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ECONOMICS OF WASTEWATER COLLECTION NETWORKS

by

Jarir S. Dajani

and

Robert S. Gemmell

Northwestern University

Evanston, Illinois

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UNIVERSITY OF ILLINOIS

WATER RESOURCES CENTER

2535 Hydrosystems Laboratory

Urbana, Illinois 61801

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

SYNOPSIS

Urban public services are a significant component of the overall urban system. Increasing urbanization is, to some extent, a manifestation of man's attempt to reap the maximum obtainable benefits from the collective supply of these services. One common characteristic of these services is that they are composed of both nodal and network components: the first representing the production and processing functions, the second representing the delivery or collection functions. In spite of the importance of these services, very little is known about the nature of their cost functions. This is especially true regarding their network components.

This study addresses the question of understanding the nature of the cost functions of a service network with a public works content. The provision of wastewater collection services is used to demonstrate how technological relationships and principles of micro-economics can be used to generate normative cost functions for such service networks. In doing so, the study explores both the demand for the service, as measured by parameters of urban development, and the supply of the service, as determined by the basic technology of providing it.

A wastewater collection network is first broken down into its basic component: the sewerline or link satisfying a linear demand. The economics of sewerline design are investigated and the application of optimization concepts is explored. Inputs and outputs of the wastewater collection process are identified. A sewerline cost equation is empirically obtained from actual bid information.

The concept of optimization is then explored with respect to overall collection networks. Present design methodology and recent developments in both network layout and design are explored. The problem of the optimal choice of a mix of diameters and slopes for a given network, and a specific set of economic and technological inputs, is fitted to a separable convex programming framework, for which a global optimal solution can be obtained using existing commercial computer programs.

The nature of an areally distributed demand is dependent on the type of urban development generating it. Population, area and density are basic parameters for the measurement of urban settlements. Following a review of relevant research methodologies and concepts, a 160-Acre experimental module is presented as a basis for the development of normative network cost models. Different population densities and subdivision patterns can be superimposed on this module in a controlled environment.

Minimum-cost wastewater collection networks were designed and their costs estimated for these theoretical modules. Relationships between cost, area, population and density were developed. Using these relationships together with treatment plant cost information developed by others, such items as the tradeoffs between network and nodal costs, the minimum-cost size of total service area, and the overall service cost functions are explored.

The implications of the different methodologies and models presented in this study are finally presented. The concepts of optimizing the design of a technologically based urban service by rigorously incorporating cost as an input to the design process is stressed. Such conceptual frameworks as the development of service cost functions as tools of the urban systems planning process, the use of normative cost relationships as guides in system design, the understanding of the cost implications of urban land-use patterns, the derivation of sound cost allocation formulae and the economic determination of optimal system planning horizons are outlined.

URBAN PUBLIC SERVICES

Urban public service form a set of urban sub-systems which has a marked influence on every other sub-system in the city.[1] These services also affect the life of every urbanite. Their existence is an outstanding manifestation of the economics of human concentration. Attention is presently being centered on the development of theories which determine the optimum location of these services on the basis of economic and social principles and objectives.

Urban public services share many common characteristics. Of most interest here is their composition of a combination of nodal (points, vertices) and network components. Each of these components has its own cost function, which in turn contributes to the overall cost function of the total service. This general function determines the optimum location of the facilities concerned. The spatial aspects of these cost functions are still far from being understood, and many of the determinants of these functions are not yet even quantifiable. A grasp of these aspects is a prerequisite to the development of any theory of public facility location.

The cost functions of nodal facilities are easier to develop than those for network facilities. They bear more of a resemblance to industrial production functions with which the economist is familiar. The cost of these facilities usually decrease with the increase in the quantity of service produced, and thus with the area and density of the facility's service area, until a certain limit is reached. This is in direct comparison with the case of internal returns to scale in classical micro-economic theory.

Network costs offer a more difficult situation, especially since they are affected by the form and structure of the areas being served. Economies of scale may be offset by diseconomies of dispersion, agglomeration, or spatial arrangement and pattern. Each city has different spatial characteristics, describing its shape, size, pattern and density distribution. Unless these variations can be measured in some general way, no overall theory can be meaningfully developed. It is to the problems involved in the development of these spatial cost functions that this research basically addresses itself.

It is felt, however, that an attempt to study these problems in the context of a certain public service would have its payoffs and advantages. It would develop a framework and a methodology which could be extended to other services. It will also set the stage for finding the answers to some of the questions which relate to the specific service to be studied. We have chosen to study a public works oriented service, namely municipal wastewater collection and disposal. It is our belief that the basic approach suggested herein can be fruitfully applied to other systems of the urban physical infrastructure, such as electricity, gas, water, telephones, urban communication systems, and the like.

The planning and growth of our urban settlements today, is to a large extent, constrained by existing technologies of utilities and services. We must understand and analyze the costs of existing methods of supplying these services, in order to be able to evaluate any breakthrough in technology which may be forthcoming.

Wastewater Collection Systems

A survey of existing literature shows that empirical studies of the statistical type have been used to study the cost behavior of sewage treatment plants of different sizes and types. Some empirical relationships have been developed to estimate the costs of sewer lines, but no attempt is made to understand how the cost of these lines behave when they are combined to form networks serving populations of given spatial characteristics. An extensive literature review indicates that no studies exist relating the technical, spatial, and economic aspects of wastewater collection networks. The absence of such studies precludes the possibility of developing any general sewerage cost model, which integrates the various components of the system into a unified framework.

A study by the Business and Defense Services Administration estimates that $37.4 billion will be needed by the year 1980 for the construction of facilities for the collection and treatment of municipal wastes.[2] The United States Public Health Service estimates that $700 million will be needed annually during the seventies. The Federal Water Pollution Control

Administration's estimates range between \$26 and \$29 billion over the coming five years. Whatever the actual figures may be, there is no doubt that they will be of such magnitude as to form a major drain on the public budget, and thus affect the pockets of millions of individuals. There is no question that efficient use of funds is called for in order to achieve the highest levels of cost effectiveness and management efficiency.

The above estimates are a reflection of the increasing rates at which cities grow. The construction of new suburbs, satellites, and new towns emphasizes the need for efficiency in the planning of new municipal facilities, the management and extension of existing facilities, as well as the development of new technologies for meeting the increasing demand at lower costs.

The Business and Defense Service Administration estimates are broken down among the two basic components of municipal wastewater disposal systems: collection and treatment, as well as among the different types of municipal service demands. The breakdown shows that about 63 percent of the total projected capital outlays are allocated to collection networks. Lawrence estimates that the average capital investment cost for an overall sewerage system is about \$200 per capita, three quarters of which represents collection system costs, and one quarter covers the costs of treatment and disposal facilities.[3]

In spite of the large capital outlays which are involved in the network component of municipal wastewater disposal projects, the methods of design remain very intuitive, and no real attempt is generally made to generate and evaluate alternative solutions and designs. It is only recently that the relationship between sewerage systems and community development began to be appreciated, and their study within a systems framework thought of. A better understanding of existing systems will help improve their design methodologies and thus their total social effectiveness.

Existing technology, together with the historic, economic, and institutional constraints associated with it, remain to be taken for granted, and no real effort is being undertaken to evaluate the benefits that may accrue from the introduction of new technologies, and thus from the relaxation of some of these constraints.

Empirical work has shown definite economies of scale in the nodal components of wastewater disposal systems, such as Treatment Plants and Pumping Stations.[4] Very little, however, is known about the cost function of the collection network. A lack of understanding of the nature of this function will naturally stand as a block in the way of intelligently evaluating alternative system designs or urban forms. Unless these functions are clearly defined and understood, it will not be possible to evaluate the impact of technological innovations and breakthroughs, which are long overdue in a service which has undergone no basic change in centuries.

While some work is being undertaken on locating treatment facilities on a regional basis, the determination of the number, size and location of treatment plants in a metropolitan area remains subject to rules of thumb and intuition.[5] Naturally, no method of analysis could be developed until the cost behavior of networks is completely understood.

If we hope to be able to replace some of the present intuitive judgments in municipal utility design by well-grounded scientific and economic criteria, a step must be taken in the direction of understanding these network cost functions within the context of the existing and proposed varieties of urban forms, patterns and structures.

Objectives of the Study

It is recognized that this research will not answer all the questions in the area of wastewater collection and disposal costs, let alone public service costs and the economics of population concentration in general. The research results reported herein will answer some of these questions, keeping in sight the ultimate objective of being able to integrate the cost characteristics of the various components of a system into a unified cost model. This model, when developed, will make possible the selection of the site or the family of sites which will minimize the cost of municipal sewage disposal in the metropolitan area. It is also hoped that this research will privide an insight into the trade-offs between network and nodal sizes and costs, which will be useful in the analysis of the feasibility of forthcoming technological innovations.

The basic objectives of this study are to explore the nature of the cost determinants of wastewater collection lines and networks, to investigate the methodology and economics of network design, to explore the cost behavior of these networks under different conditions of urban form and structure and to rigorously develop a framework for a predictive cost model for wastewater collection facilities.

The cost function of an urban public service is the summation of the cost functions of the network and the nodal components of the service. Developing these functions and understanding their implications, can prove to be an important tool in the analysis, formulation and implementation of urban public policy. The basic hypothesis central to this research is that the direct unit network cost (annual and capital) of a certain service is a function of the technology of the service as well as of the characteristics of the urban area to be served. Other costs are incurred and benefits derived as a result of the interaction of the system in question with other systems in the city.

SEWERLINE COSTS

The capital cost of installing a wastewater collection system is dependent on a number of factors. This chapter will attempt to introduce some of these variables and to isolate and analyze the most important determinants of these costs.

A wastewater collection system is made up of a number of physical components, namely: pipelines, appurtenances and pumping stations. Where the topography is favorable, a simple gravity system can be built, and no pumping costs will be incurred. In such cases the construction cost will be totally made up of pipeline and appurtenance costs.

The output of a sewer system is dependent on factors internal to existing sewer flow technology. These factors are determined by the flow process functions of open-channel hydraulics. The output of "installed flow capacity" of the system is a function of the shape, size, material and slope of the different pipeline sections which combine to make up the total network. It is measured in terms of the number of units of flow per unit of time. The inputs necessary to produce these outputs include construction, maintenance and operation, financing, and engineering inputs. The first of these items is by far the most important. It is closely related to flow and length, while the other items are independent of both of these measures of output. Construction inputs usually determine both financing and engineering costs. Financing costs are also a function of the magnitude and complexity of the project. Maintenance and operating costs in a gravity flow system are of a small magnitude and are not related to the quantity of flow. Isard and Coughlin have not allowed for any such costs in their study.[6] Others have estimated annual sewer maintenance costs at a fixed amount per mile of pipe.[7]

The following discussion will establish a generalized mathematical model to estimate the inputs (costs) needed to produce a wastewater collection service of a certain output (capacity). Inputs and outputs will be related on the basis of technological construction and flow functions in order to obtain an insight into the costs of producing this urban service.

Construction Costs

As mentioned above, the largest single component of the construction costs of a sewerage network is the cost of supplying and installing the different pipe sections which make up the network. A study based on average national figures estimates that 85 percent of the cost of gravity sewer systems is devoted to excavation, pipe supply and installation. The remaining 15 percent covers the cost of manholes.[8] These estimates are based on the most common method of construction, namely that of laying a pipe in pre-excavated open-cut trenches.

An analysis of the cost of the excavation and pipe components can form the basis of predictive model for pipeline costs. Construction cost estimates were prepared on the basis of figures obtained for the 1970 CE Cost Guide[9] for pipes with diameters ranging between 8 and 24 inches in diameter, placed in different types of soil at depths of up to 20 feet. Data were collected for five sewerage contracts for which bids have been prepared by contractors. Unit costs as estimated by the engineers, the lowest bidder, the second lowest bidder, and the highest bidder were obtained for each of the contracts (excluding two for which there were only two bidders). Together with unit costs, the respective diameters and depths of the pipes were obtained for a total of 18 cases. Sample sizes ranged between 22 and 68. Three of the contracts specified concrete pipes while the other two specified vitrified clay pipes. No information on the properties of the soil was available.

