, resulted in a ten percent increase in output at Lake View.

The daily cost model and marginal cost allocation scheme can efficiently deal with a very large number of pumping stations and, as such, it can be used for regional allocation and it can be used to set general operating policies for multiple-plant systems. Where the operation of the existing regional system is not centrally coordinated, but physical interconnection is possible such as in the Chicago Metropolitan Area, the benefits of central control can be readily determined from the proposed scheme. Since marginal cost analysis produces the most efficient ratio of outputs given total demand, this knowledge can lead to cost savings and better use of the available capacity. The quadratic cost function serves as a basis for generalized, regional allocation.

While the quadratic cost model can easily deal with the forced constraint of demand (i.e., $\sum_{i=1}^{n} Q_i$ = total demand), system constraints, or limitations, are not so clearly handled since the model is based on mean daily output and average energy requirements. The physical limitation of the system, such as pumping capacity or delivery tunnel capacity, are better analyzed by considering maximum hourly demand or some similar, critical event. Nevertheless, limits can be placed on the probable,

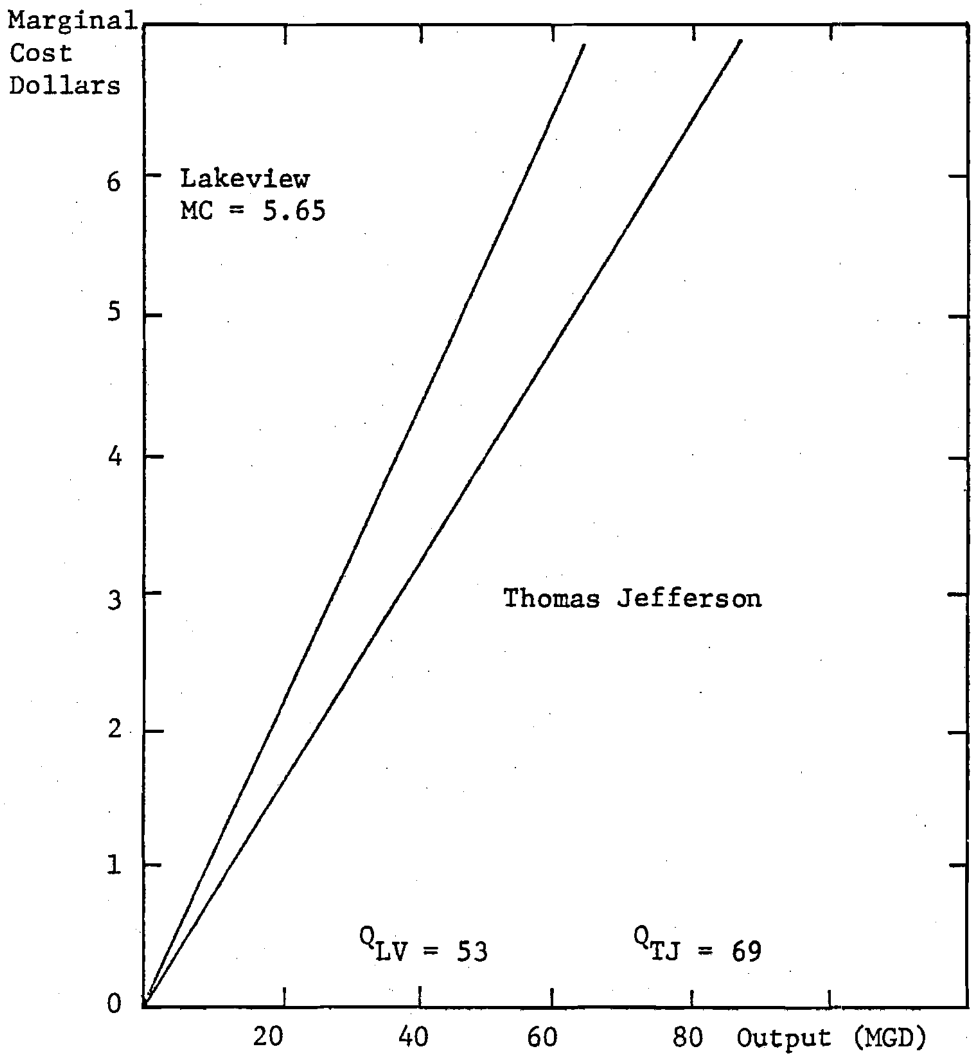


FIGURE IV-11

MARGINAL COSTS

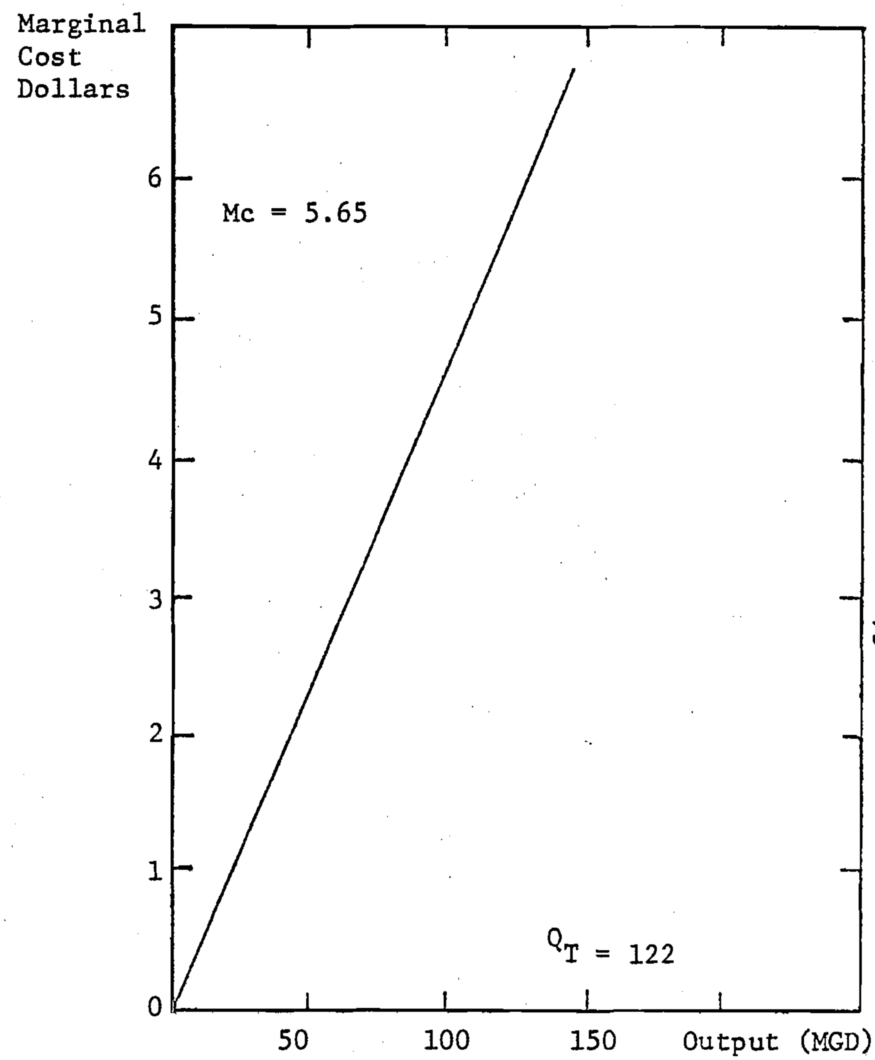


FIGURE IV-12

AGGREGATE FUNCTION FOR MARGINAL COSTS

maximum daily output that various pumping stations can deliver. These limits then become additional forced constraints. Because of the demand constraint, pumping stations which have not reached their maximum limit will operate at higher marginal costs than for those stations which have reached their physical limit. Consider, for example, the coordinated operation of three pumping stations. The Lagrangean function is

$$L = a_1Q_1^2 + a_2Q_2^2 + a_3Q_3^2 + b - \lambda_1(-C_1 + Q_1 + Q_2 + Q_3)$$
$$- \lambda_2(-C_2 + Q_1 + S_1) - \lambda_3(-C_3 + Q_2 + S_2) - \lambda_4(-C_4 + Q_3 + S_3)$$

where C_i i= 2,3,4 is the capacity limit of the i-1 pumping station, S_i, i = 2,3 is the slack variable associated with the capacity constraint, and C_1 is the total demand. The partial derivatives and minimizing conditions are

$$\frac{\partial L}{\partial Q_1} = 2a_1Q_1 - \lambda_1 - \lambda_2 = 0$$

$$\frac{\partial L}{\partial Q_2} = 2a_2Q_2 - \lambda_1 - \lambda_3 = 0$$

$$\frac{\partial L}{\partial Q_3} = 2a_3Q_3 - \lambda_1 - \lambda_4 = 0$$

$$\frac{\partial L}{\partial S_1} = -\lambda_2 = 0$$

$$\frac{\partial L}{\partial S_2} = -\lambda_3 = 0$$

$$\frac{\partial L}{\partial S_3} = -\lambda_4 = 0$$

$$\frac{\partial L}{\partial \lambda_1} = -C_1 + Q_1 + Q_2 + Q_3 = 0$$

$$\frac{\partial L}{\partial \lambda_2} = -C_2 + Q_1 + S_1 = 0$$

$$\frac{\partial L}{\partial \lambda_3} = -C_3 + Q_2 + S_2 = 0$$

$$\frac{\partial L}{\partial \lambda_4} = -C_4 + Q_3 + S_3 = 0$$

If the third pumping station reaches its limit, given C_1, then $S_3 = 0$ since $Q_3 = C_4$. With $Q_3 = C_4$ and $\lambda_i = 0$ (i = 2,3,4,) the minimizing set of equations becomes

$$2a_1Q_1 - \lambda_1 = 0$$

$$2a_2Q_2 - \lambda_1 = 0$$

$$-C_1 + Q_1 + Q_2 + C_4 = 0 \ .$$

Consequently, marginal cost conditions hold for the first and second pumping stations but not for the third station. Nevertheless, the third pumping station remains in operation since its marginal costs are less than for the other two and it assists in the overall minimizing process through is contribution to total demand. This point is demonstrated by the solution of the above set of equations for λ_1 . $\lambda_1 = 2a_1a_2 (C_1 - C_4)/(a_1 + a_2)$. Obviously, the marginal cost, λ_1, would be greater if C_4 or Q_3 was not included, and the greater λ_1 is, the greater total cost will be.

