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RELIABILITY OF RESERVOIR OPERATION UNDER HYDROLOGIC UNCERTAINTY

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ABSTRACT

Sensitivity analyses were performed to examine typical stochastic programming (SP) modeling issues for a hypothetical single reservoir system. The elements considered include the partitions of inflow and storage states, the hydrologic characteristics of inflows, the types of system performance functions, and the tradeoffs between conflicting objectives. Simulation studies were conducted to verify the modeling outcomes and to provide insights for possible improvements of the system performance. Results from these analyses show that (1) both the numbers and the discrete increment values of the inflow and storage states affect an SP model's accuracy; (2) the uncertainty associated with the coefficients of variation of the inflows consistently has a greater impact on the system performance than the influence of the serial correlations; (3) in a sample study with flood control being the only objective, the use of either a convex function or a concave function alone for flood damages will not lead to an optimal operation policy which always prevents excessive flood release when there exists some unused storage space in the reservoir; (4) the preferences between the conflicting objectives have been shown to affect both the expected system performance and the individual operation decisions; and (5) modification of the discrete optimal solution, using a simple interpolation scheme, may improve the reservoir performance without resorting to a more complex model.

A case study of Lake Shelbyville, Illinois was conducted based on the findings of sensitivity analyses for the hypothetical reservoir system using SP. An ad hoc approach was used to estimate accurately the agricultural and property damages in the optimization procedure. The optimal pool levels of Lake Shelbyville in the summer months were found to be roughly 2 to 5 ft lower than the current target level which is 599.7 ft. When the summer pool was forced to reach this target level using a penalty function approach in the SP model, the annual expected damages would increase by 9%. Generally, it would take more than one month for Lake Shelbyville to resume the summer pool from the winter drawdown level. Therefore, a transition period longer than one month between the winter drawdown and the summer recovery of lake levels is recommended for consideration if future modification is made in the rule curve.

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I. INTRODUCTION

1.1 General

Reservoirs are physical storage spaces naturally or artificially created which function to modify the temporal pattern of inflow quantities into a more desirable outflow pattern. The configuration and size of a reservoir system impose the primary constraints on how much surface runoff can be controlled in a river basin. In addition, regional hydrology plays a critical role in determining how well the water stored in reservoirs may be regulated. For more than two decades, researchers have adopted mathematical programming techniques and optimization procedures to enhance reservoir system operation. Parallel to this development there has been an expansion of knowledge in stochastic hydrology which involves generation of synthetic data bases and prediction of future hydrologic conditions. The merging of mathematical programming and stochastic hydrology has led to innovative studies of reservoir operation under hydrologic uncertainty (Yeh, 1985).

Depending upon their respective applicabilities, various optimization and simulation techniques, together with engineering experience and judgment, may be coordinated in the modeling process. One example is to employ a long-term planning model to determine the steady-state storage and release policies for each operating period, which in turn serve as the goals to be attained for a short-term, real-time reservoir control model. In this manner, distinctive priorities and restrictions can be considered in a detailed modeling framework for different operation levels. The Tennessee Valley Authority (TVA) adopted the above approach in a weekly multipurpose planning model for its immense reservoir system facilities

(Giles and Wunderlich, 1981; Shane and Gilbert, 1982; Gilbert and Shane, 1982).

Reservoir models based on mathematical programming were originally developed to assist decision makers in effectively utilizing valuable water resources. However, when many facets of a reservoir system are considered, complexity in these models increases. It is not uncommon therefore to find these models too complicated to provide much help in comprehending a reservoir system's characteristics (Beard, 1973; Helweg et al., 1982; Friedman et al., 1984; and Rogers and Fiering, 1986).

Despite the complex nature of real reservoir systems, however, mathematical models have proved worthy for use under certain operation conditions. One programming technique that has not received much attention for real-world reservoir operation is stochastic programming (SP) (Gablinger and Loucks, 1970; Askew, 1975; Gal, 1979; Houck and Datta, 1981).

1.2 Stochastic Programming (SP) Model

A typical SP model is used to find the optimal steady-state operation policies (e.g., the storage and release levels at a given time period of year) as discrete functions of the current storage and inflow states of a reservoir system. These optimal policies, which constitute a Markov decision process, are generally determined by a recursive procedure which incorporates either temporal or spatial correlations of natural streamflows into the optimization of expected future returns. A concise mathematical expression of SP typically used in a reservoir study is given in Sec. 3.2.2.1.

The large size of an SP model as well as the implicit assumption of complete knowledge of future hydrologic events under statistically stationary conditions usually limit the SP technique to theoretical developments rather than practical applications. SP models, nevertheless, offer several advantages over either deterministic models or simulation models in certain respects. Long-term steady-state operation conditions can generally be obtained from an SP model. In addition, an SP model would yield all the optimal decisions for operating a reservoir under the various combinations of reservoir storage and inflow conditions. Therefore, SP models can be used for screening purposes to eliminate those alternative decisions that are clearly inferior (Loucks et al., 1981). A recent study by Wang and Adams (1986) extends the use of SP for real-time optimization of the hydropower generation facilities in operating the Dan-River-Issue Reservoir in China.

The usefulness of the SP approach for reservoir operation has been more often than not overshadowed by its drawbacks as emphasized in most of the literature. Some of the limitations might be relaxed either by improving the solution algorithm or by finding a more accurate and robust predictive model for future hydrologic conditions. On the other hand, SP models could also be simplified through careful manipulations in the modeling procedure to yield desirable information for the operation of a reservoir system. The general purpose of this research is to investigate the modeling obstacles associated with the SP formulation and to resolve them to the extent that facilitates the use of these modeling tools in practical applications for optimal reservoir control.

1.3 Objectives and Scope of the Research

This study examines several common issues encountered when using stochastic programming in both the model formulation and the practical application phases. Only a single reservoir system with multiple purposes is considered. It is thought that a comprehensive study of the single reservoir system may also provide useful insights for operation of more complex reservoir systems.

In the first phase of investigation the SP model formulation was examined regarding:

(a) the effects of different partitions of the storage and the inflow states on the final result obtained;

(b) the effects of hydrologic parameter uncertainty on the expected system performance;

(c) the variation in optimal decisions as a result of using different measures for reservoir performance;

(d) continuous decisions versus discrete decisions in the determination of optimal operation policies.

The findings observed in these analyses were used in a case study for Lake Shelbyville, a real reservoir system in Illinois. In contrast to the usual application of SP models to the optimization of either hydropower generation or water supply where the returns could be repeated over time, the case study extends the use of SP models for systems in which returns are in general not repeatable but cumulative -- e.g. agricultural losses. A modified SP model was developed to accommodate this specific situation. The performance of Lake Shelbyville, as evaluated by the losses of agricultural revenues and recreation benefits, property damages, and by the changes in pool levels and releases, was investigated.

A closing summary of the findings of this study discusses the pros and cons of using SP models for reservoir study. A recommendation is provided for consideration if future modification is made in the regulation of the Lake Shelbyville system. Possible extensions of the current research on SP models are also addressed.

II. LITERATURE REVIEW

2.1 Mathematical Programming in Water Resource Systems

The conventional planning process "comprises the collection of needed information, the preparation of a tentative plan based upon analysis of this information, and the search for an optimal plan by modification of the tentative plan through analysis" (Reedy, in Maass et al., p. 300, 1962). It is basically a trial-and-error procedure usually involving repetition of similar tedious computations. Mathematical programming or optimization models contribute to the planning and operation processes mainly in the efficiency of searching for the best scheme among all the feasible choices. In addition, sensitivity analysis of the outcomes may be performed more easily using mathematical programming models.

Since the Harvard Water Group published its pioneering study (Maass et al., 1962), many mathematical programming models have been developed for water resources systems. Prompted by the early successes in both military and industrial implementations, those programming concepts were widely acclaimed for systematically assessing water resources related problems, such as finding the best design of a multiple purpose reservoir (Hall, 1964), or controlling the water quality in a river basin (Liebman and Lynn, 1966). Meanwhile, the capacity and the availability of fast digital computers have been increased significantly in the past two decades, which further accelerated the progress in using mathematical programming models in the real-time control of water resource systems.

Reservoirs are probably the systems in water resources that have been explored most using mathematical programming. The mass balance law constitutes the main skeleton of a mathematical programming model for reservoir study. The simple linear relationship between the incoming and the outgoing flows makes the modeling process generally straightforward. Assuming that the characteristics associated with a reservoir system can be modeled using mathematical expressions, there are, however, some factors that significantly complicate the modeling and solution procedures. The natural hydrologic process is stochastic; so there are uncertainties in the flow records used for describing the movement of waters. In addition, measures of reservoir performance, whether in monetary terms or not, generally introduce nonlinearities into a model, making it more difficult to formulate and solve than a simple linear model.

Most reservoir models attempt to accommodate these complexities in one way or another, while at the same time keeping model size within practical limits. In the following sections, the evolution of stochastic programming models is reviewed, starting with a discussion of deterministic models. Various strategies for handling hydrologic uncertainties, nonlinearities, and other issues related to optimal reservoir controls are also addressed.

2.2 Deterministic Models

The basic components of a reservoir model are the mass conservation of water as well as the storage and the release constraints. If the sequence of external water flowing into a reservoir system is explicitly specified in a mathematical programming framework, it is generally referred as a deterministic model. The historical flow record is commonly used either to provide critical period hydrologic information or to generate synthetic flow series for use in deterministic models. It is assumed that the past flow records are sufficient to reflect the general hydrologic conditions for the corresponding river basin.

Hall et al. (1968) presented a monthly operation model of the Shasta Dam in northern California for a period of low flows from 1928 to 1934. The objective is to maximize the total income from the sales of both water and energy during that critical period. The monthly schedule of releases could be determined from the optimization model to compute the firm water and the firm energy provided by the Shasta Dam. Harboe et al. (1970) proposed a two-stage optimization procedure for the Folsom Reservoir and Power Plant, also in northern California. Given a certain contract level of the annual firm water supply from the reservoir, the maximum annual firm energy production was determined for a critical period of 12 years. Then, that maximum firm energy production was used as a constraint in a dynamic programming model to determine the maximum total energy output -- including both firm and dump energy production -- based on 50 years of historical monthly flow data. Tradeoffs between the firm water supply and the firm energy production can also be obtained by varying the contract levels of the annual firm water supply.

Using the historical low flow record to repeat the drought phenomenon of a watershed can sometimes be conservative. Hall et al. (1969), as well as Askew et al. (1971), examined the flow records of 26 river basins throughout the continental U.S. Large numbers of equally likely hydrographs of the same length as the historical record were generated. The critical periods based on synthetic data were used to obtain the yields from the river basins and compared to those of the historical records. They showed that the generated records as a whole had significantly less severity than the historical records of the same length. Therefore, unless the objective is to protect against the worst drought that is similar in scale as that recorded in history, the expected performance of a reservoir system may be underestimated. Generally, the historical flows are used to illustrate the improvement of the reservoir performance using mathematical programming.

In contrast to the limited applications of deterministic models for long-term planning purposes, incorporation of these models for real-time, optimal reservoir control is widely advocated by system analysts. The reasons are twofold. First, within a shorter time period, the hydrologic uncertainty might be small because of better forecasting and monitoring of the surrounding physical environment. Second, the capacity of a reservoir system is generally large enough so that real-time operation decisions can be adjusted in a relatively short time period to respond to the changes of inflow volumes. Hence, the assumption of a totally known hydrologic future might be more appropriate for a short time span.

In the study of a single hypothetical reservoir, Croley (1974) showed that results very close to the optimum might be obtained with only a few operation horizons being considered in an optimization model. The results indicated that the hydrologic conditions in the remote future would be practically irrelevant to the near-term reservoir operation decisions. For real-time reservoir controls, the operation horizons are usually restricted to either hours or days, for which reliable hydrologic information can generally be obtained (Panel on Weather and Climate, 1977).

Becker and Yeh (1974) developed an optimization model for the real-time operation of a multiple reservoir system. The methodology adopts a form of dynamic programming (DP) for selecting an optimal reservoir storage policy for a specified number of policy periods, and a linear programming (LP) routine is used for the optimization within each period. An energy surplus of 35% over the contract level for a 12-month period could be achieved for the Shasta-Trinity system in northern California. Yazicigil et al. (1983) also proposed an LP model for the real-time release schedule for the Green River Basin Reservoirs system in Kentucky. The objective is to minimize the total penalties on the deviations from both the target storage and the target release levels aggregated over the four reservoirs. An operation horizon of 1 to 5 days was used with inflows generated from a separate forecast model. The penalty functions were assumed convex, and allowed to vary to represent most of the goals and priorities of the reservoir regulation authority. The study showed that the penalty could be reduced by 45.8% compared to historical operations.

The performance of deterministic, real-time reservoir operation models may vary to a certain extent depending upon the forecast models and the penalty functions used. Datta and Burges (1984), as well as Can and Houck (1985), explored these issues extensively and had the following conclusions. First, short-term forecasts appear to be desirable for real-time reservoir operations whenever the forecasts can yield reduced variance (uncertainty) of actual streamflows. Second, the rate of improvement in reservoir performance decreases as the operation horizon increases. Beyond some finite time period, the extension of the operation horizon has a negligible effect on the results obtained. Third, depending upon the performance criterion used, the value of a forecast varies. Datta and Burges (1984) showed that although the overall losses could be reduced with improved forecast information, the average storage variance which affects wildlife habitats and recreational pool uses was essentially unchanged. They also warned that a bad forecast may very well offset all the expectations in terms of losses and benefits.

As the name implies, deterministic models require a relatively high degree of certainty about the information used. One may expect to obtain relatively stable results from a real-time operation model under normal hydrologic conditions, provided that it is based upon certain long-term operating rules determined a priori. For long-term planning, however, a deterministic model would be restricted mainly to a preliminary study of a reservoir system.

2.3 Stochastic Models

Stochastic models should be considered as complements of deterministic models rather than substitutes for them in a reservoir study. The basic idea is to incorporate hydrologic uncertainty into a mathematical programming model using temporal or spatial (or both) correlations of natural streamflows. In doing so a wide variety of flow situations can be modeled, and the expected performance of a reservoir system may be obtained.

Little (1955) pioneered the concept of using probabilistic methods for reservoir operation. The hydropower generation of a single reservoir system was treated as an inventory problem, with the inflows, not the outflows, as random variables. The operation horizon was divided into a finite number of successive time intervals between which the river flows are characterized as a simple Markov process. For each time interval, the release decision based on a certain combination of inflow and storage can be determined by a recursive procedure that optimizes the expected returns of future operations. Little's work laid the foundation for subsequent research on stochastic programming (SP) models mainly in two respects. First, the Markov assumption is useful to correlate streamflows in a straightforward and very simple manner in the SP model. Second, the proposed optimization procedure -- currently called stochastic dynamic programming (SDP) -- proved to be the most efficient solution algorithm to date for this type of problem. The recursive equation used by Little (1955) is expressed in the form of an integral which is computationally feasible only for simplified reservoir models with continuous return functions. A more general approach is to transform the continuous inflow and storage variables into discrete units.

Gablinger and Loucks (1970) explored the various ways of formulating SP models and compared their performances based on the respective optimization results. Both linear programming (LP) and dynamic programming (DP) were used for three different versions of the stochastic model. The first model defines the release policy as a function of the current storage and net inflow states. The second model differs from the first by replacing the current net inflow by the immediate past inflow information. The third model examines the transient characteristics of the release decisions when the reservoir has not reached a steady state in the early stages of operation. They showed that the LP and the DP formulations are actually duals of each other for the same problem; and both yield identical solutions. The first model results in less expected deviations from the target storages and releases than the second model due to the inclusion of the current, updated flow information. As expected, when the discount rate is increased the release policy in transient stages tends to hedge less for future periods in order to meet current operation priorities.

The steady-state probability associated with a specific optimal release policy can be calculated from the results of a stochastic programming model. Sometimes, an undesirably high chance of failures may result since risk and reliability considerations are not explicitly included in the model. Askew (1974a, b; 1975) proposed a penalty function approach to control the risk and reliability levels of the reservoir operations. A penalty is attached to the recursive function as a reduction from the net benefits. By varying the value of the penalty the probability of failure of the reservoir system can be controlled.

Another unique modeling technique which explicitly brings risk and reliability into consideration was first proposed by ReVelle et al. (1969) for reservoir planning. A reliability programming (RP) model with chance constraints is constructed by transforming selected reliability indices of reservoir performance into the cumulative probabilities of seasonal inflows. A deterministic model is then formulated by converting these cumulative probabilities into the corresponding inflow volumes which must be satisfied together with other physical constraints. The release commitments are usually expressed as linear functions of the storage, the inflow information, and a set of artificial variables which serve as surrogate decision variables for the releases. After a reservoir problem is solved using the RP model, the seasonal releases can be determined based on these linear decision rules (LDR).

Studies on the RP approach have been conducted by many researchers. ReVelle and Kirby (1970) and ReVelle and Gundelach (1975) examined the LDRs based on various reservoir performance criteria. Nayak and Arora (1971) and Eastman and ReVelle (1973) extended the RP model to multi-reservoir systems. Gundelach and ReVelle (1975) also developed a general algorithm for formulating and solving the family of RP models. Houck et al. (1980) included economic returns and hydropower production as alternative objective measures in RP models -- in contrast to the common approach of minimizing reservoir capacity. The original formulation as proposed by ReVelle et al. (1969) and its many variations generally suffer from the problem of being overconservative for the specified risk and reliability levels because they do not account for correlations among the flows (Loucks, 1970; Loucks and Dorfman, 1975; Luthra and Arora, 1976; Stedinger et al., 1983; and Stedinger, 1984). Recently developed modifications of RP models have demonstrated significant improvements in handling this problem by incorporating the necessary covariance structure between the successive inflows (Houck and Datta, 1981; Joeres et al., 1981).

Stochastic programming and reliability programming adopt distinctive concepts as well as methodologies to account for the hydrologic uncertainty of natural streamflows. The flexibility of SP models allows for various considerations of the expected reservoir performance based on the storages and the inflows. On the other hand, RP models place major emphasis on the reliability aspect of reservoir operation to achieve certain performance criteria. In comparison, the SP formulations may have wider scope as well as greater potential for practical applications than the RP models for the following reasons. First, although the theories of RP for a single reservoir for long-term planning purposes were well developed, extensions to multireservoir systems might be limited because the system reliability would be hard to define to reflect the aggregate effects of the individual reservoir performances in the system. Moreover, to represent the correlations among streamflows properly, discretization of flows is inevitable. Thus, the size of an RP model can easily grow much larger than that of models with no intraperiod flow correlations.

Other stochastic modeling approaches were usually developed either for a specific reservoir operation environment or by adopting a different mathematical programming technique. Maidment and Chow (1981) presented a state variable model in conjunction with the SP formulation for a single reservoir operation. The model allows continuous inflows within the dynamic programming procedure, thereby allowing precise application of chance constraints within the optimization. However, the assumption of a normal inflow distribution is relatively restrictive. Bras et al. (1983) developed a closed loop control procedure for the real-time monthly operation of the High Aswan Dam in Egypt. Their model differs from the conventional SP formulation in that the current inflows are conditioned on the forecast inflows rather than on the steady-state Markov transition probabilities. Turgeon (1980) proposed both a one-at-a-time method and an aggregation/decomposition method to solve an SP problem indirectly for large multireservoir systems.

Whatever the variations of stochastic models may be, either the Markov assumption or chance constraints are included as basic components to account for hydrologic uncertainties. Although stochastic models incorporate probabilistic considerations in the optimization procedure, the resulting operation policy, once executed, becomes deterministic. Thus, stochastic models are most useful for evaluating the consequences from applying the optimal operation policies under various inflow situations. Considerable judgment must be exercised, however, about the future hydrologic conditions before any release decision is made.

2.4 Optimization Techniques

Optimization models for reservoir planning, design, or operation have usually been linear programming (LP) or dynamic programming (DP) formulations. LP models require linearity in both the objective function and the constraints. Techniques are available (e.g., in Loucks et al., pp. 57-62, 1981) to develop linear approximations of the nonlinear functions of either hydropower production or economic returns. However, the increased number of variables and constraints due to linearization greatly hampers computational efficiency in finding the optimal solution. LP models are mostly solved by commercially available computer codes. Sometimes, preparation of the data base for LP codes could be more time-consuming than execution of the optimization algorithm itself.

The size of a reservoir model also increases when the planning or operation horizon is increased. Since both natural inflows and water demands generally exhibit cyclic patterns, an LP formulation of a reservoir model may include many similar constraints with only minor differences in parameter values. Relatively, DP may be more efficient for reservoir study when multiple periods are considered (Hall et al., 1968). Recursive sets of equations are fundamental to DP; the optimal solution can be obtained by sequentially optimizing the recursive equations. Roefs and Guitron (1975) explored these issues and concluded that DP is preferred for most reservoir planning and operation situations. Yakowitz (1982) provided a rather comprehensive review of the various applications of DP models in water resource systems. Other techniques such as decomposition or successive approximation (Turgeon, 1981; Houck and Cohon, 1978), however, might be used jointly with LP or DP for multireservoir systems.

DP, in contrast to LP, does not have a standard mathematical form. Transformation of a reservoir problem into a DP model is sometimes difficult in terms of selection of state variables, or determination of recursive equations. Although a DP model with continuous state variables can be in theory solved, almost all the applications are in the discrete form for easy formulation and for convenience in coding computer programs.

The number of feasible decisions that must be evaluated within a stage might increase tremendously with finer increments of the discrete state variables in a DP model. Various strategies have been developed to accelerate the recursive optimization process. For example, discrete differential dynamic programming (DDDP) formalized by Heidari et al. (1971) is an "iterative technique in which the recursive equation of dynamic programming is used to search for an improved trajectory." DDDP has been reported to be very effective in the analysis of various water resource system problems (Chow et al., 1975). Liebman and Lynn (1966) earlier included this concept in the solution algorithm of a DP model for the optimal control of water quality in the Willamette River, Oregon.

Trott and Yeh (1973) also used the same concept as DDDP, which they called incremental dynamic programming (IDP), with successive approximation to determine the size of individual reservoir to be built in a multireservoir system. A nonoptimal solution might result, however, when adopting the DDDP technique (Turgeon, 1982). Nopmongcol and Askew (1976) proposed a modified version of IDP, named multilevel incremental dynamic programming (MIDP), which was claimed to handle IDP better in searching for the global optimum. It should be noted that none of the abovementioned modified approaches is very useful within the stochastic DP framework in that they are deterministic models; and a sequence of known flows must be specified.

Beside LP and DP, other techniques also exist for reservoir modeling. Windsor (1977) presented a mixed integer linear programming model for the capacity expansion of power plants in a pumped-storage system. The integer variables were used for describing the possible discrete states of installation of future power generation facilities. Martin (1983) made successive linear approximations to a nonlinear multireservoir system and used an efficient network solution algorithm throughout the course of optimization. Sigvaldason (1976) also adopted the out-of-kilter algorithm (Ford and Fulkerson, 1962) for deriving the optimal release schedules for a 48-reservoir network in the Trent River system in Ontario, Canada. Klemeš (1979) developed a unique "stretched thread" method based on the concept of mass curve of streamflows. He claimed that in some occasions the proposed method is computationally more efficient than either the LP or the DP formulations.

Selection of a mathematical model for reservoir study requires careful judgment from a modeler. The efficiency of the solution algorithm is only one of the many considerations that may affect the modeling results. The quality of data used in mathematical programming models is as important as the models themselves. Quite often the biggest challenge to modelers is to collect the data and incorporate them into the rigid programming framework.

2.5 Comments

Reservoir operation models have proliferated during the past two decades. Mathematical programming techniques help identify efficiently the best scheme among various alternative operation plans. By taking advantage of efficient optimization procedures, system analysts are able to incorporate more factors related to optimal reservoir controls than they are using a conventional trial-and-error approach.

Stochasticity can be included in mathematical programming frameworks to evaluate a reservoir system's performance under hydrologic uncertainty. However, a tremendous data base might be required to characterize a reservoir system properly when using stochastic models. The effectiveness and practicality of mathematical programming models could easily be hampered because of their size as well. The analysts should exercise judgment to utilize a model's flexibility rather than be restricted by its rigid mathematical structure.

Determination of the proper objective function is crucial for an optimization model to describe a decision maker's will as well as the system's physical reality adequately. However, the definition and selection of objective functions are common problems in the planning and design of a reservoir system that are not unique to constructing optimization models. Future socio-economic and political changes may affect the optimal decisions that are based on the current reservoir operation goals as described by the mathematical model. Multiobjective programming techniques (e.g. Cohon and Marks, 1975) may be useful in providing broader insights in operation situations.

Finally, a significant reason that reservoir optimization models are seldom implemented for real-world reservoir systems is the lack of coordination among different groups of people with various backgrounds and interests. These models are restricted in that they lack multi-disciplinary inputs to the model formulation (Changnon, 1985). Even if a reservoir operation model could be developed, to deliver and utilize effectively the information revealed by a model might be just as difficult as the task of developing the model itself.

III. EVALUATION OF STOCHASTIC PROGRAMMING MODEL

3.1 Purpose

Stochastic programming (SP) models are commonly used to evaluate the optimal expected performance of a reservoir system under steady-state operation. Much work has been done to develop efficient solution algorithms and to examine the effects of using various operation criteria and performance measures. The modeling results might be biased, however, if an SP model is improperly constructed. Klemes (1977) showed that the level of partitions of inflow and storage states could affect the accuracy of the expected reservoir performance. He also pointed out that to discretize continuous variables could distort the optimal decisions, and that the distortion might not be easily recognized. Therefore, it is important to realize the potential shortcomings of using SP models, and to avoid misunderstandings that may lead to erroneous conclusions about reservoir operation.

In Section 3.3, sensitivity analysis is performed for SP models to investigate the various causes which can result in misleading or even conflicting operation decisions. A few selected cases are further studied using Monte Carlo simulation to examine the consequences of following different operation practices under hydrologic uncertainty. The findings observed in these analyses provide useful insights for constructing an operation model for a reservoir system based on the SP framework.

3.2 A Hypothetical Single Reservoir System

To explore the fundamental issues related to system performance for a wide range of reservoir operation conditions, a hypothetical single reservoir system was used. This hypothetical system was configured so that both the physical characteristics and the inflow-storage relationship are reasonably close to reality and analogous to those of Lake Shelbyville, Illinois, which is considered for the case study in Chapter 4. Throughout the sensitivity analysis the values of system components as well as the inflow characteristics are varied within appropriate ranges to compare the system's response under different model settings and operation requirements.

3.2.1 Definitions

3.2.1.1 Storage State

Let S_c be the active storage capacity of a single reservoir to regulate outflow release. The storage space which cannot be utilized freely, such as the dead storage or the conservation storage for water supply and navigation usages, is not considered as part of the active storage. For explicit stochastic programming the active storage is discretized to represent the finite number of possible storage states for reservoir control.

Klemeš (1977) summarized two distinct methods for dividing the storage space based on several earlier studies conducted by other researchers (Savarenskiy, 1940; Moran, 1954; Venetis, 1969; and Doran, 1975). Figure 3.1 illustrates the difference between the two ways for defining storage states. The first scheme (Savarenskiy, 1940; and Doran, 1975) treats each of the equally divided zones of S_c as a state interval with increment ΔS; the corresponding storage level is defined at the center of this interval and called the state mark. In addition, both ends of the active storage, denoting respectively the full and the empty states, are

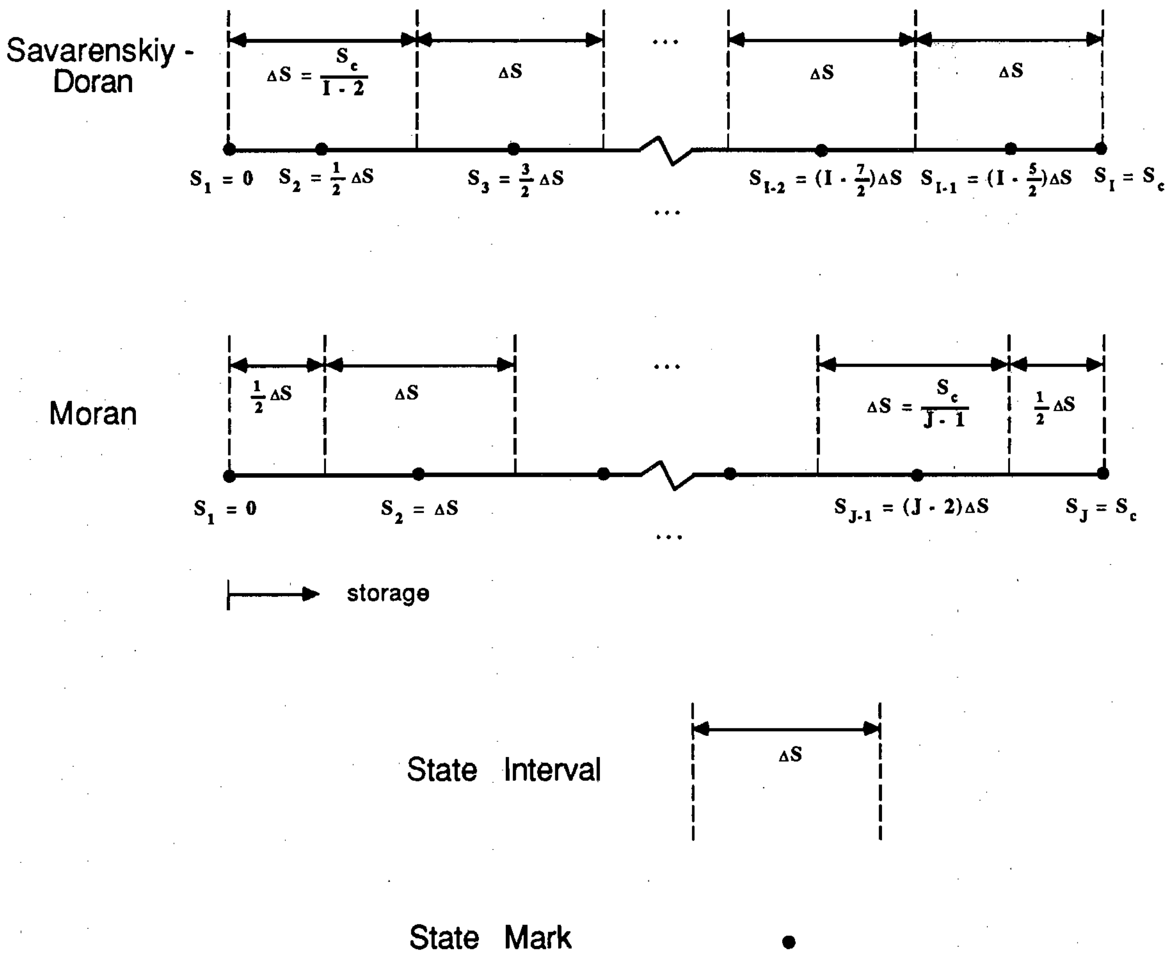

Figure 3.1 Illustration of Savarenskiy-Doran's and Moran's Schemes for Defining Discrete Storage States.

defined separately with zero state intervals. Thus, the discrete storage states can be defined by: $S_1 = 0$; $S_i = (i - 3/2)\Delta S$, for $2 \leq i \leq I-1$; and $S_I = (I - 2)\Delta S = S_c$. In the second scheme (Moran, 1954), the state boundaries and the state marks are interchanged in position as compared to those of the first scheme. That is, the state marks are defined at the boundaries of the equally divided storage zones in the first scheme; and the midpoints of these storage zones are reversely treated as the boundaries of the corresponding storage states. The only two exceptions are the full and the empty states with their respective state marks residing on one side of the state boundaries. As a result, the storage states are defined by: $S_1 = 0$ and $S_j = (j - 1)\Delta S$, for $2 \leq j \leq J-1$, and $J = I-1$ if the ΔS's are equal in both cases.

Because of its definition, Moran's scheme tends to overestimate the probabilities of both emptiness and fullness of the storage states (Doran, 1975; and Klemeš, 1977). It should be realized that the severity of this overestimation depends not only upon the way that the storage state is defined but also upon its relationship to the inflow volumes. From the analysis by Klemeš (1977), it can be concluded that as long as the storage increment ΔS is significantly less than the total range of possible inflow volumes, the distortion in probability estimation of strict emptiness and fullness caused by storage discretization would be minimal in SP modeling.

The release policy also could have certain impacts on how the storage states might vary from period to period. Both Doran's and Klemeš' studies examined the consequences of improper discretization schemes for storage states based on simple release policies. Such policies include the "standard policy" which dictates a constant release and the policy in which release is a function of either the initial storage or the mean inflow volume. None of the above policies conforms to the general SP framework in which the optimal release policy is a function of both the initial storage and the stochastic inflow. Because of the possible transitions of inflow states between successive periods, the storage state is unlikely to be trapped in either the empty or the full states given a reasonably sized active storage space. Thus, care should be exercised before Doran's and Klemeš' findings can be generalized to typical SP models.

Finally, a real reservoir system can be regarded as being in an unsatisfactory state far before the storage reaches strict emptiness or fullness. Therefore, for practical reasons the emphasis on accurate probability estimation of strict emptiness and fullness might not be as crucial as that of proper definition of threshold storage levels which differentiate the satisfactory states from the unsatisfactory states in reservoir operations. In the subsequent analysis, the Savarenskiy-Doran scheme is adopted for its overall consistent representation of storage states in the center of respective storage zones.

3.2.1.2 **Inflow State**

Some special features are uniquely associated with the partition of the inflow variable as opposed to that of the storage variable. First, the range of an inflow distribution is hard to determine. Second, inflow distributions tend to be skewed; and the probabilities of normal inflow states are usually much greater than those of the extreme inflow states. Finally, an inflow distribution varies periodically in time. Thus, the partition of inflow states needs special attention for each month or season of a year in the discrete SP model. It is beyond the scope of this study to investigate which inflow distribution might be the most appropriate to use; rather, the emphasis is placed upon the different partition precisions that could affect the modeling results. Therefore, the following assumptions are made in order to proceed with the analysis.

The lognormal distribution is used for inflows in the hypothetical system because it can represent diverse distribution patterns of the inflows and is easy to manipulate mathematically. Unless otherwise specified, the lower bound of the inflow distribution is assumed to be zero. The upper bound is defined to be the probable maximum inflow (PMI) which could be derived from the probable maximum precipitation (PMP) (Stallings et al., 1986) in a river basin (Singh, 1977). Since an upper bound is specified, the inflow distribution is actually "truncated" at the upper end. The original probability mass beyond the upper bound is then redistributed within the feasible range in proportion to the likelihoods of occurrence of individual inflow events so that the total probability mass equals one for the truncated distribution.

The bounded distribution is then divided into a finite number of inflow states as illustrated in Fig. 3.2. The discrete inflow state or state mark is defined at the center of each inflow zone or state interval. The probability of each inflow state is integrated through the corresponding inflow zone, and is assumed to be concentrated at the discrete inflow state. Therefore, the inflow states are defined by $Q_m = (m - 1/2)\Delta Q$, for $1 \leq m \leq M$; and ΔQ is the state increment with $M\Delta Q = PMI$. The corresponding steady-state probabilities are PQ_m, for $1 \leq m \leq M$.

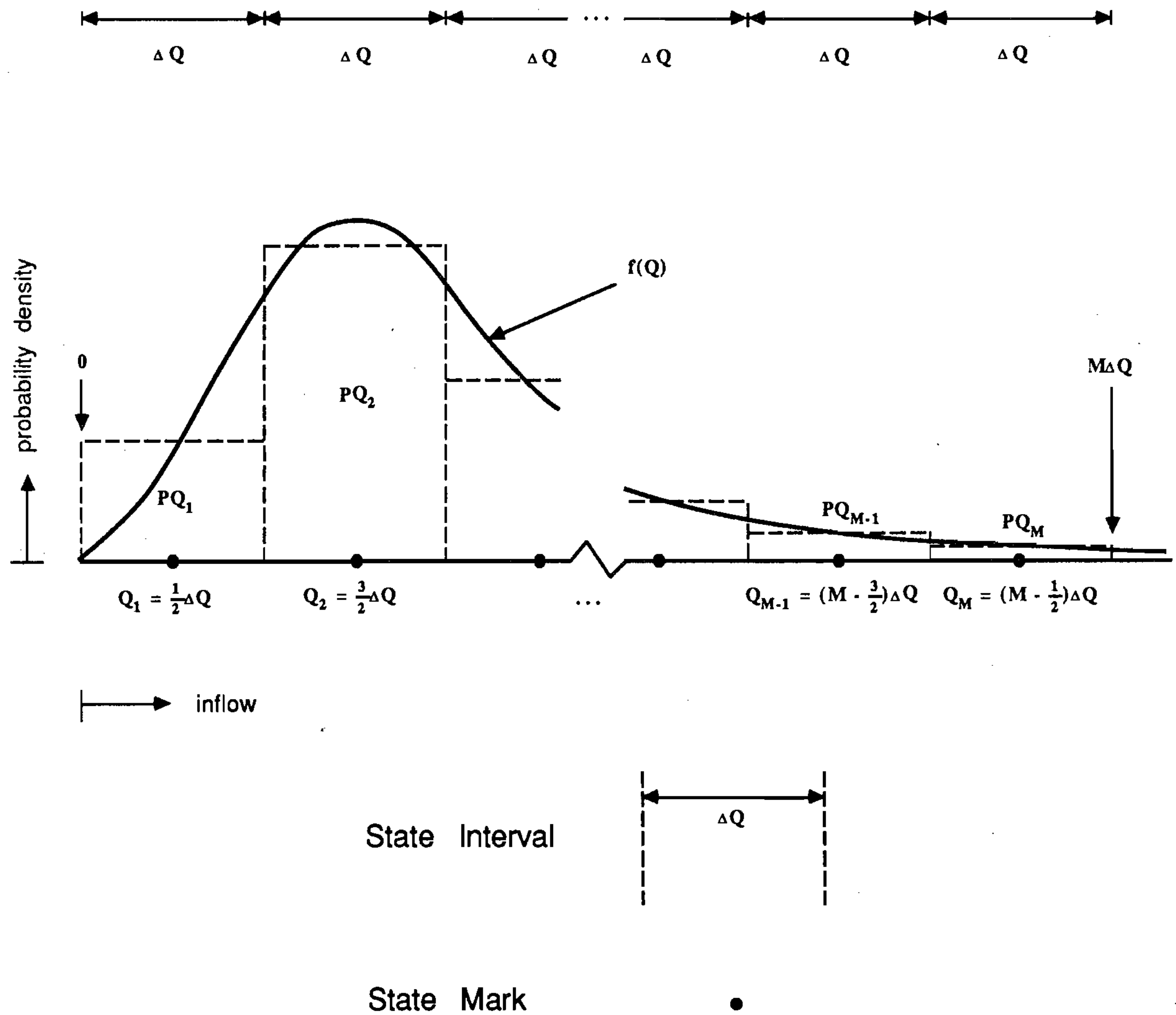

Figure 3.2 Definition of Inflow States with the Corresponding Steady State Probabilities, PQ_m.

