WRC RESEARCH REPORT NO. 122

OPTIMAL OPERATION OF FLOOD CONTROL SYSTEMS

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FINAL REPORT: VOLUME II

Project No. A-079-ILL

This project was partially supported by the U. S. Department of the Interior in accordance with the Water Resources Research Act of 1964, P.L. 88-379, Agreement No. 14-31-0001-6014.

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March 1977

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ABSTRACT

OPTIMAL OPERATION OF FLOOD CONTROL SYSTEMS

The management and control of multiple-reservoir flood control systems is studied. Our objective is to devise operating policies which minimize flood damages as determined by the flood peaks. Methodologies are presented that employ dynamic programming and stochastic dynamic programming for the optimal operation of multiple-reservoir flood control systems with deterministic and stochastic inflows, respectively. The methodologies are applied to a number of real-world problems involving river basins in Illinois and elsewhere. We then study the effects of parametrically varying a number of the input parameters. A Markov renewal flood synthesis model and a methodology for determining the optimal capacity of a new flood control reservoir are also presented.

Feng, C-J and Morin, T. L.

OPTIMAL OPERATION OF FLOOD CONTROL SYSTEMS

University of Illinois Water Resources Center Report No. 122

KEYWORDS -- flood control/reservoirs/dynamic programming/operation/water resource system

PREFACE

This report is one of the two volumes comprising the completion report for Project No. A-079-ILL. This volume deals with the operation of flood control systems, whereas the other volume deals with the planning of flood control systems. The authors would like to acknowledge useful discussions with E. Çinlar, R. Gemmell and S. Pliska.

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CHAPTER I

INTRODUCTION

1.1 Reservoir System Management and Flood Control

Despite the substantial expenditures on structural measures for flood protection, flood damages continue to increase. For example, at the local level on the upper Salt Creek Watershed in Illinois the annual damages are projected [41] to increase from $412,600 in 1970 to $853,000 in 1995. The increase is even more dramatic at the national level, where the total annual flood damage potential is estimated to increase from $1.7 billion in 1966 to $5.0 billion in 2020 on the basis of the current status of flood control improvements and the projected conditions of floodplain use [86]. These dramatic increases have given rise to increased awareness and interest in floodplain management and control. This is precisely the issue to which this study is addressed.

1.2 Review of Relevant Research

Several researchers have applied linear decision rules in discrete time parameter models in order to determine the optimal capacity of a flood control reservoir. ReVelle [75] developed a chance-constrained model for determining the optimal size and operating policy of a multi-purpose reservoir. Subsequently, ReVelle and Kirby [74] developed the model more rigorously, modified it to include evaporation loss, and used the basic model with four performance objectives, given the reservoir size. Eastman and ReVelle [25] developed a chance constrained model for minimizing the required capacity subject to chance constraints for water supply, recreation and flood control. Nayak and Arora [62] considered a chance constraint formulation of multiple-reservoir system in order to find the optimal reservoir capacity. Recently, Sobel [80] determined the optimal reservoir capacity by minimizing the maximum stored quantity and specified operating rules for multiple-reservoir systems in the case of independent inflow, assuming that the outflow of upstream reservoirs does not affect the downstream reservoirs.

There are also some papers which propose the use of linear decision, chance-constrained models for reservoir management, such as Morton [59], in which the demand for water is stochastic. Eisel [26] developed a model for a single reservoir which demonstrates the mathematical complexities resulting from the convolution problem in chance-constrained programming. Young and Asce [91] developed recursive algorithm in order to find the optimal operating policies for a single reservoir which minimize a strictly convex quadratic loss function while satisfying pre-specified target releases. They also studied the effects of stochastic inflows using a Monte Carlo approach. Roefs and Bodin [76] considered the multiple-reservoir system operation problem with energy production considerations. For other operating models see [87, 88]. However, none of the foregoing models consider the possibility of overflow which may be very important in flood control. More importantly, most models are for long term operation instead of individual storms.

There is also a body of relevant literature on descriptive mathematical models of reservoir systems. Çinlar and Pinsky [16, 17] considered the case where the release function is assumed to be arbitrary and the input process to be an additive process. They solved the storage equation and obtained a number of properties. Çinlar [13] generalized the model to the continuous time and state space case with arbitrary release functions. One of the basic features in the input process is the dependence on environmental factors. Çinlar [14] also studied one of the most general cases, in which the release function is an arbitrary continuous nondecreasing function vanishing at the origin, the environment process is a Markov Process and the input process is a nonstationary additive process defined on the environment process. Moran's [50, 51] model considered a constant environment input process and constant release rules. Bather [3] supposed a controlled Brownian motion process as a model of a reservoir. More recently, Pliska [70] considered the water level in a reservoir as a controlled diffusion process on a compact interval of the real line. These control models also did not consider the overflows.

Additional relevant literature will be cited and reviewed in the body of study where appropriate.

1.3 Purposes and Outline of the Study

The purposes of this study are:

(1) To formulate mathematical programming models for

a) the optimal control of both parallel and series multiple-reservoir flood control systems for both single-storm and long-term operation with both deterministic and stochastic inflows.

b) the determination of the optimal capacity of a new reservoir to add to an existing flood control system.

(2) To develop methods for sloving these models.

(3) To implement the solution methodologies on a digital computer.

(4) To illustrate and assess the practicality and feasibility of the solution methodologies using data from a number of real-world examples.

These purposes are realized in the following manner.

In Chapter II, mathematical models of the optimal operation of both single-reservoir systems and parallel and series multiple-reservoir flood control systems with the deterministic inflows are formulated. Dynamic programming is used to determine the optimal release policies. The dynamic programming algorithms are coded in FORTRAN IV and implemented on a digital computer. Several real-world examples are solved to illustrate the methodology.

In Chapter III, mathematical models of the optimal operation of both single-reservoir flood control systems and parallel and series multiple-reservoir systems with stochastic inflows are formulated. Stochastic Dynamic Programming is used to determine optimal release policies under the assumptions that the input probabilities are time independent and time-dependent respectively. The dynamic programming algorithms are implemented on a digital computer and used to solve several real-world examples. The effectiveness of these policies is compared with both actual policies and standard policies using both historical flow data and simulated flow data. A stochastic flood synthesis model is also presented.

The effect of varying various flood control parameters is studied in Chapter IV. A number of characterizations of optimal flood control policies are derived.

In Chapter V, the optimal expansion of flood control systems is modeled. Dynamic Programming is used to determine the optimal capacity of additional reservoirs in both the parallel and series cases. The dynamic programming algorithms are implemented on a digital computer and several real-world examples are solved to illustrate the solution methodologies.

The study concludes with a summary and recommendations for future research in Chapter VI.

CHAPTER II

DETERMINISTIC INFLOW

2.1 Introduction

In this chapter, the basic reservoir control model is formulated in §2.2 and §2.3, assuming a given input hydrograph. The linear programming and mixed-integer programming solution methodologies are reviewed and critically evaluated in §2.4. Dynamic programming algorithms, which overcome the shortcomings of these other methodologies, are presented in §2.5. The algorithms are implemented on Northwestern University's CDC 6600 and used to solve several real-world examples.

2.2 The Basic Model

Figure 2.1 depicts the physical structure of the hypothetical river basin systems under investigation. The extension of procedure to handle other system topologies should be immediately apparent.

For the series system shown in the Figure 2.1a, the basic assumptions are that:

(1) the water released from reservoir 1 at time t arrives at reservoir 2 at time t, i.e., instantaneous flow;

(2) each reservoir spillway is designed in such a way that it allows for any kind of release rate;

(3) the capacity C_2 of the downstream reservoir exceeds that C_1 of the upstream reservoir;

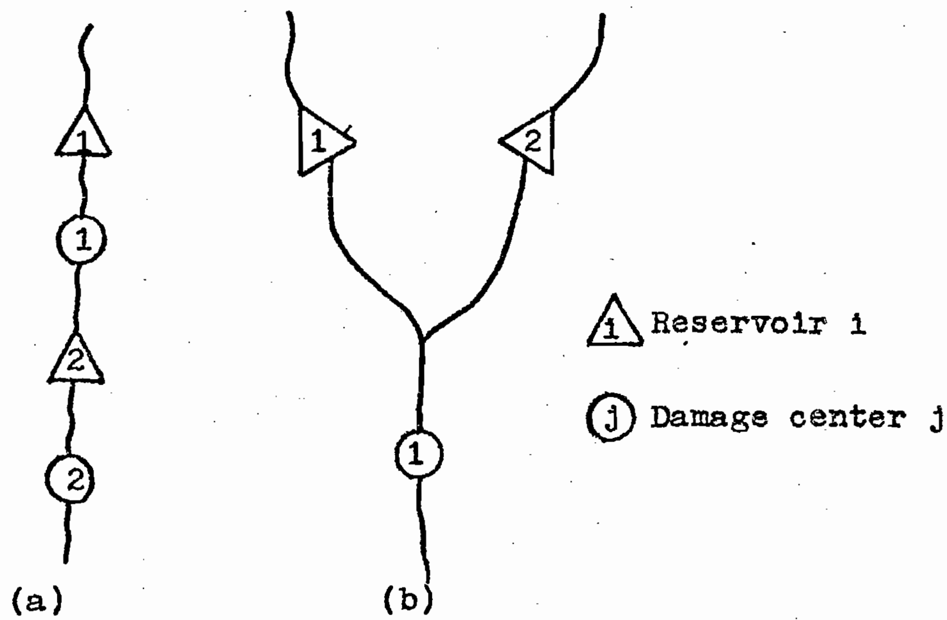


Figure 2.1 Proposed Multiple-reservoir flood control system. (a)Series (b)Parallel.

(4) the releases and inflows occur simultaneously.

The storage equation for the series system can be written as

$$Z_t^1 = Z_o^1 + \int_0^t Y^1(s)ds - \int_0^t r^1(s)ds \qquad t \geq 0, \tag{2.1}$$

and

$$Z_t^2 = Z_o^2 + \int_0^t Y^2(s)ds + \int_0^t (r^1(s) - r^2(s))ds \qquad t \geq 0, \tag{2.2}$$

in which $r^i(s)$ is the release rate of i^{th} reservoir at time s, $\{Y^i(t):t \geq 0\}$ is the deterministic input process of i^{th} reservoir, Z_t^i is the storage of the i^{th} reservoir at time t, and C_i is the storage capacity of the i^{th} reservoir.

The object is to determine $\{r^i(t) | i = 1, 2\}$ in order to

$$\underset{\{r^1(t), r^2(t)\}}{\text{Min}} \sum_{i=1}^{2} U_i(\hat{r}^i) \tag{2.3}$$

subject to the constraints

(1) $$0 \leq Z_t^i \leq C_i \qquad t \geq 0, \ i = 1, 2, \tag{2.4}$$

(2) $$0 \leq r^i(t) \qquad t \geq 0, \ i = 1, 2. \tag{2.5}$$

Here $U(\cdot)$ is the damage cost function and $\hat{r}^i$ is the peak release rate of reservoir i, i.e., $\hat{r}^i = \max_{t \geq 0} r^i(t)$.

For the parallel system shown in Figure 2.1b, the basic assumptions are as follows.

(1) The water released from reservoirs 1 and 2 arrives at the damage center at the same time.

(2) Each reservoir spillway is designed in such a way that it allows for any kind of release rate.

(3) The releases and inflows occur simultaneously.

The storage equations for the parallel system can be written as:

$$Z_t^1 = Z_o^1 + \int_o^t Y^1(s)ds - \int_o^t r^1(s)ds \qquad t \geq 0, \qquad (2.6)$$

and

$$Z_t^2 = Z_o^2 + \int_o^t Y^2(s)ds - \int_o^t r^2(s)ds \qquad t \geq 0. \qquad (2.7)$$

The object is to determine $\{r^i(t) | i = 1, 2\}$ in order to

$$\underset{\{r^1(t) + r^2(t)\}}{\text{Min}} U(r) \qquad (2.8)$$

subject to the constraints:

(1) $$0 \leq A_t^i \leq C_i \qquad t \geq 0, \; i = 1, 2, \qquad (2.9)$$

(3) $$0 \leq r^i(t) \qquad t \geq 0, \; i = 1, 2. \qquad (2.10)$$

Here $U(\cdot)$ is the damage cost function at the damage center, r is the peak flow rate at the damage center, and

$$r = \max_t \{(r^1(t) + r^2(t))\}.$$

Estimating potential flood damages is an important problem in planning water resources projects. Unfortunately, however, there is a paucity of published data for use by engineers and planners in making such damage estimates. However, it is reasonable to assume that flood damage can be modeled as a monotone nondecreasing function of the depth of flood water [42]. On some streams the stage-discharge curve is roughly parabolic [33], yielding an equation of the type

$$r = d(h - a)^b, \tag{2.11}$$

where r is the discharge (cfs),

a, b, d, are constants, and

h is the stage (ft).

Here we can see that the damage cost U is a monotone nondecreasing function of discharge rate.

The exact amount of flood damage may also depend on the time of year, the velocity of flow, the depth and duration of inunduation as well as the nature and quantity of silt deposited. It will be assumed in this study, however, that the critical factor is the peak discharge and the effect of all other factors are of secondary importance and may be neglected. We will also assume that we know or can estimate the damage cost function U(r).

2.3 Continuous Time Process Approximated by Discrete Time Process

The continuous time process can be approximated by a discrete time process. From the basic storage equation:

$$Z(t) = Z(0) + \int_0^t Y(s)ds - \int_0^t r(s)ds \tag{2.12}$$

where Y(s) is the input rate, we have

$$\dot{Z}(t) = Y(t) - r(t) \tag{2.13}$$

Recall that the object is to determine r(t), which satisfies $Z(t) \geq 0$, $Z(t) \leq C$ for $0 \leq t \leq T$, where T is the duartion of flood hydrograph, so as to minimize the return

$$I = U(\max_t r(t)). \tag{2.14}$$

The value of the continuous-time state variable Z(t) at the discrete time

$$t = i\Delta \qquad\qquad i = 0, 1, 2, \ldots$$

can be approximated by a discrete-time variable Z_i satisfying the difference equation

$$Z_{i+1} = Z_i + \frac{dZ(i\Delta)}{dt}\,\Delta. \tag{2.15}$$

This approximation is illustrated on Figure 2.2.

The difference equation can also be written as

$$Z_{i+1} = Z_i + (Y(i\Delta) - r(i\Delta))\Delta. \tag{2.16}$$

Then, if Δ is sufficiently small, we have

$$Z_i \approx Z(i\Delta). \tag{2.17}$$

Now, consider a discrete-time decision process with state variable W_i, and decision variable v_i. The transformation of state is

$$W_{i+1} = W_i + (Y_i - v_i)\Delta. \tag{2.18}$$

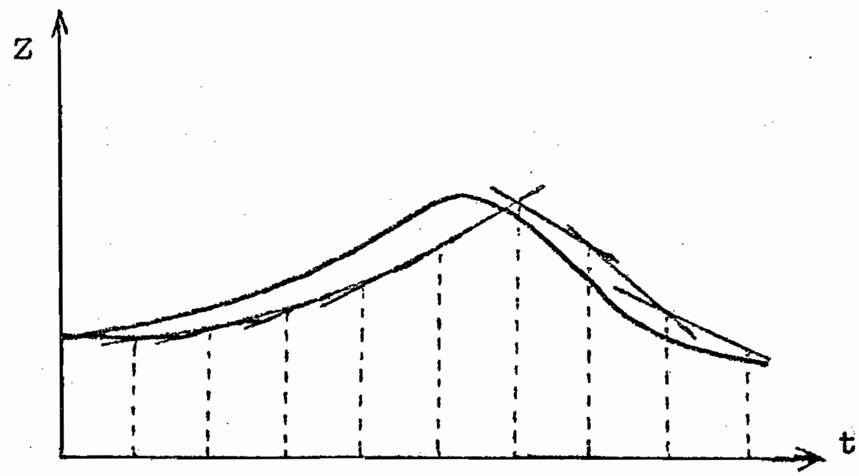


Figure 2.2 Approximating a Continuous-time Variable by a Discrete-time Variable

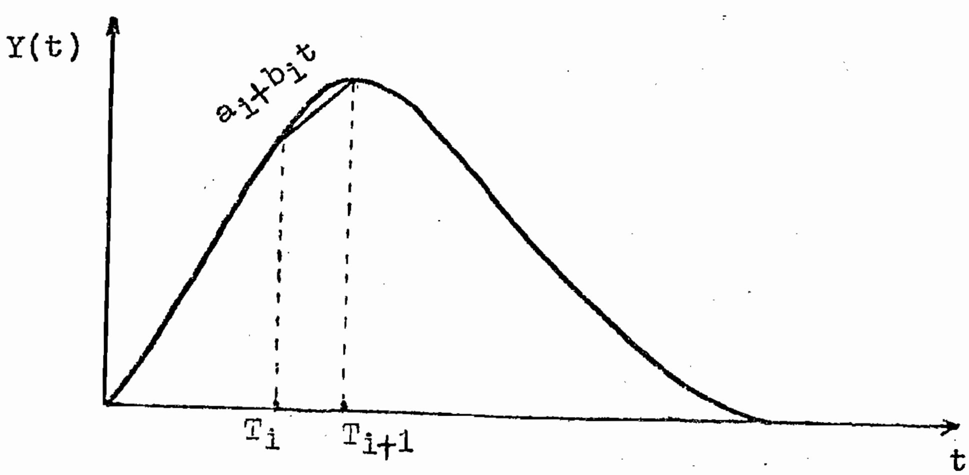


Figure 2.3 Piecewise Linearization of Hydrograph

The object is to select v_o, v_1, ... $v_{T/\Delta}$ so as to minimize the return function

$$J = U(\max(v_o, v_1, v_2, \ldots, v_{T/\Delta})) \tag{2.19}$$

subject to the constraints

$$W_i \geq 0, \quad W_i \leq C \qquad i = 1, 2, \ldots \tag{2.20}$$

Notice that the discrete-time decision variable v_i takes the same value as the continuous-time decision variable $r(t)$ at $t = i\Delta$. This discrete time process is such that as Δ goes to zero, the state variable W_i approaches the discrete time variable Z_i. But as Δ goes to zero, the approximation (2.17) also become an equality and so

$$W_i \rightarrow Z(i\Delta) \qquad \text{as } \Delta \rightarrow 0, \tag{2.21}$$

and we have

$$\lim_{\Delta \rightarrow 0} J = \lim_{\Delta \rightarrow 0} (\max(v_o, v_1, v_2, \ldots, v_{T/\Delta})) = I \tag{2.22}$$

Thus, the discrete-time process tends to behave like the continuous-time process as the discrete time interval goes to zero.

The procedure is to divide the time interval into N + 1 stages and piecewise linearize the hydrograph as shown in Figure 2.3. We determine the N + 1 cut points, in such a way that all the critical points of the hydrograph are contained. Then in the interval ΔT_i, the input is

$$Q_i = a_i \Delta T_i + \frac{b_i}{2} (\Delta T_i)^2. \quad (2.23)$$

If we denote $\sum_{j=0}^{n} \Delta T_j = T_n$, then the total release in time interval ΔT_n is

$$R_n = \int_{T_{n-1}}^{T_n} r(t)dt = r_n \Delta T_n.$$

The state transformation is

$$Z_{n+1} - Z_n = Q_n - R_n \qquad n = 0, 1, 2, \ldots N \quad (2.24)$$

The objective function is

$$\underset{\{(r_o, r_1, \ldots, r_n)\}}{\text{Min}} \{\max\ (U(r_o), U(r_1), \ldots U(r_N))\} \quad (2.25)$$

and the constraints are

$$0 \le Z_n \le C \qquad n = 0, 1, \ldots, N$$

$$0 \le R_n \qquad n = 0, 1, \ldots, N$$

$$0 \le Q_n \qquad n = 0, 1, \ldots, N$$

2.4 Evaluation of Previous Solution Approaches

Windsor [87] suggested a linear programming method to solve a similar problem using the piecewise linearization of the damage cost function. Consider the following three cases.

(a) Convex U(r)

If U is a convex function of r, as in Figure 2.4a, then we have

$$r_n = d_1\lambda_{1,n} + d_2\lambda_{2,n} + \ldots + d_m\lambda_{m,n} \qquad n = 0, 1, \ldots, N \quad (2.26)$$

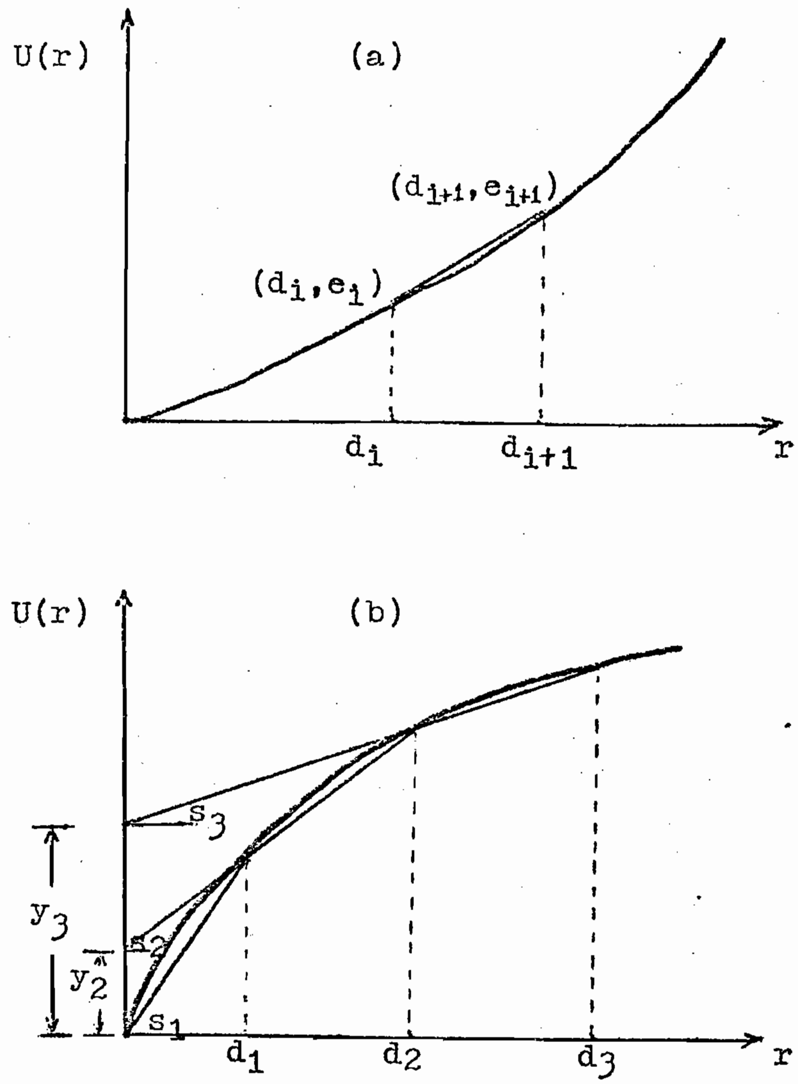


Figure 2.4 Damage Cost Functions and Piecewise linear Approximations (a)Convex (b)Concave.

$$r = d_1\lambda_1 + d_2\lambda_2 + \ldots + d_m\lambda_m, \tag{2.27}$$

$$U(r) = e_1\lambda_1 + e_2\lambda_2 + \ldots + e_m\lambda_m. \tag{2.28}$$

The problem becomes

$$\underset{\{(\lambda_1,\lambda_2,\ldots,\lambda_m)\}}{\text{Min}} \{\sum_i (e_i\lambda_i)\} \tag{2.29}$$

subject to

$$Z_{n+1} - Z_n + (d_1\lambda_{1,n} + d_2\lambda_{2,n} + \ldots + d_m\lambda_{m,n})\Delta T = Q_n \qquad n = 0, 1, \ldots N \tag{2.30}$$

$$d_1\lambda_1 + d_2\lambda_2 + \ldots + d_m\lambda_m - (d_1\lambda_{1,n} + \ldots + d_m\lambda_{m,n}) \geq 0$$

$$n = 0, 1, \ldots N \tag{2.31}$$

$$0 \leq Z_n \leq C \qquad n = 0, 1, \ldots N \tag{2.32}$$

This is a linear programming problem.

(b) U(r) Concave

If U is a concave function of r, then we cannot use the piecewise linear approximation that was employed in the convex case. For example, if $d_3 < r < d_4$ as Figure 2.5, then r can be a convex combination of d_3 and d_4. But it can also be a convex combination of d_1 and d_5 or d_2 and d_5. In the linear programming process, we will find a variable which yields minimum the cost value. Hence in this case, we will follow the straight line joining (d_1, e_1) and (d_5, e_5) which obviously introduces a significant error.

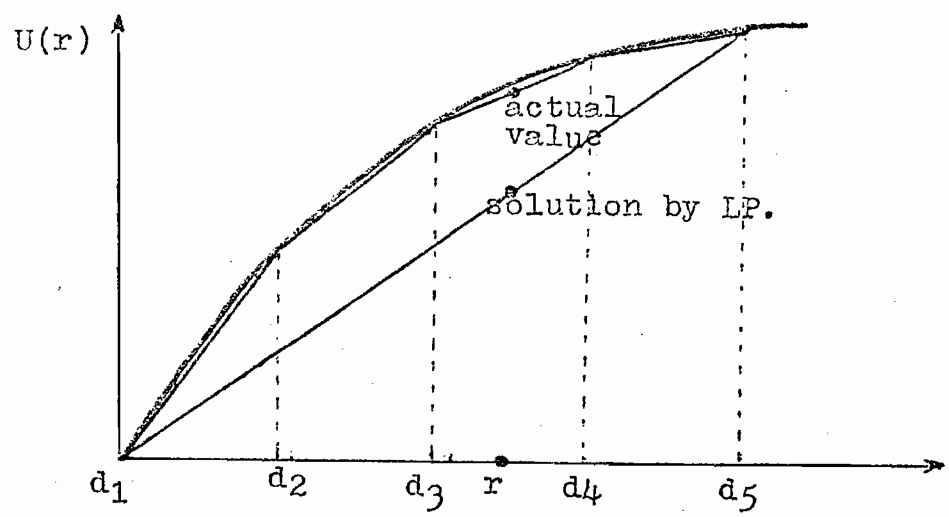


Figure 2.5 Illustration of the error incurred for a concave curve approximated by convex piecewise linearization.

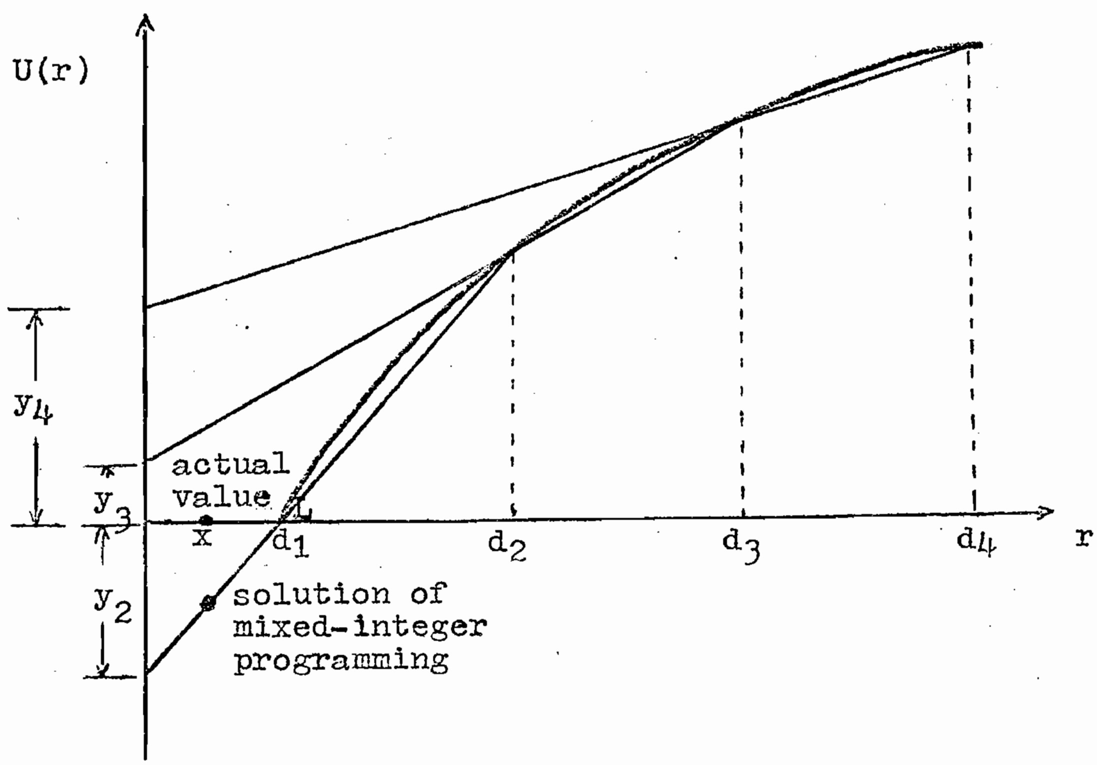


Figure 2.6 Illustration of convex concave mixed cost function and error introduced by piecewise linearization.

Windsor [88] suggested another method of linearization for this case. As shown in Figure 2.4b, define a set of auxiliary variables $h_{1,n}$, $h_{2,n}$, ..., $h_{m,n}$ and h_1, h_2, ..., h_m such that

$$0 \le h_{i,n} \le d_i \lambda_{i,n} \qquad i = 1, 2, \dots m \tag{2.33}$$

$$0 \le h_i \le d_i \lambda_i \qquad i = 1, 2, \dots m \tag{2.34}$$

where

$$\begin{aligned} &\lambda_i = 0 \quad \text{or} \quad 1 \quad \text{and} \\ &\lambda_{i,n} = 0 \text{ or } \quad 1. \end{aligned} \tag{2.35}$$

The cost function becomes

$$\sum_{i=1}^{m} s_i h_i + \sum_{i=1}^{m} y_i \lambda_i \tag{2.36}$$

In this linearization, every point is a convex combination of the two nearest points only.

The problem becomes

$$\underset{\substack{(h_1, h_2, \dots h_m) \\ (\lambda_1, \lambda_2, \dots \lambda_m)}}{\text{Min}} \left\{ \sum_{i=1}^{m} s_i h_i + \sum_{i=1}^{m} y_i \lambda_i \right\} \tag{2.37}$$

subject to

$$Z_{n+1} - Z_n + (h_{1,n} + h_{2.n} + \dots + h_{m,n}) \Delta T = Q_n \quad n = 0, 1, \dots, N \tag{2.38}$$

$$\lambda_{1,n} + \lambda_{2.n} + \dots + \lambda_{m,n} = 1 \qquad n = 0, 1, \dots, N \qquad (2.39)$$

$$0 \le h_{i,n} \le d_i\lambda_{i,n} \qquad \begin{matrix} n = 0, 1, \dots, N \\ i = 1, 2, \dots, m \end{matrix} \qquad (2.40)$$

$$0 \le h_i \le d_i\lambda_i \qquad i = 1, 2, \dots, m \qquad (2.41)$$

$$h_1 + h_2 + \dots + h_m - (h_{1,n} + h_{2,n} + \dots + h_{m,n}) \ge 0$$

$$n = 0, 1, \dots, N \qquad (2.42)$$

The peak release rate is

$$r = h_1 + h_2 + \dots + h_m.$$

This is a mixed-integer programming problem [88]. The solution method has a major shortcoming in that it only can find the optimal peak release rate. But we still do not know how to control the release rate. Another flaw is that the method only be employed to solve the pure concave damage function case. Finally, computational limitations will limit the size of the problems that can be solved. (See [28] and [31]).

(c) U(r) Mixed Convex and Concave

Sometimes we have a mixed convex and concave damage cost function. In the simplest case if the release rate is below some value, say L (flood stage), it will not cause a flood. So we will not incur any damage. The cost function is illustrated as Figure 2.6. For example, if we are at a point x, $0 \le x \le L$, then we want to follow the line of the r-axis. But the mixed-integer programming solution will follow a line which yields the minimum value of the objective function. Then it will follow the line that has negative cost value. This is an obvious error. Furthermore, there is also another difficulty in that for a release rate between 0 and L, the linear and mixed-integer programming solution will not guarantee choice of the biggest value of release rate which has the same return. But in a flood control problem, we need to ensure that the storage space as large as possible to accommodate the flood flow during the next stage.

In order to overcome these difficulties we propose a solution by dynamic programming.

2.5 Dynamic Programming Approach

2.5.1 Single-Reservoir Flood Control Systems

In the dynamic programming approach we divide the time horizon into N equal time intervals ΔT and linearize the cost function, whether convex, concave, or mixed, by piecewise linearization. We also assume that in each small time interval ΔT, the release rate is constant. The storage at stage n after release (at the beginning of stage n + 1) Z_{n+1} is

$$Z_{n+1} = Z_n + Q_n - r_n \Delta T \qquad (2.43)$$

This is also the transition function for dynamic programming. Since the constraints are $Z_n \geq 0$, $Z_n \leq C$, for all n, the upper bound for r_n when at state Z_n with input Q_n is

$$Ub(Z_n, Q_n) = (Z_n + Q_n)/\Delta T, \qquad (2.44)$$

and the lower bound of r_n is

$$Lb(Z_n, Q_n) = \max\{0, (Z_n + Q_n - C)/\Delta T\}. \tag{2.45}$$

Define $f_n(Z_n)$ as the minimum cost from stage n (time $n\Delta T$) to stage N (time T), if the storage level is Z_n at time $n\Delta T$ (stage n). Then, invoking Bellman's principle of optimality yields the following functional equation of dynamic programming

$$f_n(Z_n) = \min_{r_n \in R(Z_n, Q_n)} \{\max (U(r_n), f_{n+1}(Z_n + Q_n - r_n\Delta T))\}$$

$$n = 0, 1, \ldots, N \tag{2.46}$$

with the boundary condition

$$f_{N+1}(Z_{N+1}) = 0, \tag{2.47}$$

where

$$R(Z_n, Q_n) = [\max \{0, (Z_n + Q_n - C)/\Delta T\}, (Z_n + Q_n)/\Delta T]. \tag{2.48}$$

The dynamic programming algorithm was coded in FORTRAN IV and implemented on Northwestern University's CDC 6600. A macro-flow diagram of the algorithm is shown on Figure 2.7 and the FORTRAN program appears in Appendix A1. We note that the digital computer provides a powerful tool for executing the tedius computational work. One value of the tool comes through its rapid and low cost determination of the economic consequences of alternative policies. The decision maker can readily obtain the information he or she needs to predict the

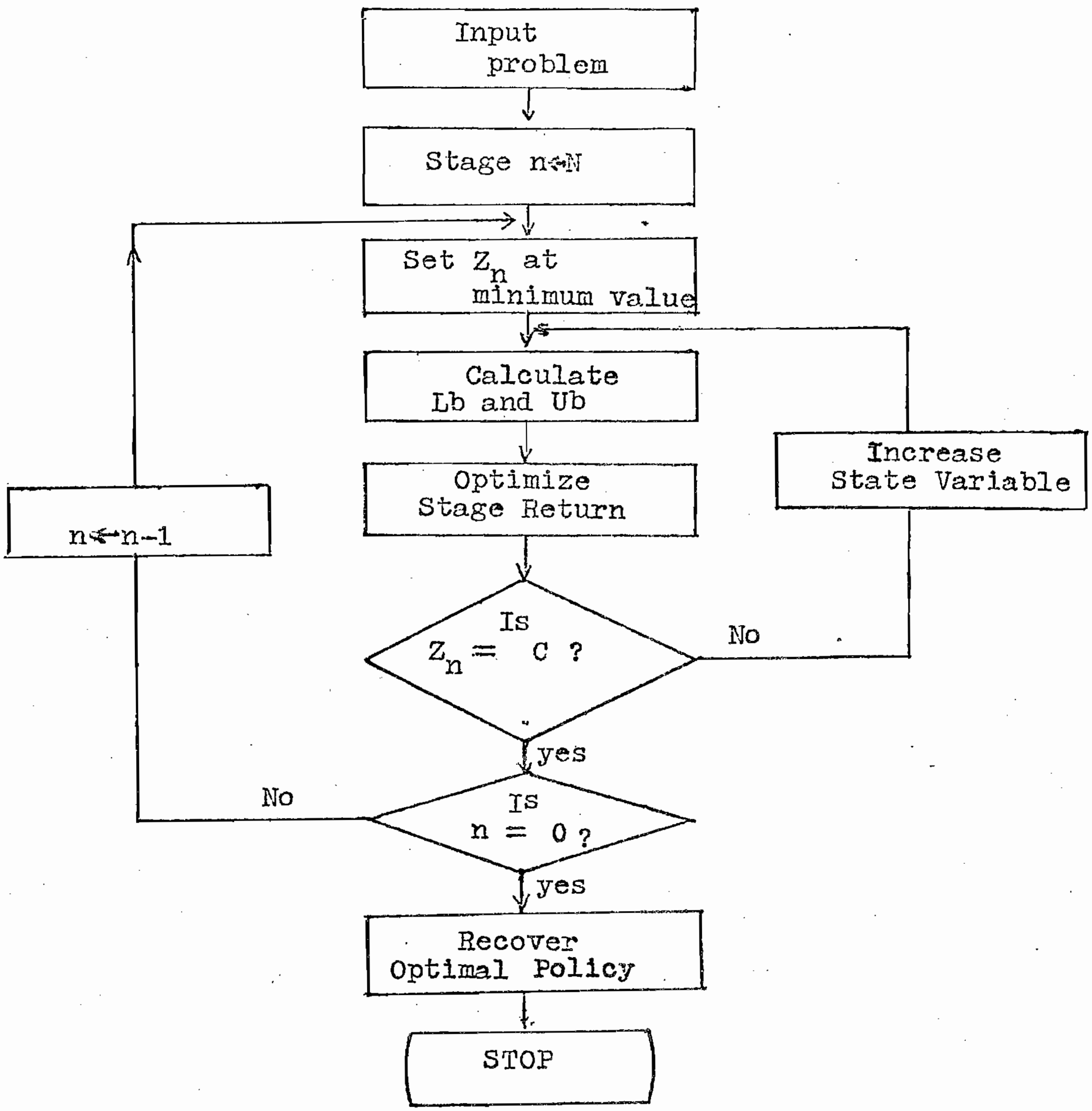


Figure 2.7 Macro-Flow Diagram of the Dynamic Programming Algorithm for a Single-Reservoir Flood Control System.

consequences of his or her choices.

We also note that with any discrete model of a process in which the state variables vary continuously there is always a problem associated with deciding on the class interval scheme upon which the discrete version of the process is to be based. This is the problem of how the accuracy of the results is related to both the size of the discrete unit and the geometry of the discretization scheme. The smaller the discrete unit is the closer the discrete approximation approaches the exact continuous situation and so the greater the reliability of the results. However, smaller time units require larger computation time and the solution process may be very expensive, if it is possible at all. In actual processes, the hydrological data are discrete time and discrete value over a considerably large interval. Hence, we need to choose the size of discrete unit based on a compromise between the accuracy desired and the quantity of time and money available for the analysis.

2.5.2 An Example

The Crooked Creek Reservoir on Crooked Creek River, Pennsylvania was constructed by the Pittsburgh flood commission [69]. The horizontal area vs. elevation curve is given on Figure 2.10. The capacity of the reservoir is 3,310,000,000 ft^3 and the dam is 918 ft. high. A historical flood hydrograph from [85] is shown in Figure 2.9. When the flood is coming, the initial storage in the reservoir was 840 ft. high and the volume was 220,000,000 ft^3. The actual release rule is a function of the water level of the reservoir which is shown on Figure 2.11. The maximum allowable release rate which will not cause

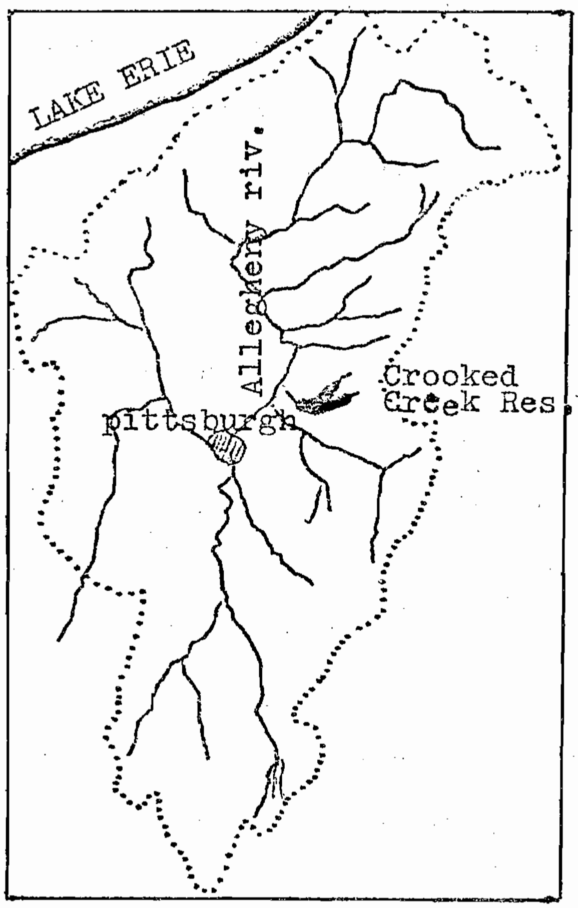


Figure 2.8 Crooked Creek Reservoir for protection of Pittsburgh, Pennsylvania.

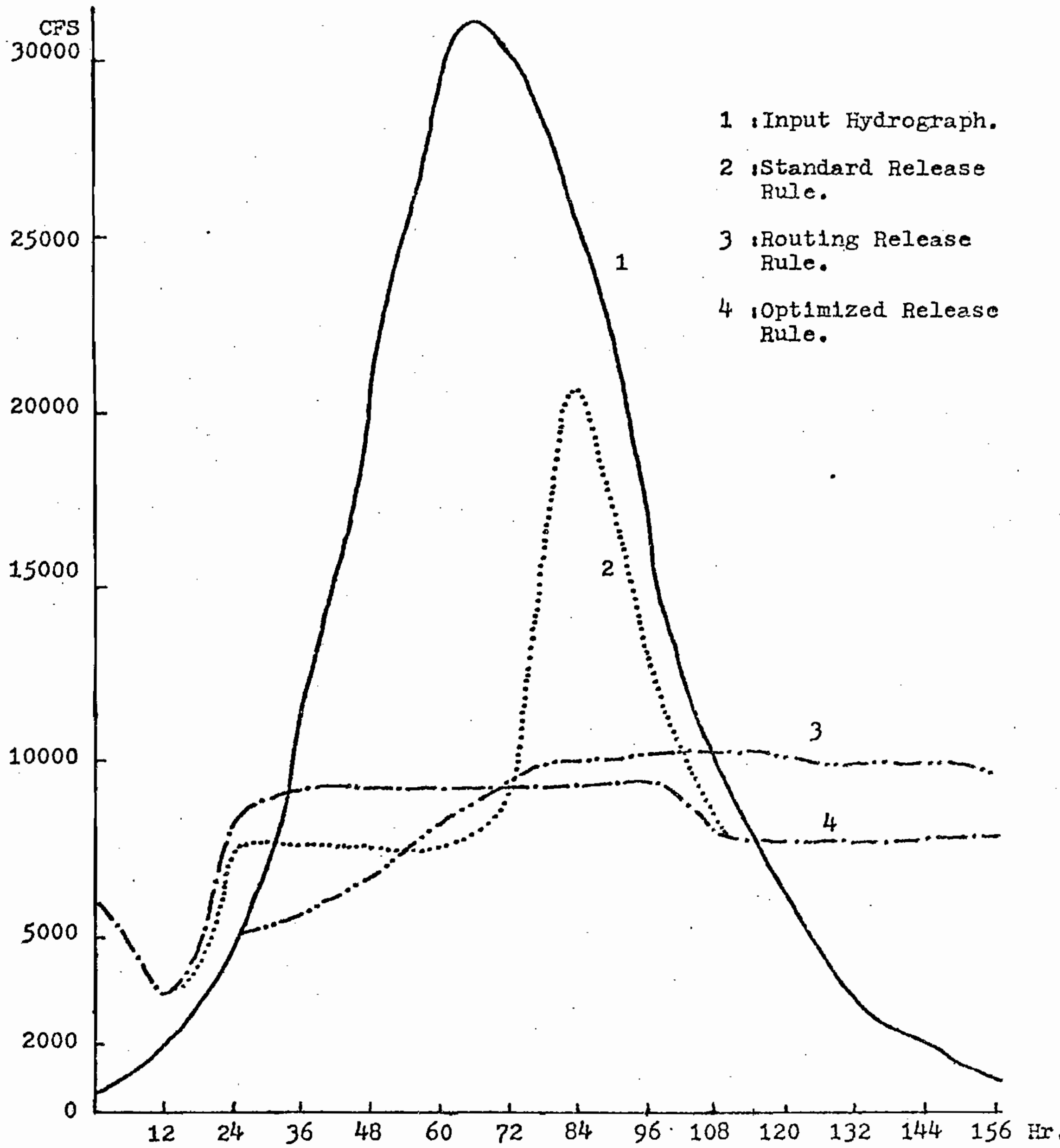


Figure 2.9 Input Hydrograph and Release Rates.

a flood is 7,500 cfs. The damage cost function is given as Figure 2.12. The "Standard" release rule [60] is shown on Figure 2.13.

The optimal release rule determined by dynamic programming is displayed on Figure 2.9. From the Figure 2.9, we can find that using optimal release rule the reservoir absorbed the peak flow. Furthermore, the optimal release rule also performed better than both the currently used routing release rule which is as a function of the water levels, and the standard release rule, which is as a function of water storage.

By integrating the area to the left of the curve on Figure 2.10, we can obtain the storage volume when the elevation of the reservoir is z. We can then plot the volume vs. elevation curve S, volume with unit of 6 hr.-sec.-ft., as on Figure 2.15, we can also plot the curve of $S - \frac{Q\Delta t}{2}$ and $S + \frac{Q\Delta t}{2}$ on each side of S and calculate the input of each interval Δt, and then plot $I\Delta t$ on curve S. In this manner we obtain the elevation of each stage and the release rate of each stage. These computations are summarized in Table 2.1.

The optimal release rule, the currently used routing release rule and standard release rule for operating Crooked Creed Reservoir are shown on Figure 2.9. The initial flood cost is $5.26 million, the routing rule cost is $0.864 million, the standard release rule cost is $3.876 million, and the optimal release rule cost is $0.758 million. From this we can see that the stationary, Standard Policy, did not absorb the flood peak effectively. The computing time to determine the optimal policies was 122 CPU seconds on Northwestern University's CDC 6600.

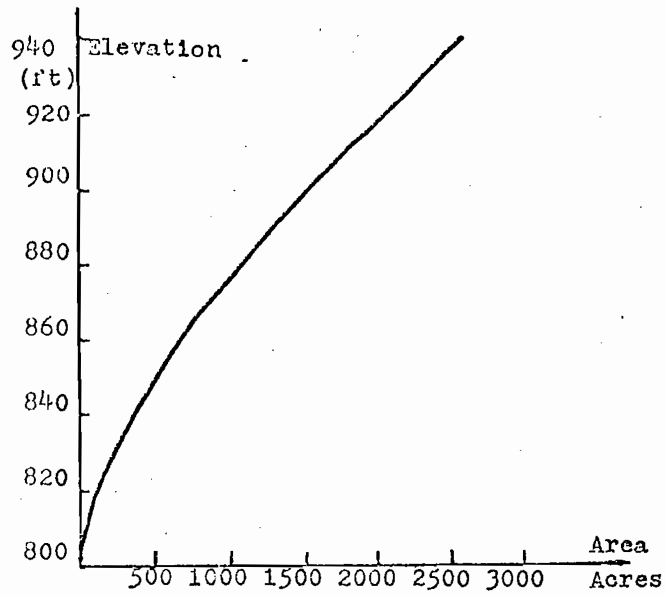


Figure 2.10 Horizontal area vs Elevation curve of Crooked Creek Reservoir.

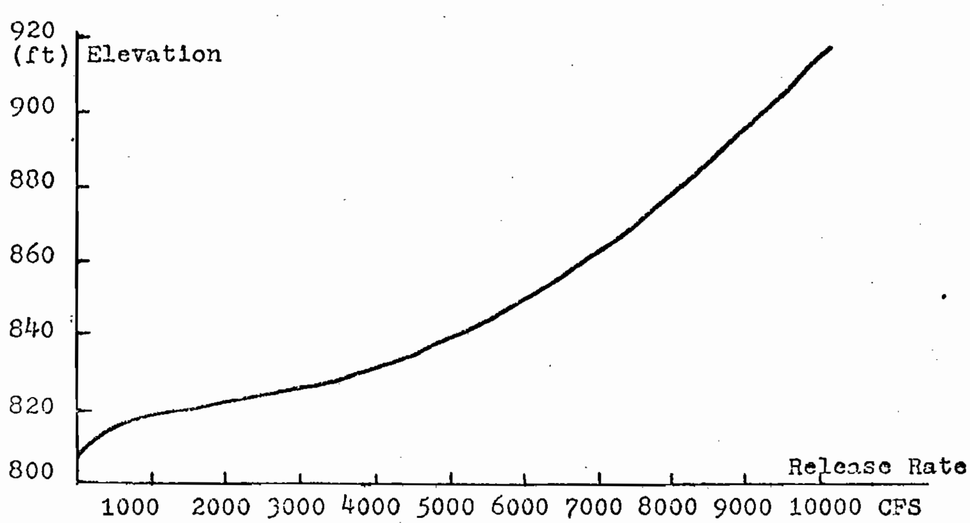


Figure 2.11 Routing release rate of Crooked Creek Reservoir.