Hourly Cost Model

In determining the cost of supplying the maximum hourly demand and in evaluating the capability of the transportation system to meet that critical demand, the system-capacity function only serves as a reference for judging whether service requirements can be met. The head-capacity function, the headloss in the primary delivery system (pipes or tunnels bringing water to the pumping station) and, in the case of two or more pumping stations operating in the same distribution network, the energy boundary relationship between pumping stations control the output levels, the service pressure, and the variable cost of transportation. The general mathematical model, or set of equations, describing energy requirements and costs is as follows:

$$\text{power costs} = .0346 \sum_{i=1}^{n} q_i h_i / e_i \qquad \text{for } n \text{ pumping stations}$$

$$\text{total demand} = \sum_{i=1}^{n} q_i$$

$$\text{head-capacity} = f_i(q_i) \qquad i=1,\ldots,n$$

$$\text{efficiency-capacity} = g_i(q_i) \qquad i=1,\ldots,n$$

$$\text{dynamic head} = f_i(ph_i, sh_i, vh_i) \qquad i=1,\ldots,n$$

$$\text{pressure head } (ph_i) = f_i(q_1, q_2, \ldots q_n, ph_1, \ldots ph_{i-1}, ph_{i+1}, \ldots ph_n) \qquad i=1,\ldots,n$$

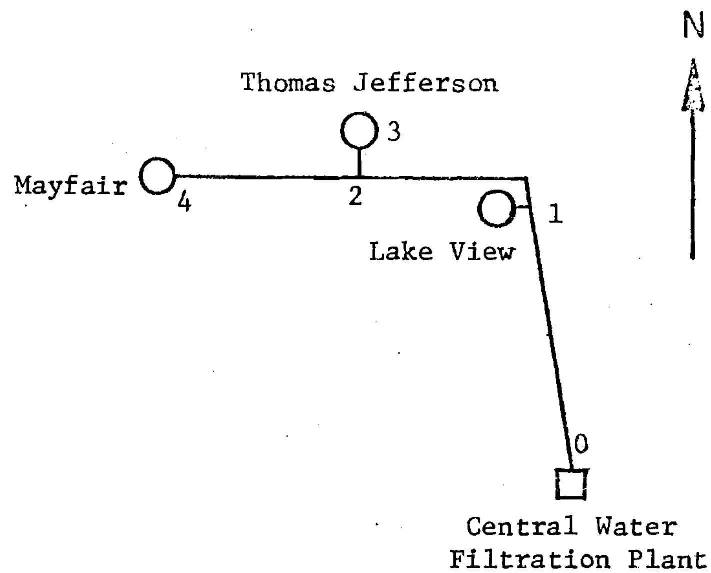
where ph_i is the pressure head at the i^{th} station, sh_i is the suction head and vh_i is the velocity conversion head.

An example of a limiting physical condition can be found in the North Tunnel Zone of Chicago's water supply system. Headlosses in the delivery tunnel due to peak summer demands are becoming sufficiently large that the suction lift at the Mayfair Pumping Station is approaching the permissible limit. In fact, the projected maximum hourly demands (2) for this tunnel zone indicate that the limit will be exceeded by 1980.

When water is subjected to a pressure less than its vapor pressure, it boils and vapor pockets are entrained in the liquid (43). Upon reaching a region of higher pressure (the pressure side of the pump), these pockets of air collapse. The process is called cavitation and it results in a reduction of pump efficiency and it can cause physical damage to the pump. For centrifugal pumps, the practical suction lift is fifteen feet. The critical elevation at the Mayfair Pumping Station in reference to the Chicago datum is -13.5 feet. This elevation constitutes the most critical physical constraint to supplying water through the North Tunnel Zone, when considering the 1980 projected conditions.

To analyze the effects of this constraint on the allocation of pumping station output, a cost model was developed based on the general model given above. The characteristics of the tunnel system are given in Figure IV-13. The head-capacity functions used in this model are the same as those developed earlier in this chapter. Other relevant physical data for the pumping stations are given in Table IV-2 and the parameterized cost model is given in Table IV-3. Parameter values were supplied by the City of Chicago except for those used in the energy boundary relationship which are based on an independent Hardy Cross analysis of the distribution network.

To test the validity of the model, operating data for two peak hours (8:00 pm, July 11, 1972, and 2:00 pm, August 18, 1972) were compared to simulated values (Table IV-4 gives a listing of both sets of



Node	
0	Central Water Filtration Plant
1	Lake View Pumping Station
2	Junction
3	Thomas Jefferson Pumping Station
4	Mayfair Pumping Station

Link	Length (ft)	Diameter (ft)	k	n
0-1	28,000	16	2.92×10^{-5}	2.0
1-2	9,000	12	1.32×10^{-4}	1.76
2-3	200	8	4.71×10^{-5}	2.0
2-4	13,000	12	1.50×10^{-4}	1.89

FIGURE IV-13

TUNNEL CHARACTERISTICS

Table IV-2

PUMPING STATION DATA[1,2]

Lake View	
No. of Pumps	3
Unit Capacity	35
Total Capacity	105
Pump Elevation[1]	-11.21
Curb Elevation	+ 8.42
Gage Elevation[2]	- 0.55
Thomas Jefferson	
No. of Pumps	4
Unit Capacity	40
Total Capacity	160
Pump Elevation	+ 1.0
Curb Elevation	+16.5
Gage Elevation	+ 1.0
May Fair	
Critical Suction Elevation	-13.5

1. All elevations are relative to Chicago datum.

2. Gages are set to read curb pressure in pounds per square inch.

Table IV-3

SYSTEM MODEL: LAKE VIEW--THOMAS JEFFERSON
MAXIMUM HOUR 1980 = 550 MGD

Total Power Costs

$$PC = 3.14\ q_{1v}h_{1v}/e_{1v} + 3.14q_{tj}h_{tj}/e_{tj}$$

Total Output

$$q_T = q_{1v} + Q_{tj} + q_{mf}$$

Head-Capacity (three pumps on)

$$h_{1v} = 203 - .665q_{1v}$$

$$h_{tj} = 257 - .645q_{tj}$$

Efficiency-Capacity

$$e_{1v} = .78$$

$$e_{tj} = .76$$

Total Dynamic Head

$$h_{1v} = ph_{1v} + sh_{1v} + .000246q_{1v}^2$$

$$h_{tj} = ph_{tj} + sh_{tj}$$

Energy Boundary

$$ph_{1v} = ph_{tj} + .000592(q_{1v} + q_{tj})q_{1v} - .000525(1_{1v} + q_{tj})q_{tj} - Z_{1v} + Z_{tj}$$

Suction Head

$$sh_{1v} = Z_{1v} - (Z_o - .0000292q_T^2)$$

$$sh_{tj} = Z_{tj} - [Z_o - .0000292q_T^2 - .000032(q_{tj} + q_{mf})^{1.76} - .0000471q_{tj}^2]$$

Tunnel Constraint

$$H_{L_{mf}} \geq .0000292q_T^2 + .000132(q_{tj} + q_{mf})^{1.76} + .000150q_{mf}^{1.89}$$

Table IV-4

OBSERVED AND SIMULATED SYSTEM PERFORMANCE

Peak Hour: July 11, 1972--8:00 pm		Peak Hour: August 18, 1972--2:00 pm	
Observed	Simulated	Observed	Simulated
three pumps on		three pumps on	
EL_o = 5.00 ft	Given	EL_o = +5.6	Given
EL_1 = 1.50 ft	-.93 ft	EL_1 = -2.0	+.50
Q_1 = 108 mgd	Given	Q_1 = 114	Given
Sta. Press. = 51 psi	51 psi	Sta. Press. = 49 psi	50 psi
SH_1 = -9.71 ft	-10.3 ft	SH_1 = -9.21	-11.7
PH_1 = 137 ft	138 ft	PH_1 = 133	135
VH_1 = 2.87 ft	2.87 ft	VH_1 = 3.19	3.19
TDH_1 = 130 ft	131 ft	TDH_1 = 127	127
PC_{1v} = $621	not avail.	PC_{1v} = $610	
EL_2 = -5.0	-5.22		
two pumps on		two pumps on	
Q_2 = 98 mgd	Given	EL_2 = -3.5	-3.1
Sta. Press. = 47 psi	46 psi	Q_2 = 99	Given
SH_2 = 6.00	6.22	Sta. Press. = 46	47 psi
PH_2 = 124	123	SH_2 = 4.5	4.1
TDH_2 = 130	129	PH_2 = 122	124
PC_{tj} = $582	not avail.	TDH_2 = 126	128
EL_3 = -8.5	-8.93	PC_{tj} = $569	
Q_3 = 245 mgd	Given	EL_3 = -4.0	-5.6
		Q_3 = 205	Given