Simple multiple regression equations utilizing the internal diameter of the pipe (D) and the average depth of excavation (X), both in feet, as the independent variable, and cost per foot length (C) in dollars, were developed for each subset of the data described above. All data groups fit the general form

$$C = a + bD^2 + cX^2 \tag{1}$$

These equations were tested for significance using the F-test. The null hypothesis $b = c = 0$ was tested against the alternative hypothesis that b and/or c are not equal to zero. The students' t-distribution was examined for each of the regression coefficients at the 5 per cent level of significance. Observed values of t exceeded the critical value for both variables in all cases.

Each construction company has a somewhat different bidding procedure and a special bidding strategy which depends on such factors as its work load, growth and profit objectives, and probabilities of award in the face of existing competition. Bidders may present unbalanced bids in order to capitalize on expectations of changes in quantities. They may bid high on items which are to be completed early in the contract in order to obtain larger payments at the beginning of the work and thus reduce their need for borrowed capital.[10] In spite of these and other causes of possible inconsistency in bid pricing, the structural form of the general model suggested above has exhibited a high degree of dependability in predicting the values of the bid unit prices in all 18 cases for which regression curves of the unit costs on the squares of both the diameters and the depths were obtained. These results are shown in Table 1. The values of the constants and the regression coefficients vary appreciably, reflecting different local conditions and bidding strategies. The coefficient of the diameter term assumes a consistently larger value than that of the trench depth term. The consistency of the structural form of the model underlines the possibility of applying a new set of tools to the design, estimating, bidding and contract management aspects of the wastewater collection service.

TABLE 1

Cost Models for Concrete and Vitrified Clay Pipes Laid in Open-Cut Trenches, Based on Actual Bid Prices*

D and X are in feet; C is in \$/ft.

Pipe Material	Sample Size	Bid**	Cost Model	R^2
Contract No. 1 Concrete Pipe	22	1	$-5.04 + 0.46X^2 + 5.99D^2$ t = 15.37 t = 5.47	0.926
		2	$1.034 + 0.032X^2 + 3.47D^2$ t = 21.78 t = 6.19	0.963
		3	$-6.39 + 0.089X^2 + 6.67D^2$ t = 12.72 t = 2.62	0.903
		4	$-0.81 + 0.088X^2 + 9.53D^2$ t = 24.31 t = 7.25	0.970
Contract No. 2 Concrete Pipe	57	1	$-0.524 + 0.037X^2 + 5.33D^2$ t = 25.73 t = 23.03	0.953
		2	$-0.957 + 0.054X^2 + 3.37D^2$ t = 23.84 t = 7.70	0.917
		3	$-0.757 + 0.078X^2 + 3.30D^2$ t = 28.33 t = 6.15	0.938
		4	$2.42 + 0.026X^2 + 4.64D^2$ t = 18.09 t = 16.50	0.911

See notes on following page.

TABLE 1 (Cont.)

Pipe Material	Sample Size	Bid**	Cost Model	R^2
Contract No. 3 Vitrified Clay Pipe	57	1	$-0.138 + 0.031X^2 + 5.36D^2$ t = 26.26 t = 23.56	0.955
		2	$-1.228 + 0.081X^2 + 3.70D^2$ t = 25.93 t = 6.20	0.927
		3	$-7.08 + 0.096X^2 + 6.02D^2$ t = 22.12 t = 7.18	0.906
Contract No. 4 Concrete Pipe	68	1	$3.442 + 0.060X^2 + 2.87D^2$ t = 19.70 t = 10.89	0.894
		2	$2.319 + 0.068X^2 + 1.56D^2$ t = 26.47 t = 23.09	0.954
		3	$4.516 + 0.015X^2 + 2.19D^2$ t = 25.06 t = 4.46	0.912
		4	$4.191 + 0.054X^2 + 2.01D^2$ t = 14.61 t = 10.34	0.843
Contract No. 5 Vitrified Clay Pipe	40	1	$-2.91 + 0.064X^2 + 6.77D^2$ t = 30.65 t = 21.44	0.973
		2	$2.620 + 0.058X^2 + 2.59D^2$ t = 18.40 t = 10.75	0.922
		3	$1.620 + 0.071X^2 + 2.16D^2$ t = 14.54 t = 6.13	0.867

*This analysis is based on data obtained from the records of Greeley and Hansen Engineers, Chicago.

**Bid numbers indicate: (1) Engineers' Estimate, (2) Low Bidder, (3) Second Lowest Bidder, and (4) Highest Bidder.

Technological Relationships

The basic form of the process function of flow in an open-channel gravity pipe is

$$Q = V A \tag{2}$$

where Q is the total output of the pipe in cubic feet per second,

V is the mean velocity of flow when the pipe is running full, in feet per second,

A is the cross-sectional area of the pipe in square feet.

The velocity of flow in a full sewer pipe is usually limited by standards to a minimum of 2 fps (feet per second) and a maximum of 10 fps. A velocity of 2 fps when running full ensures that the velocity would be 1 fps when the sewer is less than 17 percent full. This latter velocity, called the self-cleansing velocity, is the minimum needed to prevent sedimentation of sludge and light mineral matter.[11] The upper limit is fixed to avoid excessive erosion of the invert of the pipe. The velocity of flow is a function of the roughness of the pipe (the rougher the pipe the slower the flow), of the slope of the pipe (the steeper the slope the faster the flow), and of its shape and size. Many formulae exist for the determination of the velocity of open-channel flow. One commonly used relationship is the "Manning Formula."[12] This formula is

$$V = \frac{1.49}{n} R^{2/3} S^{1/2} \tag{3}$$

where V is the mean velocity of flow in fps,

R is the hydraulic radius of the pipe in feet. It equals the area of the pipe divided by its wetted perimeter. This is a measure of both the pipe's shape and size.

S is the slope of the pipe in feet/ft.,

and n is the roughness coefficient of the pipe.

Equation (2) and (3) above can be rewritten as

$$Q = k_1 D^2 V \qquad (4)$$

$$V = \frac{k_2}{n} D^{2/3} S^{1/2} \qquad (5)$$

where D is the internal (nominal) diameter of the pipe, and k_1 and k_2 are factors which depend on the shape of the pipe and the design standards being used. In the case of a circular pipe running full, the hydraulic radius is 1/4 of the pipe diameter, $k_1 = \Pi/4$, and $k_2 = 0.59$. If the pipe is designed for less than full flow, if its shape is other than circular, or if metric units are used, these constants must be adjusted accordingly.

Combining the above two equations, the following terms are obtained for both the capacity and the diameter:

$$Q = \frac{k_1 k_2}{n} D^{8/3} S^{1/2} \qquad (6)$$

and

$$D = \left(\frac{n}{k_1 k_2}\right)^{3/8} Q^{3/8} S^{-3/16} \qquad (7)$$

The Quantity of Flow

The total quantity of flow for which a sewerline is designed, is based on the peak expected demand which is to be accomodated by that line. In a predominantly residential area, demand is measured by the number of people being served and by the peak expected volume of wastewater generated by each person.

The average volume of per capita wastewater generation is usually slightly less than the average quantity of water supplied to the individual. This is due to losses resulting from leakage, lawn sprinkling, and similar uses. Both average water consumption figures and wastewater generation rates must be estimated on the basis of their ultimate expectations during the design period of the system in question.

The volume of flow varies continuously throughout the day. It is also expected to vary by the day of the week and the month of the year. Each line must be designed to handle the expected peak flow. The ratio of the peak flow to the average daily flow is significantly affected by the size of the line, i.e., the total number of people it serves. As a line gets larger and serves a larger number of generators, the probability that the peak flows of these generators coincide is reduced, and so is the ratio of the peak to the average daily flows.

Many methods exist for the estimation of this ratio. They vary from simply following regulatory standards as to the minimum acceptable design flow rates for each type of line to the development of curves and equations, relating the peak and average flows. All the mathematical models which have been developed exhibit an inverse exponential relationship between the ratio of the two flows and the population being served. The actual parameters used vary, and records of existing systems are rarely complete enough to permit making good estimates of the sanitary sewage component of the peak flow.

A simple form of this relationship has been used by Babbit.[13] He suggests that

$$M = \frac{5}{P^{1/5}} \tag{8a}$$

where M is the ratio of the maximum flow to the average daily flow,

and P is the population served in thousands.

The ratio given above is not to exceed 5 nor to be less than 1.5. This limits the use of the above model to population figures ranging from one thousand to 412 thousand people. Lines serving larger demands must be designed for a peak flow equal to 1.5 times the average daily flow.

Equation (2-8) may be rewritten as

$$Q = 5Q_{av}/P^{1/5} \tag{8b}$$

$$= 1.37(Q_{av})^{4/5} \quad f^{1/5} \tag{8c}$$

where Q and Q_{av} represent the peak and average daily flows in cubic feet per second, respectively; and f represents the average flow in gallons per capita per day.

The above equation can be used to write all the above process functions in terms of average daily flow, which can be directly related to, and calculated from, the number of people being served. For very large (more than 421 thousand persons) and for very small (less than one thousand persons) populations, a linear relationship would exist between Q and Q_{av}, as mentioned above.

While the value of the average daily per capita flow will vary by locale, climate, socio-economic characteristics and living patterns of people, a figure of 100 gallons per capita per day is a likely average figure. For the sake of simplicity, this figure will be used in subsequent discussions of the cost functions. This, of course, does not reduce the generality of the model, since any figure can actually be used. Equation (8c) can thus be reduced to the form

$$Q = 3.45Q_{av}^{4/5} \qquad \text{for } 1 \le P \le 412 \tag{9a}$$

$$Q = 1.5Q_{av} \qquad \text{for } P \ge 412 \tag{9b}$$

$$Q = 5Q_{av} \qquad \text{for } P \le 1 \tag{9c}$$

where Q, Q_{av} and P are as defined above.

Cost Functions and Sewer Design

The relationship between the above input and output functions and their implications on sewer design will be investigated in this section. It will be shown that while one single mathematical model can be used in estimating the cost of a network link on the basis of physical construction inputs, no such single continuous function can be developed to relate physical inputs and flow outputs. This is basically due to two design limitations, namely the limitation on a minimum size pipe and the limitation on allowable velocity ranges. The incorporation of these constraints into the flow functions will result in a noncontinuous cost function, composed of four discrete portions. Each portion is associated with a certain range of flows. On the basis of the above analysis, it is possible to identify the different ranges over which these cost sub-functions are defined. It is also possible to determine the mathematical structure and the economic properties of these sub-functions.

All the cost functions discussed in this chapter are long-range. They reflect the situation where a planner or designer can select among the total variety of feasible choices available to him that one which minimizes his total cost. The long run total cost curves derived below will be designated (TC) and will determine the cost of transporting a quantity of flow of Q cubic feet per second, a distance of L feet. Unit and marginal costs can be derived from the total cost function by dividing it by Q and calculating its slope, respectively.

From Eqs. (1) and (7) above, the general form of the total cost function can be derived:

$$TC = b\left(\frac{n}{k_1 k_2}\right)^{3/4} Q^{3/4} S^{-3/8} L + (a + cX^2)L \qquad (10)$$

So long as the value of the exponent of Q is less than unity in the total cost function, and negative in the unit cost function, there are economies of scale in the construction of the facility in question. This indicates that as the total cost of the facility increases with size, its unit cost decreases.

The usual sewer design problem is one in which the demand to be satisfied is given. It is usually exogenously determined on the basis of the spatial distribution of the population to be served in terms of both volume and flow Q and length of transmission L. The designer is faced with the problem of choosing the slope and diameter of the pipe in such a way as to satisfy the demand at the minimum cost, while satisfying the design standards of pipe diameter and velocity of flow. The choice of a larger diameter will allow a flatter slope and thus a trading-off of excavation costs for pipe costs. The extent of this trade-off will depend on the ratio of the coefficients b and c in Eq. (1) above. Since this is a case of predetermined flow, the choice of slope automatically determines the diameter. The development and calibration of the basic cost equation (1) above prior to the undertaking of actual design or estimating activities is imperative, if a minimum cost design is desired. It is also evident that in a situation where a certain demand Q is to satisfied, the minimization of the cost per unit of Q will result in the overall most economical solution, as will the minimization of the total cost function.

