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MATHEMATICAL MODELS FOR USE IN PLANNING REGIONAL WATER RESOURCES AND ENERGY SYSTEMS

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ABSTRACT

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Existing and projected energy facilities will, in the near future, place major demands on the country's water resources. These demands compete with many other uses of the resources, including municipal and industrial uses, navigation, irrigation, and water quality maintenance. The possible development of coal conversion facilities presents another potential water demand. Complex public sector problems such as: 1) the extent and development of coal conversion capacity, 2) interbasin transfer of water, 3) cooling technologies for large energy facilities, 4) diversion of Lake Michigan water, and 5) allowable withdrawal and consumptive uses of river water, all arise from the interlocking nature of the water resources-energy system.

Although mathematical models cannot solve these problems directly, they can be useful in gaining insight into major issues associated with policy alternatives. With the aid of such models, quantitative trends such as costs and water development patterns associated with each decision alternative can be more readily identified.

In this report, mathematical models are presented for use in planning a regional allocation of water for energy facilities as well as for other water uses. These models include components for the interrelated water and energy subsystems. The use of these models in conjunction with other existing models in order to provide a better picture of the overall system is discussed. Since the models use widely available computer codes, they are practical and easy to utilize. Example applications are presented, with a discussion of computational results.

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CHAPTER 1: SUMMARY, DISCUSSION, AND CONCLUSIONS

1.1 INTRODUCTION

It has become clear that the United States must develop new ways to meet its growing energy needs. Pollution control regulations have led to increased demands for clean-burning fuels such as petroleum and natural gas. The domestic reserves of these fuels, however, will become increasingly inadequate, and the cost and reliability of foreign supplies is uncertain.

Much attention is currently being given to utilizing the vast coal reserves in the Ohio River Basin area. Since much of this coal has a high sulfur content, major research efforts are underway to examine alternative technologies for converting it to clean-burning gaseous, liquid, and solid fuels. In addition to coal, these processes require large supplies of water for cooling and to provide the hydrogen used in forming the synthetic fuels. This project has produced two staff reports containing background information on the various processes and their water requirements (see Stout, 1974).

In addition to vast coal resources, the Ohio River Basin and nearby regions offer abundant water resources. There are, however, other existing and potential demands for these resources, including municipal and industrial uses, electricity generation, navigation, irrigation, and the maintenance of water quality. Currently, water withdrawals for electricity generation account for the largest percentage by far of the total water withdrawals in the Ohio River Basin. Because the development of a large-scale coal-conversion industry would exert a major demand for water, it may necessitate new policies and regulatory programs at the state level for managing water resources and allocating them among the various uses. For example, legislation to establish a water permit system was introduced in the Illinois General Assembly (House Bill 1786) in 1975; this legislation would also establish a Water Resource Authority to plan and regulate the use of the state's water resources.

The water-resources and energy systems of a region are interlocking in nature because of the major water demands for energy facilities. The study described here has examined some of the relationships and interactions between those systems. The objectives of the study have been:

(1) To develop a mathematical model of a regional water-resources system and the interlocking energy-production system;
(2) To develop an optimization procedure for use in planning a regional allocation pattern for supplying water to major energy facilities as well as for other water needs;
(3) To demonstrate the impact that the development of a large energy industry could have on the allocation of water resources in a large region such as the Ohio River Basin.

The planning model is intended to serve as a tool for evaluating some aspects of the major policy issues in a large region such as a state. Examples of those issues are: (1) Should a large coal-conversion industry be developed and, if so, to what extent? (2) Should high-quality groundwater be used for nonmunicipal purposes? (3) Should interbasin transfers of water be allowed? (4) Which cooling technologies should be used for large energy facilities? (5) Should additional Lake Michigan water be diverted to the state of Illinois? (6) What water withdrawals and consumptive use should be allowed for the major rivers?

Issues such as these are complex public-sector planning problems, and they cannot, of course, be resolved directly by the use of a mathematical model. Quantitative screening models, however, can be very useful in gaining insights into the major issues associated with policy alternatives. Furthermore, quantitative analysis techniques can be used to evaluate specific planning alternatives. Although judgment and preliminary calculations can be used--and in many cases are necessary--to evaluate a decision alternative, in many instances a practical mathematical model can be used to look at more of the system components, giving a clearer picture of quantitative trends (e.g., cost and water development patterns).

The mathematical model (outlined below and in the next chapter) was developed with the above criteria in mind. The basic model was designed as a preliminary screening model capable of providing quantitative estimates of some of the important trends and trade-offs associated with major policy issues. The model can be evaluated using linear programming, and the necessary computer programs are widely available. Also, the method is practical to use in terms of computer costs and man-hour requirements, a characteristic that is essential if it is to be used frequently within an ongoing planning activity.

The model was applied to example data for the state of Illinois which is typical of a large region in the Ohio River Basin. This application demonstrates the practicality and utility of the screening model for examining the water resources and energy systems for such a region.

1.2 OVERVIEW OF LINEAR PROGRAMMING MODEL

A number of mathematical programming models have been developed for use in planning the allocation of a region's water resources (e.g., Young and Pisano, 1970; Texas Water Development Board, 1970, 1971, and 1974; King *et al*, 1972; and Reynolds *et al*, 1973).

Also, many mathematical models have been developed for use in planning electrical systems (e.g., Gately, 1970; Anderson, 1972; Farrar and Woodruff, 1973; and Sawey, 1973). The screening model presented here deals with the water resources system and the pipeline gas and electrical systems.

The linear programming model contains the following components:

(1) Water source points, which include the major river segments, groundwater aquifers, and potential reservoir sites,
(2) Coal source points,
(3) Municipal and industrial water demand (MID) points (excluding energy industry demand points), which also take into account rural demands,
(4) Candidate electrical energy supply points, which are also potential water and coal demand points,
(5) Potential pipeline gas supply points, which are also potential water and coal demand points,
(6) Electrical energy demand points,
(7) Pipeline gas demand points, and
(8) Municipal and industrial coal demand points (excluding energy industry demand points).

The fundamental interrelationships between the water-resources and energy systems are illustrated in Figure 1.1. As indicated in the figure, the model is designed so that subregions, such as counties, are used as municipal and industrial water demand points and as electrical energy and pipeline gas supply points. Thus, locations are not chosen for individual power plants; rather, power plant capacities are located in the subregions. Also, the figure does not indicate the return flows of water to the source points, although these flows are considered in the mathematical model.

The model presented in this report is static in the sense that one planning period (e.g., 25 years) is used. A multiperiod formulation would require a straightforward extension, a possibility discussed in Appendix A.

The decision variables, listed in Chaper 2 represent (1) the amounts of water, coal, electricity, and pipeline gas transported from supply points to demand points, and (2) the electricity and gas generating capacities located at each supply point. Electric-plant capacity is subdivided into nuclear and coal-fired plant capacities because the fuel and water requirements are different.

A complete list of the model's parameters is also given in Chapter 2. They specify (1) the amounts of coal and water resources available for development at the various source points and (2) the demand levels for additional water, pipeline gas, and electricity. Other paramenters are used to match the levels of the resources (inputs) and the products (outputs) at the energy facilities. For this purpose, it is assumed in the basic model that a cooling technology is prespecified for each subregion (supply point). This assumption is not necessary, however; and the model can be modified to allow for alternative cooling technologies, as discussed in Chapter 3.

The objective function, which is to be minimized, is the sum of the site-dependent supply costs of water and the transportation costs for water, coal, electricity, and gas. The supply costs differ for river, groundwater, and reservoir sources because the facilities required are different (e.g., dams, wells) and because different unit processes are necessary for water treatment. The cost factors are described in more detail in Chapter 2.

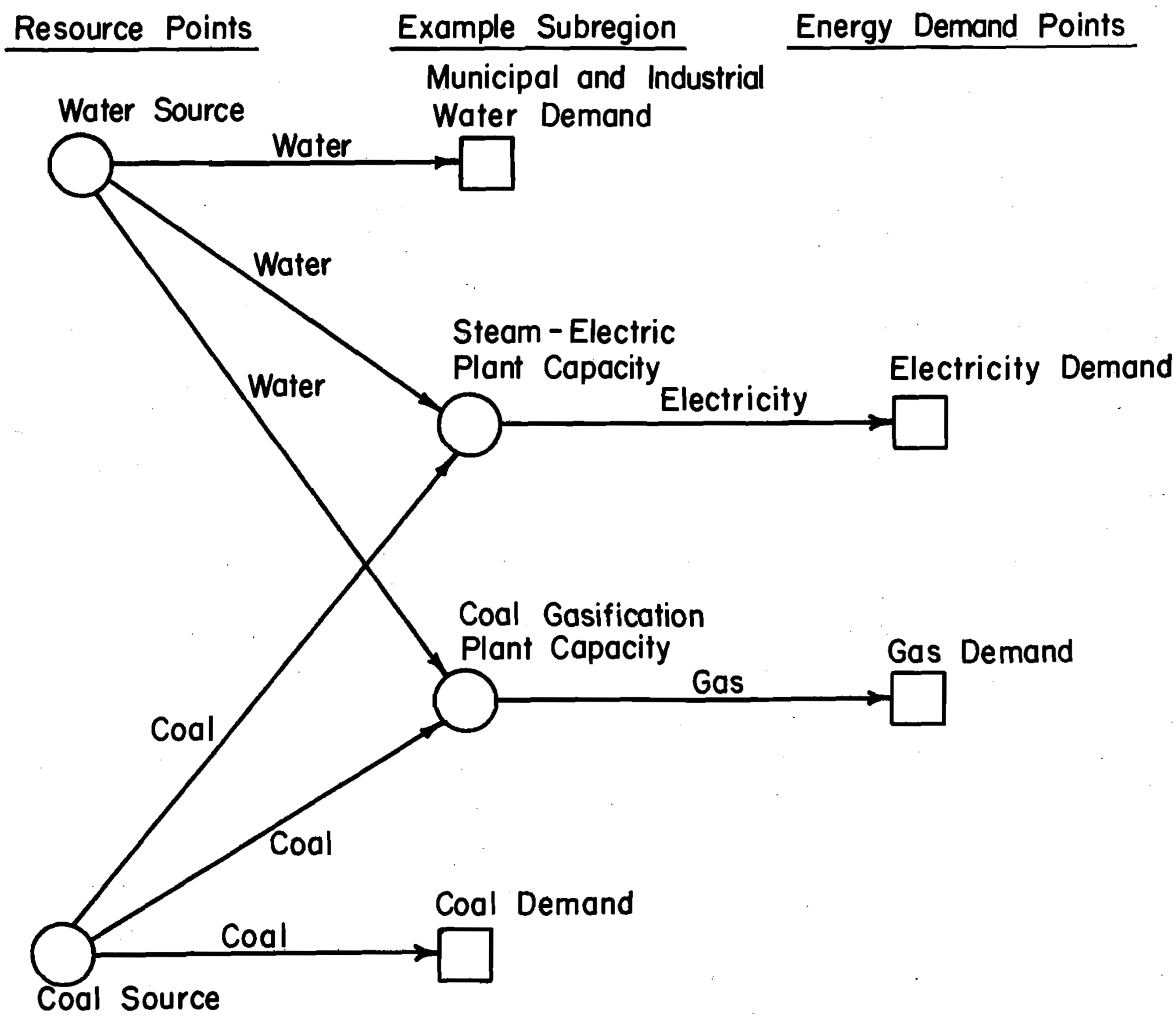

Figure 1.1 Some of the Interrelationships Within a Water Resources-Energy System

For the purpose of model development, the cost of mining and pretreating coal has been assumed to be site-independent. This assumption implies that the costs are fixed and therefore nonoptimizable. Although in this study mining costs are not considered, in an actual application of the model, costs of this type could be added to the cost of transporting coal from each of the various source points.

A similar assumption was made about the costs of constructing and operating power plants. These costs are very important for steam-electric plants when the mix of different plant types is planned. Mathematical programming models have also been used to help solve that problem (see the references cited above). Those models, however, often do not consider the effect of plant location, since many of the major costs are site-independent. Because the model presented here does consider some of the factors dependent on location (e.g., water supply and transmission costs), it could be used in conjunction with those other planning models to provide a more complete evaluation.

Likewise, the costs of constructing and operating large coal gasification (or coal conversion) plants are largely site-independent and will be evaluated when they are planned. The model presented here could, however, be used to take into account any of the site-dependent costs which apply uniformly to each unit of plant capacity located in a given subregion.

1.3 DISCUSSION AND CONCLUSIONS

Throughout the study the main challenge was to develop a model that would capture as much realism as possible and yet stay within the bounds of practicality. One main approach to this end was to use subregions, such as counties, as "sites" for locating energy-facility capacities rather than individual plants. This approach has intuitive appeal and is consistent with current energy-systems planning. Siting studies are typically carried out hierarchically, the first step being to screen subregions within a larger region and the second step being to examine in detail specific sites within the attractive subregions. A preliminary screening model, used in the first step of such a procedure, would be used in conjunction with other planning models and analysis methods.

This approach (the screening of subregions) is very important because it decreases the required number of mathematical constraints, greatly reducing computer running time and the difficulty in interpreting solutions. It also lowers the number of variables, reducing data preparation requirements. When applied to example data for the entire state of Illinois, roughly 400 constraints and 4,000 variables were required using one model variation, and 500 constraints and 7,500 variables were required using another variation. A model of this size is practical, since very simple utility computer programs can be used to generate most of the program data base and since the computer expenses are affordable (from $20 to $100 per example run, with the more costly runs being for parameterization purposes).

As mentioned above, the screening model presented here is designed for use in conjunction with other planning methods and models. Population projections and economic models would be needed to estimate the demand levels for water, electricity, gas, and other coal conversion products. The desired mix of nuclear and fossil-fuel steam-electric plants could be temporarily assumed, based on utility plans made with the aid of mathematical programming models (see, for example, Gately, 1970, and Anderson, 1972). The screening model could then be used along with planning judgement to examine issues on a regional basis, such as for an entire state.

Specific regional patterns could be selected and evaluated in more detail using a simulation model for the entire region. In addition, each subsystem (water, pipeline gas, and electricity) could be evaluated individually. For example, the electrical utilities could reevaluate their models for selected regional plans. Finally, more detailed analyses could be made on a subregional basis to evaluate individual sites for facilities such as power plants, coal conversion plants, reservoirs, and groundwater well fields and to evaluate individual transmission routes for electricity and pipeline routes for water and gas. Optimization and simulation models might be used for any of these analyses. For example, the thermal pollution aspects of specific sites can be evaluated using a model such as the one presented by Shiers and Marks (1973). As another example, if cooling towers are selected for a given power plant, then mathematical models could be used to determine the minimum cost design (Croley *et al.*, 1975). After using the screening model, however, detailed data for individual sites and routes and refined analysis steps would be needed only for the alternatives selected.

As discussed in the introductory section, the planning model has been designed as a means of gaining insights into the trends and trade-offs associated with major policy issues. The objective function of the model addresses the region-wide economic-efficiency objective. The model, however, is designed for performing the most basic type of multiobjective analyses by examining different scenarios and by using sensitivity and parameterization techniques.

As examples, consider the policy issues listed in Section 1.1. By varying the required pipeline gas production incrementally from one extreme--complete importation--to the other--a high level of exportation, the impact on the water resources and coal development can be illustrated. The incremental solutions would indicate the increases in the regional cost (for the cost factors included in the model) and would illustrate the secondary effects on water resources development, municipal water supply, and the electrical power system. In a similar way, the model can be used to study the effects of varying the allowable interbasin transfers, and also the allowable Lake Michigan diversion, and allowable withdrawals from rivers, to examine the sensitivity of the solutions to different cooling water regulations.

The basic linear programming (LP) model and the two variations used for the example application to Illinois are described in Chapter 2. The example solutions illustrate the utility of the model for screening a large region to identify subregions where it may be desirable to locate large energy facilities. Furthermore, a sensitivity analysis can be used to identify the changes in the solution which would result from small changes in parameter values. These changes specify different planning alternatives which can be examined in more detail. Also, example parameterization runs were used to demonstrate the utility of the model for examining the effects of changing parameter values over a wide range.

The Illinois application highlights the major impact that the development of a large energy industry could have on the allocation of the water resources of a large region. In all cases, the cooling water costs represented a very large percentage of the total system costs. Furthermore, a parameterization run that incrementally increased the demand levels for pipeline gas showed that the water allocation patterns changed significantly at different levels. This example underscores the model's capability for examining the secondary impacts that may result from a large change in one component of the interlocking water-resources and energy systems.