System-Head Relationship: $PH_1 - PH_2 = K_1 Q_T Q_1 - K_2 Q_T Q_2 - Z_1 + Z_2$

$K_1 = .000592 \quad Z_1 = -11.21$

$K_2 = .000525 \quad Z_2 = +1.0$

Date	Time	Observed	Simulated
7/11/72	8:00 pm	13.0 ft	12.8 ft
8/18/72	2:00 pm	11.0 ft	13.5 ft

values) and the comparison is close. The error in determining the water elevation in the suction well may be caused by the difficulty of accurately measuring the water level due to surges in the tunnel system as well as to the fact that one measurement is used to define the average hourly water level. However, the error is minor in that it is only 1.6 percent of the total dynamic head.

The maximum hourly demand anticipated by the year 1980 for the North Tunnel Zone is 550 MGD (see Table IV-8). Of this total demand, the forecast calls for Lake View to supply 78 MGD, Thomas Jefferson to supply 109 MGD and Mayfair to supply 368 MGD. If the distribution system is operated in accordance with this allocation, the suction lift constraint at Mayfair will be violated. In order to determine whether the projected total demand can be supplied without violating the suction constraint and still meet the service pressure constraint, the above model or set of equations was used to simulate the transportation system and to generate the requisite output for each pumping station. The above equation set, excluding the power cost function, were solved simultaneously for a feasible allocation. Since the equations are non-linear, their simultaneous solution is complicated. Geometric programming was used as the solution technique (see Chapter III).

The application of geometric programming requires that inequality constraints replace equality constraints. The transformed set of equations, the primal geometric program, is given in Table IV-5. The dual program is given in Table IV-6.

There are two distinct advantages in using geometric programming. It deals directly with the non-linear functions without linear approxi-

Table IV-5

PRIMAL PROGRAM

System Model

Lake View--Thomas Jefferson

(Program A)

Minimize Power (Variable Cost)

$$4.02\ Q_{1v}H_{1v} + 4.15\ Q_{tj}H_{tj}$$

Subject to

Total Output

$$.00182\ Q_{1v} + .00182\ Q_{tj} + .00182\ Q_{rt} \geq 1$$

Head-Capacity

$$.00328\ Q_{1v} + .00439\ H_{1v} \geq 1$$

$$.00251\ Q_{tj} + .00389\ H_{tj} \geq 1$$

Total Dynamic Head

$$H_{1v}X_1^{-1} + 19.21\ X_1^{-1} \leq 1$$

$$PH_{1v}X_1^{-1} + 8.83\ X_1^{-1} + .00246\ Q_{1v}^2X_1^{-1} \geq 1$$

$$Q_{tj}X_2^{-1} + Q_{mf}X_2^{-1} \leq 1$$

$$H_{tj}X_3^{-1} + 7.00X_3^{-1} \geq 1$$

$$PH_{tj}X_3^{-1} + 8.83X_3^{-1} + .00132X_2^{1.76}X_3^{-1} + .0000471Q_{tj}^2X_3^{-1} \leq 1$$

System-Head

$$PH_{1v}X_4^{-1} + .00525Q_{tj}^2X_4^{-1} \leq 1$$

$$PH_{tj}X_4^{-1} + .000592Q_{1v}^2X_4^{-1} + .0000670Q_{1v}Q_{tj}X_4^{-1} + 12.21X_4^{-1} \geq 1$$

Tunnel Constraint

$$.0000104X_2^{1.76} + .0000118Q_{mf}^{1.89} \leq 1$$

Table IV-6

DUAL PROGRAM

System Model

Lake View--Thomas Jefferson

(Program B)

Maximize

$$\nu(\delta,\alpha) = \{[(\frac{4.02}{\delta_1})^{\delta_1}(\frac{4.15}{\delta_2})^{\delta_2}][(\frac{1}{\delta_{10}})^{\delta_{10}}(\frac{12.21}{\delta_{11}})^{\delta_{11}}(\frac{1}{\delta_{15}})^{\delta_{15}}(\frac{1}{\delta_{16}})^{\delta_{16}}$$

$$(\frac{1}{\delta_{19}})^{\delta_{19}}(\frac{8.83}{\delta_{20}})^{\delta_{20}}(\frac{.000132}{\delta_{21}})^{\delta_{21}}(\frac{.0000771}{\delta_{22}})^{\delta_{22}}(\frac{1}{\delta_{23}})^{\delta_{23}}$$

$$(\frac{.000525}{\delta_{24}})^{\delta_{24}}(\frac{.0000104}{\delta_{24}})^{\delta_{29}}(\frac{.0000118}{\delta_{30}})^{\delta_{30}}][(\frac{\alpha_3^2}{.00162\ \delta_3})^{\delta_3}$$

$$(\frac{\alpha_4^2}{.00182\ \delta_4})^{\delta_4}(\frac{\alpha_5^2}{.00182\ \delta_5})^{\delta_5}(\frac{\alpha_6^2}{.00328\ \delta_6})^{\delta_6}(\frac{\alpha_7^2}{.00493\ \delta_7})^{\delta_7}$$

$$(\frac{\alpha_8^2}{.00251\ \delta_8})^{\delta_8}(\frac{\alpha_9^2}{.00389\ \delta_9})^{\delta_9}(\frac{\alpha_{12}^2}{\delta_{12}})^{\delta_{12}}(\frac{\alpha_{13}^2}{8.83\ \delta_{13}})^{\delta_{13}}$$

$$(\frac{\alpha_{14}^2}{.00246\delta_{14}})^{\delta_{14}}(\frac{\alpha_{17}^2}{\delta_{17}})^{\delta_{17}}(\frac{\alpha_{18}^2}{7.00\ \delta_{18}})^{\delta_{18}}(\frac{\alpha_{25}^2}{\delta_{25}})^{\delta_{25}}(\frac{\alpha_{26}^2}{8.83\ \delta_{26}})^{\delta_{26}}$$

$$(\frac{\alpha_{27}^2}{.000132\ \delta_{27}})^{\delta_{27}}(\frac{\alpha_{28}^2}{.0000471\ \delta_{28}})^{\delta_{28}}]\}\{[\Pi\lambda_k(\delta)^{\lambda_k(\delta)}][\Pi\lambda_1(\delta)^{\lambda_1(\delta)}]$$

Subject to

$\delta_1 + \delta_2 = 1$ $\quad\sum_{i=1}^{n} q_{ij}\delta_i$ $\quad j = 1,2,\ldots,11$ $\quad n$ = number of terms in the j^{th} constraint

Here, $\lambda_k(\delta) = \sum_{i=1}^{n} \delta_i$ $\quad k = 4,6,8,9,$ and 11

$\lambda_1(\delta) = \sum_{i=1}^{n} \delta_i$ $\quad 1 = 1,2,3,5,7,$ and 10 $\quad a_{ij}$ is the exponent matrix.

mations and it provides sensitivity-analysis as a direct result of its log transformation of the dual problem (see Chapter III). A unique advantage of GEPROG, the geometric programming algorithm on Northwestern University's computer facility, is its ability to handle reversed constraints (e.g., $q_T \geq q_{1v} + q_{tj} + q_{mf}$), slack constraints and degeneracy (25)(11). In fact, the North Tunnel Zone problem is the first practical reversed constraint problem solved using GEPROG.

The solution of the dual and primal problem and the values of the primal and dual variables are given in Table IV-7. If the output at Lake View is equal to 98 MGD and the output at Thomas Jefferson is equal to 169 MGD, the critical suction lift at Mayfair will not be exceeded. A comparison of the projected output levels with the output levels generated by the mathematical model (see Table IV-8) indicates a need to increase output at Lake View and Thomas Jefferson and for a corresponding decrease in output at Mayfair if the projected maximum hourly demand is to be met without further capital costs. The tunnel hydraulic grade lines for both projected and simulated conditions are given in Figure IV-14.

