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Continuous Thickening of Non-Ideal Suspensions

Richard I. Dick
and
Ali R. Javaheri

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UNIVERSITY OF ILLINOIS
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2535 Hydrosystems Laboratory
Urbana, Illinois 61801

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ABSTRACT

CONTINUOUS THICKENING OF NON-IDEAL SUSPENSIONS

The purpose of this study was to investigate the steady state continuous thickening of non-ideal suspensions such as sludges from water and wastewater treatment plants. Suspensions of high grade calcium carbonate, activated sludges, water softening sludges, and suspensions of fine glass beads were used. The suspensions were thickened in a closed, continuous, pilot, thickening system.

The solids flux theory was used successfully for predicting performance of the steady state continuous thickener from batch settling velocities of the suspensions. For the optimal performance of the thickener, effective stirring of concentration layers near the underflow level was found to be an absolute necessity. This was to prevent dilute solids from reaching the thickener bottom. Homogeneous distribution of the feed over the area of the tank was also essential.

The feed concentration generally did not affect the thickening function. However, the interaction between thickening and clarification functions of the tank was established through the magnitude of feed concentrations. As the feed concentration decreased with fixed solids loading, the overflow velocity increased, and the clarity of overflow generally deteriorated.

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LIST OF FIGURES

NOTATION

Symbol	Quantity	Dimensions
A	Area	L^2
C	Suspended solids concentration	F/L^3
C_o	Initial suspended solids concentration	F/L^3
C_{L1}	First critical or limiting suspended solids concentration	F/L^3
C_{L2}	Second critical suspended solids concentration	F/L^3
C_f	Feed suspended solids concentration	F/L^3
C_{of}	Overflow suspended solids concentration	F/L^3
C_u	Underflow suspended solids concentration	F/L^3
D	Dilution factor (i.e. weight of liquid per unit weight of solids)	-
D_∞	Ultimate dilution factor	-
G	Solids handling capacity; solids flux; batch flux; total flux	F/L^2T
G_B	Solids flux in batch settling test	F/L^2T
G_L	Limiting solids handling capacity; limiting solids flux	F/L^2T
G_T	Total solids flux	F/L^2T
h	Height	L
h_o	Initial height of suspension	L
h_f	Height of feed level	L
h_{of}	Height of overflow level	L
h_{th}	Height of thick bed in continuous thickener	L
$\bar{K}$	Rate constant in compression zone	T^{-1}
M	Weight of solids per unit area	F/L^2

and the clear liquid remains behind at the top of the basin. The concentrated bottom slurry and the clear liquid are then separated manually or mechanically. The operation then is repeated with new feed slurry. In continuous thickening, the basin has a feed inlet, an underflow outlet, and an overflow outlet. The inlet receives the feed suspension and distributes it over the area of the basin. The bed receives the incoming feed and separates it into two fractions. The solids fraction travels downward and is taken out by the underflow outlet. The remaining clear liquid fraction travels upward above the bed and is taken out by the overflow outlet. An inventory of separated solids is maintained at the bottom of continuous thickeners.

Batch thickening is not a steady state operation. Continuous thickening may or may not be at steady state. Temporal changes may take place in the concentration and flow rate of feed suspensions, underflow slurry, overflow liquid, and thickening characteristics of the feed suspension. If these temporal changes do not exist, the operation reaches a steady state.

The analysis of steady state continuous thickening of certain non-ideal suspensions was the major subject of this study.

1.2 — Objective and Scope of the Study

The objective of this study was to investigate continuous thickening of non-ideal suspensions and evaluate a rational approach for design and operation of steady state continuous thickeners.

Suspensions of calcium carbonate, activated sludges, water softening sludges and a suspension of fine glass beads were used in

the experiments. The first three were typical non-ideal and compressible suspensions. The glass beads were incompressible but were not of uniform size.

All suspensions were continuously thickened in a closed pilot thickening system. Batch settling experiments were also performed with the suspensions. The solids flux theory, which is based on the analysis of the characteristic batch settling velocities of suspensions and the operational features of a continuous thickener, was used as the basic means of data analysis.

2. THEORY OF GRAVITY THICKENING

2.1 — Introduction

A schematic diagram of a continuous thickener is shown in Figure 1. A suspension with solids concentration of C_f (weight per unit volume) flows into the thickening basin at a rate Q_f. The thickened suspension is taken out at concentration C_u and flow rate Q_u. The clear separated fluid with (normally) negligible solids concentration, C_{of}, flows from the top of the thickener at a rate, Q_{of}. Assuming essentially complete removal of solids, the product C_fQ_f or C_uQ_u is the solids loading on the thickener. Also, $\frac{Q_f}{A}$, $\frac{Q_u}{A}$, and $\frac{Q_{of}}{A}$ are called hydraulic feed rate, underflow rate or velocity, and overflow rate, respectively. Here A refers to the area of thickener, and $Q_{of} = Q_f - Q_u$.

The principal problem in the continuous thickening of a particular suspension is the following: How much area, A, and volume, V, is required to thicken the suspension with given concentration, C_f, and flow rate, Q_f, to a final required underflow concentration, C_u? Factors influencing these determinations are considered in the following sections.

2.2 — Area Required for Thickening

Coe and Clevenger (1916) argued that in a batch or continuous thickener, any layer of suspension has a specific solids handling capacity. The solids handling capacity is the mass of solids that a unit area of the layer can pass to the layer below. The limiting layer with

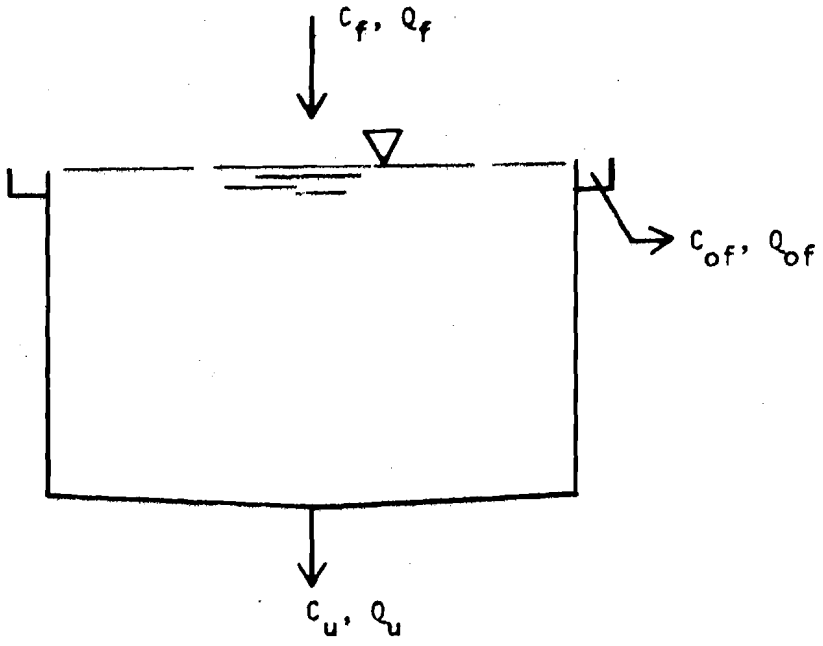

FIGURE 1. SCHEMATIC DIAGRAM OF CONTINUOUS THICKENER

minimum solids handling capacity, G_L, should be identified and considered for area determination. They showed that the capacity of a layer of any concentration to discharge its solids may be calculated from

$$G = \frac{V}{\frac{1}{C} - \frac{1}{C_u}} \tag{2.1}$$

where

G = solids handling capacity of a layer of concentration C_i

C = concentration of the layer

C_u = concentration of thickened sludge

V = settling velocity of the layer

The required area of a continuous thickener then is

$$A = \frac{Q_f C_f}{G_L} \tag{2.2}$$

where G_L is the minimum value of G obtained by applying Equation (2.1) to all concentrations of sludge which could exist in the thickener.

Yoshioka, et al. (1957) and Hasset (1958) proposed the solids flux theory for determining required area for a thickener. They argued that in a continuous thickener any concentration layer travels downward due to its own settling velocity, V, and also due to downward underflow velocity, V_u. The total solids flux through any layer then is

$$G = CV + CV_u \tag{2.3}$$

The first term, CV, is the flux due to settling of the solids relative

to the liquid, and the second term, CV_u, is the solids flux due to the underflow velocity. The flux through the thickener may be written as

$$G = \frac{Q_u}{A} C_u = C_u V_u \tag{2.4}$$

Combining Equations (2.3) and (2.4) one can obtain

$$G = \frac{V}{\frac{1}{C} - \frac{1}{C_u}} \tag{2.5}$$

which is the same as Equation (2.1) developed by Coe and Clevenger in 1916.

It should be noted that in the derivation of the solids handling capactiy equations, it was assumed that feed solids are homogeneously distributed across the thickener and that horizontal concentration gradients do not exist in any layer of the bed including the underflow layer.

The graphical presentation of Equations (2.3), (2.4), and (2.5) simplifies analysis of continuous thickeners. Figure 2 shows results using a suspension of high grade calcium carbonate. The batch flux curve (i.e. CV vs C) was obtained using data from multiple batch tests. The solids transport due to a specific underflow velocity, V_u, is equal to the product CV_u. The summation of the two curves is shown as $G = CV + CV_u$. Point B on the curve is the limiting solids handling capacity, G_L. A horizontal line drawn through point B intersects the total flux curve at point A which corresponds to the concentration, C_{L2}. This concentration might exist in a continuous thickener since it has the same limiting solids handling capacity, G_L. If concentrations less than C_{L2} should exist in the thickener, they would limit the capacity

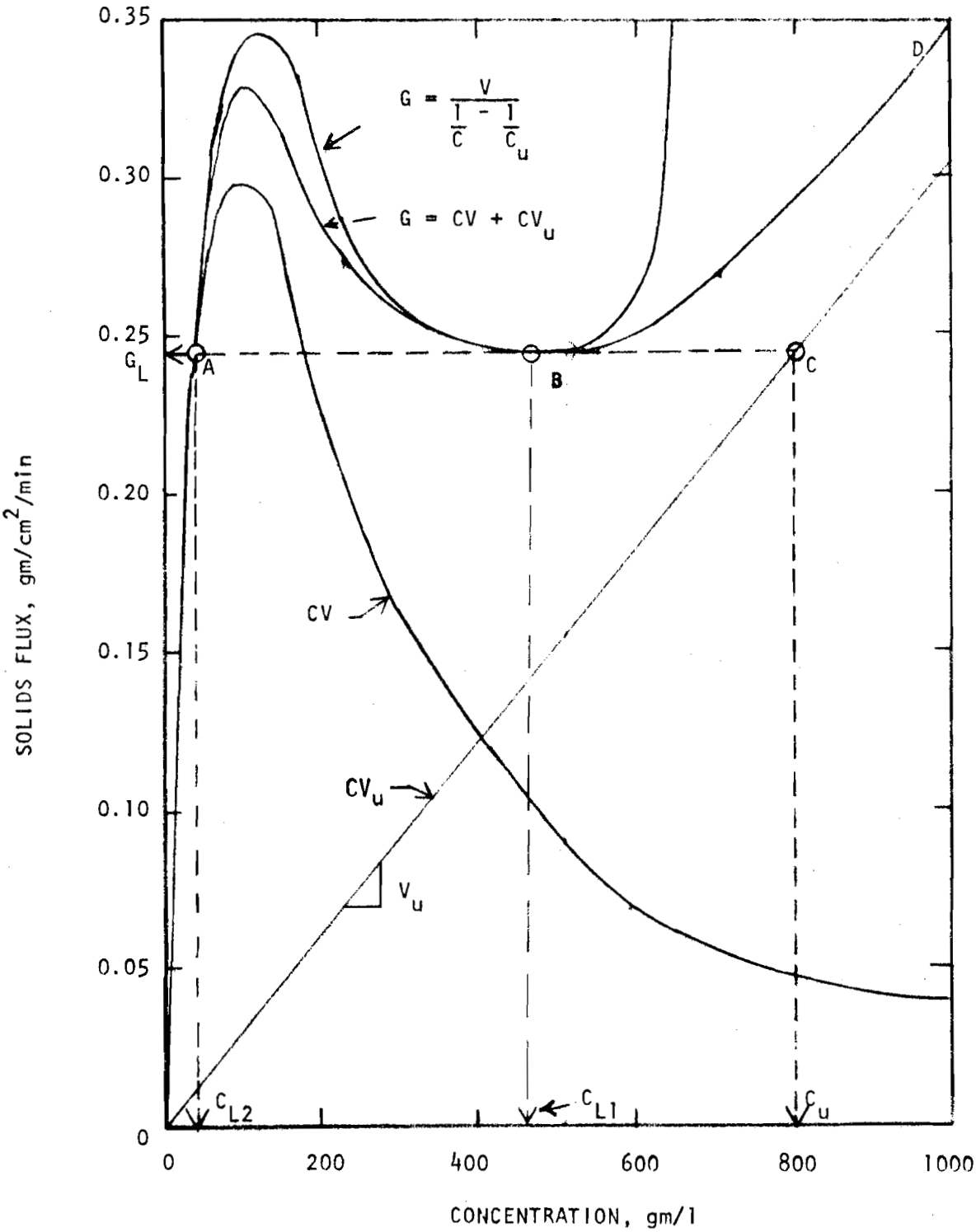

FIGURE 2. GRAPHICAL ANALYSIS OF SOLIDS THICKENING AND TRANSPORT IN A CONTINUOUS THICKENER

of the thickener since its solids handling capacity would be less than G_L. The underflow concentration must be that of point C because at the underflow level, $C_u = G_L/V_u$.

The solids handling capacity curve calculated from Equation (2.5) also is shown in Figure 2. Because G in Equation (2.4) was taken as the actual solids flux through the thickener, Equation (2.5) is valid only for the concentrations of solids which exist in the thickener (Dick, 1970a). For this reason, the plot of G according to Equation (2.5) does not coincide with the plot of G according to Equation (2.3) except at points A and B.

Graphical analysis of flux data can be simplified (Dick, 1970a) by drawing a line from G_L tangent to the flux curve to intersect the concentration axis at C_u. This graphical method of performance prediction is illustrated in Figure 3 for an underflow concentration of 800 mg/l. Comparison with Figure 2 will illustrate the validity of the approach.

From 1916 to 1952 no great contribution was made to the theoretical aspects of thickening. In 1952 Kynch formulated the mechanism of batch thickening for ideal suspensions. His work lead to a more thorough understanding of the conclusions that Coe and Clevenger (1916) made about the existence of the limiting solids handling capacity.