The same example is also used to illustrate how the dual variables (or shadow prices) give the change in the objective function which would result from a unit change in the requirement imposed by each constraint. If the demand level for pipeline gas at one load center is increased by a small amount, then the corresponding dual variable gives the unit cost of this change. The unit cost takes into account the secondary effects on all of the components considered in the model as well as the costs directly related to supplying the pipeline gas. A marginal cost curve, or supply curve, can be plotted using values of this dual variable for all of the different solutions obtained from the parameterization run. The example curve given in Section 2.3.3 shows significant increases in the marginal cost of supplying large amounts of pipeline gas from coal conversion.

Chapter 3 discusses two extensions of the model which consider alternative cooling technologies in each subregion. One extension considers a fixed water requirement for each alternative technology; the resulting modified linear programming model has been demonstrated for the state of Illinois. Since different mixes of cooling systems were found under different conditions, alternative cooling systems, if they are to be considered, should be included in the preliminary screening model. The second extension allows the water requirement to vary for each of the different cooling alternatives. The resulting mathematical program has nonlinear constraints but can be evaluated using separable programming. This approach is demonstrated for a 14-county region in western Illinois and it is shown that good solutions can be found within a reasonable amount of computer time.

Since this type of planning model is designed for frequent use, the possibility of using a more efficient netwok algorithm for solving the mathematical program was examined; the approach is discussed in Chapter 4. With one modification, the first variation of the model for the state of Illinois was written as a transshipment problem using the concept of equivalent flows to express water, pipeline gas, and electricity as a single flow commodity (in water units). The transshipment formulation was evaluated for the Illinois example using the out-of-kilter algorithm, and it was observed that network algorithms are much more efficient than the simplex algorithm for solving the given formulation. Additional research is underway in this area since more efficient solution methods increase the practical limit on the problem size. This capability may be important for disaggregating some of the supply or demand points, for considering large regions, or for modifying the model to account for several time periods. The latter modification is straightforward and is discussed in Appendix A.

Thus, this study has developed a basic planning model with several variations for use in examining the interlocking water-resources and energy systems of a large region. The example applications demonstrate the practicality of the method and illustrate the impact of the energy system on the water-resources system. It should be emphasized that the solutions, which were obtained for demonstration purposes, reflect many assumptions and therefore should not be considered as suggesting specific recommendations for the state of Illinois. Future research, however, will be aimed at using the model as part of a program to investigate some of the planning issues in the Great Lakes area and in the Ohio River Basin. The existing model deals only implicitly with water quality constraints for the waterways. For example, minimal flow requirements are implied by the withdrawal and consumption limits for rivers. The model does not, however, consider explicit water quality standards, and it is recommended that future research address that factor as well.

CHAPTER 2: BASIC LINEAR PROGRAMMING MODEL AND EXAMPLE APPLICATION TO ILLINOIS

2.1 INTRODUCTION

This chapter describes a general linear programming (LP) model of the regional water resources and energy systems and illustrates its use through example applications to data for the state of Illinois. The general LP model would most likely be modified in a unique way for a specific application, as demonstrated by the examples.

2.2 MODEL DESCRIPTION

2.2.1 Notation

The notation used in this chapter is given below:

a. *Indices*

i = groundwater supply point
j = riverwater supply segment
k = reservoir water supply point
m = subregion, such as a county
a = coal supply point
p = pipeline gas demand point
q = electricity demand point

b. *Decision Variables*

d^M_{im}, d^G_{im}, d^E_{im} = amount of water sent from groundwater source i to subregion m for municipal and industrial demands (MID), gasification, and electrical power generation, respectively (gallons per minute, GPM.)

r^M_{km}, r^G_{km}, r^E_{km} = amount of water sent from reservoir k to subregion m for MID, gasification, and electrical power generation, respectively (GPM).

$w^M_{jm}, w^G_{jm}, w^E_{jm}$ = amount of water sent from river segment j to subregion m for MID, gasification, and electrical power generation, respectively (GPM).

Groups of these variables can be omitted in cases where subregion m has no MID demand or where long-distance shipments are judged impractical. All of these water allocations are incremental, since they are in addition to existing allocations.

s_m = coal-gasification plant capacity located in subregion m. This capacity is expressed in terms of the corresponding number of unit-size gasification plants (the unit-size is given by the constant GP).

t_m = coal-fired steam-electric plant capacity located in subregion m, expressed as number of unit-size plants.

t^N_m = nuclear plant capacity located in subregion m, expressed as number of unit-size plants.

These variables are continuous, implying that new plant capacities can take any nonzero value. This characteristic is acceptable since the model is designed to locate capacity, not individual plants, and since it is designed for preliminary screening purposes. For example, the subregions that are selected by the model for developing coal-gasification plant capacities might be ranked by the total plant capacity to give an indication of their relative desirability.

$x^M_{om}, x^G_{om}, x^E_{om}$ = coal transported from source h to subregion m for municipal and industrial uses, gasification, and electrical generation, respectively (tons/day).

g_{mp} = pipeline gas sent from subregion m to demand point p (standard cubic feet per day, SCFD).

e_{mq} = electricity sent from subregion m to demand point q (megawatts, MW).

c. *Constants*

D_i = amount of water available from groundwater source i (GPM).

W_m = amount of water available from river segment j (GPM).

R_k = amount of water available from reservoir k (GPM).

RG_{mj} = amount of water returned to river segment j from a unit-size coal-gasification plant in subregion m (GPM).

RE_{mj} = amount of water returned to river segment j from a unit-size coal-fired electric plant in subregion m (GPM).

RN_{mj} = amount of water returned to river segment j from a unit-size nuclear power plant in subregion m (GPM).

MID_m = amount of water required for municipal and industrial purposes in subregion m (GPM).

WG = amount of water required by a unit-size coal-gasification plant (GPM).

WE = amount of water required by a unit-size coal-fired steam-electric plant (GPM).

WEN = amount of water required by a unit-size nuclear power plant (GPM).

X_o = amount of coal available at point h (tons/day).

$MICD_m$ = amount of coal required for municipal and power industrial purposes (other than electrical power generation) in subregion m (tons/day).

CG = amount of coal required by a unit-size coal-gasification plant (tons/day).

CE = amount of coal required by a unit-size steam-electric plant (tons/day).

GP = amount of gas produced by a unit-size coal-gasification plant (SCFD).

GD_p = amount of gas demanded at demand point p (SCFD)

EP = amount of electricity produced by a unit-size steam-electric plant (MW).

ED_q = amount of electricity demanded at point q during peak periods (MW).

B_{mq} = percentage of power lost in transmission of electricity between county m and demand point q.

All supply and demand constants reflect limits on the incremental use of resources and on incremental demands for new energy facilities and additional water allocations.

2.2.2 Objective Function

The cost items that should be used with the model are those which depend most on the resources development pattern used to meet demands for water, pipeline gas, and electricity. These costs are the incremental costs that are optimizable; costs which are either sunk costs or fixed costs should not be considered, since they do not depend on the pattern of development.

a. *Cost Items.* The cost items that should be considered are listed in Table 2.1. It may be possible, however, in a particular application of the model to evaluate the cost data and eliminate those possibilities that are clearly impractical or dominated by other options. For the Illinois example, it was possible

Table 2.1 - Cost Items

Variable	Potential Cost Items
d^M_{im}, d^G_{im}, d^E_{im}	well draining, operation and maintenance; pumping, piping, and water treatment
r^M_{km}, r^G_{km}, r^E_{km}	dam construction, operation and maintenance, piping, and water treatment
w^M_{jm}, w^G_{jm}, w^E_{jm}	piping and water treatment
s_m, t_m, t^N_m	site dependent energy facility costs, such as land costs construction costs and cooling systems costs
g_{mp}	transmission pipeline construction, operation and maintenance
e_{mq}	transmission line construction (operation and maintenance assumed negligible)
x^M_{hm}, x^G_{hm}, x^E_{hm}	mining, barge, rail, or slurry transportation and pretreatment

to show that the cost of piping water in the amount needed for wet cooling towers would be less than the cost of transporting coal to the water supply point even if it were necessary to send the gas or electricity produced back to the water supply point (see Appendix B). Thus, a cost minimizing solution would never include coal shipments, and the coal shipment variables could be eliminated from the mathematical programming model. As discussed in the following description of the constraints, this result allowed several constraint sets to be combined. Although this result will not necessarily hold in other cases (for example, when once-through cooling is allowed for power plants), in any application the data should be evaluated to determine whether the size of the mathematical programming problem can be reduced.

For the Illinois examples, annual costs were used based on an 8% discount rate.* Also, this example study did not consider the existing transmission lines for electricity, pipelines for gas, nor reservoirs, wells, treatment facilities, and pipelines for water. This simplification is realistic to a large degree because most of the existing facilities are either used at capacity or committed for future use. As mentioned throughout this chapter, the model considers the major incremental product demands which must be met primarily by new, large-scale resources development which usually occurs across subregional boundaries. Local supplies of water, for example, are netted out in calculating the water demands that must be met by major new facilities. The model is designed to consider the variable costs of these major facilities. In an application of the model, however, any existing facilities which do have uncommitted capacity could be readily taken into account. The use of such facilities (for any category of the variable) could be considered as a variable and the cost coefficient would include the operation and maintenance costs but not the construction costs. An example, although unlikely, is the case in which a section of a large, interstate natural gas pipeline would be made available for intrastate use.

For demonstration purposes in the example study, none of the costs associated with coal supplies were considered; this simplification is realistic only if the mining and coal treatment costs are fixed (independent of the coal source). Also, the costs associated with constructing power facilities were assumed to be site-independent. In an application where any of these costs vary significantly, unit

*Costs were expressed in 1973 dollars using the Engineering News Record Construction Cost Index.

costs can be readily incorporated for the s_m, t_m, and t_m^N variables. All of the cost data used for the Illinois example are described in Singh *et al.* (1972), Prelin and McGough (1970), Mutschler *et al.* (1973), Wasp and Thompson (1973), and the Federal Power Commission (1971, 1974).

If any of these cost factors are included, the model must be used carefully to avoid giving an inappropriate advantage to one type of energy facility. For example, if coal shipment costs were considered but nuclear fuel costs were not, nuclear plants would appear to be advantageous relative to coal-fired plants. In this case, the total nuclear capacity should be fixed (or parameterized). In this way, the model presented here could be used to examine the location aspects of a particular level of nuclear plant capacity while another study (perhaps using other mathematical programming models) could examine the effects of alternative mixes of power plant types in developing the electrical system.

b. *Limitations of Using Linear Cost Functions.* One of the major limitations on the realism of this type of linear model is that all of the cost functions must be linear with no fixed cost component. Thus, the cost estimates must be simplified to linear form even in a case where available cost functions are nonlinear. Since most of the cost items considered here exhibit economies of scale, the cost functions are concave. These functions cannot be handled efficiently and adequately for problems of this size by available optimization methods, such as mixed integer and nonlinear programming. The approach here has been to use linear approximations, leading to two significant implications: (1) the cost estimates are less accurate and (2) the "optimality" of the mathmeatical programming solutions is in question.

One major effect is produced for each of the three commodity flow systems. Using the electrical subsystem for example, the model may find solutions where two separate transmission lines are constructed near to each other on parallel routes. Similar solutions are likely for gas pipelines and water treatment and piping facilities as well. In practice, many of these parallel commodity flows would be combined to take advantage of the ensuing economies of scale.

The model described here, however, is a preliminary screening tool for looking simultaneously at the subsystems of the water-resources and energy systems and their interrelationships. For this type of model, which is designed to look at trends and trade-offs for a large region, the many simplifications are judged reasonable. Moreover, the restrictions described above are inherent in any linear programming model and, in many cases, limit such a model's role to screening purposes. More detailed analyses, of course, should be carried out for the individual subsystems, for geographical subregions, and for a relatively small number of alternative development patterns for the entire region.

c. *Using the Model to Examine Trade-offs Between Different Objectives.* The objective function specified here addresses one planning objective: overall economic efficiency. The planning process is, of course, multiobjective in nature, and, in fact, the model is designed to look at trends and trade-offs associated with many policy issues. For example, water quality objectives are important in placing limits on withdrawals from rivers. The trade-off between regional cost and more stringent withdrawal limits could be examined, and the changes in the pattern of resources development could be observed. In general, the mathematical programming model could be used to evaluate the sensitivity of alternative solutions and to examine issues using parameterization runs (also, see the discussions in Sections 1.3 and 2.3).

In addition, it is desirable to examine the development patterns of the water, coal-conversion, and electricity subsystems and to evaluate their costs, since these subsystems are managed by a combination of private companies, regulated utilities, and governmental agencies. The model can be used as it is written to keep track of the subsystem costs. For example, simple equations can be used to sum the costs of electricity transmission and pipeline gas transmission. The water shipment costs can also be evaluated for any set of water supply variables, depending on the variation of the model used in a given application. Examples of costs for the different subsystems are given for the Illinois case in Section 2.3.

The subsystem costs could also be used in an hierarchical approach whereby priorities could be assigned to the different subsystems and trade-offs could be evaluated between them. Also, in a general multilevel approach, other subsystems such as commodity demand models could be evaluated. The scope of this project, however, has been limited to the subsystems described; the emphasis has been on developing a model (with several variations) which can be used in conjunction with other models and planning methods to examine some of the major policy issues associated with developing the interlocking water-resources and energy systems in a large region.

2.2.3 Constraints

The mathematical constraints are described below. In any given application, however, it is expected that the general formulation will be modified. Two variations are used for the example application to data for Illinois. Also, along with each constraint set the example data for Illinois are described.

a. *Water Source Points.* Constraint Sets 1, 2, and 3 limit the amount of water that can be withdrawn from each groundwater aquifer i, from each potential reservoir site k, and from each river segment j, respectively. The withdrawals for MID, coal gasification, and electricity generation are summed to give the total withdrawal from each source.

Set 1: Groundwater Supply

$$\sum_m (d^M_{im} + d^G_{im} + d^E_{im}) \leq D_i \quad \forall i$$

Set 2: Reservoir Supply

$$\sum_m (r^M_{km} + r^G_{km} + r^E_{km}) \leq R_k \quad \forall k$$

Set 3: Riverwater Supply

$$\sum_m (w^M_{jm} + w^G_{jm} + w^E_{jm}) \leq W_j + RG_{mj} \cdot s_m + RE_{mj} \cdot t_m + RN_{mj} \cdot t^N_m$$

Groundwater withdrawals are limited by the continuous net aquifer yield for each source; the reservoir withdrawals are also limited by a continuous safe yield estimate for each site.

The allowable river withdrawals are determined by the river flow and the return flows that are available for reuse. The river water availability is given by W_j, which may be expressed as a percentage of the 7-day 10-year low flow. In addition, the return flows from municipal and industrial supplies (regardless of their origin) that are discharged into river segment j or its tributaries may be available for reuse. These return flows are known if it is assumed that the municipal and industrial demands will be met, and they can be added directly to W_j. The return flows from potential coal-gasification and steam-electric plants can also be made available for reuse as shown in the constraint. For screening pruposes, these return flows should probably be considered for long river segments. Caution should be exercised, however, because of the related water quality issues.

It should also be noted that if all return flows are considered, the withdrawal limit really specifies a constraint on water consumption for each river segment. In some cases it might also be necessary to add additional constraints to ensure that enough water would be available in the river for each withdrawal. As long as W_j represents a relatively small percentage of the low flow, however, this problem would not be expected in practice.

A river, of course, is continuous, but it is appropriate to consider fairly long reaches as water supply points since the total withdrawal (or consumption) is the most important factor. The cost of transporting water to a given demand point, however, can be estimated from the part of the river nearest (or most convenient for) the given point. If the river flow conditions change significantly, then two or more segments can be used with different withdrawal limits. The limit for a given segment should also take into account all upstream return flows and should apply to the cumulative withdrawals considering all upstream segments. The constant, W_j, in Constraint Set 3 would include all upstream municipal and industrial return flows, and the variable terms would be summed for segment j and all upstream segments.

The constraint sets may be modified extensively in any given application. For example, the variables d^M_{im}, d^G_{im}, and d^E_{im} could be replaced by a single aggregate variable d_{im} if the unit costs are nearly equal. This modification was used for the initial application of the model to data for Illinois, as discussed later in this chapter. In most cases, however, the water treatment costs vary significantly for different uses, and the effect of these differences on the total supply costs should be considered in evaluating this type of modification. Also, if three sets of aggregate variables (d_{im}, r_{im}, and w_{im}) are used, then the water demand constraints, described below, could also be combined to form one aggregate constraint for each subregion.

Another example modification is also described in this chapter for the Illinois case; two types of water supply variables are used - one for the sum of MID and coal gasification process water supplies and one for the sum of all cooling water supplies. This grouping was judged to be practical because of similarities in the water quality requirements.

The water availability data for the Illinois application were obtained from several studies by the Illinois State Water Survey. River water supplies were listed by Singh and Stall (1973), and potential reservoir sites were identified by Dawes and Terstriep (1966A, 1966B) and Ackermann (1962). More recently, all of the groundwater and surface water supplies were evaluated as part of a resources inventory for coal conversion in Illinois (Smith and Stall, 1975).