Consider the design of a sewerline in an area having level terrain. Let E_1 and E_2 be the upstream and downstream depths of the sewerline to be designed, respectively. The upstream depth is usually known so that the average depth of excavation will be $E_1 + SL/2$. The slope S is defined as $(E_2 - E_1)/2$. Substituting this value into Eq. (10) above, expanding, rearranging and dropping the negligible term of the square of the slope, yields:

$$TC = b\left(\frac{n}{k_1 k_2}\right)^{3/4} Q^{3/4}\ S^{-3/8}\ L + cSL^2E_1 + (cE_1^2 + a)L \qquad (11)$$

An optimal sewerline design can be obtained by minimizing this cost subject to technological constraints on diameter and allowable ranges of velocity. This results in a series of cost functions which apply at different quantities of flow.[14]

Equation 2-11 shows that cost basically increases at a decreasing rate as the quantity of flow increases, while it increases at an increasing rate as the length increases. This implies that decreasing unit costs are realized by increasing the quantity of flow, while the opposite is true when the service area is increased, and the line is lengthened.[15] The "volume" and "distance" effects on the cost of sewer lines, and on most public utility lines for that matter, pull in opposite directions. The "volume effect" indicates decreasing unit costs with increasing total demand. The "distance effect" indicates increasing costs with decreasing density of demand.

NETWORK DESIGN

The previous chapter has presented an empirical and theoretical framework for the study of the economics of sewer line design. A series of mathematical models have been developed to estimate the cost of constructing a wastewater collection line, providing for different ranges of linearly distributed demands. The integration of a large number of lines or "links" into a network structure which serves a demand of complex area spatial distribution adds new dimensions to the cost and performance functions suggested above. This is the case in real-life urban situations to which this chapter will address itself.

The tradeoffs between the various components of sewerline design are not explicitly incorporated into the process of designing wastewater collection lines or networks at the present time. They are only indirectly considered as an input to the design process through the experience and engineering judgment of the designer. The parameters of the sewerline cost functions derived in the previous chapter are a necessary input into the design of complete sewer networks. At the present time, attempts to produce economical network designs are at best limited to the generation and evaluation of a very small number of alternative solutions. Time-consuming and costly design and cost-estimating techniques limit the feasibility of testing a large number of possible solutions.

The present methodology of sewer system design consists of three-interrelated processes: the selection of a treatment or disposal site or sites (sometimes referred to as sinks or roots), the design of a system layout and the choice of a slope and diameter combination for each link in the network.[16] The selection of the number and location of sinks is constrained by the availability and cost of land, the location of such ultimate disposal facilities as a body of water, the terrain, and the tradeoffs between system and treatment costs. The design of the system layout is usually selected in such a manner as to serve each existing and future land-use through an easily accessible sewerline. Lines are usually located in the right-of-way of the street system. Gravity flow is generally desirable, and normally follows the natural slopes of the terrain. The maximum quantities of flow in different selections of the network are estimated on the basis of future land-use water consumption and socio-economic forecasts. The slope and diameter of each link in the network are chosen from standard available alignment charts (nomographs). This choice is made consistent with standard limitations on allowable ranges of velocities and on minimum conduit diameter. It is also adjusted to conform to the commercially available discrete pipe sizes.

These three processes of design define the two main sets of choices within which alternative solutions to the sewer design problem offer themselves. The first set of choices consists of two-dimensional locational choices: it determines the location of each link in the network and the direction of flow in that link. The second set of choices defines the sizes of the different components of the system and their location in the third dimension of urban physical space: depth.

An overwhelming number of alternative solutions exist when different combinations of the overall sets of choices are considered. The designer intuitively discards alternatives which are obviously dominated, such as the case of flow against natural slopes toward an upstream located sink. He does not, however possess a simple and formal methodology for comparing marginal alternatives. Nor does he possess the analytical tools for generating an "optimal" or "minimum-cost" solution. No simple and economical method for the generation of a "good" or "near-optimal" solution and for estimating its cost exists. Partial solutions toward the optimization of each of the last two design processes discussed above have been suggested, but no unique and comprehensive answer is at hand.

This chapter outlines a solution to the problems of achieving a "good" or "near-optimal" design. The method presented herein is sequential in the sense that it starts by assuming that a network layout exists and then finds the cost of the most economical design of this layout. Once a simple and economical design and estimating procedure is available, the evaluation of a number of alternative feasible network layouts and sink locations becomes possible. The practical aspects of the method are stressed, and every effort is made to adapt it to existing and easily accessible computer soft-ware.

Network Layout

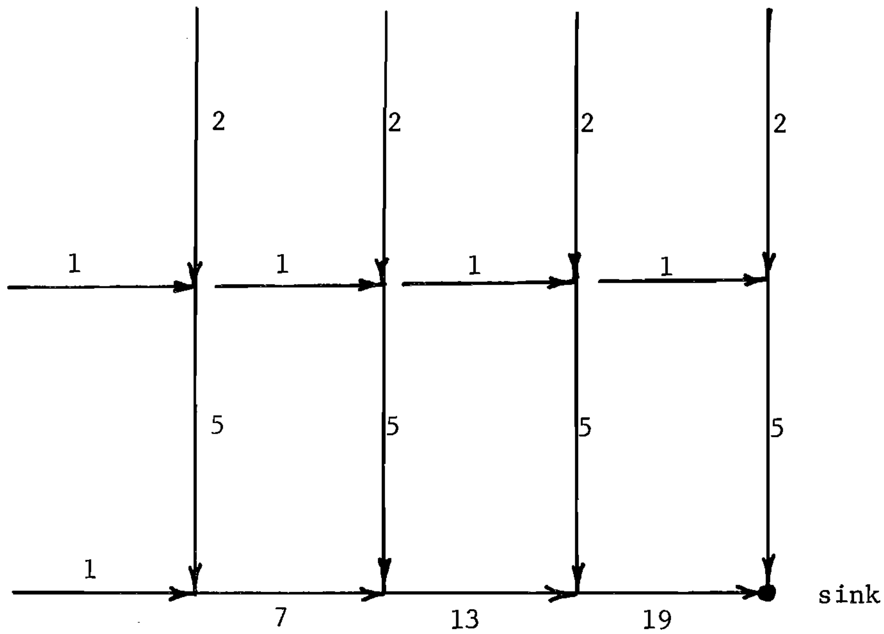
Sewer networks are man's simulation of natural drainage and runoff systems. The network representation of both of these systems has the general form known as a "finite-rooted tree."[17] This is a network which is characterized by having no circuits or closed loops within it. Sewer networks, however, differ from natural drainage networks in that they are defined over an esisting or proposed street pattern, which can be represented by a highly connected graph with a large number of loops or circuits. The sewer tree must span each and every link in the connected street network in order to supply complete service to the area being sewered. A sewer tree has two types of junctions: internal and external. An internal junction (or node) is one at which two or more branches (links, edges, or arcs) meet. An external node is one from which only one link emanates. Both sewer and natural drainage trees have the common characteristic of flowing from a number of sources into one sink. As a result, while many links may be flowing into a node, only one link will flow out of it. This characteristic eliminates the need for designating links by the double subscript X_{ij}, which is used in the familiar transportation problem.[18] One subscript suffices to denote both the node and its downstream link.

Existing literature on sewer design offers very few guidelines for the economical layout of networks. These guidelines commonly include detailed descriptions of the preferred location of sewer lines in relation to other utilities in the public right-of-way, and suggestions relating to the general directions of flow and the location and spacing of manholes.[19] Hardenbergh makes some more definite suggestions regarding network layouts:

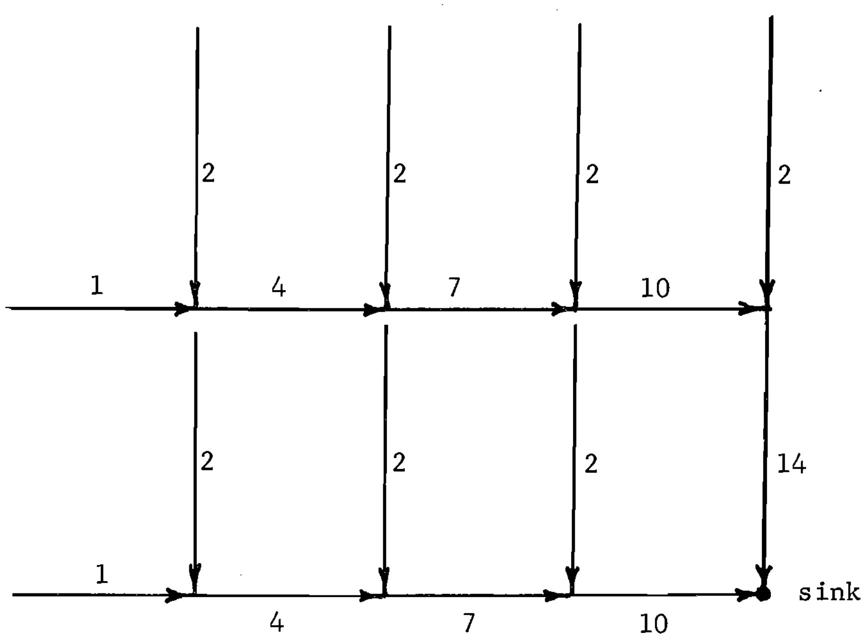
> In planning the layout for a sewerage system, it is not possible to follow rigidly any fixed procedure. However, after the general map of the area to be sewered has been studied, it will usually be found that one or two rather typical layouts, or a combination of these layouts, can be readily adapted to suit the topographic features. The best grades for the main sewers will usually be obtained when those sewers follow the natural drainage channels in a general way. But the entire system cannot always be laid out to conform solely to topography.
>
> The two fundamental layouts on which the plans for most sewerage systems are based may be termed the <u>perpendicular layout</u> and the <u>fan layout.</u> Either of these layouts may require the installation of an intercepting sewer.[20]

The two layouts suggested above are shown in Fig.1. Hardenbergh makes no further suggestions as to how to choose either of these systems, or, once chosen, how to assign links of various orders to streets. This remains subject to the judgment and experience of the designer. As will be seen in Chapter V, these may not be very significant matters in a regular grid street pattern. They do, however, assume more complex dimensions as the basic street pattern becomes less regular.

The number of possible trees which can be constructed to comprehensively cover a certain street network is overwhelmingly large.[21] Recognizing the fact that even with repid electronic computers it would be impractical to investigate every possible layout for anything but a tirvially small network, Liebman developed a heuristic method for the generation of a "good" layout. His method is based on a computer search technique which improves upon a given layout. He sites the major drawback of his method as being the lack of consideration for flow conditions and thus



Perpendicular layout



Fan Layout

Figure (1)

Typical Generic Network Layout

its limitation to networks composed of links of equal capacity or diameter.[22]

A different quick and approximate method for the generation of a sewer tree which completely spans a given street network, and which has "good" cost qualities has been suggested by Dajani.[23] While it is recognized that the outcome is not globally optimal, its logical development is expected to lead to a better approximation of optimality than present intuitive methods. The effort involved in the application of this approximate method does not deter from its utility. The method generates shortest path trees, totally spanning a given street network. It is based on the fact that length is the most important determinant of the cost of routing flow between two given points, and on the proof that relative lengths are a fairly dependable measure of relative link costs. The fact, remains however, that no method exists for the selection of an optimal layout. Rational approximations may be obtained and an "optimal" or "near-optimal" solution can then be found by designing and estimating the costs of these approximations and choosing that layout which yields the least-cost system.

Optimal System Design

Once an acceptable system layout has been generated, the designer usually proceeds to choose among the second set of alternatives open to him a combination of diameter, slope, and depth. As mentioned above, these choices are usually made with the aid of alignment charts for each individual element in the system. The interaction among the different links, the cost characteristics of the construction operations, and the tradeoffs between the cost of one link and another are usually not explicitly considered in the design. It is clear that depending on the coefficients of the pipe and excavation terms in the basic cost function a tradeoff between slope and diameter can always be made when a certain quantity of flow is being accommodated. Savings on pipe materials in one link will usually result in more costly excavations for both the link in question and for the whole subsystem downstream of it.