Kynch assumed that the velocity, V, of any particle in a layer was a function only of local concentration. Mathematical analysis of thickening predicated on this assumption lead Kynch to conclude that, during batch thickening, layers of solids at constant concentration are propogated upward at uniform velocities. Extension of Kynch's findings

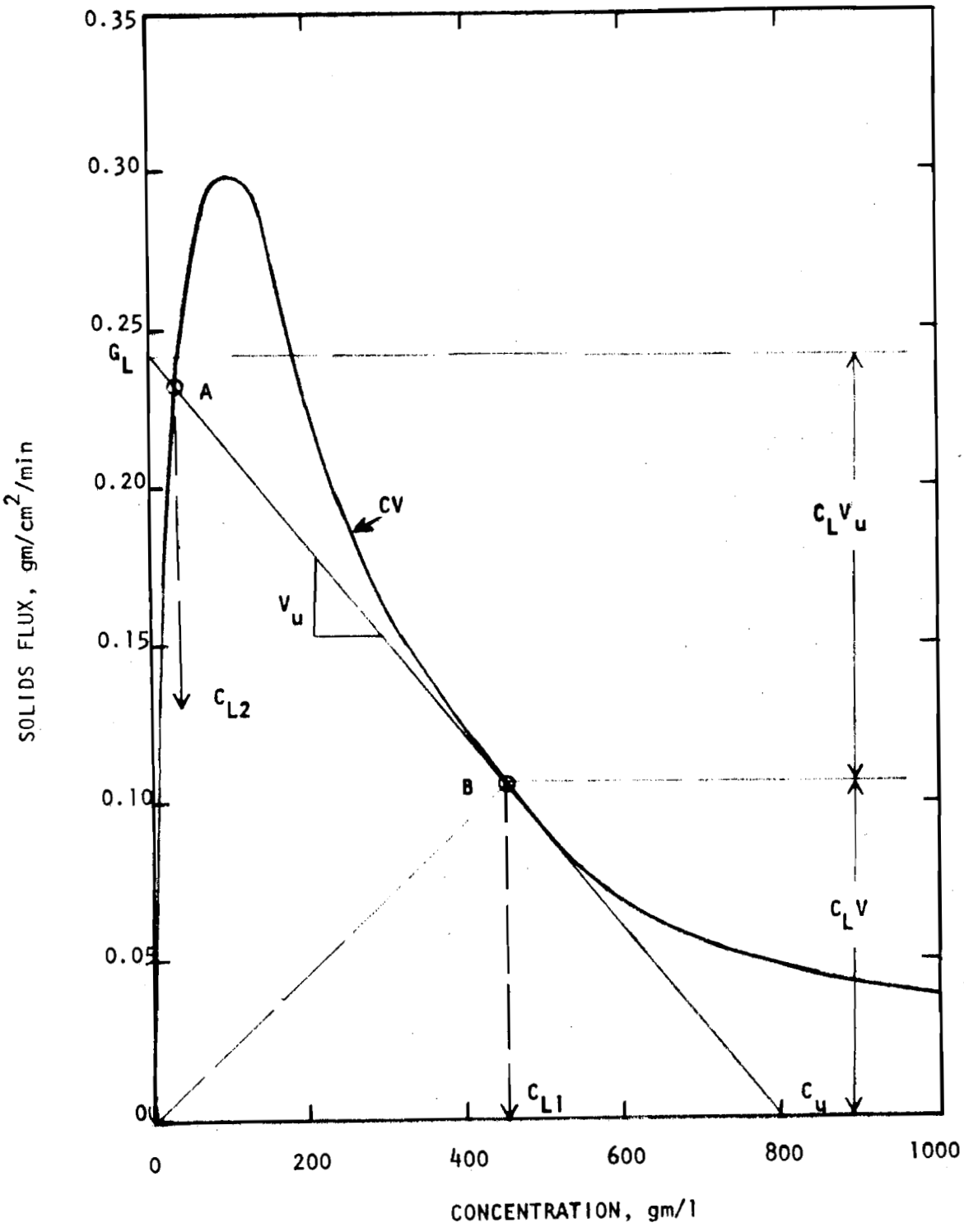

FIGURE 3. BATCH FLUX CURVE AND PERFORMANCE ANALYSIS OF CONTINUOUS THICKENER

to continuous thickening leads to the realization that at steady state conditions in continuous thickeners, the underflow velocity, V_u, exactly opposes the upward propogation velocities of sludge layers. The net effect is a stationary solids bed.

2.3 — Solids Profiles in Continuous Thickeners

Theoretical analysis of thickening provides a basis for predicting solids concentration profiles within continuous thickeners operating at steady state. Depending on the form of the batch flux curve and the magnitude of the underflow velocity, continuous and discontinuous gradients in vertical concentrations may exist. The relationship between the batch flux curve and the concentration profile is illustrated in Figures 4 and 5. For the underflow concentration, C_u, (equal to G_L/V_u), the continuous thickener will be capable of receiving the solids flux, G_L. Inside the thickener, concentrations C_{L1} and C_{L2} will prevail and there will be a sharp discontinuity between these two concentrations (see Figure 5). At the outlet of the thickener a transition would be expected between the concentrations C_{L1} and C_u.

Inspection of Figure 4 also indicates that for higher underflow concentrations, G_L and C_{L2} will decrease, and C_{L1} will increase in magnitude.

Theoretically, the height of thick bed, h_{th}, of concentration, C_{L1}, should be calculated as follows:

$$C_{L2}\,(h_f - h_{th}) + C_{L1}\,(h_{th}) + \int_0^{h_1} (C - C_{L1})\,dh = M \qquad (2.6)$$

where h_f refers to the height of feed level, h_1 is the depth of sludge

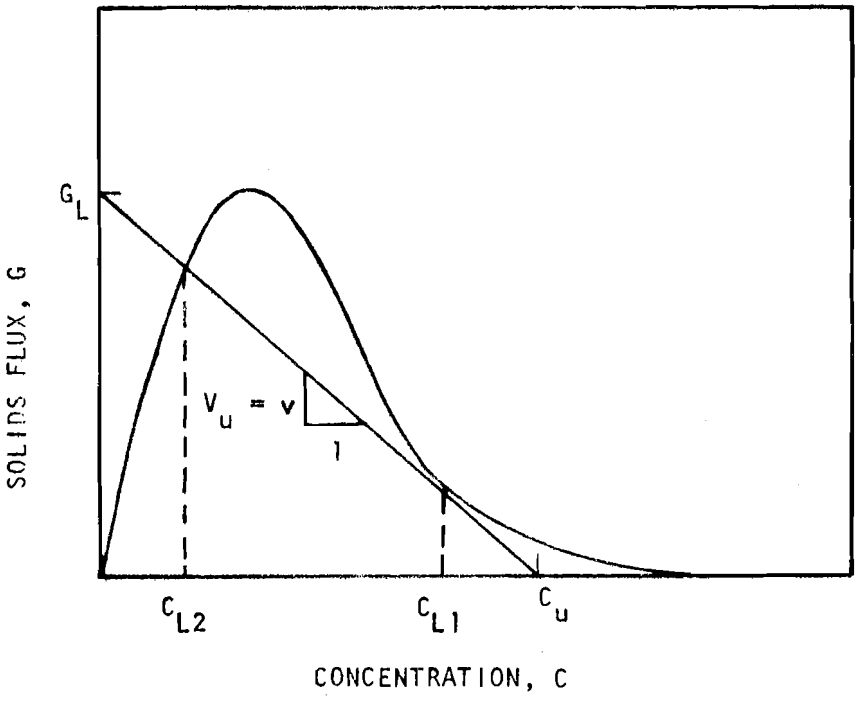

FIGURE 4. ANALYSIS OF IDEAL CONTINUOUS THICKENER TO IDENTIFY EXPECTED CONCENTRATIONS

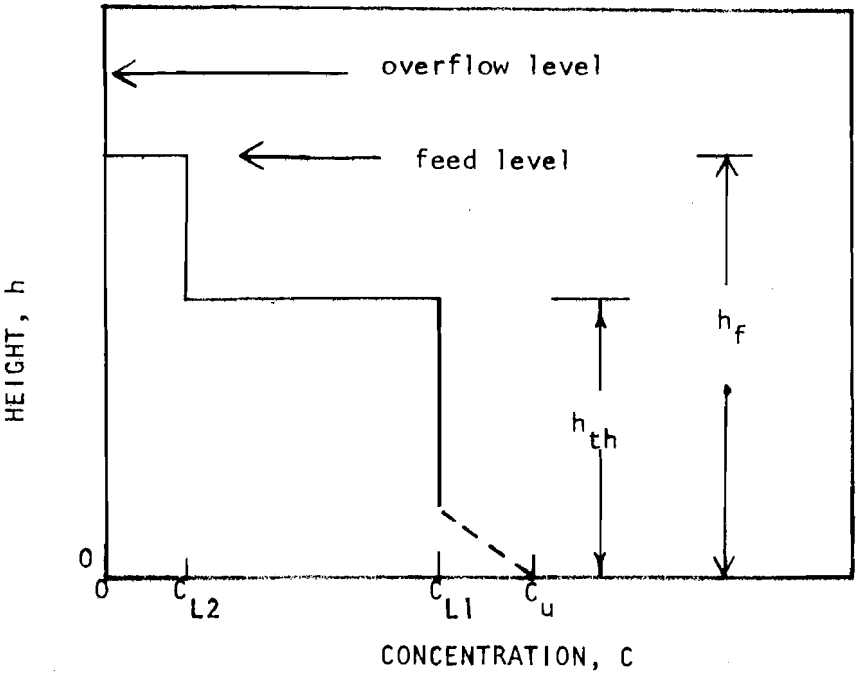

FIGURE 5. EXPECTED CONCENTRATION PROFILE IN CONTINUOUS THICKENER

of concentration greater than C_{L1}, and M is the total weight of solids stored in a unit area of the thickener. In these equations C_{L1} and C_{L2} can be determined from the batch flux curve for the required underflow concentration, C_u. M and h_f should be defined by the problem. However, there still is more than one unknown, i.e., h_{th} and h_1. So, unless further assumptions or equations can be written, h_{th} cannot directly be calculated.

2.4 — Volume Requirement

With flocculent suspensions, sludge depth (and hence thickener volume) influences settling velocity (Dick and Ewing, 1967a). Roberts (1949) hypothesized that the rate at which water is eliminated from a flocculent suspension in compression settling is at all times proportional to the amount of water remaining to be eliminated. He experimentally verified the hypothesis for two different flocculent suspensions and calculated the required compression depth in a continuous thickener. Mathematically, the hypothesis was formulated as follows:

$$-\frac{dD}{dt} = \bar{K}\,(D - D_\infty) \qquad (2.7)$$

where D is the slurry dilution (i.e., weight of liquid per unit weight of solids), D is the dilution at infinite time, and $\bar{K}$ is a rate constant. He calculated the required depth for compression using further equations based on Equation (2.7). Behn and Liebman (1963) developed a general equation for calculation of depth based on the Robert's hypothesis.

Dick (1970b) indicated that, as a practical matter, the depth of thickeners is commonly controlled by the need for storing solids

which accumulate during periods of peak loading. Another factor which influences proper thickener depth is the need for uniformly collecting thickened sludge from the bottom of the thickener.

III. EXPERIMENTAL MATERIALS, EQUIPMENT, AND PROCEDURES

3.1 — Selection of Suspensions

Characteristics of suspensions of three different kaolin clays, two different calcium carbonate precipitates, activated sludge, water softening sludge, and fine glass beads were evaluated for possible use in the study. The settling properties of two of the kaolin clays* (ASP 100 and ASP 400) deteriorated significantly under the influence of mixing, and the third kaolin clay (Satintone No. 4) exhibited very poor settling properties. Consequently, suspensions of the clays were not considered further. The rheological characteristics of two grades of calcium carbonate (low grade number 4052 and reagent grade number 4072**) were studied. The reagent grade had superior settling characteristics, and was chosen for continuous thickening experiments.

Activated sludge from the main Urbana-Champaign water pollution control plant was used. The water softening sludge was from the Urbana plant of the Northern Illinois Water Corporation where slaked lime is used to soften ground water. The glass beads had a specific gravity of 2.50. The average diameter, as measured microscopically by the manufacturer***, was 29 microns with about 90 percent of the beads falling between 20 and 40 microns in diameter.

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* Englhard Minerals and Chemicals Corporation, Menlo Park, Edison, J. J.

** Mallinkrodt Chemical Works, St. Louis, Missouri

*** Catalog Number 380, 3M Company, St. Paul, Minnesota

3.2 — Batch Settling Equipment and Procedures

3.2.1 — General

Batch settling tests were performed in cylindrical Plexiglas* columns of various diameters and in 1 liter standard graduated cylinders. Figure 6 shows schematic diagrams of the columns. The inside diameters varied between 3.18 cm and 19.00 cm. The Plexiglas columns had flat bottom plates.

Two stirrers were used. Stirrer F (Figure 6) fit inside column B, and stirrer G fit inside graduated cylinder E. The stirrers were made of vertical stainless steel rods, 0.48 cm in outside diameter. The stirrers covered the entire depth of the columns. The portions extending from the vertical rod were made of 0.32 cm rods.

3.2.2 — Settling Tests with High Grade Calcium Carbonate

A master suspension of high grade calcium carbonate was prepared by adding tap water to 22,600 grams of calcium carbonate in a plastic reservoir of 60 liters maximum capacity. The concentration of the suspension was about 450 mg/l. The reservoir contents were strongly mixed with a Lightnin Mixer (Model C12, 1780 rpm) for a period of eleven days. The settling velocities of the suspension rapidly decreased during the first few days of mixing and then remained almost constant. A short period of mixing was done at later times when samples of the suspension were taken from the main reservoir for batch or continuous settling purposes.

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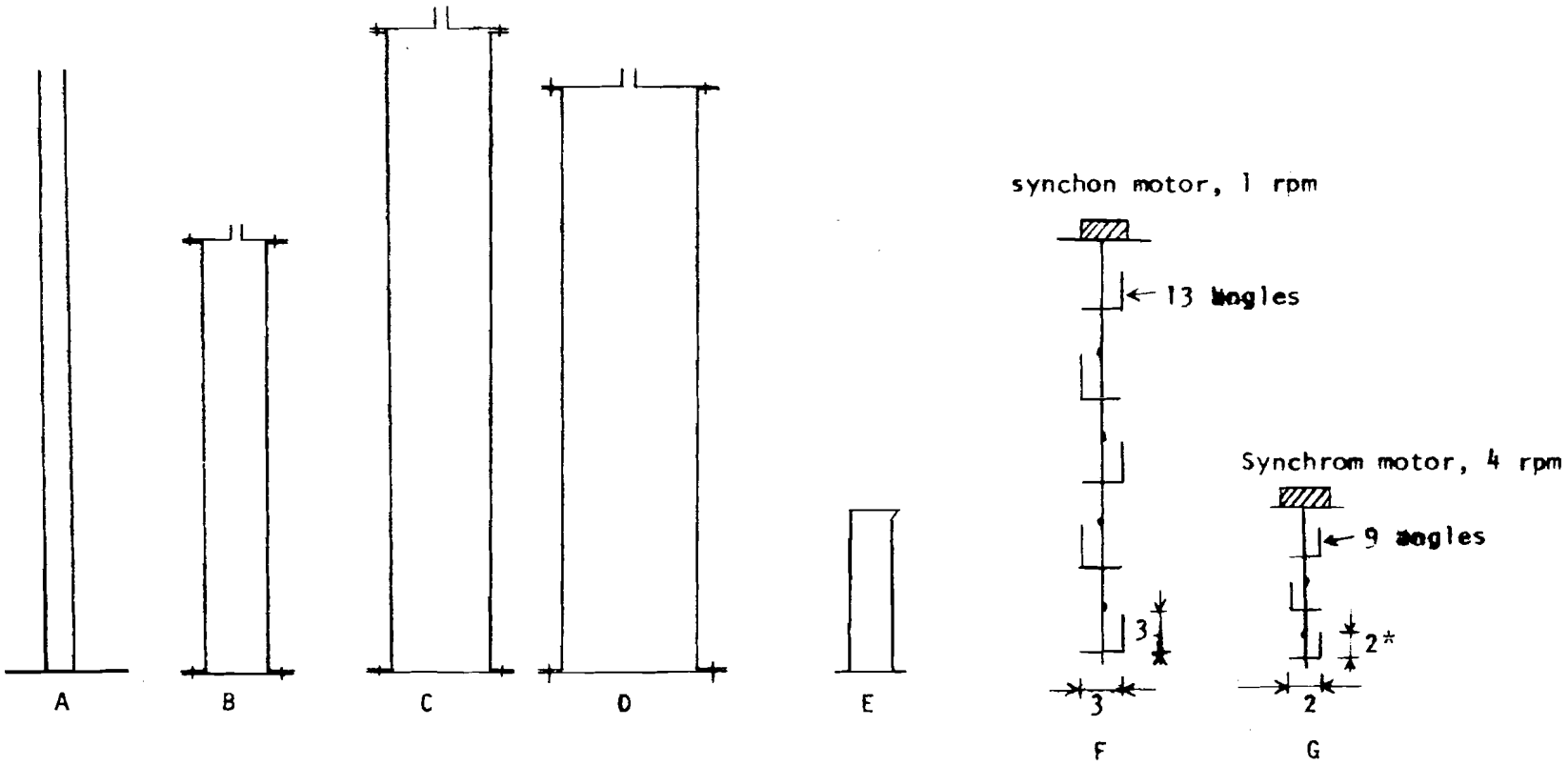
*Cast acrylic resin tubes, manufactured by Rohm and Haas Co., Philadelphia, Pennsylvania



A. 3.18 cm inside diameter; 180 cm total height
B. 8.90 cm inside diameter; 120 cm total height
C. 15.25 cm inside diameter; 180 cm total height
D. 19.00 cm inside diameter; 165 cm total height
E. 5.75 cm inside diameter; 45 cm total height (1 liter graduated cylinder)
F. Stirrer for B
G. Stirrer for E

* all dimensions in cm

FIGURE 6. SCHEMATIC DIAGRAM OF BATCH SETTLING CYLINDERS AND STIRRERS

Columns of 3.18 cm, 8.90 cm, and 15.25 cm diameter were used to investigate settling characteristics over a range of concentrations between 50 and 740 gm/l. The settling velocities obtained did not indicate a significant wall effect. The height effect was studied in the 3.18 cm diameter settling column. Very small increases in settling velocity were observed with larger initial heights. The effect of stirring on the settling velocity was studied in a 1 liter cylinder. Tests were performed under quiescent conditions and also with a stirrer rotating at 4 revolutions per minute. Two concentrations, 225 gm/l and 620 gm/l, were examined. No beneficial effect due to stirring was observed.