The example application considered seventeen major groundwater sources, six potential reservoir sites, and seven river segments. These water sources are shown on the map in Figure 2.1. The Mississippi and Ohio Rivers were modeled using three and two segments, respectively; one segment was used for each of the other two rivers. River withdrawals were initially limited to 1% of the 7-day 10-year low flow, but this percentage was parameterized to demonstrate the effect of changing the allowable withdrawals on the water-resources and energy systems.

b. *Water Demand Points*. Constraint Sets 4, 5, and 6 specify the demands for water in each subregion for MID, gasification, and electrical power generation, respectively.

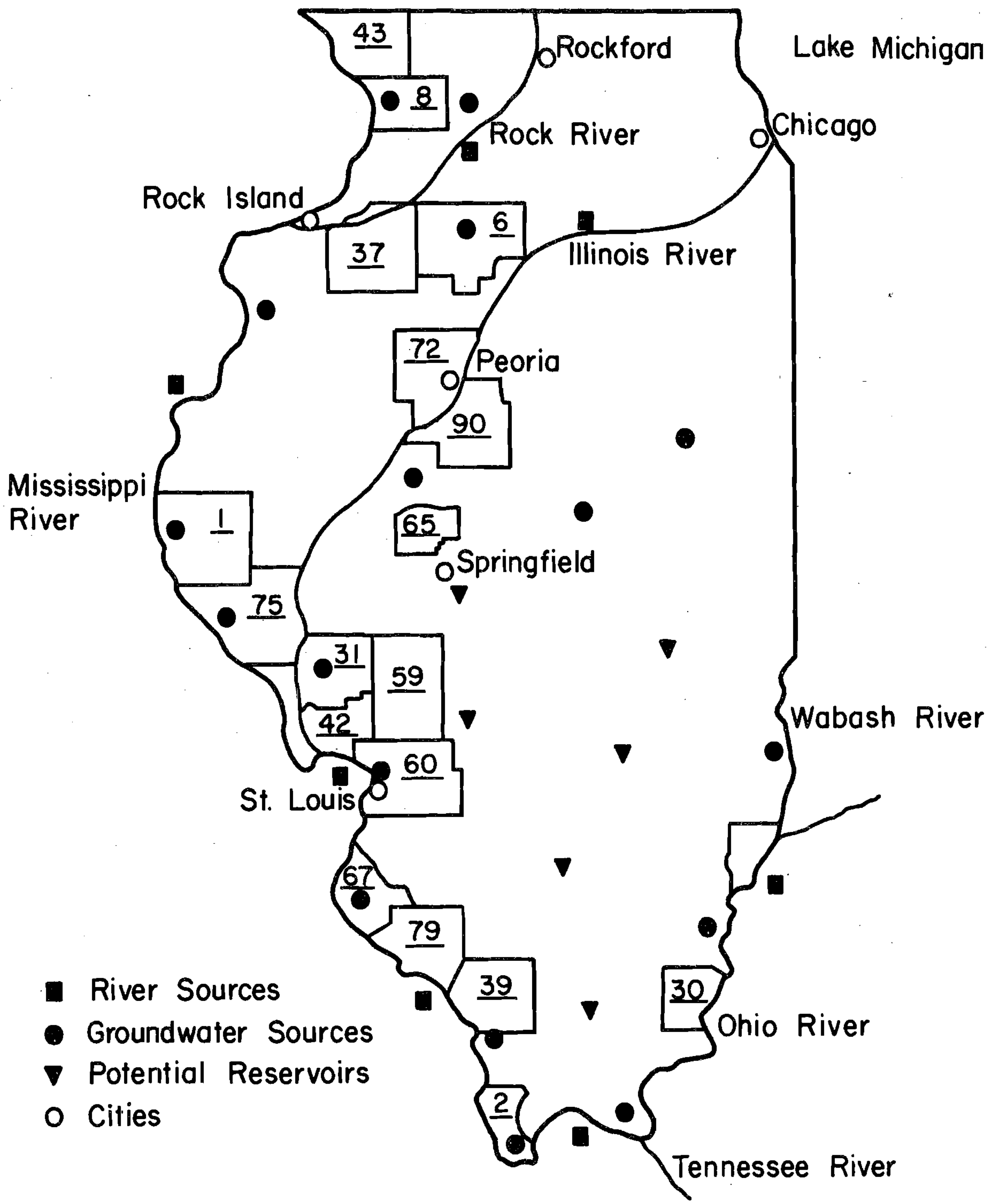

Figure 2.1 Major Water Source Points and Energy Demand
Note: Several key counties are illustrated and labelled by numbers.

Set 4: MID Water Demand

$$\sum_i d^M_{im} + \sum_j w^M_{jm} + \sum_k r^M_{km} \geq MID_m \quad \forall m$$

Set 5: Gasification Water Demand

$$\sum_i d^G_{im} + \sum_j w^G_{jm} + \sum_k r^G_{km} \geq WG \cdot s_m \quad \forall m$$

Set 6: Electrical Power Generation Water Demand

$$\sum_i d^E_{im} + \sum_j w^E_{jm} + \sum_k r^E_{km} \geq WE \cdot t_m + WEN \cdot t^N_m \quad \forall m$$

The MID_m requirement (Set 4) would specify the demand for water in subregion m. This demand is given by the deficit between future water needs (above the current usage levels) and local water supplies. For the Illinois example, MID requirements were estimated on the basis of projections for 1980 by Csallany (1972). The example used each of the 102 counties in Illinois as subregions, but MID requirements were necessary for only 44 of them where local supplies were assumed to be inadequate. (The local supply estimates were based on the water availability data from the Illinois Technical Advisory Committee on Water Resources [1967]).

The gasification water demand for a subregion is given by the process water and cooling water required for the total plant capacity located in the region. This requirement can be expressed as the water needed for a unit-size plant multiplied by the total plant capacity expressed as the equivalent number of unit-size plants. The unit-size plant designation is arbitrary and is used only to balance the resource requirements and the gas produced. Since the water requirements for a unit plant are assumed to be constant, a cooling technology must be prespecified for each subregion; this constant would vary from one subregion to another if different cooling technologies were specified. (Chapter 3 presents modifications of the model to allow for alternative cooling technologies.) Data on water requirements for coal-conversion facilities were evaluated and presented in a staff report published by Stout (1974).

For the example study, gasification water demands were needed for the 59 subregions (counties) which contain enough coal for a coal-conversion facility. (For the example data, it could be shown that water would always be shipped to coal points rather than coal to the water points because of the relative costs of transporting coal, water, and gas--see part a. of Section 2.2.2). It should be noted that the coal-gasification water constraints require an aggregate amount of water for both process water and cooling water. It might be desirable to disaggregate these variables to take into account the different water quality requirements. As mentioned in part a. of this section, for the second set of example runs for Illinois, the process water requirement was added to the MID constraint for each county, and the cooling water requirements for all of the energy facilities in each county were aggregated.

Constraint Set 6 is similar to Set 5 and ensures that the water needs for the coal-fired and nuclear power plants are met. Coal-fired plants were allowed only in the 59 counties with coal, but these constraints were needed for all counties because of the possibility of constructing nuclear plants.

For the Illinois application, it was assumed that wet cooling towers would be used for all energy facilities in the state. The constants that specify the water requirements for these facilities are given in Table 2.2.

c. *Coal Supply Points.* Constraints such as the ones in Set 7 can be used to ensure that the coal shipments from a subregion do not exceed the available supply.

Set 7: Coal Supply

$$\sum_m (x_{om}^M + x_{om}^G + x_{om}^E) \leq X_o \quad \forall\, o$$

The coal supply is limited by the amount of reserves available for deep mining and strip mining, and the technology must be considered in calculating the supply available for use. For the Illinois examples, it was assumed that strippable

Table 2.2 - Constants for Energy Facilities
Used in the Illinois Example

WG = 14,000 gallons per minute	CG = 16,000 tons per day
WE = 10,000 gallons per minute	CE = 12,000 tons per day
WEN = 15,000 gallons per minute	GP = 250 million cubic feet per day
	EP = 1,000 megawatts

coal was already committed and that deep-mined coal would be used for a major new industry. It was further assumed that one-half of the reserves could be considered for use. The example data were based on Illinois State Geological Survey estimates of reserves in each county (Hopkins and Simon, 1974). These estimates have been updated by a more recent cooperative report among Illinois agencies (Smith and Stall, 1975).

d. *Coal Demand Points*. Constraint Sets 8, 9, and 10 require that the coal demands be met for municipal and industrial uses, gasification, and electrical power generation, respectively.

Set 8: MICD Coal Demand

$$\sum_{o} x_{om}^{M} \geq MICD_m \quad \forall m$$

Set 9: Gasification Coal Demand

$$\sum_{o} x_{om}^{G} \geq CG \cdot s_m \quad \forall m$$

Set 10: Electricity Coal Demand

$$\sum_{o} x_{om}^{E} \geq CE \cdot t_m \quad \forall m$$

If all coal shipments are interchangeable, then these constraints could be combined and one coal variable for each source and subregion could be used. However, it might be desirable to use separate variables and constraints in some cases (e.g., where coal pretreatment costs vary for the different uses).

In the Illinois examples, coal requirements for MICD purposes were not considered. Furthermore, as discussed above, coal plants were only allowed in the 59 counties containing coal. (This result was cost minimizing for the case study data, but in other applications different conclusions might be reached.) The constants that specify the coal requirements for the energy facilities are given in Table 2.2. Since the costs of supplying coal were also not considered, constraint sets 7, 9, and 10 were reduced to one set of constraints limiting the level of coal use for gasification and electricity in each of the 59 counties.

It is realistic to omit the costs of supplying coal only in cases where the mining and treatment costs do not vary significantly from one subregion to another. In cases where these costs do vary significantly in practice, different unit costs can be added to shipment costs or used as cost coefficients for the plant capacities located in the respective subregions.

e. *Gas Supply Points.* Constraint Set 11 prevents the amount of pipeline gas shipped from any subregion from exceeding the total capacity of the coal conversion facilities located in that subregion.

Set 11: Gas Supply

$$\sum_p g_{mp} \leq GP \cdot s_m \quad \forall\, m$$

The total plant capacity in a subregion is expressed as the number of unit-size plants multiplied by the capacity of such a plant (see Table 2.2). For the Illinois example, these constraints were used for the 59 counties with adequate coal supplies. (It should be noted that these constraints would be met as equalities since there are costs of supplying water for plant capacities. It follows that these constraints could be eliminated from the mathematical formulation since each s_m variable in the remaining formulation can be replaced by its equivalent in terms of the g_{mp} variables as given by Set 11. This change would be expected to decrease the computational time required for solving the linear programming problems.)

f. *Gas Demand Points.* Pipeline gas demands can be specified for the major metropolitan and industrial load centers. The actual geographical location of the demand points could in some cases be determined by the location of major storage areas or the origin of a local distribution system. Constraints can be used to ensure that these demands are met as follows:

Set 12: Gas Demand

$$\sum_m g_{mp} \geq GD_p \quad \forall\, p$$

Since costs are incurred in transporting gas, these constraints would be met as equalities in an optimal solution to a mathematical programming problem.

Three demand centers (near Chicago, East St. Louis, and Peoria) were used for the Illinois examples. The gas shipments were summed over the subregions that contain coal and were allowed as "sites" for locating gasification plant capacity. The level of future coal conversion is very uncertain and depends on the level of consumption and on the levels of imports and exports. Since the level of coal conversion represents a major state policy decision, the example computer runs examined the effects of various levels of production on the degree and pattern of resources development. Parametric linear programming was used to vary this level from zero, which represents continued importion, to levels representing major exportation.

g. *Electricity Supply Points.* The total amount of electricity that can be supplied to all demand points from a given subregion cannot exceed the capacity of the power plants in that subregion. This requirement is given by Constraint Set 13.

Set 13: Electricity Supply

$$\sum_q e_{mq} \leq EP \cdot t_m + EP \cdot t_m^N \quad \forall\, m$$

The total power plant capacity in a subregion is expressed as the number of unit-size coal-fired plants and nuclear plants. Since EP represents an arbitrary unit-size plant, the same constant can be used for both nuclear and fossil-fuel plants. For the Illinois example, these constraints were written for all 102 counties, but only nuclear plants were allowed in the counties without coal. (As with Set 11, it should be noted that these constraints would be met as equalities, and it would be possible to eliminate them from the formulation. This change would require writing separate variables for electricity transmission from the coal-fired and nuclear power sources in each subregion. Each t_m and t_m^N could then be replaced by the sum of the corresponding outgoing electricity transmission variables.)

h. *Electrical Power Demand Points.* Electrical power demands are specified for the major load centers and must be supplied by the electricity flows from the various subregions. As shown below, these constraints include the fraction, B_{mq}, which gives the transmission loss between plants located at subregion m and demand point q.

Set 14: Electricity Demand

$$\sum_m (1-B_{mq})\ e_{mq} \geq ED_q \quad \forall q$$

This simple formulation, which deals only with transmission levels during periods of peak demands, was judged to be the most useful for dealing with the location of major energy facilities when using the type of preliminary screening model described here. Other formulations, however, were also examined initially. One alternative formulation (Anderson, 1972) requires the shipment of average electricity demands; that formulation specifies a total plant capacity requirement which takes into account the peak demands, and it requires the transmission line capacities to exceed the shipments under average conditions by the same peaking factor. This alternative formulation, however, allows power plants to be built for peaking purposes and does not require the necessary transmission links for these plants. Several example computer runs using the formulation indicated that major transmission requirements were omitted. For the model presented here, the transmission costs are important, since they are greater for plant capacity located farther from the demand areas. Thus, constraint set 14 was chosen as the most effective formulation; it is quite simple yet recognizes the most basic transmission requirements. Also, as discussed later in this chapter, the mathematical programming solutions are not very sensitive to the electrical power transmission costs.

For the Illinois example, estimated demands for the year 2000 were used for five load centers located near Chicago, Rockford, Rock Island, East St. Louis, and Springfield, respectively (Provenzano, 1975). These demands were assumed to represent the power needs that would be met by new plant capacity; in general, this load requirement would be based on projected usage increases plus the needed plant replacement. The power loss factors, B_{mq}, were estimated using a power loss factor of 2% per hundred miles (FPC, 1971).

i. *Limit on Nuclear Plant Capacity.* The total nuclear plant capacity can be limited by a constraint such as:

Set 15. Nuclear Plant Limit

$$\sum_m EP \cdot t_m^N \leq NUC$$

This type of constraint, whether expressed as a limit or an equality, can be used to effect a desired mix of steam-electric plants. For example, the desired level of nuclear plant capacity may be known thru the prior determination of the optimal mix of nuclear and fossil fuel plants. The actual desired level of nuclear capacity, however, is an important policy issue for the power industry and for state officials. The location-allocation model described here might be used in conjunction with other models, such as a plant-mix model and a more detailed simulation model, and with other planning and analysis methods to address the issue. For the Illinois example, the value of NUC was parameterized to examine the effect of this constraint on the development of statewide resources.

2.3 EXAMPLE APPLICATION OF THE MODEL FOR THE STATE OF ILLINOIS

2.3.1 Background

This section describes very briefly the example applications of the basic LP screening model to data for Illinois. These computer applications were undertaken for four purposes: (1) to aid in developing the model itself, (2) to demonstrate the interrelated nature of the water resources and energy systems, (3) to demonstrate, in a general way, the potential impact of a major new coal conversion industry, and (4) to demonstrate the use of the model as an aid in evaluating regional planning issues.

All of the LP examples were run using MPSX (IBM, 1971, and IBM, 1973), which is typical of widely available computer codes, on the IBM 360-75 at the University of Illinois. The application for Illinois demonstrated the practicality of the model, since the cost of individual computer runs for this large region varied from $20 for single solutions to $100 for long parameterization runs.

The first set of computer runs dealt with a 14-county region within Illinois. The main purpose was to exercise the model to identify the components which needed modifications. This work led, for example, to the final choice of the simplified formulation of the electrical distribution system.

The model was next applied to example data for the entire state of Illinois. The major water sources that were considered are shown in Figure 2.1. They include 17 large groundwater aquifers, 6 reservoir sites, and 7 river segments. The initial energy demands were set as listed in Table 2.3; these values reflect major policy issues, however, and some of the were parameterized. The variables, constraints, and cost coefficients of the model are all described in Section 2.2 above. The 102 counties of Illinois were used as subregions; the counties mentioned below are labeled by numbers in Figure 2.1.

