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ADVANCED METHODOLOGIES FOR DESIGN OF STORM SEWER SYSTEMS

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F I N A L   R E P O R T

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ABSTRACT

ADVANCED METHODOLOGIES FOR DESIGN OF STORM SEWER SYSTEMS

This report describes the development of a series of computer models capable of determining the diameter, slope and crown elevations of each sewer in a storm drainage system in which the layout and manhole locations are predetermined. The criterion for design decisions is the generation of a least-cost system. The basis for all of the models is the application of discrete differential dynamic programming (DDDP) as the optimization tool. Two important concepts are introduced as optimal model components: hydrograph routing and risks and uncertainties in designs. Three routing procedures are adopted, each with its own advantages. Expected flood damage costs are evaluated through the analysis of numerous risks and uncertainties associated with the design. This analysis permits the estimation of the probability of exceeding the capacity and the corresponding expected assessed damage of any sewer in the system. The expected damage cost is added to the installation cost to obtain the total cost which is then minimized in the DDDP procedure. Two example sewer systems are used as a basis for illustrating different aspects of the various least-cost design models and developing user guidelines.

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FOREWORD

There has been a long history of research on urban drainage problems in the Department of Civil Engineering of the University of Illinois at Urbana-Champaign. In 1887 Professor Arthur N. Talbot proposed his renowned waterway area formula which was widely used until the 1950's. More recently Professor Ven Te Chow made various significant contributions regarding rainfall frequency analysis and rainfall-runoff relationships useful in solving urban water problems.

The research study described in this report is part of an ongoing research program specifically aimed at the development of improved methods for *design* of urban storm drainage systems. In 1969 OWRR sponsored a project entitled "Methodologies for Flow Prediction in Urban Storm Drainage Systems," Project No. B-043-ILL. Under that project an improved hydraulic design model for storm sewers, the Illinois Storm Sewer System Simulation Model, was developed and the philosophy on design of urban drainage facilities was re-examined.

The present research project, entitled "Advanced Methodologies for Design of Storm Sewer Systems," OWRT Project C-4123 began on October 1, 1972. The major objective was to utilize three different concepts, namely, hydraulics, risk analysis, and optimization, to develop new sewer design methods and to demonstrate the savings that can be achieved through cost-effective design methods over the traditional design methods.

The research products of this project are the result of a team effort. The authors wish to thank those, both within and outside of the University, who contributed to the study either through their participation or in furnishing refreshing ideas. Those who were supported under the project are listed in Appendix G. The authors are grateful to Professor Jon C. Liebman of the Department of Civil Engineering for his valuable advice concerning operations research. Appreciation is also expressed for the cooperation and encouragement of Dr. Glenn E. Stout, Director, and the staff of the Water Resources Center of the University of Illinois. Special thanks are also due Mrs. Norma J. Barton and Miss Hazel Dillman for their patient, painstaking typing efforts throughout the project.

A continuous phase of this research program is currently in progress through OWRT Project B-098-ILL, "Risk Based Methodology for Cost-Effective Design of Storm Sewer System - Phase II." This study is devoted to supplementing and improving the work presented in this report. In view of the large amount of money devoted each year to sewer designs, cost-effective design models such as those developed in this research can provide substantial savings in public expenditures. However, in order to achieve the standards set in the Federal Water Pollution Control Act Amendments of 1972, P.L. 92-500, much more research is needed to formulate and implement new methods using currently available technologic knowledge and to develop new technologies as well.

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NOTATION

A = area

A_f = full pipe flow area

B = water surface width

C = cost; also, runoff coefficient; also, coefficient

C_D = expected damage cost

C_I = installation cost

C_m = manhole cost

C_p = pipe cost

c = celerity

D = decision, i.e., drop in elevation in optimization procedure

d = pipe diameter

E = elevation

E_r = acceptable error (Eq. 4.4)

F = function; also, cumulative cost function

f = Weisbach resistance coefficient

G = function

g = gravitational acceleration

H = sewer invert depth below ground surface

h = depth of flow; also, height of manhole below ground surface

I = inflow

i = rainfall intensity; also an index

i_o = referrence rainfall intensity

j = an index

K = constant; also, storage constant in Eqs. 6-10 and 6.11

k = surface roughness; also, an exponent (Eq. 5.21); also, an index

L = length of sewer

M_n = number of manholes on isonodal line n

m = an exponent (Eq. 5.21)

m_n = manhole on isonodal line n

N = number of stages in DDDP procedures; also, number

n = Manning's roughness factor; also, stage

P = probability

Q = discharge

Q_C = sewer capacity

Q_f = full pipe flow rate

Q_L = a reference discharge

Q_o = design discharge

Q_p = peak discharge

R = hydraulic radius

r_n = return for stage n

r_{ij} = coefficient of correlation

S = slope; also, state

S_f = friction slope

S_i = ith state

S_n = input state

$\tilde{S}_n$ = output state

S_o = sewer slope

SF = safety factor

s = storage

T = design period; also, return period; also, inflow hydrograph base line

$T_{m_n, m_{n+1}}$ = connection vector between manholes m_n and m_{n+1}

t = time

t_d = duration of rainfall

t_f = sewer flow travel time

V = flow velocity

W_t = time weight factor in Eq. 3.10

W_x = space weight factor in Eq. 3.10

X = inflow weight factor in Eq. 6.10

x = distance along the sewer; also, a variable

Z = $\ln(Q_C/Q_L)$, Eq. 5.3

z = elevation of invert

α = a factor or a coefficient

Δ = increment

Δ_s = state increment

δ = coefficient of variation

θ = angle between sewer axis and a horzontal plane

λ = correction factor

ν = kinematic viscosity

τ = time

Φ = central angle of water surface in sewer (Fig. 3.1)

ϕ = cumulative standard normal probability distribution

Ω = coefficient of variation

Chapter 1. INTRODUCTION

One of the vital facilities in preserving and improving the urban environment is an adequate and properly functioning stormwater drainage system. Estimates from various sources all indicate that the total cost in the Nation for construction of new storm sewers and of maintenance and operation of existing storm and combined sewer systems well exceeds one billion dollars annually. Atop of this expenditure are the tangible and intangible losses due to inadequate or improper drainage of storm water. Despite the large amount of money involved in urban storm water drainage, and contrary to the general belief of the public, the presently available technological tools are not being applied to this problem except in isolated instances. In fact, the majority of design engineers working on storm water drainage problems have not gone beyond the stage of using the widely criticized rational method and there is very little post evaluation done once the system is in the ground.

From an engineering viewpoint the drainage problem can be divided into two aspects: runoff prediction and system design. Considerable effort has been devoted in recent years to runoff prediction in urban areas, encouraged in part by the enactment of the Federal Water Pollution Control Act Amendments of 1972, P.L. 92-500. Rainfall-runoff model building has become a popular activity and a variety of such tools are now available, and the state-of-the-art of this aspect of urban drainage has been reported by Chow and Yen (1976), James F. MacLaren, Ltd. (1975), Heeps and Mein (1974), Brandstetter (1974) and McPherson (1975). However, despite the existence of such techniques they are not being extensively used. Part of the problem lies in the need for urban runoff quality-quantity data for use in model calibration. Another difficulty lies in the confusion which exists concerning which model is economically and/or technically appropriate for a specific situation.

The second aspect of the drainage problem, design methodology, has received relatively little attention. This is the subject of this report. The design of new sewer systems or for extension of existing systems may be for the purpose of urban flood mitigation, or it may be in conjunction with other pollution control facilities such as treatment plants and overflow regulators, or both. Urban storm water drainage actually consists of two distinct and sequentially connected systems; namely, the land surface drainage system from receiving the water (precipitation) to the inlet catch basins, and the sewer system downstream from the inlet catch basins. Only the latter part, the design aspect of the sewer systems, is considered in this report. Several techniques or tools have been investigated and four design models have been developed. The framework of these models is the use of a particular form of dynamic programming to perform a search for a minimum cost combination of pipe sizes, slopes and elevations. Two types of costs are considered: the installation cost and the damage cost in the event that the capacity of the system is exceeded. The latter cost is usually not formally considered in urban drainage design work.

Another design aspect that is usually given superficial attention is the consideration of uncertainties. Conventional techniques begin with the determination of a return period to be used in selecting a "design storm" or rainfall. The runoff from this design storm is then used to design the system which is then assumed to acquire the same performance return period as assumed for the rainfall. Furthermore, no additional consideration of uncertainty is given. There are, in fact, many sources of uncertainty in any design procedure and the probability of exceeding the capacity of the system should include all of them. Chapter 5 of this report presents an approach for accounting for uncertainties and thus has been adopted for use in several of the design models.

A final technique given consideration is flood routing in the sewer system. The rational method of design uses no routing since each pipe is independently designed. However, the pipes do not perform independently and flood routing techniques provide a more realistic picture of the translation and attenuation of in-system hydrographs. This can lead to more economical designs since the overall effect is to reduce the computed peak flows. Several routing techniques have been evaluated as described in Chapter 6 and incorporated in the design models.

The work described in this report represents one possible initial step in the development of a comprehensive method for design of urban drainage systems. Several of the techniques are relatively new as far as design methodology is concerned, but it is believed that their consideration has considerable merit.

Chapter 2. DESIGN PHILOSOPHY

Ideally, an optimal design method for storm sewer systems should produce a design providing maximum economic benefits, considering realistically and accurately the pertinent hydrologic, hydraulic, construction, and economic factors. The optimization should be carried out considering not solely the sewer system itself but also the drainage facilities immediately connected to and closely interfaced with it. These facilities include the land surface drainage system upstream from the sewer system and the treatment system and receiving water bodies downstream. To include the surface drainage system, treatment system, receiving water body and storm sewer system together for water quality and quality management strategic planning is sufficiently difficult in view of today's computer capability. To consider simultaneously all these systems for optimal design of layout, slope, and size of a storm sewer system is a task that has yet to be attempted. As a first step towards this general goal, it is worthwhile to first define the philosophy for optimal design of the storm sewer system itself. From the methodology development and classification viewpoint, the design philosophy can be discussed from four different aspects; namely, system optimization, uncertainties and risks, routing of sewer flow, and the constraints and assumptions involved.

2.1. System Optimization

The key points involved in the idea of optimization are the following:

(a) The optimization is carried out for the entire sewer system, including not just the sewers but also manholes, junctions, and other auxiliary facilities such as detention reservoirs, overflow devices, pumps, and other flow regulators.

(b) The objective of optimization is to produce a design of the entire sewer system providing the best benefit-cost relationship within the physical, economical, social, and environmental constraints and assumptions. Ideally the measure of benefit should include not only the tangible ones such as reduction of damages but also the intangibles such as improvement of the environmental health and reduction of risk of loss of human lives. The cost should include not only the installation cost but also other costs such as those for operation and maintenance.

(c) The optimal design of the system should give not only the sizes of the individual sewers and manholes but also the sewer slopes and layout.

2.2. Uncertainties and Risks

Uncertainties arise in almost every aspect and every factor involved in urban storm sewer systems. These uncertainties should be accounted for in an optimal design. In fact, the design methodology should be able to produce a design giving the best benefit-cost relationship with the corresponding risk levels for the sewers that are within the specified acceptable maximum risk levels for the project. In the design at least the following uncertainties should be accounted for:

(a) Hydrologic uncertainties - These include uncertainties on the accuracy of the inlet hydrographs which are the input into the sewer system, the probability of occurrence of future floods more severe than the design inlet hydrographs, and the uncertainty on the future change of the physical characteristics of the drainage basin.

(b) Hydraulic uncertainties - These include the uncertainties in the mathematical simulation model in describing the flow in the sewers and through the junctions. Particularly, if simple flow formulas such as Manning's formula and the Bernoulli equation are used, the uncertainties in using these formulas to describe unsteady flow, upstream and downstream backwater effects, sewer surcharges, and junction losses should be accounted for. Also the uncertainty on change of sewer pipe roughness with time should be considered.

(c) Material uncertainties - These include the uncertainties on the quality control of the materials used in the sewer system, such as the diameter, straightness, and uniformity of surface roughness of the sewer pipes.

(d) Construction uncertainties - These include the uncertainties in the accuracy in laying the sewer pipes, settlement of the bedding soil, and sewer deflection under load.

(e) Uncertainties on costs and damages - These include the uncertainties in the cost estimation functions for installation, operation and maintenance, damages, and changes of interest and inflation rates.

(f) Uncertainties in the expected sewer system service life and the design return period, or, more rationally, the acceptable failure risk level.

2.3. Sewer Flow Routing

Theoretically, a reliable hydraulic routing method should be used in designing sewers. Sewer flows are generally unsteady, nonuniform when the pipe is not flowing full and under pressure, and subject to backwater effects from both upstream and downstream of the pipe. As discussed by

Yen (1973), a high accuracy hydraulic routing method using the full dynamic equations for sewers and junctions accounting for flow unsteadiness and backwater effects requires a considerable amount of computer time on a large digital computer. It is most unlikely that such a sophisticated routing scheme can be incorporated within an optimization procedure to provide a new design method which is within the capabilities of existing computers. In fact, as suggested by Yen and Sevuk (1975), even for hydraulic design of sewers without accounting for uncertainties and cost optimization, the sophisticated routing scheme using the full dynamic equations for sewer system design is needed and justifiable in most cases only for the final checking of the hydraulic accuracy of the design. They showed that in view of the discrete sizes of commercially available pipes, simpler approximate hydraulic routing methods are useful in sewer designs.

Yen and Sevuk (1975) pointed out that using either the Manning's formula for sewer flow with appropriate time shifting of the hydrographs or a nonlinear kinematic wave approximation usually give acceptable designs with considerable savings in computer time. In addition, a modified kinematic wave routing scheme, called the Muskingum-Cunge method (Cunge, 1969), also give results close to those given by the nonlinear kinematic wave approximation while requiring less computer time.

It is difficult to establish a priori which, if any, routing technique is best. Therefore the above three methods have all been considered. A discussion of each is included in Chapter 6. Presumably, the ideal routing method for the optimal design should be the one that giving sufficient accuracy yet not requiring excessive computer time and capacity.

2.4. Constraints and Assumptions

In addition to the above considerations, a design methodology must include a number of constraints and assumptions which are commonly used in engineering practice such as those presented by the ASCE Urban Water Resources Research Program (1968) and ASCE and Water Pollution Control Federation (1969). The constraints and assumptions used in the various design models in this study are as follows:

(a) Free-surface flow exists for the design discharges or hydrographs, i.e., the sewer system is "gravity flow" so that pumping stations and pressurized sewers are not considered.

(b) The sewers are commercially available circular sizes no smaller than 8 in. in diameter. Flows that require pipes smaller than 8 in. in diameter can be carried by street gutters eliminating the need of sewers. The commercial sizes in inches are 8, 10, 12, from 15 to 30 with a 3 in. increment and from 36 to 120 with an increment of 6 in.

(c) The design diameter is the smallest commercially available pipe that has flow capacity equal to or greater than the design discharge and satisfies all the appropriate constraints.

(d) Storm sewers must be placed at a depth that will not be susceptible to frost, drain basements, and allow sufficient cushioning to prevent breakage due to ground surface loading. Therefore, minimum cover depths must be specified.

(e) The sewers are joined at junctions such that the crown elevation of the upstream sewer is no lower than that of the downstream sewer.

(f) To prevent or reduce permanant deposition in the sewers, a minimum permissible flow velocity at design discharge or at barely full-pipe gravity flow is specified. A minimum full-conduit flow velocity of 2 fps is required or recommended by most health departments.

(g) To prevent occurrence of scour and other undesirable effects of high velocity flow, a maximum permissible flow velocity is also specified. The most commonly used value is 10 fps. However, recent studies have shown with the quality of modern concrete and other sewer pipes the acceptable maximum velocity can be considerably higher.

(h) At any junction or manhole the downstream sewer cannot be smaller than any of the upstream sewers at that junction.

(i) The design inflows into the sewer system are the inlet hydrographs or peak discharges.

Furthermore, for the optimization models, the following additional assumptions are made:

(a) The sewer system is a dendritic network converging towards downstream.

(b) No negative slope is allowed for any sewers in the dendritic network.

(c) The direction of the flow in a sewer is uniquely determined from topographic considerations.

(d) Presumably the cost function for installation varies with geographic locations and time. For illustrative purposes a set of simple cost functions proposed by Alan M. Voorhees (1969) is adopted in this study. The pipe installation cost in dollars per linear foot of sewer, C_p is

$$C_p = 10.98d + 0.8H - 5.98 \qquad d \leq 3' \text{ and } H \leq 10'$$

$$C_p = 5.94d + 1.166H + 0.504Hd - 9.64 \qquad d \leq 3' \text{ and } H \geq 10' \qquad (2.1)$$

$$C_p = 30.0d + 4.90H - 105.9 \qquad d > 3'$$

in which d is sewer diameter in feet and H is the sewer invert depth in feet below the ground surface. The depth H of each sewer is computed as the average of the invert depths at the upstream and downstream ends of the sewer. The unit cost of a manhole, C_m, in dollars is

$$C_m = 250 + h^2 \qquad (2.2)$$

in which h is the depth of the manhole in feet which is determined by the lowest invert of the sewers joining the manhole.

Chapter 3. REVIEW OF EXISTING SEWER DESIGN METHODS

Most of the previously developed sewer design models that have been adopted in engineering practice are hydraulic design models. In the last decade a few studies have been reported dealing with design of sewers on the basis of minimum cost. A brief review of these two types of sewer design models is given in this chapter.

3.1. Hydraulic Design Models

The sewer hydraulic design models determine the sewer sizes using only hydraulic considerations. No consideration is given to cost minimization nor are risks and uncertainties accounted for. The sewer system layout is predetermined and the sewer slope generally is assumed to follow the ground slope or is specified. The basic design concept is to determine the minimum sewer size that has a capacity to carry the design discharge under full pipe gravity flow conditions.

The design models considered here are those having a built-in mechanism for determination of sewer sizes. Many of the so called "sewer design methods" are actually flow simulation or prediction methods to provide the design hydrographs. They require the layout, slope, length and size of the sewers to be known or assumed. They do not have a means for direct determination of sewer sizes. Hence, they are not regarded herein as true sewer design methods and not considered in this review.

The important features of the major sewer hydraulic design models are summarized in Table 3.1.

3.1.1. Steady Flow Methods

The most commonly used model is the rational method or its variations which can be collectively called steady flow methods. The sewer design discharge is obtained by adding the hydrographs or peak flows from the upstream sewers with or without considering lag effects. The required sewer size is subsequently computed by using the Manning, Darcy-Weisbach, Hazen-Williams, or similar formula assuming full pipe flow with a predetermined sewer slope. The adopted sewer size is the next commercially available pipe size that is equal to or greater than the required size. No routing of the flow is involved and no consideration is given to the unsteady and nonuniform nature of the sewer flow. The effect of in-line storage is neglected. Using the Manning formula, the minimum required sewer diameter d for the design discharge Q_p is (Yen and Sevuk, 1975)

$$d = (4.66 \frac{n^2}{S_o} Q_p^2)^{3/4} \tag{3.1}$$

in which n is the Manning's roughness factor and S_o is the sewer slope.

3.1.2. Chicago Hydrograph Method

This method (Tholin and Keifer, 1960) is a steady flow hydrograph routing approach which considers in-line storage. Two approaches were recommended by Tholin and Keifer. The simpler one is a time-offset scheme in which a sewer inflow hydrograph is subdivided into a number of component hydrographs, each shifted by a time equal to an assumed time of travel. The sum of these shifted component hydrographs gives the outflow hydrograph of the sewer. This technique lacks theoretical justification and the result depends on the number of component hydrographs used and consequently the solution is not necessarily unique.

The other approach considered is a storage routing scheme using Manning's formula and the continuity equation for flow in the sewer. From the hydraulic viewpoint, this is a linear kinematic-wave approximation (Yen, 1973a). In this approach, the continuity equation expressing mass conservation is written as

TABLE 3.1. Summary of Sewer Hydraulic Design Models

Model	Sewer System Input	Sewer Hydraulics	Junction Hydraulics	Backwater Effect Considered	Design Sequence in Network	Output	Ref.
Rational	Inlet peak discharges	No routing, no time lag of discharges	Continuity equation	No	Cascading	Sewer diameters and design discharges	Yen and Sevuk (1975); ASCE and WPCF (1969)
Rational	Inlet hydrographs	No routing, time lag of hydrographs	Continuity equation	No	Cascading	Sewer diameters and design discharges	Yen and Sevuk (1975)
Chicago Hydrograph	Inlet hydrographs	Storage routing or time-offset without routing	Continuity equation	No	Cascading	Sewer diameters and design hydrographs	Tholin and Keifer (1960); Yen and Sevuk (1975)
TRRL	Inlet hydrographs	Reservoir routing lagged by time of travel	Continuity equation	No	Cascading	Sewer diameters and basin runoff hydrograph	Watkins (1963)
ILLUDAS	Inlet hydrographs	Reservoir routing lagged by time of travel	Continuity equation	No	Cascading	Sewer diameters and design hydrographs	Terstriep and Stall (1974)
Kinematic Wave	Inlet hydrographs	Nonlinear kinematic wave routing	Continuity and dynamic equations	Upstream only	Cascading	Sewer diameters, discharge hydrographs and depth	Yen and Sevuk (1975)
EPA SWMM	Inlet hydrographs	Improved nonlinear kinematic wave routing	Continuity equation	Upstream and partial downstream	Cascading	Sewer diameters, design hydrographs and flow velocities	Metcalf & Eddy et al. (1971); Huber et al. (1975)
ISS	Inlet hydrographs	Dynamic wave (St. Venant eqs.) routing	Continuity and dynamic equations	Both upstream and downstream	Y-segment sequence	Sewer diameters and discharge, depth and velocity graphs	Yen and Sevuk (1975); Sevuk et al. (1973)

$$I_1 + I_2 - Q_1 + \frac{2s_1}{\Delta t} = Q_2 + \frac{2s_2}{\Delta t} \tag{3.2}$$

in which I is the inflow rate into the sewer; Q is the outflow rate at the exit of the sewer; s is the storage of water in the sewer; t is time; and subscripts 1 and 2 represent the quantities at the beginning and end of the time interval, Δt, being considered.

Following Tholin and Keifer's assumptions and using Manning's formula, the storage function can be obtained as

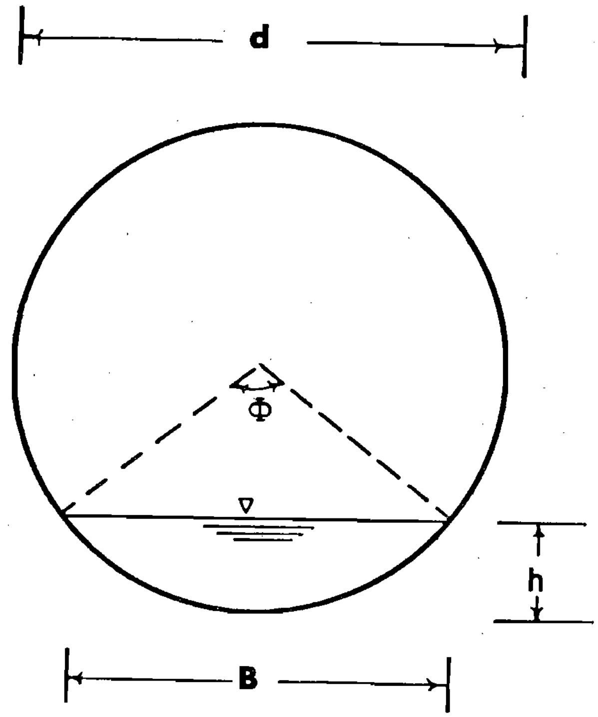
$$s = 0.143\ n^{3/5}\ S_o^{-3/10}\ L(\Phi d)^{2/5}\ (I^{3/5} + 5\ Q^{3/5}) \tag{3.3}$$

in which L is the sewer length and Φ is the central angle of the water surface as shown in Fig. 3.1. Thus, the inflow hydrograph for a sewer can be routed using Eqs. 3.2 and 3.3 to obtain the outflow hydrograph for the sewer.

The sewer design procedure for this method is essentially the same as that for the steady-flow method with the time shifting of hydrographs discussed in Section 3.1.1, except that the hydrographs are now routed through the sewers instead of simply lagged. Once the peak flow, Q_p, is evaluated for a sewer, its diameter can be determined by using Eq. 3.1.

3.1.3. Transport and Road Research Laboratory Method

The British Transport and Road Research Laboratory (TRRL) method (Watkins, 1962, 1963; Terstriep and Stall, 1969) is another steady-flow hydrograph routing method known in the United States by its original name, the RRL method. The method was developed mainly to calculate "the rates of storm runoff in sewer systems" (Watkins, 1962) although a scheme for



Flow area $A = \frac{d^2}{8} (\Phi - \sin \Phi)$

Hydraulic radius $R = \frac{d}{4} (1 - \frac{\sin \Phi}{\Phi})$

Depth $h = \frac{d}{2} (1 - \cos \frac{\Phi}{2})$

Water surface width $B = d \sin \frac{\Phi}{2}$

Fig. 3.1. Circular Sewer Flow Cross Section

sewer size computation was added. The inlet hydrographs are routed through the sewers using a reservoir routing technique. The time of travel in a sewer is computed as $t = L/V$, where L is the length of the sewer and V is the full pipe flow velocity computed by using the Darcy-Weisbach formula

$$V = \left(\frac{8gRS_f}{f}\right)^{1/2} \qquad (3.4)$$

in which g is the gravitational acceleration; R is the hydraulic radius, S_f is the friction slope, and the Weisbach resistance coefficient f is given by the Colebrook-White formula

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left[\frac{k}{14.8R} + \frac{0.89\nu}{R\sqrt{gRS_f}}\right] \qquad (3.5)$$

where k is the equivalent pipe roughness and ν is the kinematic viscosity.

The inflow hydrograph of a sewer is the combination of the outflow hydrograph from the upstream system and the local inlet hydrographs. The sewer diameter can then be computed for the peak discharge using the Darcy-Weisbach formula

$$d = \left(0.0252 \frac{f}{S_o} Q_p^2\right)^{1/5} \qquad (3.6)$$

in which d is in ft and Q_p in cfs.

The inflow hydrograph for a sewer is routed using the continuity relationship, Eq. 3.2 and a storage-discharge relationship is supplemented to Eq. 3.2 to give the outflow rate. Originally, Watkins suggested the use of the recession part of recorded runoff hydrograph to establish the storage-discharge relationship. In a later version, it was suggested to approximate this relationship using the Darcy-Weisbach formula (Eq. 3.4) with f given by Eq. 3.5 assuming instantaneously the sewer flow is steady and uniform. A linear interpolation between the values of hydraulic radius and flow area was suggested to avoid a time consuming interative solution.

Basically, from a routing viewpoint, the sewer design scheme of the TRRL method is essentially the same as that of the Chicago hydrograph method. The only difference between the two methods is that instead of using Eq. 3.3 which is based on the Manning formula, Eq. 3.4 with f being estimated by a simplified Colebrook-White formula is used to give the storage function. Consequently, the two methods give identical designs when the flow is turbulent and fully developed for which Weisbach's f and Manning's n are equivalent.

3.1.4. Illinois Urban Drainage Area Simulator (ILLUDAS)

ILLUDAS (Terstriep and Stall, 1974) is a modification of TRRL method to account for the surface runoff from pervious areas. Its sewer flow routing concept is the same as TRRL method. Since Manning's formula instead of Darcy-Weisbach's is used in the computation, the sewer design aspect of ILLUDAS is essentially the same as the Chicago hydrograph method with its storage routing scheme.

3.1.5. Kinematic Wave Method

In the nonlinear kinematic wave method, the unsteady sewer flow is described by the following two equations (Yen and Sevuk, 1975)

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \tag{3.7}$$

$$S_o = S_f \tag{3.8}$$

in which x is the distance along the sewer; A is the flow cross sectional area normal to x; and t is time. The friction slope, S_f, is approximated by Manning's or Darcy-Weisbach's formula. Equations 3.7 and 3.8 are then solved numerically with initial and upstream conditions specified. No downstream boundary condition is required and consequently downstream backwater effects cannot be accounted for.

Yen and Sevuk (1975) formulated a nonlinear kinematic wave sewer design model using a four-point noncentral implicit finite-difference numerical scheme. Manning's formula is used to evaluate S_f. The junction or manhole condition is accounted for by the continuity equation

$$Q_1 + Q_2 + Q_j - Q_3 = \frac{ds}{dt} \tag{3.9}$$

in which s = storage in the manhole or junction; Q_j = surface inflow into the junction; subscripts 1 and 2 represent the inflow sewers; and subscript 3 indicates the outflow sewer from the junction. If the storage of the manhole or junction is negligible, the right-hand side of Eq. 3.9 is equal to zero.

3.1.6. EPA Storm Water Management Model (SWMM)

The EPA SWMM (Metcalf & Eddy, Inc. et al., 1971), is a relatively comprehensive urban stormwater runoff quantity and quality prediction and management simulation model. In one of the recent SWMM modifications, a hydraulic design capability was included (Huber et al., 1975).

For sewer flows considered in the Transport Block portion of the SWMM, a modified nonlinear kinematic wave approximation is used. Continuity equation (Eq. 3.7) and Manning's formula are used with the slope assumed equal to the friction slope, and the flow is assumed to be steady within each time interval. The continuity equation is expressed in finite difference form, using the x-t plane shown in Fig. 3.2 with $\Delta x = L$ = sewer length, as follows

$$\frac{(1-W_t)(A_D-A_A) + W_t(A_C-A_B)}{\Delta t} + \frac{(1-W_x)(Q_B-Q_A) + W_x(Q_C-Q_D)}{L} = 0 \qquad (3.10)$$

in which Q is the discharge; and A is the flow cross sectional area. The time derivative is weighted W_t at the downstream station and the spatial derivative is weighted W_x at the end of Δt. Subsequently Eq. 3.10 is normalized by the full conduit flow area and discharge A_f and Q_f, respectively. By assuming steady uniform condition and using Manning's formula a single curve representing the relationship between Q/Q_f and A/A_f for the conduit can be established. With this nondimensional discharge-area curve replacing Manning's formula and the normalized (only for A and Q) form of Eq. 3.10, numerical solutions are then obtained with known initial and upstream boundary conditions using a four-point implicit finite difference scheme.

Since no downstream boundary condition is required for the solution, SWMM cannot account for the downstream backwater effect when the sewer flow is subcritical. Nevertheless, to improve the solution accuracy, an ingenuous approximation is introduced. Although the Q/Q_f - A/A_f curve is established assuming steady uniform flow, in seeking the solution the value of Q_f is actually computed by using Manning's formula with the friction slope S_f estimated by using a quasi-steady dynamic-wave approximation (Yen, 1973a) of the St. Venant equation, i.e., dropping the local acceleration term in the St. Venant momentum equation. Thus, using the x-t plane shown in Fig. 3.2, the friction slope for Q_f is

$$S_f = S_o - \frac{h_A - h_D}{L} + \frac{V_A^2 - V_D^2}{2gL} \qquad (3.11)$$

The use of Eq. 3.11 to estimate Q_f partially accounts for the downstream backwater effect at some later time steps. Nonetheless, since the downstream boundary condition is not truly accounted for, it is recommended in SWMM that for a sewer with a large downstream storage element for which the backwater effect is severe, the water surface is assumed as horizontal from the storage element going backward until it intercepts the sewer invert. Moreover, when the sewer slope is steep, presumably implying high velocity supercritical flow, the flood may simply be translated through the sewer without routing. Also, if the backwater effect is expected to be small and the sewer is circular in cross section, the gutter flow routing method may be applied to the sewer as an approximation. No hydraulic jump or drop is considered within a sewer.

In SWMM large junctions with significant storage capacity and storage facilities are called storage elements, equivalent to the case with junction storage (i.e., $ds/dt \neq 0$) discussed earlier for the kinematic wave model. Only the continuity equation (Eq. 3.9) is used in storage element routing. No dynamic equation is considered except for the cases with weir or orifice outlets. Small junctions are treated as the point-type junctions with $ds/dt = 0$ in Eq. 3.9 as described earlier in the kinematic wave model.

In the design version, small diameters are first assumed for the sewers that would ensure full pipe flow. The size of the sewer is then increased and the computation is repeated until free-surface flow occurs. This smallest commercial pipe giving free-surface flow is thus adopted as the sewer size. The sewers are designed following a downstream cascading sequence, the same as for the kinematic wave model.

3.1.7. Illinois Storm Sewer System Simulation Model (ISS Model)

The ISS model (Sevuk et al., 1973) is a highly accurate simulation model considering the unsteady and backwater effects in the sewers as well

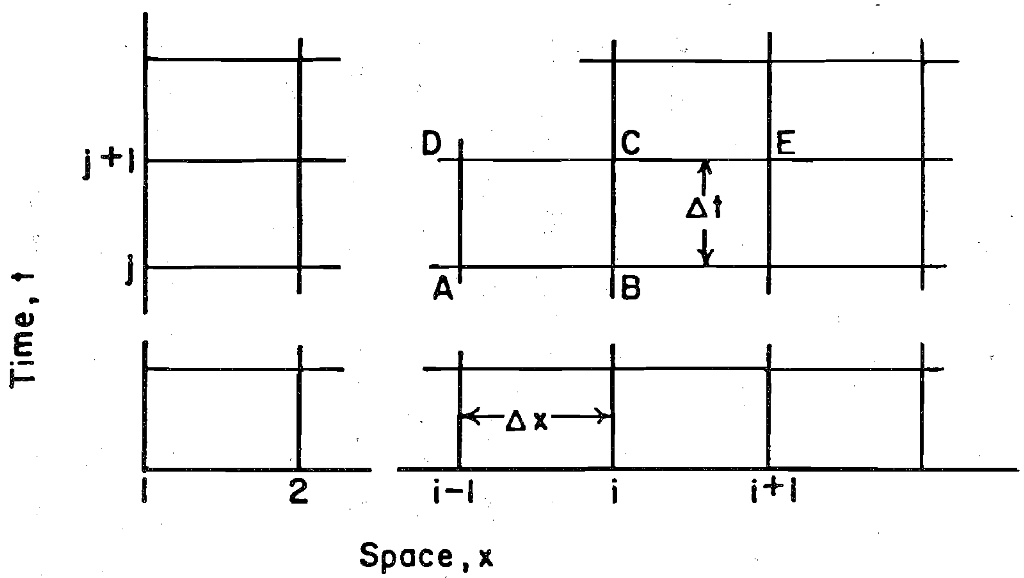


Fig. 3.2. Computational Grid for Four-Point Implicit Finite Difference Scheme

as the effects of junctions and manholes. The model can be used for design of sewer sizes as well as flow prediction. Only the design option is discussed here.

The time-varying storm runoff in gravity-flow sewers can be described mathematically by the St. Venant equations (Sevuk, 1973; Yen, 1973a,b)

$$B \frac{\partial h}{\partial t} + BV \frac{\partial h}{\partial x} + A \frac{\partial V}{\partial x} = 0 \qquad (3.12)$$

$$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + g \frac{\partial h}{\partial x} = g(S_o - S_f) \qquad (3.13)$$

in which A, B, and h are the cross sectional area, water surface width, and depth above invert of the flow in the sewer, respectively; $V = Q/A$ is the mean flow velocity; x is the distance along the sewer; t is time; S_o is the sewer slope; and S_f is the friction slope of the flow. Equation 3.12 is simply a continuity equation having a different form of Eq. 3.7. The value of S_f can be evaluated by using either Darcy-Weisbach's formula or Manning's formula. In the ISS Model, Darcy-Weisbach's formula is used and the Moody diagram is adopted to give the value of the Weisbach resistance coefficient f.

In solving Eqs. 3.12 and 3.13 for subcritical flow in a sewer, a downstream boundary condition is required which reflects the backwater effect from the downstream junction. Therefore, a junction dynamic equation is needed in addition to the junction continuity equation (Eq. 3.9). The dynamic equation for a junction is formulated by considering the continuity of the water surface at the junction. Thus, at a point-type junction with negligible storage

$$z_1 + h_1 = z_2 + h_2 = z_3 + h_3 \qquad (3.14)$$

in which z is the elevation of the sewer invert above a reference horizontal datum and h is the depth of the sewer flow at the exit or entrance of the joining sewers, and the subscripts are as defined in Eq. 3.9. At a reservoir-type junction with large storage

$$z_1 + h_1 = z_2 + h_2 = z_3 + h_3 + \frac{V_3^2}{2g} \tag{3.15}$$

For both types of junctions, if the inflowing sewer has a drop producing a free-fall of the flow, then the flow depth at the exit of that sewer is equal to the critical depth corresponding to the instantaneous discharge.

Presently the ISS Model considers the direct backwater effects of only up to three sewers at a junction or manhole. For junctions or manholes with more than three joining sewers, the additional sewers (preferably those with small backwater effects from the junction) can be treated as direct inflows, i.e., as Q_j in Eq. 3.9.

Equations 3.12 and 3.13 applied to each sewer coupled with Eqs. 3.9 and 3.14 or 3.15 for the junctions and manholes can be solved numerically with appropriate initial and boundary conditions on a large digital computer using a first-order characteristics method together with an overlapping segment scheme (Sevuk et al., 1973). The sewer system is subdivided into a number of overlapping Y-segments and solution is obtained through iterations to satisfy the junction and sewer dynamic conditions. The detailed procedure of the ISS Model for sewer size design can be found in Sevuk et al. (1973) and Yen and Sevuk (1975). It is the only sewer system hydraulic design model that accounts for both upstream and downstream backwater effects and automatically computes the reversal flow when it occurs. It is also the only model that determines the sewer diameter by maximum flow depth rather than maximum discharge. A comparison of the design of a hypothetical 14-pipe sewer system for the ISS, EPA SWMM, Kinematic wave, TRRL, Chicago hydrograph, and steady flow methods has been presented elsewhere (Yen and Sevuk, 1975).

3.2. Design Optimization Models

With the advancements in computer technology and operations research during the past quarter of a century, it is logical that attempts would be made to achieve optimization in sewer design considering the sewers as a system. In the past decade several publications have appeared dealing with optimal design of sewer systems on the basis of least cost. Linear or nonlinear cost formulas were used which can be solved by using available standard computer algorithms. These studies can be classified as (a) optimization for design of sewer slopes and sizes with predetermined layout; (b) optimization for design of sewer system layout; (c) optimization for design of sewer layout, slopes, and sizes simultaneously; and (d) optimization for sewer size only, but including other components of the overall drainage system such as treatment facilities. The important features of the major least-cost sewer system design models are summarized in Table 3.2.

3.2.1. Models for Least-Cost Design of Sewer Slopes and Sizes

Most of the models reported in the literature for least-cost design are directed at the determination of sewer sizes and slopes of a system with a specified layout. This group of models include the third to the twelfth models listed in Table 3.2. All but one use Manning's formula to determine pipe size, ignoring unsteady effects on the flow and utilizing no hydrograph routing. Often the sewer slope is expressed in terms of the upstream and downstream invert elevations and the given sewer length.

Holland (1966) developed a model for selecting sewer diameters and soil cover depths to minimize the cost of a sewer system with a given layout satisfying conventional design constraints and assumptions. In this model the objective is to minimize

$$\text{COST} = \sum_{\substack{\text{commercial} \\ \text{size pipes}}} (C_p d_i^2 + C_x X_i^2 + C_0) L_i + \sum_{\text{manholes}} C_d (E_i - E_j) \qquad (3.16)$$

subject to

$$\begin{aligned} V_{min} &\le V_i \le V_{max} \\ z_i &\ge z_c \\ d_i &\le d_j \\ V_i &= \frac{K}{n} d_i^{2/3} S_{oi}^{1/2} \\ X_i &= \frac{z_i + z_j}{2} \\ Q_i &= \alpha d_i^2 V_i \end{aligned} \qquad (3.17)$$

in which C_p, C_x, C_0 and C_j are cost coefficients of materials, excavation, laying the pipe in place, and manholes, respectively; d_i, X_i, and S_{oi} are the diameter, average depth, and slope of the i-th sewer, respectively; $E_i - E_j$ is the drop in pipe elevation between the manholes; z_i is invert depth; z_C is the minimum cover depth; V_i denotes sewer flow velocity; Q_i is flow rate; n is Manning's roughness factor, K is a constant which depends on the measurement units in which the variables are given; and α is a shape factor that relates the area to the nominal dimension of the pipe.

TABLE 3.2. Summary of Least-Cost Sewer System Design Models

Model	Optimization technique	Design	Variables: Decision	Variables: State	Sewer flow hydraulics	Consideration of commercial pipe diameters	Form of cost function	Remarks
Liebman (1967)	Heuristic procedure	Layouts	N/A	N/A	Ignored, (fixed flow assumed in each pipe	Yes	Any form of cost function	Begins with a trial layout and attempts to find cheaper layouts. All pipe diam are fixed.
Lowsley (1973)	Network theory procedure based on an implicit enumeration algorithm	Layout	N/A	N/A	Ignored, (fixed flow assumed in each pipe)	Yes, but cannot consider multiple pipe sizes	Nonlinear	Optimization based on concept of network trunk, where a trunk is that chain in a rooted spanning tree having the largest excavation and pipe cost.
Holland (1966)	Nonlinear separable programming	Diam and depth of sewers for given layout	Size and invert elevation of pipes	N/A	Manning's formula	Diam are continuous and rounded up to commercial sizes	Nonlinear	Suggest a random search as means of selecting commercial pipe sizes.
Alan M. Voorhees & Associates (1969)	Dynamic programming	Diam and depth of sewers, location of pumps, pressurized sewer mains for given layout	Size, slope, and depth of pipes		Manning's formula	Yes	Linear and nonlinear	Conceptual model intended as a long range planning tool. Never programmed or verified.
Zepp and Leary (1969)	Dynamic programming	Diam and depth of sewers, location of pumps for given layout			Manning's formula	Yes	Linear and nonlinear	Based on Voorhees model, select the least-cost alternative for each succeeding link and carries that single alternative forward as the optimal.
Deininger (1970)	Linear programming	Diam and invert elevations of sewers for given layout	Size and invert elevation of pipes	N/A	Manning's formula	No	Linear	Limited to nonbranching systems. Some constraints are nonlinear which are linearized by successive approximations.
Meredith (1971)	Dynamic programming	Diam and depth of sewers for given layouts	Drop in elevation across pipe	Invert elevations	Manning's formula	Yes	Linear and nonlinear, can incorporate other forms of cost function	For nonbranching systems only. Two approaches of handling manholes: all pipes connecting to manhole are at the same elevation, or relaxation of this elevation constraint.
Dajani and Gemmell (1971)	Convex separable programming and random sampling	Diam and depth of sewers for given layout	Two decision variables: sum and difference of upstream and downstream invert elevations of sewers	N/A	Manning's formula	Diam are continuous then use Holland's random sampling approach to select commercial sizes	Quadratic	Use equations for cost of excavation and pipe component developed by multiple regression.
Merritt and Bogan (1973)	Dynamic programming	Diam and depth of sewers and pump stations for given layout	Pipe sizes and existing invert elevation at a manhole	Pipe sizes and invert elevations	Manning's formula	Yes	Any form of cost function	Two hydraulic models: (a) conventional Manning's formula for which limiting slopes are set by full-pipe flow conditions. (b) Manning's formula in which specified maximum and minimum velocities are met at actual depths for design flowrates.

TABLE 3.2. (Continued)

Model	Optimization technique	Design	Variables: Decision	Variables: State	Sewer flow hydraulics	Consideration of commercial pipe diameters	Form of cost function	Remarks
Dajani and Hasit (1974)	Convex separable mixed integer programming	Diam and depth of sewers for given layout			Manning's formula	Yes	Piecewise linearization of Dajani and Gemmell's (1971) cost functions	Three model formulations are given. First is Dajani and Gemmell's and other two are Dajani and Hasit's formulated for fullpipe flow and partial pipe flow.
Froise, Burges, and Bogan (1975)	Dynamic programming	Diam and depth of sewers, pump station capacities, lift heights, retention basin configurations and volumes	Pipe sizes, slopes, lift stations, storage volumes, and maximum discharges	Invert elevations	Kinematic wave or dynamic wave routing using Darcy-Weisbach formula for pipe design	Yes	Any form of cost function	Extension of the earlier model by Merritt and Bogan (1973). In routing uniform flow is assumed at entrance and exit of each pipe.
Barlow (1972)	Heuristic search procedure, shortest spanning tree and shortest path through many points techniques	Layout and sewer diam to limited extent	N/A	N/A	Darcy-Weisbach's formula	Yes	Nonlinear	
Argaman, Shamir and Spivak (1973)	Dynamic programming	Diam and depth of sewers and layout to a very limited extent	Upstream and downstream invert elevations and drainage directions	Invert elevations, connectivity	Manning's formula and Pomeroy's formula	Yes	Any form of cost function	Considers main and local pipes, i.e. the upstream end either connects or doesn't connect to a manhole. Optimization of layout determines whether each pipe is local or main.
Battelle Northwest Labs (Brandstetter, Engel, and Cearlock, 1973)	Dynamic programming (modified gradient technique)	Sewer diam, regulators, treatment plants, and storage facilities for given layout and sewer slopes	Pipe sizes, storage sizes, regulator sizes	N/A	Nonlinear kinematic wave routing by characteristic method	Yes		Intended for optimum design and control of metropolitan wastewater management systems, primarily for simulation of major sewer systems components, such as trunk and interceptors, treatment plants.
MIT (Kirshen and Marks, 1974)	Linear programming	Optimal operating policy, sewer diam, storage tank sizes and treatment plant sizes	Pipe sizes, storage tank sizes, treatment plant sizes, and flow amounts	N/A	Nonlinear kinematic wave routing	Yes	Linear	The model is designed to be used interactively with EPA SWMM which determines major areas of flooding and magnitudes and quantities of overflow for use on combined sewer systems to screen control alternatives and choose the least expensive combination so that there are no overflows or excessive local flooding.

The generalized subscripts i and j refer to upstream and downstream locations respectively, with respect to a pipe. The decision variables are the pipe sizes and their invert elevations. The problem is arranged in such a manner that all the constraints are linear in terms of elevations and the objective function is nonlinear and separable. The design discharge in a given pipe is considered as constant and the diameter is computed by using the Manning formula assuming just full pipe flow. The pipe diameters which are considered as continuous variables are arbitrarily rounded up to commercially available sizes; a process that may result in a non-optimal system design. A random search around the optimum is suggested as a means of locating the best solution having commercial pipe diameters.

Alan M. Voorhees and Associates (1969) proposed a wastewater collection cost estimation model. This model was intended to be a long range planning tool to estimate the present worth of investment associated with installing and maintaining a sewer system or subsystems to serve a proposed land use configuration. Land use data is converted into expected wastewater flows which are used by the model to project the minimum cost of satisfying demands for each segment of a system using a dynamic programming algorithm to determine optimal sizes, slopes and invert elevations of the sewer pipes. At interior nodes (junctions) of the sewer system (tree), the subtree cost of all nodes for feasible elevations and pipe sizes are defined by the functional equation

$$t_j(d,z) = \sum_{i \text{ in } I_j} \min_{k \text{ in } K_j} [a_k(i,j) + t_i(d_k,z_k)] \qquad (3.18)$$

where $t_j(d,z)$ is the optimal subtree cost of node j at depth z and pipe size d; I_j is node j's set of connecting nodes; k is an index of feasible depth-pipe size combinations; K_i is the set of feasible depth-pipe size combinations at node i; and $a_k(i,j)$ is the cost of connecting node i at depth z_k by a pipe of diameter d_k to node j at (or above) depth z and size d. The output of the model includes the cover depth, slope, and size of each pipe in the system; locations of pumps; and total present worth of the system including maintenance and operation costs. The Voorhees model was developed in some details but was never programmed or verified.

Zepp and Leary (1969) developed a sewer cost estimation computer program that was patterned somewhat after the Voorhees model. Their model incorporates a limited optimization procedure in that it selects the least cost alternative for each succeeding pipe during the design and carries that alternative forward as the optimal for the system. Deininger (1970) formulated a linear programming model for the minimum cost design of sewer systems assuming the excavation and sewer costs to be linear. This formulation results in some of the constraints being nonlinear which are transformed into linear constraints by successive approximations.

Meredith (1971) developed a dynamic programming model to determine the components of minimum cost non-branching sewer systems in which only commercially available pipe sizes were considered. A simplified approach and a more realistic approach were each considered. The simplified approach assumes that the invert at the exit of the outflowing sewer is at the same elevation as the invert at the entrance of the inflow sewer joining at the same manhole. The more realistic approach relaxes this constraint. For the simplified approach each stage represents a pipe plus the downstream manhole. The input and output states at each stage i represent the invert elevations at the upstream and downstream ends of each pipe. The decision at each stage represents the drop in elevation between the two ends of the pipe. For the more realistic approach, each stage represents a single component (manhole or pipe) of the system. The decision is the drop in elevation for each stage. The recursive equation for the stage-by-stage optimization from upstream to downstream is stated as

$$F_i(S_i) = \text{Min } [r_i + F_{i-1}(S_{i-1})] \qquad i = 1, 2, \ldots \tag{3.19}$$

where $F_i(S_i)$ represents the minimum cost of the sewer system through stage i and $F_o(S_o) = 0$. The return, r_i, at each stage for the simplified approach is the cost of installation of the pipe and the downstream manhole. For the more realistic approach, the return is the cost of installation of either the pipe or the manhole. A comparison of these approaches using a cost function proposed by Alan M. Voorhees and Associates (1969) illustrated that allowing a drop across a manhole results in much cheaper sewer system designs.

Another application of dynamic programming to the optimal design of sewer systems is reported by Merritt and Bogan (1973). In this model the stages are manholes; the state variables are pipe sizes and invert elevations; and the decision variables are the pipe sizes and the existing invert elevation at the manhole. The transformation between stages is given by the Manning formula. Similar to Meredith's model, the stage-by-stage recursive optimization procedes from the upstream to the downstream end of the sewer system. Drop manholes are considered when the maximum allowable velocity constraint is exceeded. Also when a gravity flow solution violates the maximum depth constraint, a pumping station is added. Similar to the other dynamic programming applications, this model considers commercial pipe sizes.

Dajani and Gemmell (1971) developed multiple regression equations for the cost of the excavation and pipe components based upon construction bidding. The resulting general form of the cost function is

$$C = a + bd^2 + cX^2 \tag{3.20}$$

where C is the installation cost per foot of sewer; d is the sewer diameter; X is the average depth of excavation; and a, b, and c are the regression coefficients. Based upon this cost function, an overall nonlinear objective function was formulated as

$$C_{ti} = 2.35b\left(\frac{n}{K_1K}\right)^{3/4} Q_{di} \; S_{oi}^{-3/8} L_i + cX_i^2 L_i + aL_i \tag{3.21}$$

where C_{ti} is the total cost of the pipe originating from node i; n is Manning's roughness factor; K is the measurement unit constant in Manning's formula as defined in Eq. 3.17; $K_1 = Q/d^2V$; Q_{di} is the average daily flow; S_{oi} is the slope of pipe i; and L_i is the length of pipe i. By substituting in equations for the slope and depth of excavation, Eq. 3.21 is reduced to contain two decision variables which are the summation of, and the difference between, the upstream and downstream invert elevations of each pipe in the sewer network. It is interesting to note that the diameter has been eliminated as a decision variable because of relationship for the average daily flow. Six linear constraints, including the minimum allowable diameter, the minimum and maximum velocity limits, the minimum pipe cover, and diameter and invert elevation progression constraints, were formulated in terms of the two decision variables. Convex separable programming was used to solve this model and Manning's formula was used to solve for the diameter. This procedure assumes that sewer pipes are available in any theoretical size so they recommend the use of Holland's random sampling approach to select commercially available sizes.

Dajani and Hasit (1974) have extended the model of Dajani and Gemmell to account for discrete pipe sizes. The nonlinear objective function, Eq. 3.21, is linearized using piecewise linearization which adds six sets of piecewise approximation constraints for each pipe in the sewer network. In addition two sets of constraints having 0-1 integer variables are added to the formulation to obtain commercially available pipe sizes. These constraints complete the formulation as a convex-separable, mixed-integer programming problem, having the continuous variables of excavation depths and integer variables with a total of 14 constraints for each pipe in the network. Three model formulations were compared. The first is Dajani and Gemmell's (1971) model considering a continuous range of diameters and is formulated assuming full pipe flow. The second model deals with discrete commercial pipe size for partially filled pipe flow. The third model is a combination of the first two models formulated for full and partially filled flows in commercial size pipes.

Froise, Burges, and Bogan (1975) recently proposed a model to determine least-cost strategies for sewer system design using dynamic programming in conjunction with a hydraulic simulation model. This model is an extension of the earlier model developed by Merritt and Bogan (1973) for which each stage is represented by a node in the network. At each stage of the system the state variables are the hydrographs, storage volumes, pipe sizes, pump station capacities, invert elevations and solution costs. The control options or variables are maximum discharges, pipe sizes and slopes, lift stations, and storage volumes. Each pipe size is only considered at one specific slope for each state and for each quantized increment of flow. This slope is the one that results in the conduit flowing full at the maximum discharge, or, if this decision violates the minimum velocity constraint, the slope that results in the minimum allowable velocity of flow is used. When a solution violates the downstream cover depth constraints, solutions which include drop structures or pump stations at the upstream junction are selected. When the optimization phase has been completed at each stage the inlet hydrograph is routed to the next downstream stage. Either the kinematic wave or the dynamic wave equations are used for routing by an implicit numerical scheme. The selection of routing model is based upon the pipe diameter and slope. Uniform flow conditions are assumed at the upstream and downstream ends of each pipe.

3.2.2. Models for Least-Cost Selection of Sewer System Layouts

The first formal approach to the least-cost selection of the layout for sewer systems is a study by Liebman (1967). A heuristic procedure was developed which uses a simple search method for seeking improved layouts in gravity flow sewer systems. This method begins with a designer selected trial layout and attempts to find layouts having smaller total costs. The sewer diameters are assumed to be fixed and the best layout is found by the search procedure. At each step of the procedure one branch of the network is changed. The change is retained if it results in a decrease in the total cost. A major drawback with this method is that fixed values of discharge are assumed for each pipe of the system, ignoring the hydraulics of the sewer network.

Another layout model was proposed by Lowsley (1973) using a network theory procedure to obtain a layout giving minimum total cost of sewers and excavation. The algorithm is an implicit enumeration process based on the concept of a network trunk, where a trunk is defined as that chain in a rooted spanning tree having the largest excavation and pipe cost. Like Liebman's model, the sewer diameters are assumed unchanged and the hydraulics of the sewer flow is ignored. Moreover, the model cannot consider multiple pipe sizes, instead specifies minimum and maximum slopes for single pipe sizes.

3.2.3. Models for Least-Cost Design of Sewer Slopes, Sizes and Layout

Simultaneous optimal design of the sizes, slopes and layout of the sewers in a sewer system is a more complicated task than the optimal design for only the sizes, the layout, or the sizes and slopes. Only two studies have recently been reported on limited scopes of the simultaneous optimal design problem. Barlow (1972) proposed a heuristic search procedure which chooses the major trunk sewers and then uses the shortest-path-through-many-points technique and the shortest-spanning-tree technique to determine the final layout and the sewer diameters to a limited extent. The sewer slope is implicitly included by restricting it within a specified maximum and minimum and within this range using the slope computed by Manning's formula that gives just full pipe flow for the sewer diameter.

Another model proposed by Argaman, Shamir, and Spivak (1973) using dynamic programming considers both local pipes which start next to a manhole but do not connect to it and main pipes which lead out of a node (manhole). Both local and main pipes collect local drainage along their routes. The optimization of the layout only determines whether each pipe is a local or main pipe. The model is formulated to minimize over the connectivity of the network and the invert elevations of each pipe. The objective function is stated as

$$\min_{T_i, H_{ui}, H_{di}} \left[\sum_{\text{all pipes}} (C_{Pi}(d_i, H_{ui}, H_{di}, T_i) + \sum_{\text{all drops}} (C_{di}(d_i) \right] \qquad (3.22)$$

in which $T_i = 1$ when pipe i is a main pipe and $T_i = 0$ for local pipes; H_{ui} and H_{di} are the upstream and downstream invert elevations, respectively, of pipe i; d_i is the diameter of pipe i; C_{Pi} is the cost of pipe i; and C_{di} is the cost of the drop structure at the end of pipe i. The independent decision variables are the drainage directions of all nodes and upstream and downstream invert elevations of all pipes. The system is divided into stages by isonodal lines (called drainage lines by Argaman et al., 1973) which are the imaginary lines passing through all nodes having the same link-distance from the outlet. All nodes on isonodal line n can drain only to nodes on line n+1. The recursive equation for the stage-by-stage dynamic programming optimization is

$$F^*_{n+1}(H^{n+1}) = \underset{H^n}{\mathrm{Min}}\{F^*_n(H^n) + \sum_{i} \sum_{T(i,n+1)} f^*_i[H^n_i, H^{n+1}, T(i,n+1)]\} \quad (3.23)$$

in which $F^*_{n+1}(H^{n+1})$ is the minimum cost of the system from isonodal lines 1 to n+1; $f^*_i[H^n_i, H^{n+1}, T(i,n+1)]$ is the cost of the cheapest feasible pipes between node i on line n and the nodes on line n+1; H^n, H^{n+1} are vectors of quantized node elevations on isonodal lines n and n+1; H^n_i is the elevation of node i on isonodal line n; and T(i,n+1) is the vector of connectivity between node i on isonodal line n and nodes on line n+1. A large sewer system must be decomposed into smaller subsystems which are optimized separately and then recombined. This technique is not practical at the present due to limitations in computer size and computation time.

3.2.4. System Optimization Models for Design of Sewer Sizes

The optimization models that determine only the size of sewers with specified sewer layout and slopes actually are models intended to achieve optimization considering not only the sewers in the system but also other components in the overall urban drainage system, a concept that has been discussed at the beginning of Chapter 2. Understandably, because of the limitations of existing computers, the two models that have been proposed and listed at the end of Table 3.2 consider conjunctively only the selected subsystems of the overall urban drainage system and design only for the diameter of the sewers. A group of researchers at Battelle Pacific Northwest Laboratories (Brandstetter, Engel, and Cearlock, 1973) developed a model intended for optimal management of urban wastewater disposal systems, primarily for simulation of major system components such as sewer trunks, interceptors, and treatment plants for the purpose of automatic operational control of stormwater runoff. For design studies with given layout and sewer slopes, sizes of sewers, overflow storage facilities, treatment plants, and overflow treatment facilities are computed which minimizes the cost for specified constraints on the quality of overflows and treatment plant effluents. The optimization is performed through a modified gradient technique of dynamic programming. The flow is routed through sewers using the characteristics method applied to nonlinear kinematic wave equations. Downstream flow control, backwater, flow reversal, junction surcharging and sewer pressurized flow are not considered.

