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METHODS FOR GENERATING ALTERNATIVE SOLUTIONS TO WATER RESOURCES PLANNING PROBLEMS

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ABSTRACT

An optimization model is generally not a perfect representation of a complex water resources planning problem because not all important issues can be captured in a model. Optimization models can be used in a planning process to generate planning alternatives that are good and different so that the analyst and the decision maker can examine a wide range of alternatives to gain insight and understanding. This approach is called modeling to generate alternatives (MGA). Several new MGA methods, a random method, a branch and bound/screening (BBS) method and a Fuzzy HSJ method, are described. This work also provides an assessment of the potential use of these methods as well as the HSJ method for generating good and different alternative solutions; the methods are illustrated using a wastewater treatment system planning problem, which is formulated as a mixed integer programming (MIP) model.

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KEYWORDS: Water resources planning, Generating planning alternatives, Optimization models

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CHAPTER 1

INTRODUCTION

Optimization techniques have been shown to be useful for obtaining the "optimal" solution to a mathematical model. They may also be used to obtain a few second best solutions using parametric or sensitivity analysis, a set of noninferior solutions using multiobjective methods, or various solutions by *ad hoc* procedures. Many water resources planning problems are quite complex, however, and some important issues may be unmodeled. The premise of this study is that optimization models may be more useful in dealing with such problems if they can also be used to generate planning alternatives that are good with respect to the modeled objective(s) and different with respect to the decisions that they specify. Some of these alternatives may be better than others with respect to the unmodeled issues. In general, analysts and decision makers can react to these different solutions, so that better understanding and insight may be obtained. In this way, the utility of optimization models may be extended and the possibility of using an optimization model successfully may be greatly increased.

Given the premise, the objective of this work was to develop and evaluate different modeling to generate alternatives (MGA) methods for generating planning alternatives. The MGA methods developed are a random generation method, a Fuzzy Hop, Skip, Jump method and a branch and bound/screengin (BBS) method. These methods and the Hop, Skip, Jump (HSJ) method developed by Brill (1979) are evaluated using applications to typical water resources planning problems formulated as an mixed integer programming (MIP) model. These methods are shown to be able to generate various alternatives that are good with respect to modeled objective(s) and different with respect to decision variables for the example problems.

Water resources planning problems, as well as other public-sector planning problems, are usually very complex and cannot be fully represented by a mathematical model (see, e.g., Liebman, 1976, and Brill, 1979). The comlexity comes mainly from the fact that many interactive issues are involved in the planning process. Many objectives are qualitative and conflicting; many constraints are not clearly defined. Since the real planning problem usually cannot be fully represented by a mathematical model, the "optimal" solution of that model is unlikely to be the "best" solution for the planning problem (see, e.g., Deininger, 1973, and Roy, 1976). Therefore, the use of an optimization model to obtain directly the "best" solution for a planning problem is not expected to be generally successful.

Hamming (1962) said, "The purpose of computing is insight, not numbers"; Geoffrion (1976) also stated, "The purpose of mathematical programming is insight, not numbers". Beer (1966) suggested that a "model" is the basis of insight and prediction. He also stated, "The model ---- is the tool of operations research which enables alternative decisions, policies and controls to be evaluated and compared in quantitative terms. --- The whole purpose of operations research, --- is to aid the manager". If we accept that, for a complex environmental planning problem, an optimization model should be used to give insight and to aid the decision maker in a planning process, then one approach is to use optimization models to generate alternatives for further evaluation.

One drawback of the conventional public-sector planning process is that only a few alternatives are usually generated for evaluation because of manpower, computational, and time limitations. With the development of optimization models and with the help of high speed digital computers, it is possible to construct optimization models to assist in the generation of planning alternatives.

Hahn _et al._ (1973) found that few alternatives have been investigated in the traditional approach for solving regional wastewater planning problems and suggested, "It is desirable to investigate the largest possible number of meaningful alternatives". Deininger (1973) stated, "A mathematical model gives the tool to investigate more alternatives. - - - - It increases the capability of generating information that can be used in the decision-making". But few optimization models have been used specifically to generate a wide range of alternatives. Even though the feasible space of a model may contain a large number of solutions, it has usually been used to obtain the "best" solution for the planning problem and to carry out parametric analyses.

Multiobjective programming methods have received considerable attention in solving public-sector planning problems. Among them, weighting, constraint, and multiobjective simplex methods have been proposed to generate the set of noninferior solutions; the noninferior set estimation method has been developed to find an approximation of the noninferior set (see, e.g., Cohon and Marks, 1975; Cohon, 1978; and Zeleny, 1974a). One limitation of these approaches is that the planning problem must be formulated explicitly or at least implicitly with two or more objective functions. Even though many public-sector planning problems are multiobjective, not every such problem can be easily formulated mathematically because some objectives cannot be quantified. On the other hand, it may be computationally impractical to generate the whole noninferior set when there are many more than two objectives. Also, many solutions along the noninferior curve are not significantly different from one another. Steuer (1976) applied a filtering technique to select "dissimilar" solutions from the set of noninferior extreme points. However, "dissimilar" was defined only in objective space. A premise of this study is that differences with respect to decision space are also important and should be considered.

Another drawback of the set of noninferior solutions is that, since there is usually at least one objective not included in the optimization model, the noninferior solutions may not be good with respect to omitted issues, and the best solution may well be located in the "inferior" region as defined by the model rather in the noninferior set (see, e.g., Brill, 1979, and Falkenhausen, 1979). Thus, alternatives outside the noninferior set should also be searched in a planning process.

Brill and Nakamura (1978) employed a branch and bound algorithm to generate different alternatives for planning regional wastewater treatment systems. They also used an imputed value matrix to compare the generated alternatives. The limitation of their approach is that it tends to generate too many alternatives, and most of the alternatives are not significantly different from one another.

Few mathematical programming methods have been developed specifically for the purpose of generating alternatives that are different. Church and Huber (1979) used a reverse Teitz and Bart heuristic to find close to optimal solutions for maximal covering location problems. They defined difference as "the number of sites that a solution does not share with the identified optimal solution". Consequently, even though their method identifies alternative solutions which have many different locations when compared to the optimal solution, they may not be very different when compared to each other.

Falkenhausen (1979) used a heuristic evolution strategy to generate alternative solutions for a regional waste treatment system planning problem. The evolution procedure, which is based on the principle of biological evolution, generates new sets of solutions by mutation and recombination of the existing set of solutions. He also compared the evolution method to a branch and bound method and concluded that the evolution strategy provided more solutions than the branch and bound method. He also pointed out that the evolution strategies tend to generate similar solutions and it might not be easy to obtain initial solutions.

Brill (1979) suggested a mathematical programming method to generate alternative solutions. This method, called Hop, Skip, Jump (HSJ), is designed to generate planning alternatives that are good with respect to objectives included in the model and are significantly different from one another. Recently, the HSJ method has been applied to a hypothetical land use planning problem to generate land use alternatives. The results showed that the HSJ method can be used to generate numerous land use alternatives that are good with respect to the objectives included in the model and different with respect to an unmodeled issue and with respect to the decision specified (Chang et al., 1979).

Even though random generation has been used mostly to generate input information in surveying, stochastic simulation, and statistical process involving uncertainty, it can also be used to generate solutions. Rosenshine (1970) and Olson and Wright (1975) applied the random generation process to schedule police patrols. Brooks (1958, 1959) used a random method in maximum-seeking experiments and concluded that the random method is preferred to other methods when the number of experimental factors is large. The Southeastern Wisconsin Regional Planning Commission applied a similar maximum-seeking random method to generate land use planning solutions (SWRPC, 1973). The validity and the efficiency of this random search technique to generate land use plans were also evaluated by Sinha, et al. (1973).

When solutions are generated at random, if no guidance is followed (e.g., a stratified random search), it is possible to miss some good solutions in a large portion of the decision space (Brooks, 1959). Furthermore, the generation process may be very inefficient for generating a good solution. Hopkins (1975) used a random method to generate land use plans. The problem is that when there are many feasibility constraints, it is difficult to generate feasible solutions.

The advantage of using a random method in a planning process is that it can generate "unexpected" solutions to a planning problem, thus giving new insight to the problem. It may also spark the imagination and creativity of the analyst or decision maker. Whether or not a random method can be successfully applied in a planning process to generate good and different planning alternatives is dependent on whether the "randomness" or "unexpectedness" of the random process can be preserved, and on whether it can generate solutions efficiently.

The random generation method developed in this study uses the original constraints of the model to ensure the solution obtained is feasible. Targets are also set for the planning objectives included in the model to make sure that the solution obtained is good with respect to modeled objectives. An extreme point solution is located by maximizing an objective function (the sum of a set of decision variables) that is randomly generated. In this way, solutions that are feasible and good can be generated efficiently, and different solutions may be generated by randomly selecting different objective functions to be maximized.

Another approach developed in this study to generating good and different alternatives to a planning problem is first to generate many solutions using an existing computer code; then a screening process can be applied to select solutions that are good and different. For example, a branch and bound algorithm can be used to generate numerous feasible solutions that meet targets for the modeled objectives; those solutions can be screened to select a subset of solutions that are different from each other. A method of measuring differences among alternatives must be established before the screening process can be applied. This approach is called a branch and bound/screening (BBS) method. This approach will be promising if both the generating and screening processes are relatively efficient.

In addition to the various issues mentioned above, decision making in the real world is fuzzy, i.e., the objectives and constraints that are known are not precisely defined by formulations or numbers (see, e.g., Bellman and Zadeh, 1970). Consequently, as discussed above, the optimal solution to an exact mathematical model and the optimal solution to the planning problem will not likely be the same. Thus it may be possible to represent a planning problem more reasonably by a fuzzy formulation with fuzzy goals and constraints. However, since mathematical manipulations are precise, fuzzy goals and constraints must be defined precisely as a fuzzy set, and fuzzy models must be converted to "crisp" models before they can be solved (Bellman and Zadeh, 1970, and Zimmermann, 1978).

Zimmermann (1975, 1978) applied fuzzy set theory to solve linear vector-maximization problems by fuzzy linear programming. Since a few assumptions must be made to convert a fuzzy LP model to a crisp LP model that can be solved by an existing LP algorithm, and since there is most likely an objective that is not included in the model, the mathematically optimal solution is still probably not the best solution to the real problem. Furthermore, one solution will not provide much insight into the problem. The concept of fuzziness can, however, be applied as part of the above methods for generating alternatives. A Fuzzy HSJ method is proposed in this study to generate planning alternatives. It is a modification of the HSJ method and can be used to increase the flexibility of the feasible decision space and the targets for the objectives included in the model.

The general procedures of the HSJ, random, BBS, and the Fuzzy HSJ methods are described in Chapter 2. Also included are two methods of measuring differences among alternatives using decision variables. Chapter 3 describes the hypothetical applications of the HSJ, random, BBS and Fuzzy HSJ methods to a wastewater system planning problem formulated as an MIP problem. Comparisons among these methods are also included.

CHAPTER 2

GENERAL PROCEDURES

The HSJ, random, BBS, and Fuzzy HSJ methods will be examined using a water resources planning problems formulated as an MIP model. All of these methods can be used to search the whole decision space (both the noninferior and the inferior regions). These approaches and two methods of measuring differences among alternatives using decision variables are described in the following sections. A more detailed description of the HSJ method is shown elsewhere (Brill, 1979, Chang et al., 1979).

2.1 HSJ Method

A single objective (assumed here to be cost minimization) LP formulation for a planning problem is:

$$\begin{aligned} &\text{Min} \quad CX \\ &\text{s.t.} \quad AX=B \end{aligned} \tag{2.1}$$

where: X is the vector of real decision variables,

C is the cost coefficient vector for decision variables X,

A is the coefficient matrix for the constraints, and

B is the vector of right hand side values.

An initial HSJ solution can be obtained by solving formulation

2.1 directly. This solution is the optimal solution with respect to the cost objective for this particular formulation. To obtain the second HSJ solution the sum of nonzero decision variables (basic) in the first solution is minimized, subject to the original constraints. The cost is relaxed an acceptable amount in comparison to the best possible value and specified as an additional constraint. The formulation is designed to produce a solution that is "maximally different" from the initial solution. Specifically, the following formulation should be solved:

$$\begin{aligned} &\text{Min} \quad \sum_{b \in K} X_b \\ &\text{s.t.} \quad CX \leq T \\ &\qquad \text{constraint set of formulation 2.1} \end{aligned} \tag{2.2}$$

where X_b is a nonzero variable in the initial solution,

K is the set of indices of the nonzero variables in the initial solution,

CX is the cost objective function,

T is the target for cost where $T=c^*+a$ where c^* is the cost obtained by solving (2.1), and a is the amount that the target for cost is relaxed from c^*.

To obtain the third and the following alternatives, a formulation similar to formulation 2.2 can be used except that the nonzero variables in the HSJ objective function should include all of the nonzero variables in all previous solutions. This procedure continues either until no more alternatives can be obtained or until enough different alternatives have been generated. Several initial solutions could be used to generate a set of solutions; this step may be especially desirable when only a few alternatives are generated using only one initial solution.

A facility location planning problem is used here as a typical MIP planning problem. The formulation for a single objective (e.g., minimize cost) problem can be stated as follows:

$$\begin{aligned} \text{Min} \quad & C_1Y + C_2X \\ \text{s.t.} \quad & A_1Y + A_2X = B \end{aligned} \tag{2.3}$$

where Y is the vector of 0,1 decision variables associated with the existence of facilities,

X is the vector of continuous variables that specify facility capacities,

C_1 is the vector of fixed cost coefficients for the 0,1 decision variables.

C_2 is the vector of unit cost coefficients for the continuous decision variables,

A_1 is the constraint coefficient matrix for the 0,1 decision variables,

A_2 is the constraint coefficient matrix for the continuous decision variables, and

B is the vector of right hand side values.

Again, the initial HSJ solution can be obtained by solving formulation 2.3 directly. The second HSJ solution can be obtained by solving a formulation analogous to formulation 2.2 given in the linear programming case. Unlike the LP formulation, there are two kinds of variables in an MIP formulation: the continuous variables associated with the facility capacities, and the zero-one variables associated with the facility locations. Since these two kinds of variables do not have the same properties, one set of variables should be scaled if both of them should appear together in the HSJ objective function. The scaling factor, which should be based on the relative importance of facility location versus facility capacity, would be difficult to determine before alternative solutions are examined. In fact, each set of variables can be treated individually as a surrogate for difference among alternatives, and either one can be used alone in the HSJ objective function (see Section 4.2).

If a multiobjective LP or MIP formulation is used for a planning problem instead of a single objective formulation, the HSJ procedure is the same as described above, except that: (1) the initial solution can be obtained in numerous ways, including: (a) minimizing the weighted sum of all objectives subject to the original constraints, (b) minimizing one objective subject to the original constraints as well as constraints that specify targets for the other objectives, or (c) minimizing one objective subject to the original constraints without placing constraints on any other objective; and (2) targets on some or all objectives can be relaxed an acceptable amount and treated as additional constraints when solving a formulation such as 2.2.

2.2 Random Method

The random generation method developed and used in this study is designed to generate planning alternatives that are good with respect to the objectives included in the optimization model and significantly different with respect to omitted issues. The procedure is described as follows. An objective function (set of decision variables) is randomly generated, and then it is maximized subject to the original constraints. At the same time, additional constraints on modeled objective function values are added to make sure good alternatives with respect to the objectives included in the model will be obtained. Specifically, the following LP formulation is used if the problem is originally represented by formulation 2.1.

$$\begin{aligned} \text{Max} \quad & z = \sum_{k \in K} X_k \\ \text{s.t.} \quad & CX \leq T \qquad (2.4) \\ & \text{constraint set 2.1} \end{aligned}$$

where K is the set of decision variables randomly generated,

z is the sum of randomly selected decision variable values (SORD), and

T is the target for the cost objective.

Note that constraint set 2.1 reduces the entire decision space to the feasible decision space, while the target constraint further reduces the feasible decision space to a space in which all solutions are good with respect to the modeled objectives. One SORD function is then randomly generated and maximized to locate one solution on an "extreme" point in this decision space. Unlike the HSJ method, since this random method generates each solution independently, an initial solution is not required. However, the optimal solution to formulation 2.1 is still a very useful reference and can be used to set the target for (2.4).

If a planning problem is represented by an MIP model, then there are continuous and zero-one variables in the formulation as discussed in Section 2.1. Again, each kind of variable can be used alone in the random generation method to generate planning alternatives. If the planning problem is represented by a multiobjective formulation, then the target for each objective function can be specified.

2.3 A Branch and Bound/Screening (BBS) Method

The HSJ and random methods discussed above are designed to generate solutions directly that are good with respect to the modeled objectives and different with respect to omitted issues. The method discussed in this section is a two-step approach. The first step is to generate many potential solutions using an available computer code; then a screening process is applied to select a set of good and different solutions.

For an LP formulation, typical LP code such as APEX provides only one optimal solution although sensitivity or parametric analyses can provide more solutions. For a single objective MIP formulation, however, a typical code such as APEX MIP code can specify feasible solutions within a certain cost limit (specified before the execution) by the branch and bound procedure. If the limit is set within an acceptable value, all solutions obtained will be good with respect to the cost objective. Then the screening process can be used to eliminate those solutions that are similar. Thus, a set of good and different solutions can be obtained. If a multiobjective MIP formulation is used, one objective can be optimized while other objectives are constrained to meet target values to obtain a set of solutions. If necessary, each objective can be optimized in turn to obtain several different sets of solutions. The screening process can then be applied based on some method of measuring differences to select solutions that are different from one another.

In a broader sense, all methods mentioned above belong to this two-step generation method. The only difference is that if the alternatives generated by the HSJ or random method are all significantly different, the screening process is not needed. If some solutions generated are similar or too many solutions are generated and only solutions with the largest difference are needed, however, then a screening process can be used to eliminate those solutions that are similar.

2.4 Fuzzy HSJ Method

In the Fuzzy HSJ approach, the initial solution can be the same as the one used in HSJ, i.e., obtained by solving formulation 2.1. To obtain solutions that are different from the first solution, a formulation similar to (2.2) can be used, except that the objective function and the first (cost) constraint would now be fuzzy, which means that we want a solution which is "significantly" different from the first solution and the cost is "not much higher" than the cost in the initial solution. The corresponding mathematical formulation that replaces formulation 2.2 is:

$$\sum_{b \in K} X_b \underset{\sim}{\geq} z$$

$$CX \underset{\sim}{\geq} c^* \qquad (2.5)$$

constraint set 2.1

where X_b is a nonzero variable in the initial solution, z is the optimal difference level desired,

K is the set of indices of the nonzero variables in the initial solution,

CX is the cost objective function,

c* is the cost obtained by solving (2.1), and

$\gtrsim$ is the fuzzy greater than or equal to sign.