If the Lake View Pumping Station services the critical supply node (Node No. 80) under the simulated allocation, the headloss between Lake View and that node would be

$$H_{L_{1v-80}} = .00373(267)^{.824}(98)^{1.027} = 41.3 \text{ ft} .$$

Service pressure at the node would be 23.4 psi which is below the desired 30 psi. A more serious problem arises from the fact that all of the available pumping capacity at Lake View and Thomas Jefferson is

Table IV-7

SYSTEM MODEL

LAKE VIEW--THOMAS JEFFERSON

OPTIMAL SOLUTION

Primal Program A

Power = 158,000 KW Power Cost = $1740.00

Qlv = 98 mgd
Qtj = 169 mgd
Qmf = 283 mgd
Hlv = 138 ft
Htj = 148 ft
PHev = 144 ft
PHtj = 138 ft

Dual Program B

Power = 158,000

δ_1 = +.343	δ_{11} = +.000754	δ_{21} = +.000260
δ_2 = +.657	δ_{12} = -.00575	δ_{22} = +.0000475
δ_3 = -.181	δ_{13} = -.000338	δ_{23} = +.00575
δ_4 = -.310	δ_{14} = -.000462	δ_{24} = +.000725
δ_5 = -.538	δ_{15} = +.114	δ_{25} = -.00554
δ_6 = -.161	δ_{16} = +.193	δ_{26} = -.000318
δ_7 = -.348	δ_{17} = -.00594	δ_{27} = -.000156
δ_8 = -.462	δ_{18} = -.000266	δ_{28} = -.000466
δ_9 = -.651	δ_{19} = +.00554	δ_{29} = +.174
δ_{10} = +.00538	δ_{20} = +.000360	δ_{30} = +.182

Sensitivity of Constraints

λ_1 = +1.03
λ_2 = +.509
λ_3 = +1.11
λ_4 = -.00614
λ_5 = +.00614
λ_6 = -.307
λ_7 = +.00620
λ_8 = -.00620
λ_9 = -.00648
λ_{10} = +.00648
λ_{11} = -.356

Table IV-8

NORTH TUNNEL ZONE[1]--DEMAND PROJECTIONS
MAXIMUM HOUR--1980, 1990, 2000
(Chicago, Present and Future Suburbs)

	1980	1990	2000
Total	500	550	596
Lake View	72	73	74
Thomas Jefferson	108	109	110
Mayfair	320	368	412

MODEL SOLUTION

	1990
Total	550
Lake View	98
Thomas Jefferson	169
Mayfair	283

1. Taken from Report Upon Adequate Water Supply for Chicago Metropolitan Area 1969 to 2000.

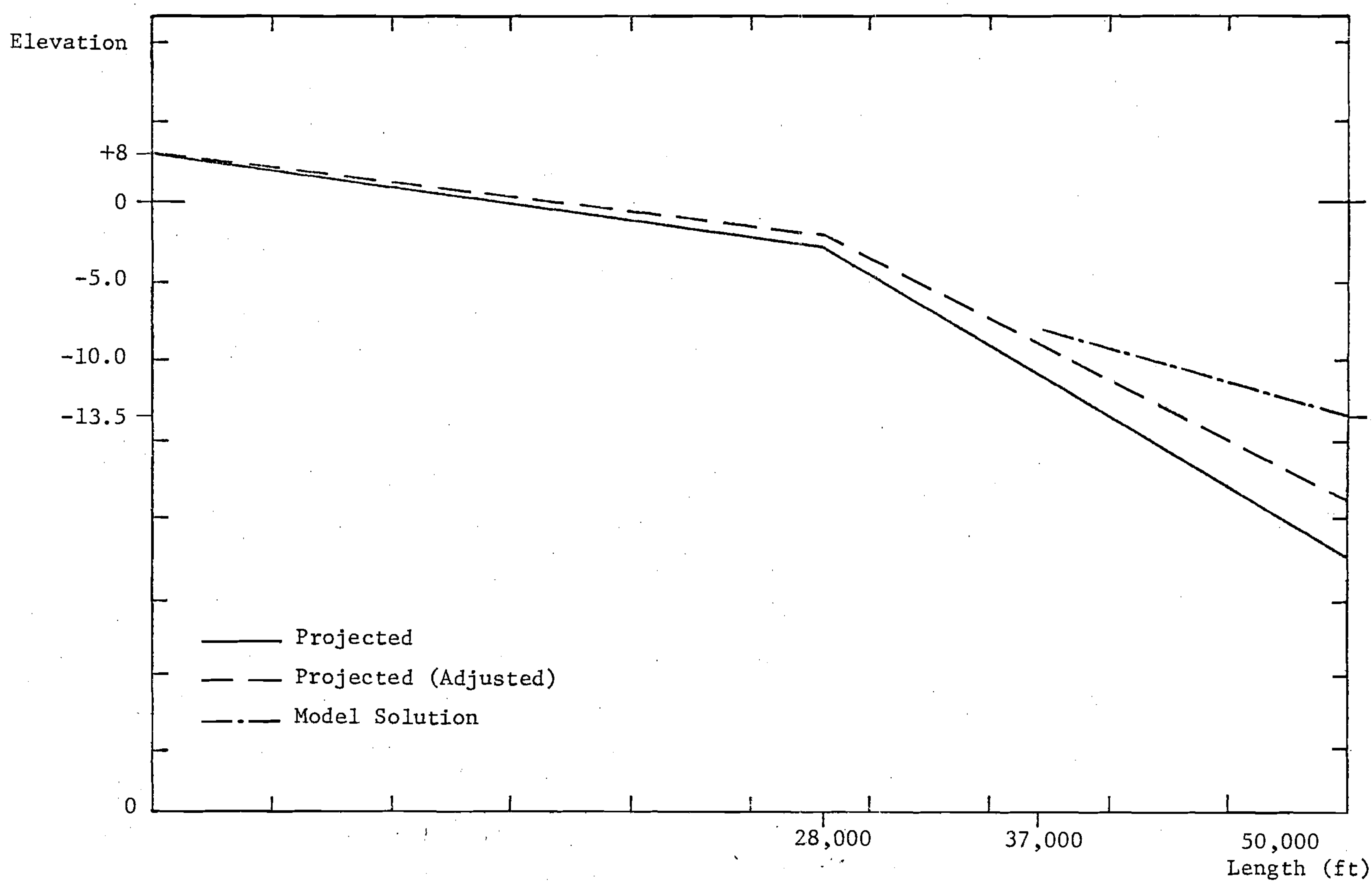


FIGURE IV-14

HYDRAULIC GRADE LINE FOR THE NORTH TUNNEL ZONE required to meet the projected demand and satisfy the tunnel constraint. Consequently, there is no reserve capacity at these two stations with which to meet such emergencies as fires or pump failures. However, emergency supplies could be obtained from adjacent pumping stations both in Chicago and the suburbs. The feasibility and cost of obtaining additional supplies in this manner can be established just as the above system operations was analyzed.

The sensitivity of power costs to a perturbation of the coefficient on the i^{th} variable term is equal to the value of the dual variable, δ_i, at the optimum point (see Table IV-7) and the sensitivity of power costs to small changes in the right-hand side of the i^{th} constraint is expressed by λ_i. For example, if total demand (q_T - 550) is reduced by ten percent in the above problem, power costs will be reduced by 10.3 percent, as noted above, since λ_1 is equal to 1.03. Similarly, a ten percent change in the value of K_{2-4} (the coefficient of the headloss term for the tunnel section between the Thomas Jefferson Pumping Station and Mayfair Pumping Station) would result in 1.82 percent change in power costs (δ_{30} = .182). Therefore, power costs are relatively insensitive to perturbations of this parameter. If the value of the parameter, K_{2-4}, were incorrect, the error introduced would be quite small in terms of the power costs. Another interpretation would be if K_{2-4} could be reduced; e.g., by enlarging the tunnel diameter, power costs would be reduced by a mere 1.82 percent. Consequently, sensitivity analysis can be used to interpret the effect of measurement error as well as the effect of physical changes in the system through the corresponding change in model parameters.

Although only one Hardy Cross analysis was made of the Lake View/ Thomas Jefferson service area, the significance of energy boundary between pumping stations in determining transportation costs appears to be minor. The sensitivity of power costs to the relaxation of the boundary constraints (constraints 8, 9, and 10) in all cases is less than .01 (i.e., $\lambda_8 = .0062$, $\lambda_9 = -.00648$ and $\lambda_{10} = +.00648$). The same conclusion is noted in the section dealing with daily costs.

The significant constraints are 1, 2, 3, 6 and 11. The first constraint is the demand constraint. The sixth constraint is a dummy constraint. The eleventh constraint models the suction lift at Mayfair. The fact that power costs are less sensitive to this constraint than to the head-capacity constraint (2 and 3) at Lake View and Thomas Jefferson is noteworthy. The original formulation of head-capacity constraint was

$$h_i = a_i - b_i q_i$$

This was transformed into an inequality constraint

$$\frac{1}{a_i} h_i + \frac{b_i}{a_i} q_i \geq 1 \text{ (see Tables IV-3 and 5)}$$

In order to relax the right-hand side of the inequality, a_i would need to change. However, a_i is the static head at zero output and, as previously demonstrated, it is a constant for a given pumping station. Consequently, b_i is the only parameter in the head-capacity relationship which can be manipulated. The sensitivity of power to perturbations of b, the slope of the head-capacity curve, at Lake View and Thomas Jefferson is given by the values of the dual variables δ_6 and δ_8 respectively. If the slope of the head-capacity curve at Lake View were decreased a small amount the increase of the objective function would be .16x the percentage decrease. Likewise, if the slope of Thomas Jefferson's head-capacity curve were decreased, the increase in power would be .46x the percentage decrease. However, these sensitivities are computed assuming that the output at each pumping station remains constant. This would not be the case if b were altered since a is constant. To determine the aggregate effect on power, the chain rule of differentiation must be employed, i.e.,

$$\frac{\partial p}{\partial b} = \left(\frac{\partial p}{\partial q}\right)\left(\frac{\partial q}{\partial b}\right) ,$$

and the partial derivatives evaluated at the optimum point. The sensitivity form of the above equation is

$$el(p/b_i) = \partial p/p \Big/ \partial b_i/b_i = c_i(h_i^2 - a_i h_i)(b_i p)$$

Evaluating this equation for Lake View and Thomas Jefferson results in $el(p/b_{lv}) = -.34$ and $el(p/b_{tj}) = -.66$. Although different in magnitude, the relative sensitivity is the same as before. Therefore, power cost would be increased less by an increasing output at Lake View; but, since all of the available pumping capacity is being used in the example problem, changing the head-capacity curve at Lake View would require capital expenditure. This involves long-run economic analysis; nevertheless, the short-run analysis provides some insight as to where additional pumping capacity might be most beneficial and that would be at Lake View.