A group of researchers at MIT developed a screening model for stormwater control (Kirshen and Marks, 1974) which was later modified and became proprietary. The model has been under considerable alternation since its initial development. However, there exists no comprehensive report which describes collectively in sufficient detail the model as used for urban stormwater drainage management purposes. The version that involves sewer design is a conjunctive optimization model which considers combined sewers, detention storage devices, and special stormwater treatment plants. The objective is to search for the least-cost solution with no overflows or excessive local flooding. Optimization is pursued through a screening technique using linear programming. For the sewer system part the layout and sewer slopes are predetermined and the design involves the determination of the sewer sizes that satisfy the objective function and specified constraints. The screening model is formulated to be used interactively with EPA SWMM which routes the flow through sewers and determines major areas of flooding and quantities of overflows. Previously, Harley et al. (1970) proposed a model to route the stormwater flow through drainage systems using a nonlinear kinematic wave scheme. Linear diffusion and dynamic wave schemes were also discussed. However, there is no published record to indicate that these hydraulic routing schemes have been adopted conjunctively with the optimization model for determination of sewer diameters.

Clearly, the sewer size design models with conjunctive optimization with other drainage facilities, as well as the optimal design models for layout, slopes and sizes of sewers, are in their early stages of development and considerable research effort is needed for practical application.

Chapter 4. APPLICATION OF OPTIMIZATION TECHNIQUES

In this chapter the optimization techniques adopted to obtain the least-cost design of sewer slopes and diameters for a sewer system are discussed. The constraints and assumptions involved in the least-cost sewer design have been described in Section 3.4. However, the cost functions given in Eq. 2.1 are only examples, other cost functions may also be used instead.

4.1. Problem Statement

The problem under consideration is how to determine the least-cost combination of sizes and slopes of the sewers and the depths of the manholes for a sewer network to collect and drain the wastewater from an urban drainage basin. Since for a given sewer length the slope depends on the end elevations of the sewer, the design variables can be considered as the diameters and upstream and downstream end crown elevations of the sewers and the depths of the manholes. Given is a set of manhole locations at various points within the drainage basin with the network layout connecting these manholes known. The design inflows into these manholes are also predetermined. The principal tasks in the development and formulation of an optimization model for the design of storm sewer systems are twofold.

(a) Representation of the set of manholes in a form suitable for digital manipulation.

(b) Selecting optimization techniques for the overall model which are flexible enough to handle design constraints and assumptions, various forms of cost functions, risk models, and hydraulic or hydrologic models, and to incorporate all design information.

These principal tasks must be considered conjunctively to arrive at a solution scheme that can be constructed efficiently and used to design large scale sewer networks.

Most storm sewer systems are converging-branch or simply tree-type systems, which are topologically characterized by

(a) a root node, i.e., the outlet of the sewer system;

(b) internal nodes which are manholes or junctions of sewers where two or more branches meet;

(c) external nodes which are manholes where only one branch is connected, i.e., manholes where a branch of the sewer system begins;

(d) branches which link nodes without forming closed paths or loops within the network, i.e., the sewers in the system.

The node-link (manhole-sewer) representation of a typical dendritic sewer system is important in formulating the optimization model.

The manner of representing manholes to describe the sewer system layout for the optimization procedures will be discussed in Sections 4.3 and 4.4. The optimization schemes developed in this study do not allow closed loops. Inflows are permitted at all manholes of the system; however it is not necessary that there be an inflow to each manhole. This is in accordance with design because some manholes are for cleaning purposes or changes in ground slopes where changes in pipe sizes may result.

A storm sewer system may consist of a large number of sewers, junctions, manholes and inlets in addition to other regulating or operational devices such as gates, valves, weirs, overflows, regulators, and pumping stations. These devices do have an effect upon the system, hydraulically dividing it into a number of subsystems. However for the sake of simplicity, at the present stage these special devices are not considered in the optimization scheme.

The basic optimization technique used to develop the storm sewer design models is discrete differential dynamic programming (DDDP). Two design models, each representing the nodes and links of a sewer system in a different manner for the optimization, have been developed. The first model considers the sewer system as a nonserial optimization problem in which the basic strategy is to decompose the converging branched system into equivalent serial subsystems for solution. This model has been described in detail by Mays and Yen (1975) so that only a brief description is given in this report. The second model considers the sewer system as a serial optimization problem such that multi-level branching sewer systems can be handled more easily (Mays and Wenzel, 1977). A detailed discussion of this second model is given in this chapter.

As a prelude to the following discussion of applications of DDDP to the design of least-cost sewer systems, it is desirable to define the terms stage, state, decision, return, and transformation and to indicate their counterparts in sewer systems.

(a) Stages: A stage is analogous to a component of the system; for the nonserial approach a stage is a sewer pipe (link) or a manhole (node), for the serial approach a stage is a set of sewers all located at the same number of links upstream from the outlet of the system.

(b) States: The states of a stage represent the variables of that stage; e.g., the states at each sewer stage are analogous to the crown elevations of the pipe or pipes, the input state S_n for a stage is the crown elevation of the upstream end of the stage, and the output state $\tilde{S}_n$ is the crown elevation of the downstream end of the stage.

(c) Decisions: The decision D_n at each stage is the elevation drop across the stage.

(d) Returns: The return r_n of a stage is analogous to the cost of installation for that stage, and also damage costs if considered.

(e) Transformation: The transformation function of a stage n defines the manner in which an input state is transformed into an output state by the decision variable given by

$$\tilde{S}_n = S_n - D_n \qquad (4.1)$$

No negative slope is allowed in the sewer system, therefore at any stage n, $S_n \geq \tilde{S}_n$.

It should be remarked here that conventionally in sewer design the invert elevation rather than the crown elevation is used in pipe design. For small slopes the former is simply the latter deducted by the pipe diameter. There is no clear advantage of using the invert elevation as many engineers thought. The invert elevation does not give directly the trench depth in construction because of the thicknesses of the pipe wall and of the bedding. In fact sometimes this is the source of error in construction as the trench is dug without considering these thicknesses of the invert and is measured erroneously. Also, using the invert elevation in the design, often the checking of the constraint of minimum soil cover requirement is forgotten. In joining sewers at a manhole, having the inverts aligned, though preference to crowns aligned, hydraulically does not offer the best performance. As pointed out by Yen et al. (1974), aligning the centerlines of the joining sewers provides an improved hydraulic performance. Since there is no difficulty in conversion between the crown and invert elevations once the diameter is known, and that the minimum soil cover depth is one of the important design constraints, the crown elevation is used because of its simplicity to be adopted as the state in the optimization.

Prior to discussing the nonserial and serial approaches of representing a sewer network for the optimization procedure, a brief description of DDDP applied to storm sewer design is given in the next section.

4.2. Selection and Description of Optimization Technique - DDDP

A review of the existing least-cost sewer design methods points out the advantages of using dynamic programming (DP) techniques over other optimization techniques for the least-cost design of sewer systems. The flexibility of DP approaches to handle various forms of cost functions, design constraints, etc. is of extreme importance. In addition, for the models described later in this report, DP has been found to be superior to other optimization techniques because of its flexibility to incorporate risk and hydraulic routing models. However, when DP is applied to large systems, there are difficulties in obtaining an optimal solution without a considerable increase in computer time (Mays and Yen, 1975). The increase in computer time is even more significant when risk and hydraulic routing models are incorporated into the optimization scheme. Consequently, other techniques based upon DP that could possibly reduce the computer time were investigated.

A special type of dynamic programming, discrete differential dynamic programming (DDDP), has been proven to be a very effective method in the analysis of various types of water resources systems (Chow et al., 1975). This method is able to overcome the chief limitations of DP, namely, the number of state variables and the level of discretization of the state variable. In the application of DP and DDDP to obtain least-cost designs of branched sewer systems, the use of DDDP has been shown to be very effective in decreasing computation time over that of DP (Mays and Yen, 1975). This is mainly due to the level of discretization of the state variable required in DP to obtain equivalent results using DDDP.

DDDP is defined by Heidari (1970) and Heidari et al. (1971) as an "iterative technique in which the recursive equation of dynamic programming is used to search for an improved trajectory among the discrete states in the neighborhood of a trial trajectory." This optimization procedure is solved through iterations of trial states and decisions to find the optimal returns (minimum cost) for a system subject to the constraints that the trial states and decisions should be within the respective admissible domain of the state and decision spaces.

In DDDP the first step is to assume a trial sequence of admissible decisions D_n called the trial policy, and the state vectors of each stage n are computed accordingly. This sequence of states within the admissible state domain for different stages is called the trial trajectory and can be designated $\bar{S}_n$ for n = 1,2,...,N where N is the total number of stages. Actually for a storm sewer system a complex network of trajectories representing the upstream and downstream crown elevations of each stage are formed. An alternative to the above procedure, which can be used for the problem presented herein, is first to assume a system of trial trajectories and then use it to compute the trial policy (i.e., a trial set of decisions or drops in crown elevations, Eq. 4.1). The procedure would first specify the initial trial states for the first and last stages of the entire sewer system. This procedure is used to compute slopes for the first trial set of pipes based on which the crown elevations, and the corresponding decisions for each stage of the system for the first trial trajectories can be computed.

Several crown elevations in the neighborhood of a trial trajectory can be introduced to form a band called a "corridor" around the trial trajectory. The corridor is defined by the input states (crown elevations k = 1,2,... at the upstream end of the stage) and the output states (crown elevations j = 1,2,... at the downstream end of the stage). The trajectory crown elevations and a given state increment Δ_s (distance between crown elevations) at the upstream and downstream ends of the stage are kept in storage so that they can be used to establish the states. For example, a corridor for a pipe connection with five states (lattice points) at both ends is shown in Fig. 4.1. The corridor is defined by the crown elevation matrix

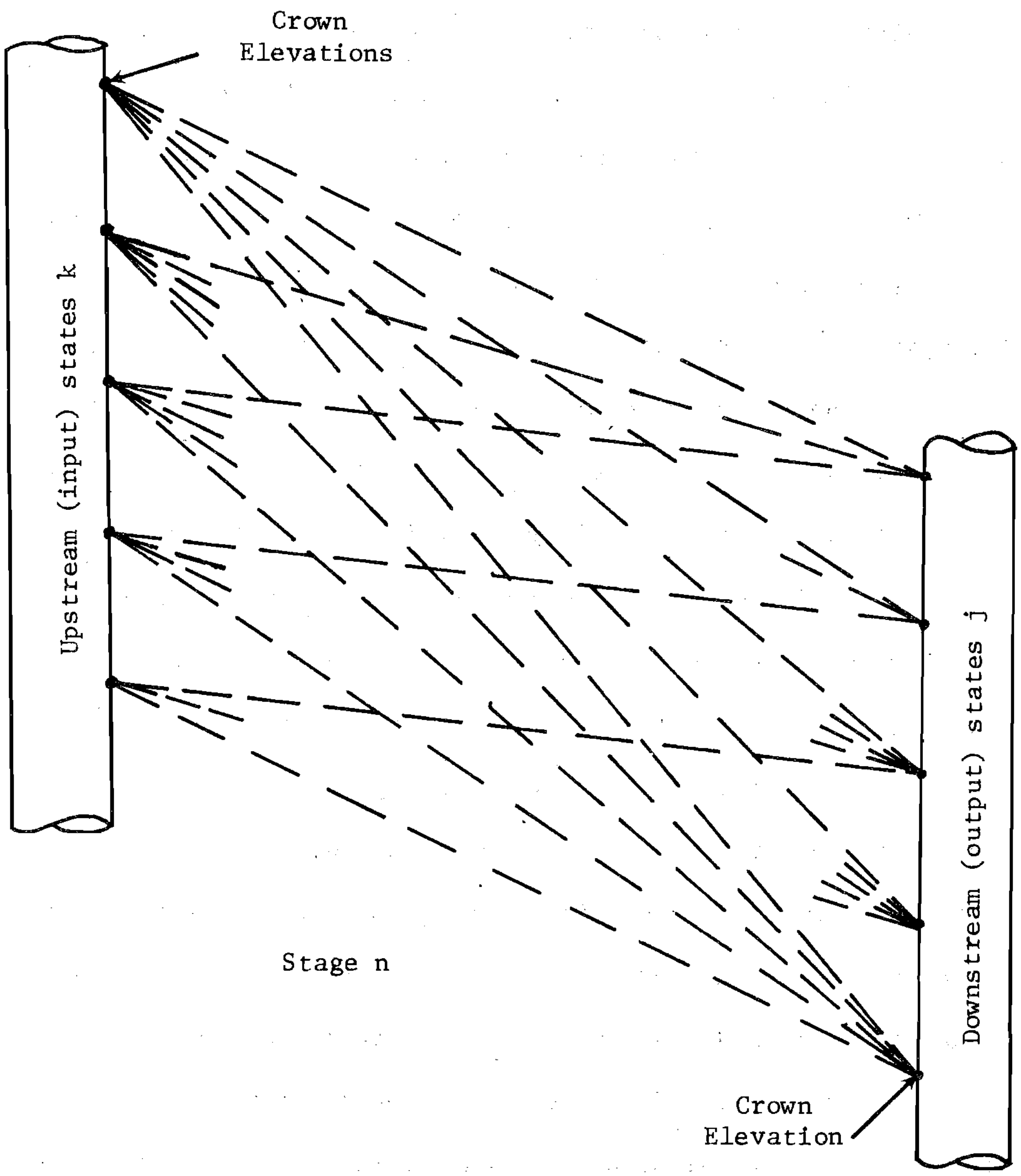
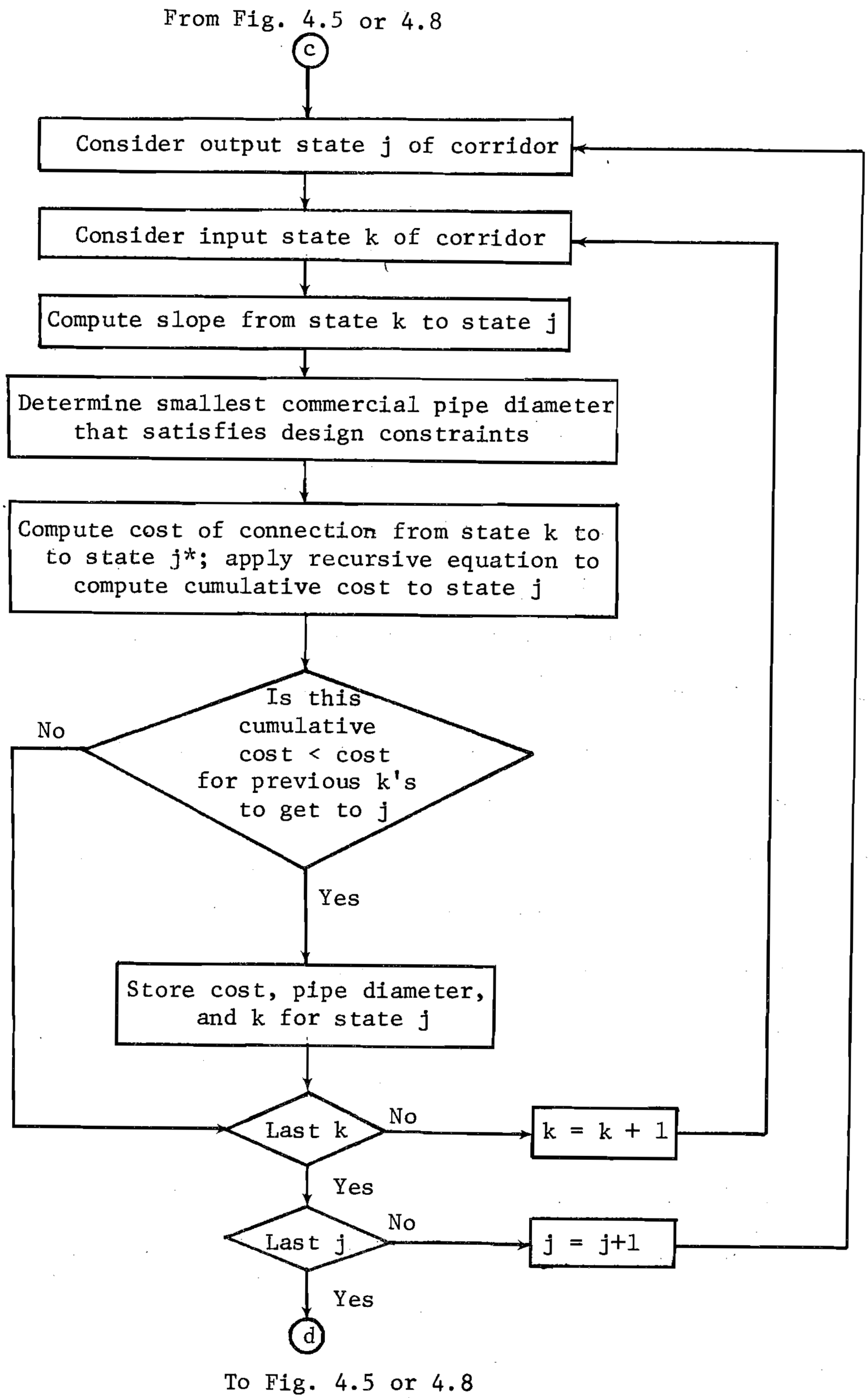


Fig. 4.1. Five-Lattice-Point Corridor Showing Drops in Crown Elevations

$$S = \begin{bmatrix} \bar{S} + 2\Delta_s \\ \bar{S} + \Delta_s \\ \bar{S} \\ \bar{S} - \Delta_s \\ \bar{S} - 2\Delta_s \end{bmatrix} \tag{4.2}$$

in which $\bar{S}$, the middle crown elevation, represents the trial trajectory. It is apparent that the crown elevations must fall within the admissible domain of the state space. The upper boundary of the domain is defined by the required minimum soil coverage of the sewers or other more restrictive constraints. If any elevations do not satisfy the minimum cover depth criteria, the state space of the corridor is shifted down maintaining the improved trajectory. Likewise, the crown elevations must not be lower than the lower boundary which is defined by the elevation of the lowest sewer in the system or preferably by other more restrictive constraints such as that imposed by minimum sewer slopes. The option also exists to constrain any elevation at the upstream or downstream end of a pipe, by simply setting $\Delta_s = 0$. This restriction is required when special control devices such as siphons or regulators are used in the sewer system, or for specified system outlet elevation.

For a storm sewer system, a complex network of corridors would be formed, i.e. a corridor for the connections between manholes at each of the stages. Once the corridor is established, the traditional dynamic programming approach is applied within the corridors at each stage (Fig. 4.2). The downstream crown elevations are varied, and for each of these, the upstream crown elevations are varied. The sewer slope and smallest commercial pipe diameter satisfying the constraints on flow, velocity, and preceding (upstream) sewer diameter are computed for each input state to the output state. Selection of the pipe diameter is performed using one of the



*For nonserial approach this is the cost of sewer pipe or manhole; for serial approach the cost of pipe plus the cost of the upstream manhole m_n for state k.

Fig. 4.2. DP Flow Chart Within Corridor

hydraulic methods and possibly the risk component discussed respectively in Chapters 5 and 6. For each feasible set of input and output states, a pipe diameter equal to the largest of the upstream pipe diameters is considered first. If the pipe is full or if the velocity exceeds the allowable maximum velocity, the next larger commercial size pipe is considered. Conversely, if the allowable minimum velocity constraint is violated, the sewer slope is too small and for the current output state the next input state is considered. The cost for the current output state of the stage is computed for each of the possible input states. This cost is added to the minimum cumulative cost upstream from the current stage that is associated with the input state k. Thus, the recursive function through state j of stage n, $F_n(\tilde{S}_n)$ is

$$F_n(\tilde{S}_n) = \min_{D_n}[r_n(S_n,D_n) + F_{n-1}(\tilde{S}_{n-1})] \tag{4.3}$$

in which D_n and r_n are the decision and return (the cost of connection between the current input and output states), respectively; $F_{n-1}(\tilde{S}_{n-1})$ is the cumulative cost upstream of the current connections; and $F_o(\tilde{S}_o) = 0$. This currently computed cost through stage n at the current downstream state j is compared to determine whether it is less than the previously computed minimum cumulative cost for state j. If so, the current cumulative cost replaces the previous minimum cumulative cost for state j as a basis for further comparison.

This procedure is repeated until all the feasible input states connecting to the output state j are considered. In other words, the slope or drop in crown elevation of the stage is determined for the output state j, among all the feasible input states to find the state k which provides the minimum cumulative cost of getting to state j. This state k and the associated costs for state j are stored for later use in the optimization procedure. After the minimum cumulative cost to the output state j of the current stage has been established, the next output state of the same stage is considered. This procedure is repeated until the minimum cumulative costs to each of the feasible output states of the stage have been computed and stored.

This algorithm continues downstream stage by stage until the DDDP computations are performed for the last stage. There now exists a set of minimum cumulative costs for each state at each stage in the system. A trace-back is now performed which begins at the last stage, selecting the output state with the minimum cumulative cost and moving upstream to the associated input state. This procedure is followed through successive upstream stages and a new trajectory is formed using the selected states of each stage. This process is called an "iteration." A new corridor is formed based on the new trajectory and this procedure is repeated beyond some iteration i which produces corridors with a sewer system design of a total system cost F_i. No further iterations with this size of corridors will produce a reduction in total system cost less than a specified tolerance. At this point in the optimization procedure, the value of Δ_s is reduced to set up new corridors in which the crown elevations or lattice points are spaced closer together. These smaller corridors are formed around the improved trajectories of the latest iteration. The iterations continue reducing Δ_s throughout the system accordingly until a specified minimum Δ_s is reached and least-cost design is obtained. A flow chart showing the DDDP procedure for sewer systems is given in Fig. 4.3.

The criterion used to determine during the computations when the magnitude of Δ_s should be reduced is based on the relative change of the minimum cost of the latest (i-th) iteration, F_i, i.e.,

$$|F_i - F_{i-1}|/F_{i-1} \leq E_r \qquad (4.4)$$

When the ratio is equal to or smaller than a specified value E_r, the increment Δ_s is reduced to one-half or any other desired fraction of its previous

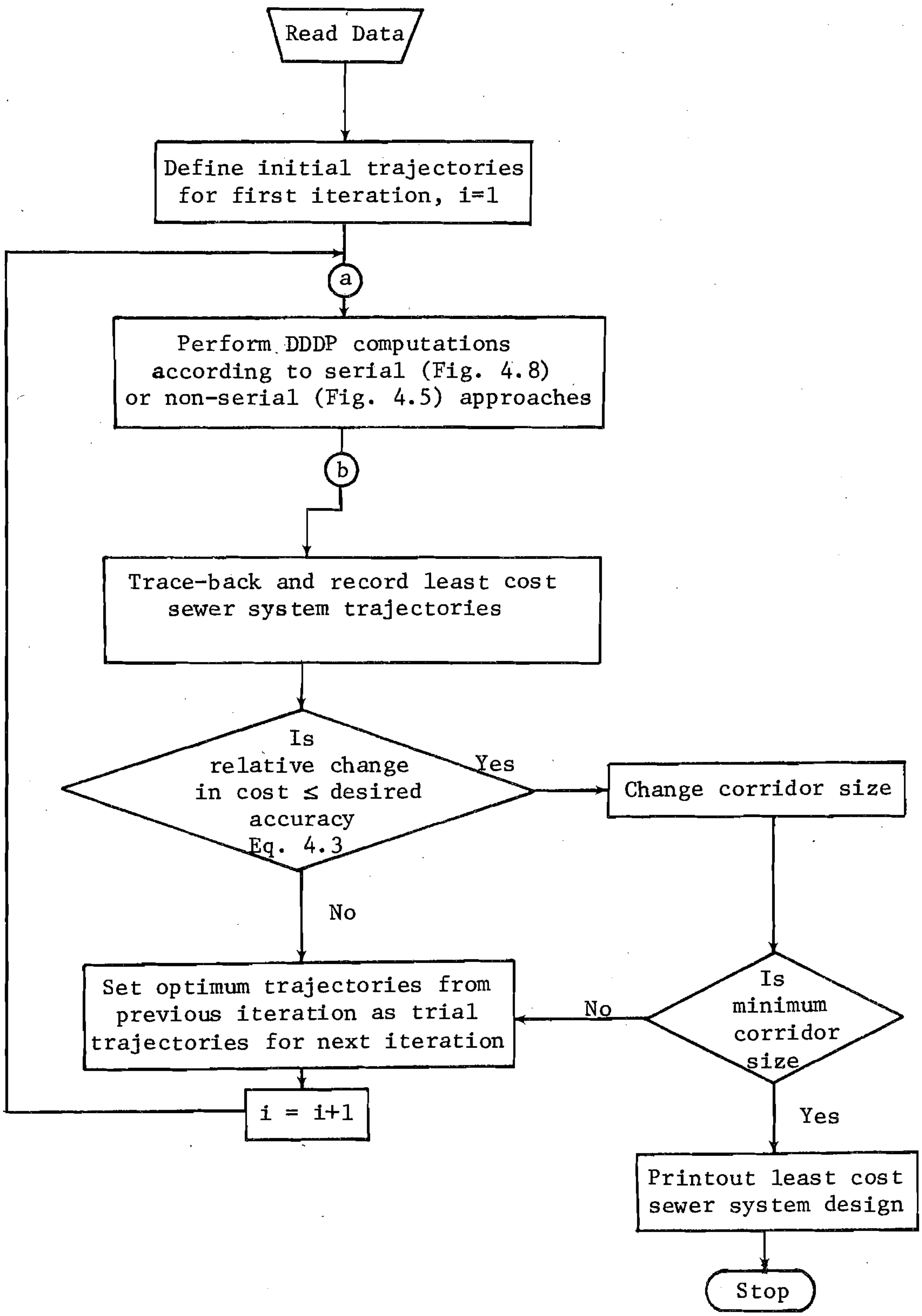


Fig. 4.3. Flow Chart of Design Optimization Procedure for Sewer Systems

value and then iterations are resumed. This procedure is repeated until Δ_s is smaller than a specified acceptable value. Obviously, appropriate selection of the initial values of Δ_s can significantly improve the efficiency of the DDDP.

There are three possible downstream boundary conditions at the last stage of a storm sewer system using DDDP. The first is when the downstream crown elevation of the system (final outlet) must be at a specific elevation, i.e., $\tilde{S}_N$ is a constant. In this case, the state increment for the downstream state of the last stage N of the system is zero. The trace-back through the system for each iteration to determine the minimum cost crown elevations of each of the upstream stages starts at the specified elevation $\tilde{S}_N$ of the final outlet. The second possible downstream condition is when the final outlet can be at any elevation, i.e., $\tilde{S}_N$ is not specified. In this case, $\Delta_s(\tilde{N})$ for the downstream states for the last stage N of the system is not zero. The trace-back through the entire sewer system for each iteration to determine the minimum cost crown elevations starts at the downstream elevation of the last stage of the system that gives the least total cost for the system. The third possible downstream condition is that $\tilde{S}_N$ is not fixed but restricted within a certain range of elevations, which defines the state space boundary for the last stage. Consequently, only those elevations of the last stage that fall within this range are considered. However, in actual computations, the latter two conditions can be treated as the first by adding an imaginary stage N+1 consisting of a pipe connection having a specified elevation for its output state and setting the cost for this imaginary stage equal to zero.

4.3. Nonserial Optimization Approach and Its Limitations

The initial approach used in this study to represent a dendritic sewer system for DDDP cost optimization decomposes the sewer system into a main chain with branches connected to it. Each branch is in turn similarly decomposed. The computations begin at the upstream end of the main chain and proceed downstream until a branch connection is reached. This branch is then considered, beginning at its upstream end, in a manner identical to the main chain, with the branch output in terms of costs, crown elevations, and flow serving as input to the main chain. The procedure then proceeds downstream until all the branches and main sewers have been considered.

Each sewer and each manhole in the system is considered as a stage. The manner in which the stages are linked is given by the incidence identity which is the relationship that the output from each stage forms the input to the next succeeding stage. The downstream (output) crown elevation of stage n must be the same as the upstream (input) crown elevation of stage n+1 given as $S_{n+1} = \tilde{S}_n$, and $S_n \geq \tilde{S}_n$ in which the equal sign applies only to manhole stages with crowns of joining sewers aligned. An example of the stage-state domains with the corridor, trial trajectory, and state space boundaries for a main or a branch for the nonserial approach is shown in Fig. 4.4.

At the manhole where a branch joins the main, the cuts dividing the branch from the main are at the downstream end of the manhole stage. Because of identical elevations at these cuts, the output elevations of the main and branch are equal, $\tilde{S}_{main} = \tilde{S}_{branch}$, which is also equal to the input elevation for the sewer main immediately downstream from the manhole. Obviously, through this manhole stage the recursive equation $F_{n-1}(\tilde{S}_{n-1})$ in Eq. 4.3 includes the minimum cumulative costs of both the main and branches.

A flow chart showing the logic for the nonserial optimization approach is given in Fig. 4.5. This figure together with the DDDP procedure described in the preceding section (Figs. 4.2 and 4.3) illustrates the nonserial optimization design model. Details of the procedure have been reported elsewhere (Mays and Yen, 1975) and are not repeated here.

There are several limitations for this nonserial optimization approach when applied to large sewer systems with many levels of branching.

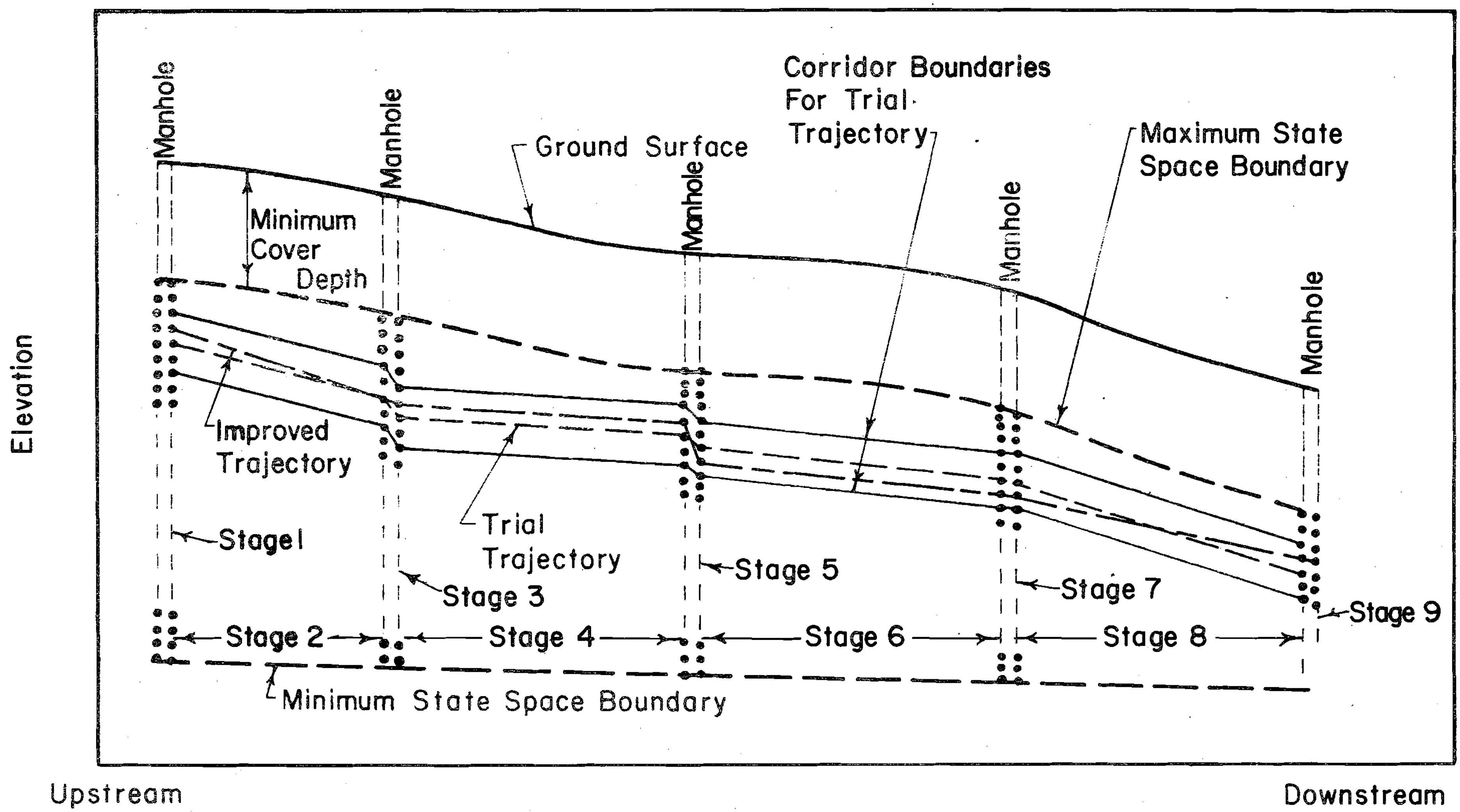


Fig. 4.4. Example of Stage-Corridor Representation for Nonserial Optimization Approach

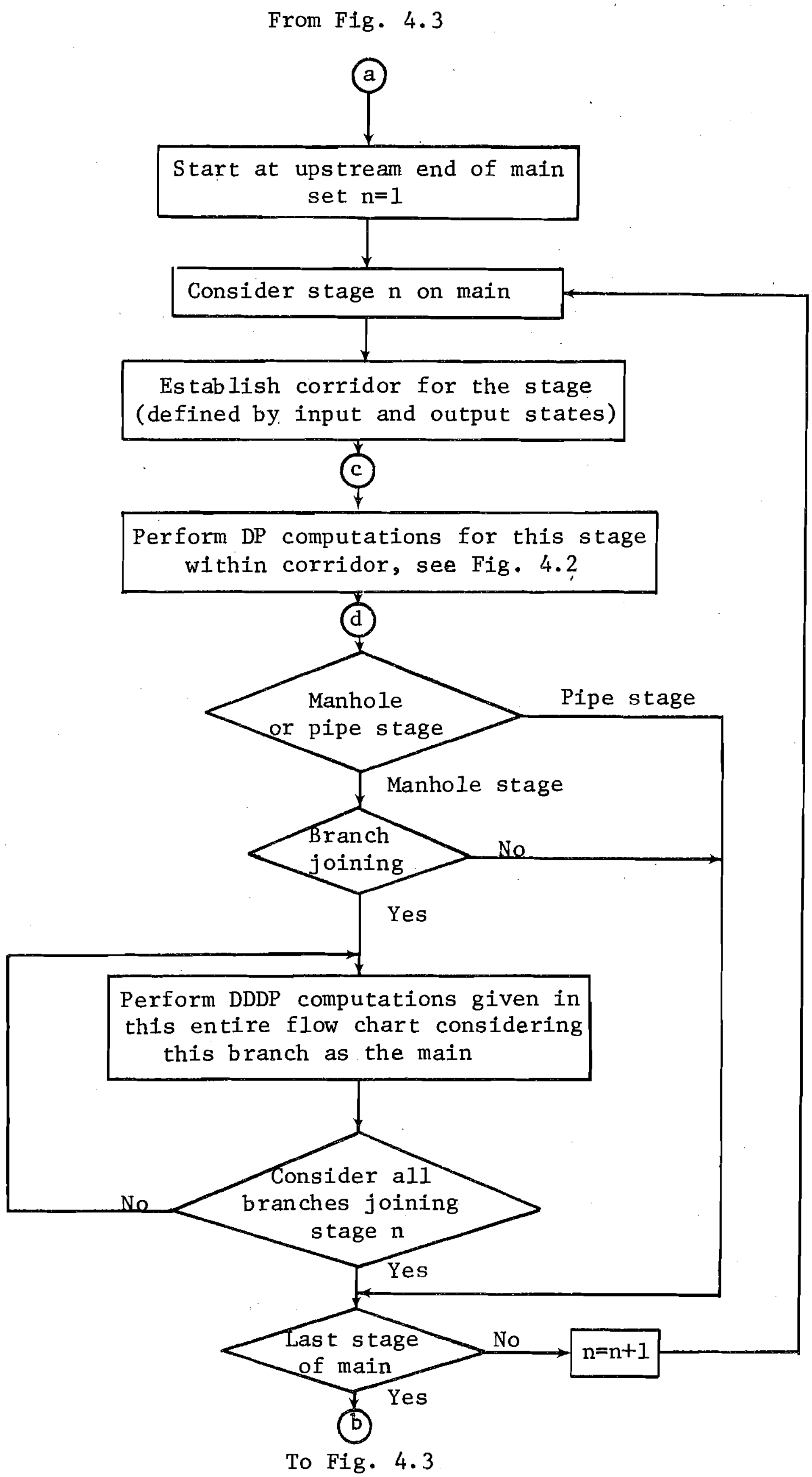


Fig. 4.5. Flow Chart for Each Iteration of Nonserial Approach

First, the computer storage requirements become a major limiting factor. Storing the information for the various levels of branching, connections of branches to other branches or to the main, etc. which are necessary for the trace-back routine results in large storage requirements. Pipe diameters for each downstream state, upstream crown elevation indexes, slopes, ground surface elevations, elevations of the trajectories, design flows, etc. must all be stored in reference to their locations in the system. These input data and computed information must be stored with respect to the pipe or manhole stage on the branch.

The large amount of computer time required is the second disadvantage of the nonserial approach when applied to large systems. The execution time is significantly increased because of the time required to retrieve information in arrays. All of the information except that needed for the DDDP stage considered in the computation could be stored on discs or tapes; however, this significantly increases computer time.

The third limitation is the difficulty in programming. It is evident from the description of what input and computed information must be stored for this approach that programming becomes a rather difficult task when several levels of branching must be considered. The manner in which the system is optimized over also results in programming difficulties. Finally, the difficulty in defining a main chain for even small networks is also a limitation of this approach.

An alternate method of using this nonserial approach would be to divide the sewer system into several smaller subsystems and compute the minimum cost designs for each and then combine them. However, because the minimum of the sums is not necessarily equal to the sum of component minimums, the result of this approach may vary considerably from the true optimal. Also, the computer time required to solve several smaller systems would be increased as compared to one larger system. Mays and Wenzel (1977) use an example sewer system that further illustrates the limitations of the nonserial approach and shows advantages of the serial approach which is described below.

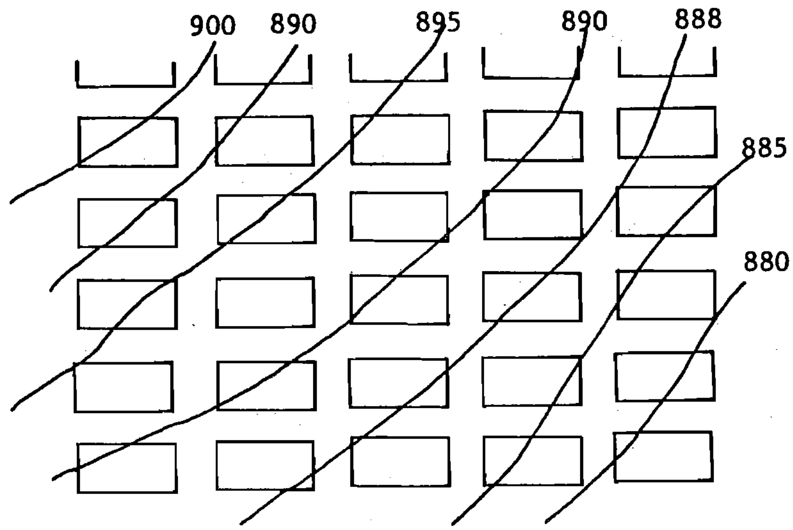
4.4. Serial Optimization Approach

4.4.1. Network Representation for Serial Optimization Approach

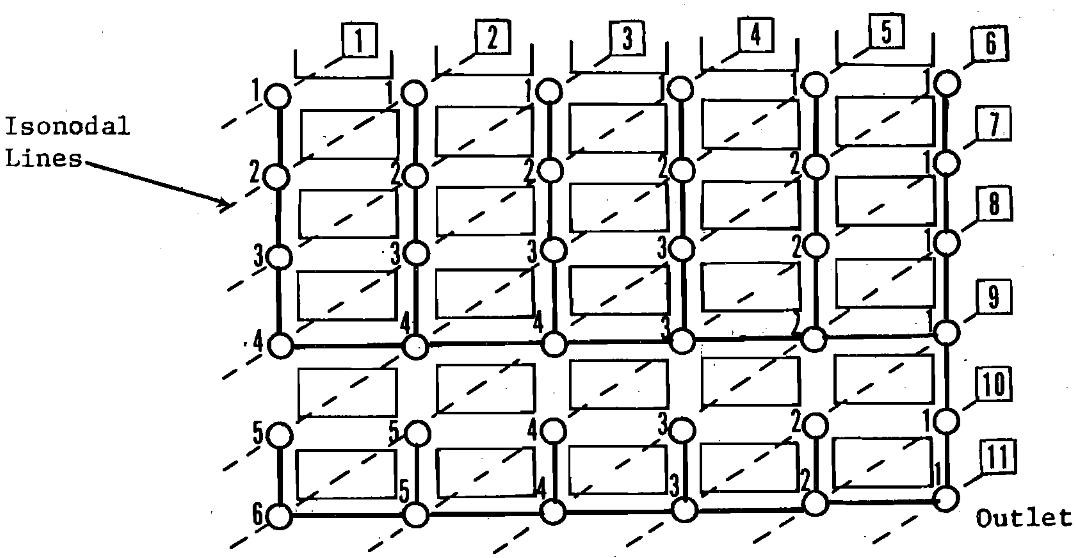
The prescribed layout of the sewer system can be represented by properly numbering the manholes (nodes) and identifying the connections between the manholes. For gravity flow systems, sewers are generally sloped towards low ground surface elevations. Hence, manholes located at higher ground elevations usually have sewer pipes connecting them to manholes at lower ground elevations. This concept gives rise to a rather simple approach as compared to the nonserial approach of representing the sewer network in a form suitable for digital manipulation in the DDDP procedure.

Imaginary lines called isonodal lines (INL) are used to divide the dendritic sewer system into stages. These lines are defined such that they pass through manholes (nodes) which are separated from the system outlet by the same number of sewers (links). Argaman et al. (1973) termed these as drainage lines; however, it is felt that "isonodal" better describes their nature and offers less chance for ambiguities and therefore this term is used throughout this report. An arbitrary stage n includes all the pipes connecting upstream manholes on INL n and downstream manholes on INL n+1. For a system with N isonodal lines and N-1 stages, the manholes on any INL n are connected to the system outlet by N-n sewers. The manholes are no longer stages as in the case of the nonserial approach. The example system shown in Fig. 4.6 is used to further illustrate this point. INL 6 passes through all the manholes which are 5 pipe-links upstream from the system outlet.

The isonodal lines divide the sewer system into stages such that the two most upstream lines form stage 1. The succeeding stages proceed



(a) Street System with Elevation Contours



(b) Layout and Isonodal Lines

Fig. 4.6. Isonodal Lines for a Simple Sewer System

downstream, each defined by adjacent upstream and downstream isonodal lines, n and n+1, for n = 1,2,...,N where N is the number of stages in the entire system. This concept of stages is illustrated in Fig. 4.6 for a simple street system. The street system and contours of elevation are shown in Fig. 4.6a. The manholes and corresponding isonodal lines for the layout are superimposed on the street system in Fig. 4.6b. The isonodal lines are constructed starting at the outlet of the system and proceeding upstream, but are numbered in the reverse order, beginning at the upstream end of the system as shown in Fig. 4.6 for the example sewer system.

The layout description for the digital manipulation is accomplished by the vector of connectivity which is easily defined for a network once the isonodal lines are established. The set of manhole connections for an arbitrary stage n is defined by a vector of connectivity between manholes on INL n and n+1. This vector of connections, given as $T_{m_n, m_{n+1}}$, represents the connection from upstream manholes m_n on INL n to downstream manholes m_{n+1} on INL n+1. Shown in Fig. 4.7 are the manhole connections for an arbitrary stage n between INL n and n+1. This matrix has as many rows as the number of connections from each manhole m_n to all the manholes on INL n+1. Considering the stage n in Fig. 4.7, for each of the three upstream manholes, m_n = 1, 2, 3, on INL n, there are four possible drainage connections, one to each downstream manhole (m_{n+1} = 1, 2, 3, 4). Each position in the vector of connectivity, either has a 1 implying connection of the manholes or a 0 implying no connection. For example, in Fig. 4.7, if the connection of manholes m_n = 3 on INL n is only to manhole m_{n+1} = 2 on INL n+1, then $T_{3,2} = 1$, $T_{3,1} = 0$, $T_{3,3} = 0$, and $T_{3,4} = 0$.

More than one manhole on INL n may have a connection to the same manhole on INL n+1 allowing for branches so that the tree type network of a storm sewer system can be defined. Also each manhole on INL n must have a connection to a manhole on INL n+1. The total vector of connectivity $\bar{T}_n$

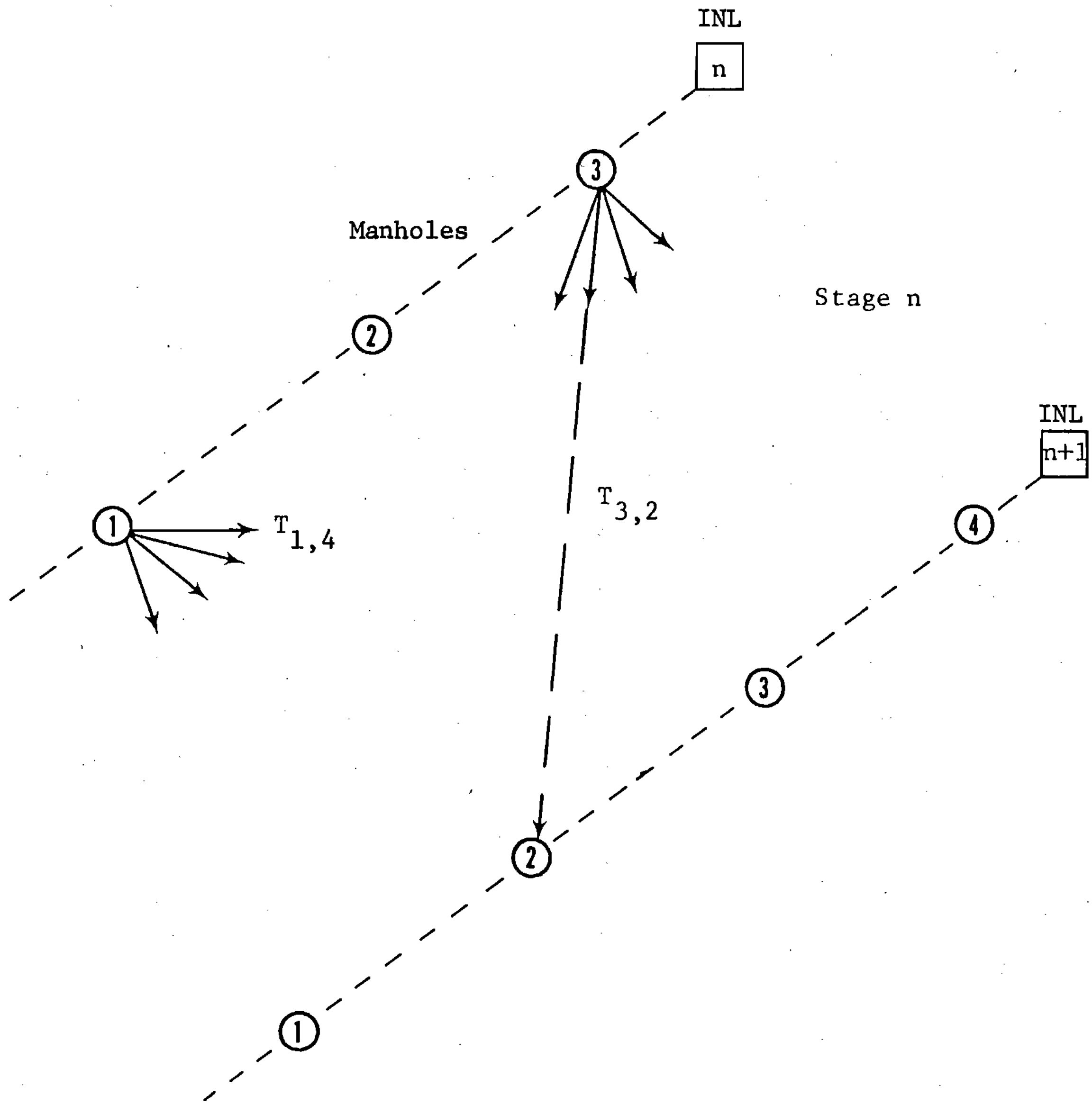


Fig. 4.7. Possible Manhole Connections

at any stage n can be defined as including all connections ($T_{m_n,m_{n+1}}$ for $m_n = 1,2,\ldots,M_n$ and $m_{n+1} = 1,2,\ldots,M_{n+1}$), where M_n and M_{n+1} are the total number of manholes on INL n and n+1, respectively.

4.4.2. System Components of Serial Approach

4.4.2.1 States — The input state vector at each stage n of the system is represented by the sets of pipe crown elevations at the downstream side of each manhole m_n along INL n. The notation for the input state vector at manhole m_n on INL n is S_{m_n}, i.e., the total state vector for INL n has $m_n = 1,2,\ldots,M_n$ sets of crown elevations where M_n is the number of manholes on INL n. For each possible pipe considered at the stage, the input states are defined as the set of crown elevations at the upstream end of the pipe. In other words, for the nonserial approach the input state vector for stage n is defined by a set of crown elevations at the upstream end of the pipe; whereas, for the serial approach the input state vector consists of a set of crown elevations at each manhole on INL n. In matrix form this can be represented for each stage as

$$S_n = \begin{bmatrix} S_1 \\ S_2 \\ \cdot \\ \cdot \\ \cdot \\ S_{m_n} \end{bmatrix} \qquad n = 1,2,\ldots,N \qquad (4.5)$$

where each position in the matrix represents a set of crown elevations (Eq. 4.2) on the downstream side of that particular upstream manhole.

The output states are the set of crown elevations at the downstream end of each pipe connection of an arbitrary stage n. The notation for the output state vector connecting manholes m_n and m_{n+1}, on INL n and n+1 respectively, is $\tilde{S}_{m_n,m_{n+1}}$ for $m_{n+1} = 1,2,\ldots,M_{n+1}$ where M_{n+1} is the total number of manholes on INL n+1. This vector, S_n, can also be represented in matrix form similar to the input states.

It should be pointed out that the output state vector for a downstream manhole at stage n can have several or no pipes connecting to it from the upstream manholes of stage n. The input state vector for the succeeding downstream stage n+1 at the same manhole must have one pipe leading from the manhole. This allows each upstream manhole at each stage to be drained to a manhole on the downstream isonodal line.

4.4.2.2. Decisions — The independent decision variable at each stage is the drop in the crown elevation from the upstream end to the downstream end for each pipe connection in the stage. The pipe diameters also involve a decision variable; however, diameters depend directly upon the slope and maximum flow rate or the risk model (discussed in Chapter 5) so that the pipe diameter is not considered as an independent decision variable. Slopes are determined by the drop in crown elevations and pipe length, and maximum flow rate is a function of the layout, slope, and pipe diameter.

The notation for the set of possible drops in crown elevations from upstream manhole m_n to downstream manhole m_{n+1} on INL n and n+1 is $D_{m_n,m_{n+1}}$. In other words, $D_{1,2}$ represents the set of possible drops in crown elevations across a stage n from manhole $m_n = 1$ on INL n to manhole $m_{n+1} = 2$ on INL n+1. The possible drops in crown elevations for this pipe connection are shown in Fig. 4.1 by the dashed lines. In the figure the input states are the crown elevations at the downstream side of a manhole on INL n, and the output states are the crown elevations at the upstream side of a manhole on INL n+1. The total decision vector represents all possible drops in crown elevations from all of the M_n manholes on INL n to the M_{n+1} manholes on INL n+1 so that the total vector is $D_{m_n,m_{n+1}}$. In matrix form the total vector for each stage n can be represented as

$$D_n = \begin{bmatrix} D_{1,1} & \cdots & D_{1,m_{n+1}} \\ \vdots & & \vdots \\ D_{2,1} & \cdots & D_{2,m_{n+1}} \\ \vdots & & \vdots \\ D_{M_n,1} & \cdots & D_{M_n,M_{n+1}} \end{bmatrix} \qquad n = 1,2,\ldots,N \qquad (4.6)$$

The drop in crown elevation is defined as the difference between the upstream crown elevation (input state) and the downstream crown elevation (output state). This defines the manner in which an input state is transformed into an output state by a decision variable, which in dynamic programming terminology is the transformation function (Eq. 4.1). More specifically, in the serial approach for a possible pipe connection between manholes m_n and m_{n+1}, the transformation function is

$$\tilde{S}_{m_n,m_{n+1}} = S_{m_n,m_{n+1}} - D_{m_n,m_{n+1}} \qquad m_n = 1,2,\ldots,M_n \qquad (4.7)$$

$$m_{n+1} = 1,2,\ldots,M_{n+1}$$

$$n = 1,2,\ldots,N$$

4.4.3. DDDP Solution Scheme for Serial Approach

The DDDP procedure for each iteration starts at the upstream end of the storm sewer system and proceeds downstream stage-by-stage as discussed in Section 4.2 and shown in Fig. 4.3. Because of the definition of the stages, they vary simply by varying the isonodal lines. Stage n is defined by the upstream and downstream INL n and n+1, whereas the next downstream stage n+1 is defined by INL n+1 and n+2. A flow chart showing the logic representing the sewer system for the serial approach for the DDDP solution scheme is given in Fig. 4.8. This flow chart together with Figs. 4.2 and 4.3 gives the DDDP serial optimization model.

As shown in Fig. 4.8, at a stage n each of the downstream manholes $m_{n+1} = 1,\ldots,M_{n+1}$ are varied, and for each of these manholes, the upstream manholes $m_n = 1,\ldots,M_n$ are varied. For each combination of upstream and downstream manholes the vector of connections is checked to see if these manholes represent a connection. If there is no connection (i.e., $T_{m,m_{n+1}} = 0$), then the next upstream manhole m_n+1 is considered for the downstream manhole m_{n+1}. Also, if there is no connection and this is the last upstream manhole ($m_n = M_n$), then the next downstream manhole $m_{n+1} + 1$ and the first upstream manhole ($m_n = 1$) are considered. For each connection the corridor is formed and DP computations are applied as shown in Fig. 4.2 and discussed in Section 4.2. The recursive equation for each pipe that represents a connection (i.e., $T_{m_n,m_{n+1}} = 1$) at stage n is

$$F_n(\tilde{S}_{m_n,m_{n+1}}) = \underset{D_{m_n,m_{n+1}}}{\text{Min}} \left[r_{m_n,m_{n+1}}(S_{m_n,m_{n+1}}, D_{m_n,m_{n+1}}) + F_{n-1}(\tilde{S}_{m_{n-1},m_n})\right] \quad (4.8)$$

where $F_n(\tilde{S}_{m_n,m_{n+1}})$ represents the minimum cost of the system that is connected to downstream manhole m_{n+1} through upstream manhole m_n and where $F_o(S_{m_o,m_1}) = 0$. This recursive equation is for only one of the possible connections at this stage. A recursive equation for the above optimization procedure considering all the connections can be represented as

$$F_n(\tilde{S}_n) = \underset{T_n}{\text{Min}} \sum \underset{D_{m_n,m_{n+1}}}{\text{Min}} \left[r_{m_n,m_{n+1}}(S_{m_n,m_{n+1}}, D_{m_n,m_{n+1}}) + F_{n-1}(\tilde{S}_{m_{n-1},m_n})\right] \quad (4.9)$$

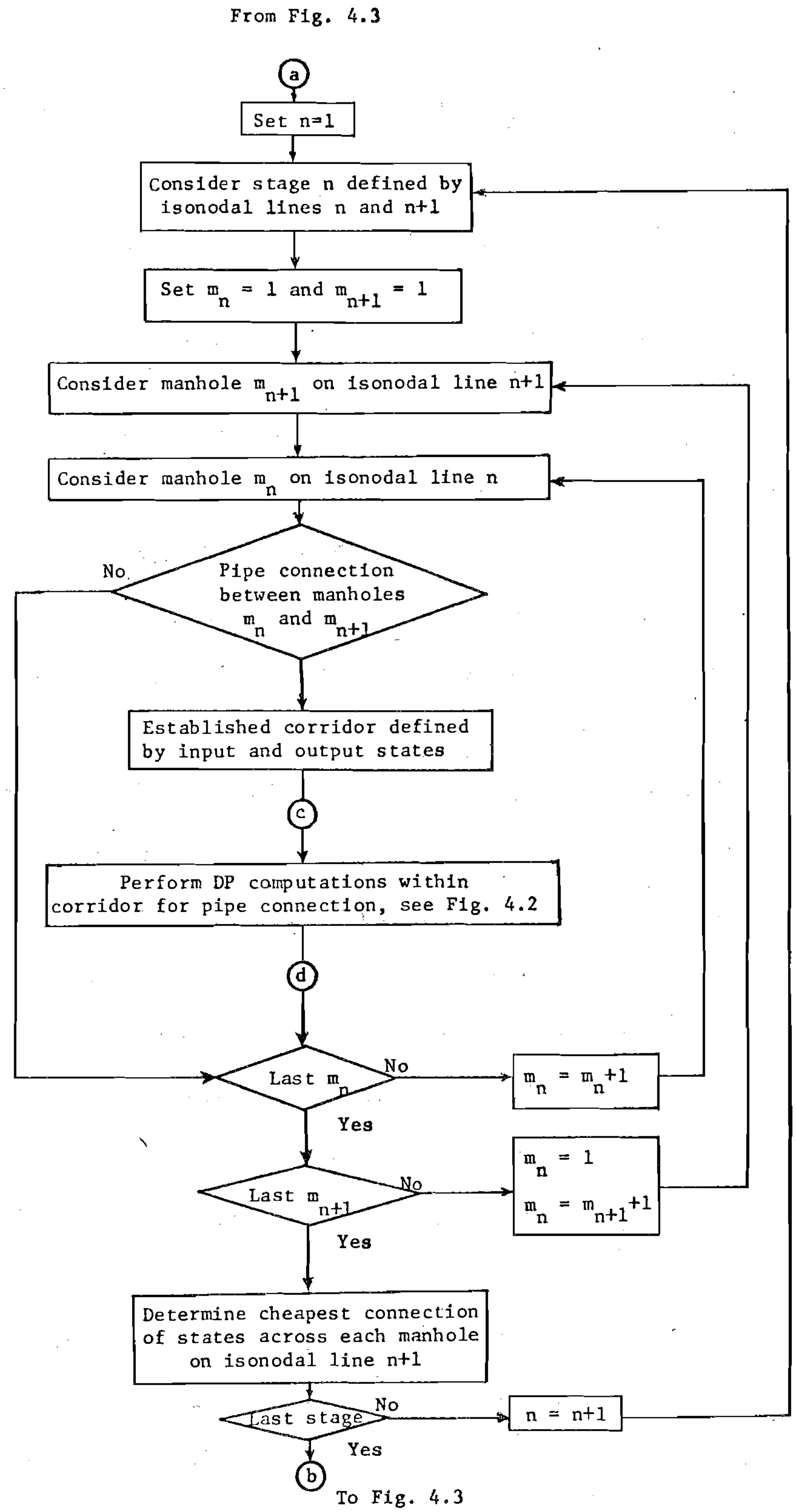


Fig. 4.8. Flow Chart for each Iteration of Serial Approach

for

$$m_{n-1} = 1,\ldots,M_{n-1}$$

$$m_n = 1,\ldots,M_n$$

$$m_{n+1} = 1,\ldots,M_{n+1}$$

$$n = 1,\ldots,N$$

where $F_n(\tilde{S}_n)$ represents the minimum cost of the entire system including all pipes and manholes through stage n, i.e., to the upstream of INL n+1.