3.2.3 — Settling Tests with Activated Sludge

Tests were performed to determine the relative significance of initial height, cylinder diameter, and stirring on the settling velocity of activated sludge. As indicated in Figure 7, there was a significant difference in the settling velocities between stirred and unstirred tests at the same initial height.

3.2.4 — Settling Tests with Softening Sludge

Softening sludge at a concentration of about 100 g/l was sheared for 5 days in the apparatus used for shearing of the calcium carbonate suspension and then effects of batch sedimentation conditions were observed. No significant diameter effects were observed. The suspensions did not appear to change its settling characteristics due to healing after the initial period of strong shearing and aging.

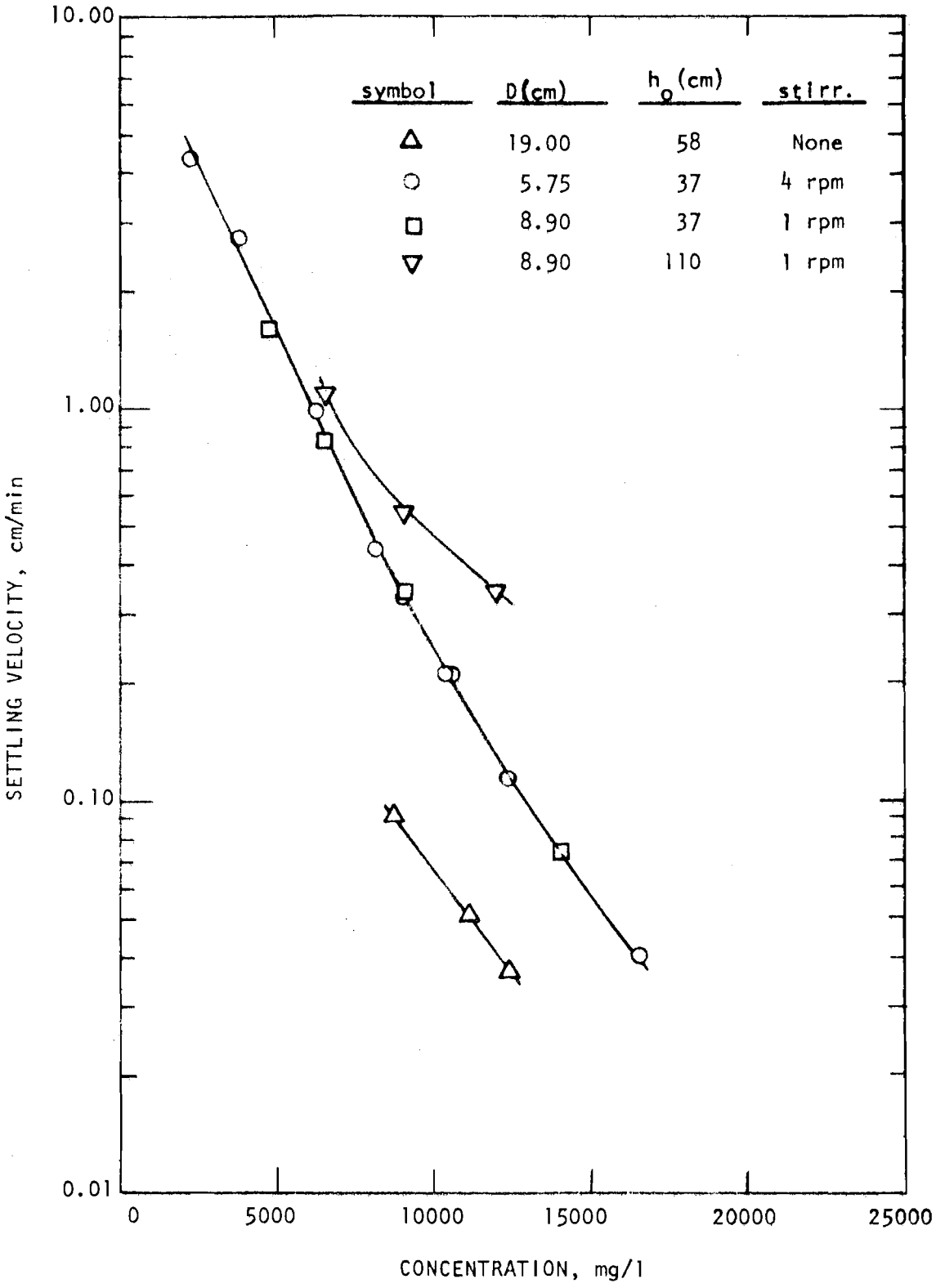


FIGURE 7. BATCH SETTLING VELOCITIES OF ACTIVATED SLUDGE SAMPLE 4

Stirring at 1 rpm did not show any beneficial effect in settling velocities. Slightly improved settling velocities could be obtained at higher initial heights but this effect was not extensively studied.

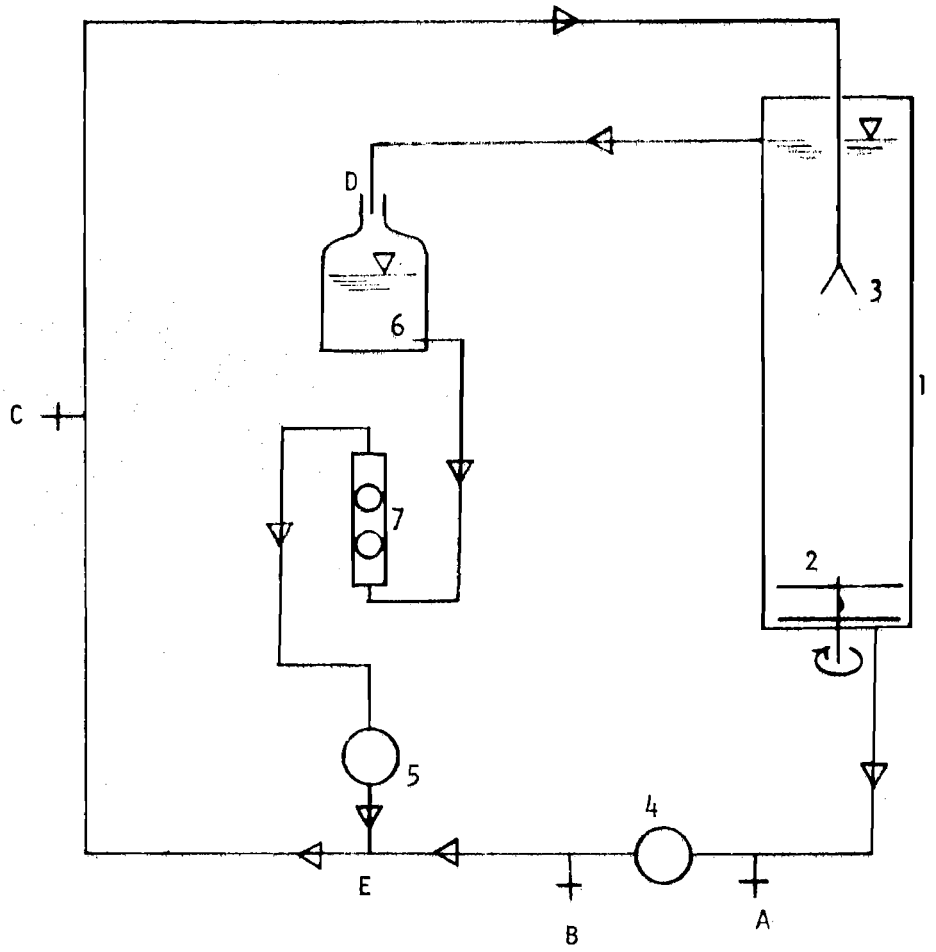
3.2.5 — Settling Tests with Glass Beads

Column diameter and the initial height of suspension have been shown to have no significant effect on the settling velocity of glass beads (Richardson and Zaki, 1954). Batch settling velocities were found to be extremely reproducible at all initial concentrations examined between 475 and 1270 gm/l. Below 450 gm/l, the interface was not clear or readable. Below the initial concentration of 400 gm/l, a bed could be seen to form at the bottom of the cylinder and it propagated upward until all solids were settled into the bed.

3.3 — Continuous Thickening Equipment and Procedures

3.3.1 — General

The pilot continuous thickener developed is shown schematically in Figure 8. It consisted of a cylindrical Plexiglas column which was provided with a bottom hole for outflow of the concentrated suspension, a small stirrer to mix the contents at the underflow level, an outlet for clear overflow liquid, and a small funnel attached to a piece of rubber gum tubing of adjustable length to distribute the feed suspension. Other components including adjustable pumps for precise control of flow rates are shown in Figure 8 and have been described in more detail by Javaheri (1971). The equipment was operated as a closed system with direct recycle of effluent and thickened underflow so that the total



1. thickening Plexiglas column
2. bottom stirrer
3. feed funnel
4. underflow pump
5. overflow pump
6. overflow reservoir
7. flowmeter

FIGURE 8. SCHEMATIC DIAGRAM OF LABORATORY CONTINUOUS THICKENING SYSTEM

amount of solids in the system during any one experimental run was constant. Four different Plexiglas thickeners with various stirrers were used for various continuous thickening experiments. Figure 9 is a schematic diagram of these thickeners and stirrers. Column A was 19.2 cm in inside diameter and 180 cm high with a 60 degree-angle plastic funnel attached to the bottom. Along the sides of the column and funnel, 29 sampling holes were placed on 6 cm centers. This column did not have a bottom stirrer.

Thickener B (Figure 9) was 15.25 cm in inside diameter and 180 cm high, and had 32 sampling holes. It also was equipped with a series of rubber stoppers drilled to accommodate small nails which could be removed to allow insertion of a syringe to take samples at any distance from the thickener wall. The bottom stirrer (B) was 5 cm high and made of 0.65 cm diameter steel rods. The underflow point was 5 cm from the center of the column.

Thickener C was 19.00 cm in inside diameter and 60 cm high. The sampling points of the column were similar to those of thickener B, and the underflow location was 5 cm from the center of the column. The bottom stirrer (C2) was made of 0.65 cm diameter steel rods with a lower portion similar to stirrer (B). Stirrer B (or C1) could also be used with thickener C.

Thickener D consisted of two pieces of 8.90 cm inside diameter Plexiglas columns attached to each other to make a total height of 180 cm. It had 27 sampling holes similar to those of thickener A but on both sides of the column. The underflow was 3 cm from the center of the cylinder. The bottom stirrer (D) was made of 0.46 diameter steel rods.

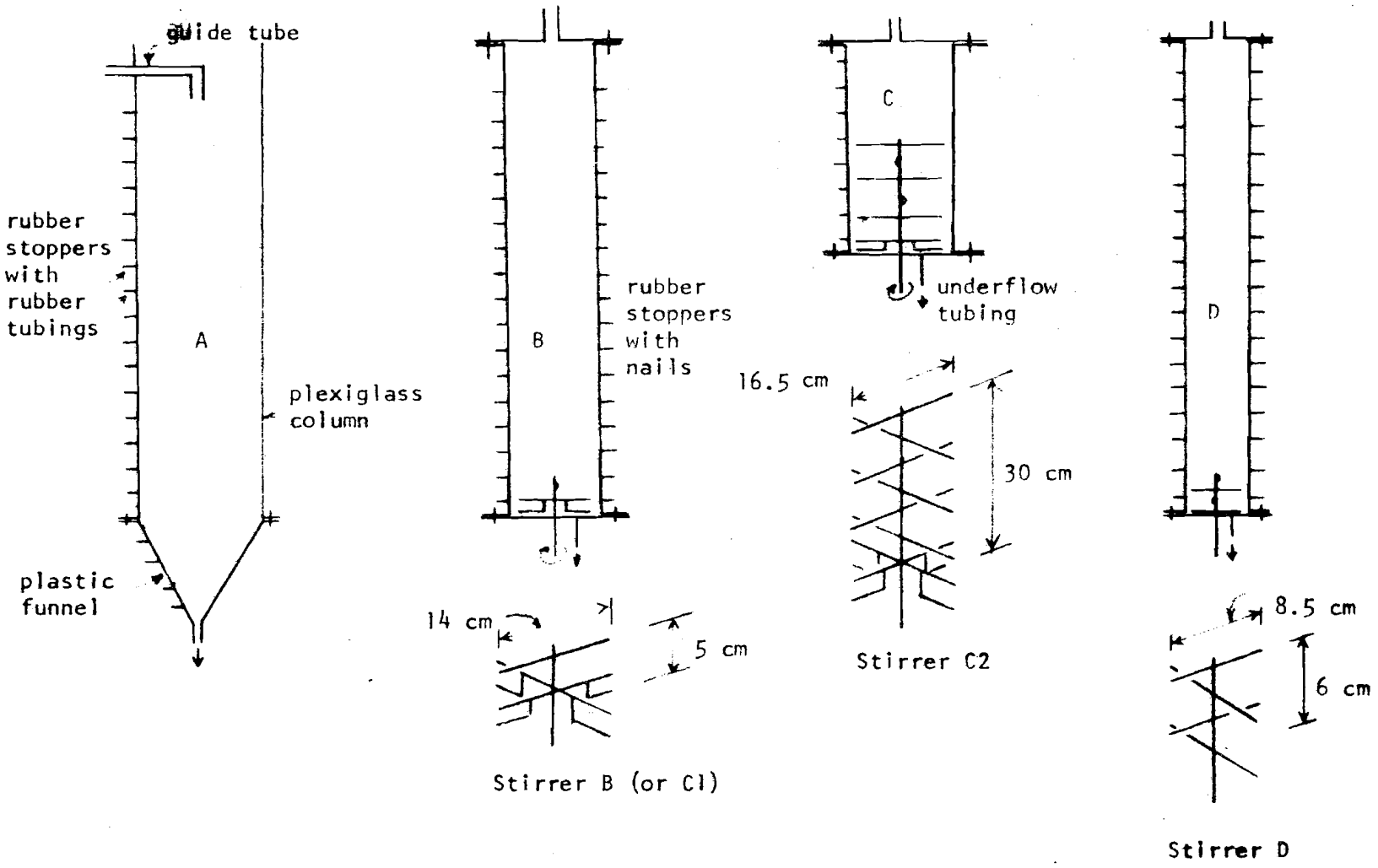


FIGURE 9. SCHEMATIC DIAGRAM OF LABORATORY THICKENING COLUMNS AND STIRRERS

3.3.2 — Monitoring of Operational Parameters

The general procedure for monitoring consisted of measuring the following parameters:

a. Concentration of underflow, C_u

b. Volumetric rate of underflow, Q_u

c. Turbidity and the suspended solids concentration of overflow, C_{of}

d. Volumetric rate of overflow, Q_{of}

e. Concentrations at various heights in the solids bed

The measurements were taken after the continuous system reached a steady state for a specific operational condition. In some experiments, excess hydrostatic heads also were measured at various elevations in the thickener.