It should be noted that inflows, unlike storage, are products of the natural complex hydrologic processes, which cannot be controlled at exact discrete levels. Thus, an inflow state used in an SP model essentially represents a range of inflow amounts within $\pm 1/2\Delta Q$ deviations from the discrete state value. At each period t, the relationship between the natural inflow $\tilde{Q}_t$ and the discrete model inflow Q_{mt} can be expressed as

$$\tilde{Q}_t = Q_{mt} + \delta_t \tag{3.1}$$

where $-1/2\Delta Q < \delta_t < 1/2\Delta Q$ is the possible inflow deviation. According to the mass balance law, the release in each period is defined by

$$R_{imjt} = S_{it} + Q_{mt} - S_{j,t+1} \tag{3.2}$$

where R_{imjt} is the release volume; S_{it} and $S_{j,t+1}$ are the initial and the final storages, respectively; and the losses are neglected. While Eq. 3.2 is used in the model, the actual release $\tilde{R}_t$ should be modified according to Eq. 3.1 as

$$R_{imjt} = S_{it} + (\tilde{Q}_t - \delta_t) - S_{j,t+1} \tag{3.3}$$

and $$\tilde{R}_t = R_{imjt} + \delta_t = S_{it} + \tilde{Q}_t - S_{j,t+1} \tag{3.4}$$

Equation 3.4 indicates that the actual release differs from the model release by the same amount as the difference between the actual and the model inflows if the discrete selection on storage state is strictly followed. Thus, it is possible that $\tilde{R}_t$ may actually violate the allowable release limits while R_{imjt} is still within the desirable release range. A practical resolution might be to restrict $\tilde{R}_t$ from violating any release limit, and to displace δ_t to the final storage $S_{j,t+1}$ which could be gradually adjusted back to one of the discrete states in the subsequent operation periods. In case that $S_{j,t+1}$ would also violate the storage constraints when absorbing δ_t, a decision must be made to weigh the relative severities of release and storage violations for a given reservoir system.

The above observation distinguishes between the implicit assumptions associated with the inflow and the storage states in discrete SP models. More discussion of the inflow and storage partitions is given in the sensitivity analyses in Sec. 3.3.2.

3.2.1.3 Markov Transition Probability Matrix

The Markov process model is widely used in engineering practice to describe natural time series. It is a conceptual model which assumes that the states of a system are governed by the previous states through certain probability laws. The transition probabilities which dictate the likelihood of transition from one state to another can be estimated in two ways depending upon the model structure and the amount of information available. They can be calculated directly from the historical data by grouping these data into the corresponding discrete state transitions. The relative frequencies of these mutually exclusive events can then be used to compute the probabilities of changing from one specific state to each of the possible states in the subsequent period. This empirical method is only feasible when the historical data base is large enough to characterize the whole spectrum of transition probabilities properly. Unfortunately, real hydrologic data bases used in water resource systems studies are rarely sufficiently large.

The second way to estimate the transition probability matrix is to use an analytical method (Wang and Adams, 1986). Let $\tilde{Q}_t$ and $\tilde{Q}_{t+1}$ be the continuous inflow variables in successive periods; and Q_{mt} and $Q_{n,t+1}$ be the corresponding inflows in the discrete domain, which represents the transition from a state m in period t to a state n in period t+1. The probability of this joint event Q_{mt} and $Q_{n,t+1}$ occurring can be expressed as

$$P^t_{mn} = \text{prob}(Q^{\ell}_{n,t+1} < \tilde{Q}_{t+1} < Q^u_{n,t+1} \mid Q^{\ell}_{mt} < \tilde{Q}_t < Q^u_{mt})$$

$$= \frac{\text{prob}(Q^{\ell}_{n,t+1} < \tilde{Q}_{t+1} < Q^u_{n,t+1} \text{ and } Q^{\ell}_{mt} < \tilde{Q}_t < Q^u_{mt})}{\text{prob}(Q^{\ell}_{mt} < \tilde{Q}_t < Q^u_{mt})} \tag{3.5}$$

where the superscripts u and ℓ denote the upper and the lower boundaries of the discrete inflow states Q_{mt} and $Q_{n,t+1}$, respectively. When written explicitly in terms of the cumulative distribution functions, Eq. 3.5 is equivalent to

$$P^t_{mn} = \frac{\psi_1}{\psi_2} \tag{3.6}$$

$$\psi_1 = F_{t,t+1}(Q^u_{mt}, Q^u_{n,t+1}) - F_{t,t+1}(Q^u_{mt}, Q^{\ell}_{n,t+1}) - F_{t,t+1}(Q^{\ell}_{mt}, Q^u_{n,t+1}) + F_{t,t+1}(Q^{\ell}_{mt}, Q^{\ell}_{n,t+1}) \tag{3.6a}$$

$$\psi_2 = F_t(Q^u_{mt}) - F_t(Q^{\ell}_{mt}) \tag{3.6b}$$

where $F_{t,t+1}(Q_t, Q_{t+1})$ and $F_t(Q_t)$ denote respectively the joint and the marginal inflow distributions. As long as the probability density functions can be mathematically defined, P^t_{mn} can be calculated readily by performing numerical integration over the desired range of inflow volumes.

To use the analytical method effectively, ordinary inflows need to be modified so that the bivariate distribution function in Eq. 3.6a can be easily obtained and evaluated. Usually, the inflow variable $\tilde{Q}$ is transformed into the standard normal variate Z by a certain transformation function, $Z = Z(\tilde{Q})$, to represent more accurately the correlation structure of the flow variables (Stedinger, 1980). In this study, a logarithmic transformation is adopted to convert the natural inflows. As a result, the transition probability P^t_{mn} can be calculated in the normal space as

$$P^t_{mn} = \frac{\psi_3}{\psi_4} \qquad (3.7)$$

$$\begin{aligned} \psi_3 = \; & G_{t,t+1}(Z^u_{mt}, Z^u_{n,t+1}) - G_{t,t+1}(Z^u_{mt}, Z^\ell_{n,t+1}) \\ & - G_{t,t+1}(Z^\ell_{mt}, Z^u_{n,t+1}) + G_{t,t+1}(Z^\ell_{mt}, Z^\ell_{n,t+1}) \end{aligned} \qquad (3.7a)$$

$$\psi_4 = G_t(Z^u_{mt}) - G_t(Z^\ell_{mt}) \qquad (3.7b)$$

where $G_{t,t+1}(Z_t, Z_{t+1})$ and $G_t(Z_t)$ are the joint and the marginal normal distributions of the transformed inflows, with $Z^u = Z(Q^u)$ and $Z^\ell = Z(Q^\ell)$. By specifying the upper limits of the two variables, the bivariate normal distribution can readily be calculated. Because a truncated distribution is assumed for this study, a final normalization scheme defined by

$$(P^t_{mn})' = \frac{P^t_{mn}}{\sum_{n=1}^{N} P^t_{mn}} \qquad \text{for all m \& t} \qquad (3.8)$$

is required to assure that $\sum_{n=1}^{N} (P^t_{mn})' = 1$.

The Markov transition probability matrix is model-dependent, as an underlying stochastic process is assumed for the historical inflow series.

Therefore, using the analytical method outlined above does not imply that more hydrologic information, in terms of the state-to-state transitions, can be extracted from the historical record. Rather, it only provides a proper working basis to derive the transition probabilities under the Markov assumption. In the following derivation, P^t_{mn} is indisputably used in place of $(P^t_{mn})'$ for representing normalized transition probabilities to simplify the mathematical form of an SP model.

3.2.2 Optimization by Stochastic Dynamic Programming

3.2.2.1 Model Formulation

Consider the control of a single reservoir system in which a year is divided into a finite number of operation periods. For each period t, let S_{it} be the initial storage level at state i, and Q_{mt} be the inflow volume of state m. The final storage level is to be determined so that the best long-term system performance can be expected, given the current inflow and storage conditions.

Assuming that the reservoir ends operation at period t = T in some year in the future, define $f^{\tau}_t(i,m)$ as the total expected value of system performance with τ periods to go, including the current period. Thus, $\tau = T-t+1$. Let B_{imjt} be the value of system performance associated with S_{it}, Q_{mt}, and a final storage volume $S_{j,t+1}$. Then with only one period remaining,

$$f^1_T(i,m) = \underset{j}{\text{maximum}}\ (B_{imjT}) \qquad \text{for all } i,m;\ j \text{ feasible} \qquad (3.9)$$

which simply selects the largest B_{imjT} value for all the feasible j's. With two periods remaining, the maximum expected system performance can be determined by

$$f_{T-1}^{2}(i,m) = \underset{j}{\text{maximum}}\ [B_{i,m,j,T-1} + \sum_{n} P_{mn}^{T-1} f_{T}^{1}(j,n)]$$
$$\text{for all } i,m;\ j \text{ feasible} \quad (3.10)$$

where P_{mn}^{T-1} is the transition probability of inflow states from $Q_{m,T-1}$ to $Q_{n,T}$. Because the outcome of inflow state $Q_{n,T}$ is uncertain when looking forward from period T-1, the system performance in period T should be weighted by the possible transitions from $Q_{m,T-1}$ to all $Q_{n,T}$'s. Thus, $f_{T-1}^{2}(i,m)$ defines the best expected performance in the final two periods for each pair of $S_{i,T-1}$ and $Q_{m,T-1}$.

Equation 3.10 can be generalized for any period t, with τ periods remaining, by the same recursive relationship as

$$f_{t}^{\tau}(i,m) = \underset{j}{\text{maximum}}\ [B_{imjt} + \sum_{n} P_{mn}^{t} f_{t+1}^{\tau-1}(j,n)]$$
$$\text{for all } i,m,t;\ j \text{ feasible} \quad (3.11)$$

Along the backward optimization procedure, not only can the function $f_{t}^{\tau}(i,m)$ be evaluated, but also the corresponding storage decision $j = j(i,m,t)$ is determined. If B_{imjt} and P_{mn}^{t} do not change for the same period from year to year, and T is selected to be long enough, the optimal storage decisions will be invariant for the same period in a year (Ross, 1970). In that case, the steady-state operation policies have been found, and the backward optimization procedure can be terminated.

After the optimal policy for the final storage is determined, the related optimal release R_{imjt} can be determined by Eq. 3.2. Under a steady-state condition, the joint probability of R_{imjt}, denoted by PR_{imjt}, will be zero for nonoptimal policies. Loucks et al. (1981, p. 326) showed that the PR_{imjt} values can be calculated by solving the following linear simultaneous equations

$$PR_{j,n,k,t+1} = \sum_{\substack{i=1 \\ j=j(i,m,t)}}^{I} \sum_{m=1}^{M_t} PR_{imjt} P_{mn}^{t} \qquad \text{for all } j,n,t \tag{3.12}$$
$$k=k(j,n,t+1)$$

$$\text{and} \quad \sum_{i=1}^{I} \sum_{m=1}^{M_t} PR_{imjt} = 1 \qquad \text{for all } t \tag{3.13}$$

One equation in Eq. 3.12 for each time period is redundant; thus after eliminating the redundant equations the number of equations in Eqs. 3.12 and 3.13 equals the number of the unknowns.

The total number of unknowns in these equations equals the product of the numbers of the storage states, the inflow states, and the time periods in a year. Thus, the size of the solution matrix may become enormous when many states and time periods are considered. Rather than trying to solve for the PR_{imjt}'s directly, the successive substitution method is recommended. That is, an arbitrary distribution for PR_{imj1} is assumed and substituted into Eq. 3.12 to calculate PR_{jnk2}. Then, the same process is repeated for the subsequent periods until the PR_{imjt}'s converge to the steady-state probabilities. The advantage of using successive substitution is that it involves only simple arithmetic operations with a much smaller matrix structure. Furthermore, the rate of convergence is generally very efficient so that only the PR_{imjt}'s in first few periods need to be recalculated.

After the steady-state probabilities are obtained, the expected annual system performance can be calculated by

$$\text{Expected Annual Performance} = \sum_{\substack{t=1 \\ j=j(i,m,t)}}^{T} \sum_{i=1}^{I} \sum_{m=1}^{M_t} B_{imjt} PR_{imjt} \tag{3.14}$$

in which T now denotes the number of operation periods in a year, and M_t is the total number of inflow states for period t, t=1,...,T.

3.2.2.2 Model vs. Reality

In a typical stochastic programming model, the system performance is usually assumed to be a function of either the release or the storage, or both. However, the optimal release calculated from the mass balance equation in an SP model does not really prescribe how this amount of outflow would be regulated within that operation period. Ideally, there could be two extreme schemes in controlling the storage and the release. For the first scheme, depicted in Fig. 3.3(a), the storage is varied at a constant rate, and the release rate r will fluctuate with the inflow rate q on a real-time operation basis. The fixed difference of q and r equals the rate of change in storage. In the second scheme, illustrated by Fig. 3.3(b), the release is held constant whenever possible, and the storage level will fluctuate as a result of the uniform release. Both schemes are based on the assumption that the total inflow volume is predicted accurately in the beginning of an operation period.

However, a real reservoir system is likely to be regulated between these two extremes, as an operator must respond to the constantly changing inflow situations. For example, the water level as a function of the storage state affects directly the various recreational and agricultural activities related to a reservoir. Therefore, it is desirable to control the storage stage according to the optimal rule with minimum fluctuations in an operation period. Meanwhile, it is also important to prevent abrupt changes of the storage level to avoid potential erosion along the

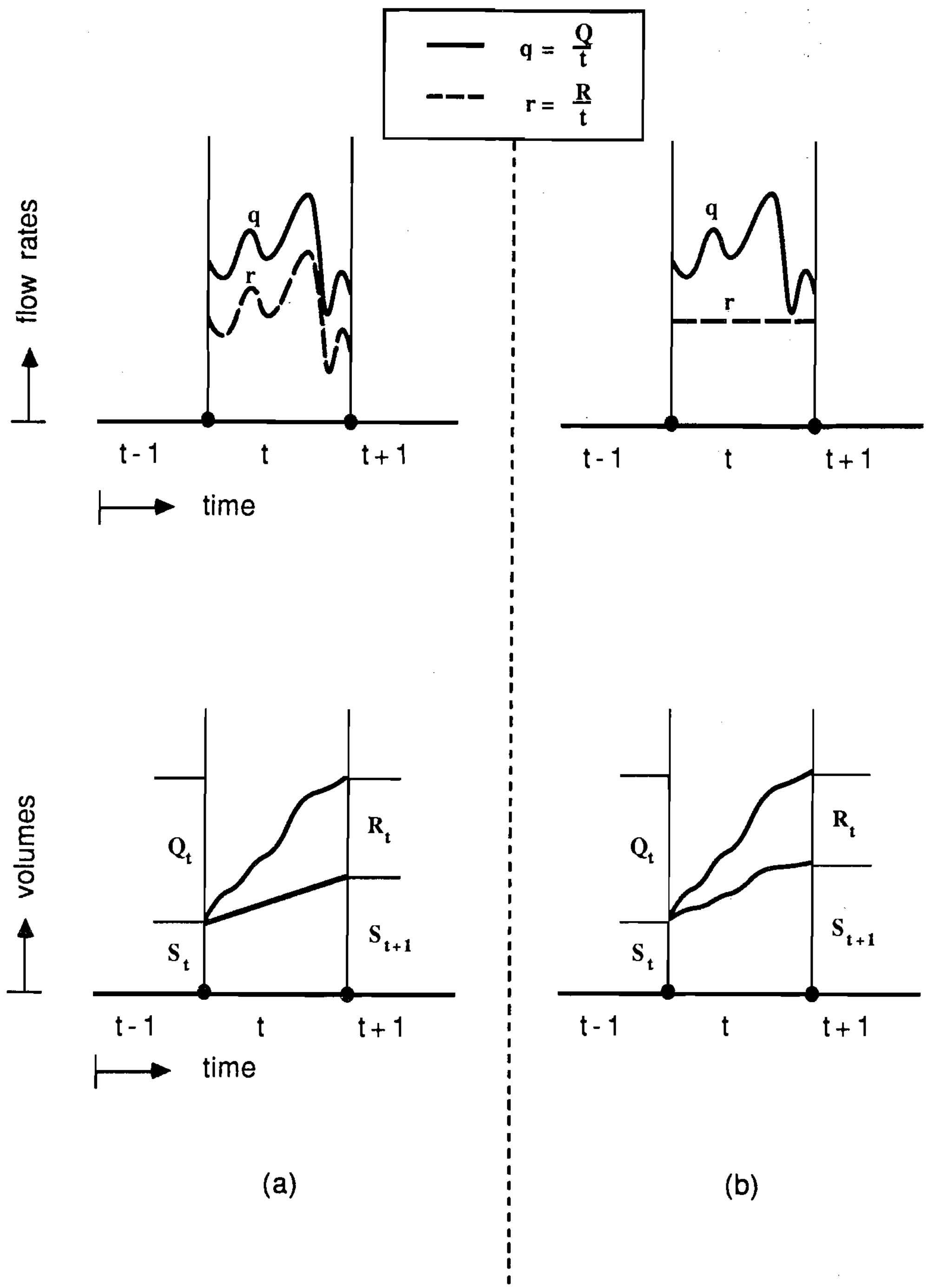

Figure 3.3 Comparison of Two Extreme Reservoir Regulation Schemes Based on the Same Discrete Policy: (a) Constant Rate of Changing from Initial Storage State to Final Storage State, and (b) Constant Release Rate.

reservoir banks. Thus, both the total storage volume and its changing rate are important in evaluating the reservoir performance.

Equally crucial is the protection of downstream floodplains, which depends largely upon the release rate from the upstream reservoir. The total release volume over a long period usually would not have great impact on flooding which is a relatively short-term phenomenon. However, for meeting water supply, irrigation, and sometimes hydropower production demands under persisting drought conditions, the total amount of water available for these purposes becomes a determining factor.

In SP modeling for steady-state reservoir operations, no explicit relationship is considered between the inflow variation and the total inflow volume for each operation period. Without perfect inflow information, SP models certainly depart from reality. As the discrete time interval is decreased, the discrepancy between model and reality maybe expected to diminish. However, for models with a longer operation period, the performance functions should be carefully selected.

3.3 Sensitivity Analysis

It has been observed in the previous sections that quite a few uncertainties as well as difficulties could exist in building a typical SP model for reservoir operation. In the following sections, sensitivity analyses are conducted for a wide variety of system conditions which can be characterized by the ratio of reservoir capacity to the mean annual inflow volume, the variations of inflows within and across the seasons, and the performance measures for system operation. Some unique features of SP modeling are demonstrated; and the optimal results discussed.

3.3.1 Basic System Setup

Given a fixed mean annual inflow volume $\bar{Q}_a$ of 50*, the hypothetical reservoir is assumed to have one of the three distinct active storage capacities S_c of 25, 50, and 100. They represent three values of the storage ratio α of 0.5, 1, and 2, which provide different levels of inflow control. A year is divided into four operation periods to reflect the seasonal variation of the inflows, and hence its influence on the intra-period release and storage regulations. The mean seasonal inflows $\bar{Q}_d$, for d = 1, 2, 3, and 4, are chosen to be 2.5, 12.5, 22.5, and 12.5, respectively, with the corresponding upper bounds of inflow volumes being $10\bar{Q}_d$. Both the correlation coefficient ρ_d and the coefficient of variation ν_d are assumed to be constants for the four seasons with values of 0.5 and 1.0, respectively. The relative magnitudes of the above inflow statistics were selected to resemble typical hydrologic conditions in central Illinois. Both ρ_d and ν_d were varied to cover a much wider range of inflow situations. Assuming the inflows are characterized by the truncated lognormal distribution, the transition probability matrix can be estimated using these inflow statistics and the analytical method described in Section 3.2.1.3. Values of the above parameters and statistics are summarized in Table 3.1.

To begin the study, only the single objective of downstream flood protection is considered. Later, a second objective of hydropower generation is added to demonstrate the tradeoffs between these two

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* Note: Since the only significance of numerical values is their relative values, no unit will be given. Both storage and inflow are in the same volumetric unit of L^3.

Table 3.1 Values of Parameters and Performance Functions Used for the Hypothetical Reservoir System and for the Corresponding Sensitivity Analysis of SP Models.

Category	Value#	Comment
Reservoir Characteristics:		
a. Active Storage Capacity, S_c	25,50,100	
Inflow Statistics:		
a. Mean Annual Inflow Volume, $\bar{Q}_a$	50	$(S_c/\bar{Q}_a$=0.5,1,2)
b. Mean Seasonal Inflow Volumes, $\bar{Q}_1$	2.5	truncated
$\bar{Q}_2$	12.5	lognormal
$\bar{Q}_3$	22.5	distribution
$\bar{Q}_4$	12.5	
c. Maximum Seasonal Inflows, $Q_{d,max}$	$10\bar{Q}_d$	d = 1,2,3,4
d. Variation of Inflows:		
1. Correlation Coefficient, $\rho_{d,d+1}$	0.3,0.5*,0.7	* typical values
2. Coefficient of Variation, ν_d	0.5,1.0*,1.5	
Partitions of States:		
a. Number of Inflow States, NQ	5,10,20	
b. Number of Storage States, NS	4,7,12,22	including two extreme states of strict emptiness and fullness
Performance Evaluation (Objectives):		
a. Flood Damage,	$(R_t-30)^\beta$, $R_t>30$; 0, $R_t\leq 30$	β = 0.5,1,2
b. Hydropower Production,	$K\{R_t,[(S_t+S_{t+1})/2]^{0.5}\}$	K{ , }: hydropower production as a function of R_t, S_t, and S_{t+1}

\# Storage and inflow are in commensurate volumetric units; flood damage and hydropower production are simplified measures of dollars and KW-hrs, respectively.

distinct objectives for different release decisions. For the flood protection objective it is assumed that based on a separate study of the downstream channel capacity, the maximum allowable release, when discharged uniformly from the reservoir, is limited to 30 in each period. Beyond this threshold value, the release would cause damages to the downstream riparian areas. A simplified cost index is used in the SP model to measure flood damages. A "damage cost" of $(R_t-30)^{\beta}$ will be induced whenever a release R_t exceeds the allowable limit, while no damage will be recorded if R_t is less than or equal to 30. β is selected to be 0.5, 1, and 2, respectively, to encompass the possible ways that the damage function may be defined (i.e., it may be concave, linear, or convex). The value of β will directly affect the optimal control policies as to how much the current system situation is weighted against future inflow and storage conditions.

3.3.2 Partitions of Inflow and Storage States

In the theoretical development of SP models, the increment values of the inflow and the storage states ΔQ and ΔS are commonly set equal to each other. For real reservoir systems, however, it is difficult to define the inflow and the storage states with exactly the same increment values. Hence it is considered appropriate in real studies for ΔQ and ΔS to be chosen with roughly the same magnitude in real studies. Under some circumstances, to set ΔQ and ΔS approximately equal in value would lead to a large number of storage states if the total storage space is much larger than the normal inflow volume within an operation period. Besides, in order to reduce the probable errors and distortions caused by discretization, the inflow and the storage states could be conservatively chosen so that the number of states might be much more than needed to characterize the system performance. There are some criteria developed from theoretical studies (Doran, 1975; and Klemeš, 1977) to determine the least number required for discretizing the inflow as well as the storage in a SP model. However, these rules simply identify the upper bounds for the numbers of states which assure stable optimal results, and sometimes these numbers may result in a very large SP model that is computationally impractical to solve.

Similar arguments can be applied to the partition of inflows in different seasons. The inflows usually exhibit seasonal variation, and the range of the inflow distribution for the wet season can be much broader than that of the dry season. Thus, an inflow partition which is considered appropriate for the wet season inflow may be too coarse for the dry season inflow if the same increment is used.

For typical SP models, the system performance would be evaluated at discrete states of either the inflow or the storage. Thus, as long as the performance measures and the resulting optimal solutions can be well represented by the selected discrete states, there seems to be no specific reason why the inflow partition cannot be varied from season to season and different from the partition of storage states. In the following analysis, an experiment is designed to test the significance of partitions of both the inflow and the storage states on the modeling results.

The basic setting defined in Section 3.3.1 is adopted for the hypothetical reservoir system. The inflow of each season is divided into NQ = 5, 10, and 20 states to represent different levels of precision with discretization. The storage is partitioned into NS = 4, 7, 12, and 22 states, with each set including the two extreme states of strictly empty and strictly full reservoirs with zero state interval. An optimization model is formulated and solved using stochastic dynamic programming to minimize the expected annual flood damage. The expected system performance can be evaluated using Eq. 3.14 for each combination of NS and NQ, for various active storage to mean annual inflow ratios $S_c/\bar{Q}_a$, and for different values of the exponent β of the damage function. The results of a number of tests incorporating these variations are summarized graphically in Figs. 3.4-3.6, and discussed in the following sections.

3.3.2.1 Effects on Expected Performance

Despite the distinct measures of the flood damage, namely the concave function with $\beta = 0.5$, the linear function with $\beta = 1$, and the convex function with $\beta = 2$, the patterns of convergence of the expected damage as a result of finer partitions of state variables are very similar in general, as described in the following paragraph. In addition, for most combinations of NS and NQ, ΔS and ΔQ are not set exactly equal, and sometimes differ in value by more than an order of magnitude. Nevertheless, the general trend of convergence of the expected flood damage is not significantly distorted by the difference between ΔS and ΔQ. The following observations and comments are considered applicable for a wide variety of performance measures taking the form of $(R_t - 30)^{\beta}$, for $R_t > 30$.

The expected damage as a function of the number of storage states NS quickly levels off as NS increases. This indicates that the optimal decisions, represented by the discrete values of the final storage, become stabilized as NS increases, and eventually would converge to the true continuous optimal decision values as the increment size gets smaller.

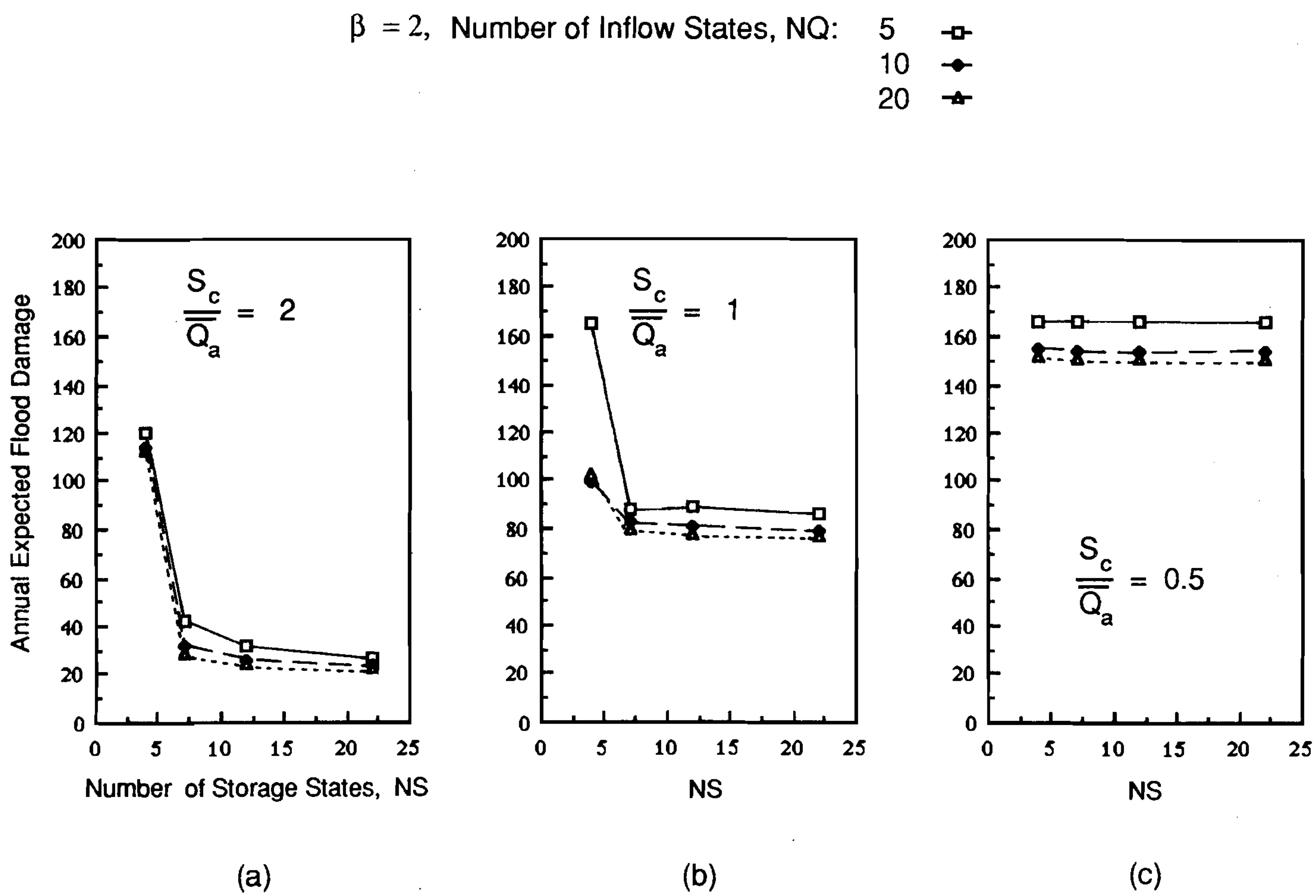


Figure 3.4 Estimation of Annual Expected Flood Damage Based on SP Model Using Different Combinations of Inflow and Storage State Partitions; Damage Is Calculated from $(R_t-30)^2$, for $R_t > 30$.

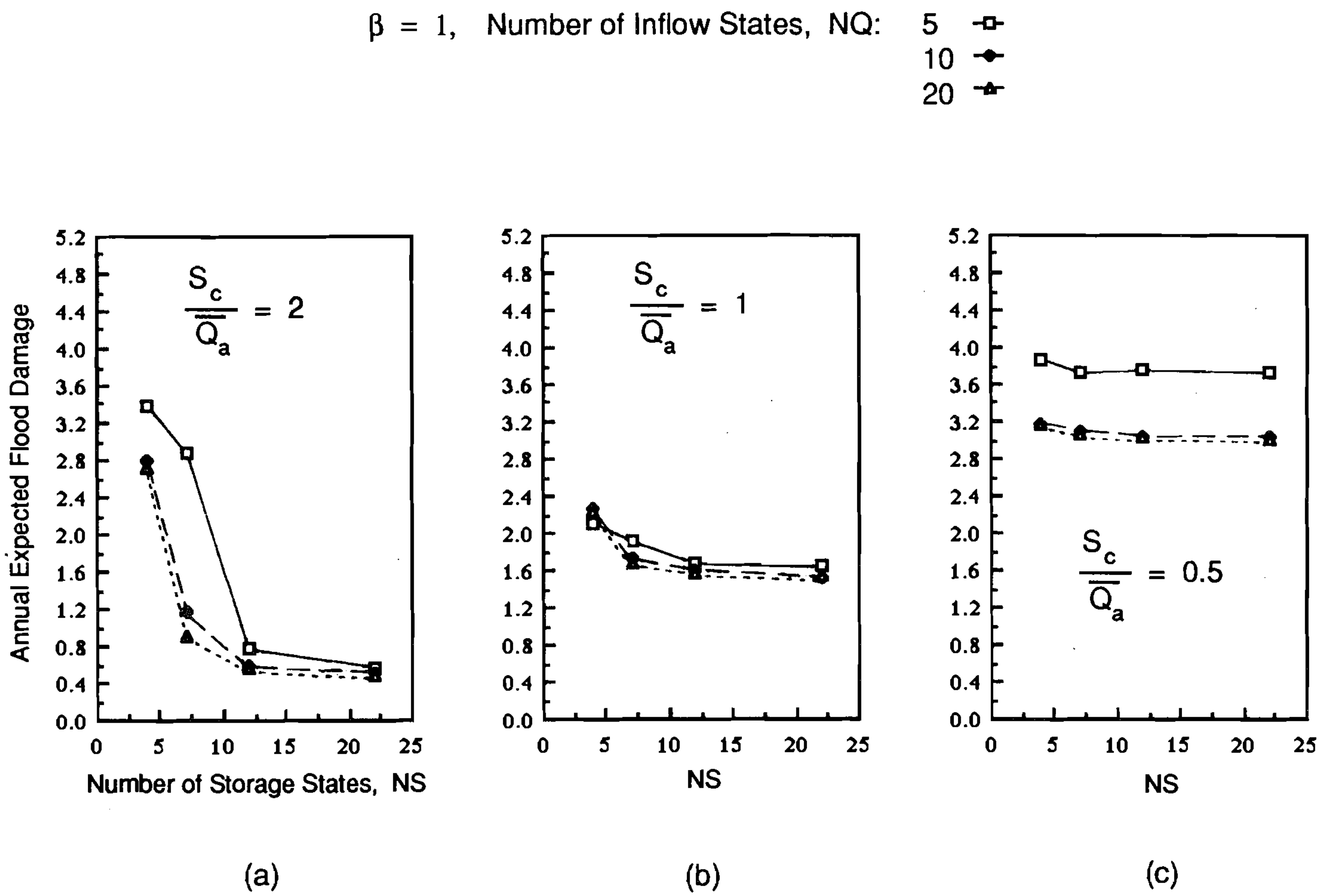


Figure 3.5 Estimation of Annual Expected Flood Damage Based on SP Model Using Different Combinations of Inflow and Storage State Partitions; Damage Is Calculated from $(R_t-30)^1$, for $R_t > 30$.

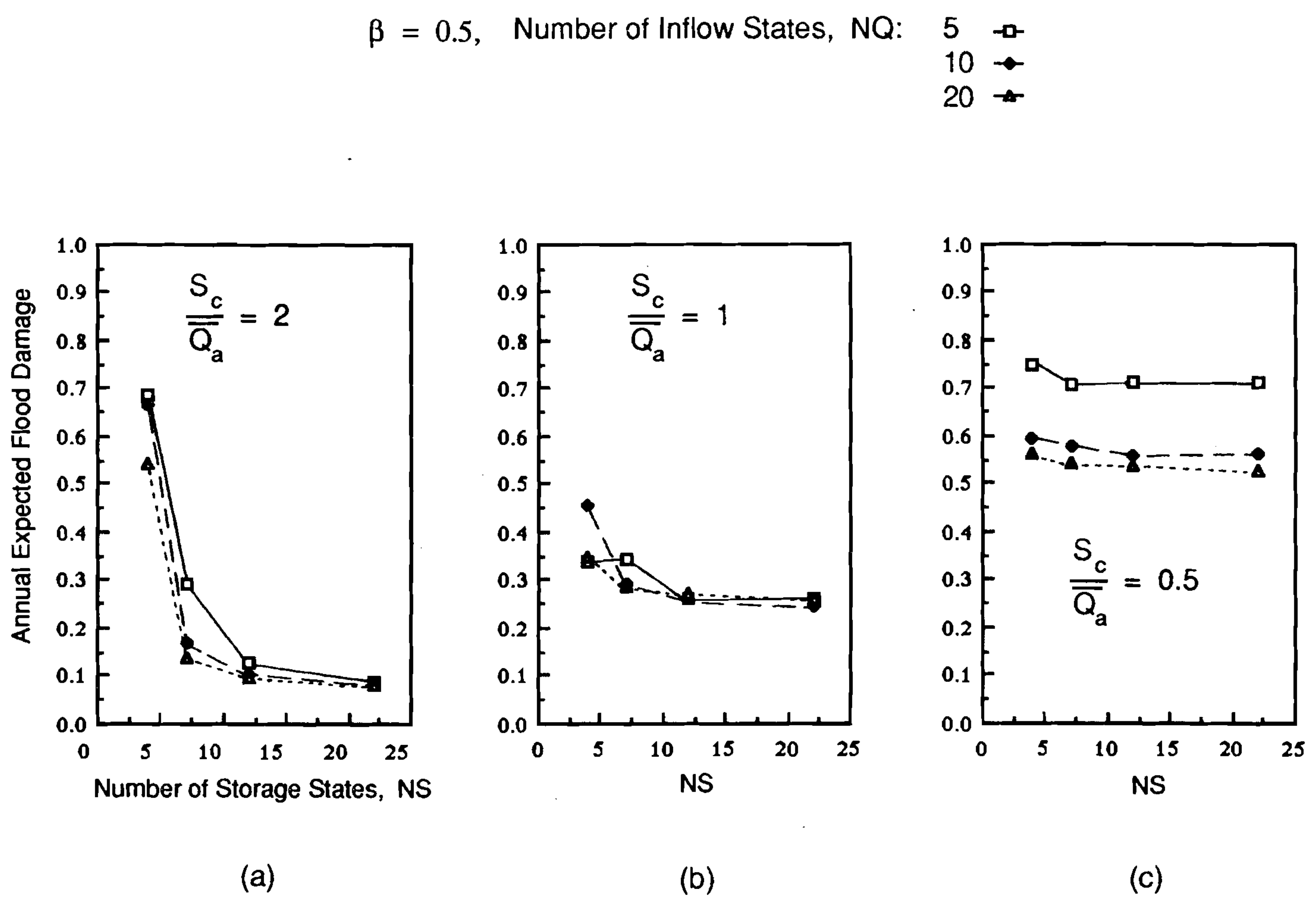


Figure 3.6 Estimation of Annual Expected Flood Damage Based on SP Model Using Different Combinations of Inflow and Storage State Partitions; Damage Is Calculated from $(R_t-30)^{0.5}$, for $R_t > 30$.