The fuzzy objective function and constraints are characterized by their membership functions, so the solution to (2.5) is the intersection of these membership functions. Since there is no fuzzy computer code that can solve (2.5) directly, formulation 2.5 must first be converted to a corresponding crisp formulation before it can be solved (Zimmermann, 1978).

A fuzzy idea is fuzzy; it cannot actually be represented by exact numbers. If it must be converted to exact numbers, assumptions must be made. One set of assumptions is as follows. First, the allowed level of difference and the allowed objective function value can be assumed to be within ranges. Second, a satisfaction (or preference) level must be assigned to each value within its range; and third, how the decision makers evaluate the satisfaction levels among different objectves should be assumed. For example, if the range of difference is set from z to z+z1, and the range of cost is from c* to c*+c, and then the satisfaction level is set to be linearly decreasing from z to z+z1 and from c* to c*+c, and if the decision makers want to maximize the minimum satisfaction of the objectives, then (2.5) can be converted to:

$$\text{Max Min } [\ (z + z1 - \sum_{b \epsilon K} X_b)/z1,\ (c^* + c - CX)/c\ ]$$
$$\text{s. t. constraint set 2.1} \qquad (2.6)$$

where z1 is the allowed range of the surrogate of difference, and c is the allowed range of the cost objective.

Formulation 2.6 can then be further converted to the following:

$$\begin{aligned} \text{Max} \quad & s \\ \text{s.t.} \quad & z + z1 - \sum_{b \epsilon K} X_b \geq z1 s \\ & c^* + c - CX \geq cs \\ & \text{constraint set 2.1} \end{aligned} \qquad (2.7)$$

where s is the satisfaction level that can be varied from 0 to 1. Formulation 2.7 can be solved by using existing LP computer codes. Additional alternatives can be obtained using a Fuzzy HSJ approach by solving a formulation analogous to (2.7) with nonzero variables from all previous solutions in the ΣX_b term.

For a planning problem represented by an MIP model, a formulation analogous to (2.7) can be used. In this case, either continuous variables or the zero-one variables can be used as a difference surrogate. If a multiobjective LP or MIP formulation is used, the initial solution can be obtained as described in Section 2.1.

Since there are many objectives, however, the allowed level of each objective must be assumed. If necessary, the weight of each objective can be attached to each objective. Furthermore, the approach can also handle the case where other constraints are fuzzy as well.

2.5 Measuring Differences among Alternatives

As discussed in Section 2.3, the measurement of differences among alternatives is necessary for eliminating solutions that are similar. The concept of differences among planning alternatives is vague and there is no perfect measure of such differences. Since the alternatives generated using the techniques presented here must be good with respect to the objectives included in the model, they must be similar with respect to those objectives. They typically are different, however, with respect to the values of the decision variables, and these differences can be measured. To calculate the difference between a new solution and all previous solutions, the pairwise differences between a new solution and all the previous solutions must be calculated first. Then the least pairwise difference can be picked as the difference between the new solution and all previous solutions.

Two methods that can be used to calculate the pairwise difference between two solutions are: (1) Calculate the number of different nonzero variables between two solutions. This method will not measure the difference caused by a change in decision variable values; for example, the difference is zero between two solutions if x1=5 in the first solution and x1=1000 in the second solution. (2) Calculate the sum of the absolute differences in decision variable values between two solutions k and k+1 as:

$$\sum_j |x_{jk} - x_{jk+1}| \qquad (2.12)$$

where x_{jk} is the jth decision variable in solution k. Scaling factors (or weights) can be applied to variables with different unit or magnitude.

The second measure mentioned above is actually the metropolitan distance between two solutions in a decision space. Note that the metropolitan distance does not differentiate between large differences in a small number of variables and small differences in a large number of variables as long as the sum of the differences is the same. In contrast, a Euclidean distance measure would put more weight on large differences in a small number of variables. For example, suppose the values of decision variables (x1,x2) are (0,10), (0,0) and (5,5) in solutions A, B and C, respectively; the difference between A and B can be viewed as large differences in a small number of variables (10 units in one variable) while the difference between A and C can be viewed as small differences in a large number of variables (5 units each in two variables). The differences between A and B, and between A and C are exactly the same, 10 units, if the metropolitan distance is used, but the differences are 10 and 7.07 units, respectively, if the Euclidean distance is used. Both the metropolitan and Euclidean measures have been tested to calculate the differences among alternatives generated by the HSJ method for the small scale land use example. The resulting profiles were similar in shape, although the actual values of differences were not the same. The metropolitan measure was chosen arbitrarily for use in the other applications.

The methods described above can be applied to MIP formulations in several ways if slight modifications are made. First, the sum of absolute numbers of different nonzero variables between two solutions can be modified to the sum of absolute different zero-one variable values between two solutions. Second, the differences in decision variable values can be modified to the differences in continuous decision variable values. If the differences in both zero-one variables and continuous variables values are used, a weighting factor could be applied to one group of values to take into account the relative importance of zero-one variables versus continuous variables.

Aother method of measuring differences used in this study is visual inspection. Note that each method has its advantages and disadvantages; none is perfect for all the cases. The common disadvantage of the above quantitative methods of measuring difference is that they will not measure how far apart, in the spatial sense, location changes are. Measurement of difference may depend on the characteristics of an individual planning problem, and they are used only to select a set of different alternatives. Each analyst or decision maker may wish to use a unique measurement of difference for this purpose.

CHAPTER 3

GENERATING PLANNING ALTERNATIVES: MIXED INTEGER PROGRAMMING MODELS

MGA techniques for a mixed integer programming (MIP) model are examined in this chapter. A regional wastewater system planning problem is used to evaluate the HSJ method, the random method, a branch and bound/screening (BBS) method, and a Fuzzy HSJ method for generating good and different planning alternatives. Some variations of the HSJ and random methods are also examined. The CDC APEX MIP code (CDC, 1979) which employs a branch and bound procedure was used to solve the MIP models.

3.1 Example Problem

One of the mixed integer formulations of a regional wastewater system planning problem with the objective of minimizing cost is:

$$\begin{aligned} \text{Min } & \sum_j FC_j Y_j + \sum_i \sum_j GC_{ij} Z_{ij} + \sum_j PC_j PQ_j + \sum_i \sum_j IC_{ij} IQ_{ij} \\ \text{s.t. } & (1)\ L_j - IQ_{jk} + IQ_{ej} = PQ_j \\ & (2)\ PQ_j \leq M_j Y_j \qquad (3.1) \\ & (3)\ IQ_{ij} \leq N_{ij} Z_{ij} \\ & Y_j,\ Z_{ij} = 0,1, \text{ all variables } \geq 0 \end{aligned}$$

where FC_j is the fixed cost for treatment plant j,

GC_{ij} is the fixed cost for interceptor from i to j,

PC_j is the cost coefficient for treatment plant j,

IC_{ij} is the cost coefficient for interceptor from i to j,

Y_j is 0,1 variable for treatment plant j, which determines whether the fixed cost for plant j should be included,

Z_{ij} is 0,1 variable for interceptor from i to j, which determines whether the fixed cost for that interceptor should be included,

PQ_j is the amount of wastewater treated at plant j,

IQ_{ij} is the amount of wastewater flow from i to j,

L_j is the amount of wastewater flow originated from j (known constant),

M_j is an upper bound for PQ (known constant),

N_{ij} is an upper bound for IQ (known constant).

This formulation will determine the locations and capacities of wastewater treatment plants and interceptors such that the total cost is minimized. The assumptions of this formulation are: (1) all treatment plants provide the same degree of treatment (e.g. secondary treatment), and (2) the cost curves for treatment plants and interceptors are concave and can be approximated using fixed charges. Note that planning regional wastewater treatment system is a complex public-sector planning problem, and there are many issues other than cost that should be considered but are not included in the model. For example, political, social, and environmental issues will also play an important role in the planning process (See, e.g., Brill and Nakamura, 1977; Brill and Nakamura, 1978.)

The example problem used in this chapter and the cost approximations for treatment plants and interceptors are from Nakamura (1977). The wastewater system network is shown in Figure 3.1. There are 15 wastewater sources, 12 candidate plant sites and 15 potential interceptor links. Flow is allowed in only one direction in all interceptors, except in the one between sources 8 and 9. Since there is an upper bound for each plant and interceptor capacity, split flows are allowed. The MIP model of this problem, formulated to minimize the total annual cost, has 28 zero-one variables, 28 continuous variables and 43 constraints.

3.2 Generating Alternatives Using the HSJ Method

3.2.1 Procedures and Results

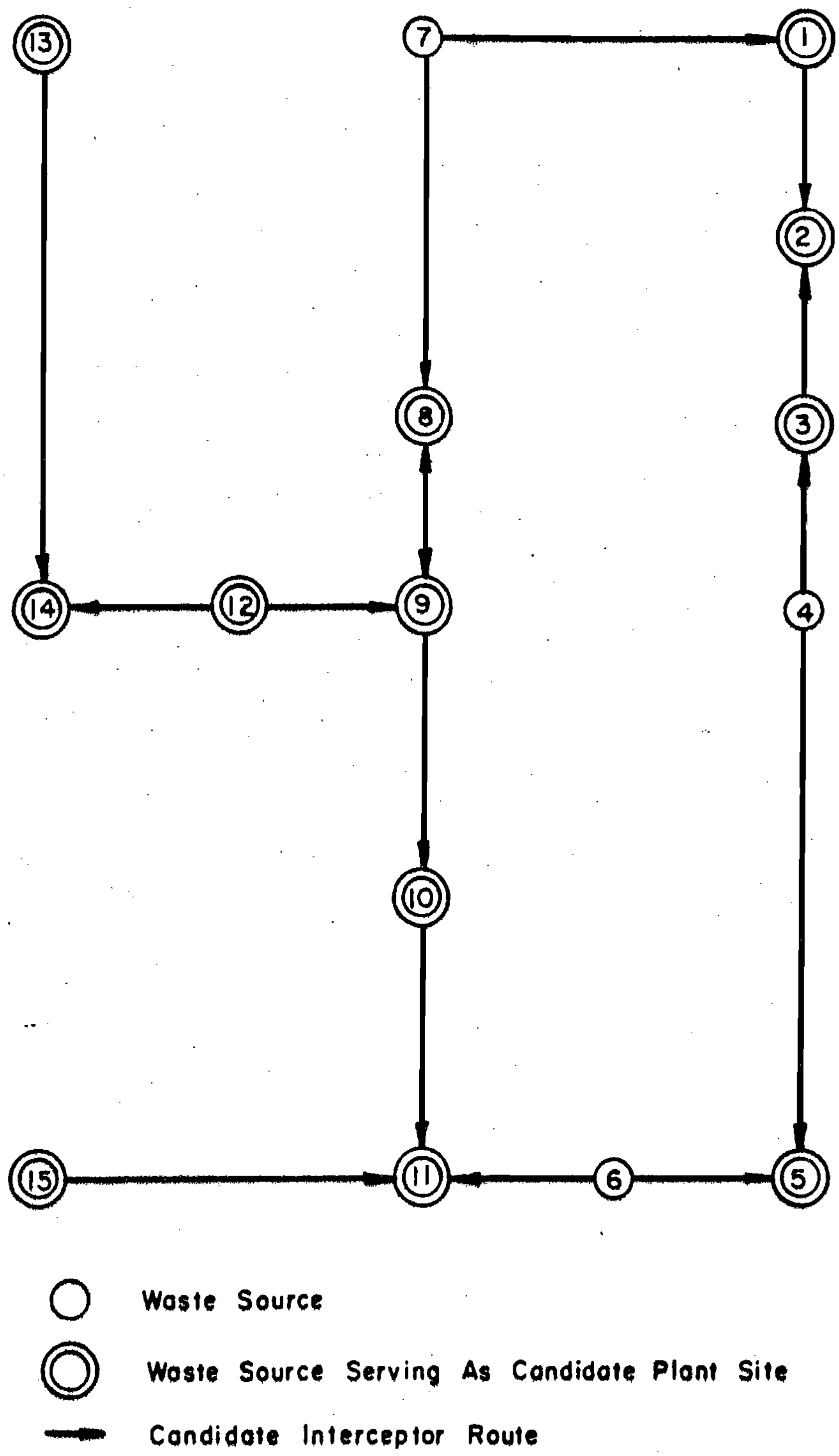

Figure 3.1 Example Wastewater System
(after Nakamura and Brill, 1977)

An initial solution, called HSJ1, was obtained by solving the MIP formulation (which minimizes the total annual cost) of the example problem. The plant locations and the cost of the initial solution are shown in Table 3.1.

To obtain a second solution, which is different from the initial solution, the sum of the nonzero variables in the initial solution is minimized in the HSJ procedure. In this problem, two different kinds of variables are included in the formulation: the zero-one variables associated with the plant and interceptor locations and the continuous variables associated with the plant and interceptor capacities. These two kinds of variables have different characteristics, and each can be used alone in the HSJ objective function as a surrogate for difference to drive HSJ iterations. The cost objective function can also be included in the objective function so that the solution obtained will not include unnecessary variables which only increase the cost of the solution; to ensure that the surrogate difference objective dominates the combined objective function, a scaling factor can be attached to either the surrogate difference objective or the cost objective.

Table 3.1 Plant and Interceptor Locations and Total Annual Costs for the Alternatives Generated by the HSJ Method

Alternative		Plant and Interceptor Locations	Annual Cost ($1000)
HSJ1	Plant	(2, 5, 11, 14)*, 8, 15	1917
	Inte.	1-2, 3-2, 4-5, 6-5, 7-8, 9-8, 10-11, 12-14, 13-14	
HSJ2	Plant	(2, 5, 11, 14), 1, 3, 9, 10, 13	2076
	Inte.	7-1, 4-3, 6-11, 8-9, 12-9, 9-10, 15-11	
HSJ3	Plant	(2, 5, 11, 14), 1, 3, 8, 9, 10, 12, 15	2108
	Inte.	7-1, 4-3, 6-5, 13-14	

* plants 2, 5, 11 and 14 must be in the solution because of constraints

For the first experiment, the continuous variables, which indicate the capacities of the plants and interceptors, were used alone in the surrogate difference objective function to drive the HSJ iterations. The cost target was arbitrarily set at $2,108,000, 10% higher than the least cost. Three alternatives, including the initial solution, were obtained when the HSJ procedure was applied. The plant and interceptor locations and the cost of these alternatives are given in Table 3.1. Note that both the number of plants and the total cost increase monotonically from HSJ1 to HSJ3. Since all continuous variables were nonzero at least once by the last solution, the HSJ procedure terminated.

The configurations of these three alternatives, as illustrated in Figure 3.2, are different. The differences are even more striking if the fact that plants 2, 5, 11, and 14 must be in the solution because of the constraints is taken into account. As shown in Table 3.1 and Figure 3.2, the minimization of the capacity of the six plants in HSJ1 drives only two plants (plants 8 and 15) out of the solution because plants 2, 5, 11, and 14 must be in the solution because of constraints. The capacity reduction of plants 2 and 5 is picked up by adding plants 1 and 3, while the capacity reduction of plants 8, 11, 14, and 15 is picked up by adding plants 9, 10, and 13. Even though there are

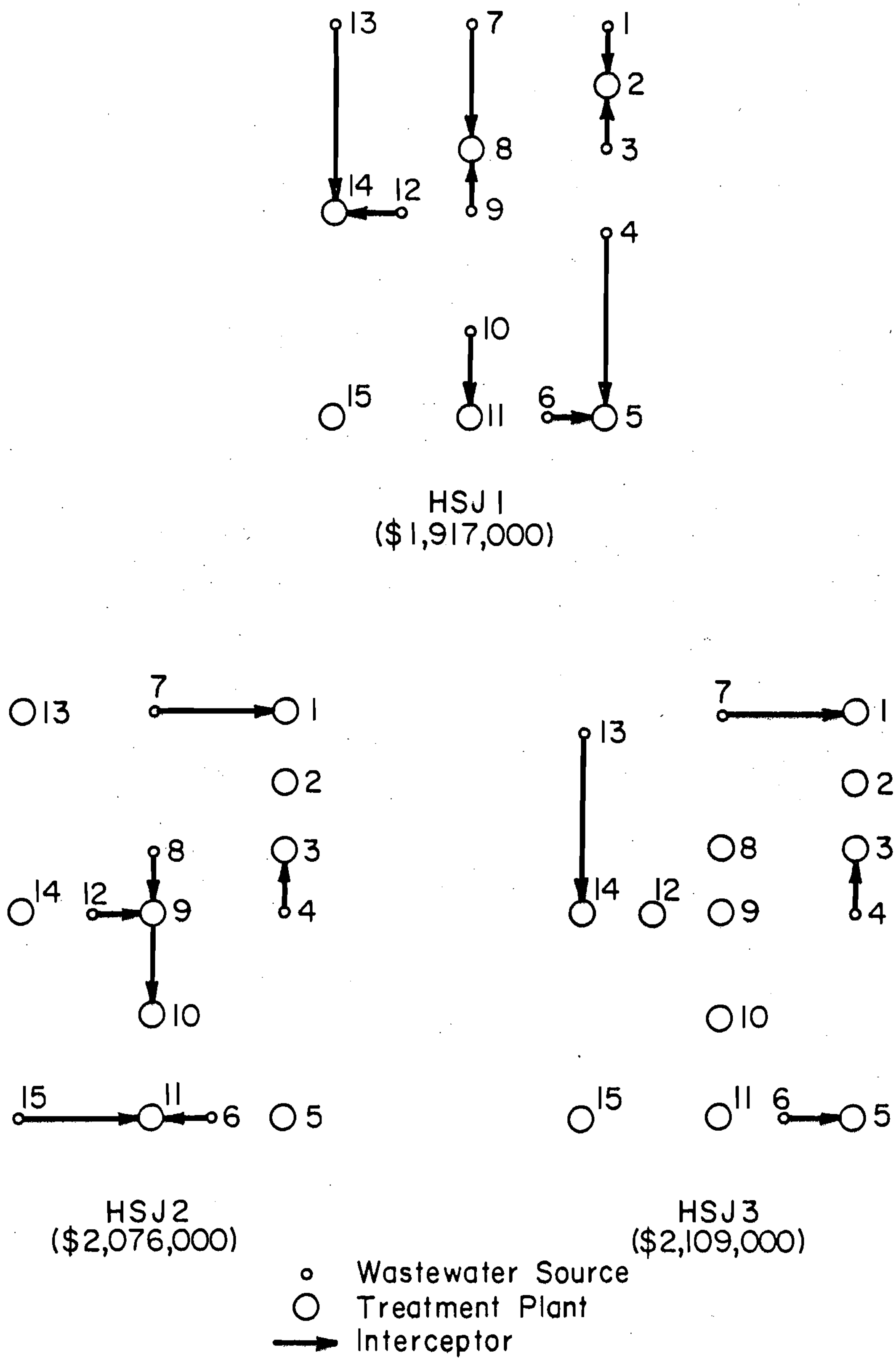

Figure 3.2 Configurations of Alternatives Generated by the HSJ Method with 10% Cost Relaxation

only two plants in HSJ1 that leave the solution, five new plants are added to HSJ2; the total capacity of the five new plants equals the reduction of the six plants in the initial solution.