2.3.2 Application Using First Modification

For the first computer runs for Illinois, one major modification was made in the basic LP model to reduce the problem size. All of the water demands in each subregion were aggregated; this change allowed the groups of water supply variables (e.g., d_{im}^{M}, d_{im}^{G}, and d_{im}^{E}) to be replaced by a single set of variables (e.g., d_{im}). Also, the water demand constraints, Sets 4, 5, and 6 were replaced by a single set using one constraint for each county. The complete mathematical formulation had 3974 variables and 368 constraints (see Table 2.4 for a breakdown by categories).

The solution, which has an objective function value of $109 million (annual cost), located the energy facility capacities as listed in Table 2.5. The coal-using facilities are listed in order of the level of coal use. The primary role of the model is for screening purposes, and, as can be seen from the table, Macoupin County is indicated to be a very desirable location for major energy facilities. This result, which was observed in all of the example runs, occurs because this subregion contains abundant coal, is near a very large water source (the Mississippi River), and is relatively near the major energy load center (Chicago). Twelve other counties were also selected for coal facilities.

The important result from the model is the ranking of the subregions--not the description of exact plant capacities. It is interesting to note, however, that

Table 2.3 - Initial Demand Levels for Pipeline Gas and Electricity

Pipeline Gas	
Load Center	Demand in million cubic feet per day
Chicago	4500
St. Louis	225
Peoria	180

Electricity	
Load Center	Demand in megawatts
Chicago	12000
St. Louis	5000
Springfield	3000
Rockford	4000
Rock Island	1000

Table 2.4 - Variables and Constraints for the First Modification of the LP Model

Variable Category	Number	Constraint Category	Number
Groundwater shipment	1734	Groundwater supply	17
Reservoir water shipment	612	Reservoir supply	6
River water shipment	714	River water supply	7
Gasification plant capacity	59	County water demand	102
Pipeline gas shipment	177	Gasification supply	59
Electric plant capacity	59	Gasification demand	3
Nuclear plant capacity	102	Electricity supply	102
Electricity transmission	510	Electricity demand	5
Subcost	7	Coal supply	59
Total	3974	Nuclear plant limit	1
		subcost calculation	7
		Total	368

Table 2.5 - Energy Facilities by Counties

County	Coal: Coal-Gasification Capacity in million cubic feet per day	Coal: Coal-Fired Electricity Capacity in megawatts
Macoupin (59)[a]	3760	
Callatin (30)		5000
Madison (60)	230	1700
Bureau (6)	530	
Henry (37)		2000
Jackson (39)		1400
Lawrence (51)		1900
Menard (65)		1800
Tazewell (90)	220	
Randolph (79)		1000
Peoria (72)	180	
Greene (31)		500
Jersey (42)		300

[a] County number as shown in Fig. 2.1.

even though continuous variables were used for plant capacities, all of them are large enough so that technically feasible plants could be considered in the corresponding subregions. Nevertheless, the social, political, and technical feasibility of locating large plant capacities in the counties selected by the model must be examined in detail for each of these counties. For example, this particular solution locates nuclear power plants in a number of southern Illinois counties, but these locations may not be technically feasible because of the earthquake potential. Through the use of a screening model, however, detailed data are needed only in cases where the issues arise. Thus, this type of model can help streamline the process of reaching a final regional plan.

To further illustrate the use of the model as a planning tool, another computer run was made to examine the issue of locating energy facilities. Constraints were aded to prevent any county from becoming a "power park" serving the major energy demand centers. The following limits were place on the total plant capacities in each subregion: coal gasification capacity, 500 million SCFD; coal-fired electric capacity, 3000 MW; and nuclear capacity, 3000 MW. The solution obtained specified many more counties for locating energy facilities--23 instead of 13 for coal facilities. Interestingly, the increase in the total cost was small--only 8%. Insights such as these can result from repeated use of the tool and can assist statewide planning.

This variation of the model was exercised extensively using sensitivity analysis and parameterization techniques. It was shown that the water supply costs consistently accounted for over half of the total cost and that the solutions were quite sensitive to changes in these cost coefficients. This component of the model was refined for the second modification described below in Section 2.3.3. The gas pipeline subsystem costs accounted for approximately 30% of the total cost, but the solutions were relatively insensitive to changes in these cost coefficients. The electrical transmission subsystem costs totaled less than 15% of the overall cost; the solutions were sensitive to these costs, although plant locations were generally unaffected. A more complete discussion of sensitivity analysis will be given in the discussion of the next modification of the LP model.

One parameterization run was used to examine the effect of changing the water demand in the Chicago area. Although the shipment of water from the distant sources considered here represents an extreme solution to meeting the needs in this major water-short area, the solutions give an indication of the high cost of such shipments (more than \$200/gpm). As this MID requirement was reduced to zero, the secondary effects on the rest of the water-resources and energy system were illustrated (see Brill *et al.*, 1975). Because of the high cost of supplying water to the Chicago area from the sources considered in the application, the municipal and industrial demands were reduced to zero for most of the examples described below. This reduction is based on the assumption that the water needs in the area will be met by local supplies (such as an increase in the allowable diversion from Lake Michigan).

2.3.3 Application Using Second Modification

As indicated above, the LP solutions obtained using the first modification of the LP model showed that the water supply costs represented the largest cost component and that the solutions were quite sensitive to these cost coefficients. Furthermore, the water supply costs were very different for different water uses because of the different treatment requirements. These differences, however, could not be considered when the water supply variables were aggregated. For the second modification, therefore, they were disaggregated to improve the realism of this model component. Since the quality requirements for cooling water are essentially the same for electricity-generation and coal-gasification facilities, those supply variables were aggregated. Also, the water supplies for meeting the MIDs and the process-water demands for coal-conversion facilities must be of higher quality, and these supply variables were aggregated. For the new formulation, the original 102 county water demands were converted to cooling water demand points. The MID-plus-process-water demands were specified using 79 additional constraints for the counties with a MID and/or coal reserves. Also, one new subsystem constraint was needed to calculate the cost for the new set of water supply variables. The number of water supply variables doubled, from 3060 to 6120. Their constraint coefficients, however, are either zero or one and the data input for the LP code was generated using simple computer programs.

The initial solution using the new formulation was different from that obtained using the first modification, but the largest energy facility capacities were located in the same counties. The new solution, however, did specify more counties for locating these capacities. The objective function value was $61 million, as compared to the $85 million value found using the first modification for the case with a zero MID for the Chicago area. This reduction represents the more realistic cost coefficients used for the water supply variables for cooling purposes. The subsystem costs are given in Table 2.6. Thus, the cooling water costs were the largest component of the water supply costs. Also, the relative values of the subsystem costs remained about the same as the ones described in the preceding section. The second modification was judged considerably more realistic than the first modification and required only 79 extra constraints.

The sensitivity analysis showed that this solution is sensitive to cost changes for some of the variables of each type--with the exception of those for the MID and process-water demands. The changes that would not greatly affect the objective function indicate alternative solutions which can be evaluated further. For example, these alternative solutions frequently indicated local shifts in supplying water for MID purposes or for locating power facilities. As a specific example, the optimal LP solution located nuclear power-plant capacity in Carroll County (No. 8), whereas the sensitivity analysis showed that shifting this capacity to Jo Daviess County (No. 43) would have little effect on the solution. Both of these counties could then be considered in the planning process.

The dual variables give the shadow prices for the incremental changes in the objective function that would result from incremental changes in the constraint limits. These values take into account all of the secondary effects in the total system. For example, as discussed above, the MID requirement was set to zero for the Chicago area. The constraint was retained, however, at the zero level, and the dual variable indicated that the cost of supplying the first incremental unit of water to this area would be $246/gpm. This result is interesting because there are many water sources considered in the model that could supply water for about $200/gpm. The total incremental cost given by the dual variables indicates that the true cost is higher when the secondary effects of precluding alternative uses of this water are considered. The sensitivity analysis indicated that this cost estimate would be valid for the first 7000 gpm of supply; a higher unit cost would occur for additional supplies.

Table 2.6 - Subsystem Costs for the Second Modification

Subsystem	Annual Cost in million dollars
Cooling water supplies	23.5
MID and process water supplies	8.1
(Total water)	(31.6)
Pipeline gas shipments	721.0
Electricity transmission	7.9

As another example, the shadow prices for the product demand constraints give the incremental cost for meeting that demand. This incremental cost takes into account all of the variable cost factors considered in the application. In practice, these variable costs could be added to the fixed costs to estimate the total cost of meeting specified gas demands. The dual variables are valid over a limited range only, however, and a more complete picture of the effect of changing a parameter can be gained by using parameterization techniques.

Many parameterization runs were made using this variation of the model, and one of them was used to examine different demand levels for pipeline gas. The first solution assumed that all gas demands would be met by importing supplies from outside the state. The total demand level (summed over all load centers) was then increased in eight steps of 2,500 million SCFD each, which corresponds to 10 unit-size plants per step, to a maximum level of 20,000 million SCFD.

The changes in the total cost and the subsystem costs are shown in Figure 2.2. It is apparent that changing the assumed level of pipeline gas production has a major impact on the entire water resources and energy systems. As expected, the costs of the cooling water and gas shipment subsystems increase greatly. The cost of the MID water supply and electricity transmission subsystems both tend to increase as the gas demands increase. At the last increment shown, representing a very high rate of gas exportation, the electricity transmission costs decrease because, for the first time, new reservoirs are needed. As a result, the water supply costs increase greatly, but some of the electric plants are located nearer the demand centers.

The dual variables for the gas demands give the incremental, or marginal, cost for supplying pipeline gas to the respective demand points. For a given load center, their values can be plotted for the different demand levels in the parameterization to give a supply curve, as shown, for example, in Figure 2.3 for the load center near Chicago.

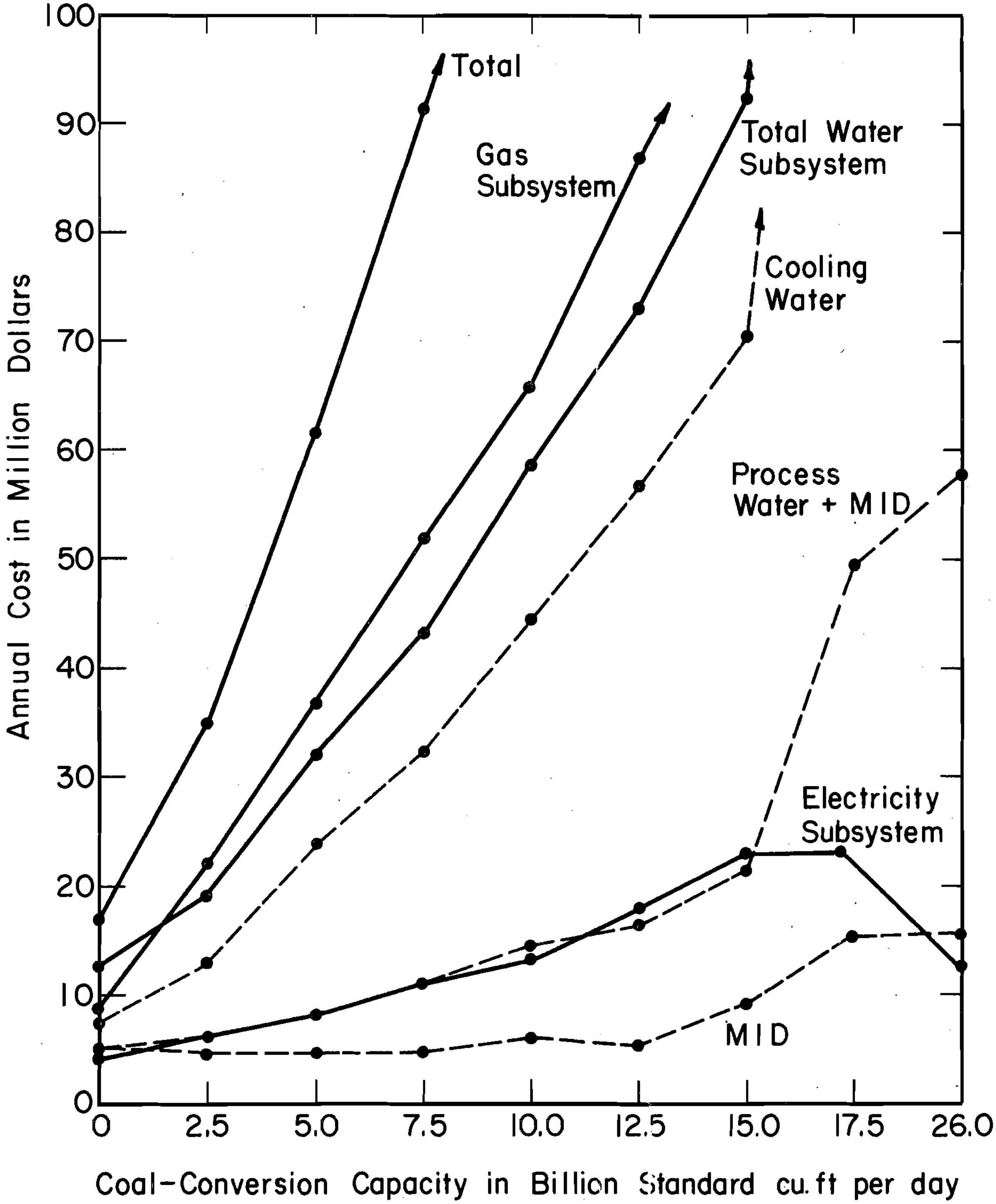

Figure 2.2 Subsystem Costs as a Function of Coal-Conversion Capacity

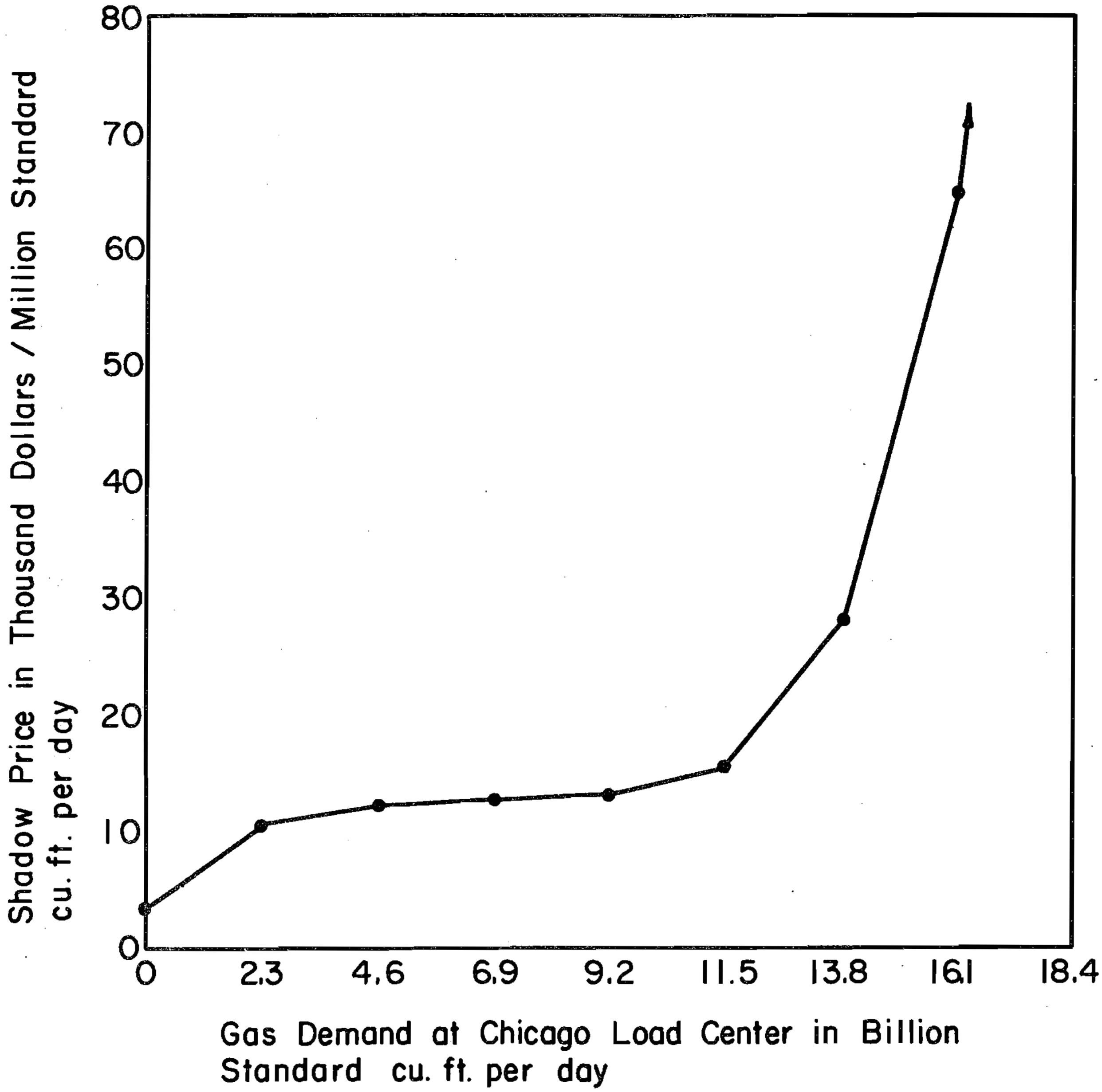

Figure 2.3 Marginal Cost for Supplying Pipeline Gas

2.4 CONCLUSION

This chapter presents a general linear programming model for use in planning regional water resources and energy systems. In any application, however, it is likely that a modified form of the model would be used. For example, in the Illinois application it was possible to eliminate the coal shipment variables after a preliminary analysis of the cost data. Many assumptions were made in this application, however, and the solutions obtained should be viewed with them in mind.