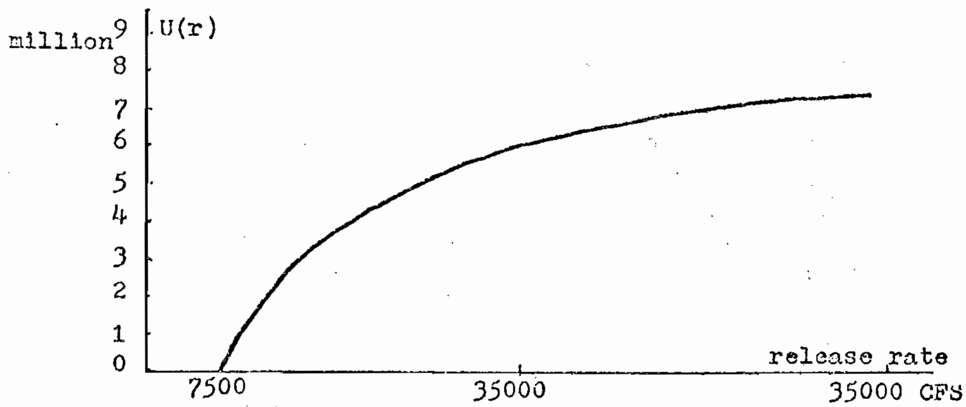


Figure 2.12 Damage cost function of Crooked Creek Reservoir

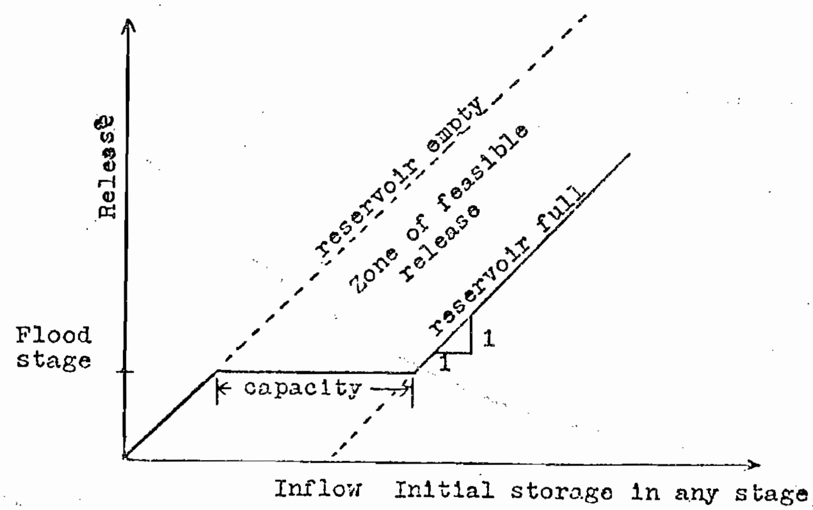


Figure 2.13 Standard release policy

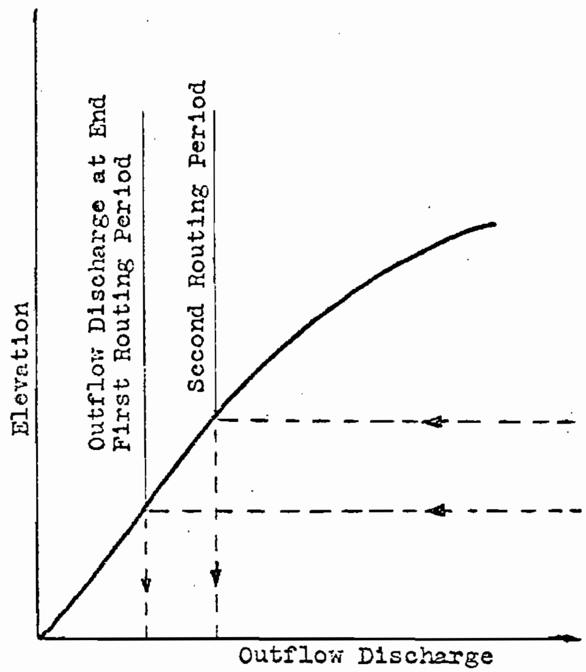


Figure 2.14 Reservoir Outflow vs Elevation

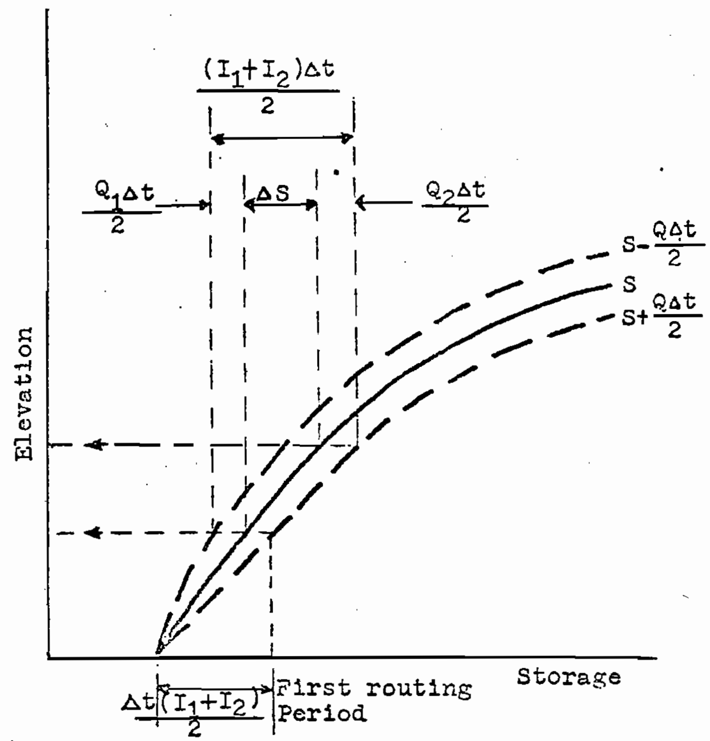


Figure 2.15 Reservoir Storage vs Elevation

Time	IΔt (6hr-sec-ft)	Pool Elevation (ft)	Out flow Rate (CFS)
24	6250	840	5020
30	9750	841.1	5020
36	13750	845	5510
42	18250	853.1	6067
48	23000	862.5	6870
54	27500	872.5	7606
60	30500	881.87	8137
66	30500	891.87	8798
72	28850	900	9225
78	26350	906.26	9553
84	22750	911.3	9796
90	18250	915	9965
96	14500	916.25	10021
102	11500	916.8	10046
108	8750	917	10055
114	6750	916.4	10028
120	5400	916.3	10023
126	3900	915.63	9993
132	2750	915	9965
138	2250	913.32	9880
144	1750	911.87	9824
150	1250	910	9740
156	1000	907.5	9592
162	1000	904.3	9450
168	1000	902.5	9356

Table 2.1 Inflow Data, Computions and Routing Outflow Results for Crooked Creek Reservoir

2.5.3 Multiple-Reservoir Flood Control Systems

Since it is the middle and lower reaches of a stream which require flood protection, flood control reservoirs must be located at the headwaters or upon the upper reaches of the stream; or in the case of larger streams upon the tributaries which contribute to the crest of the flood in the main stream. Then, too, it is upon the tributaries that reservoir sites are most likely to be found which will meet the physical and economic requirements. For this reason, a system of reservoirs, rather than a single reservoir, is necessary to control flood flows, except on small watersheds where a single reservoir at the upper end may suffice [65].

We will first analyze the case of two reservoirs in series and then the case of two reservoirs in parallel. Extention to other system topologies should be immediately apparent.

(a) Two Reservoirs in Series

For two series reservoirs the storage equations can be written

$$Z_{1,n+1} = Z_{1,n} + Q_{1,n} - r_{1,n}\Delta T \qquad n = 0, 1, \ldots, N \qquad (2.49)$$

$$Z_{2,n+1} = Z_{2,n} + Q_{2,n} + r_{1,n}\Delta T - r_{2,n}\Delta T \qquad n = 0, 1, \ldots, N \qquad (2.50)$$

$$0 \le Z_{1,n} \le C_1 \qquad n = 0, 1, \ldots, N \qquad (2.51)$$

$$0 \le Z_{2,n} \le C_2 \qquad n = 0, 1, \ldots, N \qquad (2.52)$$

where C_1 and C_2 are the capacities of reservoirs 1 and 2, respectively.

The objective function is

$$\underset{\substack{(r_{1,o},\ldots r_{1,N})\\(r_{2,o},\ldots r_{2,N})}}{\text{Min}} \{\max[(U_1(r_{1,o})+U_2(r_{2,o})),\ \ldots,\ (U_1(r_{1,N})+U_2(r_{2,N}))]\}. \tag{2.53}$$

We can also solve this problem by dynamic programming. The upper bound of $r_{1,n}$ is

$$Ub_{1,n}(z_{1,n},\ Q_{1,n}) = (Z_{1,n} + Q_{1,n})/\Delta T, \tag{2.54}$$

and the lower bound of $r_{1,n}$ is

$$Lb_{1,n}(Z_{1,n},\ Q_{1,n}) = \max\{0,\ (Z_{1,n} + Q_{1,n} - C_1)/\Delta T\}. \tag{2.55}$$

For a given $r_{1,n}$, the upper bound of $r_{2,n}$ is

$$Ub_{2.n}(Z_{2.n},\ Q_{2.n}) = (Z_{2.n} + Q_{2.n} + r_{1,n}\Delta T)/\Delta T. \tag{2.56}$$

and the lower bound of $r_{2.n}$ is

$$Lb_{2,n}(Z_{2,n},\ Q_{2,n}) = \max\ \{0,\ (Z_{2,n} + Q_{2,n} + r_{1,n}\Delta T - C_2)/\Delta T\}. \tag{2.57}$$

The transition functions are

$$Z_{1,n+1} = Z_{1,n} + Q_{1,n} - r_{1,n}\Delta T \qquad n = 0,\ 1,\ \ldots,\ N \tag{2.58}$$

$$Z_{2,n+1} = Z_{2,n} + Q_{2,n} + r_{1,n}\Delta T - r_{2,n}\Delta T \quad n = 0, 1, \ldots, N \qquad (2.59)$$

Define $f_n(Z_{1,n}, Z_{2,n})$ as the minimum cost from stage n (time nΔT) to stage N (time T) if the levels of reservoirs 1 and 2 are $Z_{1,n}$ and $Z_{2,n}$, respectively at time nΔT (stage n). Then, invoking Bellman's principle of optimality we obtain the following functional equation of dynamic programming

$$f_n(Z_{1,n},Z_{2,n}) = \underset{r_{1,n}\in R_{1,n}}{\text{Min}} \left\{ \underset{r_{2,n}\in R_{2,n}}{\text{Min}} [\max (U(r_{1.n}) + U(r_{2.n}), f_{n+1}(Z_{1,n+1},Z_{2,n+1}))]\right\}, \; n = 0, 1, \ldots N \qquad (2.60)$$

with the boundary condition

$$f_{N+1}(Z_{1,N+1},Z_{2,N+1}) = 0 \qquad (2.61)$$

in which

$$R_{1,n} = [Lb_{1,n}, Ub_{1,n}] \,,$$

and $\qquad (2.62)$

$$R_{2,n} = [Lb_{2,n}, Ub_{2,n}] \,.$$

A macro flow diagram of the dynamic programming is displayed on Figure 2.16 and the FORTRAN program appears in Appendix A2.

(b) Two Reservoirs in Parallel

For two parallel reservoirs the storage equation can be written as

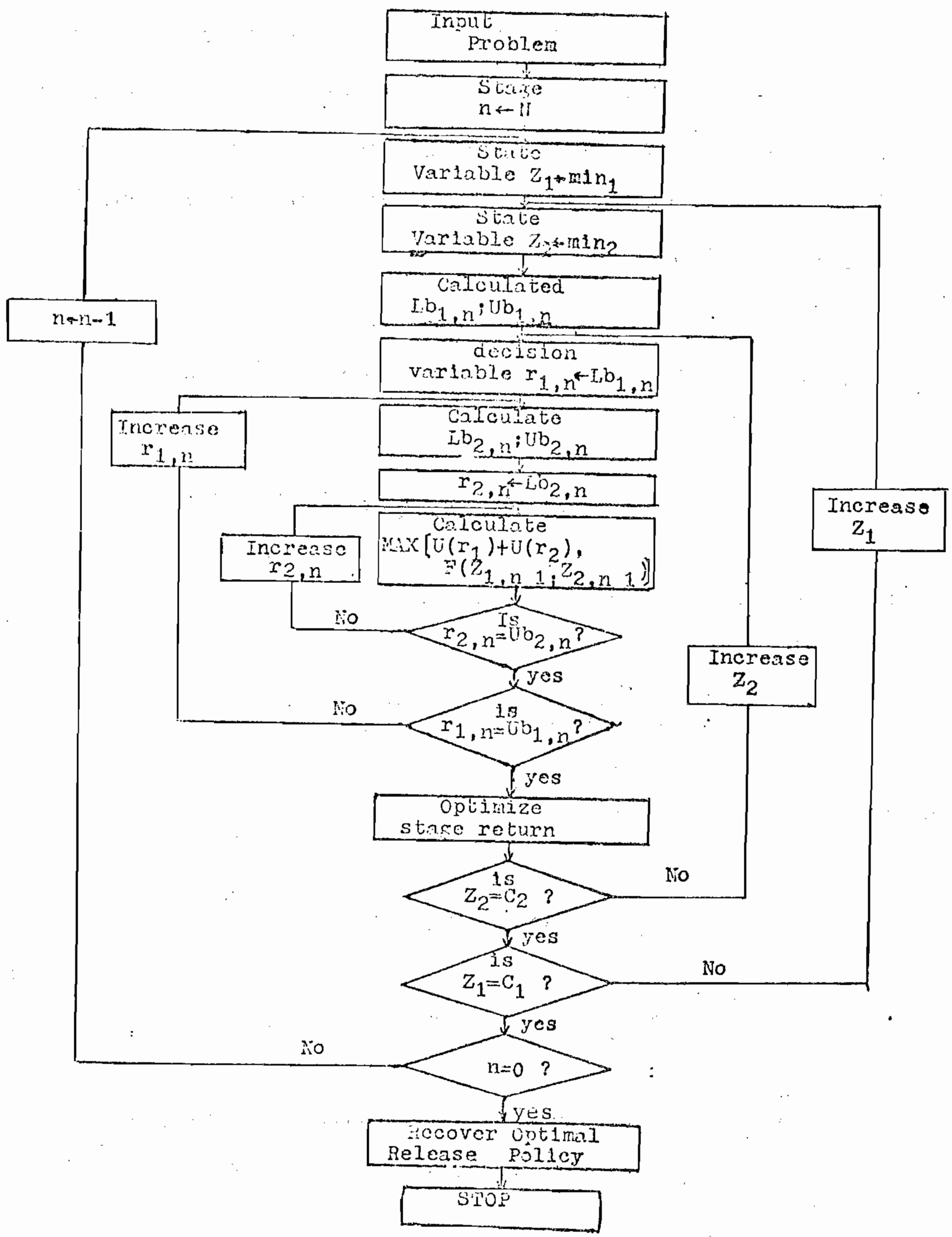


Figure 2.16 Macro Flow Diagram of the Dynamic Programming Algorithm for a two Reservoir Series Flood Control System

$$Z_{1,n+1} = Z_{1,n} + Q_{1,n} - r_{1,n}\Delta T \qquad n = 0, 1, \ldots, N, \tag{2.63}$$

$$Z_{2,n+1} = Z_{2,n} + Q_{2,n} - r_{2,n}\Delta T \qquad n = 0, 1, \ldots, N, \tag{2.64}$$

$$0 \le Z_{1,n} \le C_1 \qquad n = 0, 1, \ldots, N, \tag{2.65}$$

$$0 \le Z_{2,n} \le C_2 \qquad n = 0, 1, \ldots, N, \tag{2.66}$$

where C_1 and C_2 are the capacities of reservoirs 1 and 2, respectively. The objective function is

$$\underset{\left\{\begin{matrix}(r_{1,0},\ldots,r_{1,N})\\(r_{2,0},\ldots,r_{2,N})\end{matrix}\right\}}{\text{Min}} \{\max [U(r_{1,0} + r_{2,0}), \ldots, U(r_{1,N} + r_{2,N})]\} \tag{2.67}$$

We can also solve this problem by dynamic programming. The upper bound of $r_{1,n}$ is

$$Ub_{1,n}(Z_{1,n}, Q_{1,n}) = (Z_{1,n} + Q_{1,n})/\Delta T \tag{2.68}$$

and the lower bound of $r_{1,n}$ is

$$Lb_{1,n}(Z_{1,n}, Q_{1,n}) = \max \{0, (Z_{1,n} + Q_{1,n} - C_1)/\Delta T\}. \tag{2.69}$$

Likewise, the upper bound of $r_{2,n}$ is

$$Ub_{2,n}(Z_{2,n}, Q_{2,n}) = (Z_{2,n} + Q_{2,n})/\Delta T \tag{2.70}$$

and the lower bound of $r_{2,n}$ is

$$Lb_{2,n}(Z_{2,n}, Q_{2,n}) = \max\{0, (Z_{2,n} + Q_{2,n} - C_2)/\Delta T\}. \qquad (2.71)$$

The transition functions are

$$\begin{aligned} Z_{1,n+1} &= Z_{1,n} + Q_{1,n} - r_{1,n}\Delta T \qquad n = 0, 1, \ldots, N \\ Z_{2,n+1} &= Z_{2,n} + Q_{2,n} - r_{2,n}\Delta T \qquad n = 0, 1, \ldots, N \end{aligned} \qquad (2.72)$$

Define $f_n(Z_{1,n}, Z_{2,n})$ as the minimum cost from stage n(time $n\Delta T$) to stage N (time T) if the levels of reservoirs 1 and 2 are $Z_{1,n}$ and $Z_{2,n}$, respectively at time $n\Delta T$ (stage n). Then invoking Bellman's principle of optimality we obtain the following functional equation of dynamic programming

$$f_n(Z_{1,n}, Z_{2,n}) = \underset{r_{1,n}\in R_{1,n}}{\text{Min}} \left\{ \underset{r_{2,n}\in R_{2,n}}{\text{Min}} [\max(U(r_{1,n} + r_{2,n}), f_{n+1}(Z_{1,n+1}, Z_{2,n+1}))] \right\} \quad n = 0, 1, \ldots N \qquad (2.73)$$

with the boundary condition

$$f_{N+1}(Z_{1,N+1}, Z_{2,N+1}) = 0, \qquad (2.74)$$

in which

$$\begin{aligned} R_{1,n} &= [Lb_{1,n}, Ub_{1,n}] \\ R_{2,n} &= [Lb_{2,n}, Ub_{2,n}]. \end{aligned} \qquad (2.75)$$

A macro flow diagram of the dynamic programming algorithm is displayed on Figure 2.17 and the FORTRAN program appears in Appendix A3.

2.5.4 Multiple-Reservoir Examples

(a) Two Reservoirs in Series

The Los Angeles County Flood Control District was organized in 1915 to control floods on the Los Angeles, Hondo, and San Gabrial River [69]. There are two reservoirs, San Gabriel No. 2 (or Cogswell), and San Gabriel No. 1, in series on San Gabriel River. The basic information is displayed on Table 2.2 [49]. Suppose that we know the input hydrograph of a storm in history [69] as shown on Table 2.3 and plotted on Figure 2.20. The cost functions for damage areas 1 and 2 are plotted on Figure 2.19. The stage time interval used is ΔT = 6 hrs. The flood stage at damage center 1 is 3500 cfs, and the flood stage at damage center 2 is 5000 cfs. The optimal release rule for Cogswell Reservoir and San Gariel No. 1 Reservoir are shown on Table 2.4. Assuming no reservoirs the initial flood damage cost is $2.357 million. The optimal flood damage cost is $0. The standard release rule damage cost is also $0. Reference to Figure 2.21 shows that these two reservoirs when operated by optimal or standard release rules were completely absorbed the flood peak on San Gabriel River. On this particular example, the standard release rule was as good as the optimal release rule. The reason is that the capacities of these two reservoir flood control system is sufficient to absorb the flood peak of this given hydrograph. The computing time for determining the optimal policy was 581 seconds on Northwestern University's CDC 6600.

(b) Two Reservoir in Parallel

McDonough County, Illinois, lies wholly in the Illinois River basin.

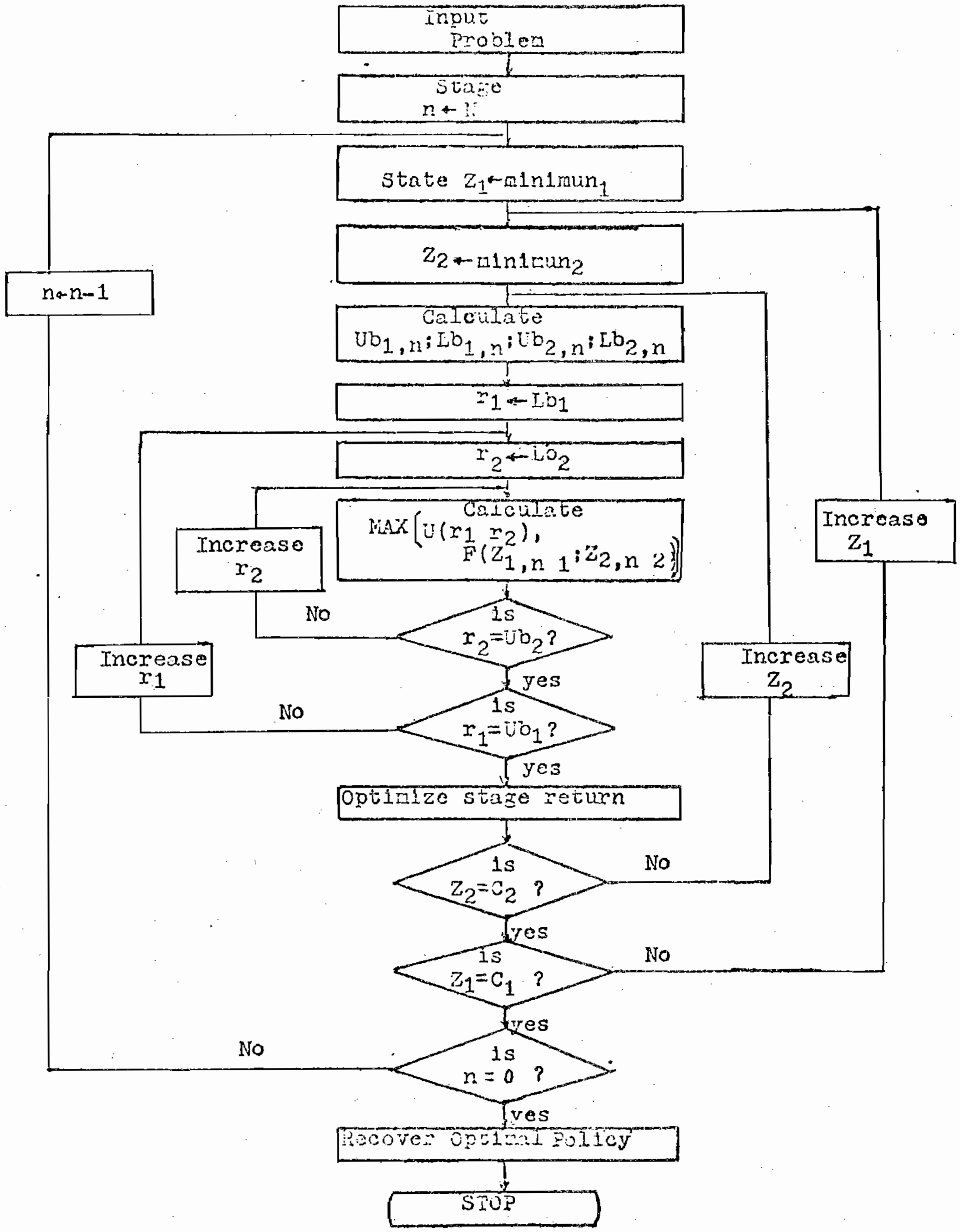


Figure 2.17 Macro Flow Diagram of the Dynamic Programming Algorithm for a Two Reservoir Parallel Flood Control System

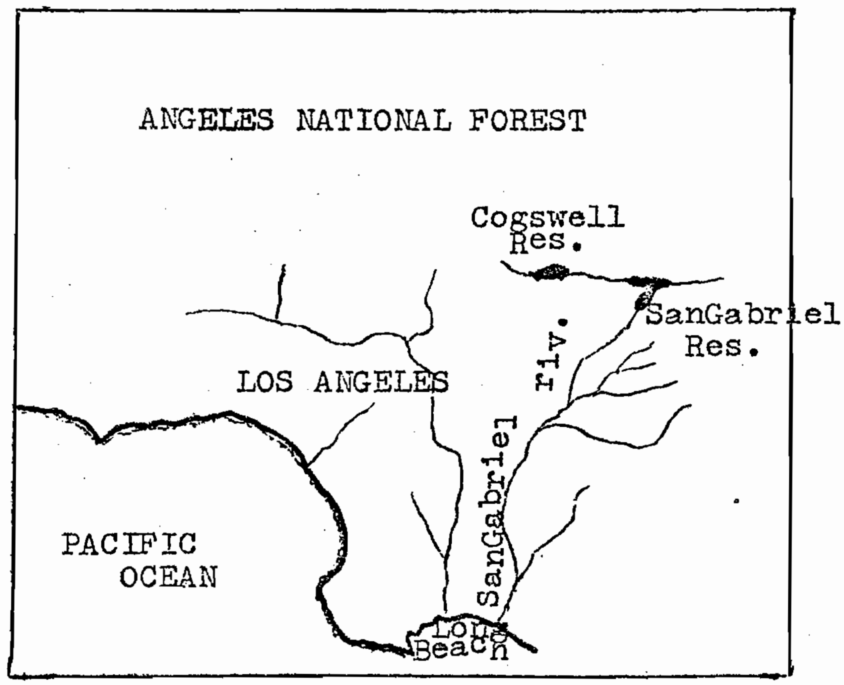


Figure 2.18 Cogswell Reservoir and San Gabriel Reservoir of the Los Angeles County Flood Control District.

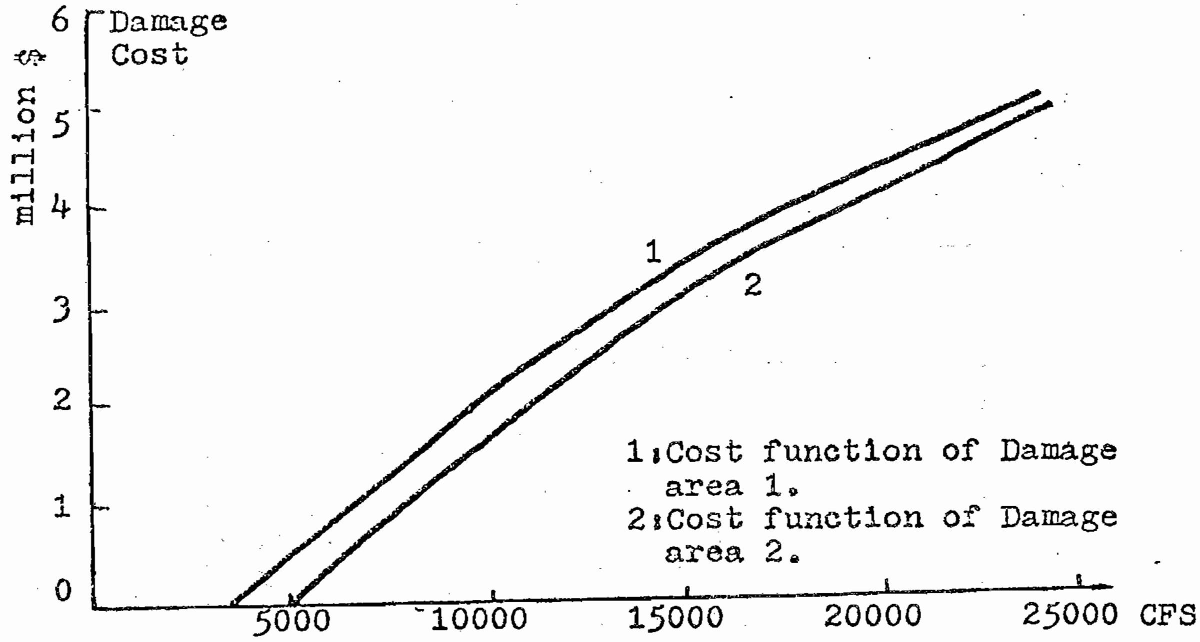


Figure 2.19 Damage cost function of Damage Center 1 and 2.

Reservoir Name	Type	High (ft)	Length Crest(ft)	Capacity (acre-ft)	Purpose
Cogswell Reservpir	Rockfill	270	620	10915	Flood control
San Gabriel Reservoir NO.1	Earthfill	285	1500	43928	Flood control

Table 2.2 Basic data of reservoirs on San Gabriel River.

Time (hr)	0	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90
CFS	0	450	1400	2400	3800	4700	5000	4800	4400	3700	3100	2500	1950	1300	720	300

Table 2.3 Input flow rate of Cogswell Reservoir and San Gabriel Reservoir.

Time (hr)	0	6	12	18	24	30	36	42	48	54	60	66	72	78	84
Optimal Release Rule of Res. 1	3500	750	1750	3000	3500	3500	3500	3500	3500	3250	2750	2000	1500	1000	500
Optimal Release Rule of Res. 2	5000	5000	3500	5000	5000	5000	5000	5000	5000	5000	5000	4000	3000	2000	1000

Table 2.4 Optimal release rate of San Gabriel River Flood Control System

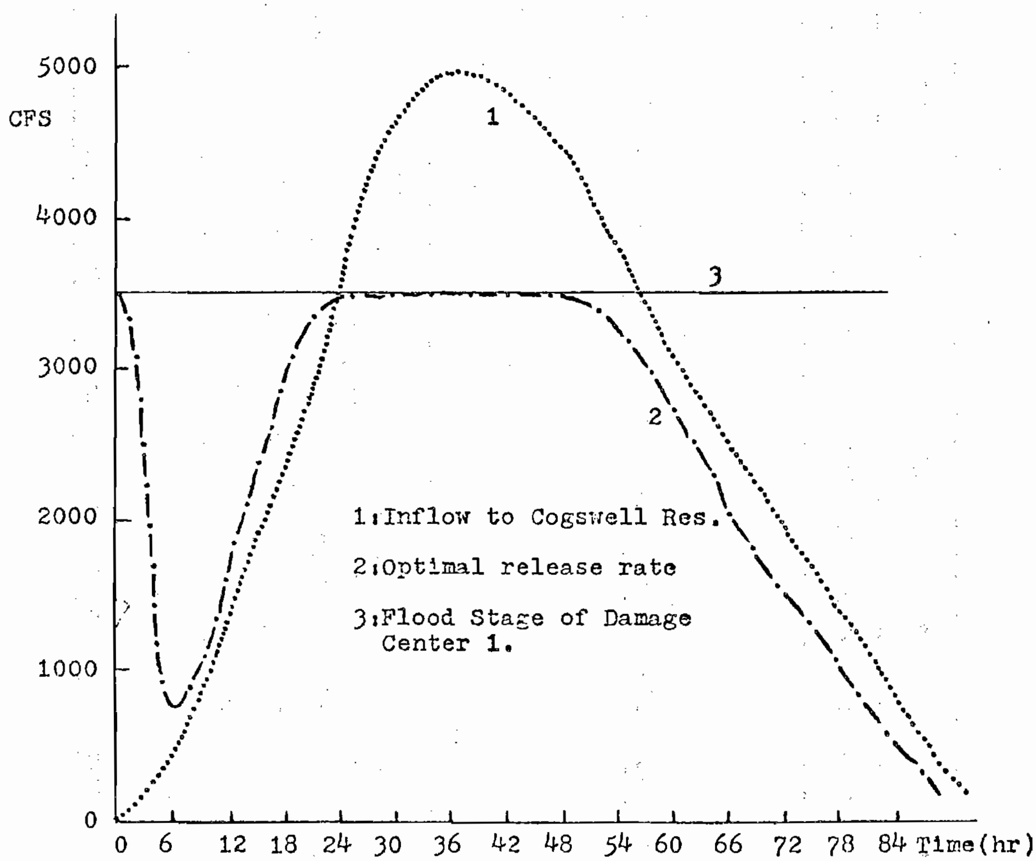
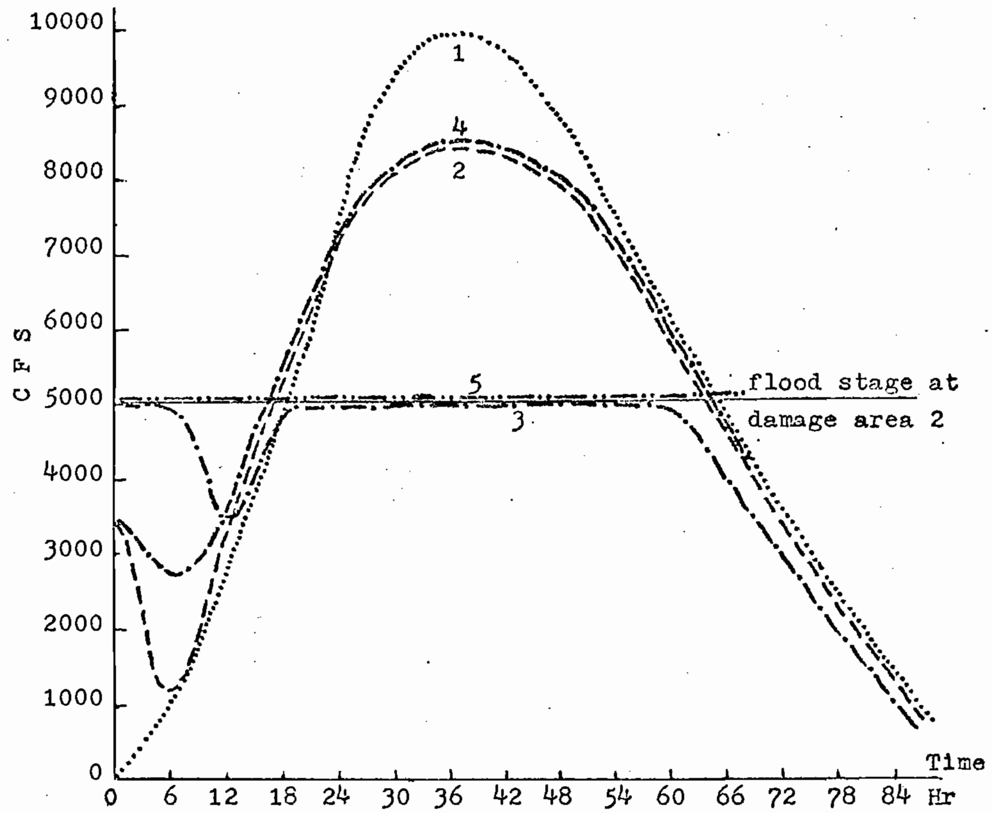


Figure 2.20 Inflow Hydrograph and Optimal Release Rate of Cogswell Reservoir.



1: Inflow rate to San Gabriel Reservoir if completely unregulated by any reservoir.
2: Inflow rate to San Gabriel Reservoir after regulated by Cogswell Reservoir with optimal policy.
3: Optimal release rate of San Gabriel Reservoir.
4: Inflow rate to San Gabriel Reservoir after regulated by Cogswell Reservoir with the standard policy.
5: Standard release rate of San Gabriel Reservoir.

Figure 2.21 Inflow Hydrograph, Optimal Release Rate and Standard Release Rate

At least 90% of the county drains southwestward through the East Fork La Moine River. Several potential reservoirs sites exist in McDonough County. In particular there are two sites: a potential reservoir site on the East Fork La Moine River 4.25 miles west and 3.75 miles north of Bushnell and another on Short Fork a tributary of the East Fork La Moine River. There positions are shown on Figure 2.22 and the basic data is displayed in Table 2.5 [21]. The input hydrograph of a typical flood is displayed in Table 2.6 and shown graphically on Figure 2.23 [79]. The flood stage of La Moine River is 2000 cfs. If no reservoirs exist, the initial flood damage is $3.04 million. If we consider that these two potential reservoirs are independently operated optimally, the damage cost is $2.26 million. The release rate of each reservoir and flow rate on damage center of La Moine River when reservoirs operated independently is displayed in Table 2.7 and shown graphically on Figure 2.23. With the whole system of parallel reservoirs is operating optimally, the release rates are as displayed in Table 2.8 and shown graphically on Figure 2.23. The optimal damage cost is $1.96 million. If we operated these systems with standard policy, the damage cost is $3.04 million. The release rate is shown graphically on Figure 2.23. In this case the standard policy cannot reduce the peak. This is due to the fact that this policy does not consider the shape of hydrograph and the interaction between the releases of these two reservoirs. The computing time to determine the optimal policy was 511 seconds on Northwestern University's CDC 6600.

From these results, we may conclude that the system of flood control reservoirs should not be operated independently.

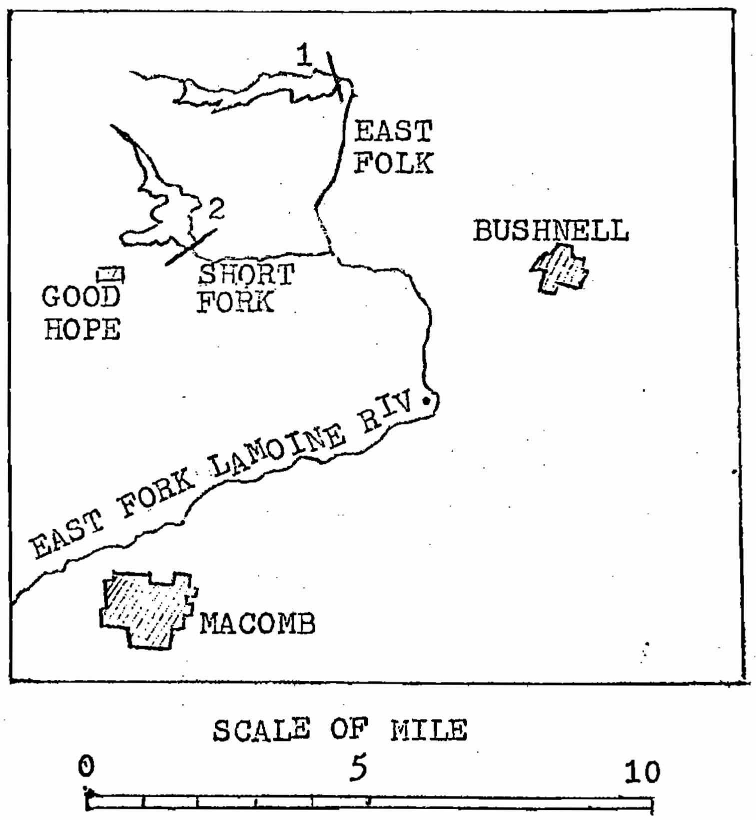


Figure 2.22 McDonough County potential reservoirs.

	Spillway Ele. (ft)	Pool Area (acres)	Capacity (ac-ft)	Depth at Dam (ft)	Watershed (sq-mi)	Purpose
Reservoir Site 1	700	336	4368	39	15.2	Flood Control
Reservoir Site 2	700	284	3040	32	6.5	Flood Control

Table 2.5 Basic Data of Reservoirs on La Moine River.

Time Stage	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Inflow at East Folk	500	800	1250	1700	2400	3000	3500	3250	2700	2200	1700	1300	1100	800	750	750	750	650
Inflow on Short Folk	725	1000	1250	1400	1800	2600	2600	2950	3100	3000	2550	2100	1400	900	750	750	750	650
La Moine	1225	1800	2500	3100	4200	5600	6100	6200	5800	5200	4250	3400	2500	1700	1500	1500	1500	1300

Table 2.6 Inflow Data on La Moine River and it's tributary

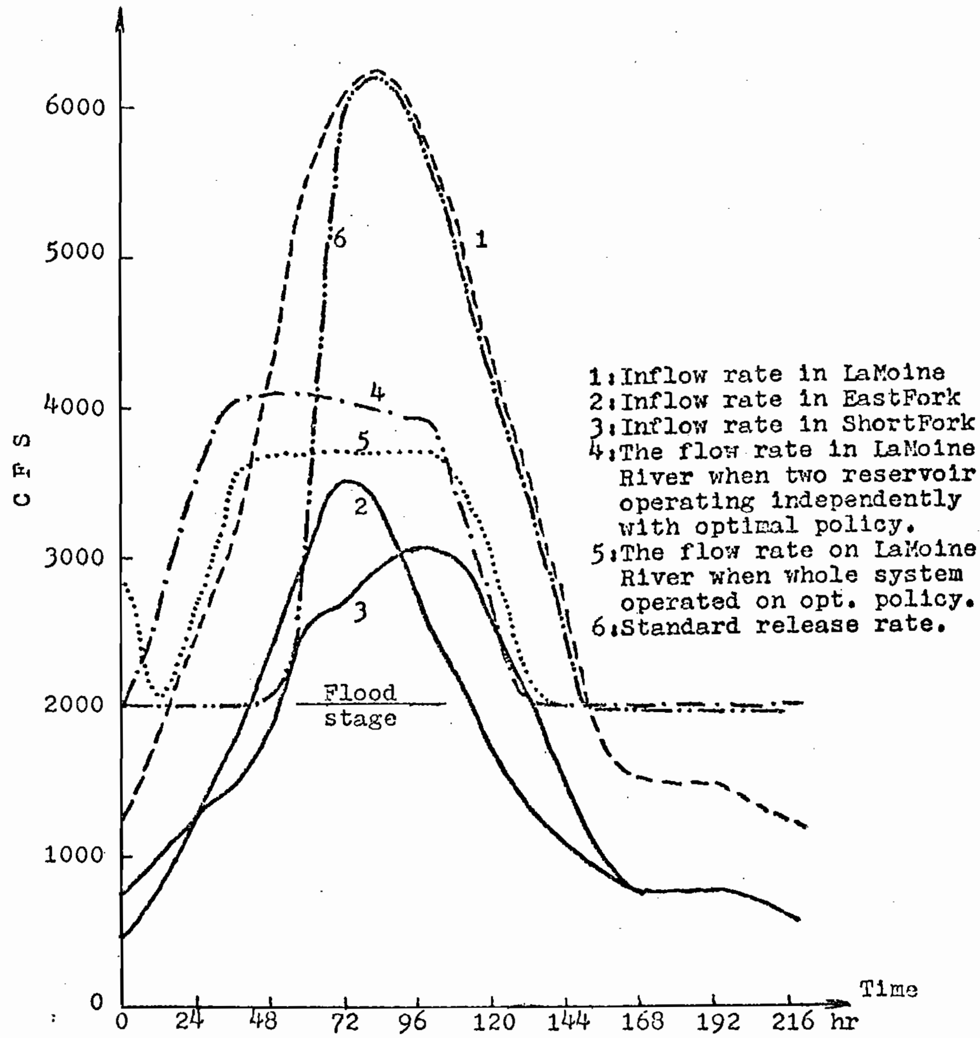


Figure 2.23 Inflow rate and Outflow rate at Damage Center of La Moine River under different operating conditions.

Time Stage	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Reservoir 1	1100	1000	1400	2000	2000	2000	2000	1900	1800	1800	1300	1000	1000	1000	1000	1000	1000	1000
Reservoir 2	1000	1100	1300	1600	2100	2100	2100	2100	2100	2100	2100	1600	1000	1000	1000	1000	1000	1000
La Moine River	2100	2700	3600	4100	4100	4100	4000	3900	3900	3900	3400	2600	2000	2000	2000	2000	2000	2000

Table 2.7 Optimal release rate and flow rate at Damage Center when each reservoir operated independently.

Time Stage	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Reservoir 1	1500	1000	1400	2000	2700	2900	1800	200	2000	1100	1100	800	1300	1600	1700	1700	700	600
Reservoir 2	1300	1100	1300	1600	1000	800	1900	3500	1700	2400	1900	1300	700	400	300	300	1300	1400
La Moine River	2800	2100	2700	3600	3700	3700	3700	3700	3700	3500	3000	2100	2000	2000	2000	2000	2000	2000

Table 2.8 Optimal release rate and flow rate at Damage Center when the whole system operated optimally.

CHAPTER III

STOCHASTIC INFLOW

3.1 Introduction

In this chapter, we first formulate a Markov Renewal flood synthesis model in §3.2. The rainfall during each storm and the times of occurrence of heavy storms is modeled as a Markov Renewal process. The rainfall quantity of the first storm of a synthesized flood and the successive occurrence times of synthesized floods are also Markov Renewal Processes. In §3.3 we develop stochastic dynamic programming algorithms to determine optimal release policies for both series and parallel multiple-reservoir flood control systems with stochastic inflows. Examples involving real-world data are provided to illustrate and assess the effectiveness of the solution methods. Finally in §3.4 we simulate the real-time use of optimal operating rules obtained in §3.3 with both historical and simulated Markov model inflow data.

3.2 Stochastic Flood Synthesis Model

In the analysis of floods, it is of interest to determine both the distribution of the times of flood occurrences and the distribution of the duration of each flood.

The relationship between precipitation and runoff is usually complex and is influenced by various storm patterns, antecedents, and basin characteristics. Because of the complexities and the frequent paucity of adequate data, many approximate formulas have been developed to relate rainfall and runoff.

Method of predicting flood-peak discharges and discharge hydrographs from rainfall events have been studied intensively. One method receiving considerable use to estimate peak discharge rates is called the unit hydrograph method. In this method it is assumed that for a given duration of rainfall, the hydrograph time base remains constant. The unit hydrograph is defined [85] as a hydrograph of direct runoff resulting from 1 in. of effective rainfall generated uniformly over the basin area at a uniform rate during a specified period of time or duration. An illustration of the derivation of the unit hydrograph is presented on Figure 3.1 [85].

Once a unit hydrograph has been developed for a basin, it can be used to obtain the surface runoff hydrographs for storm events on the basin. The runoff records can be extended then, for periods in which rainfall was measured but runoff was not. If the unit hydrograph is applied to the maximum probable rain storm for the basin, then the maximum probable flood peak may be obtained.

Application of a unit hydrograph to design rainfall excess amounts other than 1 in. is accomplished by simply multiplying the excess rainfall amount by the unit hydrograph ordinate. Since the runoff ordinate for a given duration is assumed to be directly proportional to rainfall excess. From the characteristic unit hydrograph of each watershed, we can see that there is a time lag between the flood and the rainfall duration, e.g., a 1 time-unit rainfall may take 10 time units to run through the river basin.

In application to storms of longer or smaller durations the

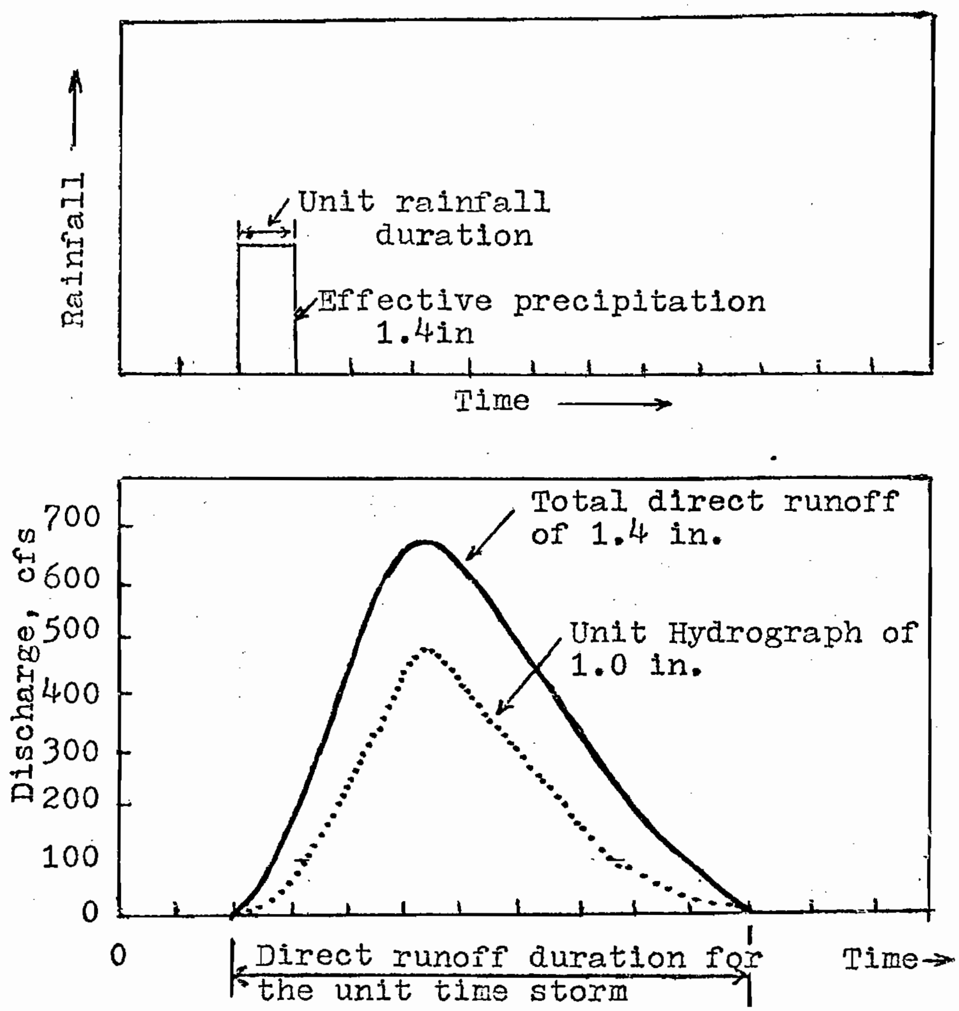


Figure 3.1 Illustration of the derivation of a unit hydrograph from an isolated storm.

method of "lagging" often used. It is based on the assumption that the linear response of the watershed is not influenced by previous storms--that is, one can superimpose storms and the results are directly additive. This is illustrated on Figure 3.2. In the 2-hr. unit hydrograph, the total rainfall is 1 inch.

In the hydrograph displayed on Figure 3.3, f_c is the maximum allowable streamflow that will not cause a flood. The portion of the hydrograph Y(t) where $Y(t) > f_c$ will cause a flood. If two storms' hydrograph durations overlap, then they may be grouped. The flood duration is a function of rainfall quantity and the rainfall duration. From the unit hydrograph, we can determine the level of total rainfall $\overline{X}$ such that when the total rainfall of a storm exceeds $\overline{X}$, it causes a flood. We shall call such storms "heavy storms".

Let the rainfall for each storm in [0, t] be a sequence of random variables I_1, I_2, ... Now consider all those heavy storms, whose total rainfall, I_i, in [0, t] exceeds $\overline{X}$. Denote by N(t) the number of heavy storms in the interval of time [0, t]. N(t) is an integer-valued variable. Assume that it satisfies the following conditions:

a) If two heavy storms occur in a sufficiently small interval, then it is considered to be a single storm.

b) The number of heavy storms over disjoint intervals are independent.

c) The probability of two storms over a time set of measure 0 is equal to zero.

Now, condition (a) says that $t \to N_t$ increases by jumps of size 1 only; and (b) says that N has independent increments. Therefore [15] (N_t) is a nonstationary Poisson process with some mean function $\Lambda(t)$. By condition (c), the expected number of storms over an interval of

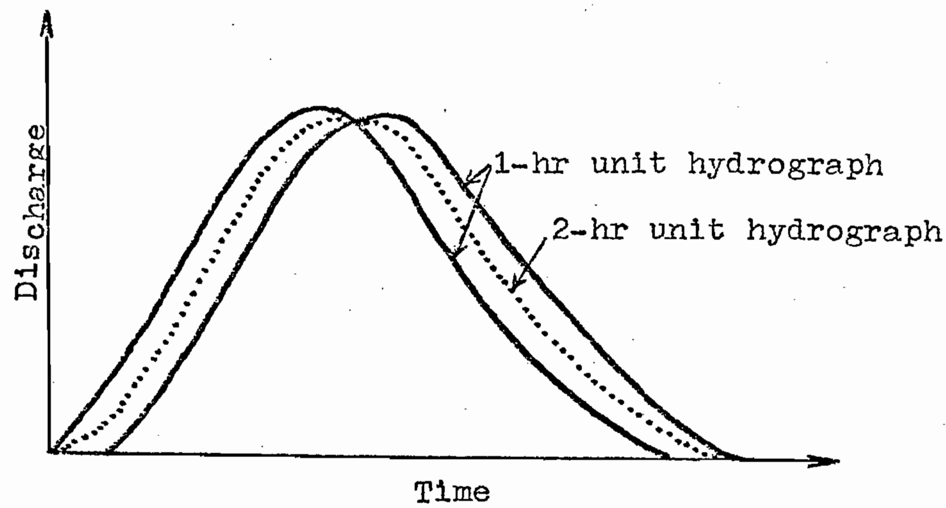


Figure 3.2 Unit hydrograph lagging procedure

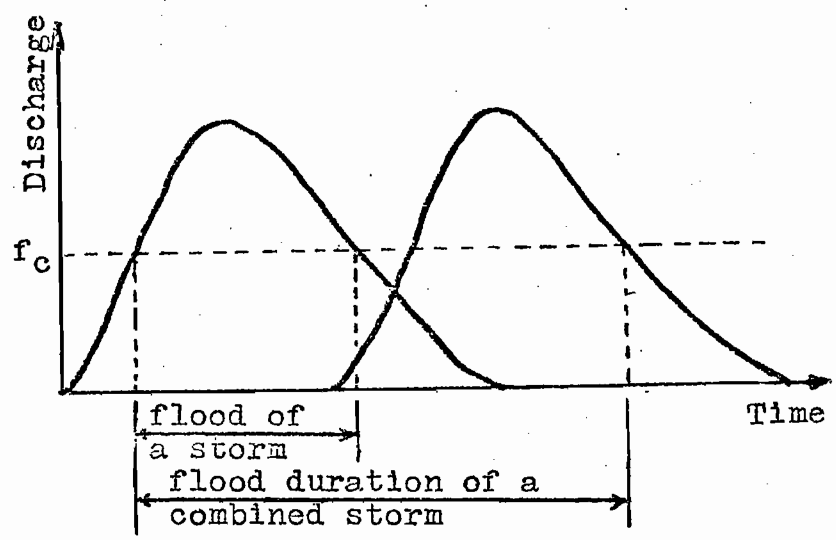


Figure 3.3 Flood duration of storms

zero Lebesgue measure is 0; and hence Λ must be absolutely continuous, that is,

$$\Lambda(t) = \int_0^t \lambda(s)ds \tag{3.1}$$

for some function λ. Then

$$P\{N_{t+u} - N_t = k\} = \frac{e^{-b}b^k}{k!}, \quad k \in N \tag{3.2}$$

where

$$b = \Lambda(t+u) - \Lambda(t) = \int_t^{t+u} \lambda(s)ds. \tag{3.3}$$

In particular, if we consider only one flood season, $\lambda(s) = \lambda$ independent of s, then N becomes a stationary Poisson process. But in general, λ should depend on s, and show a periodicity of one year.