The underflow concentration was determined by taking one or two samples, at point B of Figure 8. For this purpose point G was clamped and point B was opened immediately. Suspended solids were measured by the glass fiber filter procedure outlined by Gratteau and Dick (1968). The feed concentration was determined by taking samples at point C (Figure 8). The turbidity of overflow was determined by taking samples at point D and using a Hach* turbidimeter. The solids concentration of the overflow was determined at sampling point D. The volumetric rate of overflow was determined by measuring the overflow volume at point D for from 1 to 4 minutes.

Neglecting the overflow solids, the volumetric rate of underflow, Q_u, was calculated using the following equation:

$$(Q_{of} + Q_u)\ (C_f) = (Q_u)\ (C_u) \qquad (3.1)$$

*Model 1860, Hach Chemical Co., Ames, Iowa

where every parameter other than Q_u was determined experimentally. However, Q_u was also independently determined in some experiments. The volume of flow through point B with point G clamped was measured for from 1 to 5 minutes for this purpose and this direct determination of Q_u checked very closely with the calculated values from Equation (3.1).

Concentration profiles in the bed of solids could be determined by withdrawing samples for suspended solids analysis from the sampling points along the columns. Concentration at any elevation also could be determined rapidly using a radioisotope technique specifically designed for this purpose. The method involved passing a collimated beam of gamma radiation from a 50 millicurie Cs^{137} source through the thickener cylinder and counting the radiation penetrating the other side of the column. The source and detection equipment was mounted on a movable platform to permit solids profiles throughout the entire column depth to be determined. The equipment has been described in more detail by Javaheri (1971).

The radiation system was used in determination of concentration profiles of the solids beds in thickening of calcium carbonate suspensions, softening sludges, and glass bead suspensions. It could not be used with activated sludge because of the low density of the suspensions. Because calibration of the radiation system was influenced by exact position relative to the column, no calibration curve was used. Instead, samples for suspended solids analysis were taken at various locations in the column, and results from the radiation profiles were used to interpolate between the data points.

Piezometric pressures at different elevations of the continuous

thickener were measured by using the device shown in Figure 10. The connections between the piezometer tubes and the thickener sampling points were made with plastic tees, and point B was clamped at all times except during pressure measurement. Point A was opened only to fill the tubes with water to an elevation well above the overflow level of the thickener. For measurement of pressure at a sampling point, point B was opened. This caused a small volume of water, less than 10 ml, to flow from the tube to the thickener. In a few seconds, a balance between the pressures of the thickener suspension and the piezometer water was reached and the piezometric water elevation was then read by a sliding ruler. The pressure readings were extremely reproducible. By closing and opening point B, it was always possible to read the same pressure with variations of only 1 mm or less.

3.4 — Preliminary Continuous Thickening Experiments with Calcium Carbonate

3.4.1 — Elimination of Coring

Preliminary continuous thickening experiments with calcium carbonate were performed in column A (Figure 9) which had a sloped bottom section. Initial results did not agree well with predictions from thickening theory (Javaheri, 1971) and lead to investigation of the possibility that uniform solids collection from the thickener bottom was not being achieved.

Figure 11 shows results of a continuous thickening experiment in which concentrations were measured at the wall and at the center of thickener A to evaluate the possibility of uneven distribution of solids. These results and others indicated that a "coring" effect was created

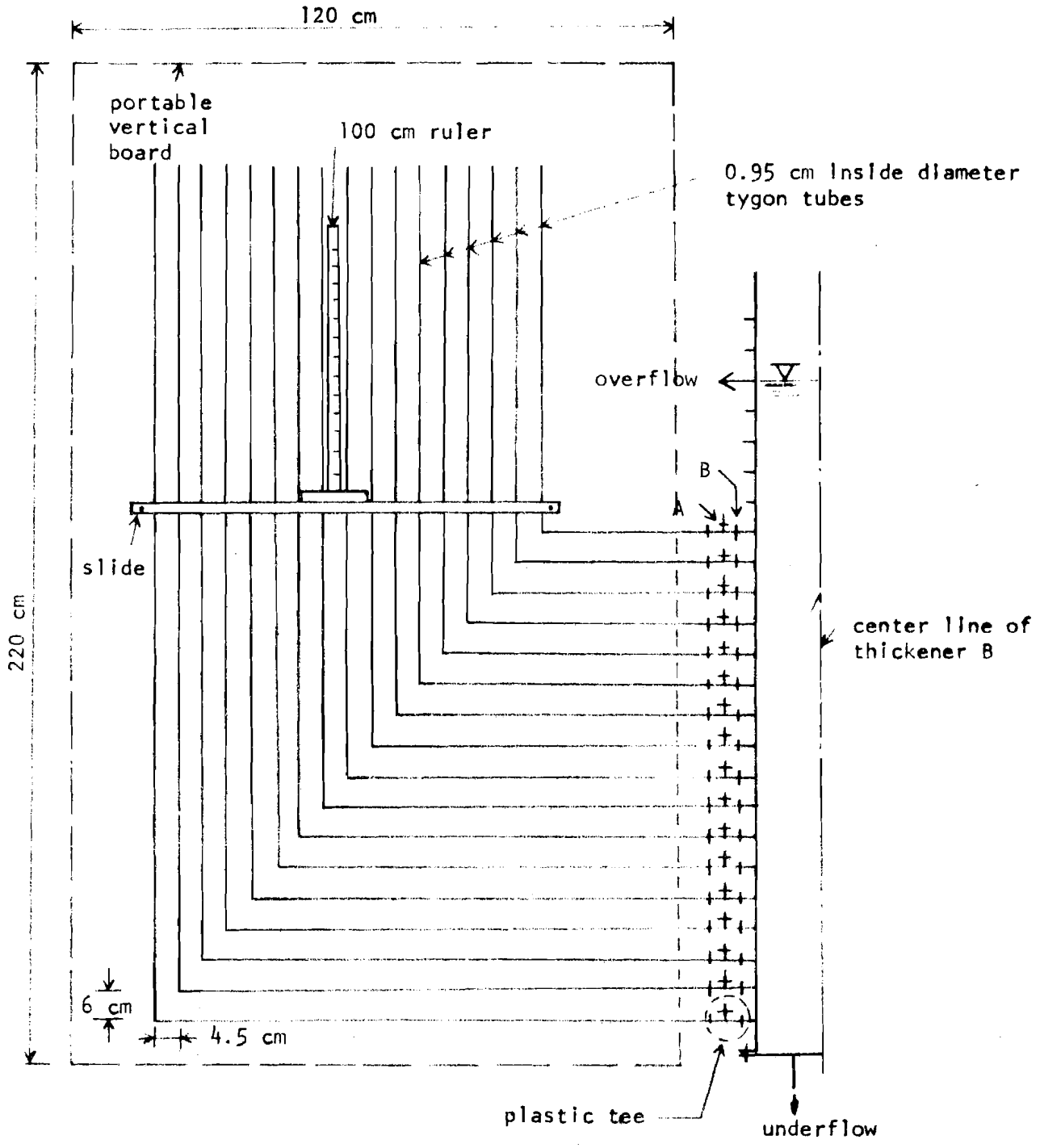


FIGURE 10. SCHEMATIC DIAGRAM OF PRESSURE MEASUREMENT APPARATUS

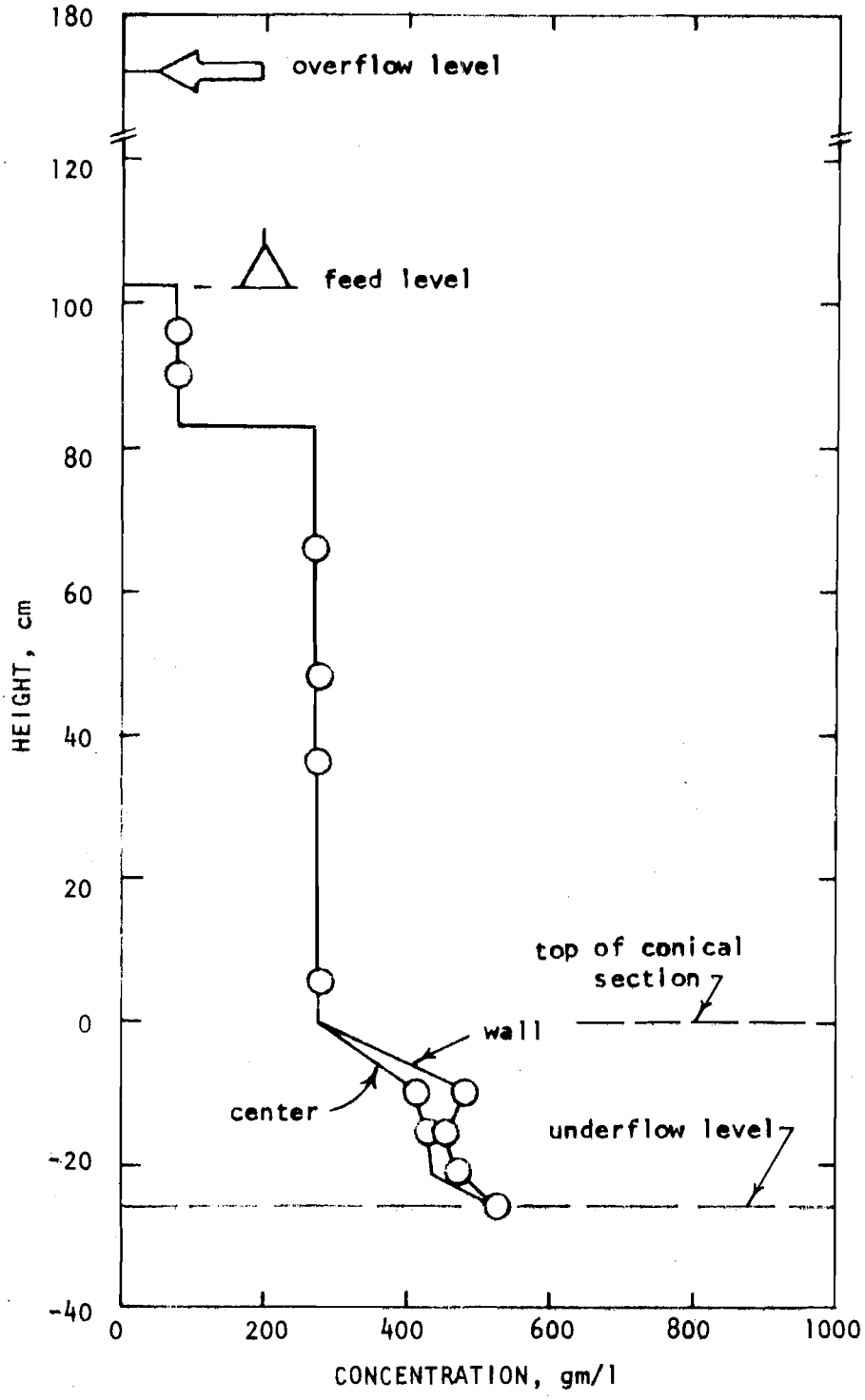


FIGURE 11. EXISTENCE OF RADIAL CONCENTRATION GRADIENTS IN THICKENER A

at the thickener bottom, and lead to development of thickener B in which stirring at the underflow level was used as a means to achieve uniform distribution of solids. The bottom stirrer (Figure 9) could be rotated at speeds from 0 to more than 100 rpm, and it was necessary to establish an optimum speed within that range. Optimum stirring was that intensity which sufficiently mixed the contents of the bottom section of the thickener and resulted in a homogeneous concentration at any horizontal layer at that section. Also, the magnitude of the induced velocities due to stirring the bottom section were to be low enough to avoid a hindering of the gravitational settling at elevations above the stirring zone.

This optimum velocity was envisioned to be a function of the physical characteristics of a given suspension.

Experiments performed with calcium carbonate and thickener B without stirring resulted in a coring action similar to that observed with thickener A (Figure 11). When the amount of stored solids was increased, the underflow was completely clogged.

By performing continuous experiments (Javaheri, 1971) with calcium carbonate at different underflow velocities and weight of stored solids, speeds between 9 and 50 rpm were found to be optimal.

3.4.2 — Establishment of Steady-State Conditions

Experiments were performed in thickener B with calcium carbonate to find the minimum amount of time required for the establishment of steady state conditions in which distribution of sludge solids concentration was constant. The solids were well mixed in the thickening column initially and then continuous operation of the thickener was

started. The underflow concentration and the sharp demarcation line between the two concentrations were measured as a function of time. Figure 12 shows two typical experiments. It was observed that the underflow concentration increased to a steady value in a matter of 4 to 5 hours and the elevation of the thick bed reached a steady state value at a slightly later time. At higher underflow velocities than those shown in Figure 12, the time for the establishment of steady state was less than 4 to 5 hours.

Extensive study of unsteady state thickening was not done; however, methods were devised to assure the measurement of the thickening parameters at the steady state conditions. Operational times were chosen at conservative values greater than 4 to 5 hours in most of the experiments. In addition, the radioisotope method was used to rapidly monitor solids concentrations at a few elevations to assure achievement of steady state conditions. Also, for most experiments, a sample of underflow was taken about one hour before final monitoring for concentration determination. The concentrations so determined agreed closely with the final determinations.

3.5 — Preliminary Continuous Thickening Experiments with Other Suspensions

Experiments were conducted with activated sludge, water softening sludge, and suspensions of glass beads to determine the optimum speed of stirring at the thickener bottom and the time required for establishment of steady-state conditions. Variations in settling properties of activated sludge in continuous thickeners also were investigated.