4.4.4. Connection of States at Manholes

For the manholes on INL n+1 which are connected by a pipe from the upstream, the connection of states across the manholes must be determined before proceding to the next downstream DDDP stage. This procedure determines the minimum total costs for each of the states on the downstream side of manhole m_{n+1} which are the input states at this manhole for the next downstream DDDP stage. This is done by varying the state on the downstream side of the manhole, and for each of these, consider each state on the upstream side of the manhole which represents a crown elevation greater than or equal to the crown elevation on the downstream side (feasible states). A set of states on the upstream side of the manhole exists for each upstream pipe that connects to the manhole so that the feasible state with the minimum cost for each pipe is chosen. The procedure is illustrated in Fig. 4.9 for a Y junction of pipes.

The sum of minimum costs for each of the pipe connections joining to the manhole is the cumulative minimum cost associated with the crown elevation on the downstream side of the manhole. The states of each connection on the upstream side of the manhole having the minimum costs for the states on the downstream are stored for later use in the trace-back routine. This

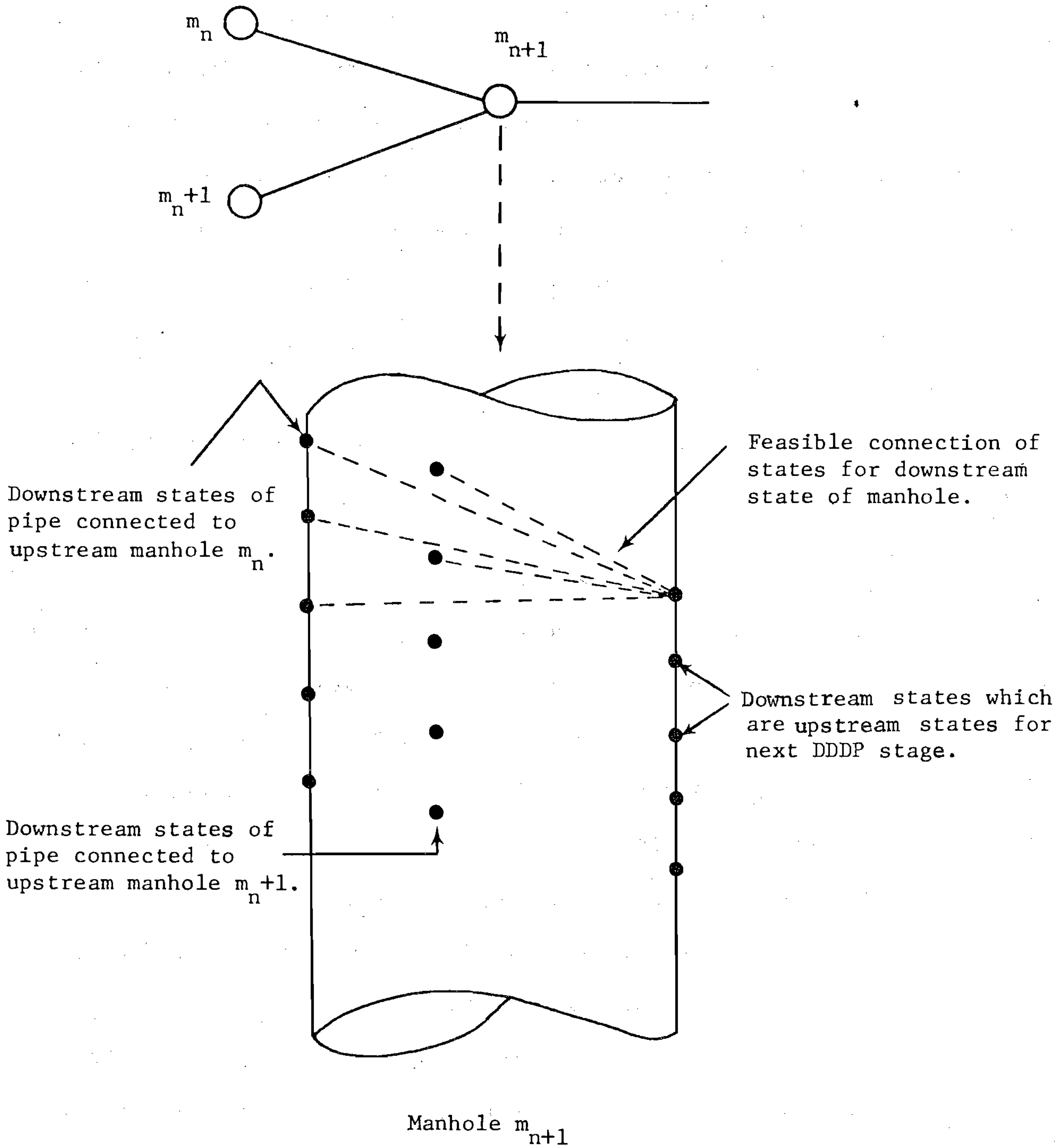


Fig. 4.9. Connectivity of States at Manhole Junctions for Serial Approach

procedure is repeated for each of the states on the downstream side of the manhole, determining the cumulative minimum cost for each state.

The above procedure for the connectivity of states at the manhole is essentially a dynamic programming procedure at each downstream manhole m_{n+1} having an upstream connection in stage n. However there is no return considered because the manhole cost is computed along with the downstream pipe in the DDDP scheme. This is done because the manhole depth, which determines the cost, cannot be computed until the downstream connecting pipe diameter is known. This procedure dictates the manner in which stage n is linked to stage n+1 through the manholes. This linkage is called the incidence identity which relates the output from each stage to the input to the succeeding stage.

4.4.5. Trace-Back Routine

After a DDDP iteration is completed and the minimum cost associated with each of the feasible output states of the last stage are established, the least-cost is determined by comparing these minimum costs for different states. A trace-back routine is performed to retrieve the least-cost design of the sewer system for this iteration. The trace-back commences at the downstream end of the system proceeding upstream stage-by-stage. At each stage the manholes on the downstream isonodal line are varied, and for each of these, the manholes on the upstream isonodal line are varied. For each combination of upstream and downstream manholes, the vector of connectivity $T_{m_n, m_{n+1}}$, is checked to see if these two manholes represent a connection of the system layout. If these manholes do not represent a connection of the layout, the next combination of manholes is considered. If the manholes do represent a connection, the trace-back continues by determining the upstream state for a known downstream state j from the stored indexes of upstream states associated with each downstream state. Remember the trace-back begins at the system outlet at which the minimum cost state (downstream) was computed. When the upstream state k at manhole m_n is found, the downstream states j for each connection to this manhole for the preceding upstream stage can be found from the stored index of connections across manholes. The trace-back at the last two stages of a system is illustrated in Fig. 4.10.

This procedure is repeated for each connection of the layout at the stage found by varying the upstream and downstream manholes. Once all connections of the layout at this stage have been considered, the next upstream stage is considered and the procedure is repeated. Each time the states at the upstream and downstream ends of the pipes are determined, the crown elevations which represent the improved trajectory are stored to be used as the trial trajectory for the next iteration of the algorithm.

4.4.6. Advantages of Serial Approach

As discussed in Section 4.3, there are several major limitations to the nonserial optimization approach when applied to large sewer systems with many levels of branching. These limitations include the large computer storage requirements and difficulty in programming. The serial approach, on the other hand, requires less storage and correspondingly less computer time as well. This is because the sewer system layout is represented by the matrix of connectivity $T_{m_n, m_{n+1}}$, which is a simpler and more general method of storing the required information than is employed in the nonserial approach, thereby facilitating the programming effort.

A further advantage is the ease of defining the system for the optimization. No matter how many levels of branching the sewer system may have, the serial approach always defines the stages by use of the isonodal lines. The superiority of the serial approach to the nonserial approach is particularly apparent when large systems with many levels of branching are considered.

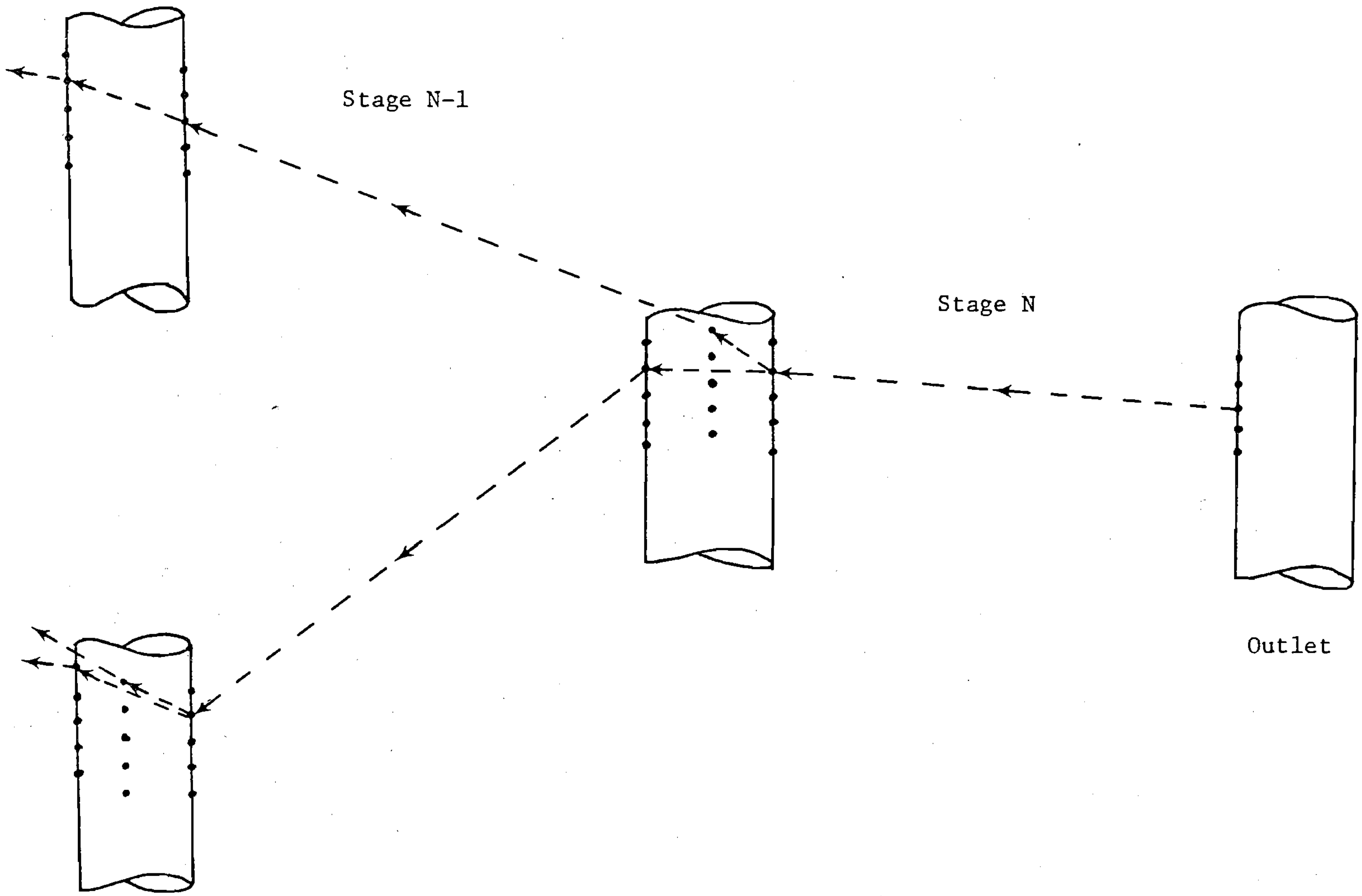


Fig. 4.10. Trace-Back at Last Two Stages of a Sewer System

Chapter 5. CONSIDERATIONS OF RISKS AND UNCERTAINTIES

Engineering designs are inevitably subject to uncertainties. The design of storm sewers is no exception. Traditionally storm sewers are designed using a deterministic approach once the design return period of rainstorm is established. After the design discharge is evaluated, the size of the sewer is determined as the smallest pipe that can convey the design discharge. No consideration is given to the uncertainties and their effect on sewer design. As mentioned in Section 2.2 and will be elaborated further later, the uncertainties involved in sewer design include hydrologic and hydraulic uncertainties, uncertainties due to construction and materials, and uncertainties on cost functions. In this chapter a method is developed to incorporate the effect of uncertainties to sewer design.

5.1. Basic Concepts and Theory

5.1.1. Risk Function

The failure of a storm sewer can be defined as the event in which the runoff or loading Q_L which is imposed on a sewer by a rainfall event exceeds the capacity, Q_C, of the sewer. In other words, the risk of failure is the probability of the event $Q_L > Q_C$; i.e.,

$$\text{Risk} = P(Q_L > Q_C) \tag{5.1}$$

Since both Q_L and Q_C in Eq. 5.1 are non-negative quantities, the probability in Eq. 5.1 is equal to $P[(Q_C/Q_L) < 1]$ or $P[\ln(Q_C/Q_L) < 0]$. Hence

$$\text{Risk} = P(Z < 0) \tag{5.2}$$

in which

$$Z = \ln(Q_C/Q_L) \tag{5.3}$$

By using the first order approximation of the Taylor's series expansion (Ang and Tang, 1975, p. 193) the mean and variance of Z are

$$\overline{Z} \approx \ln(\overline{Q}_C/\overline{Q}_L) \tag{5.4}$$

and

$$\mathrm{Var}(Z) \approx \left(\frac{\partial Z}{\partial Q_C}\right)_0^2 \mathrm{Var}(Q_C) + \left(\frac{\partial Z}{\partial Q_L}\right)_0^2 \mathrm{Var}(Q_L) = \Omega_{QC}^2 + \Omega_{QL}^2 \tag{5.5}$$

in which $\overline{Q}_C$, $\overline{Q}_L$ and Ω_{QC}, Ω_{QL} are the mean values and coefficient of variation of Q_C and Q_L respectively. The subscript 0 with the parenthesis denotes that the quantity within the parenthesis is evaluated at the mean values of the random variables. It is implicitly assumed in Eq. 5.5 that Q_C and Q_L are statistically independent of each other.

Since Q_C and Q_L are usually functions of many random variables as will be detailed in the following section, the distribution of Z is not generally easy to determine. However, it has been shown (Ang 1970; Yen and Ang, 1971) that for a risk level of 10^{-3} or larger, the risk is not sensitive to the type of distribution assumed. Hence, for simplicity, assuming Z to be normally distributed, the risk is

$$P(Q_L > Q_C) = P(Z < 0) = \phi\left[\frac{-\overline{Z}}{\sqrt{\mathrm{Var}(Z)}}\right] \tag{5.6}$$

or, from Eqs. 5.4, 5.5 and 5.6,

$$\mathrm{Risk} = \phi\left[\frac{\ln(\overline{Q}_L/\overline{Q}_C)}{(\Omega_{QL}^2 + \Omega_{QC}^2)^{1/2}}\right] \tag{5.7}$$

in which $\phi(x)$ denotes the cumulative standard normal distribution evaluated at x. The values of ϕ can be found in Appendix A for negative values of x or from tables in standard statistics reference books (e.g., Benjamin and Cornell, 1970; Ang and Tang, 1975) for positive x. Note that $\phi(-x) = 1 - \phi(x)$.

Actually, for storm sewers there are two different types of failure as discussed by Yen and Ang (1971). One is the property damage type failure which causes local flooding but involves no failure or damage in the sewer structures or change in the functioning of the sewer system. Temporarily the sewer is incapable of conveying the entire storm runoff, resulting in property damages such as flooding of basements and lowlands and interruption of traffic. The other type is a catastrophic failure which involves damage to the sewer system such that its proper functioning is no longer possible. The definition of failure as given in Eq. 5.1 essentially follows the concept of the property damage type failure. It is most unlikely that a catastrophic type failure of a storm sewer would happen before the occurrence of the property damage type failure. However, under special circumstances when it is necessary, the probability of catastrophic type failure can also be similarly evaluated through an appropriate modification of Q_C in Eq. 5.1.

5.1.2. Analysis of Component Uncertainties

Since Q_L and Q_C are both, in general, functions of other random variables, an assessment of their mean values and coefficients of variation in terms of those of the component random variables is mandatory. Suppose Q is predicted by a mathematical model G which is a function of variables x_1 to x_j. To account for any error in the prediction as a result of the model idealization, a corrective factor λ with mean $\bar{\lambda}$ and coefficient of variation Ω_λ is introduced, such that Q is expressed as

$$Q = \lambda G(x_1, x_2, \ldots . x_j) \tag{5.8}$$

By applying the first order approximation for Q,

$$\bar{Q} \approx \bar{\lambda}\bar{G}(x_1, x_2, \ldots . x_j) \tag{5.9}$$

or

$$\overline{Q} \approx \overline{\lambda} G(\overline{x}_1, \overline{x}_2, \ldots \overline{x}_j) \tag{5.10}$$

and

$$\mathrm{Var}(Q) \approx \left(\frac{\partial Q}{\partial \lambda}\right)_0^2 \mathrm{Var}(\lambda) + \sum_j \left(\frac{\partial Q}{\partial x_j}\right)_0^2 \mathrm{Var}(x_j)$$

$$+ \sum_{i \neq j} \sum \left(\frac{\partial Q}{\partial x_i}\right)_0 \left(\frac{\partial Q}{\partial x_j}\right)_0 r_{ij} [\mathrm{Var}(x_i) \mathrm{Var}(x_j)]^{1/2} \tag{5.11}$$

in which r_{ij} is the coefficient of correlation between x_i and x_j. Assuming that all the x_j's are statistically independent, and noting that $\partial Q/\partial x_j = (\partial Q/\partial G)(\partial G/\partial x_j) = \lambda(\partial G/\partial x_j)$, Eq. 5.11 can be simplified as

$$\Omega_Q^2 = \Omega_\lambda^2 + \frac{1}{\overline{G}^2} \sum_j \left(\frac{\partial G}{\partial x_j}\right)_0^2 \overline{x}_j^2 \, \Omega_{xj}^2 \tag{5.12}$$

The specific formulas to evaluate the mean and coefficient of variation for Q_L and Q_C depend on the mathematical model adopted for Q_L and Q_C, as will be illustrated later in this chapter.

5.1.3. Safety Factor

Conventionally, the sewer is designed to have a capacity Q_C exceeding the nominally required capacity Q_o. Thus, the safety factor may be defined as

$$SF = \frac{\overline{Q}_C}{Q_o} \tag{5.13}$$

The value of Q_o is the peak flow that the sewer must carry as determined by the hydrologic and/or hydraulic analysis, such as the peak discharge computed by using the rational formula for a given return period equal to the expected project life. The value of $\overline{Q}_C$ is the expected value of the capacity of the sewer of a given diameter and slope evaluated by using one of the flow formulas such as the Darcy-Weisbach or Manning formulas.

5.2. Uncertainties in Rainstorm Runoff and Sewer Capacity

As shown in Eq. 5.12, the evaluation of the component uncertainties of the design discharge Q_L and sewer capacity Q_C depends on the formulas used to compute Q_L and Q_C. To introduce the methodology and for the sake of brevity and clarity, the Manning formula is adopted to evaluate the sewer capacity, and the rational method is adopted to evaluate the design discharge. The reader should not interpret this adoption as an endorsement of the rational formula.

5.2.1. Uncertainties in Design Discharge

The rational formula, because of its simplicity, is the most commonly used formula to estimate the peak runoff rate due to rainfall. Through the years much criticism has been leveled on the rational formula and most of its drawbacks are well-known. Recently many improved and sophisticated flow simulation methods have been developed which are more satisfactory than the rational formula (e.g., see Chow and Yen, 1976). Nevertheless, as mentioned earlier, the rational formula is adopted as an example for the sake of simplicity and clarity; other methods could also be used if desired.

If the rational formula, Q = CiA, is used, where C is the runoff coefficient, i is the rainfall intensity and A is the drainage basin area, the value of Q_o in Eq. 5.13 can be computed from

$$Q_o = \overline{C}\, i_o\, \overline{A} \tag{5.14}$$

in which i_o is the reference rainfall intensity conventionally used to compute the discharge for the design return period. The bar represents, as before, the mean value of the variable.

The value of $\overline{Q}_L$ in Eq. 5.7, which is the expected value of the maximum discharge during the T year expected service life of the sewer, can also be estimated using the rational formula. However, since the rational formula represents only an approximate model, a correction factor λ_L is introduced. Thus, applying Eqs. 5.10 and 5.12 to the rational formula, one obtains

$$\overline{Q}_L = \overline{\lambda}_L \; \overline{C} \; \overline{i}_T \; \overline{A} \tag{5.15}$$

$$\Omega_{QL}^2 = \Omega_L^2 + \Omega_C^2 + \Omega_i^2 + \Omega_A^2 \tag{5.16}$$

where Ω_L, Ω_C, Ω_i and Ω_A are the coefficients of variation of the model correction factor, runoff coefficient, rainfall intensity and drainage area, respectively. The quantity $\overline{i}_T$ represents the expected maximum rainfall intensity during the T yr sewer service life and it can be evaluated from the rainfall intensity-frequency relationship using a return period equal to T yr.

5.2.2. Uncertainties in Sewer Capacity

The Manning formula is

$$Q = \frac{1.49}{n} AR^{2/3} S^{1/2} \tag{5.17}$$

in which n is Manning's roughness factor; A is the flow cross sectional area; R is the hydraulic radius; and S is the friction slope of the flow. In computing the sewer capacity, assuming gravity flow with just full pipe of diameter d, Eq. 5.17 can be written as

$$Q_C = \frac{0.463}{n} d^{8/3} S^{1/2} \tag{5.18}$$

Application of Eqs. 5.10 and 5.12 to Eq. 5.18 yields

$$\overline{Q}_C = \frac{0.463}{\overline{n}} \overline{\lambda}_m \overline{S}^{1/2} \overline{d}^{8/3} \tag{5.19}$$

$$\Omega_{QC}^2 = \Omega_m^2 + \frac{1}{4} \Omega_S^2 + 7.1 \Omega_d^2 + \Omega_n^2 \tag{5.20}$$

in which λ_m accounts for the approximation associated with the Manning formula, and Ω_{QC}, Ω_m, Ω_S, Ω_d and Ω_n are the coefficients of variation of Q_C, the model correction factor, slope, diameter and roughness, respectively.

5.3. Procedure to Establish Risk-Safety Factor Relationship

The basic procedure to establish the risk-safety factor curves for a geographic location is to use Eqs. 5.7 and 5.13 to compute the risk and safety factor. The details depend on the factors affecting Q_o, $\overline{Q}_L$, $\overline{Q}_C$, Ω_{QL}, and Ω_{QC}. The following summary is only meant for reference rather than a rigid rule, and the engineer may alter the procedure as the situation dictates.

For the drainage basin or location where the risk-safety factor relationships are to be established, the suggested procedure is as follows.

(a) Select the appropriate models to compute the sewer capacity and design discharge.

(b) Perform an analysis of uncertainties on the rainfall intensity. This involves assessment of uncertainties due to return period, duration, limited rainfall record and data reliability. For each choice of return period and duration the result usually consists of a reference rainfall intensity for the evaluation of Q_o in Eq. 5.13 and a mean intensity, together with the coefficient of variation for the estimation of $\overline{Q}_L$ and Ω_{QL}. Alternatively, if input is the sewer inflow hydrograph, perform an analysis of uncertainties on the hydrograph.

(c) Perform an analysis of uncertainties for the design discharge. This involves an assessment of factors, in addition to the rainfall intensity, contributing to the uncertainties for the design discharge. In other words, this step involves the determination of the mean and coefficient of variation for each of the component factors affecting the design discharge. The result usually consists of a set of values of Q_o, $\overline{Q}_L$ and Ω_{QL} (using, for example, Eqs. 5.14, 5.15 and 5.16) for the duration and rainfall return period which is selected as equal to the expected service life of the sewer.

(d) For an arbitrarily selected pipe size available commercially, perform an analysis of uncertainties for the sewer capacity. This involves an analysis of the uncertainties in the pipe size, roughness, straightness, construction reliability such as the slope, and the model error. The values of the mean and coefficient of variation for the factors affecting the sewer capacity are determined first. The end result consists of the values of $\overline{Q}_C$ and Ω_{QC} (using formulas such as Eqs. 5.19 and 5.20) for the pipe size considered.

(e) Compute the risk using Eq. 5.7.

(f) Compute the safety factor using Eq. 5.13.

(g) The pair of values for the risk and safety factor, computed in (e) and (f), gives one point of the risk-safety factor curve.

(h) Repeat steps (d) to (f) for a different pipe size. This will give another point on the risk-safety factor curve. Repeating this procedure for other pipe sizes will give additional points to plot the curve for the selected rainfall duration and design period.

(i) Repeat steps (c) to (h) for different rainfall durations having the same design period. The results will give curves for different durations. However, it has been found that the effect of rainfall duration is usually small and the points having different durations but the same design period can be represented by a single curve.

(j) Repeat steps (c) to (i) for different design periods, the results will complete the set of risk-safety factor curves for different expected sewer service life periods.

In view of the amount of repetitive computations involved to establish the risk-safety factor curves, it is suggested that such computations are best done on a digital computer.

5.4. Development of Risk-Safety Factor Curves

To illustrate the computational details in the process of developing the risk-safety factor curves, a drainage basin of 10 ac in size located in Urbana, Illinois is adopted as an example. The design discharge is computed by using the rational formula and the sewer capacity by the Manning formula.

5.4.1. Analysis of Uncertainties in Rainfall Intensity

The uncertainty in rainfall intensity varies with the design period T and duration t_d of the rainfall and the location and size of the drainage basin. For most drainage basins in the U.S. the relationship between the rainfall intensity, duration, and return period can be estimated from a National Weather Service atlas (Hershfield, 1963). The drainage basin considered here as an example is a 10-ac area at Urbana, Illinois and the following example computations are for T = 10 yr and t_d = 30 min.

In most locations the point rainfall intensity, i, can be expressed as

$$i = \frac{a\ T^m}{b + t_d^k} \tag{5.21}$$

in which a and b are constants and m and k are constant exponents. Equation 5.21 represents the frequency distribution of the annual maximum point rainfall intensity, i_a. At Urbana, based on the data from Hershfield (1963) for rainfall duration from 5 min to 2 hr and return period from 1 to 100 yrs, a = 120, b = 27, m = 0.175 and k = 1. Hence,

$$i = \frac{120\ T^{0.175}}{27 + t_d} \tag{5.22}$$

in which i is in in./hr, T is in yr, and t_d in min.

5.4.1.1. Effect of Design Period - In order to estimate the expected maximum discharge $\bar{Q}_L$ (Eq. 5.15) it is necessary first to estimate the expected maximum rainfall intensity, $\bar{i}_T$, for a specified rainfall duration during the extire service life of the sewer. The variance of i for the distribution expressed in Eq. 5.21 with 0.5 > m > 0 is (Yen, 1975b)

$$\mathrm{Var}(i_a) = \frac{m^2}{1 - 2m} \left(\frac{1}{1 - m} \frac{a}{b + t_d^k}\right)^2 \tag{5.23}$$

The distribution of intensity given in Eq. 5.22 is obtained from limited data and therefore there is an uncertainty in its specification due to a finite length of record. Since the rainfall intensity whose distribution is being considered consists of the largest values of the record whether the annual maximum series or annual exceedance series is used, it is reasonable to assume that the intensity i_T actually follows a Type I extreme value (Gumbel) distribution. The subscript T of i is to emphasize that each value of the intensity corresponds to a period of T yr as was expressed in Eq. 5.15. According to Benjamin and Cornell (1970), for Gumbel distribution of i_T, its expected value in T yr is

$$\bar{i}_T = i_o + 0.45 \sqrt{\mathrm{Var}\ (i)} \tag{5.24}$$

in which i_o is the value of i given by Eqs. 5.21 or 5.22 for the specified T yr period. Hence, substitution of Eqs. 5.21 and 5.23 into Eq. 5.24 yields

$$\bar{i}_T = \frac{a}{b + t_d^k} \left(T^m + \frac{0.45}{\sqrt{1 - 2m}} \frac{m}{1 - m}\right) \tag{5.25}$$

The coefficient of variation of i_T (accounting only for the effect of design period T) is $\delta_{iT} = \sqrt{\mathrm{Var}(i_T)} / \bar{i}_T$. Heaney (1971) showed that $\mathrm{Var}(i_T) \approx \mathrm{Var}(i)$. Therefore,

$$\delta_{iT} \approx \frac{\sqrt{\mathrm{Var}(i)}}{\bar{i}_T} \tag{5.26}$$

or

$$\delta_{iT} \approx \left[\left(\frac{1}{m} - 1\right)\sqrt{(1 - 2m)}\,T^m + 0.45\right]^{-1} \tag{5.27}$$

From Eq. 5.22, m = 0.175, k = 1, a = 120 and b = 27. Substitution of these values into Eqs. 5.25 and 5.27 yields $\bar{i}_T$ = 3.40 in./hr and δ_{iT} = 0.160 respectively, for T = 10 yr and t_d = 30 min.

5.4.1.2. Effect of duration - In the rational formula, the rainfall intensity is assumed to have a duration equal to the time of concentration of the drainage area upstream of the design location. This assumption on duration is not necessarily correct and the error may be considered in the modeling error later instead of here. Even if the error of this time-of-concentration assumption is discounted, there still exists a prediction error for the duration. For the rainfall intensity relationship described by Eq. 5.21,

$$\bar{i}_a = \frac{a}{(1-m)bt_d^k} \tag{5.28}$$

The effect of error in duration t_d on the rainfall intensity $\bar{i}_T$ can be obtained through first order analysis on $\bar{i}_a$ and then adjusted by a factor $\bar{i}_a/\bar{i}_T$ yielding the coefficient of variation, δ_{id}, as

$$\delta_{id} = k(1 - \frac{b}{b + \bar{t}_d^k})\delta_d \frac{\bar{i}_a}{\bar{i}_T} \tag{5.29a}$$

in which δ_d is the coefficient of variation of the duration. For the Urbana basin,

$$\delta_{id} = (1 - \frac{27}{27 + \bar{t}_d}) \delta_d \frac{\bar{i}_a}{\bar{i}_T} = \frac{\bar{t}_d \delta_d}{27 + \bar{t}_d} \frac{\bar{i}_a}{\bar{i}_T} \tag{5.29b}$$

Assuming that the estimated duration can be off by δ_d = 12.5% and for $\bar{t}_d$ = 30 min, the computed values of δ_{id} is 0.049.

5.4.1.3. Effect of Size of Area - For a given return period and duration, the average rainfall intensity tends to decrease with increasing size of area. For a small area of 10 ac as discussed in this example, the effect is relatively small. The error can be assumed as δ_{iA} = 0.001.

5.4.1.4. Effect of Limited Rainfall Record - Because of the limited number of years of rainfall record available to establish the values in the Atlas (Hershfield, 1963), statistical uncertainties exist in the estimation procedure. The contribution of these uncertainties to the overall uncertainty comes mainly from the estimation of the intensity $\bar{i}_a$ given in the Atlas. The statistical uncertainty (measured by coefficient of variation) of i_a is approximately equal to $\delta_{ia}/\sqrt{N}$ where N is the number of years of record and δ_{ia} is the coefficient of variation of i_a. The corresponding uncertainty in $\bar{i}_T$ due to limited record may be obtained as

$$\delta_{ir} = \frac{\delta_{ia}}{\sqrt{N}} \frac{i_a}{\bar{i}_T} = \frac{\sqrt{Var(i_a)}}{\sqrt{N}\ \bar{i}_T} \qquad (5.30)$$

For the example considered, from Eq. 5.23, $Var(i_a) = 0.307$ and $i_T = 3.40$ in./hr. Hence, for a 50-yr record, N = 50 and $\delta_{ir} = \sqrt{0.307}/(\sqrt{50} \times 3.4) = 0.023$.

5.4.1.5. Effects Due to Errors in Instrumentation, Data Reading and Handling, Interpolation - Available information for the example is inadequate for a detailed probabilistic analysis of the uncertainties due to these errors. Hence, the gross uncertainty in this group is subjectively estimated to be $\delta_{ie} = 0.054$.

5.4.1.6. Total Uncertainty in Rainfall Intensity - This is given by the coefficient of variation

$$\Omega_i = (\delta_{iT}^2 + \delta_{id}^2 + \delta_{ir}^2 + \delta_{iA}^2 + \delta_{ie}^2)^{1/2} \qquad (5.31)$$

Hence, for the present example with T = 10 yr and $\bar{t}_d$ = 30 min, $\Omega_i = 0.177$.

5.4.2. Analysis of Uncertainties in Design Discharge

Besides the rainfall intensity, there are other factors contributing to the uncertainty of the design discharge as indicated in Eqs. 5.15 and 5.16.

5.4.2.1. Runoff Coefficient - The weighted runoff coefficient C in the rational formula is computed from

$$C = \sum_j C_j \alpha_j \tag{5.32}$$

in which $\alpha_j = a_j/A$ where A is the total area of the drainage basin and a_j is the sub-area having a runoff coefficient C_j. There are three possible ways to account for the two factors, C_j and α_j, in estimating the uncertainty in $\overline{C}$. The first is to consider that there is no uncertainty in α_j, so the uncertainty in $\overline{C}$ comes solely from C_j. This may be a preferred approach for well defined sub-areas such as a typical city block. The second way is to consider that there is no uncertainty in C_j so that α_j is the only contributor. This approach is highly impractical since C is difficult to determine precisely and even for a particular location C depends on rainfall intensity and time. Also, for a given location the value of C changes with seasonal variation and alternation of land use. The third way is to allow for uncertainties in both C_j and α_j. This is perhaps the most common approach as in practice a drainage basin is often subdivided into certain percentages of permeable, semi-permeable and impermeable areas, or more precisely, the percentages of areas for roofs and buildings, roads, driveways, paths, lawns, woods, etc. There is uncertainty associated with the percentage of area and C used for each category. As the third approach is suitable for most locations this approach is adopted in the example. Thus, by applying a first-order analysis to Eq. 5.32,

$$\overline{C} = \sum_j \overline{C}_j \overline{\alpha}_j \tag{5.33}$$

$$\Omega_C = \frac{1}{\overline{C}} [\sum_j \overline{C}_j^2 \overline{\alpha}_j^2 (\delta_{C_j}^2 + \delta_{\alpha_j}^2)]^{1/2} \tag{5.34}$$

TABLE 5.1. Component Errors for Runoff Coefficients

Surface	Driveways and sidewalks	Roofs	Streets
α_j	0.40	0.40	0.20
$\delta_{\alpha j}$	0.10	0.10	0.10
Range of C_j*	0.75-0.85	0.75-0.95	0.70-0.95
$\bar{C}_j$#	0.800	0.850	0.825
Variability of C_j#	0.036	0.068	0.087
Prediction error of $\bar{C}_j$##	0.012	0.023	0.029
δ_{Cj}**	0.038	0.072	0.092

*Obtained from standard references; e.g., Chow (1964, p. 14.8)

#Assume uniform distribution over the range, see Appendix C for formulas; variability in terms of coefficient of variation

##In terms of coefficient of variation, assume $\bar{C}_j$ varies uniformly within the middle third of the range, see Appendix C for formulas

**δ^2_{Cj} = (variability)^2 + (prediction error)^2

Although the α_j's are somewhat dependent because they should add up to unity, statistical independence among all C_j's and α_j's are assumed here for simplicity. Besides, the effect of dependence among α_j's will diminish as j becomes large. Suppose the drainage basin considered is a highly developed area consisting of 40% roofs, 20% asphalt streets and 40% driveways and sidewalks. The analysis of uncertainties of the components contributing to Ω_C is summarized in Table 5.1. The prediction error for α_j is subjectively and conservatively assumed to be 0.10. From the values in Table 5.1 and Eqs. 5.33 and 5.34, $\overline{C} = 0.825$ and $\Omega_C = 0.071$.

5.4.2.2. Drainage Basin Area - The error in estimating drainage basin area A comes mainly from two sources: the uncertainty in determining the boundary of the drainage basin and the error in measuring the area. Usually the area is determined from a map. To obtain an idea on the magnitude of this prediction error, 34 engineering students were asked to inspect a 3-sq mi drainage basin at Urbana, Illinois, and then determine the area from a US Geological Survey 7.5-min map. The average error measured in terms of the coefficient of variation was found to be $\delta_{A1} = 0.045$. Hence, the coefficient of variation describing the estimation uncertainty associated with N persons each making one independent prediction is approximately $\delta_{A1}/\sqrt{N}$. Using $\delta_{A1} = 0.050$ and assuming the area is estimated by one engineer in the present example, the prediction error in terms of coefficient of variation is $0.050/\sqrt{1} = 0.050$. The uncertainty associated with the accuracy of the map is usually small and is assumed to have a coefficient of variation of 0.001. Accordingly $\Omega_A = (0.050^2 + 0.001^2)^{1/2} = 0.050$ for $\overline{A} = 10$ ac.

5.4.2.3. Model Uncertainty - The correction factor, λ_L, accounting for the uncertainties in the use of the rational formula to model the rainfall-runoff relationship is rather difficult to assess precisely. It is well known that the rational formula is an approximation. Even if the values of C, i, and A could be determined precisely, the rational formula can only predict the peak runoff rate approximately because of the nonlinear effects involved in the surface runoff phenomenon. The rational formula may over or underestimate the peak runoff rate depending on the conditions encountered. A preliminary analysis summarized in Appendix B gives the value $\bar{\lambda}_L = 1.0$ and $\Omega_{\lambda L} = 0.15$.

5.4.2.4. Uncertainty in Design Discharge - With the values of the mean and coefficient of variation of λ_L, C, i, and A calculated as described above, the total uncertainty in the design discharge, Ω_{QL}, can be computed using Eq. 5.16. Correspondingly, $\bar{Q}_L$ and Q_o can be computed using Eq. 5.15 and Eq. 5.14. The computed values of Q_o, $\bar{Q}_L$ and Ω_{QL} are 26.0 cfs, 28.1 cfs and 0.281, respectively, for the 10-ac Urbana basin for 10-yr design period and 30 min duration.

5.4.3. Analysis of Uncertainties in Sewer Capacity

Because sewer flows are unsteady and nonuniform, unless the St. Venant equations are used together with realistically specified initial, upstream, and downstream conditions, the sewer flow capacity cannot be accurately determined. Using the Manning formula, the error in estimating the sewer capacity is expressed by Eq. 5.20. The four parameters contributing to the uncertainty are evaluated as follows:

5.4.3.1. Effect of Pipe Roughness - The uncertainty in Manning's roughness factor comes mainly from the slimming of the pipe wall and variations in the size and distribution of the surface roughness. Other factors, such as deviation of the sewer diameter from the nominal size, have a negligible effect on the value of n. For most concrete sewer pipes n ranges from 0.013

to 0.017. Assuming a triangular distribution of n over this range with peak at the mean = 0.015 and using the formula given in Appendix C, the corresponding coefficient of variation $\Omega_n = 0.0553$.

5.4.3.2. Effect of Sewer Diameter - There are two major sources of uncertainty in the sewer diameter. One is the manufacturer's tolerance for the pipe. The other is the size reduction due to deposition, which is traditionally accounted for as change of resistance coefficient and hence not considered here. The tolerance of a sewer pipe depends on the material and the manufacturer. Assuming a tolerance of ±1.0 in. and a uniform distribution over this range for a 5-ft pipe, the mean diameter $\bar{d} = 5.0$ ft and using the formula given in Appendix C, $\Omega_d = 0.578\ (61 - 59)/(61 + 59)$ $= 0.0098$. The value of Ω_d would vary for sewers of different sizes and materials. However, since the value is relatively small, $\Omega_d = 0.010$ may be considered satisfactorily representing other conditions.

5.4.3.3. Effect of Sewer Slope - Uncertainties on sewer slope come mainly from sewer misalignment and crookedness of the pipe as well as settlement, and are worse for small slopes. A slope with a 6-in. drop in 500 ft is not uncommon for flat land as in central Illinois. Assuming an error of ± 1 in. for the 6-in. drop and a symmetric triangular distribution over this range of error, the error is $\Omega_S = 0.068$ for $\bar{S} = 0.001$.

5.4.3.4. Effect of Equation Error - Urban storm flood flows are highly unsteady and nonuniform; hence, the use of Manning's formula results in additional uncertainty. A statistical analysis of the results on storm sewer design by Yen and Sevuk (1975) indicates that $\bar{\lambda}_m = 1.1$. Assuming a triangular distribution of λ_m from 0.8 to 1.4 with the mode at 1.1, gives $\Omega_m = 0.11$.

5.4.3.5. Total Uncertainty in Sewer Capacity - With the values of mean and coefficient of variation for λ_m, S, d, and n evaluated for a 5-ft diameter concrete pipe in Urbana, the sewer capacity and the associated total uncertainty can be computed using Eqs. 5.19 and 5.20 as $\overline{Q}_C$ = 78.6 cfs and Ω_{QC} = 0.130, respectively. The computed results are summarized in Table 5.2.

5.4.4. Construction of Risk-Safety Factor Curves

Combining the values of Q_o, $\overline{Q}_L$, $\overline{Q}_C$, Ω_{QL} and Ω_{QC} estimated for the example Urbana basin for a rainfall of 30-min duration and 10-yr design period and for a 5-ft diameter concrete sewer pipe,

$$\text{Risk} = \phi\left[\frac{\ln(28.1/78.6)}{(0.248^2 + 0.130^2)^{1/2}}\right] = \phi\ (-3.674) = 0.00012 \qquad (5.35)$$

from Eq. 5.7 and the corresponding safety factor SF = 78.6/26.0 = 3.02 from Eq. 5.13. This pair of values constitutes point A on the risk-safety factor curve as shown in Fig. 5.1.

To obtain other points for the risk-safety factor curves, the above procedure is first repeated for different pipe sizes, keeping other conditions unchanged. Accordingly, $\overline{Q}_C$ and Ω_{QC} will change, leading to a set of points shown as triangles in Fig. 5.1. The entire procedure is repeated again for a different duration, say 60 min, keeping the design period unchanged, resulting in another set of points, shown as open circles in Fig. 5.1. As the effect of duration appears to be small, the risk-SF relationship for a given design period may be represented by a single curve as shown by the solid line.

The procedure can be repeated and curves for different design periods can be established. Such plots have been shown elsewhere (Tang et al., 1975) and reproduced here for T = 2, 5, 25, 50, and 100 yr as Fig. 5.2. It

TABLE 5.2. Uncertainties for an Example Sewer

Parameter	Mean	Coef. of Variation Ω	Ω^2/Ω_{QL}^2 %	$\Omega^2/(\Omega_{QL}^2 + \Omega_{QC}^2)$ %
C	0.825	0.071	8.2	6.4
i	3.40 in./hr	0.177	51.1	40.0
A	10.0 acres	0.050	4.0	3.2
λ_L	1.00	0.15	36.7	28.7
Q_L	28.1 cfs	0.248	100.0	78.3
Parameter	**Mean**	**Coef. of Variation Ω**	**$\alpha\Omega^2/\Omega_{QC}^2$ %**	**$\alpha\Omega^2/(\Omega_{QL}^2 + \Omega_{QC}^2)$ %**
n	0.015	0.0553	18.0	3.9
d	5. ft	0.010	4.2	0.9
S	0.001	0.068	6.8	1.5
λ_m	1.10	0.110	71.0	15.4
Q_C	78.6 cfs	0.130	100.0	21.7

Note: 1. α is the coefficient of the terms in Eq. 5.20.
2. Analysis is based on a 10-ac drainage area at Urbana, Ill. with $i(\text{in./hr}) = 120\ T^{0.175}/(27+t_d)$ using T = 10 yr and t_d = 30 min.

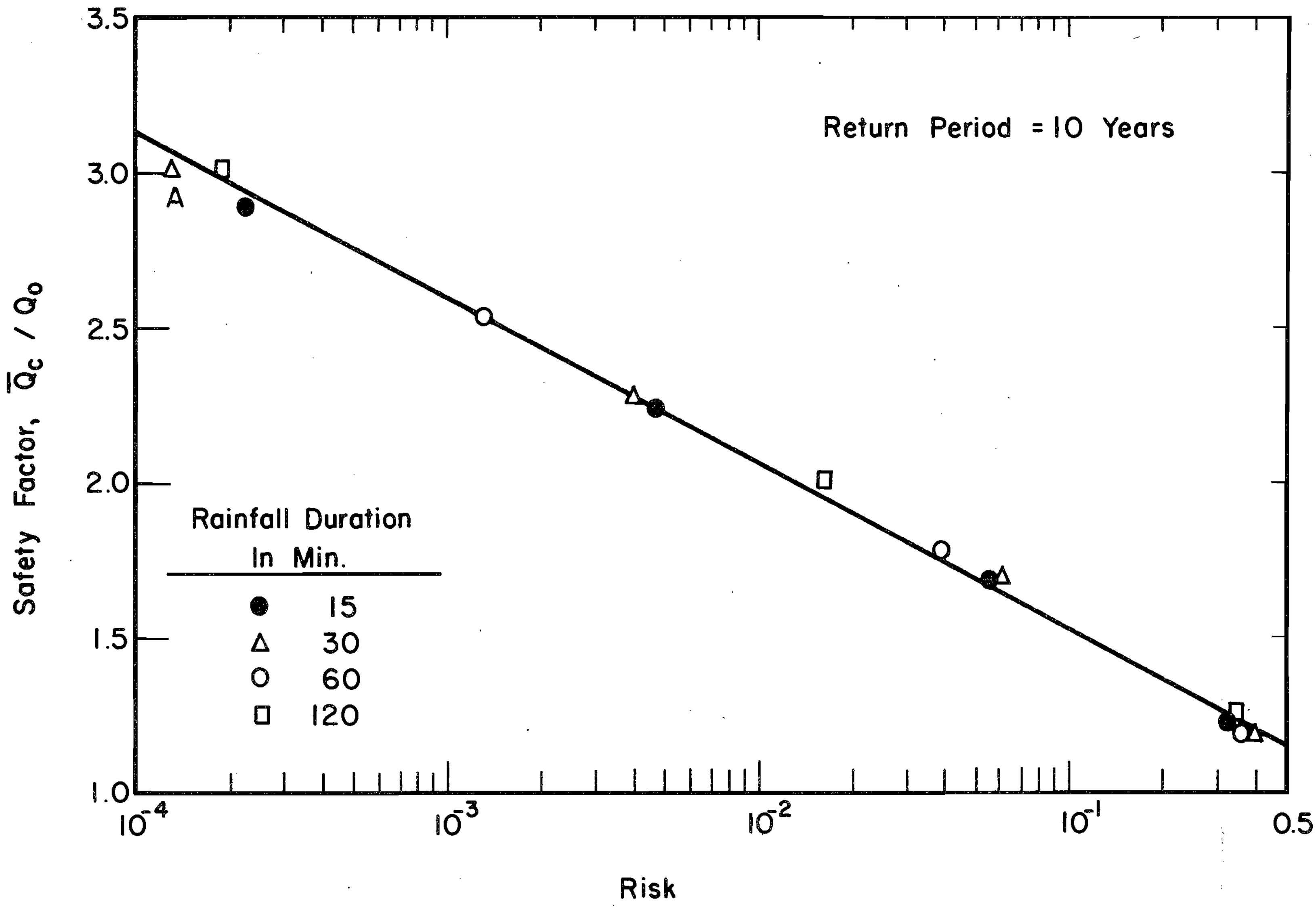


Fig. 5.1. Risk-Safety Factor Curve for 10-yr Design Period at Urbana, Illinois

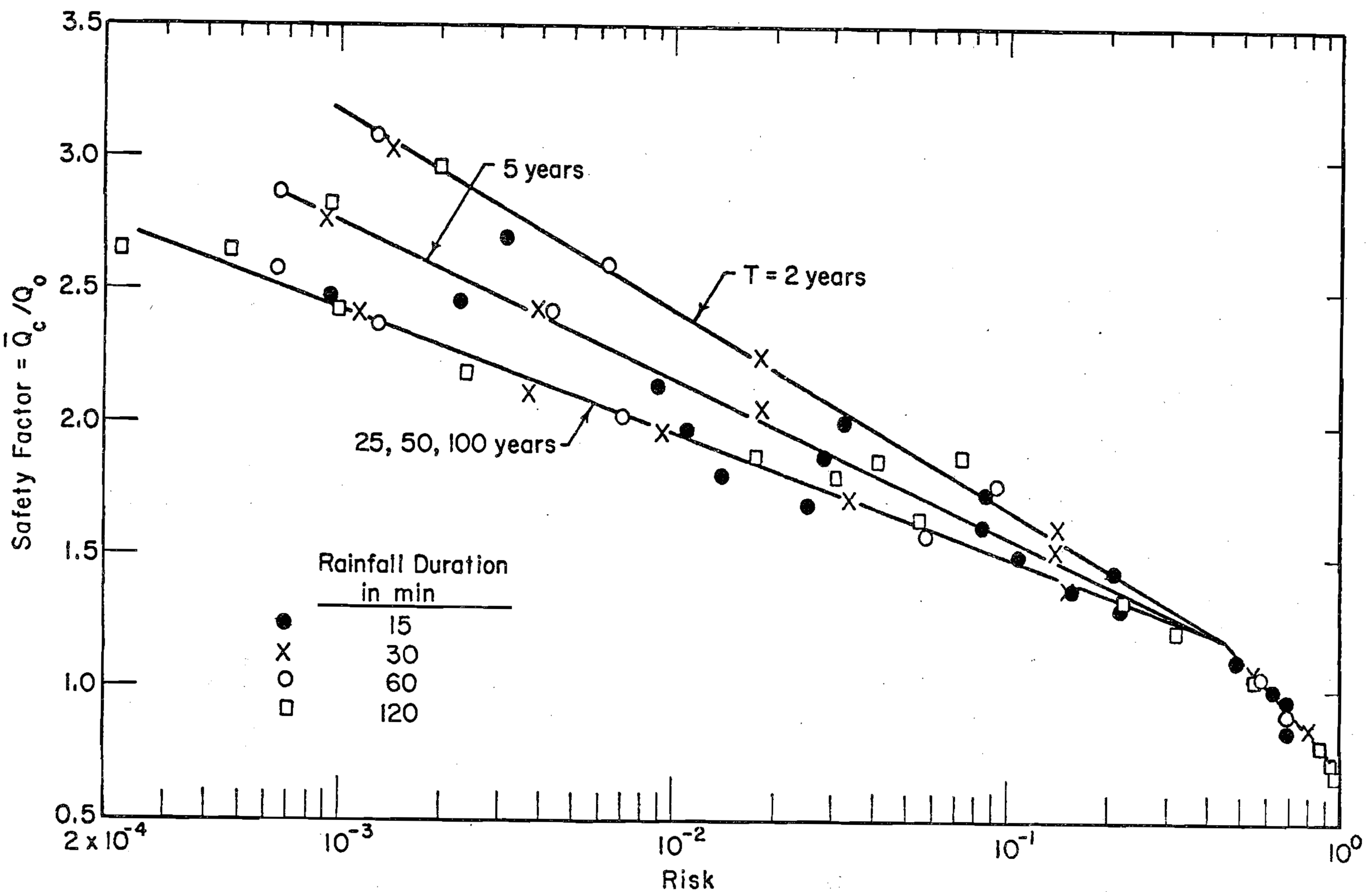


Fig. 5.2. Risk-Safety Factor Relationship for Sewer Design at Urbana, Illinois

is suggested that the repetitive computations involved be best done on a digital computer.

5.5. Use of Risk-Safety Factor Curves for Design

In using the risk-safety factor curves for design, the engineer no longer needs to determine the design return period arbitrarily. The control factor is the level of protection sought expressed as chance of failure, i.e., risk, for the expected life of the project. The return period becomes an intermediate reference parameter which is chosen to be equal to the expected service life of the sewer for convenience. This point is best illustrated by an example.

Suppose the size of the sewer is to be determined for the Urbana basin allowing a risk of failure of 2% for the 10-yr expected period of service of the sewer. Using the 10-yr curve, the required safety factor is 1.9. To determine Q_o from the rational formula as is conventionally done, it is necessary first to determine the duration which is assumed equal to the time of concentration. Various formulas and graphs have been proposed to estimate the time of concentration and all have severe limitations. Using arbitrarily the Kirpich formula (Chow, 1964, p. 14.7), which may not be valid for the condition considered, with the basin length equal to 1080 ft and average slope of 0.001, the time of concentration is evaluated as

$$t_d = 0.00013 \frac{1080^{0.77}}{0.001^{0.385}} = 0.40 \text{ hr}$$

Hence, from Eq. 5.22 with T = 10 yr and t_d = 24 min, i_o = 3.62 in./hr. With $\bar{C}$ = 0.825 and $\bar{A}$ = 10 ac, Q_o is computed using Eq. 5.14 as 29.9 cfs. Accordingly, $\bar{Q}_C$ = SF x Q_o = 1.9 x 29.9 = 56.9 cfs. The design discharge $Q_C = \bar{Q}_C/\lambda_m$ = 56.9/1.1 = 51.6 cfs. Assuming concrete pipe with roughness n = 0.016 due to slimming, the required diameter can be computed from

Eq. 5.18 as 1.91 ft. Thus a concrete pipe with a 2 ft nominal diameter is adopted.

Suppose the 2-ft sewer was laid and it was found later that the sewer could possibly be used for 50 yr instead of 10 yr. For the 50 yr expected life, the risk is higher than that of 2% of the original design, and can be calculated from

$$\frac{(SF)_a}{(SF)_b} = \frac{(\bar{Q}_C/Q_o)_a}{(\bar{Q}_C/Q_o)_b} = \frac{Q_{ob}}{Q_{oa}}$$

Thus, $(SF)_{50} = (SF)_{10}[(Q_o)_{10}/(Q_o)_{50}] = 1.9[(i_o)_{10}/(i_o)_{50}] = 1.43$

With SF = 1.43, from the 50-yr curve in Fig. 5.2, the risk is 0.12 for the 50-yr period.

Chapter 6. HYDRAULIC CONSIDERATIONS

Sewer flows produced by rainstorms vary rapidly with time, and they are subject to dynamic effects caused by the sewers and junctions. Precise mathematical simulation of such unsteady flows in a network is difficult and requires extensive computer capability. In this chapter a brief theoretical background is first presented and various routing models are reviewed. The routing methods selected for use in the design models are then discussed. These methods reflect a balance between accuracy of results and computer time required for least-cost sewer system designs.

6.1. Theoretical Considerations

Unsteady gravity flows in sewers can be represented mathematically by a pair of quasi-linear hyperbolic partial differential equations called the St. Venant equations:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \tag{6.1}$$

$$\frac{1}{gA}\frac{\partial Q}{\partial t} + \frac{1}{gA}\frac{\partial}{\partial x}\left(\beta \frac{Q^2}{A}\right) + \cos\theta \frac{\partial h}{\partial x} = S_o - S_f \tag{6.2}$$

in which Q is the discharge; t is time; x is the distance along the sewer; A and h are the cross sectional area and depth above the invert (measured normal to x) of the flow, respectively; θ is the angle between the sewer axis and a horizontal plane; $S_o = \sin\theta$ is the sewer slope; S_f is the friction slope; g is the gravitational acceleration; and β is the momentum flux correction factor which is often assumed equal to unity. Complicated as they appear, the St. Venant equations are not exact but provide a good approximate description of unsteady sewer flow (Yen, 1973b; 1975a). They can be solved numerically if one initial and two boundary conditions are specified. When the flow is supercritical, the two boundary conditions are furnished by the flow conditions at the upstream end of the sewer which can be obtained through computations for upstream sewers and junctions. When the sewer flow is subcritical, one boundary condition is given by an upstream flow condition whereas the second requires a flow condition at the downstream end of the sewer. However, at any instant of time this downstream condition is unknown since it depends on the flow conditions in the downstream manhole and the sewers connected to it. The effect of the downstream system on the flow in a sewer is called the backwater effect. One possible approach to this problem is by setting up the flow equations for all the sewers and junctions and solve them simultaneously. Such an approach is highly impractical because of the excessive computational requirements involved. Hence, alternate solution methods must be sought. The successive overlapping Y-segment scheme used in the ISS model (Sevuk et al., 1973) is one feasible approach. In that study a method for selections of sewer diameters in a network based solely on hydraulics has been developed. Further discussion on the application of the St. Venant equations to sewer flows and solution methods can be found in Yen (1973a) and Sevuk (1973) and is not repeated here. It can be concluded that with the present computer capabilities and numerical solution techniques, adoption of the complete St. Venant equations into any of the least-cost design models developed in this study is impractical. Various approximations of the St. Venant equations have been used as routing models. Each has advantages and disadvantages and its suitability depends on the specific application. A general discussion of these approximations is presented, followed by a detailed presentation of the routing analysis adopted in the design models. The diffusion wave approximation is not presented because it also requires two boundary conditions like the St. Venant equations.

6.2. Routing Methods

The St. Venant equations describe mathematically the propagation of the flood waves of the storm runoffs in sewers, and hence routing of the flow using these equations is sometimes referred to as dynamic wave routing. Because of the difficulties in solving the St. Venant equations various approximations of the equations have been proposed. From a hydraulics viewpoint, these approximations can be classified as shown in Fig. 6.1.

$$\frac{1}{gA}\frac{\partial Q}{\partial t} + \frac{1}{gA}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + \cos\theta\,\frac{\partial h}{\partial x} = S_o - S_f$$

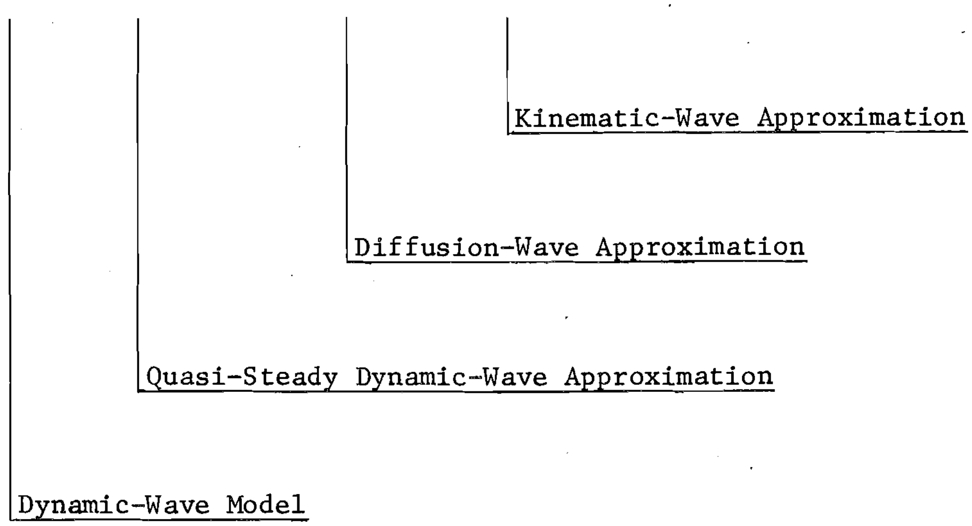


Fig. 6.1 HYDRAULIC ROUTING SCHEMES

6.2.1. Steady Flow Approximations

As shown in Fig. 6.1 the simplest among the different approximations is the kinematic wave approximation. Several versions exist within this category. The most elementary of these versions is expressed as

$$S_f - S_o = 0 \tag{6.3}$$

with S_f estimated by the Manning, Darcy-Weisbach, or similar formulas. No additional equation relating to conservation of mass is used. The result is a steady uniform flow approximation. The required sewer diameter can then be computed by using equations like Eqs. 3.1 or 3.6. This approximation has been termed steady flow methods in Section 3.1.1 and can be used with inputs consisting of either inlet catch basin hydrographs or merely inlet peak discharges. The peak inlet and upstream sewer discharges are simply added together giving no consideration to the differences in flow time in the sewers. This no-time lag version can also be applied to the case with inlet hydrographs as input instead of merely peak discharges. However, because of the negligence of possible different times of occurrence of peak discharges from different sewer lines and inlets, this steady-flow, no routing, no time-lag method tends to produce high peak flows as the computation proceeds downstream and hence results in over design.