Two systematic methods have been suggested for the optimization of sewer system design, given a network layout and a set of flow quantities. Both methods are based on the application of optimization techniques to existing design rules and methodologies. Holland has formulated the problem in the format of a non-linear programming algorithm.[24] Voorhees suggested the use of an n-stage, two parameter programming approach.[25] The first method has the advantage of neatly fitting a standard mathematical formulation. The subsequent formulation is based on Holland's development and is adapted for solution using standard programming techniques and existing computer programs.

While a number of cost sub-functions have been developed for a sewer line conveying different quantities of flow, they have all been derived from one basic function by applying the relevant technological constraints to it. This overall function has been shown to be:

$$TC_i = 2.35b \left(\frac{n}{k_1 k_2}\right)^{3/4} Q_{av(i)}^{3/5} \; S_i^{-3/8} \; L_i + cX_i^2 L_i + aL_i \qquad (12)$$

the subscript i indicates that the above cost function represents the link which originates from node i in the network. As has been mentioned earlier, one subscript completely defines a sewer network link. A further note on the notation being used will facilitate the development which follows. Symbols which are not identified by a subscript associating them with a specific link are assumed to be the same for all links. While this is usually the case, the overall model can accommodate different values for these parameters. A differentiation will be made between the invert elevations at the upstream and downstream nodes defining a link. These elevations will be denoted $\overline{E}_i$ and $\underline{E}_i$, respectively. Ground elevations at these points will be denoted $\overline{G}_i$ and $\underline{G}_i$. Another terminology which will be adopted whenever generalized subscripts are used is the numerical ordering of links in a downstream direction, i.e. link i will be just upstream of link (i + 1) and just downstream of link (i - 1). All elevations are assumed to be measured with respect to a given datum. The following analysis is written in terms of the average quantity of flow and is applicable for networks consisting of fully-utilized links serving a maximum of 412,000 persons.[26]

The overall cost of a network is the summation of all its link costs. Substituting $(\bar{E}_i - \underline{E}_i)L_i$ and $\frac{1}{2}\left[(\bar{G}_i + \underline{G}_i) - (\bar{E}_i + \underline{E}_i)\right]$ for the slope S and the depth of excavation X in Eq. (12) above, the total cost of an n-link network can be written as:

$$\sum_{i=1}^{i=n} r\left(\bar{E}_i - \underline{E}_i\right)^{-3/8} + \frac{cL_i}{4}\left(\bar{E}_i + \underline{E}_i\right)^2 + \frac{cL_i}{2}\left[\left(\bar{G}_i + \underline{G}_i\right)\left(\bar{E}_i + \underline{E}_i\right)\right] + \frac{cL_i}{4}\left[\left(\bar{G}_i + \underline{G}_i\right)^2\right] + aL_i$$

$$\text{where } r = 2.53b\left(\frac{n}{k_1 k_2}\right)^{3/4} Q_{av(i)}^{3/5}\; L_i^{11/8} \qquad (13)$$

This equation is the objective function of the sewer network optimal design problem. The only two decision variables are the summation of, and the difference between, the upstream and downstream invert elevations of each link in the network. All other entities are known and can be readily evaluated. The problem has thus been reduced to one of minimizing the value of the above objective function, subject to a certain set of technological constraints. These constraints include the minimum allowable diameter, the minimum and maximum allowable velocity limits, the minimum pipe cover, and the diameter and invert elevation progression constraints. These constraints can be shown to be linear in the two variables which appear in the objective function. The following is a development of the constraint relationships:

1. Minimum Allowable Diameter Constraint:

On the basis of Eq. (3) it can be readily shown that

$$\bar{E}_i - \underline{E}_i \leq 11.9 D_{min}^{-16/3} \left(\frac{n}{k_1 k_2}\right)^2 Q_{av(i)}^{8/5} \; L_i \tag{14a}$$

2. Allowable Velocity Range Constraint:

Substituting the value of the diameter given in Eq. (7) into Eq. (5) and replacing the slope S by the equivalent term $(\bar{E}_i - \underline{E}_i)/L_i$ yields

$$V_i = \left(\frac{k_2}{n}\right)^{3/4} k_1^{-1/4} \, Q_{av(i)}^{1/4} \, L_i^{-3/8} \left(\bar{E}_i - \underline{E}_i\right)^{3/8} \tag{15}$$

the limiting values of which are V_{min} and V_{max}. Rearranging and solving in terms of $(\bar{E}_i - \underline{E}_1)$ results in the constraint:

$$0.438 V_{min}^{8/3} \left(\frac{n}{k_2}\right)^2 k_1^{2/3} \, Q_{av(i)}^{-8/15} \, L_i \leq \bar{E}_i - \underline{E}_i \leq$$

$$0.438 V_{max}^{8/3} \left(\frac{n}{k_2}\right)^2 k_1^{2/3} \, Q_{av(i)}^{-8/15} \, L_i \tag{14b}$$

which insures that the velocity does not exceed either of its allowable limiting values.

3. Minimum Pipe Cover Constraint:

In order to secure the minimum cover specified for link i at its upstream end, the constraint

$$2\bar{G}_i + (\bar{E}_i - \underline{E}_i) - (\bar{E}_i + \underline{E}_i) \geq 2 \text{ x min. cover required} \tag{14c}$$

must be satisfied. The above constraint is equivalent to

$$\bar{G}_i - \bar{E}_i \geq \text{minimum allowable cover.} \tag{14d}$$

If the slope of the terrain is steeper than that of the pipe, then this constraint must be written twice: once for each node in the link. If, on the other hand, the slope of the terrain is flat or sloping at less than the general slopes of the pipes, then this constraint need only be applied to the external nodes of the network. This, of course, assumes a fairly regular terrain along each link.

4. Diameter and Invert Progression Constraints:

These constraints insure that the diameter of any link is at least equal to the diameter of the links which flow into it. They also set the upstream invert elevation of any link to be at most equal to the downstream invert elevation of any link flowing into it.

From the basic process functions which have been previously developed, it can be shown that in order to have $D_i \leq D_{(i-1)}$ the following constraint must be met:

$$\left(\overline{E}_i - \underline{E}_i\right) - \left(\frac{L_i}{L_{(i-1)}}\right)\left(\frac{Q_{av(i)}}{Q_{av(i-1)}}\right)^{8/5}\left(\overline{E}_{(i-1)} - \underline{E}_{(i-1)}\right) \leq 0 \qquad (14e)$$

And in order to have $\overline{E}_i \leq \underline{E}_{(i-1)}$ the constraint

$$\left(\overline{E}_i + \underline{E}_i\right) + \left(\overline{E}_i - \underline{E}_i\right) \leq \left(\overline{E}_{(i-1)} + \underline{E}_{(i-1)}\right) - \left(\overline{E}_{(i-1)} - \underline{E}_{(i-1)}\right) \qquad (14f)$$

must be satisfied.

5. Sink Constraint:

This constraint is included to insure that any laterals arriving at the sink will do so at an invert elevation equal to or above that of a main line at the sink. This will allow the lateral to flow into the main line if a single interceptor is needed. If a lateral j and a main line i meet at a sink, then

$$-\left(\overline{E}_i - \underline{E}_i\right) + \left(\overline{E}_i + \underline{E}_i\right) + \left(\overline{E}_j - \underline{E}_j\right) - \left(\overline{E}_j + \underline{E}_j\right) \leq 0 \qquad (14g)$$

The above formulation is applicable to sewer systems flowing totally by gravity in fairly regular terrain. However, it can be easily extended to include the costs of drop manholes or lift stations. Both of these appurtenances can be represented by terms in the objective function. The cost of constructing drop manholes and lift stations can be expected to vary as the square of the amount of the drop or lift.

A simplification of the above objective function can be made when the terrain is flat. This is sometimes a realistic situation and may prove useful in theoretical constructs. The simplification is achieved by assuming both of the two ground elevations associated with each link, $\overline{G}_1$ and $\underline{G}_i$, to be equal to zero and by considering the datum line from which the invert elevations of the different pipes is measured to be the ground level. Minor adjustments in the formulation of the constraint equations will also have to be made.

The problem is thus one of minimizing a non-linear objective function, subject to the linear constraints given by equations (14a) through (14 g). The objective function is separable, since it can be written as a finite sum of separate terms, each of which involves only a single choice variable. A global optimum solution can be found for this type of problem only if all of its separate terms are concave functions and it is desired to maximize the objective function or if all the separate terms are convex functions and it is desired to minimize the objective function. Such problems can be solved for constraints which are either linear functions or non-linear separable functions whose component functions are all convex.

The objective function given in Eq. (13) above is composed of the summation of three separate variable terms and two constant terms for each link in the network. The three variable terms represent the choice variables raised to the powers -3/8, 2, and 1, respectively. The first two exponents indicate strictly convex functions, while the third represents a linear relationship which is, by definition, both convex and concave. Holland has proved that an objective function of this firm is convex by showing that its Hessian matrix is Hermitian and diagonally dominant.[27] It is thus guaranteed that a global, rather than a myopic, minimum-cost solution can be obtained by solving the problem as formulated.

The solution to this programming problem is based on approximating each non-linear function by piece-wise linear segments and thus replacing the objective function in the original problem by a summation of ordinary linear functions of a new set of variables. These new variables are obtained by decomposing each of the original non-linear separate functions into a number of linear approximations. An example of the reformulation of the problem in this manner is given in Appendix 1. The solution is then obtained by the application of algorithms that use linear programming techniques.[28] Computer programs based on these algorithms are readily and commercially available.

The decomposition of the terms of the original objective function into linear approximations requires the introduction of a set of linear constraints which define the way in which each non-linear term in the objective function has been divided and the approximate value of the function at the boundaries of these linear approximations. It is this transformation which allows the use of the usual linear programming algorithms for the solution of non-linear problems. It requires the addition of three linear constraints for each separable function in the objective function.

The optimal solution to the above problem will yield the minimum total cost for any sewer network as well as the accompanying values of the choice variables $(\overline{E}_i - \underline{E}_i)$ and $(\overline{E}_i + \underline{E}_i)$. The first of these variables can be used to obtain the diameter of the pipe using Eq. (3) . Half the sume of the two variables gives the value of $\overline{E}_i$, the upstream invert elevation for each link in the network. This information completes the design of the system.

The suggested procedure assumes that sewer pipes are available in any theoretical size. This assumption is not a realistic one, since only a set of discrete pipe diameters are commercially available. The usual procedure in manual traditional methods of design is to choose the next highest commercially available diameter. This procedure could also be applied to the optimal solution obtained by the above method. Theoretically, this insures neither an optimal nor a feasible solution. Algorithms have not yet been developed for obtaining discrete solutions to such problems, which would be of the separable-integer variety. Holland has suggested the use of variations of a random sampling approach for the selection of a solution consisting of commercially available sizes. After eliminating the possibility of using a random sampling approach to search the entire region for an optimal discrete solution, he experimented with sampling about the continuous solution obtained from the separable programming with an iterative sampling technique. His experiments are limited to a small 7-link network, and he does not comment on the utility of applying these search techniques to larger, more realistic networks. He concludes that the random sampling technique has a high probability of selecting the best solution.[29]

For purposes of comparing different designs, obtaining order-of-magnitude estimates, or investigating theoretical cost structures and patterns, however, it is felt that the continuous solution is satisfactory. Further research into the solution of the separable-integer programming problem would lead to the development of some realistic relationships between the total costs obtained by the continuous and discrete solutions.

NETWORK AND SYSTEM COSTS

The previous sections presented an analysis of the costs of the basic components of wastewater collection networks, and suggested methods for obtaining an optimal (minimum-cost) design for both a sewerline and a collection network. The basic concern of this section is to investigate the different parameters of urban design which can be expected to influence the cost of a public utility, and by applying these parameters to empirical evidence, demonstrate the possibility of developing an analytic mathematical model of the functional relationships between these costs and parameters. The large numbers of variables involved, however, requires that a number of simplifying assumptions be made. These assumptions will be explicitly stated, as they are a necessary input to the proposed cost prediction model. They do not limit the validity and generality of the overall approach, since the resulting model can be calibrated for any other set of assumptions or actual situations.