Hence the time between each heavy storms are independent if and only if $\lambda(s)$ = constant, otherwise, if λ varies, time between storms are still exponential but not independent of each other [15]. The cumulative distribution function is

$$\Phi(t) = 1 - e^{-\lambda t}. \tag{3.4}$$

Let X_o, X_1, ... be the quantity of the successive heavy rainfalls and T_n be the time of the n^{th} storm. Assume the successive rainfalls form a Markov Chain. Then for each $n \in N$, the X_n is a random variable taking value in a countable set $\mathcal{S}$ (the set of all possible rainfalls) and T_n is a random variable taking values in $R^+ = [0, +\infty)$ with

$$0 \le T_o \le T_1 \le T_2 \;\ldots. \tag{3.5}$$

Assume that $\{X_n, n = 0, 1, \ldots\}$ is a Markov Chain with transition matrix $G(i, j)$. Then $\{X_n, T_n; n = 0, 1, \ldots\}$ is a Markov Renewal Process [15] with Semi-Markov Kernel over $\mathcal{S}$ as

$$Q(i, j, t) = P\{X_{n+1} = j, T_{n+1} - T_n \le t | X_n = i\}$$

$$= (1 - e^{-\lambda t})G(i, j) \qquad i, j \in \mathcal{S} \tag{3.6}$$

For each heavy storm the quantity of rainfall and duration of rainfall are both random variables. Hence from the unit hydrograph, the flood duration of each heavy storm is also a random variable. It is dependent on both the rainfall quantity and the rainfall duration, but is independent of the time which floods occur. For each fixed rainfall quantity k, the distribution of the flood duration of individual storms' rainfalls are connected together, we consider this a single storm. Hence we can assume that the time between storms is longer than the time duration of its rainfall duration. The Stochastic Flood Model is described below.

Let τ_o, τ_1, τ_2, ... be the flood durations associated with the rainfalls occurring at T_o, T_1, T_2, ... When a heavy storm with rainfall quantity i comes, it causes a flood with a time duration τ_1, a random variable with distribution $\Psi(i, \cdot)$ independent of the time the flood occurred. If another heavy storm with rainfall quantity j arrives during the previous flood period, then it will cause a flood with τ_2 units of time; τ_2 is random variable with distribution $\Psi(j, \cdot)$.

Let Y_t be the quantity of rainfall of a heavy storm whose flood period covers time t. Let L be the length of the first possible synthesized flood. Assume that at time 0, there is a heavy storm of rainfall i, which causes a flood. The probability that the flood started at 0 is still going on at time t and the last rainfall before t was of amount k is

$$f(i, t) = P_i\{Y_t = k, L > t\}. \tag{3.7}$$

Now,

$$f(i, t) = P(Y_t = k, L > t, T_1 > t | X_o = i) + P(Y_t = k, L > t, T_1 \le t | X_o = i) \tag{3.8}$$

From Figure 3.4a, we have

$$P(Y_t = k, L > t, T_1 > t | X_o = i)$$

$$= I(i, k)[1 - \sum_j Q(i, j, t)](1 - \Psi(i, t)) \tag{3.9}$$

From Figure 3.4b, we have

$$P(Y_t = k, L > t, T_1 < t | X_o = i)$$

$$= E[P(Y_t = k, L > t, T_1 < t | X_o = i, X_1, T_1)$$

$$= E[P(Y_t = k, \tilde{L} > t - T_1 | X_o = i, X_1, T_1)$$

$$= \sum_j \int_o^t Q(i, j, ds) f(j, t - s). \tag{3.10}$$

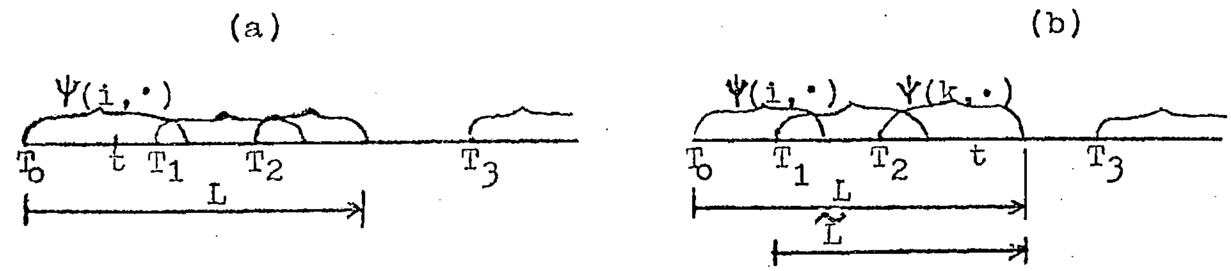


Figure 3.4 Some possible realizations of the flood process

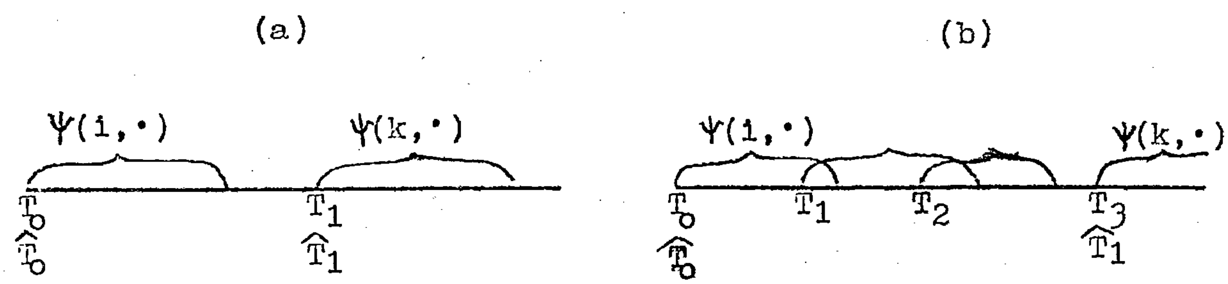


Figure 3.5 Some possible realizations of the synthesised process

Hence, we have

$$f(i, t) = g(i, t) + Q*f(i, t) \tag{3.11}$$

$$\begin{aligned} g(i, t) &= I(i, k)[1 - \sum_j Q(i, j, t)](1 - \Psi(i, t)) \\ &= I(i, k)e^{-\lambda t}(1 - \Psi(i, t)) \end{aligned} \tag{3.12}$$

This is a Markov Renewal equation [15]. From Markov renewal theory, the solution of (3.11) is

$$f = R*g \tag{3.13}$$

Where R is the Markov renewal function corresponding to Q.

Hence,

$$\begin{aligned} f(i, t) &= \sum_{\ell \in \mathcal{S}} \int_o^t R(i, \ell, ds) g(\ell, t - s) \\ &= \sum_{\ell \in \mathcal{S}} \int_o^t R(i, \ell, ds) I(\ell, k)[1 - \sum_j Q(\ell, j, t - s)](1 - \Psi(\ell, t - s)) \\ &= \sum_{\ell \in \mathcal{S}} \int_o^t R(i, \ell, ds) I(\ell, k) e^{-\lambda(t-s)}(1 - \Psi(\ell, t - s)) \\ &= \int_o^t R(i, k, ds) e^{-\lambda(t-s)}(1 - \Psi(k, t - s)). \end{aligned} \tag{3.14}$$

Also, since

$$Q(i, j, t) = G(i, j)(1 - e^{-\lambda t}), \tag{3.15}$$

R must have a special form; namely,

$$R(i, j, t) = \sum_{n=0}^{\infty} \frac{e^{-\lambda t}(\lambda t)^n}{n!} \sum_{m=0}^{n} G^m(i, j)$$

$$= \sum_{m=0}^{\infty} G^m(i, j) \sum_{n=m}^{\infty} \frac{e^{-\lambda t}(\lambda t)^n}{n!} . \qquad (3.16)$$

Hence,

$$R(i, j, dt) = \sum_{m=1}^{\infty} G^m(i, j) \frac{\lambda e^{-\lambda t}(\lambda t)^{m-1}}{(m-1)!} + I(i, j)\ \epsilon_o(dt). \qquad (3.17)$$

Now,

$$\int_o^t R(i, k, ds)e^{-\lambda(t-s)}[1 - \Psi(k, t - s)]$$

$$= I(i, k)e^{-\lambda t}[1 - \Psi(k, t)]$$

$$+ \sum_{m=1} G^m(i, k) \int_o^t \frac{\lambda e^{-\lambda s}(\lambda s)^{m-1}}{(m-1)!} e^{-\lambda(t-s)}[1 - \Psi(k, t - s)]. \qquad (3.18)$$

We next consider the process of the synthesized flood. Let $\hat{T} = 0$, $\hat{T}_1$, $\hat{T}_2$, ... be the time of successive occurences of synthesized floods and let $\hat{X}_o$, $\hat{X}_1$, $\hat{X}_2$, ... be the rainfall quantities of the first storms of each successive synthesized flood. Since (X, T) is a Markov renewal process, we have

$$P(X_{n+1} = j, T_{n+1} - T_n \leq t | X_o, \ldots X_n; T_o, \ldots T_n)$$

$$= P(X_{n+1} = j, T_{n+1} - T_n \leq t | X_n). \qquad (3.24)$$

Now

$$P(\hat{X}_{n+1} = j,\ \hat{T}_{n+1} - \hat{T}_n \le t | \hat{X}_o,\ \dots\ \hat{X}_n;\ \hat{T}_o,\ \dots\ \hat{T}_n)$$

$$= P(X_m = j,\ T_m - T_\ell \le t | X_o,\ \dots\ X_\ell;\ T_o,\ \dots\ T_\ell) \qquad (3.25)$$

for some m, ℓ, m > ℓ, where m and ℓ are stopping times of the (X, T) process and (X, T) is Markov renewal process, By the strong Markov property, we obtain

$$P(X_m = j,\ T_m - T_\ell \le t | X_o,\ \dots\ X_\ell;\ T_o,\ \dots\ T_\ell)$$

$$= P(X_m = j,\ T_m - T_\ell \le t | X_\ell)$$

$$= P(\hat{X}_{n+1} = j,\ \hat{T}_{n+1} - \hat{T}_n \le t | \hat{X}_n) \qquad (3.26)$$

Thus $(\hat{X}, \hat{T})$ is also Markov renewal process. Let $\hat{Q}$ be the semi-Markov kernel corresponding to $(\hat{X}, \hat{T})$. Let

$$h(i, t) = \hat{Q}(i, k, t)$$

$$= P(\hat{X}_{\hat{T}_1} = k,\ \hat{T}_1 - \hat{T}_o \le t | \hat{X}_{\hat{T}_o} = i) \qquad (3.27)$$

be the probability that the next flood occurs before time t and is caused by a storm with rainfall k given that at time 0 the synthesized flood occurs as a result of storm with rainfall i.

$$h(i, t) = P(\hat{X}_{\hat{T}_1} = k,\ \hat{T}_1 - \hat{T}_o \le t | \hat{X}_{\hat{T}_o} = i)$$

$$= P(\hat{X}_{\hat{T}_1} = k,\ \hat{T}_1 - \hat{T}_o \le t,\ \hat{T}_1 = T_1 | \hat{X}_{\hat{T}_o} = i) + P(\hat{X}_{\hat{T}_1} = k,\ \hat{T}_1 - \hat{T}_o \le t,\ \hat{T}_1 > T_1 | \hat{X}_{\hat{T}_o} = i)$$

(3.28)

From Figure 3.5a, we obtain

$$P(\hat{X}_{\hat{T}_1} - \hat{T}_1 - \hat{T}_o \le t,\ \hat{T}_1 = T_1 | \hat{X}_{\hat{T}_o} = i)$$

$$= P(X_{T_1} = k,\ T_1 - T_o \le t | X_{T_o} = i)$$

$$= \int_o^t Q(i,\ k,\ ds)\Psi(i,\ s), \qquad (3.29)$$

from Figure 3.5b, we obtain

$$P(\hat{X}_{\hat{T}_1} = k,\ \hat{T}_1 - \hat{T}_o \le t,\ \hat{T}_1 | \hat{X}_{\hat{T}_o} = i)$$

$$= E[P(\hat{X}_{\hat{T}_1} = k,\ \hat{T}_1 - \hat{T}_o \le t,\ \hat{T}_1 > T_1 | \hat{X}_{\hat{T}_o} = i,\ X_{T_1},\ T_1)]$$

$$= E[P(\hat{X}_{\hat{T}_1} = k,\ \hat{T}_1 - T_1 \le t - T_1,\ \hat{T}_1 > T_1 | \hat{X}_{\hat{T}_o} = i,\ X_{T_1} = j,\ T_1)]$$

$$= \sum_j \int_o^t Q(i,\ j,\ ds)(1 - \Psi(i,\ s))h(j,\ t - s). \qquad (3.30)$$

Hence, we have

$$h(i,\ t) = g(i,\ t) + \sum_j \int_o^t \overline{Q}(i,\ j,\ ds)f(j,\ t - s) \qquad (3.31)$$

where

$$\overline{Q}(i,\ j,\ ds) = Q(i,\ j,\ ds)(1 - \Psi(i,\ s)) \qquad (3.32)$$

$$g(i,\ t) = \int_o^t Q(i,\ k,\ ds)\Psi(i,\ s)$$

$$= - \int_o^t Q(i,\ k,\ ds)(1 - \Psi(i,\ s)) + \int_o^t Q(i,\ k,\ ds)$$

$$= Q(i,\ k,\ t) - \overline{Q}(i,\ k,\ t). \qquad (3.33)$$

From Markov renewal theory, the solution of (3.31) is

$$h(i, t) = \overline{R}*g(i, t)$$

where $\overline{R} = \sum_n \overline{Q}^n$ is the Markov renewal function corresponding to $\overline{Q}$ and

$$h(i, t) = \sum_j \int_0^t \overline{R}(i, j, ds)g(j, t - s)$$

$$= \sum_j \int_0^t \overline{R}(i, j, ds)[Q(j, k, t - s) - \overline{Q}(j, k, t - s)]. \quad (3.3)$$

Hence the probability that next flood occurs before time t and is caused by storm with rainfall k given that at time 0, the synthesized flood occurs as a result of storm with rainfall i is

$$\hat{Q}(i, k, t) = \sum_j \int_0^t \overline{R}(i, j, ds)[Q(j, k, t - s) - \overline{Q}(j, k, t - s)]. \quad (3.34)$$

3.3 Stochastic Decision Process

A flood control reservoir differs from a storage reservoir in that the goal is to keep the reservoir as nearly empty as possible, rather than as full as possible. Thus there is involved in the operation of a flood control reservoir a trade-off between possible downstream flood damage and the risk of subsequent flood damage due to storing water in the flood control space. Askew [2], Eastman [22], Eisel [23], Morton [46], Nayak [49, 50], have all described methods for optimizing the operation of a storage reservoir minimizing a lose function subject to chance constraints. But in flood control, we can not use the target constraints on the release rate and chance constraints on the storage volume. The constraints on flood control, such as the storage constraints and the release constraints are physical constraints and cannot be violated. The cost function in flood control depends on peak discharge rate, but not on the size of missing the target. We will describe a stochastic dynamic programming method for finding the optimal release rule in the following.

Assume that the water inflow to the reservoir is a stochastic process defined on [0, T]. At time t, the input rate Y(t) is a random variable. Then the problem is to find $\{r(t) | t \in [0, T]\}$ so as to

$$\min_{\{r(t)\}} \{\max_t E[U(r(t))]\} \tag{3.35}$$

As in Chapter II the storage equation is

$$\begin{aligned} Z_t &= Z_o + \int_o^t Y(s)ds - \int_o^t r(s)ds \\ &= Z_o + Q(t) - \int_o^t r(s)ds, \end{aligned} \tag{3.36}$$

in which Z_t is the storage at time t, $0 \le Z_t \le C$, where C is capacity of reservoir, and Q_t is the total input from time 0 to time t.

We will use a discrete time procedure to approximate Y(t). Divide the time interval [0, T] of interest into N subdivisions, each stage of time duration ΔT. Then the storage equation becomes

$$Z_n = Z_o + \sum_o^n Q_i - \sum_o^n r_i \Delta T \qquad n = 0, 1, \ldots, N \qquad (3.37)$$

$$0 \leq Z_n \leq C \qquad n = 0, 1, \ldots, N \qquad (3.38)$$

For a given cost function U(r), where r is the peak discharge rate, the objective function can be written as

$$\underset{\{(r_o,\ r_1,\ \ldots\ r_n)\}}{\text{Min}} \{\max(EU(r_o),\ EU(r_1),\ \ldots,\ EU(r_N))\}. \qquad (3.39)$$

The transition of state, however, depends not only on the current state Z_n, the decision r_n but also on a random variable Q_n. The state transition function is

$$Z_{n+1} = Z_n + Q_n - r_n \Delta T \qquad n = 0, 1, \ldots, N \qquad (3.40)$$

3.3.1 Time Dependent Inflow Rate

Let $P_n(q_n)$ be the probability density of the random variable Q_n, n = 0, 1, 2, ..., N. Define $f_n(Z_n)$ as the optimal expected value with respect to Q_n of the total cost from the n^{th} stage to the end of process, given that the process is in state Z_n at stage n. Then, invoking Bellman's principle of optimality we obtain the following functional equation of dynamic programming

$$f_n(Z_n) = \underset{r_n \in R_n(Z_n, Q_n)}{\text{Min}} \{\max(EU(r_n),\ Ef_{n+1}(Z_n + Q_n - r_n \Delta T))\}$$

$$n = 0, 1, \ldots, N \qquad (3.41)$$

with the boundary condition

$$f_{N+1}(Z_{N+1}) = 0 \tag{3.42}$$

For fixed Q_n, the lower bound of r_n is

$$Lb_n(Z_n, Q_n) = \max\{0, (Z_n + Q_n - C)/\Delta T\}, \tag{3.43}$$

The upper bound of r_n is

$$Ub_n(Z_n, Q_n) = (Z_n + Q_n)/\Delta T. \tag{3.44}$$

Since Q_n is a random variable, the interval $[Lb_n, Ub_n]$ is a random interval on R^+.

The physical meaning of Ub_n is that all the water is released in period n and there is no more water in the reservoir. Hence, when the decision is r_n where $r_n > Ub_n$ the actual release is only equal to Ub_n. Likewise, the physical meaning of Lb_n is that the release is only the overflow part of water and the reservoir is full. The release rate cannot be less than this quantity. Hence, when the decision is r_n where $r_n < Lb_n$, the actual release is Lb_n.

Thus

$$r_n = \begin{cases} Lb_n & \text{if } r_n < Lb_n \\ r_n & \text{if } r_n \in [Lb_n, Ub_n] \\ Ub_n & \text{if } r_n > Ub_n \end{cases} \tag{3.45}$$

The range of r_n is

$$R_n(Z_n, Q_n) = [\underset{Q_n}{\text{Min}}\ Lb_n(Z_n, Q_n), \underset{Q_n}{\max}\ Ub_n(Z_n, Q_n)]. \qquad (3.46)$$

A macro-flow diagram of the dynamic programming algorithm is given on Figure 3.6. The algorithm was coded in FORTRAN IV and implemented on Northwestern University's CDC 6600. The program listing appears in Appendix A4.

3.3.2 Individual Storm

Each watershed area of a reservoir has its own characteristic unit hydrograph. Let $Y'(t)$ be the ordinate of unit hydrograph at time t and M be the total rainfall excess of a specific storm. The inflow hydrograph of that storm is

$$Y(t) = Y'(t)M. \qquad (3.47)$$

In practical cases, the total rainfall of a storm is probabilistic. Suppose that we know its probability density function p(m), then in the functional recurrence equation

$$f_n(Z_n) = \min_{r_n}\{\max(EU(r_n), Ef_{n+1}(Z_n + Q_n - r_n\Delta T))\}$$

$$n = 0, 1, \ldots, N \qquad (3.48)$$

the distributions of input at each stage are no longer independent of each other; they depend on the previous stage. Hence the functional

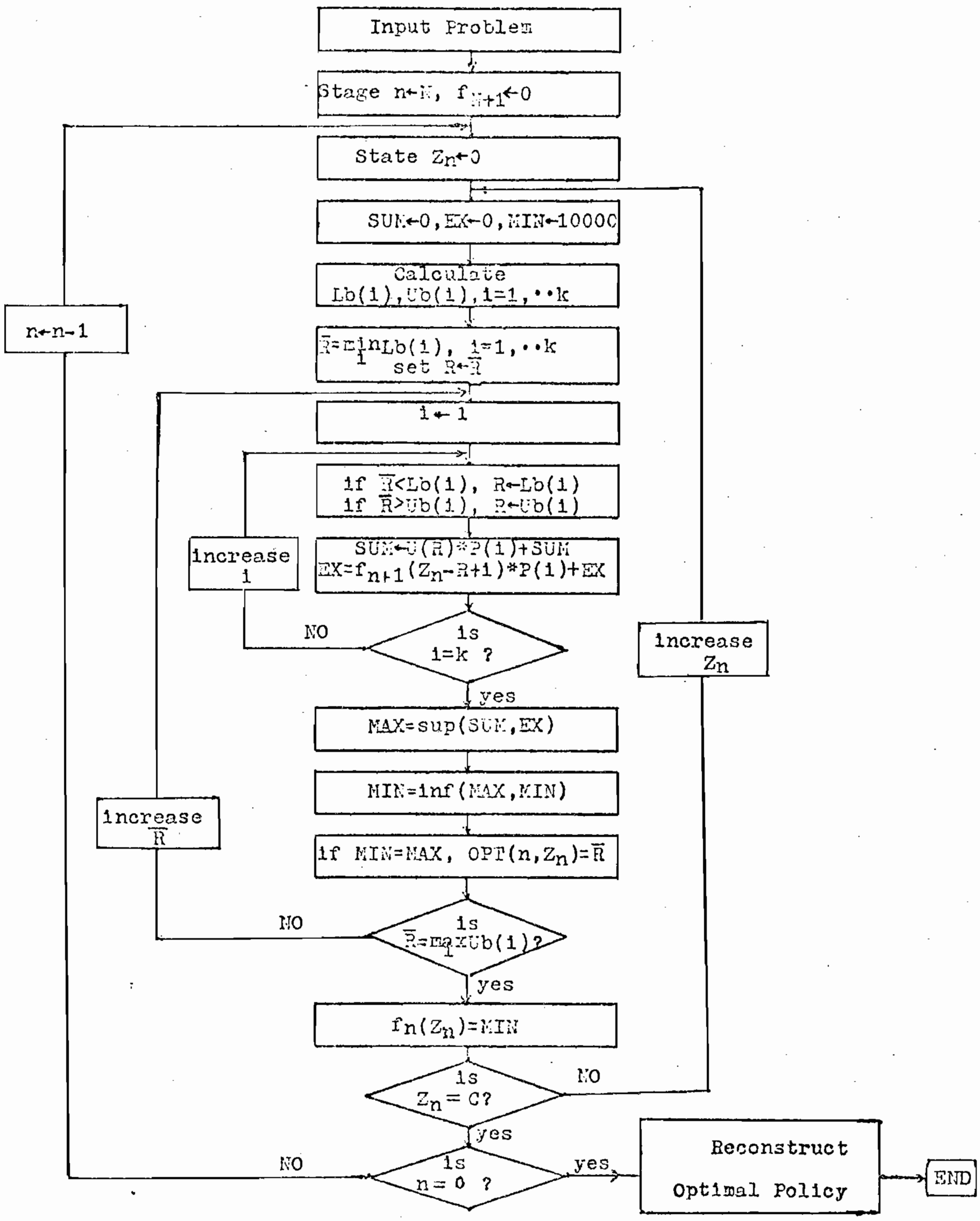


Figure 3.6 Macro Flow Diagram of the Stochastic Dynamic Programming Algorithm for Single Reservoir with Stochastic Inputs.

equation of dynamic programming changes to

$$f_n(Z_n) = \min_{r_n} \{ E_m [\max(U(r_n),\ g_{n+1}(Z_n + Q_n - r_n \Delta T, m))]\} \quad n = 0, 1, \ldots N \qquad (3.49)$$

in which $g_{n+1}(Z_{n+1}, m)$ is the optimal return from the deterministic process when N - n - 1 stages are left, and the total input rainfall is m, where m is a random variable.

$$g_n(Z_n, m) = \min_{r_n}\{\max(U(r_n), g_{n+1}(Z_n + Q_n - r_n \Delta T, m))\} \quad n = 0, 1, \ldots N \qquad (3.50)$$

$$f_{N+1}(Z_{N+1}) = 0 \qquad (3.51)$$

$$g_{N+1}(Z_{N+1}) = 0 \qquad (3.52)$$

For fixed Q_n, the lower bound of r_n is

$$Lb_n(Z_n, Q_n) = \max\{0, (Z_n + Q_n - C)/\Delta T\}, \qquad (3.53)$$

and the upper bound is

$$Ub_n(Z_n, Q_n) = (Z_n + Q_n)/\Delta T. \qquad (3.54)$$

Since $Y_n = Y_n' \cdot m$; m is random variable, hence the interval $[Lb_n, Ub_n]$ is a random interval on R^+. We have

$$r_n = \begin{cases} Lb_n & \text{if} \quad r_n < Lb_n \\ r_n & \text{if} \quad r_n \in [Lb_n, Ub_n] \\ Ub_n & \text{if} \quad r_n > Ub_n. \end{cases} \qquad (3.55)$$

3.3.3 Multiple-Reservoir Flood Control Systems

For the case of a system of two series reservoirs, assume that at time t the input rates $Y_1(t)$ and $Y_2(t)$ are both random variables. Then the problem is

$$\underset{\{r_1(t),r_2(t)\}}{\text{Min}} \{\underset{t}{\max}(EU_1(r_1(t)) + EU_2(r_2(t)))\}. \tag{3.56}$$

The storage equations are

$$Z_1(t) = Z_1(0) + \int_0^t Y_1(s)ds - \int_0^t r_1(s)ds, \tag{3.57}$$

$$Z_2(t) = Z_2(0) + \int_0^t Y_2(s)ds + \int_0^t r_1(s)ds - \int_0^t r_2(s)ds. \tag{3.58}$$

Dividing the time interval into N stages, such that each stage has time duration ΔT, the storage equations become

$$Z_{1,n} = Z_{1,0} + \sum_{j=0}^{n} Q_{1,j} - \sum_{j=0}^{n} r_{1,j}\Delta T \qquad n = 0, 1, \ldots, N \tag{3.59}$$

$$Z_{2,n} = Z_{2,0} + \sum_{j=0}^{n} Q_{2,j} + \sum_{j=0}^{n} r_{1,j}\Delta T - \sum_{j=0}^{n} r_{2,j}\Delta T \quad n = 0, 1, \ldots, N \tag{3.60}$$

The objective function can be written as

$$\underset{\left\{\begin{matrix}(r_{1,0},\ldots,r_{1,N})\\(r_{2,0},\ldots,r_{2,N})\end{matrix}\right\}}{\text{Min}} \{\underset{n}{\max}(E[U(r_{1,n}) + U(r_{2,n})])\}, \tag{3.61}$$

Define $f_n(Z_{1,n}, Z_{2,n})$ as the optimal expected value with respect to $Q_{1,n}$ and $Q_{2,n}$ of the total cost from the n^{th} stage to the end of process, given that the process is in states $Z_{1,n}$ and $Z_{2,n}$ at stage n. Then invoking Bellman's principle of optimality we obtain the following functional equation of dynamic programming

$$f_n(Z_{1,n}, Z_{2,n}) = \min_{r_{1,n}\in R_{1,n}} \left\{ \min_{r_{2,n}\in R_{2,n}} (\max(E[U_1(r_{1,n}) + U_2(r_{2,n})],\right.$$

$$\left. Ef_{n+1}(Z_{1,n+1}, Z_{2,n+1}))) \right\}, \qquad n = 0, 1, \ldots, N \qquad (3.62)$$

with the boundary condition

$$f_{N+1}(Z_{1,N+1}, Z_{2,N+1}) = 0.$$

The transition functions are

$$Z_{1,n+1} = Z_{1,n} + Q_{1,n} - r_{1,n}\Delta T \qquad n = 0, 1, \ldots, N \qquad (3.63)$$

$$Z_{2,n+1} = Z_{2,n} + Q_{2,n} + r_{1,n}\Delta T - r_{2,n}\Delta T \qquad n = 0, 1, \ldots, N \qquad (3.64)$$

in which $Q_{i,n}$ is the input to reservoir i at stage n, a random variable with probability density function $p_i(Q)$.

The decision sets are

$$R_{1,n} = [\max\{0, (Z_{1,n} + Q_{1,n} - C_1)/\Delta T\}, (Z_{1,n} + Q_{1,n})/\Delta T],$$
$$(3.65)$$
$$R_{2,n} = [\max\{0, (Z_{2,n} + Q_{2,n} + r_{1,n}\Delta T - C_2)/\Delta T\}, (Z_{2,n} + Q_{2,n} + r_{1,n}\Delta T)/\Delta T].$$

In the case of a system of two parallel reservoirs, assume that at time t, the inflow rates $Y_1(t)$ and $Y_2(t)$ are both random variables.

The objective function is

$$\underset{\{r_1(t), r_2(t)\}}{\text{Min}} \{\max_t (EU(r_1(t) + r_2(t)))\}. \tag{3.66}$$

The storage equations are

$$Z_1(t) = Z_1(0) + \int_o^t Y_1(s)ds - \int_o^t r_1(s)ds, \tag{3.67}$$

$$Z_2(t) = Z_2(0) + \int_o^t Y_2(s)ds - \int_o^t r_2(s)ds. \tag{3.68}$$

Dividing the time interval into N stages, such that each stage has time duration ΔT, the storage equations become

$$Z_{1,n} = Z_{1,0} + \sum_{j=0}^{n} Q_{1,j} - \sum_{j=0}^{n} r_{1,j}\Delta T \quad n = 0, 1, \ldots, N \tag{3.69}$$

$$Z_{2,n} = Z_{2,0} + \sum_{j=0}^{n} Q_{2,j} - \sum_{j=0}^{n} r_{2,j}\Delta T \quad n = 0, 1, \ldots, N \tag{3.70}$$

$$0 \le Z_{i,n} \le C_i \qquad i = 1, 2; \; n = 0, 1, \ldots, N \tag{3.71}$$

The objective function can be written as

$$\underset{\left\{\begin{matrix}(r_{1,0},\ldots,r_{1,N})\\(r_{2,0},\ldots,r_{2,N})\end{matrix}\right\}}{\text{Min}} \{\max_n (E[U(r_{1,n} + r_{2,n})])\}. \tag{3.72}$$

Define $f_n(Z_{1,n}, Z_{2,n})$ as the optimal expected value with respect to $Q_{1,n}$ and $Q_{2,n}$ of the total cost from the n^{th} stage to the end of process, given that the process is in states $Z_{1,n}$ and $Z_{2,n}$ at stage n. Then invoking Bellman's principle of optimality we obtain the following functional equation of dynamic programming

$$f_n(Z_{1,n}, Z_{2,n}) = \min_{r_{1,n} \in R_{1,n}} \left\{ \min_{r_{2,n} \in R_{2,n}} (\max(EU(r_{1,n} + r_{2,n}), \right.$$

$$\left. Ef_{n+1}(Z_{1,n+1}, Z_{2,n+1}))) \right\} \quad n = 0, 1, \ldots, N \qquad (3.73)$$

with the boundary condition

$$f_{N+1}(Z_{1,N+1}, Z_{2,N+1}) = 0.$$

The transition functions are

$$Z_{1,n+1} = Z_{1,n} + Q_{1,n} - r_{1,n}\Delta T \qquad n = 0, 1, \ldots, N \qquad (3.74)$$

$$Z_{2,n+1} = Z_{2,n} + Q_{2,n} - r_{2,n}\Delta T \qquad n = 0, 1, \ldots, N \qquad (3.75)$$

The decision sets are

$$R_{1,n} = [\max\{0, (Z_{1,n} + Q_{1,n} - C_1)/\Delta T\}, (Z_{1,n} + Q_{1,n})/\Delta T], \qquad (3.76)$$

$$R_{2,n} = [\max\{0, (Z_{2,n} + Q_{2,n} - C_2)/\Delta T\}, (Z_{2,n} + Q_{2,n})/\Delta T]. \qquad (3.77)$$

3.3.4 Example

Shelbyville Reservoir is situated between Shelby County and Moultrie County, Illinois, as show on Figure 3.7 [21]. The basic data are given on Table 3.1. The daily flow rate of the Kaskaskia

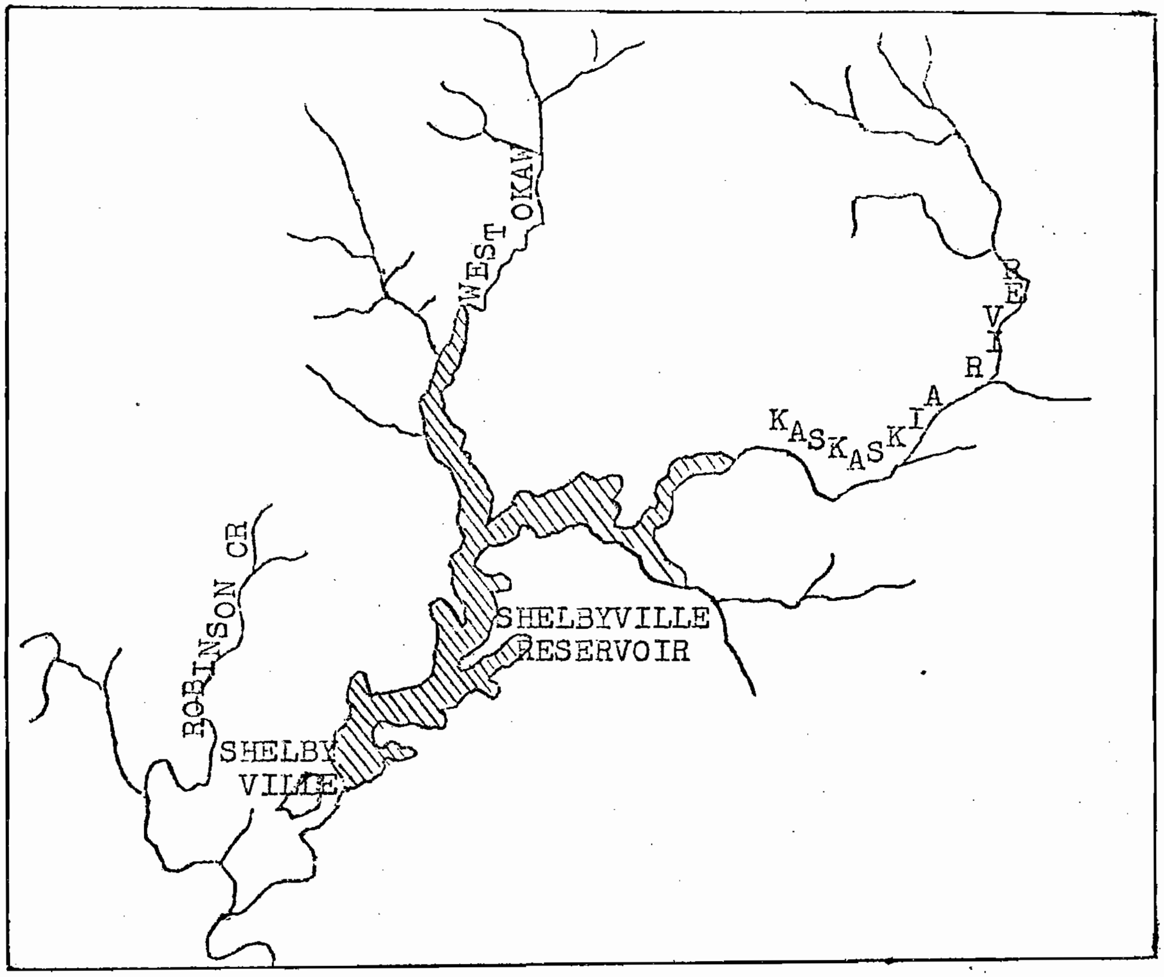


Figure 3.7 Shelbyville Reservoir, Shelby County, Illinois.

River on Shelbyville is given on Appendix B[79]. The cumulative distribution function of the stream flow rate derived from these data are presented on Table 3.2.

In this example, we will consider three kinds of flow process to approximate the actual stream process. They are (1) individual storm, (2) time independent flow rate for each stage, and (3) time dependent flow rate.

A. Individual Storm

Recall that each watershed area has its characteristic unit hydrograph. Hence, we will assume that the unit hydrograph of Shelbyville Reservoir watershed is known. The total runoff is a stochastic process. From the hydrograph of the storm on March 1911, the total runoff is 1.14 inch. The hydrograph and unit hydrograph data is present on Table 3.3 and the initial storage is 50,000,000 $ft.^3$.

Because of our emphasis on flood control, in determining the distribution of total runoff of each storm we only consider the runoff data of some rainy season. For our example, there are December, January, February, March, April, May and June. The summer and fall seasons are relatively dry. There are a total 35 data points

0.85	3.37	2.08	5.29	0.62	0.03	0.05	1.67	1.22	4.60	2.17
1.89	2.46	0.66	2.14	0.69	1.36	0.30	2.06	0.53	0.57	1.84
1.42	1.14	2.63	0.74	0.11	0.98	0.95	1.30	4.59	3.20	2.08
0.48	0.07									

The runoff can be modeled as a Gamma distribution with $\alpha = 1.25$, $r = 2$ (the test of Goodness of Fit by Chi-square Test and Komogorov Test is on Appendix C1). The expected value is $2/1.25 = 1.6$ in. The result of optimal release policy for the individual storm case is

NAME	OWNER	WATERSHED AREA(sq.mi)	HIGHT OF DAM (ft)	POOL AREA (acre)	STORAGE CAPACITY (ac-ft)
SHELBYVILLE RESERVOIR	U.S.CORPS OF ENGINEERS	1030	65	10000	22960

Table 3.1 Basic data of Shelbyville Reservoir.

Period of record	max cfs	min cfs	95%	90%	80%	70%	60%	50%	40%	30%	20%	10%	5%	0%
4.8 year	10600	0.2	17	26	61	123	238	420	720	1070	1740	3150	4600	10600

Table 3.2 Cumulative distribution of stream flow rate.

TIME (date)	Mar. 6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
Ordinate of Hydrograph	600	1360	2400	2450	2300	2050	1910	1680	1450	1150	1030	918	810	738	702	666
Ordinate of Unit graph	0	608	1521	1564	1443	1214	1091	889	687	424	319	264	126	63	31	0

Table 3.3 Ordinate of hydrograph of Shelbyville Reservoir.

present on Table 3.4. Reference to Table 3.4 reveals that the optimal release is 1500 cfs (maximum nonflood release rate) at most stages. This is because in most cases, the capacity of the reservoir is sufficient to handle any single storm flood. The initial expected damage cost is $2.385 million. For standard release rule, the expected damage cost is $1.92 million. We cannot determine the expected damage cost of the routing policy since we are lacking both the storage vs. elevation and the outflow vs elevation curves. The optimal expected damage cost is $0.5898 million. The computing time to determine the optimal policy was 158 CPU second on Northwestern University's CDC 6600.

B. Time Independent Inflow Rate

Suppose that at any time interval the flow rate is distributed as in Table 3.3. Here for a two month period (12 stages), with five days for each stage interval, the optimal release rule is as shown on Table 3.5. When the initial storage is half full the optimal expected cost is $0.56 million.

C. Time Dependent Inflow Rate

A set of historical or synthetic flow data for a stream is a sequence of numbers or values produced by a random process in a succession of time intervals; such a sequence is called a time series [60]. In general, the i^{th} member of a time series which we write X_i, is the sum of two parts [61].

$$X_i = d_i + e_i \tag{3.78}$$

Here d_i is the deterministic part. Typically, d_i might be a function

STAGE / CFS / STATE	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0×10^6		1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1050	850	700
50×10^6	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1450	1300
100×10^6		1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
150×10^6		1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
200×10^6		1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
250×10^6		1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
300×10^6		1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
350×10^6		1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
400×10^6		1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
450×10^6		1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
500×10^6		1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
550×10^6		1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
600×10^6		1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
650×10^6		1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
700×10^6		1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
750×10^6		1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
800×10^6		1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
850×10^6		1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
900×10^6		1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
950×10^6		1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
1000×10^6		1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500

Table 3.4 Optimal release rule of individual storm of Shelbyville Res.

STAGE / 10^6 Cfs / STATE	1	2	3	4	5	6	7	8	9	10	11	12	Optimal Expect Cost
0	2200	2200	2150	2100	2100	2050	2000	1900	1850	1750	1650	1500	0.541
50	2200	2200	2150	2100	2100	2050	2000	1900	1850	1750	1650	1500	0.542
100	2150	2100	2100	2050	2000	1950	1900	1850	1800	1650	1550	1500	0.544
150	2050	2000	2000	1950	1900	1850	1750	1700	1650	1550	1500	1500	0.547
200	1950	1900	1850	1800	1800	1750	1700	1650	1600	1550	1500	1500	0.549
250	1850	1850	1800	1800	1750	1750	1700	1650	1600	1550	1500	1500	0.550
300	1800	1800	1750	1750	1700	1650	1650	1600	1550	1500	1500	1500	0.551
350	1700	1700	1650	1650	1650	1600	1600	1550	1550	1500	1500	1500	0.552
400	1700	1650	1650	1650	1600	1600	1600	1550	1500	1500	1500	1500	0.552
450	1650	1600	1600	1600	1600	1550	1550	1500	1500	1500	1500	1500	0.553
500	1600	1600	1600	1600	1550	1550	1550	1500	1500	1500	1500	1500	0.560
550	1600	1600	1550	1550	1550	1550	1500	1500	1500	1500	1500	1500	0.565
600	1550	1550	1550	1550	1550	1500	1500	1500	1500	1500	1500	1500	0.569
650	1550	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	0.570
700	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	0.572
750	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	0.574
800	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	0.602
850	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	0.635
900	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	0.678
950	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	0.703
1000	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	0.724

Table 3.5 Optimal release rate with time independent inflow rate.

of the mean flow, of the variability of flow, (as measured by their standard deviation), and of previous flow such as X_{i-1}, X_{i-2}, ... The random component of the generation scheme is e_i. It is a random number drawn or sampled from a set of random numbers with a certain probability distribution or pattern. In this discussion, we will assume that d_1 depends on stage i, but not on the previous flows. In the Markovian case, we will use the limiting probability of the flow rate distribution.

We assume the average flow rate at each stage of the same month is constant. These values are presented on Table 3.6. The data are not sufficient to test the distribution of e_i; hence, we will assume the distribution of e_i is a Normal distribution [61] with mean 0 and variance as the sample variance. The problem was solved by the dynamic programming algorithm presented in Figure 3.6. The release policies for the two month periods of January-February, March-April, and May-June are presented on Table 3.7, 3.8, 3.9 respectively. The optimal expected damage cost for these three two-month periods are $1.93, $3.6, and $2.15 million respectively. The computing time for finding the optimal policy of each two-month period was 211 CPU seconds on Northwestern University's CDC 6600.

3.4 Simulation

In this section we simulate the real-time operation of flood control systems using the optimal release policies determined in §3.2 and §3.3. In this manner we can assess the feasibility and practicality of using calculated optimal release rules in real time.

Digital computer simulation is a popular and powerful tool for using

Month	Jan.	Feb.	Mar.	Apr.	May.	Jun.	July.	Aug.	Sep.	Oct.	Nov.	Dec.
Mean CFS	1114	1938	2088	2364	2208	669	672	166	447	839	485	415
Stand Deviation	560.3	969.1	1043.5	1181.7	1105.6	336.3	333	83	229	438	252	213

Table 3.6 Monthly Mean and Stand Deviation of Kaskaskia River.

Stage / State 10^6 cfs	1	2	3	4	5	6	7	8	9	10	11	12	Optimal Expected Cost
0	2200	2200	2200	2200	2200	2200	2200	2100	1950	1800	1600	1500	1.93016
50	2300	2300	2300	2300	2300	2300	2150	2000	1900	1800	1600	1500	1.93016
100	2450	2450	2450	2450	2450	2450	2100	2000	1900	1800	1650	1500	1.93016
150	2550	2550	2550	2550	2550	2550	2100	2000	1900	1800	1650	1500	1.93016
200	2650	2650	2650	2650	2650	2650	2100	2000	1900	1800	1650	1500	1.93016
250	2750	2750	2750	2750	2750	2750	2050	2000	1900	1800	1650	1500	1.93016
300	2900	2900	2950	2900	2900	2900	2050	2000	1900	1800	1700	1500	1.93016
350	3000	3000	3000	3000	3000	2550	2050	2000	1900	1800	1700	1500	1.93016
400	3100	3100	3100	2650	2300	2200	2050	2000	1900	1850	1700	1500	1.93016
450	3250	3250	2500	2250	2150	2100	2050	1950	1900	1800	1700	1500	1.93016
500	2850	2450	2250	2150	2100	2100	2050	1950	1900	1850	1700	1500	1.93016
550	2500	2300	2200	2100	2050	2050	2000	1950	1900	1800	1700	1500	1.93016
600	2350	2200	2150	2100	2050	2050	2000	1950	1900	1850	1750	1500	1.93016
650	2300	2200	2150	2050	2050	2050	2000	1950	1900	1850	1750	1500	1.93016
700	2250	2150	2100	2050	2050	2050	2000	1950	1900	1800	1700	1500	1.93016
750	2250	2150	2100	2050	2050	2050	1950	1900	1850	1800	1700	1500	1.93016
800	2200	2150	2100	2050	2050	2050	1950	1900	1850	1750	1650	1500	1.93017
850	2200	2150	2100	2050	2050	2000	1900	1850	1800	1700	1650	1500	1.93017
900	2200	2150	2100	2050	2050	2000	1850	1800	1750	1650	1550	1500	1.93018
950	2200	2150	2100	2050	2050	2000	1750	1700	1600	1550	1500	1500	1.93020
1000	2200	2150	2100	2050	2050	2000	1650	1550	1500	1500	1500	1500	1.93022

Table 3.7 Jan-Feb Optimal Release Policy for Shelbyville Reservoir.

STAGE / STATE 10^6 cfs	1	2	3	4	5	6	7	8	9	10	11	12	Optimal Expected Cost
0	4150	4150	4150	4150	4150	4150	3400	2850	2500	2150	1850	1500	3.600
50	4250	4250	4250	4250	4250	4250	3150	2750	2450	2150	1850	1500	3.600
100	4400	4400	4400	4400	4499	4400	2950	2700	2400	2150	1900	1500	3.600
150	4500	4500	4500	4500	4500	4500	2850	2650	2400	2150	1900	1500	3.600
200	4600	4600	4600	4600	4600	3400	2800	2600	2400	2150	1900	1500	3.600
250	4700	4700	4700	4700	3600	3150	2750	2600	2350	2150	1900	1500	3.600
300	4850	4850	4850	4050	3300	3000	2750	2500	2350	2150	1900	1500	3.600
350	4950	4950	4950	3500	3100	2900	2700	2500	2300	2150	1950	1500	3.600
400	5050	5050	3750	3250	2950	2850	2650	2500	2300	2150	1950	1500	3.600
450	5200	4450	3450	3100	2900	2800	2650	2450	2300	2150	1900	1500	3.600
500	5300	3850	3350	3050	2850	2750	2600	2450	2250	2150	1950	1500	3.600
550	4550	3600	3200	2950	2850	2750	2600	2400	2250	2100	1950	1500	3.600
600	4050	3450	3150	2950	2800	2750	2550	2400	2250	2100	1950	1500	3.601
650	3800	3350	3100	2900	2800	2700	2550	2400	2200	2100	1900	1500	3.603
700	3650	3300	3050	2900	2750	2700	2500	2350	2200	2100	1900	1500	3.605
750	3600	3250	3000	2850	2750	2700	2550	2350	2150	2050	1900	1500	3.607
800	3500	3200	3000	2850	2750	2650	2450	2250	2150	2000	1900	1500	3.609
850	3500	3150	2950	2850	2750	2650	2400	2200	2100	1950	1800	1500	3.612
900	3450	3150	2950	2800	2750	2650	2350	2200	2050	1900	1750	1500	3.615
950	3400	3100	2900	2800	2700	2600	2250	2100	1950	1800	1600	1500	3.617
1000	3350	3100	2900	2750	2650	2600	2150	2000	1800	1550	1500	1500	3.621

Table 3.8 Mar-Apr Optimal Release Policy for Shelbyville Reservoir.

Stage / State 10^6 cfs	1	2	3	4	5	6	7	8	9	10	11	12	Optimal Expected Cost
0	2900	2500	2200	2000	1750	1500	1300	1300	1300	1300	1300	1300	1.8351
50	2750	2450	2200	2000	1750	1500	1400	1400	1400	1400	1400	1400	1.8667
100	2650	2400	2200	2000	1800	1500	1500	1500	1500	1500	1500	1500	1.8914
150	2550	2350	2200	2000	1800	1500	1500	1500	1500	1500	1500	1500	1.9302
200	2550	2350	2150	2000	1800	1500	1500	1500	1500	1500	1500	1500	1.9596
250	2500	2300	2150	2000	1800	1500	1500	1500	1500	1500	1500	1500	1.9918
300	2450	2300	2150	2000	1850	1500	1500	1500	1500	1500	1500	1500	2.0221
350	2450	2300	2150	2000	1850	1500	1500	1500	1500	1500	1500	1500	2.0488
400	2400	2250	2150	2000	1850	1500	1500	1500	1500	1500	1500	1500	2.0897
450	2400	2250	2150	2000	1850	1500	1500	1500	1500	1500	1500	1500	2.1181
500	2350	2200	2100	2000	1850	1500	1500	1500	1500	1500	1500	1500	2.1508
550	2350	2200	2100	2000	1850	1500	1500	1500	1500	1500	1500	1500	2.1824
600	2300	2200	2100	2000	1850	1500	1500	1500	1500	1500	1500	1500	2.2177
650	2300	2200	2100	2000	1850	1500	1500	1500	1500	1500	1500	1500	2.2421
700	2250	2150	2100	2000	1850	1500	1500	1500	1500	1500	1500	1500	2.2722
750	2250	2150	2050	1950	1800	1500	1500	1500	1500	1500	1500	1500	2.2996
800	2200	2100	2000	1900	1800	1500	1500	1500	1500	1500	1500	1500	2.3346
850	2200	2100	2000	1900	1700	1500	1500	1500	1500	1500	1500	1500	2.3513
900	2150	2050	1900	1800	1650	1500	1500	1500	1500	1500	1500	1500	2.3816
950	2100	1950	1800	1700	1550	1500	1500	1500	1500	1500	1500	1500	2.3988
1000	1950	1800	1600	1500	1500	1500	1500	1500	1500	1500	1500	1500	2.4227

Table 3.9 May-June Optimal Release Policy for Shelbyville Reservoir.

in water resources planning. It allows for a rapid evaluation of system performance for a set of trial policies. The performance of a system is evaluated through a quantitative model which is a mathematical representation of the system under study. The operating policy of the system was the major concern of this study.

3.4.1 Effectiveness Parameters for Flood Control

Three effectiveness parameters for flood control which have been proposed [10] are presented below.

1. Absolute Flood Peak Reduction (ΔQ_p)

The absolute flood peak reduction, ΔQ_p is defined as

$$\Delta Q_p = Q_p - Q_{pR} \qquad \text{(cfs)}, \tag{3.79}$$

where

Q_p = flood peak for non-regulated flood, and

Q_{pR} = flood peak for flow regulated by reservoirs.

The parameter ΔQ_p would be the one used directly in flood damage mitigation calculations.

2. Relative Flood Peak Reduction (RR)

In order to facilitate an interpretation of the results obtained with different policies, it is best to normalize ΔQ_p. One normalization, called the relative flood peak reduction or RR, is defined as

$$RR = 100 \frac{\Delta Q_p}{Q_p} \% \tag{3.80}$$

This parameter isuseful in relating flood control effectiveness to system configuration.

3. Relative Flood Reduction (RC)

Another normalization of ΔQ_p called the relative flood reduction, or RC, is defined as

$$RC = 100 \frac{\Delta Q_p}{Q_p - \bar{Q}_f} \%, \tag{3.81}$$

where $\bar{Q}_f$ = maximum allowable discharge which will not cause a flood. This parameter is useful in relating policy performance with variations in $\bar{Q}_f$ induced for example by levee construction, zoning, etc.

The relation of these Effectiveness Parameters for flood control is displayed graphically on Figure 3.8.