Conclusion

Both of the methodologies described above could play an important role in regional water supply management. The daily cost model provides information on general, regional allocation of transportation capacity and the cost of this type of analysis is quite low. This model does not provide information on how to operate pumping stations other than to indicate relative levels of output. However, the peak demand, or hourly, model, which is relatively expensive to operate, can provide explicit information on operational strategies to minimize pumping costs. As in the example given above, the relative sensitivity of power costs to the head-capacity parameters can be used to judge which station should be manipulated to meet changes in demand.

Neither model seemed to be particularly sensitive to boundary conditions. The constraints describing the energy boundary, in the peak demand model, did not exhibit a major influence on power costs nor did the energy relationship considered with the daily model. Nevertheless, the market area of each pumping station is defined by the energy boundary. This definition was explicit in the peak demand model, but, with both models, the level of output is the explicit indicator of market area.

The equality of marginal costs is established between market areas (or pumping stations and their associated distribution systems) and not necessarily at the boundary between areas. Energy is consumed at a given pumping station according to the pump, distribution network, and demand characteristics, and the operation of adjacent stations. Consequently, marginal costs are a function of these factors. In a fixed distribution network, marginal costs do not appear to be a function of distance from the supply source, the pumping station.

Finally, most of the examples given in this Chapter deal with critical demand conditions. It should not be assumed that these are typical conditions. In fact, these conditions happen rarely, and then they only last for a few hours. The examples were selected to demonstrate constrained operating conditions. Most of the time, excess capacity exists at Lake View, Thomas Jefferson, and the other pumping stations. The optimal operation of these pumping stations, under non-critical demand conditions, still is predicted on the principles given above.

V. PRODUCTION FUNCTION

The variable costs of production result from energy expended to collect and transport raw water to the treatment plant (the energy used for mixing, etc., is not significant and the process of desalinization is not considered) and from the chemicals used to transform the raw water input to a quality suitable for consumption. The extent to which each of these two cost factors affects the production cost function depends on the treatment plant design, nature of the treatment process, raw water supply source, and climatic conditions (28). For example, power cost at the Central Water Filtration Plant (rapid sand filtration process with Lake Michigan the supply source) is about seventeen percent of the total of variable costs (see Table V-2) while chemical costs represent approximately eighty-three percent. Power costs at the Evanston Treatment Plant (also, rapid sand filtration process and Lake Michigan the supply source) are twenty-one percent of the variable costs and chemical costs are seventy-nine percent. However, power cost constitutes sixty-one percent and chemical costs thirty-nine percent of the total variable costs at the Des Plaines Treatment Plant. The substantial change in the relation between power and chemical costs is due to the fact that Des Plaines is a softening plant and obtains its raw water input from wells. Because the variation in treatment processes and the relative significance of each factor in the process, the production cost function will vary in form. Further discussion on this point is given later in this chapter.

There have been numerous studies (17)(22) in which production functions have been developed; however, these functions were directed toward establishing long-run cost relationships. They include fixed costs and

TABLE V-1

FACTOR PRICES

PRODUCTION FACTOR	UNIT-PRICE/PLANT		
	Central 960 MGD	Evanston 72 MGD	Des Plaines 6 MGD
1. Power $/KWH	.0118	.0111	.0109
2. Alum $/lbs.	.0232	.0250	*
3. Ammonia $/lbs.	.0228	.0750	*
4. Carbon $/lbs.	*	.100	*
5. Caustic $/lbs.	.0320	*	*
6. Chlorine $/lbs.	.0508	.0675	.125
7. Drew floc $/lbs.	*	*	.165
8. Ferrous Sulfate $/lbs.	.0132	*	*
9. Fluorine $/lbs.	.134	.0376	*
10. Lime $/lbs.	.0181	*	.0107
11. Natural gas $/ft^3	*	*	.0771

* -- signifies factor not used in production process of the indicated plant (carbon is used at the South and Central plants, although not continuously, and the period of record analyzed did not include its use)

TABLE V-2

MEAN DAILY PRODUCTION AND COSTS

Production Factor(s)	Central	Evanston	Des Plaines
1. Power $/Day	648.00	44.00	85.60
2. Chemicals $/Day	3150.00	166.00	54.80
3. Raw Water MGD	732.00	28.40	2.11
4. Variable Operation Cost $/Day	3790.00	211.00	140.00
5. Mean Output MGD	686.00	27.80	2.03
6. Power/Total Variable Costs	.171	.211	.611
7. Chemicals/Total Variable Costs	.829	.789	.389
8. Variable Operating Cost/Mean Output $/MGD	5.52	7.60	69.00

fixed operating costs as well as variable operating costs. Such functions are useful for design and plant expansion, but they cannot be used to evaluate the economic operation of existing facilities. Short-run production cost functions are necessary for the regional operating model as demonstrated in Chapter VI. Consequently, an attempt is made in the following sections to substantiate a general form for the production function and construct exemplary functions for the Central Water Filtration Plant and Evanston's treatment plant.

Operating Costs

Power: In Chapter IV, the cost required to transport water was shown to be approximated by a quadratic function in output. The underlying physical relationship was

$$PC = .0346\ qh$$

where q is output expressed in MGD and h is the total dynamic head in feet of water. In a distribution system, h is dynamic due to variation in demand and large variation in headloss. However, at some treatment plants, raw water is delivered with only minor headloss to a constant head tank or to a small reservoir whose surface elevation experiences minor variations. Therefore, head, h, is nearly constant and power cost becomes directly proportional to flow, i.e.,

$$PC = aq$$

Both the Evanston plant and the Chicago Central Water Filtration Plant are examples of this type. When the raw water source is a considerable distance from the treatment plant, headlosses may be quite large. They may contribute significantly to production costs. The Des Plaines treatment plant is a good example of this. The nature of the production cost function, undoubtedly, will vary from plant to plant depending on the supply system. For the Evanston and Central plants, the theoretical and empirical considerations indicate a linear cost function.

Chemicals: From a theoretical viewpoint, the amounts of chemicals needed to treat a quantity of water are determined by: the quality characteristics of the raw water, the desired quality characteristics of the treated water, the design and operational characteristics of the treatment facility, and the kinetics of the reactions and processes involved in the treatment. Of these, only the last two may be considered to produce variations in chemical usage, and therefore chemical costs, in response to variations in the rate at which water is processed. Most treatment plants utilize all of their facilities continuously and adjust the rate of processing to provide the amount demanded, rather than maintaining a constant process rate and bringing parts of the facility on- and off-line as needed to meet demand. If adequate storage is provided for treated water, the daily variation in processing rate can be substantially reduced or even eliminated, although variations in rate of processing would still exist from day to day or season to season.

It is interesting to note, as an aside, that the variation in rate of treatment common to most water supply facilities tends to produce a "consumer surplus" in terms of water quality, since the minimum standards for water quality are usually met during times when the processing rates are high, and exceeded (i.e., better quality) at other times. It is doubtful that the typical consumer values this surplus to any appreciable extent. If not, it is possible that cost reductions might be realized by revising the treatment plant designs and operations to avoid production of the surplus.

The kinetics of chemical reactions and physicochemical processes are influenced by a variety of factors, most notably through temperature and the raw water quality characteristics. Although much research has focused on the specific influence of one or a few of the relevant factors each of the several processes involved, the combined effect of many factors on all treatment processes in concert has received little attention.

For purely illustrative purposes, the basic water treatment process can be loosely described by the differential equation (15)

$$\pm dy/dt = K\Phi(y)$$

where y is the concentration of the substance added or removed, t is time and K is the reaction rate. If, for example, $\Phi(y) = y$ and $y = y_o$ at $t = 0$, the concentration of a subject material is:

$$y = y_o e^{\pm Kt}$$

However, t can be replaced by C_p/Q where C_p is treatment capacity (fixed in the short-run) and Q is the rate of flow through the treatment plant. Then,

$$y = y_o e^{\pm KC_p/Q}$$

In processes where y represents the concentration of chemicals added, chemical costs may be considered to be proportional to the above relationship; for example,

y = pounds of chemicals per day/million gallons per day
= pounds/million gallons or dollars/million gallons.