A considerable improvement on the above version is to consider the lag time of the hydrographs due to the travel time in the sewers. Precise evaluation of the sewer travel time is a complicated matter and can only be achieved through using dynamic wave routing. However, a simple approximation can easily be obtained by shifting the sewer inflow hydrograph without any distortion by

$$t_f = L/V \tag{6.4}$$

in which L is the length of the sewer and V is a sewer flow velocity. The velocity V can be approximated by using the Manning formula assuming just-full gravity pipe flow

$$V = \frac{0.591}{n} d^{2/3} S_o^{1/2} \tag{6.5}$$

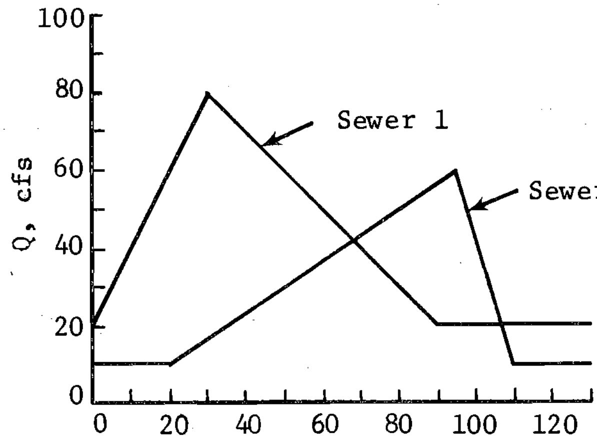
or the Darcy-Weisbach formula (Eq. 5.17), or by

$$V = \frac{4Q_p}{\pi d^2} \tag{6.6}$$

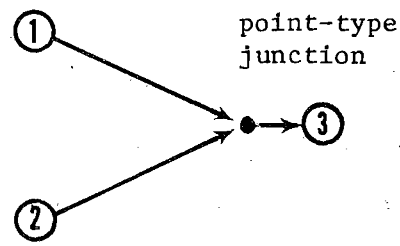
in which Q_p is the peak discharge. Use of Eq. 6.6 is preferred because it gives a smaller value of V and hence is closer to the average velocity than by Eq. 6.5. The sewer outflow is the shifted hydrograph, and these hydrographs for the sewers flowing into a junction or manhole are added linearly to the manhole direct inflow hydrograph using a common time scale according to the continuity relationship (Eq. 3.9) to produce the inflow hydrograph of the downstream sewer. An example showing the linear combination of the hydrographs of two inflowing sewers to produce the inflow hydrograph for the downstream sewer is shown in Fig. 6.2. A point-type junction with insignificant storage capacity is illustrated for simplicity. Since the sewer flow continuity equation is not considered in any form, in the hydrograph shifting as well as the no time lag version of the steady flow method the effect of sewer storage is completely ignored.

6.2.2. Linear Kinematic Wave Approximations

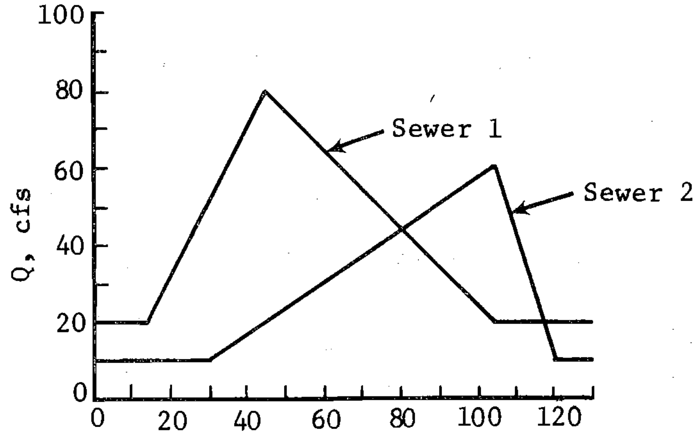
A somewhat more complex kinematic wave model utilizes a linear storage function, usually Eq. 3.2 or its variations. This is coupled with Eq. 6.3 to represent the sewer flow. The linear storage function is actually a linear approximation of the continuity equation (Eq. 6.1) and the methods using this approach can be termed as linear kinematic wave methods from a hydraulic viewpoint. Again the friction slope S_f in Eq. 6.3 is evaluated by using the Manning or Darcy-Weisbach formulas. Typical examples of linear kinematic wave routing are the Chicago Hydrograph, TRRL, and ILLUDAS methods discussed in Section 3.1. The consideration of sewer storage makes little improvement in the design results as compared to the



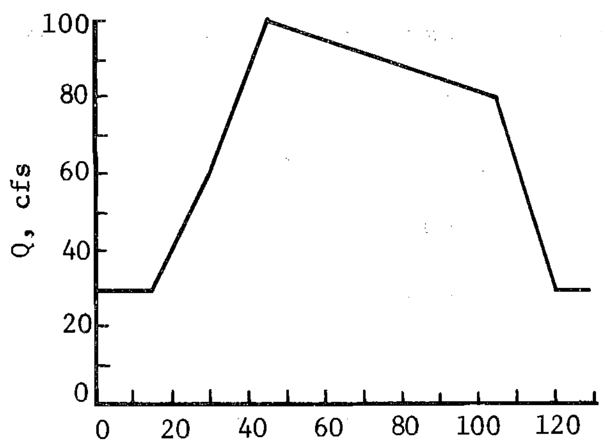
Time, min
Inflow hydrographs of sewers



Flow time in sewer 1 = 15 min
Flow time in sewer 2 = 10 min



Time, min
Outflow hydrographs of sewers



Time, min
Inflow hydrograph for sewer 3

Fig. 6.2. Shifting of Hydrographs for Steady Flow Time Lag Method

time-shifted hydrograph version of the steady flow method (Yen and Sevuk, 1975) because at design discharges the sewers are flowing nearly full.

6.2.3. Nonlinear Kinematic Wave Approximations

Basically, the nonlinear kinematic wave routing method utilizes Eq. 6.3 together with the unsteady flow continuity equation (Eqs. 6.1 or 3.12). The friction slope S_f in Eq. 6.3 is evaluated using the Manning formula

$$S_f = \frac{n^2 Q^2}{2.22 A^2} R^{-4/3} \qquad (6.7)$$

or the Darcy-Weisbach formula

$$S_f = \frac{f Q^2}{8 g R A^2} \qquad (6.8)$$

in which R is the hydraulic radius and A is the flow cross sectional area which is a function of R. The elimination of the inertial and pressure terms in the momentum equation (Fig. 6.1) eliminates one boundary condition requirement (namely, the downstream boundary condition) for numerical solution of the partial differential equation. This simplification permits the solution to proceed in the downstream direction sewer-by-sewer in sequence in a cascading manner. Thus, it considerably reduces the requirements for computer size and time as compared to the dynamic wave model. The inclusion of the unsteady flow continuity equation also accounts for sewer storage more realistically than does the linear kinematic approximation. However, elimination of the need of the downstream boundary condition also eliminates the mechanism to account for the downstream backwater effect for subcritical flows.

Even without serious backwater effects, the accuracy and applicability of the nonlinear kinematic wave approximation depends on the numerical procedure used for solution. Various finite difference numerical solution schemes have been proposed to solve the hyperbolic partial differential equations which include the nonlinear dynamic wave, quasi-steady dynamic wave, and kinematic wave approximations (Sevuk and Yen, 1973; Price, 1974; Liggett and Woolhiser, 1967). Sevuk and Yen (1973) have shown that first- and second-order method of characteristics and a four-point, noncentral, implicit scheme are superior to other finite difference schemes in solving unsteady open channel flow problems including flow in sewers. Fread (1974) also suggests the use of the four-point noncentral implicit scheme which permits independent large time and space increments, Δt and Δx, respectively, in the computations resulting in savings in computer time.

Theoretically, there is no wave attenuation for nonlinear kinematic models because the attenuation mechanism is eliminated by neglecting the inertial and pressure terms in the momentum equation. However, since some type of finite difference solution scheme is used, numerical errors are inevitably introduced. Such numerical attenuation often behaves in a manner similar to hydrodynamic attenuation, making the wave appear to be damped. Consequently, the use of a coarse grid creates the greatest apparent attenuation whereas a finer grid reduces the numerical error and hence the attenuation effect.

Several variations of the nonlinear kinematic wave routing method have been proposed. A modified scheme has been suggested in SWMM for sewer flow routing as discussed in Section 3.1. Cunge (1969) proposed a nonlinear kinematic wave method based on the Muskingum method, in which a traditional linear hydrologic storage routing method is used in channel routing. By referring to the time-space computational grid shown in

Fig. 3.2, the Muskingum routing formula can be written for the discharge at $x = (i+1)\Delta x$ and $t = (j+1)\Delta t$ as

$$Q_{i+1}^{j+1} = C_1 Q_i^j + C_2 Q_i^{j+1} + C_3 Q_{i+1}^j \tag{6.9}$$

in which

$$C_1 = \frac{KX + 0.5\Delta t}{K(1-X) + 0.5\Delta t}$$

$$C_2 = \frac{0.5\Delta t - KX}{K(1-X) + 0.5\Delta t} \tag{6.10}$$

$$C_3 = \frac{K(1-X) - 0.5\Delta t}{K(1-X) + 0.5\Delta t}$$

where K is termed as the storage constant having a dimension of time and X is a factor expressing the relative importance of inflow. Cunge showed that by taking K and Δt as constants, Eq. 6.9 is an approximate solution of the nonlinear kinematic wave equations (Eqs. 6.1 and 6.3). He further demonstrated that Eq. 6.9 can be considered as an approximate solution of a modified diffusion equation if

$$K = \frac{\Delta x}{c} \tag{6.11}$$

and

$$X = \frac{1}{2} - \frac{\varepsilon}{c\Delta x} \tag{6.12}$$

in which ε is a "diffusion" coefficient and c is the celerity of the flood peak which can be approximated as the length of the reach divided by the flood peak travel time through the reach. Assuming $K = \Delta t$ and denoting $\alpha = 1-2X$, Eq. 6.9 can be rewritten as

$$Q_{i+1}^{j+1} = \frac{2-\alpha}{2+\alpha} Q_i^j + \frac{\alpha}{2+\alpha} Q_i^{j+1} + \frac{\alpha}{2+\alpha} Q_{i+1}^j \tag{6.13}$$

In the traditional Muskingum method X and consequently α is regarded as constant. In the Muskingum method as modified by Cunge, α is allowed to vary according to the channel geometry and is computed as

$$\alpha = \frac{KQ}{S_o (\Delta x)^2 B} \tag{6.14}$$

in which B is the surface width of the flow and S_o is the sewer slope. The values of α are restricted between 0 and 1 so that C_1, C_2, and C_3 in Eq. 6.10 will not be negative.

The Muskingum-Cunge method offers two advantages over the standard nonlinear kinematic wave methods. First, the solution is obtained through a linear algebraic equation (Eq. 6.9 or Eqs. 6.13 and 6.14) instead of a partial differential equation, permitting the entire hydrograph to be obtained at successive cross sections instead of solving for the flow over the entire length of the sewer pipe for each time step as for the standard nonlinear kinematic wave method. Second, because of the use of Eq. 6.14, a limited degree of wave attenuation is included, permitting a more flexible choice of the time and space increments for the computations as compared to the standard kinematic wave method.

6.3. Selection of Routing Methods

As discussed in the preceding two sections, the computer requirements for the dynamic wave (St. Venant equations) and diffusion wave routing methods make them unsuitable for incorporation into the least-cost sewer system design models. Among the other approximate methods of routing, theoretically the nonlinear kinematic wave methods are the most accurate and sophisticated. From the view point of flow simulation for existing sewer systems, they are clearly superior to the steady-flow and linear kinematic wave approximations. However, from the view point of sewer design and because of the discrete sizes of commercial pipes, it is possible that the linear kinematic wave and steady flow routing methods may produce similar designs with less computer requirements than the nonlinear kinematic wave method. Since the relative merits of these simpler routing approximations have not been investigated when incorporated into least-cost design, they are investigated in this study. Specifically, four routing methods are considered; namely, the no time lag steady flow routing method, the hydrograph time lag method, the standard nonlinear kinematic wave method, and the Muskingum-Cunge method.

6.3.1. No Time Lag Steady Flow Method

In the no time lag version of the steady flow method the peak discharges of the joining sewers and the direct surface inflow at the manholes are simply added to give the design discharge for the following sewers. For instance, if the peak discharges of the two upstream sewers flowing into the manhole are Q_{p1} and Q_{p2}, respectively, and Q_j is the direct manhole inflow rate, then the design discharge for the downstream sewer outflowing from the manhole, Q_{p3}, is

$$Q_{p3} = Q_{p1} + Q_{p2} + Q_j \qquad (6.15)$$

As discussed in Section 6.2.1, this method is the simplest but the least accurate. Neither the wave translation time nor the wave attenuation is considered. It tends to over-design the downstream sewers and is probably unsuitable for use in practice except for very small systems. However, this method is included in the present study because of its simplicity and because it provides a simple means to illustrate how risk components can be incorporated into the least-cost design of sewer systems.

6.3.2. Hydrograph Time Lag Method

The hydrograph time lag version of the steady flow routing method has been discussed in detail in Section 6.2.1. The inflow hydrograph of a sewer is shifted without distortion by the sewer flow time t_f estimated by Eq. 6.4 to produce the sewer outflow hydrograph. The sewer flow velocity is approximated by Eq. 6.6. The outflow hydrographs of the upstream sewers at a manhole are added linearly at the corresponding times to the direct manhole inflow hydrograph to produce the inflow hydrograph for the downstream sewer as specified in Eq. 3.9 and shown in Fig. 6.2. The diameter of the sewer can then be computed using Eq. 3.1 with Q being the peak discharge of the inflow hydrograph.

Theoretically, shifting of hydrographs accounts for approximately the sewer flow translation time but offers no wave attenuation. However, because a constant time increment is used in the numerical specifications of the hydrographs in the computer program, the peak flow may not occur at an even multiple of Δt. Therefore, through linear interpolation within any Δt, a numerical attenuation may be introduced.

This method is simple, has limited computer requirements and yet provides results which are greatly improved over the no time lag version discussed in Section 6.3.1. Yen and Sevuk (1975) has shown that using this method resulted in a sewer system design which was very similar to that obtained through the more sophisticated nonlinear kinematic wave method.

6.3.3. Nonlinear Kinematic Wave Method

The theoretical background of the nonlinear kinematic wave method for sewer flow routing has been briefly presented in Sections 6.1 and 6.2.3. The basic equations used in the method and adopted in this study are

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \qquad (6.1)$$

and

$$Q = \frac{1.49}{n} S_o^{1/2} AR^{2/3} \tag{6.16}$$

in which the flow cross sectional area A and hydraulic radius R are both functions of the flow depth h. The initial condition for a sewer is defined by the initial base flow from which the flow depth and consequently A and R can be computed by using Manning's formula (Eq. 6.16) together with the geometric equations shown in Fig. 3.1. The upstream boundary condition for each sewer is specified by the inflow hydrograph of the sewer, from which A and R can be computed again using Eq. 6.16. The junction condition is the continuity relationship, Eq. 3.9.

Equations 6.1 and 6.16 are solved numerically using a four-point, noncentral, implicit finite difference scheme, proceeding sewer by sewer in the downstream direction. Within each sewer the flow for the entire pipe for a given time is determined before proceeding to the next time step. Noting that

$$B(h) = \frac{\partial A}{\partial h} \tag{6.17}$$

$$G(h) = \frac{\partial Q}{\partial h} \tag{6.18}$$

Eq. 6.1 can be rewritten as

$$B(h) \frac{\partial h}{\partial t} + G(h) \frac{\partial h}{\partial x} = 0 \tag{6.19}$$

which is the continuity equation expressed in a different form than Eq. 3.12. For partially filled circular pipes,

$$B(h) = d \sin[\cos^{-1}(1 - \frac{2h}{d})] \tag{6.20}$$

and using Manning's formula (Eq. 6.16)

$$G(h) = \frac{1.49}{n} S_o^{1/2} R^{2/3} \frac{B}{3}[5 + \frac{1}{\sin^2 \frac{\Phi}{2}} (\frac{\sin \Phi}{\Phi} - 1)]$$

$$= \frac{0.196}{n} S_o^{1/2} d^{5/3}(1 - \frac{\sin \Phi}{\Phi})^{2/3}[5 \sin \frac{\Phi}{2} + \frac{1}{\sin \frac{\Phi}{2}}(\frac{\sin \Phi}{\Phi} - 1)] \quad (6.21)$$

in which the central angle (Fig. 3.1)

$$\Phi = 2 \cos^{-1}(1 - \frac{2h}{d}) \quad (6.22)$$

By referring to the computational grid shown in Fig. 3.2, the partial derivatives in Eq. 6.19 are approximated by forward difference quotients as (Sevuk and Yen, 1973)

$$\frac{\partial h}{\partial t} = \frac{1}{2\Delta t}(h_{i,j+1} + h_{i+1,j+1} - h_{i,j} - h_{i+1,j}) \quad (6.23)$$

$$\frac{\partial h}{\partial x} = \frac{1}{\Delta x}(h_{i+1,j+1} - h_{i,j+1}) \quad (6.24)$$

The partial derivatives of the flow cross sectional area and discharge in Eqs. 6.17 and 6.18, respectively, are approximated by

$$B = 0.5(B_{i,j+1} + B_{i+1,j+1}) \quad (6.25)$$

and

$$G = 0.5(G_{i,j+1} + G_{i+1,j+1}) \quad (6.26)$$

Substitution of Eqs. 6.23 through 6.26 into Eq. 6.19 yields the implicit four-point forward difference equation

$$\frac{1}{2\Delta t}[(B_{i,j+1} + B_{i+1,j+1})(h_{i,j+1} + h_{i+1,j+1} - h_{i,j} - h_{i+1,j})]$$

$$+ \frac{1}{\Delta x}[(G_{i,j+1} + G_{i+1,j+1})(h_{i+1,j+1} - h_{i,j+1})] = 0 \qquad (6.27)$$

This equation is nonlinear only with respect to the unknown flow depth $h_{i+1,j+1}$ since $B_{i+1,j+1}$ and $G_{i+1,j+1}$ can both be expressed in terms of the depth (Eqs. 6.19 and 6.20), and it can readily be solved by using Newton's iteration method.

6.3.4. Muskingum-Cunge Method

As discussed in Section 6.2.3, the Muskingum-Cunge method (Eqs. 6.13 and 6.14) yields a solution of the nonlinear kinematic wave equation. It can also be considered as an approximate solution of a modified diffusion equation. The routing is done through solving an algebraic equation (Eq. 6.13) instead of a partial differential (or finite difference) equation. The coefficient α in Eq. 6.13 is computed by using Eq. 6.14 for each time and space point of computation since the flow width B and constant K both change with respect to time and space. The values of K are computed by using Eq. 6.11 with the celerity c evaluated by

$$c = \frac{\partial Q}{\partial A} \qquad (6.28a)$$

or for a partially filled pipe using Manning's formula

$$c = \frac{0.196}{n} S_o^{1/2} d^{2/3}(1 - \frac{\sin \Phi}{\Phi})(5 - 2 \frac{1 - \frac{\sin \Phi}{\Phi}}{1 - \cos \Phi}) \qquad (6.28b)$$

The initial flow condition is computed from the specified base flow as in the case of nonlinear kinematic wave method. The sewer system inflows are defined by the inflow hydrographs as for the upstream boundary condition of the nonlinear kinematic wave method. With the discharge known the flow depth and other geometric parameters can be computed from the geometric equations given in Fig. 3.1. The junction condition used is again the continuity relationship, Eq. 3.9.

In applying the Muskingum-Cunge method to a sewer system, the solution is obtained over the entire time period at a flow cross section before proceeding to the next cross section. The solution then proceeds downstream section by section, and then sewer by sewer, in a cascading sequence. In solving for the hydrograph at a cross section of a sewer, the computational procedure is described as follows. A computation time increment Δt_1 is determined as the roundoff value equal to or less than $\Delta x/c_1$ where c_1 is computed by using Eq. 6.28b with Φ corresponding to the depth equal to 0.6d. This Δt_1 is constant for a cross section but can vary from section to section. The hydrograph at the immediate upstream station, cross section $i\Delta x$, which has been stored at every Δt_2 time increment, is now parabolically interpolated to every Δt_1 and stored for future computations. In using Eq. 6.13 to compute the discharge Q_{j+1}^{i+1} for cross section $(i+1)\Delta x$ at specified time t_{j+1}, a reference time K is first computed by using Eq. 6.11 with the celerity c evaluated by Eq. 6.28b and Φ corresponding to the flow at the time $t_{j+1} - \Delta t_1$ at the same cross section. Subsequently α can be computed using the values of B and Q at the same time $t_{j+1} - \Delta t_1$ at the cross section. The value of Q_{i+1}^{j} for the current section $(i+1)\Delta x$ is obtained through linear interpolation for the time $t_{j+1} - K$ from the part of the hydrograph determined at previous times, whose ordinates have been stored in the computer at a time interval Δt_1. The values of Q_i^j and Q_i^{j+1} are linearly interpolated for the times $t_{j+1} - K$ and t_{j+1} from the hydrograph of the immediate upstream section for which discharge values have been stored at a time interval Δt_1. This computation of Q_{i+1}^{j+1} is repeated for the time increment of Δt_1 until the entire hydrograph for the cross section is obtained. The computed hydrograph with discharge values at Δt_1 apart are then parabolically interpolated to yield values at Δt_2 apart and stored. Obviously, the computational accuracy can be improved if α is computed as the average values of α_i^j, α_i^{j+1} and α_{i+1}^j instead of merely the last. However, such averaging would considerably increase the computation time and is not adopted at the present stage for this study.

Chapter 7. DEVELOPMENT OF DESIGN MODELS

In previous chapters detailed descriptions have been given of the optimization technique, the risk and uncertainty considerations, and the various hydraulic models. The purpose of this chapter is to bring together these concepts to develop the various least-cost sewer system design models that are listed in Table 7.1. The serial DDDP technique is the basic foundation for each of these design models incorporating various types of routing procedures with and without considering the risks. This chapter presents each of these models and discusses the interaction of the optimization component with the hydraulic and/or risk components. A detailed discussion of the application of each model through examples is presented in Chapter 8.

TABLE 7.1. Least-Cost Sewer System Design Models

Model Designation	Routing Procedure Used	Risk Analysis Incorporated
A	None	No
B-1	Hydrograph Time Lag	No
B-2	Kinematic-Wave	No
B-3	Muskingum-Cunge	No
C	None	Yes
D	Hydrograph Time Lag	Yes

7.1. Design Models Without Considering Risks

7.1.1. Model A - No Routing

The simplest design model is Model A which is essentially the serial DDDP procedure discussed in Sections 4.2 and 4.4. The hydraulic component of the model is the no time lag steady flow approach which simply consists of using Manning's formula for full-pipe flow (Eq. 3.1) to select the smallest commercial pipe diameter satisfying the constraints on flow, velocity, and preceding (immediately upstream) sewer diameters, etc. (given in Chapter 2). This size selection takes place for each feasible set of input states (upstream crown elevations) and output states (downstream crown elevations) as shown in Fig. 4.2. The sewer slope in Manning's formula is simply computed as the difference between the upstream and downstream crown elevations divided by the pipe length, i.e., for the connection of manholes m_n and m_{n+1} the slope, S_o, is

$$S_o = [S_{m_n,m_{n+1}} - \tilde{S}_{m_n,m_{n+1}}]/L \qquad (7.1)$$

The peak inflow for each sewer is computed as the sum of all inflows for connecting upstream sewers plus the direct inflow for the manhole m_n at the upstream end of the sewer being considered.

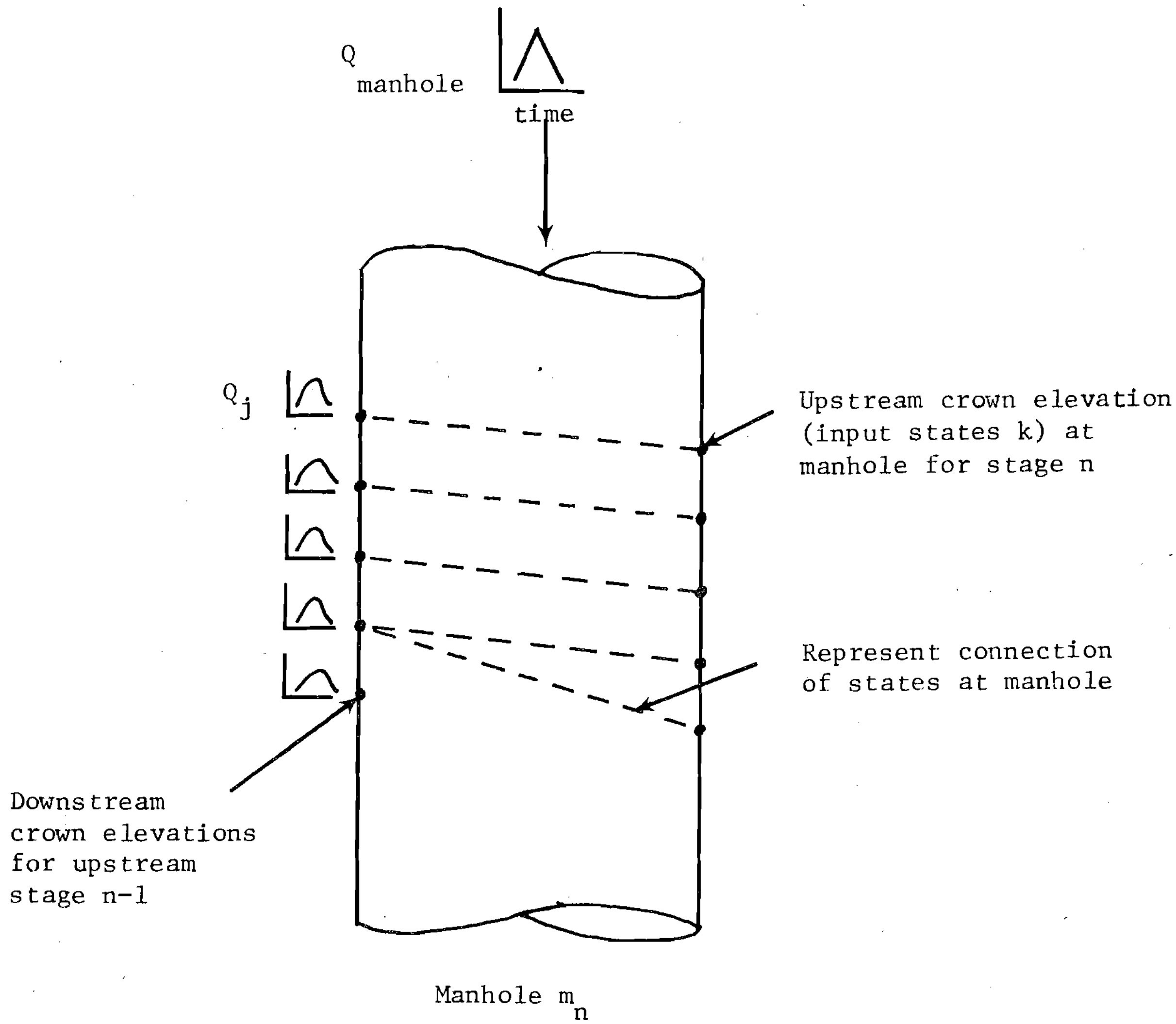
The input parameters for optimization consist of the initial trial trajectory, the number of lattice points N_p and the initial state increment Δ_s to set up the initial corridor, the reduction rate of Δ_s for successive iterations, and the acceptable error for the total system cost which determines the minimum allowable state space increment $\Delta_{s_{min}}$. Selection of these input parameters will be discussed in detail in Chapter 8. The input for the hydraulics component includes Manning's roughness factor n, allowable maximum flow velocity, and peak design inflow rates at each manhole. Other required input includes the topography, network description, sewer lengths, and allowable minimum soil cover above the crown of sewers. The flow charts for this model can be seen in Figs. 4.2, 4.3 and 4.8, and the computer program is listed in Appendix D.

7.1.2. Model B - Incorporation of Routing Techniques

The inclusion of hydrograph routing in the model permits advantage to be taken of peak attenuation and the time shift of peak pipe flows with respect to peak inlet flows. This results in more complex models as explained below.

The DDDP computations of the serial approach at each stage of the models with routing are essentially the same as described in Chapter 4 with a few modifications for each routing technique. When a routing procedure is incorporated into the optimization, there is an input hydrograph associated with each input state k (upstream crown elevation) for each pipe connection at a stage. For a sewer having no other pipes connected to its upstream manhole the inflow hydrograph associated with each state at the upstream manhole m_n is simply the inlet hydrograph of that manhole. For a sewer having other pipes connected to its upstream manhole the procedure is more complicated. Assuming the DDDP computations for a sewer network have been performed through stage n-1 and the connectivity of states at manholes on isonodal line (INL) n (Fig. 4.9) have been determined, the next step before continuing the DDDP computations for a pipe connection at stage n is to determine the inflow hydrographs for each input state at each manhole on INL n. This is accomplished for each state at a manhole m_n by adding the inflow hydrographs associated with the connecting upstream state at the manhole (i.e., with the downstream crown elevation at m_n for upstream stage n-1). This procedure is shown schematically in Fig. 7.1 and is performed for each input state at the upstream manhole.

The peak inflow for each input state is determined from the corresponding inflow hydrographs for the state. These peak inflows for each of the input states are then used in the DDDP computation as the design inflows. As described for Model A, Manning's formula for full pipe flow (Eq. 3.1) is used to select the smallest commercial pipe diameter which can handle the peak inflow associated with each input state and satisfies the design constraints.



Inflow hydrograph for each input state k to stage n

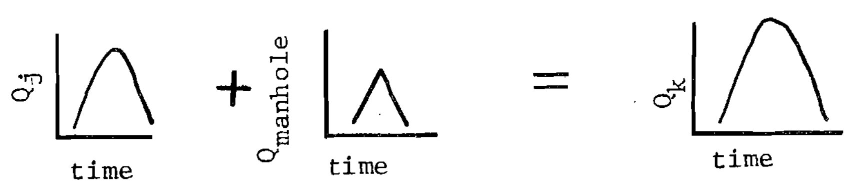


Fig. 7.1. Hydrographs for States at Manholes

A flow chart for each iteration of the DDDP solution scheme when a routing procedure is included is given in Fig. 7.2. The overall optimization procedure is the same as shown in Fig. 4.3. The DP computations within each corridor are the same as shown in Fig. 4.2.

Once the DP computations have been performed within a corridor, representing a pipe connection of manholes m_n and m_{n+1} at the stage, the minimum cumulative cost connection of upstream states to each downstream state in the corridor and the diameters of each connection are known. The next step is to perform the routing computations for the minimum cumulative cost connection to each output state j at the downstream end of the stage within the corridor. The respective inflow hydrograph associated with the input state that has a connection to the output state j is routed through the pipe. Once the routing computations are performed for each connection in the corridor, there exists an outflow (downstream) hydrograph for each output state in the corridor. If this is the only pipe that is connected to the downstream manhole m_{n+1}, its outflow hydrograph is added to the direct inflow hydrographs for manhole m_{n+1} for each output state. The resulting hydrograph serves as the inflow hydrograph for the downstream connecting pipe from manhole m_{n+1} (Fig. 7.1). If there are other upstream pipes connected to manhole m_{n+1}, each outflow hydrograph for each connection to an output state plus the direct inflow hydrograph for manhole m_{n+1} are added. This procedure is repeated for each output state. The new hydrographs for the output states serve as the respective inflow hydrographs for the downstream connecting pipe for those states. The particular hydrograph for an input state of a pipe for the next downstream stage n+1 connecting manhole m_{n+1} at its upstream are determined by the connection of states across the manhole as shown in Fig. 4.9. The computer program for design Models B-1, B-2, and B-3 is listed in Appendix E.

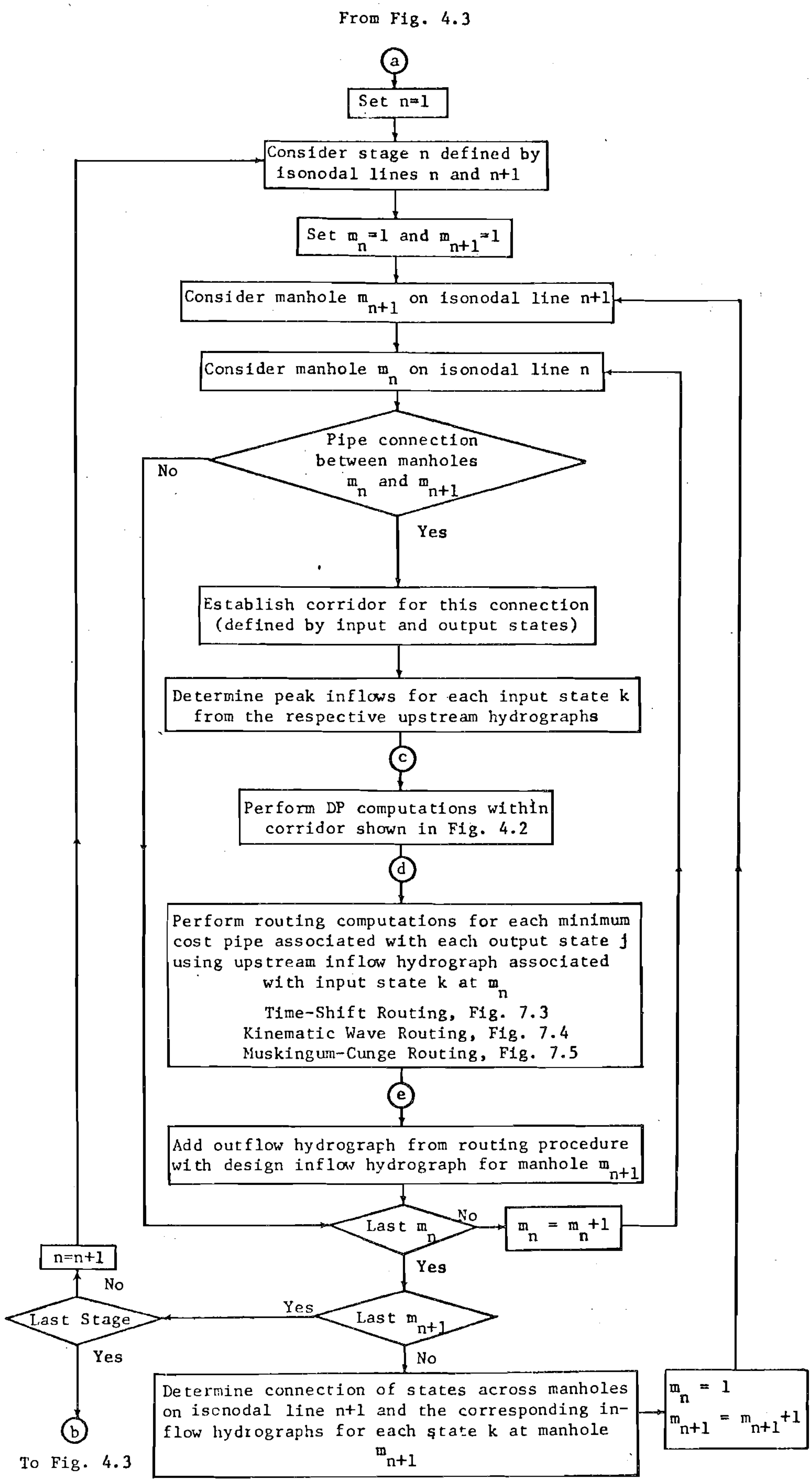


Fig. 7.2. Flow Chart of Serial DDDP Solution Scheme for Each Iteration With Routing

7.1.2.1. Model B-1: Hydrograph Time Lag Routing - The simplest of the three hydrograph routing techniques utilized is hydrograph time lagging discussed in Section 6.3.2. The design model using this routing technique is referred to as Model B-1. Yen and Sevuk (1975) have shown that this routing technique provides rather good results when a high level of accuracy is not required. The flow chart of this routing procedure is given in Fig. 7.3. The DDDP solution scheme for each iteration is the same as that described above and shown in Fig. 7.2 and the overall scheme in Fig. 4.3.

The required input data for Model B-1 includes the optimization model input parameters, the design parameters needed for Model A, and in addition, a design direct inflow hydrograph for each manhole in the sewer network. It should be pointed out that it is not necessary to have an inflow hydrograph at each manhole. The inputs for this design model are further discussed in Chapter 8.

7.1.2.2. Model B-2: Kinematic Wave Routing - A more sophisticated routing technique that is used to formulate design Model B-2 is the nonlinear kinematic wave method which has been discussed in Section 6.3.3. Because of the simplifications made in the development of the procedure, no downstream flow conditions are required, and consequently downstream backwater effects are not accounted for. Obtaining a downstream flow condition for any routing procedure in conjunction with the optimization procedure is impossible because in the optimization procedure the sewer pipes for the downstream stages have not been designed. Consequently, it would be impossible to account for any mutual backwater effects caused at the downstream end of a sewer. The four-point noncentral implicit finite-difference scheme discussed in Section 6.3.3 is adopted for the numerical computations. The DDDP solution scheme for each iteration is the same as that shown in Fig. 7.2. A simplified flow chart for the kinematic wave method is given in Fig. 7.4. The friction slope, S_f, is

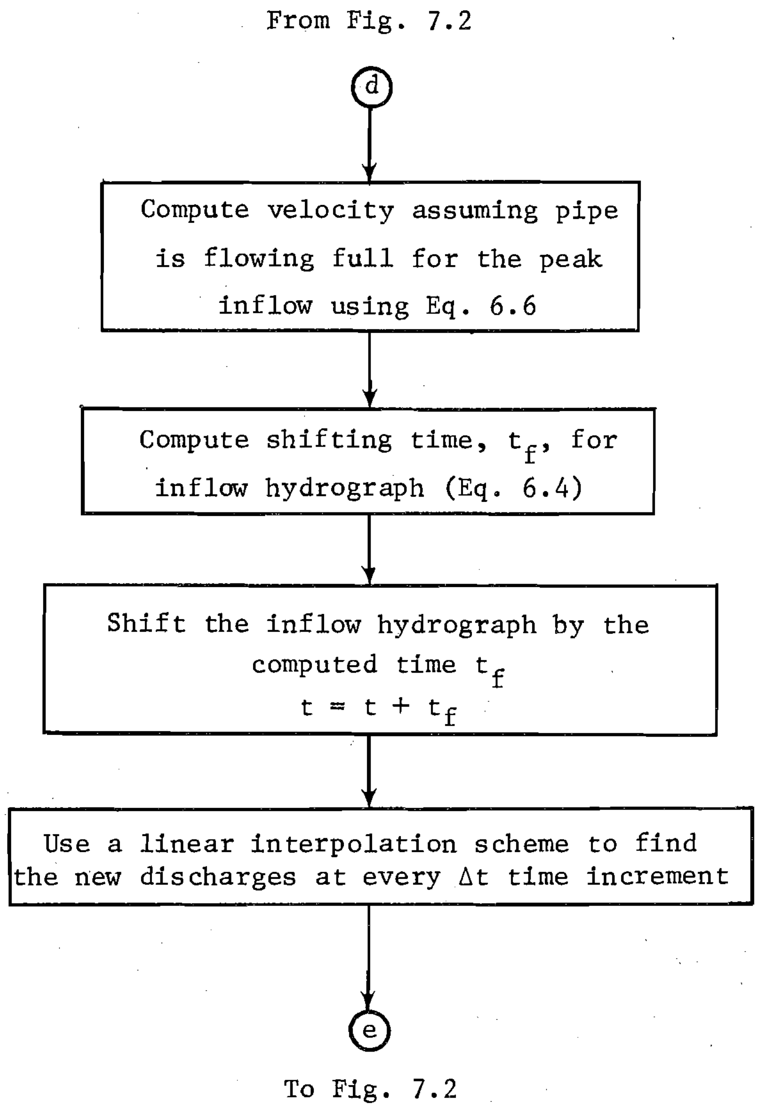
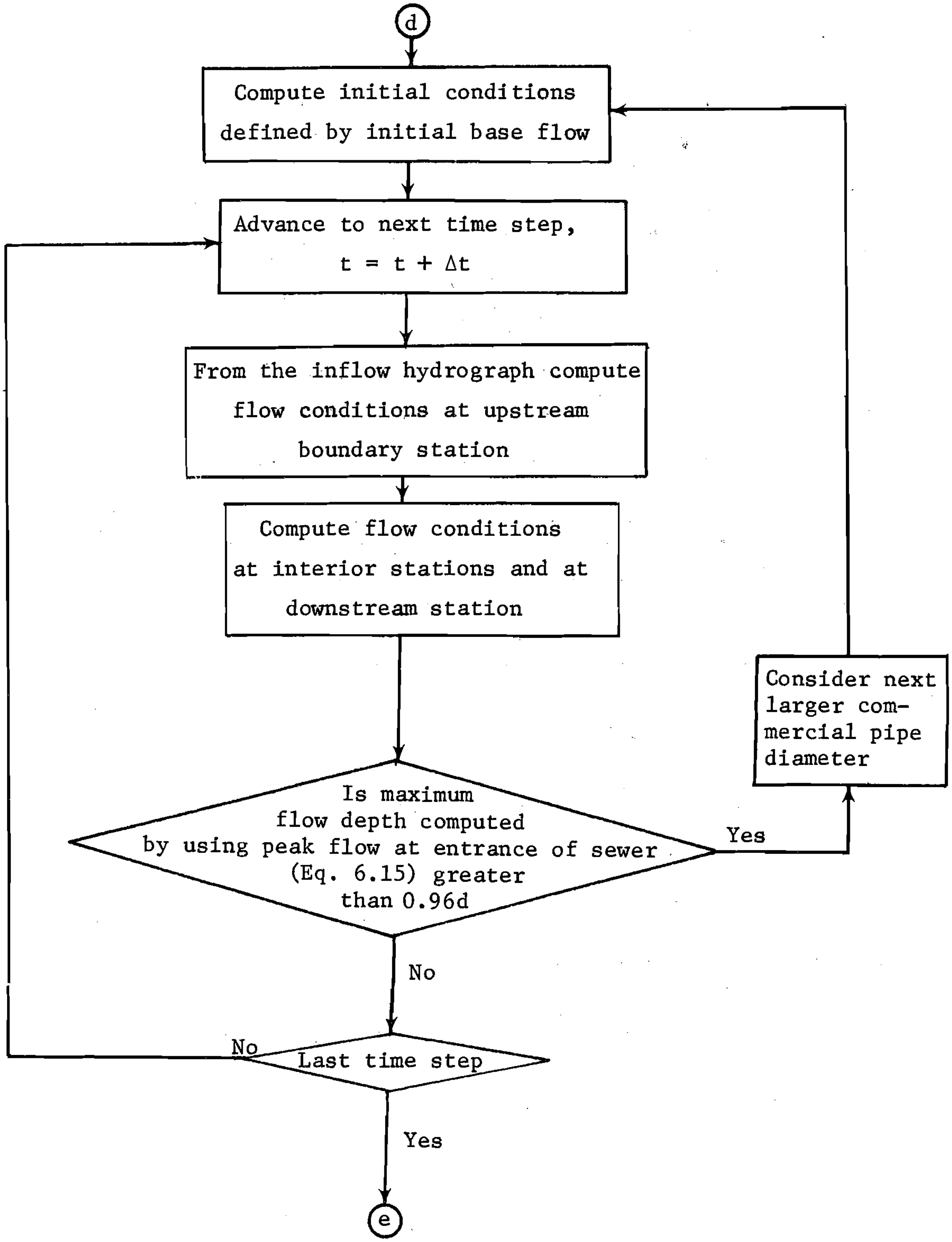


Fig. 7.3. Flow Chart for Hydrograph Time Lag Routing

From Fig. 7.2



To Fig. 7.2

Fig. 7.4. Flow Chart for Nonlinear Kinematic-Wave Routing

evaluated using the Manning formula (Eq. 6.7). In determining the pipe diameter, to avoid computational instability when the pipe is flowing nearly full, and also in view of the fact that maximum pipe flow occurs at about 0.94d, the maximum pipe flow depth is arbitrarily chosen as 0.96d instead of d.

The inputs for this design model include in addition to those for Model B-1 a specified time increment, Δt, and an allowable maximum distance increment Δx_{max}. For example, if Δx_{max} is set at 400 ft and the pipe is 1000 ft long then it is divided into three equivalent sections, each 333 ft long. There would be four grid points along the spatial axis of the finite difference grid.

7.1.2.3. Model B-3: Muskingum-Cunge Routing - The Muskingum-Cunge routing technique is used in conjunction with the optimization technique to formulate design Model B-3. The technique has been discussed in Section 6.3.4. Because this is a relatively new routing technique, a somewhat more detailed flowchart of the computational procedure is given in Fig. 7.5. This procedure is incorporated into the optimization component in a manner similar to that for the previous two routing procedures. The input data for this model is similar to that of the nonlinear kinematic wave approximation except that a Δt for the computational procedure is not required. As for the nonlinear kinematic wave approximation, the user must specify the allowable maximum length of pipe section, Δx_{max}, that is to be used in the routing of flow through each pipe. However the pipe sections are treated in a different context than in the kinematic wave procedure. For the Muskingum-Cunge routing, the entire inflow hydrograph is routed through the first upstream section, then the outflow from that section is taken as the inflow to the next downstream section of the pipe. This procedure continues in a downstream manner for each section of the pipe until the last section of

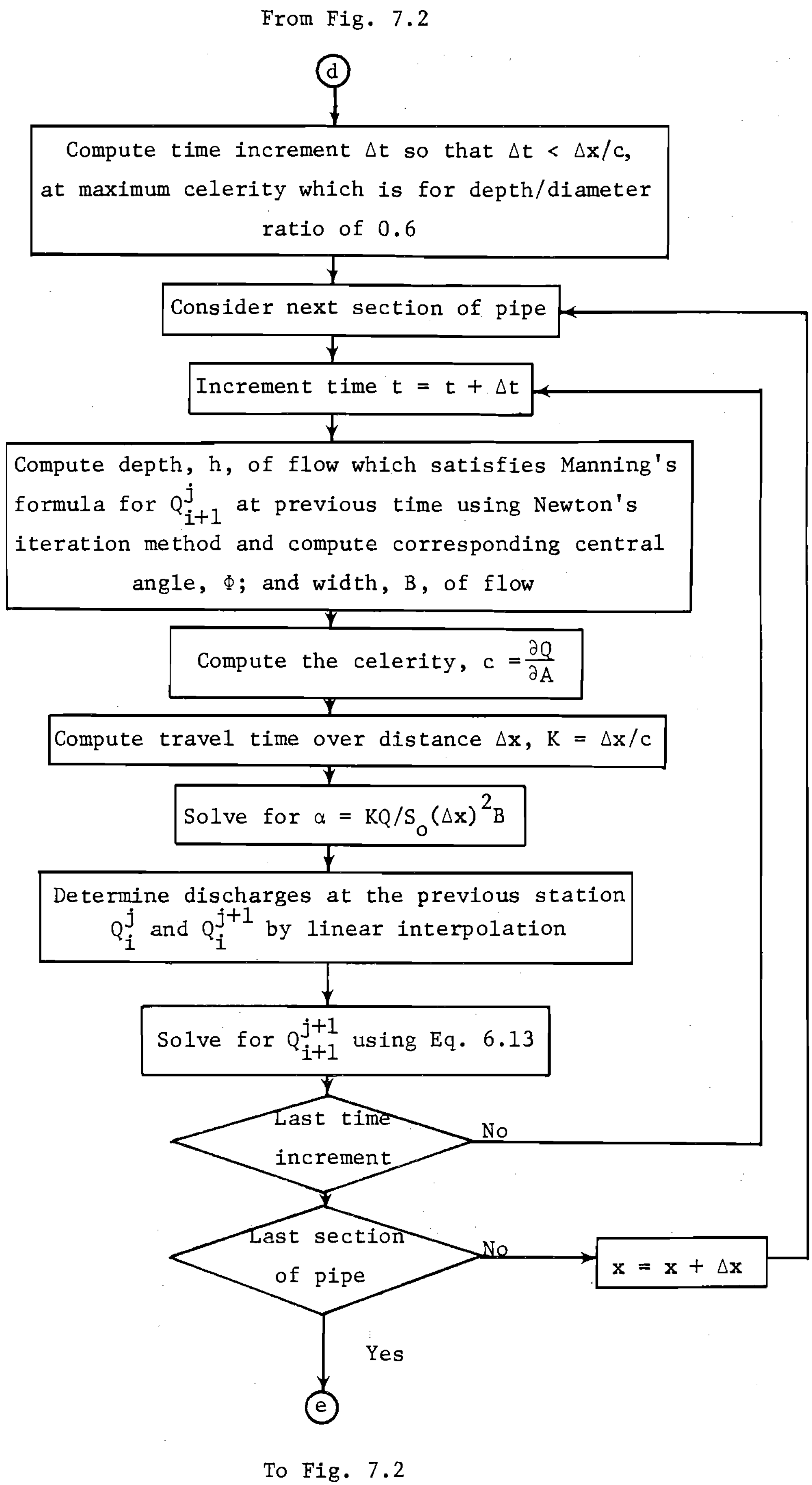


Fig. 7.5. Flow Chart for Muskingum-Cunge Routing Technique

pipe is reached. The DDDP solution scheme including this routing procedure is identical to those shown in Figs. 7.2, 4.2, and 4.3.

7.2 Design Models Incorporating Risks

A detailed description of the concepts required in considering risks and uncertainties in sewer design and the development of risk-safety factor relationships used in the risk model has been given in Chapter 5. The purpose of this section is to illustrate how the risk component is used in conjunction with the optimization. The least-cost design model without routing is referred to as Model C and is described in Section 7.2.2. Such a risk-based design model accounts for the cost interactions of the various components of sewer systems in order to maintain a tradeoff between the costs of installing the system and potential flood damages. The assessment of expected damage cost is discussed in Section 7.2.1. In addition the procedure systematically accounts for the uncertainties that cannot be avoided in sewer design. This risk-based design model is then extended to include the hydrograph time lag routing component to account for the time shifting of hydrographs. This design model is referred to as Model D and is described in Section 7.2.3.

7.2.1. Expected Damage Costs

The risk analysis in Chapter 5 provides an estimate of the probability of occurrence of the surface runoff exceeding the capacity of a pipe system during the expected service life of the project, i.e. the probability of "failure." In order to incorporate risks into a design model based on an optimization technique, the cost associated with failure of the sewer must be evaluated. This can then be added to the installation cost and the total cost minimized.

The evaluation of damage due to storm water flooding in an urban area is not an easy task. In order to provide a simple mechanism for evaluating expected damage costs (sometimes called risk damage costs), an "assessed damage value" is introduced. This is defined as the damage value associated with the area drained by a specific sewer in the event that its capacity is exceeded. The expected damage cost, C_D, is then computed as the product of the risk and the assessed damage value for the sewer, i.e., for the sewer connecting manholes m_n and m_{n+1},

$$C_D(S_{m_n,m_{n+1}}, D_{m_n,m_{n+1}}, d) = (C_F)_{m_n,m_{n+1}} \cdot [P(Q_L > Q_C)] \qquad (7.2)$$

in which $(C_F)_{m_n,m_{n+1}}$ is the assessed damage value in the event of $P(Q_L > Q_C)$ due to insufficient capacity of the sewer of diameter d. The assessed damage value is assumed to be the average damage weighted over all possible magnitudes of the flooding as well as the time of occurrences of flooding during the project service life. It is introduced as a first attempt at incorporating flood damages in a least-cost design model so that the effect of this aspect of design, which has previously been ignored, can be demonstrated, which is done in Chapter 8.

The actual determination of an assessed damage value involves considerable judgement. Some data on urban flood damage are available and can serve as guidelines (Grigg et al., 1974, 1975; Homan and Waybur, 1960). It is emphasized that this approach of accounting for potential flood damages in the design model is only a first step. A second phase of this study, OWRT project B-098-ILL, is directed in part at the development of an improved procedure for incorporating risks into the design model.

7.2.2. Model C - Risk Component Without Routing

The risk-based least-cost design model without routing has been presented by Tang, Mays, and Yen (1975) using the nonserial DDDP approach. The corresponding design model using the serial approach is very similar with modification of the recursive equations. The DDDP solution scheme is essentially that shown in Fig. 4.3 with the DP computations shown in Fig.

7.6. The major difference in the DP computations with and without the risk component lies in the determination of sewer sizes. With the risk component, for each diameter considered for a feasible set of states there is an installation cost together with a corresponding expected flood damage cost. The sum of these costs is the total expected cost. The installation cost includes the costs of the pipe and the connecting upstream manhole. The cost which is minimized in the DDDP procedure is the sum of the installation cost, C_I, and the expected damage cost, C_D, due to insufficient sewer capacity.

The decision variables include the drop in crown elevation across the pipe and the diameter of the pipe. For each possible (feasible) drop in crown elevation, a diameter of the pipe is selected which provides the minimum total expected cost for the particular drop being considered. In general, the cost of installing a sewer pipe of a specified material depends on the pipe size and depth of excavation, i.e., in terms of the optimization variable, $C_I = C_I(S,D,d)$. The amount of flood damages is related to the capacity of the pipe or the slope and diameter which in turn is related to the optimization variable, i.e., $C_D = C_D(S,D,d)$ as given in Eq. 7.2. The minimum total expected cost for the connection, which is the return in the recursive equations (Eqs. 4.7 and 4.8), is

$$r_{m_n,m_{n+1}}(S_{m_n,m_{n+1}},D_{m_n,m_{n+1}}) = \underset{d}{\text{Min}}[C_I(S_{m_n,m_{n+1}},D_{m_n,m_{n+1}},d) + C_D(S_{m_n,m_{n+1}},D_{m_n,m_{n+1}},d)] \quad (7.3)$$

A flow chart showing the risk computations to determine the diameter for each feasible set of states is given in Fig. 7.7. For each possible drop in crown elevation, first a diameter is selected which satisfies the preceding (upstream) diameter constraint. Manning's formula is then used to compute the full-flow pipe capacity, Q_C. The velocity is then computed

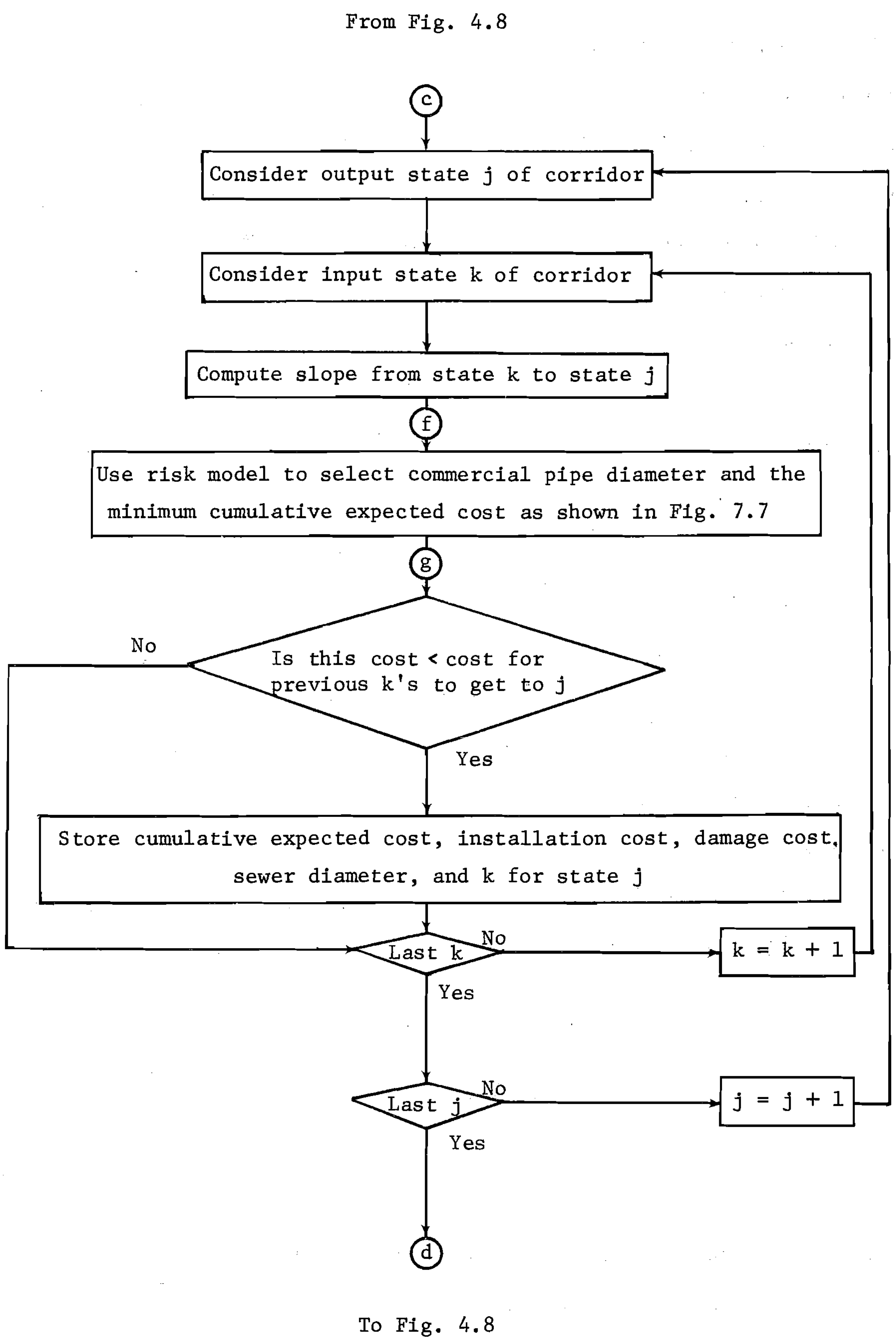


Fig. 7.6. DP Computations within Corridor Considering Risks

using Eq. 6.5 and checked to see that it satisfies the velocity constraints. If the minimum velocity constraint is not satisfied this is the last diameter considered for this pipe elevation drop. Contrarily, if the maximum velocity constraint is not satisfied the next larger commercial pipe size is considered. The safety factor SF = $\bar{Q}_C/Q_o$ (Eq. 4.13) is computed. Accordingly the risk is determined from the risk-safety factor curves (Fig. 5.2) knowing SF for the sewer diameter under consideration. The risk is subsequently used in Eq. 7.2 to evaluate C_D which in turn is used in Eq. 7.3 to obtain $r_{m_n, m_{n+1}}$. This procedure is repeated systematically considering successively larger diameters that satisfy the constraints on flow, velocity, and preceding (upstream) diameters until the safety factor is greater than 6* or the largest commercially pipe size considered. The computational procedure then returns to the DP computations (Fig. 7.6) once a diameter is selected.