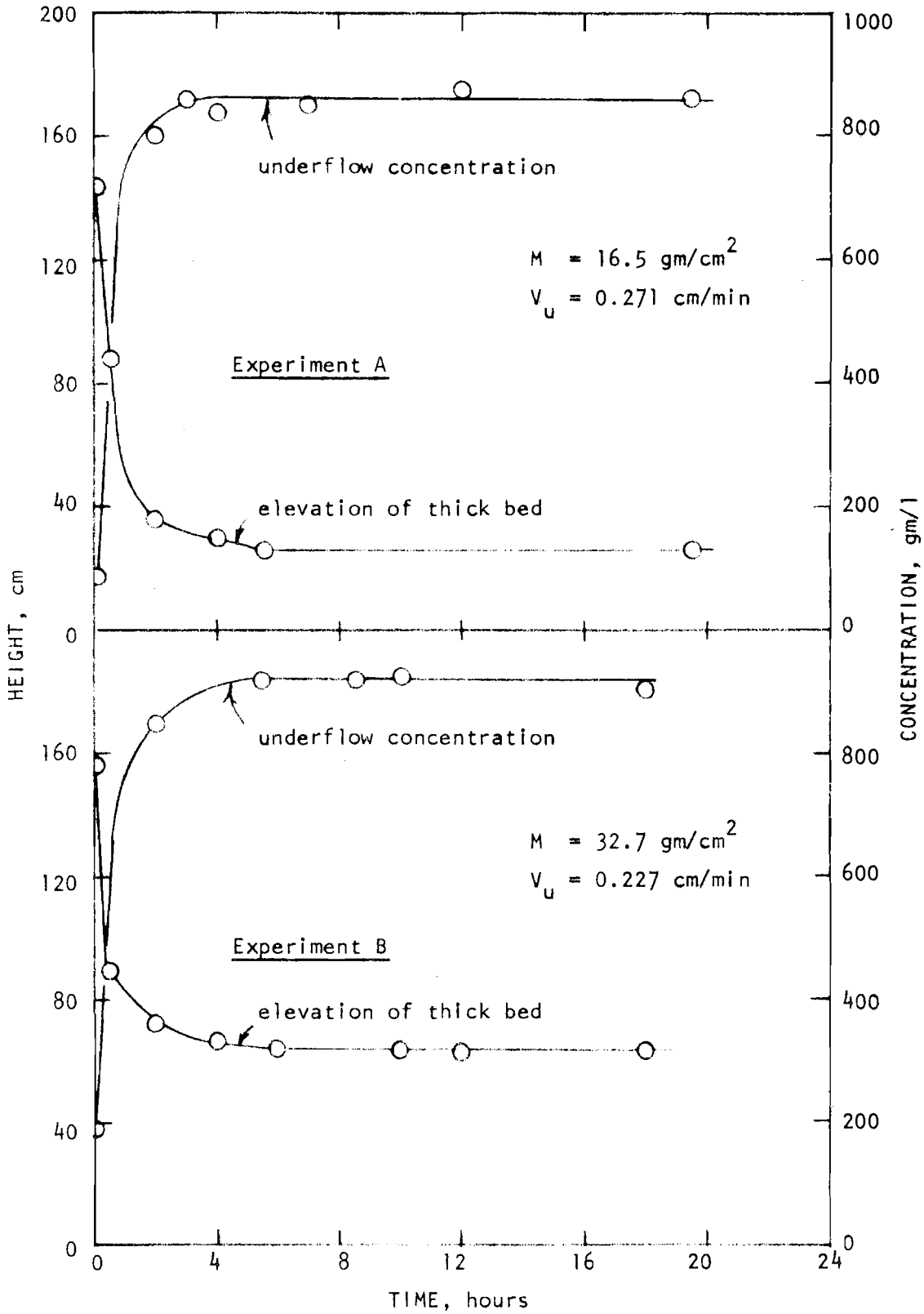


FIGURE 12. TYPICAL UNSTEADY STATE THICKENING OF CALCIUM CARBONATE

3.5.1 — Optimum Speed of Stirring

In experiments with activated sludge, a stirring speed in the range of 2 to 3 rpm was found optimal (Javaheri, 1971). At optimal speeds maximum underflow concentrations were obtained. At speeds below 2 rpm, a small degree of coring took place as evidenced by lower underflow concentration, and significant coring took place when no stirring was used. At high speeds, activated sludge above the stirring zone was disturbed, floc structure broke down, gravitational settling was adversely affected, and underflow concentrations were reduced. A stirring speed of 2 rpm also was found to produce maximum underflow concentration in experiments with softening sludges. Speeds between 10 and 20 rpm were optimum for glass bead suspensions.

3.5.2 — Time Requirement for Establishing Steady-State

Experiments similar to those illustrated in Figure 12 were performed with activated sludge, water softening sludge, and the suspension of glass beads. Periods of approximately 2 hours, 4 hours, and 1 hour, respectively, were required. Ample operation time beyond these limits was allowed for the establishment of steady-state in the experiments reported later. The thick bed usually reached a constant position inside the thickener and remained there, sometimes for hours, before the system was monitored.

3.5.3 — Variations in Settling Characteristics of Activated Sludge

To obtain meaningful continuous thickening results, it was essential that no large changes in the settling characteristics of

the activated sludge occurred. The preliminary continuous experiments indicated that small amounts of gas were produced if the sludge remained in the continuous thickener for about one day. The gas production rapidly increased after one day and lead to a partial rise of the activated sludge in the thickener. To eliminate this problem, the contents of the thickener were aerated for about 10 minutes after one or two continuous experiments, depending on their duration. This practice was found to be useful in reducing the number of bubbles traveling through the bed.

Batch settling tests were performed after continuous experiments to determine if any changes in sludge settleability occurred. For this purpose, a 1 liter graduated cylinder with a 4 rpm stirrer was used. It was concluded (Javaheri, 1971) that continuous experiments could be run for up to a week with an activated sludge before extreme deterioration of settling quality would take place.

4. RESULTS AND DISCUSSION

4.1 — General

Results of continuous thickening of four different suspensions (calcium carbonate, activated sludge, water softening sludge and glass beads) are reported in this chapter. Emphasis of the discussion is placed on mechanisms of continuous thickening, prediction of the performance of continuous thickeners from batch settling tests, and reasons for departures from the expected performance of the observed continuous thickening results.

4.2 — Continuous Thickening of Calcium Carbonate Suspension

A total of 19 continuous thickening experiments were performed with calcium carbonate. A typical solids profile is shown in Figure 13. A sharp jump from the first critical concentration to underflow concentration in the bottom 6 cm of the thickener, and a sharp demarcation line existed between the first and the second critical concentrations. This demarcation line was clearly observable through the Plexiglas wall of the thickener. The concentrations in the two concentration zones were uniform and the radioisotope method confirmed the constancy of these concentrations. Also, as described earlier, the radioisotope concentration measurements were used to interpolate between the data points in curves such as Figure 13.

The turbidity of the overflow was not affected by moving the overflow elevation to a position closer to the feed elevation. A minimum distance of about 5 cm was needed to avoid an increase in overflow turbidity which could take place due to the velocity gradients developed around

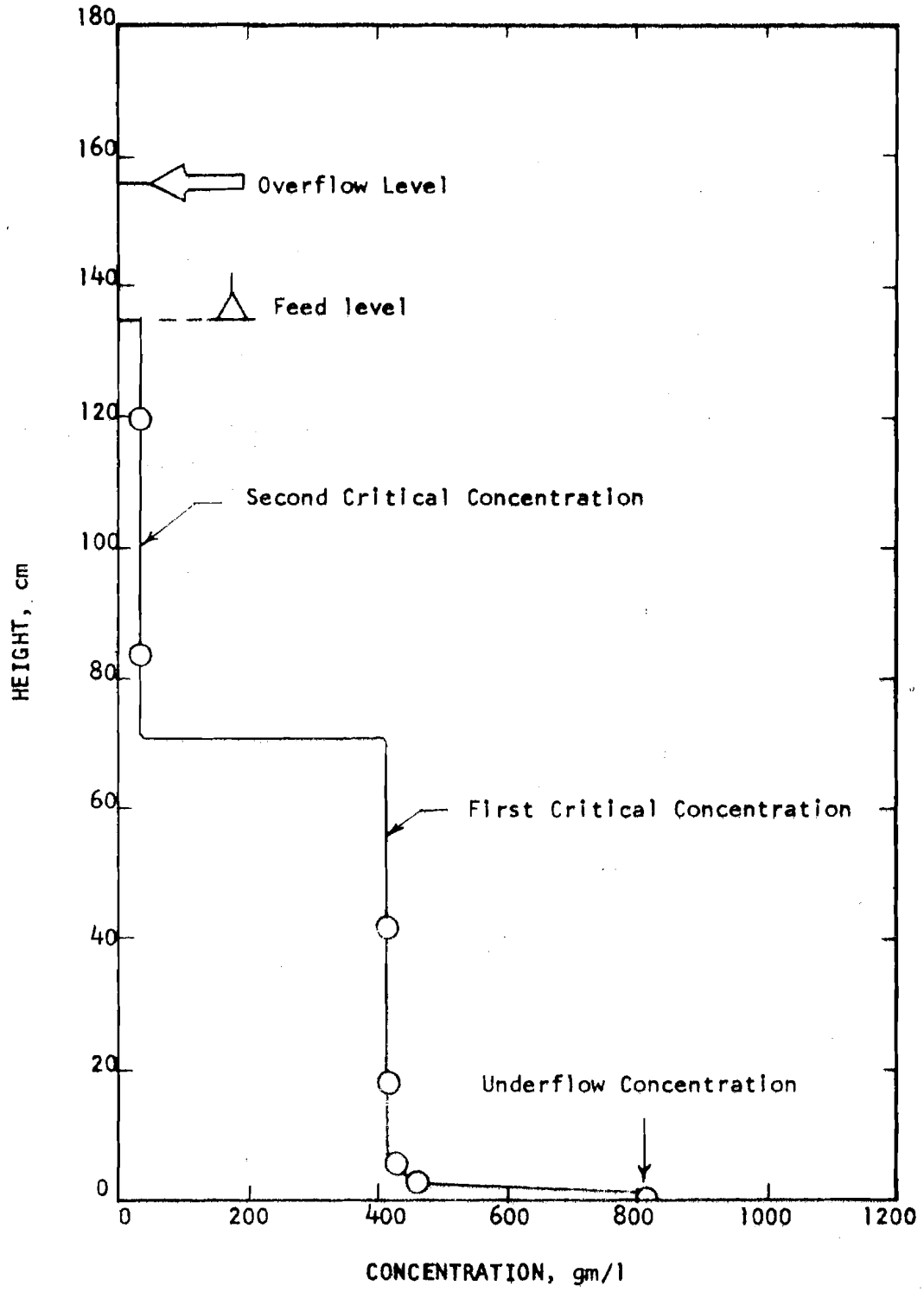


FIGURE 13. TYPICAL SOLIDS PROFILE WITH CALCIUM CARBONATE IN THICKENER B

the overflow contraction. It was possible for the limiting concentration to reach to the feed level. This is illustrated in Figure 14. The cause and significance of this is discussed in Section 4.2.3.

4.2.1 — Calculation of Settling Velocities

The settling velocities of the concentration layers, C_{L1} and C_{L2}, were calculated from the relationship

$$G = CV + CV_u \qquad (2.3)$$

Here, G, V_u, and C (i.e., C_{L1} or C_{L2}) were known. Therefore, the gravitational settling velocity, V, corresponding to the concentration, C, could be calculated. The velocities calculated by using data from the 19 calcium carbonate continuous thickening experiments are shown in Figure 15. The different symbols identify the individual experiments. Normally, each symbol appears twice in Figure 15 - one corresponding to the high velocity at concentration C_{L2} and one corresponding to the low velocity of C_{L1}. Some points have been omitted from the figure to avoid crowding.

The solid points in Figure 15 refer to settling velocities obtained in batch tests in the 3.18 cm column with an initial height of 100 cm. Dark hexagonal points refer to the batch tests performed about 3 months before the continuous experiments. The dark circles refer to batch tests performed in the middle of the continuous experiments. Generally, the batch settling characteristics of calcium carbonate did not show any significant indication of variability with time. Additional batch tests were performed during the continuous experiments and they

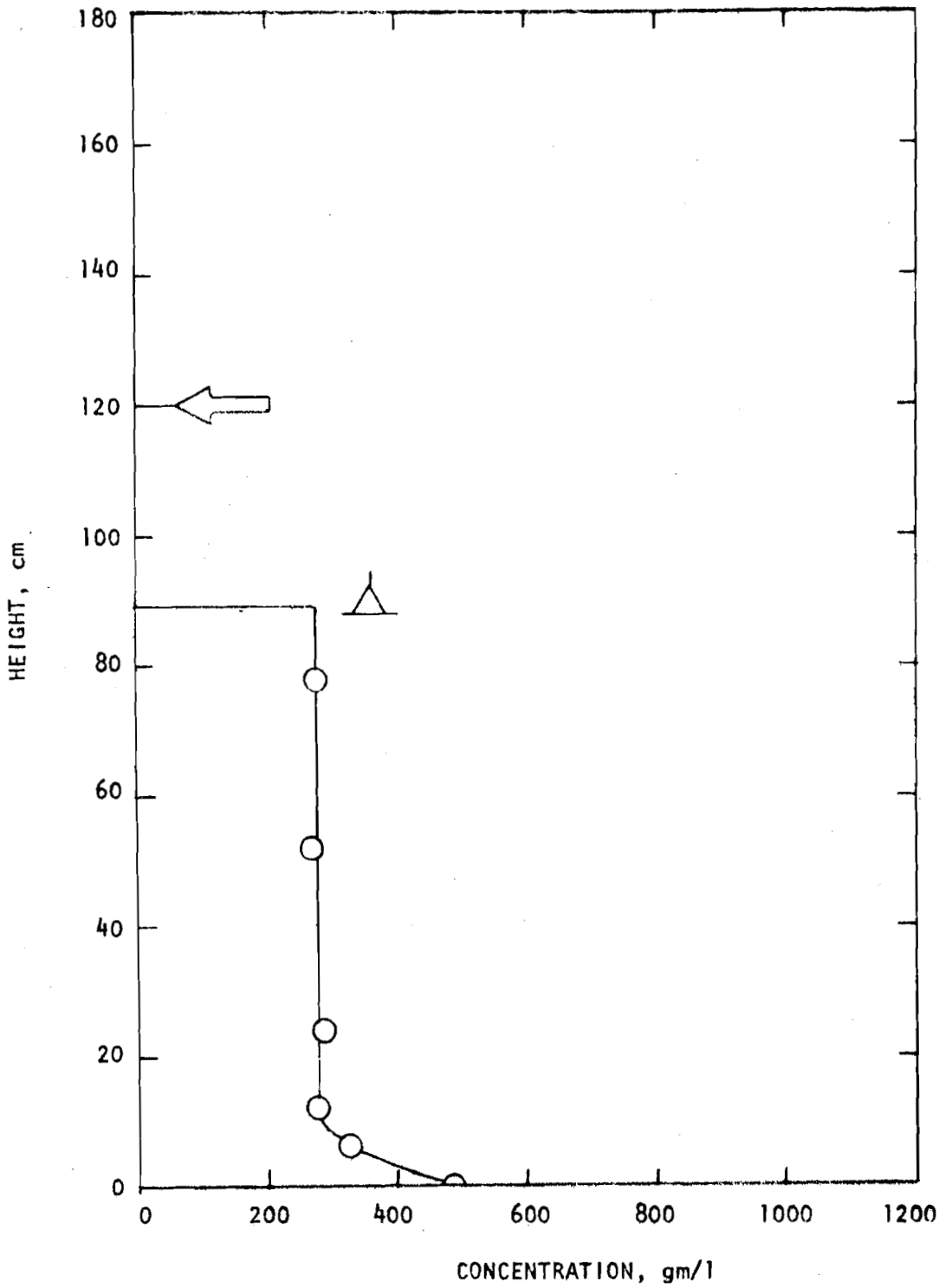


FIGURE 14. SOLIDS PROFILE OF CALCIUM CARBONATE WITHOUT SECOND CRITICAL CONCENTRATION

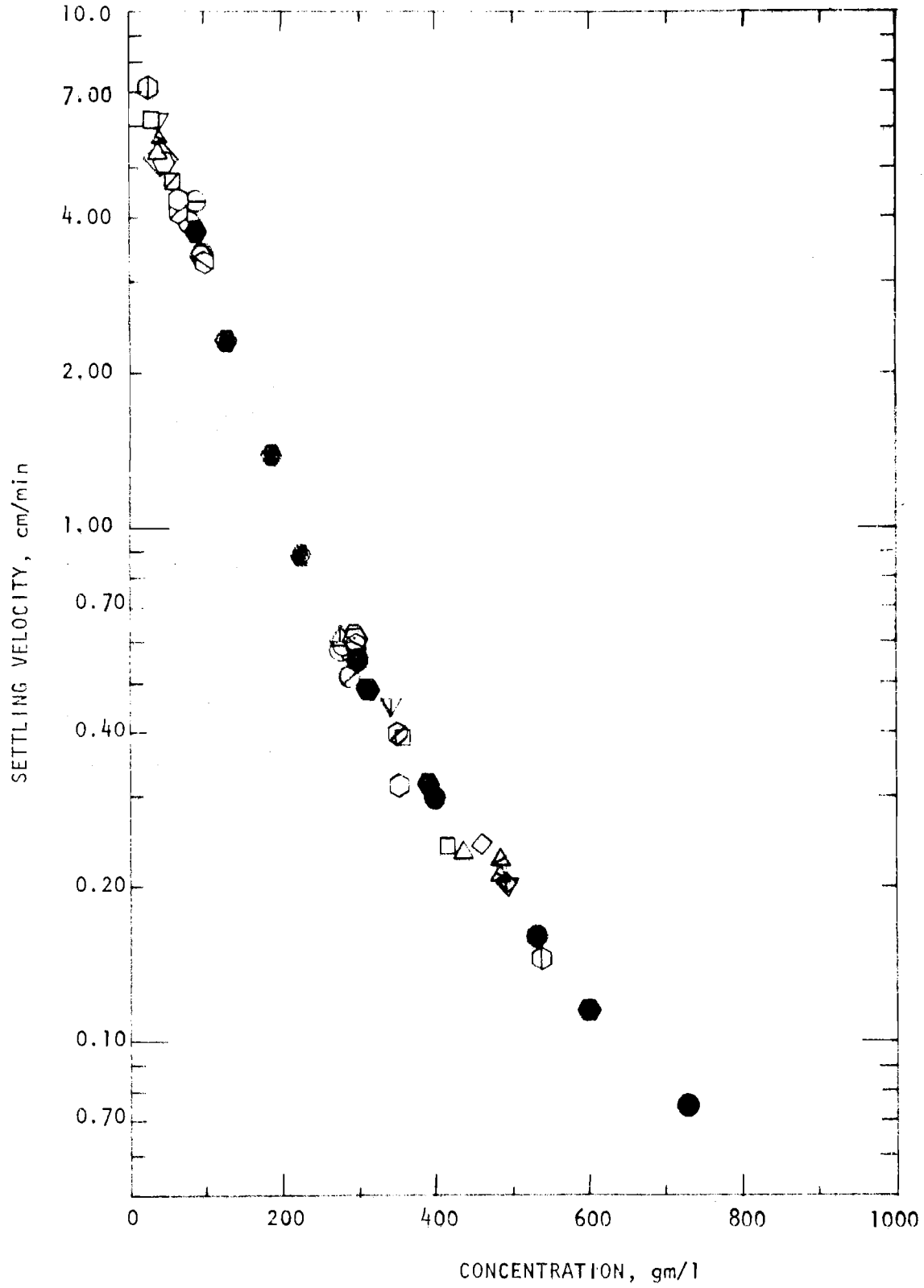


FIGURE 15. SETTLING VELOCITIES OF CALCIUM CARBONATE IN BATCH AND CONTINUOUS THICKENING

agreed with data presented in Figure 15 but were not included to avoid crowding of the figure.