The Analytical Framework

The cost of a public utility network is a function of a number of variables. Some of these variables are internal to the technology of the service in question, others are a function of conditions which are unique to a certain site and life style which the planner usually accepts as a given, while a third set involves factors which relate to the urban morphology. These latter factors can be looked upon as choice variables from the point of view of the planner or urban designer who is involved in planning a new town or studying the cost implications of alternative zoning and development policies. Some variables can be easily classified in the above manner, while others may fall in more than one category. Wastewater collection systems are no exception to other public utility networks, and their cost determinants can be classified into these three basic categories.

The main objective of this section is to explore the relationships between overall network costs and some of the main morphological elements. The theoretical structure of these relationships will be exploited by isolating the element in question and observing the effect of varying its value on network costs, while all other elements are controlled and kept constant. The general statement of the hypothesis will be carried a step further. It will be assumed that among the morphological elements affecting the systems' cost function, total population (or demand) and total service area are the two basic cost-sensitive elements. It is implied in this assumption that "density," which is a combined measure of the quantity of service and its distribution is space, is an important determinant of cost. It has been shown that link costs are sensitive to both link length and capacity, which is consistent with the above generalization pertaining to networks.

The Experiment

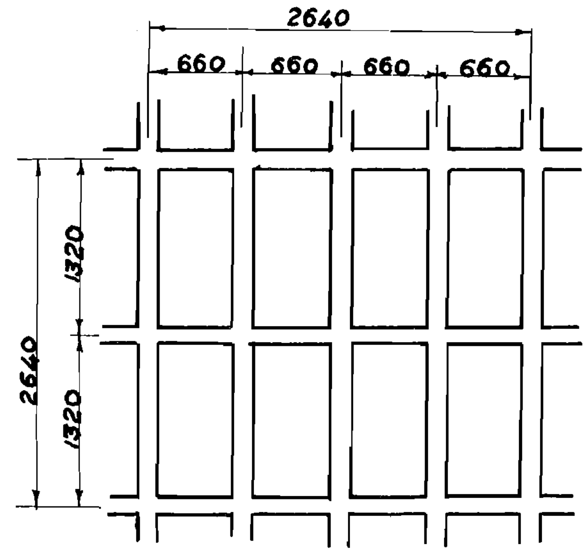
The system cost-optimization model which has been developed in the previous section, will now be used in order to design and estimate the cost of optimal wastewater collection systems for the experimental modules discussed in the aforementioned chapter. The resulting sets of optimal network costs will serve as the data base to be used for deriving overall cost functions.

A study of literature on the analysis of urban public service costs, and a review of the development characteristics of urban areas reveals that any theoretical analysis of urban development costs is frustrated by the large combinatorial problem which exists. An infinite number of possible combinations of block sizes, shapes, textures, subdivision policies and network patterns is possible. Each of these factors has some influence on the cost of development, and unless the cost-sensitivity of each factor can be isolated, no meaningful theoretical cost model can be developed. An experimental module has to be designed in order to simulate a condition where some of the more significant variables can be manipulated and studied.

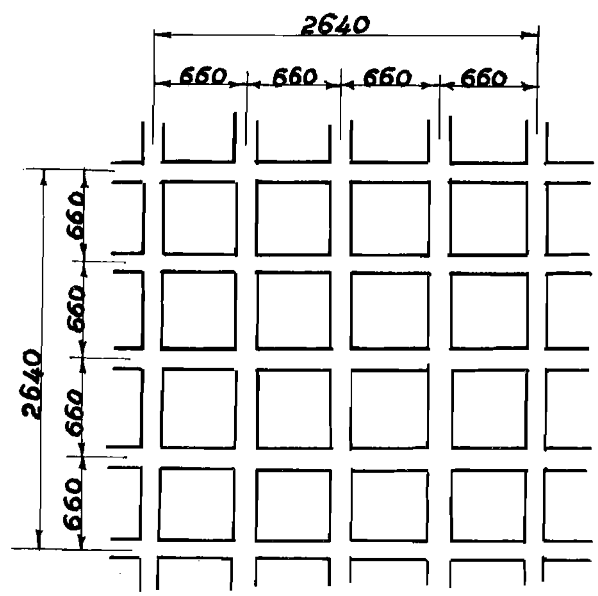
A careful review of the components of urban morphology and a study of existing investigations of the relationship between urban descriptors and service costs has suggested that an experimental module of a given shape and size, which can be subdivided into blocks of different size and on which different population densities can be superimposed, might be used to advantage. Further simplifying restrictions will have to be imposed regarding terrain, block shape and network patterns. The chosen module has the following characteristics:

1. Module Shape and Size: A square experimental module with an area of 160 acres or 0.25 square miles was chosen. The dimensions of the square are 2640 x 2640 feet. This shape and size combination has special significance in the study of settlements, since it represents the quarter section: a traditional unit of regional land subdivision. This unit has been used by researchers to investigate the costs of urban public services and utilities.[30]
2. Block (Cell) Shape and Size: The experimental module will have to be subdivided into blocks of a given shape and size in order to insure the comparability of the results. The rectangular grid pattern is by far the most common. Stone has stated that "unfortunately it is impractical to carry out theoretical studies on an adequate scale except with a parallel block layout."[31] Most studies on urban costs with spatial implications assume a rectangular "grid-iron" pattern.[32] The 160-Acre experimental module will thus be divided in two distinct fashions:
   A. into sixteen 10-Acre square blocks, and
   B. into eight 20-Acre rectangular blocks.

   Block areas are given between the centerlines of adjacent streets. These two basic forms of the module are shown in Figure 2 .
3. The simplifying assumption of flat terrain will be made. This assumption can be easily relaxed as discussed in other sections of this study.
4. Texture: A range of uniform densities will be superimposed on each of the two variations of the experimental module. The densities chosen for the analysis are 10, 25, 50, 100, 150, 250, 500, 750 and 1000 persons per gross residential acre. At these respective densities the 160-Acre module will house 1600, 4000,



(a)
160-Acre Module, 20-Acre Blocks



(b)
160-Acre Module, 10-Acre Blocks

Figure (2)

The Experimental Module

8000, 16000, 24000, 40000, 80000, 120,000 and 160,000 persons.

The following assumptions are made regarding the technological and site vectors described above:

1. All sewers are designed as circular pipes flowing full.
2. Roughness factor for all pipes is 0.0013.
3. Minimum and maximum allowable velocities are 2 and 10 feet per second respectively.
4. Minimum allowable pipe diameter (D_{min}) is 8 inches.
5. Minimum allowable pipe cover is 5 feet.
6. Flat terrain.
7. Local conditions such as soil, weather, location, relative factor prices, etc., are reflected by the cost function

   $$\text{Cost per foot} = 1.403 + 1.499\ D^2 + 0.019\ X^2$$

   where D and X are the pipe diameter and invert depth below ground level, both in feet. The cost per foot is ex pressed in Dollars. This function is based on the findings of Chapter Two, for laying plain concrete pipes in open-cut trenches in common soil.
8. Average per capita wastewater generation Q_{av} = 100 gallons per day.
9. Peak (design) flow $Q^* = 3.45\ Q_{av}^{4/5}$ for $1000 \leq P \leq 412000$

   $= 5\ Q_{av}$ for $P < 1000$
10. A given treatment plant (sink) location at the corner of the experimental module.

The use of the separable programming techniques in the design and estimation of optimal wastewater collection networks has been discussed above. The application of this method prerequires a network layout. As has been pointed out before, it is apparent that in the case of a regular grid pattern the "perpendicular" and "fan" layouts identified in Figure 1 are viable ecomical alternatives. They both allow flow from a certain location to the sink through a most direct route. In order to evaluate these two types of layout, a test was run on the 20-Acre module, in which the optimal costs of the two layouts were estimated under different density conditions. While the cost of the "perpendicular" layout has been found to be consistently less than the cost of the "fan" layout, the difference in cost is hardly significant. The two layouts can, for all practical purposes, be considered equivalent.

The optimization model, which was presented in Chapter Three applies to links which are running full. Under-utilized links must have the minimum allowable diameter, associated with a slope that would ensure a self-cleansing velocity. Such links must be separately designed and estimated, since no trade-offs between diameter and slope are possible. Under our assumptions the cut-off demand for service below which no optimization is necessary is about 1000 persons per link (or 100,000 gallons per day). At densities of 150 and 200 persons per gross acre or more, each and every link in the 20 and 10-Acre block modules respectively, will have a total load of not less than 1000 persons, and thus be fully-utilized to its design capacity. At lower gross densities the capacities of some links will be only partially utilized.

Minimum system costs were estimated for each of the two experimental modules developed at each of the nine population densities mentioned above.

In order to obtain some further data on the effect of the size of the area to be developed on network costs, a study of the feasibility of aggregating experimental modules into larger units was undertaken. A simplified approximate method which can generate optimal cost estimates for combinations of 160-Acre modules on the basis of the basic module cost was developed. Using this method, estimates were obtained for a 640-Acre square site (one square mile), subdivided into 20-Acre building blocks. The whole area was drained to a single sink located at one corner. Costs were estimated for various gross population densities.

The Basic Cost Model

The cost data which have been generated for the two experimental modules and the four-module composite area, were analyzed with the purpose of searching for underlying patterns that could be formulated into a simple and useful analytic model. This analysis has revealed a definite split of the data into two discernible subsets: one representing the costs of networks consisting of links whose design capacity has been fully-utilized, and another representing the costs of networks consisting of both fully and partially utilized links. As mentioned above, the cut-off points occur at densities of 150 and 200 persons per gross acre for 20 and 10-Acre building block subdivisions respectively. In reality, this cutoff point can also be attained by considering networks consisting of main lines only, and excluding all under-utilized links.

Regressions were fitted by the method of least squares to the three sets of data, using gross density as the independent variable. Consistent patterns which were identified among the two sets representing the 20-Acre subdivision have led to their treatment as a single set of data, with the total area being considered as a second independent variable. The results of these regressions show a conclusive relationship between cost and these variables. The relationship which resulted from applying regression analysis to the set of costs of fully-utilized networks can be expressed in any of the following three equivalent forms:

$$\begin{aligned}\text{Network Cost} &= a\, A^{\alpha}\, D^{\delta} \\ &= a\, A^{\alpha-\delta}\, P^{\delta} \\ &= a\, P^{\alpha}\, D^{\delta-\alpha} \qquad (16)\end{aligned}$$

where

a	is a constant
A	is the overall area in acres
D	is the gross population density in persons per acre
P	is the total population

α is an exponent whose value is greater than unity

δ is an exponent whose value is smaller than unity

and d is the density of population beyond which all links are utilized to their design capacity.

Specifically, for the experimental module subdivided into 20-Acre blocks, this cost model is

$$\text{Network Cost} = 41.91\ A^{1.17}\ D^{0.30} \quad \text{for } D > 150 \text{ ppa.}$$

$$(R^2 = 0.999) \qquad (17)$$

and for the module subdivided into 10-Acre building blocks:

$$\text{Network Cost} = 46.35\ A^{1.17}\ D^{0.30} \quad \text{for } D > 200 \text{ ppa}$$

$$(R^2 = 0.994) \qquad (18)$$

Both of these regressions indicate highly significant statistical relationships, i.e., less than the 0.001 level of significance. Figures (3) and (4) show the scatterograms and the fitted curves for the 20-Acre experimental modules, plotted on both arithmatic and semi-logarithmic scales.