The solutions of the model in the Illinois case demonstrated the use of the model for screening purposes. It was shown that sensitivity analysis could be used to find alternative solutions which could then be analyzed in more detail during the planning process. Also illustrated was the potential use of the dual variables for finding the incremental values of additional resources and for finding the incremental costs of additional products where all of the secondary effects considered by the model are taken into account.

In addition, parameterization techniques were demonstrated for use in examining trends and trade-offs over a range of parameter values. By increasing the levels of the demands for pipeline gas over a large range, it was shown that this level affected all other components of the model. Also, the dual variables for the demand constraints were plotted to illustrate a supply function for a specified load center.

The model presented in this chapter is a practical and versatile tool which can be used to aid the planning of regional water resources and energy systems. Furthermore, the example application demonstrated the interrelated nature of the two systems and the potential impact of a large new coal-conversion industry.

CHAPTER 3: MODEL MODIFICATIONS FOR CONSIDERING ALTERNATIVE COOLING TECHNOLOGIES

3.1 INTRODUCTION

The basic linear programming (LP) model described in Chapter 2 can be modified so that alternative cooling technologies are allowed for each subregion. One technology or a mix of them might be desirable for a given subregion. The first modification, described in Section 3.2, extends the LP model by adding a plant-capacity variable for each alternative in each subregion. A fixed water requirement is specified for each of those alternatives. The second modification, described in Section 3.3, allows the water requirement for each alternative to vary as well; this change produces a nonlinear model, and several solution approaches are discussed.

3.2 LINEAR PROGRAMMING MODEL WITH ALTERNATIVE COOLING TECHNOLOGIES

Alternative cooling technologies can be incorporated into any of the variations of the basic LP model relatively easily if one considers a constant (e.g., average) water requirement for each alternative. Considering coal-conversion facilities, for example, let:

a = index for a given cooling alternative

WG_m^a = water requirement for a unit-size coal-gasification plant in subregion m, using cooling alternative a (GPM)

s_m^a = coal-gasification-plant capacity located in subregion m, expressed as a number of unit-size plants

The basic LP model presented in Chapter 2 can be modified to allow for different gasification-plant cooling alternatives by substituting WG_m^a for WG and $\sum_a s_m^a$ for s_m. As an example, Constraint Set 5 would be rewritten as:

Set 5' Gasification Water Demand

$$\sum_i d_{im}^G + \sum_j w_{jm}^G + \sum_k r_{km}^G \geq \sum_a WG_m^a \cdot s_m^a \qquad (3.1)$$

Analogous changes are also necessary for the variables and constraints related to electric plant capacities.

It should be noted that if one were to base the selection of cooling alternatives purely on the water requirements, then there would not be any need for mathematical optimization in choosing an alternative. Because of the costs of supplying water, an optimization model would in each case choose the alternative requiring the least water. There are, however, offsetting costs; in general, cooling system costs are inversely proportional to their water requirements, as shown in Figure 3.1. Hence, there is a trade-off between cooling system costs and water supply costs which can be evaluated by the modified model described here. The cooling alternative selected for a given subregion would be determined by this trade-off as it is influenced by the availability of water and by the other water demands.

Table 3.1 gives the relative ranges of water requirements and corresponding relative costs of different cooling alternatives for coal-fired steam-electric plants. These data can be modified to apply to coal-gasification and nuclear electric plants by considering typical overall efficiencies and the heat to be dissipated.

The modified LP model was applied to data for the state of Illinois; different cooling strategies were included with their respective constant water requirements per unit power output. Once-through cooling was not considered because of the limitations and guidelines proposed by USEPA as authorized by Section 301 of the Federal Water Pollution Control Act, as amended. These guidelines preclude, in most cases, the use of once-through cooling after July 1, 1977 (MacFarlane *et al.*, 1975).

Details regarding the input data on costs, water and coal availability, energy demands, and other inputs, such as the maximum number of nuclear plants and electricity transmission losses, are described in Chapter 2.

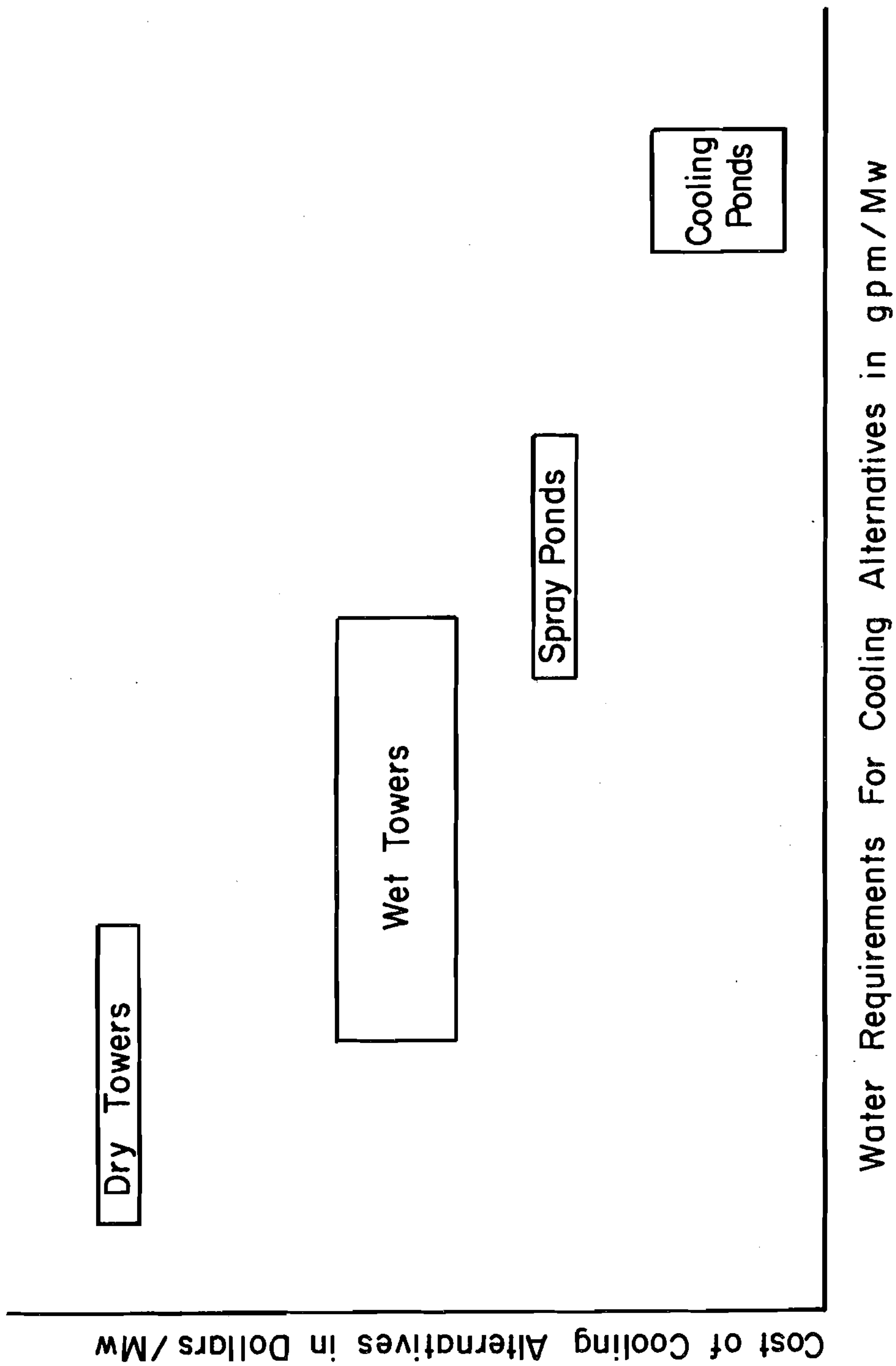

Figure 3.1 Relationship Between Cost and Water Requirements of Cooling Alternatives

Table 3.1 - Water Requirements and Costs of Cooling Alternatives

Cooling Alternative	Water Requirement in gallons per minute per megawatt	Annual Cost in thousand dollars per megawatts Source A[a]	Source B[c]
Once-Through	600[a]	1.53-2.42	.53
	600-850[b]		
	900[c]		
	966[d]		
	1150[e]		
Cooling Ponds	4.47-7.83[d]	1.96-3.0	.95
	15.7-29.5[d]		
Spray Ponds	8.39-14.6[f]	2.3-3.59	1.05
	10.2-23.6[d]		
Wet Cooling Towers	8.95-15.6[f]	4.75-6.45	1.10
	9.2-18.4[e]		
	27[c]		
	30[a]		
Dry Cooling Towers	.23[c]	6.45-8.25	2.64
	2[a]		

[a] From Dynatech (1969)
[b] From Cootner and Lof (1965)
[c] From Roa (1972)
[d] From Eggen and Livengood (1974)
[e] From Parker and Krenkel (1970)
[f] From Croley (1975)

The water requirements for different cooling alternatives were taken as the average values of the ranges given in Table 3.2. This table was prepared from the data in Table 3.1 with assumed thermal efficiencies of 40 percent for coal-fired electric plants and 65 percent for gasification plants. In addition, coal-gasification plants were assumed to require 3000 GPM of process water. This additional water requirement is assumed to have no direct effect on cooling system costs.

Two different LP runs were made for the state; in each run a parameterization was performed on the water supply costs for power generation. The LP model was identical to the second modification discussed in Section 2.3 except for an increase in the number of variables required for the different cooling alternatives; there were 7696 variables and 471 constraints. The central processing unit (CPU) time for each run, which was started with an initial LP basis, was approximately 3.62 minutes, and each run cost about $24 on the University of Illinois IBM 360/75 computer.

The two LP runs were different because only the second run considered the return flows given in the formulation (see Section 2.2). For both runs, the cooling system costs were a significant part of the overall system costs. Surprisingly, the first parameterization did not produce significantly different solutions. The second run, however, allowed more water for use throughout the region, and the following trends were observed as the cooling water treatment costs were reduced.

(1) At extremely high treatment (or supply) costs, the solution even specified the use of dry cooling towers, but at each change increment they were replaced by wet towers.

(2) The total nuclear plant capacity increased significantly (because of the large water requirement in comparison to coal-fired plants).

Also, the total nuclear plant capacity increased ten-fold from the first run to the second because much more water was available for use.

Table 3.2 - Ranges of Water Requirements for Coal-Fired Steam-Electric and Coal-Gasification Plants

Cooling Alternative	Water Requirements in thousand gallons per minute	
	Electricity in 1000 megawatts	Gasification in 250 million cubic feet per day
1. Cooling Ponds	15.7-29.5	20.2-35.4
2. Spray Ponds	10.2-23.6	14.2-29.0
3. Wet Cooling Towers	8.90-18.4	12.8-23.2
4. Dry Cooling Towers	.23-2.0	3.25-5.2

It is interesting that wet cooling towers were selected instead of less-expensive spray ponds and cooling ponds because of the limits placed on water use at each water source point. It should be noted that the dual variable associated with each water source would indicate the value of relaxing the limit on withdrawals. This estimate reflects the potential savings, including the savings in cooling systems. This issue could be examined further, in practice, using a parameterization run to vary the allowable withdrawals at the critical sources. These examples are used to illustrate the use of the model; because of the many assumptions, they do not lead to recommendations for Illinois. A general conclusion, however, is that the water supply and cooling systems play a major role in determining an efficient regional development plan.

3.3 NONLINEAR MODELS WITH ALTERNATIVE COOLING WATER REQUIREMENTS

3.3.1 A Nonlinear Formulation

The model presented in the last section assumes a constant water requirement for each cooling alternative. This may not be a good assumption especially when one considers the wide ranges of water requirements given in Table 3.2.

The main reasons why a range of water requirements (as shown in Table 3.2) exists for each alternative are:

(1) Cooling efficiencies are highly related to local atmospheric conditions; hence, it is very likely that identical systems will perform differently at different locations.

(2) The water-temperature rise encountered at the condensers varies from 10° to 30°F; therefore, in a water-rich area it may be desirable to use more water in order to have a lower temperature rise through the condenser.

Even though local conditions specify a range of water requirements for a given alternative, this fact alone is not sufficient to justify the inclusion of these ranges in an optimization model. That is, the model would always choose the lower bound of the range unless some benefits are incurred in going above it. As mentioned above, however, lower temperature rises can be achieved by using more water, producing benefits in terms of overall plant efficiency because of increased turbine output. Also, there may be less impact on the environment, since less heat will be dissipated and it will be diluted. For these reasons the modification given below is designed to consider ranges of water requirements for the different cooling alternatives.

The LP model can be modified so that the water requirements (WG_m^a) are decision variables rather than constants reflecting averages. These new decision variables would be bounded from above and below as specified by their respective range. For example, bounds are given below for coal-gasification water requirements:

$$LWG^a \leq WG_m^a \leq UWG^a \qquad \forall\, m,a \tag{3.2}$$

where

LWG^a = lower bound on the amount of water required for a unit-size gasification plant employing cooling alternative a (GPM).

Similar constraints would be introduced for nuclear and coal-fired electric facilities. It should be noted that the process water requirement for coal-gasificant facilities could be written as a separate constraint, or it could be added to the municipal and industrial demand as discussed in Chapter 2. In either of these cases, the discussion here would apply only to the cooling water requirements.

This modification greatly complicates the mathematical formulation, since the constraints for the water demands of power facilities have product terms (e.g., $WG_m^a \cdot s_m^a$), and thus the problem is now a nonlinear programming problem. Efficient, general purpose, nonlinear programming algorithms that can optimally solve a problem of this size do not exist at this time. A major effort of this study has been directed at the applied problem discussed here.

In summary, two trade-offs can be evaluated by the model. One is associated with all the cooling alternatives; i.e., an alternative requiring less water has a higher system cost. The other is associated with a specific alternative; i.e., it might be desirable to go above the lower bound of the water requirement for a given alternative.

Besides considering cost and water requirements, there is at least one more incentive for looking at different alternatives: to evaluate the "desirability" of the system in reference to the environment and society. Unfortunately, the "desirability" issue is a very subjective one. Although several attempts have been made to quantify such abstract concepts (Calvert and Heilman, 1972; Seiple, 1974; and Gruhl, 1972), the "desirability" and similar issues are still, and will remain, subjective. These issues are indirectly considered in this study, as will be discussed later. Theoretically one could assign "weighting factors" to represent these costs for various cooling alternatives; these factors would decrease with the degree of "desirability". Although the addition of such "weighting factors" would not further complicate the model, this approach is not very practical because the weights are unknown; hence it is not pursued here.

3.3.2 Using Separable Programming to Solve the Nonlinear Problem

The nonlinear program presented above has nonlinearities produced by product terms of decision variables in the cooling water constraints. In this study, separable programming is adapted to handle these nonlinearities. Separable programming is a method that makes use of a linear programming problem constructed to be a good approximation of the nonlinear problem. The data for the linear problem are obtained by evaluating the nonlinear functions of the problem on a grid of points spanning a suitable portion of the solution space. The method is by Miller (1963), and, as he points out, it is related to earlier work directed at handling convex separable linearities in the objective function by the simplex method (Charnes and Lemke, 1954; Dantzig, 1963). Miller's special contribution was to extend this approach to nonconvex problems and to apply the method to nonlinear constraints as well as to nonlinear objective functions provided that each nonlinear function is separable--that is, each can be separated into sums and/or differences of nonlinear functions of single arguments (hence the name "separable programming.")

The general expression x_1x_2 is not a nonlinear function of single variable, so it appears that it is not amenable to separable programming. However, one can make use of transformations to express product terms as sums or differences of separable functions. For example, consider:

Transformation 1

If $y = x_1 x_2$ then $\ln y = \ln x_1 + \ln x_2$ (3.3)

Transformation 2

$$x_1 x_2 = \left(\frac{x_1+x_2}{2}\right)^2 - \left(\frac{x_1-x_2}{2}\right)^2 \qquad (3.4)$$

The latter transformation is of particular interest because it can be used to express any quadratic function as the sums or differences of squares. This result follows since any product term (e.g., x_1x_2) can be written as a difference of squares as shown. Hence, any quadratic function is amenable to separable programming.