3.4.2 Results of Simulation

A. Historical Record Data

(a) Individual Storm

There were a total of seven simulation runs. The data for these runs were chosen arbitrary from the flow rate data in Appendix B. Each simulation run begins from the increasing period of the hydrograph. From these inflow data we can see that there was a single storm, but we don't known the time duration of each rainfall. The results are not bad. But this policy has been restricted to a single storm flood, because it does not permit consideration of the combination of two or more storm hydrographs. The results of the simulation of an individual storm is on Table 3.10. In Table 3.10, the first column is inflow of each stage, the second column is the optimal release rule, and the third column is the standard release rule. The values of the average

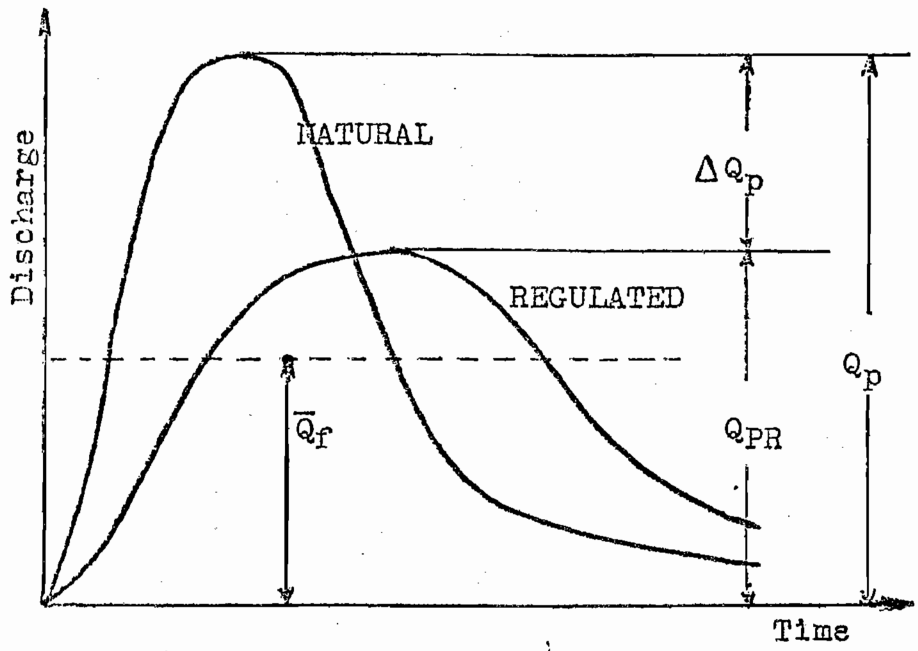


Figure 3.8 Effectiveness Parameters for flood control

	1			2			3			4			5			6			7		
Stage	IN	OPT	STAN	IN	OPT	STAN	IN	OPT	STAN	IN	OPT	STAN	IN	OPT	STAN	IN	OPT	STAN	IN	OPT	STAN
1	494	1500	1500	864	1500	1500	965	1500	1500	666	1500	1500	774	1500	1500	358	1500	1500	864	1500	1500
2	1810	1500	1500	1070	1500	1500	1070	1500	1500	1360	1500	1500	1070	1500	1500	2550	1500	1500	1230	1500	1500
3	4090	1500	1500	2800	1500	1500	1230	1500	1500	2400	1500	1500	3700	1500	1500	3380	1500	1500	1540	1500	1500
4	4650	1500	1500	3160	1500	1500	1540	1500	1500	2450	1500	1500	4310	1500	1500	3700	1500	1500	1580	1500	1500
5	3540	2796	2796	3050	1500	1500	1400	1500	1500	2300	1500	1500	4870	2936	2936	3160	1500	1500	1230	1500	1500
6	2350	2350	2350	2950	1500	1500	1270	1500	1500	2050	1500	1500	4650	4650	4650	3260	3120	3120	1860	1500	1500
7	1810	1810	1810	2850	1956	1956	1230	1500	1500	1910	1500	1500	3260	3260	3260	3380	3380	3380	2200	1500	1500
8	1680	1680	1680	2750	2750	2750	1230	1500	1500	1680	1500	1500	2850	2850	2850	3430	3430	3430	2150	1500	1500
9	1490	1500	1500	2550	2550	2550	1150	1500	1500	1450	1500	1500	2700	2700	2700	2400	2400	2400	1860	1500	1500
10	1270	1500	1500	1810	1810	1810	1030	1500	1500	1150	1500	1500	2300	2300	2300	2150	2150	2150	1680	1500	1500
11	1270	1500	1500	1400	1500	1500	918	1500	1500	1030	1500	1500	1810	1810	1810	1810	1810	1810	1490	1500	1500
12	1030	1500	1500	1360	1500	1500	846	1500	1500	918	1500	1500	1450	1500	1500	1580	1580	1580	1270	1500	1500
13	918	1500	1500	1360	1500	1500	774	1500	1500	810	1500	1500	1270	1500	1500	1360	1500	1500	1030	1500	1500
14	774	1500	1500	1270	1500	1500	738	1450	1500	738	1500	1500	1100	1500	1500	1150	1500	1500	832	1500	1500
15	846	1500	1500	882	1500	1500	702	700	880	702	1500	1500	994	1500	1500	994	1500	1500	864	1500	1500
Cost	4.49	2.60	2.60	3.07	2.53	2.53	0.09	0	0	2.09	0	0	4.65	4.5	4.5	3.65	3.36	3.36	1.68	0	0
ΔQ_p		1854	1854		410	410		40	40		950	950		220	220		270	270		700	700
RR		398%	398%		129%	129%		2.5%	2.5%		387%	387%		4.5%	4.5%		7.3%	7.3%		318%	318%
RC		587%	587%		247%	247%		100%	100%		100%	100%		6.5%	6.5%		123%	123%		100%	100%

Table 3.10 Result of Individual Storm Simulation.

effective parameters are presented on Table 3.11. Reference to Table 3.11, reveals that for real time operation the optimal policy for individual storm is quite good. The reason is that in most cases the capacity of the reservoir is sufficient to handle any single storm flood.

(b) Stage Independent Input

In simulating flood control the stage time interval can not be too long. Here we assume that the time interval is five days and the initial storage is 500,000,000 ft.^{3} (half full). The inflow data are the five days average historical data and are given from Table 3.12. The planning horizon, T, for each simulation run was two months (12 stages). The results of the simulation runs are presented in detail in Appendix D1. The average value of the effective parameters are presented on Table 3.13.

Reference to Table 3.13 reveals that for real time operation the optimal release policy is a little worse than standard release policy and both the release rules are not effective in reducing the flood peak. The reasons for this are:

1. The inflow rate is dependent upon the season, and varies significantly between wet seasons and dry seasons. However, in the simulation runs we use release policies which assumed that the distribution is stationary and we used the year-round average distribution to approximate the nonstationary flow distribution.
2. In a relatively short time interval, for example, a day, a week, or a month, the flow is clearly not time independent and stationary. If the time interval is a year, it may be time independent. For a storage reservoir, one year intervals may be appropriate. However,

	Initial flow	optimal Release Rule	Standard Release Rule
Average Damage Cost	2.817	1.855	1.855
Average RR		19.64%	19.64%
Average RC		57.46%	57.46%

Table 3.11 Average Value of Effectiveness Parameters from Simulation using Historical data with Individual Storm Inflow Assumption.

Month / 5 days Period	Jan.	Feb.	Mar.	Apr.	May.	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
1			4606	2356	4662	992	237	70	24	18	11	30
2			4870	2030	8978	807	351	50	16	13	18	27
3			4230	2106	6316	710	224	32	10	13	18	34
4			2214	1162	4302	392	130	27	12	10	25	27
5			1402	1291	3028	324	123	22	10	9	37	24
6			1104	2550	1720	211	76	22	15	9	50	30
Average			3010	1920	4730	572	187	46	15	12	26	28

Table 3.12 Five Days Average Flow Rate (1908)

Month 5 Days Period	Jan.	Feb.	Mar.	Apr.	May.	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
1	1078	2272	796	1466	1456	173	31	15	14	7324	746	846
2	810	1133	1835	3906	782	154	24	21	51	4052	702	597
3	2342	812	1648	3018	528	104	33	12	1135	1794	1628	752
4	2174	1502	840	3670	385	60	14	10	1228	1119	2552	824
5	1052	1432	569	1524	607	56	15	14	955	2056	2146	1082
6	2319	1137	550	1007	262	64	13	33	5300	1625	1255	1130
Average	1650	1400	1020	2430	657	102	21	17	1450	2950	1500	880

Table 3.12a <u>Five Days average flow rate (1911)</u>

Month 5 Days Period	Jan.	Feb.	Mar.	Apr.	May.	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
1	1070	300	4588	5354	4728	476	257	102	249		66	71
2	820	300	1328	2866	2362	312	426	82	127		122	76
3	820	300	1605	1966	1566	226	679	236	68		102	76
4	820	300	5950	1892	1346	660	529	433	63		87	63
5	820	1590	6104	2146	760	573	243	841	63		76	52
6	820	5870	4852	3526	573	396	188	292	58		76	63
Average	852	1250	4100	2960	1850	441	381	330	105		88	67

Table 3.12b <u>Five Days Average Flow Rate (1912)</u>

5 Days Period \ Month	Jan.	Feb.	Mar.	Apr.	May.	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
1	34	98	2192	372	2102	3580	682	271	32	18	87	351
2	27	406	1420	4070	2380	2272	4450	102	22	13	80	237
3	21	885	1152	5668	2808	1980	2908	68	19	13	91	770
4	32	2128	800	5374	6052	1365	3158	29	18	36	563	1344
5	80	3574	569	5998	638	576	1795	16	34	104	868	628
6	62	3560	516	3910	2420	639	506	29	27	127	563	287
Average	43	1650	1090	4230	1940	1740	2190	84	25	54	375	593

Table 3.12c Five days Average flow rate (1909)

5 days Period \ Month	Jan.	Feb.	Mar.	Apr.	May.	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
1	280	723	3386	270	593	1164	732	676	81	189	86	894
2	330	624	1860	205	1285	644	420	173	2050	867	77	392
3	1972	433	916	186	1950	385	312	80	936	475	74	344
4	4426	351	562	351	1103	256	1119	719	291	267	67	244
5	3294	433	412	317	2945	119	493	385	203	195	67	238
6	1420	2096	335	358	3514	383	436	158	260	128	2263	860
Average	1910	682	1220	281	1840	491	581	359	638	343	439	507

Table 3.12d Five Days Average flow rate (1910)

	Initial Flow	Optical Release Rule	Standard Release Rule
Average Damage Cost	2.500169	2.110559	2.030202
Average RR		13.29%	16.88%
Average RC		22.95%	35.46%

Table 3.13 Average value of Effectiveness Parameters from Simulation Using Historical data with Time Independent Inflow rate Assumption.

	Initial Flow	Optimal Release Rule	Standard Release Rule
Average Damage Cost	4.2246	3.541045	3.5523309
Average RR		13.14%	13.08%
Average RC		19.97%	19.03%

Table 3.14 Average Value of Effectivesness Parameters from Simulation Using Historical data with Time Dependent Inflow rate Assumption

in simulating the operation of flood control reservoirs the stage interval should be relatively short.

(c) Time Dependent Inflow Rate

In the simulation the historical record data on Table 3.12 was used. The results of the simulation runs for January-February, March-April, and May-June are on Appendix D2 and the average value of effective parameters are presented on Table 3.14.

In the process of determining the optimal policy, we assumed that the distribution of flow in each season was Normal with expected value equal the sample mean and variance equal to the sample variance. Here the simulation used only five years of the historical data, which did not appear to be long enough. Hence, the optimal release rule was not much more effective than the standard release rule.

Next we will use the Markovian Model to simulate the stream flow and examine the effects of the release rules.

B. Simulated Stream Flow Data

For the simplest Markovian model [61], we have

$$q_i = \beta_0 - \beta_1 q_{i-1} + e_i, \tag{3.82}$$

where

q_i is the flow rate at stage i, and

e_i is the random part of the synthetic flow.

In this Markovian flow model, we consider that e_i has a Normal distribution. We simulated the flow for three two-month periods: January-February, March-April, and May-June. Each month was divided into six stages so that there were five days in each stage interval.

In each month we assumed that the mean was constant $\bar{X}_j$. In each two month interval, we have

$$q_i = \bar{X}_1(1 - r_1) + r_1 q_{i-1} + t_i S_1 \sqrt{(1 - r_1^2)} \qquad i = 1, 2, \ldots, 6 \qquad (3.83)$$

$$q_i = \bar{X}_2(1 - r_2) + r_2 q_{i-1} + t_i S_2 \sqrt{(1 - r_2^2)} \qquad i = 7, 8, \ldots, 12 \qquad (3.84)$$

in which $\bar{X}_1$ and $\bar{X}_2$ are the means of the first and second months of each two month period, respectively, r_1 and r_2 are the correlation coefficients between stages in the first and second month period, respectively, and t_1, t_2, ... is a sequence of independent Normally distributed random numbers with mean 0 and standard deviation 1.

If q_j was negative for some j, then we used the negative q_j in the equation for q_{j+1} and discarded q_j without using it as a flow in the simulation. The correlation coefficients for January, February, March, April, May, and June, were 0.2865, 0.3639, 0.3137, 0.4742, 0.3268, and 0.3114 respectively.

The results of fifty simulated years for each two month period are presented in Appendix D3. Both the simulated and historical data were tested for identical population distribution functions (see Appendix C2 for the results). The comparison of the effective parameters is presented on Tables 3.15, 3.16, and 3.17, respectively.

From the relative flood reduction (RC), we can see that the time dependent optimal policy is much better than both the time independent optimal and the standard release policy. For the January-February and May-June periods, the time dependent optimal policy can reduce almost 40% of the flood peak. The March-April period reduction is relatively small, because the average flow rate in that period is

	Time Independent Opt. Policy	Time Dependent Opt. Policy	Standard Policy	Initial Flow
Flood Damage	2.3232	2.2374	2.5922	3.8966
Average RR	18.48%	20.912%	12.828%	
Average RC	35.454%	39.152%	25.352%	

Table 3.15 Comparision of Effectiveness Parameters (Jan.-Feb.)

	Time Independent Opt. Policy	Time Dependent Opt. Policy	Standard Policy	Initial Flow
Flood Damage	3.80328	3.4765	3.89102	4.0147
Average RR	3.402%	11.424%	1.234%	
Average RC	5.858%	17.376%	1.964%	

Table 3.16 Comparision of Effectiveness Parameters (Mar.-Apr)

	Time Independent Opt. Policy	Time Dependent Opt. Policy	Standard Policy	Initial Flow
Flood Damage	2.8686	2.47	2.8974	3.673
Average RR	9.184%	18.806%	8.541%	
Average RC	17.62%	34.15%	16.954%	

Table 3.17 Comparision of Effectiveness Parameters (May.-Jun.)

higher than the capacity of reservoir to effectively handle it.

From the previous discussion and results, we know that the use of historical data to estimate the probability of flow in each period and then determine the optimal policy is not an adequate procedure. The reason for this is that the natural conditions are highly variated and correlated. Furthermore the relation of these variations and the actually probability distributions are difficult to estimate.

The Program listing appears in Appendix A5.

CHAPTER IV

EFFECTS OF VARYING PARAMETERS

4.1 Introduction

In this chapter, we discuss the effects of varying the flood control parameters: initial storage quantity, flood stage and purposes of reservoirs. In flood control, we may adjust the water level of reservoirs when a storm has been predicted. We may also increase the flood stage by channel improvements or the construction of levees. In §4.2 and §4.3 we discuss the effects of varying initial storage quantity and flood stage for both the deterministic inflow and the stochastic inflow cases. In §4.4 we formulate a dynamic programming algorithm to determine the optimal release policies for a dual-purpose reservoir.

4.2 Effect of Varying the Initial Storage Quantity

The initial storage quantity is critical in flood control system operation. When a heavy storm has been predicted, the water level of the reservoir should be changed to some best situation in order to absorb the flood peak as much as possible.

The following results specify the "best" situation and tend to confirm our intuition.

Proposition 4.1 Consider the single reservoir, deterministic inflow flood control model of §2.2. Assume that the input quantity of water in a single time interval can not exceed the capacity C. Then the optimal damage cost is a monotone nondecreasing function of the initial storage.

Proof:

The proof is by induction. Recall from §2.5.1 that the boundary condition is

$$f_{N+1}(Z_{N+1}) = 0. \qquad \forall Z_{N+1}$$

Clearly, the result is true for N+1.

Consider stage N. Recall from (2.46) we had that

$$f_N(Z_N) = \min_{r_N \in R_N} (U(r_N)), \tag{4.1}$$

where

$$R_N = [\max\{0, (Z_N + Q_N - C)/\Delta T\}, (Z_N + Q_N)/\Delta T]. \tag{4.2}$$

By basic assumption, $U(\cdot)$ is a monotone nondecreasing function of r. Hence

$$f_N(Z_N) = U(Lb_N(Z_N, Q_N)) \tag{4.3}$$

where

$$Lb_N(Z_N, Q_N) = \max\{0, (Z_N + Q_N - C)/\Delta T\} .$$

Clearly since $Lb_N(Z_N, Q_N)$ is a monotone nondecreasing function of Z_N, $f_N(Z_N)$ is also a monotone nondecreasing function of Z_N.

Next we make the inductive assumption that the result is for $n + 1, n + 2, \ldots, N + 1$, that is, $f_{n+1}(Z_{n+1})$ is a monotone nondecreasing function of Z_{n+1}. From (2.46) we have that for the n^{th} stage

$$f_n(Z_n) = \min_{r_n \in R_n} \{\max(U(r_n), f_{n+1}(Z_n + Q_n - r_n \Delta T))\}$$

$$= \min_{r_n \in R_n} \{\max(h_1(r_n), h_2(r_n))\} \tag{4.4}$$

By the inductive assumption we have that for fixed r_n, $f_{n+1}(Z_n + Q_n - r_n \Delta T)$ is a monotone nondecreasing function of Z_n. For fixed Z_n, we have from (2.45) that

$$Lb_n(Z_n, Q_n) = \max\{0, (Z_n + Q_n - C)/\Delta T\},$$

and

$$Ub_n(Z_n, Q_n) = (Z_n + Q_n)/\Delta T.$$

There are three cases for consider i) $r_n = Lb_n$, ii) $r_n = Ub_n$, and iii) $r_n \in (Lb_n, Ub_n)$.

If $r_n = Lb_n$, then

$$h_1(r_n) = U(Lb_n(Z_n, Q_n))$$

$$= U(\max\{0, (Z_n + Q_n - C)/\Delta T)\}). \tag{4.5}$$

and

$$h_2(r_n) = f_{n+1}(Z_n + Q_n - \max\{0, (Z_n + Q_n - C)\})$$

$$= f_{n+1}(\min\{Z_n + Q_n, C\}) \tag{4.6}$$

Now if $Z_n = 0$, then

$$h_1(r_n) = U(Lb_n(0, Q_n))$$

$$= U(\max\{0, (Q_n - C)/\Delta T\})$$

$$= U(0) = 0, \qquad (4.7)$$

$$h_2(r_n) = f_{n+1}(Q_n) \geq 0. \qquad (4.8)$$

Hence

$$h_2(r_n) \geq h_1(r_n).$$

Now if $Z_n = C$, then

$$h_1(r_n) = U(Q_n/\Delta T), \qquad (4.9)$$

and

$$h_2(r_n) = f_{n+1}(\min\{C + Q_n, C\})$$

$$= f_{n+1}(C) \qquad (4.10)$$

here the value of $f_{n+1}(C)$ depends on the inflow Q_{n+1}. Hence for $r_n = Lb_n$ the relation of h_1 and h_2 depends on Q_n and Q_{n+1}.
If $r_n = Ub_n = (Z_n + Q_n)/\Delta T$, then

$$h_1(r_n) = U((Z_n + Q_n)/\Delta T), \qquad (4.11)$$

and

$$h_2(r_n) = f_{n+1}(Z_n + Q_n - Z_n - Q_n) = f_{n+1}(0). \qquad (4.12)$$

At the upper bound point the value of h_2 is the constant $f_{n+1}(0)$.

Consider $r_n \in (Lb_n, Ub_n)$. If h_1 and h_2 intersect, then this intersection is in decision region R_n. If there is no intersection then h_2 is above h_1 or h_1 is above h_2. The configuration of those curves is illustrated on Figure 4.1. If h_2 is above h_1, the $f_n(Z_n) = f_{n+1}(0)$, it is a constant. If h_2 is under h_1 the $f_n(Z_n)$ is on curve h_1. Hence, $f_n(Z_n)$ is monotone nondecreasing function of Z_n. ||

The following intuitive result is an immediate consequence of Proposition 4.1

Corollary 4.1 Consider the flood control model of §2.2. When a flood flow is predicted, the optimal operating policy is to set the water level in the reservoir as low as possible with a safe release rate.

We next consider the stochastic case.

Proposition 4.2 Suppose that the inflow quantities, Q_n, $n = 0, 1, \ldots N$, are independent random variables with known probability density $p_n(q)$. Then the optimal expected cost is a monotone nondecreasing function of the initial storage of the reservoir.

Proof:

The proof is by induction. Recall from §3.3.1 the boundary condition is $f_{N+1} = 0$. Clearly the result is true for $N + 1$.

Consider stage N. Recall from (3.41) we have that

$$f_N(Z_N) = \min_{r_N \in R_N} \{E(U(r_N))\} \tag{4.13}$$

in which, for fixed Q_N,

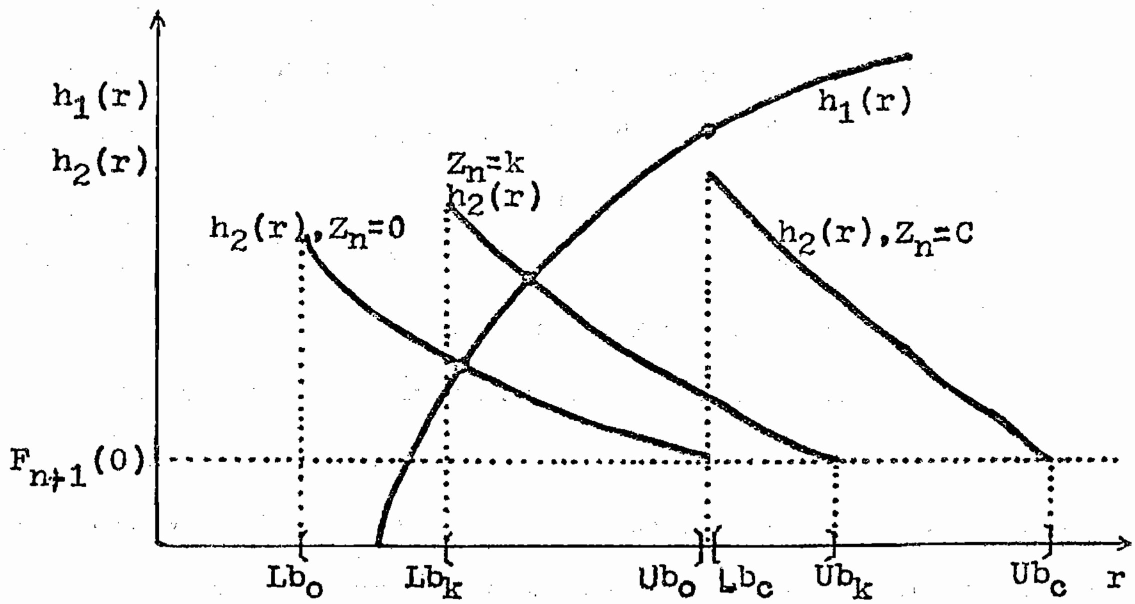


Figure 4.1 Illustration of $h_1(r)$ and the family of $h_2(r)$

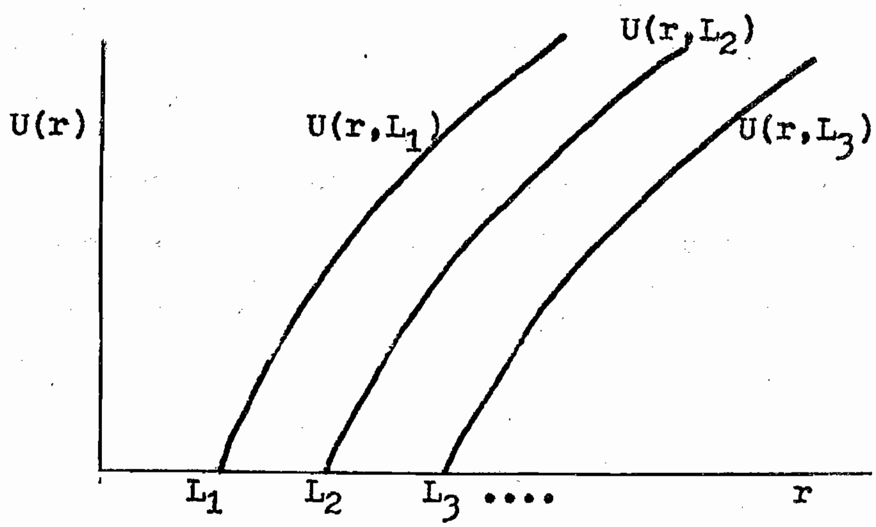


Figure 4.2 Family of damage cost functions for different flood stages

$$Lb_N(Z_N, Q_N) = \max\{0, (Z_N + Q_N - C)/\Delta T\} \qquad (4.14)$$

$$Ub_n(Z_n, Q_N) = (Z_N + Q_N)/\Delta T \qquad (4.15)$$

Now by assumption $U(\cdot)$ is a nondecreasing function of r. The notation E_{Q_N} will be used to denote the expectation with respect to the random variable Q_N. Then $E_{Q_N}(U(r_N))$ is a monotone nondecreasing function of r_N.

We can write $f_N(Z_N)$ as

$$f_N(Z_N) = \min_{r_N \in R_N} \{H_N(r_N)\},$$

where H_N denotes some monotone nondecreasing function of r_N. Recall that (3.55) the optimal decision at stage N is

$$r_N^* = \max\{L, \min[Lb_N(Z_N, Q_N^1), Lb_N(Z_N, Q_N^2), \ldots, Lb_N(Z_N, Q_N^p)]\} , \qquad (4.16)$$

where L is the flood stage and $Q_N^1, Q_N^2, \ldots, Q_N^p$ are all possible discrete values of the input at stage N. Hence, r_N^* is a monotone nondecreasing function of Z_N.

Next we make the inductive assumption that the result is true for $n + 1, n + 2, \ldots, N + 1$, that is, that $f_{n+1}(Z_{n+1})$ is a monotone nondecreasing function of Z_{n+1}. From (3.41) for n^{th} stage, we have

$$f_n(Z_n) = \min_{r_n \in R_n} \{\max(EU(r_n), Ef_{n+1}(Z_n + Q_n - r_n \Delta T))\}, \qquad (4.17)$$

Now for fixed Z_n, $E_{Q_n} F_{n+1}(\cdot)$ is a monotone nonincreasing function of r_n and $E_{Q_n} U(r_n)$ is a monotone nondecreasing function of r_n. For fixed

Q_n also, $F_{n+1}(\cdot)$ is a monotone nonincreasing function of r_n. Hence, $f_n(Z_n)$ can be written as

$$f_n(Z_n) = \min_{r_n \in R_n} \{\max(H_n^1(r_n),\ H_n^2(r_n))\}, \tag{4.18}$$

in which H_n^1 is a monotone nondecreasing function of r_n and H_n^2 is a monotone nonincreasing function of r_n. By a similar argument as in the proof of Proposition 4.1, we can obtain $f_n(Z_n)$ is a monotone nondecreasing function of Z_n.||

The following intuitive result is an immediate consequence of Proposition 4.2.

Corollary 4.2 For the flood control model of §3.3.1, when a flood flow is predicted to be coming the optimal operating policy is to set the water level in the reservoir as low as possible with a safe release rate.

We next consider the effects of varying flood stages.

4.3 Effects of Varying the Flood Stage

As a result of channel improvements or the construction of levees the flood stage will increase. These are also methods of flood control employed on some rivers. Hence, we need to consider the effects of changing the flood stage.

Suppose that the flood stage changes, but the damage cost relative to the depth of water on ground remains the same. This means the damage cost function is a family of curves which are parallel to one another as displayed on Figure 4.2.

Proposition 4.3 Consider the single reservoir, deterministic inflow flood control model of §2.2. The optimal damage cost is a monotone nonincreasing function of the flood stage.

Proof:

The proof is by induction.

Let L_i denote the flood stage, $U(r, L_i)$ denote the damage cost function when the flood stage is L_i and the release rate is r. Recall from §2.5.1 that for boundary condition is

$$f_{N+1}(Z_{N+1}, L_i) = 0. \tag{4.19}$$

Clearly the result is true for N + 1.

Consider stage N. Recall from (2.46) we have that

$$f_N(Z_N, L_i) = \min_{r_N \in R_N} \{(U(r_N, L_i)\}, \tag{4.20}$$

where the decision region $R_N = [\max\{0, (Z_N + Q_N - C/\Delta T\}, (Z_N + Q_N)/\Delta T]$ is not affected by L_i.

From (4.20) the optimal decision at stage N is

$$r_{N,L_i} = \begin{cases} Lb_N(Z_N, Q_N) & \text{if} \quad Lb_N > L_i, \\ \max\{L_i, Lb_N(Z_N, Q_N)\} & \text{if} \quad Ub_N > L_i,\ Lb_N \le L_i, \\ Ub_N(Z_N, Q_N) & \text{if} \quad Ub_N \le L_i,\ Lb_N \le L_i. \end{cases} \tag{4.21}$$

Hence, for fixed L_i, $f_N(Z_N, L_i)$ is a monotone nondecreasing function of Z_N. For fixed Z_N suppose the $L_2 > L_1$. Then from Figure 4.3, we

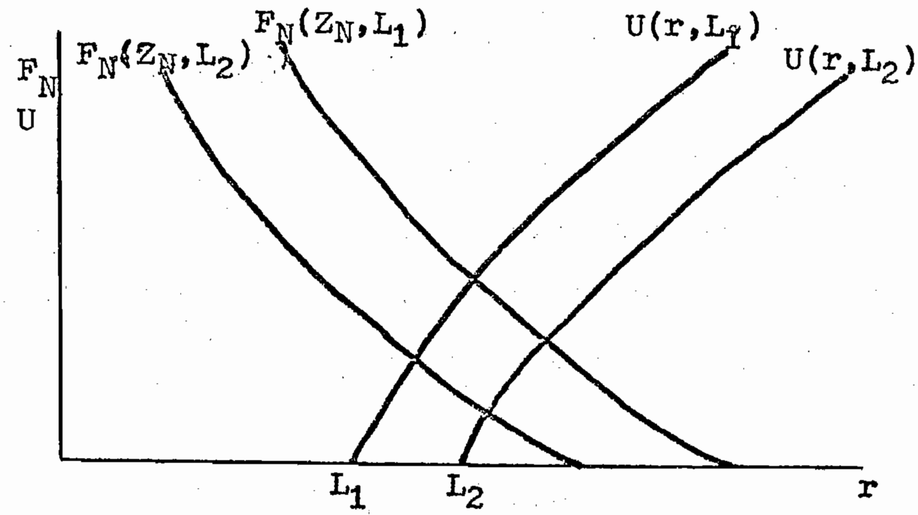


Figure 4.3 Family of curves of $F_N(Z_N, L)$ and $U(r, L)$

can see that

$$f_N(Z_N, L_2) \le f_N(Z_N, L_1).$$

Hence, $f_N(Z_N, \cdot)$ is a monotone nonincreasing function of L.

We next make the inductive assumption that the result is true for n + 1, n + 2, ..., N + 1, that is, that $f_{n+1}(Z_{n+1}, L_i)$ is a nonincreasing function of L_i. From (3.41) for the n^{th} stage, we have

$$f_n(Z_n, L_i) = \min_{r_n \in R_n} \{\max(U(r_n, L_i), f_{n+1}(Z_n + Q_n - r_n \Delta T, L_i))\}, \quad (4.22)$$

For $L_2 > L_1$

$$\max(U(r_n, L_1), f_{n-1}(Z_n - Q_n - r_n \Delta T, L_1))$$

$$\ge \max(U(r_n, L_2), f_{n+1}(Z_n + Q_n - r_n \Delta T, L_2)) \quad (4.23)$$

Hence $f_n(Z_n, \cdot)$ is a monotone nonincreasing function of L.||

We next consider the stochastic case.

Proposition 4.4 Suppose that the inflow quantities Q_n, n = 0, 1, ... N are independent random variables with known probability density $p_n(q)$. Then, the optimal expected cost is a monotone nonincreasing function of the flood stage L.

Proof:

The proof is by induction. Recall from 3.3.1 the boundary condition is $f_{N+1} = 0$.

Clearly, the result is true for N + 1.

Consider stage N. We have

$$f_N(Z_N. L) = \min_{r_N \in R_N} (E_{Q_N} U(r_N, L))$$

For fixed r_N, $U(r_N, \cdot)$ is monotone nonincreasing function of L and for fixed L, $U(\cdot, L)$ is nondecreasing function of r_N. Obviously, $E_{Q_N} U(r_N, L)$ has the same monotonicity as $U(r_N, L)$. Hence, for fixed L, $f_N(Z_N, Q_N)$ is a monotone nondecreasing function of Z_N, and for fixed Z_N, $f_N(Z_N, L)$ is a monotone nonincreasing function of L.

Next we make the inductive assumption that the result is true for $n + 1, n + 2, \ldots, N + 1$.

From (3.41), we obtain

$$f_n(Z_n, L) = \min_{r_n \in R_n} \{\max(E_{Q_n} U(r_n, L), E_{Q_n} f_{n+1}(Z_n + Q_n - r_n \Delta T, L))\}.$$

Now $E_{Q_n} U(r_n, L)$ and $E_{Q_n} f_{n+1}(Z_n + Q_n - r_n \Delta T)$ inherit the same monotonicity property for r_n, and L, as $U(r_n, L)$ and $f_{n+1}(Z_n + Q_n - r_n \Delta T, L)$, respectively. Hence for fixed Z_n, $f_n(Z_n, L)$ is a monotone nonincreasing function of L. Thus, the optimal damage cost $F_1(Z_1, L)$ is a monotone nonincreasing function of L. ||

4.4 Effects of Varying the Objective Function

Here we consider multi-purpose reservoirs which have additional functions such as recreation, water supply, hydroelectric generation, etc.

The recreation requirements impose a minimum water level constraint. However, we can eliminate this constraint simply by changing the capacity of the reservoir (i.e., use the active capacity). The minimum release constraint can be eliminated simply by changing the range of the decision variables.

For flood control and water supply we need a dual-purpose reservoir. The flood damage cost is a function of flood peak and the water supply benefit can be modeled as a sum of the nondecreasing functions of release rates at each stage. The problem is to determine r_n so as to

$$\min U(r) - \sum_{n=1}^{N} B(r_n) \tag{4.24}$$

subject to

$$Z_n - Z_{n-1} = Q_n - r_n \Delta T \qquad n = 0, 1, \ldots, N$$

$$0 \leq Z_n \leq C \qquad n = 0, 1, \ldots, N$$

where

U is the damage cost, a function of peak discharge rate,

r is the peak discharge rate,

B is the benefit function of release rate, and

r_n is the discharge rate in stage n.

We can address this problem with dynamic programming. The transition function is

$$Z_{n+1} = Z_n + Q_n - r_n \Delta T. \tag{4.25}$$

From the constraints $0 \leq Z_n \leq C$, for all n, we obtain the upper bound of r_n as

$$Ub_n(Z_n, Q_n) = (Z_n + Q_n)/\Delta T,$$

and the lower bound of r_n as

$$Lb_n(z_n, Q_n) = \max\{0, (Z_n + Q_n - C)/\Delta T\}.$$

The decision region of r_n is

$$R_n = [Lb_n, Ub_n]$$

Define $f_n(Z_n)$ as the minimum cost from stage n(time $n\Delta T$) to stage

N (time T), if the storage level is Z_n at time $n\Delta T$ (stage n). Then, invoking Bellman's principle of optimality yields the following functional equation of dynamic programming

$$f_n(Z_n) = \min_{r_n \in R_n} \{\max[U(r_n), g_{n+1}(Z_n + Q_n - r_n\Delta T)] - B(r_n)$$

$$- h_{n+1}(Z_n + Q_n - r_n\Delta T)\} \quad n = 0, 1, \ldots, N \qquad (4.26)$$

$$= \max[U(r_n^*), g_{n+1}(Z_n + Q_n - r_n^*\Delta T)] - B(r_n^*) - h_{n+1}(Z_n + Q_n - r_n^*\Delta T),$$

with the boundary condition

$$f_{N+1}(Z_{N+1}) = 0 \qquad (4.27)$$

$$g_{N+1}(Z_{N+1}) = 0 \qquad (4.28)$$

$$h_{N+1}(Z_{N+1}) = 0 \qquad (4.29)$$

where r_n^* is the optimal release rate for state Z_n of stage n and we use the notation

$$h_n(Z_n) = B(r_n^*) + h_{n+1}(Z_n + Q_n - r_n^*\Delta T), \qquad (4.30)$$

and

$$g_n(Z_n) = \max\{U(r_n^*), g_{n+1}(Z_n + Q_n - r_n^*\Delta T)\}. \qquad (4.31)$$

We note that another approach to this problem would be to optimize the two objectives simultaneously as in vector optimization [4, 30] or in bicriterion methematical programming [5, 29]. Then we would search for efficient (undominated, Pareto optimal) solutions--see also [18, 35, 44, 84].

CHAPTER V

OPTIMAL EXPANSION OF FLOOD CONTROL SYSTEMS

5.1 Introduction

There are basically three structural methods of flood protection [69]. Channel improvements increase the discharge of a stream by increasing the velocity and possibly the channel cross-section and decreasing the distance the water has to travel to reach the outlet of the watershed. This method is generally applicable only in small streams. The construction of levees also increases the discharge by increasing the depth of flow, and thereby the velocity, and by providing a flood way outside the channel. On the lower reaches of long rivers, this method affords the only sure means of flood control. Both of these methods control flooding by hastening the flow of the water from the watershed. The construction of a reservoir, gives protection in an entirely different way, namely, by retarding the flow of the water and limiting the flow to the quantity which the channel can safely carry, thereby preventing floods. There are two main requirements which must be met in establishing a reservoir site for flood protection, one is physical and the other is economic. The physical conditions of the watershed must be such that reservoirs can be constructed of sufficient size to store or retard the excess flood water. The cost of such reservoirs must be reasonable.

The problem of expanding an existing flood control reservoir system is formulated in mathematical terms by providing functions, equations, and inequalities that represent appropriate criteria and characteristics of the physical system in §5.2 and §5.3. Of interest here is the question of when a reservoir should be added to an existing system and how large the new reservoir should be. We will consider only the latter problem here. The reader is referred to [52, 55, 56] for a discussion of the timing problem. Dynamic programming is proposed as a solution method in §5.4. The methodology is illustrated using real-world data in §5.6.

We shall first consider some of the major assumptions and decisions that must be made in formulating the problem. The most significant assumptions are that:

(1) The existing system consists of a single reservoir, which can not completely prevent flood damages at the damage center.

(2) In the series case, the new reservoir is located upstream of the river. In the parallel case, the existing reservoir can handle the flood peak of its own tributary, but the release, when combined with the flow on the parallel tributary, will be sufficient to cause flood damages at the damage center.

(3) The inflow hydrograph over the live of the project is assumed to be known and deterministic.

(4) The operating policy of the system has been limited to the optimal policy for flood control.

(5) The capital cost of a reservoir is a monotone increasing function of its storage capacity [67].

5.2 Formulation of the Objective Function

The criterion that will be used for project justification will be economic evaluation.

Series New Reservoir

The objective function is a function of the storage capacity of the new reservoir. Let

$f(C_1)$ = optimal system flood damage cost with a new series reservoir of capacity C_1

$$= \min_{\left\{\begin{matrix}(r_{1,0}, r_{1,1}, \ldots, r_{1,N}) \\ (r_{1,0}, r_{1,1}, \ldots, r_{1,N})\end{matrix}\right\}} \{\max(U_1(r_{1,0}) + U_2(r_{2,0}), \ldots$$

$$U_1(r_{1,N}) + U_2(r_{2,N}))\} \qquad (5.1)$$

and

$$F = \min_{C_1} f(C_1), \qquad (5.2)$$

where

M is the maximum possible storage capacity that can be constructed,

C_1 is the storage capacity of the new reservoir, and

U_i is the damage cost function at damage center i.

We want to find C^* such that

$$C^* = \text{smallest element in } \Omega$$

where $\Omega \subseteq [0, M]$ is such that $f(C) = F$ for $C \in \Omega$

Parallel New Reservoir

Let

$f(C_1)$ = optimal system flood damage cost with a new parallel reservoir of capacity C_1

$$= \min_{\left\{\begin{matrix}(r_{1,0}, r_{1,1}, \ldots, r_{1,N}) \\ (r_{2,0}, r_{2,1}, \ldots, r_{2,N})\end{matrix}\right\}} \{\max_n (U(r_{1,n} + r_{2,n}))\} \tag{5.3}$$

and

$$F = \min_{C_1} f(C_1). \tag{5.4}$$

We want to find C* such that C* is smallest element in Ω where $\Omega \subseteq [0, M]$ is such that $f(C) = F$ for $C \in \Omega$ and U is the damage cost function at the damage center.

5.3 Constraints

Budgetary Constraints

The capital budget constraint is calculated differently in the private and public sectors. In the private sector, it is considered to be a function of the corporation current assets and current debt level, whereas in the public sector, it depends upon congressional or state water resources appropriation. By the assumption that the total cost of a reservoir is a continuous monotone increasing function of its storage capacity, the capital constraint is equivalent to

$$C_1 \leq M \tag{5.5}$$

where M is the maximum possible storage capacity that can be constructed.

Operational Constraints

For a new series reservoir, we have

$$Z_{1,n+1} = Z_{1,n} + Q_{1,n} - r_{1,n}\Delta T \qquad n = 0, 1, \ldots, N, \qquad (5.6)$$

$$Z_{2,n+1} = Z_{2,n} + Q_{2,n} + r_{1,n}\Delta T - r_{2,n}\Delta T \qquad n = 0, 1, \ldots, N, \qquad (5.7)$$

$$0 \leq Z_{1,n} \leq C_1 \qquad n = 0, 1, \ldots, N, \qquad (5.8)$$

$$0 \leq Z_{1,n} \leq C_2 \qquad n = 0, 1, \ldots, N, \qquad (5.9)$$

where

C_1 is the capacity of the new series reservoir, and

C_2 is the capacity of the existing reservoir.

For a new parallel reservoir, we have

$$Z_{1,n+1} = Z_{1,n} + Q_{1,n} - r_{1,n}\Delta T \qquad n = 0, 1, \ldots, N, \qquad (5.10)$$

$$Z_{2,n+1} = Z_{2,n} + Q_{2,n} - r_{2,n}\Delta T \qquad n = 0, 1, \ldots, N, \qquad (5.11)$$

$$0 \leq Z_{1,n} \leq C_1 \qquad n = 0, 1, \ldots, N, \qquad (5.12)$$

$$0 \leq Z_{2,n} \leq C_2 \qquad n = 0, 1, \ldots, N. \qquad (5.13)$$

5.4 Dynamic Programming Formulation

For the objective function

$$F = \min_{C_1} f(C_1)$$

we can find $f(C_1)$ by dynamic programming for each C_1. In the series case, for fixed C_1, we have

$$\min_{\left\{\begin{matrix}(r_{1,0},r_{1,1},\ldots,r_{1,N})\\(r_{2,0},r_{2,1},\ldots,r_{2,N})\end{matrix}\right\}} \{\max_n (U_1(r_{1,n}) + U_2(r_{2,n}))\}$$

$$= W_0(Z_{1,0};\ Z_{2,0};\ C_1) \tag{5.14}$$

where

$$W_n(Z_{1,n};\ Z_{2,n};\ C_1) = \min_{r_{1,n}\in R_{1,n}} \left\{ \min_{r_{2,n}\in R_{2,n}} (\max(U_1(r_{1,n}) + U_2(r_{2,n}),\right.$$

$$\left. W_{n+1}(Z_{1,n+1};\ Z_{2,n+1};\ C_1)))\right\}, \tag{5.15}$$

and

$$W_{N+1}(Z_{1,N+1};\ Z_{2,N+1};\ C_1) = 0. \tag{5.16}$$

Hence

$$F = \min_{C_1}\{W_0(Z_{1,0};\ Z_{2,0};\ C_1)\}, \tag{5.17}$$

where $Z_{1,0}$ and $Z_{2,0}$ are the initial storage of reservoirs 1 and 2, respectively.

In the parallel case, for fixed C_1, we have

$$\min_{\left\{\begin{matrix}(r_{1,0},r_{1,1},\ldots,r_{1,N})\\(r_{2,0},r_{2,1},\ldots,r_{2,N})\end{matrix}\right\}} \{\max_n (U(r_{1,n} + r_{2,n}))\}$$

$$= V_0(Z_{1,0};\ Z_{2,0};\ C_1) \tag{5.18}$$

where

$$V_n(Z_{1,n};\ Z_{2,n};\ C_1) = \min_{r_{1,n}\in R_{1,n}} \left\{ \min_{r_{2,n}\in R_{2,n}} (\max(U(r_{1,n} + r_{2,n}),\right.$$

$$\left. V_{n+1}(Z_{1,n+1};\ Z_{2,n+1};\ C_1)))\right\}, \tag{5.19}$$

and

$$V_{N+1}(Z_{1,N+1};\ Z_{2,N+1};\ C_1) = 0. \tag{5.20}$$

Hence

$$F = \min_{C_1}\{V_0(Z_{1,0};\ Z_{2,0};\ C_1)\}, \tag{5.21}$$

where $Z_{1,0}$ and $Z_{2,0}$ are the initial storage of reservoirs 1 and 2, respectively.

We next state some results which will assist our search for the optimal capacity.

Lemma 5.1 For fixed $Z_{1,0}$ and $Z_{2,0}$; $W_0(Z_{1,0},\ Z_{2,0},\ C_1)$ and $V_0(Z_{1,0},\ Z_{2,0},\ C_1)$ are both monotone nonincreasing functions of C_1.

Proof:

For any stage n, the decision set is

$$R_{1,n} = [Lb_{1,n},\ Ub_{1,n}], \tag{5.22}$$

where

$$Lb_{1,n} = \max\{0,\ (Z_{1,n} + Q_{1,n} - C_1)/\Delta T\}$$

$$Ub_{1,n} = (Z_{1,n} + Q_{1,n})/\Delta T$$

The $Ub_{1,n}$ is independent of C_1, but the $Lb_{1,n}$ is a function of C_1. If $C_1 > C_1'$, then

$$\begin{aligned} Lb_{1,n} &= \max\{0, (Z_{1,n} + Q_{1,n} - C_1)/\Delta T\} \\ &\le \max\{0, (Z_{1,n} + Q_{1,n} - C_1')/\Delta T\} \\ &= Lb_{1,n}' \end{aligned}$$

Hence

$$R_{1,n} \supseteq R_{1,n}' \tag{5.23}$$

Since the minimum value over a larger domain can be no greater than the minimum value over a smaller domain contained in the larger domain, $W_0(Z_{1,0};\ Z_{2,0};\ C_1)$ and $V_0(Z_{1,0};\ Z_{2,0};\ C_1)$ both are monotone nonincreasing functions of C. ||

Lemma 5.2 The optimal value of C* exists and can be found by binary search.

Proof:

By Lemma 5.1, $W_0(Z_{1,0};\ Z_{2,0};\ C_1)$, $V_0(Z_{1,0};\ Z_{2,0};\ C_1)$ are monotone nonincreasing functions of C_1, and the cost value is nonnegative. By assumption 5, the capital cost of a reservoir is a continuous monotone increasing function C_1 which is bounded from above. Then we can find F, where

$$F = \min\left\{f(C_1) \;\middle|\; C_1 \in [0, M]\right\}$$

Hence $\Omega = \{C_1 \in [0, M];\ f(C_1) = F\}$ is a compact set and C* exists and can be found by binary search. ||

5.5 Search for Optimal Capacity of a New Reservoir

(a) Sequential Search

Sequential search is perhaps most the straightforward and simple of the available search procedures. It consists simply of starting at some value of C, usually the boundary value, and comparing the f(C) one at a time, until either the minimum value of C such that f(C) equal 0 is found or all values of C have been searched. It is easy to code but may consume excessive amounts of computing time.

(b) Fibonacci Search

Since f(C) is monotone, it is unimodal also. If we compare the value of the function at any two points, then a finite number of points can be excluded from optimality. Clearly by making successive evaluations and comparisons, we do not need to carry out an exhaustive search to find the optimal solution. Depending upon where C* lies, search procedures generally require different number of evaluations. Under the assumption of complete uncertainty regarding the value of C*, a reasonable measure of effectiveness is to minimize the maximum number of evaluations. Let k_n be the maximum number of points in the domain so that the optimal point always may be determined with no more than n evaluations. Let K_n equal the maximum of k_n over all search procedures.

$$K_n = K_{n-1} + K_{n-2} + 1 \qquad n > 2$$

$$K_1 = 1, \qquad K_2 = 2 \tag{5.24}$$

From the table of Fibonacci Search [64], we can find the points of first two evaluation as a function of n.

(c) Binary Search

At each comparison in binary search we either find the optimal value in question or eliminate half of the region to be searched. Binary search requires that f(C) should be monotone. The procedure begins by evaluating the midpoint value of C. If f is nonnegative and f(C) is equal 0, then upper range of the C values ignored and we can evaluate the midpoint of the remaining lower half. This process continues until the search interval of C satisfied some required accuracy. If we let E be the number of evaluations and M be the number of grid points then $E_{max} \geq \log_2(M+1)$. For monotone functions, the number of iterations for each search is given in Table 5.1 [72]. Reference to Table 5.1 reveals that binary search is best for monotone functions. For this reason it was employed in our study. The macro-flow diagram of the overall optimization is on Figure 5.6.

5.6 Example

Consider the East Fork of Silver Creek in Madison County, Illinois which is shown on Figure 5.1. There is a Reservoir on Silver Lake [22]. The dam height is 30 ft. and its storage capacity is 10400 ac.-ft. The topography and geology of Madison County are generally suited to reservoir development. The East Fork of Silver Creek has a potential reservoir site located 1 mile east and 0.5 mile north of Grantfork. This is approximately 3 miles upstream from Silver Lake.

In the spring, when the ground is still frozen, the snow melts and and heavy rains are likely to occur. The flow in East Fork above the Silver Lake can get extremely high, with a typical flood peak being 4030 cfs [79]. The Silver Lake Reservoir can not absorb such flood peak completely since flood stage is 800 cfs.

The inflow hydrograph of a typical heavy storm on East Fork watershed area [79] is given on Table 5.2 and displayed on Figure 5.2. The

Number of Points	Sequential Search Max.	Sequential Search Average	Fibonacci Search	Binary Search Max.	Binary Search Average
5	5	3	4	3	2
10	10	5	5	4	3
50	50	25	8	6	5
100	100	50	10	7	6
1000	1000	500	15	10	9
10000	10000	5000	19	14	13

Table 5.1 Iteration number for Sequential, Fibonacci, and Binary Search.

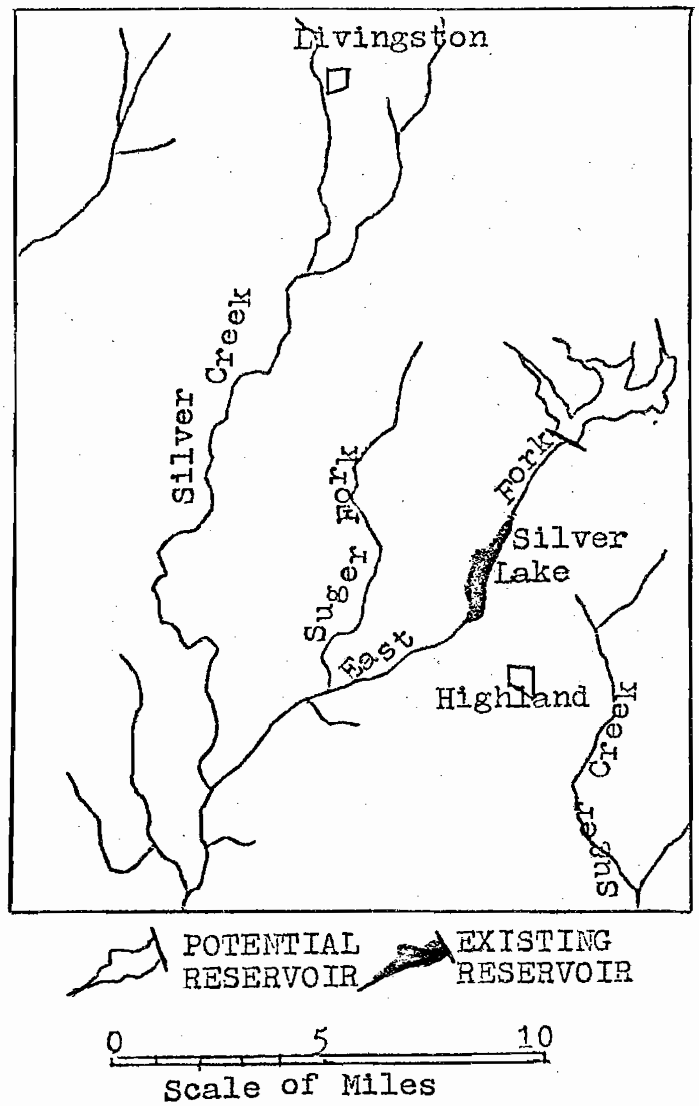


Figure 5.1 East Fork of Silver Creek.