For NS $\geq$ 12 and NQ $\geq$ 10, the change in damage index becomes negligible when considering the probable uncertainties involved in choosing the right performance function as well as obtaining accurate hydrologic models.

The significance of partitions depends upon the storage capacity as well. For $S_c/\bar{Q}_a$ = 0.5, the active storage space (S_c = 25) is barely greater than the mean inflow volume of the wet season ($\bar{Q}_a$ = 22.5). Since the damages are mostly concentrated in the wet season, the inflow variation within that period, with relatively little storage space available, would more likely contribute to the downstream flooding than that of any other season. Hence, it would be more critical to discretize the wet season inflows properly than the storage variable. This effect is illustrated in Figs. 3.4-3.6 by comparing the rates of convergence in performance evaluation for the group of curves associated with $S_c/\bar{Q}_a$ = 0.5. Increasing NS would not improve the damage estimation significantly, while NQ needs to be greater than or equal to 10 to converge closely to a stable and correct solution. For larger storage ratios, the inflow variation would most likely cause storage fluctuations in the reservoir for each operation period. As a result, for representing the proper transitions between the storage states, the partition of storage variables would be more crucial than that of inflows. It should be noted that if the number of periods considered in a year is increased, the possible amount of inflow entering the reservoir in each period will be decreased; consequently, the partition of inflows will become less important than that of the storage.

Because of the seasonal variation of inflows, the majority of flood damages would be induced in the wet season. Therefore, more emphasis should be placed on the partitions of wet season inflows than on those of dry season inflows. For example, when NQ = 10, the flow increment for period three is 22.5, while the probable maximum inflow in period one is only 25. It was found that the expected damage was essentially unchanged if many fewer inflow states were considered for the dry season, given the same increment of discrete inflows in the wet season. Thus, it seems unnecessary to divide the dry season inflows into excessively fine increments to improve drastically the estimation of the expected system performance.

When only a few inflow and storage states are considered, results that seem conflicting might occur. For example, with NS = 4, the expected annual flood damage was calculated to be larger for $S_c/\bar{Q}_a = 2$ than for $S_c/\bar{Q}_a = 1$ for all β's considered (except for $\beta = 2$ and NQ = 5). This is clearly contradictory to the intuitive understanding that a larger reservoir storage capacity should provide better protection against flooding. The result computed is mainly caused by the different precisions associated with the storage states. A storage increment of 25 is used in $S_c/\bar{Q}_a = 1$, compared to an increment of 50 in $S_c/\bar{Q}_a = 2$. The misleading damage evaluation for $S_c/\bar{Q}_a = 2$, NS = 7, NQ = 5, and $\beta = 1$, as revealed in Fig. 3.5(a), might not be easily recognized if no sensitivity analysis of the state partitions was conducted. Therefore, it would be quite risky to use an SP model in any preliminary study to screen out inferior alternatives when only a few inflow and storage states were considered.

3.3.2.2 Effects on Optimal Decisions

Although the system performance as expressed by the expected flood damage offers a direct way to evaluate the appropriateness of state partitions, it does not indicate whether the optimal decisions are properly represented by the discrete solutions. Figure 3.7 shows the typical trend of convergence of the optimal discrete solutions for increasing numbers of inflow and storage states. The discrete solutions would gradually approach a hyperplane representing the continuous decisions in 3-dimensional space. For the example case in which only flood control is considered, the decision seems approximately linear as revealed by Fig. 3.7(a). However, the optimal decisions in Fig. 3.7(b, c) with finer state increments begin to exhibit some nonlinearities. Such nonlinearities imply that a unique discrete optimal policy can not be determined simply based on the total volume of the initial storage and the current inflow. This is because there is a unique steady-state probability PR_{ijmt} associated with each pair of S_{it} and Q_{mt}, and the resulting optimal release decision depends not only on the absolute flood damage measure, but also on the chance for this event to occur. Reliability programming models mentioned in Sec. 2.3 might not capture this nonlinear feature in the optimal decision space since a linear decision rule (LDR) would usually be implied (Loucks, 1970).

For real applications where problem solution is subject to time and budget constraints, it is usually impractical to repeat the same SP model for various precisions of the state partitions in order to assure that proper results are obtained. However, it is possible to examine whether the discrete optimal solutions comply with the criteria for measuring the system performance. In Fig. 3.8 the optimal decisions are represented by the release R_t. Since R_t is directly related to the downstream flooding and is used to compute the corresponding damage, a transition on the

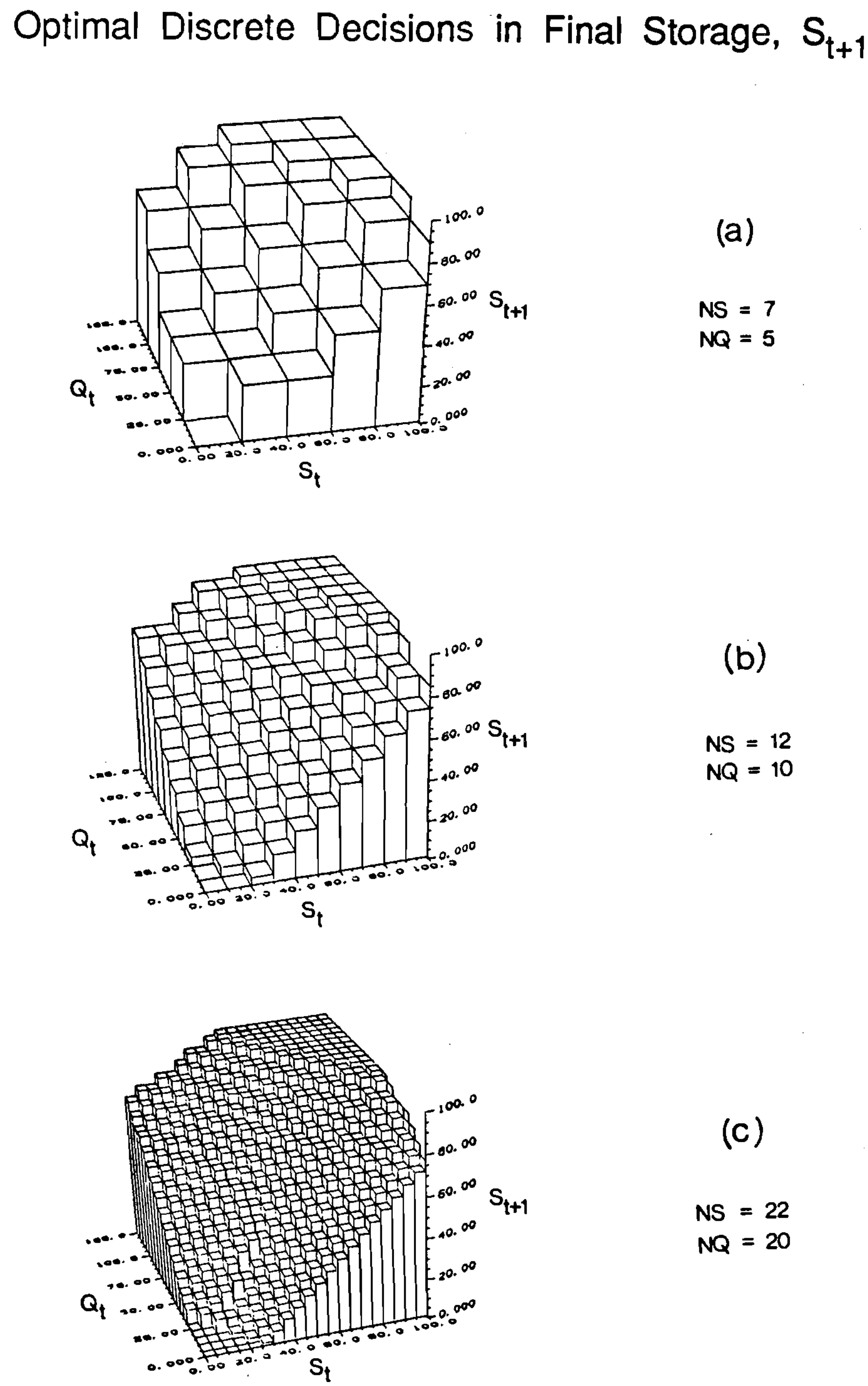


Figure 3.7 Illustration of the Optimal Discrete Decisions Expressed in the Final Storage States, S_{t+1}, for Different Partitions of Inflow and Storage States; for S_c = 100, β = 2, and t = 2.

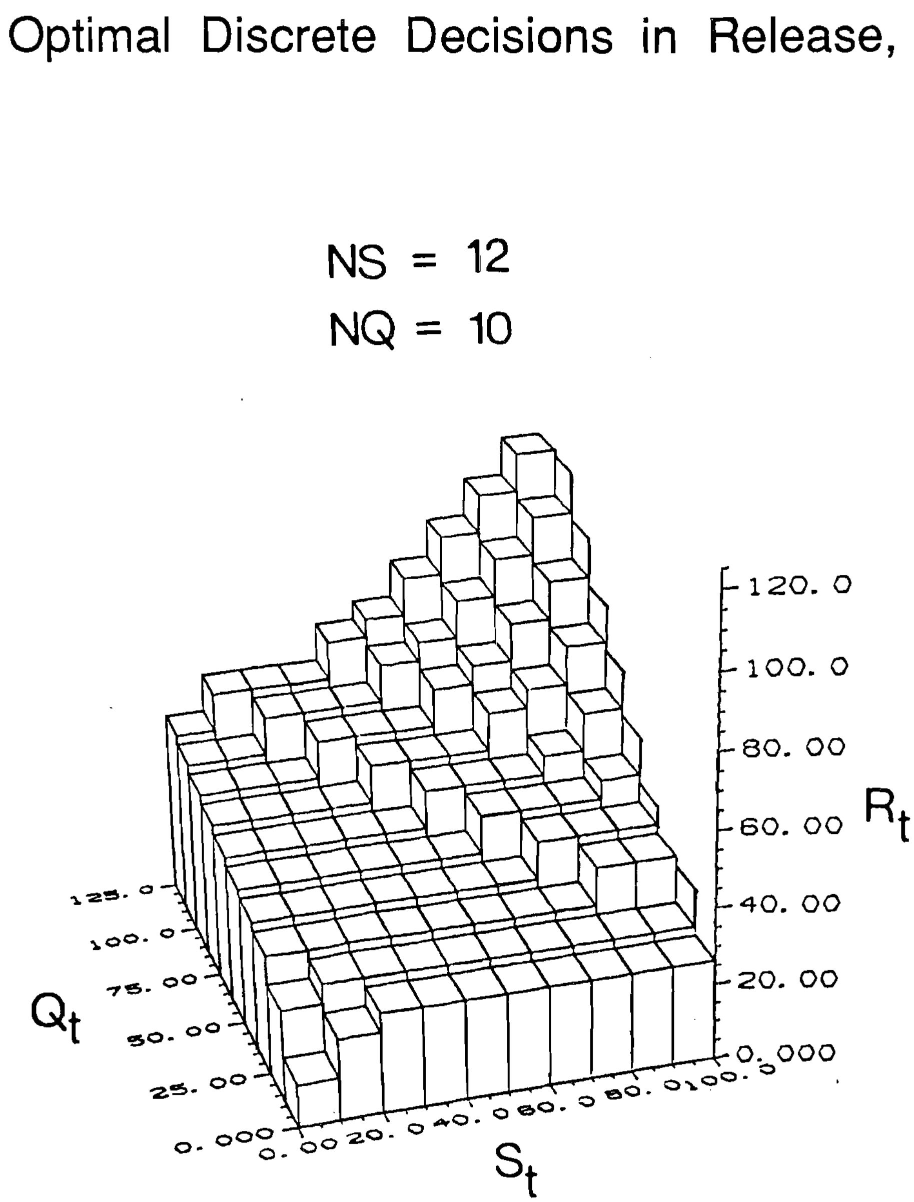


Figure 3.8 Illustration of the Optimal Discrete Decisions Expressed in Release, R_t, for NS = 12, NQ = 10, S_c = 100, β = 2, and t = 2.

hyperplane describing the optimal decision set should be expected to exist to represent the threshold value (R_t = 30) distinguishing the situations with or without flooding. That boundary can be observed in Fig. 3.8 near R_t = 30. This picture shows that consistent decisions would be made to keep excessive inflows inside the reservoir to protect against downstream flooding. Therefore, it is believed that as long as the correspondence between the optimal policies and the governing criteria for reservoir control can be demonstrated unambiguously by the optimal discrete solutions, the partitions of the states should be appropriate. On the other hand, for models with complicated performance functions or many distinct objectives, this relationship might not be easily observed with crude discretization of the states.

3.3.3 Errors in Hydrologic Parameter Estimation

Because stochastic programming models include probabilistic features of the natural hydrologic time series in the optimization procedure, errors from parameter estimation for the inflow distributions, and hence errors in the Markov transition probability matrices, may affect the evaluation of expected system performance as well as final optimal decisions. The statistics commonly used to characterize a hydrologic time series are the mean, the standard error, the skew coefficient, and the serial correlation ρ between inflows in successive time periods. For lognormally distributed random variables, the skew coefficient is a function solely of the coefficient of variation, ν, defined by the ratio of the standard error to the mean (Haan, 1977). Since for the hypothetical reservoir system the inflows are assumed approximately lognormal (although truncated at $Q_d = 10\bar{Q}_d$), the uncertainty associated with the skew coefficient is not considered separately. In addition, inflow variations within each season are represented by the coefficient of variation to reduce the number of parameter values considered in the following sensitivity analysis.

For each combination of the storage capacity and the damage function considered, the SP model for the hypothetical system was solved for ν = 0.5, 1.0, 1.5, and for ρ = 0.3, 0.5, 0.7, using NS = 12 and NQ = 10. It is believed that these ranges for ν and ρ encompass the hydrologic conditions for most real watersheds in the U.S with patterns of seasonal inflow variation similar to that of the hypothetical system. The results are illustrated in Fig. 3.9. It can be observed that as either ν or ρ changes from one extreme value to the other, the resulting estimate of the expected damage cost can vary over several orders of magnitude. Through the following regression analysis, the tremendous errors likely to be incurred in damage estimation are presented and discussed.

Let the expected damage in Fig. 3.9 be a function of both ν and ρ, for a given combination of S_c and β. The continuous damage hyperplane may be approximated by performing multiple regression on the discrete points represented by the nine combinations of ν and ρ. Assume that the regression hyperplane can be described by a second-order polynomial as

$$Y = a_0 + a_1\nu + a_2\rho + a_3\nu^2 + a_4\rho^2 + a_5\nu\rho \tag{3.15}$$

in which Y is the expected damage from the SP model; and the a_d, d = 0, 1, ...5, are the estimated coefficients from regression. To measure the relative sensitivity of Y due to a small change in either ν or ρ, take the partial derivatives of Y with respect to ν and ρ separately. After

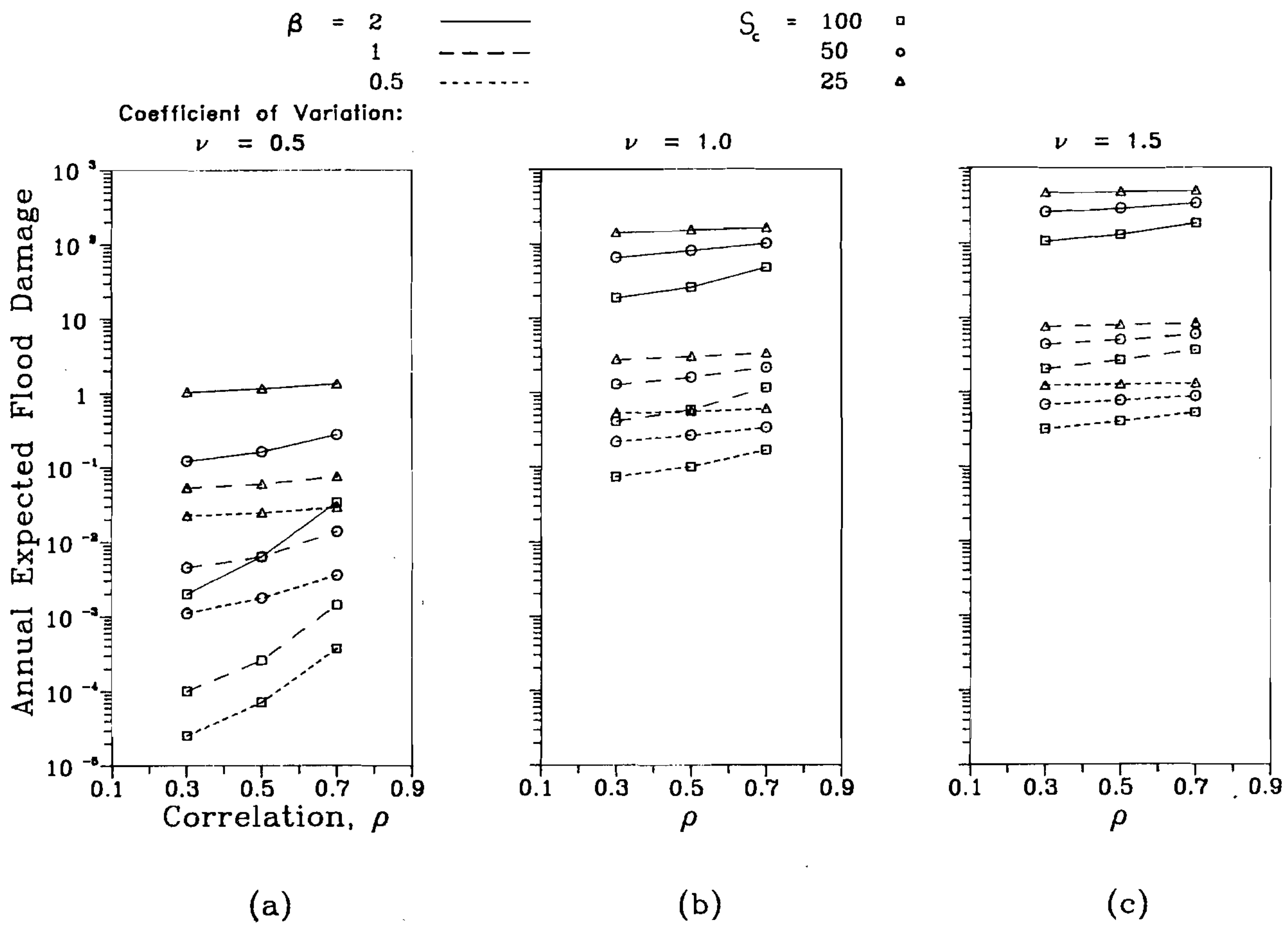


Figure 3.9 Sensitivities of Annual Expected Flood Damage as a Result of the Uncertainties in Estimating Hydrologic Parameters; ν = Coefficient of Variation, ρ = Correlation Coefficient, S_C = Storage Capacity, β = Damage Function Parameter.

normalization, the following equations can be obtained:

$$\left.\frac{\partial Y/Y}{\partial \nu/\nu}\right|_{Y=\hat{Y}(\hat{\nu},\hat{\rho})} = (a_1 + 2a_3\nu + a_5\rho)\ \frac{\nu}{Y} \tag{3.16}$$

$$\left.\frac{\partial Y/Y}{\partial \rho/\rho}\right|_{Y=\hat{Y}(\hat{\nu},\hat{\rho})} = (a_2 + 2a_4\rho + a_5\nu)\ \frac{\rho}{Y} \tag{3.17}$$

Equations 3.16 and 3.17 thus represent the percentage changes of the expected flood damage as a result of the percentage errors involved in estimating ν and ρ. Numerical results are summarized in Table 3.2 for different S_c and β values, all evaluated at $\nu = 1.0$ and $\rho = 0.5$.

The second-order polynomial regression equation for the expected damage Y, Eq. 3.15, proved to be adequate as can be shown by the value of the percentage explanation by the regression equation in the last column of Table 3.2. It also demonstrates that, regardless of the damage function used, the sensitivity of the expected flood damage depends more on the uncertainty of the variation of inflows within each period than on the serial correlation between the inflows in successive operation periods. For a storage ratio of 2.0 ($S_c = 100$, $\bar{Q}_a = 50$), the correlation coefficient would contribute an error of roughly the same value in percentage to that in damage estimation. However, for small storage capacities, the influence of accurate serial correlations quickly becomes negligible. With little or no storage space available for the incoming floods, the expected system performance would depend more upon the steady-state probability of a hydrologic event than upon the conditional probability of the previous event. Therefore, the error of the serial correlation between the seasonal inflows would have minimum effects on the evaluation of expected system performance.

Table 3.2 Relative Sensitivity of Annual Expected Flood Damage due to Errors in Estimating the Coefficient of Variation ν and the Correlation Coefficient ρ, Evaluated at $\nu = 1.0$ and $\rho = 0.5$, and NS = 12 and NQ = 10.

Storage Capacity, S_c	Damage Function Index, β	$\frac{\partial Y/Y}{\partial \nu/\nu}$	$\frac{\partial Y/Y}{\partial \rho/\rho}$	Percentage Explanation By Regression Equation
100	2.0	3.53	1.23	98.81
100	1.0	3.23	1.23	99.20
100	0.5	3.14	1.02	99.52
50	2.0	3.23	0.57	99.81
50	1.0	2.89	0.62	99.88
50	0.5	2.72	0.50	99.97
25	2.0	2.93	0.14	99.99
25	1.0	2.61	0.20	99.99
25	0.5	2.29	0.12	99.93

$$Y = a_0 + a_1\nu + a_2\rho + a_3\nu^2 + a_4\rho^2 + a_5\nu\rho$$

$$\left.\frac{\partial Y/Y}{\partial \nu/\nu}\right|_{Y=\hat{Y}(\hat{\nu},\hat{\rho})} = (a_1 + 2a_3\nu + a_5\rho)\,\frac{\nu}{Y}$$

$$\left.\frac{\partial Y/Y}{\partial \rho/\rho}\right|_{Y=\hat{Y}(\hat{\nu},\hat{\rho})} = (a_2 + 2a_4\rho + a_5\nu)\,\frac{\rho}{Y} \qquad \hat{\nu} = 1.0,\ \hat{\rho} = 0.5$$

The coefficient of variation ν measures the relative dispersion of the inflows with respect to its mean. Therefore, errors in estimating ν would alter the chance of occurrence of the extreme events. These extreme events, in turn, would affect the calculation of the expected system performance (Eq. 3.14). This explains why ν consistently affects the accuracy of the expected damage estimation no matter what storage ratio or damage function might be used. The importance of accurate estimation of the serial correlations of a hydrologic time series has been noticed and verified in many reservoir studies, while the influence associated with the variation of inflows on the system performance were generally less discussed. The numerical evidence in Table 3.2 highlights the relative importance of the two hydrologic parameters for estimating the expected system performance. Other combinations of ν and ρ values were evaluated as well; they demonstrated results similar to those shown in Table 3.2 and are not discussed further.

Care should be exercised, however, in generalizing the above assertions. For the hypothetical reservoir system, the flood damage was the only index for evaluating the system performance. For a system where the major portion of the expected performance occurs for the normal inflow conditions, e.g., the benefits from water supply, irrigation, and hydropower production, the influence of both ν and ρ might not be as significant as that of a reservoir system with flood protection as the only objective.

3.3.4 Effects of Performance Evaluation Function

Another major source of uncertainty in SP modeling besides the probable hydrologic errors is the functions used to evaluate the expected performance of a reservoir system. Many factors might contribute to errors in measuring the system performance, which involve the interpretation of the socio-economic goals, the weighting scheme between different objectives, or the uncertainty in future development of the river basin. It has been observed previously from Fig. 3.9 that tremendous uncertainty could exist in evaluating the expected flood damage when the error of the damage function, i.e., the error of β, is large. Therefore, it is important to examine the relative sensitivity of the expected damage due to errors from the damage evaluation function.

Let the expected damage in Fig. 3.9 be a function of β, and be expressed by a power function of the following form:

$$Y = c_0 + c_1 \beta^{c_2} \tag{3.18}$$

in which Y is the expected flood damage, and c_0, c_1, and c_2 are the coefficients to be estimated. For a given set of S_c, ν, and ρ values, only three Y's were obtained from the SP model using β = 2.0, 1.0, and 0.5. Therefore, the coefficients in Eq. 3.18 can be solved exactly. To measure the sensitivity of Y from β, take the partial derivative of Y with respect to β. After normalizing the differential by the values of Y and β being evaluated, one obtains

$$\left. \frac{\partial Y / Y}{\partial \beta / \beta} \right|_{\hat{Y}=\hat{Y}(\hat{\beta})} = \frac{c_1 c_2 \beta^{c_2}}{Y} \tag{3.19}$$

Table 3.3 contains the results based on Eq. 3.19 for different S_c, ν, and ρ values.

Table 3.3 Relative Sensitivity of Annual Expected Flood Damage due to Errors in Estimating the Parameter of Damage Function, for NS = 12 and NQ = 10.

Storage Capacity, S_c	Coefficient of Variation, ν	Correlation Coefficient, ρ	c_2	$\frac{\partial Y/Y}{\partial\beta/\beta}$, β = 2.0	1.0	0.5
100	0.5	0.3	4.66	4.61	3.61	0.56
100	0.5	0.5	5.06	5.01	3.77	0.41
100	0.5	0.7	4.95	4.90	3.78	0.47
100	1.0	0.3	5.78	5.76	4.84	0.49
100	1.0	0.5	5.70	5.68	4.82	0.54
100	1.0	0.7	5.57	5.55	4.85	0.70
100	1.5	0.3	5.92	5.91	5.07	0.53
100	1.5	0.5	5.83	5.82	5.04	0.58
100	1.5	0.7	5.85	5.84	5.10	0.62
50	0.5	0.3	5.12	5.08	3.99	0.47
50	0.5	0.5	5.09	5.04	3.79	0.40
50	0.5	0.7	4.68	4.62	3.60	0.54
50	1.0	0.3	5.93	5.91	5.00	0.48
50	1.0	0.5	5.91	5.89	5.00	0.49
50	1.0	0.7	5.78	5.76	4.95	0.59
50	1.5	0.3	6.12	6.11	5.25	0.49
50	1.5	0.5	6.09	6.08	5.24	0.51
50	1.5	0.7	6.04	6.02	5.24	0.55
25	0.5	0.3	5.00	4.89	2.96	0.22
25	0.5	0.5	4.95	4.85	3.00	0.23
25	0.5	0.7	4.77	4.67	3.03	0.29
25	1.0	0.3	5.90	5.94	4.91	0.42
25	1.0	0.5	5.93	5.91	4.91	0.44
25	1.0	0.7	5.86	5.84	4.89	0.47
25	1.5	0.3	6.22	6.20	5.29	0.44
25	1.5	0.5	6.19	6.18	5.28	0.46
25	1.5	0.7	6.13	6.12	5.26	0.48

$$Y = c_0 + c_1\beta^{c_2}$$

$$\left.\frac{\partial Y/Y}{\partial\beta/\beta}\right|_{\hat{Y}=\hat{Y}(\hat{\beta})} = \frac{c_1 c_2 \beta^{c_2}}{Y}\qquad \hat{\beta} = 2.0,\ 1.0,\ 0.5;\qquad \frac{\partial Y/Y}{\partial\beta/\beta}\begin{cases}\to 0 & \text{as } \beta \to 0\\ \to c_2 & \text{as } \beta \to \infty\end{cases}$$

It is interesting to note that the general trend of the relative sensitivity in estimating the expected damage is consistent for each distinct damage function considered, $\beta = 0.5, 1.0, 2.0$, for a wide spectrum of S_c, ν, and ρ values. When the quadratic function is used, an error from β will be magnified approximately 5 to 6 times in the final evaluation of the expected flood damage. For the linear damage function, this error magnifying factor ranges from 3 to 5, and still affects the evaluation of system performance to a great extent. As β is reduced below one, the impact on Y from the error in β will diminish quickly. The normalized differential in Eq. 3.19 is actually bounded by two finite numbers. When β approaches ∞, c_0 in Eq. 3.18 becomes negligible compared to $c_1\beta^{c_2}$, since c_0, c_1, and c_2 are positive coefficients for the 27 different cases listed in Table 3.3. This will lead to an upper bound of c_2 for Eq. 3.19. As β approaches zero, so does Eq. 3.19. Equation 3.19, when plotted against β, will look like an S-curve bounded by $[0, c_2]$.

The significance of the above errors associated with β depends largely upon how the expected system performance might be utilized in the overall decision making process. For instance, if the performance function happened to be an economic measure in monetary terms (e.g. Little, 1955; and Askew, 1974a, b), the consequences due to the error in calibrating β might be very critical provided that the expected system performance was compared to other alternative flood control schemes or used to justify the costs of building a new dam for flood mitigation purpose. In some cases, the operation rule curves for the storage or the targets for release are presumed known for a reservoir system. The performance index would then be represented by the penalty applied to any operation that deviates from the established rules or targets (e.g., Sigvaldason, 1976; Hashimoto et al., 1982a, b; Yazicigil et al.; and Datta and Burges, 1984). In those cases, the reservoir manager or the modeler would be more interested in the sensitivity of the optimal decisions than the numerical results based on the artificially selected penalty function.

The shape of the damage function, as characterized by the exponent β, could affect the optimal decisions under certain inflow and storage conditions. Many researchers (e.g. Klemeš, 1977; Buchanan and Bras, 1981; Hashimoto et al. 1982a; and Datta and Burges, 1984) have discussed the consequences of using a convex function as opposed to a concave function. Generally, a penalty of $|\sigma|^{\beta}$ would be attached to any deviation σ of release or storage from the target. This penalty might be one-sided; only the condition of $\sigma > 0$ or of $\sigma < 0$, but not both, is penalized. If $\beta > 1$, the penalty increases at a greater rate than $|\sigma|$. Thus, to protect against a catastrophic failure of the system it may be better to permit small deviations from the target even when enough water or storage is available to meet the current operation requirements. On the other hand, if $\beta < 1$, the penalty associated with large deviation is less emphasized; and the resulting operation decision would satisfy the current demand whenever possible.

The above assertion would not be always applicable if different aspects of a system's conditions were considered. The optimal policy obtained from the combination of $S_C = 100$, NS = 12, and NQ = 10 will be used to illustrate certain seemingly contradictory observations based upon the same argument about the penalty function. The optimal final storage states, expressed as a function of the initial storage and the current inflow states, are graphically presented in Fig. 3.10, for the four operation seasons and for the two distinct penalty functions with $\beta = 0.5$ and 2.0. The two initial storage states of $S_{9,t} = 75$ and $S_{4,t} = 25$ are considered to reflect sufficiently the general trend in the optimal decisions for either high or low initial storage conditions.

To utilize fully the available storage space inside a reservoir, it is plausible to lower the storage level whenever possible without violating the release constraint. This phenomenon can be observed from Fig. 3.10. In each operation season, if the inflows are small (e.g., $Q = Q_{1,t}$), the optimal final storage level would be lower than the initial storage level. Consequently, the release made would be larger than the total inflow amount of this operating season. However, when the release is near the threshold value 30, models using different β's might dictate different release decisions. For instance in Fig. 3.10(a) for t = 1, if the inflow state is 3, using $\beta = 2$ would result in an optimal release of $R_t = S_{4,t} + Q_{3,t} - S_{1,t+1} = 25 + 6.25 - 0 = 31.25$; whereas using $\beta = 0.5$ would yield $R_t = S_{4,t} + Q_{3,t} + S_{2,t+1} = 25 + 6.25 - 5 = 26.25$, in which the first subscript stands for the respective inflow and storage states (see Secs. 3.2.1.1 and 3.2.1.2 for definitions). This is a situation in which the previous reasoning on the influence of β on the optimal decision is valid. The release difference is magnified in the second season in that there is a higher risk of getting a large amount of inflow from the wet season than the rest of the seasons in a year. Therefore, the decision of allowing a little violation from the target might be justified to achieve better long-term flood protection of the reservoir system.

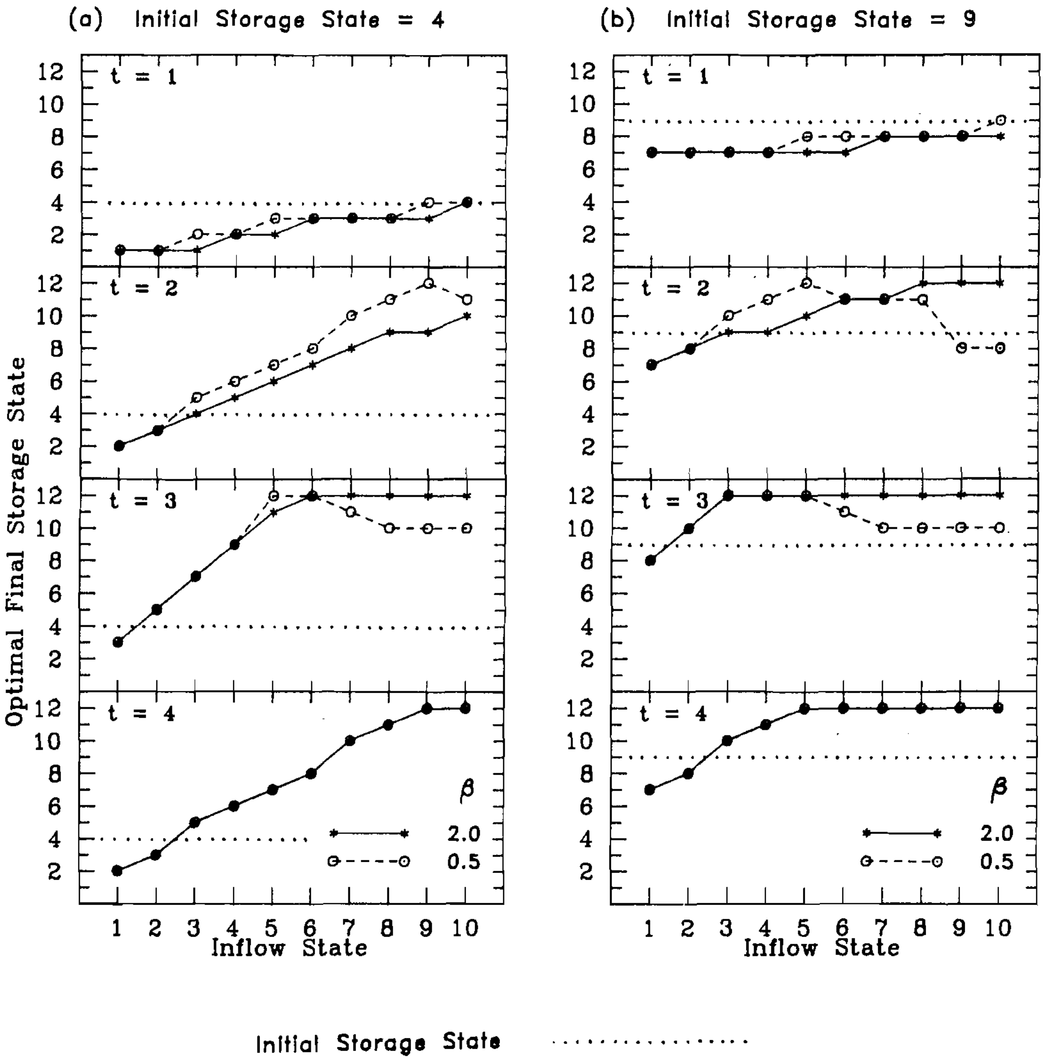


Figure 3.10 Differences in Final Storage Decisions Using Convex (β = 2) vs. Concave (β = 0.5) Penalty Functions; for S_c = 100, NS = 12, NQ = 10, and Initial Storage States of 4 and 9 (S_t = 25 and 75), Respectively.

In case of $S_t = 4$, $Q_t = 8$, and $t = 2$, the release would be $S_{4,t} + Q_{8,t} - S_{11,t+1} = 25 + 93.75 - 95 = 23.75$ for $\beta = 0.5$, as compared to that of $S_{4,t} + Q_{8,t} - S_{9,t+1} = 25 + 93.75 - 75 = 43.75$ for $\beta = 2.0$ -- a quite large release violation even though there is still considerable room for holding more flood water inside the reservoir in the current season.