A group of equations and special rules are required for each separable function in order to produce valid approximations to the nonlinear functions using piecewise-linearized segments. (This requirement is frequently referred to as "separable logic".) Enforcement of separable logic is commonly achieved by restricting the entry of certain variables to the basis. In the literature two forms of the approximating problem are given for implementing separable logic: the λ-form and the δ-form (Hadley, 1964).

By the manner in which approximations are introduced for each separable function, these functions will retain their convexity-concavity properties. Therefore, if the convexity-concavity properties of the original nonlinear program are such that any local optimum is also a global optimum, then any local optimum to the approximating problem will also be a global optimum. The latter solution will be an approximation to a point at which the original problem assumes its global optimum. Under these circumstances, as the grid linearization is made finer and finer, the solution to the approximating problem, in the limit, approaches the solution to the original problem (Hadley, 1964).

In the absence of proper convexity-concavity characteristics the solution obtained to the approximating problem, through the use of restricted basis entry, will be a local optimum (see Hadley, [1964] for a complete proof). A mixed-integer linear programming (MILP) model can be used to formulate the problem in such a way that the global optimal solution can be obtained; in this formulation separable logic is enforced through the use of binary (integer) variables instead of restricted basis entry (Hadley, 1964). The obvious drawbacks of MILP models are the need for a large number of integer variables (one for each segment) and the lack of efficient solution algorithms. The MILP approach is discussed further in Section 3.4.

For special problems, a very powerful and interesting characteristic of separable programming is that separable logic will be automatically satisfied (and need not be enforced explicitly) for the convex functions (for minimization) in the objective function or on the left-hand side of a less-than-or-equal constraint (Beale, 1970). This characteristic is important because without the explicit enforcement of separable logic, either completely or partially, computational efficiency can be significantly improved.

As mentioned earlier, the nonlinear program (NLP) in question has nonlinear constraints of the following type only:

<u>Set 5'</u> Gasification Water Demand

$$\sum \text{water supplies} \geq \sum_a WG_m^a \cdot s_m^a \quad \forall m \tag{3.5}$$

Since separable programming (SP) can utilize transformations to handle product terms, it can be used to find at least a local optimum to the NLP. A detailed convexity and concavity analysis of the above NLP is given by Velioglu (1976). He shows that, because of the constraints like those in Set 5', the constraint set of the NLP is convex in cases where an increase in s_m^a results in an increase in WG_m^a. There is no justification, however, for expecting or requiring this kind of relationship between s_m^a and WG_m^a. Therefore, solving the given NLP by separable programming would not guarantee global optimality.

3.3.3 Modification of the Objective Function

In the preceding section the objective function of the NLP was considered to be linear. It was also mentioned that, unless benefits are incurred, the model would not choose water supply levels above the lower bounds of the requirements. It is not appropriate, however, to multiply the water requirement variables (WG_m^a) by benefit coefficients because they give the requirement per unit-size plant and do not take into account the size of the plant capacities.

One way around these difficulties, using coal-gasification plant capacities as an example, is to introduce the nonlinear term, $WG_m^a \cdot s_m^a$, to the objective function. The objective function would then look like:

$$\text{Min} \quad z = \cdots + CS_m^a \cdot s_m^a - BG_m^a \cdot (WG_m^a \cdot s_m^a) + \cdots \qquad (3.6)$$

where:

CS_m^a = unit cost of the cooling system a for coal gasification plants in subregion m (\$/unit-size plant)

BG_m^a = unit benefits incurred per unit cooling water requirement for a unit-size coal gasification plant in county m using cooling alternative a (\$/GPM for a unit-size plant)

This function does not allow benefits to occur when no plants are built. In addition, total benefits would be based on the actual plant capacity and the total water usage. The new nonlinear terms introduced to the objective function would not pose any additional difficulties for the SP formulation because the necessary group of equations and special rules are already used to handle the identical nonlinear terms which appear in the constraints.

There remains unanswered the question of obtaining the unit benefits, BG_m^a. The subjectivity of abstract concepts such as "desirability" was mentioned earlier. Unfortunately, like "desirability," "benefits" are also difficult to quantify using dollar values, and even when quantified they retain their subjectivity.

In this study no attempt has been made to quantify benefits; instead, an indirect path will be demonstrated for determining the monetary levels up to which benefits would not change the mathematically optimal solution. These critical levels are evaluated using a new set of variables, u_m^a, and a new set of constraints, as follows:

$$u_m^a = (WG_m^a - LWG^a) \cdot s_m^a \qquad (3.7)$$

where

u_m^a = amount of water in excess of the lower limit of the cooling-water requirement for the coal-gasification plant capacity employing cooling alternative a in subregion m (GPM)

Similar variables are also needed for coal-fired and nuclear electric plants. Now, equation 3.6 can be replaced by:

$$\text{Min } z = \cdot\cdot\cdot + CS_m^a \cdot s_m^a - BG_m^a \cdot u_m^a \cdot\cdot\cdot \qquad (3.8)$$

From equation 3.7 it follows that:

$$WG_m^a \cdot s_m^a = u_m^a + LWG^a \cdot s_m^a \qquad (3.9)$$

Therefore, using equation 3.9, the water demand constraints for coal-gasification plants can be rewritten in linear form as:

<u>Set 5'</u> Gasification Water Demand

$$\sum \text{water supplies} \geq \sum (u_m^a + LWG^a \cdot s_m^a) \qquad (3.10)$$

Thus, one can apply separable programming to solve the NLP taking into account the unit benefit coefficients (dollars per GPM) for using an amount of water in excess of the minimum. In the case where all benefit coefficients are set to zero, then clearly in a mathematically optimal solution all water requirements would be at their lower bounds, and all excess water variables would be zero. Otherwise, unnecessary expenses would be incurred for supplying water.

In the case where benefit coefficients are set to zero, however, one can obtain an estimate of the critical values of these benefits. At the critical values, given by the reduced costs of the u_m^a variables in the optimal LP objective row, the model would select higher water usage levels. These critical values are useful because they can be compared to "ball-park" estimates for the benefits. Based on this type of comparison, selected water use variables might be permanently set to their lower limits, thereby reducing the size of the mathematical problem for future planning analyses.

3.3.4 Application of the Separable Programming Model to an Example Region

A 14-county region in the western part of the state of Illinois was chosen as the sample region for an example application of the SP model. The selected region illustrates the range of conditions typically found in Illinois. It is bounded on the west by the Mississippi River, a major water source. Flowing through it is the Illinois River, the major river internal to the state. It also contains groundwater sources and potential reservoir sites. Some counties have abundant water supplies and some have water shortages. In addition, some of the counties have large coal reserves while others do not. These resources are indicated on the map in Figure 3.2.

The county water demands and the supplies of coal and water are specified in the same way as discussed in Section 2.3. A number of simplifications were made for this example, including these: nuclear plants were not considered, water returns were not specified, electricity transmission losses were neglected, and water treatment costs were assumed to be the same for all uses. In an actual application of the model, any of these factors could easily be included.

Demands for gas and electricity were specified for two points, one inside the region, at Springfield, and the second one outside the region, at Chicago. The demand levels for electricity were based on projections made for these areas by

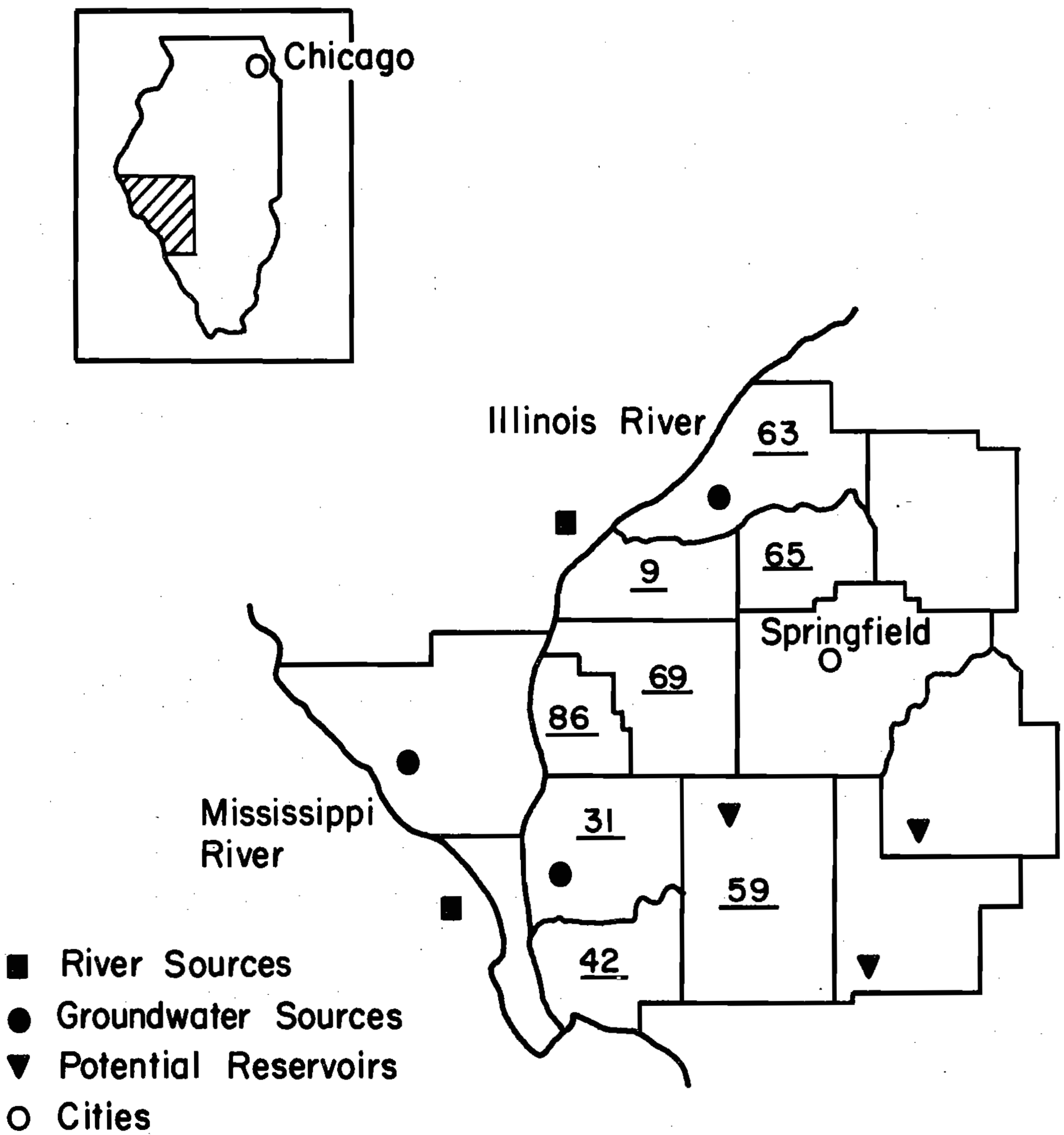

Figure 3.2 Fourteen County Example Region in Illinois

the FPC (1971), and the gas demand levels were selected on the basis of rough estimates; the aggregate demands were 6500 MW and 1350 x 10^6 SCFD, given in Table 3.1, and the water-requirement ranges for the energy facilities were as given in Table 3.2.

Assuming that a computer code with provisions for simple upper bounds is used (such as the IBM/MPSX Code used in this study [IBM, 1971]), the number of constraints required by Transformation 2 is less than that required by Transormation 1, using the δ-formulation of the SP. Therefore, Transformation 2 was used, and three piecewise linear segments were used for each nonlinear function. An example of the series of constraints required by the formulation is given by Velioglu (1976).

The program, as applied to the sample region, had 459 constraints, of which 336 were required by the approximations of the nonlinear functions. The problem had 1736 variables, of which 672 were bounded variables for the linear segments of the approximations.

The initial series of computer runs was aimed at gaining insight into the applicability of separable programming. As suggested earlier, the unit benefits for going above the minimum water requirements were initially set to zero, and the SP was solved using the restricted basis approach. The solution, which has an objective value of $78 million (annual cost), is partially presented in Table 3.3. LP solution included in that table was obtained by solving the problem with the water requirements set to their lower bounds a priori. The latter solution is the global optimal solution with an objective value of $78 million, since the unit benefits are zero.

The objective function of the local optimum obtained using SP is within 1.5 percent of that from the global optimum, although the solutions are different with respect to the size and location of the power plant capacities. The cooling methods chosen for both cases consisted of cooling ponds and spray ponds. With one exception, both solutions specified the same counties as locations for energy-facility capacities, although the coal-gasification and coal-fired electric plants were interchanged. The LP model required 599 iterations to reach the optimum in about 0.65 minutes of CPU time, whereas the SP model required twice as many interations and approximately five times as much CPU time. This difference results from the increase in the number of extreme points produced by the grid linearization in the SP model.

The second set of computer runs was directed at improving the computational efficiency of the SP approach by identifying those nonlinear functions for which the separable logic would not have to be enforced explicitly. For the case where benefit coefficients are zero, it can be shown that it is not necessary to enforce explicitly the separable logic for some of the piecewise approximations (Velioglu, 1976). In particular, this requirement can be omitted for the approximations to the second nonlinear terms of the transformations--e.g., $(wg^a_{m_2} - s^a_m)^2$--and for the analogous terms for electric plants. As expected, the CPU time was decreased (to 2.36 as opposed to 3.25 minutes). Several "violations" of the logic did appear, but each corresponded to an alternate optima which was the globally optimal solution found by LP and given in Table 3.3. Thus, it is possible to significantly reduce computing time by omitting the unnecessary requirements. In addition, it was shown that by changing the iteration policies or even the order of the data, different SP solutions were produced. This implies that different local optima are identified and can be further evaluated in the planning process.

In addition to its simplicity in application, separable programming offers another very powerful capability, parametric programming, which allows one to observe the effects on the overall problem of changes in the right-hand sides, constraint coefficients, and cost coefficients. Hence, it can play an important role in the process of evaluating major policy issues.

Example parameterizations were performed on:

(1) Cooling system costs
(2) River water availability
(3) Water supply costs for power generation
(4) Energy shipment costs

Of the four cases, the first two parameterizations produced significant

Table 3.3 - Partial Solutions to the Separable Programming and Linear Programming Models

County Name	Cooling Alternative	Number of Unit-Size Gasification Plants: Separable Programming	Number of Unit-Size Gasification Plants: Linear Programming
Scott (86)[a]	Spray Ponds	-	0.099
Cass (9)	Spray Ponds	-	0.054
Mason (63)	Spray Ponds	0.025	0.079
Menard (65)	Spray Ponds	0.066	1.07
Morgan (69)	Spray Ponds	3.91	2.85
Macoupin (59)	Spray Ponds	1.00	0.85
		Number of Unit-Size Electric Plants: Separable Programming	**Number of Unit-Size Electric Plants: Linear Programming**
Macoupin (59)	Cooling Ponds	3.91	-
Greene (31)	Cooling Ponds	0.42	-
Jersey (42)	Cooling Ponds	0.27	0.27
Scott (86)	Spray Ponds	0.13	-
Cass (9)	Spray Ponds	1.43	-
Mason (63)	Spray Ponds	0.071	-
Menard (65)	Spray Ponds	0.047	-
Morgan (69)	Spray Ponds	0.071	-
Macoupin (59)	Spray Ponds	-	5.73
Greene (31)	Spray Ponds	-	0.49

[a] County numbers as given in Figure 3.2.

changes in the cooling systems specified by the solutions. Also, different water and energy supply patterns were observed at each parameterization increment. On the other hand, Cases 3 and 4 had almost no bearing on the choice of cooling alternatives until the extremes of the parameter values were reached.

In Cases 3 and 4, however, all changes in the objective functions can be attributed to the changes directly imposed by changes in the water supply and energy shipment costs; the resource and energy shipments and the power facility sizes and locations were not affected significantly. Interestingly, the total cost for supplying municipal water to each county was relatively insensitive to changes in all four cases (Velioglu, 1976). For the example region, it was found that changes in cooling system costs have the greatest influence on the objective function, followed by energy shipment costs, river water availability and water supply costs for power generation.

So far, the benefits (e.g., BG_m^a) for going above the lower bounds of the water requirements were assumed to be zero. There were two reasons for this assumption: (1) benefit coefficients are usually not known and (2) the "goodness" of the solutions could be checked by running an equivalent LP model to find the globally optimal solution. It is important to note, however, that an SP solution has more information than the corresponding LP solution about the effect of such benefits. Specifically, the critical values are given at which these benefits would change the optimal solution. For example, when the average values for cooling-system costs given by Dynatech (1969) were used, the critical monetary levels varied from \$54 to \$214. Otherwise, it is not worthwhile to use additional water. Similarly, for Roa's (1972) costs, these levels varied from \$18 to \$165/GPM. It should be noted that the above levels are a function of plant size, location and type, and of the cooling alternative in question.