4.2.2 — Prediction of Performance from Batch Tests

A batch flux curve developed from the batch settling velocities of Figure 15 was shown as Figure 3. Graphical analysis of Figure 3 was used to predict the performance of a continuous thickener from the batch tests as shown on the "predicted performance" line in Figure 16. The experimentally determined solids flux, G, and the underflow concentration, C_u, are also plotted in Figure 16 for comparison. The standard deviation, σ, of the solids flux points from the prediction curve in Figure 16 was 0.0122 gm/cm^2/min and the coefficient of variation was 0.555.

4.2.3 — Prediction of Required Volume

The performance of the continuous thickener experiments with calcium carbonate were satisfactorily analyzed using the batch flux theory. However, the amount of stored solids and the volume below feed level differed appreciably for various experiments. Apparently, the volume of the thickener played a role in providing space for the stored solids but the variation in the amount of stored solids did not affect the performance of the thickener. Moreover, the slight height effect previously indicated in the batch settling of calcium carbonate suspensions was not observed in continuous experiments. If a height effect existed in the continuous experiments, it could not be differentiated from experimental error.

It was observed that at all operating conditions the solids distributed themselves in the thickener such that the second critical

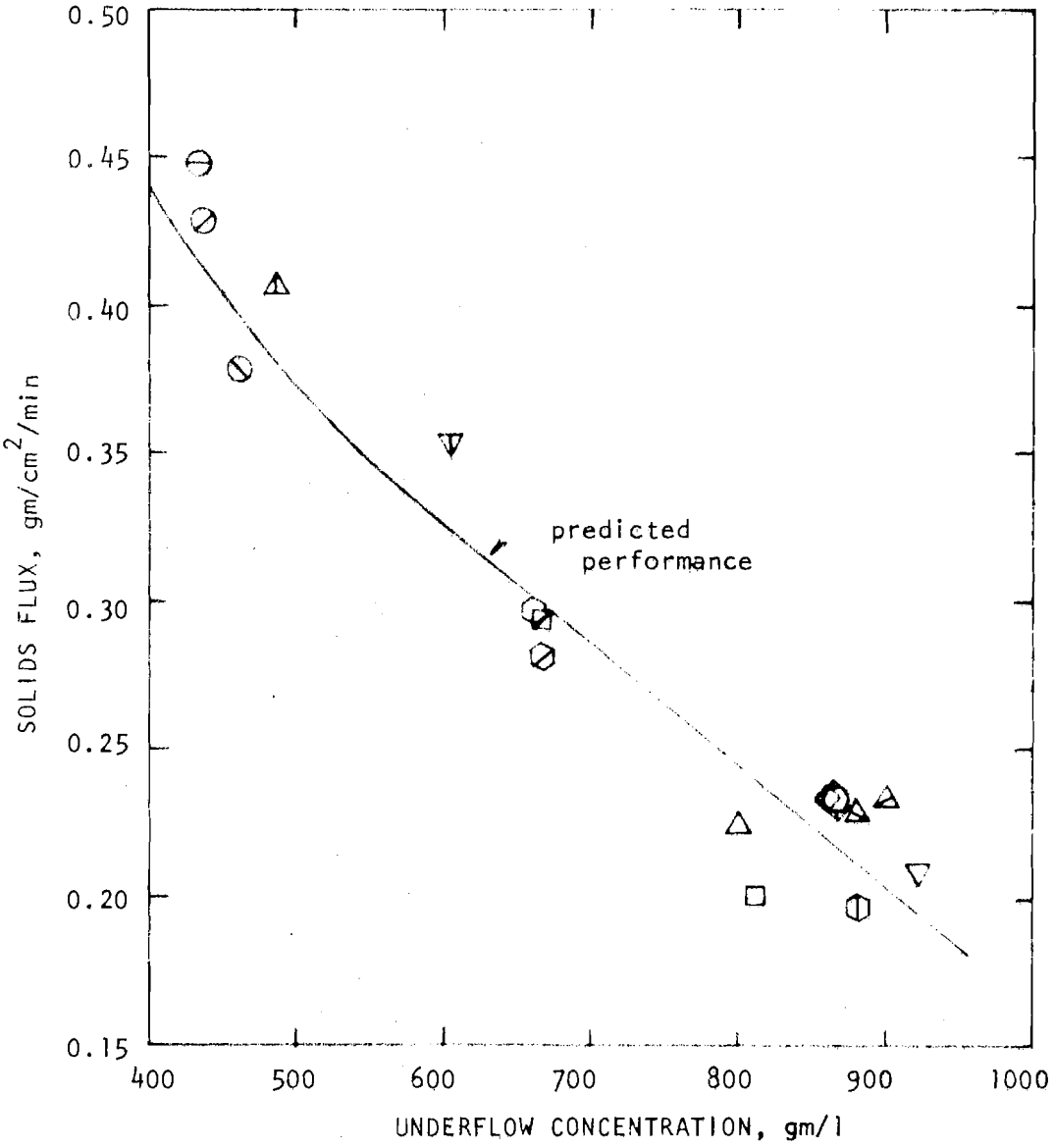


FIGURE 16. ACTUAL AND PREDICTED THICKENING OF CALCIUM CARBONATE

concentration reached to the feed level (Figure 13). Rarely did the first critical concentration reach the feed level. It would seem that this latter condition represents maximum utilization of thickener volume and that the existence of a large zone of concentration C_{L2} is an indication that optimum utilization of the available volume is not accomplished.

The height of the thick bed occupied by solids at concentration C_{L1} could be estimated as follows. Given the weight of stored solids, M, and an assumed elevation of feed level, h_f,

$$(C_{L1})(h_{th}) + (C_{L2})(h_f - h_{th}) + m = 1000\ M \qquad (4.1)$$

where m is the small amount of solids between C_{L1} and C_u at the stirring zone of the thickener and can be neglected for practical purposes. For a particular underflow concentration, C_u, the values of C_{L1} and C_{L2} can be determined from the batch flux curve. Consequently, the height of the thick bed, h_{th}, can be calculated. This method was used to predict the theoretical heights of the thick beds in the 19 continuous thickener experiments with calcium carbonate. The results are shown in Figure 17. The agreement is extremely good except for two points which correspond to low underflow concentrations.

4.2.4 — Pressures in Solids Bed

Unlike non-flocculent suspensions where the particles do not deform under contact or effective pressures, the floc particles of a flocculent calcium carbonate suspension could deform under such pressures. This deformation could possibly lead to a reduction in floc sizes, loss

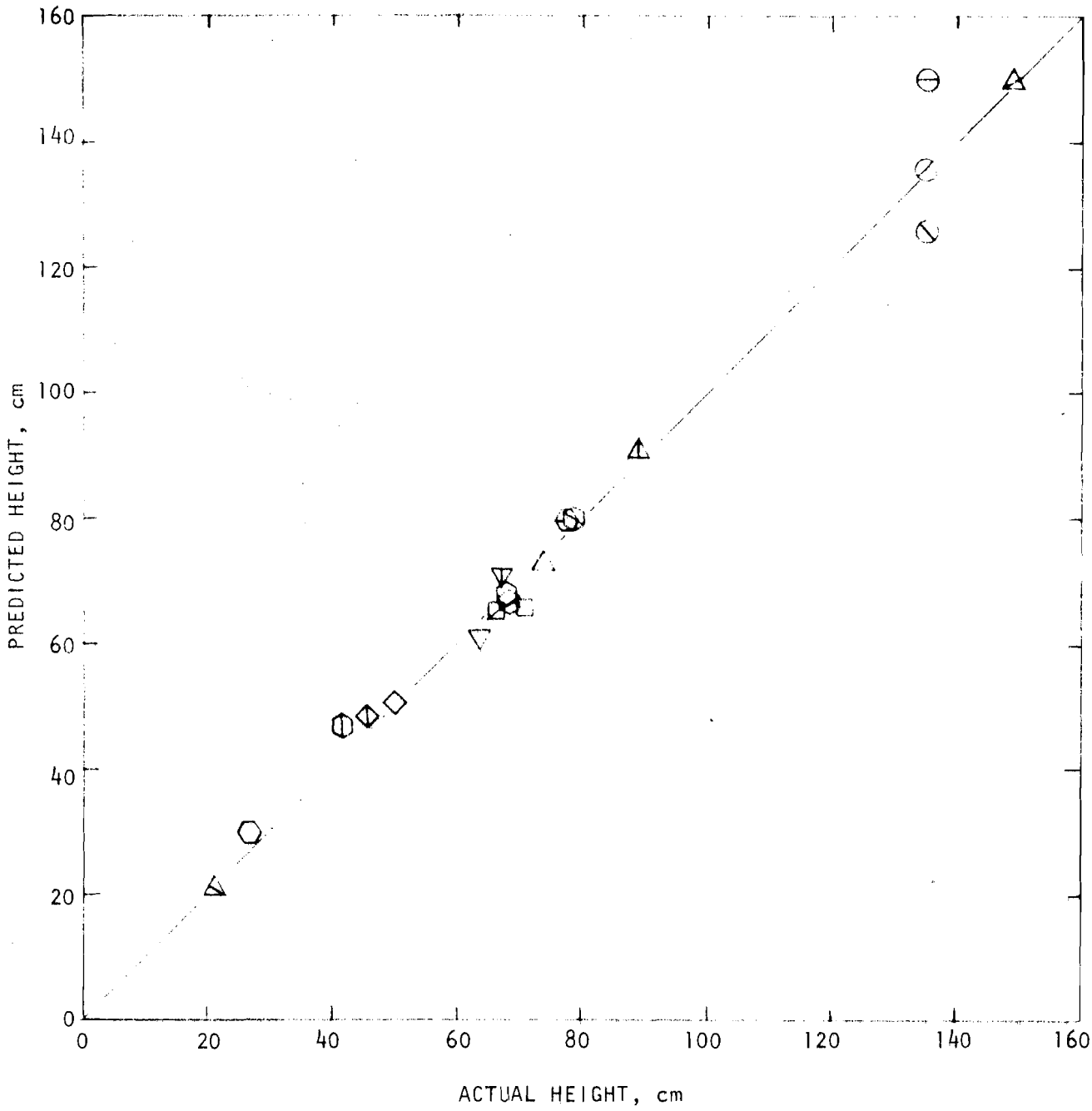


FIGURE 17. COMPARISON OF ACTUAL AND PREDICTED HEIGHT OF THICK BED IN CALCIUM CARBONATE THICKENING

of water from the floc, and increase in settling velocity. Measurement of pore water pressure was undertaken to evaluate these effects.

A description of this phenomenon is available in the consolidation theory of Terzaghi usually applied to soil problems. According to Terzaghi (Taylor, 1948), the total pressure, or stress, applied to a soil mass is counteracted by effective pressures developed between the soil particles and the pore water pressure. Hence, the total pressure, P_t, can be written as:

$$P_t = P_e + u \tag{4.2}$$

where P_e and u, respectively, refer to the effective pressure and the total pore water pressure. Increases in the total pressure on a consolidating soil mass increase the pore water pressure immediately. The time dependent dissipation of this pore water pressure due to escape of water from the soil mass and consequent increases in the magnitude of the effective pressure leads to the volume reduction or consolidation of the soil mass.

Pore water pressures were measured by the apparatus of Figure 10. Analysis of the results indicated that small effective pressures (in the order of 1 gm/cm^3) existed in the lower layers of the solids bed. In view of this low value and the absence of significant influence of sludge depth on performance, further analysis of pore pressures in continuous thickening of calcium carbonate did not seem promising.

4.2.5 — Significance of Other Thickening Parameters

The feed concentration plays a role in a thickening tank as it

affects the solids flux, G, which is equal to $C_f V_f$ (or $C_u V_u$). Changes in the feed concentration did not generally affect the thickening performance of the tank, but they significantly affected the clarification function. The increased volume of liquid associated with more dilute feeds passes through the overflow and consequently increased the turbidity of the overflow liquid. Observed overflow turbidities as a function of overflow velocity are shown in Figure 18. The relationship between turbidity and suspended solids concentration (in mg/l) in the range of 10 to 100 JTU was found to be:

$$C = 0.77 \text{ (JTU)} \tag{4.3}$$

There was little evidence that the volume between the feed level and the overflow level influenced the overflow turbidity except that solids were drawn to overflow because of local velocities around the overflow contraction when this volume was very small.

The variations in the solids and hydraulic detention times did not play significant roles in the thickening function of a sedimentation tank. Rather, the thickening performance of all calcium carbonate experiments was predicted from the solids flux theory. Hydraulic and solids detention times are not fundamental variables in this theory.