The reversed phenomenon would occur if the total volume of initial storage and inflow would be significantly greater than the storage capacity of the reservoir system. As exemplified by Fig. 3.10(b) for t = 2, before the inflow becomes too large to be regulated by the available reservoir storage, decisions based on the concave function ($\beta = 0.5$) demand to withhold the flood water as much as possible. However, beyond a certain inflow state, a release much larger than 30 is inevitable; and using the concave penalty function could lead to a decision to release more water than actually needed when some storage space was still available. This result occurs because the concave function tends to diminish the difference between large release violations of similar magnitudes. Since the release was already large, it would be advantageous to release a little bit more to spare more storage space for the next operation period. With this little extra storage space available, the small and medium floods in the next season, which would have a much higher probability of occurrence than the current large flood, might be avoided.

As shown in Fig. 3.10(a, b), the shape of the penalty function does not affect the optimal decision for the fourth season. This is because the probable maximum inflow in the oncoming dry season is only 25. Therefore, even if the reservoir is full in the beginning of season one, no release violation beyond 30 would be possible; and it would be unnecessary to reserve any storage space at the end of season four. Moreover, it is very likely that the reservoir level could be lowered because inflows much less than 30 would be experienced in seasons 1 and 2; and some storage space could be obtained by releasing an amount of 30 in those two seasons. Thus, even though the hydrologic characteristics of inflows in season 2 and 4 are identical for this hypothetical system, the optimal release decisions can be different depending upon the penalty function used, and upon the risks of flood events occurring in the subsequent periods.

The optimal decisions shown in Fig. 3.10(a, b) also demonstrate the difference between the modeling results and the decisions intuitively preferred by a real-world reservoir manager. Adopting the optimal release decisions could increase the number of small floods for $\beta > 1$, or enlarge the magnitude of major floods for $\beta < 1$. Thus, decisions resulting from the optimization model might be unacceptable to the public who generally would be inclined to avoid or minimize the present flood damage. For the example demonstrated, the preferred final storage level would probably always be the upper one of the two curves shown in each plot of Fig. 3.10, i.e., to release the flood inflows as quickly as possible without violating the non-damaging release constraint. The implications are twofold. First, neither the convex function nor the concave function alone could reflect human behavior for the entire spectrum of operation conditions. Second, decisions based on human intuition would likely be suboptimal judged strictly from the result of the mathematical programming model. The incompatibility between mathematical models and human intuition could exist, and might not be easily recognized by either the reservoir manager or the modeler.

3.3.5 Conflicting Objectives

A reservoir system is generally operated for meeting several distinct goals which may compete for the available storage space. If the reduction of downstream flood damage is of primary importance, it is often desirable to lower the storage level whenever possible so that much storage room is saved for holding probable excessive inflows in future time periods. Conversely, in order to develop the hydropower potential of a river basin effectively, the water head behind a dam is preferably kept high at all times. As a result, the storage space may be mostly occupied, with little room left for a reservoir to regulate flood waters. Objectives other than flood protection and hydropower generation would most likely lead to an optimal storage level between the two opposite extremes. Therefore, the objectives of flood protection and hydropower generation will be used in the following study to typify the tradeoffs between various objectives for operating a reservoir system.

The flood damage will be measured by the one-sided convex penalty function (β = 2.0) as previously defined for a release greater than 30. The hydropower in itself is a function of both the release rate and the head drop through the hydroelectric turbines. For simplicity, the hydropower produced is used herein to represent the benefit from selling the generated hydroelectricity to the consumers. Assume that the storage volume is represented as a simple quadratic function of the head water elevation based on the natural geomorphological features at the reservoir site; and the release is assumed to be uniform within each operation period. The hydropower production may be approximated by

$$\text{Hydropower Production} = K\{R_t,\ [(S_t + S_{t+1})/2]^{0.5}\} \qquad (3.20)$$

in which R_t is a constant release in period t; $(S_t + S_{t+1})/2$ represents the average storage level in the same period; and K{ , } is the hydropower production simplified as a function of the mean storage and the release. A conversion factor is assumed to be included in K implicitly for making hydropower commensurate in units with the flood damage. Various combinations of weights, W_f for flood damage and W_p for hydropower benefit, are considered to represent the relative priorities between the two objectives in an SP model for operating the hypothetical single reservoir system.

3.3.5.1 Effects on Expected Performance

Figure 3.11 illustrates the tradeoffs between the expected values of annual flood damage and annual hydropower production from the SP model for three different reservoir sizes. When flood mitigation is the only concern ($W_f/W_p = \infty$) for reservoir control, the expected annual flood damage can be reduced rapidly by increasing the reservoir size so that most small and medium flood events are eliminated. Beyond a certain reservoir capacity, e.g. for $S_c > 100$, further improvement of flood protection is rather limited, and is bounded by an upper limit of the expected damage of zero. The expected hydropower production increases gradually as the storage capacity is increased. This is because a larger reservoir with more storage space is able to withhold a greater amount of flood water, which in turn will increase the water head to produce more hydroelectricity. Nonetheless, the rate of increase in hydropower production diminishes eventually as the storage capacity approaches infinity when all releases can be completely controlled.

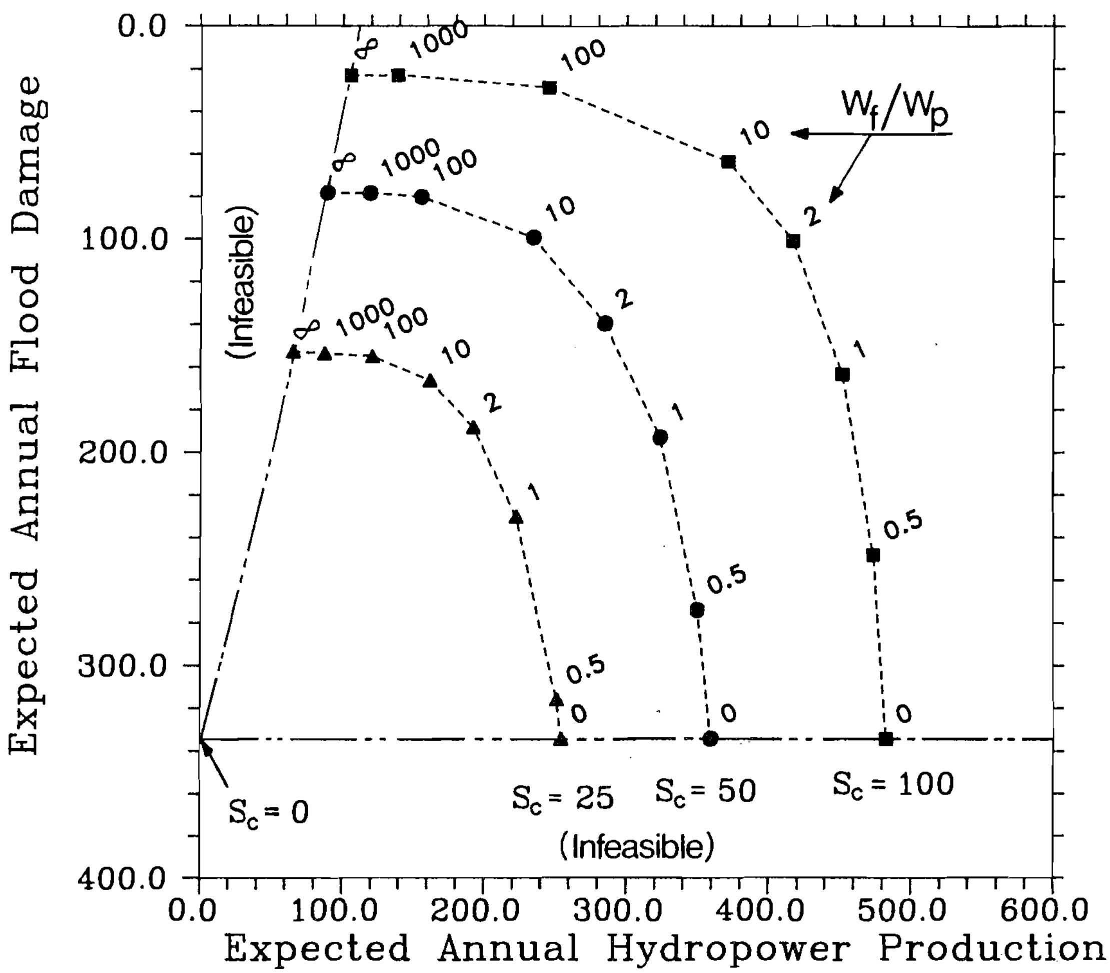


Figure 3.11 Tradeoffs between Flood Protection and Hydropower Production with Different Reservoir Sizes, S_c = 25, 50, and 100; W_f and W_p Are Weights Assigned to the Respective Objectives.

On the other hand, if hydropower generation is the only function that a reservoir serves, increasing reservoir size would not reduce the expected flood damage at all since the reservoir would be kept full at all times. Unlike the flood protection case, the amount of hydropower which could be generated increases monotonically with the reservoir storage. The effect of hydrologic flood routing through a reservoir, which can somewhat reduce the peak flow rate even with a full reservoir, is neglected here.

Between the extremes are the tradeoffs with various levels of emphasis or preference on one objective versus the other. In theory, for a specific weight ratio W_f/W_p there exists a set of utility contours reflecting a decision maker's preferences on various objectives. The one that is tangent to the Pareto frontier would yield the maximum utility subject to the system's constraints, and the tangent point would indicate the tradeoff in the performances of the different objectives. Note that the example illustrated in Fig. 3.11 assumed a uniform, unchanged weight ratio throughout a year. In reality, the preference for meeting each objective might vary with time. For instance, in the wet season the chance of having large flood inflows is higher than that in the dry season. Therefore, more emphasis should be put on the flood protection objective rather than on hydropower production. The Pareto frontier might then be pushed even farther outward to yield better expected annual performances of the two objectives than those based on uniform weight ratio. Difficulties could arise, however, in the determination of proper weight ratios not only between the objectives but also between the successive operation seasons.

3.3.5.2 Effects on Steady-state Probability of Storage

The tradeoff between the objectives as reflected by the expected performance is only one way to comprehend the modeling results related to a system's characteristics. For the reservoir study, the information associated with the optimal states of storage in each operation period could be more meaningful in revealing what might be expected had a specific rule been followed. Figure 3.12 shows the variations of the steady-state probability of the initial storage for each period based on different ratios of the weights for flood protection and hydropower generation. The reservoir capacity considered herein is 100.

As expected, the storage remains at the empty state most of the time if the preference is completely biased toward flood protection. The occasional large floods may cause the storage to stay at higher stages for certain extended periods, which explains why the steady-state probability is not concentrated totally at the empty state. The storage distribution in the fourth period is broader than those of the other periods because it follows the wettest season in a year. When the preference is gradually shifted from flood protection to hydropower generation, the steady-state probability of storage is re-distributed toward the higher levels. Eventually as W_f/W_p becomes zero, the steady-state storage would be locked at the full state.

By varying W_f/W_p from ∞ to 100, which only caused a little shift of the steady-state probability distribution in two of the four periods, the expected hydropower production could be more than doubled with the expected flood damage only increased by a minimal percentage (see Fig. 3.11). This phenomenon indicates that as long as the storage level is kept low in

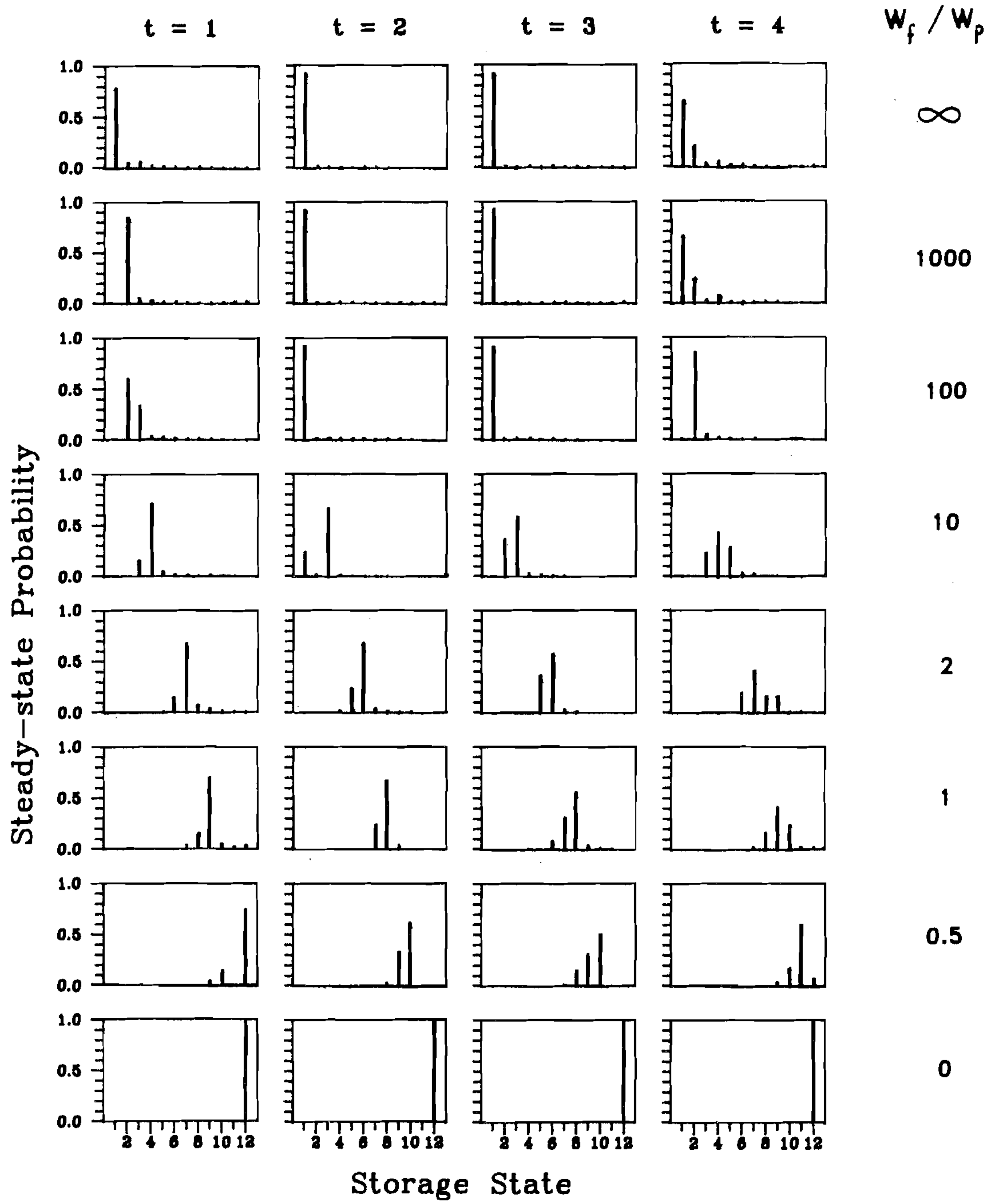


Figure 3.12 Steady-state Storage Distributions Based on Various Levels of Preferences on Flood Protection and Hydropower Generation; S_c = 100.

the wet season to avoid most flood damages, the storage levels in other periods might be allowed to remain high to gain greater water head for hydropower generation. The reservoir generally can hold more water in the fourth period than any other period since the oncoming inflows in the dry season pose no threat of downstream flooding. However, subject to the lower storages in period 3 for flood control, the storage in period 4 can not resume the full state in the long run.

For a relatively large reservoir, $S_c/\tilde{Q}_a = 2$ in this case, the steady-state storages are mostly confined within a smaller finite range between the two extreme states, i.e., full and empty. This indicates that the expected storage level for each period might be used as the storage target for short-term operation, without greatly affecting the long-term reservoir performance. The steady-state probability histogram of storage thus provides useful information as to what the storage level would be in balancing the two opposite objectives.

3.3.5.3 Effects on Optimal Decisions

Both Figs. 3.11 and 3.12 demonstrate the consistent pattern of change in either the expected performance or the steady-state probability distribution when the relative preferences for reservoir operation change gradually between the two objectives. However, such consistency might not always be observed for certain optimal decisions in the transition of preferences for the objectives. This can be illustrated using Fig 3.13, which contains the optimal decisions for the final storage level for three initial storage states, S_{it} = 3, 7, and 10, and for three weight ratios $W_f/W_p = \infty$, 1, and 0, respectively.

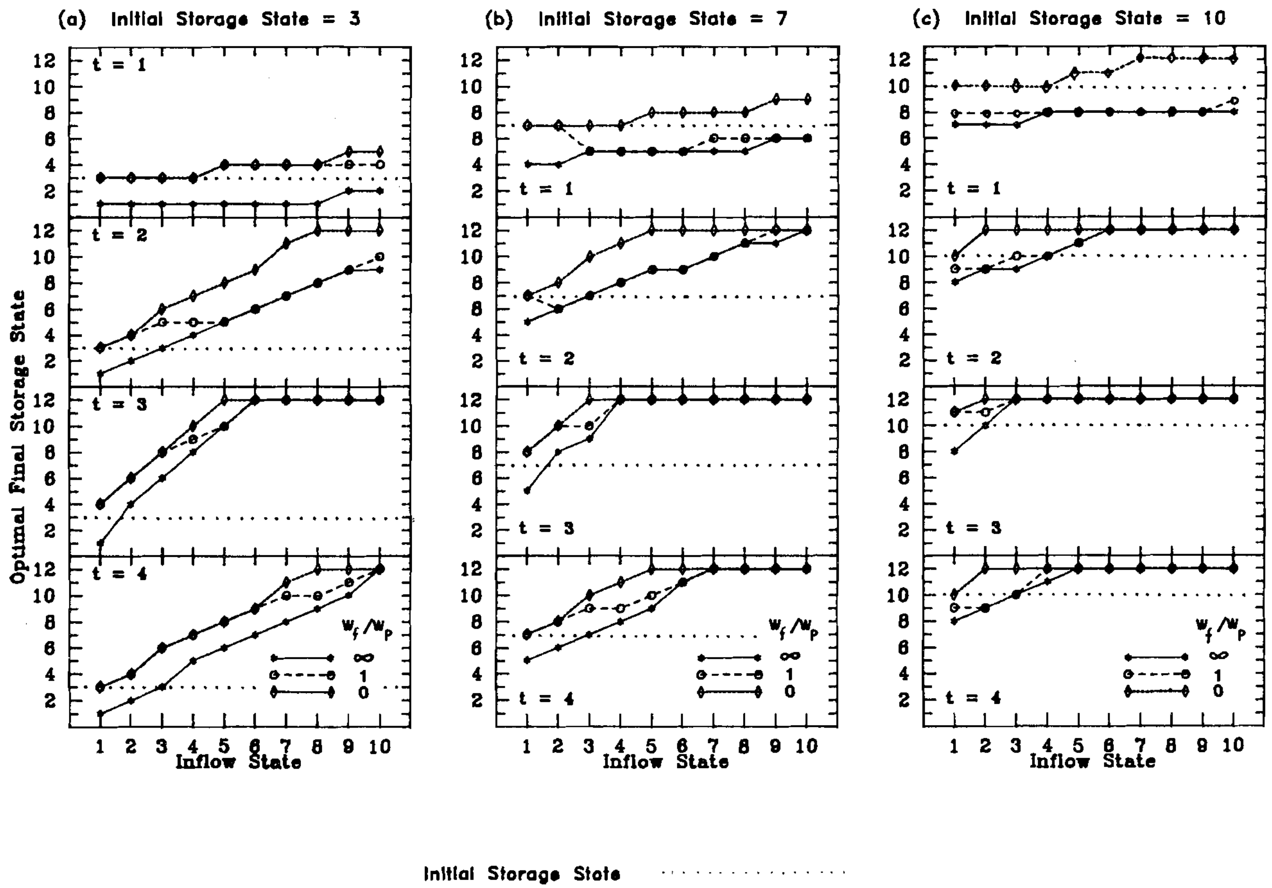


Figure 3.13 Transition of the Optimal Decisions for Final Storage States as the Preferences for Objectives Shift from Flood Protection to Hydropower Generation; for $S_c = 100$, $\beta = 2$, and $W_f/W_p = \infty$, 1, and 0.

Although the optimal final storage state is monotonically nondecreasing as the inflow state increases, with either flood protection or hydropower generation being the sole objective, the underlying causes are totally different. For the flood protection objective, the final storage level is forced to rise as the inflow is increased because only a portion of the large inflow that does not cause downstream flooding is allowed to pass through the dam, while the rest is withheld. For hydropower generation considerations, however, all inflows will be kept inside the reservoir, provided that there is still some storage room available, to raise the water head eventually to the full state. Therefore, the optimal decision on the final storage for $W_f/W_p = 0$, as shown in Fig. 3.13, actually represents a transient condition of the storage before attaining steady state. Once the storage reaches the full state, the reservoir would be kept full ever after.

In Fig. 3.13, the corridor of the optimal decisions bounded by the two extremes would contain all the possible storage decisions for any combination of preference values assigned to the two objectives. The upper left region above the curve for $W_f/W_p = 0$ is infeasible for the final storage since the sum of the initial storage and the current inflow is less than the discrete final storage volume. The lower right region below the curve for $W_f/W_p = \infty$, though feasible, would always be inferior to the decisions between the two extreme curves. Consequently, a majority of the combinations of initial storage and current inflow states may be eliminated in the optimization procedure for a stochastic dynamic programming model to reduce the computation time in searching for the optimum.

The optimal decisions for the balanced objectives ($W_f/W_p = 1$ in the example), unlike the two binding extreme storage curves, might not be always monotonically nondecreasing as the inflow is increased. In Fig. 3.13(a) for t = 1 and 4, the initial storage would be low with the inflows in the following seasons posing no major threat to downstream flooding. Hence, the optimal final storage level could be kept mostly at the highest possible state for generating more hydropower in the successive periods. In the case where t = 2, because the wettest season in a year immediately follows, the flood protection objective would begin to affect the decisions on storage. This phenomenon becomes pronounced if the inflow in period 2 is higher than normal. Since the inflows are assumed positively correlated, it is more likely that in period 3 an unusually large inflow would also occur. As a result, the decision on storage would gradually shift from the higher extreme to the lower extreme. When the initial storage level is high as illustrated in Fig. 3.13(c), there may not be much storage room available for holding the future flood waters. A little gain in the water head would not contribute much more hydropower production, while the consequences from a major flood due to less storage space might be very severe.

If the initial storage state is moderate, and the current inflow is small, as shown in Fig. 3.13(b), it would be better to keep this inflow totally within the reservoir for reaching a higher water head in the next period. The marginal gain in hydropower production based on this decision would more than compensate for the marginal loss in flood damage. Beyond a certain inflow volume, the release decision as determined by the SP model would be reversed drastically to lower the storage level and to reserve more storage space when the inflow is increased. As one of the characteristics in SP modeling, the optimal decision for a certain combination of initial storage and inflow state is determined by evaluating the expected longterm system performance as a result of making such a decision. For the above case, the loss of future hydropower production revenue could be compensated by releasing more water in the current operation period and by the extra benefit of reducing future potential flood damages. Thus, the gain in terms of the flood damage reduction might offset the loss of hydropower production by shifting the water inventory between successive operation periods.

Although all of these phenomena can be explained judging by the expected performance of a specific decision as opposed to the others, their existence is not always predictable before the SP model is solved. To avoid possible misinterpretation of the modeling results, different aspects of the operation objectives as well as the dependence of optimal decision on the current system status should be considered. The modeling results which seem contradictory to intuition might not always be erroneous and should be interpreted carefully.

3.4 Simulation Study

One of the major advantages of SP modeling is that the steady-state probability of the system being in any specific state as well as the overall expected performance can be derived directly from the optimal solution. However, the consequences caused by the optimal policy may not be easily captured by simple reasoning about the long-term effects of the reservoir system based on steady-state operation conditions. For instance, what would be the meaning of the probability of occurrence of an extreme flood event in the real world? Although the chance for this event occurring may be very slim, the consequences can be catastrophic; and the expected flood damage due to this event may be significant when compared to that from minor floods. Moreover, the actual performance of a real-world reservoir system will largely depend upon the hydrologic events that actually occur within its service life span. And the eventual system performance in a relatively limited time period might deviate noticeably from what had been calculated using the model.

System analysts can use simulation techniques to collect important information when an optimization model fails to capture certain characteristics of a system under a prespecified operation environment. Furthermore, simulation can be used to extend the findings from an optimization model to reveal the real-time response of the system under various operation schemes. In the following analysis, an application of Monte Carlo simulation is presented; it was used to complement the understanding of operations of the hypothetical reservoir system based on the optimal policy determined by the SP model. Variations of the optimal policy were also considered to examine the flexibility in implementing those optimal decisions.

3.4.1 Simulation Design

The hypothetical reservoir system in the simulation study contains the basic setup defined in Sec. 3.3.1, with $S_c = 100$, $\bar{Q}_a = 50$, $\nu = 1.0$, and $\rho = 0.5$. Only the flood protection objective is considered, using the one-sided quadratic cost function for measuring the flood damage caused by any release greater than 30 ($\beta = 2$). Ten sets of sixty-year seasonal inflows were generated based on the truncated lognormal distribution assumed for the four seasons. Each of the ten synthetic inflow databases was used to simulate natural runoffs entering the reservoir, and the system's operation was simulated for sixty years according to the optimal rules determined by the SP model. To eliminate the initial storage effect of the reservoir, the first ten years of simulation results were discarded, leaving fifty years of simulated record to be analyzed for each set. Inflow information was assumed known at the beginning of each operation period. Evaluation of a reservoir system performance under various levels of inflow predictability is beyond the scope of this research and can be found in a study by Datta and Burges (1984).

To study the effects of partitions of both the inflow and the storage states on the simulation results, three different levels of precision are considered, i.e. (NQ, NS) = (5, 7), (10, 12), and (20, 22). As discussed in Sec. 3.3.2.2, the set of optimal decisions of final storages obtained from a discrete SP model would approach a continuous hyperplane in 3-dimensional space if the increments of both the discrete inflow and storage states became smaller. Therefore, it is interesting to compare the simulation results based on both a discrete operating rule and a continuous rule, to examine the relative importance of the partitions in the modeling stage as opposed to the implementation stage.

In the case of the discrete rule, the final storage level in the simulation is restricted to one of the finite storage states as defined in the SP model. For the continuous operation rule, a simple bilinear interpolation scheme is used to calculate the optimal final storage level in continuous space. Figure 3.14 presents this bilinear interpolation scheme graphically. Let $\tilde{S}_t$ and $\tilde{Q}_t$ be the continuous initial storage and current inflow variables respectively bounded by the neighboring discrete

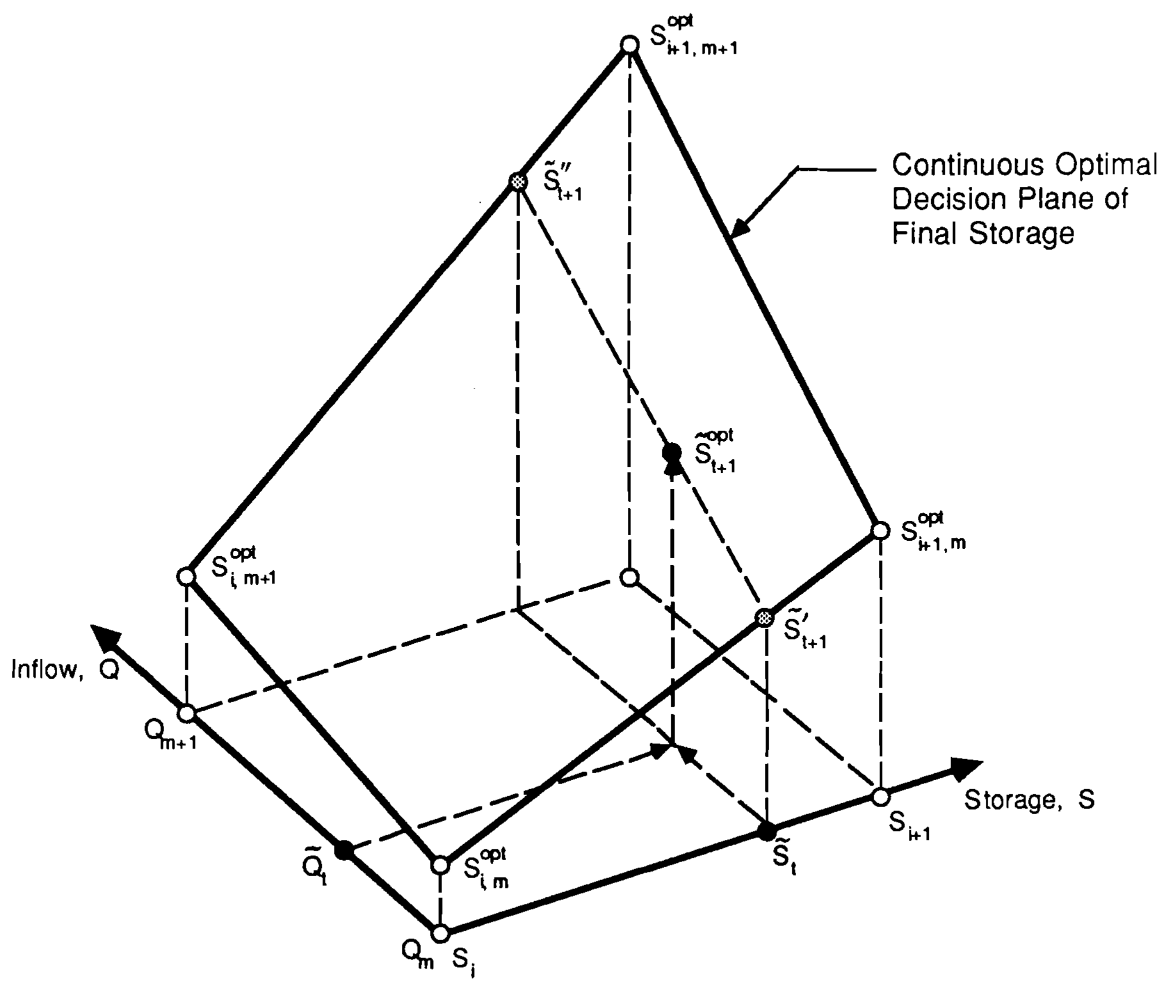


Figure 3.14 A Simple Bilinear Interpolation Scheme for Deriving the Continuous Rule from the Discrete Rule for the Optimal Final Storage Level.

states, i.e., $S_i < \tilde{S}_t < S_{i+1}$ and $Q_m < \tilde{Q}_t < Q_{m+1}$. Moreover, define S_{xy}^{opt} to be the optimal final storage level based on the discrete states S_x and Q_y, in which x = i and i+1, and y = m and m+1. The time subscript for the discrete storage and inflow states is neglected in the following derivation without losing any generality. When performing linear interpolation of the continuous final storage on the discrete initial storages, one obtains

$$\tilde{S}'_{t+1} = S_{im}^{opt} + (S_{i+1,m}^{opt} - S_{im}^{opt}) \frac{\tilde{S}_t - S_i}{S_{i+1} - S_i} \quad \text{for } Q_m \qquad (3.21)$$

$$\tilde{S}''_{t+1} = S_{i,m+1}^{opt} + (S_{i+1,m+1}^{opt} - S_{i,m+1}^{opt}) \frac{\tilde{S}_t - S_i}{S_{i+1} - S_i} \quad \text{for } Q_{m+1} \qquad (3.22)$$

Carrying out the interpolation between $\tilde{S}'_{t+1}$ and $\tilde{S}''_{t+1}$ on the discrete inflows results in

$$\tilde{S}_{t+1}^{opt} = \tilde{S}'_{t+1} + (\tilde{S}''_{t+1} - \tilde{S}'_{t+1}) \frac{\tilde{Q}_t - Q_m}{Q_{m+1} - Q_m} \qquad (3.23)$$

It can be proved that the order of this linear interpolation, whether first on the discrete initial storages or on the discrete inflows, does not affect the computation of the optimal final storage in the continuous space.

Several indices were selected to evaluate different aspects of the system's performance based either on the discrete rule or on the continuous rule with various levels of precision of the inflow and the storage states. The first index counts the total number of flood occurrences in each simulation run, which indicates the frequency of disturbances to the downstream area caused by flooding in the 50-year operation period. The second index calculates the annual average flood damage, which is compared to that directly estimated from the SP model. The third index, termed as the total forced release volume, accumulates any excessive flood release only for the amount beyond 30. Generally, an operation policy leading to less total forced release would represent a better performance of a flood control reservoir. The last index simply records the maximum flood release for each 50-year simulation period.

In the following sections, only the results which have demonstrated significant variations among the ten simulations will be discussed in detail. Other general observations such as the relative frequencies of the storage states, which did not differ greatly from simulation to simulation and were similar to what was anticipated from the SP model, are not elaborated upon in the ensuing comparison studies.

3.4.2 General Results

The simulation results are summarized in Table 3.4(a, b, c, and d). For all combinations of state partitions and operating rules, the indices show tremendous variations of the flood control by the reservoir system. The release hydrographs of the different simulation runs show that both of the large damages found in simulation runs 7 and 9 in Table 3.4(b) were respectively caused by a single catastrophic event. Had those extreme floods not occurred, the reservoir would have been pretty much in control of the incoming floods based on the optimal policy determined by the SP model. Note that none of the annual average flood damages measured from the ten simulation runs came close to those estimated from the optimal solutions of the SP model. In the majority of cases, the average damages for the 50-year operation period were far less than those predicted by the model result. However, the occurrence of only a few extreme events (two in five hundred years in this example) raises the average flood damage to

Table 3.4 Comparisons between Simulation Results Based on a Discrete Rule vs. a Continuous Rule for Four Performance Indices. Simulation Period = 50 Years, with 10 Distinct Simulation Runs.

(a) Total Number of Flood Occurrences

Simulation Runs	Discrete Rule NQ = 5 NS = 7	Discrete Rule NQ = 10 NS = 12	Discrete Rule NQ = 20 NS = 22	Continuous Rule NQ = 5 NS = 7	Continuous Rule NQ = 10 NS = 12	Continuous Rule NQ = 20 NS = 22
1	21	11	19	7	3	8
2	14	19	17	6	6	10
3	9	8	7	4	4	4
4	8	7	7	2	3	3
5	13	10	12	5	3	8
6	6	2	7	3	1	0
7	18	18	15	8	9	9
8	10	10	13	3	4	5
9	14	13	11	6	5	9
10	9	12	14	7	7	8
Average	12.2	11.0	12.2	5.1	4.5	6.4

(b) Annual Average Flood Damage

Simulation Runs	Discrete Rule NQ = 5 NS = 7	Discrete Rule NQ = 10 NS = 12	Discrete Rule NQ = 20 NS = 22	Continuous Rule NQ = 5 NS = 7	Continuous Rule NQ = 10 NS = 12	Continuous Rule NQ = 20 NS = 22
1	54.84	17.09	1.88	12.90	7.86	1.48
2	20.63	6.05	7.00	3.81	2.22	4.24
3	28.41	7.49	1.82	18.64	7.11	3.84
4	8.77	2.54	1.55	2.86	1.16	1.38
5	14.45	2.58	0.63	0.39	0.29	0.58
6	1.69	1.81	0.62	0.79	0.12	0
7	181.64	162.12	178.79	139.05	162.14	138.69
8	5.97	2.19	1.50	0.04	0.03	0.15
9	111.76	76.47	73.46	91.92	73.32	75.51
10	8.24	6.12	2.48	4.42	1.25	0.92
Average	43.64	28.45	26.97	27.48	25.55	22.68
Expected	(42.08)	(25.92)	(22.66)	(42.08)	(25.92)	(22.66)

Table 3.4 (continued)

(c) Total Forced Release Volume Which Causes Flooding (50 Years)

Simulation Runs	Discrete Rule NQ = 5 NS = 7	Discrete Rule NQ = 10 NS = 12	Discrete Rule NQ = 20 NS = 22	Continuous Rule NQ = 5 NS = 7	Continuous Rule NQ = 10 NS = 12	Continuous Rule NQ = 20 NS = 22
1	155.9	55.5	34.3	43.2	23.6	19.6
2	99.1	57.2	57.5	28.5	18.2	27.9
3	80.6	42.3	18.4	35.8	21.6	16.5
4	52.2	27.7	17.7	13.4	9.2	10.5
5	89.2	30.2	16.0	9.0	6.0	12.8
6	19.1	13.4	13.4	8.6	2.5	0
7	200.2	158.3	150.1	149.9	138.1	136.1
8	41.4	28.1	25.1	2.0	2.3	4.7
9	151.7	98.8	77.3	87.0	72.0	77.8
10	51.6	44.4	33.5	28.2	13.5	15.7
Average	94.1	55.6	44.3	40.6	30.7	32.2

(d) Maximum Flood Release

Simulation Runs	Discrete Rule NQ = 5 NS = 7	Discrete Rule NQ = 10 NS = 12	Discrete Rule NQ = 20 NS = 22	Continuous Rule NQ = 5 NS = 7	Continuous Rule NQ = 10 NS = 12	Continuous Rule NQ = 20 NS = 22
1	71.3	56.3	34.1	53.4	49.6	35.3
2	45.7	40.7	43.2	39.1	39.1	43.2
3	60.8	45.8	38.3	60.3	48.7	43.7
4	41.1	36.1	36.1	41.9	37.5	38.1
5	41.0	35.8	33.2	33.2	33.0	32.9
6	37.3	37.3	33.5	36.0	32.5	-
7	118.6	113.6	121.1	95.0	113.6	100.0
8	40.6	35.6	35.1	31.3	31.1	31.9
9	99.6	89.6	89.6	96.7	89.6	90.5
10	41.9	41.9	36.9	42.4	37.4	34.9
Maximum	118.6	113.6	121.1	96.7	113.6	100.0

a value close to that predicted by the model. Hence, extreme events generally distort the perception of the average system performance, and the direct modeling results sometimes do not provide sufficient information about a system's behavior for various operation concerns. It might be more proper to classify the system based on a few distinct operation conditions such as no flooding, small flooding, medium flooding, large flooding and extreme flooding states to account for the various impacts of these events on the overall system performance.