To observe the effect on overall problem behavior of using nonzero benefit coefficients, a run was made where the benefit levels were arbitrarily set 20% higher than the critical values found in the SP solution in the counties where power facilities were specified. The new solution was found using SP, and as expected it was observed that assigning sufficiently high benefits to these variables did indeed affect the size and location of power plants as well as the cooling alternatives used and produced water usage levels greater than their lower limits (Veglioglu, 1976)

If benefit coefficients are not considered, one need not worry about the degree of approximation introduced by grid linearization, because no excess water (u_m^a) will be used. The water demand constraints (equation 3.10) guarantee the shipment of the minimum amount of water to every subregion with power facilities. In this case, then, the exact values of the water requirements (WG_m^a), as determined by the model, are not important. They should equal their lower bounds and are treated accordingly, even if the linear piecewise approximations produce slightly different values. If the benefits of extra water are to be considered, however, the extra water usage is important. As shown by Velioglu (1976), the values obtained from the model were suitable for planning purposes, since they were within five percent of those calculated from the s_m^a and WG_m^a values.

Incidentally, the water demand constraints also imply that any feasible solution to the approximate problem is also a feasible solution to the original problem, since these are the only constraint sets that tie the approximations to the original problem and since these constraints ensure that at least the minimum water requirements are met in each subregion.

3.4 USING MIXED INTEGER PROGRAMMING TO SOLVE THE NONLINEAR PROGRAMS

To develop an understanding of the computational performance of the separable programming algorithm using restricted basis entry as compared to the mixed interger formulation given by Hadley (1964), a hypothetical, 3-county region with three water supply points and two energy demand points was considered. A very small region was used because of the computing limitations posed by the integer variables.

The example problem was solved as (1) a linear program (LP), (2) a separable program using restricted basis entry (SP), and (3) a mixed integer program using integer variables instead of restricted basis entry (MIP). The mixed integer programs were solved using an optimal feature of the IBM/MPSX code (IBM, 1973). A substantial increase was observed in the number of constraints, variables and the required CPU time in going from the LP to the SP method and from the SP to the MIP method. The following observations were made:

(1) SP (with restricted basis entry) may indeed yield a "very good" solution within a reasonable amount of computer time as compared to LP.

(2) MIP is very problem dependent and may require a considerable amount of time to reach the optimal solution. (In one case, for the very small example problem, the optimal solution to the MIP formulation was not found after 12 minutes of CPU time.) Thus it may be impractical even for very small problems, and the expected running times increase exponentially for larger problems. It is also possible to use a mix of restricted basis entry and linear variables to enforce separable logic. This approach, however, leads to a significant trade-off between computation time and the attainment of solutions better than those found using SP alone. Details regarding these example runs and the observation described above are given by Velioglu (1976).

3.5 CONCLUSION

Section 3.2 demonstrated that it is very practical to allow for alternative cooling technologies in a linear programming model to be used in planning a water resource-energy system. Since different mixes of cooling systems were found under different conditions for the sample application for Illinois, it is recommended that these systems be included at the preliminary screening stage of a planning activity if they are to be considered.

In section 3.3, it was shown that a nonlinear formulation can be used to consider different levels of water usage for each cooling alternative. A separable programming approach to solving this problem was shown to be practical even for a large region; the method was demonstrated for a 14-county region in western Illinois.

For this type of problem a detailed analysis of the convexity and concavity properties is of utmost importance. Even though the problem may not have a convex objective function (for minimization) and a convex feasible solution space, one can still take advantage of the convex portions of the objective function and/or the feasible region if they can be identified. For the separable programming algorithm it is not necessary to explicitly enforce separable logic for the piecewise linear approximations of the convex portions of the objective function (for minimization) and/or of the solution space. Omitting this requirement leads to a significant improvement in computational efficiency for the problem considered here.

The separable logic for the piecewise linear approximations can be effectively enforced by using restricted basis entry, and this approach was observed to yield "good" solutions within a reasonable amount of time for both large and small problems. In contrast, however, as shown in Section 3.4, the mixed integer programming approach was observed to be impractical because it requires long computation times even for very small problems.

CHAPTER 4: A NETWORK FORMULATION

4.1 INTRODUCTION

This chapter presents a network solution method for the basic model described in Chapters 1 and 2. Network algorithms have proven to be far more computationally efficient than the simplex method for solving network problems, leading to potential cost reductions which are especially important in solving large problems (e.g., multiperiod) or solving the same problem many times with different input data.

Water resources systems have been modeled as network problems before. For instance, Hamdan and Meredith (1975) report that a network algorithm solved their network of groundwater and surface-water systems in one-fifteenth of the computer time required by the simplex method. The Texas Water Development Board (1970, 1971, 1974) also applied network algorithms in the models they used in developing the Texas Water Plan.

Because of the strict form required for network formulation, not all aspects of the system under consideration here can be included in the model. It will be shown, however, that these aspects may either be omitted from consideration because they have little effect on the system or may be dealt with heuristically.

The network algorithm used here was the Share Out-of-Kilter Network (OKA) Routine available from IBM (1967) and described by Clasen (1968). A brief overview of the OKA and network formulations is followed by formulation of the basic model in Sections 4.2 and 4.3. The concept of expressing electrical power and pipeline gas flows in terms of water equivalents is presented. Section 4.4 discusses the advantages and the disadvantages of using the OKA and compares the computational results obtained by linear programming with those from the out-of-kilter algorithm for the example application for Illinois.

4.2 NETWORK FORMULATION

The OKA is a network-optimizing algorithm. The type of network considered here, shown in Figure 4.1, consists of a set of numbered points called nodes, i,

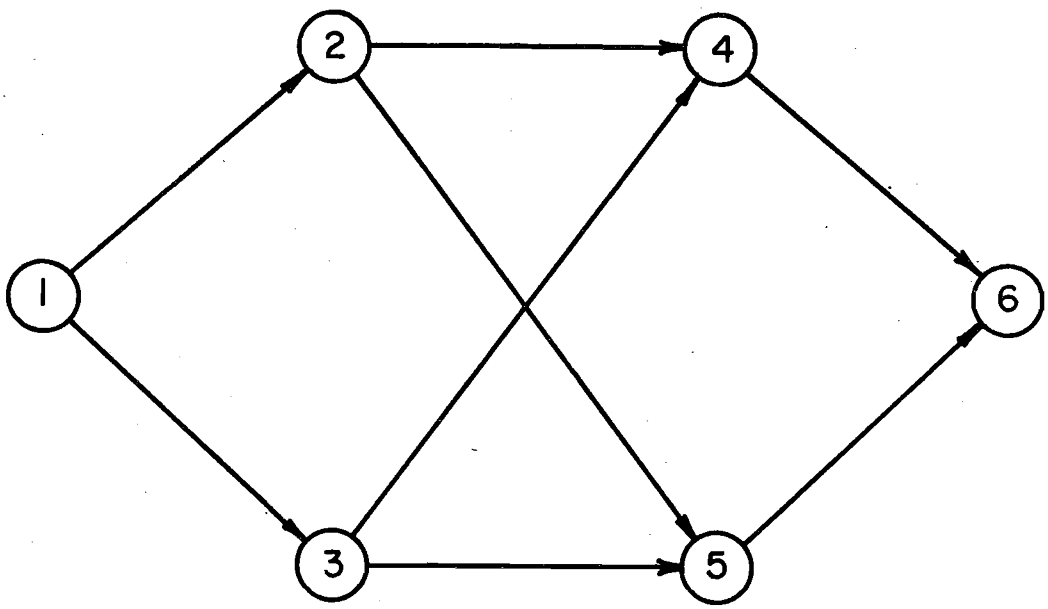

Figure 4.1 Example Network

and a set of directed line segments called arcs, (i, j), that connect pairs of nodes, i and j. These arcs can carry the flow of a commodity such as water in the direction of the arrows. Each arc is assigned a unit cost and a lower and upper capacity for the flow in that arc. For the example network, nodes 1 and 6 are called the super source and super sink, respectively. The network problem considered here consists of routing a known amount, X, of a commodity from the super source through the network to the super sink in such a way that the total cost is minimized and the capacity bounds on each arc are not violated. As discussed in Section 4.3, it is important that all flows be of a single commodity.

In a general mathematical formulation of the network, the total cost to be minimized can be written as:

$$\text{Min } z = \sum_a C_a f_a$$

where C_a = unit cost of arc a

f_a = flow on arc a

N = number of nodes

Three sets of constraints apply. The first set consists of N constraints, one for each node. These constraints insure that Kirchoff's law is not violated for each node; that is, the amount of flow into the node is equal to the amount of flow out. The second set pertains to the lower bounds on the flow in each arc. Those lower bounds can be used to ensure that demands, such as these for electricity or pipeline gas, will be met; for example, a lower bound on arc (4, 5) in Figure 4.1 could be interpreted as a demand at node 4. The third constraint set refers to the upper bound on the flow in each arc. Mathematically, these constraints may be written as:

Set 1

$$\sum_i f_{ij} - \sum_k f_{jk} = 0 \qquad \forall \text{ nodes } j \qquad (4.2)$$

Set 2

$$f_{ij} \geq L_{ij} \qquad \forall \text{ arcs } (i,j) \qquad (4.3)$$

Set 3

$$f_{ij} \leq U_{ij} \qquad \forall \text{ arcs } (i,j) \qquad (4.4)$$

Our water-resources and energy system (see Figure 5.2) involves flows of four different commodities: water, gas, electricity and coal. A network formulation to be solved by a network algorithm, however, must have flows that can be represented by one commodity. This restriction on the formulation is a result of Kirchoff's law as expressed in the first constraint set: that which flows into a node (other than the source or sink) must also flow out of the node. If water, gas, and electricity flows are all to be incorporated into the desired network formulation, they must be converted to equivalent flows of a single commodity.

The flow of electrical power from node j to node k can be expressed as the amount of water required to produce that flow. For example, the production of x megawatts (MW) of electricity is assumed to require K•x gallons per minute (GPM) of water, where K is a constant. As discussed in Section 2.2, arbitrary constants were used to match inputs and outputs of energy facilities. Using the data from Table 2.1, K would be 15000/1000 (GPM/MW) if nuclear capacity is assumed. With similar reasoning, an electrical power demand, x MW, could be expressed in water units as K•x. The continuity constraint for electric facilities in subregion j could be written as:

$$\sum_{i} f^{W}_{ij} - \sum_{k} f^{E}_{jk} = 0 \qquad (4.5)$$

The first term sums the water flows from the water sources into subregion j, and the second term gives the electricity flows out of subregion j to electrical power demand points. All flows are expressed as water equivalents (GPM). It is also necessary to adjust the cost coefficients on the electricity flows, since they are expressed in GPM units. If the original unit cost is C_{ij} (\$/MW), then the unit cost, C^E_{ij}, for f^E_{ij} would be $(1/K)C_{ij}$. A continuity equation must also be written for each electrical demand node, j, as follows:

$$\sum_i f^E_{ij} - f^E_{js} = 0 \qquad (4.6)$$

The first term sums all electricity flows (in GPM equivalents) into demand node j, and the second term routes this flow to the super sink, s.

In a similar manner, the flow of pipeline gas (standard cubic feet per day, SCFD) can be expressed in terms of the water required to produce the gas. The conversion constant, K', based on the data from Table 2.1, would be 14000/250 x 10^6 (GPM/SCFD). The continuity equation for gasification facilities in subregion j would be:

$$\sum_i f^W_{ij} - \sum_k f^G_{jk} = 0 \qquad (4.7)$$

The first term sums the water flows from water sources into subregion j, and the second term sums the gas flows out of subregion j to gas demand points. All flows are expressed as water equivalents. If the original unit cost for piping gas is C_{ij} (\$/SCFD), then the unit cost C^G_{ij}, for f^G_{ij} could be $(1/K')C_{ij}$. The continuity equation for a gas demand node j would be:

$$\sum_i f^G_{ij} - f^G_{js} = 0 \qquad (4.8)$$

The first term sums all gas flows (GPM equivalents) into j, and the second term routes this flow to the super sink.

A unit-size gasification plant is assumed to require 3000 GPM for process water and 11000 GPM for cooling water, giving a total water requirement of 14000 GPM. Since process water costs more to develop (because of more stringent treatment requirements), the cost, C_m, of supplying water to gasification facilities is the weighted average of C_p and C_c:

$$C_m = 3/14\ C_p + 11/14\ C_c \tag{4.9}$$

where C_p and C_c are process-water and cooling-water development costs, respectively.

Note that the use of equivalents may be applied to any system where many different commodities are produced from a set of resources of which only one limits production. The procedure is the same as described above; each product flow is expressed as the flow of the limiting resource required to manufacture that product. Since all resources except the limiting resources are abundant, they need not be considered. Of course, every solution should be checked to insure that the abundant resources are not overdrawn. Even if they are, a heuristic method could modify the solution to make it feasible.

The complete network mathematical formulation of the water-resources and energy system is given below. The objective function sums the costs for the flows of water, electricity and gas (all in water equivalents). The sets of arcs for water, electricity, and gas flows are represented by A_W, A_E, and A_G, respectively. The nodes are illustrated in Figure 4.2 for each constraint set as indicated by the capital letters in parentheses.

$$\text{Min } z = \sum_{A_W} C_{ij}^W f_{ij}^W + \sum_{A_E} C_{ij}^E f_{ij}^E + \sum_{A_G} C_{ij}^G f_{ij}^G \tag{4.10}$$

Such that:

$$\sum_i f_{ij}^W - \sum_k f_{jk}^W = 0 \quad \forall \text{ nodes representing water source points and MID points (A,B,C, and D)} \tag{4.11}$$

$$\sum_i f_{ij}^W - \sum_k f_{jk}^E = 0 \quad \forall \text{ nodes representing electricity supply points (E)} \tag{4.12}$$

$$\sum_i f^W_{ij} - \sum_k f^G_{jk} = 0 \quad \forall \text{ nodes representing gasification supply points (F)} \tag{4.13}$$

$$\sum_i f^E_{ij} - f^E_{js} = 0 \quad \forall \text{ nodes representing electricity demand points (G)} \tag{4.14}$$

$$\sum_i f^G_{ij} - f^G_{js} = 0 \quad \forall \text{ nodes representing gas demand points (H)} \tag{4.15}$$

and each flow, f_{ij}, is within the range given by the upper and lower limits, U_{ij} and L_{ij}, respectively. Where f^W_{ij}, f^E_{ij}, and f^G_{ij} represent flows of water, electricity, and gas, respectively, all expressed in water units (GPM). U^W_{ij} (L^W_{ij}), U^E_{ij} (L^E_{ij}), and U^G_{ij} (L^G_{ij}) refer to the upper (lower) capacities of arcs that carry water, electricity, and gas respectively. This formulation yields a network representation of the system in terms of a single commodity, water, and it can be solved using the out-of-kilter algorithm or any other transshipment algorithm. A detailed example is described below to illustrate the formulation.

The static model for Illinois is illustrated in Figure 4.2. It has 245 nodes and 6922 arcs. Nodes 1 and 245 indicate the super source and super sink, respectively. The arcs leaving node 1 have upper bounds that reflect the availability of water at the different water sources. The water sources, SW1, SW2, SW3, represent the upper, middle, and lower segments of the Mississippi River, respectively. They must be handled differently from the other water sources, since the water available in the two downstream segments is equal to the sum of the water available in each of those river segments plus upstream water supplies that have not been withdrawn. Arcs (SW1, SW2) and (SW2, SW3) must therefore be included to represent the transport of available upstream water to the downstream segments of the river. The set of arcs from node 1 to the thirty different water sources constitutes class 1.

Class 2 arcs are those from the water sources to the counties. A county may be represented by 1, 2 or 3 nodes, each representing one of three possible water demands within the county: MID, coal gasification, and electrical power generation.