The minimization process, on the other hand, drives all of the nine interceptors in HSJ1 out of the solution and adds a new set of seven interceptors to HSJ2. HSJ2 is obtained by minimizing the sum of the capacities of both plants and interceptors that appear in HSJ1, and the decreases or increases in plant capacities are closely related to the decreases or increases in interceptor capacities. For example, the minimization of the sum of the capacities of plants 2, 5, and 8 and interceptors 1-2 (source 1 to source 2), 3-2, 4-5, and 7-8 brings plants 1 and 3 and interceptors 7-1 and 4-3 into the solution.

HSJ3 is obtained by minimizing all nonzero continuous variables that appear in HSJ1 or HSJ2. Since the first two HSJ solutions specify all interceptors, including the capacities of interceptors in the objective function is actually an attempt to drive all possible interceptors out of the solution. In other words, the minimization tries to locate as many plants as possible to reduce the capacity of interceptors. (For a large size problem, where only a small portion of the interceptors are specified in the first several solutions, this may not be true.)

As shown in Table 3.1 and Figure 3.2, all potential locations of treatment plants have been selected in HSJ3 except plant 13. Accordingly, all interceptors which can be driven out of the solution have been driven out except interceptor 13-14. The tendency of adding plants and removing interceptors from the solution can also be shown by the fact that the number of plants increases from 6 for HSJ1 to 9 for HSJ2 and 11 for HSJ3 while the number of interceptors dereases from 9 for HSJ1 to 7 for HSJ2 and 4 for HSJ3. It can be seen from Figure 3.2 that HSJ3 is significantly different from HSJ1 and HSJ2 even though it does not have a completely different set of facility locations. Note that HSJ2 is more decentralized than HSJ1, while HSJ3 is even more dispersed. The more decentralized solution is less cost-effective, but it may be good with respect to other issues (e.g., impact on the water quality, flexibility of future recycling and reuse).

The pairwise differences between any two of these three alternatives, as measured by the sum of different plant and interceptor capacities and by the sum of different plant and interceptor locations, are shown in Table 3.2. The least value in each row is taken as the difference between the solution (the solution in that row) and all previous solutions. Thus the difference among these alternatives, using the first measure,

Table 3.2 Differences among the Three Alternatives Generated by the HSJ Method

Alternative	HSJ1	HSJ2	HSJ3
HSJ1	0		
HSJ2	157(23)*	0	
HSJ3	110(14)	74(11)	0

* a (b)
  a: measured by the sum of different plant and interceptor capacities (cfs).
  b: measured by the sum of different plant and interceptor locations (unit); the addition or removal of any plant or interceptor contributes 1 to the sum.

decreases from 152 cfs for HSJ2 to 74 cfs for HSJ3. Similarly, the difference as measured by the sum of different plant and interceptor locations decreases from 23 for HSJ2 to 11 for HSJ3.

The total waste flow to be treated is 79.5 cfs. Since plants must be located at sources 2, 5, 11, and 14 to treat at least the waste flow originating at those points, the flow that can be shifted around is 52.4 cfs. Thus the maximal possible difference as measured by the sum of different plant and interceptor capacities is 4X52.4=209.6 cfs if the flow from one source goes through only one interceptor. The maximal possible difference as measured by the sum of different plant and interceptor locations is 24 -- 8 for plant locations and 16 for interceptor locations. Compared to the maximal possible difference that can be obtained, the differences among these alternatives using each method of measuring differences are significant. The drastically different patterns of plant and interceptor locations for the alternatives are potentially useful because the decision maker can compare them not only with respect to the objectives that are not included in the model, but also with respect to cost. By examining these alternatives, the decision maker may be able to find overlooked issues and gain more insight and better understanding of the problem.

3.2.2 Sensitivity of the Differences Among Alternatives with Respect to the Target for the Cost Objective

If the target for the cost objective is restricted to $2,013,000 (5% more than the optimal cost), then five alternatives are obtained using the HSJ approach. Since the fifth alternative is exactly the same as the fourth one, the HSJ procedure terminates. The plant and interceptor locations and the cost of the four different alternatives are given in Table 3.3; the configurations are shown in Figure 3.3. There are four new plants (plants 1, 3, 9 and 10) and four new interceptors (interceptors 7-1, 4-3, 8-9, and 15-11) in HSJA2 in comparison to the initial solution, HSJA1, while plants 8 and 15 and interceptors 1-2, 3-2, 4-5, 7-8, 9-8, and 10-11 in the initial solution leave the solution. Even though the number of plants leaving the initial solution is the same for the 5% and 10% cases, the number of new plants and new interceptors in the second solution and the number of interceptors leaving the initial solution all decrease in the 5% case because of the cost constraint. The number of new plants with respect to all previous solutions is reduced to one for HSJA3 and for HSJA4. Similarly, the number of new interceptors with respect to all previous solutions is reduced to two and zero for HSJA3 and HSJA4, respectively. The reduction in the number of new plants and interceptors is an indication that the

Table 3.3 Plant and Interceptor Locations and Total Annual Costs for the Alternatives Generated by the HSJ Method with 5% Cost Relaxation

Alternative		Plant and Interceptor Locations	Annual Cost ($1000)
HSJA1	Plant	(2, 5, 11, 14)*, 8, 15	1917
	Inte.	1-2, 3-2, 4-5, 6-5, 7-8, 9-8, 10-11, 12-14, 13-14	
HSJA2	Plant	(2, 5, 11, 14), 1, 3, 9, 10	2010
	Inte.	7-1, 4-3, 6-5, 8-9, 12-14, 13-14, 15-11	
HSJA3	Plant	(2, 5, 11, 14), 8, 10, 12, 14	2013
	Inte.	1-2, 3-2, 4-5, 6-11, 7-8, 9-10, 13-14	
HSJA4	Plant	(2, 5, 11, 14), 3, 8, 10, 13, 15	2013
	Inte.	1-2, 4-3, 6-5, 7-8, 9-8, 12-14	

* plants 2, 5, 11 and 14 must be in the solution because of constraints

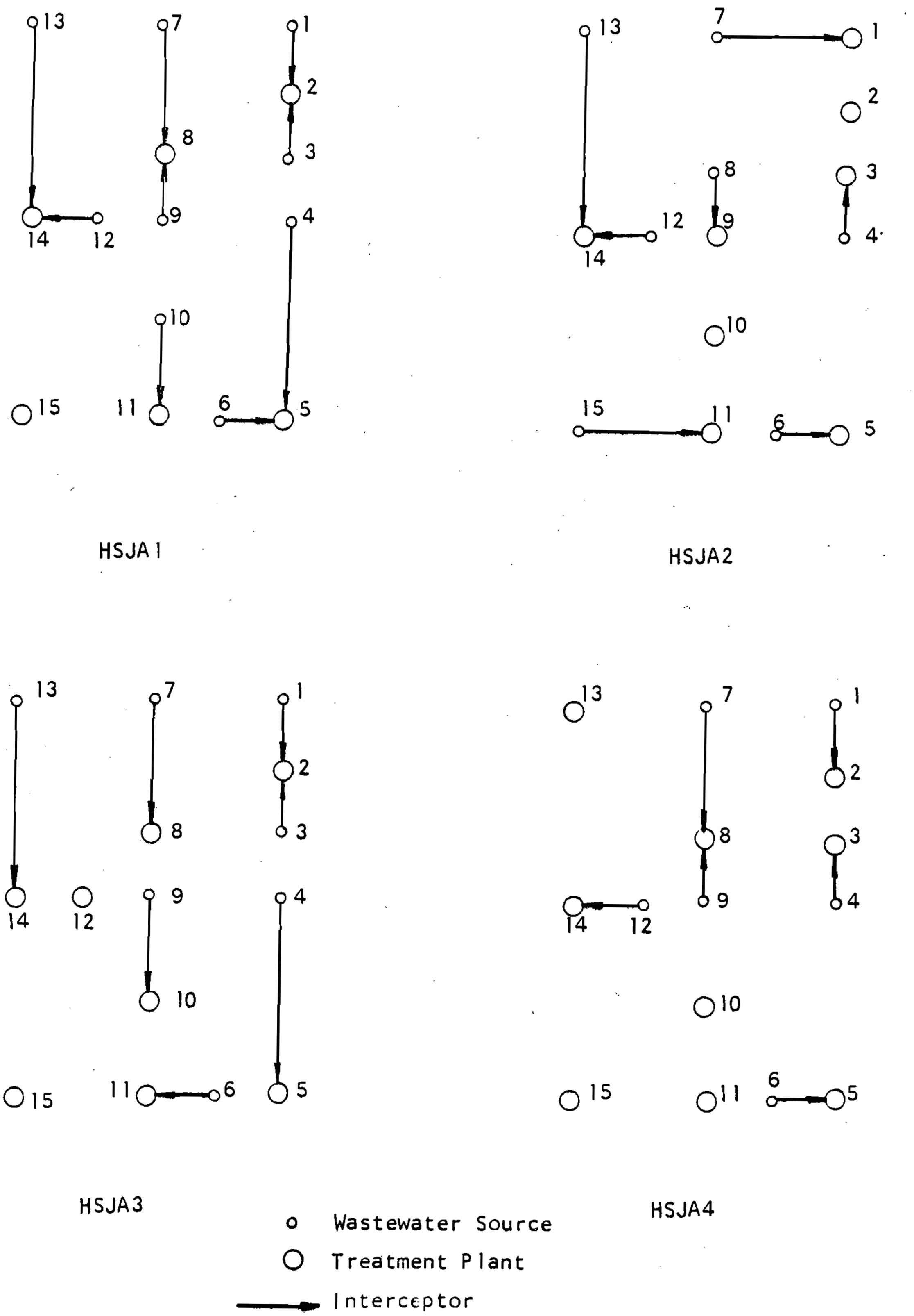

Figure 3.3 Configurations of Alternatives Generated by the HSJ Method with 5% Cost Relaxation

difference between the new alternative and all previous alternatives decreases as additional new alternatives are generated. The cost of alternatives HSJA2 through HSJA4 is close to the target so that the ability to generate different alternatives is limited by the cost constraint.

It can be seen from Figure 3.3 that the four alternatives are different from one another. For example, HSJA1 has the least number of plants (6 plants), while HSJA4 has the largest number of plants (9 plants); and the plants are evenly distributed, i.e., the same number of plants are located in east, west, and central part of the region. Note that the number of plants tends to increase (6 for HSJA1, 8 for HSJA2 and HSJA3, and 9 for HSJA4) while the number of interceptors tends to decrease (9 for HSJA1, 7 for HSJA2 and HSJA3, and 6 for HSJA4). Thus, more decentralized solutions were obtained. However, this tendency is much less pronounced than in the 10% case.

The pairwise differences between any two of the four alternatives, as measured by the sum of different plant and interceptor capacities and by the sum of different plant and interceptor locations, are shown in Table 3.4. Again, the least value in each row is taken as the difference between the new solution and all previous solutions. Thus the difference among

Table 3.4 Differences among the Three Alternatives Generated by the HSJ Method with 5% Cost Relaxation

Alternative	HSJA1	HSJA2	HSJA3	HSJA4
HSJA1	0			
HSJA2	118(16)*	0		
HSJA3	66(8)	119(18)	0	
HSJA4	66(8)	74(12)	67(12)	0

* a (b)
a: measured by the sum of different plant and interceptor capacities (cfs).
b: measured by the sum of different plant and interceptor locations (unit); the addition or removal of any plant or interceptor contributes 1 to the sum.

these alternatives, as measured by the sum of different plant and interceptor capacities, decreases from 118 cfs for HSJA2 to 66 cfs for HSJA3 and HSJA4, respectively. Then it drops to zero for HSJA5. The difference among these alternatives as measured by the sum of different plant and interceptor locations drops from 16 for HSJA2 to 8 for HSJA3 and HSJA4, respectively; then it drops to zero for HSJA5. Both methods of measuring difference indicate that there are significant differences among the first four alternatives and no differences between the fifth one and the fourth one. But the levels of difference among alternatives are less significant than in the 10% case.

If the target on the cost objective is relaxed to $2,300,000 (20% more than the optimal cost), then three alternatives are obtained by the HSJ approach. The second alternative is exactly the same as that obtained with 10% cost relaxation, indicating that further relaxation of the cost objective from 10% to 20% of the optimal cost does not have any effect in obtaining the second solution. The third alternative with cost $2,125,000, however, cannot be obtained in the 10% relaxation case. It is interesting to see that plants are located in 12 potential locations for the third solution. Consequently all interceptors, except those constrained to be in the solution, are out of the solution. The cost of the third solution is 11.1% higher than that of the initial solution. The differences between the third solution in the 20% case and that in the 10% case is that in the 20% case, because of the further cost relaxation, the HSJ objective function value is further reduced by eliminating interceptor 13-14 and adding plant 13 to treat the waste originating at that source.

The differences among the sets of alternatives generated in the 5%, 10%, and 20% relaxation cases, as measured by the sum of different plant and interceptor capacities, are plotted in Figure 3.4; the differences, as measured by the sum of different plant and interceptor locations, are plotted in Figure 3.5. It can be seen that the differences measured by both methods have the same trend for each of the three relaxation cases. The difference for HSJ2 is the same in the 10% and 20% cases. The difference decreases rapidly for HSJ3, and then the procedure stops because no different alternative can be generated. For the 5% case, the difference is lower than that of 10% and 20% cases for HSJ2 -- 118 cfs versus 157 cfs as measured by the sum of different plant and interceptor capacities and 16 versus 23 as measured by the sum of different plant and interceptor locations. For HSJ3, the difference in the 5% case is slightly lower than that for the 10% and 20% cases. Unlike the 10% and 20% cases, however, the HSJ iterations continue and generate HSJ4, which has the same level

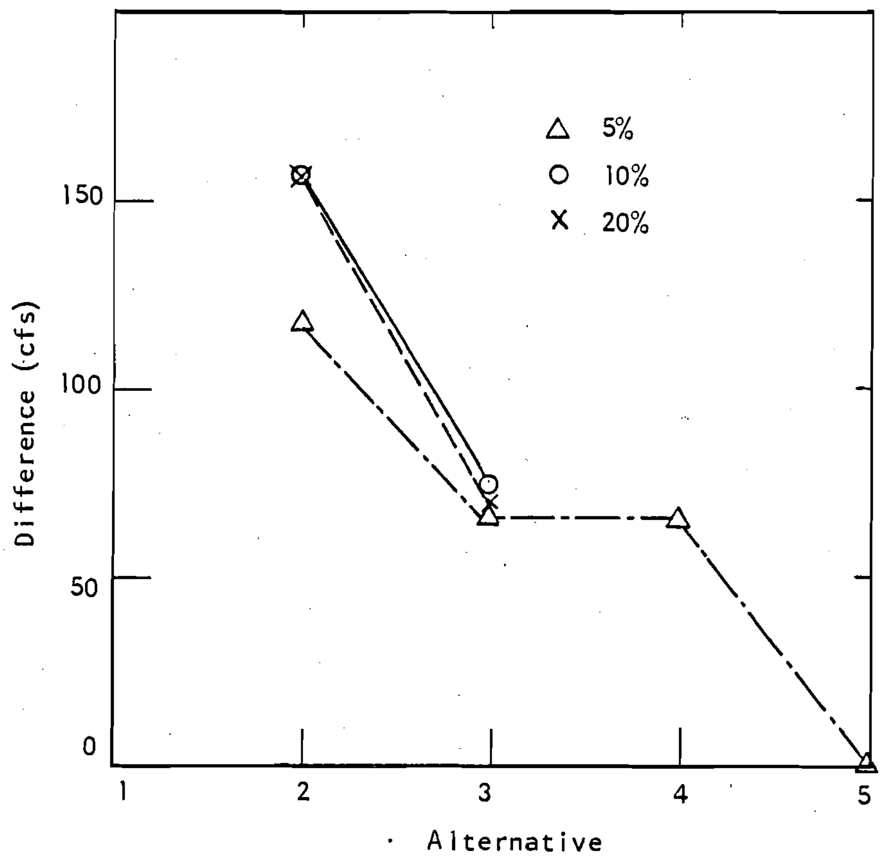

Figure 3.4 Differences among Alternatives Generated by the HSJ Method as Measured by the Sum of Different Plant and Interceptor Capacities

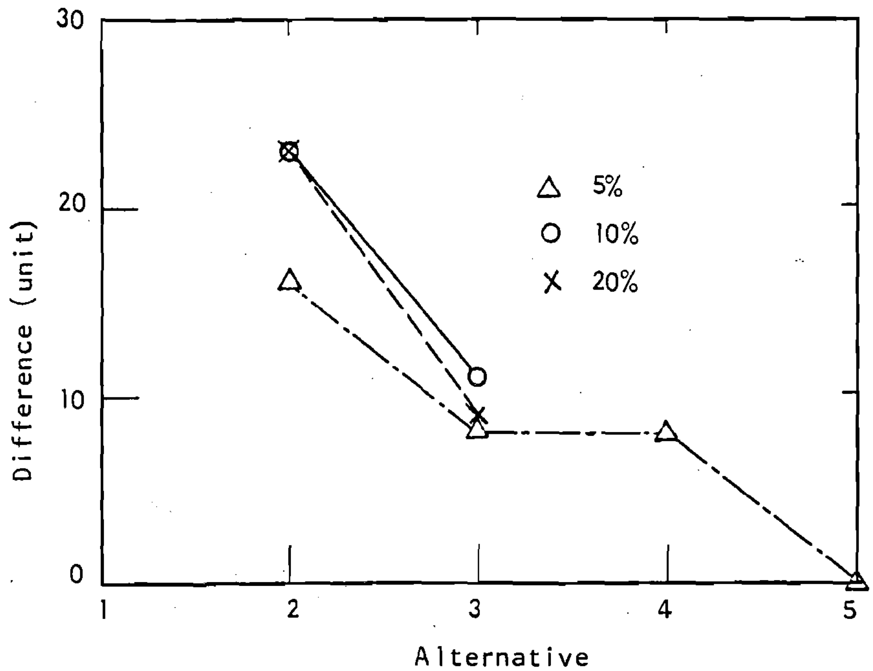

Figure 3.5 Differences among Alternatives Generated by the HSJ Method as Measured by the Sum of Different Plant and Interceptor Locations

of difference as HSJ3. The fifth alternative, however, is the same as HSJ4, so the difference is zero.