Increasing the number of partitions of storage and inflow states generally improves the accuracy of modeling results as can be observed from the average values of the four indices of the ten simulation runs. Beyond a certain precision level for defining the storage and inflow variables ($NQ \geq 10$ and $NS \geq 12$ in this example), most of the critical performance indices would not change significantly. It is interesting that the total number of flood occurrences in the long run is greater for $NQ = 20$ and $NS = 22$ than for $NQ = 10$ and $NS = 12$ (reflected by the average values in Table 3.4(a)). This phenomenon can be explained by the quadratic damage function used in this SP model. As discussed in Sec. 3.3.4, the convex damage function could lead to an optimal policy of releasing more water than needed during some periods to reduce the chance of future damage caused by an extreme flood. With coarser increments of the discrete storage and inflow states, this property might not be well represented by the optimal result. However, with increasing storage and inflow states, the SP model is able to capture this phenomenon more accurately.

Both storage and inflow are essentially continuous variables in the real world. The fact that they were approximated by a finite number of states in a discrete model should not restrict the implementation of the optimal results in the real-time operation of a reservoir system. As can be observed from Table 3.4(a, b, c, and d), the continuous operation rule based on a simple linear interpolation scheme for determining the optimal discrete policy significantly improves the system performance in comparison to the discrete rule. Among others, the greatest improvement came from the reduction of the total number of floods in the 50-year simulation period (Table 3.4(a)). Roughly half of the minor floods are avoided if the release decisions are allowed to take intermediate values between the discrete values. The total forced release in each simulation run is also greatly reduced because of the elimination of the minor floods using the continuous rule (Table 3.4(c)). The maximum flood release (Table 3.4(d)), however, is the least affected index if the continuous rule is used.

The annual average flood damage for NQ = 5 and NS = 7 was reduced significantly using the continuous rule. This damage would have been greatly overestimated had it been calculated directly from the discrete modeling results. In essence, the continuous rule based on a coarser partition of the state variables would lead to a comparable or even better system performance than that simulated by the discrete rule based on a finer partition. Thus, when solving a discrete reservoir model further partitions of the state variables can be avoided by a straightforward interpolation scheme on the discrete optimal policy.

3.4.3 A Modified Policy for Improved Reservoir Performance

From the simulation results, it has been observed that the magnitudes of most flood events lay within a limited range above the nonflooding release criterion. The reservoir was able to withhold totally an excessive inflow without significantly affecting the available storage space for the successive operation periods. The extreme floods were isolated events in comparison to the small and medium floods. Therefore, it might be possible to modify the optimal release policy near the threshold value so that the small and medium floods could be eliminated without causing adverse effects during the future operation of the system.

An experiment was designed to test the sensitivity of releases near the threshold volume of 30. Simulations were repeated for the reservoir system using the previously defined continuous rule, except that a release was restricted to 30 whenever the calculated release volume was less than $30 + \delta R$, where δR is a tolerance factor for release adjustment. δR was selected to be 2.5, 5, 10, and 20 to monitor the changes in system performance. The simulation results are contained in Table 3.5(a, b, c, and d) for NQ = 10 and NS = 12.

The use of this release tolerance δR up to a value of 10 improved the overall system performance as reflected by the various performance indices. The small and even the medium floods were eliminated depending upon the tolerance value used in the simulation. The improvement persists with increasing δR until the tolerance is greater than 10. Beyond this tolerance level, the artificial restriction imposed upon the release in the long run leads to greater average flood damage (Table 3.5(b)) as well as larger maximum flood magnitude (Table 3.5(d)). This phenomenon might

Table 3.5 Comparison among Simulation Results Based on the Modified Continuous Rule with Corrections Made on the Releases within a Tolerance Value of δR beyond the Threshold Release of 30. Simulation Period = 50 Years, with 10 Distinct Simulation Runs. NQ = 10; NS = 12.

(a) Total Number of Flood Occurrence

Simulation Runs	δR 0	2.5	5	10	20
1	3	5	2	1	1
2	6	4	2	1	1
3	4	2	1	1	1
4	3	1	1	1	1
5	3	3	0	0	0
6	1	0	0	0	0
7	9	7	3	3	2
8	4	0	0	0	0
9	5	4	3	2	2
10	7	2	1	0	0
Average	4.5	2.8	1.3	0.9	0.8

(b) Annual Average Flood Damage

Simulation Runs	δR 0	2.5	5	10	20
1	7.86	2.84	1.92	1.12	1.12
2	2.22	2.96	2.25	3.70	2.94
3	7.11	3.58	3.25	3.25	2.49
4	1.16	1.13	1.13	0.57	0.57
5	0.29	0.55	0	0	0
6	0.12	0	0	0	0
7	162.14	162.44	161.40	161.40	259.20
8	0.03	0	0	0	0
9	73.32	75.00	75.00	73.98	73.98
10	1.25	1.34	1.09	0	0
Average	25.55	24.98	24.60	24.40	34.03

Table 3.5 (continued)

(c) Total Forced Release Volume Which Causes Flooding (50 Years)

Simulation Runs	δR 0	2.5	5	10	20
1	23.6	24.4	13.8	7.5	7.5
2	18.2	22.9	14.5	13.6	12.1
3	21.6	16.8	12.7	12.7	11.2
4	9.2	7.5	7.5	5.3	5.3
5	6.0	9.1	0	0	0
6	2.5	0	0	0	0
7	138.1	141.4	127.0	127.0	121.2
8	2.3	0	0	0	0
9	72.0	80.1	76.8	70.4	70.4
10	13.5	10.9	7.4	0	0
Average	30.7	31.3	26.0	23.7	22.8

(d) Maximum Flood Release

Simulation Runs	δR 0	2.5	5	10	20
1	49.6	38.4	37.4	37.5*	37.5*
2	39.1	39.1	39.1	43.6	42.1*
3	48.7	42.7	42.7	42.7	41.2*
4	37.5	37.5	37.5	35.3*	35.3*
5	33.0	33.0	-	-	-
6	32.5	-	-	-	-
7	113.6	113.6	113.6	113.6	143.6
8	31.1	-	-	-	-
9	89.6	89.9	89.9	89.9	89.9
10	37.4	37.4	37.4	-	-
Maximum	113.6	113.6	113.6	113.6	143.6

* Reservoir full.

be attributed to the discrete approximation of the storage and the inflow states. Because the state variables were represented by a few finite discrete values, distortions exist in the optimal discrete solution. This distortion cannot be totally eliminated even when the continuous rule is used, since the continuous rule is based on simple linear interpolation on the discrete rule, which can be biased in the first place. By allowing a tolerance on the release above the non-flooding ceiling, the distortion and uncertainty involved in a discrete model can somewhat be adjusted.

In this experimental study, the increments of storage and wet season inflow used in SP model are 10 and 22.5 respectively. Thus, the error introduced for the storage and inflow would be roughly half the state increment, i.e., 5 and 11.25. Since the optimal release is computed by the mass balance equation, the uncertainty associated with the state variables with larger increment would be transmitted to the release as well. Hence, a tolerance value near 11.25 should be expected to lead to the best system performance; and this has been observed in Table 3.5 for $\delta R = 10$. As a check, a separate experiment has been performed with both the numbers of storage and inflow states doubled in the SP model. It was found that the breakthrough value of δR shifts to 5, which still conforms to the above assertion.

The numerical evidence provided in Tables 3.4 and 3.5 demonstrates that the direct optimal solution obtained from an SP model sometimes would not lead to the best overall system performance. Although the modeling results would ideally become more accurate by using finer state increments for the variables, the increasing computation burden might limit the number of states ultimately included in a discrete model. The system performance can perhaps be improved as much or more, however, without resorting to a more complex model if the major system characteristics can be more accurately described by modifying the discrete optimal solution.

3.5 Summary

A typical SP model for reservoir study generally involves the discretization of continuous storage and inflow variables, the estimation of a Markov transition probability matrix, and the selection of adequate measures for evaluating the system performance. The major advantage of using SP models is that the resulting optimal steady-state operation policy implicitly accounts for the future hydrologic uncertainty, and the long-term performance of a reservoir system can be assessed without resorting to laborious simulation studies. However, because of a general lack of complete information related to a reservoir system, some distortions and errors will be introduced in the modeling procedure. The series of sensitivity analyses that has been conducted demonstrates the possible causes of errors at the various stages in formulating a typical SP model, and the uncertainties involved in the evaluation of the expected system performance and the corresponding optimal operation policy.

The partition of storage and inflow states has been shown to affect directly the precision of the optimal release policy. Extremely large releases are likely to occur as isolated events in the real-time operation of the reservoir system. These extreme events are relatively insensitive to the increments of the state variables as compared to the smaller releases. Nevertheless, these rarely occurring extreme events can have a major impact on the estimation of expected system performance in an SP model. Even if the expected system performance appears to be stabilized with the models of finer state partitions, the distortions associated with the small or medium releases might not be completely diminished. On the other hand, the discrete optimal decisions would normally provide fundamental insights as to how a system's objectives and constraints might be captured by the model solution. It has been shown by the simulation results that it could be more efficient to eliminate the model's distortion by directly adjusting the discrete optimal result rather than using the same SP model with more states.

The uncertainty associated with estimation of the commonly used second order hydrologic parameters, specifically the coefficient of variation ν and the correlation coefficient ρ of inflows, generally affects the accuracy of the expected system performance. The coefficient of variation has a significant impact on the modeling result regardless of the reservoir size and performance function used in the sensitivity analysis. Comparatively, the influence of serial correlation of inflows on the expected system performance depends largely on the storage capacity of a reservoir system. For reservoir systems with storage ratios less than 2, the uncertainty associated with ν can cause significantly larger errors in the flood damage estimation than the uncertainty with ρ. For even smaller reservoirs, the uncertainty associated with ρ would be practically immaterial to the expected performance estimated from the SP model.

The shape of the performance function for a reservoir system has long been recognized in the literature to affect directly the resulting optimal policy of an SP model. In the flood protection example, the convex and the concave damage functions led to drastically different optimal decisions on the releases depending upon the initial storage, the current inflow, and the future hydrologic conditions. For all the possible system conditions, the convex or the concave function alone does not result in a consistent operation policy which complies with human intuition towards short-term interests rather than long-term benefits. Hence, the control strategy of a reservoir system based on human intuition would likely be suboptimal if judged from the strict result of a mathematical programming model.

The tradeoffs between the competing objectives in reservoir operation can be examined by the changes in certain system characteristics, such as the expected reservoir performance or the steady-state distribution of the storage, as a result of the shifting emphases on the various objectives. Although such indices related to the lumped system performance might demonstrate a clear and consistent pattern of tradeoffs in objective space, they usually failed to provide detailed information about the changes in decision space due to the varying system preferences. It has been shown that when boiled down to the level of individual discrete decisions, not only the relative preferences of the objectives but also the current system status would affect the optimal operation policy for the long-term best system performance.

Finally, simulation studies are shown to be useful in complementing the SP modeling results. Simulation would be necessary to help identify the implicit distortions involved in SP modeling, which might not be very evident by looking at the direct optimal results. Despite the many uncertainties embedded in SP models, this modeling technique provides a comprehensive assessment of the example reservoir system's response to the varying hydrologic inputs in steady-state operation conditions. It is critical for a modeler to perceive the sources of potential errors, and to filter out the useful information contained in the optimal solution to effectively apply an SP model for reservoir study. Frequently, modifications are needed to make the strict SP modeling results closer to reality without worsening the system performance in the long run.

IV. A CASE STUDY OF LAKE SHELBYVILLE, ILLINOIS

4.1 Purpose

The objective of conducting the case study of this chapter is to extend the findings of SP modeling from a hypothetical reservoir system to a real reservoir system. The flexibility of the SP formulation allows for the consideration of various operation rules in response to the ever-changing reservoir operation conditions, such as those for Lake Shelbyville, Illinois. In addition, a modified stochastic dynamic programming (SDP) model is presented to measure properly the agricultural and property damages due to flooding in consecutive crop-growing months. The sensitivity of the performance of Lake Shelbyville, judged from the losses of agricultural revenues and recreational benefits, property damages, as well as the changes in pool levels and outflow releases is investigated and discussed.

4.2 System Description

4.2.1 Background

Lake Shelbyville and Carlyle Lake are the two major man-made reservoirs in the Kaskaskia River Basin, Illinois, and were created by damming the Kaskaskia River near Shelbyville and Carlyle (Fig. 4.1). The U.S. Army Corps of Engineers has been regulating both reservoirs since their completion in 1970 and 1967, respectively. Other major basin-wide water resources projects include the construction of the Kaskaskia Navigation Channel (completed in November, 1974) downstream from Fayetteville to the Mississippi River, the authorization of six levee districts between Shelbyville and Carlyle, and the completion of New Athens local protection

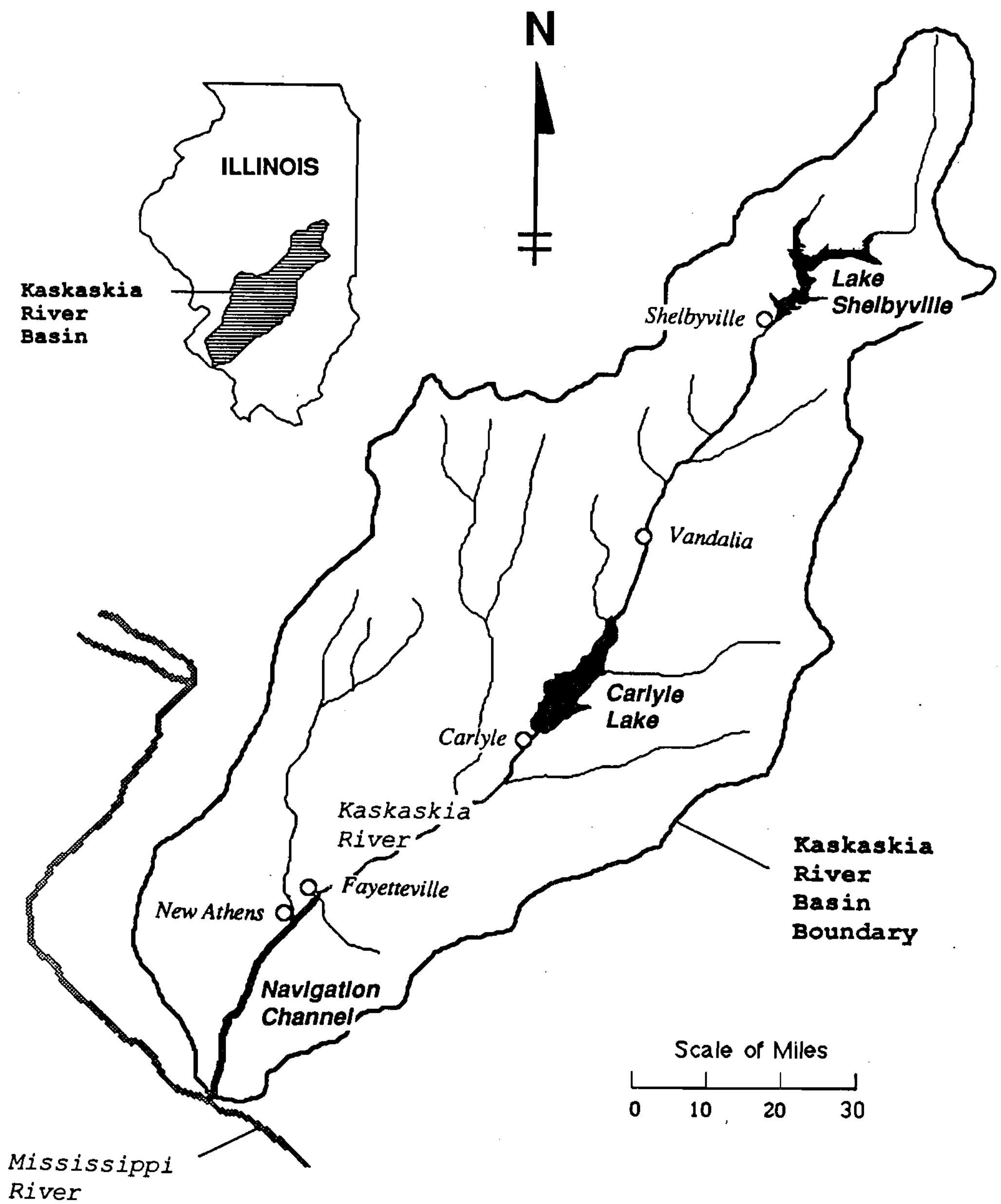


Figure 4.1 Locations of the Major Water Resources Developments in the Kaskaskia River Basin, Illinois.

project. These developments jointly serve the region for flood control, recreation, navigation, water supply, and fish and wildlife conservation purposes (Corps of Engineers, 1977; 1983a, b).

Lake Shelbyville and Carlyle Lake together control the runoff from about half of the total drainage area of the Kaskaskia River Basin above Carlyle. While the design capacity of Lake Shelbyville is 1.7 times the mean annual inflow above Shelbyville, the storage ratio of Carlyle Lake to the mean annual inflow above Carlyle is only 0.84. About half of either lake's total storage space is devoted to withholding the projected flood which equals 60% of the spillway design storm. Additional storage space (surcharge storage) can be created when the tainter gates are fully opened; and the maximum surcharge storage can increase to as much as 32% and 23% of the total storage space available in Lake Shelbyville and Carlyle Lake, respectively. The joint-use storage reserved for the combined downstream navigation and water supply release accounts for 18% of the total storage space in both lakes. In recent years the urban areas and local industries in the Lower Kaskaskia River Basin have not developed to their projected growths, which might demand the full amount of the joint-use reserve. So the present operations of both lakes are mostly centered around the flood control, recreation, and fish and wildlife conservation issues.

Despite the large storage capacities of both lakes, their operation rules have been modified several times since the reservoirs were placed in operation. The major reasons for causing these changes are summarized below (Corps of Engineers, 1983c):

(a) The maximum flood release from each lake is governed by the downstream channel capacity. Some of the proposed local levees were never built, restricting the maximum allowable release to a lower rate than originally planned.

(b) In the severe flood years 1973-1974, the control of both lakes based on the then-existing operation guidelines caused both upstream and downstream flooding which could have been either avoided or mitigated had a better rule been used.

(c) The Kaskaskia River Navigation Channel was placed in operation on a limited basis beginning in July 1976. The target winter pool level was then raised from 590.0 to 596.0 ft so that enough water would be available to augment the flow in the Navigation Channel when a drought occurred.

(d) The increasing emphasis on the recreation and the fish and wildlife conservation uses, accompanied by the economic developments of the private businesses surrounding the lake areas necessitates constant meetings between the Corps of Engineers and the local interest groups. These meetings have usually led to modifications of the existed operation rules.

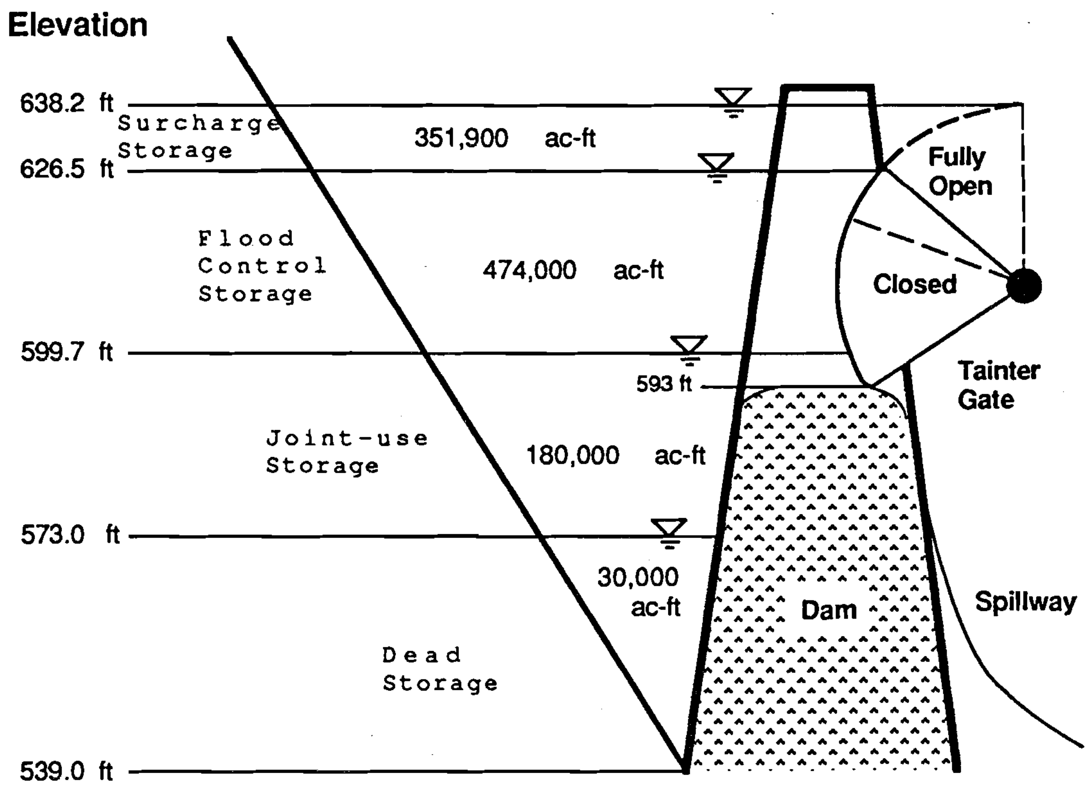
The above reasons typify the problems often encountered in the development of a river basin. Because of the stochastic nature of hydrologic events and the unpredictable future of socio-economical changes, managing valuable water resources within a river basin inevitably needs constant re-assessment of the existing regulation and control plans.

Although the lakes are located serially along the Kaskaskia River, the storage and release criteria of each are constrained essentially by the local recreation activities on and near the lake, and by the respective downstream channel capacities for confining the flood releases. Thus, either lake can be modeled as a single reservoir system without distorting very much the designated functions of the two reservoirs for regional flood protection and economic development. In the ensuing sections, a stochastic dynamic programming model is developed for Lake Shelbyville to assist in the evaluation of alternative plans for the changing operation environment in the Upper Kaskaskia River Basin.

4.2.2 Physical Settings

Lake Shelbyville controls the surface runoff from the Upper Kaskaskia River Basin which encompasses a total drainage area of 1,054 square miles in East-central Illinois. The storage space created by the Shelbyville Dam is 1,035,900 ac-ft and is divided into four major storage zones which include dead storage, joint-use storage, flood control storage, and surcharge storage (Fig. 4.2). The dead storage space is used to trap the sediments brought in by the upstream and local inflows, and is essentially ineffective in the overall flood control plan. The joint-use pool of 180,000 ac-ft is contracted to be apportioned to the federal and the state storages of 155,000 ac-ft and 25,000 ac-ft, respectively. The federal storage is to be used for downstream flow augmentation during the drought period; and the state storage is reserved mainly for regional water supply purposes (Corps of Engineers, 1964). In order to retain the full joint-use pool of Lake Shelbyville, the tainter gates must be completely closed. The flood control storage is designed to withhold 60% of the net runoff from the standard project storm occurring in the Upper Kaskaskia River Basin. This was felt to be the minimum storage space required to meet the

Lake Shelbyville



- Total Storage = 1,035,900 ac-ft

Figure 4.2 Major Storage Zone Divisions of Lake Shelbyville.

downstream flood control criteria (Corps of Engineers, 1962). When the tainter gates are fully opened, the surcharge pool is created with an additional 351,900 ac-ft of storage space available for holding the incoming flood. Figure 4.2 also shows the top elevations of the respective storage zones.

The land both upstream and downstream from the lake is mostly devoted to agriculture use for growing corn, soybeans, and a limited amount of wheat. Protection of the riparian agricultural lands from being flooded is the primary function of the lake. While a non-damaging release rate of 1,800 cfs or less does not affect the downstream crop production in the growing season, the maximum release in the dormant season was allowed to be as high as 4,500 cfs from 1975 to 1983 for lake levels below the top of the flood control pool, i.e. 626.5 ft m.s.l. (mean sea level). Overland flooding in the upstream lake area can be induced, however, when the lake level is above 610.0 ft.

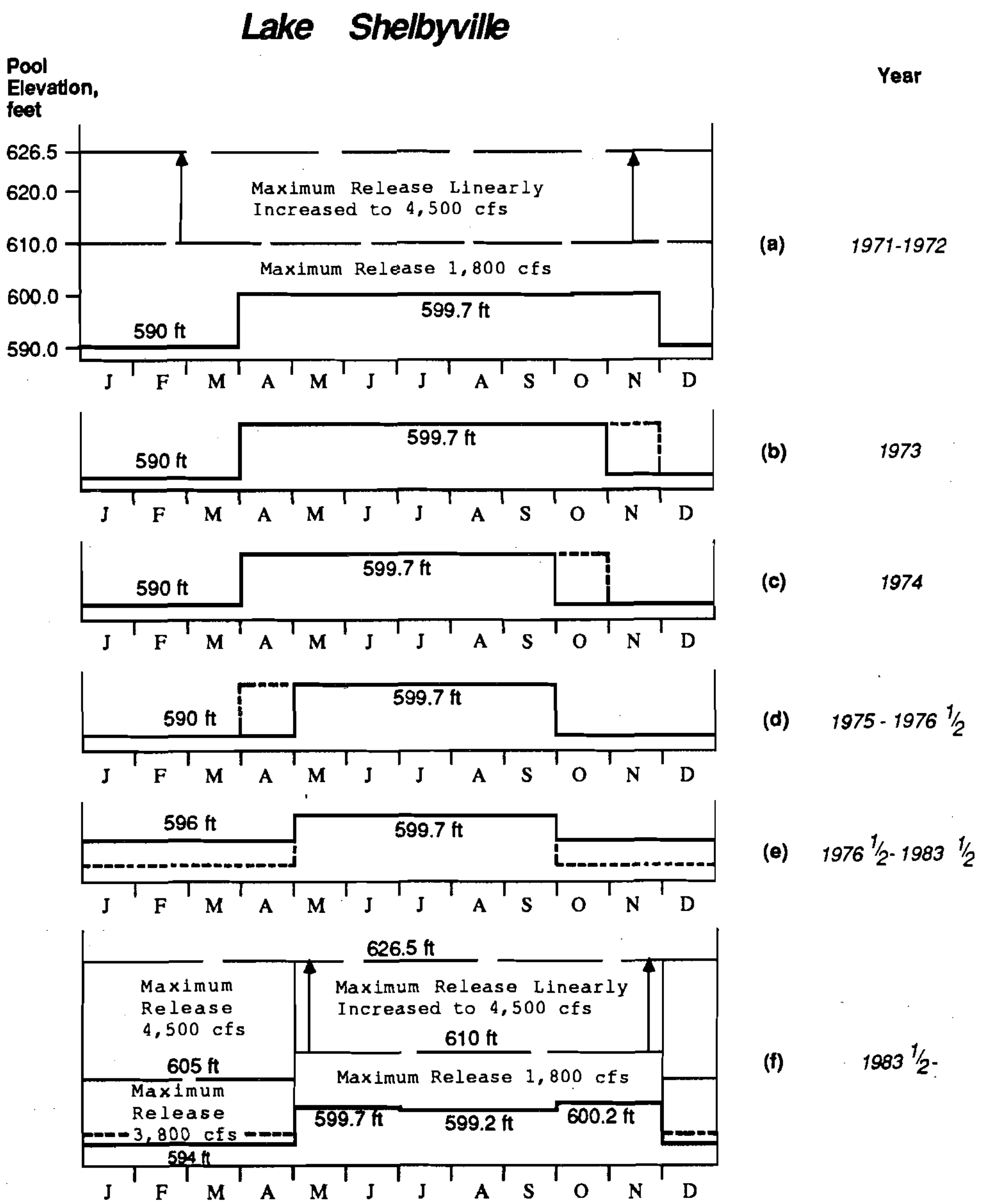
Various facilities have been built along the lakefront as well as in the lake for fishing, boating, and waterskiing activities that take place mostly in the summer. Although a pool level ranging between 589.0 and 602.0 ft would not significantly reduce the number of tourists coming to the lake area (Singh et al., 1975), the summer lake level is conveniently chosen to be at the top of the joint-use pool of 599.7 ft. The goal is thereby to maintain this pool level whenever possible without causing downstream flooding in the summer tourist season.

4.2.3 Operation History and Previous Studies

The control of Lake Shelbyville can be categorized into four major areas: (a) the release rate, (b) the change of release rate, (c) the lake level, and (d) the change of lake level. These control regulations are not only interrelated but also time-varying. In the crop-growing season, the flood control release (maximum allowable release) is 1,800 cfs when the lake level is below 610.0 ft. If the lake level exceeds 610.0 ft up to the top of the flood control pool, the release ceiling of 1,800 cfs is lifted linearly to 4,500 cfs. In the dormant season, the flood control release was set at 4,500 cfs for lake levels below 626.5 ft during 1975-1983. In December 1983, the flood control release was reduced to 3,800 cfs for lake levels below 605.0 ft. When the lake level exceeds 626.5 ft, it is required to pass the flood at a higher releasing rate than the inflow rate up to the spillway capacity in order to bring the pool down below 626.5 ft. The minimum release is always 10 cfs regardless of the time of year. Changes of release or storage rates are restricted so that serious bank erosion both within and downstream from the lake can be avoided (Corps of Engineers, 1983a, EXHIBITS D1-3). In addition to other rules, a change of release rate greater than 500 cfs per day is always prohibited.

The lake is generally controlled to meet the desired pool level whenever possible without violating other release and storage requirements. The release is constrained mainly by the channel capacity; and the storage rule curve has experienced several modifications over the past sixteen years. The evolution of the current operation rule curve is summarized in Fig. 4.3.

The initial operation plan of Lake Shelbyville was re-evaluated in 1969, even before the completion of the dam, in response to numerous complaints about the regulation of Carlyle Lake which was built and placed



- Beginning in Oct. 1975, the maximum winter release was increased to 4,500 cfs for lake levels above the winter target level.
- Dash lines depict the previous rule curve before a change was made.
- Minimum release = 10 cfs.

Figure 4.3 Evolution of the Operation Rule Curve of Lake Shelbyville.

in operation in April 1967. Among the nine alternative plans considered, the one shown in Fig. 4.3(a) was selected, dictating a summer pool level of 599.7 ft from April 1 to November 30, and a winter drawdown to 590.0 ft for the rest of a year. The maximum allowable release had been restricted to 1,800 cfs for the entire year for lake levels below 610.0 ft before the later change made in 1975. It should be noted that in the transitions between the two pool levels the regulation would not be as strict as that implied by the graphs in Fig. 4.3. Actually the lake pool would be gradually shifted from one seasonal level to another with all the release constraints met. Thus, in Fig. 4.3(a), beginning on April 1 the release from the lake would be restricted to a minimum of 10 cfs until the lake level was raised to the summer pool level. On the other hand, beginning on December 1 the release would be larger than the upstream inflow, yet less than 1,800 cfs, until the lake was lowered to the winter pool level.

From October 1972 to September 1974, the Kaskaskia River Basin experienced the two wettest hydrologic years in its recorded history, which prompted another modification of the lake operation rule. After a transition period of two and half years, the winter drawdown period of Lake Shelbyville was lengthened from October 1 to May 1, in an attempt to increase the flood storage during the spring planting and seeding season. In addition, the maximum allowable release in the dormant season was increased to 4,500 cfs for faster discharge of the flood inflow. In the summer of 1976, the lake never regained the desired level of 599.7 ft because of the deep drawdown of the winter pool and the limited surface runoff entering the lake in that season. The pool actually stayed below 594.0 ft for the entire summer in 1976. To reduce the gap between the summer pool and the winter pool, while not sacrificing very much of the flood control function provided by the winter drawdown, the winter pool was changed to 596.0 ft beginning in the latter half of 1976. The time of making this change coincided with the beginning operation of the Kaskaskia River Navigation Channel. An increase of the winter pool level was also desirable to provide adequate amount of release for augmenting the flow in the Navigation Channel when a drought occurred. It was concluded that a moderate degree of satisfaction for the summer recreational endeavors could be maintained at 596.0 ft even when the summer pool would not reach the desired elevation of 599.7 ft (Corps of Engineers, 1983a).

For the next six years after 1976, the weather in the Kaskaskia River Basin was relatively mild; and the operation rule curve remained essentially unchanged. The most recent modification was made in late 1983 based on the operation experience of the Corps of Engineers as well as on constructive criticisms from the local business groups and farmers. The winter drawdown period was shortened from October 1 back to December 1 because the 1975 lengthening of the drawdown period had not provided measurable improvement in the spring flood control. In addition, waterfowl hunting could be improved by extending the summer pool to November. The winter target level was lowered to 594.0 ft; and the maximum allowable release was reduced to 3,800 cfs for pool levels below 605.0 ft for better spring flood control. Minor adjustments including a 6-in drawdown on July 1 and a 1-ft pool raise on October 1 were also made to improve fishery and waterfowl habitat.

The sixteen-year operation history of Lake Shelbyville so far has involved three major transition periods. The 1970-1976 transition was the adaptive period during which both the unusual wet years 1973-1974 and the dry summer of 1976 were encountered in the basin -- an unusual combination of hydrologic events. Owing to the impacts of these events, a more acceptable operation plan evolved and was adopted for the next transition period between late 1976 and 1983. The third period began in late 1983 when the era of fine-tuning the rule curve commenced in order to meet the needs of local interest groups in the basin.

Besides the efforts of the Corps of Engineers in conducting economic and hydrologic studies of Lake Shelbyville and Carlyle Lake in the first transition period, two separate studies of the operation of both lakes were conducted jointly by the Division of Water Resources, Illinois Department of Transportation and the Illinois State Water Survey (Singh et al. 1975; and Singh, 1977). In the 1975 report, Singh et al. used discrete differential dynamic programming (DDDP) to determine the optimal operation plan which minimizes the agricultural, recreation, and property damages in both lake areas in a 24-year period from 1942 to 1965. A simulation study based on the findings from the DDDP model revealed that to maintain a summer pool level of 595.0 ft would induce less long-term agricultural and recreation losses than those achieved by the Corps' plan of a 599.7 ft summer pool. Differences in other aspects between the two plans were comparatively insignificant to the overall reservoir performance. In the 1977 follow-up report, Singh extended the previous simulation study and suggested a winter pool ranging from 590.0 to 594.0 ft and a summer pool from 593.5 to 597.0 ft for different levels of the downstream water supply and navigation demands. Although the Corps of Engineers adjusted the winter pool somewhat following these reports, they never adopted the plan to lower the summer pool below 599.7 ft because of the already established water-related facilities in the lake area.

The selection of proper summer and winter target levels as well as the respective periods for maintaining these pools is considered the core of proper operation of the Lake Shelbyville system. Thus, in the following study, the issue of improving reservoir performance is mainly concentrated on the determination of target pool levels. Most of the economic data prepared by Singh et al. (1975) and Singh (1977) were used in constructing the stochastic dynamic programming model; and the hydrologic data were updated through 1982.

4.3 System Characteristics

4.3.1 Hydrology

A continuous daily flow record of the Kaskaskia River at Shelbyville is available from the United States Geological Survey (in Water-supply Papers for the Upper Mississippi River Basins before 1960; and in Water Resources Data for Illinois since 1961) beginning in water year 1941. The flows were natural runoffs before the Shelbyville Dam was completed in August, 1970 and have been controlled releases ever since. The monthly and annual flow statistics can be computed directly from the daily data before August, 1970. After the completion of the dam the monthly and the annual flows at Shelbyville need to be corrected to account for the lake storage effects, and for consistency with the earlier uncontrolled flow conditions. The annual flow data are contained in Table 4.1.

It is observed that, in the last 42-year recorded history, two severe drought situations occurred (in 1941 and 1953-1955) in the Upper Kaskaskia River Basin. Moreover, the river basin has had large inflows

Table 4.1 Annual Flow Data for the Kaskaskia River at Shelbyville, Illinois.

Water Year	cfs	ac-ft/yr
1941	174	126,000
1942	1,358	983,000
1943	1,207	874,000
1944	662	481,000
1945	815	590,000
1946	915	662,000
1947	963	697,000
1948	675	490,000
1949	805	583,000
1950	1,657	1,200,000
1951	1,169	847,000
1952	915	664,000
1953	338	245,000
1954	36.1	26,000
1955	291	211,000
1956	467	339,000
1957	1,212	878,000
1958	953	690,000
1959	707	512,000
1960	611	444,000
1961	486	352,000
1962	997	722,000
1963	300	217,000
1964	458	332,000
1965	486	352,000
1966	523	379,000
1967	837	606,000
1968	1,113	808,000
1969	741	536,000
1970	932*	675,000*
1971	545*	395,000*
1972	672*	488,000*
1973	1,754*	1,270,000*
1974	1,950*	1,412,000*
1975	1,085*	785,000*
1976	655*	476,000*
1977	419*	303,000*
1978	1,264*	915,000*
1979	1,344*	973,000*
1980	453*	329,000*
1981	797*	577,000*
1982	1,319*	955,000*
Average	837	606,000

A water year begins in October of the previous year and ends in September of the indicated year.