4.3 — Continuous Thickening of Activated Sludge

Thickening characteristics of four different samples of activated sludge were examined in the continuous thickeners B and C. A total of 35 continuous thickener experiments were conducted. Batch settling velocities were measured to permit prediction of the performance of the continuous

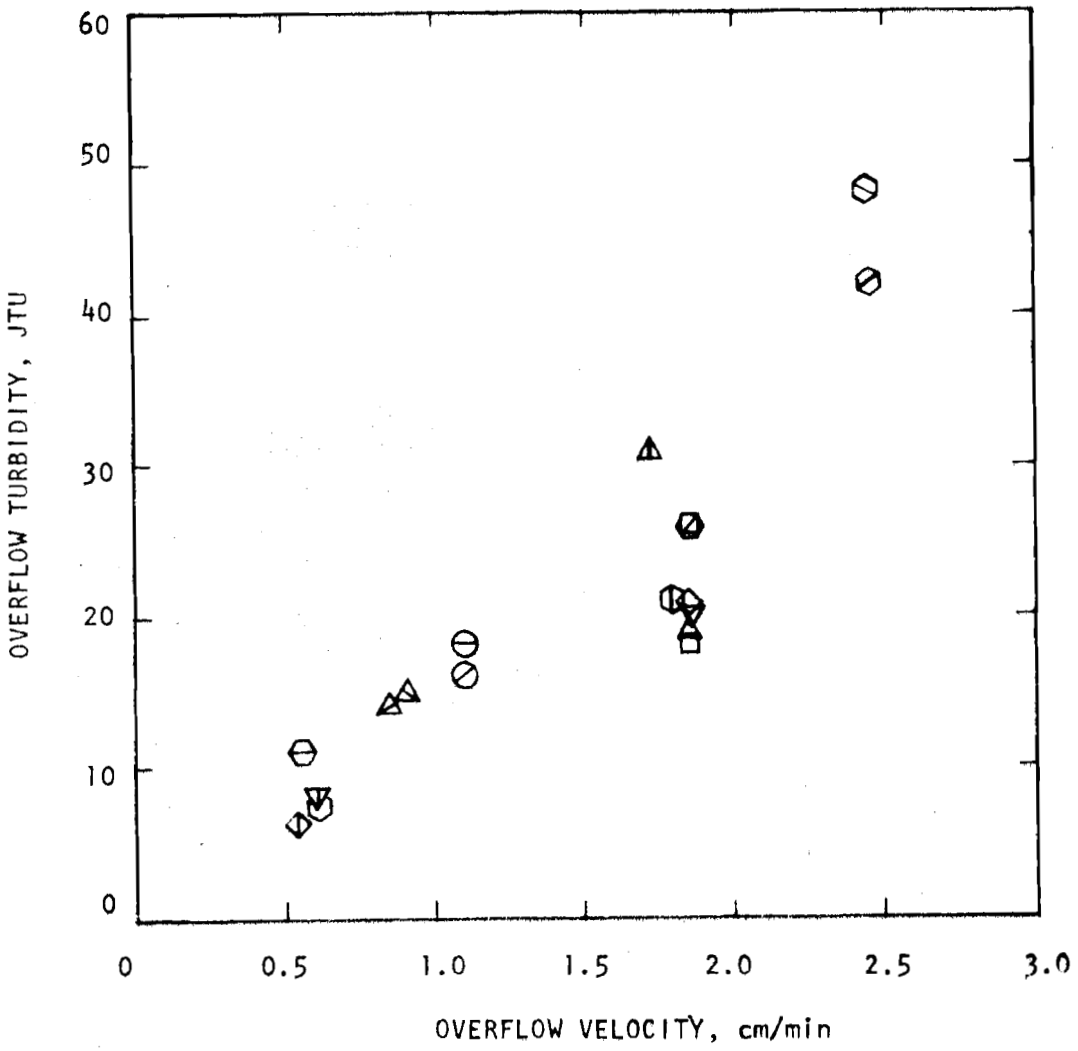


FIGURE 18. CLARIFICATION OF THE CALCIUM CARBONATE SUSPENSION

thickeners.

4.3.1 — Suspended Solids Profiles

Figure 19 shows a typical solids profile. The profile is very similar to those obtained by Albrecht, et al. (1966) in a large activated sludge settling tank. Similarity also exists between the shapes of the profile in Figure 19 and that obtained by Comings, et al. (1940). The thick bed did not have a single concentration as was the case with calcium carbonate. Instead, vertical concentration gradients existed in the bed and no definite first critical concentration could be defined. These vertical gradients were less pronounced when the underflow concentration was decreased due to the higher underflow velocity as illustrated in Figure 20. The vertical gradients in concentrations of the thick bed are considered to be due to the highly flocculent nature of activated sludge.

As with calcium carbonate, when the solids content of the thickener increased, or the feed funnel was lowered, it was possible for the second critical concentration not to exist at all.

4.3.2 — Prediction of Performance from Batch Tests

Comparison of batch settling velocities in stirred 1 liter cylinders with those measured in larger columns (Javaheri, 1971) indicated that for the activated sludge used in these experiments, the smaller, more convenient test cylinder could be used. Batch flux curves were plotted from these measurements and the graphical method of batch flux analysis (Figure 3) was then used to predict underflow concentrations for all of the experiments. Comparisons of predicted and actual concentrations are shown in

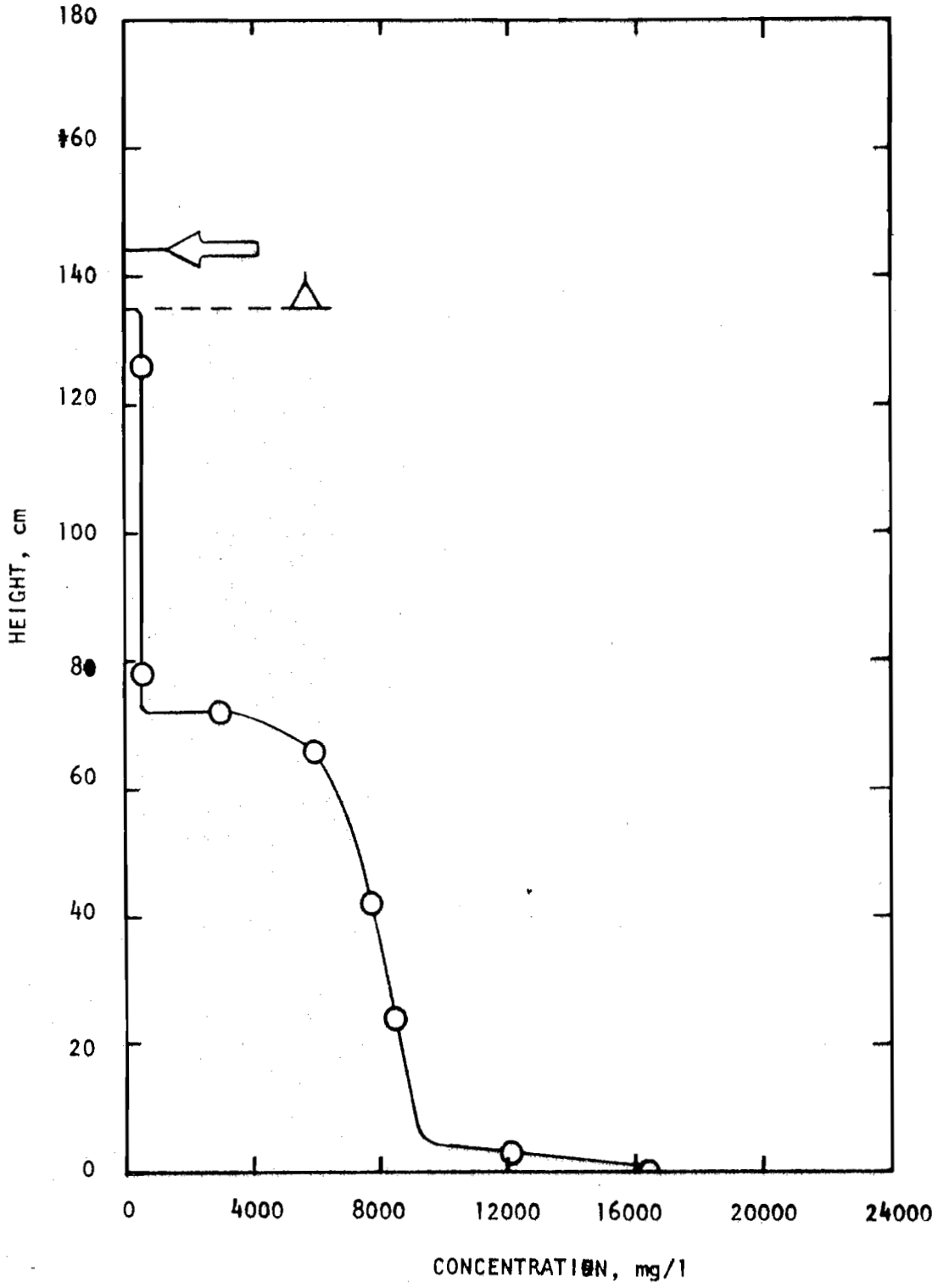


FIGURE 19. TYPICAL SOLIDS PROFILE OF ACTIVATED SLUDGE AT LOW UNDERFLOW VELOCITY

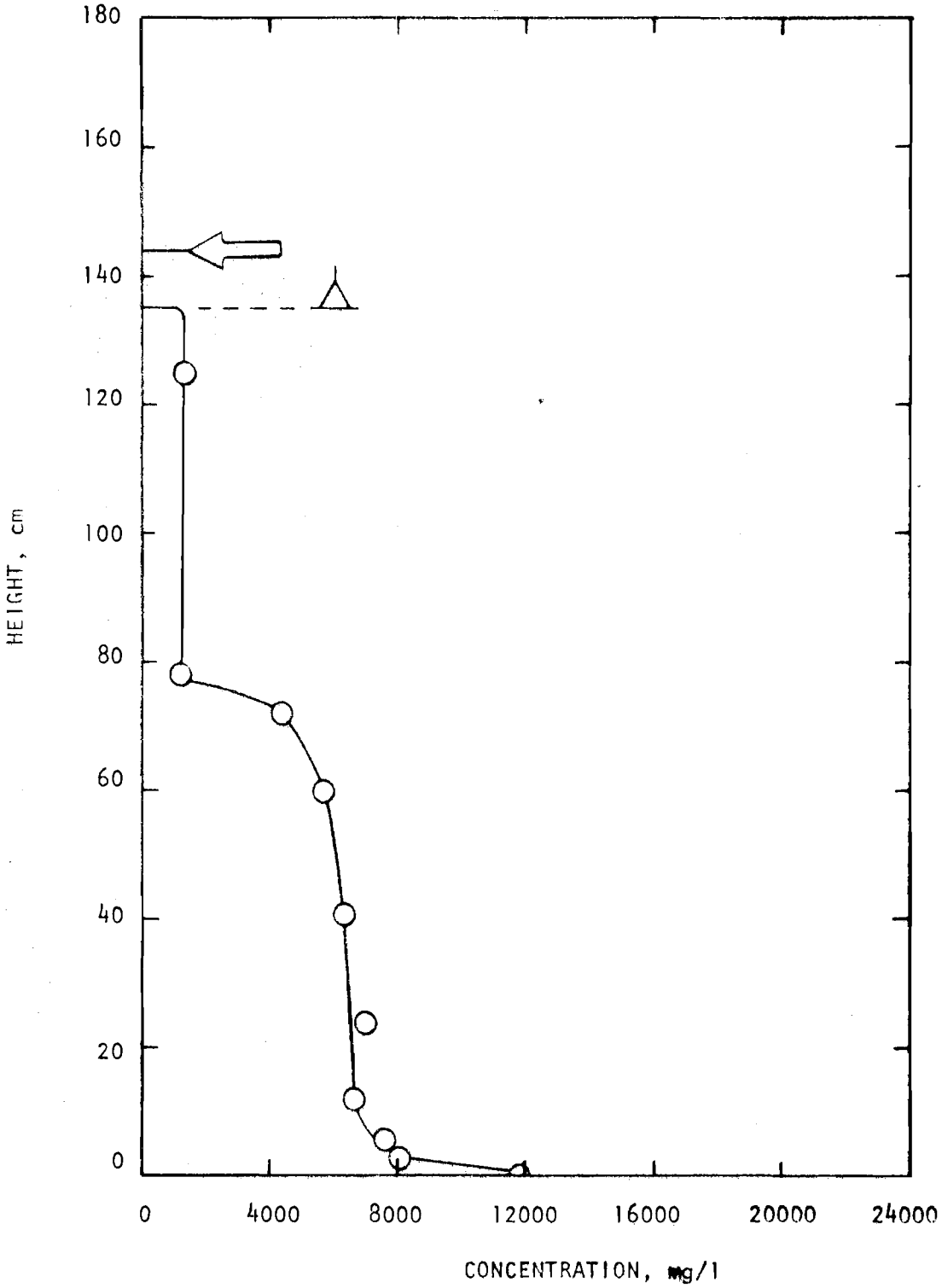


FIGURE 20. TYPICAL SOLIDS PROFILE OF ACTIVATED SLUDGE AT RELATIVELY HIGH UNDERFLOW VELOCITY

Figures 21 and 22. Most of the continuous tests were performed with stirrer B and, in general, results were overestimated by the batch flux curves. However, as shown in Figure 22, in the presence of the longer stirrer C the performance estimation was better. Better agglomeration of sludge was observed with the long stirrer. Floc sizes were more regular and somewhat larger than those experiments where the small stirrer, B, was used. It is not obvious whether the improved performance with stirring of the bed was due to the possible collapse of bridge networks (as advocated by Vesilind, 1968) in the continuous thickener, or to a true beneficial effect. Continuous thickening experiments with very large tanks are required to answer this question.

4.3.3 — Prediction of Required Volume

The amount of stored solids, the height of the thick solids bed, the solids and hydraulic detention times, were not closely related to the thickening performance of activated sludge. However, enough volume was required for storing the solids. Although the activated sludge used in these experiments settled well and was not as "compressible as many," there was little indication that benefits could be achieved by using deep sludge depths to increase settling velocity.

Equation (4.1) which was used for prediction of h_{th} in calcium carbonate experiments, also was tried with activated sludges. It was assumed that the actual area of the thick bed in the solids profile could be estimated by a rectangle with width of C_{L1} and height of h_{th}. Concentrations C_{L1} and C_{L2} were obtained from batch flux curves for experimental values of G. Unfortunately, he predicted