The risk procedure shown in Fig. 7.7 and described above allows the risk, $P(Q_L > Q_C)$, to vary freely for different sewers. The optimization produces not only the least cost design but also specifies the associated risks. Another approach presented by Tang, Mays, and Yen (1975) is to design for an acceptable maximum risk level, i.e., each sewer pipe for a connection is designed for the same minimum safety factor. The acceptable maximum risk level can vary for different connections in the network. This procedure can be incorporated in the risk model shown in Fig. 7.7.

The inputs for Model C include the optimization model parameters, ground surface elevations, design inflows for each manhole, pipe lengths, Manning's roughness factor, coefficients for the risk-safety factor relationship, and assessed damage values for each pipe connection in the sewer network. The computer program for Model C is listed in Appendix D.

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*A safety factor of 6 has been arbitrarily selected because the installation cost for the pipe would be so high that the corresponding diameter would never result in the minimum total expected cost.

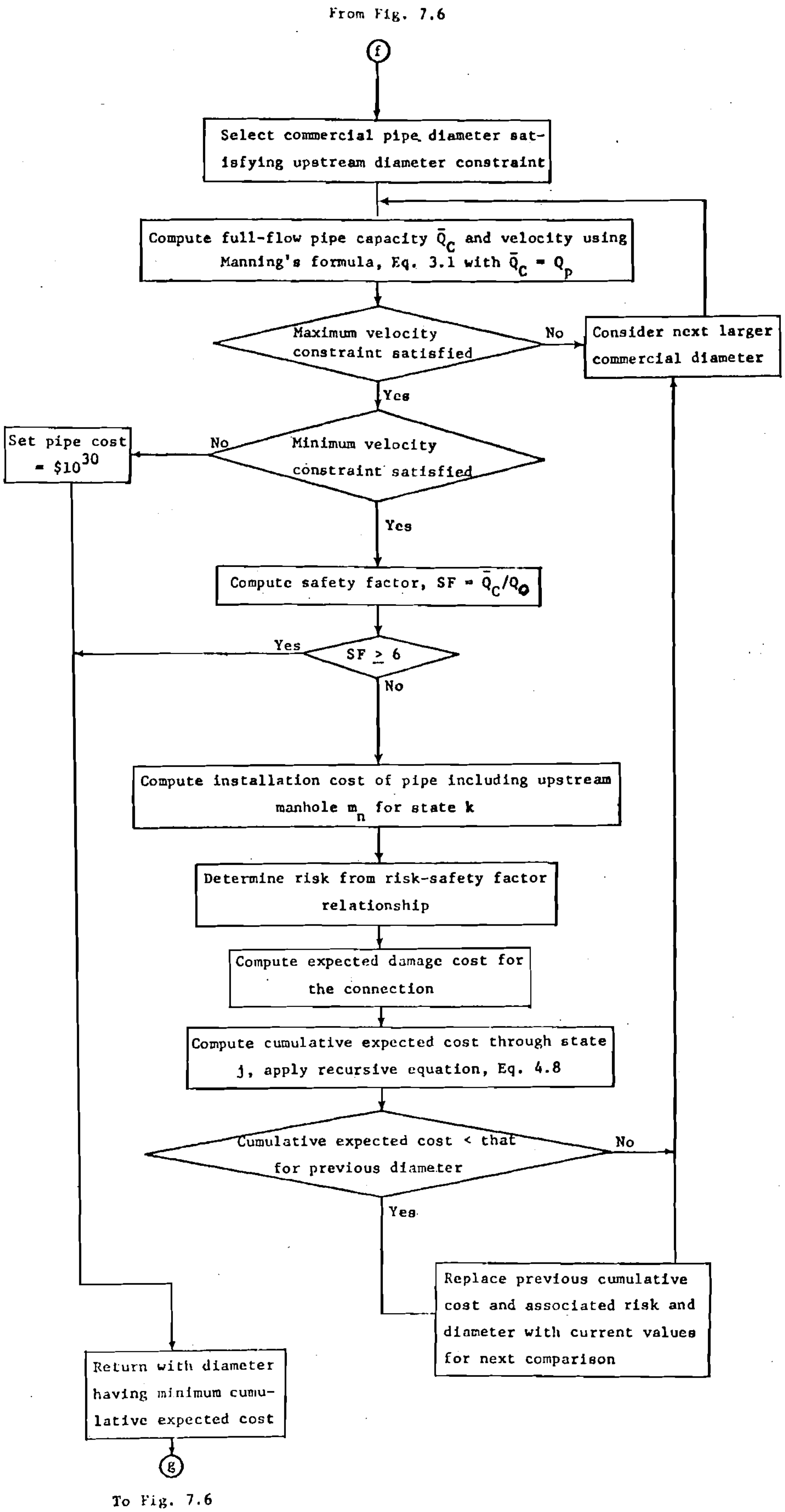

Fig. 7.7. Flow Chart for Sewer Diameter Selection Considering Risks

7.2.3. Model D - Risk Component With Hydrograph Time Lag Routing

Model C can be extended to include the hydrograph time lag routing component. The result is referred to as Model D. The DDDP solution scheme at each stage is shown in Fig. 7.2 with the DP computations using risk as shown in Fig. 7.6. The risk model shown in the flow chart in Fig. 7.7 applies to this design model and the routing scheme shown in Fig. 7.3 also applies. The required input for this design model is similar to that for Model C with the addition of inflow hydrographs at each manhole. The computer program for Model D is listed in Appendix E.

Chapter 8. EXAMPLE APPLICATIONS OF DESIGN MODELS

The purpose of this chapter is to demonstrate the applications of the least cost sewer system design models described in the previous chapter. In order to provide guidelines for the selection of the models and an appreciation of their effect on the resulting design two examples are presented. The first is a hypothetical example which is used primarily for a sensitivity analysis. The second example is an actual sewer system taken from ASCE (1969) Manual No. 37 and is presented to further illustrate the various models.

8.1. Model Input Parameters

As described in Chapter 7, each of the models employs an optimization component. A particular model may also employ a routing and/or a risk component as well. Each component requires certain input information, and to a certain extent, the results and the computational efficiency depend on this input data.

The DDDP procedure is used in all of the models. The four optimization decision parameters affecting this procedure are listed as follows.

(a) The number of lattice points, N_p, defining the number of states at each end of a system link; i.e., the number of possible crown elevations within a corridor at each end of a sewer.

(b) The initial state increment, Δ_{s_1}, which is the distance between possible crown elevations at each end of a sewer for the first iteration.

(c) The initial trial trajectory used to establish the location of the corridors within the state space (range of possible crown elevations) for the first iteration.

(d) The reduction rate of the state increment Δ_s for successive iterations which determines the corridor width for subsequent iterations.

If a routing component is used additional parameters may be required. The nonlinear kinematic wave routing procedure requires the specification of both a distance increment, Δx, and a time increment, Δt. The Muskingum-Cunge procedure requires the specification of a distance increment only, while the hydrograph time lag routing requires no additional input decision parameter specifications.

The risk component requires an analysis of uncertainties as described in Chapter 5. This results in a set of risk-safety factor curves which is the input required by the risk component of the design models.

The constraints pertinent to the design models are discussed in Section 2.4. The cost functions used in this study are Eq. 2.1 for the sewers and Eq. 2.2 for the manholes.

8.2. Example I

8.2.1. Sewer System Description

Example I is a branched system used previously by Yen and Sevuk (1975) containing 14 sewers, 14 manholes and a single free-fall outlet. The layout and isonodal lines dividing the system into 6 stages and manhole numbers are shown in Fig. 8.1. The sewer lengths, ground elevations and specified crown elevations at various locations are given in Table 8.1. The latter were included to demonstrate that the design models can handle the situation where elevation constraints exist at arbitrary points in the system. The Manning roughness factor n is assumed equal to 0.0133 for all the sewers. In this example a minimum soil cover depth of 8 ft is used as well as minimum and maximum velocities of 2 and 10 fps, respectively.

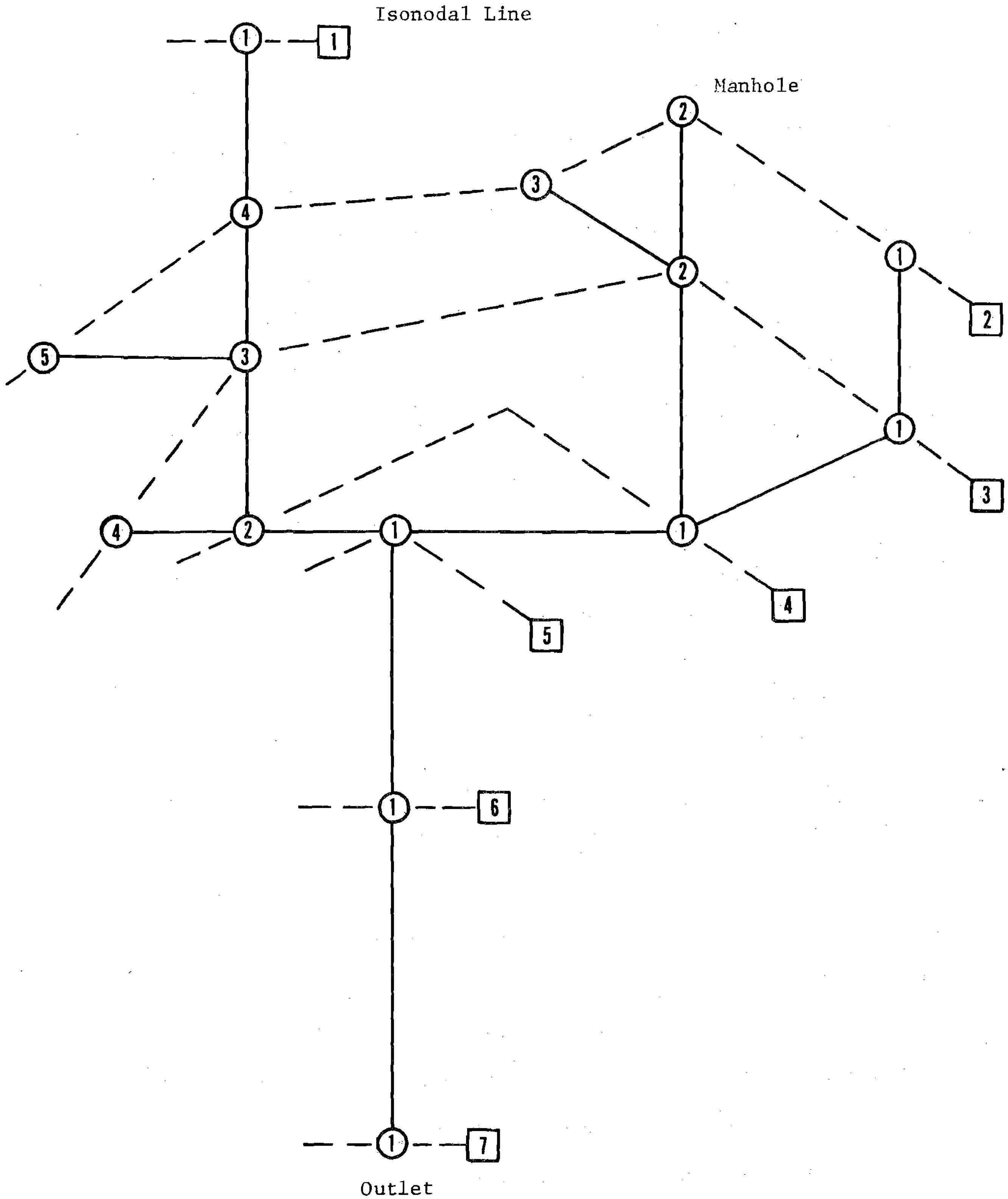

Fig. 8.1. Example I Sewer System Layout with Isonodal Lines

TABLE 8.1. Example I Layout Data

Isonodal Line	Manhole	Ground Elevation ft	Downstream Manhole	Pipe Length ft	Required Crown Elevation at Pipe Exit ft
1	1	421.2	4	510	
2	1	417.3	1	320	403.3
	2	417.5	2	600	
	3	416.1	2	450	
	4	418.7	3	360	
	5	421.4	3	460	405.8
3	1	414.8	1	700	
	2	414.7	1	800	
	3	417.3	2	380	
	4	417.7	2	294	403.5
4	1	414.6	1	1000	
	2	415.0	1	410	398.0
5	1	413.1	1	1210	
6	1	412.9	1	1320	
7	1	400.0			(outlet)

The inflow hydrographs at the manholes are assumed to be symmetrical triangles with a base flow. The numerical inflow data are given in Table 8.2. With the exception of Models C and D (Table 7.1), any method can be adopted for developing these inflow hydrographs. The risk-safety factor curves used in Models C and D were based in part on an analysis of the rational method, implying that this was the method used to determine the peak inflows. The hydrographs all have a common time scale but the initial rise time varies as shown in Table 8.2 and Fig. 8.2.

It should be emphasized that this example is hypothetical. Its purposes are to demonstrate the various least-cost design models and to illustrate their sensitivity to the various input parameters.

8.2.2. Optimization Component Parameter Sensitivity

The parameters used in the optimization procedure which must be specified as input are listed in Section 8.1. In order to illustrate the effects of these parameters on the minimum cost design, the Example I sewer system is designed using Model A with different numbers of lattice points and initial state increments, i.e., for various initial corridor widths. The results are summarized in Table 8.3. In establishing the values for these parameters it must first be recognized that they are interdependent in relation to their effect on the final design and the rate that the models converge to that design. For example, if a small corridor width is chosen in conjunction with an initial trial trajectory which is far from the optimal region, additional iterations are required to move the trajectory into the optimal or near-optimal region. It is also possible under such circumstances that the model solution converges to a design which is far from the global optimal (Mays and Yen, 1975). The corridor width, the number of lattice points of the corridor,

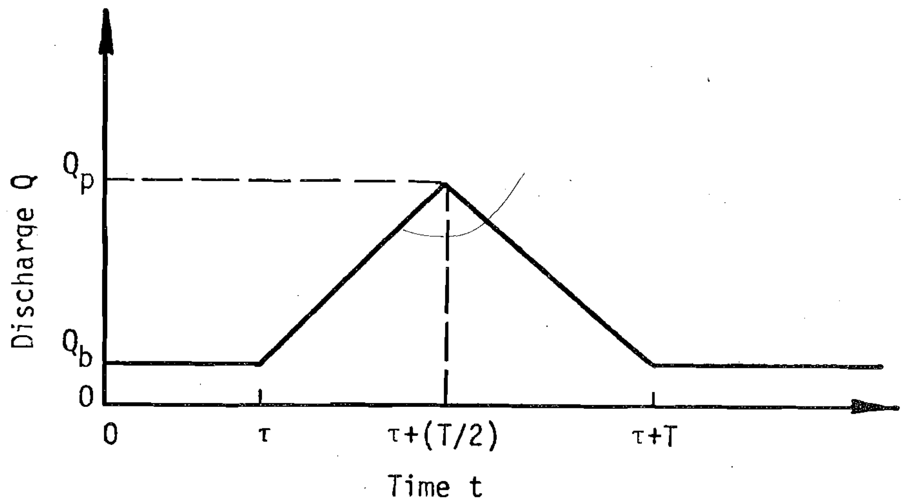

Fig. 8.2. Definition of Inflow Hydrograph Parameters

TABLE 8.2. Example I Inflow Hydrograph Data

Isonodal Line	Manhole	Initial Rise Time τ sec	Base Time T sec	Baseflow Q_b cfs	Peak Flow Q_p cfs
1	1	480	1200	1.0	11
2	1	240	1200	1.0	11
	2	120	720	0.5	5
	3	0	960	1.0	9
	4	0	720	0.5	5
	5	360	1200	1.0	11
3	4	240	1200	1.0	11
4	1	120	960	1.0	9
6	1	600	1200	1.0	11

TABLE 8.3. Results for Example I Using Model A

Number of Lattice Points	Initial State Increment ft	Initial Corridor Width ft	Installation Cost $	Execution Time sec
3	1.00	2	502,614	2.79
	2.00	4	474,510	2.94
	3.00	6	473,704	3.35
	4.00	8	475,986	3.19
	5.00	10	473,227	2.95
5	0.50	2	537,321	2.59
	1.00	4	474,190	5.37
	1.50	6	475,463	5.20
	2.00	8	482,568	
	2.50	10	473,081	5.65
	3.00	12	473,227	5.77
	3.50	14	476,991	6.69
	4.00	16	482,619	6.82
	4.50	18	472,322	6.14
	5.00	20	472,909	5.84
7	0.33	2	506,082	3.44
	0.50	3	499,096	6.92
	0.67	4	474,752	7.05
	1.00	6	474,370	7.90
	1.33	8	472,871	7.54
	1.67	10	474,285	8.62
	2.00	12	475,070	8.72
	2.33	14	475,085	8.17
	2.67	16	475,199	9.15
	3.00	18	472,623	9.12
	3.33	20	472,771	11.05
	4.00	24	475,123	11.39
9	0.25	2	505,947	5.85
	0.50	4	474,190	13.60
	0.75	6	472,592	11.32
	1.00	8	472,295	14.02
	1.25	10	472,608	13.17
	1.50	12	475,002	13.22
	1.75	14	472,223	14.74
	2.00	16	475,070	15.47
	2.25	18	475,002	14.52
	2.50	20	474,936	13.99
	2.75	22	472,463	14.19
	3.00	24	475,002	13.85

N_p, and the state increment, Δ_s, are interrelated; i.e., the corridor width is equal to $(N_p-1)\Delta_s$, and only two of these parameters can be independently specified. A small initial corridor width can be produced by a combination of a small number of lattice points and a small initial state increment.

The effect of choosing a bad initial trial trajectory can be reduced if a large initial corridor width, i.e., a large number of lattice points and/or a large initial state increment, is used. In essence, the better the initial trajectory the smaller the required number of lattice points and the initial state increments, or simply, the smaller the initial corridor width which can be used. Small state increments with many lattice points can be used also to establish a corridor width. This can result in improved convergence; however, increasing the number of lattice points increases the computation time. Computation time can be reduced by increasing the reduction rate of the state increment Δ_s at each iteration; however, too large a reduction rate of Δ_s may cause the model to miss the optimal region thus not providing the minimum cost design. Choosing a large initial state increment and a large reduction rate of Δ_s may be advantageous. However, when the initial state increment is too large resulting in a large initial corridor width, unnecessary computations are performed in regions of the state space far from the optimal.

Because of the mutual dependence of the above parameters, the following strategy is used. Based upon computer runs of several examples using various reduction rates of Δ_s and the results of studies by Mays and Yen (1975) and Mays (1976), it was concluded that the best reduction rate of Δ_s is 1/2. Also, instead of continuously reducing Δ_s until a minimum specified increment is reached, the following procedure is recommended: after several successive reductions of Δ_s at the rate of 1/2 (e.g., after five iterations), then the size of Δ_s is increased by some multiple of its current value. For the remaining iterations Δ_s is reduced at a rate of 1/2 until the specified minimum value is reached. It should be kept in mind that for Δ_s to be reduced after an iteration, the cost criterion, Eq. 4.4, must be satisfied. It has been found that after 5 iterations, increasing the state increment, Δ_s, to a value of 2 or 3 times its present value is most satisfactory. This procedure has resulted in good convergence to a minimum cost solution.

The results of several designs for various example sewer systems show that the minimum-cost solutions normally have pipe slopes somewhat parallel to the ground surface slopes. Choosing initial trial trajectories having crown elevations sufficiently below the required minimum soil cover depth so that the top of the corridor either follows or is close to the minimum soil cover depth line is advisable. A general guideline in selecting initial crown elevations at the upstream and downstream side of each manhole is

$$E_{d\ min} \geq \bar{S}_{m_n} \geq E_{d\ min} - 0.5(N_p - 1)\Delta_{s_1} \qquad (8.1)$$

in which $E_{d\ min}$ is the elevation corresponding to the minimum cover depth at manhole m_n; $\bar{S}_{m_n}$ is the crown elevation for the initial trial trajectory at manhole m_n; N_p is the number of lattice points used; and Δ_{s_1} is the initial state increment selected. In applying this guideline the optimization component computes the elevation of the top of the initial corridor based on the prescribed values of $\bar{S}_{m_n}$, N_p and Δ_{s_1}. If this elevation exceeds $E_{d\ min}$ the value of $\bar{S}_{m_n}$ is lowered by an integer multiplier of Δ_{s_1} such that the entire initial corridor is below $E_{d\ min}$.

In order to evaluate the sensitivity of the initial corridor width and the number of lattice points within the corridor to designs for the Example I system, the results of installation costs and computer execution time listed in Table 8.3 are plotted in Figs. 8.3 and 8.4 for the initial corridor widths ranging from 2 to 24 ft and for numbers of lattice points, N_p, equal to 3, 5, 7, and 9. It should be noted that for the runs with N_p = 3 and 5, the maximum initial corridor widths are limited. This is because for $\Delta_{s_1} \gtrsim 6$ ft there exists in the system at least one corridor which cannot satisfy all of the design constraints within the feasible set of states.

In observing the trends of the installation cost shown in Fig. 8.3 it is seen that the cost drops rapidly with increasing initial corridor width regardless of the number of lattice points used when the initial corridor width is less than the average drop of elevation of the sewers. For the sake of simplicity the average sewer elevation drop can be estimated as the nominal sewer drop, which is computed as the difference in elevation between the highest manhole ground elevation on INL 1 and the ground elevation at the system outlet, divided by the number of sewers in between. For the example system this nominal sewer drop is (421.2-400.0)/7 = 3.03 ft. When the initial corridor width is greater than nominal sewer drop the installation cost levels off and fluctuates within 1% of the computed minimum cost of 472,223 (except two points for N_p = 5) with no further apparent trend. In other words the computed installation cost of the design depends mainly on the initial corridor width which should be chosen greater than the average elevation drop of the sewer. The fluctuation of the computed system installation costs is due partly to the fact that discrete commercial pipe sizes are used and partly that the DDDP procedure cannot guarantee global optimality. For a given initial corridor width, the computed installation costs vary randomly for the values of N_p and Δ_{s_1} used. Therefore, for a specified initial corridor width, the preferred number of lattice points used within the corridor is determined by

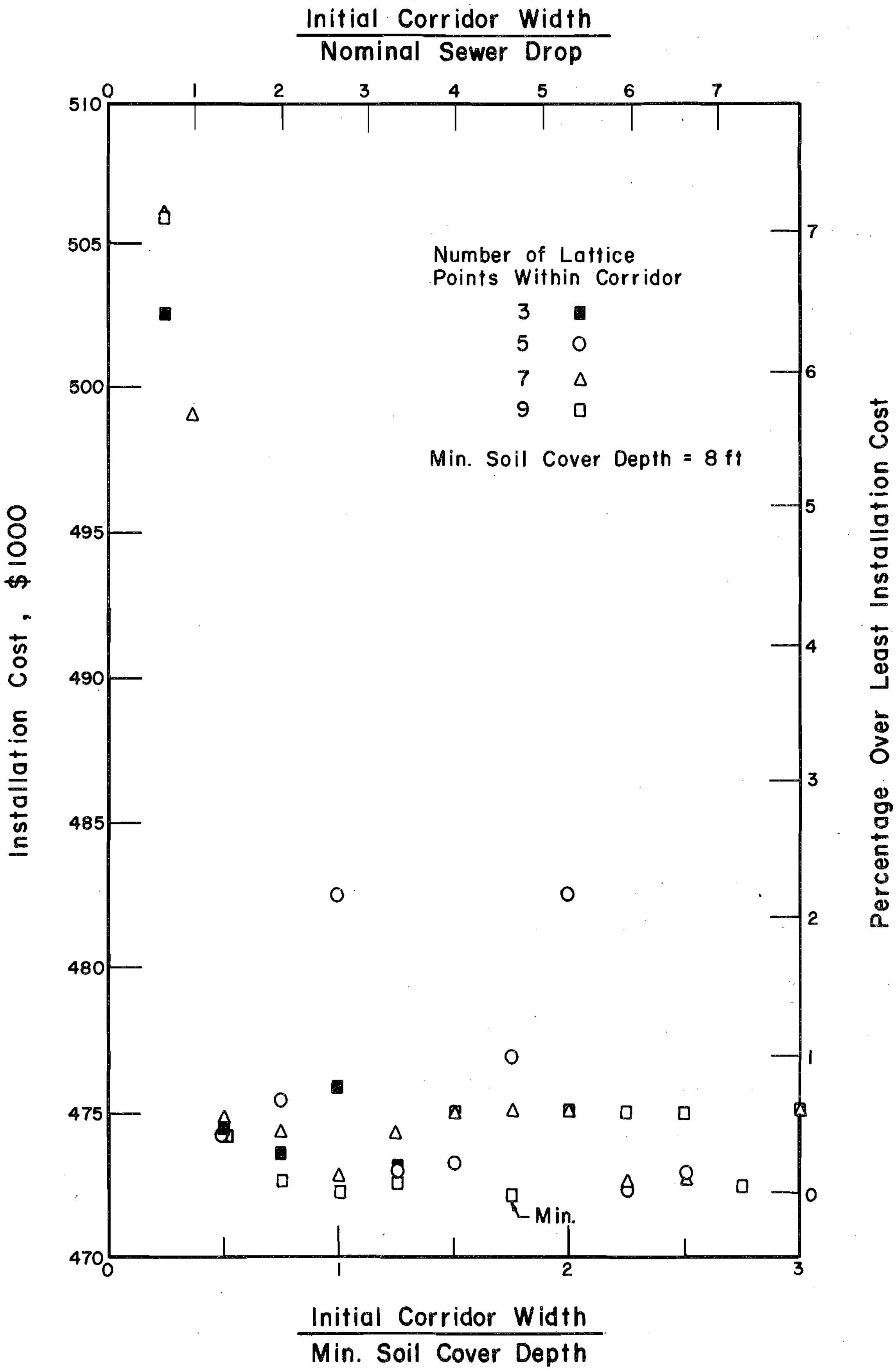

Fig. 8.3. Sensitivity of Design to Initial Corridor Width and Number of Lattice Points

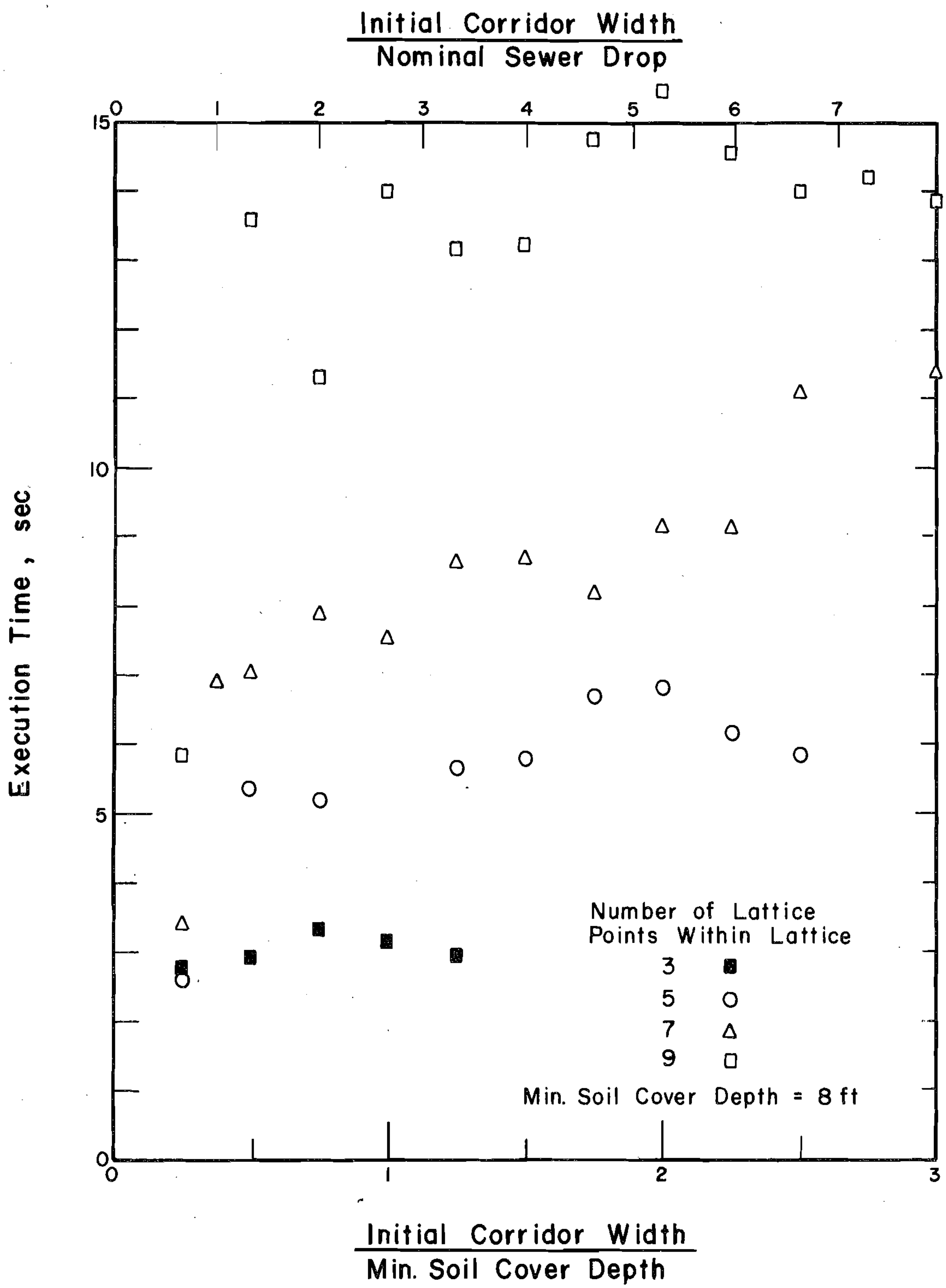

Fig. 8.4. Variations of Computer Execution Time with Initial Corridor Width and Number of Lattice Points

the computer execution time. As shown in Fig. 8.4, the execution time depends mainly on the number of lattice points for initial corridor widths greater than the nominal sewer drop, and for the example, averages about 1.2 sec per lattice point. It can also be observed that the execution time tends to increase slightly with increasing initial corridor width. Thus, it can be concluded from these results and experience with other examples that an initial corridor width of two to five times the nominal sewer drop with 3 to 7 lattice points usually provides good results whereas using 9 or more lattice points merely increases execution time without significant improvement in design. With the initial corridor width and number of lattice points chosen, the value of initial state increment, Δ_{s_1}, can be determined accordingly. Of course it should be recognized that the design constraints will have some effect but those used in this example are typical.

In order to verify the above conclusion on sensitivity to the optimization input parameters for more sophisticated least-cost design models, the Example I sewer system was tested by using the other models listed in Table 7.1. The results for Models B-1, B-2, and B-3 which incorporate routing by using the hydrograph time lag, kinematic wave, and Muskingum-Cunge methods, respectively, are summarized in Table 8.4 and plotted in Figs. 8.5 and 8.6. For all these models 7 lattice points forming the corridor were used, and the maximum distance increment for computations, Δx_{max}, along each sewer was 800 ft. For Models B-1 and B-2 the routing time increment Δt was 120 sec. The Example I sewer system was also designed by using Models C and D listed in Table 7.1 incorporating the risk component, again using 7 lattice points, and for a design service period of 25 years. The risk-safety factor curves described in Section 5.4 for Urbana, Illinois are assumed applicable to this example. The least-cost system designs for Model C were performed using the assumed assessed damage cost scales given in Table 8.5 and the results are

TABLE 8.4. Results for Example I Using Routing Components

		Model B-1 Hydrograph Time Lag		Model B-2 Kinematic Wave		Model B-3 Muskingum-Cunge	
Initial Corridor Width ft	Δs_1 ft	Installation Cost $	Ececu-tion time sec	Installation Cost $	Execu-tion time sec	Installation Cost $	Execu-tion time sec
2	0.33	465,401	5.3	453,317	46.6	415,061	195.3
3	0.50	457,774	10.6	433,332	97.7	415,061	115.3
4	0.67	448,298	11.2	433,213	88.4	408,964	115.6
6	1.00	433,016	12.4	425,951	102.4	406,925	143.5
8	1.33	433,735	12.2	426,679	98.5	407,442	149.3
10	1.67	434,405	14.0	427,176	116.6	392,850	163.4
12	2.00	433,016	13.4	417,601	115.5	438,797	186.6
14	2.33	430,987	13.8	418,044	111.7	352,777	166.3
16	2.67	429,915	13.6	370,744	110.6	351,285	165.5
18	3.00	434,652	14.2	426,093	115.8	394,428	173.1
20	3.33	433,404	15.6			392,810	183.3
24	4.00	433,069	14.8			392,994	187.2

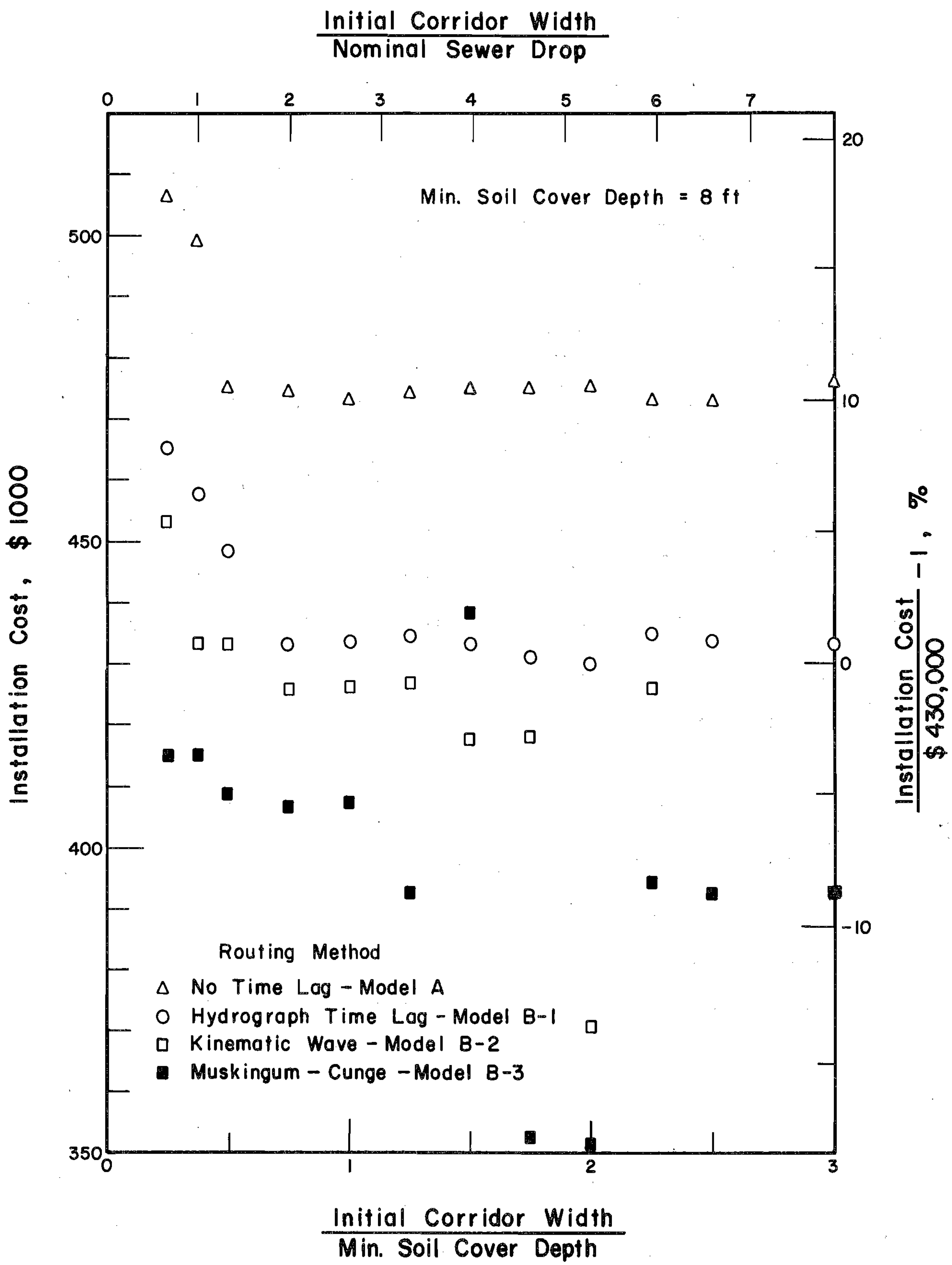

Fig. 8.5. Sensitivity of Design to Initial Corridor Width with Routing

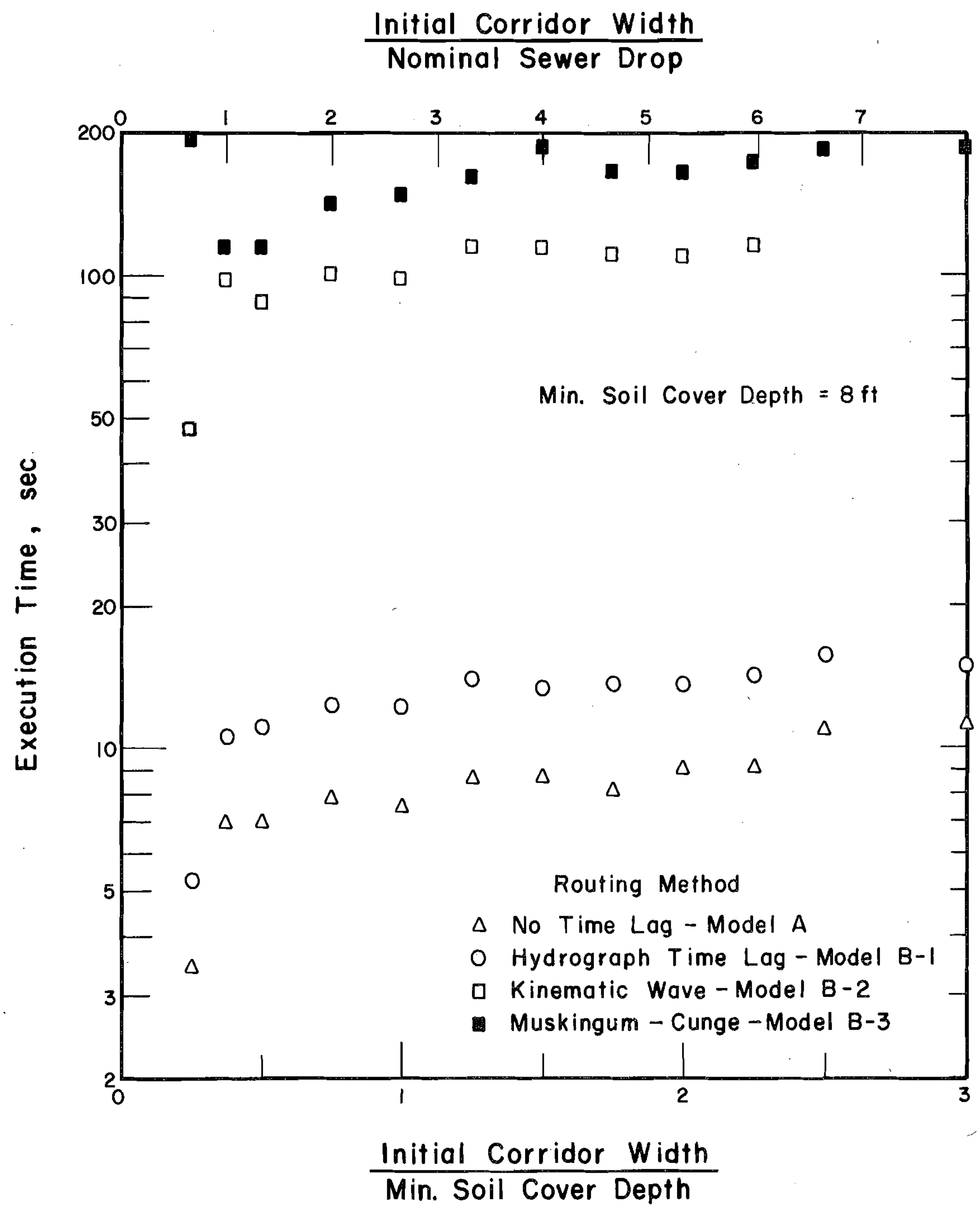

Fig. 8.6. Computer Execution Time for Designs with Routing

presented in Table 8.6. The results for Model D using the assessed damage scale A listed in Table 8.5 are summarized in Table 8.7. As can be seen from these two tables and from Figs. 8.5 and 8.6, the conclusions drawn from Model A on the effects of the initial corridor width and the number of lattice points in forming the corridor (and hence the magnitude of the state increment) apply to the more sophisticated least-cost system design models as well.

TABLE 8.5. Hypothetical Assessed Damage Scales

Sewer Length ft	Scale A	Scale B	Scale C
0 - 300	50,000	5,000	500
301 - 600	100,000	10,000	1,000
601 - 900	150,000	15,000	1,500
901 - 1200	200,000	20,000	2,000
> 1200	300,000	30,000	3,000

However, as shown in Fig. 8.5, the fluctuation of the system cost for different initial corridor widths for Models B-2 and B-3 are clearly more than for Models A and B-1. At first glance, such appreciable fluctuations make it less certain that using a selected pair of initial corridor width and number of lattice points together with the selected initial trial trajectory would produce a design that is reasonably close to the global optimum. Actually, a careful examination of the resulting designs reveals that the fluctuations are caused mainly by changes in the size of one or two sewers. Because of the discrete sizes of commercial

TABLE 8.6. Results for Example I Using Model C With Risk Component

Initial Corridor Width ft	Δs_1 ft	Assessed Damage Scale A: Installation Cost $	Expected Damage Cost $	Total Cost $	Execution time sec	Assessed Damage Scale B: Installation Cost $	Expected Damage Cost $	Total Cost $	Execution time sec	Assessed Damage Scale C: Installation Cost $	Expected Damage Cost $	Total Cost $	Execution time sec
2	0.33	663,307	22,394	685,701	7.1	528,723	52,048	580,771	6.4	506,083	11,085	517,168	6.3
3	0.50	628,712	30,683	659,395	13.7	494,889	57,672	552,561	12.8	499,095	11,484	510,579	12.3
4	0.67	632,815	21,601	654,416	13.4	496,125	53,858	549,983	12.5	474,752	11,453	486,205	12.2
6	1.00	614,823	28,311	643,134	15.0	508,423	39,340	547,763	14.1	474,177	11,531	485,708	14.0
8	1.33	615,094	28,060	643,154	13.7	507,996	39,629	547,625	14.0	472,942	11,460	484,402	15.2
10	1.67	614,802	28,410	643,212	17.5	508,449	39,320	547,769	16.4	474,773	11,508	486,281	15.6
12	2.00	614,823	28,311	643,134	16.1	500,688	50,025	550,713	16.2	474,877	11,553	486,430	15.2
14	2.33	615,287	27,968	643,255	16.3	507,963	39,569	547,532	15.4	475,085	11,518	486,603	14.8
16	2.67	615,227	28,137	643,364	15.3	508,006	39,618	547,624	15.5	475,270	11,467	486,737	14.9
18	3.00	614,728	28,500	643,228	15.2	508,465	39,304	547,769	17.3	472,781	11,501	484,282	16.8
24	4.00	614,813	28,311	643,124	16.3	508,423	39,340	547,763	16.5	475,120	11,540	486,660	16.7

pipes, for a small system like that of Example I, the change of the size of one sewer may produce an appreciable change in the cost. The cost change is particularly noticeable if the sewer size is greater than 30 in., since 6 in. size increments would then be used and the cost (computed by Eq. 2.1) increases more rapidly with pipe size. This is indeed the case for the Example I system as all the large cost fluctuations are due to the change of a sewer from 36 in. to 42 in. or vice verse. Nonetheless, it is expected that for a large sewer system the system cost would fluctuate much less with respect to the initial corridor width and number of lattice points used and the resulting design would be reasonably close to the global optimum.

TABLE 8.7. Results for Example I Using Model D With Risk and Hydrograph Time Lag Routing Components

Initial Corridor Width ft	Δ_{s_1} ft	Installation Cost $	Expected Damage Cost $	Total Cost $	Execution Time sec
2	0.33	521,438	65,011	586,449	8.7
3	0.50	528,613	23,229	551,842	17.1
4	0.67	530,427	21,183	551,610	15.5
6	1.00	530,379	21,071	551,450	17.6
8	1.33	535,887	24,307	560,194	18.4
10	1.67	530,170	21,342	551,512	21.1
12	2.00	530,379	21,071	551,450	21.7
14	2.37	530,211	21,315	551,526	20.1
16	2.67	536,087	24,253	560,340	19.2
18	3.00	529,822	21,649	551,471	20.0
20	3.33	530,108	21,410	551,518	22.1
24	4.00	530,379	21,071	551 450	21.3

8.2.3. Comparison of Example I Results Using Various Design Models

8.2.3.1. Effect of Routing on Design - The system costs for the Example I sewer system designed by using the various models listed in Table 7.1 have been presented in Figs. 8.3 to 8.6 and Tables 8.3, 8.4, 8.6 and 8.7. A comparison of these costs provides some interesting and useful information. As can be seen in Fig. 8.5 the models incorporating the routing component always produce designs with a considerable lower total cost than the corresponding models without routing. This result is expected in view of the discussion presented in Section 6.3. The routing procedure can phase the upstream and local inflow hydrograph peaks such that the peak of their sum is less than the sum of their individual peaks.

Among the three routing models, the Muskingum-Cunge method usually provides the best results because it partially accounts for the sewer storage and the peak discharge attenuation, whereas the hydrograph time-lag shifting method usually produces highest cost designs. This indeed is the case as can be seen from Fig. 8.5. However, the reduction of system cost between Models B-3 and B-1 is only a few percent whereas the computer execution time is increased by one order of magnitude (Fig. 8.6). In view of the fact that none of the three routing methods is exact the least cost design should be checked hydraulically (when economically justified) using a more reliable hydraulic model such as the ISS Model (Sevuk et al., 1973) and readjusted if necessary.

The much simpler Model B-1 appears to be just as useful as the slightly more accurate Model B-3, with the preference depending primarily on the particular design situation. Conversely, Model B-2 usually produces a design very close to that by Model B-1 whereas the computer execution time of the former is one order of magnitude higher. Consequently Model B-2 appears to be least useful. Moreover, it should be emphasized that the quantitative differences of the designs using different models are a function of the system size and inflow hydrographs. Therefore it would be misleading to quantitatively discuss cost savings as a function of design model based on one example.

8.2.3.2. Hydraulic Design vs. Least-Cost Design - The designs of the Example I sewer system using different least-cost design models are summarized in Tables 8.8, 8.9 and 8.10 giving the diameters, slopes, and crown elevations of the sewers. The designs presented in these tables as examples were obtained by using an initial corridor width of 6 ft with 7 lattice points to form the corridors.

Since the Example I system was used by Yen and Sevuk (1975) for the hydraulic design of sewer sizes using the same inflow hydrographs, it would be interesting to compare their results using the no time lag, hydrograph time lag, and nonlinear kinematic wave routing methods to the results of Models A, B-1, and B-2, respectively. The comparison indicates that with the exception of one sewer each for the hydrograph time lag and nonlinear kinematic wave routings, all the sewers in the least-cost designs are equal or smaller than the corresponding sewers in the hydraulic designs. Since the sewers in the example hydraulic design are generally buried deeper under the ground surface, clearly the total cost of the sewer system is lower for the least-cost design than the hydraulic design. However, the costs for the hydraulic designs are not given here because a fair comparison cannot be made. In Yen and Sevuk's designs there are drops specified at the exits of certain sewers whereas in the present study only the crown elevations are specified at these locations. The existence of the drops reduces the slope of the sewers resulting in larger diameters and hence increasing the cost. However, it is estimated that even with the same constraints, the least-cost design models would produce a lower cost design than the hydraulic model.

TABLE 8.8. Least-Cost Designs of Example I Sewer System without Considering Risks

Upstream Isonodal Line	Upstream Manhole	Downstream Manhole	Crown Elevations Upstream ft	Crown Elevations Downstream ft	Sewer Slope	Sewer Diameter in.
			Design Using Model A			
6	1	1	394.38	389.75	0.00350	48
5	1	1	397.60	394.38	0.00267	48
4	1	1	402.06	399.31	0.00275	36
	2	1	403.50	402.06	0.00351	36
3	1	1	403.80	402.06	0.00248	24
	2	1	405.70	402.44	0.00408	24
	3	2	405.80	404.06	0.00457	30
	4	2	407.70	404.31	0.01152	18
2	1	1	409.30	405.63	0.01148	18
	2	2	409.50	405.70	0.00633	15
	3	2	408.10	406.57	0.00339	21
	4	3	410.63	406.75	0.01076	21
	5	3	413.40	408.06	0.01160	18
1	1	4	413.20	410.63	0.00505	21
			Design Using Model B-1			
6	1	1	393.44	387.38	0.00459	42
5	1	1	397.60	393.44	0.00344	42
4	1	1	403.00	399.63	0.00337	30
	2	1	403.50	400.44	0.00747	30
3	1	1	403.80	403.00	0.00114	27
	2	1	406.57	403.81	0.00345	24
	3	2	405.80	404.13	0.00441	30
	4	2	407.70	404.31	0.01152	18
2	1	1	409.30	405.63	0.01148	18
	2	2	409.50	406.70	0.00467	18
	3	2	408.10	406.57	0.00339	21
	4	3	410.63	406.69	0.01094	21
	5	3	413.40	408.06	0.01160	18
1	1	4	413.20	410.63	0.00505	21

TABLE 8.8. (Continued)

Upstream Isonodal Line	Upstream Manhole	Downstream Manhole	Crown Elevations		Sewer Slope	Sewer Diameter in.
			Upstream ft	Downstream ft		
Design Using Model B-2						
6	1	1	394.69	391.63	0.00232	42
5	1	1	397.60	394.69	0.00241	42
4	1	1	403.00	399.44	0.00356	27
	2	1	403.50	400.88	0.00640	30
3	1	1	403.80	403.00	0.00114	27
	2	1	405.70	403.44	0.00283	24
	3	2	405.80	404.19	0.00424	30
	4	2	407.70	404.31	0.01152	18
2	1	1	409.30	405.63	0.01148	18
	2	2	409.50	405.70	0.00633	15
	3	2	408.10	406.57	0.00339	21
	4	3	410.63	407.06	0.00990	21
	5	3	413.40	408.06	0.01160	18
1	1	4	413.20	410.63	0.00505	21
Design Using Model B-3						
6	1	1	393.88	388.56	0.00402	42
5	1	1	397.60	393.88	0.00308	42
4	1	1	402.25	398.88	0.00337	30
	2	1	403.50	400.44	0.00747	30
3	1	1	403.80	402.25	0.00221	24
	2	1	405.70	403.38	0.00291	24
	3	2	405.80	404.13	0.00441	30
	4	2	407.70	404.31	0.01152	18
2	1	1	409.30	405.63	0.01148	18
	2	2	409.50	405.70	0.00633	15
	3	2	408.10	406.57	0.00339	21
	4	3	410.63	406.44	0.01163	21
	5	3	413.40	408.06	0.01160	18
1	1	4	413.20	410.63	0.00505	21

TABLE 8.9. Least-Cost Designs of Example I Sewer System Using Model C

Upstream Isonodal Line	Upstream Manhole	Downstream Manhole	Crown Elevations Upstream ft	Crown Elevations Downstream ft	Sewer Slope	Sewer Diameter in
			Using Damage Scale A			
6	1	1	392.69	384.44	0.00625	54
5	1	1	397.60	392.69	0.00406	54
4	1	1	402.94	398.44	0.00450	42
	2	1	403.50	398.75	0.01159	36
3	1	1	403.80	402.94	0.00123	36
	2	1	406.57	402.94	0.00455	30
	3	2	405.80	403.50	0.00605	36
	4	2	407.70	404.38	0.01131	24
2	1	1	409.30	405.31	0.01246	24
	2	2	409.50	406.57	0.00488	21
	3	2	408.10	406.57	0.00339	27
	4	3	410.51	405.81	0.01306	27
	5	3	413.40	408.31	0.01106	24
1	1	4	413.20	410.51	0.00527	27
			Using Damage Scale B			
6	1	1	394.38	386.50	0.00597	48
5	1	1	397.60	394.38	0.00267	48
4	1	1	402.56	397.63	0.00494	36
	2	1	403.50	399.75	0.00915	36
3	1	1	403.80	402.56	0.00177	30
	2	1	406.39	402.56	0.00478	27
	3	2	405.80	403.69	0.00556	36
	4	2	407.70	403.88	0.01301	21
2	1	1	409.30	404.63	0.01461	21
	2	2	409.50	406.39	0.00519	18
	3	2	408.10	406.39	0.00381	24
	4	3	410.45	405.81	0.01288	24
	5	3	413.40	407.63	0.01255	21
1	1	4	413.20	410.45	0.00539	24
			Using Damage Scale C			
6	1	1	394.38	389.75	0.00350	48
5	1	1	397.60	394.38	0.00267	48
4	1	1	402.88	400.13	0.00275	36
	2	1	403.50	402.06	0.00351	36
3	1	1	403.80	402.88	0.00132	27
	2	1	406.57	403.31	0.00408	24
	3	2	405.80	404.06	0.00457	30
	4	2	407.70	404.31	0.01152	18
2	1	1	409.30	405.63	0.01148	18
	2	2	409.50	406.70	0.00467	18
	3	2	408.10	406.57	0.00339	21
	4	3	410.63	406.75	0.01076	21
	5	3	413.40	408.06	0.01160	18
1	1	4	413.20	410.63	0.00505	21

TABLE 8.10. Least-Cost Designs of Example I Sewer System Using Model D

Upstream Isonodal Line	Upstream Manhole	Downstream Manhole	Crown Elevations Upstream ft	Crown Elevations Downstream ft	Sewer Slope	Sewer Diameter in.
			Using Damage Scale A			
6	1	1	393.19	386.44	0.00511	48
5	1	1	397.60	393.19	0.00365	48
4	1	1	402.94	399.31	0.00362	36
	2	1	403.50	398.94	0.01113	36
3	1	1	403.80	402.94	0.00123	36
	2	1	406.45	402.94	0.00439	30
	3	2	405.80	403.50	0.00605	36
	4	2	407.70	404.38	0.01131	24
2	1	1	409.30	405.31	0.01246	24
	2	2	409.50	406.45	0.00508	21
	3	2	408.10	406.45	0.00367	27
	4	3	410.45	406.25	0.01167	27
	5	3	413.40	408.31	0.01106	24
1	1	4	413.20	410.45	0.00539	27
			Using Damage Scale B			
6	1	1	393.81	388.63	0.00393	42
5	1	1	397.60	393.81	0.00313	42
4	1	1	402.69	399.69	0.00300	36
	2	1	403.50	400.44	0.00747	36
3	1	1	403.80	402.69	0.00159	30
	2	1	406.25	402.75	0.00437	27
	3	2	405.80	403.81	0.00523	36
	4	2	407.70	403.88	0.01301	21
2	1	1	409.30	404.63	0.01461	21
	2	2	409.50	406.25	0.00542	18
	3	2	408.10	406.25	0.00411	24
	4	3	410.45	405.81	0.01288	24
	5	3	413.40	407.63	0.01255	21
1	1	4	413.20	410.45	0.00539	24
			Using Damage Scale C			
6	1	1	393.44	387.38	0.00459	42
5	1	1	397.60	393.44	0.00344	42
4	1	1	403.00	399.63	0.00337	30
	2	1	403.50	400.44	0.00747	30
3	1	1	403.80	403.00	0.00114	27
	2	1	406.57	403.81	0.00345	24
	3	2	405.80	404.13	0.00441	30
	4	2	407.70	404.31	0.01152	18
2	1	1	409.30	405.63	0.01148	18
	2	2	409.50	406.70	0.00467	18
	3	2	408.10	406.57	0.00339	21
	4	3	410.63	406.69	0.01094	21
	5	3	413.40	408.06	0.01160	18
1	1	4	413.20	410.63	0.00505	21

As can be seen from Table 8.8, with rare exceptions, the sewer sizes for Model A without routing are equal or greater than the corresponding sewers designed by models with routing. This is particularly obvious for downstream sewers. Comparison between Models C and D (Tables 8.9 and 8.10) yields the same conclusion. However, the designs by the three models with routing are almost the same. There are no more than two sewers different in size between any two of the designs from Models B-1, B-2, and B-3. This again indicates that unless the hydrograph attenuation effect is very important, Models B-2 and B-3 may not offer significant improvement in design over Model B-1 while requiring considerably more computer time.

8.2.3.3. Effect of Considering Risks in Design - Since the present study provides the first models to incorporate the risk component into a least-cost design, it is of considerable interest to examine the effect of risks on the design. Presented in Table 8.11 are the risks for each of the sewers in the system assuming a design service life of 25 years for each of the designs using the six models. Even though Models A and B do not include the risk component in the optimization procedure, the implicit risk for each sewer associated with the least-cost designs can be calculated using the same 25-yr risk-safety factor curve as employed in the designs using Models C and D. The sewer capacity, $\overline{Q}_C$, is calculated by using Manning's formula, Eq. 5.18, with $S = S_o$. The safety factor is then computed as $\overline{Q}_C/Q_o$ with $Q_o = Q_p$. Subsequently the risk is obtained through the 25-yr risk-safety factor relationship. If the service life of the sewers are different, the computed risks will also vary.

The designs using Models C and D vary with the design service life and the assessed damage values. Without specifying the maximum acceptable risk, Models C and D each produces a least-cost design together

TABLE 8.11. Risks Associated with Example I Designs Using Various Models

Isonodal Line	Manhole		Design Model									
			A	B-1	B-2	B-3	C			D		
	u/s	d/s					Scale A	Scale B	Scale C	Scale A	Scale B	Scale C
6	1	1	0.617	0.619	0.615	0.609	0.0192	0.264	0.617	0.0155	0.619	0.619
5	1	1	0.612	0.615	0.618	0.610	0.0367	0.612	0.612	0.0328	0.617	0.615
4	1	1	0.613	0.617	0.617	0.618	0.0111	0.215	0.613	0.0045	0.048	0.617
	2	1	0.603	0.609	0.615	0.611	0.0177	0.050	0.603	0.0047	0.039	0.609
3	1	1	0.618	0.610	0.617	0.617	0.0055	0.085	0.619	0.0034	0.070	0.610
	2	1	0.610	0.608	0.610	0.605	0.0116	0.103	0.610	0.0052	0.052	0.608
	3	2	0.615	0.611	0.617	0.604	0.0151	0.022	0.615	0.0095	0.023	0.611
	4	2	0.617	0.617	0.617	0.617	0.0041	0.059	0.617	0.0041	0.059	0.617
2	1	1	0.619	0.619	0.619	0.619	0.0024	0.037	0.619	0.0024	0.037	0.619
	2	2	0.614	0.163	0.614	0.614	0.0037	0.112	0.163	0.0029	0.095	0.163
	3	2	0.617	0.617	0.617	0.617	0.0102	0.093	0.617	0.0069	0.069	0.617
	4	3	0.615	0.616	0.616	0.611	0.0037	0.071	0.615	0.0032	0.057	0.616
	5	3	0.614	0.614	0.614	0.614	0.0047	0.068	0.614	0.0047	0.068	0.617
1	1	4	0.619	0.618	0.618	0.618	0.0083	0.113	0.618	0.0074	0.113	0.618
Average			0.615	0.582	0.616	0.613	0.0110	0.140	0.582	0.0076	0.140	0.582

Note: u/s = upstream, d/s = downstream

with a set of associated risks for each of the sewers. This design is the minimum cost design among all the least-cost designs for different risk levels for the specified project life. Moreover, for a given assessed damage scale, if the expected sewer life is 50 yr instead of 25 yr (used in Tables 8.9, 8.10, and 8.11), the least-cost designs would have larger pipes with higher installation costs to offset the increase in expected damage costs.