* Flows were corrected for lake storage effects.

more frequently than in the previous year since the completion of the dam. A comparison between the flow records before and after the construction of the Shelbyville Dam is provided in Table 4.2. The flows in March, August, and December accounted for 59% of the total difference between the mean annual flows of the two periods. Moreover, 65% of the difference between the mean annual flows was made up of the differences of flows in the winter drawdown period from December to April. Several statistical tests were conducted to examine if these differences are significant.

The monthly flows are assumed to be log-normally distributed based on the Kolmogorov-Smirnov Test at a 95% confidence level. The means and standard deviations of the log-transformed flows are also listed in Table 4.2. The t-statistic and the F-statistic (Miller and Freund, 1977) are used to test the difference between the means and between the standard errors of the normal sample distributions. At a confidence level of 95%, the differences would be insignificant for $-2.02 < t < 2.02$, and for $F < 2.14$. As a result of the test, only the August flows exhibit a significant difference between the two samples. No apparent reason can be linked to the August abnormality; and the physical characteristics of the Upper Kaskaskia River Basin in general are not considered to have undergone a fundamental change since 1941. Thus, the distinction between the mean annual flows for the two sample periods might be solely attributed to the natural randomness of the runoff process in this river basin.

The monthly evaporation coefficients within Lake Shelbyville range from 0.68 inch in January to 6.02 inches in July (Corps of Engineers, 1983a, Table 1). The evaporation can be as low as 0.9% of the mean monthly flow in February; and as high as 50% in September. However, the

Table 4.2 Comparison of the Mean Flow Statistics of the Kaskaskia River at Shelbyville before and after the Construction of the Dam.

	Mean Flow $\bar{Q}$, x 10^3 ac-ft			Logarithm of Flow, $Log_{10}(Q)$					
				Mean			Stan. Dev.		
	1941-1970	1971-1982	1941-1982	1941-1970	1971-1982	t*	1941-1970	1971-1982	F#
Oct	13.4	18.2	14.8	3.28	3.59	-0.89	0.97	1.21	0.64
Nov	20.1	26.3	21.5	3.69	3.74	-0.13	0.90	1.29	0.48
Dec	33.5	60.4	41.2	3.96	4.35	-1.37	0.85	0.85	1.00
Jan	56.4	66.4	59.3	4.25	4.37	-0.42	0.78	0.81	0.93
Feb	70.0	83.2	73.8	4.48	4.67	-0.88	0.69	0.52	1.76
Mar	65.6	127.4	83.2	4.67	4.97	-1.90	0.49	0.38	1.65
Apr	87.8	104.1	92.0	4.81	4.89	-0.00	0.37	0.37	0.99
May	79.5	83.7	80.7	4.73	4.78	-0.30	0.40	0.39	1.07
Jun	69.3	74.9	70.9	4.68	4.70	-0.12	0.39	0.40	0.93
Jul	37.7	48.8	40.9	4.28	4.49	-0.15	0.56	0.44	1.56
Aug	11.8	39.6	19.7	3.66	4.43	-4.26	0.57	0.40	2.02
Sep	5.2	15.2	8.0	3.22	3.27	-0.16	0.67	1.55	0.19
Annual	550.3	748.2	606.0						

* The t-statistic is insignificant within ± 2.02 at a 95% confidence level.

\# The F-statistic is insignificant when less than 2.14 at a 95% confidence level.

Note: The Corps of Engineers (1983a, EXHIBIT A) also recorded the mean annual inflows as
(a) 570,000 ac-ft for 1930-1970,
(b) 758,000 ac-ft for 1971-1976.

annual evaporation is only about 5.7% of the mean annual inflow. The lake evaporation in general plays a very small role in the current performance of Lake Shelbyville for flood control and recreation.

4.3.2 Flood Control

Flood control was one of the major reasons for the construction of the Shelbyville Dam. The primary purpose of flood control is to minimize the agricultural losses and the property damages in the bottomlands along the Kaskaskia River Valley downstream from Shelbyville. Upstream damage began to accumulate, however, when the lake level rose above 610.0 ft after the completion of the Dam. The crop-growing season is considered to begin on May 1, and to end when the harvest is completed; the earliest possible time that this would occur is October. The spring rainstorms which affect the soil moisture and hence the net basin runoff are most likely to occur between April and June when field preparation and crop planting are underway. The extra storage space spared by the winter draw-down is used to hold back the spring floods and to protect the downstream agricultural land.

The Corps of Engineers suggested the Flood Hydrograph-Damage Integration (FHDI) Method (Cochran, 1960) for estimating flood damages in agricultural lands. Both the loss of direct production investment (DPI) at the time of flooding and the loss of income (LI) are included in the damage estimation. The values of DPI and LI for a typical acre are obtained by multiplying the values for various crops in that acre by the respective fractions of the acre for each crop, and adding the products. Singh et al. (1975) explained the FHDI method in great detail and provided the resulting Lake Shelbyville area agricultural damage data which is summarized in Table 4.3. The property damage data pertaining to farmsteads, roads, and farm fences or other structures are also listed in Table 4.3. While the damage to property may be caused by flooding at any time in a year, the flood damage to crops is assumed to occur only in the growing season between May and October. To compute the total agricultural and property damages of the flooded lands, the area-elevation and the area-release relationships for the upstream and downstream reaches in the Lake Shelbyville area are needed and therefore included in Table 4.3.

4.3.3 Recreation

Recreation activities are assumed to take place mainly in the lake area behind the Shelbyville Dam. Singh et al. (1975) divided the most popular recreation activities into seven major categories -- camping, picnicking, swimming, boating, skiing, fishing, and hunting. All the activities except hunting take place mainly in the summer tourist season from May to September. Table 4.4 shows the year-round distribution of visitors for each recreation activity on a monthly basis. The visitor distribution data were obtained from the Corps field office at Lake Shelbyville for years 1971-1974 (Singh et al., 1975). The monthly distribution pattern of each activity was assumed unchanged; and the total visitor numbers were estimated for 1975. The total expenditure of each visitor in the lake area is a highly variable parameter depending upon the kind of activities involved, and the intensities of the recreational facilities used. Corps of Engineers (Singh et al., 1975) used a price of $3.00 per visitor per day for both fishing and hunting, and $1.50 per visitor per day for the rest of the recreation activities in the Lake

Table 4.3 Pertinent Agricultural and Property Data, with Complementary Area-Elevation and Area-Release Relationships for the Upstream and the Downstream Reaches in the Lake Shelbyville Area.

	Above Shelbyville*	Below Shelbyville**
Crop damages, dollars/acre		
Corn	24.30	88.20
Soybeans	10.36	42.00
Wheat	2.77	
Total	37.43	130.20
Property damages, dollars/acre	0.42	0.30
Area(A) - Elevation(H) (ac) (ft)	$A = 0.02063H^3 - 33.1140H^2 + 17859.4H - 3239336.2$	
Area(A) - Release(R) ($\times 10^3$ ac) ($\times 10^3$ cfs)		$A = 0$, $R \leq 1.8$
		$A = 0.1278R^3 - 1.4212R^2 + 7.1821R - 9.0454$, $1.8 < R \leq 4.5$
		$A = -0.035R^2 + 0.975R + 2.580$, $4.5 < R \leq 7.0$
		$A = -0.018R^2 + 0.620R + 4.245$, $7.0 \leq R$

* For area flooded above 610.0 ft.

** For area flooded in bottomlands (release > 1,800 cfs).

Table 4.4 Distributions of Visitors for Various Recreation Activities.

Month	Visitor Distribution*						
	Camping	Picnicking	Swimming	Boating	Skiing	Fishing	Hunting
Oct	0.0572	0.0424	0	0.0457	0.0214	0.0586	0.2075
Nov	0.0178	0.0181	0	0.0145	0	0.0199	0.5876
Dec	0.0074	0.0077	0	0.0050	0	0.0101	0.2049
Jan	0.0061	0.0059	0	0.0025	0	0.0067	0
Feb	0.0075	0.0104	0	0.0049	0	0.0096	0
Mar	0.0205	0.0246	0	0.0183	0	0.0314	0
Apr	0.0586	0.0677	0.0165	0.0739	0.0199	0.0717	0
May	0.1309	0.1425	0.1639	0.1514	0.2053	0.1548	0
Jun	0.1544	0.1654	0.2325	0.1786	0.2192	0.1701	0
Jul	0.1935	0.1867	0.2547	0.1901	0.2053	0.1716	0
Aug	0.2045	0.1849	0.2174	0.1826	0.1954	0.1658	0
Sep	0.1416	0.1437	0.1150	0.1325	0.1335	0.1297	0
Total	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Visitors (/year)	365,000	130,000	400,000	300,000	60,000	350,000	35,000
Expenditure** ($/visitor /day)	1.50	1.50	1.50	1.50	1.50	3.00	3.00

* For years 1971-1974.
** For year 1975.

Shelbyville area, based on the recommended data from the Water Resources Council (1973). Although both the number of visitors and their expenditures may change through the years, the data provided by Corps of Engineers are used herein in order to compare the results from the stochastic programming model to those of the previous studies.

The damage to recreation is measured by the loss of visitors in the lake area because of the undesirable pool levels for the various recreation activities (Singh, 1975). Table 4.5 lists the lake levels at which recreation damages would occur, as well as the percent loss of visitors per foot change of the lake level. All the camping and picnicking areas along the lake are located above the top of the flood control pool, 626.5 ft; and the visitors involved in these two activities would not be affected by the change of lake level. For each of the other activities, there are high and low threshold levels between which the visitors would feel indifferent to the change of lake level. However, when the lake level rises or falls beyond those ranges, fewer tourists would be expected to visit the lake area. For example, the lowest level that all swimmers in the lake area would tolerate is 589.0 ft. If the lake falls below this level, a 5% decrease in the expected number of swimmers would be incurred for an additional one foot drop of the lake level. However, only a maximum of 50% of the total expected swimmers could be turned away when the lake drops below 579.0 ft. Thus, the recreational loss of a certain activity due to an undesirable lake level can be estimated by multiplying the expected expenditure per visitor per day by the number of visitors lost in that particular day.

Table 4.5 Percent of Recreation Loss per Foot of Change in Lake Level, Lake Shelbyville.

Activity	Low Level,* feet	Loss, %/ft	Max Loss, %	High Level,** feet	Loss, %/ft	Max Loss, %
Camping		No loss			No loss	
Picnicking		No loss			No loss	
Swimming	589	5	50	603	8.3	70
Boating	585	25	100	610	8.3	39
Skiing	585	25	100	610	8.3	39
Fishing	585	15	75	610	5	75
Hunting	589	10	100	602	3.5	95

* Lowest lake level below which recreational damage occurs.
** Highest lake level above which recreational damage occurs.

4.3.4 Priority between Flood Control and Recreation

Although the damage to agriculture, property, and recreation, in monetary terms, should not be the sole criterion in evaluating a reservoir's performance, it provides a convenient starting basis for building a basic mathematical model. Other operation criteria may later be incorporated into the basic model in further analysis of the reservoir system. It is hard to perceive the relative weights of the various damages directly from the information provided in Tables 4.3-4.5. With a little arithmetic manipulation of the same data important insights can be obtained as to how the system may respond under hydrologic uncertainty.

Figures 4.4 (a, b, and c) show the damage costs as functions of the storage levels in the lake, and of the release rates downstream from the dam. The summer season for agricultural activities extends from May to October; whereas the summer season for recreation activities begins in May and lasts until September. The curves are plotted at selected discrete storage and release levels which will be used in the following stochastic dynamic model. It can be seen from Fig. 4.4(a) that in the summer the potential recreation damages cause the major concerns in reservoir operation if the lake level drops too low. On the other hand, both agricultural and recreation damages have roughly the same influence on the control of high lake levels. The relative importance of recreation damages varies in time. In winter, although the recreation activities decrease significantly, the damages to those activities are still the prevailing factors for overall lake control. Property damage is on all accounts minimal as compared to damages to agricultural and recreation activities. A public which is properly informed about the flood control

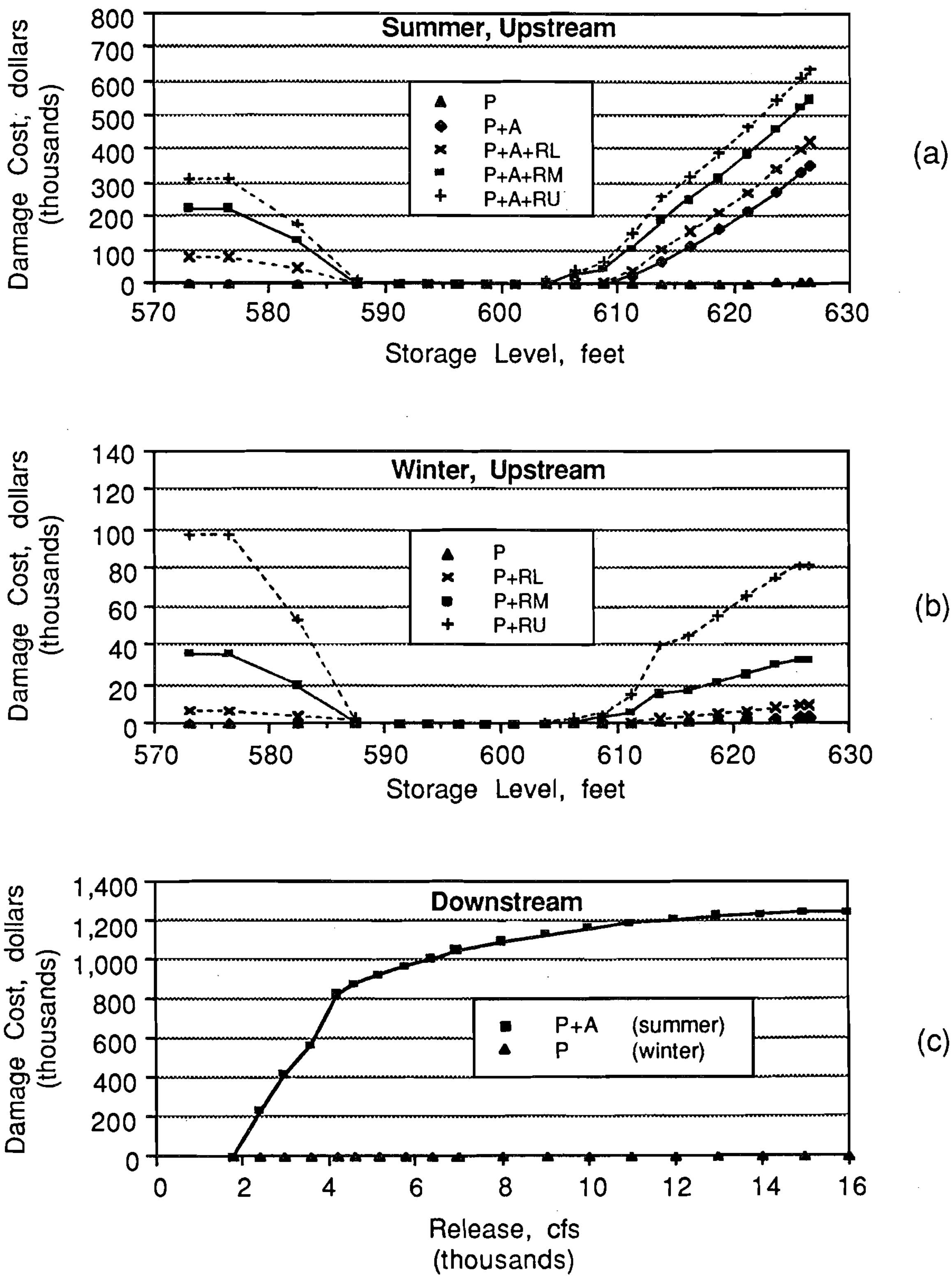


Figure 4.4 Flood Damages and Recreation Losses as Functions of Lake Levels and Releases at Lake Shelbyville for Upstream and Downstream Reaches, and for Summer and Winter Periods; P = Property, A = Agriculture, RL = Lower Limit on Recreation, RM = Mean for Recreation, and RU = Upper Limit on Recreation.

zones both within and below the lake areas could reduce the potential property damage by not building permanent facilities in this region.

The channel capacity downstream from Lake Shelbyville is 1,800 cfs. For flows greater than 1,800 cfs, most of the agricultural lands along the Kaskaskia River Valley are quickly flooded due to the mild transverse slope of the floodplain, which results in roughly uniform increases in the agricultural damage as the flow increases. Since the Kaskaskia River Valley is mostly a geomorphologically depressed area relative to the neighboring higher agricultural lands, the flood flows would be mostly confined within the valley. This explains the decreasing rate of agricultural losses as the flow increases above 4,500 cfs (Fig. 4.4(c)).

From the damage curves in Figs. 4.4(a, b, and c), it can be inferred that the best storage levels at Lake Shelbyville should lie somewhere between 589.0 ft and 602.0 ft, a range in which no damage would occur. In addition, the desirable summer and winter pools might not stay at fixed levels due to large seasonal variation of the upstream inflows. The current summer pool target of 599.7 ft might be a little bit too high, when judging from the flood damages and recreation losses, since choosing a lower summer pool target would reduce the risk of reaching either high storage levels or high downstream releases.

In the following sections, a stochastic dynamic programming model is used to explore the tradeoffs of storages between the summer and the winter seasons, as well as the tradeoffs of damages between the upstream and the downstream reaches, in the Lake Shelbyville area. A penalty function approach is adopted to account for operation criteria not commensurate in units with the economic measures used in the objective function. Certain unmodeled issues are discussed based on the modeling results.

4.4 Stochastic Dynamic Programming

4.4.1 Partitions of Inflow and Storage States

The definition of discrete inflows as discussed in Sec. 3.2.1.2 and illustrated in Fig. 3.2 is used for partitioning the monthly runoffs into Lake Shelbyville. The monthly inflows can be assumed to be lognormally distributed based on the Kolmogorov-Smirnov test at a 95% confidence level. The lower limits of the monthly flows are assumed zero; while the information about the upper limits is provided by Singh (1977, p. 46) through elaborate analyses of the various physical, meteorological, and hydrologic characteristics of the Upper Kaskaskia River Basin. The flow state increment or class interval of all months is taken to be 35,000 ac-ft, which is of roughly the same order of magnitude as the storage increments defined later in this section. The inflow statistics and state partitions are summarized in Table 4.6.

Column (1) lists the estimated probable maximum inflows for 12 months, which were derived from the respective probable maximum precipitations in the Upper Kaskaskia River Basin, and then adjusted by the proper basin fraction indices (Singh, 1977). Column (2) contains the recorded maximum inflows between 1941 and 1982 at Shelbyville. The flows in Columns (1) and (2) are expressed in cfs-month, representing the total flow volume accumulated in a month at the indicated mean flow rate in cfs. The recorded maxima in months of lower inflows (July-December) in the 42-year period deviate relatively farther from their estimated maximum

Table 4.6 Statistics and Partitions of Upstream Inflows of Lake Shelbyville for Use in the Stochastic Dynamic Programming Model.

Month	(1) Probable Maximum Inflow,* cfs-month	(2) Recorded Maximum Inflow,** cfs-month	(3) Maximum Discrete Inflow, ac-ft	(4) Logarithm of Inflow** Mean	(5) Logarithm of Inflow** Stan. Dev.	(6) Logarithm of Inflow** Corr.	(7) Number of States***
Oct	7,400	2,031 (1969)	455,000	3.37	1.04		13
						0.167	
Nov	7,700	2,493# (1972)	455,000	3.71	1.01		13
						0.221	
Dec	7,900	3,143 (1967)	490,000	4.07	0.86		14
						0.308	
Jan	9,700	7,097 (1950)	595,000	4.29	0.78		17
						0.213	
Feb	8,400	4,033 (1951)	455,000	4.54	0.65		13
						0.240	
Mar	8,000	5,720# (1979)	490,000	4.76	0.47		14
						0.095	
Apr	10,800	4,848 (1944)	630,000	4.84	0.37		18
						-0.121	
May	12,400	6,527 (1943)	770,000	4.75	0.39		22
						0.274	
Jun	12,200	4,608 (1957)	735,000	4.69	0.39		21
						0.055	
Jul	9,000	3,043 (1942)	560,000	4.34	0.53		16
						-0.087	
Aug	6,100	2,347 (1958)	385,000	3.88	0.64		11
						0.043	
Sep	3,700	774# (1972)	210,000	3.24	0.98		6
						0.144	

* Data taken from Singh (1977, Table 26).
** Based on monthly flow record in 1941-1982; number in parentheses represents the year when the maximum was recorded.
*** State increment = 35,000 ac-ft.
\# Adjusted for lake storage effects.

probable values, which could be due to the generally higher skewness of the inflow distributions during this period. The historical monthly maxima were recorded rather randomly through the years, with a tendency of occurring more frequently in wet years. Moreover, the recorded maximum monthly inflows are mostly much larger than the maximum non-damaging rate, 1,800 cfs, which indicates the importance of the flood protection function provided by Lake Shelbyville.

The maximum discrete inflows were chosen to be multiples of the state increment, 35,000 ac-ft, in the neighborhood of the respective probable maximum inflows. Column (7) gives the number of states partitioned for each month, with the class mark defined to be the center of each class interval. The Markov transition probability matrix between inflows of each of the 12 pairs of adjoining operation periods was estimated by the technique discussed in Sec. 3.2.1.3; and the flow statistics were calculated based on the 1941-1982 record.

The effective storage space of Lake Shelbyville, totaling 654,000 ac-ft, consists of the joint-use pool and the flood control pool between 573.0 ft and 626.5 ft (Fig. 4.2). When the lake level is below the top of the flood control pool, 626.5 ft, the outflow up to 4,500 cfs is released through the two sluice gates at the bottom of the dam. However, the sluice gates are closed completely as the lake level rises above the flood control pool; and control of the release is taken over by the tainter gates. A surcharge pool is then created. Although the surcharge pool adds up to 351,000 ac-ft of extra storage space to the lake, that storage may not exist for a long time because the regulation requires the lake to level be lowered as quickly as possible whenever the lake is above 626.5

ft (Corps of Engineers, 1983a). The spillway can discharge a maximum of 121,000 cfs which is at least 10 times the probable maximum inflow of any month at Lake Shelbyville. Thus, it can be reasonably assumed that although the surcharge storage may exist temporarily, it is practically irrelevant to the normal operation of the reservoir system and can be excluded from consideration in the reservoir model.

The partition of the effective storage in Lake Shelbyville is illustrated in Table 4.7. Twenty storage states are considered, including the two extreme states of emptiness and fullness with zero class intervals. The state boundaries and the state marks are expressed in elevations for easy comprehension of the relationship between the states and the crucial control levels of the lake. The storage states are defined in such a way that the state intervals are of the same order of magnitude as the increment of the inflow states. According to the discussions in Sec. 3.3.2, the partitions of the inflow and storage states for the Lake Shelbyville system should yield stable results from the discrete optimization model.

4.4.2 Basic Model Formulation

The reservoir model developed herein for the Lake Shelbyville system parallels closely that for the hypothetical reservoir system previously presented in Sec. 3.2.2.1. The optimal operation policy is determined solely by the recursive equation shown in Eq. 4.1 to minimize the long-term damage to this reservoir system. No other release and storage constraints are considered in the model. For the Lake Shelbyville system, a year is divided into 12 monthly operation periods. Let C_{imjt} be the damage cost that accrues in time period t, which is associated with the

Table 4.7 Partition of Effective Storage Space in Lake Shelbyville for Use in the Stochastic Dynamic Programming Model.

State Number	State Boundary (Elevation, ft)	State Mark (Elevation, ft)	State Interval (Volume, ac-ft)
	573		
1		573	0
	573		
2		576.5	26,400
	580		
3		582.5	26,700
	585		
4		587.5	34,300
	590		
5		591.25	20,300
	592.5		
6		593.75	22,300
	595		
7		596.25	24,500
	597.5		
8		598.6	23,400
	599.7		
9		601.1	32,500
	602.5		
10		603.75	31,900
	605		
11		606.25	34,900
	607.5		
12		608.75	37,900
	610		
13		611.25	41,100
	612.5		
14		613.75	44,100
	615		
15		616.25	47,200
	617.5		
16		618.75	50,300
	620		
17		621.25	53,900
	622.5		
18		623.75	57,900
	625		
19		625.75	36,800
	626.5		
20		626.5	0
	626.5		

* Storage (S, ac-ft) expressed as a function of pool elevation (H, ft):

$$S = 1.283H^3 - 2105.6775H^2 + 1152441.635H - 210338184.5$$

initial storage S_{it}, the inflow Q_{mt}, and the final storage $S_{j,t+1}$. Then, the minimum expected damage $f_t^\tau(i,m)$ to the system with only τ periods remaining can be determined by the following recursive equation:

$$f_t^\tau(i,m) = \underset{j}{\text{minimum}} \left[C_{imjt} + \sum_n P_{mn}^t f_{t+1}^{\tau-1}(j,n)\right] \quad \text{for all } i,m,t;\ j \text{ feasible} \qquad (4.1)$$

in which P_{mn}^t is the transition probability of the discrete inflows changing from Q_{mt} to $Q_{n,t+1}$ in successive periods. In Eq. 4.1, $f_t^\tau(i,m)$ is determined by finding the final storage state(s) which minimizes the sum of the current damage and the expected total damage in the future τ-1 operation periods. The optimization procedure expressed by Eq. 4.1 starts at some time in the remote future, and proceeds backward through a finite number of periods until the optimal storage decisions of any period in a year become invariant for two consecutive years. The final storage state(s) so determined represent the optimal steady-state operation policy for any feasible combination of S_{it} and Q_{mt} in period t.

The damage cost C_{imjt} incurred in a certain time period, as implied by the state indices i, m, and j, is a function of the initial storage, the inflow, and the final storage. Essentially, C_{imjt} comprises five distinct components: the recreation loss, C_{imjt}^r, the agricultural losses in both the upstream and the downstream reaches, C_{imjt}^{au} and C_{imjt}^{ad}, and the property losses in these two reaches, C_{imjt}^{pu} and C_{imjt}^{pd}. Thus, Eq. 4.1 can be written more explicitly as

$$f_t^\tau(i,m) = \underset{j}{\text{minimum}} \left[C_{imjt}^r + C_{imjt}^{au} + C_{imjt}^{ad} + C_{imjt}^{pu} + C_{imjt}^{pd} + \sum_n P_{mn}^t f_{t+1}^{\tau-1}(j,n)\right] \quad \text{for all } i,m,t;\ j \text{ feasible} \qquad (4.2)$$

As discussed in Secs. 4.3.2-4.3.4, each of the five damage components is expressed as a function of either the lake level (thus the lake storage) or the downstream release, which are related via the following mass balance equation:

$$R_{imjt} = S_{it} + Q_{mt} - S_{j,t+1} - E_{ijt}(S_{it}, S_{j,t+1}) \qquad (4.3)$$

in which R_{imjt} is a feasible release in ac-ft. E_{ijt} is the total evaporation loss in period t, which is assumed to be a function of only the initial and the final storages and is defined by

$$E_{ijt}(S_{it}, S_{j,t+1}) = (E_{it}(S_{it}) + E_{j,t+1}(S_{j,t+1})) / 2 \qquad (4.4)$$

Because all of the terms on the right-hand side of Eq. 4.3 take on only discrete values, only a finite number of values may be obtained for R_{imjt}.

The recreation damage in each month is calculated from decreases in the number of visitors and their expected expenditures in the lake area due to the unfavorable lake conditions in comparison to normal recreation activity levels. The lake level is assumed to increase or decrease linearly in time for different initial and final storage states within an operation period. Thus the percent loss of visitors, if any, is considered time-dependent and should be computed as such. A schematic representation of the evaluation of the recreation losses in the recursive equation is provided in Fig. 4.5.

According to the loss information contained in Table 4.5, the lake level can be divided into five mutually exclusive zones, each of which involves a different way for estimating the recreation damage. Taking boating activity as an example, the five zones are separated by the four

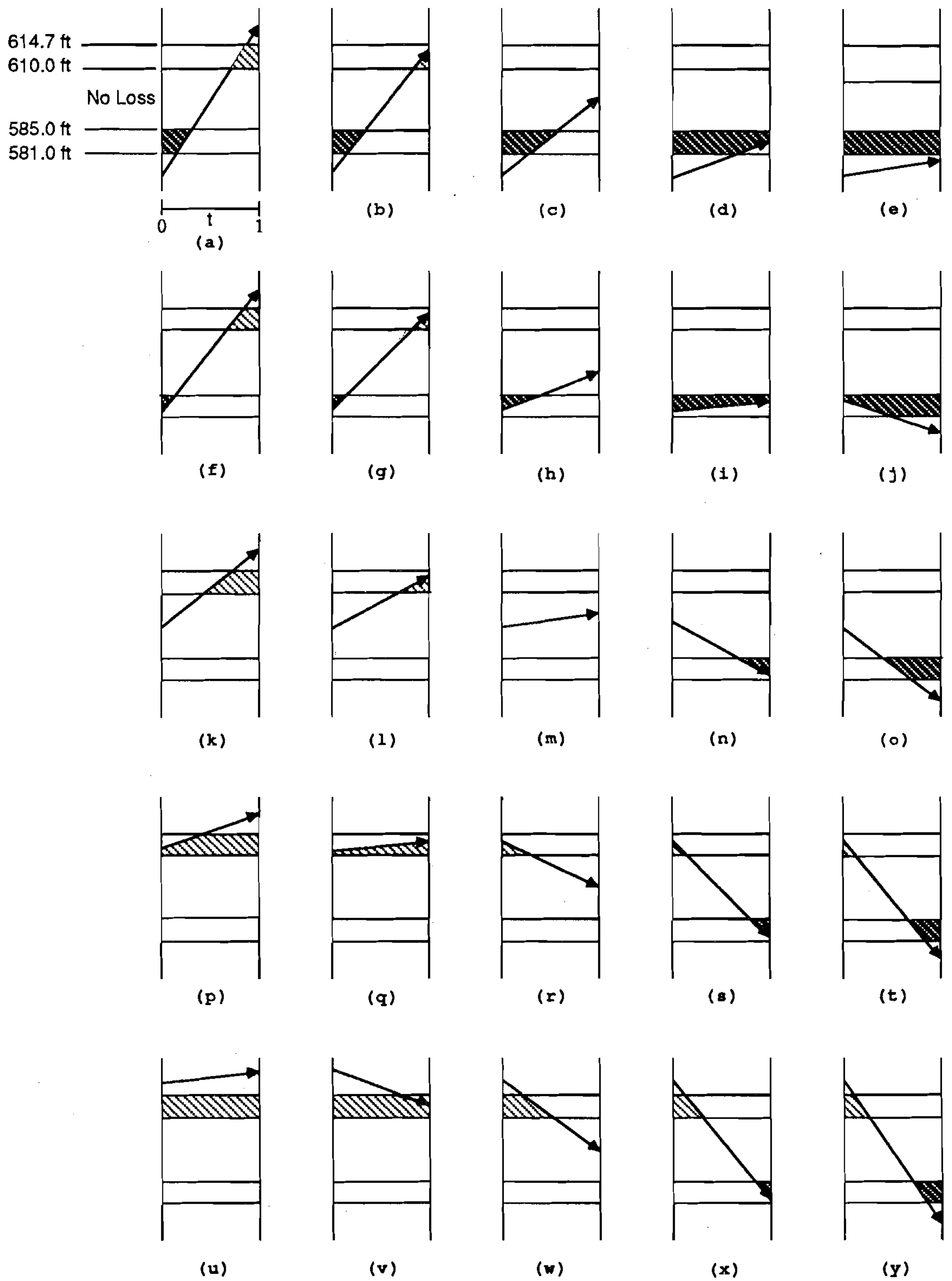


Figure 4.5 Schematic Representation of Evaluating the Recreation Losses in the Recursive Equation (Eq. 4.1) Using Boating Activity as an Example.

levels of 581.0 ft, 585.0 ft, 610.0 ft, and 614.7 ft. For a lake level below 581.0 ft, all boating activity ceases; and the loss of boaters is 100%. For the range of 581.0-585.0 ft, the percent loss of visitors is an inverse linear function of the increasing lake level. No loss of visitors is incurred for a lake level between 585.0 ft and 610.0 ft. The loss of visitors increases linearly from 0% to 39% as the lake level rises from 610.0 ft to 614.7 ft. Beyond 614.7 ft, the loss of visitors stays at the maximum 39%.

For each plot in Fig. 4.5, the vertical axis shows the lake elevation; and the horizontal axis represents the time span of one month. There are 5^2 distinct combinations of the initial and the final storage states (or lake levels), from which various patterns of the losses in time are displayed. In each plot the shaded areas, after being multiplied by the proper loss rates (25% and 8.3% per foot of change in lake level for the darker area and the lighter area, respectively) and summed, equal the percent loss of expected visitors in a month. Multiplying this percent loss by the total visitors in that month and by the expected individual expenditure ($1.50 per visitor) yields the damage to boating activity in the month due to the change of the lake level. It is assumed that visitors to the lake area are uniformly distributed within a month. Recreation losses on swimming, skiing, fishing, and hunting activities can be evaluated in a similar manner except that the five lake zones may be defined differently for each activity. There is no loss associated with either camping or picnicking at any lake level; and the total recreation loss accumulated in a month is simply the arithmetic sum of those losses pertaining to swimming, boating, skiing, fishing, and hunting.

The above method for computing recreation damage is considered applicable to any month in a year, and independent of the recreation activities before or after the current operation period. This is because the damage is counted on the basis of loss of the expected daily expenditures of individual visitors to the lake area. In addition, the loss of visitors on a certain day can be reasonably assumed to be a function of the lake level only, and to be unrelated to the number of visitors in the previous days.

4.4.3 Modification for Unrepeatable Damages

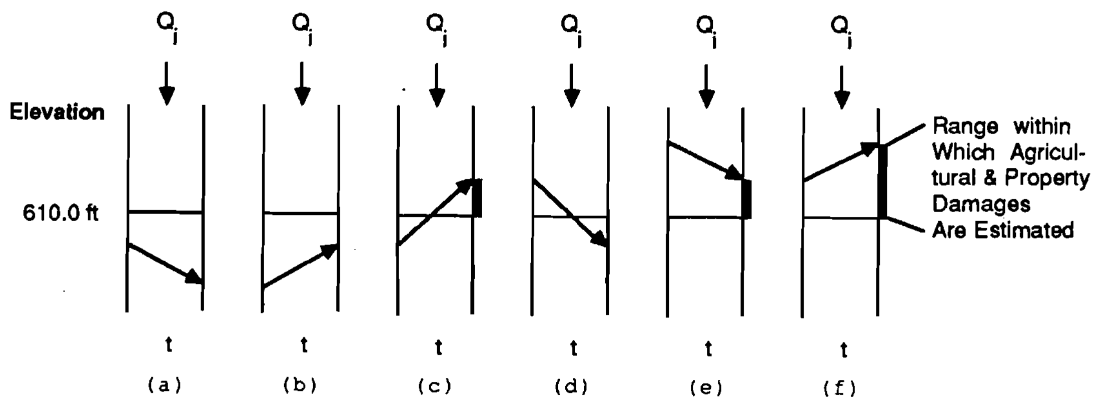
In evaluating agricultural and property damages, care should be exercised in defining the recursive equation so that these damages are not overestimated. Consider the following example in which the pool rises above the damage level of 610.0 ft to 615.0 ft during the third month of the current crop-growing season. The agricultural damage in this month would be estimated from the loss of direct production investment and the loss of income of the crops in the flooded area. If in the remaining growing season the pool stays below 615.0 ft, no additional cropland would be flooded; and the total agricultural damage in the upstream area would equal that accrued in the third month only. On the other hand, should the pool rise further above 615.0 ft at any time during the remaining growing season, extra crop damage would be added, but only for the additional cropland flooded above 615.0 ft. In other words, neither agricultural nor property damages can be counted more than once for the same flooded area within a growing season. The damages to the downstream cropland should be evaluated in a similar way by considering the releases in different months of the growing season. The storage and release thresholds for incurring property damages in both reaches are the same as those for the agricultural losses.

Because the typical recursive equation defined in stochastic dynamic programming (Eq. 4.2) does not consider the effects of the previous storage or release conditions in the calculation of the current agricultural or property losses, the agricultural damage might very well be overestimated. Since it has been assumed previously that the pool would increase or decrease uniformly with time in each month, the lake would reach the highest level at either the beginning or the end of that month. For consistency in the backward optimization procedure, the agricultural damage is assumed to be evaluated at the end of each month in the growing season.

Method I in Fig. 4.6 shows that using the typical recursive equation, the crop damages would be counted for lake levels above 610.0 ft (indicated by the black bar) during that month. Some of the agricultural damages would be duplicated if the lake level would stay above 610.0 ft for more than one month in the growing season. This would lead to an overestimate of the expected agricultural damage in the long run. To resolve this problem, an improved approach can be used.