Time (hr)	0	12	24	36	48	60	72	84	96	108	120	132	144	156	168
Input hydrograph of Silver Lake	0	600	1380	2450	3700	4030	3750	3150	2500	1900	1220	800	450	150	0
Input due to the watershed of potential Res.	0	400	930	1550	2400	2530	2530	2000	1600	1250	800	550	350	100	0
Input due to watershed between two reservoir	0	200	450	900	1300	1500	1500	1150	900	650	420	250	100	50	0

Table 5.2 Inflow Data for East Fork

Iteration	1		2		3		4		5		6		7	
Capacity	5000		2500		1250		625		937		781		859	
	1	2	1	2	1	2	1	2	1	2	1	2	1	2
Stage														
1	600	600	600	600	600	600	600	900	400	800	300	900	200	900
2	600	600	600	600	0	600	600	900	400	800	450	900	500	900
3	1200	600	1200	600	600	600	900	900	800	800	900	900	1400	900
4	1800	600	1500	600	1500	600	1800	900	1600	800	2400	900	1800	900
5	2400	600	1800	600	1800	600	2100	900	2800	800	2250	900	2200	900
6	2400	600	1800	600	1800	600	2100	900	2000	800	2100	900	2200	900
7	0	600	2100	600	2100	600	2100	900	2200	800	2400	900	2400	900
8	0	600	0	600	1500	600	1200	900	1800	800	1950	900	1900	900
9	300	600	300	600	300	600	1200	600	600	800	750	900	800	900
10	600	600	600	600	600	600	600	600	800	800	900	900	1000	900
11	900	600	900	600	900	600	600	600	1000	800	1050	750	1100	800
12	900	600	900	600	300	600	300	600	400	800	600	750	700	800
13	1200	600	900	600	0	600	0	600	200	800	150	750	200	800
14	1200	600	0	0	0	600	0	600	0	800	0	750	0	800
Damage Cost at center		0		0		0		0.125		0		0.125		0.125

Table 5.3 Summary of Results of the Computions for each Iteration

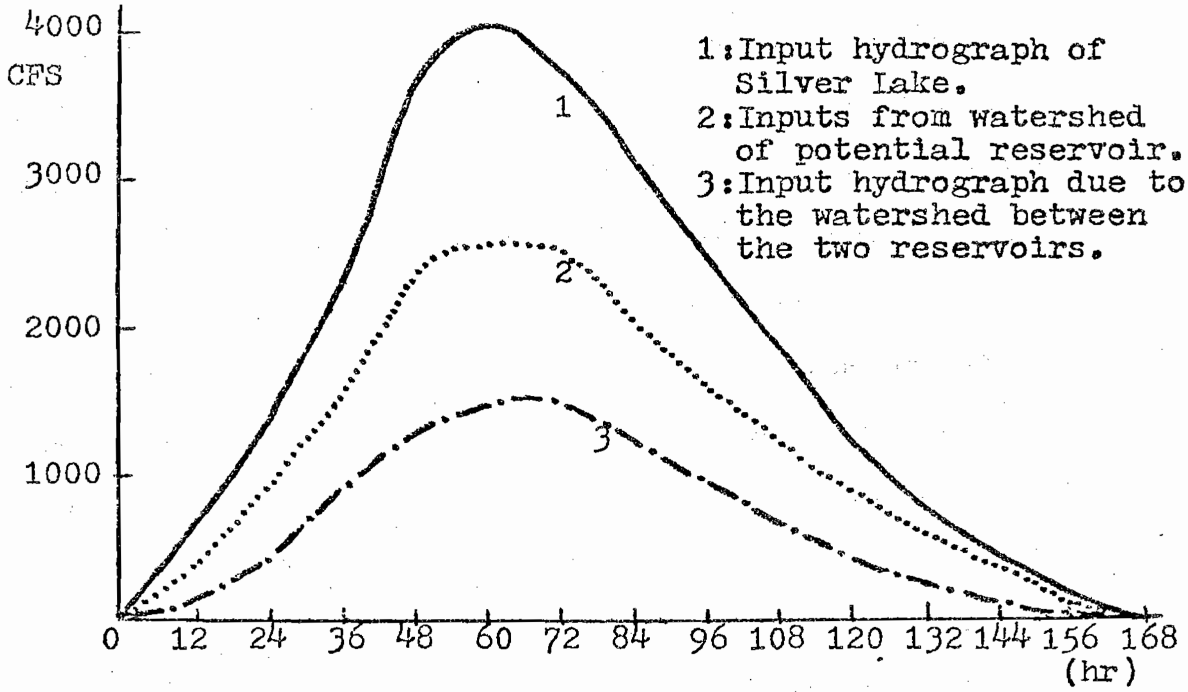


Figure 5.2 Input Hydrograph of East Fork.

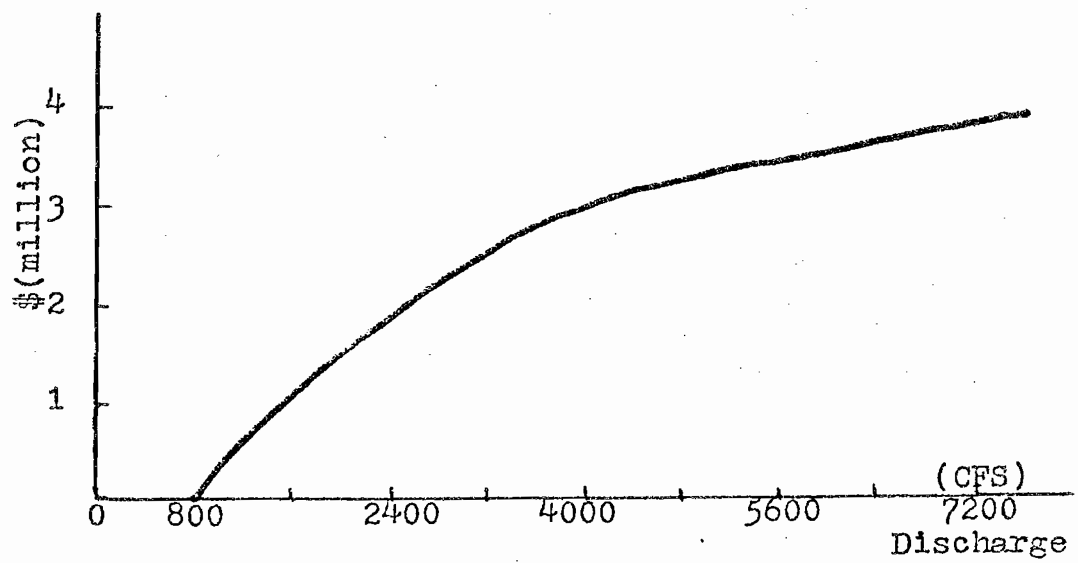


Figure 5.3 Damage Cost Function at East Fork Damage Center.

cost function at the East Fork damage center is displayed on Figure 5.3. Since there is a mountain area between these two reservoirs, there is no direct damage cost due to the proposed reservoir releases. The initial flood cost is $2.98 million. The optimal release policy for Silver Lake Reservoir is displayed on Figure 5.4. The optimal damage cost is $0.875 million and the flood peak is not completely absorbed.

The grid size used in the search procedure was 100 ac.-ft. This yields an optimal capacity of the proposed reservoir of 937 ac.-ft. The flood damage would be $0. The computing for the optimal solution is 645 seconds on Northwestern University's CDC 6600. The storage capacity vs. optimal damage cost is displayed on Figure 5.5. The stage time interval ΔT used was 12 hr. The results of the computations for each of the seven iterations aregiven on Table 5.3. For each iteration, column 1 is the release rate of the proposed reservoir with capacity C_1, for which we want to optimize and column 2 is the release rate of reservoir 2 which already exists. The program listing appears in Appendix A5.

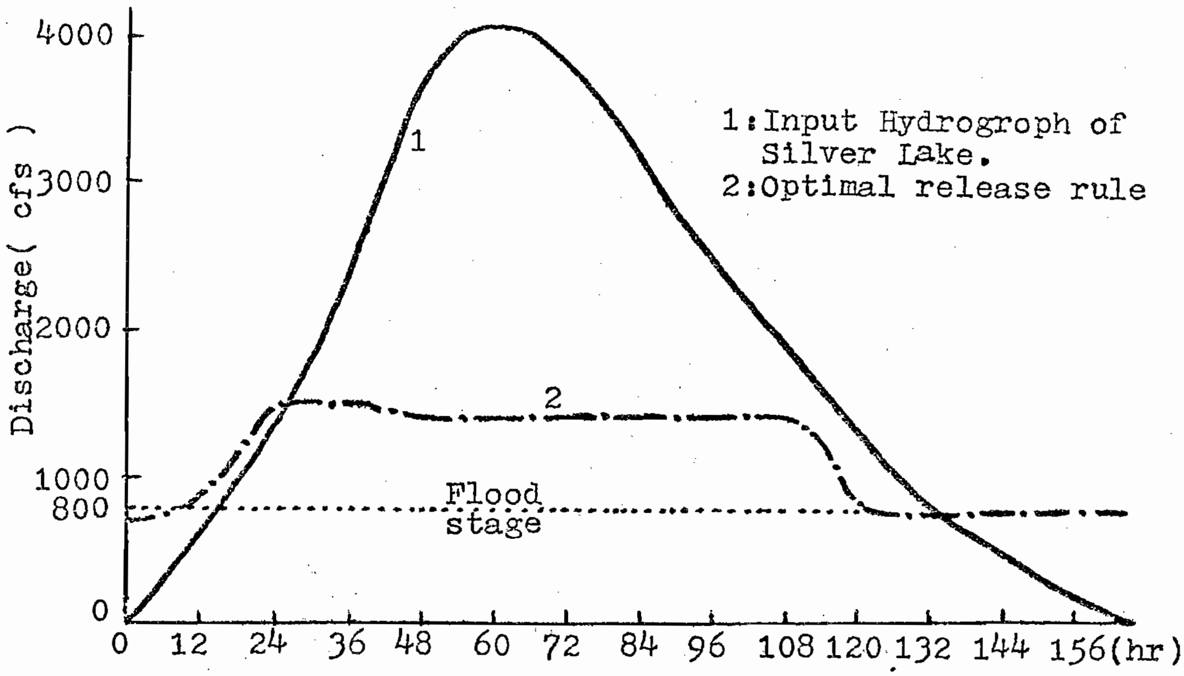


Figure 5.4 Optimal Release Rule for Silver Lake.

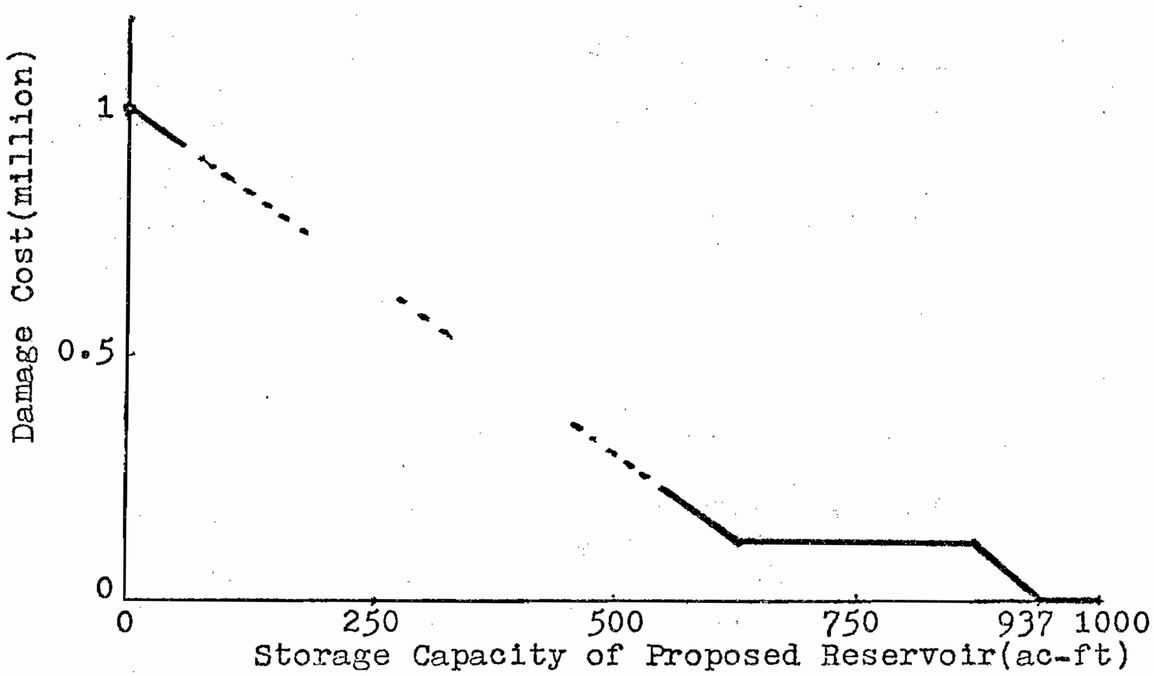


Figure 5.5 Storage Capacity vs. Optimal Damage Cost.

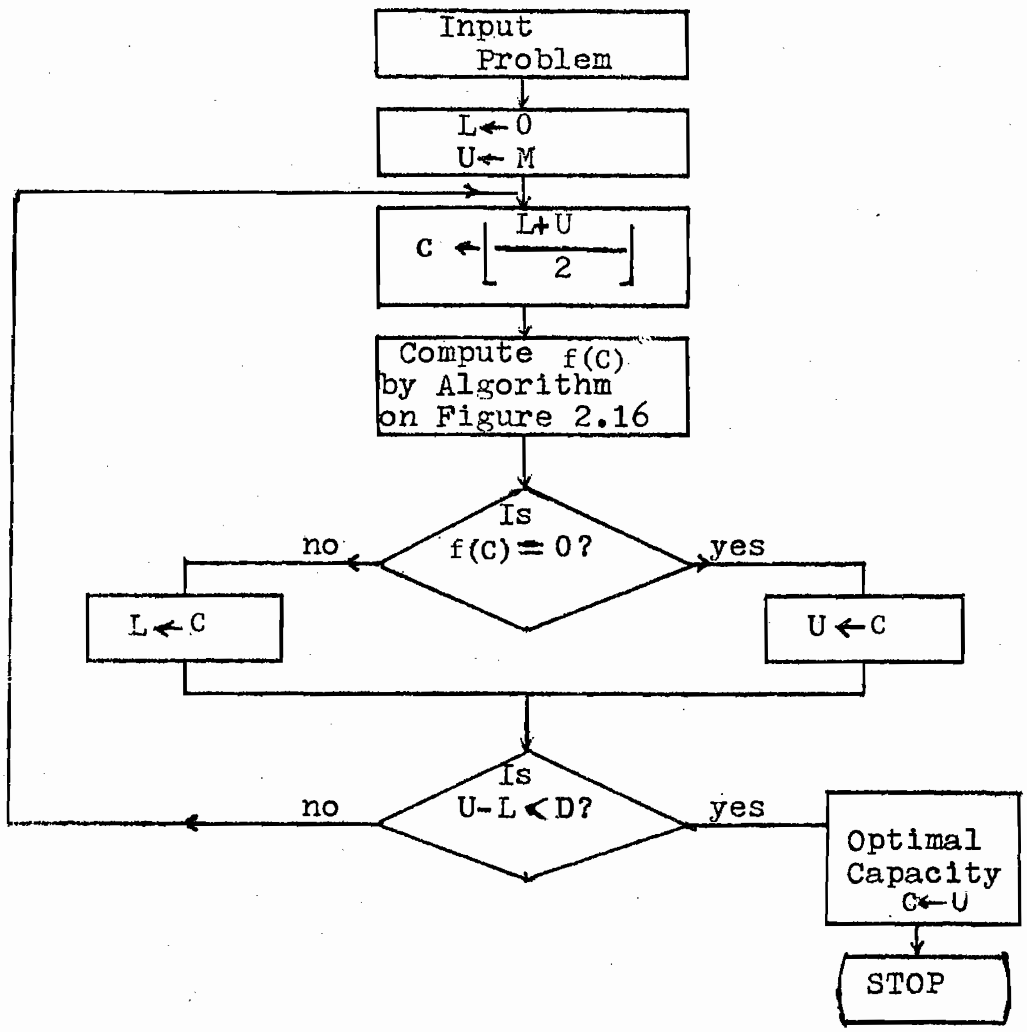


Figure 5.6 Macro-Flow Diagram for Finding the Optimal Capacity of the Additional Reservoir of a Single Reservoir Flood Control System.

CHAPTER VI

SUMMARY

6.1 Summary

Methodologies have been developed for determining the optimal operating policies for single and multiple-reservoir flood control systems in Chapters II and III. Dynamic programming and stochastic dynamic programming were used as the optimization tools. This approach has overcome both the difficulties of the convexity requirements for the linear programming approach and the physical constraint validity of the chance-constrained linear programming approach. Therefore, it offers a potentially powerful method in the analysis of flood control systems.

This paper also modeled synthesized floods as Markov renewal processes in Chapter III. From our study of the effects of changes in the initial storage quantity in Chapter IV, we know that when a flood is coming, it is optimal to lower the water level as much as possible.

The expansion of existing flood control systems was modeled in Chapter V. We determined the minimal capacity of a new reservoir, which furnishes the reduction of predicted flood peak, by dynamic programming, under the assumption that an optimal operation policy is employed.

Real-world examples have been included both to illustrate and to explicate the results of the theoretical discussions. The use of the derived optimal operating rules in real-time was also simulated using both historical flow rate data and simulated flow rate data.

The optimal release policy performed as well or better than both the standard release policy and the routing release policy.

6.2 Directions for Future Research

The uncertainty of future hydrometeorological conditions occurring within a riverbasin confounds the decision-making process when applied to the management of water resources systems. The stochastic nature of water resources suggests continued research in application of stochastic programming methods to the operation of these systems. There are several recommendation can be drawn directly or indirectly from this research. First, efforts should be made to further develop techniques for estimating flood damage cost functions. Second, efforts should be also be directed toward refining and developing techniques for predicting the surface-runoff from precipitation. Third, development of the ability to accurately predict long-range stream flow pattern and distribution should be enhanced. Fourth, development of more efficient algorithms to solve the problems would be desirable. In this regard, the recent incorporation [58] of branch-and-bound methods in dynamic programming to reduce computation and storage requirements which proved useful in a number of problems [56, 57], should be a fruitful avenue of approach.

APPENDIX A

PROGRAM LISTINGS

Appendix A1

Program Listing (Single Reservoir Flood Control System Deterministic Inflow)

```
      PROGRAM TEST (INPUT,OUTPUT)
      DIMENSION AN(20),BN(20),E(30),INPT(20),OPT(20,300),RELES(20,300),
     1FF(20,300),ZZ(20,300)
      DATA(AN(I),I=1,14)/1000.0,2000.0,3000.0,6000.0,10000.0,16000.0,
     125000.0,30000.0,30000.0,21000.0,12000.0,5000.0,2000.0,0.0/
      DATA(BN(I),I=1,14)/-0.02315,-0.02315,-0.02315,-0.06944,-0.0926,
     1-0.1389,-0.2083,-0.11574,0.0,0.2083,0.2083,0.162,0.06944,0.0463/
      DATA(E(I),I=1,30)/0.0,0.0,0.0,1.8,2.9,3.7,4.4,4.9,5.3,5.7,6.1,6.3,
     16.5,6.7,6.9,7.0,7.1,7.2,7.3,7.4,7.5,7.6,7.7,7.8,7.9,8.0,8.1,8.2,
     28.3,8.4/
      DATA NTG,DT,DS,DR,DU/14,43200.0,2000000.0,200.0,3800.0/
      DO 9 N=1,NTG
      INPT(N)=AN(N)*DT+0.5*BN(N)*DT**2
    9 CONTINUE
      C=3310560000.0
      INITIAL=224952000.0/DS+1
      KMAX=C/DS+1
      KTG=0
  100 KTG=KTG+1
      KK=1
      IF (KTG.EQ.NTG) KK=INITIAL
   79 CONTINUE
      P=((KK-1)*DS+INPT(KTG))/DT
      BUP=AMAX1(P,0.0)
      BLO=AMAX1(P-C/DT,0.0)
      IUP=BUP/DR+1
      ILO=BLO/DR+2
      OPT(KTG,KK)=100000.0
      DO 10 I=ILO,IUP
      CALL CURVF(E,I,DR,DU,VALUE)
      IF (KTG.NE.1) GO TO 31
      TEMP=VALUE
      IF (OPT(KTG,KK).LT.TEMP) GO TO 30
      J=I
      OPT(KTG,KK)=TEMP
   30 CONTINUE
      GO TO 10
   31 CONTAIN=(KK-1)*DS-(I-1)*DR*DT+INPT(KTG)
      ISTATE=CONTAIN/DS+1
      IF (ISTATE.GE.KMAX) ISTATE=KMAX
      TEMP=AMAX1(VALUE,FF(KTG-1,ISTATE))
      IF (OPT(KTG,KK).LT.TEMP) GO TO 10
      J=I
      OPT(KTG,KK)=TEMP
   10 CONTINUE
      RELES(KTG,KK)=(J-1)*DR
      XINPT=(KK-1)*DS
      FF(KTG,KK)=OPT(KTG,KK)
      ZZ(KTG,KK)=XINPT-RELES(KTG,KK)*DT+INPT(KTG)
      PRINT 2, FF(KTG,KK),RELES(KTG,KK),XINPT,KTG
    2 FORMAT (*     VALUE  =  *,F10.5,*  ,    RELESE  RATE =  *,F12.5,*  S
     1TATE  =*,F14.2,*  AT   STAGE   *,I5)
      IF (KK.GE.KMAX.OR.KTG.EQ.NTG) GO TO 48
      KK=KK+1
      GO TO 79
   48 CONTINUE
      IF(KTG.NE.NTG) GO TO 100
```

```
      PRINT 5, FF(NTG,INITIAL)
    5 FORMAT(*   OPTIMAL  VALUE  *,F14.5)
      KCT=NTG
      PRINT 6,KCT,RELES(NTG,INITIAL)
      OLD=(INITIAL-1)*DS
      NEW=ZZ(NTG,INITIAL)
      KZZ=NEW/DS+1
      DO 803 I=2,NTG
      KCT=NTG-I+1
      PRINT 6,KCT,RELES(KCT,KZZ)
      OLD=NEW
      NEW=ZZ(KCT,KZZ)
      KZZ=NEW/DS+1
    6 FORMAT(*   AT  STAGE  *,I5,*   RELESE  RATE  IS  *,F15.5)
  803 CONTINUE
      END
```

```
      SUBROUTINE  CURVE(E,ID,DR,DU,VALUE)
      DIMENSION E(30)
      L=((ID-1)*DR)/DU+1
      VALUE=E(L)+((E(L+1)-E(L))/DU)*((ID-1)*DR-(L-1)*DU)
      RETURN
      END
```

Appendix A2

Program Listing (Two Reservoir Series Flood Control System with Deterministic Inflow)

```
      PROGRAM TEST(INPUT,OUTPUT)
      DIMENSION AN(2,17),NEW(2),KZZ(2),OLD(2),FF(20,20,20),
     1RELES1(20,20,20),RELES2(20,20,20),ZZ1(20,20,20),ZZ2(20,20,20),
     2KMAX(2),INITIAL(2),KK(2),R(2),G(44),H(44),U(2),IR(2),C(2),Z(2),
     3DU(2),INPT(2,16)
      DATA AN/0.0,0.0,300.0,300.0,720.0,720.0,1300.0,1300.0,1950.0,
     11950.0,2500.0,2500.0,3100.0,3100.0,3700.0,3700.0,4400.0,4400.0,
     24800.0,4800.0,5000.0,5000.0,4700.0,4700.0,3800.0,3800.0,2400.0,
     32400.0,1400.0,1400.0,450.0,450.0,0.0,0.0/
      DATA G/0.0,0.0,1.3,2.6,3.0,3.7,4.4,5.0,5.5,6.0,6.5,6.9,7.3,7.7,
     18.1,8.4,8.8,9.2,9.4,9.7,10.0,10.2,10.4,10.6,10.8,11.0,11.2,11.4,
     211.5,11.7,11.8,11.9,12.0,12.1,12.2,12.3,12.4,12.5,12.6,12.7,12.8,
     312.9,13.0,13.1/
      DATA H/0.0,0.0,1.8,3.0,4.0,5.0,5.7,6.5,7.2,7.7,8.3,8.8,9.2,9.7,
     110.1,10.4,10.7,11.0,11.3,11.5,11.7,11.9,12.1,12.3,12.4,12.5,12.6,
     212.7,12.8,12.9,13.0,13.1,13.2,13.3,13.4,13.5,13.6,13.7,13.8,13.9,
     314.0,14.1,14.2,14.3/
      DATA NTG,DT,DS,DR,DU(1),DU(2)/16,21600.0,100000000.0,250.0,
     113500.0,5000.0/
      DO 9 I=1,2
      DO 9 N=1,NTG
      INPT(I,N)=0.5*(AN(I,N)+AN(I,N+1))*DT
    9 CONTINUE
      C(1)=500000000.0
      C(2)=1900000000.0
      KMAX(1)=C(1)/DS+1
      KMAX(2)=C(2)/DS+1
      INITIAL(1)=100000000.0/DS+1
      INITIAL(2)=200000000.0/DS+1
      KTG=0
  100 KTG=KTG+1
      KK(1)=1
      KK(2)=1
      IF(KTG.NE.NTG) GO TO 79
      KK(1)=INITIAL(1)
      KK(2)=INITIAL(2)
   79 CONTINUE
            P=((KK(1)-1)*DS+INPT(1,KTG))/DT
      IUP=MIN1(6000.0,P)/DR+1
      ILO= MAX1(P-C(1)/DT,0.0)/DR+2
      IF(P-C(1)/DT.LE.0.0) ILO=1
   63 IR(1)=ILO
      FF(KTG,KK(1),KK(2))=10000.0
   62 R(1)=(IR(1)-1)*DR
      Z(1)=(KK(1)-1)*DS+INPT(1,KTG)-R(1)*DT
      CALL CURVE(G,IR(1),DR,DU(1),VALUE)
      U(1)=VALUE
      Q=((KK(2)-1)*DS+INPT(2,KTG)+R(1)*DT)/DT
      LUP= MIN1(12000.0,Q)/DR+1
      LLO= MAX1(Q-C(2)/DT,0.0)/DR+2
      IF(Q-C(2)/DT.LE.0.0) LLO=1
      IR(2)=LLO
   61 R(2)=(IR(2)-1)*DR
      Z(2)=(KK(2)-1)*DS+INPT(2,KTG)+R(1)*DT-R(2)*DT
      CALL CURVE(H,IR(2),DR,DU(2),VALUE)
```

```
      U(2)=VALUE
      IF (KTG.NE.1) GO TO 31
      TEMP=U(1)+U(2)
      IF (FF(KTG,KK(1),KK(2)).LT.TEMP) GO TO 30
      RELES1(KTG,KK(1),KK(2))=R(1)
      RELES2(KTG,KK(1),KK(2))=R(2)
      FF(KTG,KK(1),KK(2))=TEMP
   30 CONTINUE
      GO TO 10
   31 IA=Z(1)/DS+1
      IB=Z(2)/DS+1
      TEMP=AMAX1(U(1)+U(2),FF(KTG-1,IA,IB))
      IF(FF(KTG,KK(1),KK(2)).LT.TEMP) GO TO 10
      RELES1(KTG,KK(1),KK(2))=R(1)
      RELES2(KTG,KK(1),KK(2))=R(2)
      FF(KTG,KK(1),KK(2))=TEMP
   10 CONTINUE
      IF(IR(2).GE.LUP) GO TO 51
      IR(2)=IR(2)+1
      GO TO 61
   51 IF(IR(1).GE.IUP) GO TO 52
      IR(1)=IR(1)+1
      GO TO 62
   52 CONTINUE
      XIN1=(KK(1)-1)*DS
      XIN2=(KK(2)-1)*DS
      ZZ1(KTG,KK(1),KK(2))=XIN1-RELES1(KTG,KK(1),KK(2))*DT+INPT(1,KTG)
      ZZ2(KTG,KK(1),KK(2))=XIN2-RELES2(KTG,KK(1),KK(2))*DT
     1+RELES1(KTG,KK(1),KK(2))*DT+INPT(2,KTG)
      PRINT 3,KTG,XIN1,XIN2,FF(KTG,KK(1),KK(2)),RELES1(KTG,KK(1),KK(2)),
     1RELES2(KTG,KK(1),KK(2))
    3 FORMAT(*   STAGE=*,I3,*  STATE 1=*,F15.1,*  STATE 2=*,F15.1,/,
     120X,*  VALUE=*,F9.6,*  RELES RATE 1=*,F10.1,*  RELES RATE 2=*,
     2F10.1)
      IF(KK(2).GE.KMAX(2).OR.KTG.EQ.NTG) GO TO 53
      KK(2)=KK(2)+1
      GO TO 63
   53 IF(KK(1).GE.KMAX(1).OR.KTG.EQ.NTG) GO TO 54
      KK(1)=KK(1)+1
      KK(2)=1
      GO TO 79
   54 CONTINUE
      IF(KTG.NE.NTG) GO TO 100
      PRINT 5,FF(NTG,INITIAL(1),INITIAL(2))
    5 FORMAT(*   OPTIMAL VALUE= *,F9.6)
      KCT=NTG
      PRINT 6,KCT,RELES1(NTG,INITIAL(1),INITIAL(2)),
     1RELES2(NTG,INITIAL(1),INITIAL(2))
      OLD(1)=(INITIAL(1)-1)*DS
      OLD(2)=(INITIAL(2)-1)*DS
      NEW(1)=ZZ1(NTG,INITIAL(1),INITIAL(2))
      NEW(2)=ZZ2(NTG,INITIAL(1),INITIAL(2))
      KZZ(1)=NEW(1)/DS+1
      KZZ(2)=NEW(2)/DS+1
      DO 803 I=2,NTG
```

```
      KCT=NIG-I+1
      PRINT 6,KCT,RELES1(KCT,KZZ(1),KZZ(2)),RELES2(KCT,KZZ(1),KZZ(2))
      OLD(1)=NEW(1)
      OLD(2)=NEW(2)
      NEW(1)=ZZ1(KCT,KZZ(1),KZZ(2))
      NEW(2)=ZZ2(KCT,KZZ(1),KZZ(2))
      KZZ(1)=NEW(1)/US+1
      KZZ(2)=NEW(2)/US+1
    6 FORMAT( *    AT STAGE *,I3,*  RELES RATE 1= *,F10.1,*RELES RATE 2=*
     1,F10.1)
  803 CONTINUE
      END
```

```
      SUBROUTINE CURVE(Y,ID,DR,DU,VALUE)
      DIMENSION Y(44)
      L=((ID-1)*DR)/DU+1
      VALUE=Y(L)+((Y(L+1)-Y(L))/DU)*((ID-1)*DR-(L-1)*DU)
      RETURN
      END
```

Appendix A3

Program Listing (Two Reservoir Parallel Flood Control System with Deterministic Inflow)

```
      PROGRAM TEST(INPUT,OUTPUT)
      DIMENSION AN(2,19),NEW(2),KZZ(2),OLO(2),FF(20,20,20),
     1RELES1(20,20,20),RELES2(20,20,20),ZZ1(20,20,20),ZZ2(20,20,20),
     2KMAX(2),INITIAL(2),KK(2),R(2),G(29),IR(2),C(2),Z(2),INPT(2,20)
      DATA AN/625.0,625.0,650.0,650.0,750.0,750.0,750.0,750.0,750.0,
     1750.0,800.0,900.0,1100.0,1400.0,1300.0,2100.0,1700.0,2550.0,
     22200.0,3000.0,2700.0,3100.0,3250.0,2950.0,3500.0,2600.0,3000.0,
     32600.0,2400.0,1800.0,1700.0,1400.0,1250.0,1250.0,800.0,1000.0,
     4500.0,725.0/
      DATA G/0.0,0.0,0.0,0.0,0.0,0.75,1.3,1.8,2.2,2.5,2.7,2.9,3.1,3.3,
     13.5,3.7,3.9,4.0,4.1,4.2,4.3,4.4,4.5,4.6,4.7,4.8,4.9,5.0,5.1/
      DATA NTG,DT,DS,DR,DU/18,43200.0,20000000.0,100.0,500.0/
      DO 9 I=1,2
      DO 9 N=1,NTG
      INPT(I,N)=0.5*(AN(I,N)+AN(I,N+1))*DT
    9 CONTINUE
      C(1)=200000000.0
      C(2)=140000000.0
      KMAX(1)=C(1)/DS+1
      KMAX(2)=C(2)/DS+1
      INITIAL(1)=3
      INITIAL(2)=2
      KTG=0
  100 KTG=KTG+1
      KK(1)=1
      KK(2)=1
      IF(KTG.NE.NTG) GO TO 79
      KK(1)=INITIAL(1)
      KK(2)=INITIAL(2)
   79 CONTINUE
            P=((KK(1)-1)*DS+INPT(1,KTG))/DT
      IUP=P/DR+1
      ILO= MAX1(P-C(1)/DT,0.0)/DR+2
      IF(P-C(1)/DT.LE.0.0) ILO=1
   63 CONTINUE
      Q=((KK(2)-1)*DS+INPT(2,KTG)          )/DT
      LUP=Q/DR+1
      LLO= MAX1(Q-C(2)/DT,0.0)/DR+2
      IF(Q-C(2)/DT.LE.0.0) LLO=1
      IR(1)=ILO
      FF(KTG,KK(1),KK(2))=10000.0
   62 R(1)=(IR(1)-1)*DR
      IR(2)=LLO
      Z(1)=(KK(1)-1)*DS+INPT(1,KTG)-R(1)*DT
   61 R(2)=(IR(2)-1)*DR
      Z(2)=(KK(2)-1)*DS+INPT(2,KTG)          -R(2)*DT
      JJ=(R(1)+R(2))/DR+1
      CALL CURVE(G,JJ,DR,DU,VALUE)
      U=VALUE
      IF (KTG.NE.1) GO TO 31
      TEMP=U
      IF (FF(KTG,KK(1),KK(2)).LT.TEMP) GO TO 30
      RELES1(KTG,KK(1),KK(2))=R(1)
      RELES2(KTG,KK(1),KK(2))=R(2)
      FF(KTG,KK(1),KK(2))=TEMP
```

```
 30 CONTINUE
    GO TO 10
 31 IA=Z(1)/DS+1
    IB=Z(2)/DS+1
    TEMP=AMAX1(U,FF(KTG-1,IA,IB))
    IF(FF(KTG,KK(1),KK(2)).LT.TEMP) GO TO 10
    RELES1(KTG,KK(1),KK(2))=R(1)
    RELES2(KTG,KK(1),KK(2))=R(2)
    FF(KTG,KK(1),KK(2))=TEMP
 10 CONTINUE
    IF(IR(2).GE.LUP) GO TO 51
    IR(2)=IR(2)+1
    GO TO 61
 51 IF(IR(1).GE.IUP) GO TO 52
    IR(1)=IR(1)+1
    GO TO 62
 52 CONTINUE
    XIN1=(KK(1)-1)*DS
    XIN2=(KK(2)-1)*DS
    ZZ1(KTG,KK(1),KK(2))=XIN1-RELES1(KTG,KK(1),KK(2))*DT+INPT(1,KTG)
    ZZ2(KTG,KK(1),KK(2))=XIN2-RELES2(KTG,KK(1),KK(2))*DT
   1+INPT(2,KTG)
    PRINT 3,KTG,XIN1,XIN2,FF(KTG,KK(1),KK(2)),RELES1(KTG,KK(1),KK(2)),
   1RELES2(KTG,KK(1),KK(2))
  3 FORMAT(*   STAGE=*,I3,*  STATE 1=*,F15.1,*  STATE 2=*,F15.1,/,
   120X,*  VALUE=*,F9.6,*  RELES RATE 1=*,F10.1,*  RELES RATE 2=*,
   2F10.1)
    IF(KK(2).GE.KMAX(2).OR.KTG.EQ.NTG) GO TO 53
    KK(2)=KK(2)+1
    GO TO 63
 53 IF(KK(1).GE.KMAX(1).OR.KTG.EQ.NTG) GO TO 54
    KK(1)=KK(1)+1
    KK(2)=1
    GO TO 79
 54 CONTINUE
    IF(KTG.NE.NTG) GO TO 100
    PRINT 5,FF(NTG,INITIAL(1),INITIAL(2))
  5 FORMAT(*   OPTIMAL VALUE= *,F9.6)
    KCT=NTG
    PRINT 6,KCT,RELES1(NTG,INITIAL(1),INITIAL(2)),
   1RELES2(NTG,INITIAL(1),INITIAL(2))
    OLD(1)=(INITIAL(1)-1)*DS
    OLD(2)=(INITIAL(2)-1)*DS
    NEW(1)=ZZ1(NTG,INITIAL(1),INITIAL(2))
    NEW(2)=ZZ2(NTG,INITIAL(1),INITIAL(2))
    KZZ(1)=NEW(1)/DS+1
    KZZ(2)=NEW(2)/DS+1
    DO 803 I=2,NTG
    KCT=NTG-I+1
    PRINT 6,KCT,RELES1(KCT,KZZ(1),KZZ(2)),RELES2(KCT,KZZ(1),KZZ(2))
    OLD(1)=NEW(1)
    OLD(2)=NEW(2)
    NEW(1)=ZZ1(KCT,KZZ(1),KZZ(2))
    NEW(2)=ZZ2(KCT,KZZ(1),KZZ(2))
    KZZ(1)=NEW(1)/DS+1
    KZZ(2)=NEW(2)/DS+1
  6 FORMAT( *   AT STAGE *,I3,*  RELES RATE 1= *,F10.1,*RELES RATE 2=*
   1,F10.1)
803 CONTINUE
    END
    SUBROUTINE CURVE(Y,ID,DR,DU,VALUE)
    DIMENSION Y(29)
    L=((ID-1)*DR)/DU+1
    VALUE=Y(L)+((Y(L+1)-Y(L))/DU)*((ID-1)*DR-(L-1)*DU)
    RETURN
    END
```

Appendix A4

Program Listing (Single Reservoir Flood Control System with Stochastic Inflow)

```
      PROGRAM DAM(INPUT,OUTPUT)
      DIMENSION P(35),ILB(35),IUB(35),E(32),F(30,300),OPT(30,300)
      INTEGER W(35)
      DATA (P(I),I=1,10)/0.0228,0.0669,0.0919,0.1498,0.1915,0.1915,
     10.1498,0.0619,0.0396,0.0228/
      DATA(W(I),I=1,12)/2208,2208,2208,2208,2208,2208,669,669,669,669,
     1669,669/
      DATA(E(I),I=1,32)/0.5,0.0,0.0,1.8,2.9,3.7,4.4,4.9,5.3,5.7,6.1,6.3,
     16.5,6.7,6.9,7.0,7.1,7.2,7.3,7.4,7.5,7.6,7.7,7.8,7.9,8.0,8.1,8.2,
     28.3,8.4,8.45,8.5/
      DATA NTG,DT,DS,DR,DU,IP/13,432000.0,5000000.0,50.0,750.0,10/
      C=160000000.0
      KMAX=C/DS+1
      N=NTG+1
    4 N=N-1
      K=1
    5 CONTINUE
      IF(N.EQ.NTG) GO TO 49
      S=13500.0
      DO 21 J=1,IP
      Y=(K-1)*DS+(W(N)*2/IP)*J*DT
      IUB(J)=MAX1(Y/DT,0.0)/DR+1
      ILB(J)=MAX1((Y-C)/DT,0.0)/DR+2
   21 CONTINUE
      IR=ILB(1)
   30 CONTINUE
      SUM=0.
      EX=0.
      DO 31 J=1,IP
      LR=IR
      IF(IR.LT.ILB(J)) LR=ILB(J)
      IF(IR.GT.IUB(J)) LR=IUB(J)
      CALL CURVE(E,LR,DR,DU,VALUE)
      SUM=VALUE*P(J)+SUM
      IZ=((K-1)*DS-(LR-1)*DR*DT+(W(N)*2/IP)*J*DT)/DS+1
      EX=F(N+1,IZ)*P(J)+EX
   31 CONTINUE
      A=AMAX1(SUM,EX)
      B=AMIN1(A,B)
      IF(B.EQ.A) OPT(N,K)=IR
      IF(IR.GE.IUB(IP)) GO TO 40
      IR=IR+1
      GO TO 30
   40 F(N,K)=B
      STATE=(K-1)*DS
      RELES=(OPT(N,K)-1)*DR
      PRINT 90,N,STATE,RELES,F(N,K)
   90 FORMAT(*       STAGE=*,I3,*    STATE=*,F12.1,*     RELES RATE=*,
     1F10.1,*    STAGE OPT VALUE=*,F10.5)
      GO TO 51
   49 F(N,K)=0.
   51 IF(K.GE.KMAX) GO TO 50
      K=K+1
      GO TO 5
   50 IF(N.LE.1) GO TO 60
      GO TO 4
   60 CONTINUE
       SUBROUTINE CURVE(E,ID,DR,DU,VALUE)
       DIMENSION E(32)
       L=((ID-1)*DR)/DU+1
       VALUE=E(L)+((E(L+1)-E(L))/DU)*((ID-1)*DR-(L-1)*DU)
       RETURN
       END
```

Appendix A5

Program Listing (Simulation)

```
      PROGRAM SIMU(INPUT,OUTPUT)
      DIMENSION NR(20,40),MR(20,40),Z(20),IZ(20),E(32),X(20),IX(20),
     1W(20),IW(20),Q(12),MEAN(12),VAR(12),COR(12),R(15)
      DATA US,NTG,NST,C,DT,DU/50000000.0,12,21,1000000000.0,432000.0,
     1750.0/
      DATA(E(I),I=1,32)/0.0,0.0,0.0,1.8,2.9,3.7,4.4,4.9,5.3,5.7,6.1,6.3,
     16.5,6.7,6.9,7.0,7.1,7.2,7.3,7.4,7.5,7.6,7.7,7.8,7.9,8.0,8.1,8.2,
     28.3,8.4,8.45,8.5/
      DATA MEAN/6*2088,6*2364/
      DATA VAR/6*1044.0,6*1182.0/
      DATA COR/6*0.3137,6*0.4142/
      Z(1)=500000000.0
      IZ(1)=11
      X(1)=Z(1)
      W(1)=Z(1)
      IX(1)=IZ(1)
      IW(1)=IZ(1)
      DO 10 J=1,21
      READ 1,(NR(I,J),I=1,12)
   10 CONTINUE
      DO 11 J=1,21
      READ 1,(MR(I,J),I=1,12)
   11 CONTINUE
    1 FORMAT(12I4)
      T=1500*DT
      A=T+C
      S=MEAN(1)
      DO 99 K=1,100
      PRINT 5,K
      PRINT 3
    5 FORMAT(*     THE   *,I4,*    RUN   *)
    3 FORMAT(5X,*STAGE*,5X,*INPUT*,5X,*DEPEN*,5X,*INDEP*,5X,*STAND*)
      L=1
      IY=0
      JY=0
      KY=0
      CALL GGNOR(4,15,R)
      DO 21 M=1,NTG
      IF(M.NE.1) GO TO 51
      Q(M)=MEAN(M)*(1-COR(M))+COR(M)*S       +VAR(M)*R(M)*(1-COR(M)**2)
     1**0.5
   40 IF(Q(M).GT.0) GO TO 42
      Q(M)=MEAN(M)*(1-COR(M))+COR(M)*Q(M)+VAR(M)*R(NTG+L)*(1-COR(M)**2)
     1**0.5
      L=L+1
      GO TO 40
   51 Q(M)=MEAN(M)*(1-COR(M))+COR(M)*Q(M-1)+VAR(M)*   R(M)*(1-COR(M)**2)
     1**0.5
      IF(Q(M).GT.0.) GO TO 42
      Q(M-1)=Q(M)
      R(M)=R(NTG+L)
      L=L+1
      GO TO 51
   42 BU=Z(M)/DT+Q(M)
      AU=X(M)/DT+Q(M)
```

```
      BL=AMAX1(BU-C/DT,0.0)
      AL=AMAX1(AU-C/DT,0.0)
      IR=NR(M,IZ(M))
      JR=MR(M,IX(M))
      IF(IR.LT.BL) IR=BL
      IF(IR.GT.BU) IR=BU
      Z(M+1)=Z(M)+Q(M)*DT-IR*DT
      IZ(M+1)=Z(M+1)/DS+1
      IY=MAX0(IY,IR)
      IF(JR.LT.AL) JR=AL
      IF(JR.GT.AU) JR=AU
      X(M+1)=X(M)+Q(M)*DT-JR*DT
      IX(M+1)=X(M+1)/DS+1
      JY=MAX0(JY,JR)
      B=W(M)+Q(M)*DT
      IF(B.LE.T) GO TO 31
      IF(B.GT.T.AND.B.LE.A) GO TO 32
      KR=(B-C)/DT
      W(M+1)=C
      GO TO 33
   32 KR=T/DT
      W(M+1)=B-T
      GO TO 33
   31 KR=B/DT
      W(M+1)=0
   33 CONTINUE
      IW(M+1)=W(M+1)/DS+1
      KY=MAX0(KY,KR)
      PRINT 2,M,Q(M),IR,JR,KR
    2 FORMAT(5X,I3,8X,F8.1,3X,I6,3X,I6,3X,I6)
   21 CONTINUE
      CALL CURVE(E,IY,DU,G)
      CALL CURVE(E,JY,DU,H)
      CALL CURVE(E,KY,DU,F)
      PRINT 4,G,H,F
    4 FORMAT(*     COST   *,17X,F7.5,3X,F7.5,3X,F7.5)
      S=Q(NTG)
   99 CONTINUE
      END


      SUBROUTINE CURVE(E,IY,DU,VALUE)
      DIMENSION E(35)
      L=IY/DU+1
      VALUE=E(L)+((E(L+1)-E(L))/DU)*(IY-(L-1)*DU)
      RETURN
      END
```

Appendix A6

Program Listing(Optiaml Capacity of Series New Reservoir)

```
      PROGRAM OPT(INPUT,OUTPUT)
      INTEGER RIGHT,CENTER
      X=43560.0
      D=30000000.0
      DR=300.0
      M=10000
      W=M*X
      LEFT=0
      RIGHT=M
      CALL DAM(W,V,D,DR)
      IF(V.GT.0.0) GO TO 400
      PRINT 300,M,V
  300 FORMAT(*   CAPACITY *,I7,*  COST *,F10.7)
  500 IF(RIGHT-LEFT.LE.100) GO TO 400
      IF(RIGHT-LEFT.GT.800) GO TO 501
      D=13600000.0
      DR=200.0
  501 CONTINUE
      IF(RIGHT-LEFT.GT.400) GO TO 502
      D=6700000.0
      DR=150.0
  502 CONTINUE
      IF(RIGHT-LEFT.GT.200) GO TO 503
      D=3390000.0
      DR=100.0
  503 CONTINUE
      CENTER=(LEFT+RIGHT)/2
      W=CENTER*X
      CALL DAM(W,V,D,DR)
      PRINT 300,CENTER,V
      IF(V.EQ.0.0) GO TO 201
      LEFT=CENTER
      GO TO 500
  201 RIGHT=CENTER
      GO TO 500
  400 PRINT 301 ,RIGHT,V
  301 FORMAT(*   OPYIMAL CAPACITY IS*,  I7,* DAMAGE COST IS*, F11.7)
      END

      SUBROUTINE CURVE(Y,ID,DR,DU,VALUE)
      DIMENSION Y(44)
      L=((ID-1)*DR)/DU+1
      VALUE=Y(L)+((Y(L+1)-Y(L))/DU)*((ID-1)*DR-(L-1)*DU)
      RETURN
      END
```

```
      SUBROUTINE DAM(W,V,D,DR)
      DIMENSION AN(2,15),NEW(2),KZZ(2),OLD(2),FF(20,20,20),
     1RELES1(20,20,20),RELES2(20,20,20),ZZ1(20,20,20),ZZ2(20,20,20),
     2KMAX(2),INITIAL(2),KK(2),R(2),G(20),      U(2),IR(2),C(2),Z(2),
     3DU(2),INPT(2,16)
      DATA AN/0.0,0.0,100.0,50.0,350.0,100.0,550.0,250.0,800.0,450.0,
     11250.0,650.0,1600.0,900.0,2000.0,1150.0,2350.0,1400.0,2530.0,
     21500.0,2400.0,1300.0,1550.0,900.0,920.0,450.0,400.0,200.0,0.0,0.0/
      DATA G/0.0,0.0,1.0,1.8,2.5,3.0,3.2,3.4,3.6,3.8,4.0,4.1,4.2,4.3,
     14.4,4.5,4.6,4.7,4.8,4.9/
      DATA NTG,DT,DS,DU(1),DU(2)/14,43200.0,30000000.0,800.0,800.0/
      C(1)=W
      C(2)=450000000.0
      DO 9 I=1,2
      DO 9 N=1,NTG
      INPT(I,N)=0.5*(AN(I,N)+AN(I,N+1))*DT
    9 CONTINUE
      C(1)=W
      KMAX(1)=C(1)/D +1
      KMAX(2)=C(2)/DS+1
      INITIAL(1)=2
      INITIAL(2)=3
      KTG=0
  100 KTG=KTG+1
      KK(1)=1
      KK(2)=1
      IF(KTG.NE.NTG) GO TO 79
      KK(1)=INITIAL(1)
      KK(2)=INITIAL(2)
   79 CONTINUE
            P=((KK(1)-1)*D +INPT(1,KTG))/DT
      IUP=MIN1(3500.0,P)/DR+1
      ILO= MAX1(P-C(1)/DT,0.0)/DR+2
      IF(P-C(1)/DT.LE.0.0) ILO=1
   63 IR(1)=ILO
      FF(KTG,KK(1),KK(2))=10000.0
   62 R(1)=(IR(1)-1)*DR
      Z(1)=(KK(1)-1)*D +INPT(I,KTG)-R(1)*DT
      U(1)=0.
      Q=((KK(2)-1)*DS+INPT(2,KTG)+R(1)*DT)/DT
      LUP= MIN1( 5000.0,Q)/DR+1
      LLO= MAX1(Q-C(2)/DT,0.0)/DR+2
      IF(Q-C(2)/DT.LE.0.0) LLO=1
      IR(2)=LLO
   61 R(2)=(IR(2)-1)*DR
      Z(2)=(KK(2)-1)*DS+INPT(2,KTG)+R(1)*DT-R(2)*DT
      CALL CURVE(G,IR(2),DR,DU(2),VALUE)
      U(2)=VALUE
      IF (KTG.NE.1) GO TO 31
      TEMP=U(1)+U(2)
      IF (FF(KTG,KK(1),KK(2)).LT.TEMP) GO TO 30
      RELES1(KTG,KK(1),KK(2))=R(1)
      RELES2(KTG,KK(1),KK(2))=R(2)
      FF(KTG,KK(1),KK(2))=TEMP
   30 CONTINUE
```

```
      GO TO 10
   31 IA=Z(1)/D +1
      IB=Z(2)/DS+1
      TEMP=AMAX1(U(1)+U(2),FF(KTG-1,IA,IB))
      IF(FF(KTG,KK(1),KK(2)).LT.TEMP) GO TO 10
      RELES1(KTG,KK(1),KK(2))=R(1)
      RELES2(KTG,KK(1),KK(2))=R(2)
      FF(KTG,KK(1),KK(2))=TEMP
   10 CONTINUE
      IF(IR(2).GE.LUP) GO TO 51
      IR(2)=IR(2)+1
      GO TO 61
   51 IF(IR(1).GE.IUP) GO TO 52
      IR(1)=IR(1)+1
      GO TO 62
   52 CONTINUE
      XIN1=(KK(1)-1)*D
      XIN2=(KK(2)-1)*DS
      ZZ1(KTG,KK(1),KK(2))=XIN1-RELES1(KTG,KK(1),KK(2))*DT+INPT(1,KTG)
      ZZ2(KTG,KK(1),KK(2))=XIN2-RELES2(KTG,KK(1),KK(2))*DT
     1+RELES1(KTG,KK(1),KK(2))*DT+INPT(2,KTG)
    3 FORMAT(*   STAGE=*,I3,*  STATE 1=*,F15.1,*  STATE 2=*,F15.1,/,
     120X,*  VALUE=*,F9.6,*  RELES RATE 1=*,F10.1,*  RELES RATE 2=*,
     2F10.1)
      IF(KK(2).GE.KMAX(2).OR.KTG.EQ.NTG) GO TO 53
      KK(2)=KK(2)+1
      GO TO 63
   53 IF(KK(1).GE.KMAX(1).OR.KTG.EQ.NTG) GO TO 54
      KK(1)=KK(1)+1
      KK(2)=1
      GO TO 79
   54 CONTINUE
      IF(KTG.NE.NTG) GO TO 100
      PRINT 5,FF(NTG,INITIAL(1),INITIAL(2))
    5 FORMAT(*   OPTIMAL VALUE= *,F9.6)
      V=FF(NTG,INITIAL(1),INITIAL(2))
      KCT=NTG
      PRINT 6,KCT,RELES1(NTG,INITIAL(1),INITIAL(2)),
     1RELES2(NTG,INITIAL(1),INITIAL(2))
      OLD(1)=(INITIAL(1)-1)*D
      OLD(2)=(INITIAL(2)-1)*DS
      NEW(1)=ZZ1(NTG,INITIAL(1),INITIAL(2))
      NEW(2)=ZZ2(NTG,INITIAL(1),INITIAL(2))
      KZZ(1)=NEW(1)/D +1
      KZZ(2)=NEW(2)/DS+1
      DO 803 I=2,NTG
      KCT=NTG-I+1
      PRINT 6,KCT,RELES1(KCT,KZZ(1),KZZ(2)),RELES2(KCT,KZZ(1),KZZ(2))
      OLD(1)=NEW(1)
      OLD(2)=NEW(2)
      NEW(1)=ZZ1(KCT,KZZ(1),KZZ(2))
      NEW(2)=ZZ2(KCT,KZZ(1),KZZ(2))
      KZZ(1)=NEW(1)/D +1
      KZZ(2)=NEW(2)/DS+1
    6 FORMAT( *   AT STAGE *,I3,*  RELES RATE 1= *,F10.1,*RELES RATE 2=*
     1,F10.1)
  803 CONTINUE
      RETURN
      END
```

APPENDIX B

FLOW RATE DATA OF KASKASKIA RIVER AT SHELBYVILLE, ILLINOIS

Appendix B

Flow Rate Data of Kaskaskia River at Shelbyville, Ill.