The effect of the risk component can perhaps be more clearly seen when cost figures are examined. As can be seen from Table 8.11, by comparing the design of Model A to those by Model C, and Model B-1 to Model D, respectively, the general effect of including the risk component in design is to lower the risks by increasing the sewer capacities (and hence increasing the installation costs) to offset the expected damage costs, so that the total cost of the system is minimized. The installation, expected damage, and total costs for the designs using Models A, B-1, C and D are summarized in Table 8.12 for comparison. The damage costs for Models A and B-1, which do not include the risk component, were computed using the risks determined in Table 8.11. In fact, Models A and B-1 can be considered respectively as extensions of Models C and D with an assessed damage scale equal to zero. Note that the risk values for many of the sewers for the Model C design using damage scale C are identical to those of the Model A design, with the average risk being slightly lower.

To design a sewer system for a given drainage area to serve for an expected period, the higher the assessed damage values (e.g., Scale A for Model C in Table 8.11), the smaller is the risk of the least-cost design. However, if the damage costs are small (e.g., Scale C of Model C or Model A in Table 8.11), it is economically justified to use high risk designs; i.e., smaller pipes. This can further be illustrated by comparing the costs for Model C or D designs listed in Table 8.12 using the three different

TABLE 8.12. Cost Comparison for Example I Designs

Model	Cost	Assessed Damage Scale A	Assessed Damage Scale B	Assessed Damage Scale C
A	Install	$ 474,370	$ 474,370	$ 474,370
	Damage	1,198,000	119,800	11,980
	Total	1,672,370	594,170	486,350
B-1	Install	433,016	433,016	433,016
	Damage	1,154,000	115,400	11,540
	Total	1,587,016	548,416	444,556
C	Install	614,823	508,423	474,177
	Damage	28,311	39,340	11,531
	Total	643,134	547,763	485,708
D	Install	530,379	457,465	433,016
	Damage	21,071	45,163	11,537
	Total	551,450	502,628	444,553

assessed damage scales. As the assessed damage values decrease from Scale A to Scale C, both the installation and the total costs decrease. Theoretically, the expected damage cost should also be decreasing monotonically if the pipe sizes were continuous. Correspondingly, the expected damage cost would occupy a smaller percentage of the total cost and the installation cost would occupy an increasing percentage. However, because of the discrete sizes of commercial pipes, there are fluctuations with respect to the general trend of the expected damage cost as shown in Table 8.12 for Models C and D. This also means that for any sewer in the system the risk chosen by the optimization phase may vary over a wide range. Supposedly, the installation cost for the Model A design is equivalent to the case of the Model C design with zero assessed damage values. Hence, the installation cost for Model A design should be slightly less than that for Model C using assessed damage scale C. However, as shown in Table 8.12, the installation cost for the latter is $474,177 whereas that for Model A is $474,370. The reason of this discrepancy is that DDDP does not guarantee global optimality; and as shown in Fig. 8.3, the minimum installation cost for Model A is actually around $472,000, which is about one-half percent lower than the value given in Table 8.12.

Table 8.12 also illustrates the disadvantage of not considering damage costs in design. For example, comparing Models B-1 and D using Scale A, the installation cost for the Model B-1 design is $433,016 which is lower than the $530,379 cost resulting from Model D. However, for the 25-yr design service period the expected damage cost associated with the Model B-1 design is $1,154,000 which is considerably higher than $21,071 for the Model D design. This shows that damage costs are particularly important when the assessed damage values are high and when the expected service life is long.

8.3. Example II

To further illustrate the application of the design models another sewer system is chosen as Example II. This is the sewer system used to illustrate the rational method in ASCE (1969, p. 54) Manual No. 37 and is familiar to many engineers involved in storm drainage design.

8.3.1. Sewer System Description

The layout of the Example II sewer system is reproduced in Fig. 8.7, and its isonodal lines and manholes are shown in Fig. 8.8 together with the corresponding manhole notation used in Fig. 8.7.

The input data for the sewer system required by the design models are summarized in Table 8.13. The Manning roughness factor n is 0.013 for

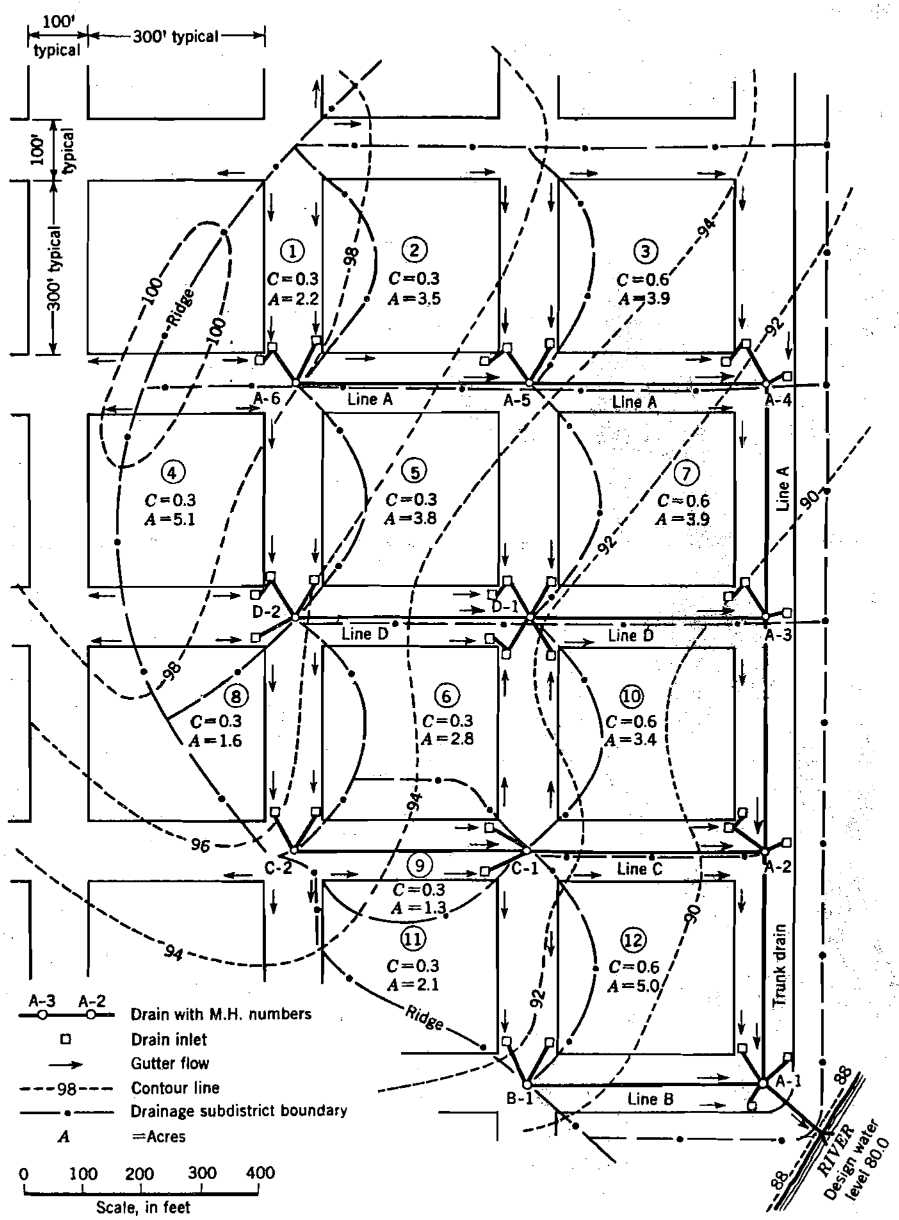

Fig. 8.7. Example II Sewer System Layout (ASCE, 1969)

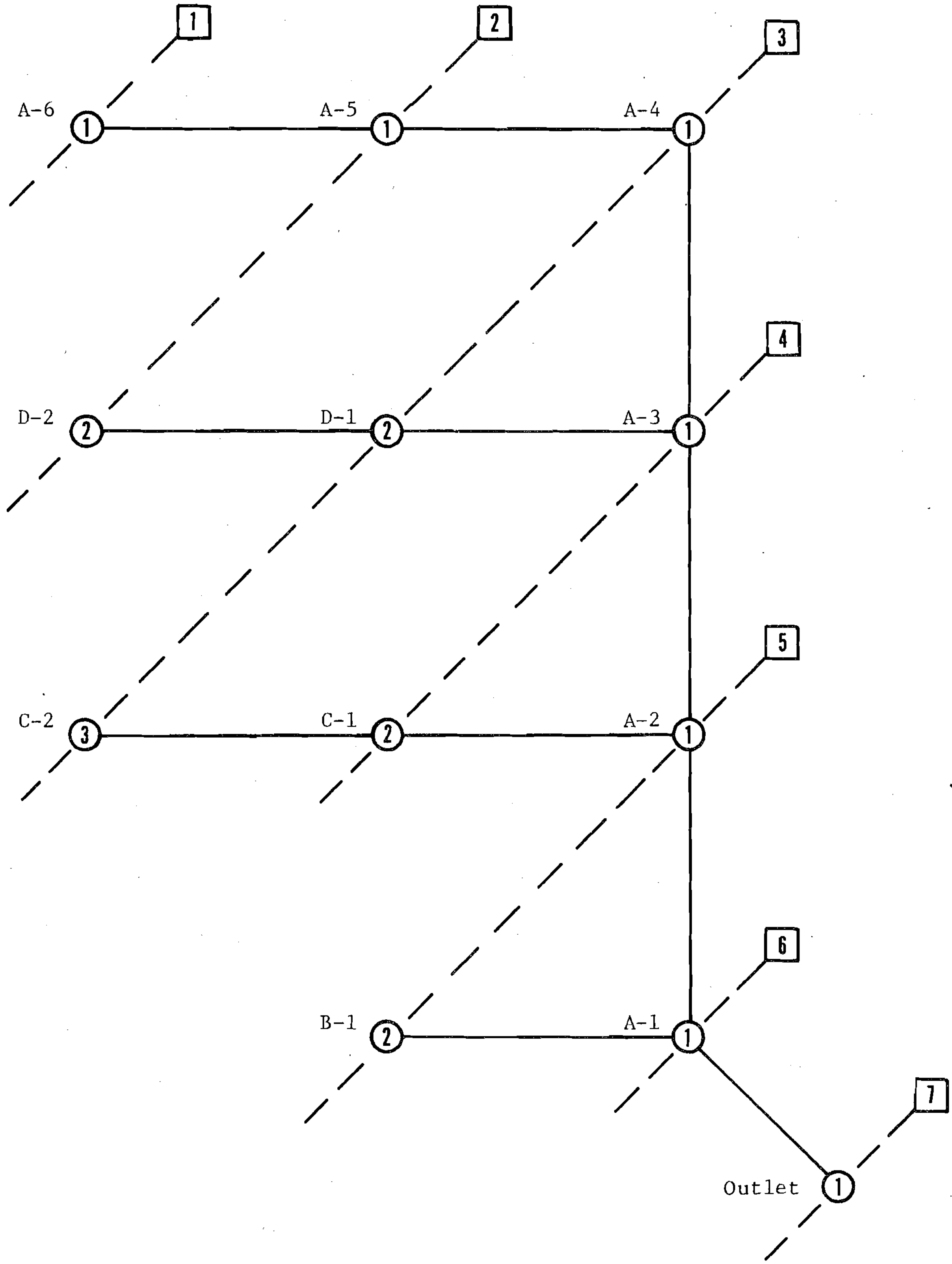

Fig. 8.8. Example II Isonodal Lines

TABLE 8.13. Example II Sewer System Input Data

Isonodal Line	Manhole Number	Ground Elev ft	Downstream Manhole Number	Sewer Length ft	Peak Inflow
1	1	98.4	1	400	2.0
2	1	94.9	1	400	3.1
	2	96.2	2	400	4.7
3	1	91.8	1	400	6.6
	2	92.3	1	400	1.2
	3	94.6	2	400	1.5
4	1	89.7	1	400	10.1
	2	92.7	1	400	1.0
5	1	89.5	1	400	5.1
	2	91.6	1	400	2.0
6	1	88.5	1	125	7.1
7	1	88.0			

all the sewers. The peak inflows are in part taken as the design flows calculated in Table XIII of ASCE (1969) Manual No. 37. However, in that table only the design of Line A in Fig. 8.7 is given. The direct inflows for manholes in Lines B, C and D are computed using the rational formula. For design Models B and D with routing, all the manhole direct inflow hydrographs are assumed to be symmetric and triangular in shape (Fig. 8.2) with a constant base flow Q_b = 0.1 cfs, a base time T = 2400 sec and initial rise time τ = 0. Only the peak flow rate Q_p varies as given in Table 8.13.

In addition, in the design the minimum soil cover depth above the sewer crown is 3.5 ft. The allowable maximum sewer flow velocity is 10 fps and the minimum is 2 fps. For Models B and D, Δx_{max} is 400 ft and Δt is 120 sec. For all the models, the initial corridor width for the optimization procedure is 12 ft with 7 lattice points (and hence initial state increment = 2.0 ft). The nominal sewer elevation drop is (98.4 - 88.0)/6 = 1.7 ft which is considerably smaller than the initial corridor width used. The reduction rate of Δ_s is 1/2.

Again only commercial size pipes are considered in the designs. However, a minimum diameter of 12 in. instead of 8 in. is used since this constraint was imposed in the ASCE design. In applying Models C and D (Table 7.1) to Example II sewer system, for simplicity a constant value of $10,000 is assumed for the assessed damage cost instead of a scale that varies with sewer length for Example I.

8.3.2. Example II Results

The results for this example illustrate the same trends as shown by Example I. The sewer sizes, slopes and crown elevations of the least-cost designs using Models A and B are given in Table 8.14 and Models C and D in Table 8.15. The traditional design using the rational method as given in ASCE (1969) Manual 37 is summarized in Table 8.16 for comparison. In

TABLE 8.14. Least-Cost Designs of Example II Sewer System without Considering Risks

Upstream Isonodal Line	Upstream Manhole	Downstream Manhole	Crown Elevations		Sewer Slope	Sewer Diameter in.
			Upstream ft	Downstream ft		
			Design Using Model A			
6	1	1	84.25	83.00	0.01000	36
5	1	1	85.38	84.25	0.00281	36
	2	1	88.10	85.00	0.00775	12
4	1	1	86.07	85.38	0.00175	36
	2	1	89.20	86.00	0.00800	12
3	1	1	88.30	86.07	0.00556	21
	2	1	88.80	86.20	0.00650	18
	3	2	91.10	89.20	0.00475	12
2	1	1	91.40	88.30	0.00775	15
	2	2	92.70	88.80	0.00975	15
1	1	1	94.90	91.40	0.00875	12

TABLE 8.14. (Continued)

Upstream Isonodal Line	Upstream Manhole	Downstream Manhole	Crown Elevations		Sewer Slope	Sewer Diameter in.
			Upstream ft	Downstream ft		
			Design Using Model B-1			
6	1	1	83.75	83.00	0.00600	36
5	1	1	84.69	83.75	0.00234	36
	2	1	88.10	85.00	0.00775	12
4	1	1	86.20	84.69	0.00378	30
	2	1	89.20	86.00	0.00800	12
3	1	1	88.30	86.20	0.00525	21
	2	1	88.80	86.20	0.00650	18
	3	2	91.10	89.20	0.00475	12
2	1	1	91.40	88.30	0.00775	15
	2	2	92.70	88.80	0.00975	15
1	1	1	94.90	91.40	0.00875	12
			Design Using Model B-2			
6	1	1	83.69	83.00	0.00550	36
5	1	1	84.63	83.69	0.00234	36
	2	1	88.10	85.00	0.00775	12
4	1	1	86.20	84.63	0.00394	30
	2	1	89.20	86.00	0.00800	12
3	1	1	88.30	86.20	0.00525	21
	2	1	88.80	86.20	0.00650	18
	3	2	91.10	89.20	0.00475	12
2	1	1	91.40	88.30	0.00775	15
	2	2	92.70	88.80	0.00975	15
1	1	1	94.90	91.40	0.00875	12
			Design Using Model B-3			
6	1	1	83.50	83.00	0.00400	36
5	1	1	84.38	83.50	0.00219	36
5	2	1	88.10	85.00	0.00775	12
4	1	1	85.82	84.38	0.00362	30
4	2	1	89.20	86.00	0.00800	12
3	1	1	88.30	86.20	0.00525	21
3	2	1	88.80	85.82	0.00744	15
3	3	2	91.10	89.20	0.00475	12
2	1	1	91.40	88.30	0.00775	15
2	2	2	92.70	88.80	0.00975	15
1	1	1	94.90	91.40	0.00875	12

TABLE 8.15. Least-Cost Designs of Example II Sewer System Considering Risks

Upstream Isonodal Line	Upstream Manhole	Downstream Manhole	Crown Elevations		Sewer Slope	Sewer Diameter in.
			Upstream ft	Downstream ft		
Design Using Model C						
6	1	1	84.06	83.00	0.00850	42
5	1	1	85.44	84.06	0.00344	42
	2	1	88.10	84.06	0.01009	12
4	1	1	86.20	85.44	0.00191	42
	2	1	88.76	86.00	0.00691	15
3	1	1	88.30	86.20	0.00525	27
	2	1	88.80	86.20	0.00650	21
	3	2	91.10	88.76	0.00584	12
2	1	1	91.40	88.30	0.00775	18
	2	2	92.70	88.80	0.00975	18
1	1	1	94.90	91.40	0.00875	12
Design Using Model D						
6	1	1	84.13	83.00	0.00900	42
5	1	1	85.50	84.31	0.00297	42
	2	1	88.10	84.13	0.00994	12
4	1	1	86.20	85.50	0.00175	42
	2	1	89.20	85.50	0.00925	12
3	1	1	88.30	86.20	0.00525	24
	2	1	88.80	86.20	0.00650	18
	3	2	91.10	89.20	0.00475	12
2	1	1	91.40	88.30	0.00775	18
	2	2	92.70	88.80	0.00975	18
1	1	1	94.90	91.40	0.00875	12

TABLE 8.16. Design of Example II Sewer System as Given in ASCE Manual 37

Upstream Isonodal Line	Upstream Manhole	Downstream Manhole	Crown Elevations		Sewer Slope	Sewer Diameter in.
			Upstream ft	Downstream ft		
6	1	1	83.55	83.05	0.0040	36
5	1	1	85.15	83.55	0.0040	36
	2	1	85.15	81.55	0.0090	12
4	1	1	86.25	85.05	0.0040	30
	2	1	86.75	83.15	0.0090	12
3	1	1	87.90	90.30	0.0060	21
	2	1	88.05	86.55	0.0070	18
	3	2	89.55	86.75	0.0070	12
2	1	1	91.00	87.40	0.0090	15
	2	2	91.80	87.80	0.0100	15
1	1	1	94.35	90.75	0.0090	12

the Model C and D designs the assessed damage value is $10,000 for each sewer as mentioned previously. It is also assumed that the 25-yr risk-safety factor curve developed in Section 5.4 for Urbana, Illinois is directly applicable to this example without adjustment. The risks associated with the designs using the six least-cost design models as well as the rational method design are given in Table 8.17. The implicit risks for the designs of Models A and B and the ASCE design are computed in a manner as described in Section 8.2.3.3 for Example I. The installation, expected damage, and total costs for the designs are listed in Table 8.18. The same installation cost functions (Eqs. 2.1 and 2.2) and assessed damage value used in Models C and D together with the risks listed in Table 8.17 are used to determine the expected damage costs for the designs of Models A, B and the ASCE rational method.

Considering installation costs only, it is again seen from Table 8.18 that any of the routing techniques lowers this cost whereas the inclusion of the risk component (Models C and D) increases it. By comparing the installation cost of the ASCE design with those for Models A or B, it is clear that the least-cost design models indeed produce improved designs. It should be noted that the total costs and risks (Table 8.17) are also lower for the least-cost designs than the ASCE design. Actually, the savings in installation cost from the ASCE design is considerably more because in the latter design the 30 in. and 36 in. sewer from INL 4 and 6, respectively, are flowing full (Q_C/Q_p = 0.94 and 0.95, respectively) and the next larger size pipes should have been used.

When the associated risks are considered in design, the improvement in total cost and risks of the design is even more significant as demonstrated by the results of Models C and D. In this example, Model D produces a design with a 24% cost savings over the conventional procedure presented in ASCE Manual 37, and the corresponding improvement in the probability of failure from 32% for the latter to 3.25% for the former.

TABLE 8.17. Risks Associated with Example II Designs Using Various Models

Isonodal Line	Manhole		Design Model						ASCE
	u/s	d/s	A	B-1	B-2	B-3	C	D	
6	1	1	0.099	0.247	0.369	0.483	0.0053	0.0005	0.670
5	1	1	0.617	0.591	0.614	0.588	0.0426	0.0232	0.453
	2	1	0.071	0.047	0.047	0.047	0.0236	0.0159	0.039
4	1	1	0.612	0.610	0.617	0.612	0.0653	0.0356	0.685
	2	1	0.310	0.103	0.133	0.107	0.0040	0.0588	0.212
3	1	1	0.609	0.554	0.581	0.550	0.0124	0.0771	0.572
	2	1	0.139	0.088	0.083	0.610	0.0036	0.0794	0.105
	3	2	0.051	0.028	0.028	0.028	0.0208	0.0283	0.009
2	1	1	0.515	0.402	0.460	0.423	0.0210	0.0088	0.451
	2	2	0.205	0.177	0.177	0.177	0.0029	0.0023	0.192
1	1	1	0.044	0.028	0.028	0.028	0.0438	0.0282	0.033
Average			0.297	0.262	0.285	0.332	0.0223	0.0325	0.311

Note: u/s = upstream, d/s = downstream

TABLE 8.18. Cost Comparison for Example II Designs

Model	Execution Time Sec	Cost $		
		Installation	Damage	Total
A	5.4	69,062	32,714	101,776
B-1	11.3	67,001	28,768	95,769
B-2	198.2	67,036	31,383	98,419
B-3	151.5	66,107	36,533	102,264
C	14.2	79,904	2,452	82,356
D	18.4	75,900	3,580	79,480
ASCE	-	70,087	34,700	104,787

The effect of the risk component in design is also seen by the significant reduction in damage costs. It should be emphasized that the $10,000 assigned damage value used is arbitrary and that the effect of the risk component depends on this value as well as on the service life of the sewers. Nevertheless, the importance of including the risk concept in the design process is illustrated.

In example I in terms of sewer sizes, the designs using the least-cost design models are very similar. This is also the case for Example II. Actually the diameters of the sewers are identical for designs using Models B-1, B-2, and the ASCE rational method. However, because the corresponding sewers have different slopes for different designs, the installation costs and associated risks are different. Model B-3 produces a design which differs from Models B-1 and B-2 in sewer sizes by only one sewer: a 15-in. pipe instead of a 18-in. pipe from INL 3, resulting in a lower installation cost but higher risk. The Model A design also differs from Model B-1 design by only one sewer size: a 36-in. pipe instead of a 30-in. from INL 4, resulting in a higher installation cost.

In this example the hydrograph time lag routing method used in Model B-1 resulted in the lowest total cost design as well as the lowest average risk and shortest computer execution time among the three routing models. However, this result is for a small, relatively simple system and it is misleading to draw general conclusions from it. An opposite trend has been observed in Fig. 8.5 for the more complicated Example I sewer system. For the Example II system a difference in the size of one sewer or a significant change of one slope would be enough to change the relative cost. Nevertheless, the results further demonstrate that for least-cost designs using routing and under normal circumstances, the hydrograph time lag technique is prefered because of its relatively short computer execution time.

Chapter 9. CONCLUSIONS AND RECOMMENDATIONS

Several concepts and techniques have been investigated and incorporated into a set of least-cost design models for determining the sizes and slopes of the sewers in a network. The major concepts are:

(a) The application of discrete differential dynamic programming (DDDP) as the basis for flexible least-cost design models.

(b) The inclusion of risk analysis in the design procedure.

(c) The inclusion of expected flood damage costs as part of the total project cost.

(d) The use of flood routing procedures to account for attenuation and lag of in-system hydrographs.

9.1. Conclusions

With recent advancement in computer capabilities, numerical analysis, and optimization techniques, improved methods far superior to the traditional methods for design of sewer systems can be developed. The five least-cost design models developed in this study are examples of such improved new models. These five models all are based on the DDDP approach for optimization but they incorporate different factors in decision making, i.e., routing and/or design risks, as listed in Table 7.1. In application the user may select a model that is most suitable for a particular situation.

The following general conclusions can be made based on the experience gained in this study.

(a) The use of DDDP together with the serial approach described in Chapter 4 provides an efficient basic tool for a least-cost design model for design of sewer systems. The design results include the crown elevations and slope in addition to the diameter of the sewers.

(b) It is possible to analyze the uncertainties involved in the design process and to summarize their effect in terms of the risk or probability of exceeding the capacity of a sewer. Subsequently, the risk can be incorporated in the design of the sewers. This is accomplished through the establishment of the risk-safety factor relationship for the drainage area considered as described in Chapter 5.

(c) By incorporating the risks in the design, the engineer no longer needs to arbitrarily choose the design return period. Instead, the expected service life of the project is first determined. The models will then proceed accordingly to produce a design that gives the lowest total cost, and the result will also specify the risks of the sewers for this least-cost design. The models can also be modified to design with specified expected service life together with maximum acceptable risk of failure during this period.

(d) With the risks of failure evaluated, it is possible to include expected flood damages as part of the total system cost. The design models then produce a design which gives the best trade-off between the installation cost and expected damage cost. The use of such design models requires as an initial step the recognition that flood damage costs are an important consideration and therefore must be included as part of the total system cost.

(e) Incorporation of flood routing into the design models results in a lowering of the cost of the sewer system. This is mainly due to the lag effect in which the peaks of the inlet and in-system hydrographs are out of phase. For large systems the attenuation of the peaks may also become important. The often-used method of simply adding runoff peaks is not necessarily conservative yet results in expensive designs without reducing the overall risk as illustrated in Table 8.17. Normally, for design purposes, the hydrograph time lag procedure described in Section 6.3.2 and incorporated into Models B-1 and D is adequate and is recommended because it requires very little computation time and provides reasonable results. However, in sewer systems where hydrograph attenuation in the sewers is of greater concern and the sewer system is relatively large, either the nonlinear kinematic wave or Muskingum-Cunge procedures may be used, with the latter preferred. More sophisticated routing techniques such as solving the St. Venant equations do not appear practical for use in least-cost design models because of their extensive computer requirements.

(f) Since DDDP does not guarantee a global optimum, to a certain degree, the designs of the sewer systems depend on the optimization parameters used in the DDDP procedure; namely, the location of the initial trial trajectory, the width of the initial corridor enclosing the trial trajectory, the number of lattice points within the corridor (or the initial state increment within the corridor), and the reduction rate of the state increment during iterations. Based on the experience obtained in this research project, it has been found that the closer the presupposed initial trial trajectory is to the true optimal trajectory, the better the resulted design. Since the downstream sewers are usually more expensive because they are larger and buried deeper than the upstream ones, consequently often it is advantageous for the downstream sewers to use an initial trial trajectory with steeper slopes than ground slopes. The preferred initial corridor width is two to five times the ave age elevation drop of the sewers. In addition the use of 5 or 7 lattice points together with a reduction rate of the state increment equal to 1/2 is recommended.

9.2. Recommendations for Future Studies

The sewer system design models presented in this report are only a first step towards the goal of optimal design for entire urban drainage systems. Consequently modification, refinement, and ramification of these models on the basis of experience gained through extensive field applications are most desirable. The proposed design models are clearly more rational than the traditionally used sewer design methods. However, their use requires a recognition of the several concepts involved in sewer system design in addition to the conventional ones so that full advantage can be taken of the capabilities of the design models. Conversely, in view of the limited manpower in government design offices and engineering firms, it is fully realized that there is an urgent need for the development of a user's manual for the design model so that the maximum use of the results of this study can be achieved. This manual should provide a clear guide to the use of the various models so as to make it as easy as possible for the design engineers to obtain results with a minimum amount of effort and investment of time.

Among the numerous possible future studies as a result of this investigation, the following deserve immediate attention.

(a) Since the cost of a sewer system depends on the layout of the sewers, and engineers often do have a limited degree of freedom in selecting the layout, it is desirable to include the layout selection in the optimization procedure. Such a design model without considering the risks and routing has been developed under the partial support of this research project (Mays, 1976). Extension of Mays' model to include routing and risk consideration is being investigated.

(b) For the design models presented in this report inlet hydrographs must be independently developed. It is desirable to have an optional procedure for generating them; i.e., a surface hydrology model is needed. The user should have the option of providing his own hydrographs or utilizing the hydrologic model.

(c) In this study the effects of the hydraulics and costs of all appurtenances and special structures in a sewer system except manholes have been excluded from consideration. For example, in-line detention reservoirs are receiving increased attention and there is a need to consider their effectiveness and use in design.

(d) The effectiveness of the least-cost design bviously depends on the reliability of the cost functions of the sewer system components. Published information on such cost functions are meager and inadequate. Gathering and systematic analysis of the scattered data appeared in the literature such as Engineering News-Records and information from contractors should be carried out to provide such functions.

(e) The example results of Models C and D clearly demonstrate that the assessed damage values due to insufficient sewer capacity is an important factor in determining the least-cost design. The damage value obviously varies with the amount of water during the peak flow period that the sewer cannot carry. At present considerable judgement is required in establishing the assessed damage value. A procedure is needed to estimate damage costs in a more well defined manner. This task is being undertaken under OWRT project B-098-ILL which is the continued phase of this research project.

(f) In this study the risk component has been incorporated in the least-cost design models which have no routing or with hydrograph time lag. Further research is needed to incorporate the risk component in the design models using other hydraulic routing techniques.

(g) Although under normal conditions the costs of operation and maintenance contribute little to the least-cost design of the sewer system, under certain circumstances these costs may become an influential factor. Therefore, inclusion of these costs in the design models should be considered.

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APPENDIX A

VALUES OF CUMULATIVE NORMAL DISTRIBUTION FUNCTION

$$\phi(x) = \int_{-\infty}^{x} p(x)dx, \quad \phi(-x) = \int_{x}^{\infty} p(x)dx$$

-x		0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.	50000	49601	49202	48803	48405	48006	47608	47210	46812	46414
0.1		46017	45620	45224	44828	44433	44038	43644	43251	42858	42465
0.2		42074	41683	41294	40905	40517	40129	39743	39358	38974	38591
0.3		38209	37828	37448	37070	36693	36317	35942	35569	35197	34827
0.4		34458	34090	33724	33360	32997	32636	32276	31918	31561	31207
0.5		30854	30503	30153	29806	20460	29116	28774	28434	28096	27760
0.6		27425	27093	26763	26435	26109	25785	25463	25143	24825	24510
0.7		24196	23885	23576	23270	22965	22663	22363	22065	21770	21476
0.8		21186	20897	20611	20327	20045	19766	19489	19215	18943	18673
0.9		18406	18141	17879	17619	17361	17106	16853	16602	16354	16109
1.0		15866	15625	15386	15151	14917	14686	14457	14231	14007	13786
1.1		13567	13350	13136	12924	12714	12507	12302	12100	11900	11702
1.2		11507	11314	11123	10935	10749	10565	10383	10204	10027	98525
1.3	0.0	96800	95098	93418	91759	90123	88508	86915	85343	83793	82264
1.4		80757	79270	77804	76359	74934	73529	72145	70781	69437	68112
1.5		66807	65522	64255	63008	61780	60571	59380	58208	57053	55917
1.6		54799	53699	52616	51551	50503	49471	48457	47460	46479	45514
1.7		44565	43633	42716	41815	40930	40059	39204	38364	37538	36727
1.8		35930	35148	34380	33625	32884	32157	31443	30742	30054	29379
1.9		28717	28067	27429	26803	26190	25588	24998	24419	23852	23295
2.0		22750	22216	21692	21178	20675	20182	19699	19226	18763	18309
2.1		17864	17429	17003	16586	16177	15778	15386	15003	14629	14262
2.2		13903	13553	13209	12874	12545	12224	11911	11604	11304	11011
2.3		10724	10444	10170	99031	96419	93867	91375	88940	86563	84242
2.4	0.0^{2}	81975	79763	77603	75494	73436	71428	69469	67557	65691	63872
2.5		62097	60366	58677	57031	55426	53861	52336	50849	49400	47988
2.6		46612	45271	43965	42692	41453	40246	39070	37926	36811	35726
2.7		34670	33642	32641	31667	30720	29798	28901	28028	27179	26354
2.8		25551	24771	24012	23274	22557	21860	21182	20524	19884	19262
2.9		18658	18071	17502	16948	16411	15889	15382	14890	14412	13949
3.0		13499	13062	12639	12228	11829	11442	11067	10703	10350	10008
3.1	0.0^{3}	96760	93544	90426	87403	84474	81635	78885	76219	73638	71136
3.2		68714	66367	64095	61895	59765	57703	55706	53774	51904	50094
3.3		48342	46648	45009	43423	41889	40406	38971	37584	36243	34946
3.4		33693	32481	31311	30179	29086	28029	27009	26023	25071	24151
3.5		23263	22405	21577	20778	20006	19262	18543	17849	17180	16534
3.6		15911	15310	14730	14171	13632	13112	12611	12128	11662	11213
3.7		10780	10363	99611	95740	92010	88417	84957	81624	78414	75324
3.8	0.0^{4}	72348	69483	66726	64072	61517	59059	56694	54418	52228	50122
3.9		48096	46148	44274	42473	40741	39076	37475	35936	34458	33037
4.0		31671	30359	29099	27888	26726	25609	24536	23507	22518	21569
4.1		20658	19783	18944	18138	17365	16624	15912	15230	14575	13948
4.2		13346	12769	12215	11685	11176	10689	10221	97736	93447	89337
4.3	0.0^{5}	85399	81627	78015	74555	71241	68069	65031	62123	59340	56675
4.4		54125	51685	49350	47117	44979	42935	40980	39110	37322	35612
4.5		33977	32414	30920	29492	28127	26823	25577	24386	23249	22162
4.6		21125	20133	19187	18283	17420	16597	15810	15060	14344	13660
4.7		13008	12386	11792	11226	10686	10171	96796	92113	87648	83391
4.8	0.0^{6}	79333	75465	71779	68267	64920	67131	58693	55799	53043	50418
4.9		47918	45538	43272	41115	39061	37107	35247	38476	31792	30190

APPENDIX B

MODEL ERROR FOR THE RATIONAL FORMULA

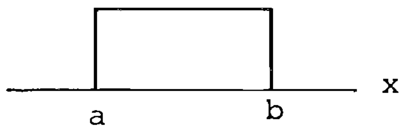
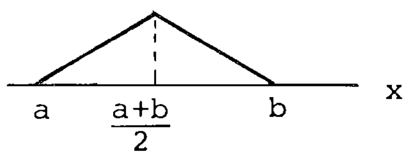
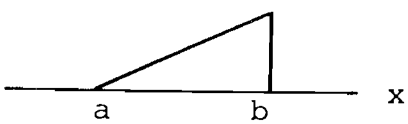
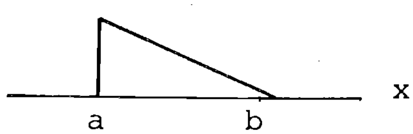
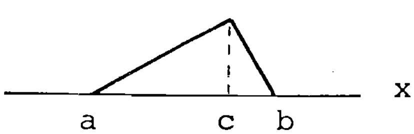
A comparative study on storm sewer runoff simulation models (Chow and Yen, 1975) gave the peak discharges in cfs from the Chicago Oakdale Avenue drainage basin for four independent rainstorms as follows:

	Rainstorm	May 19, 1959	July 2, 1960	April 29, 1963	July 7, 1964
(1)	Recorded	7.2	17.5	6.7	9.6
(2)	ISS	7.4	15.4	6.7	9.9
(3)	Rational	9.1	13.7	5.8	10.0
(4)	$= \frac{(1)}{(3)}$	0.79	1.28	1.16	1.04
(5)	$= \frac{(2)}{(3)}$	0.81	1.12	1.16	1.01

Accordingly, comparing the rational formula to the recorded data, the mean value of λ is $\Sigma(\text{Col.}4)/4 = 1.05$. Var $= \Sigma(\text{Col.}4-1.05)^2/(N-1) = 0.0459$ and the corresponding coefficient of variation $= \sqrt{\text{Var}}/\text{Mean} = 0.205$. Likewise, comparing with the ISS model, mean $\lambda = \Sigma(\text{Col.}5)/4 = 1.02$, Var $= \Sigma(\text{Col.}5-1.02)^2/(N-1) = 0.0243$ and coefficient of variation $= \sqrt{\text{Var}}/\text{Mean} =$ 0.153. Assuming the average of these two gives an indication of the reliability of the rational method, it follows that for the rational method $\bar{\lambda}_L = 1.03$ and coefficient of variation $= (0.205+0.153)/2 = 0.176$. However, neither the recorded data nor the ISS model is absolutely accurate. Moreover, the computed coefficient of variation actually accounts for more than the modeling error because other uncertainties such as those due to C and A, and to certain extent i, are also included. In view of the uncertainties evaluated in Section 5.4.2, it is reasonably to adopt $\Omega_{\lambda L} = 0.15$ with $\bar{\lambda}_L = 1.0$.

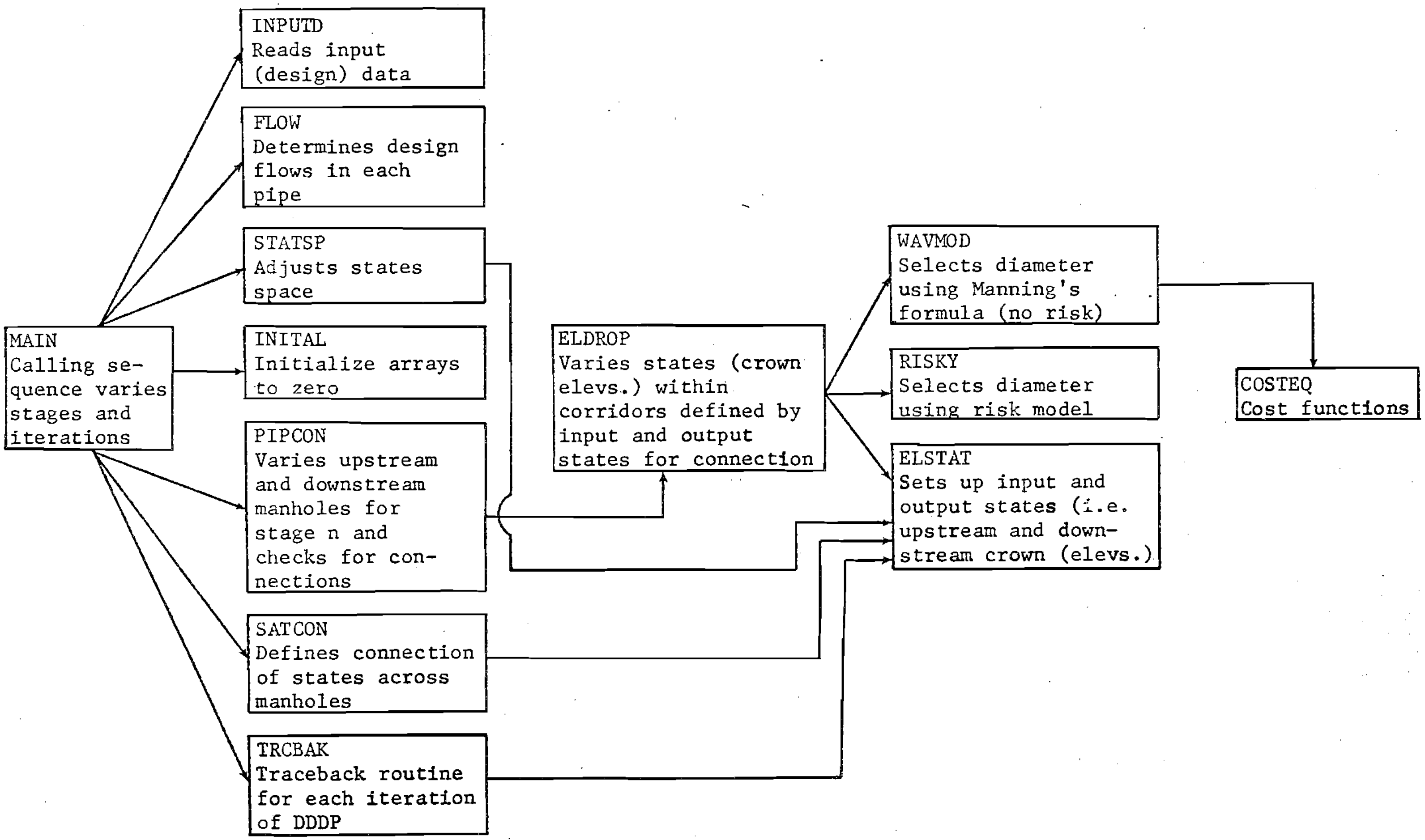
APPENDIX C

STATISTICS OF FIVE SIMPLE DISTRIBUTIONS

	Mean	Coefficient of variation
	$\bar{x} = 0.5(a+b)$	$\delta = 0.578 \frac{b-a}{b+a}$
	$\bar{x} = 0.5(a+b)$	$\delta = 0.408 \frac{b-a}{b+a}$
	$\bar{x} = 0.333(a+2b)$	$\delta = 0.707 \frac{b-a}{2b+a}$
	$\bar{x} = 0.333(2a+b)$	$\delta = 0.707 \frac{b-a}{b+2a}$
	$\bar{x} = \frac{1}{3}(a+b+c)$	$\delta = \left[\frac{1}{2} - \frac{1}{6\bar{x}^2}(ab+bc+ca)\right]^{\frac{1}{2}}$