Figures 3.4 and 3.5 indicate that the differences among alternatives are not greatly affected by relaxing the cost from 10% to 20% of the optimal cost. When the relaxation is restricted to 5% of the optimal cost, however, more alternatives are generated, but the difference is smaller between the first two alternatives. Note that for HSJ3, the difference is slightly higher for the 10% relaxation than that for the 20% relaxation, indicating that the relaxation from 10% to 20% does not necessarily increase the difference among alternatives as measured by these two methods of measuring differences. This result indicates slight inconsistency between the HSJ surrogate difference objective function and the two methods used for measuring differences.

3.2.3 Variations of the HSJ Method

In Sections 3.2.1 and 3.2.2, continuous variables alone were used to drive HSJ iterations. This section discusses using the zero-one variables associated with the plant and interceptor locations in the surrogate difference objective function. Again, the target on the cost objective was set at $2,108,000 (10%

relaxation). Three alternatives were obtained, and the plant and interceptor locations and the cost of each alternative are shown in Table 3.5. The first two alternatives are exactly the same, respectively, as those obtained by minimizing the sum of continuous variables (see Section 3.2.1). The third alternative, however, has a lower cost than the corresponding alternative obtained using continuous variables alone. The difference between alternatives 1 and 3 is that in alternative 1 the wastewater flow originated from source 12 is routed to plant 14 for treatment while in alternative 3 the same flow is treated at plant 12. The differences between the first and the third alternatives, as measured by the sum of different plant and interceptor capacities and by the sum of different locations, are 20 cfs and 2, respectively. Both measures indicate the difference between alternatives 1 and 3 is relatively insignificant.

The third alternative (HSJC3) obtained by minimizing the sum of continuous variables as shown in Section 3.2.1, however, is significantly different from the first alternative with respect to the locations of both plants and interceptors. Compared to that alternative, the third alternative obtained by minimizing the sum of zero-one variables has fewer plants (7 vs. 11) but has more interceptors (8 vs. 4). However, the total number of plants

Table 3.5 Plant and Interceptor Locations and Total Annual Costs for the Alternatives Generated by the HSJ Method Using Zero-One Variables in the HSJ Objective Function

Alternative		Plant and Interceptor Locations	Annual Cost ($1000)
HSJC1	Plant	(2, 5, 11, 14)*, 8,15	1917
	Inte.	1-2, 3-2, 4-5, 6-5, 7-8, 9-8, 10-11, 12-14, 13-14	
HSJC2	Plant	(2, 5, 11, 14), 1, 3, 9, 10, 13	2076
	Inte.	7-1, 4-3, 6-11, 8-9, 12-9, 9-10, 15-11	
HSJC3	Plant	(2, 5, 11, 14), 8, 12, 15	1964
	Inte.	1-2, 3-2, 4-5, 6-5, 7-8, 9-8, 10-11, 13-14	

* plants 2, 5, 11 and 14 must be in the solution because of constraints

and interceptors is the same (15) for both alternatives. Since the zero-one variables associated with plants and interceptors are equally weighted, the minimization of zero-one variables simply reduces the total number of facilities that appear in previous solutions. Thus, when the total number of such plants and interceptors cannot be reduced, the solution with 15 facilities and the lowest cost ($1,964,000) is obtained. The minimization of continuous variables in the HSJ objective function, however, tends to reduce the number of interceptors further to reduce the total capacity of facilities even though that will also increase the number of plants.

The above results show that even though the zero-one variables are related to the values of the corresponding continuous variables, the use of zero-one variables alone in the HSJ objective function may not necessarily generate the same set of alternatives as that generated by using continuous variables. Also, the use of zero-one variables in the objective function does not produce a driving force to reduce the capacities of plants and interceptors if those capacities cannot be reduced to zero. Thus, this approach appears to be less effective than the use of continuous variables in the HSJ objective function.

Instead of minimizing the sum of nonzero variables in all previous solutions for each HSJ iteration, maximizing the sum of zero variables in all preceeding solutions was also examined using continuous variables and zero-one variables in the HSJ objective function. In each case, three alternatives were generated. The plant and interceptor locations and the cost for each set of alternatives are shown in Table 3.6. Note that the third alternative (HSJD3) is similar to the initial solution. HSJD3 is actually formed by adding plant 12 and eliminating interceptor 12-14 from the initial solution. The inability to obtain a different third alternative results from the fact that only one variable (plant 12) has not appeared in previous solutions, the maximization of the capacity of plant 12 simply adds that plant to treat its own waste and removes interceptor 12-14. The minimization of nonzero variables, however, as shown in Section 3.2.1 drives most of the interceptors out of the solution so that the total capacity is reduced and more difference is obtained.

A problem with maximizing the sum of nonbasic (zero) variables to drive the HSJ iterations is that some unnecessary variables may come into the solution; as a result the cost of the solution is unnecessarily increased. As an example, the plant locations of alternative HSJD2 are exactly the same as the plant locations of

Table 3.6 Plant and Interceptor Locations and Total Annual Costs for the Alternatives Generated by the HSJ Method by Maximizing the Sum of Zero-One Nonbasic Variables

Alternative		Plant and Interceptor Locations	Annual Cost ($1000)
I. Maximizing Zero Continuous Variables			
HSJD1	Plant	(2, 5, 11, 14)*, 8,15	1917
	Inte.	1-2, 3-2, 4-5, 6-5, 7-8, 9-8, 10-11, 12-14, 13-14	
HSJD2	Plant	(2, 5, 11, 14), 1, 3, 9, 10, 13	2085
	Inte.	7-1, 4-3, 6-11, 8-9, 12-9, 9-8, 9-10, 15-11	
HSJD3	Plant	(2, 5, 11, 14), 8, 12, 15	1964
	Inte.	1-2, 3-2, 4-5, 6-5, 7-8, 9-8, 10-11, 13-14	
II. Maximizing Zero 0,1 Variables			
HSJE1	Plant	(2, 5, 11, 14)*, 8, 15	1917
	Inte.	1-2, 3-2, 4-5, 6-5, 7-8, 9-8, 10-11, 12-14, 13-14	
HSJE2	Plant	(2, 5, 11, 14), 1, 3, 9, 10, 12, 13	2108
	Inte.	1-2, 3-2, 4-3, 7-1, 6-11, 8-9, 9-10, 15-11	
HSJE3	Plant	(2, 5, 11, 14), 8, 15	1937
	Inte.	1-2, 3-2, 4-5, 6-5, 7-8, 9-8, 10-11, 12-14, 13-14	

* plants 2, 5, 11 and 14 must be in the solution because of constraints

alternative HSJ2 obtained by minimizing the basic continuous variables (see Tables 3.9 and 3.1). However, in HSJD2, the capacity of the interceptor 8-9 is 7.9 cfs and the capacity of interceptor 9-8 is 1.0 cfs. It is obvious that 1.0 cfs of the capacity of each interceptor and the zero-one variable for interceptor 9-8 are unnecessary. For another example, in HSJE2, the capacity of the interceptor from source 12 to source 9 is zero, but the zero-one variable associated with that interceptor capacity is one. All these unnecessary variables enter the solution because they will increase the HSJ objective function value. The presence of the cost objective in the overall objective function does not prevent unnecessary variables from entering the solution because the cost objective has been multiplied by a small scaling factor to avoid affecting the surrogate difference objective. These unnecessary variables will increase the cost of the solution obtained but will not produce important differences. So if the sum of nonbasic variables is used to drive the HSJ iterations, one more step is needed to remove unnecessary variables after the solution is obtained. For such cases, there appears to be no advantage to maximizing the sum of nonbasic variables in comparison to minimizing the sum of basic variables.

Since the capacities of the interceptors are related to the capacities of the plants, it may be simpler to use the variables associated with plant capacities alone to drive the HSJ iterations. If the sum of all nonzero plant capacities in all previous solutions is minimized, four alternatives are obtained. The fourth alternative is exactly the same as the initial solution. The plant and interceptor locations and the cost of the three alternatives are shown in Table 3.7. Since only the variables associated with plant capacities are used in the HSJ objective function, not all interceptors in the initial solution are removed (interceptor 6-5 is still in HSJF2). As discussed in Section 3.2.1, if continuous variables associated with both plants and interceptors are used in the HSJ objective function, all interceptors in the initial solution are removed. The third alternative (HSJF3) has one more plant than the initial solution (HSJF1), indicating the difference between these two alternatives may be small.

The pairwise difference between any two alternatives for these three alternatives, as measured by the sum of different plant and interceptor capacities and by the sum of different locations, were calculated. The difference as measured by the sum of different plant and interceptor capacities is 154 cfs for HSJF2, then it drops sharply to 20 cfs for HSJF3. The difference as

Table 3.7 Plant and Interceptor Locations and Total Annual Costs for the Alternatives Generated by the HSJ Method Using Continuous Variables Associated with Plant Only in the HSJ Objective Function

Alternative		Plant and Interceptor Locations	Annual Cost ($1000)
HSJF1	Plant	(2, 5, 11, 14)*, 8, 15	1917
	Inte.	1-2, 3-2, 4-5, 6-5, 7-8, 9-8, 10-11, 12-14, 13-14	
HSJF2	Plant	(2, 5, 11, 14), 1, 3, 9, 10, 13	2068
	Inte.	7-1, 4-3, 7-1, 6-5, 8-9, 9-10, 12-9, 15-11	
HSJF3	Plant	(2, 5, 11, 14), 8, 12, 15	1964
	Inte.	1-2, 3-2, 4-5, 6-5, 7-8, 9-8, 10-11, 13-14	

* plants 2, 5, 11 and 14 must be in the solution because of constraints

measured by the sum of different plant and interceptor locations has the same trend; it starts with 21 for HSJF2 and drops sharply to 2 for HSJF3. The difference for HSJF3 is much lower than that for HSJ3 (obtained by minimizing the sum of all nonzero continuous variables), indicating that the use of the continuous variables associated with plant capacities alone is less powerful than the use of all continuous variables.

If maximizing the sum of nonbasic variables instead of minimizing the sum of basic variables is applied using only the plant capacity variables, then exactly the same results are obtained. Since the total wastewater flow to be treated is fixed, the total reduction of capacities of some plants must cause an equal increase in the total capacities of other plants. Hence the minimization of the sum of basic variables is equivalent to the maximization of the sum of nonbasic variables. Note that in this case, the maximization of the sum of nonbasic variables will not bring in any unnecessary variables as discussed in the previous section if the cost objective is included in the objective function with a scaling factor.

As another experiment, the zero-one variables associated with plant locations alone were used in the HSJ objective function. Three alternatives were obtained, but the differences are much less than those obtained using the other HSJ objective functions.

3.2.4 Summary Results

The above results show that the HSJ method can be used to generate good alternatives that are significantly different from one another. The number of alternatives generated is limited and is dependent on the cost target specified. If a relatively tight target is specified, the difference among alternatives is reduced but more alternatives were obtained. For this example problem, the use of continuous variables associated with both plants and interceptors as the surrogate difference in the HSJ objective function is more effective than the use of continuous variables associated with plants only or the use of zero-one variables associated with both plants and interceptors in obtaining different alternatives. Also, minimizing the nonzero variables in previous solutions is more efficient than maximizing the zero variables because the latter approach may introduce unnecessary variables into the solution.

3.3 Generating Alternatives Using the Random Method

3.3.1 Overview

One obvious method to generate alternatives for this wastewater system planning problem is to select plant locations randomly from the potential plant locations. Then the capacity of each plant and the locations and sizes of the interceptors can be judgementally assigned. One problem with this approach is that the potential solution is often infeasible. Also, if by chance a feasible solution is generated, it may not be good with respect to the objective included in the model. If the interceptor locations are also generated randomly, the chance of obtaining a feasible solution is even less. The likelihood of feasibility can be increased if the locations where plants must be located are recognized.

For the example problem, there are 12 candidate plant locations. The number of total possible solutions is 4,095 with respect to plant locations alone. If the four locations where plants must be located are recognized, however, the total number of possible solutions is drastically reduced to 255. The APEX MIP code which employs a branch and bound search obtained 32 solutions that are good with respect to the cost objective -- within 10% of the optimal cost, but eight of them have the same plant locations. On the other hand, there may be good solutions that cannot be found by the APEX code because it will not continue to search along a branch where an integer solution has been found.

Assuming that there are 32 good solutions, the probability of finding one randomly is approximately 1/8. Thus, approximately 40 random solutions may be required to provide five good solutions; the probability of generating five good solutions consecutively is only .000023. Furthermore, there is no guarantee that all five good solutions are different from one another. When the four plant locations where plants must be located are not recognized, the probability of obtaining one good solution is 1/128 (32/4095). Then 640 random solutions may be required to provide approximately five good solutions, and it is almost impossible (1 in 34,000 million) to generate five good solutions consecutively. So if no guidance is followed, the random generation method may be very inefficient for generating feasible and good solutions.

For the MIP example probelm, a random generation method like the one described in Section 2.2 can be used. A set of continuous decision variables is randomly generated, and then the sum of the randomly generated decision variables (SORD) is maximized. The cost objective is constrained to meet a specified target. The number of randomly generated continuous decision variables can be fixed or randomly determined.

3.3.2 Results of the Random Method

For the first experiment, the number of continuous decision variables to be randomly generated was arbitrarily fixed at four (there are 28 continuous decision variables). The target for the cost objective was specified at $2,108,000 (10% greater than the optimal cost). Ten alternatives were obtained by maximizing the sum of the 10 sets of decision variables. The plant and interceptor locations and the cost of each alternative are given in Table 3.8. The cost of these alternatives does not vary significantly; the highest cost ($2,018,000) is only 3.7% higher than the lowest cost ($1,943,000) and is only 5.3% higher than the optimal cost ($1,917,000). Note that alternative 3 is exactly the same as alternative 2. Even though the probability of randomly generating the same set of decision variables is very small (in this case .0022), the probability that the same two solutions result from the optimization step may be much larger because of the constraints.

Since there are 28 continuous decision variables, and since each alternative is obtained by maximizing the sum of four different randomly specified variables, the probability that a certain variable is not specified in 10 alternatives is .214 ($(24/28)^{10}$). This probability can be reduced by increasing the number of

Table 3.8 Plant and Interceptor Locations and Total Annual Costs for the Alternatives Generated by the Random Method

Alternative		Plant and Interceptor Locations	Annual Cost ($1000)
RM1	Plant	(2, 5, 11, 14)*, 3, 9, 15	1991
	Inte.	1-2, 4-3, 6-5, 7-8, 8-9, 10-11, 12-9, 12-14, 13-14	
RM2	Plant	(2, 5, 11, 14), 9, 10, 12	1997
	Inte.	1-2, 3-2, 4-5, 6-5, 7-8, 8-9, 9-10, 15-11, 13-14	
RM3	Plant	(2, 5, 11, 14), 9, 10, 12	1997
	Inte.	1-2, 3-2, 4-5, 6-5, 7-8, 8-9, 9-10, 15-11, 13-14	
RM4	Plant	(2, 5, 11, 14), 8, 10	1990
	Inte.	1-2, 3-2, 4-3, 4-5, 7-1, 6-5, 7-8, 9-8, 9-10, 12-9, 15-11, 13-14	
RM5	Plant	(2, 5, 11, 14), 9, 13, 15	1943
	Inte.	1-2, 3-2, 4-3, 4-5, 6-5, 7-8, 8-9, 10-11, 12-14	
RM6	Plant	(2, 5, 11, 14), 3, 8, 13, 15	1988
	Inte.	1-2, 4-3, 6-11, 7-8, 9-10, 10-11, 12-14	
RM7	Plant	(2, 5, 11, 14), 8, 15	1946
	Inte.	1-2, 3-2, 4-5, 6-5, 7-8, 9-8, 10-11, 12-14, 15-11, 13-14	
RM8	Plant	(2, 5, 11, 14), 3, 8, 15	1953
	Inte.	1-2, 3-2, 4-3, 6-5, 7-8, 9-8, 10-11, 12-14, 13-14	
RM9	Plant	(2, 5, 11, 14), 3, 9, 10	2018
	Inte.	1-2, 3-2, 4-3, 6-5, 7-8, 8-9, 9-10, 12-9, 15-11, 13-14	
RM10	Plant	(2, 5, 11, 14), 1, 8, 10	1976
	Inte.	1-2, 3-2, 4-5, 7-1, 6-11, 9-8, 12-14, 13-14, 15-11	

* plants 2, 5, 11 and 14 must be in the solution because of constraints

randomly specified continuous variables or by increasing the number of alternatives generated.

Note that the number of plants specified by each alternative is between 6 and 8. It at first seems that they are not very different from one another. However, since plants 2, 5, 11, and 14 must be used, the differences among the alternatives are greater. For example, in addition to the four required plants, alternative 4 locates plants at sources 8 and 10, while alternative 5 specifies locations 9, 13, and 15 -- a completely different set. The number of interceptors for the ten alternatives, on the other hand, varies from 7 to 12. Interceptor 1-2 appears in each alternative, while interceptors 3-2, 6-5, 7-8, and 13-14 appear in at least eight of the ten alternatives. Interceptors 7-1 and 6-11 appear in only two alternatives.