KASKASKIA RIVER AT SHELBYVILLE, ILL.

LOCATION.—Chain gage in sec. 8, T. 11 N., R. 4 E. of the third principal meridian, at highway bridge in the eastern edge of Shelbyville, Shelby County, Ill., just above the Chicago & Eastern Illinois Railway bridge and Big Four Railroad bridge.

DRAINAGE AREA.—1,030 square miles.

RECORDS AVAILABLE.—Feb. 25, 1908 to Sept. 30, 1912; Nov. 1, 1912 to Dec. 31, 1912; Aug. 11, 1914 to Dec. 5, 1914, when finally discontinued.

EXTREMES.—Maximum discharge recorded, 10,600 sec.-ft., May 8, 1908 (gage height, 25.8 feet); minimum discharge recorded, 0.2 sec.-ft., Oct. 4-7, 1914 (gage height, 4.7 feet).

REMARKS.—Stage-discharge relation affected by ice during winter months. Shifting channel but measuring section was at a pool and control was considered permanent. During high water discharge relation was probably affected by backwater caused by drift lodging at the two railroad bridges below the gaging station.

DAILY DISCHARGE, IN SECOND-FEET, OF KASKASKIA RIVER AT SHELBYVILLE, ILL., FOR 1908-1912.

Day.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1908												
1			4,260	2,400	2,500	1,030	256	90	34	25	8	34
2			4,480	3,000	2,300	956	224	73	25	18	10	34
3			4,990	2,500	1,810	918	256	73	25	18	10	34
4			4,760	2,300	7,820	1,110	224	58	18	18	13	25
5			4,540	1,580	8,780	956	224	58	18	13	13	25
6			4,420	1,450	8,480	846	256	45	18	13	18	25
7			4,650	1,400	8,720	738	358	45	18	13	18	25
8			4,870	2,350	10,600	666	392	45	18	13	18	25
9			5,100	2,450	9,260	810	426	58	13	13	18	25
10			5,330	2,500	7,820	956	324	73	13	13	18	34
11			5,220	2,350	7,220	918	290	73	10	13	18	34
12			4,650	2,250	6,800	774	290	58	10	13	18	34
13			4,090	2,150	6,380	666	224	73	10	13	18	34
14			3,760	2,100	5,850	630	192	58	10	13	18	34
15			3,430	1,680	5,330	562	130	58	10	10	18	34
16			3,160	1,450	4,990	460	130	45	10	10	25	34
17			2,550	1,150	4,700	426	109	45	13	10	25	25
18			2,050	1,110	4,260	358	90	45	13	10	25	25
19			1,630	1,070	4,140	324	160	34	13	10	25	25
20			1,680	1,030	3,480	392	160	34	13	10	25	25
21			1,490	994	3,000	358	160	34	13	10	25	25
22			1,450	956	3,480	358	130	25	10	10	34	25
23			1,400	918	3,160	324	130	25	10	8	34	25
24			1,360	1,680	2,900	290	109	25	10	8	45	25
25		3,760	1,310	1,910	2,600	290	90	25	10	8	45	18
26		4,990	1,190	2,200	2,350	256	90	25	10	10	58	18
27		5,330	1,070	2,500	2,250	224	73	25	10	10	58	18
28		5,100	994	2,700	1,630	160	73	18	13	10	45	25
29		4,370	1,150	2,800	1,540	192	58	18	18	8	45	25
30			1,150	2,650	1,360	224	73	25	25	8	45	34
31			1,070		1,190		90	25		8		34

DAILY DISCHARGE, IN SECOND-FEET, OF KASKASKIA RIVER AT SHELBYVILLE, ILL., FOR 1908-1912—Continued.

Day.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1909												
1	34	[a]40	3,000	392	3,260	4,140	426	358	34	18	109	392
2	34	90	2,600	392	1,860	3,820	290	324	34	18	90	358
3	34	90	2,100	358	1,810	3,720	290	256	34	18	90	358
4	34	109	1,770	358	1,810	3,380	256	224	34	18	73	324
5	34	160	1,490	358	1,770	2,850	2,150	192	25	18	73	324
6	[a]30	192	1,310	4,700	1,580	2,550	2,700	130	25	13	73	290
7	[a]30	324	1,360	4,650	1,540	2,350	5,500	109	18	13	90	[a]250
8	[a]25	460	1,450	4,200	1,580	2,050	5,740	90	18	13	90	[a]230
9	[a]25	494	1,540	3,700	3,600	1,860	5,100	90	25	13	73	[a]216
10	[a]25	562	1,440	3,100	3,600	2,550	3,210	90	25	13	73	[a]200
11	[a]20	630	1,310	3,000	3,540	2,550	2,550	90	25	13	58	[a]200
12	[a]20	774	1,230	4,040	3,430	2,050	2,550	73	18	13	90	426
13	[a]20	882	1,190	6,740	2,550	1,770	3,540	73	18	13	90	666
14	[a]25	1,030	1,070	7,580	2,420	1,720	3,030	58	18	13	109	1,070
15	[a]25	1,110	956	6,980	2,100	1,810	2,850	45	18	13	109	1,490
16	[a]25	1,190	846	6,440	1,680	1,540	2,550	34	18	13	426	1,580
17	[a]25	1,490	1,030	5,910	1,310	1,910	2,900	34	18	13	392	1,540
18	[a]30	2,100	738	5,500	1,150	1,450	3,480	25	18	25	666	1,450
19	34	2,650	702	4,930	1,030	1,150	3,600	25	18	58	666	1,150
20	45	3,210	666	4,090	882	774	3,260	25	18	73	666	[a]1,000
21	58	3,350	630	5,620	528	630	2,550	25	18	73	738	[a]850
22	73	3,430	562	6,440	702	596	2,450	18	18	58	956	[a]700
23	90	3,480	494	6,980	666	596	1,860	13	18	130	918	[a]600
24	90	3,760	562	5,560	596	528	1,450	13	58	130	882	[a]520
25	90	3,820	596	5,450	702	528	666	13	58	130	846	[a]470
26	73	3,820	630	5,220	1,360	738	702	25	34	130	702	[a]400
27	73	3,600	596	4,590	2,200	702	630	18	25	130	630	[a]350
28	73	3,260	494	3,600	2,100	666	528	18	25	130	562	[a]300
29	[a]60	-------	494	3,540	2,200	562	426	34	25	130	494	[a]250
30	[a]50	-------	460	2,600	2,900	528	392	34	25	130	426	[a]220
31	[a]40	-------	426	-------	3,760	-------	358	45	-------	109	-------	[a]200
1910												
1	[a]250	774	3,700	290	392	1,450	738	1,150	58	208	66	1,420
2	[a]250	702	3,160	290	358	1,270	774	937	90	176	90	1,050
3	[a]300	738	3,260	256	596	1,110	882	630	66	160	100	720
4	[a]300	702	3,350	256	738	994	494	375	90	176	82	684
5	[a]300	702	3,430	256	882	994	774	290	100	224	90	596
6	[a]300	666	2,400	256	846	846	630	224	1,150	579	82	511
7	[a]300	630	2,150	224	1,230	702	545	273	1,630	975	90	341
8	[a]300	666	1,810	224	1,540	596	409	160	3,180	1,070	82	341
9	[a]350	596	1,580	160	1,580	562	290	100	2,500	937	66	375
10	[a]400	562	1,360	160	1,230	494	224	109	1,840	774	66	392
11	[a]500	528	1,150	160	1,860	494	224	90	1,450	613	66	341
12	738	460	994	192	2,200	426	176	82	1,030	511	82	307
13	1,800	324	994	192	2,150	392	409	90	756	460	90	341
14	3,160	426	774	192	1,860	324	392	66	666	392	66	375
15	3,600	426	666	192	1,680	290	358	73	777	307	66	358
16	3,380	426	630	324	1,490	290	774	494	426	392	66	307
17	2,900	358	596	358	1,270	256	1,050	720	341	273	66	273
18	4,260	290	562	358	1,030	256	1,470	994	290	240	73	208
19	5,330	358	528	358	882	256	1,270	756	208	224	66	208
20	6,260	324	494	358	846	224	1,030	630	192	208	66	224
21	4,540	494	426	358	774	160	846	358	176	256	66	176
22	3,650	460	426	324	1,070	109	579	307	145	208	66	176
23	3,380	460	426	324	3,700	109	409	290	160	176	73	208
24	2,700	392	392	290	4,310	109	358	545	176	192	66	341
25	2,200	358	392	290	4,870	109	273	426	358	145	66	290
26	1,860	358	358	290	4,650	109	160	290	307	160	66	208
27	1,490	2,550	358	324	3,260	256	160	208	273	145	3,510	145
28	1,230	3,380	358	358	2,850	392	176	192	256	145	3,600	756
29	1,110	-------	324	426	2,700	494	256	100	240	130	2,420	1,520
30	994	-------	324	392	2,300	666	756	90	224	100	1,720	1,360
31	882	-------	290	-------	1,810	-------	1,110	66	-------	90	-------	1,170

Note.—(*a*) Estimated. Ice effect, Jan. 6-18, Jan. 29-Feb. 1, and Dec. 7-11, 20-31, 1909; also Jan. 1-11, 1910.

DAILY DISCHARGE, IN SECOND-FEET, OF KASKASKIA RIVER AT SHELBYVILLE, ILL., FOR 1908-1912—Concluded.

Day.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1911												
1	1,150	2,650	918	494	1,810	208	22	12	16	8,450	918	918
2	1,190	2,750	846	494	1,680	192	45	13	12	8,000	810	882
3	a1,100	2,550	774	702	1,470	160	30	16	12	7,430	596	846
4	a1,000	1,810	738	1,720	1,270	145	34	18	13	6,590	666	810
5	a950	1,400	702	3,920	1,050	160	22	18	16	6,150	738	774
6	a910	1,360	666	6,440	956	145	25	22	22	5,560	774	702
7	a860	1,310	1,360	3,920	846	160	25	25	40	4,650	702	494
8	a810	1,270	2,400	3,260	756	160	22	22	82	4,090	666	528
9	a760	882	2,450	3,160	702	145	25	18	73	3,260	666	596
10	a714	846	2,300	2,750	666	160	22	16	40	2,700	702	666
11	666	956	2,050	2,350	613	130	90	12	273	2,300	738	702
12	494	882	1,910	1,910	562	120	22	12	720	2,050	1,720	738
13	1,810	774	1,680	1,680	511	109	18	12	2,180	1,720	1,860	738
14	4,090	494	1,450	4,040	494	90	18	12	1,450	1,540	1,960	774
15	4,650	956	1,150	5,100	460	73	16	13	1,050	1,360	1,860	810
16	3,540	1,070	1,030	4,820	409	66	13	12	1,250	1,190	1,810	810
17	2,350	1,230	918	4,260	392	73	13	9	1,520	1,150	2,700	846
18	1,810	1,540	810	3,430	358	52	12	9	1,130	1,070	2,800	810
19	1,680	1,810	738	3,160	341	58	18	9	1,110	994	2,850	774
20	1,490	1,860	702	2,680	426	52	16	10	1,130	1,190	2,600	882
21	1,270	1,720	666	2,150	975	58	18	9	864	1,630	2,500	1,030
22	1,270	1,540	596	1,700	630	58	18	7	720	2,050	2,350	1,150
23	1,030	1,400	562	1,450	562	52	12	5.5	579	2,100	2,200	810
24	918	1,270	528	1,230	477	58	13	22	494	2,200	1,960	1,230
25	774	1,230	494	1,090	392	52	12	25	2,120	2,300	1,720	1,100
26	846	1,230	528	994	324	90	13	30	2,980	2,450	1,540	1,270
27	1,070	1,150	596	864	240	82	12	40	2,780	2,100	1,400	1,190
28	2,800	1,030	562	918	224	73	13	30	5,070	1,580	1,230	1,150
29	3,160	-------	562	918	273	45	13	22	6,920	1,400	1,110	1,070
30	3,050	-------	562	1,340	256	30	12	25	8,750	1,190	994	994
31	2,950	-------	494	-------	256	-------	13	16	-------	1,030	-------	1,110
1912												
1	1,110		5,680	5,220	5,500	562	257	122	122	-------	51	63
2	1,070		5,620	5,040	4,990	528	231	105	182	-------	63	63
3	1,070		4,820	5,270	4,700	462	284	105	231	-------	63	76
4	1,030		3,920	5,040	4,310	430	257	90	399	-------	63	76
5			2,900	6,200	4,140	399	257	90	312	-------	90	76
6			1,860	3,920	3,100	369	284	90	206	-------	105	76
7			1,450	3,260	2,650	340	312	76	160	-------	122	76
8			1,070	2,650	2,200	312	284	76	105	-------	140	76
9			1,110	2,350	2,050	284	257	76	90	-------	122	76
10			1,150	2,150	1,810	257	994	90	76	-------	122	76
11		a300	882	2,050	1,680	257	562	105	76	-------	105	76
12			738	2,000	1,580	231	430	160	72	-------	105	76
13			596	1,960	1,540	231	369	206	67	-------	105	76
14			1,720	1,960	1,400	206	1,150	340	63	-------	105	76
15			4,090	1,860	1,540	206	882	369	63	-------	90	76
16			4,870	1,810	1,550	562	596	284	63	-------	90	76
17	a820		5,850	1,770	1,490	702	562	231	63	-------	90	63
18			6,090	1,720	1,360	596	528	312	63	-------	90	63
19			6,440	1,960	1,190	666	495	528	63	-------	90	63
20			6,500	2,200	1,110	774	462	810	63	-------	76	51
21			6,740	2,500	956	810	349	1,580	63	-------	76	51
22		1,190	6,680	2,350	810	666	257	956	63	-------	76	51
23		1,150	6,320	2,150	774	528	231	774	63	-------	76	51
24		1,270	5,680	2,050	666	462	206	528	63	-------	76	51
25		2,750	5,100	1,680	596	399	182	369	63	-------	76	63
26		5,160	4,820	2,000	528	340	399	666	63	-------	76	63
27		5,500	4,540	2,550	528	702	312	399	63	-------	76	63
28		6,440	3,700	3,380	596	340	76	206	63	-------	76	63
29		6,380	5,390	4,370	562	312	63	182	51	-------	76	63
30		-------	5,390	5,330	630	284	140	160	51	-------	76	63
31		-------	5,270	-------	596	-------	140	140	-------	-------	-------	63

Note.—(*a*) Estimated. Ice effect, Jan. 3-10, 1911, and Jan. 5-Feb. 21, 1912. Gage not read in October, 1912.

DAILY DISCHARGE, IN SECOND-FEET, OF KASKASKIA RIVER AT SHELBYVILLE, ILL., FOR THE PERIOD AUG. 10 TO DEC. 5, 1914.

Day.	Aug.	Sept.	Oct.	Nov.	Dec.	Day.	Aug.	Sept.	Oct.	Nov.	Dec.
1914						1914					
1		35	.5	12	22	16	1.5	12	1.5	12	
2		35	.5	12	22	17	1.5	12	1.5	12	
3		35	.5	12	22	18	1.5	4.5	1.5	12	
4		35	.2	12	22	19	.5	4.5	4.5	12	
5		68	.2	12	22	20	.5	4.5	4.5	12	
6		774	.2	12		21	.5	1.5	4.5	12	
7		462	.2	12		22	.5	1.5	4.5	12	
8		110	.5	12		23	.5	.5	4.5	12	
9		68	1.5	12		24	.5	.5	4.5	12	
10	.5	50	4.5	12		25	.5	.5	4.5	12	
11	50	35	4.5	12		26	.5	.5	4.5	22	
12	35	22	1.5	12		27	1.5	.5	4.5	22	
13	22	22	1.5	12		28	50	.5	4.5	22	
14	12	22	1.5	12		29	50	.5	12	22	
15	4.5	12	1.5	12		30	50	.5	12	22	
						31	50		12		

Note.—Discharge determined from a rating curve which is not well defined.

MONTHLY DISCHARGE OF KASKASKIA RIVER AT SHELBYVILLE, ILL., FOR 1908-1912.

[Drainage area, 1,030 square miles.]

Month.	Discharge in second-feet.				Run-off—depth in inches on drainage area.
	Mean daily maximum.	Mean daily minimum.	Mean.	Mean per square mile.	
1908					
February (25-29)	5,330	3,760	4,710	4.57	.85
March	5,330	994	3,010	2.92	3.37
April	3,000	918	1,920	1.86	2.08
May	10,600	1,190	4,730	4.59	5.29
June	1,110	160	572	.555	.62
July	426	58	187	.182	.21
August	90	18	45.5	.044	.05
September	34	10	14.7	.014	.02
October	25	8	11.8	.011	.01
November	58	8	26.4	.026	.03
December	34	18	27.8	.027	.03
1909					
January	90		43.4	.042	.05
February	3,820		1,650	1.60	1.67
March	3,090	426	1,090	1.06	1.22
April	7,580	358	4,230	4.12	4.60
May	3,760	528	1,940	1.88	2.17
June	4,140	528	1,740	1.69	1.89
July	5,740	256	2,190	2.13	2.46
August	358	13	83.9	.081	.09
September	58	18	25.4	.025	.03
October	130	13	54.3	.053	.06
November	956	58	375	.364	.41
December	1,580		593	.576	.66
The year	7,580		1,168	1.13	15.31

MONTHLY DISCHARGE OF KASKASKIA RIVER AT SHELBYVILLE, ILL., FOR 1908-1912—Concluded.

[Drainage area, 1,030 square miles.]

Month.	Discharge in second-feet.				Runoff—depth in inches on drainage area.
	Mean daily maximum.	Mean daily minimum.	Mean.	Mean per square mile.	
1910					
January	6,360		1,910	1.86	2.14
February	3,380	290	682	.662	.69
March	3,700	290	1,220	1.18	1.36
April	426	160	281	.273	.30
May	4,870	358	1,840	1.79	2.06
June	1,450	109	491	.477	.53
July	1,470	160	581	.564	.65
August	1,150	66	359	.349	.40
September	3,180	58	638	.619	.69
October	1,070	90	343	.333	.38
November	3,600	66	439	.426	.48
December	1,520	145	507	.492	.57
The year	6,360	58	774	0.751	10.25
1911					
January	4,650	494	1,650	1.60	1.84
February	2,850	494	1,400	1.36	1.42
March	2,450	494	1,020	.990	1.14
April	6,540	494	2,430	2.36	2.63
May	1,810	224	657	.638	.74
June	208	30	102	.099	.11
July	90	12	21.2	.021	.02
August	40	5.5	16.8	.016	.02
September	8,730	12	1,450	1.41	1.57
October	8,450	994	2,950	2.86	3.30
November	2,850	596	1,500	1.46	1.63
December	1,270	494	880	.854	.98
The year	8,750	5.5	1,173	1.14	15.40
1912					
January			852	.827	.95
February	6,440		1,250	1.21	1.30
March	6,740	596	4,100	3.98	4.59
April	6,200	1,080	2,960	2.87	3.20
May	5,500	528	1,850	1.80	2.08
June	810	206	441	.428	.48
July	1,150	63	381	.370	.43
August	1,580	76	330	.320	.37
September	399	51	105	.102	.11
November	140	51	88.2	.086	.10
December	76	51	66.9	.065	.07

MONTHLY DISCHARGE OF KASKASKIA RIVER AT SHELBYVILLE, ILL., FOR THE PERIOD AUG. 10 TO NOV. 30, 1914.

[Drainage area, 1,030 square miles.]

Month.	Discharge in second-feet.				Runoff—depth in inches on drainage area.
	Mean daily maximum.	Mean daily minimum.	Mean.	Mean per square mile.	
1914					
August (10-31)	50	.5	15.2	.015	.01
September	774	.5	61.0	.059	.07
October	12	.2	3.38	.0033	.004
November	22	12	13.7	.013	.01

APPENDIX C

HYPOTHESIS TEST RESULTS

Appendix C1

Test for runoff distribution of Kaskaskia River basin at Shelbyville.

A. Chi-Square Test

The data are from the 1908-1914 stormy seasons (January, February, March, April, May, June, and December). There are total of 35 data points.

0.85	3.37	2.08	5.29	0.62	0.03	0.05	1.67	1.22	4.60
2.17	1.89	2.46	0.66	2.14	0.69	1.36	0.30	2.06	0.53
0.57	1.84	1.42	1.14	2.63	0.74	0.11	0.98	0.95	1.30
4.59	3.20	2.08	0.48	0.07					

Hypothesis

H_o: The runoff distribution is a Gamma distribution with $\alpha = 1.25$, $r = 2$

$$\Gamma(r, \alpha) = \frac{\alpha}{\Gamma(r)} (\alpha x)^{r-1} e^{-\alpha x}$$

Category(in)	f_i Observed	Gamma Distribution	e_i Expected frequency
0 -0.5	6	0.104	3.46
0.51-1	9	0.2259	7.9
1.01-1.5	5	0.2014	7.05
1.51-2	3	0.1505	5.267
2.01-2.5	6	0.1071	3.784
2.51-3	1	0.0703	2.46
3.01-3.5	2	0.0448	1.568
3.51-4	0	0.0278	0.973
4.01-	3	0.0682	2.387

Test statistic

$$\chi^2 = \sum_{i=1}^{m} \frac{(f_i - e_i)^2}{e_i}$$

$$= 6.659$$

Decision:

The test statistic χ^2 is smaller than the 0.05 significance level quantile of Chi-square distribution $\chi^2_{0.05,8} = 15.507$. Therefore, we accept H_o.

B. Kolmogorov-Smirnov Test

Category	F*(x)	S(x)	|F*(x)-s(x)|
0 -0.5	0.104	0.1714	0.0674
0.51-1	0.3299	0.4285	0.1036
1.01-1.5	0.5313	0.5713	0.04
1.51-2	0.6818	0.6571	0.0247
2.01-2.5	0.7889	0.8285	0.0396
2.51-3	0.8592	0.8571	0.00215
3.01-3.5	0.904	0.914	0.01
3.51-4	0.9318	0.914	0.0178
4.01-	1	1	0

Hypothesis:

H_o: $F(x) = F^*(x)$ for all x

H_1: $F(x) \neq F^*(x)$ for at least one value of x

Test Statistic:

$$T = \sup_x |F^*(x) - S(x)|$$

Decision Rule:

Reject H_o at the level of significance α if the test statistic T exceeds the $1 - \alpha$ quantile $w_{1-\alpha}$ on table of the Kolmogorov Test Statistic. For this test, $T = 0.1036$ for $\alpha = 0.05$ and the critical value is 0.43. Since $T < 0.43$, we accept H_o.

Appendix C2

Test of the Simulation Inflow Data and the Historical Data

A. K-Sample Slippage Test [19]

Hypothesis:

H_o: All simulation runs and the historical data have identical population distribution functions.

H_1: Some populations tend to furnish larger observed values than other populations.

Test Statistic:

The test statistic is evaluated by comparing the "largest" sample with the "smallest" sample, as follows. In each of the K-samples, find the largest of the n observations and denote the extreme (largest) value by Z_i, i = 1, 2, ... K. Compare the Z_i's, the extremes, and find the largest Z_i, denote by $Z^{(K)}$, the largest of all the observed values, is called the sample of rank K, or the largest sample. The sample from which $Z^{(1)}$ comes is called the sample of rank 1, or the smallest sample. The test statistic T equals the number of observations from the sample of rank K, which is greater than $Z^{(1)}$, the largest observation in the sample of rank 1.

Decision Rule:

Reject H_o at the level of significance α if T exceeds the $1 - \alpha$ quantile $w_{1-\alpha}$, as given by the Table C2.1 [19]. In this test, K = 55, n = 12, the following steps were followed in analyzing the data.

(1) An asterisk was placed at the upper right corner of the extreme (greatest) value in each sample.

(2) The smallest extreme was underlined once and the greatest extreme was underlined twice. Shown on Table C2.3, C2.4, C2.5.

The smallest values of m such that $P(M(1,k) \geq m) \leq \alpha$, where the first, second, and third values given for each k, number of samples, and n, sample size, correspond to $\alpha = .05$, $\alpha = .01$, and $\alpha = .001$ in that order. Absence of a value in the table means there is no m such that $m \leq n$ and $P(M(1, k) \geq m) \leq \alpha$, for that particular choice of k, n and α.

k \ n	4	5	6	7	8	9	10	12	14	16	18	20	25	30	35	40	∞
2	4	4, 5	5, 6	5, 6, 7	5, 6, 8	5, 6, 8	5, 7, 8	5, 7, 9	5, 7, 9	5, 7, 9	5, 7, 10	5, 7, 10	6, 7, 10	6, 8, 10	6, 8, 10	6, 8, 11	6, 8, 11
3		5	5, 6	5, 6	5, 7, 8	6, 7, 8	6, 7, 9	6, 7, 9	6, 8, 10	6, 8, 10	6, 8, 10	6, 8, 10	6, 8, 11	6, 8, 11	6, 8, 11	6, 9, 11	7, 9, 12
4		5	5, 6	6, 7	6, 7, 8	6, 7, 9	6, 7, 9	6, 8, 9	6, 8, 10	6, 8, 10	7, 8, 11	7, 8, 11	7, 9, 11	7, 9, 11	7, 9, 12	7, 9, 12	7, 10, 13
5		5	6, 6	6, 7	6, 7, 8	6, 7, 9	6, 8, 9	6, 8, 10	7, 8, 10	7, 8, 10	7, 9, 11	7, 9, 11	7, 9, 11	7, 9, 12	7, 9, 12	7, 9, 12	8, 10, 13
6		5	6, 6	6, 7	6, 7, 8	6, 8, 9	6, 8, 9	7, 8, 10	7, 8, 10	7, 9, 11	7, 9, 11	7, 9, 11	7, 9, 12	7, 9, 12	7, 10, 12	8, 10, 12	8, 10, 14
7		5	6	6, 7	6, 7, 8	6, 8, 9	7, 8, 9	7, 8, 10	7, 9, 10	7, 9, 11	7, 9, 11	7, 9, 11	7, 9, 12	8, 10, 12	8, 10, 12	8, 10, 12	8, 11, 14
8		5	6	6, 7	6, 7	7, 8, 9	7, 8, 9	7, 8, 10	7, 9, 11	7, 9, 11	7, 9, 11	7, 9, 11	8, 10, 12	8, 10, 12	8, 10, 13	8, 10, 13	8, 11, 14
9		5	6	6, 7	6, 7	7, 8, 9	7, 8, 9	7, 8, 10	7, 9, 11	7, 9, 11	7, 9, 11	8, 9, 12	8, 10, 12	8, 10, 12	8, 10, 13	8, 10, 13	9, 11, 14
10		5	6	6, 7	6, 8	7, 8, 9	7, 8, 10	7, 9, 10	7, 9, 11	7, 9, 11	8, 9, 11	8, 10, 12	8, 10, 12	8, 10, 13	8, 10, 13	8, 10, 13	9, 11, 15
11			6	6, 7	7, 8	7, 8, 9	7, 8, 10	7, 9, 10	7, 9, 11	8, 9, 11	8, 9, 12	8, 10, 12	8, 10, 12	8, 10, 13	8, 10, 13	8, 10, 13	9, 11, 15
12			6	6, 7	7, 8	7, 8, 9	7, 8, 10	7, 9, 10	7, 9, 11	8, 9, 11	8, 10, 12	8, 10, 12	8, 10, 12	8, 10, 13	8, 10, 13	8, 11, 13	9, 11, 15
13			6	6, 7	7, 8	7, 8, 9	7, 8, 10	7, 9, 10	8, 9, 11	8, 9, 11	8, 10, 12	8, 10, 12	8, 10, 12	8, 10, 13	8, 11, 13	9, 11, 13	9, 12, 15
14			6	6, 7	7, 8	7, 8, 9	7, 8, 10	7, 9, 10	8, 9, 11	8, 9, 11	8, 10, 12	8, 10, 12	8, 10, 13	8, 10, 13	9, 11, 13	9, 11, 13	9, 12, 15
15			6	6, 7	7, 8	7, 8, 9	7, 8, 10	7, 9, 10	8, 9, 11	8, 9, 11	8, 10, 12	8, 10, 12	8, 10, 13	8, 11, 13	9, 11, 13	9, 11, 14	9, 12, 15
16			6	6, 7	7, 8	7, 8, 9	7, 8, 10	8, 9, 10	8, 9, 11	8, 10, 11	8, 10, 12	8, 10, 12	8, 10, 13	9, 11, 13	9, 11, 13	9, 11, 14	9, 12, 15
17			6	6, 7	7, 8	7, 8, 9	7, 8, 10	8, 9, 11	8, 9, 11	8, 10, 12	8, 10, 12	8, 10, 12	8, 10, 13	9, 11, 13	9, 11, 13	9, 11, 14	9, 12, 15
18			6	7, 7	7, 8	7, 8, 9	7, 8, 10	8, 9, 11	8, 9, 11	8, 10, 12	8, 10, 12	8, 10, 12	9, 10, 13	9, 11, 13	9, 11, 14	9, 11, 14	10, 12, 16
19			6	7, 7,	7, 8	7, 8, 9	7, 9, 10	8, 9, 11	8, 9, 11	8, 10, 12	8, 10, 12	8, 10, 12	9, 11, 13	9, 11, 13	9, 11, 14	9, 11, 14	10, 12, 16
20			6	7, 7	7, 8	7, 8, 9	7, 9, 10	8, 9, 11	8, 9, 11	8, 10, 12	8, 10, 12	8, 10, 12	9, 11, 13	9, 11, 13	9, 11, 14	9, 11, 14	10, 12, 16

Table C2.1 <u>Quantile of The K-Sample Slippage Test Statistic</u>

Simulation Runs

Stage	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	258	617	796	731	1160	1282	1612	1848	937	2207	1167	649	62	745	230	1110	1625	988	527
2	903	1223	1058	573	438	2805*	1797	1784	1071	1663	1504	1042	787	2191	523	1555	1864	542	427
3	209	149	1150	363	850	1720	1315	1347	1479	1595	1827	1218	1117	1775	928	2055	1073	1439	1070
4	1373	882	411	317	1254	1080	1618	1553	1964	1697	918	1014	1540	260	1411	1362	1521	1351	307
5	590	716	1265	2223	492	944	942	958	1327	1510	1005	1376	1931	648	823	1399	2259	2127	517
6	1579	1234	353	1923	1456	874	1087	1439	1757	1178	1627	929	1417	208	79	415	1933	1173	183
7	1009	2283	2573	4159*	2379	2384	1645	2545	1121	1799	4257*	1874	675	749	1070	1095	2372	2098	1818
8	2115	1729	3665*	2041	3675*	1416	2072	3631*	2489	1046	3042	3415*	1039	666	377	2727	2037	3101*	2964
9	2521	2393*	1405	2118	3104	1960	1956	3371	3796*	1743	2388	758	1564	1610	232	4217*	2621	1458	3131*
10	2727*	1838	1601	2002	1808	3300	1223	2241	1321	2631	2023	2872	2378	2808*	2392*	2051	3159*	1581	2037
11	2301	783	2203	1657	1999	2744	3828*	1139	1814	3744*	2380	2063	2538	2351	1409	2639	523	2806	2730
12	786	1214	3486	2157	2208	2383	3811	711	735	3578	578	3034	2642*	2104	1226	902	902	1519	3072

Simulation Runs

Stage	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38
1	856	1939	256	1296	638	641	1180	1185	1445	1065	1139	825	1128	1327	763	996	856	1272	1483
2	1496	788	1147	444	1342	1328	1322	884	1053	831	2478	1350	1008	962	411	484	373	1673	1151
3	2013	1335	1994	57	2016	495	780	2491	1374	1131	2315	1087	1305	1624	785	1317	866	1576	1000
4	1772	1332	1803	1672	1584	849	1149	1539	829	1836	1912	1443	939	1027	912	2044	1195	399	835
5	1909	2050	600	1409	1099	399	1731	1033	676	1839	2941*	1013	822	419	1034	1065	1380	864	735
6	1267	1977	1357	1625	513	1377	1320	1464	1005	2210	2591	1053	1213	347	1290	1211	536	768	338
7	2899	1728	2159	2124	1846	1886	1793	3499	315	2182	2732	2561	2995	814	1244	2249	1535	1343	1074
8	2978*	1794	1750	3468	2447	1900	2349	3546*	1441	3202*	1467	2189	1026	2055	1048	2672	1942	2025	3360*
9	1327	1266	1303	3756*	1609	2311	2432*	2559	1954	2430	1960	2720*	2044	2332	3431*	3006	1528	2702*	1901
10	533	1908	1850	3502	709	2865	2358	1667	1974	2656	1810	2505	2669	3819*	1409	3637*	3514*	2116	2036
11	396	4041*	2339	1517	2767*	2872*	1433	2739	3401*	2509	1472	1390	3224*	3275	2414	1342	2496	1121	2579
12	694	2833	3175*	661	2409	2120	1118	2265	2360	2031	2876	2434	1206	2163	2599	2445	3369	2378	1962

	Simulation Runs												Historical Data			
Stage	39	40	41	42	43	44	45	46	47	48	49	50	1	2	3	4
1	807	873	1117	653	1638	908	1669	492	814	375	258	629	34	280	1075	1070
2	942	1484	1157	1487	411	1399	1510	1595	1294	613	843	503	27	320	810	820
3	826	1312	666	400	2170	845	1601	1105	1809	1566	1146	1761	21	1972	2342*	820
4	1026	1575	739	400	1325	1020	1165	988	1646	937	857	1099	32	4426*	2174	820
5	1245	907	1021	1457	2099	479	2345	720	916	892	1199	864	80	3294	1052	820
6	1256	1663	932	3909	935	1610	1012	1285	1298	1026	1477	634	62	1420	2319	820
7	71	2519	2668	5077*	874	1184	1595	2453	2320	2305	2600*	972	98	723	2272	300
8	123	2392	3886*	4730	1749	2828	1550	3127*	2508	2279	1172	3000	406	624	1133	300
9	2749	2636	1577	2720	1705	2126	2438	1348	2525*	2309	2007	3115	885	433	812	300
10	3097*	3203*	1265	1417	671	3687*	2485	2625	198	2733*	2444	2270	2128	351	1502	300
11	2516	2428	1538	2496	3442*	1806	2424	2722	1793	2293	2436	3202*	3574*	433	1432	1590
12	1513	786	1911	1918	1975	2633	3235*	2142	2392	2399	1079	2302	3560	2096	1137	5870*

Table C2.3 Hypothetical results of Slippage Test(Jan.-Feb.)

Simulation Runs

Stage	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	390	589	2461	1840	1910	2410	2653	1211	2891	2054	1056	1067	1995	2542	2967	390	1179	1523
2	1620	2094	2271	1209	752	5242*	3119	1828	3036	2685	1878	3104	1671	2177	3186	1620	2279	1977
3	339	237	2242	706	1509	3303	2488	2707	3502	2859	2254	3347*	842	1321	3947	339	259	2149
4	2503	1591	812	573	2307	2083	2905	3666	1788	3241*	1899	1089	1722	2740	2631	2503	1609	783
5	1106	1316	2342	4098	927	1789	1821	2526	1896	2056	2570	2560	3693	2112	2657	1106	1322	2333
6	2924	2282	674	3631	2706	1640	2691	3308	3038	3046	1757	2490	2085	1720	816	2924	2283	671
7	1644	3045*	3112	5542*	3237	3055	3439	1860	5541*	2127	2472	1272	4082*	3457*	1377	1644	3046*	3111
8	2678	2249	4455*	2843	4604*	1836	4564*	3165	3993	1932	4197*	1452	3694	882	3260	2678	2249	4454*
9	3112	2949	1849	2733	3930	2397	4250	4671*	3098	1702	1078	2826	1776	3167	5109*	3112	2949	1849
10	3362*	2290	1980	2515	2354	3991	2887	1781	2574	380	3455	2025	969	3252	2651	3362*	2290	1980
11	2869	1001	2666	2064	2489	3409	1498	2242	2707	1591	2561	2867	1759	2013	3265	2869	1001	2666
12	1044	1436	4219	2622	2711	2979	889	934	584	3184	3697	2082	3453	2468	1201	1044	1436	4219

Simulation Runs

Stage	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
1	2226	2409	1914	1777	1244	2035	1913	2080	1129	982	1565	985	2392	1920	2184	2176	2723	569
2	851	5241*	2901	2006	280	1883	2042	2908	1589	778	2767	2456	842	1158	2469	1650	1999	641
3	1519	3303	1629	2763	232	846	1279	3860	1592	1962	3767	567	91	3241	1478	4631*	2576	1210
4	2310	2083	2541	3685	1685	1723	2727	2604	1868	572	3266	2658	3061	2078	2140	2943	1574	1806
5	928	1789	2498	2532	1071	3693	2107	2648	2526	934	3612	1773	2645	1629	3232*	1980	1265	2095
6	2707	1640	3292	3309	2209	2085	1718	813	1730	314	2428	1632	3055	1186	2502	2748	1861	1237
7	3237	3055	3370*	1861	683	4082*	3457*	1376	3236*	2131	3826*	2631	3007	1289	2515	4587	625	2933*
8	4604*	1836	3154	3166	2933	3694	882	3260	2183	3542	3793	3097	4373	3602	2980	4526	1754	2619
9	3930	2397	765	4672*	4992*	1776	3167	5109*	2255	3829*	1779	3300*	4701*	3821	3030	3304	2345	2808
10	2354	3991	1548	1781	3112	969	3252	2651	340	2573	712	622	4410	2853	2927	2162	2393	409
11	2489	3409	3318	2242	3490	1759	2013	3265	1101	3351	450	1154	2033	3928*	1814	3352	4104*	2009
12	2711	2979	628	934	3827	3453	2468	1201	2743	3782	672	2357	887	2900	1378	2817	2955	405

Stage	Simulation Runs 37	38	39	40	41	42	43	44	45	46	47	48	49	50	Historical Data 1	2	3	4	5
1	2236	1590	2112	1754	2490	1876	2062	2539	1558	1638	1717	948	519	1445	4606	2192	3386*	796	4588
2	1563	2523	1891	1571	1820	914	2241	1814	1767	2763	2611	2956	1552	563	4870*	1420	1860	1835	1328
3	368	2050	2438	2280	3032	2431	1318	3660	1543	2470	1062	2088	2123	3115	4230	1152	916	1648	1605
4	2696	2703	1771	3553*	1952	3815	3175*	3687	1907	2959	1904	1859	1605	2050	2214	800	562	840	5950
5	1945	1919	1539	2727	801	2044	2672	2231	2322	1732	901	1352	2231	2572	1402	569	412	569	6104*
6	3438	1975	2256	2032	630	2281	2011	3349	2355	3106	2975	2384	2762	3201*	1104	516	335	550	4852
7	3992*	3328	3892	1062	994	3015	2549	4221*	431	3482	1860	3268	3517*	3049	2356	372	270	1466	5354
8	3020	2794	1453	2258	2422	3374	1591	449	209	3117	3542	3933*	1643	1300	2030	4070	205	3906*	2866
9	1731	3365*	2507	1736	2807	3737	3767	2443	3222	3311	2691	1796	2479	558	2106	5668	186	3018	1966
10	1523	3116	3249	2041	4619*	4496*	2217	2779	3750*	3962*	4503*	3203	2985	1248	1162	5374	351	3610	1892
11	2959	1777	3947*	3023	4077	1803	2730	3202	3126	3063	2337	3349	2996	2215	1291	5998*	317	1524	2146
12	1934	2949	1592	3116	2764	2993	1290	1267	1924	1068	3238	2676	1387	1083	2550	3910	358	1007	3526

Table C2.4 Hypothetical results of Slippage Test (Mar.-Apr.)

Simulation Runs

Stage	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	2047	2032	3151	1091	2336	2996	3452	1257	143	3229	3228	1524	668	2118	2031	2988	818	2203
2	624	183	3577*	2120	2982	3106	3458*	2015	1476	3592*	3543	4295*	3108	1930	2159	792	3232	3071
3	2755	1887	2668	2916	3637*	3117	2774	2339	2158	3401	3338	3583	3493*	894	1356	2178	3305*	4084*
4	1471	1483	3233	3898*	1895	3465*	906	2014	3023	2011	2120	622	1164	1803	2867*	2110	2030	2783
5	3626*	3651	1927	2703	2002	2207	2546	2713*	3833*	3187	865	1254	2691	3883*	2234	3205*	2442	2816
6	2165	4742*	2160	3509	3203	3226	1174	1867	2876	980	3671*	385	2634	2225	1821	2417	2197	882
7	1134	1383	1114	1239	2278	1268	421	1129	938	880	1804	386	907	1673	1413	900	1108	629
8	182	767	885	1137	1236	746	243	1336	592	818	624	288	602	1149	312	106	1207	1045
9	1102	957	722	1395	853	548	702	257	629	650	586	604	899	471	1048	674	1311	1486
10	765	581	429	425	695	124	316	1030	896	1178	798	1000	579	268	946	587	949	657
11	1421	22	1360	632	894	523	807	700	892	901	28	801	833	542	537	838	887	898
12	1363	507	1293	247	244	965	165	1050	907	887	1157	714	572	1030	704	578	1368	279

Simulation Runs

Stage	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
1	1237	1950	2580	1621	2124	545	2420	2507	1330	2030	2274	2889*	2770	1971	2349	2232	1856	2519
2	1680	1078	1339	2902	1169	2202	2555	896	2632	1227	1740	2128	3242*	1692	1654	2001	1658	2717*
3	1678	2792	2759	3978*	1293	3908*	585	87	3983*	3382*	4874*	2725	2594	2323*	390	2573*	2400	1957
4	1966	2632	2890*	3474	2272	3621	3179*	3194	3204	2203	3144	1676	3158	2619	2818	1879	3748*	1857
5	2662*	4211*	2232	3839	3037*	1282	1335	2797	2243	1727	2120	1337	2136	1332	2057	1627	2903	2084
6	1837	2414	1529	2599	1889	2667	1067	3235*	1055	1253	2909	1955	1646	2371	3678*	2375	2167	1778
7	1350	1338	833	1664	1127	1411	353	1531	933	666	1943	592	1184	1104	1901	1639	764	1151
8	708	1258	411	1207	491	792	392	1441	939	1169	1413	679	691	590	1027	495	826	864
9	670	523	27	477	514	504	241	1344	566	1097	901	741	873	707	505	773	541	714
10	80	561	315	193	575	671	1204	1188	245	761	561	701	707	606	448	945	612	822
11	369	991	430	162	548	823	769	471	996	1101	956	1192	703	8	884	1112	888	684
12	849	503	1019	304	172	1101	724	212	825	763	766	785	598	626	531	374	876	503

	Simulation Runs														Historical Data				
Stage	37	38	39	40	41	42	43	44	45	46	47	48	49	50	1	2	3	4	5
1	1987	447	2592	2184	2939*	1675	2292	2517	1020	1625	788	1507	577	1824	4642	2102	593	1456*	4728*
2	971	1424	1839	2514	2314	1870	2318*	1947	3107*	2536	1175	546	1627	2754	8928*	2380	1285	782	2362
3	2550	2660	1733	2000	2000	1630	1345	775	2217	3574*	3032*	3246	2231	1703	6316	2808	1950	528	1506
4	4024*	2209	2752	2217	1659	2007	1320	2241	1970	3307	1881	2170	1696	2009	4302	6052*	1103	385	1346
5	2188	976	3258*	3338*	1443	2448	1945	2069	1431	1885	1767	2718	2350	955	3028	638	2945	607	760
6	2420	3011*	2115	2106	653	2491*	1818	2695*	2507	2572	2015	3386*	2917	3121*	1720	2420	3514*	262	573
7	1389	1496	787	623	563	631	1396	1073	1477	1447	1313	1570	1625	1166	992	3580	1164	173	476
8	1101	766	619	785	1255	255	1480	1241	1262	1051	932	533	594	1228	807	2272	644	154	312
9	1080	620	292	360	654	1067	529	834	481	916	834	223	770	786	710	1980	358	104	226
10	1255	759	56	637	719	1093	435	687	933	49	952	428	869	1299	392	1365	256	60	660
11	412	388	638	1109	891	849	545	678	936	659	776	691	839	583	324	576	119	56	573
12	849	732	305	654	661	497	674	545	719	845	818	312	350	907	211	639	383	64	396

Table C2.5 Hypothetical results of Slippage Test (May-June)

(3) The number of values in the sample with the greatest extreme that exceeded the smallest extreme was determined to be statistic T.

(4) From Table C2.1 find out the critical region of size α = 0.05 was determined. The results of these tests is presented below on the Table C2.2.

	January-February	March-April	May-June
$Z^{(K)}$	5870	6104	8978
$Z^{(1)}$	2342	2933	1456
$W_{1-\alpha}$	8	8	8
T	1	6	6
Decision	Accept H_o	Accept H_o	Accept H_o

Table C2.2 Result of K-Sample Slippage Test

B. The Friedman Test [20]

Hypothesis:

H_o: The data from each simulation run and the historical data are equally likely.

H_1: At least one data block tends to yield larger observed value value than at least one other.

Test Statistic:

The Friedman test statistic is defined as [20]

$$T = \frac{12}{bk(k+1)} \sum_{j=1}^{k} R_j^2 - 3b(k+1).$$

The rank assigned to each run of simulation and historical data of each season period is presented on Tables C2.7, C2.8, and C2.9.

Decision Rule:

Reject the null hypothesis at the level α if the Friedman test statistic T exceeds the $1 - \alpha$ quantile of a Chi-Square random variable with k - 1 degree of freedom. In this test, b = 12, k = 55, and $\alpha = 0.05$. The results of the test are presented on Table C2.6.