These cost functions are characterized by an increasing cost as both the density (or population size) and the area increase. This absolute increase in cost, however, is associated with a decreasing rate of growth with respect to density (or population size), keeping the area unchanged. It is also associated with an increasing rate of growth as the area (or total population) is increased, keeping the density unchanged. In other words, these cost functions confirm that economies of scale exist with respect to both population density and size, given a fixed area of service. This indicates a situation of continuously decreasing unit costs, as the volume of service

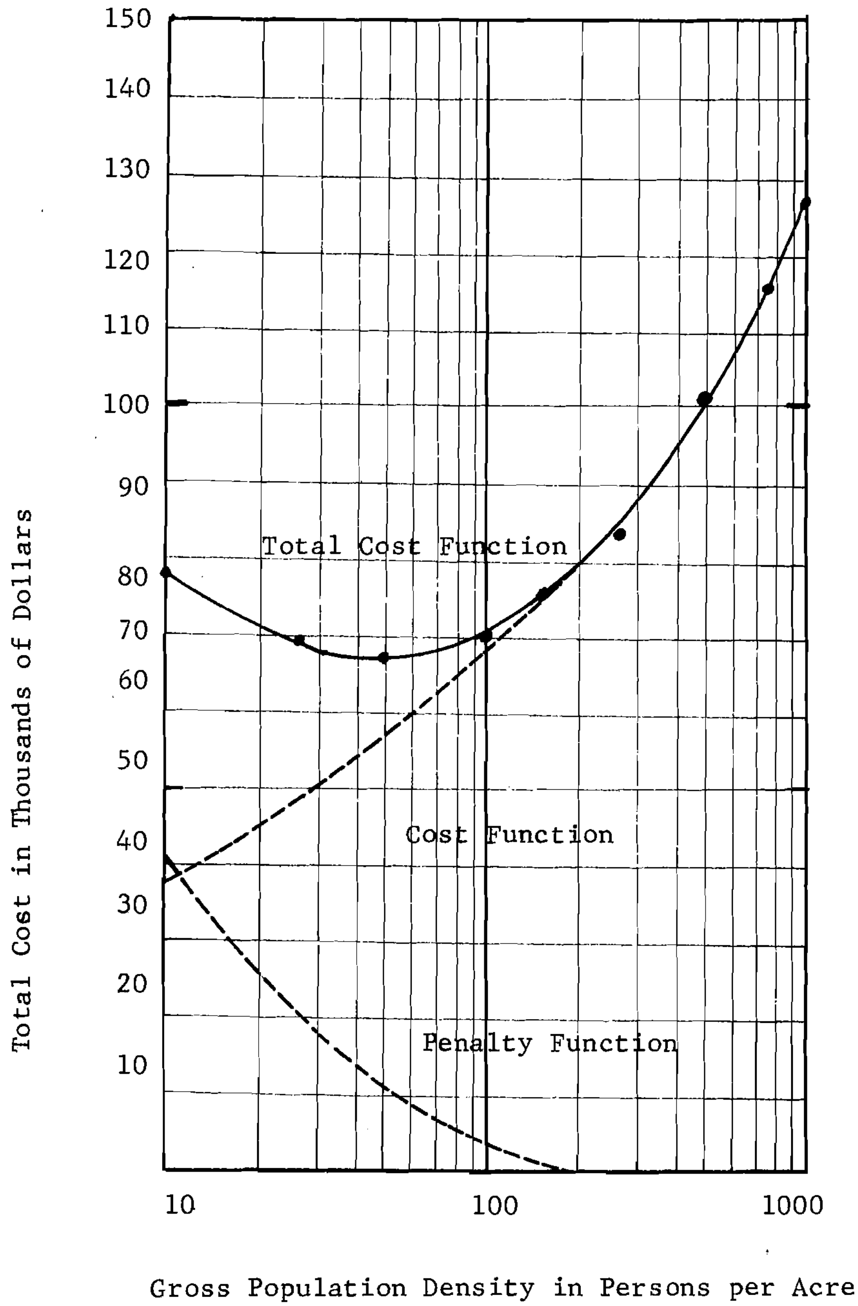


Figure (3)

Cost and Penalty Curves Experimental Module Divided into 20-Acre Blocks.

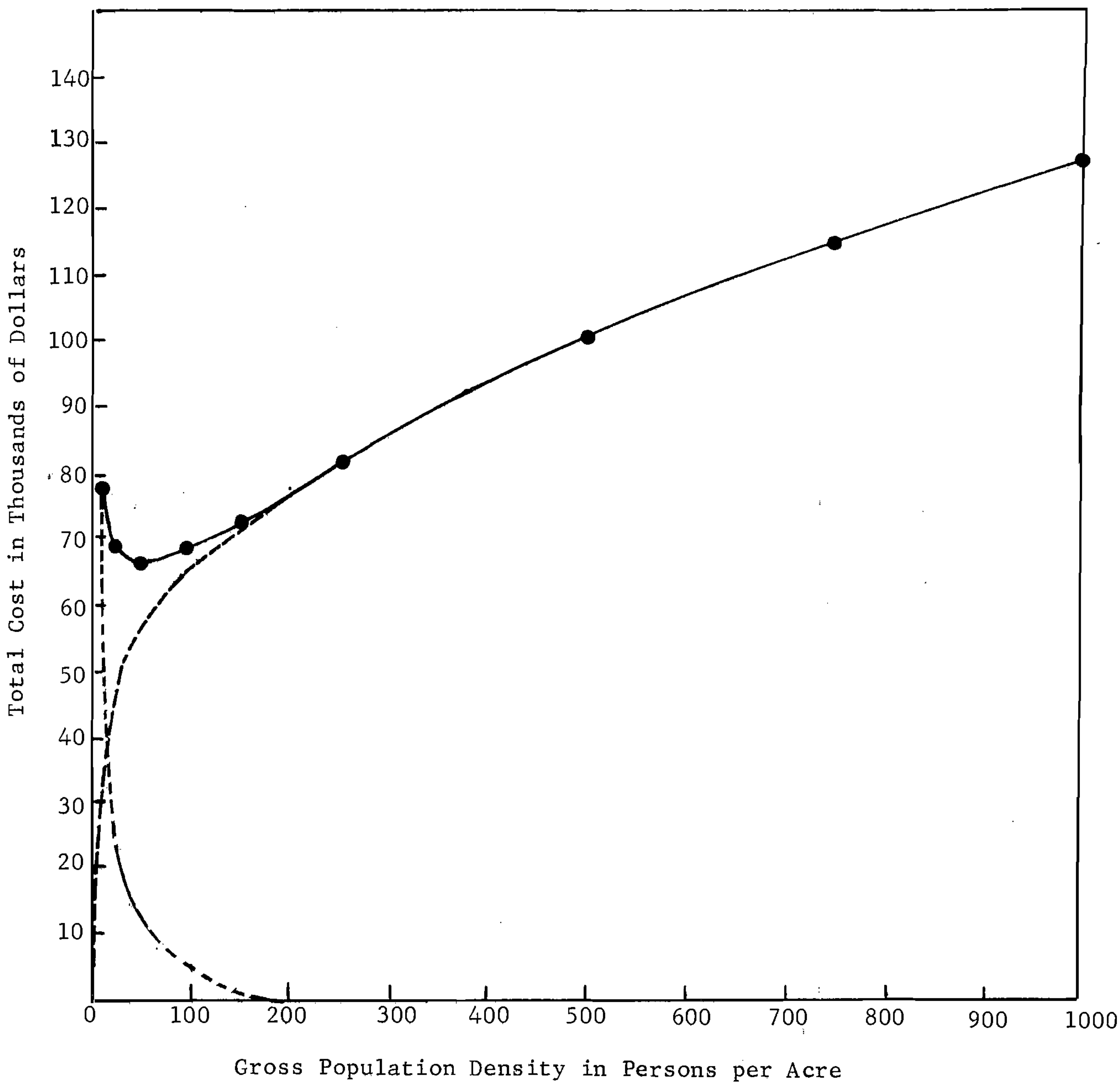


Figure (4)

Cost and Penalty Curves
Experimental Module Divided Into
20-Acre Blocks

supplied is increased within given boundaries. The positive fractional values of the exponents of both the population size and density terms in Equation (16) above, and thus the negative fractional exponents which result when average of unit cost functions are derived from these equations, indicate this relationship. Diseconomies of scale, on the other hand, are indicated whenever a larger area having the same population density is served by the same sink. The cost per capita in this case will be continuously rising as the service area is expanded, and as new areas are developed at the same population densities. In contrast to the situation discussed above, the exponent of the area term in the above equations has a value exceeding unity. Both the average and marginal costs of adding an acre at the same population densities are growing at a diminishing rate.

The basic functional forms given in Equation (16) above are limited to networks composed of links flowing at their full capacity. It can be hypothesized that in the absence of any design standards, such as these limiting the minimum allowable pipe diameter, these relationships could be seen to continue to hold at the lower extreme of population densities as well. Technological constraints, however, in the form of standards and design criteria, do not permit the diameter of the pipe to fall below an allowable minimum, with the result that minimum-size pipes may be designed to flow at less than half of their capacity and thus may require larger slopes. These constraints are reflected in an increase in cost at lower densities. This increase, over and above the costs estimated by the theoretical model given above, will be designated by a "penalty function." A regression was fitted to the respective penalties obtained by subtracting the costs calculated by Equations(17) and(18) above from the actual optimal costs. The regression has indicated another powerful predictive relationship of the following general mathematical form:

$$\text{Penalty} = A^{\alpha} (bD^{-\gamma} - c) \qquad (19\text{ a})$$

$$= bP^{-\gamma} A^{\alpha+\gamma} - c\, A^{\alpha} \qquad (19\text{ b})$$

$$= P^{\alpha} D^{-\alpha} (bD^{-\gamma} - c) \qquad (19\ c)$$

for $D \leq d$

where b and c are the regression coefficient and constant, respectively, γ is an exponent with a value of less than unity and all other symbols are as defined above.

The results of the regression for the 20-Acre block subdivision are given by:

$$\text{Penalty} = A^{1.17} (418\ D^{-0.3} - 92) \qquad \text{for } D \leq 150 \text{ ppa}$$

$$(R^2 = 0.992 \qquad (20)$$

and for the 10-Acre subdivision:

$$\text{Penalty} = A^{1.17} (813\ D^{-0.40} - 111) \qquad \text{for } D \leq 200 \text{ ppa}$$

$$(R^2 = 0.991) \qquad (21)$$

Statistics describing the power of the regression are given in Table (3). All the results are significant at the 0.001 level. The penalty curves as well as the overall cost curves for the 20-Acre subdivision are also shown in Figures (3) and (4).

The total cost functions obtained by adding the theoretical and penalty functions described above, indicate that there exists a gross population density at which a given area can be developed, in order to result in a minimum total or per acre cost of wastewater collection facilities. By virtue of the simplicity and differentiability of these functions, it is possible to calculate this minimum-cost density by differentiating and equating to zero. It can be shown to be about 40 and 88 persons per acre for the 20 and 10-Acre subdivisions, respectively. A premium has to be paid, if the above two modules are to be developed at either a higher or a lower density than indicated by these theoretical optima.

The limited data on which this study is based gives a numerical indication of the effect on network costs of an increase in the area of the average building block. A larger linear footage of pipe is needed to completely service an area which is divided into smaller building blocks. An inverse relationship can thus be expected to exist between network cost and block size. This relationship is clear from equations (20) and (21). These findings substantiate the contention that beyond a certain level of population density, economies of scale can be attained.

Extensions of the Basic Model

The studies reported in this section have demonstrated the possibility of developing a network cost function on the basis of a design which is prepared in line with a cost-minimization policy. The effect of such morphologic characteristics as population size and density, area of development and average area of subdivision were investigated. The shape of the development with respect to the location of the treatment plant is another factor which is expected to have a bearing on the overall cost function. A central treatment plant location, for instance, would be expected to result in savings in network costs, albeit at the expense of more expensive land and undesirable environmental consequences. The areal distribution of the population is another factor which influences the cost structure. An analysis has been attempted, whereby four different spatial distributions of a given population were superimposed on the experimental module, and optimal cost figures were generated. The interference of the density distribution effect and the effect of the penalty function has made it very difficult to derive a simple overall mathematical model. It should be possible, however, to generate such a model for networks comprising totally of fully utilized links.

The other two factors which have been assumed to be constant throughout the foregoing analysis, namely technological and site characteristics, can be easily varied within the framework described above. This will allow their inclusion into the model and the evaluation of their effect on overall cost.

As the area served by the network grows, two additional elements might have to be considered in the network cost function, namely: pumping stations and additional feeder lines. The effect of these two factors on the basic cost functions will be discussed below.