However, as Q increases, K must increase if y is to be maintained at a standardized level. The variable relationship between K, Q, and y appears not to have a usable theoretical basis at present. That is to say, a great deal of research has been done in the field of treatment kinetics (15), but most of the useful mathematical formulations describing treatment processes are empirical.

An example empirical relationship is one dealing with disinfection. This process can be approximated by

$$y^n t_p = A \tag{9}$$

where y is the concentration of the disinfectant (e.g., chlorine), n is a measure of the order of the reaction, t_p is the time required to effect a constant percentage kill of organisms, and A is a constant depending on the type of organism to be killed. Again, the time factor can be expressed in terms of plant capacity and flow rate; therefore

$$y = \left(\frac{A}{C_p}\right)^{1/n} Q^{1/n}$$

Since y is the concentration of chlorine, it can be expressed as lbs per day per MGD or dollars per day per MGD; therefore, the cost of chlorination, C_c is

$$C_c = A'Q^{2.16}$$

when n = .86. While this equation indicates a non-linear relationship between chemical costs and output, it oversimplifies the process of chlorination and it is by no means representative of the other chemical processes. For example, the above relationship does not include the effects of temperature or pH on the disinfection process, yet they both influence the concentration and quantity of chlorine required to meet the safety standards for drinking water. In addition, pH is influenced by the application of other chemicals used in the treatment process.

Further, given a set of output standards, the application of chemicals in the treatment process depends on the quality of the raw water input. From an overall operational standpoint, the rate of chemical application is not likely to be directly proportional to the quality of the raw water input. Once the effect of the input water quality has been assessed, there is some trade-off between time and chemicals as indicated by the chlorination relationship, but this trade-off appears to be achieved only in the lower ranges of output. Thus, there is little theoretical or practical evidence which would indicate what the form of the production cost function should be.

Data Collection and Analysis

The factors of production at the Central Water Filtration Plant are chlorine, aluminum sulfate, ferrous sulfate, lime, caustic, fluoride, ammonia, and energy for low lift pumping. Daily chemical costs per MG produced are calculated and recorded at the plant as is power consumption. Multiplying daily production levels by unit chemical cost and adding the product of kilowatts consumed times unit energy cost results in the daily variable operating costs. Carbon is used in the treatment process; however, it is used only as conditions warrant. Since carbon is used only about half of the time, and it usually is not required during the summer peak demand periods, this factor was excluded, and the days when carbon was used were not included in the regression analysis. Factor prices are listed in Table V-1. One hundred forty-seven daily observations were used in the regression analysis. Mean production was 686 MGD. The standard deviation was 110 MGD. The minimum production level was 293 MGD, and the maximum level was 1,120 MGD, resulting in a production range of 822 MGD. Mean variable operating costs per day were $3,790, and the mean chemical and power costs were $3,150 and $638, respectively.

Daily chemical costs for the Evanston plant were determined much as those for the Central Water Filtration Plant. Chemicals used in the treatment process are ammonia, chlorine, fluoride, aluminum sulfate, and carbon. The recorded daily dosages were multiplied by their unit cost and then summed to obtain the daily total chemical cost. However, power consumption for low lift pumping presented a somewhat different problem. The only data that were available were the hours a particular pump operated during the day, monthly power consumption, and power costs. To arrive at the daily power consumption, the monthly power consumed was prorated among the pumps according to their rated capacity and to the number of hours they were in service during the month. The head against which each pump operated was about the same and remained constant for the period of investigation. The allocative scheme appeared to work quite well, and the unit cost per hour of operation for each pump was thereby established. Multiplying these values by the appropriate hours each pump was in operation resulted in daily power cost. By adding the chemical cost and the power cost, daily variable operating costs were determined. One hundred twenty-five observations were used. Mean output was 27.8 MGD with a standard deviation of 5.60 MGD. The minimum output was 18.6 MGD, and the maximum was 49.2 MGD. This resulted in a range of 30.6 MGD. Mean variable operating cost was $211, and its standard deviation was $54.23. Mean chemical cost was $166 and mean power cost was $44.50.

Cost Functions

Several different functions were tested using least-squares linear regression on cost data from Central and Evanston.

Based on the F statistic all but three forms were rejected. The surviving forms were:

$$TC = A + BQ$$
$$TC = AQ^B$$
$$TC = A + BQ^2$$

The correlation coefficient for the first and third forms were always greater than for the second form, the log transformation. Consequently, the second form was dropped from consideration; however, it is interesting to note that the exponent, B, in each case, was only slightly greater than one (i.e., $B_{cf} = 1.10$, $B_{cv} = 1.14$). Finally, the value of the A term in the quadratic expression is quite large in comparison to that of the linear model. For Evanston, $A_{ev} = 11.6$ for the linear form, while $A_{ev} = 121.0$ for the quadratic. The same magnitude of difference was observed between the two models for the Central Water Filtration Plant. This along with the lack of other support was the basis for rejecting the quadratic form.

For each of the above regression analyses, the F statistic was not significant, the Kolmogorov-Smirnov statistic was not significant (i.e., the residuals can be assumed to be normally distributed), there was no correlation between the residuals and the independent variable, but the Durban-Watson statistics were significant as expected (see Chapter IV). After serial correlation was removed, the treatment cost functions for Evanston and Central were:

$$TC_{ev} = 11.6 + 7.18\ Q$$

$$TC_{cf} = 19.6 + 5.37\ Q$$

The correlation coefficient for TC_{ev} was .82 and that for TC_{cf} was .81.

The confidence interval for TC_i was

$$(A_i + B_iQ_i) \pm (t_{\varepsilon/2}\sigma_\mu)\ \frac{1}{n} + (Q_i - \overline{Q})^2 / \sum_{i=1}^{n} (Q - \overline{Q})^2$$

For example, if Evanston were operating at 50 MGD the 95 percent confidence interval would be

$$\pm 1.96\sigma_\mu\ \frac{1}{125} + (50 - 27.6)/3630$$

where the standard deviation, σ_μ, is computed from the residuals of the lagged model,

$$TC_{ev_i} = r\ TC_{ev_{i-1}} + \alpha + \beta\ (Q_{ev_i} - rQ_{ev_{i-1}}) + \varepsilon_i.$$

After considerable algebraic manipulations and the summation of a geometric series, $\sigma_\mu = \sigma_\varepsilon \sqrt{1/(1 - r^2)}$, where r is the coefficient of the first-order autoregressive scheme (see Chapter IV). In the example, $r = .701$ and $\sigma_\mu = 17.8$; therefore, $\sigma_\mu - 1.40\ \sigma_\varepsilon = 25.0$ (this is less than the standard deviation of the residuals for the un-lagged model). This results in the confidence interval of ± 5.83 which is ± 2% of the predicted treatment cost, $TC_{ev_{50}} = \$371.00$.

If the cost model could include the additional information of TC_{i-1} and Q_{i-1} the confidence interval would be reduced. The inclusion of this information would be useful for predicting cost, but cannot be used to describe the production cost function.

In the next chapter, the transportation and production functions are combined to form an example objective function of a regional cost model. The objective function is minimized by means of a Lagrangian Function and the importance of the production function in the regional cost model is analyzed.

VI. REGIONAL MODEL

Cost Model of a Regional Water Supply System

Variable operating costs for a water supply system composed of one treatment plant and one pumping station (e.g., Evanston's system) can be represented by the general equation:

$$VOC_i = a_i + b_i Q_i + a_j + b_j q_j^2$$

where Q_i is production output and q_j is pumping output. Equality between production and distribution does not imply a system without storage. Rather, it indicates that output is either directly distributed or transported through the distribution system to storage within the system. For a one plant multiple-pumping station system (e.g., Central Filtration Plant and the seven associated pumping stations),

$$VOC_i = a_i + b_i Q_i + \sum_{j=1}^{n} (a_j + b_j q_j^2)$$

where $Q_i = \sum_{j=1}^{n} q_j$ and n = the number of pumping stations.
The regional model for m-treatment plants and n-pumping stations is:

$$RC = \sum_{i=1}^{m} (a_i + b_i Q_i) + \sum_{j=1}^{n} (a_j + b_j q_j^2) \quad .$$

The previous analysis indicated that the quadratic functions adequately represented the variable transportation costs (see Chapter IV) and that a linear model adequately represented production costs—power and chemicals (see Chapter V). However, since the most significant economic characteristic of the water supply industry is declining average costs, a further test as to whether the selected functions model the economic behavior of the industry is to determine the slope of the average variable cost curve for each plant and pumping station. If all curves are negatively sloped throughout the range of operation, any combination of these functions would result in declining average costs without even considering fixed and fixed variable costs. The addition of fixed costs is considered later.

The average variable treatment costs, for the general case, is

$$AVTC_i = \frac{a_i}{Q_i} + \frac{b_i Q_i}{Q_i} = \frac{a_i}{Q_i} + b_i$$

The slope of this function is

$$\partial AVTC_i / \partial Q_i = - a_i / Q_i^2 \quad .$$

The slope will always be negative (for the linear model) if $a_i > 0$ since Q_i is squared and, in any event, is always greater than or equal to zero. Further, there is no reason to believe that a_i should ever be less than zero; at zero output, variable costs should be equal to or greater than zero.