The pairwise differences between any two of the ten alternatives, as measured by the sum of different plant and interceptor capacities and by the sum of different plant and interceptor locations, are shown in Table 3.9. Note that the alternatives have been reordered according to the difference measured by the sum of different plant and interceptor capacities. The least value in each row is taken as the difference between the new

Table 3.9 Differences among the Ten Alternatives Generated by the Random Method

	Alternative									
	6	9	10	5	1	4	8	2	7	3
6	0									
9	131 (14)*	0								
10	90 (15)	73 (15)	0							
5	72 (9)	72 (11)	81 (16)	0						
1	56 (9)	82 (7)	115 (18)	44 (6)	0					
4	117 (15)	44 (7)	42 (8)	105 (14)	127 (14)	0				
8	53 (10)	99 (11)	46 (12)	41 (8)	71 (6)	73 (10)	0			
2	129 (17)	24 (5)	65 (12)	72 (10)	99 (12)	48 (8)	98 (14)	0		
7	65 (11)	99 (13)	38 (8)	57 (8)	93 (10)	65 (8)	22 (4)	87 (10)	0	
3	129 (17)	24 (5)	65 (12)	72 (10)	99 (12)	48 (8)	98 (14)	0 (0)	87 (10)	0

* a (b)

a: measured by the sum of different plant and interceptor capacities (cfs).

b: measured by the sum of different plant and interceptor locations (unit); the addition or removal of any plant or interceptor contributes 1 to the sum.

solution and all previous solutions and is plotted in Figure 3.6. The difference as measured by the sum of different capacities decreases sharply from 131 cfs for the second alternative to 73 cfs for the third alternative and remains approximately the same for the fourth alternative. After that, the difference decreases to 44 cfs for the fifth alternative, then it gradually decreases to zero for the 10th alternative.

The difference as measured by the sum of different plant and interceptor locations, on the other hand, is not strictly monotonically decreasing. It starts with 14 for the second alternative and then increases to 15 for the third alternative. In general, however, even though there exist some discrepancies, the two methods of measuring difference indicate the same trend.

The configurations of the four most different alternatives, as measured by each measure are shown in Figure 3.7. It can be observed that these four alternatives are quite different from one another. For example, in addition to the plants at sources 2, 5, 11, and 14, alternative 6 locates plant at sources 3, 8, 13, and 15; while alternative 9 locates plants at sources 3, 9, and 10. Some of the interceptor locations are also different. For example, alternative 6 has seven interceptors while alternative 9 has ten. Among them, four interceptors appear in

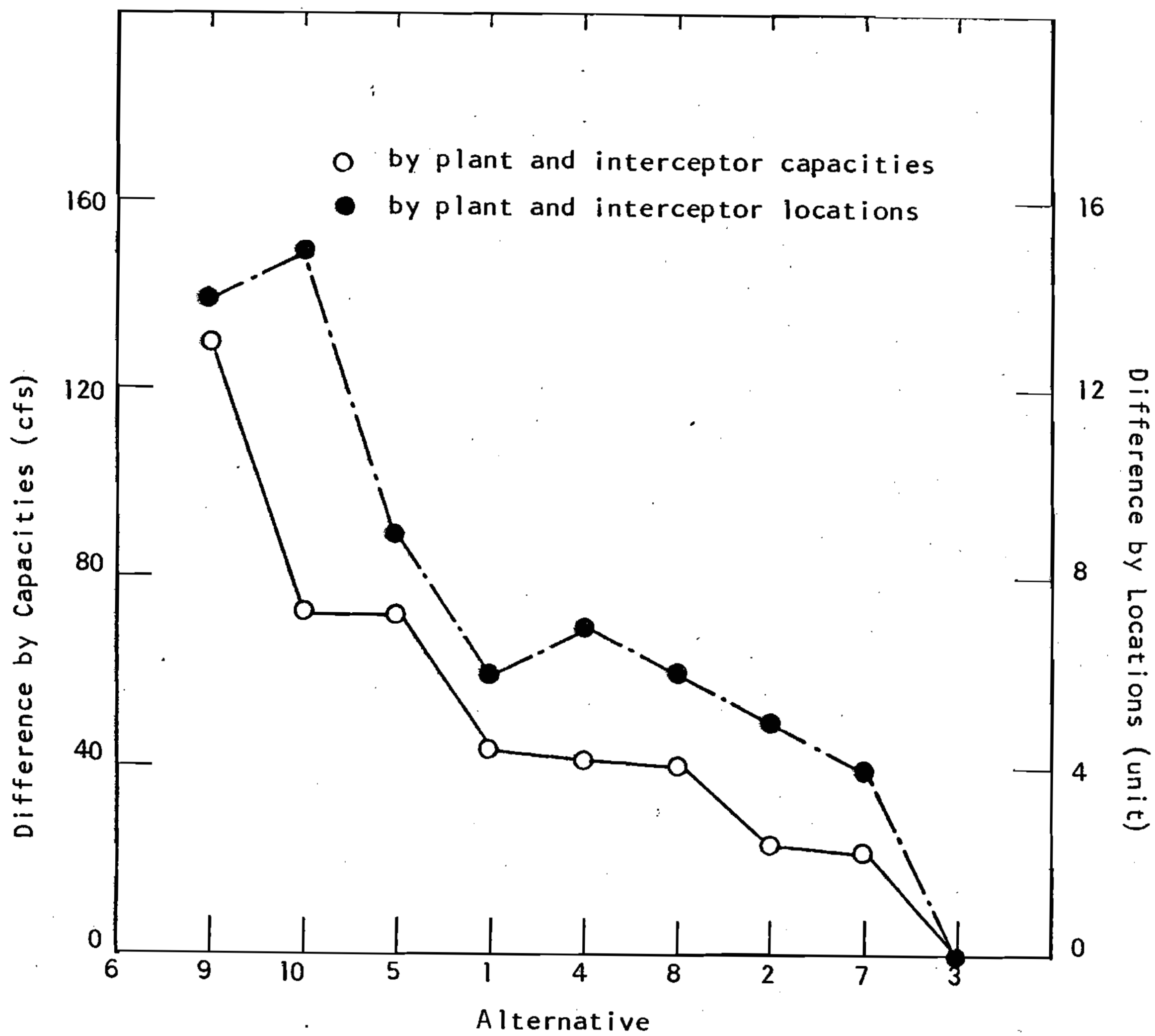

Figure 3.6 Differences among Ten Alternatives Generated by the Random Method

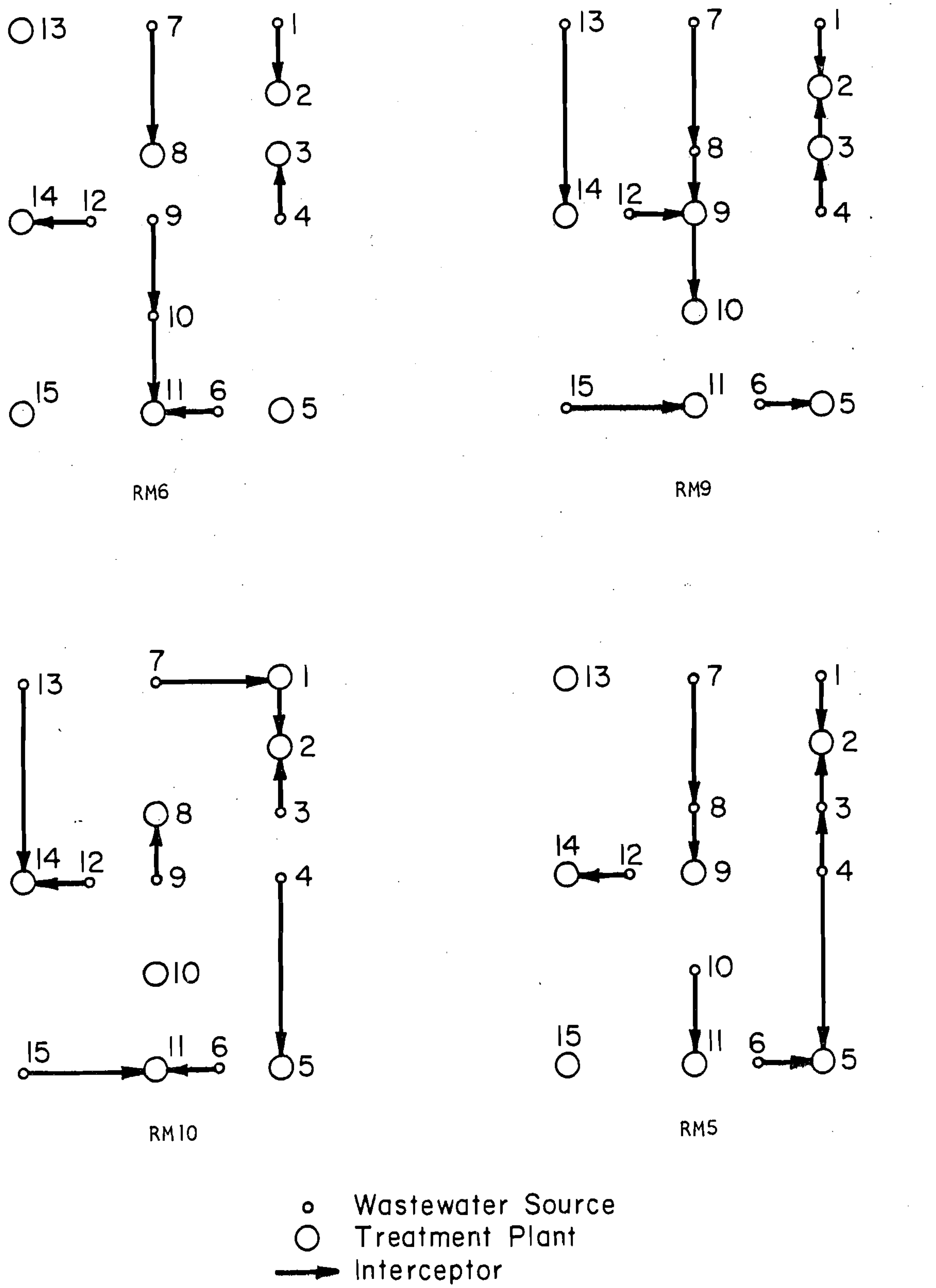

Figure 3.7 Configurations of the Four Alternatives Generated by the Random Method with 10% Cost Relaxation

each solution. Note, in alternatives 5 and 9, there are some split flows. In alternative 9, for example, although a plant is located at source 3, there is still an interceptor that transports some of the waste flow from source 3 to source 2. In alternative 5, one part of the waste flow originated from source 4 is shifted to node 3 then to plant 2, the other part is shifted to plant 5.

3.3.3 Variations of the Random Method

As discussed in Secton 3.2.3, in the HSJ approach the maximization of nonbasic variables may introduce unnecessary variables in the solution that increase the cost. This phenomenon may also occur in the random generation method if the sum of randomly generated decision variable values, SORD, is maximized. This result occurs, for example, for interceptors that allow flow in both directions (e.g., interceptor 8-9 and 9-8 in the example problem); if the decision variables associated with such an interceptor are maximized, flow can result in both directions. Of course, constraints can be specified to allow flow in only one direction, but that would increase the complexity of the model. If the SORD function is minimized instead of maximized, then unnecesssary variables will not appear in the solution. Ten alternatives were randomly generated by minimizing the sum of ten sets of randomly specified continuous variables. As before, the number of variables selected was four, and the cost target was $2,108,000.

The plant and interceptor locations and the cost of each alternative are shown in Table 3.10. Alternative 2 is exactly the same as alternative 1 even though the randomly specified variables are not exactly the same. The number of plants varies from 6 to 9, and there are two alternatives with 9 plants. In the maximization case, no solutions with more than 8 plants were obtained. The number of interceptors varies from 6 to 9, indicating a reduction in the number of interceptors compared to the maximization case (7 to 12 interceptors). The reduction may be caused in part by the fact that split flows are less likely in the minimization case. For example, alternative 7 obtained in the maximization case (Table 3.8) and alternative 1 obtained in the minimization case (Table 3.10) have the same plant locations. Alternative 7 includes interceptor 15-11 which shifts part of the flow from source 15 to source 11, while for alternative 1 that interceptor is not in the solution.

The pairwise differences between any two of the ten alternatives, as measured by the sum of different plant and interceptor capacities and by the sum of different plant and interceptor

Table 3.10 Plant and Interceptor Locations and Total Annual Costs for the Alternatives Generated by the Random Method by Minimizing the Sum of Decision Variables

Alternative		Plant and Interceptor Locations	Annual Cost ($1000)
RN1	Plant	(2, 5, 11, 14)*, 8, 15	1924
	Inte.	1-2, 3-2, 4-5, 6-11, 7-8, 9-8, 10-11, 12-14 13-14	
RN2	Plant	(2, 5, 11, 14), 8, 15	1924
	Inte.	1-2, 3-2, 4-5, 6-11, 7-8, 9-8, 10-11, 12-14, 13-14	
RN3	Plant	(2, 5, 11, 14), 3, 8, 15	1953
	Inte.	1-2, 3-2, 4-3, 6-5, 7-8, 9-8, 10-11, 12-14, 13-14	
RN4	Plant	(2, 5, 11, 14), 1, 3, 8, 15	1988
	Inte.	4-3, 7-1, 6-5, 9-8, 10-11, 12-14, 13-14	
RN5	Plant	(2, 5, 11, 14), 1, 3, 8, 12, 15	2036
	Inte.	4-3, 7-1, 6-5, 9-8, 10-11, 13-14	
RN6	Plant	(2, 5, 11, 14), 8, 10	1937
	Inte.	1-2, 3-2, 4-5, 6-5, 7-8, 9-8, 12-14, 15-11, 13-14	
RN7	Plant	(2, 5, 11, 14), 3, 9, 10, 13, 15	2062
	Inte.	1-2, 3-2, 4-3, 6-11, 7-8, 8-9, 9-10, 12-9	
RN8	Plant	(2, 5, 11, 14), 1, 9, 15	1996
	Inte.	3-2, 4-5, 7-1, 6-5, 8-9, 9-10, 10-11, 12-9, 13-14	
RN9	Plant	(2, 5, 11, 14), 1, 8, 15	1868
	Inte.	3-2, 4-5, 7-1, 6-5, 9-8, 10-11, 12-14, 13-14	
RN10	Plant	(2, 5, 11, 14), 8, 12, 12, 15	1901
	Inte.	1-2, 3-2, 4-5, 6-5, 7-8, 9-8, 10-11	

* plants 2, 5, 11 and 14 must be in the solution because of constraints

locations, were calculated. Again, the order of the alternatives has been rearranged according to the first difference measurement, and the least value in each row is plotted in Figure 3.8. The difference, as measured by the sum of different plant and interceptor capacities, starts from 148 cfs for the second alternative, then it sharply drops to 88 cfs for the third alternative. After that, the difference decreases approximately linearly to zero for the tenth alternative. The difference as measured by the different plant and interceptor locations shows approximately the same shape.

The differences, as measured by the sum of different plant and interceptor capacities, for the ten alternatives generated by maximizing the SORD function is also plotted in Figure 3.8. Even though the alternatives generated by minimization of SORD have slightly larger differences up to the fifth alternative, while the alternatives generated by maximization of SORD have slightly larger differences from the sixth to the ninth alternative, the two profiles show the same trend and are similar. The minimization approach appears as effective as the maximization approach and has less chance of generating solutions with split flows.

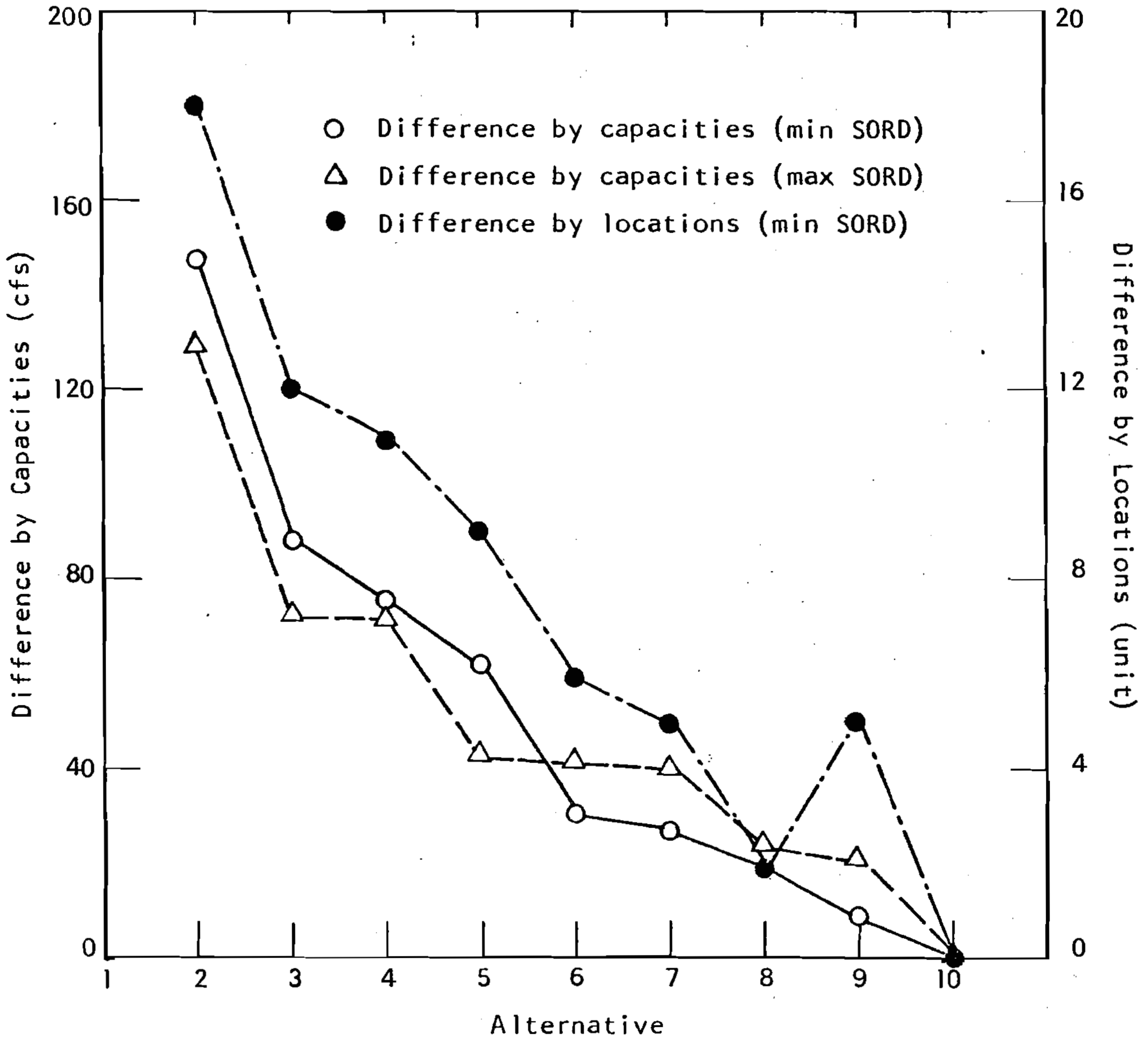

Figure 3.8 Differences among Ten Alternatives Generated by the Random Method Using Minimization of the SORD Function

The configurations of the four most different alternatives based on the two measurements of difference are shown in Figure 3.9. These alternatives are quite different from one another. For example, in addition to plants 2, 5, 11, and 14, which are constrained to be in the solution, alternative 6 locates plants at sources 8 and 10, while alternative 8 locates plants at sources 1, 9, and 15. These two alternatives are similar, however, with respect to the facilities associated with sources 4, 5, 6, 12, 13, and 14.