	January-February	March April	May-June
T	66.73	72.03	73.101
$W_{1-\alpha}$	72.04	73.2	73.2
Decision	Accept H_o	Accept H_o	Accept H_o

Table C2.6 Results of Friedman Test

	Simulation Runs																											
Stage	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
1	6	11	20	17	38	43	48	52	28	54	39	15	2	18	3	34	49	29	10	24	53	4	44	13	14	40	41	45
2	22	33	28	13	7	54	50	49	29	47	43	26	15	52	11	45	51	12	6	42	16	30	8	37	36	35	21	27
3	4	3	25	5	14	41	29	32	35	38	45	26	22	43	16	50	19	34	18	48	31	47	2	49	7	9	54	33
4	34	14	7	4	29	24	43	40	51	46	16	20	39	2	35	33	37	32	3	47	31	48	45	42	12	26	38	10
5	7	11	35	50	5	23	22	24	36	42	25	37	45	9	16	39	51	49	6	46	47	8	40	32	2	43	28	10
6	42	27	7	48	39	14	22	38	47	24	45	15	36	4	2	8	49	23	3	29	50	34	44	9	35	33	40	18
7	11	35	45	52	39	40	21	43	15	24	53	27	5	7	12	13	38	29	25	49	22	31	30	26	28	23	51	4
8	28	17	51	25	52	13	27	50	34	9	42	47	8	6	3	37	24	43	39	40	20	19	48	33	21	31	49	14
9	37	33	11	27	47	24	22	50	53	19	32	4	14	17	1	54	40	12	49	9	7	8	52	16	30	35	39	21
10	41	18	14	22	16	49	7	29	9	38	23	45	32	43	33	25	47	13	24	4	20	19	50	6	44	31	15	21
11	24	4	22	16	20	43	53	6	19	52	27	21	36	26	9	38	3	44	40	1	54	25	13	42	45	11	41	49
12	6	14	50	27	29	34	53	4	5	52	1	45	42	24	15	8	9	17	46	3	43	47	2	37	25	11	30	32
R_j	262	220	315	306	335	402	397	417	361	445	391	328	296	251	156	384	417	337	289	342	394	320	378	342	299	328	447	285

	Simulation Runs																						Historical Data			
Stage	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	1	2	3	4
1	51	37	23	36	45	19	30	25	42	47	21	26	35	16	50	27	51	9	22	8	5	12	1	7	33	32
2	19	53	38	25	24	4	9	3	48	31	23	40	32	41	5	39	44	46	34	14	20	10	1	2	17	18
3	23	52	20	27	40	10	30	15	37	17	12	28	8	6	51	13	39	21	44	36	24	42	1	46	53	11
4	49	50	36	18	23	15	52	28	5	11	22	41	8	6	30	21	27	19	44	17	13	25	1	54	53	9
5	44	53	26	15	3	29	31	38	18	13	34	20	27	41	48	4	52	12	21	19	33	17	1	54	30	14
6	51	53	21	26	6	31	25	10	12	5	28	46	16	54	17	43	19	30	32	20	41	11	1	37	52	13
7	32	48	44	50	8	17	33	19	18	14	1	42	47	54	9	16	20	41	37	36	46	10	2	6	34	3
8	45	15	29	7	26	10	36	22	23	46	1	32	53	54	18	38	16	44	35	30	12	41	4	5	11	2
9	34	23	43	26	31	51	46	13	42	20	45	41	15	44	18	28	36	10	38	29	25	48	6	3	5	2
10	39	17	36	40	54	10	52	51	27	26	46	48	8	11	5	53	35	37	1	42	34	30	28	3	12	2
11	34	12	8	47	48	28	7	33	5	37	35	30	14	32	50	18	29	39	17	23	31	46	51	2	10	15
12	22	44	38	13	28	40	39	49	33	20	16	7	18	19	21	41	48	26	35	36	10	31	51	23	12	54
R_j	423	457	362	330	336	264	390	306	310	287	284	401	281	378	322	341	416	334	360	310	294	323	148	242	322	175

Table C2.7 The rank assignmemt of Friedman Test(Jan-Feb)

Simulation Runs

Stage	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
1	2	5	46	25	27	45	49	14	51	33	10	11	31	42	52	1	13	17	40	44	29	24	15	32	28	35	12	3
2	17	35	38	10	4	55	51	23	49	29	27	50	26	36	52	18	39	31	7	54	46	33	1	28	34	47	16	5
3	6	3	32	9	20	46	38	40	49	42	33	48	10	18	53	5	4	31	21	47	25	41	2	11	16	52	23	27
4	33	10	6	3	31	28	43	51	17	48	21	8	14	42	37	34	12	4	32	27	35	52	13	15	41	36	20	2
5	11	13	38	54	6	22	24	40	26	30	44	43	53	33	47	10	14	37	7	23	39	42	9	52	32	46	41	8
6	41	29	7	54	37	14	36	50	44	45	19	34	24	17	9	42	30	6	38	14	49	51	26	25	16	8	18	1
7	13	28	34	55	36	31	41	15	54	18	21	8	49	42	11	14	29	33	37	32	40	17	5	50	43	10	35	19
8	22	17	51	25	54	12	53	33	46	14	48	7	41	4	35	22	17	50	55	13	32	34	27	42	5	35	16	38
9	33	28	12	24	46	18	48	49	32	5	4	27	8	35	53	34	29	13	47	19	3	50	52	9	36	54	16	45
10	43	22	16	27	24	44	34	13	29	3	45	18	7	41	30	44	23	17	25	49	12	14	37	8	42	31	1	28
11	34	3	29	20	26	48	8	23	31	10	28	33	11	17	42	35	4	30	27	49	44	24	50	12	18	43	5	46
12	11	21	54	31	33	41	7	8	3	45	50	26	47	28	15	12	21	55	34	42	4	9	52	48	29	16	35	51
R_j	266	214	363	337	344	404	422	359	421	322	350	313	321	355	426	271	235	324	370	413	358	391	289	332	340	413	238	278

Simulation Runs / Historical Data

Stage	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	Historical Data 1	2	3	4	5
1	19	9	43	30	38	37	50	4	41	20	36	23	47	26	34	48	18	21	22	7	3	16	55	39	53	6	54
2	45	40	6	9	41	19	32	3	14	42	30	15	22	8	37	21	20	44	43	48	13	2	53	12	25	24	11
3	51	8	1	45	19	55	39	15	7	28	36	34	43	35	17	50	22	37	13	29	30	44	54	14	12	26	24
4	49	38	46	26	29	44	9	18	39	40	16	50	24	54	47	53	23	45	22	19	11	25	30	5	1	7	55
5	51	21	19	18	50	25	12	31	28	27	17	49	4	29	48	35	36	20	5	15	34	45	16	2	1	3	55
6	33	13	46	11	35	39	20	12	53	21	27	23	5	28	22	52	31	47	43	32	40	48	10	3	2	4	55
7	46	24	26	9	22	52	4	25	48	38	47	7	6	27	23	51	3	44	16	39	45	30	20	2	1	12	53
8	43	30	49	40	28	52	11	21	29	24	8	19	20	37	9	3	2	31	39	45	10	6	15	47	1	44	26
9	10	38	51	44	31	39	17	26	6	41	22	7	25	42	43	20	37	40	23	11	21	2	15	55	1	30	14
10	6	5	51	33	35	20	26	4	11	38	40	19	54	52	21	32	47	48	53	39	36	10	9	55	2	46	15
11	2	6	19	51	15	47	54	16	36	13	52	38	53	14	32	41	40	39	25	45	37	22	7	55	1	9	21
12	5	27	6	38	19	37	40	2	25	39	23	44	36	43	18	17	24	13	46	32	20	14	30	53	1	10	49
R_j	360	259	363	354	362	456	314	177	337	371	344	328	339	395	351	423	303	429	350	361	300	264	314	342	101	221	432

Table C2.8 The rank assignment of Friedman Test(Mar-Apr)

Stage	Simulation Runs 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
1	44	27	50	21	36	49	53	11	1	53	51	15	6	29	26	48	9	52	10	22	42	16	30	3	38	39	12	25
2	3	1	52	28	43	45	50	27	15	53	51	54	47	25	30	5	48	44	19	8	13	42	9	31	38	6	39	11
3	32	15	30	38	48	40	34	24	20	44	42	47	45	6	9	21	49	53	12	35	33	51	7	50	4	1	52	43
4	8	9	46	52	17	48	3	23	40	22	26	2	5	13	38	25	24	36	18	34	39	49	32	50	43	44	45	28
5	50	51	18	37	20	27	34	38	52	46	4	8	36	54	29	47	32	41	35	55	23	53	45	9	7	40	30	15
6	25	55	24	51	47	48	9	18	42	6	53	2	39	28	16	32	27	5	17	31	11	38	19	40	8	49	7	10
7	27	37	24	32	54	33	4	26	19	15	51	3	17	50	41	16	23	9	36	35	14	49	25	40	2	45	18	11
8	3	26	33	40	46	24	4	51	15	30	19	6	17	41	7	1	43	37	23	49	10	44	11	28	9	53	35	42
9	50	45	33	53	40	19	29	6	24	25	21	22	42	10	46	28	51	54	27	16	1	11	15	13	5	52	20	49
10	37	22	15	13	31	5	11	48	41	50	38	47	21	9	44	23	45	27	4	18	10	6	20	29	52	51	7	36
11	55	2	54	21	43	13	32	27	42	45	3	31	34	15	14	35	39	44	8	48	11	6	17	33	29	12	49	50
12	53	19	54	7	6	47	2	50	45	44	52	32	22	49	30	23	55	8	41	17	48	9	3	51	33	5	39	35
R_j	387	309	432	393	431	398	265	349	356	433	411	269	301	329	330	304	445	390	250	368	260	374	233	377	268	397	353	355

Stage	Simulation Runs 29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	Historical Data 1	2	3	4	5
1	34	46	45	23	37	33	20	41	24	2	43	31	47	18	35	40	9	17	7	14	4	19	54	28	5	13	55
2	21	29	49	20	17	26	18	40	7	14	22	36	32	23	33	24	46	37	10	2	16	41	55	35	12	4	34
3	54	31	28	37	2	27	25	17	26	29	14	18	19	11	8	5	22	46	39	41	23	13	55	36	16	3	10
4	41	11	42	33	37	14	51	16	53	29	35	30	10	20	6	31	19	47	15	27	12	21	54	55	4	1	7
5	24	11	25	10	21	14	42	23	26	6	43	49	13	33	19	22	12	17	16	39	31	5	44	2	43	1	3
6	43	20	12	29	54	30	26	14	33	45	23	22	4	35	15	41	36	37	21	50	44	46	13	34	52	1	3
7	53	7	31	22	52	48	12	28	38	44	13	8	6	10	39	21	43	42	34	46	47	30	20	55	29	1	5
8	52	21	22	14	36	12	31	32	39	25	18	27	48	5	54	47	50	38	34	13	16	45	29	55	20	2	8
9	43	34	41	30	14	36	18	32	48	23	7	9	26	47	17	39	12	44	38	3	35	37	31	55	3	2	4
10	18	32	33	24	17	43	25	39	53	35	2	26	34	49	16	30	42	1	46	14	40	54	12	55	8	3	28
11	47	53	28	1	38	52	40	25	10	9	22	51	41	37	16	24	46	23	30	26	36	20	7	19	5	4	18
12	36	37	24	25	20	13	43	18	42	34	10	27	28	16	29	21	31	40	38	11	12	45	4	36	14	1	15
R_j	426	332	380	268	345	348	351	325	399	259	257	334	308	304	287	345	368	389	328	286	316	376	378	435	216	36	190

Table C2.9 The rank assignment of Friedman Test(May-June)

APPENDIX D

SIMULATION RESULTS

Stage	1 IN	1 OPT	1 STAN	2 IN	2 OPT	2 STAN	3 IN	3 OPT	3 STAN	4 IN	4 OPT	4 STAN	5 IN	5 OPT	5 STAN	6 IN	6 OPT	6 STAN
1	4606	3448	3448	4642	3484	3840	237	1394	1394	24	1181	1181	11	1168	1168	34	1191	1191
2	4870	4870	4870	8987	8978	8978	351	351	351	16	16	16	18	18	18	27	27	27
3	4230	4230	4230	6316	6316	6316	224	224	224	10	10	10	18	18	18	21	21	21
4	2214	2214	2214	4302	4302	4302	130	130	130	12	12	12	25	25	25	32	32	32
5	1402	1500	1500	3082	3082	3082	124	124	124	10	10	10	37	37	37	80	80	80
6	1104	1500	1500	1720	1720	1720	76	76	76	15	15	15	50	50	50	62	62	62
7	2356	1862	1862	992	1500	1500	70	70	70	18	18	18	30	30	30	98	98	98
8	2030	2030	2030	807	1500	1500	50	50	50	13	13	13	27	27	27	406	406	406
9	2106	2106	2106	710	1500	1500	32	32	32	13	13	13	34	34	34	885	885	885
10	1162	1500	1500	392	716	715	27	27	27	10	10	10	27	27	27	2128	1750	1500
11	1291	1500	1500	324	324	324	22	22	22	9	9	9	24	24	24	3574	1637	1887
12	2550	2003	2003	221	221	221	22	22	22	9	9	9	30	30	30	3560	3560	3560
COST	4.65	4.65	4.65	6.49	6.49	6.49	0	0	0	0	0	0	0	0	0	3.51	3.49	3.49
ΔQ_p		0	0		0	0		-	-		-	-		-	-		14	14
RR		0	0		0	0		-	-		-	-		-	-		3.9%	3.9%
RC		0	0		0	0		-	-		-	-		-	-		6.7%	6.7%

Stage	7 IN	7 OPT	7 STAN	8 IN	8 OPT	8 STAN	9 IN	9 OPT	9 STAN	10 IN	10 OPT	10 STAN	11 IN	11 OPT	11 STAN	12 IN	12 OPT	12 STAN
1	2192	1600	1500	2102	1600	1500	682	1600	1500	32	1189	1189	87	1244	1244	280	1437	1437
2	1420	1500	1500	2380	1734	1824	4450	2374	2474	22	22	22	80	80	80	330	330	330
3	1152	1500	1500	2888	2888	2888	2908	2908	2908	19	19	19	91	91	91	1972	1972	1500
4	800	1500	1500	6052	6052	6052	3158	3158	3158	18	18	18	563	563	563	4426	2111	2583
5	569	1140	1290	638	1500	1500	1795	1785	1795	34	34	34	868	868	868	3294	3294	3294
6	516	516	516	2420	1558	1558	506	1500	1500	27	27	27	563	563	563	1420	1500	1500
7	372	372	372	3580	3580	3580	271	1500	1500	18	18	18	351	351	351	732	1500	1500
8	4070	1900	1755	2272	2272	2272	102	194	193	13	13	13	237	237	237	624	1500	1500
9	5668	5523	5668	1980	1980	1980	68	68	68	13	13	13	770	770	770	433	1024	1023
10	5374	5374	5374	1365	1500	1500	29	29	29	36	36	36	1344	1344	1344	351	351	351
11	5998	5998	5998	576	1500	1500	16	16	16	104	104	104	628	628	628	433	433	433
12	3910	3910	3910	639	1500	1500	29	29	29	127	127	127	287	287	287	2096	1500	1500
COST	5.3	5.3	5.3	5.33	5.33	5.33	4.35	3.07	3.07	.0.	.0	.0	0	0	0	4.38	3.21	3.21
ΔQ_p		0	0		0	0		1292	1292		-	-		-	-		1132	1132
RR		0	0		0	0		29%	29%		-	-		-	-		256%	256%
RC		0	0		0	0		438%	438%		-	-		-	-		387%	387%

Appendix D1 Results from Simulation of Historical data with Time independent stage input assumption

Stage	13 IN	13 OPT	13 STAN	14 IN	14 OPT	14 STAN	15 IN	15 OPT	15 STAN	16 IN	16 OPT	16 STAN	17 IN	17 OPT	17 STAN	18 IN	18 OPT	18 STAN
1	3386	2228	2228	593	1600	1500	732	1500	1500	51	1238	1238	86	1244	1244	1078	1600	1500
2	1860	1860	1860	1285	1435	1500	420	709	709	2050	2050	1500	77	77	77	810	1445	1500
3	916	1500	1500	1950	1950	1500	312	312	312	936	936	1486	74	74	74	2342	2150	1500
4	562	1500	1500	1103	1103	1500	1119	1119	1119	291	291	291	67	67	67	2174	2100	1500
5	412	1205	1204	2945	2100	1500	493	493	493	203	203	203	67	67	67	1052	1318	1500
6	335	335	335	3514	2044	2732	436	436	436	260	260	260	2263	2050	1500	2319	2050	1500
7	270	270	270	1164	1500	1500	676	676	676	189	189	189	894	1107	1500	2272	1900	1900
8	205	205	205	644	1500	1500	173	173	173	861	861	861	392	392	549	1133	1650	1500
9	186	186	186	385	1500	1500	80	80	80	457	457	457	344	344	344	812	936	1500
10	351	351	351	256	264	263	719	719	719	267	267	267	244	244	244	1502	1502	1500
11	317	317	317	119	119	119	385	385	385	195	195	195	238	238	238	1432	1432	1500
12	358	358	358	383	383	383	158	158	158	126	126	126	860	860	860	1137	1137	1500
COST	3.31	1.79	1.79	3.45	1.44	2.5	0	0	0	0.81	0.81	0	1.81	0.81	0	1.93	1.4	0.96
ΔQ_p		1158	1158		1414	782		-	-		0	550		213	763		192	442
RR		342%	342%		402%	222%		-	-		0	268%		104%	337%		8.2%	188%
RC		614%	614%		702%	388%		-	-		0	100%		279%	100%		228%	525%

Stage	19 IN	19 OPT	19 STAN	20 IN	20 OPT	20 STAN	21 IN	21 OPT	21 STAN	22 IN	22 OPT	22 STAN	23 IN	23 OPT	23 STAN	24 IN	24 OPT	24 STAN
1	796	1600	1500	1456	1500	1500	31	1188	1188	14	1171	1171	746	1600	1500	1070	1600	1500
2	1835	2000	1500	782	1500	1500	24	24	24	51	51	51	702	1005	1005	820	1447	1500
3	1648	1836	1500	528	673	923	33	33	33	1135	1135	1135	1628	1628	1500	820	820	820
4	840	840	1500	358	358	358	14	14	14	1228	1228	1228	2552	2100	1500	820	820	820
5	564	564	840	601	601	601	15	15	15	955	955	955	2146	1900	1500	820	820	820
6	550	550	550	262	262	262	13	13	13	5300	2985	2985	1255	1650	1500	820	820	820
7	1466	1466	1466	173	173	173	15	15	15	7324	7324	7324	846	1149	1500	300	300	300
8	3906	1900	1591	154	154	154	21	21	21	4052	4052	4052	597	597	1500	300	300	300
9	3018	2709	3018	104	104	104	12	12	12	1764	1764	1764	752	752	776	300	300	300
10	3670	3670	3670	60	60	60	10	10	10	1119	1500	1500	824	824	824	300	300	300
11	1534	1534	1534	56	56	56	14	14	14	2056	1675	1675	1082	1082	1082	1590	1590	1500
12	1007	1500	1500	64	64	64	33	33	33	1625	1625	1625	1130	1130	1130	5870	3500	3645
Cost	3.84	3.61	3.61	0	0	0	0	0	0	6.01	6.01	6.01	2.24	1.44	0	5.23	3.49	3.58
ΔQ_p	236	236			-	-	-	-	-		0	0		452	1052		2320	2225
RR		6%	6%		-	-		-	-		0	0		17.7%	41.2%		39.5%	37.9%
RC		9.8%	9.8%		-	-		-	-		0	0		43%	100%		531%	509%

Stage	25 IN	25 OPT	25 STAN	26 IN	26 OPT	26 STAN	27 IN	27 OPT	27 STAN	28 IN	28 OPT	28 STAN	29 IN	29 OPT	29 STAN
1	4588	3430	3430	4728	3570	3570	257	1411	1411	249	1406	1406	66	1223	1223
2	1328	1500	1500	2362	2362	2362	426	426	426	127	127	127	122	122	122
3	1605	1500	1500	1566	1566	1566	679	679	679	68	68	68	102	102	102
4	5950	5883	5883	1346	1550	1550	529	529	529	63	63	63	87	87	87
5	6104	6104	6104	760	1500	1500	243	243	243	63	63	63	76	76	76
6	4852	4852	4852	573	1500	1500	188	188	188	58	58	58	76	76	76
7	5354	5354	5354	476	970	970	102	102	102	1	1	1	71	71	71
8	2899	2899	2899	321	321	321	82	82	82	1	1	1	76	76	76
9	1966	1966	1966	226	226	226	236	236	236	3	3	3	76	76	76
10	1892	1892	1892	660	660	660	433	433	433	3	3	3	63	63	63
11	2146	2146	2146	573	573	573	841	841	841	5	5	5	52	52	52
12	3526	3526	3526	396	396	396	292	292	292	8	8	8	63	63	63
COST	5.35	5.35	5.35	4.55	3.51	3.51	0	0	0	0	0	0	0	0	0
ΔQ_p		0	0		1159	1159		-	-		-	-		-	-
RR		0	0		245%	245%		-	-		-	-		-	-
RC		0	0		358%	358%		-	-		-	-		-	-

Stage	1 IN	1 OPT	1 STAN	2 IN	2 OPT	2 STAN	3 IN	3 OPT	3 STAN	4 IN	4 OPT	4 STAN
1	34	1191	1191	280	1437	1437	1078	2235	1500	1070	2227	1500
2	27	27	27	330	330	330	810	810	1500	820	820	1500
3	21	21	21	1972	1972	1500	2342	2200	1500	820	820	867
4	32	32	32	4426	2200	2583	2174	2300	1500	820	820	820
5	80	80	80	3294	3205	3294	1052	1068	1500	820	820	820
6	62	62	62	1420	2100	1500	2319	2200	1500	820	820	820
7	98	98	98	723	2000	1500	2272	2150	1889	300	300	300
8	406	406	406	624	982	1500	1133	1374	1500	300	300	300
9	885	885	885	433	433	1014	812	812	1500	300	300	300
10	2128	1800	1500	351	351	351	1502	1502	1500	300	300	300
11	3574	1650	1887	433	433	433	1432	1432	1500	1590	1590	1500
12	3560	3497	3560	2096	1500	1500	1137	1137	1500	5870	3555	3645
COST	3.51	3.43	3.49	4.38	3.12	3.21	1.93	1.87	0.93	5.23	3.49	3.59
ΔQ_p		77	14		1221	1132		142	453		2315	2225
RR		2.1%	0.4%		27.6%	25.5%		6%	19.3%		39.4%	37.9%
RC		3.7%	0.7%		41.7%	38.7%		16.8%	53.7%		52.9%	50.9%

Appendix D2.1 Results from Simulation using Historical data(Jan.-Feb.) with Assumption of time Dependent Inflow Rate.

Stage	1 IN	1 OPT	1 STAN	2 IN	2 OPT	2 STAN	3 IN	3 OPT	3 STAN	4 IN	4 OPT	4 STAN	5 IN	5 OPT	5 STAN
1	4606	4870	3448	2192	3349	1500	3386	3386	2228	796	1953	1500	4580	5300	3430
2	4807	4600	4870	1420	1420	1500	1860	2860	1860	1835	1835	1500	1328	1773	1500
3	4230	4850	4230	1152	1152	1500	916	916	1500	1648	1648	1500	1605	1605	1500
4	2214	2327	2214	800	800	1500	562	562	1500	640	640	1500	5950	4150	5883
5	1402	1402	1500	569	569	1290	412	412	1204	569	569	645	6104	5589	6104
6	1104	1104	1500	516	516	516	335	335	335	550	550	550	4852	4852	4852
7	2356	2356	1862	372	372	372	270	270	270	1466	1466	1466	5354	5354	5354
8	2030	2030	2030	4070	2850	1755	205	205	205	3906	2850	1591	2866	2866	2866
9	2106	2106	2106	5668	4573	5668	186	186	186	3018	2300	3018	1966	1966	1966
10	1162	1162	1500	5374	5374	5374	351	351	351	3670	3129	3670	1892	1892	1892
11	1291	1291	1500	5998	5998	5998	317	317	317	1524	1524	1524	2146	2146	2146
12	2550	1500	2003	3910	3910	3910	358	358	358	1007	1500	1500	3536	3536	3536
Cost	4.65	4.65	4.65	5.29	5.29	5.29	3.31	3.31	1.75	3.84	3.03	3.61	5.35	5.08	5.35
ΔQ_p		0	0		0	0		0	1158		777	236		515	0
RR		0	0		0	0		0	34.2%		19.9%	6%		8.4%	0
RC		0	0		0	0		0	61.4%		32.3%	9.8%		11.2%	0

Appendix D2.2 Results from Simulation Using Historical data(Mar.-Apr.) with Assumption of time Dependent Inflow Rate.

	1			2			3			4			5		
Stage	IN	OPT	STAN	IN	OPT	STAN	IN	OPT	STAN	IN	OPT	STAN	IN	OPT	STAN
1	4642	3484	3484	2102	2350	1500	593	1750	1500	1456	1500	1500	4728	3570	3570
2	8978	8978	8978	2380	2300	1824	1285	1285	1500	782	1445	1500	2362	2362	2362
3	6316	6316	6316	2808	2150	2808	1950	1950	1500	588	988	983	1566	1600	1566
4	4302	4302	4302	6052	5384	6052	1103	1103	1500	358	358	358	1346	1700	1500
5	3028	3028	3028	638	1500	1500	2945	1750	1500	607	607	607	760	1800	1500
6	1720	1720	1720	2420	1558	1558	3515	2392	2732	262	262	262	573	1460	1500
7	992	1500	1500	3580	3580	3580	1164	1500	1500	173	173	173	476	476	969
8	807	1500	1500	2272	2272	2272	644	1500	1500	154	154	154	312	312	312
9	710	1500	1500	1980	1980	1980	385	1500	1500	104	104	104	226	226	226
10	392	715	715	1365	1500	1500	265	273	273	60	50	60	660	660	660
11	324	324	324	576	1500	1500	119	119	119	56	56	56	573	573	573
12	211	211	211	639	1500	1500	383	383	383	64	64	64	396	396	396
Cost	6.49	6.49	6.49	5.33	4.97	5.33	3.45	2.01	2.51	0	0	0	4.55	3.51	3.51
ΔQ_p		0	0		668	0		1120	782		0	0		1158	1158
RR		0	0		111%	0		319%	222%		0	0		245%	245%
RC		0	0		146%	0		445%	383%		0	0		358%	358%

Appendix D2.3 Results from Simulation using Historical data(May-June) with Assumption of time Dependent Inflow Rate.

Stage	1 IN	1 INDP	1 DEPN	1 STAN	2 IN	2 INDP	2 DEPN	2 STAN	3 IN	3 INDP	3 DEPN	3 STAN	4 IN	4 INDP	4 DEPN	4 STAN	5 IN	5 INDP	5 DEPN	5 STAN
1	258	1415	1415	1415	617	1600	1774	1500	796	1600	1953	1500	731	1600	1888	1500	1160	1600	2317	1500
2	903	904	904	903	1228	1402	1228	1500	1058	1412	1059	1500	573	862	574	962	438	1155	438	1255
3	209	209	209	209	149	149	149	151	1150	1150	1150	1162	363	363	363	363	850	851	851	850
4	1373	1374	1314	1373	882	883	883	882	411	412	414	411	317	318	318	317	1254	1254	1254	1254
5	590	590	590	590	716	717	717	716	1265	1265	1265	1265	2223	2100	2200	1500	492	493	493	492
6	1579	1580	1580	1500	1234	1234	1234	1234	353	354	354	353	1923	2047	1947	1500	1465	1465	1465	1465
7	1009	1009	1009	1089	2283	2000	2200	1500	2578	2000	2200	1500	4159	2000	2200	2991	2379	2000	2200	1500
8	2115	1900	2100	1500	1729	1850	1814	1500	3665	1929	2000	2429	2041	1886	1900	2041	3675	1741	2000	2240
9	2521	1850	1950	1500	2393	1850	1950	1500	1405	1500	1800	1500	2118	2119	1950	2118	3104	3104	2645	3104
10	2727	1500	1800	2048	1838	1500	1800	1500	1601	1508	1800	1507	2002	2002	2002	2002	1808	1808	1808	1808
11	2301	2100	1750	2301	783	1500	1265	1500	2203	2203	1750	2203	1657	1657	1657	1657	1999	2000	2000	1999
12	786	1500	1500	1500	1214	1500	1214	1500	3486	3486	3076	3486	2157	2158	2158	2157	2208	2209	2209	2208
Cost		1.44	1.44	1.875		1.2	1.68	0		3.42	2.98	3.42		1.58	1.68	2.89		3.01	2.38	3.01
ΔQ_p		627	627	426		393	193	893		179	589	179		2001	1959	1168		571	1030	571
RR		23%	23%	15.6%		16.4%	8%	37.3%		4.9%	16.1%	4.9%		48.1%	47.1%	28%		15.5%	28%	15.5%
RC		51%	51%	34.7%		44%	21.6%	100%		8.2%	27.2%	8.2%		75.2%	73.7%	44%		26.2%	47.3%	26%

Stage	6 IN	6 INDP	6 DEPN	6 STAN	7 IN	7 INDP	7 DEPN	7 STAN	8 IN	8 INDP	8 DEPN	8 STAN	9 IN	9 INDP	9 DEPN	9 STAN	10 IN	10 INDP	10 DEPN	10 STAN
1	1282	1600	2440	1500	1612	1600	2770	1500	1848	1600	2850	1500	937	1600	2095	1500	2207	1600	2850	1500
2	2805	1700	2200	1500	1797	1600	1797	1500	1784	1500	1939	1500	1071	1567	1072	1500	1663	1500	2178	1500
3	1720	1500	2326	1651	1315	1550	1315	1550	1347	1550	1348	1500	1479	1479	1479	1500	1595	1500	1595	1500
4	1080	1500	1080	1500	1618	1600	1619	1500	1553	1550	1553	1500	1964	1965	1965	1500	1697	1500	1698	1506
5	944	1500	945	1500	942	1600	943	1500	958	1550	958	1500	1327	1327	1327	1500	1510	1500	1511	1510
6	874	1550	875	1500	1087	1582	1088	1500	1439	1600	1439	1500	1757	1757	1757	1500	1178	1500	1178	1500
7	2384	1700	2200	1500	1645	1645	1645	1500	2545	1650	2200	1500	1121	1122	1122	1500	1799	1500	1799	1500
8	1416	1500	1601	1500	2072	1900	2072	1500	3631	2950	2000	3450	2489	1900	2100	1500	1046	1500	1047	1500
9	1960	1500	1950	1500	1956	1850	1950	1500	3371	3371	3034	3371	3796	2074	1900	2789	1743	1500	1743	1500
10	3300	2563	1800	2961	1223	1502	1230	1500	2241	2241	2241	2241	1321	1500	1550	1500	2631	2316	1800	2399
11	2744	2744	1940	2744	3828	1650	1600	2943	1139	1500	1500	1500	1814	1635	1650	1635	3744	3744	2262	3744
12	2383	2384	2383	2383	3811	3678	3725	3811	711	1500	1500	1500	735	1500	1500	1500	3578	3578	3578	3578
COST		2.52	2.08	2.84		3.62	3.67	3.76		3.3	2.94	3.38		1.37	1.44	2.59		3.69	3.52	3.69
ΔQ_p		556	860	339		150	103	17		260	597	181		1724	1696	1007		0	166	0
RR		16.8%	26%	10.3%		3.9%	2.7%	0.4%		7.1%	16.4%	5%		45.4%	44.7%	265%		0%	4.4%	0%
RC		30.9%	478%	183%		6.4%	4.4%	0.7%		12.2%	28%	8.5%		75%	73.9%	43.8%		0%	7.4%	0%

Table D3.1 Simulation Result of Jan-Feb Flow

	11				12				13				14				15			
Stage	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN
1	1167	1600	2324	1500	649	1600	1806	1500	62	1220	1220	1220	745	1600	1902	1500	230	1387	1387	1387
2	1504	1800	1505	1500	1042	1249	1043	1349	787	787	787	787	2191	2100	2192	1500	523	524	524	523
3	1827	2000	1827	1500	1218	1218	1218	1218	1117	1117	1117	1117	1775	2000	1775	1500	928	928	928	928
4	918	1174	918	1500	1014	1014	1014	1014	1540	1541	1541	1500	260	430	261	1500	1411	1412	1412	1411
5	1005	1005	1005	1500	1376	1377	1377	1376	1931	1931	1931	1500	648	649	649	779	823	823	823	823
6	1627	1627	1627	1500	929	930	930	929	1417	1417	1417	1500	208	208	208	208	79	80	80	79
7	4251	2000	2200	2149	1874	1874	1874	1500	675	675	675	1063	749	750	750	749	1070	1070	1070	1070
8	3042	2985	2785	3042	3415	1900	2100	1500	1039	1040	1040	1039	666	666	666	666	377	378	378	377
9	2388	2388	2388	2388	758	1500	1900	1500	1564	1564	1564	1500	1610	1611	1611	1500	232	232	232	232
10	2023	2023	2023	2023	2872	1500	1800	2105	2378	1750	1800	1500	2808	1750	1800	1500	2392	1750	1800	1500
11	2380	2382	2382	2380	2063	1895	1700	2063	2538	1500	1650	1500	2351	1500	1700	1500	1409	1500	1650	1500
12	578	1500	1500	1500	3034	3034	2329	3034	2642	1995	1795	2308	2104	1700	1500	2060	1226	1500	1500	1500
COST		2.88	2.59	2.95		2.94	1.92	2.94		1.19	1.03	1.89		1.44	1.66	1.34		0.6	0.72	0
ΔQ_p		1356	1466	1209		381	1086	381		647	711	334		708	616	748		642	592	892
RR		31.9%	345%	284%		11.1%	318%	11.1%		245%	269%	126%		252%	219%	266%		268%	247%	372%
RC		493%	533%	44%		199%	567%	199%		567%	623%	292%		54%	471%	572%		72%	664%	100%

	16				17				18				19				20			
Stage	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN
1	1110	1600	2267	1500	1625	1600	2782	1500	988	1600	2145	1500	527	1600	1684	1500	865	1600	2022	1500
2	1555	1850	1556	1500	1864	1600	1864	1500	542	1088	543	1188	427	512	428	612	1496	1919	1497	1500
3	2055	2000	2055	1500	1073	1550	1074	1500	1439	1440	1440	1439	1070	1071	1071	1070	2013	2013	2013	1500
4	1362	1791	1363	1500	1521	1650	1521	1500	1351	1351	1351	1351	307	307	307	307	1722	1722	1722	1500
5	1399	1400	1400	1500	2259	1650	2200	1500	2127	2100	2128	1500	517	517	517	517	1909	1910	1910	1500
6	415	415	415	1500	1933	1500	1993	1619	1173	1210	1173	1500	183	184	184	183	1267	1268	1268	1500
7	1095	1096	1096	1152	2372	1942	2200	2372	2098	2000	2099	1500	1818	1818	1818	1500	2899	2000	2200	2017
8	2727	1900	2100	1500	2073	2073	2000	2073	3101	1900	2100	1685	2964	1900	2100	1500	2978	1563	2000	2978
9	4217	2730	2530	3130	2621	2621	1900	2621	1458	1500	1900	1500	3131	1881	1900	2599	1327	1500	1900	1500
10	2051	2051	2051	2051	3159	3159	1850	3159	1581	1500	1800	1540	2037	2038	1819	2037	533	1500	1639	1500
11	2639	2636	2636	2636	523	1500	1500	1500	2806	1833	1650	2806	2730	2730	2730	2730	396	1500	397	1500
12	902	1500	1500	1500	904	1500	1500	1500	1519	1519	1500	1519	3072	3072	3072	3072	694	767	694	766
COST		2.5	2.36	3.04		3.07	2.58	3.07		1.44	1.55	2.62		2.98	2.98	2.98		1.23	1.68	2.87
ΔQ_p		1487	1581	1087		0	377	0		1001	956	295		59	59	59		978	778	0
RR		352%	375%	258%		0	119%	0		323%	308%	9.5%		1.9%	1.9%	1.9%		328%	261%	0
RC		547%	582%	40%		0	227%	0		625%	597%	184%		3.6%	3.6%	3.6%		662%	525%	0

	21				22				23				24				25			
Stage	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN
1	1939	1600	2850	1500	265	1422	1422	1422	1296	1600	2454	1500	633	1600	1796	1500	641	1600	1798	1500
2	788	1550	1034	1500	1147	1148	1148	1147	444	1298	444	1398	1342	1538	1342	1500	1328	1526	1328	1500
3	1335	1750	1336	1500	1994	1995	1995	1500	57	58	58	57	2016	2017	2017	1500	495	496	496	622
4	1332	1652	1332	1500	1803	1804	1804	1500	1672	1672	1672	1500	1584	1584	1584	1500	849	850	850	849
5	2050	2051	2051	1500	600	600	600	1398	1409	1410	1410	1500	1099	1099	1099	1500	399	399	399	399
6	1977	1977	1977	1500	1357	1357	1357	1357	1625	1626	1626	1500	513	514	514	852	1377	1378	1378	1377
7	1728	1728	1728	1500	2159	2000	2160	1500	2124	2000	2124	1500	1846	1846	1846	1500	1886	1886	1886	1500
8	1794	1795	1795	1500	1750	1900	1751	1500	3468	1900	2100	1986	2447	1900	2100	1500	1900	1900	1900	1500
9	1266	1266	1266	1500	1303	1314	1303	1500	3756	3135	2811	3756	1609	1600	1900	1500	2311	1850	1950	1500
10	1908	1750	1800	1500	1850	1750	1800	1500	3502	3502	3502	3502	709	1266	766	1500	2865	1550	1800	2150
11	4041	1885	1835	4005	2339	1650	1600	1500	1517	1517	1517	1517	2767	1650	1600	1500	2872	2336	1985	2872
12	2833	2833	2833	2833	3175	1651	1651	2764	661	1500	1500	1500	2409	1500	1500	1973	2120	2120	2120	2120
COST		2.65	2.68	3.94		1.2	1.58	2.55		3.44	3.44	3.71		1.24	1.44	1.14		1.93	1.49	2.71
ΔQ_p		1208	1191	36		1175	1115	411		253	253	0		748	667	794		536	752	0
RR		299%	295%	0.9%		37%	351%	129%		6.7%	6.7%	0		27%	241%	287%		187%	262%	0
RC		475%	469%	1.4%		701%	666%	245%		112%	112%	0		59%	526%	627%		391%	548%	0

	26				27				28				29				30			
Stage	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN
1	1180	1600	2338	1500	1185	1600	2342	1500	1445	1600	2602	1500	1065	1600	2222	1500	1139	1600	2296	1500
2	1322	1800	1322	1500	884	1626	884	1500	1053	1650	1054	1500	831	1454	832	1500	2478	1800	2200	1500
3	780	1040	780	1440	2491	2150	2200	1500	1374	1781	1375	1500	1131	1131	1131	1185	2315	1500	2450	1775
4	1149	1150	1150	1149	1539	1881	1831	1500	829	829	829	1360	1836	1837	1837	1500	1912	1738	2057	1912
5	1731	1731	1731	1500	1033	1033	1033	1500	676	676	676	676	1839	1839	1839	1500	2941	2941	2200	2941
6	1320	1321	1321	1500	1464	1465	1465	1500	1005	1006	1006	1005	2210	2050	2200	1500	2591	2592	2900	2591
7	1793	1793	1793	1500	3499	2000	2200	1940	315	315	315	315	2182	2000	2193	1500	2732	2732	2100	2732
8	2349	1900	2100	1500	3546	2730	2530	3546	1441	1442	1442	1441	3202	1850	2100	2955	1467	1500	1950	1500
9	2432	1650	1900	1500	2559	2559	2559	2559	1954	1850	1950	1500	2430	1811	1900	2430	1960	1928	1900	1928
10	2358	1500	1800	2171	1667	1668	1668	1667	1974	1750	1800	1500	2656	2656	1974	2656	1810	1810	1800	1810
11	1433	1500	1700	1500	2739	2740	2740	2739	3410	1550	1600	2015	2509	2511	2510	2509	1472	1500	1650	1500
12	1118	1500	1500	1500	2265	2265	2265	2265	2360	2227	2027	2360	2031	2031	2031	2031	2876	2849	1500	2849
COST		0.96	1.93	1.61		2.52	2.52	3.48		1.74	2.32	1.96		2.39	2.18	2.83		2.81	2.75	2.81
ΔQ_p		532	94	261		806	806	0		1174	799	1041		546	692	274		0	41	0
RR		219%	3.9%	107%		227%	227%	0		345%	235%	306%		171%	216%	7.7%		0	1.4%	0
RC		571%	101%	28%		394%	394%	0		618%	42%	548%		321%	407%	161%		0	2.8%	0

	31				32				33				34				35			
Stage	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN
1	825	1600	1983	1500	1128	1600	2286	1500	1327	1600	2484	1500	768	1600	1925	1500	996	1600	2154	1500
2	1350	1733	1350	1500	1008	1694	1008	1500	962	1700	963	1500	411	737	412	837	484	1038	484	1138
3	1087	1088	1088	1421	1305	1306	1306	1500	1624	1771	1624	1500	785	785	785	785	1317	1317	1317	1316
4	1443	1444	1444	1443	939	939	939	1039	1027	1028	1028	1500	912	913	913	912	2044	2044	2044	1500
5	1013	1013	1013	1013	822	822	822	822	419	419	419	516	1034	1034	1034	1033	1065	1065	1065	1500
6	1053	1054	1054	1053	1213	1214	1214	1213	347	348	348	347	1290	1290	1290	1290	1211	1212	1212	1320
7	2561	2000	2200	1500	2995	2000	2200	1500	814	814	814	814	1244	1245	1245	1244	2249	2000	2200	1500
8	2189	1650	2000	1500	1026	1550	1821	1500	2055	1900	2055	1500	1048	1048	1048	1048	2672	1850	2100	1500
9	2720	1506	1900	2156	2044	1600	1950	1500	2332	1850	1950	1500	3431	1850	1950	1500	3006	1764	1900	2613
10	2505	2506	1800	2506	2669	1500	1800	1920	3819	2143	1888	2892	1409	1500	1850	1500	3637	3637	3051	3637
11	1390	1500	1650	1500	3224	2995	1874	3224	3275	3275	3275	3275	2414	1591	1700	1941	1342	1500	1500	1500
12	2434	2324	1936	2324	1206	1500	1500	1500	2163	2164	2164	2163	2599	2600	2041	2599	2445	2289	2289	2288
COST		2.17	1.86	2.17		2.89	1.85	3.14		3.19	3.19	3.19		2.31	1.3	2.31		3.58	2.95	3.58
ΔQ_p		214	520	214		229	938	0		544	544	544		831	1390	832		0	586	0
RR		7.9%	191%	7.9%		7.1%	291%	0		142%	142%	142%		242%	405%	242%		0	161%	0
RC		17.5%	426%	17.5%		133%	544%	0		235%	235%	235%		43%	72%	43%		0	274%	0

	36				37				38				39				40			
Stage	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN
1	856	1600	2013	1500	1272	1600	2429	1500	1483	1600	2640	1500	807	1600	1965	1500	873	1600	2030	1500
2	373	787	374	887	1673	1700	1674	1500	1151	1650	1152	1500	942	1308	943	1408	1484	1914	1484	1500
3	866	866	866	866	1576	1750	1576	1500	1000	1542	1000	1500	826	826	826	826	1312	1312	1312	1500
4	1195	1195	1195	1195	399	1029	400	1500	835	835	836	1128	1026	1026	1026	1026	1575	1576	1576	1500
5	1380	1381	1381	1380	864	864	864	934	735	735	735	735	1245	1245	1245	1245	907	907	907	1309
6	536	536	536	536	768	768	768	767	338	339	339	338	1256	1256	1256	1256	1663	1663	1663	1500
7	1535	1535	1535	1500	1343	1343	1343	1343	1074	1074	1074	1074	71	72	72	72	2519	2000	2200	1500
8	1942	1900	1942	1500	2025	1900	2025	1500	3366	1900	2100	1500	123	124	124	123	2392	1650	2000	1500
9	1528	1570	1528	1500	2702	1850	1950	1500	1901	1500	1900	1500	2749	1850	1950	1500	2636	1582	1900	2395
10	3514	1750	1800	1706	2112	1500	1800	1529	2086	1639	1850	2039	3097	1681	1800	2031	3203	3203	2335	3203
11	2496	1947	1987	2496	1121	1500	1700	1500	2576	2577	1766	2576	2516	2517	2298	2516	2428	2429	2429	2428
12	3369	3369	3369	3369	2378	1500	1500	1999	1962	1962	1962	1962	1513	1513	1513	1513	786	1500	1500	1500
COST		3.29	3.29	3.29		0,96	2.06	1.2		2.28	2.37	2.28		2.19	1.87	2.19		3.12	2.06	3.12
ΔQ_p		145	145	145		802	273	703		789	726	790		580	799	581		0	774	0
RR		4.1%	4.1%	4.1%		297%	101%	26%		234%	216%	234%		187%	258%	187%		0	241%	0
RC		7.2%	7.2%	7.2%		667%	227%	585%		423%	389%	423%		363%	500%	363%		0	454%	0

	41				42				43				44				45			
Stage	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN
1	1117	1600	2275	1500	658	1600	1815	1500	1638	1600	2796	1500	908	1600	2066	1500	1669	1600	2826	1500
2	1157	1832	1157	1500	1487	1702	1487	1500	411	1600	411	1500	1399	1865	1399	1500	1510	1600	1511	1500
3	666	667	667	1099	400	400	401	703	2170	2150	2171	1500	845	846	846	1311	1601	1600	1601	1500
4	739	740	740	739	400	400	400	400	1325	1354	1326	1500	1020	1020	1020	1020	1165	1600	1165	1500
5	1021	1021	1021	1021	1457	1457	1457	1457	2099	2100	2100	1500	479	479	479	479	2345	1700	2200	1500
6	932	932	932	932	3909	2050	2200	1594	935	935	935	1500	1610	1610	1610	1500	1012	1550	1158	1500
7	2668	2000	2200	1500	5077	4622	4472	5077	874	874	874	1500	1184	1184	1184	1294	1595	1600	1595	1500
8	3886	2239	2039	2739	4730	4730	4730	4730	1749	1750	1750	1500	2828	1900	2100	1500	1550	1600	1551	1500
9	1577	1577	1577	1577	2720	2720	2721	2720	1705	1705	1705	1500	2126	1500	1900	1500	2438	1550	1950	1730
10	1265	1500	1500	1500	1417	1500	1500	1500	671	671	671	1240	3687	2928	2328	3327	2485	1816	1800	2485
11	1538	1500	1650	1500	2496	2413	2413	2413	3442	1650	1600	1500	1806	1806	1806	1806	2424	2424	1700	2424
12	1911	1714	1564	1714	1918	1919	1919	1919	1975	1500	1503	1603	2633	2633	2634	2633	3235	3235	2819	3235
COST		1.77	1.84	2.52		4.55	4.55	4.78		1.56	2.6	0.25		2.79	2.36	3.25		3.15	2.64	3.15
ΔQ_p		1647	1611	1147		347	347	0		1292	646	1840		759	1053	360		0	409	0
RR		424%	415%	295%		6.8%	6.8%	0		375%	188%	535%		206%	286%	9.8%		0	126%	0
RC		69%	675%	481%		9.7%	9.7%	0		665%	333%	947%		347%	481%	165%		0	236%	0

	46				47				48				49				50			
Stage	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN
1	492	1600	1649	1500	814	1600	1971	1500	378	1536	1536	1500	258	1408	1408	1408	629	1600	1786	1500
2	1595	1645	1595	1500	1294	1666	1295	1500	613	613	613	649	843	843	843	843	503	689	503	789
3	1105	1105	1105	1350	1809	1810	1810	1500	1566	1566	1566	1500	1146	1146	1146	1146	1761	1761	1761	1500
4	988	988	988	988	1646	1646	1646	1500	937	937	937	1004	857	857	857	857	1099	1099	1099	1360
5	720	720	720	720	916	916	916	1500	892	892	892	892	1199	1199	1200	1199	864	864	864	864
6	1285	1285	1285	1285	1298	1298	1298	1436	1026	1026	1026	1026	1477	1477	1477	1477	634	634	634	634
7	2453	2000	2200	1500	2320	2000	2200	1500	2305	2000	2200	1500	2600	2000	2200	1500	972	972	972	972
8	3127	1700	2000	1766	2508	1850	2000	1500	2279	1850	2100	1500	1172	1650	1573	1500	3000	1900	2100	1500
9	1348	1500	1900	1500	2525	1500	1900	2039	2309	1550	1900	1580	2007	1850	1950	1500	3115	1901	1900	2301
10	2625	2040	1800	2473	198	1500	1453	1500	2733	1914	1800	2733	2444	1650	1800	1500	2270	2270	2072	2270
11	2722	2722	2063	2722	1793	1500	1600	1500	2293	2293	1700	2293	2436	1500	1700	2346	3202	3202	3202	3202
12	2142	2142	2142	2142	2392	1500	1500	1500	2399	2399	2307	2399	1079	1500	1500	1500	2302	2302	2302	2302
COST		2.49	1.68	2.49		1.2	1.68	1.29		2.02	1.88	2.51		1.2	1.68	1.94		3.11	3.11	3.11
ΔQ_p		405	927	405		525	325	486		334	426	0		600	400	254		0	0	0
RR		13%	296%	13%		208%	129%	192%		122%	156%	0		231%	154%	9.8%		0	0	0
RC		249%	57%	249%		512%	317%	472%		271%	345%	0		545%	364%	231%		0	0	0

	1				2				3				4				5			
Stage	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN
1	390	1547	1547	1500	589	1600	1747	1500	2461	1600	3618	1500	1840	1600	2997	1500	1910	1600	3068	1500
2	1620	1621	1621	1500	2094	2200	2094	1500	2271	1974	2271	2074	1209	1550	1210	1500	752	1550	753	1500
3	339	339	339	507	237	278	237	1078	2242	2242	2242	2242	706	1600	706	1500	1509	1800	1509	1500
4	2503	2100	2504	1500	1591	1591	1591	1500	812	1500	812	1500	573	737	574	987	2307	1950	2307	1500
5	1106	1511	1107	1500	1316	1317	1317	1407	2342	1655	2342	1655	4098	2100	4098	1783	927	1665	928	1500
6	2924	2050	2924	1500	2232	2050	2282	1500	674	1500	675	1500	3631	3315	3631	3631	2706	2050	2707	1500
7	1644	1600	1645	1500	3045	1900	3045	1513	3112	2288	3113	2287	5542	5542	3400	5542	3237	1700	3237	3194
8	2678	1550	2678	2543	2249	1500	2249	2248	4455	4455	2850	4455	2843	2843	2671	2843	4604	4484	2850	4604
9	3112	2845	2500	3112	2949	2761	2500	2948	1849	1849	2200	1849	2733	2733	2733	2733	3930	3930	3371	3930
10	3362	3362	2150	3362	2290	2290	2150	2290	1980	1981	2150	1980	2515	2515	2515	2515	2354	2354	2354	2354
11	2869	2869	2379	2869	1001	1500	1591	1500	2666	2666	1900	2666	2064	2064	2064	2064	2489	2489	2489	2489
12	1044	1500	1500	1500	1436	1500	1436	1500	4219	4219	3755	4219	2622	2622	2622	2622	2711	2711	2711	2711
COST		3.29	2.79	3.29		2.55	2.95	2.82		4.36	3.71	4.36		5.05	4.03	5.05		4.39	3.29	4.47
ΔQ_p		0	438	0		284	0	97		0	700	0		0	1425	0		120	1233	0
RR		0	13%	0		9.3%	0	3.2%		0	157%	0		0	258%	0		2.6%	268%	0
RC		0	235%	0		184%	0	6.3%		0	237%	0		0	353%	0		3.9%	398%	0

	6				7				8				9				10			
Stage	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN
1	2410	1600	3567	1500	2653	1600	3810	1500	1211	1600	2368	1500	2891	1734	4048	1734	2054	1600	3212	1500
2	5242	4895	4150	5995	3119	3014	3119	3114	1828	1800	1829	1500	3036	3036	3036	3036	2685	1982	2685	2082
3	3303	3303	3450	3303	2488	2489	2489	2488	2707	1750	2707	1590	3502	3503	3503	3502	2859	2860	2859	2859
4	2083	2083	3030	2083	2905	2905	2905	2905	3668	3108	3668	3668	1788	1788	1788	1788	3241	3241	3242	3241
5	1789	1790	1790	1789	1821	1821	1821	1821	2526	2527	2527	2526	1896	1896	1896	1895	2056	2056	2056	2056
6	1640	1640	1640	1640	2691	2692	2692	2691	3308	3308	3308	3308	3038	3038	3038	3038	3046	3046	3047	3046
7	3055	3055	3055	3055	3439	3439	3400	3439	1860	1860	1860	1860	5544	5541	3400	5541	2127	2127	2127	2127
8	1836	1836	1836	1836	4564	4564	2850	4564	3165	3165	2850	3165	3993	3993	3819	3993	1932	1932	1932	1932
9	2397	2397	2397	2397	4250	4250	3689	4250	4671	4671	2673	4671	3098	3098	3098	3097	1702	1703	1703	1703
10	3991	3991	2150	3991	2887	2887	2887	2887	1781	1781	1782	1781	2574	2575	2575	2574	380	1500	380	1500
11	3409	3409	2937	3409	1498	1500	1500	1500	2242	2242	2242	2242	2707	2707	2707	2707	1591	1500	1591	1500
12	2979	2979	2979	2979	889	1500	1500	1500	934	1500	1500	1500	584	1500	1500	1500	3184	2157	1500	2156
COST		4.66	4.07	4.73		4.44	3.76	4.44		4.51	3.61	4.51		5.05	3.98	5.05		3.16	3.16	3.16
ΔQ_p		347	1092	247		0	754	0		0	1003	0		0	1493	0		0	0	0
RR		6.6%	208%	4.7%		0	165%	0		0	215%	0		0	269%	0		0	0	0
RC		9.3%	292%	6.6%		0	238%	0		0	316%	0		0	369%	0		0	0	0

Table D3.2 Simulation Result of March-April Flow

	11				12				13				14				15			
Stage	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN
1	1056	1600	2214	1500	1067	1600	2225	1500	1995	1600	3153	1500	2342	1600	2499	1500	2967	1810	4125	1810
2	1878	1850	1878	1500	3104	1850	3104	1514	1871	1500	1871	1500	2177	1762	2178	1862	3186	3186	3186	3186
3	2252	1800	2252	1500	3347	2912	3347	3347	842	1500	842	1500	1321	1500	1821	1500	3947	3947	3947	3947
4	1899	1600	1900	1500	1089	1500	1089	1500	1722	1600	1722	1500	2740	2562	2740	2561	2631	2631	2631	2631
5	2570	1651	2570	2501	2560	2150	2561	2150	3693	2767	3693	2967	2112	2112	2412	2112	2657	2657	2657	2657
6	1757	1758	1757	1757	2490	2490	2490	2490	2085	2085	2085	2085	1720	1720	1920	1720	816	1500	816	1500
7	2472	2472	2473	2472	1272	1500	1272	1500	4082	4082	3400	4082	3457	3457	3400	3457	1377	1500	1378	1500
8	4197	4197	2850	4197	1452	1500	1452	1500	3694	3694	2600	3694	882	1500	941	1500	3260	2456	2850	2455
9	1078	1500	2250	1500	2826	2551	2500	2551	1776	1776	2150	1776	3167	2550	2500	2547	5109	5109	3205	5109
10	3455	3034	2150	3033	2025	2025	2150	2025	969	1500	2100	1500	3252	3252	2150	3252	2651	2651	2651	2651
11	2561	2561	1950	2561	2865	2866	1850	2865	1759	1500	1900	1500	2013	2013	1900	2013	3265	3265	3265	3265
12	3697	3697	3475	3697	2082	2082	1500	2082	3453	3183	1500	3182	2468	2468	2073	2468	1201	1500	1500	1500
COST		4.12	3.41	4.12		2.77	3.27	3.27		4.01	3.64	4.01		3.39	3.33	3.39		4.81	4.05	4.81
ΔQ_p		0	722	0		481	0	0		0	682	0		0	57	0		0	982	0
RR		0	172%	0		144%	0	0		0	167%	0		0	1.6%	0		0	192%	0
RC		0	268%	0		26%	0	0		0	264%	0		0	2.9%	0		0	272%	0

	16				17				18				19				20			
Stage	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN
1	390	1547	1547	1500	1179	1600	2337	1500	1523	1600	1681	1500	2226	1600	3384	1500	2409	1600	3566	1500
2	1620	1621	1621	1500	2279	1800	2279	1500	1977	1600	1977	1500	851	1500	852	1500	5241	4893	4150	4993
3	339	339	339	507	295	1511	295	1500	2149	1550	2150	1500	1519	1600	1519	1500	3303	3303	3450	3303
4	2503	2100	2504	1500	1609	1609	1609	1500	783	1500	783	1500	2310	1600	2310	1500	2083	2083	3029	2083
5	1106	1511	1107	1500	1322	1323	1323	1500	2333	1550	2334	1610	928	1500	929	1500	1789	1789	1790	1789
6	2924	2050	2924	1500	2283	2050	2284	1500	671	1500	671	1500	2707	1587	2707	1887	1640	1640	1640	1640
7	1644	1600	1645	1500	3046	1900	3046	1858	3111	2093	3111	2283	3237	3237	3237	3237	3055	3055	3055	3055
8	2678	1550	2678	2543	2249	1500	2250	2249	4454	4454	2850	4454	4604	4604	2850	4604	1836	1836	1836	1836
9	3112	2845	2500	3112	2949	2764	2500	2949	1849	1849	2200	1848	3930	3930	3370	3930	2397	2397	2397	2397
10	3362	3362	2150	3362	2290	2290	2150	2290	1980	1980	2150	1980	2354	2354	2354	2354	3991	3991	2150	3991
11	2869	2869	2379	2869	1001	1500	1591	1500	2666	2666	1900	2666	2489	2489	2490	2489	3409	3409	2937	3409
12	1044	1500	1500	1500	1436	1500	1436	1500	4219	4219	3755	4219	2711	2711	2711	2711	2979	2979	2979	2979
COST		3.29	2.8	3.29		2.55	2.95	2.83		4.36	3.71	4.36		4.47	3.10	4.47		4.66	4.07	4.73
ΔQ_p		0	438	0		282	0	97		0	699	0		0	1220	0		348	1091	248
RR		0	13%	0		9.3%	0	3.2%		0	157%	0		0	265%	0		6.6%	208%	4.7%
RC		0	235%	0		182%	0	6.3%		0	237%	0		0	393%	0		9.3%	292%	6.6%