As the system becomes larger, and thus the individual lines deeper, the designer will have to choose between extending the system at still costlier excavation and installing pumping stations. These stations have the effect of reducing the depth of the system. They can either release the flow to a gravity sewer or to a pressure pipe. The latter is usually the case when the flow is being transmitted long distances for ultimate disposal. The decision, in this case, is made by comparing the cost function of the proposed line with that of a comparable gravity line, and choosing the most economical of the two. Transmission lines constitute an additional term in the overall service cost function, but they do not affect collection network costs, unless they are gravity lines performing a collection function as well. In this case the pumping (lift) station has the effect of splitting the total service area into two sub-areas, serviced by two independent networks connected at the lift station. A completely new network starts downstream of the lift station. The cost function of this network is similar to that of the first network. It differs from it in only one respect: it is constructed at shallower depths. Excavation costs per foot of trench increases at an increasing rate as the depth increases. As the downstream system is raised, the depth of excavation is the only variable which undergoes change. As the area served by downstream network grows, the savings resulting from decreasing the depth of the system increase exponentially.

A new cost function for the overall system can be derived from the original function, by subtracting this excavation saving exponential function and adding the cost of installing and operating the pumping station. The cost of pumping the flow accumulating from a network is a function of the area served, the population density of the area, the criteria used in pump design and the head which the pump is required to lift the sewage. Pumping costs do exhibit economies of scale and it has been shown that the capital cost of pumping 10 mgd is only about 4 times as much as that of pumping 1 mgd, at a head of 50 feet.[33] Figure 7 shows a set of conceptual curves for a case where the areas served by the first and second networks are A_1 and A_2 respectively. It is clear that the second network must have an area in excess of A_3 in order to justify the split in the system resulting from the additional lift station.

The second effect on large system cost is that which results from the fact that large trunks (in excess of about 3 feet in diameter) are not usually used as collecting sewers. Areas in which such lines pass are ordinarily served by a parallel collection line, connected to the major main at one or more points. The additional cost of these parallel collectors can be assumed to be a fixed amount per unit of length of trunk lines with a diameter in excess of 3 feet. This length is expected to increase slightly as the area served increases. The effect of this factor is superimposed on Figure 5, where it is assumed that the first 36" line appears after an area A_4 has been served. The exact nature of each of these two effects on network cost functions can be ascertained within the context of a given system.

Integrated Service Costs

Network costs are only part of the total service picture. The other part consists of the costs of nodal facilities, namely sewage treatment plants. These costs are independent of the spatial parameters affecting network costs. They are a function of the size of the plant and the degree and type of treatment.

A number of studies have been made of the costs of sewage treatment plants. A concensus exists that all treatment processes exhibit significant economies of scale. It has been found that the nature of the cost function is such that both average and marginal costs decline as the size of the plant increases, and that the marginal cost of treatment is considerably lower than the average cost. These findings have led some analysts to conclude that there exist

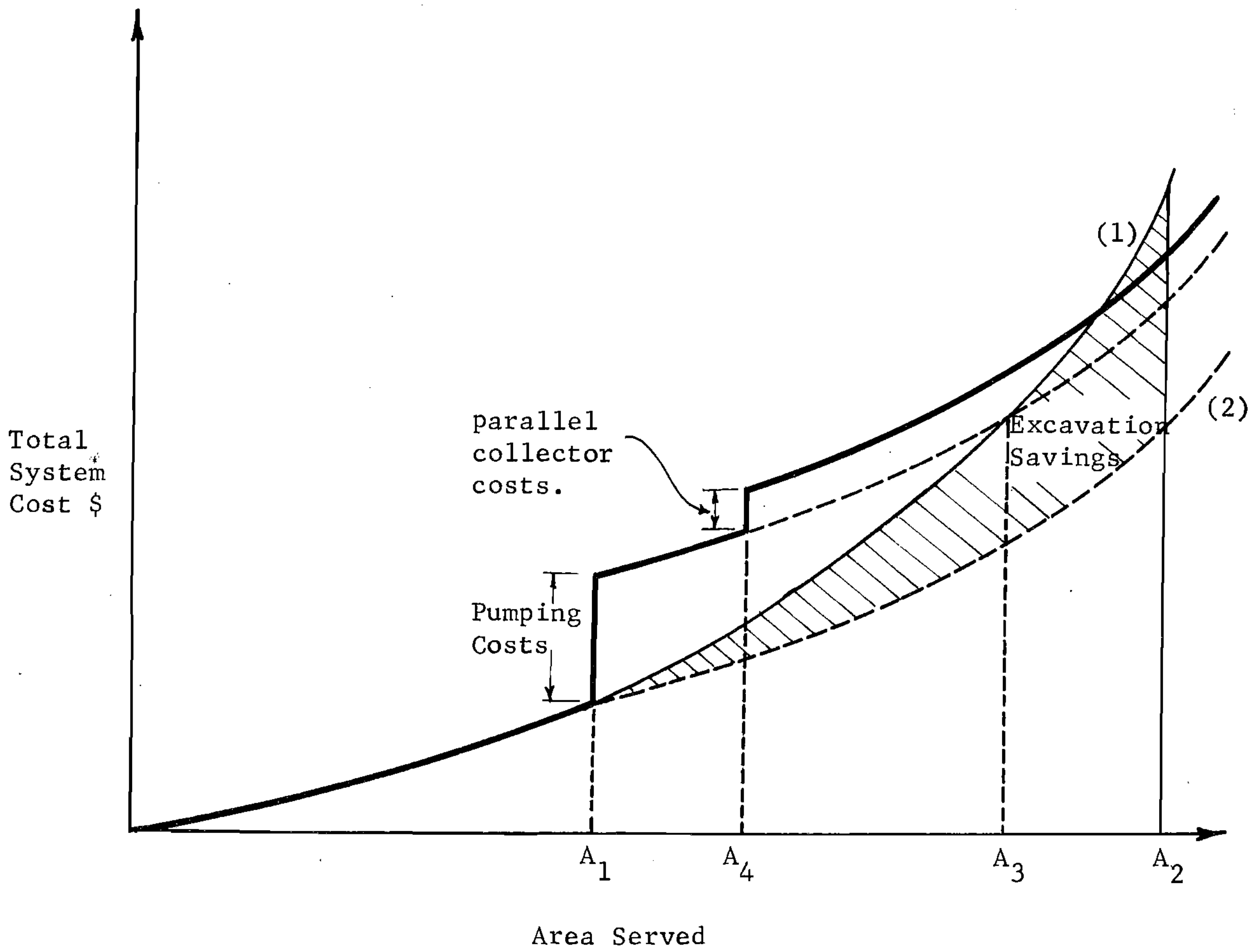


Figure (5)

The Effect of Lift Stations and Parallel Collector Lines
on Network Cost Functions

substantial economies of scale yet remaining at facilities larger than those which have been studied.[34]

Most studies on the economics of wastewater treatment facilities have concluded that the cost functions of the facilities can be expressed by the simple general format

$$\text{Total Cost (in Dollars)} = rP^{s}$$

where P is the population served (in persons) (2-2)

r is a coefficient

and s is the economy of scale factor with a value of less than unity.

This relationship is frequently expressed in logarithmic form. Shah and Reid have noted that economies of scale affect the unit construction cost of different types of secondary treatment facilities by a factor which changes very little from type to type.[35] Table (4) lists cost functions which are calculated by reducing the relationships obtained by the U. S. Public Health Service and by Robert Smith, to the general format of equation (22) . Smith's values are the adjusted average of four treatment plant cost studies. The validity of all of these studies as well as that of most other similar studies does not go beyond treatment plants serving a population in excess of 100-200 thousand persons. Few studies included plants serving as much as one million persons. Variations of the format given above exist. Regional, temporal, local and technical differences account for variations in the values of the parameters obtained by the different researchers. It should be clear that the cost functions list in Table (2) represent two different sets of conditions, and thus are not necessarily readily comparable. These functions are listed to indicate the general format and order of magnitude of the relationships involved.

Equation (16) has suggested the existence of diseconomies of network scale, as the population served increases at a given density, while equation (22) implies that economies of treatment scale exist

Table (2)

Total Cost Functions
of Treatment Facilities

Type	U. S. Public Health Service	Robert Smith
Primary Treatment	$480\ P^{0.69}$	$530\ P^{0.70}$
Trickling Filters	$760\ P^{0.66}$	$320\ P^{0.80}$
Activated Sludge	$450\ P^{0.72}$	$440\ P^{0.77}$

Sources: Calculated from information published in:

1. U. S. Public Health Service, Modern Sewage Treatment Plants - How Much Do They Cost? (Washington, D. C., Government Printing Office, 1964).

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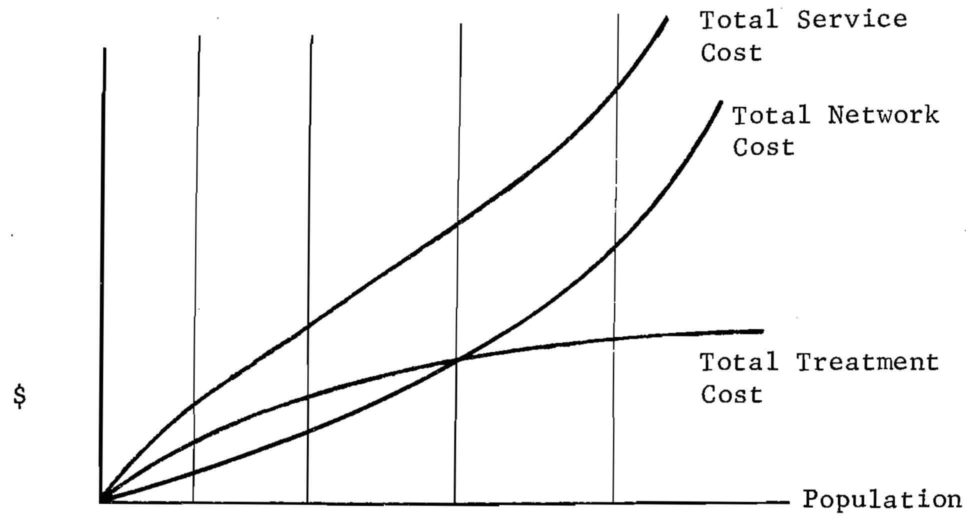
as the population served increases. In the planners' search for an overall minimum cost system, the tradeoffs between nodal and network costs in an integrated construction cost model must be investigated. Such an analysis has not been possible because of lack of information concerning network costs. In attempting to develop a complete economic analysis of the system, however, a problem arises with respect to the ranges of validity of both treatment and network cost functions. Both have been derived and have been shown to hold within a limited population range. The following analysis assumes that these cost functions can be extrapolated to cover the untested ranges of higher populations.

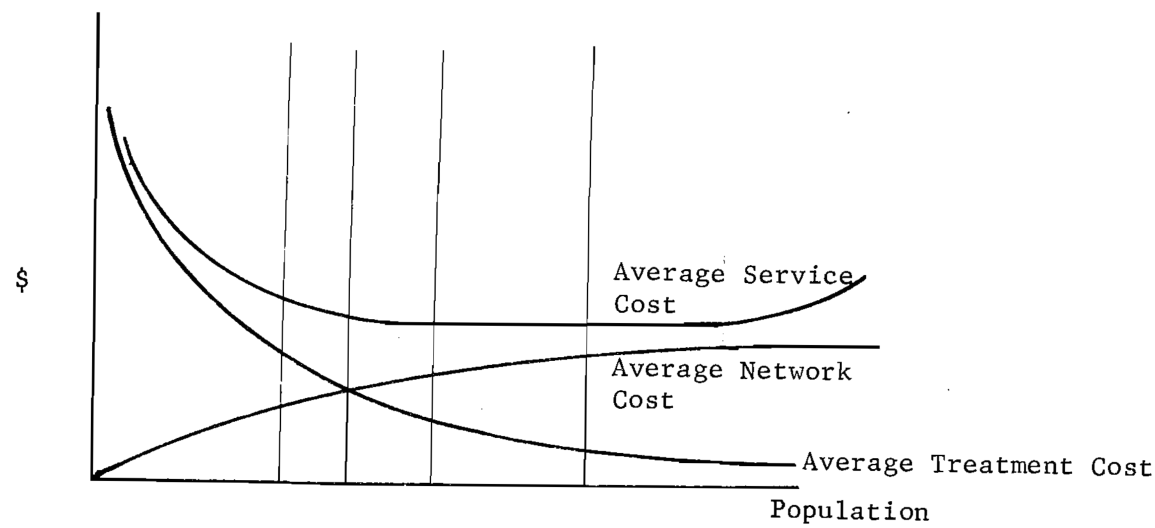
Treatment plant operation, repair and maintenance costs can be classified as either long-run or short-run. The first type is that which is affected by the size of the plant and should thus be an input into the system planning process. The second is a function of the volume of flow passing through the plant in a given period of time, once it is put in operation. Operation, repair and maintenance costs could be included in the analysis by adding the present worth of their total outlays over the life of the facility to the present worth of the network and nodal construction costs.