APPENDIX D

COMPUTER PROGRAM LISTING FOR DESIGN MODELS A AND C



Subroutine Flowchart of ILSSD1 for Design Models A and C

```
C***********************************************************************
C                                                                      *
C*                   III   L      SSSS   SSSS   DD     I               *
C*                    I    L      S      S      D D   II               *
C*                    I    L      S      S      D  D   I               *
C*                    I    L      SSSS   SSSS   D  D   I               *
C*                    I    L         S      S   D  D   I               *
C*                    I    L         S      S   D D    I               *
C*                   III   LLLL   SSSS   SSSS   DD    III              *
C*                                                                     *
C***********************************************************************
C*                                                                     *
C*     ILSSDI (ILLINOIS STORM SEWER DESIGN) MODEL IS USED FOR DESIGNING *
C*     STORM SEWER SYSTEMS WITH GIVEN LAYOUTS WITH OR WITHOUT RISK     *
C*     CONSIDERATIONS.                                                 *
C*                                                                     *
C***********************************************************************
C*                                                                     *
C*              VARIABLES IN THE OPTIMIZATION SCHEME                   *
C*                                                                     *
C***********************************************************************
C
C       ***********************************************************
C       *                                                         *
C       *    MELEV(NDL,MNU,J)      INDEXES OF K ELEVATIONS FROM DOWNSTREAM*
C       *                          MANHOLE FOR ELEVATION J TO UPSTREAM    *
C       *                          MANHOLE MNU ON ISONODAL LINE NDL.      *
C       *    DIAM(NDL,MNU,J)     ARE THE PIPE DIAMETERS.                *
C       *    FUNC(J,MNU,MND) ARE THE COSTS AT DOWNSTREAM ELEVATION J FOR *
C       *                    CONNECTION TO MND FROM UPSTREAM MANHOLE MNU.*
C       *                    THIS IS A CUMULATIVE SUM OF COSTS FOR ALL   *
C       *                    UPSTREAM CONNECTING PIPES AND MANHOLES.     *
C       *    TCN(NDL,MNU,MND) IS THE VECTOR OF CONNECTIVITY FOUND AFTER  *
C       *                     OPTIMIZATION OVER THE SEWER NETWORK AFTER  *
C       *                  EACH ITERATION.                               *
C       *    MJTOJJ(NDL+1,MNU,JJ)      IS THE INDEX OF UPSTREAM ELEVATIONS*
C       *                              J ACROSS THE MANHOLE MND ON ISO-  *
C       *                              NODAL LINE NDL+1 THAT REFERS TO   *
C       *                              UPSTREAM MANHOLE MNU ON DRAINAGE  *
C       *                              LINE NDL.                         *
C       *    FCUP(JJ,MAN) ARE THE UPSTREAM (AT MANHOLE MAN) COSTS FOR    *
C       *                 CROWN ELEVATIONS JJ.                           *
C       *                                                                *
C       ***********************************************************
        COMMON/AREA1/NELEV,AN,VMAX,DSTATE,NDLT,COVMIN,DSMIN,NWRIT
        COMMON/AREA2/PIPLTH,QINF,T
        COMMON/AREA3/MELEV,DIAM
        COMMON/AREA4/FUNC,PIPFLO
        COMMON/AREA5/GELEV,ELVMID,ELVMUP,MN
        COMMON/AREA6/MJTOJJ
        COMMON/AREA8/FCUP
        COMMON/AREA9/DAMCST,RISKS
        COMMON/AREA10/ MODEL
        COMMON/AREA11/IDSTAT
        COMMON/AREA12/ ALPHA1,ALPHA2,BETA1,BETA2
        DIMENSION PIPLTH(10,5),QINF(10,5)
        INTEGER*2 MELEV(10,5,9),DIAM(10,5,9)
        DIMENSION FUNC(9,5,5),PIPFLO(10,5)
        DIMENSION GELEV(10,5),ELVMID(10,5),ELVMUP(10,5,5),MN(10)
        INTEGER*2 MJTOJJ(10,5,9)
        INTEGER*2 T(10,5,5)
        DIMENSION FCUP(9,5)
        DIMENSION DAMCST(10,5),RISKS(10,5,9)
        INTEGER*2 IDSTAT(10,5,2)
        DIMENSION ELH(2,5)
        NWRIT = 2
        NWRIT = 0
        RV = 0.2
        CALL INPUTD
        CALL FLOW
        WRITE(6,1300)
        NDLT1 = NDLT - 1
C       **************************************
C       * VARY ITERATION OF DDDP OPTIMIZATION *
C       **************************************
           DO 500 ITER=1,10
           IF(ITER. EQ .5) DSTATE = 4.*DSTATE
           IF(DSTATE. LT .DSMIN) GO TO 510
           CALL STATSP
           CALL INITAL(NELEV,NDLT,MN)
              DO 400 NDL=1,NDLT1
              CALL PIPCON(NDL,ELH,ITER)
              IF(NDL. EQ .NDLT1) GO TO 410
              CALL SATCON(NDL)
  400         CONTINUE
  410      CONTINUE
C          ********************************************
C          * CALL TRACEBACK TO FIND MINIMUM COST DESIGN *
C          ********************************************
           CALL TRCBAK(TCOST,ITER)
           IF(ITER. EQ .1) GO TO 440
C          ************************************************************
C          *  CRITERIA TO CHANGE DSTATE FOR THE NEXT ITERATION.       *
C          *     TCOST = MINIMUM COST OF THE SYSTEM DESIGN FOR THIS ITER. *
C          *     TCST = MINIMUM COST OF THE SYSTEM DESIGN FOR THE PREVIOUS*
C          *            ITERATION.                                    *
C          *     ERRCOS = RELATIVE CHANGE IN COST ALLOWED FOR SUCCESSIVE *
C          *              ITERATIONS BEFORE DSTATE IS CHANGED.        *
C          ************************************************************
           RV = RV - 0.02
           IF(RV. LE .0.02) RV = 0.02
           ERRCOS = RV*TCST
           IF(ABS(TCOST-TCST). LE .ERRCOS) GO TO 440
           TCST = TCOST
           GO TO 500
  440      CONTINUE
           TCST = TCOST
           DSTATE = DSTATE/2.
  500      CONTINUE
  510   CONTINUE
 1300   FORMAT(1H1)
        STOP
        END
```

```
0001          SUBROUTINE INPUTO
        C     ****************************************************************
        C*    *                                                              *
        C     *  THIS SUBROUTINE READS IN ALL THE INPUT DATA                 *
        C*    *                                                              *
        C     ****************************************************************
0002          COMMON/AREA1/NELEV,AN,VMAX,DSTATE,NDLT,COVMIN,DSMIN,NWRIT
0003          COMMON/AREA2/PIPLTH,QINF,T
0004          COMMON/AREA5/GELEV,ELVMID,ELVMUP,MN
0005          COMMON/AREA9/DAMCST,RISKS
0006          COMMON/AREA10/ MODEL
0007          COMMON/AREA11/IOSTAT
0008          COMMON/AREA12/ ALPHA1,ALPHA2,BETA1,BETA2
0009          DIMENSION PIPLTH(10,5),QINF(10,5)
0010          DIMENSION GELEV(10,5),ELVMID(10,5),ELVMUP(10,5,5),MN(10)
0011          INTEGER*2 T(10,5,5)
0012          DIMENSION DAMCST(10,5),RISKS(10,5,9)
0013          INTEGER*2 IOSTAT(10,5,2)
        C     ****************************************************************
        C*    *                                                              *
        C     *      MODEL IS THE MODEL DESIGNATION: = 2 NO RISK             *
        C     *                                      = 3 INCLUDES RISK       *
        C     *                                                              *
        C     *     AN IS MANNING'S ROUGHNESS FACTOR                         *
        C     *     VMAX IS THE MAXIMUM PERMISSIBLE VELOCITY ALLOWED IN A PIPE *
        C     *     DSTATE IS THE DISTANCE BETWEEN CROWN ELEVATIONS AT A MANHOLE*
        C     *     COVMIN IS THE MINIMUM COVER DEPTH FOR EACH PIPE.         *
        C     *     DSMIN IS THE MINIMUM DISTANCE BETWEEN CROWN ELEVATIONS   *
        C     *           THIS IS THE CRITERIA TO STOP THE OPTIMIZATION .    *
        C     *     NELEV IS THE NUMBER OF CROWNS ELEVATIONS (LATTICE POINTS) *
        C     *           AT EACH MANHOLE TO ESTABLISH CORRIDORS.            *
        C     *     NDLT IS THE TOTAL NUMBER OF ISONODAL LINES.  THE NUMBER  *
        C     *           OF DDDP STAGES BEING (NDLT-1).                     *
        C     *     MN(NDL) IS THE NUMBER OF MANHOLES ON ISONODAL LINE NDL.  *
        C     *     ALPHA1, ALPHA2, BETA1, BETA2 ARE CONSTANTS  FOR THE RISK- *
        C     *           SAFETY FACTOR RELATIONSHIP.                        *
        C*    *                                                              *
        C     ****************************************************************
        C     ******************
        C     * READ INPUT DATA *
        C     ******************
0014          READ(5,1100) MODEL
0015          READ(5,1300) AN,VMAX,COVMIN,DSTATE,DSMIN
0016          READ(5,1500) NELEV,NDLT,MANT
0017          READ(5,2000) (MN(I),I=1,NDLT)
0018          READ(5,1600) ALPHA1,ALPHA2,BETA1,BETA2
        C     ****************************************************************
        C*    *                                                              *
        C     *     GELEV(NDL,MAN) ARE THE GROUND ELEVATIONS AT MANHOLE MAN ON *
        C     *                    ISONODAL LINE NDL.                        *
        C     *     ELVMUP(NDL,MNU,MND) ARE THE ELEVATIONS OF THE MIDDLE LATTICE*
        C     *                         POINTS(CROWN ELEVATIONS) ON THE UPSTREAM*
        C     *                         SIDE OF MANHOLE MND ON ISONODAL LINE NDL*
        C     *                         THAT CONNECTS TO UPSTREAM MANHOLE MNU ON*
        C     *                         ISONODAL LINE NDL-1.                 *
        C     *     ELVMID(NDL,MAN) ARE THE ELEVATIONS OF THE MIDDLE LATTICE *
        C     *                     POINTS(CROWN ELEVATIONS) AT DOWNSTREAM SIDE *
        C     *                     OF MANHOLE MAN ON ISONODAL LINE NDL.     *
        C     *     PIPLTH(NDL,MNU)     ARE PIPE LENGTHS FROM MNU TO MND WHERE *
        C     *                         MNU IS ON ISONODAL LINE NDL.         *
        C     *     QINF(NDL,MAN) ARE THE MANHOLE INFLOWS FROM SURFACE.      *
        C     *     T(NDL,MNU,MND) IS THE ARRAY OF POSSIBLE CONNECTIONS WITH *
        C     *                    UPSTREAM MANHOLE MNU ON ISONODAL LINE NDL. *
        C*    *                                                              *
        C     ****************************************************************
0019          DO 100 N=1,NDLT
0020            MAN = MN(N)
0021            READ(5,2500) (GELEV(N,I),I=1,MAN)
0022            READ(5,2500) (ELVMUP(N,I,1),I=1,MAN)
0023            READ(5,2500) (ELVMID(N,I),I=1,MAN)
0024            READ(5,4000) (PIPLTH(N,I),I=1,MAN)
0025            IF(MODEL. LE .2) GO TO 90
0026            READ(5,5000) (DAMCST(N,I),I=1,MAN)
0027     90     CONTINUE
0028            READ(5,3000) (QINF(N,I),I=1,MAN)
0029            READ(5,2000) (IOSTAT(N,I,1),I=1,MAN)
0030            READ(5,2000) (IOSTAT(N,I,2),I=1,MAN)
0031    100   CONTINUE
0032          NDLT1 = NDLT-1
0033            DO 500 KI=1,NDLT1
0034            MNU = MN(KI)
0035            MND = MN(KI+1)
0036              DO 400 N=1,MNU
0037              READ(5,2000) (T(KI,N,M),M=1,MND)
0038    400       CONTINUE
0039    500   CONTINUE
        C     ************************
        C     * WRITE OUT INPUT DATA *
        C     ************************
0040          WRITE(6,6700)
0041          WRITE(6,6000) NELEV,NDLT,MANT
0042          WRITE(6,6100) DSTATE,COVMIN,AN,VMAX
0043          WRITE(6,6400)
0044            DO 650 N=1,NDLT
0045            MAN = MN(N)
0046              DO 625 I=1,MAN
0047              WRITE(6,6300) N,I,GELEV(N,I),QINF(N,I)
0048    625       CONTINUE
0049    650   CONTINUE
0050          WRITE(6,6450)
0051          DO 700 KI=1,NDLT1
0052            KII = KI + 1
0053            WRITE(6,6500) KI,KII
0054            MNU = MN(KI)
0055            MND = MN(KI+1)
0056            DO 675 N=1,MNU
0057              DO 660 M=1,MND
0058                IF(T(KI,N,M). NE .1) GO TO 660
0059                WRITE(6,6600) N,M,PIPLTH(KI,N)
0060    660       CONTINUE
0061    675     CONTINUE
0062    700   CONTINUE
0063          WRITE(6,6700)
0064          WRITE(6,7000)
0065            DO 750 N=1,NDLT
0066            M = MN(N)
0067            IF(N. NE .1) MU = MN(N-1)
0068            WRITE(6,7100) N
0069              DO 740 MM=1,M
```

```
      IF(N. EQ .1) GO TO 735
      DO 730 MMM=1,MO
      ELVMUP(N,MMM,MM) = ELVMUP(N,1,MM)
730   CONTINUE
735   CONTINUE
      WRITE(6,7200) MM,ELVMUP(N,1,MM),ELVMID(N,MM)
740   CONTINUE
750   CONTINUE
C     ***********************************
C     * FORMAT STATEMENTS FOR INPUT DATA *
C     ***********************************
1000  FORMAT(5F10.0)
1100  FORMAT(I5)
1500  FORMAT(3I5)
1600  FORMAT(4F10.0)
2030  FORMAT(40I2)
2500  FORMAT(14F5.0)
3000  FORMAT(14F5.0)
4000  FORMAT(14F5.0)
5000  FORMAT(14F5.0)
C     ***********************************************
C     * FORMAT STATEMENTS FOR PRINTOUT OF INPUT DATA *
C     ***********************************************
6000  FORMAT(5X,'LATTICE POINTS =',I2,/,
     $       5X,'ISCNODAL LINES =',I3,/,
     $       5X,'NUMBER OF MANHOLES =',I3)
6100  FORMAT(5X,'INITIAL LATTICE POINT INCREMENT =',F4.2,/,
     $       5X,'MINIMUM COVER DEPTH =',F4.2,/,
     $       5X,'MANNINGS  ROUGHNESS FACTOR =',F7.4,/,
     $       5X,'MAXIMUM ALLOWABLE VELOCITY =',F6.2)
6300  FORMAT(17X,I2,10X,I2,7X,F7.2,6X,F6.2)
6400  FORMAT(//,5X,'MANHOLE INFORMATION:',/,10X,'ISCNODAL LINE   MANHOLE
     $   GROUND ELEV.    INFLOW')
6450  FORMAT(1H1,//,5X,'POSSIBLE CONNECTIONS')
6500  FORMAT(5X,'BETWEEN ISCNODAL LINES ',I2,' AND',I2)
6600  FORMAT(10X,'UPS MAN ',I2,' TO DNS MAN ',I2,': PIPE LENGTH =',F5.0,
     $' FT')
6700  FORMAT(1H1)
7000  FORMAT     (//,5X,'INITIAL CROWN ELEVATIONS TO ESTABLISH FIRST TRIA
     $L TRAJECTORY')
7100  FORMAT(/,10X,'ISCNODAL LINE ',I2)
7200  FORMAT(12X,'MANHOLE NO. ',I2,': CROWN ELEV. ARE ',F6.2,'FT AT UPS
     $AND ',F6.2,' FT AT DNS')
      RETURN
      END
```

```
      SUBROUTINE STATSP
C     ***************************************************************
C     * THIS SUBROUTINE CHECKS THE STATE SPACE (CROWN ELEVATIONS) FOR *
C     * FOR EACH OF THE POSSIBLE CONNECTIONS INTO AND OUT OF EACH     *
C     * MANHOLE IN THE SYSTEM.  THE PURPOSE IS TO REDEFINE THE STATE  *
C     * ALWAYS MAINTAINING THE IMPROVED TRAJECTORY BUT ELIMINATING    *
C     * DUPLICATE CROWN ELEVATIONS.                                   *
C     ***************************************************************
C
      COMMON/AREA1/NELEV,AN,VMAX,DSTATE,NDLT,COVMIN,DSMIN,NWRIT
      COMMON/AREA2/PIPLTH,QINF,T
      COMMON/AREA5/GELEV,ELVMID,ELVMUP,MN
      COMMON/AREA11/IDSTAT
      DIMENSION PIPLTH(10,5),QINF(10,5)
      DIMENSION GELEV(10,5),ELVMID(10,5),ELVMUP(10,5,5),MN(10)
      INTEGER*2 T(10,5,5)
      INTEGER*2 IDSTAT(10,5,2)
      DIMENSION ELH(2,9)
      NNEL = NELEV/2 + 1
      NNEE = NELEV - 1
      JEQMN = NNEL - 1
      DO 1000 NDL=1,NDLT
      MANT = MN(NDL)
        DO 900 MAN=1,MANT
        IF(NDL. EQ .NDLT) GO TO 530
        IF(IDSTAT(NDL,MAN,2). EQ .0) GO TO 530
C     ***************************************************************
C     *  COMPUTE CROWN ELEVATIONS ON DOWNSTREAM SIDE OF MANHOLE.     *
C     ***************************************************************
        ELMID = ELVMID(NDL,MAN)
        NNUD = 2
        ELMAXD = GELEV(NDL,MAN) - COVMIN
        CALL ELSTAT(NNUD,NELEV,NNEL,DSTATE,ELMID,ELH,ELMAXD,NDL,MAN,2)
        JEQ = 0
          DO 500 JJ=2,NELEV
500       IF(ELH(2,JJ). EQ .ELH(2,JJ-1)) JEQ = JJ - 1
        IF(JEQ. EQ .0) GO TO 530
        IF(JEQ. LE .JEQMN) ELVMID(NDL,MAN) = ELH(2,NNEL+JEQ)
        IF(JEQ. LE .JEQMN) GO TO 530
        JE = 0
          DO 520 JJQQ=NNEL,NNEE
          JE = JE + 1
          XJE = JE
          IF(JEQ. NE .JJQQ) GO TO 520
          ELVMID(NDL,MAN) = ELH(2,NELEV) - XJE*DSTATE
          GO TO 525
520       CONTINUE
525     CONTINUE
530     CONTINUE
        IF(NDL. EQ .1) GO TO 900
C     ***************************************************************
C     * VARY MANHOLES ON UPSTREAM ISCNODAL LINE (NDL-1) TO CHECK     *
C     * FOR POSSIBLE CONNECTIONS TO MANHOLE MAN FOR WHICH CROWN      *
C     * ELEVATIONS ARE BEING CHECKED TO REDEFINE STATE SPACE FOR     *
C     * UPSTREAM END OF MAN.                                         *
C     ***************************************************************
        NNUD = 1
        ELMAXU = GELEV(NDL,MAN) - COVMIN
        MNUT = MN(NDL-1)
```

```
              IF(IDSTAT(NEL,MAN,1). EQ .0) GO TO 900
                 DO 600 MNU=1,MNUT
                 TI = T(NDL-1,MNU,MAN)
                 IF(TI. EQ .0.0) GO TO 600
C
C                **********************************************************
C                *  COMPUTE CROWN ELEVATIONS ON UPSTREAM SIDE OF MANHOLE   *
C                *  MAN FOR THE POSSIBLE CONNECTION FROM UPSTREAM MANHOLE  *
C                *  (MNU) ON NDL-1.                                        *
C                **********************************************************
                 ELMID = ELVMUP(NDL,MNU,MAN)
                 CALL ELSTAT(NNUD,NELEV,NNEL,DSTATE,ELMID,ELH,ELMAXU,NDL,MAN,
     $           1)
                 JEQ = 0
                    DO 560 KK=2,NELEV
  560               IF(ELH(1,KK). LE .ELH(1,KK-1)) JEQ = KK - 1
                 IF(JEQ. EQ .0) GO TO 590
              IF(JEQ. LE .JEQMN) ELVMUP(NDL,MNU,MAN) = ELH(1,NNEL+JEQ)
              IF(JEQ. LE .JEQMN) GO TO 590
              JE = 0
                 DO 570 JJQQ=NNEL,NNEE
                 JE = JE + 1
                 XJE = JE
                 IF(JEQ. NE .JJQQ) GO TO 570
                 ELVMUP(NDL,MNU,MAN) = ELH(1,NELEV) - XJE*DSTATE
                 GO TO 575
  570            CONTINUE
  575         CONTINUE
  590            CONTINUE
  600            CONTINUE
  900         CONTINUE
 1000    CONTINUE
C        **********************************************************
C        **********************************************************
         RETURN
         END
```

```
         SUBROUTINE SATCUN(NDL)
C        **********************************************************
C        * THIS SUBROUTINE DEFINES THE CONNECTIVITY ACROSS THE MANHOLES  *
C        *    ON ISONODAL LINE NDL+1 FOR THE NEXT OPTIMIZATION STAGE     *
C        *    BETWEEN (NDL+1) AND (NDL+2).  ALSO DEFINED ARE THE COSTS,  *
C        *    FCUP(JJ,MAND) FOR EACH MANHOLE ON ISONODAL LINE (NDL+1).   *
C        *    THESE ARE THE UPSTREAM COSTS FOR THE NEXT OPTIMIZATION STAGE.*
C        **********************************************************
         COMMON/AREA1/NELEV,AN,VMAX,DSTATE,NDLT,COVMIN,DSMIN,NWRIT
         COMMON/AREA2/PIPLTH,CINF,T
         COMMON/AREA4/FUNC,PIPFLC
         COMMON/AREA5/GELEV,ELVMID,ELVMUP,MN
         COMMON/AREA6/MJTCJJ
         COMMON/AREA8/FCUP
         DIMENSION PIPLTH(10,5),CINF(10,5)
         DIMENSION FUNC(9,5,5),PIPFLC(10,5)
         DIMENSION GELEV(10,5),ELVMID(10,5),ELVMUP(10,5,5),MN(10)
         INTEGER*2 MJTCJJ(10,5,9)
         INTEGER*2 T(10,5,5)
         DIMENSION FCUP(9,5)
         DIMENSION ELMAN(2,9),KMAN(3)
         NNEL = NELEV/2 + 1
         MNDT = MN(NDL+1)
         NDL1 = NDL + 1
         IF(NWRIT. LT .2) GO TO 90
         WRITE(6,6100) NDL1
         WRITE(6,6500)
   90    CONTINUE
C           **********************************************************
            DO 500 MAND=1,MNDT
C           **********************************************************
C           * NEXT COMPUTE ELEVATIONS AT DOWNSTREAM SIDE OF MANHOLE MAND  *
C           **********************************************************
            ELMID = ELVMID(NDL+1,MAND)
            NNUD = 2
            NDLN = NDL + 1
            ELMAXD = GELEV(NDL+1,MAND) - COVMIN
            CALL ELSTAT(NNUD,NELEV,NNEL,DSTATE,ELMID,ELMAN,ELMAXD,NDLN,MAND
     $      ,2)
  100       CONTINUE
            DO 125 JP=1,NELEV
            FCUP(JP,MAND) = 0.0
  125       CONTINUE
            MNUT = MN(NDL)
               DO 400 MANU=1,MNUT
               IF(T(NDL,MANU,MAND). NE .1) GO TO 400
C              **********************************************************
C              *  COMPUTE ELEVATIONS AT UPSTREAM SIDE OF MANHOLE, MAND.   *
C              **********************************************************
               ELMID = ELVMUP(NDL+1,MANU,MAND)
               NNUD = 1
               NDLN = NDL + 1
               ELMAXU = GELEV(NDL+1,MAND) - COVMIN
               CALL ELSTAT(NNUD,NELEV,NNEL,DSTATE,ELMID,ELMAN,ELMAXU,NDLN,
     $         MAND,1)
  130          CONTINUE
C              **********************************************************
C              * LOOK AT EACH POSSIBLE ELEVATION ON DOWNSTREAM SIDE OF    *
C              *    MANHOLE MAND.                                        *
```

```
C              ***********************************************************
                   DO 300 JJ=1,NELEV
                   FT = 1.0E25
                   KKK = 0
C                  *******************************************************
C                  * LOOK AT EACH POSSIBLE ELEVATION ON UPSTREAM SIDE OF  *
C                  *    MANHOLE, MANU, TO FIND NUMBER OF FEASIBLE CONNEC- *
C                  *    TIONS.                                             *
C                  *******************************************************
                      DO 150 KK=1,NELEV
                      IF(ELMAN(1,KK). GE .ELMAN(2,JJ)) KKK = KK
      150             CONTINUE
                   IF(KKK. EQ .0) GO TO 290
C                  *******************************************************
C                  * NOW OF THE FEASIBLE POSSIBLE CONNECTIONS FIND THE    *
C                  *    CHEAPEST.                                          *
C                  *******************************************************
                   KK4 = 0
                      DO 200 KMIN=1,KKK
                      FT1 = FUNC(KMIN,MANU,MAND)
                      IF(FT1. GE .1.0E25) GO TO 200
                      IF(FT1. LE .FT) KK4 = KMIN
                      IF(FT1. LE .FT) FT = FT1
      200             CONTINUE
                   FCUP(JJ,MAND) = FCUP(JJ,MAND) + FT
                   MJTOJJ(NDL+1,MANU,JJ) = KK4
                   IF(NWRIT. LT .2) GO TO 280
                   WRITE(6,6200) MANU,MAND,KK4,JJ,FCUP(JJ,MAND)
      280          CONTINUE
                   GO TO 300
      290          CONTINUE
                   FCUP(JJ,MAND) = 1.0E25
                   KK4 = 0
                   IF(NWRIT. LT .2) GO TO 300
                   WRITE(6,6200) MANU,MAND,KK4,JJ,FCUP(JJ,MAND)
      300          CONTINUE
                DO 350 JCHECK=1,NELEV
                IF(MJTOJJ(NDL+1,MANU,JCHECK). NE .0) GO TO 400
      350       CONTINUE
                ELVMID(NDL+1,MAND)    = ELVMID(NDL+1,MAND)    - DSTATE/2.
                ELMID = ELVMID(NDL+1,MAND)
             NDLN = NDL + 1
                NNUD = 2
                CALL ELSTAT(NNUD,NELEV,NNEL,DSTATE,ELMID,ELMAN,ELMAXD,NDLN,
        $       MAND,2)
                GO TO 130
      400       CONTINUE
C               **********************************************************
      500    CONTINUE
C               **********************************************************
C        ****************************************************************
C               **********************************************************
     6100 FORMAT(//,15X,'MANHOLE CONNECTIONS FOR ISONODAL LINE',I2)
     6200 FORMAT(20X,I3,7X,I3,4X,I3,2X,I2,2X,F10.0)
     6500 FORMAT(//,20X,'UP MAN    DN MAN    KK   JJ   COST AT JJ',/)
          RETURN
          END



          SUBROUTINE PIPCON(NDL,ELH,ITER)
C         ***************************************************************
C*                                                                      *
C*        THIS SUBROUTINE VARIES THE POSSIBLE CONNECTIONS AT EACH DDDP  *
C*        STAGE BY VARYING THE MANHOLES ON THE UPSTREAM AND DOWNSTREAM  *
C*        ISONODAL LINE OF THE STAGE AND CHECKING THE VECTOR OF POSSIBLE *
C*                                                                      *
C         ***************************************************************
          COMMON/AREA1/NELEV,AN,VMAX,DSTATE,NDLT,COVMIN,DSMIN,NWRIT
          COMMON/AREA2/PIPLTH,CINF,T
          COMMON/AREA3/MELEV,DIAM
          COMMON/AREA4/FUNC,PIPFLO
          COMMON/AREA5/GELEV,ELVMID,ELVMUP,MN
          COMMON/AREA6/MJTOJJ
          COMMON/AREA8/FCUP
          DIMENSION PIPLTH(10,5),CINF(10,5)
          INTEGER*2 MELEV(10,5,5),DIAM(10,5,9)
          DIMENSION FUNC(9,5,5),PIPFLO(10,5)
          DIMENSION GELEV(10,5),ELVMID(10,5),ELVMUP(10,5,5),MN(10)
          INTEGER*2 MJTOJJ(10,5,9)
          INTEGER*2 T(10,5,5)
          DIMENSION FCUP(9,5)
          DIMENSION ELH(2,9),EL(2,9)
          IF(NWRIT. LT .2) GO TO 20
          WRITE(6,1300)
          WRITE(6,1400)
          WRITE(6,1500)
          WRITE(6,1600) ITER
          WRITE(6,1700) NDL
          WRITE(6,1500)
          WRITE(6,1400)
       20 CONTINUE
          NNEL = NELEV/2 + 1
          MNUT = MN(NDL)
          MNDT = MN(NDL+1)
          DO 60 MNU=1,MNUT
             DO 50 MND=1,MNDT
                DO 40 J=1,NELEV
                FUNC(J,MNU,MND) = 1.0E25
       40       CONTINUE
       50    CONTINUE
       60 CONTINUE
C            ********************************************************
C            * VARY MANHOLES ON DOWNSTREAM ISONODAL LINE FOR STAGE NDL *
C            ********************************************************
             DO 200 MND =1,MNDT
C               ******************************************************
C               * VARY MANHOLES ON UPSTREAM ISONODAL LINE FOR STAGE NDL *
C               ******************************************************
                DO 100 MNU=1,MNUT
                IF(T(NDL,MNU,MND). EQ .0) GO TO 100
                ELMID = ELVMUP(NDL+1,MNU,MND)
                NNUD = 2
                NDLN = NDL + 1
                ELMAXD = GELEV(NDL+1,MND) - COVMIN
                CALL ELSTAT(NNUD,NELEV,NNEL,DSTATE,ELMID,ELH,ELMAXD,NDLN,
        $       MND,1)
                FLL = ELVMID(NDL,MNU)
                NNUD = 1
```

```
      ELMAXU = GELEV(NDL,MNU) - COVMIN
      CALL ELSTAT(NNUD,NELEV,NNEL,DSTATE,ELL,ELF,ELMAXU,NDL,MNU,2)
      IF(NWRIT. LT .2) GO TO 95
      WRITE(6,1000) MNU,MND,NCL
         DO 90 NP=1,NELEV
         WRITE(6,1100) ELH(1,NP),ELH(2,NP)
   90    CONTINUE
   95 CONTINUE
      CALL ELDROP(NDL,MNU,MND,ELMID,NNEL,ELH)
  100 CONTINUE
C     ******************************************************
C     * END VARIATION OF MANHOLES ON UPSTREAM ISONODAL LINE *
C     ******************************************************
  200 CONTINUE
C     ********************************************************
C     * END VARIATION OF MANHOLES ON DOWNSTREAM ISONODAL LINE *
C     ********************************************************
 1000 FORMAT(//,5X,'CORRIDOR FROM UPSTREAM MANHOLE ',I2,' TO DOWNSTREAM
     $MANHOLE',I2,' AT STAGE ',I2,/,15X,'ELEVATIONS: UPSTREAM  DOWNSTREA
     $M')
 1100 FORMAT(28X,F6.2,5X,F6.2)
 1300 FORMAT(1H1)
 1400 FORMAT(15X,'********************')
 1500 FORMAT(15X,'*',18X,'*')
 1600 FORMAT(15X,'*  ITERATION ',I2,4X,'*')
 1700 FORMAT(15X,'*  DDDP STAGE ',I3,'  *')
      RETURN
      END
```

```
      SUBROUTINE ELDROP(NDL,MNU,MND,ELMID,NNEL,ELH)
C     *****************************************************************
C*    THIS SUBROUTINE VARIES THE INPUT STATES (UPSTREAM CROWN ELEVS.)  *
C*    AND THE OUTPUT STATES (DOWNSTREAM CROWN ELEVS.) IN THE CORRIDOR  *
C*    TO CONSIDER EACH DECISION (DROP IN ELEVATION ACROSS THE PIPE)    *
C     *****************************************************************
      COMMON/AREA1/NELEV,AN,VMAX,DSTATE,NDLT,COVMIN,DSMIN,NWRIT
      COMMON/AREA2/PIPLTH,CINF,T
      COMMON/AREA3/MELEV,DIAM
      COMMON/AREA4/FUNC,PIPFLC
      COMMON/AREA5/GELEV,ELVMID,ELVMUP,MN
      COMMON/AREA6/MJTOJJ
      COMMON/AREA8/FCUP
      COMMON/AREA9/DAMCST,RISKS
      COMMON/AREA10/ MODEL
      COMMON/AREA12/ ALPHA1,ALPHA2,BETA1,BETA2
      DIMENSION PIPLTH(10,5),CINF(10,5)
      INTEGER*2 MELEV(10,5,9),DIAM(10,5,9)
      DIMENSION FUNC(9,5,5),PIPFLC(10,5)
      DIMENSION GELEV(10,5),ELVMID(10,5),ELVMUP(10,5,5),MN(10)
      INTEGER*2 MJTOJJ(10,5,9)
      INTEGER*2 T(10,5,5)
      DIMENSION FCUP(9,5)
      DIMENSION DAMCST(10,5),RISKS(10,5,9)
      DIMENSION ELH(2,9)
      DIMENSION DIAUP(9)
      PIPL = PIPLTH(NDL,MNU)
      IF(MODEL. GE .3) LAMS = DAMCST(NDL,MNU)
C     *****************************************************************
C     *  ROUTINE TO RECOVER LARGEST UPSTREAM DIAMETER AT UP/S MANHOLE  *
C     *****************************************************************
      DO 50 KLL=1,NELEV
   50 DIAUP(KLL) = 0.0
      IF(NDL. EQ .1) GO TO 72
      MNNN = MN(NDL-1)
      ITKON = 0
        DO 70 M2UP=1,MNNN
        IF(T(NDL-1,M2UP,MNU). EQ. 0) GO TO 70
        ITKON = 100
          DO 60 KK=1,NELEV
          JUP = MJTOJJ(NDL,M2UP,KK)
          IF(JUP. EQ .0) GO TO 60
          DCHECK = DIAM(NDL-1,M2UP,JUP)/12.
          IF(DIAUP(KK). EQ. 0.0) GO TO 55
          IF(DCHECK. GT. DIAUP(KK)) DIAUP(KK) = DCHECK
          GO TO 60
   55     DIAUP(KK) = DCHECK
   60     CONTINUE
   70   CONTINUE
        IF(ITKON. GT .0) GO TO 80
   72   CONTINUE
          DO 75 KK=1,NELEV
   75     FCUP(KK,MNU) = 0.0
   80 CONTINUE
      IF(NWRIT. GE .2) WRITE(6,2500)
C     *********************************************
C     * VARY DOWNSTREAM STATES (CROWN ELEVATIONS) *
C     *********************************************
         DO 500 J=1,NELEV
```

```
      IF(J.EQ.1) GO TO 90
      IF((ELH(2,J). EQ .ELH(2,J-1)) GO TO 410
   90 CONTINUE
      FMIN = 1.0E25
C     ****************************************
C     * VARY UPSTREAM STATES (CROWN ELEVATIONS) *
C     ****************************************
         DO 400 K=1,NELEV
         IF(ELH(1,K). LE .ELH(2,J)) GO TO 400
         IF(FCUP(K,MNU). GE .1.0E25) GO TO 400
         DROPPE = ELH(1,K) - ELH(2,J)
         SLO = DROPPE/PIPL
         QPIPE = PIPFLO(NDL,MNU)
         DIALSP = DIAUP(K)
         IF(MODEL. GE. 3) GO TO 150
         CALL WAVMOD(SLO,AN,VMAX,QPIPE,DIALSP,DIAT,RISK)
         IF(DIAT. EQ .99.) GO TO 400
         DPMANU = GELEV(NDL,MNU) - ELH(1,K) + DIAT
         DEPINV = (GELEV(NDL,MNU) - ELH(1,K) + GELEV(NDL+1,MND) -
     $         ELH(2,J))/2. + DIAT
         CALL COSTEQ(DEPINV,DIAT,PIPL,COST,DPMANU)
         GO TO 180
  150    CONTINUE
         DP1 = GELEV(NDL,MNU) - ELH(1,K)
         DP2 = (GELEV(NDL,MNU) - ELH(1,K) + GELEV(NDL+1,MND) -
     $         ELH(2,J))/2.
         CALL RISKY(SLO,QPIPE,PIPL,DIALSP,DP1,DP2,DIAT,COST,DAMS,
     $         RISK)
         DEPINV = DP2 + DIAT
         IF(DIAT. EQ .0.0) GO TO 400
  180    CONTINUE
         IF(NDL. EQ .1) FCUP(K,MNU) = 0.0
         FUNCT = COST + FCUP(K,MNU)
         IF(NWRIT. LT .2) GO TO 200
         WRITE(6,2150) J,K,DIAT,QPIPE,SLO,PIPL,DEPINV,COST,FUNCT,RISK
  200    CONTINUE
         IF(FMIN. LT .FUNCT) GO TO 400
         FMIN = FUNCT
         FUNC(J,MNU,MND) = FUNCT
         RISKS(NDL,MNU,J) = RISK
         DIAM(NDL,MNU,J) = DIAT*12.
         MELEV(NDL,MNU,J) = K
  400    CONTINUE
C     ****************************************
C     * END OF VARIATION OF UPSTREAM STATES *
C     ****************************************
  410 CONTINUE
      IF(MELEV(NDL,MNU,J). EQ .0) FUNC(J,MNU,MND) = 1.0E25
  420 CONTINUE
  500 CONTINUE
C    ******************************************
C    * END OF VARIATION OF DOWNSTREAM STATES *
C    ******************************************
C    ********************
C    * CONSTRAINT CHECK *
C    ********************
     DO 600 JK=1,NELEV
     IF(MELEV(NDL,MNU,JK). NE .0) GO TO 650
 600 CONTINUE
     ELMID = ELMID + DSTATE/4.
     NNUD = 2
        NDLN = NDL + 1
     ELMAXD = GELEV(NDL+1,MND) - CCVMIN
     CALL ELSTAT(NNUD,NELEV,NNEL,DSTATE,ELMID,ELH,ELMAXD,NDLN,MND,1)
     ELVMUP(NDL+1,MNU,MND) = ELMID
     GO TO 80
 650 CONTINUE
2100 FORMAT(5X,2I5,2F6.2,F8.4,2F8.2,2F10.0)
2150 FORMAT(5X,2I5,2F6.2,F8.4,2F8.2,2F10.0,F10.5)
2500 FORMAT(/,9X,'J',4X,'K',3X,'DIA',3X,'FLOW',2X,'SLOPE',2X,'PIPL',3X,
    $  'DEPINV',6X,'COST',6X,'FUNCT',4X,'RISK')
     RETURN
     END
```

```
     SUBROUTINE FLOW
     COMMON/AREA1/NELEV,AN,VMAX,DSTATE,NDLT,CCVMIN,DSMIN,NWRIT
     COMMON/AREA2/PIPLTH,QINF,T
     COMMON/AREA4/FUNC,PIPFLO
     COMMON/AREA5/GELEV,ELVMID,ELVMUP,MN
     DIMENSION PIPLTH(10,5),QINF(10,5)
     DIMENSION FUNC(9,5,5),PIPFLO(10,5)
     DIMENSION GELEV(10,5),ELVMID(10,5),ELVMUP(10,5,5),MN(10)
     INTEGER*2 T(10,5,5)
     NDLT1 = NDLT - 1
     NDL = 1
     IF(NWRIT. GE .2) WRITE(6,800)
     MNUT = MN(1)
     DO 100 MNU=1,MNUT
     QDES = QINF(1,MNU)
     PIPFLO(1,MNU) = QDES
     IF(NWRIT. LT .2) GO TO 100
     WRITE(6,1000) NDL,MNU,PIPFLO(NDL,MNU)
 100 CONTINUE
        DO 500 NDL=1,NDLT1
        MNUT = MN(NDL)
        MNDT = MN(NDL+1)
           DO 400 MND=1,MNDT
           TTT = 0.0
           QDES = QINF(NDL+1,MND)
           DO 300 MNU=1,MNUT
           TT = T(NDL,MNU,MND)
           IF(TT. NE .1.0) GO TO 300
           TTT = TT + TTT
           IF(TTT. EQ .1.0) PIPFLO(NDL+1,MND) = QDES +
    $      PIPFLO(NDL,MNU)
           IF(TTT. GT .1.0) PIPFLO(NDL+1,MND) = PIPFLO(NDL+1,MND) +
    $      PIPFLO(NDL,MNU)
 300       CONTINUE
           IF(TTT. EQ .0.0) PIPFLO(NDL+1,MND) = QDES
           ND = NDL + 1
           IF(NWRIT. LT .2) GO TO 400
           WRITE(6,1000) ND,MND,PIPFLO(NDL+1,MND)
 400       CONTINUE
 500    CONTINUE
 800 FORMAT(//,10X,'ISCNODAL LINE    MANHOLE    FLOW FROM MANHOLE')
1000 FORMAT(16X,I3,10X,I3,10X,F7.1)
     RETURN
     END
```

```
      SUBROUTINE INITAL(NELEV,NDLT,MN)
C     ****************************************************************
C     *  THIS SUBROUTINE INITIALIZES EACH LOCATION OF THE ARRAYS:    *
C     *  DIAM, MELEV, MJTUJJ, TCN, PIPFLO, TO ZERO AFTER EACH ITERATION*
C     *  OF THE LOOP MODEL.                                          *
C     ****************************************************************
      COMMON/AREA3/MELEV,DIAM
      COMMON/AREA4/FUNC,PIPFLO
      COMMON/AREA6/MJTUJJ
      INTEGER*2 MELEV(10,5,9),DIAM(10,5,9)
      DIMENSION FUNC(9,5,5),PIPFLO(10,5)
      INTEGER*2 MJTUJJ(10,5,9)
      DIMENSION MN(10)
      NDLT1 = NDLT - 1
      DO 500 N=1,NDLT1
      MNU = MN(N)
      MND = MN(N+1)
      IF(MNU. GT .MND) MANMAX = MNU
      IF(MND. GT .MNU) MANMAX = MND
         DO 300 NU=1,MNU
            DO 200 J=1,NELEV
            DIAM(N,NU,J) = 0
            MELEV(N,NU,J) = 0
            MJTUJJ(N,NU,J) = 0
  200       CONTINUE
  300    CONTINUE
  500 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE ELSTAT(NNUD,NELEV,NNEL,DSTATE,ELL,ELH,ELMAX,NDL,MAN,NN)
C     ****************************************************************
C     *  THIS SUBROUTINE IS USED TO ESTABLISH THE CROWN ELEVATIONS   *
C     *  (LATTICE POINTS) AT THE UPSTREAM OR DOWNSTREAM ENDS OF A LOOP *
C     *  CORRIDOR GIVEN THE ELEVATION OF THE MIDDLE CROWN ELEVATION AND*
C     *  DSTATE (THE DISTANE BETWEEN CROWN ELEVATIONS).              *
C     ****************************************************************
      COMMON/AREA11/IDSTAT
      INTEGER*2 IDSTAT(10,5,2)
      DIMENSION ELH(2,9)
      DS = DSTATE
      IF(IDSTAT(NDL,MAN,NN). EQ .0) DS = 0.0
      JEQMN = NNEL - 1
      ELH(NNUD,NNEL) = ELL
        DO 50 JE=1,JEQMN
        XJE = JE
        J = JEQMN - JE + 1
   50   ELH(NNUD,J) = ELL + XJE*DS
      JEQP = NNEL + 1
      JE = 0
        DO 75 J=JEQP,NELEV
        JE = JE + 1
        XJE = JE
   75   ELH(NNUD,J) = ELL - XJE*DS
      DO 100 N=1,NELEV
      IF(ELH(NNUD,N). GT .ELMAX) ELH(NNUD,N) = ELMAX
  100 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE COSTEC(D,DIAT,PIPL,COST,DPMANU)
      IF(DIAT. GT .3.) GO TO 200
      IF(D. GT .10.) GO TO 100
      COST = 10.98*DIAT + 0.8*D - 5.98
      GO TO 300
  100 CONTINUE
      COST = 5.94*DIAT + 1.166*D + 0.5*D*DIAT - 9.64
      GO TO 300
  200 CONTINUE
      COST = 30.*DIAT + 4.9*D - 105.9
  300 CONTINUE
      COST = COST*PIPL + 250. + DPMANU*DPMANU
      RETURN
      END
```

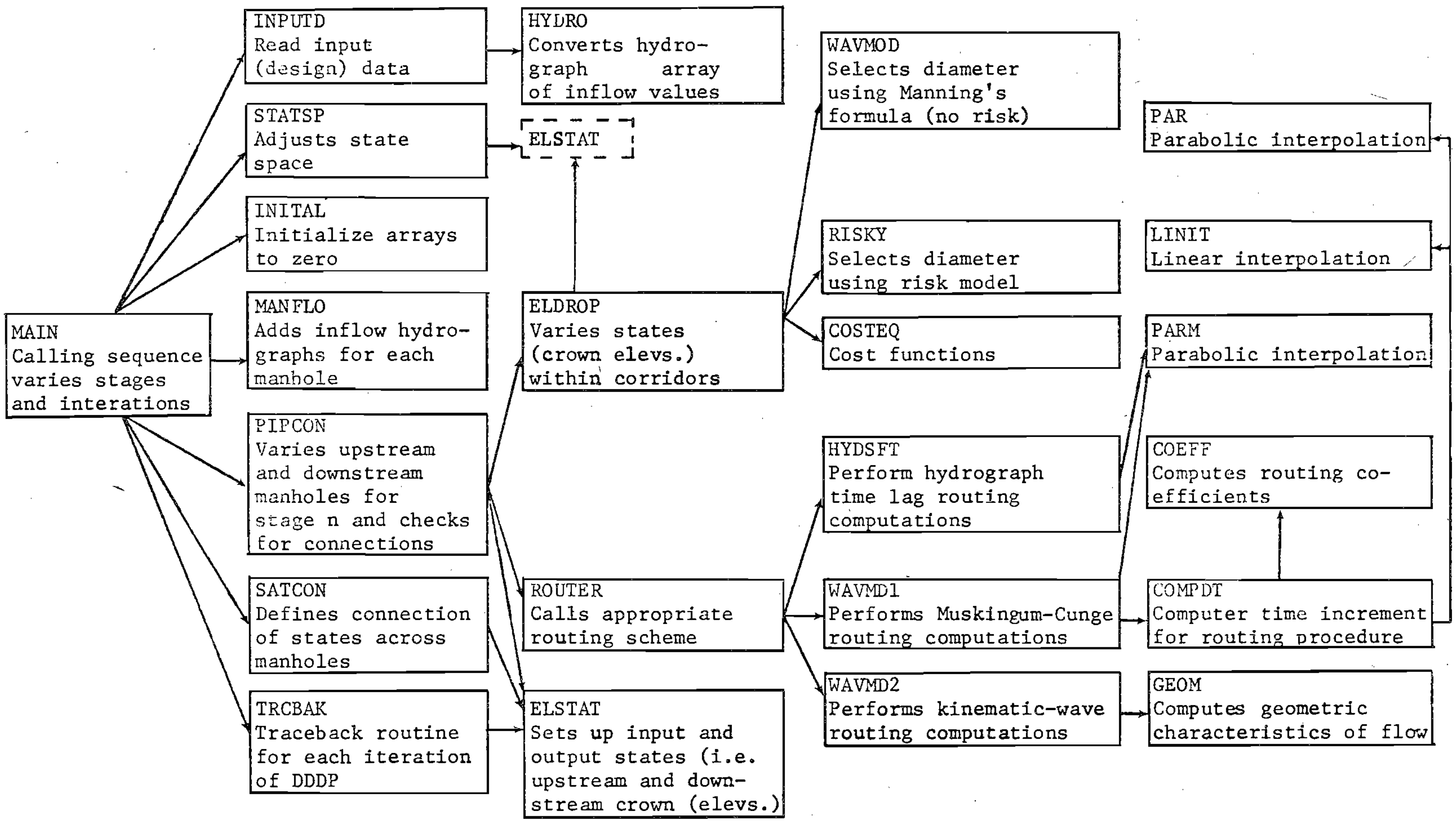
```
      SUBROUTINE WAVMOD(SLO,AN,VMAX,CPIPE,DIALSP,DIAT,RISK)
C     ***************************************************************
C     *  THIS SUBROUTINE DETERMINES THE MINIMUM SIZE COMMERCIAL DIA-  *
C     *  METER THAT CAN BE USED FOR A SET OF UPSTREAM AND DOWNSTREAM  *
C     *  CROWN ELEVATIONS REPRESENTING A DROP IN ELEVATION OF A PIPE  *
C     *  ACROSS A STAGE BETWEEN AN UPSTREAM AND DOWNSTREAM ISONODAL   *
C     *  LINE.  MANNING'S EQUATION FOR FULL PIPE FLOW IS USED.  THE   *
C     *  DIAMETER IS CHOSEN SUCH THAT THE MAXIMUM AND MINIMUM         *
C     *  PERMISSIBLE VELOCITY CONSTRAINTS ARE SATISFIED.              *
C     *                                                               *
C     *     PDIA(M) IS THE SET OF COMMERCIAL PIPE SIZES IN INCHES.    *
C     *     S2FPS(M) IS THE SET OF MINIMUM ALLOWABLE SLOPES FOR EACH  *
C     *              COMMERCIAL PIPE SIZE IN ORDER TO SATISFY THE     *
C     *              MINIMUM PERMISSIBLE VELOCITY.                    *
C     ***************************************************************
      COMMON/AREA12/ ALPHA1,ALPHA2,BETA1,BETA2
      DIMENSION PDIA(23),S2FPS(23)
      DATA PDIA/8.,10.,12.,15.,18.,21.,24.,27.,30.,36.,42.,48.,54.,60.,
     $       66.,72.,78.,84.,90.,96.,102.,114.,120./
      DATA S2FPS/.0034,.0024,.0020,.0015,.0012,.0010,.0008,.0007,.0006,
     $       .0006,.0005,.0004,.0004,.0003,.0003,.0003,.0002,.0002,
     $       .0002,.0002,.0002,.0001,.0001/
      DATA NPDIA/23/
      EFF0 = SQRT(SLO)/AN
      EFF1 = 0.5903*EFF0
      EFF2 = 0.463*EFF0
      EFF3 = 2.667
      EFF4 = 0.667
         DO 100 M=3,NPDIA
         DIAMET  = PDIA(M)/12.
         IF(DIAMET. LT .DIALSP) GO TO 100
         IF(SLO. LT .S2FPS(M)) GO TO 100
         VEL = EFF1*DIAMET**EFF4
         IF(VEL. GT .VMAX) GO TO 100
         QFLOW = EFF2*DIAMET**EFF3
         IF(QFLOW. LT .CPIPE) GO TO 100
         SF = QFLOW/CPIPE
         IF(SF. GT .8) GO TO 150
         IF(SF. LE .1.2) RISK = EXP(ALPHA1 - BETA1*SF)
         IF(SF. GT .1.2) RISK = EXP(ALPHA2 - BETA2*SF)
         GO TO 150
  100    CONTINUE
      DIAT = 99.
      GO TO 160
  150 CONTINUE
      DIAT = DIAMET
  160 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE RISKY(SLO,CPIPE,PIPL,DIALSP,DP1,DP2,DIAT,TCOST,DAMS,
     $                 RISK)
C     ***************************************************************
C     *  THIS SUBROUTINE DETERMINES THE MINIMUM SIZE COMMERCIAL DIA-  *
C     *  METER THAT CAN BE USED FOR A SET OF UPSTREAM AND DOWNSTREAM  *
C     *  CROWN ELEVATIONS REPRESENTING A DROP IN ELEVATION OF A PIPE  *
C     *  ACROSS A STAGE BETWEEN AN UPSTREAM AND DOWNSTREAM DRAINAGE   *
C     *  LINE.  MANNING'S EQUATION FOR FULL PIPE FLOW IS USED.  THE   *
C     *  DIAMETER IS CHOSEN SUCH THAT THE MAXIMUM AND MINIMUM         *
C     *  PERMISSIBLE VELOCITY CONSTRAINTS ARE SATISFIED.              *
C     *                                                               *
C     *     PDIA(M) IS THE SET OF COMMERCIAL PIPE SIZES IN INCHES.    *
C     *     S2FPS(M) IS THE SET OF MINIMUM ALLOWABLE SLOPES FOR EACH  *
C     *              COMMERCIAL PIPE SIZE IN ORDER TO SATISFY THE     *
C     *              MINIMUM PERMISSIBLE VELOCITY.                    *
C     ***************************************************************
      COMMON/AREA1/NELEV,AN,VMAX,DSTATE,NDLT,COVMIN,DSMIN,NWRIT
      COMMON/AREA12/ ALPHA1,ALPHA2,BETA1,BETA2
      DIMENSION PDIA(23),S2FPS(23)
      DATA PDIA/8.,10.,12.,15.,18.,21.,24.,27.,30.,36.,42.,48.,54.,60.,
     $       66.,72.,78.,84.,90.,96.,102.,114.,120./
      DATA S2FPS/.0034,.0024,.0020,.0015,.0012,.0010,.0008,.0007,.0006,
     $       .0006,.0005,.0004,.0004,.0003,.0003,.0003,.0002,.0002,
     $       .0002,.0002,.0002,.0001,.0001/
      DATA NPDIA/23/
      EFF0 = SQRT(SLO)/AN
      EFF1 = 0.5903*EFF0
      EFF2 = 0.463*EFF0
      EFF3 = 2.667
      EFF4 = 0.667
      RISKMN = 1.0
      DIATMN = 0.0
      COSTMN = 1.0E25
      NDIA = 0
         DO 100 M=3,NPDIA
         DIAMET  = PDIA(M)/12.
         IF(DIAMET. LT .DIALSP) GO TO 100
         IF(SLO. LT .S2FPS(M)) GO TO 100
         IF(NDIA. EQ .6) GO TO 120
         NDIA = NDIA + 1
         QFLOW = EFF2*DIAMET**EFF3
         IF(QFLOW..LT .CPIPE) GO TO 100
         VEL = EFF1*DIAMET**EFF4
         IF(VEL. GT .VMAX) GO TO 100
         SF = QFLOW/CPIPE
         IF(SF. GT .6.) GO TO 100
         DPMANU = DP1 + DIAMET
         DEPINV = DP2 + DIAMET
         CALL COSTEQ(DEPINV,DIAMET,PIPL,COST,DPMANU)
         IF(SF. LE .1.2) RISK = EXP(ALPHA1 - BETA1*SF)
         IF(SF. GT .1.2) RISK = EXP(ALPHA2 - BETA2*SF)
         COST = COST + RISK*DAMS
         IF(COST. GT .COSTMN) GO TO 100
         COSTMN = COST
         DIATMN = DIAMET
         RISKMN = RISK
  100    CONTINUE
  120 CONTINUE
      TCOST = COSTMN
      DIAT = DIATMN
      RISK = RISKMN
      RETURN
      END
```

```
      SUBROUTINE TRCBAK(TCOST,ITER)
C     ***************************************************************
C     *                                                             *
C     * THIS SUBROUTINE IS A TRACEBACK ROUTINE PERFORMED AT THE END OF *
C     *   EACH DDDP ITERATION TO IDENTIFY THE IMPROVED STORM SEWER  *
C     *   SYSTEM LAYOUT AND DESIGN.  THE TRACEBACK STARTS AT THE LAST *
C     *   STAGE WHICH IS AT THE DOWNSTREAM END BETWEEN THE LAST TWO *
C     *   ISONODAL LINES AND PROCEEDS UPSTREAM STAGE-BY-STAGE.  AT EACH*
C     *   STAGE OF THE SYSTEM THE MINIMUM COST CROWN ELEVATIONS,    *
C     *   SLOPES, AND DIAMETERS ARE IDENTIFIED FOR THE MINIMUM COST *
C     *   CONNECTIONS.                                              *
C     *                                                             *
C     ***************************************************************
      COMMON/AREA1/NELEV,AN,VMAX,DSTATE,NDLT,CCVMIN,DSMIN,NWRIT
      COMMON/AREA2/PIPLTH,CINF,T
      COMMON/AREA3/MELEV,DIAM
      COMMON/AREA4/FUNC,PIPFLO
      COMMON/AREA5/GELEV,ELVMID,ELVMUP,MN
      COMMON/AREA6/MJTCJJ
      COMMON/AREA9/DAMCST,RISKS
      COMMON/AREA10/ MODEL
      DIMENSION PIPLTH(10,5),CINF(10,5)
      INTEGER*2 MELEV(10,5,9),DIAM(10,5,9)
      DIMENSION FUNC(9,5,5),PIPFLC(10,5)
      DIMENSION GELEV(10,5),ELVMID(10,5),ELVMUP(10,5,5),MN(10)
      INTEGER*2 MJTOJJ(10,5,9)
      INTEGER*2 T(10,5,5)
      DIMENSION DAMCST(10,5),RISKS(10,5,9)
      DIMENSION KCU(5),KDD(5),ELH(2,9)
      NNEL = NELEV/2 + 1
      WRITE(6,1000) ITER
      WRITE(6,2000)
      NDLT1 = NDLT - 1
      TCOST = 0.0
      TDAMS = 0.0
      DO 900 N=1,NDLT1
      NDL = NDLT - N
      MNUT = MN(NDL)
      MNDT = MN(NDL+1)
C     ***********************************************
        DO 600 MND=1,MNDT
        TTT = 0.0
          DO 500 MNU=1,MNUT
          TT = T(NDL,MNU,MND)
          IF(T(NDL,MNU,MND). NE .1) GO TO 490
          TTT = TTT + TT
          IF((NDL+1). NE .NDLT) GO TO 300
          FTMIN = FUNC(1,MNU,MND)
             DO 250 JE=1,NELEV
             IF(FUNC(JE,MNU,MND). LE .FTMIN) J = JE
             IF(FUNC(JE,MNU,MND). LE .FTMIN) FTMIN = FUNC(JE,MNU,MND)
  250        CONTINUE
          TCOST = TCOST + FUNC(J,MNU,MND)
          IF((NDL+1). EQ .NDLT) GO TO 310
  300     CONTINUE
          JJ = KDU(MND)
          J = MJTOJJ(NDL+1,MNU,JJ)
  310     CONTINUE
          K = MELEV(NDL,MNU,J)
          KDD(MNU) = K
          ELMID = ELVMID(NDL,MNU)
          NNUD = 1
          ELMAXU = GELEV(NDL,MNU) - CCVMIN
          CALL ELSTAT(NNUD,NELEV,NNEL,DSTATE,ELMID,ELH,ELMAXU,NDL,
     $    MNU,2)
          ELMID = ELVMUP(NDL+1,MNU,MND)
          NNUD = 2
          NDLN = NDL + 1
          ELMAXD = GELEV(NDL+1,MND) - CCVMIN
          CALL ELSTAT(NNUD,NELEV,NNEL,DSTATE,ELMID,ELH,ELMAXD,NDLN,
     $    MND,1)
          SLO = (ELH(1,K) - ELH(2,J))/PIPLTH(NDL,MNU)
          DIA = DIAM(NDL,MNU,J)
          QPIPE = PIPFLC(NDL,MNU)
          RISK = RISKS(NDL,MNU,J)
          IF(MODEL. LE .2) GO TO 320
          TDAMS = TDAMS + RISK*DAMCST(NDL,MNU)
  320     WRITE(6,4100) NDL,MNU,MND,ELH(1,K),ELH(2,J),SLO,DIA,QPIPE,
     $                   RISK
          ELVMID(NDL,MNU)    = ELH(1,K)
          IF(TT. GT .0.0) ELVMUP(NDL+1,MNU,MND) = ELH(2,J)
  490     CONTINUE
          IF(TT. EQ .0.0) ELVMUP(NDL+1,MNU,MND) = ELVMID(NDL+1,MND)
  500     CONTINUE
  600   CONTINUE
      DO 650 M=1,MNUT
      KDU(M) = KDD(M)
      KDD(M) = 0
  650 CONTINUE
  900 CONTINUE
      WRITE(6,5000) TCOST
      IF(MODEL. GE .3) WRITE(6,5100) TDAMS
 1000 FORMAT(1H1,//,15X,'TRACE BACK RESULTS OF MINIMUM COST DESIGN: ITER
     $ATION ',I2)
 2000 FORMAT(//,15X,'STAGE',5X,'UP. MAN',3X,'DN. MAN',6X,'UP. ELEV',2X,
     $ 'DN. ELEV',3X,'SLOPE',6X,'DIAMETER',4X,'FLOW',5X,'RISK')
 4000 FORMAT(15X,I5,5X,I5,5X,I5,5X,2F10.2,F10.5,F10.0,F10.2)
 4100 FORMAT(15X,I5,5X,I5,5X,I5,5X,2F10.2,F10.5,F10.0,F10.2,F12.5)
 5000 FORMAT(//,15X,'OPTIMUM STORM SEWER SYSTEM COST',F12.0,' DOLLARS')
 5100 FORMAT(/,15X,'EXPECTED DAMAGE COST IS',F12.0,' DOLLARS')
      RETURN
      END
```