21

Stage	IN	INDP	DEPN	STAN
1	1914	1600	3072	1500
2	2901	2058	2901	2158
3	1629	1629	1629	1629
4	2541	2542	2542	2541
5	2498	2498	2498	2498
6	3292	3292	3292	3292
7	3370	3370	3370	3370
8	3154	3154	2850	3154
9	765	1500	1069	1500
10	1548	1500	1549	1500
11	3318	2632	1850	2632
12	628	1500	1500	1500
COST		3.29	3.29	3.29
ΔQ_p		0	0	0
RR		0	0	0
RC		0	0	0

22

Stage	IN	INDP	DEPN	STAN
1	1777	1600	2935	1500
2	2006	1600	2006	1500
3	2763	2189	2763	2389
4	3685	3685	5685	3685
5	2532	2532	2532	2532
6	3309	3310	3310	3309
7	1861	1861	1861	1861
8	3166	3166	2850	3166
9	4672	4672	2673	4672
10	1781	1782	1782	1781
11	2242	2242	2242	2242
12	934	1500	1500	1500
COST		4.51	3.63	4.51
ΔQ_p		0	986	0
RR		0	211%	0
RC		0	311%	0

23

Stage	IN	INDP	DEPN	STAN
1	1244	1600	2402	1500
2	280	1032	280	1182
3	232	233	233	232
4	1685	1635	1685	1500
5	1071	1071	1071	1257
6	2209	2050	2209	1500
7	683	842	683	1392
8	2933	1900	2850	1500
9	4992	3712	2762	4111
10	3112	3112	3112	3112
11	3490	3490	3490	3490
12	3827	3827	3827	3827
COST		3.77	3.77	3.77
ΔQ_p		1165	1165	1165
RR		233%	233%	233%
RC		334%	334%	334%

24

Stage	IN	INDP	DEPN	STAN
1	2035	1600	3139	1500
2	1884	1500	1884	1500
3	846	1500	846	1500
4	1723	1550	1724	1500
5	3693	2875	3693	3025
6	2085	2085	2086	2086
7	4082	4082	3400	4082
8	3694	3694	2600	3694
9	1776	1777	2150	1776
10	969	1500	2100	1500
11	1759	1500	1900	1500
12	3453	3183	1500	3182
COST		4.01	3.64	4.01
ΔQ_p		0	389	0
RR		0	9.5%	0
RC		0	151%	0

25

Stage	IN	INDP	DEPN	STAN
1	1913	1600	3070	1500
2	2042	1550	2042	1500
3	1279	1500	1279	1500
4	2727	2154	2727	2304
5	2107	2108	2108	2107
6	1718	1718	1718	1718
7	3457	3457	3400	3457
8	882	1500	939	1500
9	3167	2549	2500	2549
10	3252	3252	2150	3252
11	2013	2013	1900	2013
12	2468	2468	2037	2468
COST		3.39	3.33	3.39
ΔQ_p		0	57	0
RR		0	1.6%	0
RC		0	2.9%	0

26

Stage	IN	INDP	DEPN	STAN
1	2080	1600	3240	1500
2	2908	2234	2908	2334
3	3860	3860	3860	3860
4	2604	2604	2604	2604
5	2648	2648	2648	2648
6	813	1500	813	1500
7	1376	1500	1377	1500
8	3260	2451	2850	2451
9	5109	5109	3205	5109
10	2651	2651	2651	2651
11	3265	3265	3265	3265
12	1201	1500	1500	1500
COST		4.8	3.8	4.8
ΔQ_p		0	1249	0
RR		0	244%	0
RC		0	346%	0

27

Stage	IN	INDP	DEPN	STAN
1	1129	1600	2286	1500
2	1589	1850	1589	1500
3	1592	2000	1593	1500
4	1868	1887	1868	1500
5	2526	2100	2526	1548
6	1739	1850	1739	1739
7	3236	1900	3236	3236
8	2183	1511	2183	2183
9	2255	2256	2256	2255
10	340	1500	341	1500
11	1101	1500	1101	1500
12	2743	1500	1500	1500
COST		1.81	3.15	3.15
ΔQ_p		980	0	0
RR		303%	0	0
RC		565%	0	0

28

Stage	IN	INDP	DEPN	STAN
1	982	1600	2139	1500
2	778	1317	778	1417
3	1962	1962	1962	1500
4	572	572	572	1035
5	934	934	934	934
6	314	314	314	314
7	2131	2000	2131	1500
8	3542	1900	2850	1859
9	3829	3290	2350	3829
10	2573	2573	2431	2573
11	3351	3351	3351	3351
12	3782	3782	3782	3782
COST		3.73	3.73	3.77
ΔQ_p		47	47	0
RR		1.2%	1.2%	0
RC		2%	2%	0

29

Stage	IN	INDP	DEPN	STAN
1	1565	1600	2722	1500
2	2767	1600	2767	1675
3	3767	3742	3768	3767
4	3266	3266	3266	3266
5	3612	3612	3612	3612
6	2428	2428	2429	2428
7	3826	3826	3400	3826
8	3793	3793	2650	3793
9	1779	1780	2200	1779
10	712	1500	1861	1500
11	450	1500	450	1500
12	672	1149	672	1149
COST		3.77	3.72	3.77
ΔQ_p		0	58	0
RR		0	1.5%	0
RC		0	2.5%	0

30

Stage	IN	INDP	DEPN	STAN
1	985	1600	2143	1500
2	2456	1900	2456	1500
3	567	1600	568	1500
4	2658	2100	2658	1500
5	1773	1750	1774	1500
6	1632	1750	1632	1500
7	2631	1700	2631	2547
8	3097	2245	2850	3097
9	3390	3390	2400	3390
10	622	1500	1771	1500
11	1154	1500	1154	1500
12	2357	1500	1500	1500
COST		3.22	2.68	3.22
ΔQ_p		0	450	0
RR		0	136%	0
RC		0	25%	0

	31				32				33				34				35			
Stage	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN
1	2392	1600	3549	1500	1920	1600	3077	1500	2184	1600	3141	1500	2176	1600	3333	1500	2723	1600	1881	1500
2	842	1500	843	1500	1158	1550	1159	1500	2469	1895	2469	1995	1650	1500	1650	1500	1999	1965	1999	1999
3	91	1383	91	1483	3243	2013	3241	2163	1478	1500	1478	1500	4631	4201	4150	4301	2576	2577	2577	2576
4	3061	2100	3061	1500	2078	2078	2078	2078	2140	2118	2140	2118	2943	2943	3425	2943	1574	1574	1574	1574
5	2645	1600	2645	1892	1629	1630	1630	1629	3232	3232	3232	3232	1980	1980	1981	1980	1265	1500	1265	1500
6	3055	2748	3055	3055	1186	1500	1187	1500	2502	2502	2502	2502	2748	2748	2743	2748	1861	1626	1861	1626
7	3007	3007	3007	3007	1289	1500	1289	1500	2515	2515	2515	2515	4587	4587	3400	4587	625	1500	625	1500
8	4373	4373	2850	4373	3602	3079	2850	3078	2980	2980	2850	2980	4526	4526	3399	4526	1754	1500	1754	1500
9	4701	4701	3911	4701	3821	3821	2350	3821	3030	3030	2450	3030	3304	3304	3304	3304	2345	1725	2345	1725
10	4410	4410	4410	4410	2853	2853	2762	2853	2927	2927	2150	2927	2162	2162	2162	2162	2393	2394	2150	2393
11	2033	2033	2033	2033	3929	3929	3929	3929	1814	1814	1950	1814	3352	3352	3352	3352	4104	4104	2033	4104
12	887	1500	1500	1500	2900	2900	2900	2900	1378	1500	1500	1500	2817	2817	2817	2817	2955	2955	2955	2955
Cost		4.53	4.32	4.53		3.87	3.87	3.87		3.15	3.15	3.15		4.46	4.07	4.46		4.03	3.82	4.03
ΔQ_p		0	291	0		0	0	0		0	0	0		44	481	44		0	223	0
RR		0	6.2%	0		0	0	0		0	0	0		1%	104%	1%		0	5.4%	0
RC		0	9.1%	0		0	0	0		0	0	0		1.4%	154%	1.4%		0	8.6%	0

	36				37				38				39				40			
Stage	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN
1	569	1600	1726	1500	2236	1600	3393	1500	1590	1600	2747	1500	2112	1600	3270	1500	1754	1600	2911	1500
2	641	768	642	868	1563	1500	1564	1500	2523	1600	2524	1500	1891	1500	1891	1500	1571	1600	1571	1500
3	1210	1211	1211	1210	368	1500	368	1500	2050	1806	2050	2006	2438	2185	2438	2285	2280	1550	2280	1500
4	1806	1806	1806	1500	2696	1750	2697	1500	2703	2703	2703	2703	1771	1771	1771	1771	3553	3251	3553	3501
5	2095	2095	2095	1500	1945	1500	1945	1653	1919	1919	1919	1919	1539	1539	1539	1539	2727	2727	2727	2727
6	1237	1237	1237	1500	3488	3291	3489	3489	1975	1975	1976	1975	2256	2256	2257	2257	2032	2032	2032	2032
7	2933	2000	2933	1500	3992	3992	3400	3992	3328	3328	3328	3328	3892	3892	3400	3892	1062	1500	1062	1500
8	2619	1550	2619	2377	3020	3021	2600	3020	2794	2794	2794	2794	1453	1500	1945	1500	2258	1821	2259	1820
9	2808	2496	2500	2808	1731	1731	2300	1731	3365	3365	2500	3365	2507	2461	2500	2461	1736	1736	1736	1736
10	409	1500	718	1500	1523	1523	1968	1523	3116	3116	2150	3116	3249	3249	2150	3249	2041	2041	2041	2041
11	2009	1500	1850	1500	2959	2959	1850	2959	1777	1777	1900	1777	3947	3947	2740	3947	3023	3023	1850	3023
12	405	1500	566	1500	1934	1934	1500	1934	2949	2950	2343	2949	1592	1592	1592	1592	3116	3116	1947	3115
Cost		2.16	2.8	2.62		3.93	3.42	3.93		3.29	3.25	3.29		3.88	3.33	3.88		3.17	3.49	3.43
ΔQ_p		437	0	125		0	503	0		0	37	0		0	547	0		302	0	52
RR		149%	0	4.3%		0	126%	0		0	1.1%	0		0	138%	0		8.5%	0	1.5%
RC		305%	0	8.1%		0	202%	0		0	2%	0		0	224%	0		147%	0	2.5%

	41				42				43				44				45			
Stage	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN
1	2490	1600	3647	1500	1867	1600	3033	1500	2062	1600	3219	1500	2539	1600	3697	1500	1559	1600	2716	1500
2	1820	1553	1821	1653	914	1550	914	1500	2241	1564	2242	1645	1814	1596	1814	1696	1767	1600	1767	1500
3	3032	3032	3032	3032	2431	1750	2432	1500	1318	1500	1318	1500	3660	3660	3660	3660	1543	1550	1544	1500
4	1952	1953	1953	1952	3815	2980	3816	3380	3175	2994	3175	2993	3687	3687	3687	3687	1907	1550	1907	1500
5	801	1500	801	1500	2044	2044	2044	2044	2672	2672	2672	2672	2231	2231	2231	2231	2322	1642	2322	1942
6	630	1500	630	1500	2281	2282	2282	2281	2011	2011	2011	2011	3349	3349	3349	3349	2355	2355	2355	2355
7	994	1650	995	1500	3015	3015	3015	3015	2549	2549	2550	2549	4221	4221	3400	4221	431	1500	431	1500
8	2422	1900	2422	1500	3374	3374	2850	3374	1591	1591	1591	1591	449	1500	1271	1500	209	1455	209	1455
9	2807	1600	2500	1656	3737	3737	2400	3737	3767	3767	2500	3767	2443	1500	2443	1500	3222	1850	2500	1500
10	4619	4125	2612	4619	4496	4496	4043	4496	2217	2217	2150	2217	2779	2672	2150	2671	3750	2808	2158	3157
11	4077	4077	4077	4077	1803	1803	1803	1803	2730	2730	1950	2730	3202	3202	1900	3202	3126	3126	3126	3126
12	2764	2764	2764	2764	2993	2994	2994	2993	1290	1500	1500	1500	1267	1500	1500	1500	1924	1924	1924	1924
Cost		4.05	4.01	4.48		4.4	3.97	4.4		3.72	3.13	3.72		4.14	3.64	4.14		3.03	3.03	3.07
ΔQ_p		494	542	0		0	453	0		0	548	0		0	524	0		624	624	593
RR		107%	117%	0		0	101%	0		0	145%	0		0	124%	0		166%	166%	158%
RC		158%	174%	0		0	151%	0		0	242%	0		0	193%	0		27.7%	27.7%	264%

	46				47				48				49				50			
Stage	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN
1	1638	1600	2795	1500	1717	1600	2875	1500	948	1000	2105	1500	519	1600	1676	1500	1445	1600	2602	1500
2	2763	1643	2763	1743	2611	1600	2611	1671	2956	1900	2957	1500	1552	1629	1553	1500	563	1566	564	1500
3	2470	2471	2471	2470	1602	1573	1602	1602	2088	1500	2088	1835	2123	2124	2124	1500	3115	2150	3115	1500
4	2959	2959	2959	2959	1904	1905	1905	1904	1859	1694	1859	1859	1605	1605	1605	1500	2050	1650	2051	1517
5	1732	1732	1732	1732	901	1500	901	1500	1352	1500	1352	1500	2231	2100	2231	1500	2572	1624	2573	2572
6	3106	3106	3106	3106	2975	2378	2976	2377	2384	2237	2384	2236	2762	2050	2762	2138	3201	3201	3201	3201
7	3482	3482	3400	3482	1860	1860	1861	1860	3268	3269	3269	3268	3517	2047	3400	3517	3049	3049	3050	3049
8	3117	3117	2850	3117	3542	3542	2850	3542	3933	3933	2850	3933	1643	1643	1761	1643	1300	1500	1300	1500
9	3311	3311	2400	3311	2691	2692	2350	2691	1796	1797	2300	1796	2479	2479	2479	2479	558	1500	558	1500
10	3962	3962	2909	3962	4503	4503	3223	4503	3203	3203	2150	3203	2985	2985	2150	2985	1248	1500	1248	1500
11	3063	3063	3063	3063	2337	2337	2337	2337	3349	3349	2668	3349	2996	2996	1950	2996	2215	1500	1850	1500
12	1068	1500	1500	1500	3238	3238	3238	3238	2676	2676	2676	2676	1387	1500	1500	1500	1083	1500	1449	1500
Cost		3.89	3.33	3.89		4.4	3.15	4.4		3.87	3.19	3.87		2.89	3.33	3.45		3.11	3.11	3.11
ΔQ_p		0	562	0		0	1265	0		0	664	0		521	117	0		0	0	0
RR		0	142%	0		0	281%	0		0	169%	0		148%	3.3%	0		0	0	0
RC		0	228%	0		0	421%	0		0	273%	0		258%	5.8%	0		0	0	0

	1				2				3				4				5			
Stage	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN
1	2647	1600	2350	1500	2032	1600	2350	1500	3161	2004	2360	2004	1891	1600	2350	1500	1336	1600	2350	1500
2	624	1500	2079	1500	183	1500	1022	1500	3577	3577	3231	3577	2120	1550	2300	1500	2982	2561	2250	2661
3	2755	1770	2200	1870	1887	2100	1888	1500	2668	2668	2668	2668	2916	2621	2150	2771	3637	3637	3199	3637
4	1471	1500	2200	1500	1483	1543	1483	1500	3233	3234	3234	3234	3898	3898	2869	3898	1895	1895	1895	1895
5	3626	3598	1750	3598	3651	2100	1750	2079	1927	1927	1927	1927	2703	2703	2703	2703	2002	2002	2002	2002
6	2165	2165	1754	2165	4742	3978	4328	4742	2160	2160	2160	2160	3509	3509	3509	3509	3203	3203	3203	3203
7	1134	1500	1500	1500	1383	1500	1500	1500	1114	1500	1500	1500	1239	1500	1500	1500	2278	2278	2279	2279
8	182	1500	1500	1500	767	1500	1500	1500	885	1500	1500	1500	1137	1500	1500	1500	1236	1500	1500	1500
9	1102	1600	1500	1500	957	1500	1500	1500	722	1500	1500	1500	1395	1500	1500	1500	853	1500	1500	1500
10	765	900	1000	999	581	1500	1500	1500	429	965	965	965	425	1500	1500	1500	695	1500	1500	1500
11	1421	1421	1300	1421	22	28	28	28	1360	1360	1300	1360	632	1146	1146	1146	894	1495	1495	1495
12	1363	1363	1400	1363	507	507	507	507	1293	1293	1300	1293	247	247	247	247	244	244	244	244
Cost		3.54	1.95	3.54		3.91	4.24	4.56		3.51	3.15	3.51		3.84	3.44	3.84		3.58	3.12	3.58
ΔQ_p		28	1276	28		764	414	0		0	343	0		0	390	0		0	434	0
RR		0.8%	352%	0.8%		161%	8.7%	0		0	9.6%	0		0	10%	0		0	119%	0
RC		1.3%	60%	1.3%		236%	128%	0		0	165%	0		0	163%	0		0	203%	0

	6				7				8				9				10			
Stage	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN
1	2996	1839	2350	1839	3452	2295	2350	2295	1257	1600	2350	1500	143	1300	1300	1300	3229	2072	2350	2072
2	3106	3106	2595	3105	3458	3458	3403	3458	2015	1700	2080	1500	1479	1479	1479	1479	3592	3592	3314	3592
3	3117	3117	3117	3117	2774	2775	2775	2774	2389	1600	2200	1504	2158	2150	2158	1500	3401	3402	3402	3401
4	3465	3465	3465	3465	906	1500	1500	1500	2014	1618	2000	2014	3023	2100	2000	1500	2011	2011	2011	2011
5	2207	2207	2207	2207	2546	1953	1953	1953	2713	2713	1750	2713	3833	2450	2542	3700	3187	3188	3188	3187
6	3226	3226	3226	3226	1174	1500	1500	1500	1867	1867	1500	1867	2876	2877	2877	2876	980	1500	1500	1500
7	1268	1500	1500	1500	421	1500	1500	1500	1129	1500	1500	1500	938	1500	1500	1500	880	1500	1500	1500
8	746	1500	1500	1500	243	1154	1154	1154	1336	1500	1500	1500	592	1500	1500	1500	818	1500	1500	1500
9	548	1500	1500	1500	702	702	702	702	257	1500	1257	1500	629	1476	1476	1475	650	1144	1144	1144
10	124	502	502	502	316	316	316	316	1030	1550	1030	1500	896	896	896	896	1178	1178	1178	1178
11	523	523	523	523	807	807	807	807	700	719	700	768	892	892	892	892	901	901	901	901
12	965	965	965	965	165	168	168	168	1050	1050	1050	1050	907	907	907	907	887	887	887	887
Cost		3.39	3.39	3.39		3.39	3.33	3.39		2.48	1.95	2.48		2.72	2.72	3.65		3.53	3.33	3.53
ΔQ_p		0	0	0		0	55	0		0	363	0		956	956	133		0	190	0
RR		0	0	0		0	1.6%	0		0	134%	0		249%	249%	3.5%		0	5.3%	0
RC		0	0	0		0	2.8%	0		0	299%	0		41%	41%	41%		0	8.7%	0

Table D3.3 Simulation Results of May-June Flow

Stage	11 IN	11 INDP	11 DEPN	11 STAN	12 IN	12 INDP	12 DEPN	12 STAN	13 IN	13 INDP	13 DEPN	13 STAN	14 IN	14 INDP	14 DEPN	14 STAN	15 IN	15 INDP	15 DEPN	15 STAN
1	3328	2070	2350	2070	1524	1600	2350	1500	668	1600	1926	1500	2118	1600	2350	1500	2031	1600	2350	1500
2	3543	3543	3264	3543	4295	3062	2400	3162	3108	2200	2500	1500	1980	1500	2250	1500	2159	1500	2300	1533
3	3338	3339	3339	3338	3583	3582	3495	3583	3493	2312	2150	3112	894	1500	1551	1500	1356	1500	2054	1500
4	2120	2120	2120	2120	622	1500	1500	1500	1164	1500	1900	1500	1803	1500	1804	1500	2867	2657	2000	2723
5	865	1500	1500	1500	1254	1550	1850	1500	2691	2365	1850	2365	3383	3424	1750	3524	2234	2234	1850	2234
6	3671	3036	3036	3036	385	1528	1228	1500	2634	2635	2377	2634	2225	2225	2044	2225	1821	1821	1500	1821
7	1804	1804	1804	1304	386	386	386	464	907	1500	1500	1500	1673	1674	1674	1673	1413	1500	1500	1500
8	624	1500	1500	1500	288	288	288	288	602	1500	1500	1500	1149	1500	1500	1500	312	1500	1500	1500
9	586	1500	1500	1500	604	604	604	604	899	1550	1500	1500	471	1500	1500	1500	1048	1500	1349	1500
10	798	1325	1325	1324	1000	1000	1000	1000	579	754	804	803	268	1204	1204	1204	946	1536	946	1500
11	28	28	28	28	801	801	801	801	833	833	833	833	542	542	542	542	537	537	537	537
12	1157	1157	1157	1157	714	714	714	714	572	572	572	572	1030	1030	1030	1030	704	704	704	704
Cost		3.48	3.26	3.48		3.52	3.43	3.52		2.36	2.17	3.02		3.35	1.95	3.46		2.39	1.95	2.49
ΔQ_p		0	204	0		712	800	712		858	993	381		459	1533	359		210	517	144
RR		0	5.8%	0		166%	186%	166%		246%	284%	109%		118%	395%	9.2%		7.3%	18%	5%
RC		0	10%	0		255%	286%	255%		43%	498%	191%		193%	643%	151%		154%	378%	105%

Stage	16 IN	16 INDP	16 DEPN	16 STAN	17 IN	17 INDP	17 DEPN	17 STAN	18 IN	18 INDP	18 DEPN	18 STAN	19 IN	19 INDP	19 DEPN	19 STAN	20 IN	20 INDP	20 DEPN	20 STAN
1	2986	1829	2350	1829	818	1600	1975	1500	2203	1600	2350	1500	1237	1600	2350	1500	1950	1600	2350	1500
2	792	1500	2150	1500	3232	2000	2500	1500	3071	2518	2250	2618	1680	1800	1724	1500	1078	1500	1836	1500
3	2178	1500	2200	1500	3805	3098	2223	3698	4084	4084	3603	4084	1678	1800	1679	1500	2792	1600	2200	1663
4	2110	2081	2000	2081	2030	2031	2031	2030	2783	2783	2783	2783	1996	1800	1966	1500	2682	2646	2000	2682
5	3205	3205	1800	3205	2442	2442	2442	2442	2816	2816	2816	2816	2662	1700	1750	2067	4211	4211	3171	4211
6	2417	2417	2030	2417	2197	2197	2197	2197	882	1500	1500	1500	1837	1500	1500	1837	2414	2414	2414	2414
7	900	1500	1500	1500	1108	1500	1500	1500	629	1500	1500	1500	1350	1500	1500	1500	1338	1500	1500	1500
8	106	1500	1500	1500	1207	1500	1500	1500	1045	1550	1500	1500	708	1500	1500	1500	1258	1500	1500	1500
9	674	996	996	996	1311	1500	1500	1500	1486	1800	1500	1500	670	1500	980	1500	523	1500	1500	1500
10	587	587	587	587	949	1500	1500	1500	657	666	1016	1015	80	330	80	625	561	1496	1496	1496
11	838	838	838	838	887	1500	1500	1500	898	898	898	898	369	369	369	369	991	991	991	991
12	578	578	578	578	1368	1500	1500	1500	279	280	280	279	849	849	849	849	503	503	503	503
Cost		3.12	1.95	3.12		3.01	2.17	3.64		4.01	3.54	4.01		0.72	1.95	1.36		4.13	3.08	4.13
ΔQ_p		0	855	0		707	1305	107		0	481	0		862	312	595		0	1040	0
RR		0	267%	0		186%	343%	2.8%		0	118%	0		324%	117%	224%		0	247%	0
RC		0	501%	0		307%	566%	4.6%		0	187%	0		742%	269%	512%		0	384%	0

	21				22				23				24				25			
Stage	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN
1	2580	1600	2350	1500	1621	1600	2350	1500	2124	1600	2350	1500	545	1600	1703	1500	2420	1600	2350	1500
2	1389	1500	2200	1500	2902	1766	2350	1866	1169	1500	2101	1500	2202	2200	2203	1500	2555	2217	2200	2317
3	2759	2471	2150	2571	3978	3978	2644	3978	1293	1550	1294	1500	3908	2150	2200	2499	585	1500	2100	1500
4	2890	2890	2000	2890	3474	3475	3475	3474	2272	1600	2000	1500	3621	3171	3015	3621	3179	2265	2000	2265
5	2232	2232	1994	2232	3839	3839	3839	3839	3037	2491	1800	2741	1282	1500	1500	1500	1335	1500	1850	1500
6	1529	1529	1530	1529	2599	2599	2599	2599	1889	1889	1500	1889	2667	2449	2449	2449	1067	1500	1500	1500
7	833	1500	1500	1500	1664	1664	1664	1664	1127	1500	1500	1500	1411	1500	1500	1500	353	1500	655	1500
8	411	1500	1500	1500	1207	1500	1500	1500	491	1500	1500	1500	792	1500	1500	1500	392	965	392	964
9	27	587	587	587	477	1500	1500	1500	514	1448	1033	1448	504	1500	1500	1500	241	241	241	241
10	315	315	315	315	193	1193	1193	1193	575	575	575	575	671	1195	1195	1194	1204	1204	1204	1204
11	430	430	430	430	162	162	162	162	548	548	548	548	823	823	823	823	769	769	769	769
12	1019	1019	1019	1019	304	304	304	304	172	172	172	172	1101	1101	1101	1101	724	724	724	724
Cost		2.74	1.95	2.74		3.91	3.78	3.91		2.15	1.95	2.52		3.08	2.92	3.56		1.82	1.95	1.89
ΔQ_p		0	540	0		0	139	0		546	687	296		737	1459	287		914	829	862
RR		0	18.7%	0		0	3.5%	0		18%	22.6%	9.7%		18.8%	37.3%	7.3%		28.8%	26.1%	27.1%
RC		0	38.8%	0		0	5.6%	0		35.5%	44.7%	19.3%		30.6%	60.6%	11.9%		54.4%	49.4%	51.3%

	26				27				28				29				30			
Stage	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN
1	2507	1600	2350	1500	1330	1600	2350	1500	2030	1600	2350	1500	2274	1600	2350	1500	2889	1731	2350	1731
2	896	1500	2200	1500	2632	1700	2450	1500	1227	1500	2065	1500	1740	1500	2250	1500	2128	2128	2150	2128
3	87	1548	98	1500	3983	3489	2200	3789	3382	2382	2200	2482	4878	4632	3132	4732	2725	2725	2100	2725
4	3194	2100	2000	1500	3204	3204	2993	3204	2203	2203	2000	2203	3144	3144	3144	3144	1676	1676	1700	1676
5	2797	1600	1850	2325	2243	2243	2243	2243	1727	1727	1850	1727	2120	2120	2120	2120	1337	1500	1550	1500
6	3235	3212	3062	3235	1055	1500	1500	1500	1253	1500	1500	1500	2909	2910	2910	2910	1955	1793	1704	1792
7	1531	1531	1531	1531	933	1500	1500	1500	666	1500	1500	1500	1943	1943	1943	1943	592	1500	1500	1500
8	1441	1500	1500	1500	939	1500	1500	1500	1169	1500	1352	1500	1413	1500	1500	1500	679	1500	1500	1500
9	1344	1500	1500	1500	566	1309	1309	1309	1097	1550	1098	1500	901	1500	1500	1500	741	1328	1328	1328
10	1188	1500	1500	1500	245	245	245	245	761	1213	761	1262	561	1500	1500	1500	701	701	701	701
11	471	1500	1500	1500	996	996	996	996	1101	1101	1101	1101	956	1500	1500	1500	1192	1192	1192	1192
12	212	974	974	974	825	825	825	825	763	763	763	763	766	916	916	915	785	785	785	785
Cost		3.12	2.97	3.15		3.42	2.89	3.73		1.99	1.95	2.14		4.49	3.05	4.55		2.49	1.95	2.49
ΔQ_p		23	173	0		494	990	194		1000	1032	900		242	1730	142		164	539	164
RR		0.7%	5.3%	0		12.4%	24.9%	4.9%		29.6%	30.5%	26.6%		5%	35.5%	2.9%		5.7%	18.6%	5.7%
RC		1.3%	10%	0		19.9%	39.9%	7.8%		53.1%	54.8%	47.8%		7.2%	51.3%	4.2%		11.8%	38.8%	11.8%

	31				32				33				34				35			
Stage	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN
1	2700	1612	2350	1612	1971	1600	2350	1500	2349	1600	2350	1500	2232	1600	2350	1500	1856	1900	2350	1500
2	3247	3242	2505	3242	1692	1500	2300	1500	1654	1500	2250	1500	2001	1500	2250	1576	1658	1550	2300	1500
3	2594	2594	2594	2594	2823	2229	2200	2329	390	1500	951	1500	2573	2549	2150	2573	2400	1608	2200	1758
4	3158	3158	3158	3158	2619	2619	2000	2619	2818	1650	2000	1554	1879	1880	2000	1879	3748	3748	2000	3748
5	2137	2137	2137	2136	1332	1500	1850	1500	2057	1862	1850	2057	1627	1627	1850	1627	2903	1904	2200	1758
6	1646	1646	1646	1645	2371	2204	1500	2203	3678	3678	2390	3678	2375	2375	1500	2375	2167	2167	2168	2167
7	1184	1500	1500	1500	1104	1500	1500	1500	1901	1902	1902	1901	1639	1640	1500	1639	746	1500	1500	1500
8	691	1500	1500	1500	590	1500	1500	1500	1027	1500	1500	1500	495	1500	1500	1500	826	1500	1500	1500
9	873	1500	1500	1500	707	1500	1170	1500	505	1500	1500	1500	773	1500	1500	1500	541	1446	1446	1446
10	707	1270	1270	1270	606	823	606	823	448	1295	1295	1295	945	1529	1100	1500	612	613	613	612
11	703	703	703	703	8	8	8	8	884	884	884	884	1112	1113	1113	1141	888	888	888	888
12	598	598	598	598	626	626	627	626	531	531	531	531	374	374	374	374	876	876	876	876
Cost		3.16	3.07	3.16		2.34	1.95	2.34		3.62	2.01	3.62		2.24	1.95	2.27		3.69	2.25	3.69
ΔQ_p		0	84	0		204	473	204		0	1288	0		24	223	0		0	1188	0
RR		0	2.6%	0		7.2%	168%	7.2%		0	35%	0		0.9%	8.7%	0		0	317%	0
RC		0	4.8%	0		154%	358%	154%		0	591%	0		2.2%	208%	0		0	528%	0

	36				37				38				39				40			
Stage	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN	IN	INDP	DEPN	STAN
1	2519	1600	2350	1500	1987	1600	2350	1500	447	1600	1604	1500	2592	1600	2350	1500	2184	1600	2350	1500
2	2717	2479	2200	2579	971	1500	1766	1500	1424	1429	1425	1500	1839	1674	2200	1774	2514	1942	2250	2042
3	1957	1957	2050	1956	2550	1650	2200	1500	2660	2150	2200	1500	1733	1733	2150	1733	2000	2000	2100	2000
4	1887	1887	1950	1887	4024	3626	2060	3876	2209	1800	2000	1500	2752	2752	2000	2752	2217	2217	2000	2217
5	2084	2084	1850	2084	2188	2189	2189	2188	976	1650	1646	1500	3258	3258	2319	3258	3338	3338	2397	3338
6	1778	1778	1500	1778	2420	2419	2419	2419	3011	1950	1500	2072	2115	2115	2115	2115	2106	2107	2107	2106
7	1151	1500	1500	1500	1389	1500	1500	1500	1496	1500	1500	1500	787	1500	1500	1500	623	1500	1500	1500
8	864	1500	1500	1500	1101	1500	1500	1500	766	1500	1500	1500	619	1500	1500	1500	785	1500	1500	1500
9	714	1500	1500	1500	1080	1500	1500	1500	620	1191	1395	1599	292	1014	1014	1013	360	1083	1083	1083
10	822	1369	1255	1368	1255	1500	1500	1500	759	760	760	1457	56	56	56	56	637	637	638	637
11	684	684	684	684	412	1500	1500	1500	388	388	388	388	638	638	638	638	1109	1109	1109	1109
12	503	503	503	503	849	904	904	904	732	732	732	732	305	305	305	305	654	654	654	654
Cost		2.13	1.95	2.28		3.57	2.05	3.82		1.56	1.68	1.37		3.17	1.95	3.17		3.26	2.01	3.26
ΔQ_p		238	367	138		398	1605	148		510	460	588		0	908	0		0	941	0
RR		8.8%	135%	5%		9.9%	399%	3.7%		192%	173%	221%		0	279%	0		0	282%	0
RC		169%	302%	113%		158%	636%	5.9%		44%	397%	507%		0	516%	0		0	512%	0

Stage	41 IN	41 INDP	41 DEPN	41 STAN	42 IN	42 INDP	42 DEPN	42 STAN	43 IN	43 INDP	43 DEPN	43 STAN
1	2939	1781	2350	1781	1675	1600	2350	1500	2292	1600	2300	1500
2	2314	2314	2150	2314	1870	1600	2350	1500	2318	1854	2250	1954
3	2000	2000	2000	2000	1630	1550	1633	1500	1345	1500	2100	1500
4	1659	1660	1900	1659	2007	1500	2000	1526	1320	1500	1735	1500
5	1443	1500	1850	1500	2448	2224	1750	2448	1945	1612	1750	1612
6	653	1500	1500	1500	2491	2491	1500	2491	1818	1819	1500	1818
7	563	1500	981	1500	631	1500	1500	1500	1396	1500	1500	1500
8	1255	1650	1265	1500	255	1500	1084	1500	1480	1500	1500	1500
9	645	726	645	875	1067	1270	1068	1269	529	1500	920	1500
10	719	719	719	719	1093	1093	1093	1093	435	1500	436	1500
11	891	891	891	891	894	894	894	894	545	701	545	701
12	661	661	661	661	497	497	497	497	674	674	675	674
Cost		1.89	1.95	1.89		2.15	1.95	2.15		0.85	1.95	1.09
ΔQ_p		625	589	625		0	141	0		464	18	364
RR		21.3%	20%	21.3%		0	5.7%	0		20%	0.8%	157%
RC		434%	409%	434%		0	142%	0		571%	2.2%	445%

Stage	44 IN	44 INDP	44 DEPN	44 STAN	45 IN	45 INDP	45 DEPN	45 STAN
1	2517	1600	2350	1500	1020	1600	2177	1500
2	1974	1735	2200	1835	3107	1900	2500	1500
3	775	1500	1875	1500	2217	1686	2150	2186
4	2241	1516	2000	1516	1970	1971	2000	1970
5	2069	2069	1800	2069	1431	1500	1800	1500
6	2695	2695	1500	2695	2507	2439	1500	2438
7	1073	1500	1500	1500	1477	1500	1500	1500
8	1241	1500	1500	1500	1262	1500	1500	1500
9	834	1500	1500	1500	481	1500	1500	1500
10	687	1500	1042	1500	933	1500	939	1500
11	678	831	679	830	936	1407	937	1406
12	545	545	545	545	719	719	719	719
Cost		2.45	1.95	2.45		2.08	2.17	2.08
ΔQ_p		0	345	0		668	607	668
RR		0	128%	0		21.5%	19.5%	21.5%
RC		0	289%	0		41.6%	37.8%	41.6%

Stage	46 IN	46 INDP	46 DEPN	46 STAN	47 IN	47 INDP	47 DEPN	47 STAN	48 IN	48 INDP	48 DEPN	48 STAN
1	1625	1600	2350	1500	788	1600	1945	1500	1507	1600	2350	1500
2	2536	1600	2350	1504	1175	1521	1176	1500	546	1600	860	1500
3	3574	3379	2150	3574	3032	2150	2200	1500	3246	2150	2200	1500
4	3307	3307	3036	3307	1881	1650	2000	1500	2170	1600	2000	1812
5	1885	1885	1885	1885	1767	1600	1850	1500	2718	2080	1850	2718
6	2572	2572	2572	2572	2015	1550	1500	2003	3386	3386	3157	3386
7	1447	1500	1500	1500	1313	1500	1500	1500	1570	1570	1571	1570
8	1051	1500	1500	1500	932	1500	1500	1500	533	1500	1500	1500
9	916	1500	1500	1500	834	1500	1227	1500	223	1500	1500	1500
10	49	1280	1280	1280	952	1280	953	1500	428	499	499	499
11	659	660	660	660	776	776	776	1123	691	692	692	691
12	845	845	845	845	818	818	818	818	312	312	312	312
Cost		3.3	2.94	3.51		1.56	1.68	1.21		3.31	3.07	3.31
ΔQ_p		195	538	0		882	832	1029		0	229	0
RR		5.5%	151%	0		291%	274%	339%		0	6.8%	0
RC		9.4%	259%	0		576%	543%	672%		0	121%	0

Stage	49 IN	49 INDP	49 DEPN	49 STAN	50 IN	50 INDP	50 DEPN	50 STAN
1	577	1600	1735	1500	1824	1600	2350	1500
2	1627	1762	1627	1500	2754	1822	2300	1922
3	2231	2150	2200	1500	1703	1703	2150	1703
4	1696	1778	1728	1500	2009	2010	2000	2009
5	2350	2100	1750	1500	955	1500	1605	1500
6	2917	1950	1500	2742	3121	2577	1500	2577
7	1625	1500	1500	1625	1166	1500	1500	1500
8	594	1500	1500	1500	1228	1500	1500	1500
9	770	1157	1500	1500	786	1500	1500	1500
10	869	869	1376	1500	1299	1500	1500	1500
11	839	840	840	888	583	1379	686	1378
12	350	350	350	350	907	907	907	907
Cost		1.56	1.68	3.52		2.28	1.95	2.28
ΔQ_p		767	717	175		544	771	544
RR		263%	246%	6%		174%	247%	174%
RC		541%	506%	124%		336%	476%	336%

REFERENCES

1. Asce, M., and James, L. D., "Computers in Flood Control Planning" J. Hydraulic Div., ASCE. 95, 1859-1869, (1969).

2. Askew, A. J., "Chance-Constrained Dynamic Programming and the Optimization of Water Resources System" Water Resources Research, 10, 1099-1105, (1974).

3. Bather, J. A., "A Diffusion Model for the Control of Dam" J. Appl. Prob., 5, 55-71, (1974).

4. Benson, H. P., and Morin, T. L., "The Vector Maximization Problem: Proper Efficiency and Stability" SIAM Journal on Appl. Math., 32 64-72, (1977).

5. Benson, H. P., and Morin, T. L., "A Bicriterion Mathematical Programming Approach to Nutritional Planning in Developing Nations" Joint National ORSA/TIMS Meeting, Las Vegas, November, 1975.

6. Brater, E. F., Sangal, S., and Sherill, J. D., "Seasonal Effects in Flood Synthesis" Water Resources Research, 10, 441-445, (1974).

7. Becker, L., and William, W. G., "Optimization of Real Time Operation of a Multiple-Reservoir System" Water Resources Research, 10, 1107-1112, (1974).

8. Bodin, L. D., and Roefs, T. G., "A Decomposition Approach to Nonlinear Programs as Applied to Reservoir System" J. Network Anal., 1, 59-73, (1971).

9. Brockwell, P. J., and Chung, K. L., "Empty Times of a Dam with Stable Input and General Release Function" J. Appl. Prob., 12, 212-217, (1975).

10. Chang, T. P., and Toebes, G. H., "Initial Results, from Upper Wabash Simulation Model" Water Resources Research Center, Purdue University, March, 1973.

11. Chernoff, H., and Moses, L. E., Elementary Decision Theory, Wiley, New York, 1959.

12. Çinlar, E., "Markov Renewal Theory" Adv. Appl. Prob., 1, 123-187, (1969).

13. Çinlar, E., "On Dams with Continuous Semi-Markovian Input" J. Math. Analysis and Appl., 35, 434-448, (1971).

14. Çinlar, E., "Theory of Continuous Storage with Markov Additive Inputs and General Release Rule" J. Appl. Prob., 43, 207-231, (1973).

15. Çinlar, E., Introduction to Stochastic Process, Prentice-Hall Inc., Englewood Cliffs, New Jersey, 1975.

16. Çinlar, E., and Pinsky, M., "A Stochastic Integral in Storage Theory" Z. Wahrscheinlichkeitstheoric Verw. Geb., 17, 227-240, (1971).

17. Çinlar, E., and Pinsky, M., "On Dams with Additive Inputs and a General Release Rule" J. Appl. Prob., 9, 422-429, (1972).

18. Cohen, J. L., and Marks, D. H., "A Review and Evaluation of Multiobjective Programming Techniques" Water Resources Research, 11, 208-220, (1975).

19. Conover, W. J., "Two K-Sample Slippage Tests" J. of the American Stat. Assoc., 63, 614-626, (1968).

20. Conover, W. J., Practical Nonparametric Statistics, John Wiley & Sons Inc., New York (1971).

21. Dawes, J. H., and Terstriep, M. L., "Potential Surface Water Reservoir of South-Center Illinois" Report of Investigation, 54 State of ILL. Dept. of Registration and Education, 1966.

22. Dawes, J. H., and Terstriep, M. L., "Potential Surface Water Reservoir of North-Center Illinois" Report of Investigation, 56 State of ILL. Dept. of Registration and Education, 1966.

23. Day, J. C., "A Recursive Programming Model for Nonstructural Flood Damage Control" Water Resources Research, 6, 1262-1271, (1970).

24. Doran, D. G., "An Efficient Transition Definition for Discrete State Reservoir Analysis: The Divided Interval Technique" Water Resources Research, 11, 867-873, (1975).

25. Eastman, J., and ReVelle, C., "Linear Decision Rule in Reservoir Management and Design 3. Direct Capacity Determination and Intra-seasonal Constraints" Water Resources Research, 9, 29-42, (1973).

26. Eisel, L. M., "Chance Constrained Reservoir Model" Water Resources Research, 8, 339-347, (1972).

27. Faddy, M. J., "Optimal Control of Finite Dams: Discrete (2-stage) Output Procedure" J. Appl. Prob., 11, 111-121, (1974).

28. Garfinkel, R. S., and Nemhauser, G. L., Integer Programming, Wiley, New York, 1972.

29. Geoffrion, A. M., "Solving Bicriterion Mathematical Programs" Operation Research, 15, 39-54, (1967).

30. Geoffrion, A. M., "Proper Efficiency and the Theory of Vector Maximization" J. Math. Anal. and Appl., 22, 618-630, (1968).

31. Geoffrion, A. M., and Marsten, R. E., "Integer Programming Algorithms: A Framework and State-of-the-Art Survey", Management Science, 18, 465-491, (1972).

32. Ghosal, A., Some Aspects of Queueing and Storage Systems, Springer-Verlag, New York, 1970.

33. Gray, D. M., Handbook on the Principles of Hydrology, Water Information Center Inc., 1973.

34. Grigg, N. S., and Helweg, O. J., "State-of-the-Art of Estimating Flood Damage in Urban Areas" Water Resources Bulletin, 11, 376-390, (1975).

35. Haimes, Y. Y., and Hall, W. A., "Multiobjectives in Water Resources Systems Analysis: The Surrogate Worth Trade Off Method" Water Resources Research, 10, 615-624, (1974).

36. Hufschmidt, M. M., and Fiering, M. B., Simulation Techniques for Design of Water Resource System, Macmillian, New York, 1967.

37. Jackson, B. B., "Markov Mixture Models for Dorought Length" Water Resources Research, 11, 64-74, (1975).

38. Kirby, W., "On the Random Occurrence of Major Floods" Water Resources Research, 5, 778-784, (1969).

39. Klemes, V., "Probability Distribution of Outflow from a Linear Reservoir" J. Hydrology, 21, 305-314, (1974).

40. Laugbein, W. B., and Asce, A. M., "Queueing Theory and Water Storage" J. Hydraulic Div., ASCE., HY5, 1811-1821, (1958).

41. Lanyon, R., "Flood Plain Management in Metropolitan Chicago" Civil Engineering, 44, 79-81, (1974).

42. Linsley, R. K., Kohler, M A , and Paulhus, J. H , Hydrology for Engineers, Mcgraw Hill, New York, 1975.

43. Lloyd, E. H., "A Probability Theory of Reservoirs with Serially Correlated Inputs" J. of Hydrology, 1, 99-128, (1963).

44. Loucks, D. P., Conflict and Choice: Planning for multiple Objectives, in Economy Wide Models and Development Planning Oxford University Press, New York, 1975.

45. Loucks, D. P., and Dorfman, P. J., "An Evaluation of some Linear Decision Rules in Chance-Constrained Model for Reservoir Planning and Operation" Water Resources Research, 11, 777-782, (1975).

46. Mawer, P. A., and Thorn, D., "Improved Dynamic Programming Procedures and Their Practical Application to Water Resource System" Water Resources Research, 10, 274-287, (1974).

47. Meier, W. L., and Beightler, C. S., "An Optimization Method for Branching Multistage Water Resources System" Water Resources Research, 3, 645-652, (1967).

48. Mejia, J. M., Egli, P., and Leclerc, A., "Evaluating Multi-Reservoir Operating Rules" Water Resources Research, 10, 183-190, (1974).

49. Mermel, T. W., Register of Dams in the United States, McGraw-Hill, New York, 1958.

50. Moran, P. A., The Theory of Storage, Methuen, London, 1958.

51. Moran, P. A., "A Theory of Dams with Continuous Input and General Release Rule" J. Appl. Prob., 6, 88-99, (1969).

52. Morin, T. L., "Optimal Sequencing of Capacity Expansion Projects" J. Hydraulics Div. ASCE., 99, 1600-1622, (1973).

53. Morin, T. L., "Solution of some Combinational Optimization Problems in Water Resources Development" Engineering Optimization, 1, 155-167, (1975).

54. Morin, T. L., "Multidimensional Sequencing Rule", Operation Research, 23, 576-580, (1975).

55. Morin, T. L., and Esogube, A. M. O., "The Imbedded State Space Approach to Reducing Dimensionality in Dynamic Programs of Higher Dimensions", J. Math. Analysis & Applications, 48, 801-810(1974).

56. Morin, T. L., and Marsten, R. E., "A Hybrid Dynamic Programming Branch-and-Bound Approach to a Class of Sequencing Problems" Joint National ORSA/TIMS Meeting, Las Vegas, November, 1975.

57. Morin, T. L., and Marsten, R. E., "An Algorithm for Nonlinear Knapsack Problems", Management Science, 22, 1147-1158, (1976).

58. Morin, T. L., and Marsten, R. E., "Branch-and-Bound Strategies for Dynamic Programming" Operation Research, 24, 611-627, (1976).

59. Morton, L., "Conditional Chance-Constrainted Model for Reservoir Control" Water Resources Research, 9, 937-948, (1973).

60. Myron, B. F., Streamflow Synthesis Harvard University, Cambridge, Mass., 1967.

61. Myron, B. F., and Barbara, B. J., Synthetic Streamflows, American Geophysical Union, Washington, D. C., 1971.

62. Nayak, S. C., and Arora, S. R., "Optimal Capacity for a Multi-Reservoir System Using the Linear Decision Rule" Water Resources Research, 7, 485-498, (1971).

63. Nayak, S. C., and Arora, S. R., "Linear Decision Rule: A Note on Control Volume Being Constant" Water Resources Research, 10, 637-645, (1974).

64. Nemhauser, G. L., Introduction to Dynamic Programming, John Wiley & Sons Inc., New York, 1966.

65. O'Laoghaire, D. T., and Himmelblau, D. M., Optimal Expansion of a Water Resource System, Academic Press, New York, 1974.

66. Pakes, A. G., "On Dam with Markovian Input" J. Appl. Prob., 10, 317-329, (1973).

67. Parker, A. D., Planning and Estimating Dam Construction, McGraw-Hill, New York, 1971.

68. Phatarfod, R. M., and Mardia, K. V., "Some Results for Dams with Markovian Inputs" J. Appl. Prob., 10, 166-180, (1973).

69. Pickels, G. W., Drainage and Flood-Control Engineering, McGraw-Hill, New York, (1941).

70. Pliska, S. R., "A Diffusion Process Model for the Optimal Operation of a Reservoir System" J. Appl. Prob., 12, 859-863, (1975).

71. Prabhu, N. U., Queues and Inventories, John Wiley & Sons Inc., New York, 1965.

72. Price, C. E., "Table Lookup Technique" Computer Survey, 3, 49-65, (1971).

73. Rao, R. A., and Chenchayya, B. T., "Probabilistic Analysis and Simulation of the Short Time Increment Rainfall Process" Purdue University, Water Resources Research Center, West Lafayette, Indiana, 1974.

74. ReVelle, C., and Kirby, W., "Linear Decision Rule in Reservoir Management and Design, 2, Performance Optimization" Water Resources Research, 6, 1033-1044, (1970).

75. ReVelle, C., Joeres, E., and Kirby, W., "The Linear Decision Rule in Reservoir Management and Design, 1, Development of the Stochastic Model" Water Resources Research, 5, 767-777, (1969).

76. Roefs, T. G., and Bodin, L. D., "Multireservoir Operation Studies" Water Resources Research, 6, 410-420, (1970).

77. Shane, R. M., Lynn, W. R., and Asce, M., "Mathematical Model for Flood Risk Evaluation" J. of Hydraulics Div., ASCE., 8, 4119-4128, (1964).

78. Schmidt, J. W., and Taylor, R. E., Simulation and Analysis of Industrial Systems, Irwin, Inc., Homewood, Illinois, 1970.

79. Smith, W. M., Stream Flow Data of Illinois, State of Illinois, Div. of Waterways, Dept. of Public Works and Buildings, 1937.

80. Sobel, J. M., "Reservoir Management Models" Water Resources Research, 11, 767-776, (1975).

81. Srinivasan, S. K., "Analytic Solution of a Finite Dam Governed by a General Inputs" J. Appl. Prob., 11, 134-144, (1974).

82. Todorovic, P., "On Some Problems Involving Random Numbers of Random Variables" Annals of Math. Statistics, 41, 1059-1063, (1970).

83. Todorovic, P., "A Stochastic Model for Flood Analysis" Water Resources Research, 6, 1641-1648, (1970).

84. Vemuri, V., "Multiple-Objective Optimization in Water Resource System" Water Resources Research, 10, 44-48, (1974).

85. Viessman, H., and Knapp, Introduction to Hydrology, Academic Press, New York, 1974.

86. Water Resources Council The Nations Water Resources the First National Assessment of the Water Resources Council, U. S. Government Printing Office, Washington, D. C., 1968.

87. Windsor, J. S., "Optimal Model for the Operation of Flood Control System" Water Resources Research, 9, 1219-1226, (1973).

88. Windsor, J. S., "A Programming Model for the Design of Multireservoir Flood Control System" Water Resources Research, 11, 30-36, (1975).

89. Yeo, G. F., A Finite Dam with Variable Release Rate" J. Appl. Prob., 12, 205-211, (1975).

90. Yeo, G. F., "A Finite Dam with Exponential Release" J. Appl. Prob., 11, 122-133, (1974).

91. Young, G. K., and Asce, A. M., "Finding Reservoir Operating Rules" J. Hydraulics Div., ASCE , 93, 297-321, (1967).