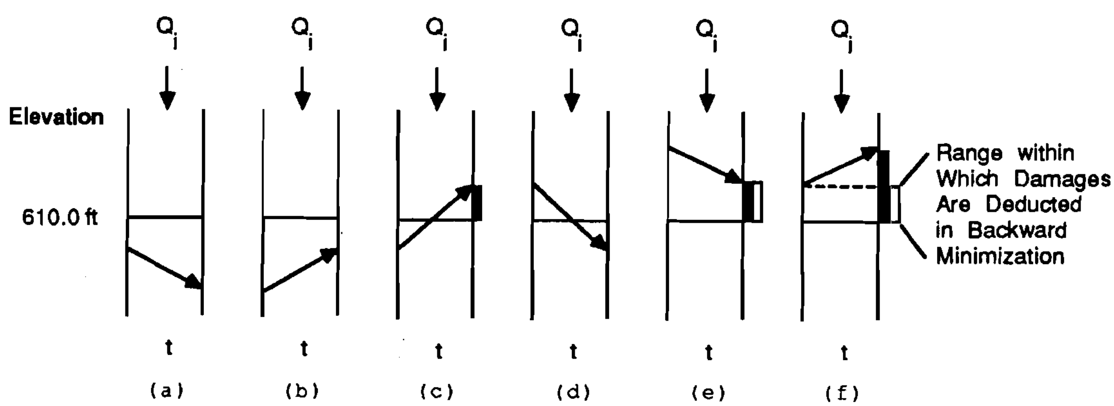
As demonstrated by Method II(e & f) in Fig. 4.6, the agricultural damage would be calculated only for areas which have not been flooded in the immediately previous month. When translated to the recursive equation in backward optimization, this concept would involve adding a nonpositive correction term to the corresponding current damage in Eq. 4.2. In case of (a), (b), (c), or (d), the recursive equation should be identical for both Method I and II; and no modification would be necessary. When the

Method I



(2 x 3^1 possible cases)

Method II



(2 x 3^1 possible cases)

Figure 4.6 Estimation of the Upstream Agricultural and Property Damages:
Method I -- Damage Independent of the Conditions in Previous Months;
Method II -- Damage Calculated Only for the Additional Flooded Area Caused by Lake Level Higher Than in the Previous Month.

lake exceeds the damage level in two consecutive months, the damage which had been counted in the later month in the backward optimization procedure should be deducted completely or partially for the range of lake levels represented by the white bars in Method II(e & f), respectively.

For each combination of S_{it}, Q_{mt}, and $S_{j,t+1}$, there are N inflow states, $Q_{n,t+1}$, in month t+1 to be considered in order to weigh the damage corrections properly in month t according to the likelihood of $Q_{n,t+1}$ occurring. The damage C^{au}_{imjt} in Eq. 4.2 (and similarly for C^{pu}_{imjt}) should be replaced by the modified damage D^{au}_{imjt} as

$$D^{au}_{imjt} = C^{au}_{imjt} - \sum_{n} P^{t}_{mn} c^{au}_{imjnk,t,t+1}(S_{it}, Q_{mt}, S_{j,t+1}, Q_{n,t+1}, S_{k,t+2}) \quad \text{for all } i,m,t;\ j \text{ feasible} \qquad (4.5)$$

in which $c^{au}_{imjnk,t,t+1}$ is the damage correction. Note that in backward optimization, $S_{k,t+2}$ is a known state as a function of $S_{j,t+1}$ and $Q_{n,t+1}$. Incorporating Eq. 4.5 into the computer code involves only the comparison of $S_{j,t+1}$ and $S_{k,t+2}$, and the corresponding damage correction, which can be easily accomplished without major increase of computing time and memory requirements. The same concept of damage correction can be applied to the evaluation of downstream agricultural and property losses, with the damage threshold being the release of 1,800 cfs.

These damage corrections for agricultural and property losses are appropriate based on two assumptions. First, the lake level does not exceed the damage threshold (610.0 ft) for crop losses in more than two consecutive months in the growing season; nor more than 2 consecutive months in a year for property losses. Second, in these periods the lake does not rise beyond 610.0 ft at two or more disjoint months. Given the relatively large storage capacity of Lake Shelbyville, and the expected normal pool level far below 610.0 ft, it is believed that the above assumptions are a reasonable approximation of reality. To assure the adequacy of this assertion, however, in the ensuing sections the optimization result from a more elaborate modification of the damage function (Method III shown in Fig. 4.7) is compared to that from Method II. Using Method III, the current damage is corrected according to all possible lake level combinations in the next two months, rather than those only in the immediate next month as represented by Method II. It can be observed from Fig. 4.7 that except for (l) and (o), the damage correction for 16 of the 18 scenarios of Method III is properly accounted for by simply adopting Method II. The probability of each of the remaining two cases occurring is extremely small so that the overestimation of damages in these two cases would contribute an insignificant effect to the resulting optimal operation policy and the long-term expected performance of a single reservoir system.

For a complicated reservoir model with many inflow and storage states, using Method III significantly increases the computation time as compared to that using Method II since the inflow combinations would be multiplied by the number of inflow states of the additional month considered for the damage correction. As a general approach, the magnitude of the expected errors in damage estimation for cases (l) and (o) in Fig. 4.7 should be calculated. If those errors are minimal as compared to other damages, simply using Method II would be adequate. Otherwise, the more elaborate Method III should be used.

Method III

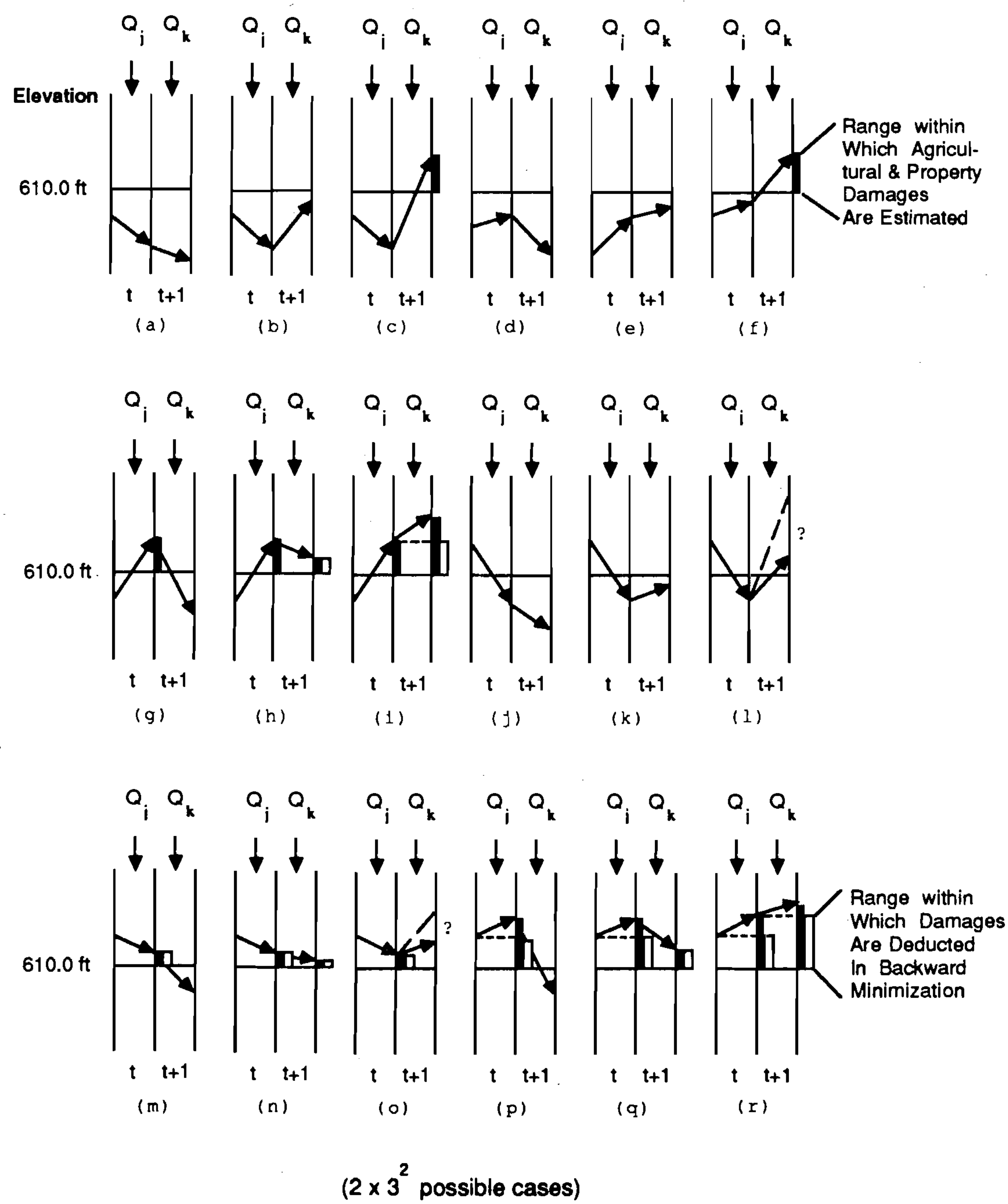


Figure 4.7 Estimation of the Upstream Agricultural and Property Damages: Method III -- Damage Calculated Only for the Additional Flooded Area Caused by Lake Level Higher Than in the Previous Two Months.

4.5 Analysis

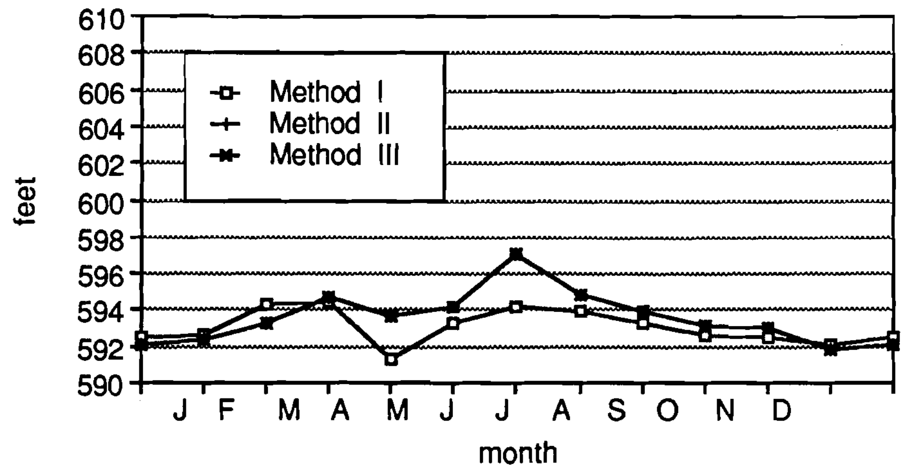
The stochastic dynamic programming model for the Lake Shelbyville system was constructed based on the 1941-1982 inflow record. The effective storage capacity of the system is considered to consist of the joint-use storage and the flood control storage. A maximum flood control release of 1,800 cfs is enforced when the lake level is below 610.0 ft in the crop-growing season from May to October. This maximum allowable release is raised linearly to 4,500 cfs as the lake level rises from 610.0 ft to 626.5 ft. No release is allowed to exceed 4,500 cfs for lake levels below 626.5 ft. Only the economic losses which can be estimated in monetary terms are included in the objective function. The three different ways of dealing with the agricultural and property losses in the backward minimization procedure are first compared based on the respective modeling results. This provides a means to determine the most efficient way to model the unrepeatable damages for the Lake Shelbyville system.

The backward minimization was started in December; and it took 16 iterations of the backward search to find the optimal steady-state operation policy, regardless of the method used for estimating the unrepeatable damages. This rate of convergence to the optimal solution is considered reasonably fast since at least 13 iterations of the backward search must be completed to verify the state of convergence. The expected total annual damage obtained as a result of using Method I is $53,573, an overestimate by roughly 45% of the total damages of $36,983 or $36,979 estimated respectively by Method II and III. The monthly mean storage levels (measured in the beginning of each month) and the monthly expected damages calculated from the optimal results are plotted in Fig. 4.8 based on the three methods.

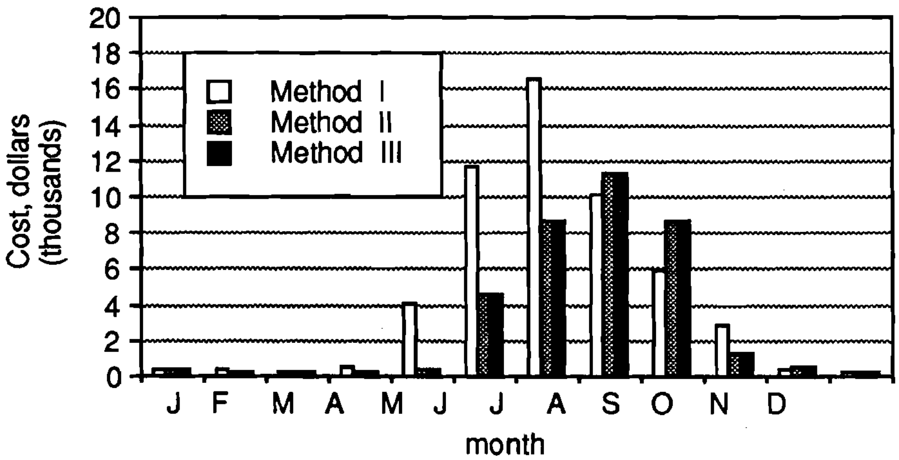
It is apparent that Method I greatly overestimated the monthly damages in the beginning months of the crop-growing season. This is because major spring floods are likely to keep either the lake level above 610.0 ft or the release rate greater than 1,800 cfs for this extended period. Because of these overweighted damages in the objective function, the solution based on Method I tends to drive the summer mean lake levels lower than those of Method II and III. As a result, more storage space can be spared to hold the spring floods and to reduce the risks in reaching high storage and release levels. In the later months of the crop-growing season, the expected damages calculated using Methods II and III are greater than those estimated using Method I. This is because the mean storage levels are higher in these months as a result of using Methods II and III. Since the recreation losses were adequately estimated by all methods, the monthly damages consisting mostly of the recreation losses would increase as the lake level rises. Therefore, using Method I would not only distort the resulting damage distribution but also affect the mean monthly storage levels.

The differences between the optimal results from Method II and III, on the other hand, are found to be negligible on all accounts. The CPU times for the models running on the Cyber 175 computer at the University of Illinois were 13.41, 14.52, and 29.78 seconds, respectively, for the three methods. Since using Method II for evaluating the agricultural and property damages does not significantly increase the model's complexity and the associated computation effort, and yields adequate modeling

(a) Mean Lake Level



(b) Expected Damage Distribution



	Method I	Method II	Method III
Expected Annual Damage	$53,573	$36,983	$36,979

Figure 4.8 Comparison of the Optimal Results from Models Based on Three Different Methods of Estimating the Unrepeatable Damages:
I - Damage Independent of the Conditions in Previous Months;
II - Damage Calculated Only for the Additional Flooded Area in the Next Month;
III - Damage Calculated Only for the Additional Flooded Areas in the Next two Months.

results, it was adopted for the detailed analysis of the Lake Shelbyville system in the following sections.

4.5.1 Basic Results

Table 4.8 summarizes the expected performance of the Lake Shelbyville system following the optimal steady-state operation policy. As expected, the monthly mean lake levels lie between 589.0 and 602.0 ft, within which range the various upstream damages can be avoided. When the major agricultural and recreation activities remain dormant from December to April, the concern of incurring upstream damage due to the high lake levels is minimal. As a result, the lake starts accumulating the excessive winter inflows gradually in order to prevent downstream properties from being flooded; and the monthly mean lake levels increase steadily during this period. The rising trend of the mean lake level is interrupted in May when the resumed agricultural and recreation activities are more sensitive to both the high lake levels and the high release rates than in the preceding months. By lowering the lake level in this period more storage space can be reserved; and the risk of reaching either a high lake level or a high release rate may be reduced. Except for the sudden dip of the mean lake level in May, the spring floods in April-June keep the mean lake level rising until reaching the peak in July. Then a period of decline in the lake level starts due to small inflows to the lake in the remaining months. Thus, the variation of the monthly mean lake levels is affected mainly by the expected inflow volume and the relative economic values of the various activities conducted in each month.

The monthly lake level distributions are in general highly skewed toward the high values. Several statistical measures are listed in Table

Table 4.8 Expected System Performance of Lake Shelbyville Based upon the Optimal Results from the Stochastic Dynamic Programming Model with the Mean Summer Lake Level Unconstrained.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Month	Mean Lake Level* feet	Standard Deviation of Lake Level, feet			Prob. of Lake Level			Prob. of Release Greater Than	
		SD_L^{**}	SD_U^{**}	SD^{**}	Drop. below 590.0 ft	Ris. above 599.7 ft	Ris. above 610.0 ft	1,800 cfs	4,500 cfs
Jan	592.2	1.24	7.46	2.25	.0022	.0557	0	.0994	0
Feb	592.4	1.41	6.80	4.03	.0018	.0867	0	.0971	0
Mar	593.3	2.21	8.59	5.26	.0016	.1249	0	.1605	0
Apr	594.7	2.45	6.90	4.49	.0013	.1668	0	.2697	0
May	593.6	2.39	4.63	3.36	.0007	.0406	0	.0047	0
Jun	594.2	2.75	10.74	5.71	.0005	.1484	.0322	.0060	.0008
Jul	597.0	3.73	13.02	6.70	.0004	.1967	.0589	.0125	.0008
Aug	594.9	3.65	13.20	7.31	.0003	.1459	.0639	.0048	0
Sep	593.9	2.86	16.29	6.94	.0026	.1121	.0505	.0009	0
Oct	593.2	2.13	17.72	5.97	.0025	.0880	.0378	.0007	0
Nov	593.0	2.03	17.23	6.39	.0024	.0627	.0393	.0660	0
Dec	591.9	1.07	7.71	3.16	.0023	.0373	0	.0592	0

* Mean lake level at the beginning of the indicated month.

** SD_L: standard deviation of all possible lake levels lower than the monthly mean (Eq. 4.6).

SD_U: standard deviation of all possible lake levels higher than the monthly mean (Eq. 4.8).

SD: standard deviation of all possible lake levels at the beginning of the indicated month.

Table 4.8 (Continued)

Month	(10) Damage Distribution by Month, % Agri.	(11) Recr.	(12) Prop.	(13) Damage Distribution by Category, % Agri.	(14) Recr.	(15) Prop.	(16) Total	(17) Expected Monthly Damage, dollars
Jan	0	1.0	11.0	0	76.7	23.3	100.0	357
Feb	0	0.9	6.1	0	84.2	15.8	100.0	289
Mar	0	0.5	12.7	0	56.8	43.2	100.0	221
Apr	0	0.3	28.6	0	23.8	76.2	100.0	284
May	3.7	0.3	0.1	79.7	20.1	0.2	100.0	432
Jun	20.5	9.9	0.8	42.0	57.9	0.1	100.0	4,577
Jul	31.8	21.2	1.9	34.4	65.4	0.2	100.0	8,683
Aug	31.6	31.1	3.8	26.1	73.6	0.3	100.0	11,376
Sep	7.3	29.9	1.0	7.9	92.0	0.1	100.0	8,707
Oct	5.1	3.0	0.5	37.7	62.0	0.3	100.0	1,300
Nov	0	1.1	29.3	0	57.8	42.2	100.0	524
Dec	0	0.8	4.2	0	86.5	13.5	100.0	233
Total Damage, %	100.0	100.0	100.0					
Total Damage, $/year	9,395	26,833	755					36,983

4.8 to describe the variation of lake levels in each month. First, two statistics SD_L and SD_U are introduced to describe the average dispersion of the lake levels respectively below and above the monthly mean. Let H_n be the lake level in state n and P_n be the corresponding probability of occurrence. Then SD_L and SD_U can be calculated by

$$SD_L = \sqrt{\frac{\sum_{n=1}^{N'} P_n(H_n - \hat{H})^2}{\sum_{n=1}^{N'} P_n}} \tag{4.6}$$

$$SD_U = \sqrt{\frac{\sum_{n=N'+1}^{N} P_n(H_n - \hat{H})^2}{\sum_{n=N'+1}^{N} P_n}} \tag{4.7}$$

in which $\hat{H}$ is the mean; N is the total number of storage (or lake level) states; and N' and N'+1 are the states separated by the mean. The commonly defined standard deviation SD can then be related to SD_L and SD_U by

$$SD = \sqrt{SD_L^2 \sum_{n=1}^{N'} P_n + SD_U^2 \sum_{n=N'+1}^{N} P_n} \tag{4.8}$$

By comparing SD_l and SD_U it is noted that the uncertainty associated with the lake level in each month is mostly attributed to the high lake levels as a result of large floods. The probability of the lake dropping below 590.0 ft is significantly less than that of the lake rising above the current summer operation pool, 599.7 ft, in any month of the year.

The lake level could rise above 599.7 ft with probabilities ranging from 3.73% in December to 19.67% in July. Therefore, it would not be a rare event for the lake level to exceed the summer pool of 599.7 ft in the normal operation of the Lake Shelbyville system.

Based on the optimal operation policy, the lake most likely would not rise above 610.0 ft from December to May. There would be some chance, however, for the lake to exceed this threshold level in the crop-growing season when it would be desired to avoid the upstream agricultural damages due to the floodings in the area above 610.0 ft. This phenomenon can be explained by the economic tradeoff between the upstream and the downstream income losses during this period. Table 4.9(a, b) contains the marginal costs to the upstream and the downstream areas as a result of a unit change in either the storage volume or the release volume per month. These marginal costs were derived directly from the cost data contained in Tables 4.3-4.5.

Consider the case in the summer growing season when the lake level is at 610.0 ft, and the inflow rate is a bit greater than 1,800 cfs. If the operation criterion of the lake were based solely on the economic values of the various activities, then the inflow would be passed only at a maximum rate of 1,800 cfs, forcing the lake to rise beyond 610.0 ft, since comparing the marginal costs in both reaches would favor this operation with less overall damages. For example, the marginal damage to the upstream reach at a lake level of 610.0 ft in July is $2.83/ac-ft, which is less than the marginal damage, $3.29/ac-ft, to the downstream reach. By the same token, if similar situations occurred in the winter months the preference in operation would be reversed in contrast to the

Table 4.9 Economic Tradeoff between the Damage in the Upstream and the Downstream Reaches for the Lake Shelbyville System.

(a) Marginal Costs for the Upstream Reach

Lake Level, feet	Marginal Cost per Ac-ft Increase in Storage, $/ac-ft											
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
					(Summer Growing Season)							
602.5	0	0	0	.01	.11	.15	.17	.14	.08	.02	.07	.02
605.0	0	0	0	.05	.50	.70	.76	.65	.34	.07	.19	.07
607.5	0	0	0	.10	1.01	1.44	1.58	1.35	.71	.12	.33	.11
610.0	.30	.31	.33	.55	1.99	2.63	2.83	2.49	1.52	.53	.76	.46
612.5	1.13	1.14	1.27	1.73	3.78	4.58	4.82	4.41	3.12	1.64	1.74	1.33
615.0	2.10	2.12	2.32	2.96	5.28	6.12	6.37	5.95	4.54	2.84	2.84	2.34
617.5	2.95	2.98	3.20	3.89	6.24	7.06	7.29	6.89	5.49	3.78	3.78	3.22
620.0	3.76	3.79	4.06	4.80	7.22	8.03	8.25	7.86	6.45	4.70	4.68	4.06
622.5	4.54	4.58	4.88	5.67	8.11	8.90	9.11	8.74	7.34	5.58	5.53	4.87
625.0	6.53	6.58	6.98	7.98	11.02	11.98	12.23	11.77	10.07	7.89	7.81	6.95

(b) Marginal Costs for the Downstream Reach

Release Rate, cfs	Marginal Cost per Ac-ft/month Increase in Release, $/ac-ft	
	Winter (Nov.-Apr.)	Summer (May-Oct.)
1,800	.0088	3.29
2,400	.0147	5.51
3,600	.0479	17.86
4,200	.0865	32.25
4,600	.0628	23.43
5,200	.0663	24.75
5,800	.0696	25.98
6,400	.0727	27.12
7,000	.0455	16.98
8,000	.0474	17.69
9,000	.0491	18.32
10,000	.0506	18.87

Note: 1 cfs = 59.5 ac-ft/month, for 30 days in a month.

practice adopted in the summer since the marginal cost for damages in the downstream reach is so low (e.g., $0.0088/ac-ft at 1,800 cfs). The selection of 610.0 ft and 1,800 cfs as the example coincides with the thresholds of the lake level and of the release rate below which no agricultural and property damages would occur.

Based on the economic data used, the optimal operation policy determined by the stochastic programming model clearly leans towards adopting a more stringent rule for release control rather than trying to prevent the upstream damages due to high lake levels. This can be observed by comparing the probabilities in Columns (7) and (8) of Table 4.8. For each month in the summer growing season, the probability of a release greater than 1,800 cfs is much less than that of the lake level rising above 610.0 ft. Under either condition some agricultural damages would be incurred. The marginal cost information provided in Table 4.9 supports, to a certain degree, the current practice of release control at Lake Shelbyville, where in the summer the maximum release is increased beyond 1,800 cfs only for lake levels above 610.0 ft. However, the maximum summer release appears to be bounded by 3,600 cfs for lake levels below the top of the flood control pool, 626.5 ft. The small probabilities of releases greater than 4,500 cfs observed in June and July (Table 4.8, Column (9)), on the other hand, are attributed to the operation constraint prohibiting the lake from rising beyond 626.5 ft under any circumstances.

In the second half of Table 4.8, the expected damages are expressed both by the distributions in time for the agricultural loss, the recreation loss, and the property loss, and by the distributions among the three different losses in each month. The percentages of the various damages are comparatively more important than the corresponding expected values because the former reflect the significance of each of the distinct activities relative to one another. The expected damages may be misleading in that the damage incurred in most months is much less than indicated sincea a few catastrophic events account for most of the weights in calculating the expected values (as discussed in Sec. 3.4.2.).

Recreation damage accounts for 72.5% of the expected annual damage; whereas agricultural loss contributes only 25.4% of the expected damage (Table 4.8). These proportions result because the chance of the lake level being between 602.0 ft and 610.0 ft, within which only upstream recreation damages occur, is much greater than the probability of the lake rising above 610.0 ft for each month. However, agricultural damage may be underestimated by the stochastic programming model since both the inflows and the releases were assumed uniform within a month. Thus, the peaks of the inflow or the release rate in a month are not captured by the model nor properly reflected by the monthly mean values. Nevertheless, this simplification of the model should not alter the resulting optimal operation policy greatly because the damage from agriculture plays a predominant role in determining the tradeoffs between the storage and the release during the critical conditions in the summer. As long as these marginal costs are properly estimated, the general trend in controlling the lake level and the release is not greatly distorted.

Although the statistics associated with the monthly lake levels and releases were calculated based on the optimal operation policy under the ideal steady-state conditions, their values generally reflect the tendencies which could be expected in the normal control of the Lake Shelbyville system under the current operation rules. The monthly mean lake levels determined from the stochastic programming model are within the ranges proposed in the two previous studies (Singh et al. 1975; and Singh, 1977). If these means were treated as the monthly targets, then they would be roughly 2 to 5 feet lower than the current rule for the control of the lake level. The following section presents an additional analysis to examine the possible changes of the optimal operation policy if the summer pool is forced to meet the current target of 599.7 ft.

4.5.2 Penalty Function

The basic results discussed above were obtained from the stochastic programming model defined in Sec. 4.4; it uses the economic values of the various activities as the criteria for operating the Lake Shelbyville system. A requirement for meeting the summer pool level of 599.7 ft can be added to the model by modifying the objective function (the recursive equation) using the penalty function approach (Askew, 1974a, b; 1975). For each of the crop-growing months, a one-sided quadratic penalty function PF defined by

$$PF = W(H - 599.7)^2 \qquad H < 599.7 \text{ ft} \quad . \qquad (4.9)$$

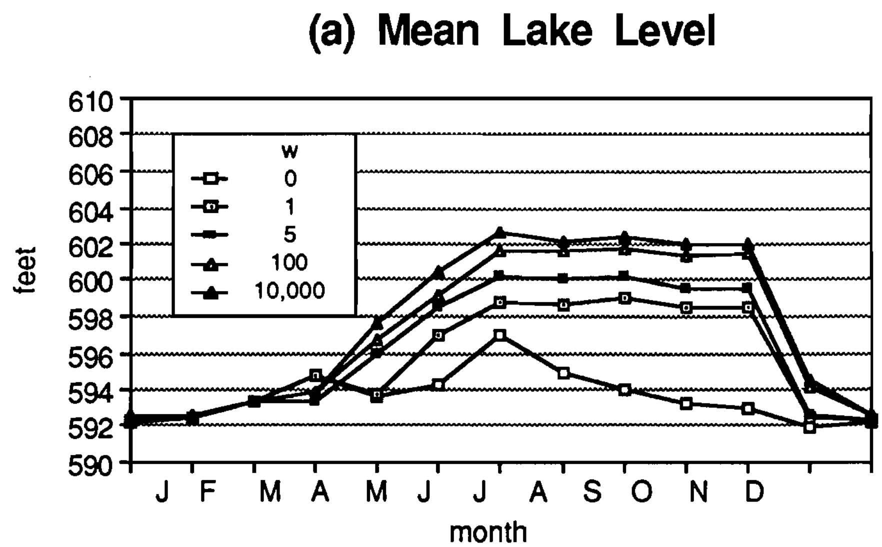
is included in the recursive equation, which penalizes any lake level H (as a function of storage) less than the desired level of 599.7 ft. W is the weighting parameter to be determined so that in the long run the expected lake levels in the summer months match the current target.

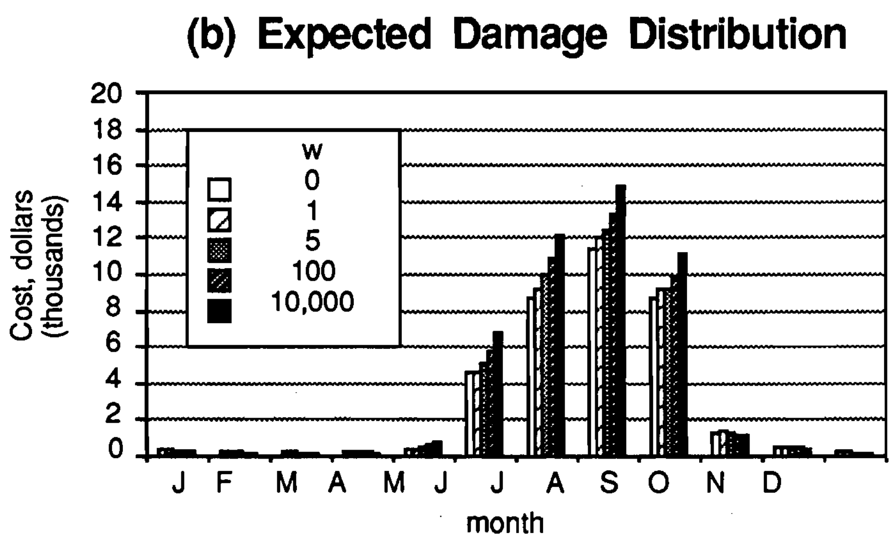
The weighting parameter was varied within a wide range of values between 0 and 10,000 to monitor the changes of the monthly mean lake levels. To begin with, each of the values of W = 0, 1, 100, and 10,000 was used in the modified stochastic programming model; and the corresponding mean lake level in the period from May to October was calculated. A semi-log plot of W against the mean lake level was created; and the weight corresponding to mean lake level of 599.7 ft can be obtained by using simple linear interpolation. It was shown that a weighting parameter of approximately 5 leads to the desired target of the mean summer pool in the long run.

Figure 4.9 compares the relative changes in the expected lake performance when different weighting parameters are used. It is interesting to note that even using a rather small value (W = 1) would greatly increase the mean lake levels in the summer months. The summer pool would keep rising with the increasing penalty on lake levels below 599.7 ft; and the optimal operation policy would remain unchanged for W > 10,000. In comparison the pool levels in the January-March period are almost unaffected by the pronounced changes in the summer pools when different penalty values are used. Moreover, despite the consistent increase of the expected damages in the summer months, these damage values do not change as greatly as the monthly lake levels if W is changed from 0 to 5. Thus, the Lake Shelbyville system may be considered rather flexible in terms of the selection of the operation rule curve. More discussions are provided in the following section specifically for the case of W = 5.

4.5.3 Further Analysis of Expected Lake Performance

Table 4.10 contains information about the expected system perform-





W =	0	1	5	100	10,000
Expected Annual Damage	$36,983	$38,837	$40,315	$43,200	$48,049

Figure 4.9 Comparison of the Expected Performances of the Lake Shelbyville System Using the Penalty Function Approach; W is the Weighting Parameter Defined in Eq. 4.9.

Table 4.10 Expected System Performance of Lake Shelbyville Based upon the Optimal Results from the Stochastic Dynamic Programming Model with the Mean Summer Lake Level Constrained to Be 599.7 ft.

Month	(1) Mean Lake Level* feet	(2) Standard Deviation of Lake Level, feet: SD_L**	(3) SD_U**	(4) SD**	(5) Prob. of Lake Level: Drop. below 590.0 ft	(6) Ris. above 599.7 ft	(7) Ris. above 610.0 ft	(8) Prob. of Release Greater Than: 1,800 cfs	(9) 4,500 cfs
Jan	592.4	1.34	7.15	3.64	.0019	.0668	0	.1046	0
Feb	592.5	1.40	6.72	4.06	.0016	.0905	0	.0974	0
Mar	593.5	2.20	8.56	5.59	.0014	.1268	0	.1315	0
Apr	593.9	2.52	7.64	4.43	.0011	.1677	0	.2545	0
May	595.9	4.59	1.76	2.59	.0006	.0406	0	.0061	0
Jun	598.6	4.16	4.91	5.02	.0004	.1703	.0358	.0061	.0008
Jul	600.2	3.28	10.07	5.45	.0003	.2496	.0691	.0139	.0009
Aug	600.1	2.95	12.89	5.80	.0003	.1810	.0745	.0057	0
Sep	600.2	2.62	11.07	4.92	.0025	.2125	.0569	.0001	0
Oct	599.6	2.04	13.57	4.86	.0024	.1106	.0418	.0008	0
Nov	599.5	1.96	16.67	5.26	.0023	.0870	.0472	.0923	0
Dec	592.6	1.58	6.56	2.71	.0021	.0700	0	.0744	0

* Mean lake level at the beginning of the indicated month.

** SD_L: standard deviation of all possible lake levels lower than the monthly mean (Eq. 4.6).

SD_U: standard deviation of all possible lake levels higher than the monthly mean (Eq. 4.7).

SD: standard deviation of all possible lake levels at the beginning of the indicated month.

Table 4.10 (Continued)

Month	(10) Damage Distribution by Month, %	(11)	(12)	(13) Damage Distribution by Category, %	(14)	(15)	(16)	(17) Expected Monthly Damage, dollars
	Agri.	Recr.	Prop.	Agri.	Recr.	Prop.	Total	
Jan	0	0.7	10.8	0	69.6	30.4	100.0	287
Feb	0	0.6	5.8	0	79.4	20.6	100.0	226
Mar	0	0.3	12.0	0	48.8	51.2	100.0	190
Apr	0	0.2	24.2	0	20.6	79.4	100.0	247
May	4.1	0.3	0.1	84.3	15.5	0.2	100.0	527
Jun	19.9	10.4	0.9	42.2	57.7	0.1	100.0	5,140
Jul	31.7	22.9	2.1	34.4	65.4	0.2	100.0	10,028
Aug	31.8	31.3	4.2	27.7	72.0	0.3	100.0	12,480
Sep	7.4	29.5	1.1	8.7	91.2	0.1	100.0	9,272
Oct	5.0	2.4	0.5	44.1	55.6	0.3	100.0	1,236
Nov	0	0.8	33.4	0	45.1	54.9	100.0	492
Dec	0	0.5	5.0	0	78.5	21.5	100.0	189
Total Damage, %	100.0	100.0	100.0					
Total Damage, $/year	10,882	28,623	810					40,315

ance similar to that listed in Table 4.8, except that the mean summer lake level is forced to increase to 599.7 ft. Although the penalty is included in the recursive equation for each of the crop-growing months, the lake generally cannot reach the target level until July. This is because the winter drawdown of the lake level is significantly lower than the summer pool in the steady-state condition. As a result, when it is desired to raise the lake to 599.7 ft beginning on May 1, there is probably not enough inflow to raise the lake level.

Although the average monthly inflow in May ideally should be enough to raise the lake from the winter pool of 592.4 ft to a summer pool level around 599.7 ft, there could be certain factors which might cause a delay in raising the lake level. First, the monthly inflows at Shelbyville are generally positively skewed. Since the majority of inflows are likely to be less than the monthly mean, there is much less than a 50% chance that the difference between the winter and the summer pool level will be completely eliminated by the inflow in May. Second, a minimum release from the lake is generally required, and some of the inflows may be passed through the dam under occasional flood conditions. Finally, evaporation may also partially offset the net monthly inflow amount. In Fig. 4.9 (a), it can be observed that the patterns of the rising rates of the mean lake level from May to July are roughly the same for the various nonzero penalties used. This pattern implies that on the average the lake level could only be increased by roughly 3 feet from May to June, and by 2 feet from June to July, under normal inflow and operation conditions.

In comparison to the monthly mean lake levels derived from the stochastic programming model, the operation rule curves that have been used for the Lake Shelbyville system (Fig. 4.3) are too simplistic to reflect the system's response to the natural inflows. A ten-foot winter drawdown mandated in the early operation plans could force more than a two month period for raising the lake level back to the summer pool. This effect was not initially observed because of the unprecedented floods which occurred in 1973-1974. The effect later become apparent in 1975-1976 when the inflow amounts were less than normal. Thus, rather than adopting the commonly used dichotomous scheme to divide the summer and the winter target operation pool levels, it would be more appropriate to define a third (or fourth) period in a year for the transition between the winter and the summer pools. For example, a two-month linear transition between the pools from April 1 to June 1 may be recommended for Lake Shelbyville to control the lake level better in the transition season.