The average variable power costs can be represented by

$$AVPC_j = a_j / Q_j + (b_j Q_j^2) / Q_j = (a_j / Q_j) + Q_j b_j$$

and the slope of this function is

$$\partial AVPC_j / \partial Q_j = -a_j / Q_j^2 + b_j$$

The same arguments as were given for the production function apply to the term a_j / Q_j^2. Consequently, this term will always be negative. However, b_j will be positive if the results, both theoretical and empirical, from the analysis of Evanston, Lake View, and Thomas Jefferson can be extended to the general case. The sign of the slope for pumping costs varies with the level of operation (see Table VI-1).

TABLE VI-1

SLOPES OF THE AVERAGE COST CURVES

Facility Treatment Plant:	b	$\frac{-a}{Q_{min}^2}$	$\frac{-a}{Q_m^2}$	$\frac{-a}{Q_{max}^2}$	Q_{min}	Slope at Q_m	Q_{max}
Central		-.000227	-.000047	-.0000137	-.000227	-.000047	-.0000137
Evanston		-.0335	-.0150	-.00479	-.0335	-.0150	-.00479
Pumping Station:							
Evanston	.102	-.237	-.0902	-.0397	-.135	+.0118	+.0623
Lake View	.0551	-.103	-.0510	-.0242	-.0480	+.00410	+.0309
Thomas Jefferson	.0408	-.0884	-.0330	-.0153	-.0476	+.00776	+.0255

Combining the average variable treatment and power cost for Evanston results in

$$AVOC_{ev} = 11.6/Q_{ev} + 7.18 + 76.9/Q_{ev} + .102\ Q_{ev}$$

and the slope of this function is

$$\partial AVOC_{ev}/\partial Q_{ev} = -11.6/Q_{ev}^2 - 76.9/Q_{ev}^2 + .102$$

The average variable costs decrease from $Q_{min_{ev}}$ to $Q_{ev} = 29.4$. Beyond 29.4 MGD, the average costs increase. However, fixed operating costs (i.e., labor, lighting, heating, etc.) are not included in the example functions or the values listed in Table VI-1. Such costs, when added to a_i, would increase a_i from eight to nine hundred percent. This increase would be more than sufficient to cause the slope of the average operating cost function to be negative throughout the range of operation.

Marginal Costs

The marginal cost of an additional amount of output (production or transportation) is the first derivative of the particular cost function. The general form of the marginal cost function for the exemplary set of facilities is $MC_j = 2b_{qj}$ for transportation costs and $MC_i = b_i$ for treatment costs. The second derivative, the slope of the marginal cost function, is $2b_j$ or zero.

In all cases, the slope of the marginal cost function is positive and constant for transportation and zero for production. The marginal cost of transportation increases at a constant rate with increases in output while the marginal cost of production remains constant with increases in output. However, transportation and production cost functions combine to form the cost function for a system; and therefore, the combined marginal cost function indicates increasing marginal cost with increasing output.

Conditions a and b (page III-5), which partially define the minimum cost solution for the multiple-plant firm, state: a) if two or more plants (systems) are operated simultaneously, the rates of output in all plants (systems) must be such as to equate their marginal costs and b) no plant (system) should be kept idle if its marginal costs at zero output are lower than the marginal costs of any other plant (system) at its actual rate of production. Since the marginal costs for each test element and example cost function can be made arbitrarily low by selecting a correspondingly small output condition b is not relevant. Obviously, if the output of a particular element is insignificant, it can be eliminated on that basis. The remaining three conditions (page III-5) can be ignored because no system will be operated at its minimum point of marginal costs and no test element has a negative marginal costs function in the feasible range of operations. Therefore, the equality of marginal costs is the only relevant condition for minimization.

Constrained Minimization

Again, the regional cost model is

$$RC = \sum_{i=1}^{m} (a_i + b_i Q_i) + \sum_{i=1}^{n} (a_j + b_j q_j^2)$$

where Q_i is production output of the i^{th} plant and q_j is output from the j^{th} pumping station. To this, two types of constraints need to be added. The first type of constraint is one to insure that the demand for water throughout the region is satisfied. Simply, it is $TD = \sum_{i=1}^{m} Q_i$ or $TD = \sum_{j=1}^{n} q_j$, where TD is total regional demand. The second type of constraint that needs to be incorporated into the model is one that insures that the physical capacity of each element in the regional system is not exceeded, i.e., $Q_i \leq$ maximum capacity, or $q_j \leq$ maximum capacity. The capacity constraints require the introduction of an additional variable--a slack variable. The slack variable permits the capacity constraint to be written as a strict equality constraint (e.g., $Q_i - S_i$ = maximum capacity, where S_i is the slack variable or the amount of unused capacity).

Minimization of the constrained regional model can be achieved by means of a Lagrangian function (5)(31)(48). The relevant Lagrangian function is:

$$L(Q,q,s,\lambda) = \sum_{i=1}^{m} (a_i + b_i Q_i) + \sum_{j=1}^{n} (a_j + b_j q_j^2) + \sum_{k=1}^{P} {}_p g^p(Q,q,S),$$

where λ_p is the Lagrange multiplier of the p^{th} constraint and $g^p(Q,q,S)$ represents the p^{th} constraint. The minimum solution to $L(Q,q,S,\lambda)$ is unique if its Hessian (the matrix of the second order partial derivatives) is positive definite (31). If $L(Q,1,S,\lambda)$ is strictly convex, its Hessian is positive definite (31). Therefore, to insure minimization, $L(Q,q,S,\lambda)$ must be strictly convex.

Strict convexity is defined as: $f(Q + 1) - f(Q) > f(Q) - f(Q - 1)$. And again, q can be directly substituted for Q. Analyzing the general function:

$$a_i + b_i(Q_i + 1)^2 - (a_i + b_iQ_i^2) > a_i + b_iQ_i^2 - [a_i + b_i(Q_i - 1)^2]$$

$$a_i + b_iQ_i^2 + 2b_iQ_i + b_i - a_i - b_iQ_i^2 > a_i + b_iQ_i^2 - a_i - b_iQ_i^2 + 2b_iQ_i - b_i$$

$$b_i > 0$$

The condition that b_i be strictly greater than zero is satisfied by each element in the test system. A similar proof can be given to show that each of the constraints is convex. Finally, the sum of strictly convex and convex functions is strictly convex (31). Since $L(Q,q,S,\lambda)$ is strictly convex, a unique minimum is insured.

To demonstrate that the derivatives are continuous, consider the example first order derivative: $\delta c/\delta Q = 2bQ$. A function defined in $\{Q_o - h < x < Q_o th\}$ is continuous at Q_o if for each $\varepsilon > o$ there is a $\delta, o < \delta \leq h$ for which $|f(Q) - f(Q_o)| < \varepsilon$ if $|Q - Q_o| < \delta$. Assuming $h=1$, $|Q-Q_o|<1$. Further, $|f(Q)-f(Q_o)| = |2bQ-2bQ_o| = |2b|$ $|Q-Q_o|$. If, $|Q-Q_o|$ is selected to be $|Q-Q_o| < \varepsilon/|2b|$, then $|2bQ-2bQ_o| < \varepsilon$. Now, $\delta = \min[1, \varepsilon/|2b|]$ and the definition is satisfied. Since δ is independent of Q_o, the derivative is uniformly continuous (31). Similar tests can be applied to other functions.

Example Solution

To demonstrate the use of the model, a subset of the metropolitan water supply system was analyzed. The Central Filtration Plant and Evanston's treatment plant were selected as the representative supply sources. Lake View and Thomas Jefferson Pumping Stations and Evanston's pumping station were used to represent transportation facilities. In this example the demand for water in these service areas was assumed to be 195 MGD. Besides Lake View and Thomas Jefferson, Central supplies five other pumping stations in the North Tunnel Zone which were assumed to draw 955 MGD. Since the service areas of these five stations were not included in the analysis, 955 MGD remained a constant demand on Central's output.

The Lagrangian function for the two-plant and three-pumping-station model is:

$$\begin{aligned} L(Q,q,S,\lambda) = {} & 19.6 + 5.37\, Q_{cf} + 11.6 + 7.18\, Q_{et} + 99.2 + .00551\, q_{1v}^{2} \\ & + 156 + .0408\, q_{tj}^{2} + 76.9 + .102\, q_{ep}^{2} + \lambda_1(955 + q_{ev}q_{tj} - Q_{cf}) \\ & + \lambda_2(195 - q_{1v} - q_{tj} - q_{ep}) + \lambda_3(q_{ep} - Q_{et}) \\ & + \lambda_4(1500 - Q_{cf} - S_1) + \lambda_5(150 - Q_{et} - S_2) \\ & + \lambda_6(105 - q_{1v} - S_3) + \lambda_7(160 - q_{tj} - S_4) \\ & + \lambda_8(124 - q_{ep} - S_5) \end{aligned}$$

where S_i $i = 1, 2, 3, 4, 5$, are slack variables. The condition for minimization is that the partial derivatives of $L(Q,q,S,\lambda)$ with respect to each variable equal zero. Since:

$$\partial L(Q,q,S,\lambda) \;/\; \partial S_i = \lambda_j$$

$$i = 1, 2, 3, 4, 5, \text{ and}$$
$$j = 4, 5, 6, 7, 8$$

and since the derivatives must equal zero,

$\lambda_j = 0$

$j = 4, 5, 6, 7, 8.$

This reduced the problem to thirteen unknowns and thirteen linear equations. The coefficient matrix is given in Table VI-2.