The preceeding analysis can be summarized in a fashion analogous to the familiar theory of cost in microeconomics. The long-run planning decisions concerning the provision of a wastewater collection and treatment service are not dissimilar to those facing an enterpreneur embarking on a major investment.

The long-run total cost function for the complete system can be derived by adding the total cost functions for the network and plant components. The average and marginal costs for each component and for the total system can also be derived. These relationships are shown in Figure 6. The average total cost has the usual U-shape. The large fairly flat portion indicates that diseconomies of scale may not be incurred until the size of the system becomes very large. An example calculation of this size using Robert Smith's estimating formula for activated sludge treatment facilities and the network cost model developed above for a 10-Acre subdivision as given by equations



(a) Long-run Total Cost Curves



(b) Long-run Average Cost Curves

Figure (6)

Long-run Cost Curves

(18) and (21) , has been performed using gross development densities of 10, 25, 50, 100 and 150 persons per acre. The total population served at the minimum average cost point are given in Table 3. These calculations do not consider the effects of pumping or parallel collection line on the network cost functions.

While the figures used in the above exercise are by no means universal, and should be only considered as part of an hypothetical example, some general tentative conclusions can be drawn from them. The most obvious conclusion being that the minimum-cost size of a wastewater collection and treatment service is very sensitive to the density of development in the area being served. This would suggest the need for refining the basic network cost model. The value of such refinements, however, is partly offset by the fact that the integrated unit cost curve contains a fairly large flat portion. For the case of a gross density of 25 persons per acre, for example, the minimum per capita cost is $69.6 for a population of 125,300 persons, and it goes up to about $69.7 at populations of 100,000 and 200,000. While even such small variations in unit cost can result in substantial savings in the overall capital cost of the project, the marginal sensitivity of unit cost to size around the minimum suggests and determines the limits on time, effort and resources which should be spent on determining the size of the optimal service area.

It should be noted, that the hypothetical figures derived above, imply the assumption that the economies and diseconomies of scale which have been observed in treatment and network costs continue *ad infinitum*. The fact that this is not necessarily true, as shown by such effects as pumping and parallel collectors in the case of networks and by technological limitations and possible diseconomies in the case of treatment plants, emphasizes the need for further investigating the behavior of the basic cost functions.

Table (3)

An Illustrative Example
of Hypothetical Population and Area
Resulting in Minimum Per Capita Costs

Gross Population Density	Minimum Unit Cost of Service	
	Population	Area (Sq. Miles)
10	4,500	0.70
25	125,300	7.83
50	1,275,600	39.84
100	10,366,200	161.97
150	32,337,000	336.84

CONCLUSIONS

This study has been directed toward the exploration of the interface between public service systems and patterns of urban developments. In doing so, it has focused on a single service comprising of a significant public works component, namely urban wastewater collection. Patterns of urban development determine the demand function for a public service, while the technology of providing the service determines its supply function. A planner seeks to reconcile supply and demand in the most socially desirable manner. One common approach to gauging social desirability in a resource conscious society is to find a solution resulting in the minimum cost per capita. This approach has been critized on the grounds that it implicitly assumes outputs to be constant, and thus is only concerned with the explicit minimization of inputs.[36] The assumption of fixed outputs is not an unrealistic one in the context of urban public utilities, where the major concern is with the provision of adequate service. Within such a context, the provision of a deficient service often causes large indirect costs to users, and is thus unacceptable, while the provision of an excessive supply might not produce any additional benefits.

In exploring the interface between service technology and urban structure, the study has focused on two basic concepts which are considered to be central to the rational design of integrated urban systems. These two concepts are:

1. The concept of "optimization" which generally describes the process of obtaining the "best" solution. For the purposes of this study, the "best" solution is defined to be the "minimum-cost" solution. No theoretical generalizations can be made about an urban service unless a normative most efficient solution can be obtained. Optimization is inherent in the development of "production functions" which

define, on the basis of a given technology, the maximum amount of output that can be obtained from a given quantity of input.

2. The concept of the "cost function" which maps the minimum monetary input required to produce a given quantity of output. A cost function is generated from the production function, and implies that the problem of optimum input combinations has been solved. The cost function can be written in terms of units of demand, such as population, and area or in terms of intensity of demand such as population density. Whichever the case, the cost function bridges the gap between service technology and urban form and structure. If it could be derived, then it should prove to be a useful tool for the planner, designer and decision-maker in the public sector.

The following paragraphs will briefly describe the methodology and findings of the study. The practical implications of these findings and suggested areas of extensions and further research are also outlined.

The Optimization Model

The methodology of urban wastewater collection network design has not undergone any basic changes in a long time. It basically involves laying out a network along existing and proposed street systems, and then designing each link in the network as a separate element. The design is based on certain hydraulic relationships and is constrained by design criteria and standards. This methodology seeks to approach optimality through the "professional judgment" and "experience" of the designer. It does not rigorously incorporate the economics of the system into the design process. The first problem which thus had to be attacked was that of consciously approaching the optimum in the design process itself. This involved a detailed analysis of the technology and economics of flow in gravity sewers. Data on the cost of construction of sewerlines were collected. They were found to fit a mathematical model composed of a linear combination of the squares of conduit diameter and depth of excavation. On the basis of this relationship as well as the hydraulics of flow and design criteria, a series of cost functions was developed to indicate the cost behavior of a single sewerline accomodating different levels of demand. The mathematical derivation of these functions is, of course, based on a optimal design.

The next step has been to investigate the question of system optimality. These functions were used as a basic input to a non-linear programming model which generates a minimum-cost design and a cost estimate for a chosen wastewater collection network or for a set of alternative network layouts. This model has been adapted for solution by commercially available computer programs.

In an era of increasing demand for urban services, and the accompanying phenomena of increasing costs and scarcity of resources, the importance of the existence of such an operational tool can be hardly underestimated. It:

1. Allows the testing of alternative wastewater collection network and system designs. The time and costs involved in designing and estimating such systems have hitherto limited this activity to a very small number of alternatives at the very best. By far the most common practice involves designing and estimating a single system. The analysis of a larger number of alternatives can conceivably result in appreciable savings. Alternatives can be defined in their spatial, temporal or technological contexts.
2. Allows the analysis of the wastewater collection cost implications of alternative urban designs and the mapping of network cost functions for varying design parameters.
3. Allows the rigorous incorporation into the design process of such important local consideration as relative factor prices, different technological capabilities of the construction industry, subsurface conditions and other economic, geographic

and technological features of the service area.

4. Allows the investigation of the cost implications of wastewater collection design standards and criteria. This includes such requirements as the minimum and maximum allowable velocities, the peak/average flow ratio, the minimum allowable pipe diameter and the minimum allowable pipe cover.
5. Allows the investigation of the cost implications of alternative technological options, such as the use of different pipe material, and the installation of lift stations and parallel collector lines.

The present research suggests a sequential network design process, whereby a "good network layout is selected, and the optimal design for this network is generated. Further research is warranted into the problem of simultaneously selecting the minimum-cost layout and design. Further possible refinements include the solution of the design problem as an integer non-linear programming problem. No algorithms are as yet available to make such a refinement possible.

Network Cost Functions

A second basic output of this research has been to demonstrate the possibility of generating wastewater collection cost functions for a given area on the basis of local information. These cost functions map the locus of all points relating minimum network cost and some parameter of urban structure. When this locus was mapped for cost vs. population density, it was found that a minimum-cost density existed (see Figure 3). This is true because each system is composed of both fully and partially utilized links. The first type exhibits economies of scale, while existence of the second adds to this basic cost function a penalty function which decreases as the density increases. The minimum-cost dinsity seems to lie at values which are of common practical significance and to increase as the average building-block area decreases (or as the street density in feet per acre increases).

Total network costs for a given population density are shown to exhibit diseconomies of scale as the total area (or population) served is increased. This indicates a continuously increasing unit cost as the size of a constant-density service area is increased.

The basic cost functions which have been developed for both links and networks spell out the nature of the complementarity that exists between the effects on cost of the two outputs of wastewater collection network, namely: quantity of flow and area of service. At a given level of investment tradeoffs exist between these two measures of system output.

The fact that network cost functions can be generated for a given area has a number of implications for the planning, design and management of these networks. Among these implications are the following:

1. If the population density in an area for which a mass wastewater collection network is desired is less than the minimum-cost density, then serious consideration should be given to non-conventional cost-saving methods. These include the possibility of designing the system for some higher density or relaxing the standard velocity constraints. The benefits derived from such actions must be weighed against the cost of some remedial action such as flushing and cleaning.
2. If both existing and future cost-density points lie on the rising portion of the curve, then the designer must choose his planning horizon in such a way as to balance the economies of scale and the cost of money effects. Such a balance will determine the optimal planning horizon, and thus the economical amount of additional capacity to be built into the system.
3. While the development of cost functions can contribute to the efficient planning and design of wastewater collection network, it can also contribute to solving the problem of finding a formula for the equitable allocation of costs among the bene-

ficiaries of the system. The existence of an operational methodology for generating designs, estimates and cost functions allows for the partitioning of the total network cost between present and future users, as well as among users with different locations and waste generation rates. Such a three-dimensional split of cost can for the basis for an equitable distribution of costs among system users, both present and potential. It also allows a fresh look at the multiplicity of existing wastewater disposal pricing structures.

4. While it is realized that the cost of wastewater collection is by no means the only, let along the most important, consideration is land use planning, yet a rigorous analysis of the cost implications of various land use patterns is vital. Cost functions such as those developed in this dissertation form an interface between land-use and public utility network planning. They provide the tools for analyzing some cost and performance implications of land-use and zoning policies.

Further research into the characteristics of network cost functions should also be directed towards the quantitative determination of the changes in network costs resulting from the installation of lift-stations which incorporate such factors as the effect of various spatial density distributions of demand will conceivably add to the practical value of the findings reported herein.

Integrated Service Cost Functions

While no previous analysis of network cost functions existed, similar cost relationships have been empirically established by a number of investigators for central treatment facilities. These two functions can be added to obtain an integrated service cost function.

While per capita network costs have been shown to increase at a decreasing rate as the area (or population) is increased at a given density, per capita treatment costs indicate a continuous decrease with increasing size. The resulting overall unit cost curve has the conventional u-shape, which indicates the existence of a minimum-cost size for a wastewater disposal service area. The large flat portion of the curve suggests an insensitivity of unit cost to size around the minimum. This study also shows that this size is highly sensitive to the population density in the area being served. The minimum-cost size of the service area drops appreciably in low density developments. This suggests the need for incorporating this type of analysis of the alternative courses of action in regional wastewater collection and disposal planning studies. At high densities, however, the economies of treatment scale tend to almost counteract the diseconomies of network scale, with the resulting minimum-cost size assuming fairly large theoretical values.

The integrated service cost function can also be used to determine whether a mass wastewater collection network is economically justifiable. This involves the development of the cost function of individual disposal systems (such as septic tanks). The point of intersection of the two curves determines the cut-off point beyond which a mass system is viable. Similar analyses of new technologies can also be undertaken.

Finally, it should be pointed out, that the concepts which have been pursued in this report and applied to the urban wastewater collection function, should be equally applicable to other urban public utilities and services. The comprehension of the cost behavior of the basic elements which make up the intermingling fabric of urban service networks and terminals is a necessary prerequisite to the understanding of the interaction between land-use and urban services. Such an understanding is a vital component of the rational approach to urban systems planning.

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