In the previous experiment, continuous variables associated with the capacities of both plants and interceptors were randomly specified to generate alternatives. Since the capacity of an interceptor is related to the capacity of a plant, it may be more efficient to use only the variables associated with the capacities of the plants. To examine this approach, the number of randomly specified variables was fixed at four, and the target for the cost objective was specified at $2,108,000. Ten alternatives were generated by maximizing ten sets of variables. The level of differences among these alternatives, as measured by the sum of different plant and interceptor capacities and by the sum of different locations, was comparable to that of the alternatives obtained in using variables associated with both plant and interceptor capacities.

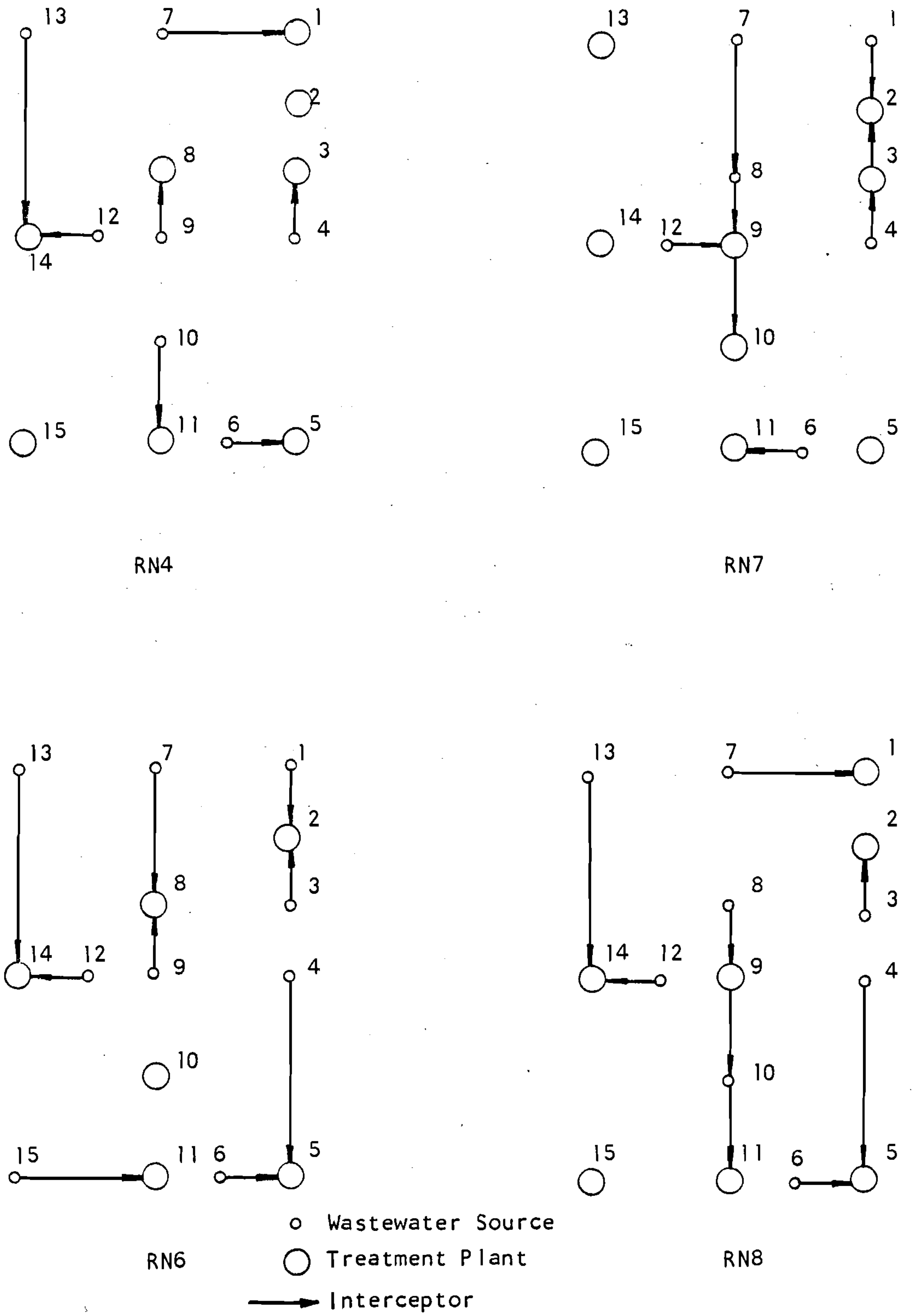

Figure 3.9 Configurations of the Four Alternatives Generated by the Random Method Using Minimization of the SORD Function

3.3.4 Summary

The above results show that the random generation method can be used to generate many alternatives that are good and significantly different for the example problem. The minimization of the sum of randomly generated decision variable values, SORD, can generate different alternatives as effectively as the maximization approach. The minimization approach also has the advantage of not generating solutions with unnecessary variables in them. The use of continuous variables associated with plant capacities alone in the SORD objective function also generated different solutions effectively.

3.4 Generating Alternatives Using a BBS Method

3.4.1 Introduction

The HSJ and random methods are designed to generate good and different solutions directly. In contrast, the approach discussed in this section is a two-step approach. The first step is to generate many potential solutions using an available computer code, and then to apply a screening process to select a set of "good" and "different" solutions.

For a single objective mixed integer programming (MIP) formulation, the APEX MIP code can be used to obtain feasible solutions within a certain limit of the objective function value (specified before execution). If the limit is set within an acceptable value, then all solutions obtained will be good with respect to the objective in the model. The solutions obtained, however, may be similar and a screening process is needed to select a set of different solutions. The APEX MIP code, available for the CDC Cyber, implements a branch-and-bound process, and it will specify only solutions that can be obtained using the corresponding mathematical steps. Not all solutions will be obtained; for example, once a node solution is integer only that solution will be specified, although there may be alternatives along that branch.

3.4.2 Procedures and Results

By setting the cost limit at $2,108,700 (10% higher than the cost of the least cost solution), 278 nodes were explored and 32 feasible solutions were obtained using the APEX MIP code. The computer time used was 18.3 seconds on the CDC Cyber 175 at the University of Illinois. The plant locations and the total cost of each solution are listed in Table 3.11. All of these solutions are feasible and acceptable with respect to the cost

Table 3.11 Plant Locations and Total Annual Costs for Alternatives Generated by APEX MIP Code

Solution	Plant Locations	Annual Cost ($1000)
1	(2, 5, 11, 14)*, 8, 15	1917
2	(2, 5, 11, 14), 9, 15	1920
3	(2, 5, 11, 14), 8, 15	1924
4	(2, 5, 11, 14), 8, 10	1937
5	(2, 5, 11, 14), 3, 9, 15	1957
6	(2, 5, 11, 14), 8, 10	1938
7	(2, 5, 11, 14), 9, 10	1941
8	(2, 5, 11, 14), 1, 8, 15	1948
9	(2, 5, 11, 14), 1, 9, 15	1950
10	(2, 5, 11, 14), 8, 9, 15	1949
11	(2, 5, 11, 14), 3, 8, 15	1953
12	(2, 5, 11, 14), 8, 9, 10	1969
13	(2, 5, 11, 14), 1, 8, 15	1956
14	(2, 5, 11, 14), 3, 8, 15	1960
15	(2, 5, 11, 14), 1, 8, 10	1969
16	(2, 5, 11, 14), 1, 9, 10	1970
17	(2, 5, 11, 14), 3, 8, 10	1973
18	(2, 5, 11, 14), 3, 8, 10	1974
19	(2, 5, 11, 14), 1, 3, 9, 15	1986
20	(2, 5, 11, 14), 3, 8, 9, 15	1985
21	(2, 5, 11, 14), 1, 8, 9, 10	2001
22	(2, 5, 11, 14), 1, 8, 10	1970
23	(2, 5, 11, 14), 1, 8, 9, 15	1981
24	(2, 5, 11, 14), 1, 3, 8, 15	1985
25	(2, 5, 11, 14), 3, 9, 10	1977
26	(2, 5, 11, 14), 1, 3, 8, 15	1992
27	(2, 5, 11, 14), 3, 8, 9, 10	2006
28	(2, 5, 11, 14), 1, 3, 9, 10	2006
29	(2, 5, 11, 14), 1, 3, 8, 9, 10	2037
30	(2, 5, 11, 14), 1, 3, 8, 10	2005
31	(2, 5, 11, 14), 1, 3, 8, 10	2006
32	(2, 5, 11, 14), 1, 3, 8, 9, 15	2017

* plants 2, 5, 11 and 14 must be in the solution because of constraints

objective; the highest cost is only 6.2% greater than the least cost.

Among these 32 solutions, there are eight pairs that have exactly the same plant locations but a different cost because of the different capacities of some plants and interceptors. For example, the difference between solutions 1 and 3 is that in solution 1 the waste flow originating from node 9 goes to plant 8, while in solution 3 that flow goes to plant 11. This same difference occurs for the other pair of solutions with the same plant locations where the cost difference is $7,000. In Table 3.11, if the cost difference between two solutions with the same plant locations is $1,000, then the waste flow originating from node 9 goes to plant 10 for the more costly alternative and to plant 8 for the less costly alternative.

The pairwise differences among these 32 alternatives, as measured by the sum of different plant and interceptor capacities as well as by the sum of different plant and interceptor locations, were calculated. Then the pairwise differences were arranged so that the least pairwise difference between a new solution and all previous solutions decreases (or remains the same) as new solutions are added. The plant and interceptor locations and the cost for the first ten alternatives, using the differences as measured by the sum of plant and interceptor capacities, are listed in Table 3.12.

The pairwise differences, as measured by the sum of different plant and interceptor capacities as well as by the sum of different plant and interceptor locations for the ten alternatives, are shown in Table 3.13. The least value in each row is plotted in Figure 3.10. The difference as measured by the sum of different plant and interceptor capacities, drops sharply from 80 cfs for the second alternative to 40 cfs for the third alternative, then it decreases slightly to 38 cfs for the next alternative. After that, it drops to 24 cfs for the fifth alternative and then decreases smoothly to 9 cfs for the tenth alternative. The differences among these ten alternatives, as measured by the sum of different plant and interceptor locations, shows roughly the same trend.

The configurations of the four most different alternatives (alternatives 1, 2, 3, and 4) are shown in Figure 3.11. All four alternatives are the same with respect to facility locations that serve sources 12, 13 and 14. Furthermore, interceptors 1-2 and 3-2 appear in each solution. Nevertheless, these four alternatives appear to be different from one another. For example, in addition to the four required plants (2, 5, 11, and

Table 3.12 Plant and Interceptor Locations and Total Annual Costs for Ten Alternatives Generated by the G&S Method

Alternative		Plant and Interceptor Locations	Annual Cost ($1000)
AP1	Plant	(2, 5, 11, 14)*, 9, 10	1941
	Inte.	1-2, 3-2, 4-3, 6-5, 7-8, 8-9, 12-14, 15-11, 13-14	
AP2	Plant	(2, 5, 11, 14), 1, 3, 8, 15	1992
	Inte.	1-2, 3-2, 4-3, 7-1, 6-5, 9-10, 10-11, 12-14, 13-14	
AP3	Plant	(2, 5, 11, 14), 1, 3, 8, 10	2005
	Inte.	1-2, 3-2, 4-3, 7-1, 6-5, 9-8, 12-14, 15-11, 13-14	
AP4	Plant	(2, 5, 11, 14), 3, 9, 15	1957
	Inte.	1-2, 3-2, 4-3, 6-5, 7-8, 8-9, 10-11, 12-14, 13-14	
AP5	Plant	(2, 5, 11, 14), 8, 15	1917
	Inte.	1-2, 3-2, 4-5, 6-5, 7-8, 9-8, 10-11, 12-14, 13-14	
AP6	Plant	(2, 5, 11, 14), 8, 10	1938
	Inte.	1-2, 3-2, 4-5, 6-5, 7-8, 9-10, 12-14, 15-11, 13-14	
AP7	Plant	(2, 5, 11, 14), 3, 8, 9, 15	1985
	Inte.	1-2, 3-2, 4-3, 6-5, 7-8, 10-11, 12-14, 13-14	
AP8	Plant	(2, 5, 11, 14), 1, 8, 9, 10	2001
	Inte.	1-2, 3-2, 4-5, 7-1, 6-5, 12-14, 15-11, 13-14	
AP9	Plant	(2, 5, 11, 14), 1, 3, 9, 10	2006
	Inte.	1-2, 3-2, 4-3, 7-1, 6-5, 8-9, 12-14, 15-11, 13-14	
AP10	Plant	(2, 5, 11, 14), 1, 9, 15	1949
	Inte.	1-2, 3-2, 4-5, 7-1, 6-5, 8-9, 10-11, 12-14, 13-14	

* plants 2, 5, 11 and 14 must be in the solution because of constraints

Table 3.13 Differences among the Ten Alternatives Generated by the G&S Method

	Alternative									
	1	2	3	4	5	6	7	8	9	10
1	0									
2	80 (14)*	0								
3	40 (10)	52 (6)	0							
4	42 (7)	38 (7)	72 (11)	0						
5	70 (8)	24 (8)	46 810)	38 (7)	0					
6	32 (4)	55 (10)	19 (8)	75 (11)	49 (6)	0				
7	65 (9)	16 (5)	48 (9)	23 (2)	14 (5)	51 (9)	0			
8	25 (5)	54 (9)	14 (5)	67 (12)	50 (9)	13 (5)	46 (10)	0		
9	10 (6)	70 (8)	29 (4)	42 (7)	76 (14)	39 (10)	62 (9)	25 (5)	0	
10	42 (7)	38 (7)	72 (11)	9 (6)	34 (7)	71 (11)	29 (8)	57 (6)	42 (7)	0

* a (b)

a: measured by the sum of different plant and interceptor capacities (cfs).

b: measured by the sum of different plant and interceptor locations (unit); the addition or removal of any plant or interceptor contributes 1 to the sum.