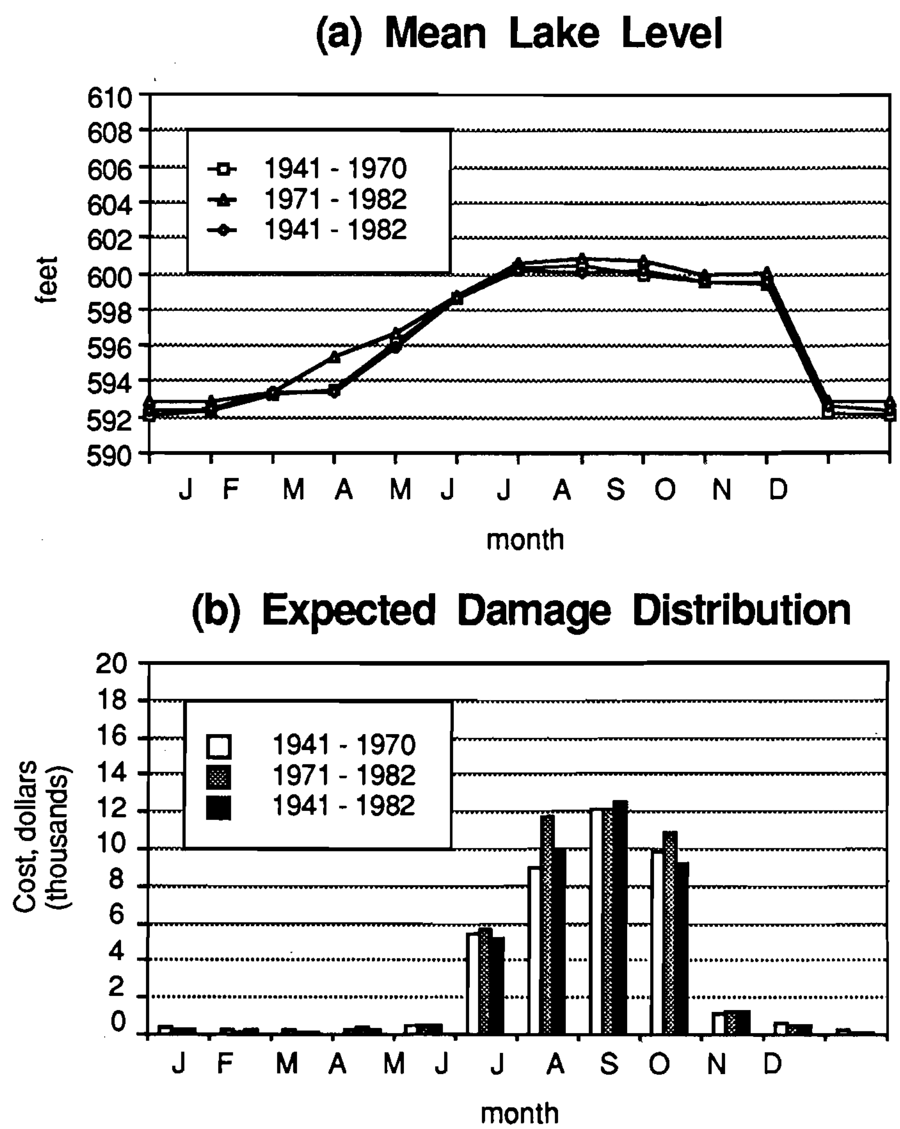
The lake level can be sharply reduced in November-December since the normal inflows in this period are small, and the maximum allowable release can be significantly increased after the harvest. If because of unmodeled issues (e.g., rapid change in lake level causes significant bank erosion within and below the lake), the winter drawdown should not be executed at a rate faster than that reflected by the model, a transition period could also be added to smooth the change between the summer and the winter pools.

The overall performance of the Lake Shelbyville system does not exhibit much change whether the penalty function is used or not. For the modified model with the penalty function, the elevated mean lake levels are accompanied by an increase in the probabilities of reaching high lake levels as can be seen in the Columns (6) and (7) in Tables 4.8 and 4.10.

It is interesting that by raising the summer pools roughly 5 ft, the probabilities of the summer lake levels rising above 599.7 ft would be increased only from an average of 12.4% to 16.8%. The expected annual damage would be increased by only 9% as a result of the higher summer pools, and the increased damages would be fairly evenly distributed by percentage through time and among the three categories.

Finally, the modeling results are found to be insensitive to the hydrologic data used. Figure 4.10 compares the expected system performance (W = 5) based on the historical inflow records for the periods of 1941-1982, 1941-1970, and 1971-1982, respectively. Although the 1971-1982 data resulted, as expected, in higher mean lake levels and expected damages, the general patterns of the monthly statistics do not vary from one another significantly. Moreover, the deviations among the performances based on different hydrologic data sets are not as great as those caused by the mandatory increase in the summer to a 599.7 ft level from the optimal target levels which would be 2 to 5 feet lower. Thus, the wet spell experienced in the Upper Kaskaskia Basin in the 1971-1982 period should not greatly affect the long-term performance of the Lake Shelbyville system.

The above observations indicate that the performance of Lake Shelbyville would in general not be seriously impaired by keeping the summer pool at 599.7 as compared to the theoretically more economical plan with the mean lake level in summer being 5 ft lower. The summer pool level of 599.7 ft has been mandated since the completion of the Shelbyville Dam; and the facilities in the lake area have been built for the past 16 years conforming to this expected summer pool. Therefore, although keeping the



Year	1941 - 1970	1971 - 1982	1941 -1982
Expected Annual Damage	$39,878	$43,725	$40,315

Figure 4.10 Comparison of the Expected Performances of the Lake Shelbyville System Using the Three Historical Data Sets of Monthly Inflows: (a) 1941-1982, (b) 1941-1970, (c) 1971-1982.

current summer pool would be considered sub-optimal based on the economic analysis using the stochastic programming model, retaining this level in the future operations of the system should be an acceptable practice unless the major functions of Lake Shelbyville are drastically changed. On the other hand, the transition from the winter pool to the summer pool has caused some problems in the past operation history of Lake Shelbyville. As a result, the lake rule curve has been modified frequently. It seems that the lake cannot recover to the summer pool level from the winter drawdown in a very short transition period. A period of two months for transition between the seasons is likely to improve the control of the lake level either in providing the necessary storage space for withholding the winter floods or in attaining the summer pool level in time for the beginning of the summer recreation season.

4.6 Summary

Lake Shelbyville has controlled the surface runoffs from the Upper Kaskaskia River Basin, Illinois, since August 1970 when the Shelbyville Dam was completed. It serves as a multi-function reservoir for flood control, recreation, fish and wildlife conservation, navigation, and water supply purposes, though the latter two functions have never been fully utilized. While the design capacity of the Lake is 1.7 times the mean annual inflow above Shelbyville, which is considered a rather large storage ratio, the operation rules for Lake Shelbyville have been modified a few times in the past 16 years. Those modifications were made partially because both the regulating agency and the local residents typically needed some time to adapt to the physical and socio-economical changes brought about by the newly-built dam. The unusual weather conditions experienced in the 1973-1974 period added to the complexity of defining a robust operation policy in the control of the summer and the winter pools. This led to five modifications of the operation rules in the six-year period from 1970 to 1976.

Two comprehensive studies conducted jointly by the Illinois State Water Survey and the State Division of Water Resources (Singh et al. 1975; and Singh, 1977) provided detailed analyses of the optimal joint operation of the Lake Shelbyville and Carlyle Lake system in the Kaskaskia River Basin. It was shown (Singh et al., 1975, Table 20) that the agricultural damages that occurred in 1973-1974 from the Shelbyville-Carlyle reach were reduced by 62%, as compared to the damages which would have occurred had the Shelbyville Dam not existed. It was also observed that the functions of Lake Shelbyville could be much improved if the summer pool levels were lowered by 2.7 to 6.2 ft (Singh, 1977, Table 39) depending upon the levels of demands from the water supply and navigation uses. The findings in these two reports, nevertheless, did not affect the later decision made by the regulating agency to retain the summer pool at 599.7 ft.

Because the various hydrologic conditions were implicitly included in the model presented here, the expected performance of the Lake could be perceived rather easily. However, careful preparation of the hydrologic and the economic data, and the incorporation of this information into the stochastic dynamic programming framework, are critical to the validity of the optimal results obtained. Although the inflow data in the 1971-1982 period seemingly implied a drastic change of the hydrologic characteristics after the dam was built, the results from the statistical tests showed that the change could be attributed more to the natural randomness of the hydrologic process than to a basic change of land use in the Upper Kaskaskia River Basin. More important is the fact that for the Lake Shelbyville system the optimal operation policy is little affected by the hydrologic data used.

The computations of the agricultural, recreation, and property damages reflect the real operation conditions with reasonable accuracy. Recreation activities generally take place on a daily basis. Therefore, the possible damages to recreation as estimated by the losses of visitors per day should vary in time if the lake level changes within a month. On the other hand, the damage to either agriculture or property is assumed to depend upon the greatest area flooded in the crop-growing season. Conventional stochastic programming models do not provide a direct link between the current operation decision and that in the future disjoint operation periods. An *ad hoc* approach was proposed in Sec. 4.4.3 to resolve this problem by adjusting these damages according to the system's condition in the previous month. Numerical evidence showed that optimal results would quickly converge with adjustments in only one additional month in the backward optimization procedure. Without this modification, not only would the expected damages be overestimated, but also the distributions of the lake levels would be distorted.

The expected lake performance as a result of the optimal operation policy resembles that already found in the previous studies by Singh et al. (1975) and Singh (1977). Based on the economic data used for estimating the various damages, the optimal operation policy determined by the model would lean to adopting a more stringent rule in the release control rather than trying to prevent upstream damages. By comparing the marginal cost data (Table 4.9), the maximum allowable release in the summer would be limited to 3,600 cfs for better long-term system performance.

A penalty function approach was used to monitor the changes in lake performance if the summer pool level of 599.7 ft is desired. The imposition of the penalty function would cause a 9% increase in the expected annual damage, and would raise the summer mean lake levels by roughly 5 ft. With a target summer pool of 599.7 ft, the probabilities of the lake exceeding this level in June-September would be greater than 20% on the average. Moreover, it would not be a rare event for the lake to rise above 610.0 ft at which upstream agricultural and property damages would start to occur, adding to recreation damages that would already have occurred. Since these events would be highly related and would occur in clusters in successive months, the actual chance of the lake exceeding 599.7 ft or 610.0 ft in the whole summer season would not be much greater than the chance of that occurring in a single month.

The most important observation from the above analysis is that it would generally take more than one month for Lake Shelbyville to resume the summer pool from the winter drawdown. The historical operation record (Corps of Engineers, 1983a) also supports this assertion. Therefore, a transition period between the winter drawdown and the summer recovery in the lake level is recommended for consideration of future modifications of the rule curve. An extended transition period between the two seasons should represent the actual response of the lake to the inflows better than the rigid transition currently adopted.

Although the stochastic programming model is only a screening model, the information provided would be generally valuable. The biases and uncertainties embedded in the modeling results could be reduced by carefully selecting the modeling parameters. A better understanding of the system operating conditions and the corresponding constraints could also help in constructing such a model with appropriate details. For the Lake Shelbyville system, the stochastic dynamic programming model as proposed in the above sections proved to be a flexible and efficient tool for analyzing the system performance.

V. SUMMARY AND RECOMMENDATIONS

5.1 Summary of Studies

This study has examined the common issues encountered when using stochastic programming (SP) in both the model formulation and the practical application phases. A hypothetical single reservoir system was used first to study typical SP modeling issues which may be obscurely defined or difficult to resolve in the complex modeling process of a real reservoir system. Sensitivity analyses were performed to monitor the changes of performance of the hypothetical system as a result of varying modeling parameters. The parameters or elements considered in these analyses include the partitions of inflow and storage states, the hydrologic characteristics of inflows, the types of system performance functions, and the tradeoffs between conflicting objective functions. Simulation studies were conducted to verify the modeling results and to provide insights for possible improvements of performance of the hypothetical system.

An SP model for the Lake Shelbyville system in Illinois was then developed as a case study to help identify the major factors which may affect the operation strategies of Lake Shelbyville, specifically the operation rule curve. The computation of agricultural, recreation, and property damages was considered; and an *ad hoc* approach (generally applicable in studying the operations of other single reservoir systems, however) was used to estimate accurately the agricultural and property damages in the optimization procedure. The performance of Lake Shelbyville was evaluated considering the losses of agricultural revenues and recreation benefits, property damages, and the changes in pool levels and outflow releases.

The following two sections summarize and discuss the findings of using SP for the hypothetical reservoir system as well as for the Lake Shelbyville study. Recommendations are provided for consideration if future modifications of the operation rule curve are made for Lake Shelbyville.

5.2 Discussion

5.2.1 Typical SP Modeling Issues

A typical SP model for reservoir study generally involves the discretization of continuous storage and inflow variables, the estimation of a Markov transition probability matrix, and the selection of adequate measures for evaluating the system performance. It has been shown that not only the numbers but also the discrete increment values of the inflow and storage states affect the modeling results. The partitions of inflow and storage states with $NQ \geq 10$ and $NS \geq 12$ would lead to stable optimal results for the hypothetical reservoir system, regardless of the relative storage capacity to mean annual inflow ratios ($S_c/\bar{Q}_a = 0.5, 1, 2$) used. The number of storage states could be further reduced without much affecting the expected system performance for small $S_c/\bar{Q}_a$ ratios ($S_c/\bar{Q}_a \leq 1$). The partitions of dry season inflows would have minimal impact on the optimal decisions provided that the overall system performance is mainly affected by the wet season inflows.

Extremely large releases are likely to occur as isolated events in the real-time operation of a reservoir system. These extreme events are relatively insensitive to the increments of the state variables as compared to the smaller releases. Nevertheless, these rarely occurring extreme events can have a major impact on the estimation of expected system performance. The small-scale flooding caused by the discrete release policy are largely attributed to the distortions in discretizing the continuous storage and inflow variables. According to the results of simulation studies, these occurrences may be eliminated by allowing a proper tolerance value for the discrete releases, and the overall system performance can be improved when compared to that estimated directly from the SP model.

The uncertainty associated with estimation of the coefficient of variation ν and the correlation coefficient ρ of inflows generally affects the accuracy of the expected system performance. The influence from serial correlations of the inflows depends largely upon the storage capacity of a reservoir system. For smaller reservoirs (e.g., $S_c/\bar{Q}_a \leq 0.5$), the uncertainty associated with ρ is practically immaterial to the expected system performance estimated from an SP model. Nevertheless, for all combinations of storage capacities and performance functions evaluated, the uncertainty of the coefficients of variation of inflows has a consistent and greater impact on the system performance than the influence of the serial correlations.

In the sample study with flood control being the only objective, convex and concave performance functions could lead to different optimal release decisions depending upon the initial storage, the current inflow, and the future hydrologic conditions. Under the convex performance function assumption, it is desirable to reduce the magnitude of an extreme flood event through prior regulation, even at a cost in the current period. On the other hand, it would not be justified to release more water than necessary when the extreme event occurs. The opposite argument can be applied to the concave function, in which case the small floods are always avoided; and the large floods may not be reduced as much even when some storage space is still available in the reservoir. Therefore, neither the convex function nor the concave function alone in an SP framework can simultaneously maximize short-term and long-term benefits, for the entire spectrum of reservoir operation conditions.

The tradeoffs between the conflicting objectives in reservoir operation can be examined by the changes in certain system characteristics, such as the expected reservoir performance or the steady-state distribution of the storage, as a result of the shifting emphases on the various objectives. Although such indices related to the lumped system performance might demonstrate a clear and consistent pattern of tradeoffs in objective space, they usually failed to provide detailed information about the changes in decision space due to the varying system preferences. In the flood control vs. hydropower generation example, it has been observed that both the relative preferences between the two objectives and the current inflow and storage conditions would affect the optimal operation decisions. An individual release decision is mainly determined by comparing the long-term marginal gain for one objective to the long-term marginal loss for the other objectives as a result of taking a certain control action in the reservoir storage and release. The consequences of these long-term marginal gains and losses to the expected system performance, however, may not be easily inferred before an SP model is solved.

Finally, simulation studies are shown to be useful and crucial in complementing the SP modeling results. Simulation would be necessary to help identify the implicit distortions involved in SP modeling, which might not be very evident by looking directly at the optimal results. Despite the many uncertainties embedded in SP models, this modeling technique provides a comprehensive assessment of the example reservoir system's response to the varying hydrologic inputs in steady-state operation conditions. It is critical for a modeler to perceive the sources of potential errors, and to filter out the useful information contained in the optimal solution to apply an SP model effectively for reservoir study. Frequently, modifications are needed to make the strict SP modeling results closer to reality.

5.2.2 Lake Shelbyville Operation

The purposes of using SP for the case study of Lake Shelbyville are twofold. First, it was intended to demonstrate that with carefully selected modeling parameters and an understanding of the system's operating history, the SP model can be a flexible and effective tool for analyzing a real-world reservoir system like Lake Shelbyville. Second, and more importantly, it was used to provide insights about the response of the Lake Shelbyville system to the natural inflows in steady-state operation condition. By studying the long-term average system performance using an SP model (e.g., the monthly pool levels), it was possible to make suggestions for future adjustment of the rule curve.

The findings from the SP modeling results, which agree fairly well with the two earlier studies reported by Singh et al. (1975) and Singh (1977), suggest that the functions of Lake Shelbyville could be improved if the summer pool levels were lowered by roughly 2 to 5 feet. The current summer pool level of 599.7 ft has been mandated since the completion of the Shelbyville Dam in 1970. When the summer pool was forced to be 599.7 ft in the SP model by adopting a penalty function approach, the annual expected damage increased by 9%. Whether or not this increase in potential damage is significant to the regulating agency and the local interest groups may depend upon other factors that cannot be expressed quantitatively in an SP model. These considerations may include the difficulty of altering the existing recreation facilities, the degradation of scenic values due to the exposure of additional bare banks around the lake, and the importance of "maintaining stability and consistency in a project regulation schedule over an extended period of time to those local people who must live with the project" (Corps of Engineers, 1983).

With a target summer pool of 599.7 ft, the probability of the lake level exceeding this value in June-September period is greater than 20% on average. In addition, it would not be a rare event for the lake to rise above 610.0 ft in the summer (with more than 5% chance of occurrence); at that level upstream agricultural and property damages start to occur. Therefore, even with the considerably large storage capacity of Lake Shelbyville, it is not possible to control the lake level always within the non-damaging range.

The economic tradeoff between the damages incurred in the upstream and the downstream reaches may be examined by comparing the marginal costs for both reaches for a unit change in the storage or release volume. A release rate greater than 3,600 cfs in the summer crop-growing months would cause more damages in the downstream reach than would be compensated by the damage reduction in the upstream reach. For Lake Shelbyville, the maximum allowable release in the summer could therefore be limited to 3,600 cfs for better long-term system performance.

The most important observation from the case study was that it would take more than one month for Lake Shelbyville to reach the summer pool from the winter drawdown. It is believed that some of the damages and inconveniences in operating Lake Shelbyville for the past 16 years might be caused by the rigid transition of the rule curve between the summer and winter pools. This type of rule curve has been followed since the completion of the dam. An extended transition period between the two seasons should reflect more closely the actual response of the lake to the inflows than the current regulation. Therefore, a transition period between the winter drawdown and the summer recovery of the lake level is recommended for consideration if future modification is made in the rule curve.

In developing the SP model for Lake Shelbyville, the computations of different losses were carefully examined to validate the use of SP and to reduce the potential biases that may be induced in the optimization procedure. The incorporation of the penalty function approach also extended the capabilities of typical SP models so that a certain desired performance could be achieved from the reservoir system. Although the SP model is only a screening model, the information provided is generally valuable. With carefully selected modeling parameters and an understanding of the operating history, the SP model proved to be a flexible and effective tool for analyzing the Lake Shelbyville system.

5.3 Proposed Future Research

The studies reported in this research were intended to accomplish two distinct research goals which are (a) to explore the issues commonly encountered when an SP model is used for a reservoir study, and (b) to use SP effectively in a case study of a real-world system. The first part of this research, though quite comprehensive in terms of the major modeling issues of SP, has been limited to a hypothetical single reservoir system. It would be desirable to extend the findings based on a single reservoir system using SP to a multiple reservoir system.

In the analysis of conflicting objectives for the hypothetical system, it has been observed that, if inferior solutions could be identified using information about the nature of the reservoir problem, the backward optimization efforts required in stochastic dynamic programming might be reduced significantly. One of the most common objections to using SP models for reservoir study is the large size of the models. It would be interesting to evaluate the possible ways, both mathematically and intuitively, for simplifying the computation burden associated with the SP models.

The case study of Lake Shelbyville in this study can be treated as complementary to the previous two studies for the same system. Because of the budget and time constraints, the same analysis was not performed for Carlyle Lake; and the joint operation of both lakes was not studied. It is believed that similar conclusions could be made about the operation of Carlyle Lake. Finally, it would be interesting to study the rule curves for many of the reservoirs in the Midwestern area to examine the merit of using a more flexible definition for the transitions between different operating seasons.

REFERENCES

Askew, A. J., W. W-G. Yeh, and W. A. Hall, "A Comparative Study of Critical Drought Simulation," Water Resources Research, Vol. 7, No. 1, pp. 52-62, February 1971.

Askew, A. J., "Optimum Reservoir Operating Policies and the Imposition of a Reliability Constraint," Water Resources Research, Vol. 10, No. 1, pp. 51-56, February 1974a.

Askew, A. J., "Chance-Constrainted Dynamic Programming and the Optimization of Water Resource Systems," Water Resources Research, Vol. 10, No. 6, pp. 1099-1106, December 1974b.

Askew, A. J., "Use of Risk Premiums in Chance-Constrained Dynamic Programming," Water Resources Research, Vol. 11, No. 6, pp. 862-866, December 1975.

Beard, L. R., "Status of Water Resource Systems Analysis," Journal of the Hydraulics Division, ASCE, Vol. 99, No. HY4, pp. 559-565, April 1973.

Becker, L., and W. W-G. Yeh, "Optimization of Real Time Operation of a Multiple-Reservoir System," Water Resources Research, Vol. 10, No. 6, pp. 1107-1112, December 1974.

Bras, R. L., R. Buchanan, and K. C. Curry, "Real Time Adaptive Closed Loop Control of Reservoirs with the High Aswan Dam as a Case Study," Water Resources Research, Vol. 19, No. 1, pp. 33-52, February 1983.

Buchanan, R. B., and R. L. Bras, "Study of a Real-Time Adaptive Closed Loop Control Algorithm for Reservoir Operation," Technical Report 265, Department of Civil Engineering, Ralph M. Parsons Lab., MIT, Cambridge, Mass., July 1981.

Can, E. K., and M. H. Houck, "Problems with Modeling Real-Time Reservoir Operations," Journal of Water Resources Planning and Management, ASCE, Vol. 111, No. 4, pp. 367-381, October 1985.

Changnon, S. A., Jr., "Research Agenda for Floods to Solve Policy Failure," Journal of Water Resources Planning and Management, ASCE, Vol. 111, No. 1, pp. 54-64, January 1985.

Chow, V. T., D. P. Maidment, and G. W. Tauxe, "Computer Time and Memory Requirement for DP and DDDP in Water Resources Systems Analysis," Water Resources Research, Vol. 11, No. 5, pp. 621-628, October 1975.

Cochran, A. L., Flood Hydrograph-Damage Integration Method of Estimating Flood Damage in Agricutural Areas, Dept. of Army, Office of the Chief Engineers, Washington D.C., 1960.

Cohon, J. L., and D. H. Marks, "A Review and Evaluation of Multiobjective Programming Techniques," Water Resources Research, Vol. 11, No. 2, pp. 208-220, April 1975.

Corps of Engineers, Shelbyville Reservoir, Design Memorendum No. 1: Hydrology and Hydraulic Analysis, U.S. Army Corps of Engineers, St Louis District, 1962.

Corps of Engineers, Shelbyville and Carlyle Reservoirs Design Memorandum: Low Flow Regulation (Joint Operation), U.S. Army Corps of Engineers, St Louis District, 1964.

Corps of Engineers, Master Reservoir Regulation Manual -- Kaskaskia River Basin, Illinois, U.S. Army Corps of Engineers, St Louis District, 1977.

Corps of Engineers, Lake Shelbyville, Appendix A to Master Reservoir Regulation Manual -- Kaskaskia River Basin, Illinois, U.S. Army Corps of Engineers, St Louis District, 1983a.

Corps of Engineers, Carlyle Lake, Appendix B to Master Reservoir Regulation Manual -- Kaskaskia River Basin, Illinois, U.S. Army Corps of Engineers, St Louis District, 1983b.

Corps of Engineers, Background and Position Paper, Kaskaskia River Basin, U.S. Army Corps of Engineers, St Louis District, 13, October 1983c.

Croley, T. E., II, "Sequential Deterministic Optimization in Reservoir Operation," Journal of the Hydraulics Division, ASCE, Vol. 100, No. HY3, pp. 443-459, March 1974.

Datta B., and S. J. Burges, "Short-Term, Single, Multipurpose Reservoir Operation: Importance of Loss Functions and Forecast Errors," Water Resources Research, Vol. 20, No. 9, pp. 1167-1176, September 1984.

Doran, D. G., "An Efficient Transition Definition for Discrete State Reservoir Analysis: The Divided Interval Technique," Water Resources Research, Vol. 11, No. 6, pp. 867-873, December 1975.

Eastman, J., and C. ReVelle, "Linear Decision Rule in Reservoir Management and Design, 3. Direct Capacity Determination and Intraseason Constraints," Water Resources Research, Vol. 9, No. 1, pp. 29-42, February 1973.

Ford, L. R., and D. R. Fulkerson, Flows in Networks, Princeton University Press, New Jersey, 1962.

Friedman, R., C. Ansell, S. Diamond, and Y. Y. Haimes, "The Use of Models for Water Resources Management, Planning, and Policy," Water Resources Research, Vol. 20, No. 7, pp. 793-802, July 1984.

Gablinger, M., and D. P. Loucks, "Markov Models for Flow Regulation," Journal of the Hydraulics Division, ASCE, Vol. 96, No. HY1, pp. 165-181, January 1970.

Gal, S., "Optimal Management of Stochastic Multireservoir Water Supply System," Water Resources Research, Vol. 15, No. 4, pp. 737-749, August 1979.

Gilbert, K. C., and R. M. Shane, "TVA Hydro Scheduling Model: Theoretical Aspects," Journal of the Water Resources Planning and Management Division, ASCE, Vol. 108, No. WR1, pp. 21-36, March 1982.

Giles, J. E., and W. O. Wunderlich, "Weekly Multipurpose Planning Model for TVA Reservoir System," Journal of the Water Resources Planning and Management Division, ASCE, Vol. 107, No. WR2, pp. 495-511, October 1981.

Gundelach, J., and C. ReVelle, "Linear Decision Rule in Reservoir Management and Design, 5. A General Algorithm," Water Resources Research, Vol. 11, No. 2, pp. 204-207, April 1975.

Haan, C. T., Statistical Methods in Hydrology, The Iowa State University Press, Ames, Iowa, 1977.

Hall, W. A., "Optimal Design of a Multiple-Purpose Reservoir," Journal of the Hydraulics Division, ASCE, Volume 90, Number HY4, pp. 141-149, April 1964.

Hall, W. A., W. S. Butcher, and A. Esogbue, "Optimization of the Operation of a Multiple-Purpose Reservoir by Dynamic Programming," Water Resources Research, Vol. 4, No. 3, pp. 471-477, June 1968.

Hall, W. A., A. J. Askew, and W. W-G. Yeh, "Use of the Critical Period in Reservoir Analysis," Water Resources Research, Vol. 5, No. 6, pp. 1205-1215, December 1969.

Harboe, R. C., F. Mobasheri, and W. W-G. Yeh, "Optimal Policy for Reservoir Operation," Journal of the Hydraulics Division, ASCE, Vol. 96, No. HY11, pp. 2297-2308, November 1970.

Hashimoto, T., J. R. Stedinger, and D. P. Loucks, "Reliability, Resiliency and Vulnerability Criteria for Water Resources System Performance Evaluation," Water Resources Research, Vol. 18, No. 1, pp. 1-20, February 1982a.

Hashimoto, T., D. P. Loucks, and J. R. Stedinger, "Robustness of Water Resources Systems," Water Resources Research, Vol. 18, No. 1, pp. 21-26, February 1982a.

Heidari, M., V. T. Chow, P. V. Kokotovic, and D. D. Meredith, "Discrete Differential Dynamic Programming Approach to Water Resources Systems Optimization," Water Resources Research, Vol. 7, No. 2, pp. 273-282, April 1971.

Helweg, O. J., R. W. Hinks, and D. T. Ford, "Reservoir System Optimization", Journal of the Water Resources Planning and Management Division, ASCE, Vol. 108, No. WR2, pp. 169-179, June 1982.

Houck, M. H., and J. L. Cohon, "Sequential Explicitly Stochastic Linear Programming Models: A Proposed Method for Design and Management of Multipurpose Reservoir Systems," Water Resources Research, Vol. 14, No. 2, pp. 161-169, April 1978.

Houck, M. H., J. L. Cohon, and C. ReVelle, "Linear Decision Rule in Reservoir Design and Management, 6. Incorporation of Economic Efficiency Benefits and Hydroelectric Energy Generation," Water Resources Research, Vol. 16, No. 1, pp. 196-200, February 1980.

Houck, M. H., and B. Datta, "Performance Evaluation of a Stochastic Optimization Model for Reservoir Design and Management with Explicit Reliability Criteria," Water Resources Research, Vol. 17, No. 4, pp. 827-832, August 1981.

Joeres, E. F., G. J. Seus, and H. M. Engelmann, "The Linear Decision Rule (LDR) Reservoir Problem with Correlated Inflows, 1. Model Development," Water Resources Research, Vol. 17, No. 1, pp. 18-24, February 1981.

Klemes, V., "Discrete Representation of Storage for Stochastic Reservoir Optimization," Water Resources Research, Vol. 13, No. 1, pp. 149-158, February 1977.

Klemes, V., "Storage Mass-Curve Analysis in a Systems-Analytic Procedure," Water Resources Research, Vol. 15, No. 2, pp. 359-370, April 1979.

Liebman, J. C., and W. R. Lynn, "The Optimal Allocation of Stream Dissolved Oxygen," Water Resources Research, Vol. 2, No. 3, pp. 581-591, June 1966.

Little, J. D. C., "The Use of Storage Water in a Hydroelectric System," Journal of the Operations Research Society of America, Vol. 3, No. 2, pp.187-197, May 1955.

Loucks, D. P., "Some Comments on Linear Decision Rules and Chance Constraints," Water Resources Research, Vol. 6, No. 2, pp. 668-671, April 1970.

Loucks, D. P., and P. J. Dorfman, "An Evaluation of Some Linear Decision Rules in Chance-Constrained Models for Reservoir Planning and Operation," Water Resources Research, Vol. 11, No. 6, pp. 777-782, December 1975.

Loucks, D. P., J. R. Stedinger, and D. A. Haith, Water Resource Systems Planning and Analysis, Prentice-Hall, Inc., New Jersey, 1981.

Luthra, S. S., and S. R. Arora, "Optimal Design of Single Reservoir System Using Delta Release Policy," Water Resources Research, Vol. 12, No. 4, pp. 606-612, August 1976.

Maass, A., M. M. Hufschmidt, R. Dorfman, H. A. Thomas, Jr., S. A. Marglin, and G. M. Fair, Design of Water-Resource Systems, Harvard University Press, Cambridge, Massachusetts, 1962.

Maidment, D. R., and V. T. Chow, "Stochastic State Variable Dynamic Programming for Reservoir Systems Analysis," Water Resources Research, Vol. 17, No. 6, pp. 1578-1584, December 1981.

Martin, Q. W., "Optimal Operation of Multiple Reservoir Systems," Journal of Water Resources Planning and Management, ASCE, Vol. 109, No. 1, pp. 58-74, January 1983.

Miller, I., and J. E. Freund, Probability and Statistics for Engineers, 2nd Edition., Prentice-Hall, Inc. Englewood Cliffs, NJ, 1977.

Moran, P. A. P., "A Probability Theory of Dams and Storage Systems," Australian Journal of Applied Science, Vol. 5, pp. 116-124, May 1954.

Nayak, S. C., and S. R. Arora, "Optimal Capacities for a Multireservoir System Using the Linear Decision Rule," Water Resources Research, Vol. 7, No. 3, pp. 485-498, June 1971.

Nopmongcol, P., and A. J. Askew, "Multilevel Incremental Dynamic Programming," Water Resources Research, Vol. 12, No. 6, pp. 1291-1297, December 1976.

Panel on Weather and Climate, Climate Change and Water Supply, Studies in Geophysics, National Academy of Sciences, Washington D.C., 1977.

ReVelle, C., E. Joeres, and W. Kirby, "The Linear Decision Rule in Reservoir Management and Design. 1, Development of the Stochastic Model," Water Resources Research, Vol. 5, No. 4, pp. 767-777, August 1969.

ReVelle, C., and W. Kirby, "Linear Decision Rule in Reservoir Management and Design, 2. Performance Optimization," Water Resources Research, Vol. 6, No. 4, pp. 1033-1044, August 1970.

ReVelle, C., and J. Gundelach, "Linear Decision Rule in Reservoir Management and Design, 4. A Rule That Minimizes Output Variance," Water Resources Research, Vol. 11, No. 2, pp. 197-203, April 1975.

Roefs, T. G., and A. R. Guitron, "Stochastic Reservoir Models: Relative Computational Effort," Water Resources Research, Vol. 11, No. 6, pp. 801-804, December 1975.

Rogers, P. P., and M. B Fiering, "Use of Systems Analysis in Water Management," Water Resources Research, Vol. 22, No. 9, pp. 146S-158S, August Supplement 1986.

Ross, S. M., Applied Probability Models with Optimization Applications, Holden-Day, San Francisco, California, 1970.

Savarenskiy, A. D., "Metod rascheta regulirovaniya stoka," Gidrotokh. Stroit., No. 2, pp. 24-28, 1940.

Shane, R. M., and K. C. Gilbert, "TVA Hydro Scheduling Model: Practical Aspects," Journal of the Water Resources Planning and Management Division, ASCE, Vol. 108, No. WR1, pp. 1-19, March 1982.

Sigvaldason, O. T., "A Simulation Model for Operating a Multipurpose Multireservoir System," Water Resources Research, Vol. 12, No. 2, pp. 263-278, April 1976.

Singh, K. P., J. B. Stall, C. Lonnquist, and R. H. Smith, Analysis of the Operation of Lake Shelbyville and Carlyle Lake to Maximize Agricultural and Recreation Benefits, Contract Report, Illinois State Water Survey, Champaign, Illinois, 30, June 1975.

Singh, K. P., Water Supply from Shelbyville and Carlyle Lakes and Their Optimal Joint Operation, Report of Investigation No. 84, Illinois State Water Survey, Champaign, Illinois, 1977.

Stallings, E. A., A. G. Cutworth, E. M. Hansen, and W. A. Styner, "Evolution of PMP Cooperative Studies," Journal of Water Resources Planning and Management, ASCE, Vol. 112, No.4, pp. 516-526, October 1986.

Stedinger, J. R., "Fitting Log Normal Distribution to Hydrologic Data," Water Resources Research, Vol. 16, No.3, pp. 481-490, June 1980.

Stedinger, J. R., B. Sale, and D. Pei, "Multiple Reservoir System Screening Models," Water Resources Research, Vol. 19, No. 6, pp. 1383-1393, December 1983.

Stedinger, J. R., "The Performance of LDR Models for Preliminary Design and Reservoir Operation," Water Resources Research, Vol. 20, No. 2, pp. 215-224, February 1984.

Trott, W. J., and W. W-G. Yeh, "Optimization of Multiple Reservoir System," Journal of the Hydraulics Division, ASCE, Vol. 99, No. HY 10, pp. 1865-1884, October 1973.

Turgeon, A., "Optimal Operation of Multireservoir Power Systems with Stochastic Inflows," Water Resources Research, Vol. 16, No. 2, pp. 275-283, April 1980.

Turgeon, A., "A Decomposition Method for the Long-Term Scheduling of Reservoirs in Series," Water Resources Research, Vol. 17, No. 2, pp. 1565-1570, December 1981.

Turgeon, A., "Incremental Dynamic Programming May Yield Nonoptimal Solutions," Water Resources Research, Vol. 18, No. 6, pp. 1599-1604, December 1982.

Venetis, C., "A Note on Moran's Probability Theory of Dams and Storage Systems," Water Resources Research, Vol. 5, No. 3, pp. 884-887, August 1969.

Wang, D., and B. J. Adams, "Optimization of Real-Time Reservoir Operations with Markov Decision Processes," Water Resources Research, Vol. 22, No. 3, pp. 345-352, March 1986.

Water Resources Council, Water and Related Land Resources: Establishment of Principles and Standards for Planning, Federal Register, Volume 38, Number 174, Part III, 1973.

Windsor, J. S., "Pumped Storage Optimization in Generation Systems," Journal of the Water Resources Planning and Management Division, ASCE, Vol. 103, No. WR1, pp. 99-109, May 1977.

Yakowitz, S., "Dynamic Programming Applications in Water Resources," Water Resources Research, Vol. 18, No. 4, pp. 673-696, August 1982.

Yazicigil, H., M. H. Houck, and G. H. Toebes, "Daily Operation of a Multipurpose Reservoir System," Water Resources Research, Vol. 19, No. 1, pp. 1-13, February 1983.

Yeh, W. W-G., "Reservoir Management and Operations Models: A State-of-the-Art Review," Water Resources Research, Vol. 21, No. 12, pp. 1797-1818, December 1985.