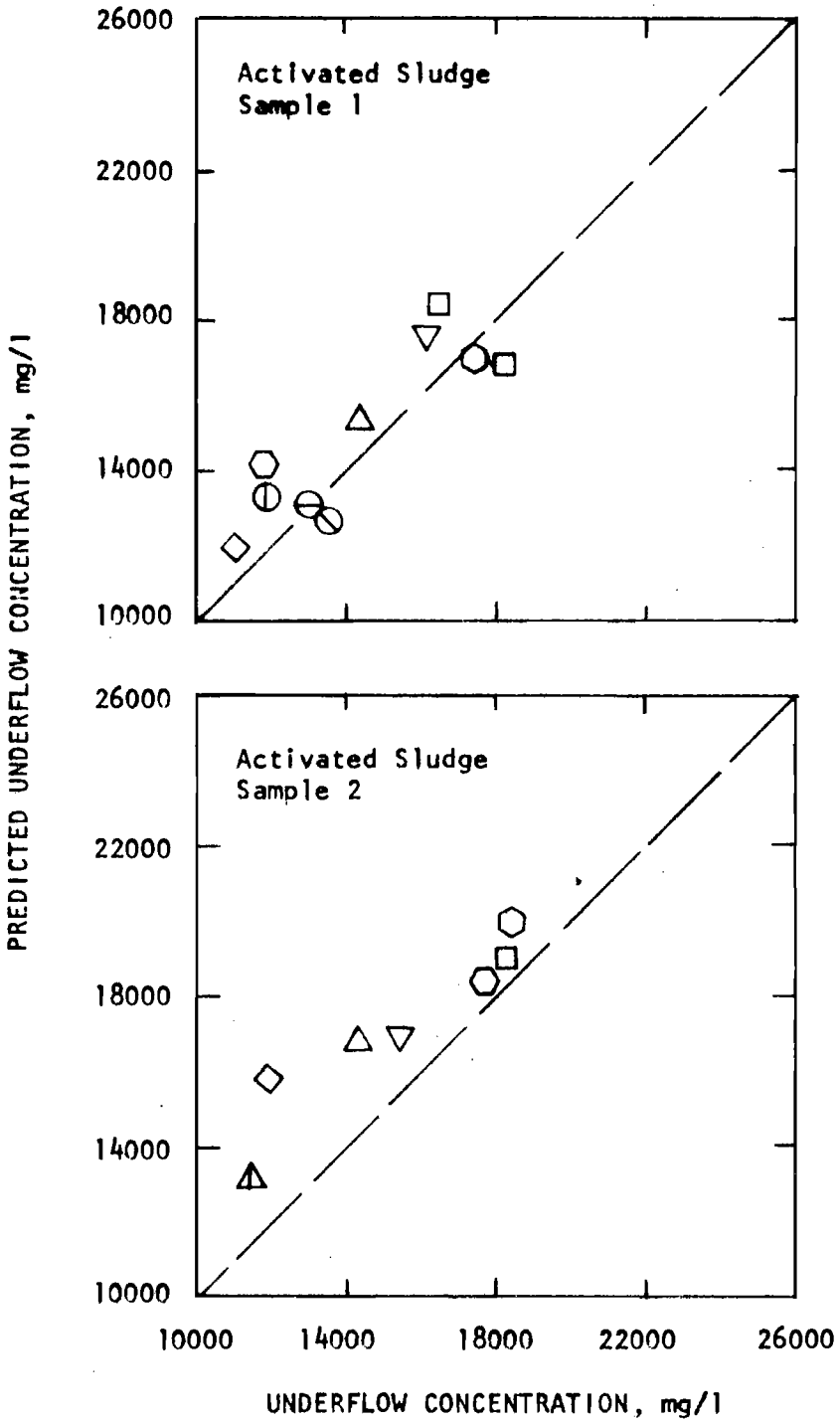


FIGURE 21. COMPARISON OF ACTUAL AND PREDICTED UNDERFLOW CONCENTRATIONS FOR SLUDGE SAMPLES 1 and 2

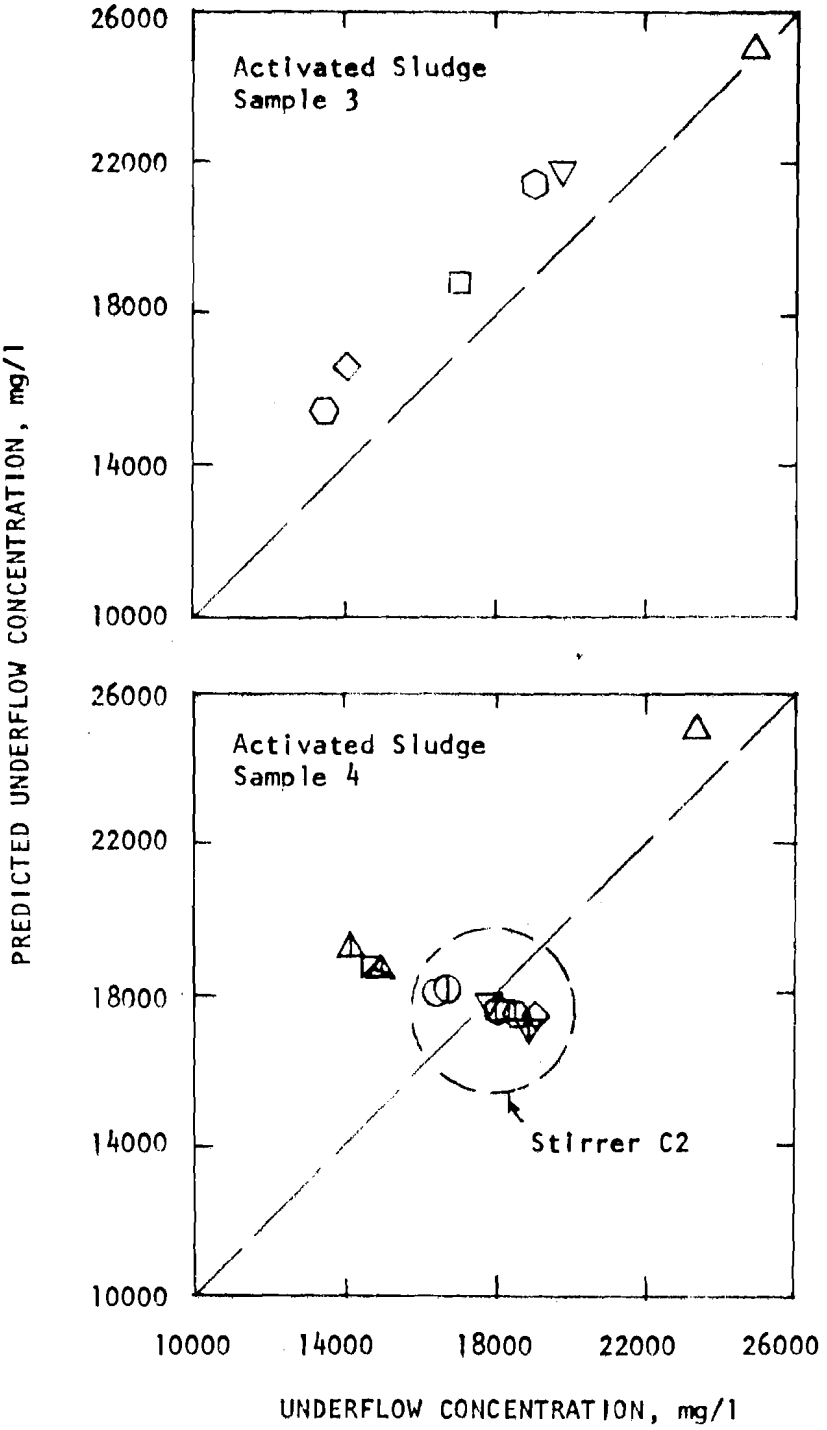


FIGURE 22. COMPARISON OF ACTUAL AND PREDICTED UNDERFLOW CONCENTRATIONS FOR SLUDGE SAMPLES 3 and 4

values of h_{th} were much smaller than the actual values. The measured values of C_{L1} from batch flux curves usually corresponded to concentrations at lower portions of the thick beds and were too large for the assumption of a rectangular shape for the thick bed.

4.3.4 — Significance of Other Thickening Parameters

The feed concentration, feed rate, and overflow rate played roles in activated sludge experiments similar to those for calcium carbonate. The concentration under the feed level (i.e. second critical concentration) was independent of the feed concentration. The second critical concentration followed the flux theory and increased consistently for practical purposes with increasing underflow velocities.

The overflow rate did not influence the overflow turbidity in a consistent manner due to changing clarification characteristics of the sludges. No definite correlation was found between the turbidity or solids concentration of the overflow and overflow rates. The solids content of the overflow liquids was in the range of 20 to 135 mg/l. The solids concentration of supernatant of activated sludges in a batch test was found to be very low (i.e. about 5 mg/l) immediately after the collection of samples. The concentrations in the supernatant which separated at later times were appreciably higher. Saunders (1971) examined a sample of sludge from the same source under the microscope at various times after the sludge was brought to the laboratory. He found that the sludge microorganisms became more dispersed with extended periods of time (i.e. several days), and the sizes of the floc particles decreased. Apparently, some of the colloidal matter in the supernatant

caused higher concentrations in the overflow. Probably the changing size characteristics of the colloidal matter also affected the light scattering characteristics in the turbidity measurements.

4.4 — Continuous Thickening of Water Softening Sludges

Twelve continuous thickening experiments were performed with two different sludge samples from the Northern Illinois Water Corporation. Accidental loss of one of the samples prior to development of the batch flux curve limited the extent of data interpretation. However, sufficient results (see Javaheri, 1971), were obtained to confirm that solids flux theory could be used for design of ideal continuous thickeners receiving similar softening sludges.

4.5 — Continuous Thickening of Glass Beads

Sixteen continuous thickening experiments with glass bead suspensions were performed to investigate thickening characteristics of incompressible suspensions. Typical solids profiles are shown in Figure 23. Beyond the point of maximum flux, the batch flux curve was nearly linear. This accounted for the usual solids profiles in Figure 23 and for the pronounced differences in parts A and B of the figure.

The amount of stored solids was not found to be an important parameter in thickening of glass beads. This is consistent with the solids flux theory and the settling nature of incompressible suspensions. The overflow turbidity did not correlate well with the overflow velocity. Clarification of the overflow seemed to be time dependent. One significant observation was that at relatively high overflow rates, i.e. above

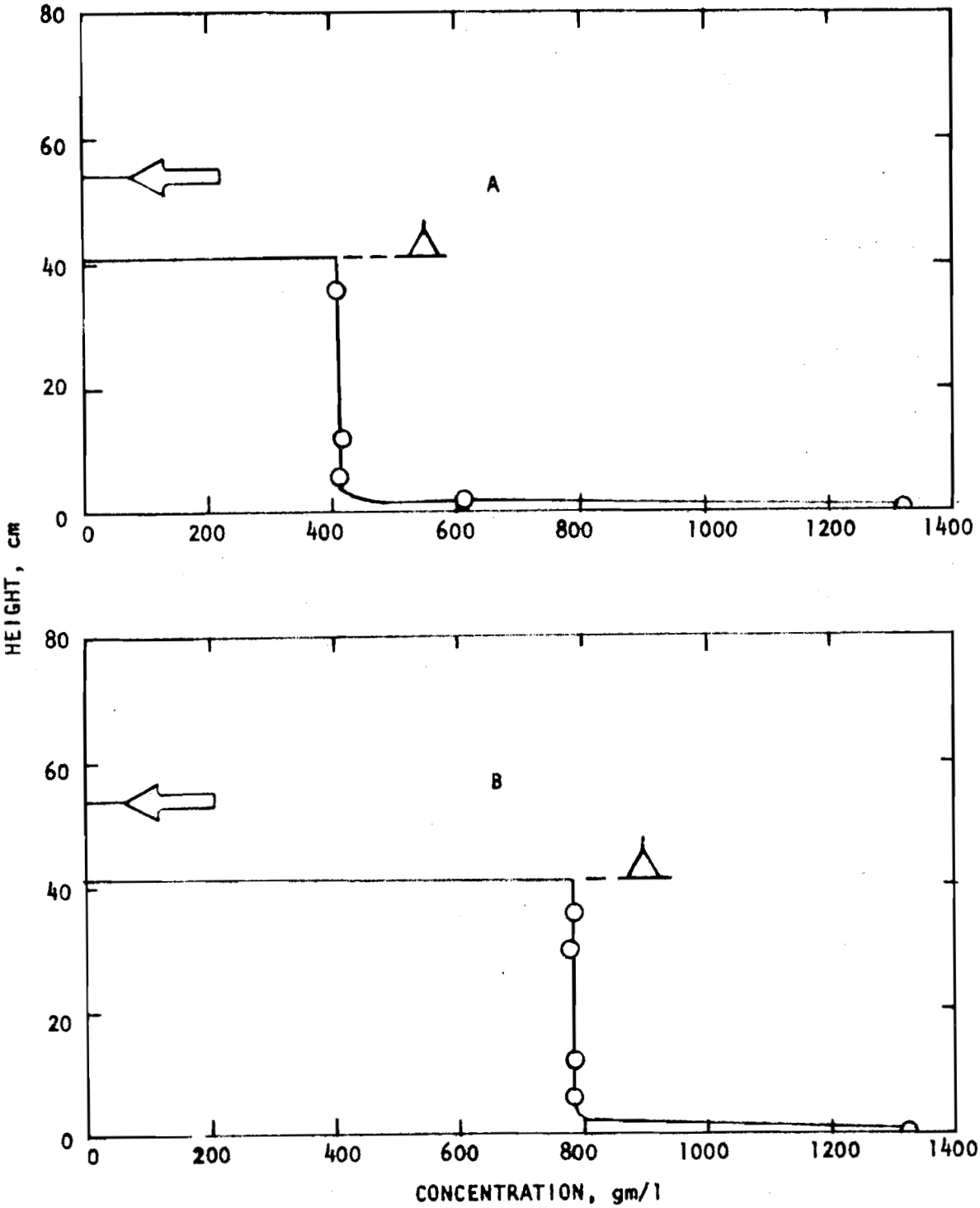


FIGURE 23. TYPICAL SOLIDS PROFILES FOR GLASS BEADS IN THICKENER B

3.0 $cm^3/cm^2/min$, some of the solids bed began to fluidize. The fluidized bed rapidly expanded and led to unsatisfactory clarification. The thickening function was not adversely affected by this phenomenon. The reason for fluidization was thought to be variations in sizes of glass beads. Smaller beads probably did not settle under the feed level. Feed concentration did not play a role in the thickening function of the thickener in the experiments. However, at lower feed concentrations, the overflow rate was relatively high for the given solids flux loading.

4.6 — Summary of Continuous Thickening Experiments

The outcome of continuous thickening experiments could be predicted by the application of the graphical technique based on ideal thickening theory. The particle size distribution of the glass bead suspension did not cause significant deviations from the expected performance. This was in agreement with findings of Shannon et al. (1963).

Fluidization of the solids bed sometimes took place with glass bead suspensions. It is thought that the particle size of the suspension, the particle size distribution, the particle density and the degree of aggregation between the particles should affect the fluidization phenomenon. These four properties of suspension particles were not known quantitatively for the calcium carbonate suspension, activated sludges, and softening sludges. Fluidization did not occur with these non-ideal suspensions.

Predictions based on application of solids flux theory became somewhat less accurate as the suspensions deviated from an incompressible state by exhibiting a flocculent and compressible nature. The degree of compressibility was evidenced by the magnitude of the height effect in

batch settling experiments. The calcium carbonate suspensions and softening sludges were less flocculent than the activated sludges. Consequently, the performance prediction of compressible suspensions was the most accurate in the case of the calcium carbonate suspension and the least accurate in the case of the activated sludges.

5. SUMMARY AND CONCLUSIONS

In carefully controlled continuous thickening experiments with a variety of suspensions, good agreement was obtained between observed thickening performance and the predictions of fundamental thickening theory. Whereas the degree of precise agreement with theory deteriorated as suspensions became more compressible, still basic agreement was attained. These results suggest that the depth of compressible sludge in continuous thickeners may not be as important as is commonly considered.

It was observed that non-ideal hydraulic conditions in continuous thickeners readily produced significant departures from theoretical thickening performance. This may be an important factor limiting the performance of full scale thickeners. In such thickeners, mixing at the thickener bottom, use of multiple underflow points, and/or improved distribution of feed solids may be necessary.

Specifically, it may be concluded that:

1. The closed continuous thickening system developed was a convenient means of laboratory thickening of suspensions. It enabled the establishment of steady-state in a reasonable amount of time so that numerous experiments could be performed without great difficulties.

2. In the continuous thickening experiments, effective stirring of the bottom layers of the solids bed near the underflow was an absolute necessity. Insufficient stirring resulted in low underflow concentrations. The intensity of stirring required depended on the type of suspension.

3. Relatively intensive stirring did not adversely affect the continuous thickening performance with suspensions of calcium carbonate, water softening sludge, and suspensions of glass beads. However, intensive

stirring of activated sludges above and beyond the optimal intensity sheared the sludge flocs in the solids bed and adversely affected the process of thickening.

4. Stirring of the thick bed of activated sludges in continuous thickening experiments at the optimal intensity resulted in higher underflow concentrations. However, by further experimentation at a large diameter thickener, it should be shown whether this was a true effect or an artifact due to small diameter of the pilot thickener.

5. Small effective pressures were found to exist in the bed of solids in continuous experiments with calcium carbonate. The settleability of the suspensions was not significantly affected by these pressures. Consequently, the consolidation theory of Terzaghi was not applied to explain the continuous thickening behavior of calcium carbonate suspensions.

6. The performance of the continuous thickener and the shape of the solids profile in the thickener could be predicted with reasonable accuracy from the batch flux curve when the solids flux theory was used as a tool of analysis. The predictions were, by and large, more accurate for the suspension of calcium carbonate, water softening sludge, and the suspension of glass beads. The predictions for activated sludge experiments were troublesome due to laboratory difficulties of obtaining realistic batch settling velocities and the complex activated sludge settling properties.

7. The thickening performance measured by the magnitude of underflow concentration depended on the magnitude of weight of solids applied per unit area and per unit of time, and the characteristic settling properties of the suspension. Performance did not depend significantly

upon the weight of stored solids, height of the solids bed, solids detention time, hydraulic detention time, or feed concentration.

8. The volume of the thickener below feed level depended upon the mass of stored solids and the settling properties of the suspension. Volumes were accurately predicted from the known weight of solids and batch flux curves for calcium carbonate and activated sludge experiments.

9. Clarity of the overflow deteriorated consistently with an increase in overflow rates for the calcium carbonate experiments. Meaningful clarification data could not be obtained with other suspensions due to specific experimental difficulties.

10. Thickening and clarification processes were found to interact through the magnitude of the feed concentration. One of these functions will limit the design if a preset performance criteria for both functions has to be met in the settling tank.

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