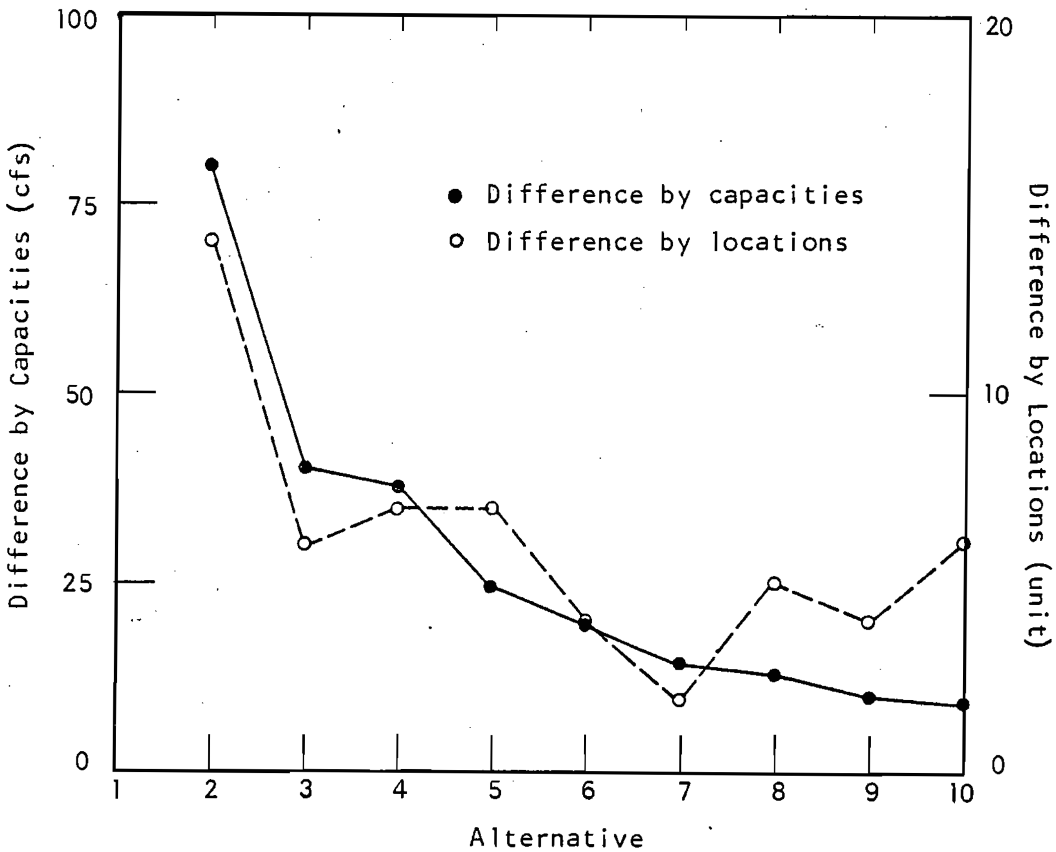

Figure 3.10 Differences among Ten Alternatives Generated by the G&S Method

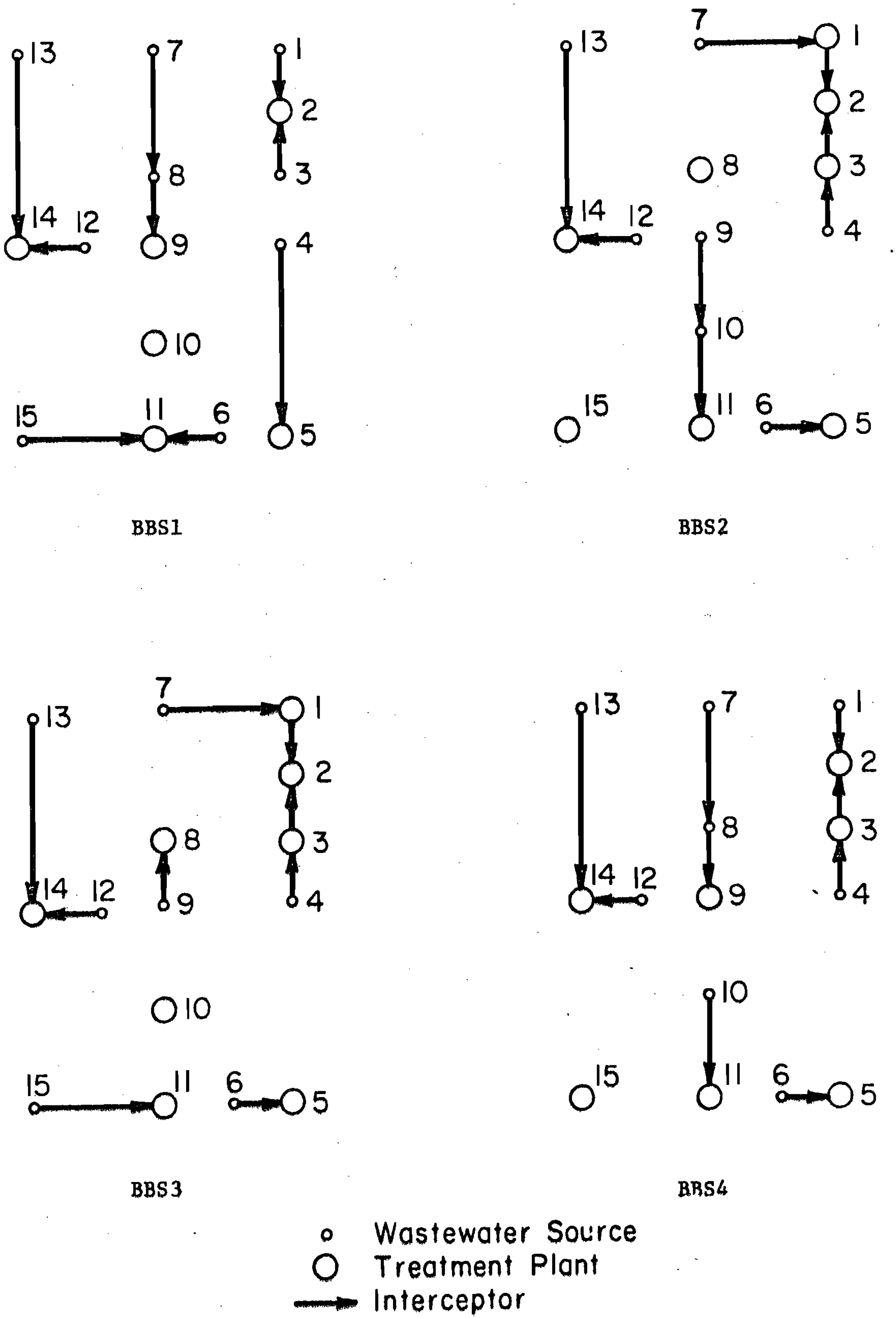

Figure 3.11 Configurations of Four Alternatives Generated by the G&S Method

14), alternative 1 locates plants at sources 9 and 10, while alternative 2 locates plants at sources 1, 3, 8, and 15. The difference between these two alternatives, however, is smaller than the difference between the two most different alternatives obtained by the HSJ or the random generation method (see Figures 3.2 and 3.7).

The ten most different alternatives obtained using the sum of different plant and interceptor locations as the criterion were also examined. The ten alternatives were not the same, only six alternatives, 1, 5, 7, 9, 26 and 28, appear in both sets, but the differences among alternatives are comparable.

3.5 Generating Alternatives Using the Fuzzy HSJ Method

3.5.1 Overview

Since objective function values and constraints usually cannot be precisely defined by formulations or numbers, a "fuzzy" representation of those objective function and constraint values has been suggested for obtaining the optimal solution. For example, when the HSJ method was used in Section 3.2 to generate alternatives, the cost objective was relaxed to $2,108,000 to obtain maximally different alternatives. It is usually not easy to determine the proper level of relaxation for the cost objective. If it is relaxed too much, large differences among alternatives may be obtained, but all of them may have only relatively fair performance with respect to the cost objective. On the other hand, if it is relaxed too little, the alternatives obtained will be good with respect to the cost objective, but less differences among alternatives will be expected. Furthermore, some constraints in the exact formulation may be too restrictive or rigid; since the feasible space is too limited, some good alternatives may be excluded.

In this section, the fuzzy concept is incorporated into the HSJ approach to generate alternatives that are good and different. The general procedure is described in Section 2.4.

3.5.2 Application of the Fuzzy HSJ Approach

An initial solution is obtained by solving the original MIP formulation of the example problem. Before searching for an alternative which is different from the first solution and good with respect to the cost objective, a few assumptions, such as the following, must be made. First the satisfaction level is assumed to change linearly with respect to the value of cost and surrogate difference, respectively. The satisfaction level is

1.0 if the surrogate difference (sum of decision variable values in the objective function after optimization) is 0 cfs and linearly decreases to 0 if the surrogate difference is 119 cfs. Note that a value of zero for the surrogate difference indicates a completely different solution. The same, or a very similar solution, is obtained if the sum of nonzero variables in the initial solution is not reduced at all. For the cost objective, the satisfaction level is 1.0 if the cost is the same as that in the optimal initial solution ($1,917,000) and decreases linearly to 0 if the cost is $2,108,000. Second, it is assumed that the objective is to maximize the minimum satisfaction level for surrogate difference and cost. The formulation to be solved to obtain the second alternative is :

$$\begin{aligned} &\text{Max} \quad s \\ &\text{s.t.} \sum_{i \in K} X_i + 119s \leq 119 \qquad\qquad (3.2) \\ &\quad \text{cost}\ (X) + 191{,}000s \leq 2{,}108{,}000 \\ &\quad \text{original constraint set} \end{aligned}$$

Where K is a set of indicies of continuous variables that are nonzero in the initial solution and cost (X) is the cost objective function.

To ensure that a cost effective solution is obtained, the cost function was also included in the objective function with a small scaling factor so that it will not affect the s value in the solution. The third and the following alternatives were obtained by solving formulations similar to formulation 3.2 except that the ΣX_i term includes the nonzero variables from all previous solutions.

As before, only the continuous variables associated with the capacities of plants and interceptors were included in the surrogate difference objective function. Five alternatives were obtained before all continuous decision variables were in the solution. The plant and interceptor locations and the cost of each alternative are shown in Table 3.14. The highest cost ($2,060,000 for FHSJ5) is only 7.5% higher than the least cost ($1,917,000 for FHSJ1). The number of plants increases from 6 for FHSJ1 to 8, 9, and 10 for FHSJ2, FHSJ3, and FHSJ4, respectively, then it decreases to 9 for FHSJ5. The number of interceptors, however, tends to decrease. It starts with 9 interceptors for FHSJ1, then it decreases to 7, 6, and 5 for FHSJ2, FHSJ3, and FHSJ4, respectively. Finally, it increases to 6 for FHSJ5. This tendency was also observed for the alternatives generated by the HSJ method, as discussed in Section 3.2.1.

Table 3.14 Satisfaction Levels, Plant and Interceptor Locations and Annual Costs for the Alternatives Generated by the Fuzzy HSJ Method

Alter.	Satisfaction Level		Plant and Interceptor Locations	Annual Cost ($1000)
FHSJ1	-	Plant	(2, 5, 11, 14)*, 8, 15	1917
		Inte.	1-2, 3-2, 4-5, 6-5, 7-8, 9-8, 10-11, 12-14, 13-14	
FHSJ2	.5153	Plant	(2, 5, 11, 14), 1, 3, 9, 10	2010
		Inte.	7-1, 4-3, 6-5, 8-9, 12-14, 13-14, 15-11	
FHSJ3	.2924	Plant	(2, 5, 11, 14), 3, 8, 10, 12, 15	2045
		Inte.	1-2, 4-3, 6-5, 7-8, 9-10, 13-14	
FHSJ4	.2479	Plant	(2, 5, 11, 14), 1, 3, 8, 10, 13, 15	2052
		Inte.	4-3, 7-1, 6-11, 9-8, 12-14	
FHSJ5	.2378	Plant	(2, 5, 11, 14), 3, 8, 9, 10, 15	2060
		Inte.	1-2, 4-3, 6-5, 7-8, 12-9, 13-14	

* plants 2, 5, 11 and 14 must be in the solution because of constraints

The configurations of these five alternatives are shown in Figure 3.12. It can be seen that alternatives 1, 2, 3, and 4 are quite different. Alternative 5, however, is similar to alternative 3. The difference between the two is that the plant located at source 12 in alternative 3 shifts to source 9 in alternative 5 and that interceptor 9-10 in alternative 5 shifts to 12-9.

The differences among these five alternatives, as measured by the sum of different plant and interceptor capacities and by the sum of different locations, are shown in Table 3.15. The least value in each row by the first measure is plotted in Figure 3.13. The difference, as measured by the sum of different capacities, decreases approximately linearly from 118 cfs for FHSJ2 to 29 cfs for FHSJ5. The difference as measured by the sum of different locations, on the other hand, decreases sharply from 16 for FHSJ2 to 10 for FHSJ3. After that, it remains at 10 for FHSJ4, then it drops to 4 for FHSJ5. Both methods of measuring difference indicate a decrease in differences among alternatives when additional alternatives are generated. The same effect was also observed with the HSJ approach. The satisfaction level shown in Table 3.14 also indicates the decrease in differences and can be used to stop the generation process.

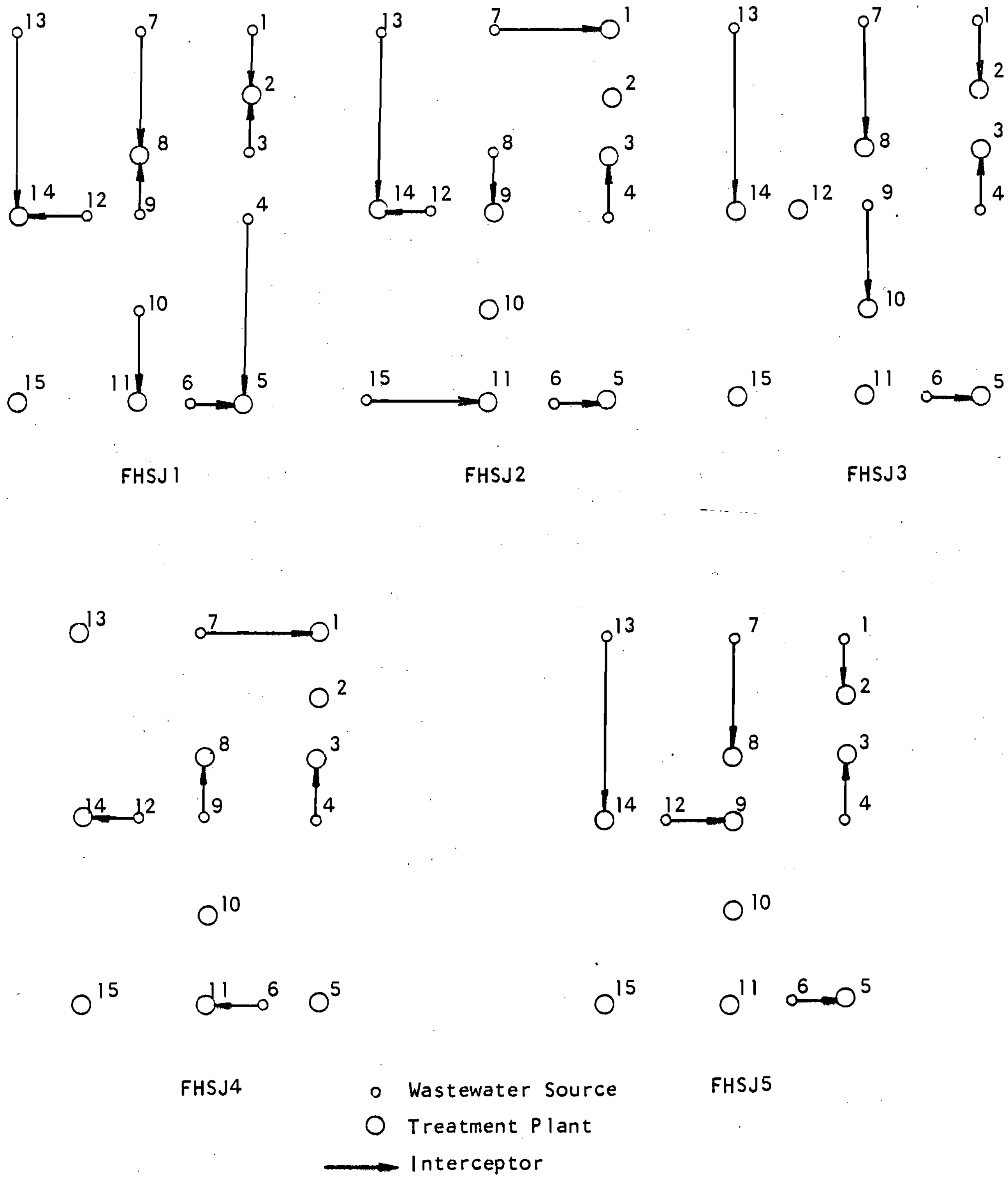

Figure 3.12 Configurations of the Five Alternatives Generated by the Fuzzy HSJ Method

Table 3.15 Differences among the Five Alternatives Generated by the Fuzzy HSJ Method

Alternative	FHSJ1	FHSJ2	FHSJ3	FHSJ4	FHSJ5
FHSJ1	0				
FHSJ2	118(16)*	0			
FHSJ3	93(10)	88(12)	0		
FHSJ4	90(14)	57(10)	58(12)	0	
FHSJ5	96(10)	72(10)	29(4)	62(12)	0

* a (b)

a: measured by the sum of different plant and interceptor capacities (cfs).

b: measured by the sum of different plant and interceptor locations (unit); the addition or removal of any plant or interceptor contributes 1 to the sum.

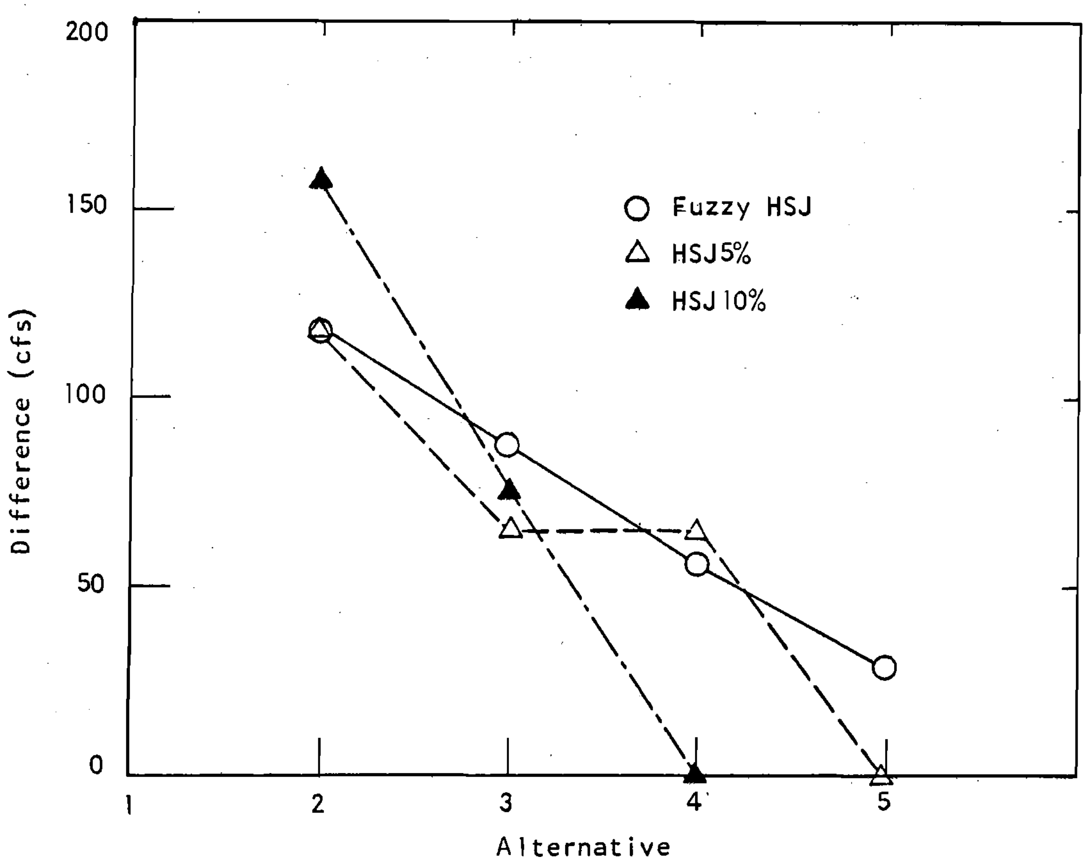

Figure 3.13 Differences among the Alternatives Generated by the Fuzzy HSJ Method

The differences as measured by the sum of different plant and interceptor capacities among the alternatives generated by the HSJ method with 5% and 10% relaxation of the least cost are also shown in Figure 3.13. It can be seen that the second alternative generated by the Fuzzy approach is the same as that generated by HSJ5% and thus has the same difference. But it has less difference and a better objective function value than that generated in the HSJ10% case. The third alternative, however, has more difference than the alternatives generated by HSJ5% and HSJ10%. Its objective function value is better than that in HSJ10% case but worse than that in the HSJ5% case. While HSJ5% and HSJ10% generated four and three different alternatives, respectively, Fuzzy HSJ generated five. The number of alternatives that can be generated, however, is dependent on the specified satisfaction level. As another experiment, the satisfaction level of the cost objective was changed so that s=1.0 for $1,917,000 and s=0 for $2,300,000; then only three alternatives were obtained.

3.5.3 Summary

The Fuzzy HSJ approach generated several good and different alternatives for this example problem. The difference plot is similar to that in the HSJ approach. However, there is a balance between the surrogate difference and the cost target for the fuzzy approach. Depending on the satisfaction level specified, the fuzzy approach can locate a solution with more difference at the expense of cost or choose a less costly solution at the expense of surrogate difference.