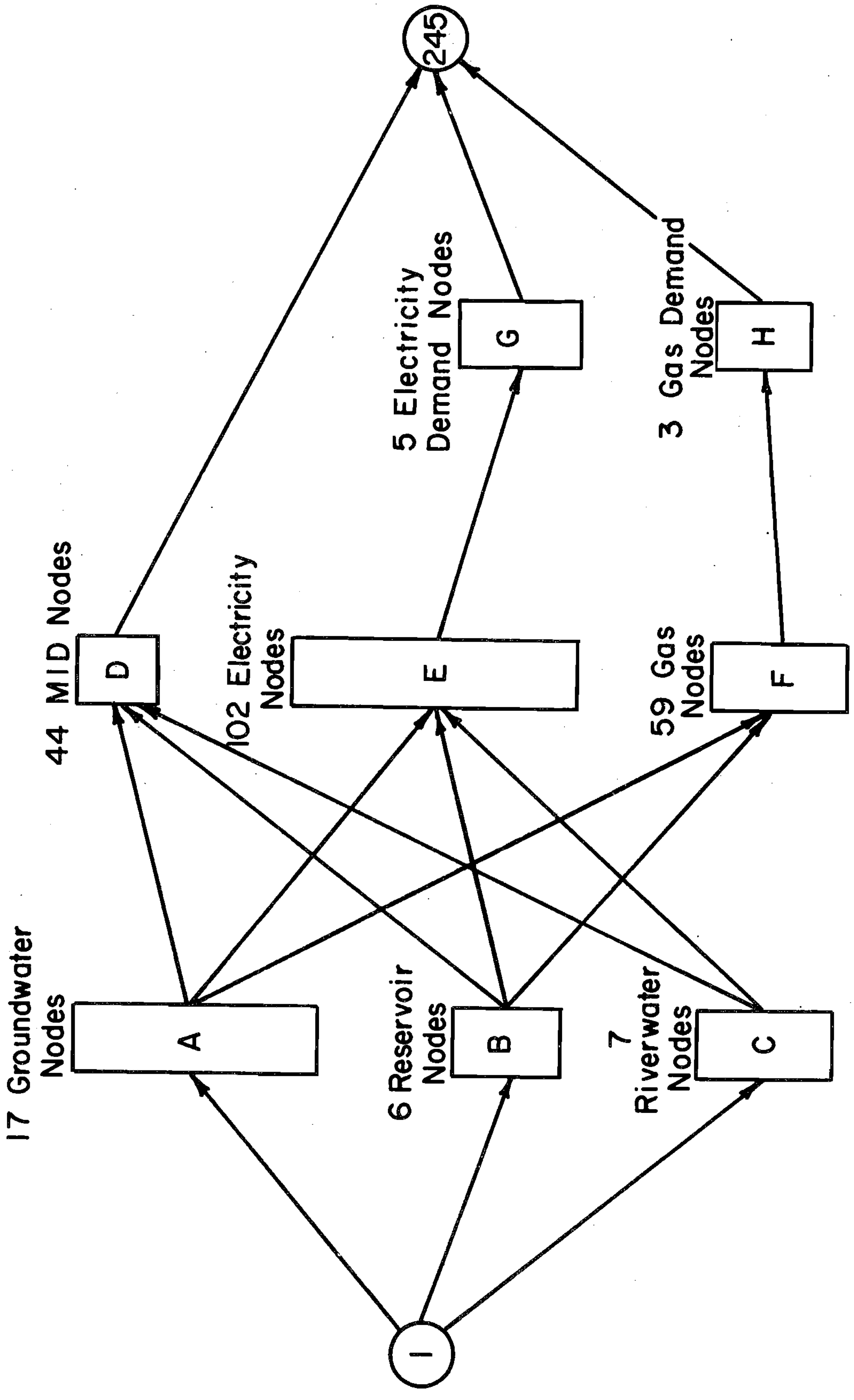

Figure 4.2 Network Model for Illinois Application

All counties are potential locations for electrical generating capacity, 59 are suitable for coal gasification, and 44 will require additional MID for 1980. Thus, 205 nodes are required to represent 102 counties.

From the counties, the gas and electricity nodes route water-equivalent flows of power to the gas and electricity demand nodes, respectively. The MID nodes send their flows to the super sink. This set of arcs constitutes class 3. The arcs from the demand nodes to the super sink have lower bounds which require that the water, gas and electricity demands be met. The costs and bounds for all of the arcs in the network are summarized in Table 4.1.

4.3 DISCUSSION AND CONCLUSIONS

The advantage of network algorithms over the simplex method is that they are far more efficient computationally, resulting in reduced computer times and costs. The OKA is easy to use, requiring only a small deck of cards coded in Fortran. There is, however, a disadvantage: since the mathematical form required by network algorithms is a special form of the LP problem, certain constraints from the more general LP model presented earlier cannot be incorporated into the OKA formulation. Two possible responses to this situation are:

(1) If the OKA solution satisfied the constraint in question, the OKA solution can be accepted as satisfactory.

(2) If the OKA solution violates the constraint in question, a heuristic method may be used to modify the OKA solution in such a way that the constraint is not violated.

The heuristic solution may not be optimal, but ideally it will be a good solution attained with far more computational efficiency than that required by LP methods. Furthermore, such heuristics are appropriate for a simplified preliminary screening model.

There are two constraint sets that cannot be incorporated into the OKA formulation and are violated by the OKA solutions. Since only one commodity can be considered, constraints cannot be added to limit directly the supply of coal in each

Table 4.1 - Summary of Network Data

Class	Arc (i,j): i	Arc (i,j): j	Lower Bound, L(i,j)	Upper Bound, U(i,j)	Cost, C(i,j)
1	super source	water source	0	water availability at source i	0
2	water source	MID node	0	∞	water supply cost
	water source	electricity supply node	0	∞	water supply cost
	water source	gas supply node	0	∞	water supply cost
3	MID	super sink	MID	MID	0
	electricity supply node[a]	electricity demand node	0	∞	electricity transmission cost
	gas supply node[a]	gas demand node	0	∞	gas transmission cost
4	electricity demand node	super sink	electricity demand	∞	0
	gas demand node	super sink	gas demand	∞	0

[a] flows must be in water equivalents

county or to allow more than one type of electrical generation. (Limits can, however, be placed on gasification and electrical power generation separately). The omission of these constraints in the formulation is not as serious as it may seem. Since Illinois has abundant coal reserves, we found that the coal supply constraints were tight in only four or five counties. In fact, in some counties the plant capacities were so large that they might be reduced because of other social, political, or environmental considerations. A practical approach, which would readily fit into the network model, would be to impose limits on the total plant capacity of each type in each subregion.

An easy and computationally efficient solution method may be to modify the network algorithm solution with a heuristic step if the network algorithm yields a solution which is optimal but is not really feasible because of the extra constraints.

Future research should be directed toward developing alternative solution procedures more fully. Several alternative approaches should be examined. For instance, it can be shown that a network-with-gains algorithm can be used to incorporate both nuclear and coal-fired electricity capacity into the network formulation. A network with gains (see Jewell, 1962; Johnson, 1966; Maurras, 1972) is a more generalized network where the continuity of flow at a node is expressed as:

$$\sum_i K_{ij} f_{ij} - \sum_k f_{jk} = B_j \tag{4.16}$$

If the network solution is infeasible because of limits on coal use or other constraints, another approach is to use the dual simplex algorithm starting with the network solution. This approach would produce the optimal solution, and the computation time would most likely be significantly less than that required by the oridinary LP method. Also, heuristic methods can be used to provide good, feasible solutions in many cases. Several possible heuristics exist. For example, if coal constraints are needed but are violated in a network solution, a feasible solution could be obtained by simply interchanging some of the nuclear and coal-fired electricity capacities or by moving the excess capacity to nearby subregions where coal is available. These heuristic changes could be made with very simple calculations.

In some cases, changes in the network solution may lead to the optimal solution which would be obtained using LP. For example, the LP model allows gasification facilities to draw process water from one source and cooling water from another source. On the other hand, although the OKA will allow these facilities to draw water from more than one source, each source must send a mix of cooling and process water in the same proportion as the mix required by a unit plant. In the example application comparing the LP and the OKA methods, this split-flow issue arose once in Madison county. The LP solution calls for the Madison county gasification facilities to withdraw process water from a groundwater source and cooling water from the Mississippi River. Since the groundwater is of higher quality, it makes sense to use it for process water while the surface water is used for cooling. The OKA solution, on the other hand, specified that the Madison county gasification facilities would withdraw water from the same two sources, but cooling and process water were withdrawn from each. This situation can easily be remedied by heuristically reassigning flows.

The first variation of the basic LP model was run for the Illinois case using MPSX (IBM, 1971 and IBM, 1973) on the IBM 360/75 at the University of Illinois. Starting without a basis, it required approximately seven minutes to execute at a cost of \$36. The IBM SHARE OKA Routine solved the network in 3.8 minutes at a cost of \$18. The reduction in time and cost is significant, and the difference is even more dramatic than it appears from this comparison because the IBM SHARE code used is outdated. As indicated by Hultz *et al.* (1976) there are at least two codes which are vastly superior. In particular, results from several runs in which various codes were tested on large transshipment problems indicate that SUPERK, a more recent OKA code, solved the test problems in about one-fourth of the time required by IBM SHARE. Another code, PNET-I, a special primal simplex algorithm, required only approximately one-tenth of the time required by SHARE. Based on this empirical evidence, it appears that the network described here could be solved using available computer codes with solution times vastly improved over those indicated. For instance, the $18 cost might be reduced to approximately $2 if PENT-I were used. This saving becomes important if we consider larger problems such as the multiperiod problem or the water-energy systems of more than one state.

Present research is investigating in detail several possible heuristic methods of modifying the OKA solutions. The use of a network algorithm and heuristic methods will very likely lead to relatively efficient methods of evaluating a water-resources and energy system for preliminary screening purposes.

APPENDIX A: MULTIPERIOD PROBLEMS

The planning horizon used in a given application can be divided into several time periods, so larger demands for water, electrical power, and pipeline gas can be specified for each successive period. The mathematical models can be readily extended to handle several time periods, and in some cases the additional realism may be worth the extra computing expenses. This appendix describes how the general linear programming model and the network model can be extended, using two time periods as an example. The same approach, however, can be taken for any number of periods.

One set of variables (as outlined in Chapter 2) is used to describe the commodity flows and energy capacities for the first period, and a similar set of variables is used for the second time period. For example, d_{im} and d'_{im} can be used to specify the amount of groundwater transported from source i to subregion m during the first and second periods, respectively.

Supply constraints for each commodity are written as in Chapter 2. For instance, the supply constraint for groundwater source i would be:

$$\sum_m (d_{im} + d'_{im}) \leq D_i \tag{A.1}$$

The total of the two withdrawal rates must be less than or equal to the supply available at source i.

One constraint is also needed for the new demand for each product in each time period. For example, if MID1 and MID2 represent the MID demands for additional water in each period, the the following constraints are needed:

$$\sum_i d_{im} + \sum_j w_{jm} + \sum_k r_{km} \geq \text{MID1} \tag{A.2}$$

$$\sum_i d'_{im} + \sum_j w'_{jm} + \sum_k r'_{km} \geq \text{MID2} \tag{A.3}$$

Similar constraints are needed for energy-facility water demands and for electricity and pipeline gas demands.

The number of variables required in a two-period formulation would be approximately double the number required for a single-period formulation. The number of constraints would increase by less than double because new constraints are needed only for the demands. The cost coefficients, of course, would be discounted for the second time period.

The multiperiod problem can be formulated as a network problem by duplicating part of the network for each period. Since the resources are assumed constant over time, their nodes are drawn only once. The nodes representing subregions and demands, however, must be duplicated. Flows during the first period circulate through the original set of nodes and arcs, while flows during the second period are restricted to the second set of nodes and arcs. Part of a multiperiod network for a two-period problem is shown in Figure A.1. The river-water nodes represent resources and are drawn once. Flows for the two periods are routed in parallel through coal-gasification nodes and gas demand nodes to the super sink. The complete network would be drawn in a similar way to include all of the categories of nodes and arcs as given in Figure 4.2. As discussed in Chapter 4, all flows would be considered in terms of water equivalents.

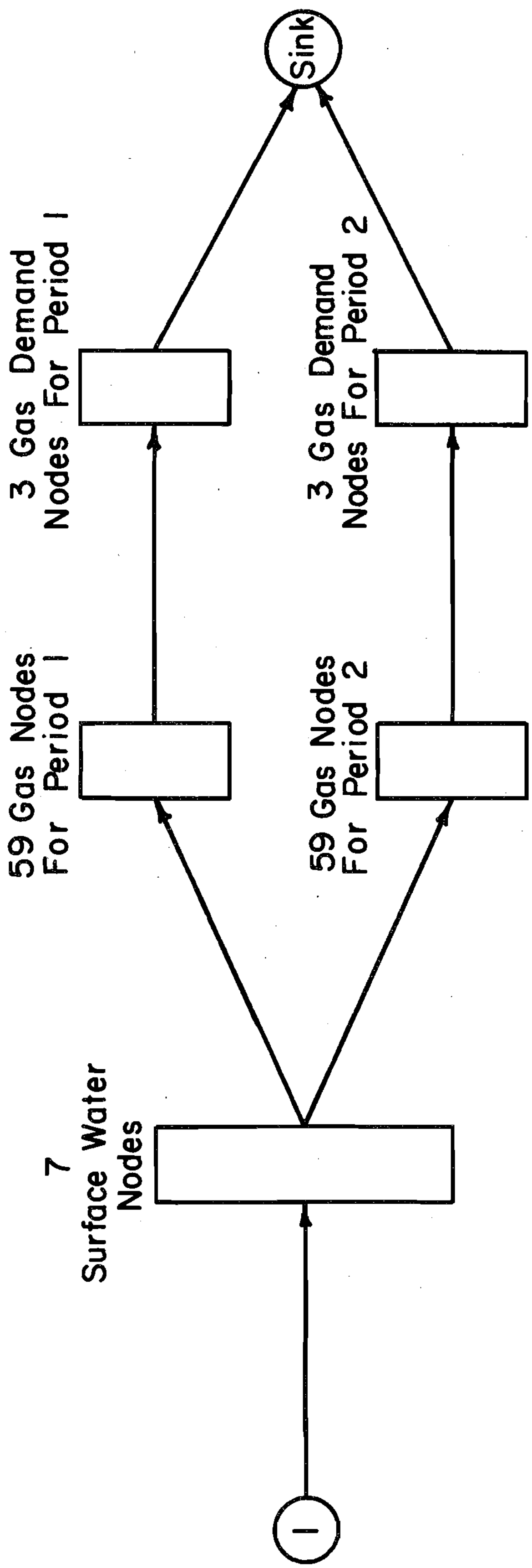

Figure A.1 A Subnetwork for the Multiperiod Problem

APPENDIX B: ANALYSIS OF COSTS OF TRANSPORTING COAL, WATER, PIPELINE GAS, AND ELECTRICITY

The models presented here were demonstrated using the assumption that coal from within Illinois would not be transported to new power plants located on waterways. Instead, it was assumed that water would be piped, if necessary, to power plants located at the coal source. This assumption followed an economic analysis of the costs of transporting the resources, water and coal, and the product, pipeline gas or electricity. The water requirements were based on the use of cooling towers (or spray ponds or cooling ponds) rather than once-through cooling; it would not be economical, of course, to pipe water long distances for once-through cooling.

For example, the costs of transporting water, coal, and pipeline gas for an arbitrary unit-size plant were evaluated using the following cost equations (given in annual cost form using 1973 dollars).

Coal (1600 tons per day)

$$C_c = .20(365)(L^{.5})(16000\ \text{tons/day}) \qquad (B.1)$$

Water (22000 gpm)

$$C_w = 2.4(L)(22000\ \text{gpm}) \qquad (B.2)$$

Pipeline Gas (250 million SCFD)

$$C_g = 56(L)(250\ \text{million SCFD}) \qquad (B.3)$$

where in each case L is the distance (miles) that the given commodity is transported. The three equations are derived from Mutschler et al. (1973), Singh et al. (1972), and the Federal Power Commission Bureau of Natural Gas (1974), respectively. It can readily be shown that, using these cost estimates, for small values of L it would be more costly to transport coal from any point A to any water supply point B than it would be to transport water from B to A and to transport the product from A back to B, if necessary. This result holds for any value of L less than about 300 miles using the equations shown. (This distance exceeds the distances considered in the example study.) It should be noted that the above comparison assumed a rather large water requirement of 22000 gpm for a unit-size coal-gasification plant; a lower requirement, such as the 14000 gpm listed in Table 2.2 for the example problem, would strengthen the result.

Similar results were reached after an analysis for coal-fired steam-electric power plants. The water and coal requirements and the electricity produced by an arbitrary unit-size plant are given in Table 2.2, and Equation B.4 was used to estimate electricity transmission costs.

$$\frac{\text{Electricity Transmission}}{\text{(MW)}} \qquad C_e = 4.4(L) \qquad \text{(B.4)}$$

Thus, it was assumed in the example application of the models that coal would not be transported significant distances within Illinois to plants located at water sources. In other cases, of course, the relevant cost functions and the critical values of L (such as the 300 miles mentioned above) could be different; and, if necessary, coal shipments could be readily incorporated into the models. The additional variables and linear constraints that would be needed are presented in the general formulation given in Chapter 2. Such a change might be necessary when considering once-through cooling, significantly different cost relationships, or very long transportation distances.

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