Table VI-2

COEFFICIENT MATRIX FOR TOTAL VARIABLE COSTS

Q_{cf}	Q_{et}	q_{ev}	q_{tj}	q_{ep}	γ_1	γ_2	γ_3	B
0	0	0	0	0	0	-1.0	0	-5.37
0	0	0	0	0	0	0	-1.0	-7.18
0	0	0.11	0	0	-1.0	1.0	0	0
0	0	0	-0.0816	0	-1.0	1.0	0	0
0	0	0	0	0.204	-1.0	0	1.0	0
0	0	1.0	1.0	1.0	0	0	0	195.
1	0	-1.0	-1.0	0	0	0	0	955.
0	-1.0	0	0	1.0	0	0	0	0

The simultaneous solution of these equations yields the following results:

$Q_{cf} = 1121$
$Q_{et} = 29$
$q_{ev} = 71$
$q_{tj} = 95$
$q_{ep} = 29$

Marginal cost = \$13.10

The constraints are all satisfied: total demand = 71 + 95 + 29 = 195, demand by other pumping stations in the North Tunnel Zone = 1121-71-95 - 955.

To demonstrate the importance of the production function in the regional cost model, the above total demand was allocated using only the transportation cost functions for Evanston, Lake View, and Thomas Jefferson. The coefficient matrix is given in Table VI-3.

Table VI-3

COEFFICIENT MATRIX FOR TRANSPORTATION COSTS

q_{ev}	q_{tj}	q_{ep}	γ	B
.11	0	0	-1.0	0
0	0.0816	0	-1.0	0
0	0	0.204	-1.0	0
1.0	1.0	1.0	0	195.0

The resulting allocation is as follows:

$q_{ev} = 67$

$q_{tj} = 91$

$q_{ep} = 36$

Marginal Cost of Transportation = \$7.40.

The inclusion of the production function causes a 20 percent shift in production at Evanston and increases the output at Lake View by six percent. The economic disadvantage of Evanston's treatment plant relative to the Central Water Filtration Plant (i.e., marginal cost coefficient for Evanston is greater than that for Central) has to be balanced by the marginal costs of transportation. Consequently, both production and transportation cost functions are important in regional allocation and cost minimization.

VII. SUMMARY AND CONCLUSIONS

The water supply facilities of metropolitan areas are usually grouped into separate but adjacent systems that are owned and operated by municipal governments. Allocation of production and transportation (distribution) is determined by the demand contained within municipal boundaries and is thereby geopolitically dictated. When an individual supply system has more than one treatment plant or pumping station, production and transportation are allocated according to the design capacities of plants and pumps and the hydraulic characteristics of the distribution network. If a regional allocative scheme based on municipal boundaries is economically efficient, this efficiency has been achieved largely by chance.

At any given time, some systems will have more capacity than required to meet their needs while other systems will have insufficient capacity. The excess capacity of one system could be used to augment the system that lacks capacity and thereby improve the utilization of treatment, transportation, and storage facilities of both systems. Hydraulic considerations play an important role in determining the use of treatment plants, pumps, reservoirs, and distribution networks; however, in any complex supply system, there exist a great number of ways to combine these facilities in order to meet the demands for water. The problem is to minimize the total cost of providing potable water, given a multiple-plant, regional water supply system. In this report, cost minimization is dealt with only in terms of short-run economic considerations. Optimal operation or utilization of the regional supply system is the central issue and not the development or expansion of the system; but such short-run economic considerations are also an essential ingredient in long-range planning.

The quantitative relationships between costs of both transportation and production and the quantity of water are defined, and a methodology is presented for combining these relationships to create an objective function that describes the variable cost of regional water supply. This function incorporates production and transportation costs as well as the constraints of meeting water demands and capacity limitations. Using established techniques, the objective function is minimized, and marginal costs among service areas are equated.

The nature of the transportation cost function is shown to vary with the period of operation. At a given point in time, when the number of pumps in service is fixed, the cost function is defined by the head-capacity relationship (pump characteristics). When the number of pumps in service is permitted to vary as over the period of a day, the cost function is defined by the system-capacity relationship (network and service characteristics). The daily cost function can be closely approximated by a quadratic function of the form $PC = a + bQ^2$ where a and b are parameters that are estimated by least-squares linear regression from existing operating data. Thus, the marginal cost function is linear and passes through the origin, implying that all pumping facilities whose costs can be represented by this quadratic function should remain in operation over the entire demand range if costs are to be minimized. However, examination of the head-capacity relationship (see Figure IV-10) reveals that, as output is reduced, an increasing amount of energy goes to maintain system pressure. Consequently, at or near zero output, operating costs are not zero; so the use of the quadratic form of the transportation function must be restricted to the upper operating range. The head-capacity relationship serves as the basis for estimating power costs for the transportation of hourly or instantaneous demand. The system-capacity function serves only as a basis for judging whether service conditions can be met. The interrelationship between pumping stations (i.e., $H_L=K(Q_i+Q_j)^n Q_i^{m-n}$) is not significant in estimating daily power costs.

Geometric programming is used to solve the non-linear equations describing the energy requirements for transporting water through delivery tunnels and the distribution network. In this model capacity constraints (e.g., suction lift) are explicitly represented by the appropriate hydraulic equations. A direct output of the geometric programming procedure used is an analysis of the sensitivity of the objective function to small changes in each of the parameters associated with it.

The results of the example analysis indicate that the energy boundary constraints and the total dynamic head constraints were not significant in determining power costs. The head-capacity constraint and the equality-of-demand constraint were the most significant. One practical conclusion resulting from this analysis is that, if additional pumping capacity is to be introduced at either Lake View or Thomas Jefferson Pumping Stations, the least increase in power cost per unit of output would be realized by the placement of additional pumping capacity at Lake View.

The difference between the daily cost model and the instantaneous or peak hourly cost model is a matter of perspective. The daily cost model reflects the operating cost required to meet service conditions while the instantaneous model indicates the cost of delivering a given quantity of water for a particular set of facilities disregarding service conditions.

The power costs to collect and transport raw water to the treatment plant (part of the production costs) are linear with respect to flow rate if the headloss in the collector system is small. For systems that are dependent on ground water, the power cost function for collection would have the same form as that for distribution. Although there is some indication that chemical costs are non-linear with respect to output, the practical operation of water treatment plants indicates that chemicals are commonly applied in direct proportion to the flow rate. Thus, where the power costs function is linear, the best estimation of production costs for purposes of allocation is the function $TC = a+bQ$ where a and b are estimated by least-squares linear regression from operating data. For operating purposes, however, the best predictor of production costs is the auto-regressive model $TC_i = a' + b' (Q_i - rQ_{i-1}$. The confidence interval for this function is smaller than that for the model $TC = a + bQ$.

The regional daily cost model can deal efficiently with large sets of pumping facilities and treatment plants. Both production and transportation costs significantly contribute to effectuating marginal cost allocation. When the marginal costs of production are constant, the difference between the marginal costs of the various treatment plants must be compensated by the level of marginal costs at which the associated pumping facilities are operated.

While marginal cost allocation indicates possible changes in the allocation of production and transportation, for a given demand, the cost savings occurring from such changes will be small in comparison to the total fixed and fixed operating costs. However, since marginal cost analysis produces the most efficient ratio of outputs given total demand, this knowledge can lead to better use of the existing capacity which could result in the delay or avoidance of additional capitalization. It can also serve as a tool for planning, especially in the analysis of operating efficiency for proposed capital additions or expansions of water systems.

Appendix A

LIST OF SYMBOLS

a	computed regression coefficient (intercept)
α	general regression coefficient (intercept)
AC	estimated average variable operating cost in dollars per MGD
APC	estimated average variable pumping costs in dollars per MGD
ATC	estimated average variable treatment costs in dollars per MGD
b	computed regression coefficient (slope)
β	general regression coefficient (slope)
cf	Chicago's Central Water Filtration Plant
D	Kolmogorov-Smirnov statistic
d_L, d_u	Durbin-Watson statistics
dp	Des Plaines' Water Treatment Plant
E	efficiency coefficient
et	Evanston's water treatment plant
ep	Evanston's pumping station
F	F statistic, $[(\beta/S)^2]$
H	water delivery pressure in feet of water
kwh	kilowatt hours
λ	Lagrange multiplier
$\lambda(\delta)$	$\Sigma\ \delta_i$
lv	Chicago's Lake View Pumping Station
m	mean of sample variable
max	maximum observation of sample variable
MC	estimated variable operating marginal cost in dollars per MG
MG	million gallons
MGD	million gallons per day

min	minimum observation of sample variable
PC	estimated variable pumping or power costs in dollars
Q	production in million gallons per day
q	transportation in million gallons per day
r	sample correlation coefficient or coefficient for the first-order auto-regressive scheme
S	standard error of the estimate
t	t statistic, $[(\beta - 0)/S]$
TC	estimated variable treatment costs in dollars
TD	total regional demand for water
tj	Chicago's Thomas Jefferson Pumping Station

Appendix B

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