3.6 Comparisons among the HSJ, Random, and BBS Methods

Three criteria were used to compare the three methods for generating planning alternatives for the example wastewater treatment system planning problem. They are performance with respect to the objective function value included in the model (total annual cost), differences among the alternatives generated, and computational requirements.

3.6.1 Comparisons Based on the Performance with Respect to the Objective Function Value

The total annual cost for each set of alternatives generated by the HSJ, random, and BBS methods are shown in Table 3.16. The first set of alternatives (HSJ10) was obtained by the HSJ method with the cost target set at $2,108,000 (10% higher than the optimal cost). The second and the third sets of alternatives were obtained by the HSJ method with the cost target set at

Table 3.16 Total Annual Costs for the Alternatives Generated by the HSJ, Random and G&S Methods

Method	No. of Alter.	Annual Cost for Each Alternative ($1000)	Mean ($1000)	Std. Dev. ($1000)
HSJ10	3	1917, 2076, 2108	2034	102
HSJ5	4	1917, 2010, 2013, 2013	1988	48
HSJ20	4	1917, 2076, 2125	2039	109
Random	10	1991, 1997, 1997, 1990, 1943, 1988, 1946, 1953, 2018, 1976	1980	25
G&S	10	1941, 1992, 2005, 1957, 1917, 1938, 1985, 2001, 2006, 1949	1969	32

$2,013,000 and $2,300,000, respectively (5% and 20%, respectively higher than the least cost). The initial solution for each of the three sets is the same -- the minimal cost solution of the original model. The fourth and fifth sets of alternatives were obtained by the random and BBS methods, respectively, with the cost target set at $2,108,000. The alternatives generated by the BBS method have the lowest average annual cost, followed by the alternatives generated by the random method. Even though the cost target for these two sets of alternatives is $2,108,000, all alternatives generated have a lower cost. The alternatives generated by the HSJ method with the same cost target have a higher average annual cost ($2,034,000) and higher standard deviation ($102,000). The high variation is caused by the low cost of the initial solution and the high cost (close to the target value) for the other solutions. The average annual cost for the alternatives generated by the HSJ method with a cost target of $2,300,000 (HSJ20) is only slightly higher than that for HSJ10. The average annual cost for the alternatives generated by the HSJ method with cost target of $2,013,000 (HSJ5) is not much higher than that obtained in the random and BBS cases. Since all alternatives generated are within the cost target specified and have similar costs, they all are considered good with respect to the cost objective. The better performance of the random and BBS methods occurs because the cost target is not binding, a problem dependent situation.

3.6.2 Comparisons Based on the Differences among Alternatives

Difference, as measured by the sum of different plant and interceptor capacities (absolute sum of differences between continuous decision variables), was used as a criterion for comparing the sets of alternatives. This criterion is by no means perfect and the result should be interpreted judgemently. The differences among alternatives for each set of alternatives are plotted in Figure 3.14. The order of the alternatives has been rearranged for the alternatives generated by the random and BBS methods so that the largest difference will appear first and the difference among alternatives decreases monotonically. HSJ20 is excluded because the difference profile of HSJ20 is similar to HSJ10.

From Figure 3.14, it can be seen that alternatives generated by the HSJ method with cost target of $2,108,000 (HSJ10) have the largest difference for the second and the third alternatives. However, the HSJ iterations stop after that because all variables have been in the solution at least once, and no more solutions can be obtained without restarting the procedure. The difference among alternatives generated by the random method is smaller than that of HSJ10 for the second alternative, and is only slightly smaller for the third alternative. The random method, however,

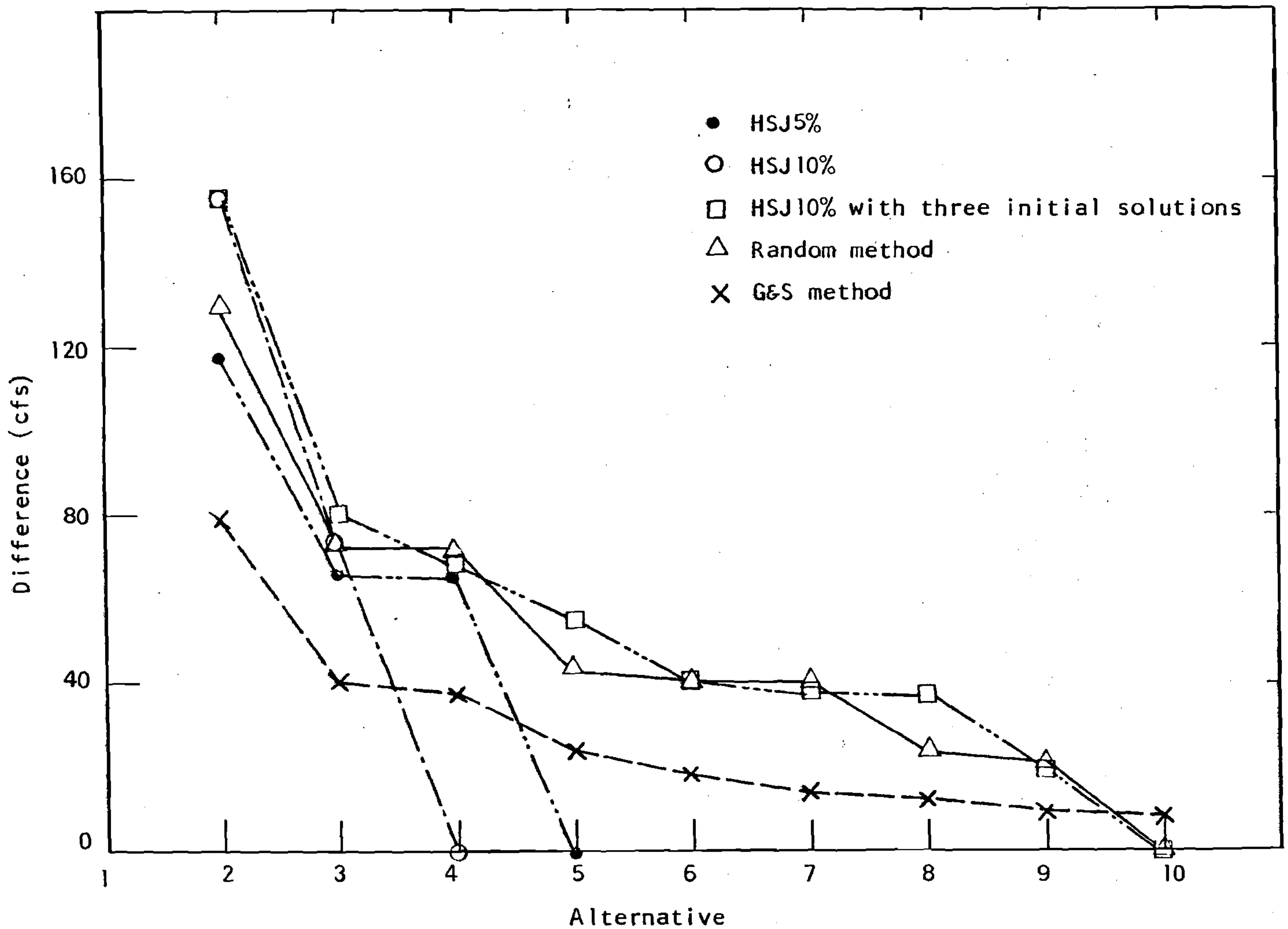

Figure 3.14 Differences among Alternatives for the Alternatives Generated by the HSJ, Random and G&S Methods

continues to generate more alternatives with considerable differences. The alternatives generated by the HSJ method with cost target of $2,013,000 (HSJ5) have slightly smaller differences than those of the random method for the second, third and fourth alternatives. After that, the HSJ iterations stop. The BBS also generates a large number of alternatives. However, the differences among alternatives are considerably smaller than those of the random method.

Note that additional solutions can be generated using the HSJ approach by using a different initial solution. For this example problem, ten solutions were obtained using three initial solutions (obtained from the branch and bound tree). The difference profile of these ten solutions is also plotted in Figure 3.14. Even though higher differences among the HSJ solutions is observed for the second, third, fifth and eighth alternatives, the overall level of differences among these HSJ alternatives is similar to that of the random method.

It is interesting to observe that the three methods yield sets of alternatives that are different in different ways. For the example problem, the HSJ method tends to generate alternatives that differ in the degree of centralization. The random method tends to generate alternatives with a similar number of plants and interceptors, but with these facilities in different locations. The BBS method tends to generate alternatives in which some part of the wastewater system is constant but combined with different arrangements of the remainder of the system. Any of these types of difference might be useful in exposing the range of alternatives and previously unmodeled objectives.

3.6.3 Comparisons Based on Computational Effort

For the HSJ method, the initial solution is obtained by solving the original formulation 3.1 using the APEX MIP code. For each additional alternative, the nonzero continuous variables in previous solutions must be identified, the original data file must be modified accordingly, and then the modified file must be solved. The modification step can be accomplished either by directly changing the original data file if the problem size is small (e.g., the size of this example problem) or by creating a revised file using a simple FORTRAN program which uses a previous solution as input. The revised file can then be directly provided to the APEX code as another input file. When the problem size is large, the latter approach is much more efficient. The computer time required for creating a revised file is trivial. The computer time used to solve the revised problem to obtain a new alternative for the example problem was approximately the same as that used to solve the original problem, from 1.16 to 6.14 computer seconds with an average of 2.72 seconds on the CDC Cyber 175 at the University of Illinois. The computer time for a HSJ solution with a relatively tight cost target constraint was usually higher than that for a HSJ solution with a more relaxed constraint. For example, the average computer time for a HSJ solution with cost target of $2,300,000 was 1.54 seconds, while for a solution with cost target of $2,013,000 it was 4.45 seconds.

For the random generation method, no initial solution is needed, and each alternative is generated independently. However, the minimal cost solution would probably be desirable for use as a reference in specifing the cost target. To generate an alternative, a set of decision variables must be generated, and then the appropriate modification of the original data file can be made. The modified file is then solved. The generation of the decison variables and the modification of the original file can be accomplished at the same time by using a simple FORTRAN program to create a revised file. The revised file and the original data file are then used as two input files to the APEX code to obtain an alternative. The computer time to create a revised file is trivial. The computer execution time to obtain an alternative for the example problem varied from 1.59 to 4.68 seconds with an average of 2.36 seconds.

The BBS method, as implemented for the example problem, was to use the APEX code to generate many solutions within the specified range of the least cost solution, and then a screening process was used to select several solutions that are significantly different from one another. The cost target was specified at $2,108,000, and 32 solutions were obtained. Ten alternatives were selected by the screening process. The computer execution time used was 18.3 seconds for generating the 32 solutions. The secreening process employed a FORTRAN program to calculate the difference between each pair of solutions and then to tablulate all solutions in an order such that the largest difference appears first and the differences decrease monotonically. Since ten solutions were selected, the computer time required for each solution is calculated as 18.3/10=1.83 seconds. The computer time required to calculate the differences among alternatives and then to tabulate the alternatives is dependent on the number of decision variables in the problem and the total number of solutions generated. For this example problem, the execution time was only 0.2 seconds.

In summary, the average computer time required for generating one solution is shown in Table 3.17. The average computer time is smallest for the BBS method, followed by the random method, and the HSJ method. However, the difference is not very significant

Table 3.17 Computer Time Used for Generating One Alternative Using the HSJ, Random and G&S Methods

Method	Mathematical Program Requirement Range	Mean	Other Requirements
HSJ	1.16 - 6.14*	2.72	create revised file .2
Random	1.59 - 4.68	2.36	create revised file .2
G&S	1.83	1.83	screening .2

* seconds on CDC Cyber 175 computer at the University of Illinois

for this example problem. The total computing requirement, however, were quite modest for each technique for this example problem; in general, computing requirements are expected to be highly problem dependent. For larger MIP problems, the computing requirement may be substantial or may even not converge.

CHAPTER 4

FINAL REMARKS

This study was designed to provide an assessment of the potential use of several modeling to generate alternatives (MGA) methods -- the HSJ, random, BBS, and Fuzzy HSJ methods -- for generating good and different planning alternatives. Since an optimization model is not a perfect representation of a complex real world planning problem, the optimal solution to the model is not necessarily the best solution to the problem. The optimal solution will provide limited insight and cannot illustrate characteristics of other good solutions to help the analyst or decision maker learn about the problem and identify overlooked issues. A premise of this work is that optimization models can be used, in some cases, in a planning process to generate alternatives that are good and different so that the analyst and decision maker can examine the range of choice to gain insight and understanding. An example wastewater system planning problem formulated by using a mixed integer programming model has been used to examine the four generation methods.

The HSJ method can be used to generate good alternatives that are significantly different from one another. The level of differences among alternatives is higher for the first several alternatives and decreases for additional alternatives. Also, the number of alternatives generated is dependent on the targets specified. If the targets are stringent, more alternatives can be obtained, but the level of differences among them is decreased.

The difference between the the LP and MIP formulations is that there are two sets of decision variables in the MIP formulation: the continuous variables and the zero-one variables. Each set can be used alone to drive the HSJ iterations. For the MIP example problem, the use of continuous variables appeared to be more effective in generating different alternatives than the use of zero-one variables. The use of the continuous variables associated with plant capacities alone, however, was less effective than the use of all of the continuous variables. The maximization of zero variables appearing in all previous solutions is conceptually the same as the minimization of nonzero variables, but less effective because it tends to include unnecessary variables in the solution.

Since the total number of decision variables is fixed, the number of alternatives that can be generated by the HSJ method is also fixed if only one initial solution is used. For the MIP example problem, only three to four alternatives were generated by the

HSJ method. To increase the number of alternatives, the HSJ objective function can be modified to include only some of the nonzero variables (those variables with large values) in previous solutions. Of course, additional alternatives can also be obtained by using another initial solution as illustrated in Section 3.6.2. There are other variations of the HSJ method. For example, weights could be applied to selected variables, and constraints can be used to retain the attractive parts of a certain alternative.

The random generation method developed in this study can be used to generate good and different alternatives efficiently. It generated many alternatives with considerable differences among them. It is possible that two alternatives randomly generated are exactly the same even though the decision variables randomly specified may be different. The possibility of generating the same alternative is larger for a small planning problem than for a large problem. Since the random method generates alternatives independently, when a solution is duplicated, it can be ignored and the process can be continued.

There are many potential variations of the proposed random generation method. For example, the sum of randomly generated decision variables could be minimized instead of maximized. This approach has the advantage of excluding unnecessary variables from the solution when applied to MIP formulations, as discussed in Section 3.3.3. Also, the decision variables could be divided into categories, and a certain number of decision variables could be randomly generated from each category. Furthermore a weight could be attached to each variable.

The BBS method is a two-step method. It uses the APEX code to generate many good solutions, then it uses a screening process to select those solutions that are different from each other. Thus, the differences among alternatives depend on how many solutions are generated. Since the APEX code does not generate all feasible solutions within the specified cost target, however, it cannot be expected that maximally different alternatives will be provided.

The example results show that BBS method can provide many different alternatives, but that the level of differences among alternatives is less than in the HSJ and random cases. However,as discussed in Section 3.6.2, these three methods yield sets of alternatives that are different in different ways. It is assumed that higher levels of differences among alternatives may offer more insight to the analyst or decision makers. In some cases, however, alternatives with higher levels of differences among them using a particular method of measuring differences may not necessarily provide more insight to a certain analyst or decision maker than alternatives with lower levels of differences.

The Fuzzy HSJ method, a modification of the HSJ method, is designed to increase the flexibility of the HSJ approach. The results show that the alternatives generated by the Fuzzy HSJ method may be different from those generated by the HSJ method, but the differences among the alternatives have the same general trend as in the HSJ case. The method is more flexible in specifying the targets but requires more information from the decision maker. The fuzzy approach can also be incorporated into other generation methods.

There is no generally applicable criterion for the number of different alternatives that should be provided to the analyst or decision maker. Five to ten different alternatives, for example, may be reasonable. The analyst and the decision maker can examine these alternatives to look for insights and understanding, and additional alternatives can be generated as desired.

Even though some specific issues related to the MGA methods examined in this study (e.g., levels of targets, number of decision variables specified in the SORD function in random method) would vary for different applications, the general procedures outlined here can be applied to other planning problems formulated as LP or MIP models. It is not possible, however, to generalize about outcomes. For some problems, for example, there may be very few alternatives that can be generated within specified targets, for other problems, there may be many alternatives, but they may not be very different. Nevertheless, such insights may enhance understanding of the particular problem at hand. Further experiments to examine the general behavior of these methods and the application of these methods to problems modeled using other mathematical formulations (e.g., dynamic programming) are needed.

The results of this study suggest that the four MGA methods can be used in a complex water resources planning process to generate planning alternatives that are good and different. The results are strengthened somewhat by the fact that realistic data were used for these example problems. Through MGA approaches, the use of an optimization model in a planning process is no longer limited to obtaining the "optimal" solution or a few "second best" solutions (or parametric analyses). Using MGA methods, the decision maker or the analyst can examine various solutions to gain insight and understanding and to find overlooked issues. If an unmodeled objective is found and quantified, it may be possible to include it in the model. Then the generation process can be started again.

Since the human mind is usually preoccupied with past experience and intuition, it may be difficult for a human to synthesize innovative solutions, especially for a complex planning problem. Optimization models, however, can be used to generate different or unexpected solutions in a defined decision space. A wide range of such solutions may spark the imagination of the analyst or decision maker, and aid the analyst in considering overlooked issues of the problem and in synthesizing better solutions. A decision based on this information may be better than a decision made by examining only the "optimal" solution or a few second best solutions.

Note that the generation of good and different alternatives is not designed to replace the generation of noninferior solutions or sensitivity analysis, but to provide additional insight. The MGA approaches are not applicable to problems which can be clearly defined by an optimization model. Furthermore, no methods used in this study can be guaranteed to find the set of the most different solutions in a defined decision space because differences cannot be perfectly defined and the generation process is limited.

The common disadvantage of the quantitative methods of measuring differences among alternatives used in this study is that they cannot measure how far, in the spatial sense, the changes are made. Even though any measurement of differences among alternatives may be problem dependent and imperfect, further exploration in this area is needed. Further research on how to present these different solutions to the decision maker so that he can perceive the differences among these alternatives and find them useful in the decision making process is also needed.

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