APPENDIX E

COMPUTER PROGRAM LISTING FOR DESIGN MODELS B-1, B-2, B-3, and D



Subroutine Flowchart of ILSSD2 for Design Models, B-1, B-2, B-3 and D

```
C*****************************************************************
C*                                                               *
C*              III    L        SSSS    SSSS    DD      22       *
C*               I     L        S       S       D D    2  2      *
C*               I     L        S       S       D  D      2      *
C*               I     L        SSSS    SSSS    D  D     2       *
C*               I     L           S       S    D  D    2        *
C*               I     L           S       S    D D    2         *
C*              III    LLLL     SSSS    SSSS    DD     2222      *
C*                                                               *
C*****************************************************************
C       ISSD2 (ILLINOIS STORM SEWER DESIGN) MODEL IS USED FOR DESIGNING  *
C       STORM SEWER SYSTEMS WITH GIVEN LAYOUTS USING A ROUTING SCHEME    *
C       FOR THE HYDRAULICS WITH OR WITHOUT RISK CONSIDERATIONS.          *
C*****************************************************************
C*                                                               *
C*          IMPORTANT VARIABLES IN THE OPTIMIZATION              *
C*                                                               *
C*****************************************************************
C
C     ************************************************************
C     *                                                          *
C     *    MELEV(NDL,MNU,J)      INDEXES OF K ELEVATIONS FROM DOWNSTREAM*
C     *                          MANHOLE FOR ELEVATION J TO UPSTREAM  *
C     *                          MANHOLE MNU ON ISONODAL LINE NDL.    *
C     *    DIAM(NDL,MNU,J)     ARE THE PIPE DIAMETERS.               *
C     *    FUNC(J,MNU,MND) ARE THE COSTS AT DOWNSTREAM ELEVATION J FOR *
C     *                    CONNECTION TO MND FROM UPSTREAM MANHOLE MNU.*
C     *                    THIS IS A CUMULATIVE SUM OF COSTS FOR ALL  *
C     *                    UPSTREAM CONNECTING PIPES AND MANHOLES.    *
C     *    TCN(NDL,MNU,MND) IS THE VECTOR OF CONNECTIVITY FOUND AFTER *
C     *                     OPTIMIZATION OVER THE SEWER NETWORK AFTER *
C     *                    EACH ITERATION.                            *
C     *    MJTOJJ(NDL+1,MNU,JJ)        IS THE INDEX OF UPSTREAM ELEVATIONS*
C     *                                J ACROSS THE MANHOLE MND ON ISO-  *
C     *                                NODAL LINE NDL+1 THAT REFERS TO   *
C     *                                UPSTREAM MANHOLE MNU ON DRAINAGE  *
C     *                                LINE NDL.                         *
C     *    FCUP(JJ,MAN) ARE THE UPSTREAM (AT MANHOLE MAN) COSTS FOR      *
C     *                 CROWN ELEVATIONS JJ.                             *
C     *                                                          *
C     ************************************************************
      COMMON/AREA1/NELEV,AN,VMAX,DSTATE,NDLT,COVMIN,DSMIN,NWRIT
      COMMON/AREA2/PIPLTH,CINF,T
      COMMON/AREA3/MELEV,DIAM
      COMMON/AREA4/FUNC,PIPFLO
      COMMON/AREA5/GELEV,ELVMID,ELVMUP,MN
      COMMON/AREA6/MJTCJJ
      COMMON/AREA8/FCUP
      COMMON/AREA9/DAMCST,RISKS
      COMMON/AREA10/ ALPHA1,ALPHA2,BETA1,BETA2
      COMMON/AREA11/IDSTAT
      COMMON/HYDRO1/ DX,TLMT,MODEL,NDT,DT
      COMMON/HYDRO2/ QIN,QOUT
      COMMON/HYDRO3/ QMAN
      DIMENSION PIPLTH(10,5),CINF(10,5)
      INTEGER*2 MELEV(10,5,5),DIAM(10,5,9)
      DIMENSION FUNC(9,5,5),PIPFLO(10,5)
      DIMENSION GELEV(10,5),ELVMID(10,5),ELVMUP(10,5,5),MN(10)

      INTEGER*2 MJTCJJ(10,5,9)
      INTEGER*2 T(10,5,5)
      DIMENSION FCUP(9,5)
      DIMENSION DAMCST(10,5),RISKS(10,5,9)
      INTEGER*2 IDSTAT(10,5,2)
      DIMENSION QIN(5,9,60),QOUT(5,9,60)
      DIMENSION QMAN(10,5,60)
      DIMENSION ELH(2,9)
      NWRIT = 2
      NWRIT = 0
      RV = 0.2
      CALL INPUTD
      WRITE(6,1300)
      NDLT1 = NDLT - 1
C     ****************************************
C     * VARY ITERATION OF DDDP OPTIMIZATION *
C     ****************************************
         DO 500 ITER=1,10
         IF(ITER. EQ .5) DSTATE = 4.*DSTATE
         IF(DSTATE. LT .DSMIN) GO TO 510
         CALL STATSP
         CALL INITAL(NELEV,NDLT,MN)
            DO 400 NDL=1,NDLT1
            CALL MANFLO(NDL,MN)
            CALL PIPCON(NDL,ELH,ITER)
            IF(NDL. EQ .NDLT1) GO TO 410
            CALL SATCON(NDL)
  400       CONTINUE
  410    CONTINUE
C        ********************************************
C        * CALL TRACEBACK TO FIND MINIMUM COST DESIGN *
C        ********************************************
         CALL TRCBAK(TCOST,ITER)
         IF(ITER. EQ .1) GO TO 440
C        *******************************************************
C        *  CRITERIA TO CHANGE DSTATE FOR THE NEXT ITERATION.  *
C        *     TCOST = MINIMUM COST OF THE SYSTEM DESIGN FOR THIS ITER. *
C        *     TCST = MINIMUM COST OF THE SYSTEM DESIGN FOR THE PREVIOUS*
C        *            ITERATION.                                *
C        *     ERRCOS = RELATIVE CHANGE IN COST ALLOWED FOR SUCCESSIVE *
C        *              ITERATIONS BEFORE DSTATE IS CHANGED.    *
C        *******************************************************
         RV = RV - 0.02
         IF(RV. LE .0.02) RV = 0.02
         ERRCOS = RV*TCST
         IF(ABS(TCOST-TCST). LE .ERRCOS) GO TO 440
         TCST = TCOST
         GO TO 500
  440    CONTINUE
         TCST = TCOST
         DSTATE = DSTATE/2.
  500    CONTINUE
  510 CONTINUE
 1300 FORMAT(1H1)
      STOP
      END
```

```
      SUBROUTINE INPUT
C     ****************************************************************
C*                                                                   *
C     *  THIS SUBROUTINE READS IN ALL THE INPUT DATA                 *
C*                                                                   *
C*********************************************************************
      COMMON/AREA1/NELEV,AN,VMAX,DSTATE,NDLT,COVMIN,DSMIN,NWRIT
      COMMON/AREA2/PIPLTH,CINF,I
      COMMON/AREA5/GELEV,ELVMID,ELVMUP,MN
      COMMON/AREA9/DAMCST,RISKS
      COMMON/AREA10/ ALPHA1,ALPHA2,BETA1,BETA2
      COMMON/AREA11/IDSTAT
      COMMON/HYDAL1/ DX,TLMT,MODEL,NDT,DT
      COMMON/HYDRC3/ QMAN
      DIMENSION PIPLTH(10,5),CINF(10,5)
      DIMENSION GELEV(10,5),ELVMID(10,5),ELVMUP(10,5,5),MN(10)
      DIMENSION DAMCST(10,5),RISKS(10,5,9)
      INTEGER*2 T(10,5,5)
      INTEGER*2 IDSTAT(10,5,2)
      DIMENSION QMAN(10,5,60)
C     ****************************************************************
C     *     MODEL IS THE MODEL DESIGNATION:                          *
C     *           = 0 HYDROGRAPH LAG NO RISK                         *
C     *           = 1 MUSKINGUM-CUNGE ROUTING NO RISK                *
C     *           = 2 KINEMATIC-WAVE ROUTING NO RISK                 *
C     *           = 5 HYDROGRAPH LAG ROUTING WITH RISK               *
C     *   AN IS MANNING'S ROUGHNESS FACTOR                           *
C     *   VMAX IS THE MAXIMUM PERMISSIBLE VELOCITY ALLOWED IN A PIPE *
C     *   COVMIN IS THE MINIMUM COVER DEPTH FOR EACH PIPE.           *
C     *   DSTATE IS THE DISTANCE BETWEEN CROWN ELEVATIONS AT A MANHOLE*
C     *   DSMIN IS THE MINIMUM DISTANCE BETWEEN CROWN ELEVATIONS     *
C     *         THIS IS THE CRITERIA TO STOP THE OPTIMIZATION .      *
C     *   NELEV IS THE NUMBER OF CROWNS ELEVATIONS (LATTICE POINTS)  *
C     *         AT EACH MANHOLE TO ESTABLISH CORRIDORS.              *
C     *   NDLT IS THE TOTAL NUMBER OF ISONODAL LINES.  THE NUMBER    *
C     *         OF DDDP STAGES BEING (NDLT-1).                       *
C     *   MN(NDL) IS THE NUMBER OF MANHOLES ON ISONODAL LINE NDL.    *
C     *   ALPHA1, ALPHA2, BETA1, BETA2 ARE CONSTANTS  FOR THE RISK-  *
C     *         SAFETY FACTOR RELATIONSHIP.                          *
C     *  NDT = NUMBER OF TIME INCREMENTS + 1                         *
C     *  DT = TIME INCREMENTS FOR INFLOW HYDROGRAPHS AT MANHOLES,SEC *
C     *  TLMT = TOTAL TIME FOR ROUTING COMPUTATIONS, SEC             *
C     *  DX = DISTANCE INCREMENT ALONG PIPES, FT                     *
C     ****************************************************************
C     *******************
C     * READ INPUT DATA *
C     *******************
      READ(5,1100) MODEL
      READ(5,1000) AN,VMAX,COVMIN,DSTATE,DSMIN
      READ(5,1200) DT,TLMT,DX,NDT
      READ(5,1500) NELEV,NDLT,MANT
      READ(5,2000) (MN(I),I=1,NDLT)
      READ(5,1600) ALPHA1,ALPHA2,BETA1,BETA2
C     ****************************************************************
C     *                                                              *
C     *   GELEV(NDL,MAN) ARE THE GROUND ELEVATIONS AT MANHOLE MAN ON *
C     *                  ISONODAL LINE NDL.                          *
C     *   ELVMUP(NDL,MNU,MND) ARE THE ELEVATIONS OF THE MIDDLE LATTICE*
C     *                  POINTS(CROWN ELEVATIONS) ON THE UPSTREAM*
C
C     *                  SIDE OF MANHOLE MND ON DRAINAGE LINE NDL*
C     *                  THAT CONNECTS TO UPSTREAM MANHOLE MNU ON*
C     *                  ISONODAL LINE NDL-1.                        *
C     *   ELVMID(NDL,MAN) ARE THE ELEVATIONS OF THE MIDDLE LATTICE   *
C     *                  POINTS(CROWN ELEVATIONS) AT DOWNSTREAM SIDE *
C     *                  OF MANHOLE MAN ON ISONODAL LINE NDL.        *
C     *   PIPLTH(NDL,MNU)    ARE PIPE LENGTHS FROM MNU TO MND WHERE  *
C     *                  MNU IS ON ISONODAL LINE NDL.                *
C     *    DAMCST(N,I) IS THE ASSESSED DAMAGE COST IN THE EVENT OF   *
C     *                  FAILURE OF THE PIPE DRAINING MANHOLE I ON ISO-*
C     *                  NODAL LINE N.                               *
C     *   T(NDL,MNU,MND) IS THE ARRAY OF POSSIBLE CONNECTIONS WITH   *
C     *                  UPSTREAM MANHOLE MNU ON ISONODAL LINE NDL.  *
C     *  QP = PEAK INFLOW,CFS.                                       *
C     *  QB = BASE FLOW, CFS.                                        *
C     *  TAU = STARTING TIME WHEN HYDROGRAPH STARTS,SEC.             *
C     *  DUR = DURATION OF THE HYDROGRAPH, SEC.                      *
C     *                                                              *
C     ****************************************************************
      DO 100 N=1,NDLT
      MAN = MN(N)
      READ(5,2500) (GELEV(N,I),I=1,MAN)
      READ(5,2500) (ELVMUP(N,1,I),I=1,MAN)
      READ(5,2500) (ELVMID(N,I),I=1,MAN)
      READ(5,4000) (PIPLTH(N,I),I=1,MAN)
      READ(5,2000) (IDSTAT(N,I,1),I=1,MAN)
      READ(5,2000) (IDSTAT(N,I,2),I=1,MAN)
      IF(MODEL. LE .2) GO TO 100
      READ(5,5000) (DAMCST(N,I),I=1,MAN)
  100 CONTINUE
      NDLT1 = NDLT-1
         DO 500 KI=1,NDLT1
         MNU = MN(KI)
         MND = MN(KI+1)
            DO 400 N=1,MNU
            READ(5,2000) (T(KI,N,M),M=1,MND)
  400       CONTINUE
  500 CONTINUE
C
C     READ IN HYDROGRAPH INFORMATION
C
         DO 600 NDL=1,NDLT1
         MNT = MN(NDL)
            DO 550 M=1,MNT
            READ(5,5500) TAU,DUR,QB,QP
            CALL HYDRG(NDL,M,TAU,DUR,QB,QP)
  550       CONTINUE
  600    CONTINUE
C     ************************
C     * WRITE OUT INPUT DATA *
C     ************************
      WRITE(6,6700)
      WRITE(6,6000) NELEV,NDLT,MANT
      WRITE(6,6100) DSTATE,COVMIN,AN,VMAX,DX
      WRITE(6,6400)
         DO 650 N=1,NDLT
         MAN = MN(N)
            DO 625 I=1,MAN
            WRITE(6,6300) N,I,GELEV(N,I)
```

```
  625       CONTINUE
  650 CONTINUE
      WRITE(6,6450)
      DO 700 KI=1,NDLT1
         KII = KI + 1
         WRITE(6,6500) KI,KII
         MNU = MN(KI)
         MND = MN(KI+1)
         DO 675 N=1,MNU
            DO 660 M=1,MND
               IF(T(KI,N,M). NE .1) GO TO 660
               WRITE(6,6600) N,M,PIPLTH(KI,N)
  660       CONTINUE
  675    CONTINUE
  700 CONTINUE
      WRITE(6,6700)
      WRITE(6,7000)
         DO 750 N=1,NDLT
         M = MN(N)
         IF(N. NE .1) MU = MN(N-1)
         WRITE(6,7100) N
            DO 740 MM=1,M
            IF(N. EQ .1) GO TO 735
            DO 730 MMM=1,MU
            ELVMUP(N,MMM,MM) = ELVMUP(N,1,MM)
  730       CONTINUE
  735       CONTINUE
            WRITE(6,7200) MM,ELVMUP(N,1,MM),ELVMID(N,MM)
  740       CONTINUE
  750    CONTINUE
      WRITE(6,8400)
         DO 900 NDL=1,NDLT1
         MNT = MN(NDL)
         WRITE(6,8500) NDL
            DO 850 MAN=1,MNT
            WRITE(6,8600) MAN
            IF(NDT. LE .20) N = NDT
            IF(NDT. GT .20) N = 20
            WRITE(6,8650) (CMAN(NDL,MAN,I),I=1,N)
            IF(NDT. LE .N) GO TO 850
            N1 = N+1
            IF((N+20). GE .NDT) N = NDT
            IF((N+20). LT .NDT) N = N+20
            WRITE(6,8650) (CMAN(NDL,MAN,I),I=N1,N)
            IF(NDT. LE .N) GO TO 850
           N1 = N+1
            IF((N+20). GE .NDT) N = NDT
            IF((N+20). LT .NDT) N = N+20
            WRITE(6,8650) (CMAN(NDL,MAN,I),I=N1,N)
  850       CONTINUE
  900    CONTINUE
      IF(MODEL. EQ .0) WRITE(6,9100)
      IF(MODEL. EQ .1) WRITE(6,8700)
      IF(MODEL. EQ .2) WRITE(6,8800)
      IF(MODEL. EQ .3) WRITE(6,8700)
      IF(MODEL. EQ .4) WRITE(6,8800)
      IF(MODEL. EQ .5) WRITE(6,9100)
      IF(MODEL. LE .2) WRITE(6,8900)
      IF(MODEL. GE .3) WRITE(6,9000)
C     ************************************
C     * FORMAT STATEMENTS FOR INPUT DATA *
C     ************************************
 1000 FORMAT(5F10.0)
 1100 FORMAT(I5)
 1200 FORMAT(3F5.0,I5)
 1500 FORMAT(3I5)
 1600 FORMAT(4F10.0)
 2000 FORMAT(40I2)
 2500 FORMAT(14F5.0)
 4000 FORMAT(14F5.0)
 5000 FORMAT(14E5.0)
 5500 FORMAT(4F5.0)
C     ***********************************************
C     * FORMAT STATEMENTS FOR PRINTOUT OF INPUT DATA *
C     ***********************************************
 6000 FORMAT(5X,'LATTICE POINTS =',I2,/,
     $       5X,'ISCNODAL LINES =',I3,/,
     $       5X,'NUMBER OF MANHOLES =',I3)
 6100 FORMAT(5X,'INITIAL LATTICE POINT INCREMENT =',F4.2,/,
     $       5X,'MINIMUM COVER DEPTH =',F4.2,/,
     $       5X,'MANNINGS  ROUGHNESS FACTOR =',F7.4,/,
     $       5X,'MAXIMUM ALLOWABLE VELOCITY =',F6.2,/,
     $       5X,'SPATIAL INCREMENT FOR ROUTING =',F5.0,' FT')
 6300 FORMAT(17X,I2,10X,I2,7X,F7.2)
 6400 FORMAT(//,5X,'MANHOLE INFORMATION:',/,10X,'ISCNCDAL LINE   MANHOLE
     $   GROUND ELEV.')
 6450 FORMAT(1H1,//,5X,'POSSIBLE CONNECTIONS')
 6500 FORMAT(5X,'BETWEEN ISCNODAL LINES ',I2,' AND',I2)
 6600 FORMAT(10X,'UPS MAN ',I2,' TO DNS MAN ',I2,': PIPE LENGTH =',F5.0,
     $' FT')
 6700 FORMAT(1H1)
 7000 FORMAT    (//,5X,'INITIAL CROWN ELEVATIONS TO ESTABLISH FIRST TRIA
     $L TRAJECTORY')
 7100 FORMAT(/,10X,'ISCNODAL LINE ',I2)
 7200 FORMAT(12X,'MANHOLE NO. ',I2,': CROWN ELEV. ARE ',F6.2,'FT AT UPS
     $AND ',F6.2,' FT AT DNS')
 8400 FORMAT(1H1,///,10X,'MANHOLE INFLOW INFORMATION')
 8500 FORMAT(10X,'ISCNODAL LINE ',I2)
 8600 FORMAT(10X,'MAN: ',I2)
 8650 FORMAT(17X,20F5.1)
 8700 FORMAT(///,10X,'MUSKINGUM-CUNGE MODEL IS USED FOR FLOW ROUTING')
 8800 FORMAT(///,10X,'KINEMATIC-WAVE MODEL IS USED FOR FLOW ROUTING')
 8900 FORMAT(/,10X,'NO RISK')
 9000 FORMAT(/,10X,'RISK INCLUDED')
 9100 FORMAT(///,10X,'HYDROGRAPH SHIFTING IS USED FOR FLOW ROUTING')
      RETURN
      END
```

```
      SUBROUTINE STATSP
C     ****************************************************************
C     *  THIS SUBROUTINE CHECKS THE STATE SPACE (CROWN ELEVATIONS) FOR *
C     *  FOR EACH OF THE POSSIBLE CONNECTIONS INTO AND OUT OF EACH    *
C     *  MANHOLE IN THE SYSTEM.  THE PURPOSE IS TO REDEFINE THE STATE *
C     *  ALWAYS MAINTAINING THE IMPROVED TRAJECTORY BUT ELIMINATING   *
C     *  DUPLICATE CROWN ELEVATIONS.                                  *
C     ****************************************************************
      COMMON/AREA1/NELEV,AN,VMAX,DSTATE,NDLT,COVMIN,DSMIN,NWRIT
      COMMON/AREA2/PIPLTH,CINF,T
      COMMON/AREA5/GELEV,ELVMID,ELVMUP,MN
      COMMON/AREA11/IDSTAT
      DIMENSION PIPLTH(10,5),CINF(10,5)
      DIMENSION GELEV(10,5),ELVMID(10,5),ELVMUP(10,5,5),MN(10)
      INTEGER*2 T(10,5,5)
      INTEGER*2 IDSTAT(10,5,2)
      DIMENSION ELH(2,9)
      NNEL = NELEV/2 + 1
      NNEE = NELEV - 1
      JEQMN = NNEL - 1
      DO 1000 NDL=1,NDLT
      MANT = MN(NDL)
         DO 900 MAN=1,MANT
         IF(NDL. EQ .NDLT) GO TO 530
         IF(IDSTAT(NDL,MAN,2). EQ .0) GO TO 530
C        ************************************************************
C        *  COMPUTE CROWN ELEVATIONS ON DOWNSTREAM SIDE OF MANHOLE.  *
C        ************************************************************
         ELMID = ELVMID(NDL,MAN)
         NNUD = 2
         ELMAXD = GELEV(NDL,MAN) - COVMIN
         CALL ELSTAT(NNUD,NELEV,NNEL,DSTATE,ELMID,ELH,ELMAXD,NDL,MAN,2)
         JEQ = 0
            DO 500 JJ=2,NELEV
  500       IF(ELH(2,JJ). EQ .ELH(2,JJ-1)) JEQ = JJ - 1
         IF(JEQ. EQ .0) GO TO 530
         IF(JEQ. LE .JEQMN) ELVMID(NDL,MAN) = ELH(2,NNEL+JEQ)
         IF(JEQ. LE .JEQMN) GO TO 530
         JE = 0
            DO 520 JJQQ=NNEL,NNEE
            JE = JE + 1
            XJE = JE
            IF(JEQ. NE .JJQQ) GO TO 520
            ELVMID(NDL,MAN) = ELH(2,NELEV) - XJE*DSTATE
            GO TO 525
  520       CONTINUE
  525    CONTINUE
  530    CONTINUE
         IF(NDL. EQ .1) GO TO 900
C        ************************************************************
C        *  VARY MANHOLES ON UPSTREAM ISONODAL LINE (NDL-1) TO CHECK  *
C        *  FOR POSSIBLE CONNECTIONS TO MANHOLE MAN FOR WHICH CROWN   *
C        *  ELEVATIONS ARE BEING CHECKED TO REDEFINE STATE SPACE FOR  *
C        *  UPSTREAM END OF MAN.                                      *
C        ************************************************************
         NNUD = 1
         ELMAXU = GELEV(NDL,MAN) - COVMIN
         MNUT = MN(NDL-1)
         IF(IDSTAT(NDL,MAN,1). EQ .0) GO TO 900
            DO 600 MNU=1,MNUT
            TT = T(NDL-1,MNU,MAN)
            IF(TT. EQ .0.0) GO TO 600
C
C           *********************************************************
C           *  COMPUTE CROWN ELEVATIONS ON UPSTREAM SIDE OF MANHOLE  *
C           *  MAN FOR THE POSSIBLE CONNECTION FROM UPSTREAM MANHOLE *
C           *  (MNU) ON NDL-1.                                       *
C           *********************************************************
            ELMID = ELVMUP(NDL,MNU,MAN)
            CALL ELSTAT(NNUD,NELEV,NNEL,DSTATE,ELMID,ELH,ELMAXU,NDL,MAN,
     $      1)
            JEQ = 0
               DO 560 KK=2,NELEV
  560          IF(ELH(1,KK). EQ .ELH(1,KK-1)) JEQ = KK - 1
            IF(JEQ. EQ .0) GO TO 590
         IF(JEQ. LE .JEQMN) ELVMUP(NDL,MNU,MAN) = ELH(1,NNEL+JEQ)
         IF(JEQ. LE .JEQMN) GO TO 550
         JE = 0
            DO 570 JJQQ=NNEL,NNEE
            JE = JE + 1
            XJE = JE
            IF(JEQ. NE .JJQQ) GO TO 570
            ELVMUP(NDL,MNU,MAN) = ELH(1,NELEV) - XJE*DSTATE
            GO TO 575
  570       CONTINUE
  575    CONTINUE
  590       CONTINUE
  600       CONTINUE
  900    CONTINUE
 1000 CONTINUE
C     ****************************************************************
C     ****************************************************************
      RETURN
      END
```

```
      SUBROUTINE SATCON(NDL)
C     ****************************************************************
C     * THIS SUBROUTINE DEFINES THE CONNECTIVITY ACROSS THE MANHOLES *
C     *    ON ISONODAL LINE NDL+1 FOR THE NEXT OPTIMIZATION STAGE    *
C     *    BETWEEN (NDL+1) AND (NDL+2).  ALSO DEFINED ARE THE COSTS, *
C     *    FCUP(JJ,MAND) FOR EACH MANHOLE ON ISONODAL LINE (NDL+1).  *
C     *    THESE ARE THE UPSTREAM COSTS FOR THE NEXT OPTIMIZATION STAGE.*
C     ****************************************************************
      COMMON/AREA1/NELEV,AN,VMAX,DSTATE,NDLT,COVMIN,DSMIN,NWRIT
      COMMON/AREA2/PIPLTH,QINF,I
      COMMON/AREA4/FUNC,PIPELC
      COMMON/AREA5/GELEV,ELVMID,ELVMUP,MN
      COMMON/AREA6/MJTOJJ
      COMMON/AREA8/FCUP
      DIMENSION PIPLTH(10,5),QINF(10,5)
      DIMENSION FUNC(9,5,5),PIPELC(10,5)
      DIMENSION GELEV(10,5),ELVMID(10,5),ELVMUP(10,5,5),MN(10)
      INTEGER*2 MJTOJJ(10,5,9)
      INTEGER*2 I(10,5,5)
      DIMENSION FCUP(9,5)
      DIMENSION ELMAN(2,9),KMAN(3)
      NNEL = NELEV/2 + 1
      MNDT = MN(NDL+1)
      NDL1 = NDL + 1
      IF(NWRIT. LT .2) GO TO 90
      WRITE(6,6100) NDL1
      WRITE(6,6500)
   90 CONTINUE
C        ************************************************************
         DO 500 MAND=1,MNDT
C        ************************************************************
C        * NEXT COMPUTE ELEVATIONS AT DOWNSTREAM SIDE OF MANHOLE MAND *
C        ************************************************************
         ELMID = ELVMID(NDL+1,MAND)
         NNUD = 2
         NDLN = NDL + 1
         ELMAXD = GELEV(NDL+1,MAND) - COVMIN
         CALL ELSTAT(NNUD,NELEV,NNEL,DSTATE,ELMID,ELMAN,ELMAXD,NDLN,MAND
     $   ,2)
  100    CONTINUE
         DO 125 JP=1,NELEV
         FCUP(JP,MAND) = 0.0
  125    CONTINUE
         MNUT = MN(NDL)
            DO 400 MANU=1,MNUT
            IF(T(NDL,MANU,MAND). NE .1) GO TO 400
C           *********************************************************
C           *  COMPUTE ELEVATIONS AT UPSTREAM SIDE OF MANHOLE, MAND.  *
C           *********************************************************
            ELMID = ELVMUP(NDL+1,MANU,MAND)
            NNUD = 1
            NDLN = NDL + 1
            ELMAXU = GELEV(NDL+1,MAND) - COVMIN
            CALL ELSTAT(NNUD,NELEV,NNEL,DSTATE,ELMID,ELMAN,ELMAXU,NDLN,
     $      MAND,1)
  130       CONTINUE
C           *********************************************************
C           * LOOK AT EACH POSSIBLE ELEVATION ON DOWNSTREAM SIDE OF   *
C           *    MANHOLE MAND.                                        *
C           *********************************************************
               DO 300 JJ=1,NELEV
               FT = 1.0E25
               KKK = 0
C              ******************************************************
C              * LOOK AT EACH POSSIBLE ELEVATION ON UPSTREAM SIDE OF  *
C              *    MANHOLE, MANU, TO FIND NUMBER OF FEASIBLE CONNEC- *
C              *    TIONS.                                            *
C              ******************************************************
                  DO 150 KK=1,NELEV
                  IF(ELMAN(1,KK). GE .ELMAN(2,JJ)) KKK = KK
  150             CONTINUE
               IF(KKK. EQ .0) GO TO 290
C              ******************************************************
C              * NOW OF THE FEASIBLE POSSIBLE CONNECTIONS FIND THE    *
C              *    CHEAPEST.                                         *
C              ******************************************************
               KK4 = 0
                  DO 200 KMIN=1,KKK
                  FT1 = FUNC(KMIN,MANU,MAND)
                  IF(FT1. GE .1.0E25) GO TO 200
                  IF(FT1. LE .FT) KK4 = KMIN
                  IF(FT1. LE .FT) FT = FT1
  200             CONTINUE
               FCUP(JJ,MAND) = FCUP(JJ,MAND) + FT
               MJTOJJ(NDL+1,MANU,JJ) = KK4
               IF(NWRIT. LT .2) GO TO 280
               WRITE(6,6200) MANU,MAND,KK4,JJ,FCUP(JJ,MAND)
  280          CONTINUE
               GO TO 300
  290          CONTINUE
               FCUP(JJ,MAND) = 1.0E25
               KK4 = 0
               IF(NWRIT. LT .2) GO TO 300
               WRITE(6,6200) MANU,MAND,KK4,JJ,FCUP(JJ,MAND)
  300          CONTINUE
            DO 350 JCHECK=1,NELEV
            IF(MJTOJJ(NDL+1,MANU,JCHECK). NE .0) GO TO 400
  350       CONTINUE
            ELVMID(NDL+1,MAND)    = ELVMID(NDL+1,MAND)    - DSTATE/2.
            ELMID = ELVMID(NDL+1,MAND)
            NNUD = 2
         NDLN = NDL + 1
            CALL ELSTAT(NNUD,NELEV,NNEL,DSTATE,ELMID,ELMAN,ELMAXD,NDLN,
     $      MAND,2)
            GO TO 130
  400       CONTINUE
C        ************************************************************
  500    CONTINUE
C        ************************************************************
C     ***************************************************************
C        ************************************************************
 6100 FORMAT(//,15X,'MANHOLE CONNECTIONS FOR ISONODAL LINE',I2)
 6200 FORMAT(20X,I3,7X,I3,4X,I3,2X,I2,2X,F10.0)
 6500 FORMAT(//,20X,'UP MAN   DN MAN   KK  JJ  COST AT JJ',/)
      RETURN
      END
```

```
0001          SUBROUTINE RISKY(SLO,QPIPE,PIPL,DIALSP,DP1,DP2,DIAT,TCOST,DAMS,
           $               RISK)
      C     *************************************************************
      C     *  THIS SUBROUTINE DETERMINES THE MINIMUM SIZE COMMERCIAL DIA-  *
      C     *  METER THAT CAN BE USED FOR A SET OF UPSTREAM AND DOWNSTREAM  *
      C     *  CROWN ELEVATIONS REPRESENTING A DROP IN ELEVATION OF A PIPE  *
      C     *  ACROSS A STAGE BETWEEN AN UPSTREAM AND DOWNSTREAM DRAINAGE   *
      C     *  LINE.  MANNING'S EQUATION FOR FULL PIPE FLOW IS USED.  THE   *
      C     *  DIAMETER IS CHOSEN SUCH THAT THE MAXIMUM AND MINIMUM         *
      C     *  PERMISSIBLE VELOCITY CONSTRAINTS ARE SATISFIED.              *
      C     *                                                               *
      C     *     PDIA(M) IS THE SET OF COMMERCIAL PIPE SIZES IN INCHES.    *
      C     *     S2FPS(M) IS THE SET OF MINIMUM ALLOWABLE SLOPES FOR EACH  *
      C     *            COMMERCIAL PIPE SIZE IN ORDER TO SATISFY THE       *
      C     *            MINIMUM PERMISSIBLE VELOCITY.                      *
      C     *************************************************************
0002          COMMON/AREA1/NELEV,AN,VMAX,DSTATE,NDLT,COVMIN,DSMIN,NWRIT
0003          COMMON/AREA10/ ALPHA1,ALPHA2,BETA1,BETA2
0004          DIMENSION PDIA(23),S2FPS(23)
0005          DATA PDIA/8.,10.,12.,15.,18.,21.,24.,27.,30.,36.,42.,48.,54.,60.,
           $          66.,72.,78.,84.,90.,96.,102.,114.,120./
0006          DATA S2FPS/.0034,.0024,.0020,.0015,.0012,.0010,.0008,.0007,.0006,
           $          .0006,.0005,.0004,.0004,.0003,.0003,.0003,.0002,.0002,
           $          .0002,.0002,.0002,.0001,.0001/
0007          DATA NPDIA/23/
0008          EFF0 = SQRT(SLO)/AN
0009          EFF1 = 0.5903*EFF0
0010          EFF2 = 0.463*EFF0
0011          EFF3 = 2.667
0012          EFF4 = 0.667
0013          RISKMN = 1.0
0014          DIATMN = 0.0
0015          COSTMN = 1.0E25
0016          NDIA = 0
0017             DO 100 M=3,NPDIA
0018             DIAMET  = PDIA(M)/12.
0019             IF(DIAMET. LT .DIALSP) GO TO 100
0020             IF(NDIA. EQ .6) GO TO 120
0021             NDIA = NDIA + 1
0022             IF(SLO. LT .S2FPS(M)) GO TO 100
0023             QFLOW = EFF2*DIAMET**EFF3
0024             IF(QFLOW. LT .QPIPE) GO TO 100
0025             VEL = EFF1*DIAMET**EFF4
0026             IF(VEL. GT .VMAX) GO TO 100
0027             SF = QFLOW/QPIPE
0028             DPMANU = DP1 + DIAMET
0029             DEPINV = DP2 + DIAMET
0030             CALL COSTEQ(DEPINV,DIAMET,PIPL,COST,DPMANU)
0031             IF(SF. LE .1.2) RISK = EXP(ALPHA1 - BETA1*SF)
0032             IF(SF. GT .1.2) RISK = EXP(ALPHA2 - BETA2*SF)
0033             COST = COST + RISK*DAMS
0034             IF(COST. GT .COSTMN) GO TO 100
0035             COSTMN = COST
0036             DIATMN = DIAMET
0037             RISKMN = RISK
0038    100      CONTINUE
0039    120   CONTINUE
0040          TCOST = COSTMN
0041          DIAT = DIATMN
0042          RISK = RISKMN
0043          RETURN
0044          END
```

```
0001          SUBROUTINE PIPCON(NDL,ELH,ITER)
      C       **************************************************************
      C*                                                                   *
      C*      THIS SUBROUTINE VARIES THE POSSIBLE CONNECTIONS AT EACH DDDP *
      C*      STAGE BY VARYING THE MANHOLES ON THE UPSTREAM AND DOWNSTREAM *
      C*      ISONODAL LINE OF THE STAGE AND CHECKING THE VECTOR OF POSSIBLE *
      C*      CONNECTIONS FOR PIPES IN THE LAYOUT.                          *
      C*                                                                   *
      C       **************************************************************
0002          COMMON/AREA1/NELEV,AN,VMAX,DSTATE,NDLT,COVMIN,DSMIN,NWRIT
0003          COMMON/AREA2/PIPLTH,QINF,T
0004          COMMON/AREA3/MELEV,DIAM
0005          COMMON/AREA4/FUNC,PIPFLO
0006          COMMON/AREA5/GELEV,ELVMID,ELVMUP,MN
0007          COMMON/AREA6/MJTOJJ
0008          COMMON/AREA8/FCUP
0009          DIMENSION PIPLTH(10,5),QINF(10,5)
0010          INTEGER*2 MELEV(10,5,9),DIAM(10,5,9)
0011          DIMENSION FUNC(9,5,5),PIPFLO(10,5)
0012          DIMENSION GELEV(10,5),ELVMID(10,5),ELVMUP(10,5,5),MN(10)
0013          INTEGER*2 MJTOJJ(10,5,9)
0014          INTEGER*2 T(10,5,5)
0015          DIMENSION FCUP(9,5)
0016          DIMENSION ELH(2,9),EL(2,9)
0017          IF(NWRIT. LT .2) GO TO 20
0018          WRITE(6,1300)
0019          WRITE(6,1400)
0020          WRITE(6,1500)
0021          WRITE(6,1600) ITER
0022          WRITE(6,1700) NDL
0023          WRITE(6,1500)
0024          WRITE(6,1400)
0025     20   CONTINUE
0026          NNEL = NELEV/2 + 1
0027          MNUT = MN(NDL)
0028          MNDT = MN(NDL+1)
0029          DO 60 MNU=1,MNUT
0030            DO 50 MND=1,MNDT
0031              DO 40 J=1,NELEV
0032              FUNC(J,MNU,MND) = 1.0E25
0033     40       CONTINUE
0034     50     CONTINUE
0035     60   CONTINUE
      C       ************************************************************
      C       * VARY MANHOLES ON DOWNSTREAM ISONODAL LINE FOR STAGE NDL *
      C       ************************************************************
0036            DO 200 MND =1,MNDT
      C         ************************************************************
      C         * VARY MANHOLES ON UPSTREAM ISONODAL LINE FOR STAGE NDL *
      C         ************************************************************
0037              DO 100 MNU=1,MNUT
0038              IF(T(NDL,MNU,MND). EQ .0) GO TO 100
0039              ELMID = ELVMUP(NDL+1,MNU,MND)
0040              NNUD = 2
0041              NDLN = NDL + 1
0042              ELMAXD = GELEV(NDL+1,MND) - COVMIN
0043              CALL ELSTAT(NNUD,NELEV,NNEL,DSTATE,ELMID,ELH,ELMAXD,NDLN,
           $      MND,1)
0044              ELL = ELVMID(NDL,MNU)
```

```
            NNUD = 1
            ELMAXU = GELEV(NDL,MNU) - COVMIN
            CALL ELSTAT(NNUD,NELEV,NNEL,DSTATE,ELL,ELH,ELMAXU,NDL,MNU,2)
            IF(NWRIT. LT .2) GO TO 95
            WRITE(6,1000) MNU,MND,NDL
               DO 90 NP=1,NELEV
               WRITE(6,1100) ELH(1,NP),ELH(2,NP)
   90          CONTINUE
   95       CONTINUE
            CALL ELDROP(NDL,MNU,MND,ELMID,NNEL,ELH)
            PIPL = PIPLTH(NDL,MNU)
            CALL ROUTER(NDL,MNU,MND,PIPL,ELH)
  100       CONTINUE
C          ******************************************************
C          * END VARIATION OF MANHOLES ON UPSTREAM ISCNODAL LINE *
C          ******************************************************
  200    CONTINUE
C       ******************************************************
C       * END VARIATION OF MANHOLES ON DOWNSTREAM ISONODAL LINE *
C       ******************************************************
 1000 FORMAT(//,5X,'CORRIDOR FROM UPSTREAM MANHOLE ',I2,' TO DOWNSTREAM
     $MANHOLE',I2,' AT STAGE ',I2,/,15X,'ELEVATIONS: UPSTREAM  DOWNSTREA
     $M')
 1100 FORMAT(28X,F6.2,5X,F6.2)
 1300 FORMAT(1H1)
 1400 FORMAT(15X,'********************')
 1500 FORMAT(15X,'*',18X,'*')
 1600 FORMAT(15X,'*   ITERATION ',I2,4X,'*')
 1700 FORMAT(15X,'*   DDDP STAGE ',I3,'  *')
      RETURN
      END



      SUBROUTINE ELDROP(NDL,MNU,MND,ELMID,NNEL,ELH)
C     ******************************************************************
C*    THIS SUBROUTINE VARIES THE INPUT STATES (UPSTREAM CROWN ELEVS.)  *
C*    AND THE OUTPUT STATES (DOWNSTREAM CROWN ELEVS.) IN THE CORRIDOR  *
C*    TO CONSIDER EACH DECISION (DROP IN ELEVATION ACROSS THE PIPE)    *
C     ******************************************************************
      COMMON/AREA1/NELEV,AN,VMAX,DSTATE,NDLT,COVMIN,DSMIN,NWRIT
      COMMON/AREA2/PIPLTH,CINF,T
      COMMON/AREA3/MELEV,DIAM
      COMMON/AREA4/FUNC,PIPFLO
      COMMON/AREA5/GELEV,ELVMID,ELVMUP,MN
      COMMON/AREA6/MJTOJJ
      COMMON/AREA8/FCUP
      COMMON/AREA9/DAMCST,RISKS
      COMMON/HYDRO1/ DX,TLMT,MODEL,NDT,DT
      COMMON/HYDRO2/ QIN,QOUT
      DIMENSION PIPLTH(10,5),QINF(10,5)
      INTEGER*2 MELEV(10,5,5),DIAM(10,5,9)
      DIMENSION FUNC(9,5,5),PIPFLO(10,5)
      DIMENSION GELEV(10,5),ELVMID(10,5),ELVMUP(10,5,5),MN(10)
      INTEGER*2 MJTOJJ(10,5,9)
      INTEGER*2 T(10,5,5)
      DIMENSION FCUP(9,5)
      DIMENSION DAMCST(10,5),RISKS(10,5,9)
      DIMENSION QIN(5,9,60),QOUT(5,9,60)
      DIMENSION ELH(2,9)
      DIMENSION DIAUP(9),QPEAK(9)
C     ******************************************************************
C
C
C
C     DETERMINE PEAK DOWNSTREAM INFLOW FROM MANHOLE MNU&FOR EACH CROWN
C     ELEVATION K
         DO 40 K=1,NELEV
         QP = 0.0
         QPEAK(K) = 0.0
            DO 30 NT=1,NDT
            QCHECK = QIN(MNU,K,NT)
            IF(QP. LT .QCHECK) QP = QCHECK
   30       CONTINUE
         QPEAK(K) = QP
   40    CONTINUE
      PIPL = PIPLTH(NDL,MNU)
      IF(MODEL. GE .3) DAMS = DAMCST(NDL,MNU)
C     ******************************************************************
C
C     *  ROUTINE TO RECOVER LARGEST UPSTREAM DIAMETER AT UP/S MANHOLE  *
C
      DO 50 KLL=1,NELEV
   50 DIAUP(KLL) = 0.0
      IF(NDL. EQ .1) GO TO 72
      MNNN = MN(NDL-1)
      ITKON = 0
         DO 70 M2UP=1,MNNN
         IF(T(NDL-1,M2UP,MNU). EQ. 0) GO TO 70
         ITKON = 100
            DC 60 KK=1,NELEV
            JUP = MJTOJJ(NDL,M2UP,KK)
            IF(JUP. EQ .0) GO TO 60
            DCHECK = DIAM(NDL-1,M2UP,JUP)/12.
```

```
            IF(DIAUP(KK). EQ. 0.0) GO TO 55
            IF(DCHECK. GT. DIAUP(KK)) DIAUP(KK) = DCHECK
            GO TO 60
   55       DIAUP(KK) = DCHECK
   60       CONTINUE
   70    CONTINUE
C     ****************************************************************
         IF(ITKON. GT .0) GO TO 80
   72    CONTINUE
            DO 75 KK=1,NELEV
   75       FCUP(KK,MNU) = 0.0
   80 CONTINUE
      IF(NWRIT. GE .2) WRITE(6,2500)
C     ******************************************
C     * VARY DOWNSTREAM STATES (CROWN ELEVATIONS) *
C     ******************************************
         DO 500 J=1,NELEV
         IF(J.EQ.1) GO TO 90
         IF(ELH(2,J). EQ .ELH(2,J-1)) GO TO 410
   90    CONTINUE
         FMIN = 1.0E25
C        ******************************************
C        * VARY UPSTREAM STATES (CROWN ELEVATIONS) *
C        ******************************************
            DO 400 K=1,NELEV
            IF(ELH(1,K). LE .ELH(2,J)) GO TO 400
            IF(FCUP(K,MNU). GE .1.0E25) GO TO 400
            DROPPE = ELH(1,K) - ELH(2,J)
            SLO = DROPPE/PIPL
            QPIPE = QPEAK(K)
            DIALSP = DIAUP(K)
            IF(MODEL. GE. 3) GO TO 150
            CALL WAVMOD(SLO,AN,VMAX,QPIPE,DIALSP,DIAT,RISK)
            IF(DIAT. EQ .99.) GO TO 400
            DPMANU = GELEV(NDL,MNU) - ELH(1,K) + DIAT
            DEPINV = (GELEV(NDL,MNU) - ELH(1,K) + GELEV(NDL+1,MND) -
     $               ELH(2,J))/2. + DIAT
            CALL COSTEC(DEPINV,DIAT,PIPL,COST,DPMANU)
            GO TO 180
  150       CONTINUE
            DP1 = GELEV(NDL,MNU) - ELH(1,K)
            DP2 = (GELEV(NDL,MNU) - ELH(1,K) + GELEV(NDL+1,MND) -
     $            ELH(2,J))/2.
            CALL RISKY(SLO,QPIPE,PIPL,DIALSP,DP1,DP2,DIAT,COST,DAM$,
     $                 RISK)
            DEPINV = DP2 + DIAT
            IF(DIAT. EQ .0.0) GO TO 400
  180       CONTINUE
            IF(NDL. EQ .1) FCUP(K,MNU) = 0.0
            FUNCT = COST + FCUP(K,MNU)
            IF(NWRIT. LT .2) GO TO 200
            WRITE(6,2150) J,K,DIAT,QPIPE,SLO,PIPL,DEPINV,COST,FUNCT,RISK
  200       CONTINUE
            IF(FMIN. LT .FUNCT) GO TO 400
            FMIN = FUNCT
            FUNC(J,MNU,MND) = FUNCT
            RISKS(NDL,MNU,J) = RISK
            DIAM(NDL,MNU,J) = DIAT*12.
            MELEV(NDL,MNU,J) = K
  400       CONTINUE
C        ****************************************
C        * END OF VARIATION OF UPSTREAM STATES *
C        ****************************************
  410    CONTINUE
         IF(MELEV(NDL,MNU,J). EQ .0) FUNC(J,MNU,MND) = 1.0E25
  420    CONTINUE
  500    CONTINUE
C     ******************************************
C     * END OF VARIATION OF DOWNSTREAM STATES *
C     ******************************************
C     ********************
C     * CONSTRAINT CHECK *
C     ********************
      DO 600 JK=1,NELEV
      IF(MELEV(NDL,MNU,JK). NE .0) GO TO 650
  600 CONTINUE
      ELMID = ELMID + DSTATE/4.
      NNUD = 2
         NOLN = NDL + 1
      ELMAXD = GELEV(NDL+1,MND) - CCVMIN
      CALL ELSTAT(NNUD,NELEV,NNEL,DSTATE,ELMID,ELH,ELMAXD,NDLN,MND,1)
      ELVMUP(NDL+1,MNU,MND) = ELMID
      GO TO 80
  650 CONTINUE
 2100 FORMAT(5X,2I5,2F6.2,F8.4,2F8.2,2F10.0)
 2150 FORMAT(5X,2I5,2F6.2,F8.4,2F8.2,2F10.0,F10.5)
 2500 FORMAT(/,9X,'J',4X,'K',3X,'DIA',3X,'FLOW',2X,'SLOPE',2X,'PIPL',3X,
     $  'DEPINV',6X,'COST',6X,'FUNCT',4X,'RISK')
      RETURN
      END
```

```
      SUBROUTINE INITAL(NELEV,NDLT,MN)
C     ****************************************************************
C     *  THIS SUBROUTINE INITIALIZES EACH LOCATION OF THE ARRAYS:    *
C     *  DIAM, MELEV, MJTOJJ, ICN, PIPELU, TO ZERO AFTER EACH ITERATION*
C     *  OF THE DDDP MODEL.                                          *
C     ****************************************************************
      COMMON/AREA3/MELEV,DIAM
      COMMON/AREA4/FUNC,PIPELU
      COMMON/AREA6/MJTOJJ
      INTEGER*2 MELEV(10,5,9),DIAM(10,5,9)
      DIMENSION FUNC(9,5,5),PIPELU(10,5)
      INTEGER*2 MJTOJJ(10,5,9)
      DIMENSION MN(10)
      NDLT1 = NDLT - 1
      DO 500 N=1,NDLT1
      MNU = MN(N)
      MND = MN(N+1)
      IF(MNU. GT .MND) MANMAX = MNU
      IF(MND. GT .MNU) MANMAX = MND
         DO 300 NU=1,MNU
            DO 200 J=1,NELEV
            DIAM(N,NU,J) = 0
            MELEV(N,NU,J) = 0
            MJTOJJ(N,NU,J) = 0
  200       CONTINUE
  300    CONTINUE
  500 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE ELSTAT(NNUD,NELEV,NNEL,DSTATE,ELL,ELH,ELMAX,NDL,MAN,NN)
C     ****************************************************************
C     *  THIS SUBROUTINE IS USED TO ESTABLISH THE CROWN ELEVATIONS   *
C     *  (LATTICE POINTS) AT THE UPSTREAM OR DOWNSTREAM ENDS OF A DDOP *
C     *  CORRIDOR GIVEN THE ELEVATION OF THE MIDDLE CROWN ELEVATION AND*
C     *  DSTATE (THE DISTANE BETWEEN CROWN ELEVATIONS).              *
C     ****************************************************************
      COMMON/AREA11/IDSTAT
      INTEGER*2 IDSTAT(10,5,2)
      DIMENSION ELH(2,9)
      JEQMN = NNEL - 1
      ELH(NNUD,NNEL) = ELL
      DS = DSTATE
      IF(IDSTAT(NDL,MAN,NN). EQ .0) DS = 0.0
         DO 50 JE=1,JEQMN
         XJE = JE
         J = JEQMN - JE + 1
   50    ELH(NNUD,J) = ELL + XJE*DS
      JECP = NNEL + 1
      JE = 0
         DO 75 J=JEQP,NELEV
         JE = JE + 1
         XJE = JE
   75    ELH(NNUD,J) = ELL - XJE*DS
      DO 100 N=1,NELEV
      IF(ELH(NNUD,N). GT .ELMAX) ELH(NNUD,N) = ELMAX
  100 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE COSTEQ(D,DIAT,PIPL,COST,DPMANU)
      IF(DIAT. GT .3.) GO TC 200
      IF(D. GT .10.) GO TO 100
      COST = 10.98*DIAT + 0.8*D - 5.98
      GO TO 300
  100 CONTINUE
      COST = 5.94*DIAT + 1.166*D + 0.5*D*DIAT - 9.64
      GO TO 300
  200 CONTINUE
      COST = 30.*DIAT + 4.9*D - 105.9
  300 CONTINUE
      COST = COST*PIPL + 250. + DPMANU*DPMANU
      RETURN
      END
```

```
      SUBROUTINE WAVMOD(SLO,AN,VMAX,QPIPE,DIALSP,DIAT,RISK)
C     ****************************************************************
C     *  THIS SUBROUTINE DETERMINES THE MINIMUM SIZE COMMERCIAL DIA-  *
C     *  METER THAT CAN BE USED FOR A SET OF UPSTREAM AND DOWNSTREAM  *
C     *  CROWN ELEVATIONS REPRESENTING A DROP IN ELEVATION OF A PIPE  *
C     *  ACROSS A STAGE BETWEEN AN UPSTREAM AND DOWNSTREAM DRAINAGE   *
C     *  LINE.  MANNING'S EQUATION FOR FULL PIPE FLOW IS USED.  THE   *
C     *  DIAMETER IS CHOSEN SUCH THAT THE MAXIMUM AND MINIMUM         *
C     *  PERMISSIBLE VELOCITY CONSTRAINTS ARE SATISFIED.              *
C     *                                                               *
C     *     PDIA(M) IS THE SET OF COMMERCIAL PIPE SIZES IN INCHES.    *
C     *     S2FPS(M) IS THE SET OF MINIMUM ALLOWABLE SLOPES FOR EACH  *
C     *              COMMERCIAL PIPE SIZE IN ORDER TO SATISFY,THE     *
C     *              MINIMUM PERMISSIBLE VELOCITY.                    *
C     ****************************************************************
      COMMON/AREA10/ ALPHA1,ALPHA2,BETA1,BETA2
      DIMENSION PDIA(23),S2FPS(23)
      DATA PDIA/8.,10.,12.,15.,18.,21.,24.,27.,30.,36.,42.,48.,54.,60.,
     $          66.,72.,78.,84.,90.,96.,102.,114.,120./
      DATA S2FPS/.0034,.0024,.0020,.0015,.0012,.0010,.0008,.0007,.0006,
     $           .0006,.0005,.0004,.0004,.0003,.0003,.0003,.0002,.0002,
     $           .0002,.0002,.0002,.0001,.0001/
      DATA NPDIA/23/
      EFF0 = SQRT(SLO)/AN
      EFF1 = 0.5903*EFF0
      EFF2 = 0.463*EFF0
      EFF3 = 2.667
      EFF4 = 0.667
         DO 100 M=3,NPDIA
         DIAMET  = PDIA(M)/12.
         IF(DIAMET. LT .DIALSP) GO TO 100
         IF(SLO. LT .S2FPS(M)) GO TO 100
         VEL = EFF1*DIAMET**EFF4
         IF(VEL. GT .VMAX) GO TO 100
         QFLOW = EFF2*DIAMET**EFF3
         IF(QFLOW. LT .QPIPE) GO TO 100
         SF = QFLOW/QPIPE
         IF(SF. GT .8.) GO TO 150
         IF(SF. LE .1.2) RISK = EXP(ALPHA1 - BETA1*SF)
         IF(SF. GT .1.2) RISK = EXP(ALPHA2 - BETA2*SF)
         GO TO 150
  100    CONTINUE
      DIAT = 99.
      GO TO 160
  150 CONTINUE
      DIAT = DIAMET
  160 CONTINUE
      RETURN
      END

      SUBROUTINE TRCBAK(TCOST,ITER)
C     ****************************************************************
C     *                                                               *
C     *  THIS SUBROUTINE IS A TRACEBACK ROUTINE PERFORMED AT THE END OF *
C     *    ACH DDDP ITERATION TO IDENTIFY THE IMPROVED STORM SEWER    *
C     *    SYSTEM LAYOUT AND DESIGN.  THE TRACEBACK STARTS AT THE LAST *
C     *    STAGE WHICH IS AT THE DOWNSTRAM END BETWEEN THE LAST TWO   *
C     *    ISONODAL LINES AND PROCEEDS UPSTREAM STAGE-BY-STAGE.  AT EACH*
C     *    STAGE OF THE SYSTEM THE MINIMUM COST CROWN ELEVATION,      *
C     *    SLOPES, AND DIAMETERS ARE IDENTIFIED FOR THE MINIMUM COST  *
C     *    CONNECTIONS.                                               *
C     *                                                               *
C     ****************************************************************
      COMMON/AREA1/NELEV,AN,VMAX,DSTATE,NDLT,COVMIN,DSMIN,NWRIT
      COMMON/AREA2/PIPLTH,CINF,T
      COMMON/AREA3/MELEV,DIAM
      COMMON/AREA4/FUNC,PIPFLO
      COMMON/AREA5/GELEV,ELVMID,ELVMUP,MN
      COMMON/AREA6/MJTOJJ
      COMMON/AREA9/DAMCST,RISKS
      COMMON/HYDRO1/ DX,TLMT,MODEL,NDT,DT
      DIMENSION PIPLTH(10,5),CINF(10,5)
      INTEGER*2 MELEV(10,5,9),DIAM(10,5,9)
      DIMENSION FUNC(9,5,5),PIPFLO(10,5)
      DIMENSION GELEV(10,5),ELVMID(10,5),ELVMUP(10,5,5),MN(10)
      INTEGER*2 MJTOJJ(10,5,9)
      INTEGER*2 T(10,5,5)
      DIMENSION DAMCST(10,5),RISKS(10,5,9)
      DIMENSION KOU(5),KOD(5),ELH(2,9)
      NNEL = NELEV/2 + 1
      WRITE(6,1000) ITER
      WRITE(6,2000)
      WRITE(6,2100)
      NDLT1 = NDLT - 1
      TCOST = 0.0
      TDAM$ = 0.0
      DO 900 N=1,NDLT1
      NDL = NDLT - N
      MNUT = MN(NDL)
      MNDT = MN(NDL+1)
C     ****************************************************
         DO 600 MND=1,MNDT
         TTT = 0.0
            DO 500 MNU=1,MNUT
            TT = T(NDL,MNU,MND)
            IF(T(NDL,MNU,MND). NE .1) GO TO 490
            TTT = TTT + TT
            IF((NDL+1). NE .NDLT) GO TO 300
            FTMIN = FUNC(1,MNU,MND)
               DO 250 JE=1,NELEV
               IF(FUNC(JE,MNU,MND). LE .FTMIN) J = JE
               IF(FUNC(JE,MNU,MND). LE .FTMIN) FTMIN = FUNC(JE,MNU,MND)
  250          CONTINUE
            TCOST = TCOST + FUNC(J,MNU,MND)
            IF((NDL+1). EQ .NDLT) GO TO 310
  300       CONTINUE
            JJ = KOU(MND)
            J = MJTOJJ(NDL+1,MNU,JJ)
  310       CONTINUE
```

```
            K = MELEV(NDL,MNU,J)
            KDD(MNU) = K
            ELMID = ELVMID(NDL,MNU)
            NNOD = 1
            ELMAXU = GELEV(NDL,MNU) - COVMIN
            CALL ELSTAT(NNOD,NELEV,NNEL,DSTATE,ELMID,ELH,ELMAXU,NDL,
     $      MNU,2)
            ELMID = ELVMUP(NDL+1,MNU,MND)
            NNOD = 2
            NDLN = NDL + 1
            ELMAXD = GELEV(NDL+1,MND) - COVMIN
            CALL ELSTAT(NNOD,NELEV,NNEL,DSTATE,ELMID,ELH,ELMAXD,NDLN,
     $      MND,1)
            SLO = (ELH(1,K) - ELH(2,J))/PIPLTH(NDL,MNU)
            DIA = DIAM(NDL,MNU,J)
            RISK = RISKS(NDL,MNU,J)
            IF(MODEL. LE .2) GO TO 320
            TDAMS = TDAMS + RISK*DAMCST(NDL,MNU)
 320        WRITE(6,4100) NDL,MNU,MND,ELH(1,K),ELH(2,J),SLO,DIA,RISK
            ELVMID(NDL,MNU)  = ELH(1,K)
            IF(TT. GT .0.0) ELVMUP(NDL+1,MNU,MND) = ELH(2,J)
 490        CONTINUE
            IF(TT. EQ .0.0) ELVMUP(NDL+1,MNU,MND) = ELVMID(NDL+1,MND)
 500        CONTINUE
 600     CONTINUE
      DO 650 M=1,MNUT
      KOU(M) = KDD(M)
      KDD(M) = 0
 650  CONTINUE
 900  CONTINUE
      WRITE(6,5000) TCCST
      IF(MODEL. GE .3) WRITE(6,5100) TDAMS
 1000 FORMAT(1H1,//,15X,'TRACE BACK RESULTS OF MINIMUM COST DESIGN: ITER
     $ATION ',I2)
 2000 FORMAT(//,15X,'STAGE',5X,'UP. MAN',3X,'DN. MAN',6X,'UP. ELEV',2X,
     $ 'DN. ELEV',3X,'SLOPE',6X,'DIAMETER',4X,'RISK')
 2100 FORMAT(50X,'(FT)',6X,'(FT)',18X,'(IN)')
 4000 FORMAT(15X,I5,5X,I5,5X,I5,5X,2F10.2,F10.5,F10.0)
 4100 FORMAT(15X,I5,5X,I5,5X,I5,5X,2F10.2,F10.5,F10.0,F12.5)
 5000 FORMAT(//,15X,'OPTIMUM STORM SEWER SYSTEM COST',F12.0,' DOLLARS')
 5100 FORMAT(/,15X,'EXPECTED DAMAGE COST IS',F12.0,' DOLLARS')
      RETURN
      END
```

```
      SUBROUTINE HYDRO(NDL,M,TAU,DUR,QB,QP)
C***********************************************************************
C*                                                                     *
C*    THIS SUBROUTINE CONVERTS A SYMMETRIC TRIANGULAR HYDROGRAPH       *
C*       (DESCRIBED BY TAU,DUR,QB,AND QP) INTO AN ARRAY OF INFLOW      *
C*       VALUES EVERY DT SECONDS.                                      *
C*                                                                     *
C***********************************************************************
      COMMON/AREA1/NELEV,AN,VMAX,DSTATE,NDLT,COVMIN,DSMIN,NWRIT
      COMMON/HYDRO1/ DX,TLMT,MODEL,NDT,DT
      COMMON/HYDRO3/ QMAN
      DIMENSION QMAN(10,5,60)
      IF(DUR. EQ .0.) GO TO 600
      THALF = TAU + DUR/2.
      TEND = TAU + DUR
      TIM = 0.0
      DO 500 NT=1,NDT
      IF(TIM. LE .TAU) QMAN(NDL,M,NT) = QB
      IF(TIM. LE .TAU) GO TO 490
      IF(TIM. GE .TEND) QMAN(NDL,M,NT) = QB
      IF(TIM. GE .TEND) GO TO 490
      IF(TIM. LE .THALF) QX = (TIM-TAU)*(QP-QB)/(THALF-TAU) + QB
      IF(TIM. GT .THALF) QX = (TEND-TIM)*(QP-QB)/(TEND-THALF) + QB
      QMAN(NDL,M,NT) = QX
 490  CONTINUE
      TIM = TIM + DT
 500  CONTINUE
      GO TO 700
 600  CONTINUE
      DO 650 NT=1,NDT
 650  QMAN(NDL,M,NT) = 0.0
 700  CONTINUE
      RETURN
      END
```

```
      SUBROUTINE ROUTER(NDL,MNU,MND,PIPL,ELH)
C     ******************************************************************
C*                                                                     *
C     *  THIS SUBROUUINE CALLS THE APPROPRIATE ROUTING SCHEME          *
C*                                                                     *
C     ******************************************************************
      COMMON/AREA1/NELEV,AN,VMAX,DSTATE,NDLT,COVMIN,DSMIN,NWRIT
      COMMON/AREA3/MELEV,DIAM
      COMMON/HYDRO1/ DX,TLMT,MODEL,NDT,DT
      COMMON/HYDRO2/ QIN,QOUT
      INTEGER*2 MELEV(10,5,9),DIAM(10,5,9)
      DIMENSION QIN(5,9,60),QOUT(5,9,60)
      DIMENSION ELH(2,9)
      DIMENSION QT(60),QMT(60)
      DO 200 J=1,NELEV
      K = MELEV(NDL,MNU,J)
      IF(K. EQ .0) GO TO 200
      DROPPE = ELH(1,K) - ELH(2,J)
      SLO = DROPPE/PIPL
      DIA = DIAM(NDL,MNU,J)/12.
         DO 100 NT=1,NDT
 100     QT(NT) = QIN(MNU,K,NT)
      NN = PIPL/DX + 1
      IF(NN. EQ .1) NN = 2
      DUR = TLMT
      NDSM = 2
C
C     CALL SUBROUTINE WAVMD1 OR WAVMD2 TO PERFORM FLOW ROUTING
C
      IF(MODEL. EQ .0) GO TO 125
      IF(MODEL. EQ .1) GO TO 110
      IF(MODEL. EQ .2) GO TO 120
      IF(MODEL. EQ .3) GO TO 110
      IF(MODEL. EQ .4) GO TO 120
      IF(MODEL. EQ .5) GO TO 125
 110  CALL WAVMD1(DIA,PIPL,AN,NN,SLO,TLMT,DUR,NDT,DT,QMT,QT,DMAX,
     $            VMAX,NROUT,NDSM,MRISK,LSPD,NCHECK)
```

```
      GO TO 130
  120 CALL WAVMD2(DIA,PIPL,AN,NN,SLO,TLMT,DUR,NDT,DT,QMT,QT,DMAX,
     $            VMXR,NROUT,NDSM,MRISK,LSPD,NCHECK)
      GO TO 130
  125 CALL HYDSFT(DIA,PIPL,AN,SLO,TLMT,DUR,NDT,DT,QMT,QT)
  130 CONTINUE
      TIM = 0.0
      IF(NWRIT. GE .6) WRITE(6,2050)
         DO 150 NT=1,NDT
         IF(NWRIT. GE .6) WRITE(6,2100) TIM,QT(NT),QMT(NT)
         TIM = TIM + DT
  150    QOUT(MNU,J,NT) = QMT(NT)
  200 CONTINUE
 2050 FORMAT(//,10X,'TIME',5X,'INFLOW',4X,'OUTFLOW')
 2100 FORMAT(5X,3F10.1)
      RETURN
      END
```

```
      SUBROUTINE MANFLO(NDL,MN)
C*************************************************************************
C*                                                                       *
C*    THIS SUBROUTINE ADDS THE INFLOW HYDROGRAPHS AT A MANHOLE TO        *
C*    DETERMINE THE OUTFLOW HYDROGRAPHS FOR EACH STATE FROM THE MANHOLE*
C*    I. E. EACH MANHOLE ON AN ISONODAL LINE IS CONSIDERED.              *
C*                                                                       *
C*************************************************************************
      COMMON/AREA1/NELEV,AN,VMAX,DSTATE,NDLT,COVMIN,DSMIN,NWRIT
      COMMON/AREA2/PIPLTH,CINF,T
      COMMON/AREA3/MELEV,DIAM
      COMMON/AREA6/MJTOJJ
      COMMON/HYDRO1/ DX,TLMT,MODEL,NDT,DT
      COMMON/HYDRO2/ QIN,QOUT
      COMMON/HYDRO3/ QMAN
      DIMENSION PIPLTH(10,5),QINF(10,5)
      INTEGER*2 T(10,5,5)
      INTEGER*2 MELEV(10,5,9),DIAM(10,5,9)
      INTEGER*2 MJTOJJ(10,5,9)
      DIMENSION QIN(5,9,60),QOUT(5,9,60)
      DIMENSION QMAN(10,5,60)
      DIMENSION QDUMB(60)
      DIMENSION MN(10)
      MANT = MN(NDL)
      IF(NDL. GT .1) MNUPT = MN(NDL-1)
      DO 200 MAN=1,MANT
         DO 150 K=1,NELEV
            DO 100 NT=1,NDT
            QDUMB(NT) = QMAN(NDL,MAN,NT)
  100       CONTINUE
         IF(NDL. EQ .1) GO TO 130
            DO 120 MNUP=1,MNUPT
            IF(T(NDL-1,MNUP,MAN). EQ .0) GO TO 120
            J = MJTOJJ(NDL,MNUP,K)
            IF(J. EQ .0) GO TO 120
               DO 110 NT=1,NDT
               QDUMB(NT) = QDUMB(NT) + QOUT(MNUP,J,NT)
  110          CONTINUE
  120       CONTINUE
  130    CONTINUE
            DO 140 NT=1,NDT
            QIN(MAN,K,NT) = QDUMB(NT)
  140       CONTINUE
  150    CONTINUE
  200 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE HYDSFT(DIA,PIPL,AN,SLO,TLMT,DUR,NDT,DT,QMT,QT)
C     ******************************************************************
C     * THIS SUBROUTINE PERFORMS THE HYDROGRAPH SHIFT METHOD OF FLOW   *
C     *  ROUTING FOR PIPE FLOW.                                        *
C     ******************************************************************
      DIMENSION QT(60),QMT(60),T(300),Q(300)
      PI = 3.14
C
C     STEP 1: COMPUTE PEAK INFLOW
      QP = 0.0
         DO 20 N=1,NDT
         QCHECK = QT(N)
         IF(QP. LT .QCHECK) QP = QCHECK
   20    CONTINUE
C
C     STEP 2: COMPUTE VELOCITY
      VEL = 4.*QP/(PI*DIA*DIA)
C
C     STEP 3: COMPUTE SHIFTING TIME
      TSHIFT = PIPL/VEL
C
C     STEP 4: SHIFT HYDROGRAPH
      T(1) = 0.0
      Q(1) = QT(1)
      T(2) = DT + TSHIFT
      Q(2) = QT(2)
      QBASE = Q(1)
      NDT2 = NDT + 2
         DO 40 NT=3,NDT2
         IF(NT. LE .NDT) Q(NT) = QT(NT)
         IF(NT. GT .NDT) Q(NT) = QBASE
         T(NT) = T(NT-1) + DT
   40    CONTINUE
      JT = NDT + 2
      CALL PARM(TLMT,NDT,JT,QMT,T,Q,DUR,DT)
         DO 50 NT=1,NDT
   50    IF(QMT(NT). LT .QBASE) QMT(NT) = QBASE
      RETURN
      END
```

```
      SUBROUTINE HAVMDL(DIAM,ALNTH,AN,NN,SO,TLMT,DUR,MT,DDT,QMT,QI,
     *                  DMAX,VMXR,NROUT,NDSM,MRISK,ISTD,NCHECK)
C*********************************************************************
C*                                                                   *
C*         *******MUSKINGUM-CUNGE METHOD********                     *
C*                                                                   *
C*    THIS  PROGRAM COMPUTES THE DOWNSTREAM DISCHARGE FOR A CIRCULAR *
C*       PIPE GIVEN THE INFLOW HYDROGRAPH AND PIPE CHARACTERISTICS   *
C*                                                                   *
C*********************************************************************
C*                                                                   *
C*      DIAM = DIAMETER IN FT                                        *
C*      ALNTH = PIPE LENGTH IN FT                                    *
C*      SO = PIPE SLOPE IN FT/FT                                     *
C*      AN = MANNING'S N                                             *
C*      NN = NUMBER OF SECTIONS, EACH DX IN LENGTH                   *
C*      T( ) = TIME, SEC                                             *
C*      QOUT( ) = DISCHARGE HYDROGRAPH AT DOWNSTREAM END OF PIPE     *
C*                EVERY DT SECS, COMPUTED BY THE  CUNGE MUSKINGUM    *
C*      QI( ) = TOTAL MANHOLE HYDROGRAPH FROM MAIN PROGRAM, EVERY DDT*
C*      NINF = NUMBER OF DT TIME INCREMENTS FOR MUSKINGUM METHOD, QX *
C*      DT = TIME INCREMENT BASED UPON K FOR CUNGE MUSKINGUM METHOD  *
C*      DDT = TIME INCREMENT OF MANHOLE FLOW QI( ) WHICH READ INTO   *
C*             COMPUTER PROGRAM ON THE MAIN PROGRAM                  *
C*                                                                   *
C*********************************************************************
      DIMENSION QX(300),QOUT(300),T(300),QI(60),QMT(60),PDIA(24)
      DATA PDIA/8.,10.,12.,15.,16.,18.,21.,24.,27.,30.,36.,42.,48.,54.,
     $     60.,66.,72.,78.,84.,90.,96.,102.,114.,120./
      DEPMIN = 0.0
      DEPMAX = 0.98*DIAM
      NROUT = 0
      NCHECK = 0
      ANN = NN
      DX = ALNTH/(ANN-1.)
      NX = NN - 1
      IF(NDSM. EQ .0) WRITE(6,2100) DIAM,ALNTH,SO,AN,NX
   10 CONTINUE
      NDIACK = 0
      CALL CCMPDT(QI,QX,DIAM,DX,SO,AN,DDT,NINF,TLMT,DT,QB,DUR,MT,TBASE)
      DMAX = 0.0
      VMXR = 0.0
      DO 200 I = 1,NN
      QB = QX(1)
      TI = 0.
      QOUT(1) = QX(1)
      T(1) = 0.
      QF = QX(2)
      NDT = 1
      NKI = 0
  100 CONTINUE
      CALL  COEFF(DIAM,NDT,CF,SO,DX,CEL,XK,C1,C2,C3,QFULL,AN,DMAX,VMXR,
     *            NDIACK)
      IF(NDIACK. EQ .100) GO TO 300
      NDT = NDT + 1
      TI = TI + DT
      T(NDT) = TI
      IF(TI. LE. TBASE) GO TO 145
      IF(TI. GT .XK) GO TO 130
  120 QJB = QX(1)
      QJPB = QX(1)
      GO TO 140
  130 CONTINUE
      TT = TI - XK
      CALL LININT(TT,QJB,T,QX,QB,NKI,NINF)
      CALL LININT(TT,QJPB,T,QOUT,QB,NKI,NINF)
      NKI = 100
  140 CONTINUE
      QOUT(NDT) = C1*QJB + C2*QX(NDT) + C3*QJPB
      IF(QOUT(NDT). LT .QB) QOUT(NDT) = QB
      GO TO 148
  145 CONTINUE
      QOUT(NDT) = QB
  148 CONTINUE
      IF(NDT. LT .NINF) QF = QX(NDT+1)
      IF(NOT. LT .NINF) GO TO 100
      IF(NDSM. EQ .0) WRITE(6,2400)
      IF(NDSM. NE .0) GO TO 160
      DO 150 K=1,NINF
  150 WRITE(6,2500) T(K),QX(K),QOUT(K)
  160 CONTINUE
      DO 170 MN=1,NINF
  170 QX(MN) = QOUT(MN)
  200 CONTINUE
      GO TO 330
  300 CONTINUE
      IF(NDSM. EQ .0) WRITE(6,1050) DIAM
      DO 310 MM=1,23
      MD = MM
      DIAT = PDIA(MM)/12.
      IF(DIAM. EQ . DIAT) GO TO 320
  310 CONTINUE
  320 DIAM = PDIA(MD+1)/12.
      IF(NDSM. EQ .0) WRITE(6,1060) DIAM
      NROUT = 2
      GO TO 10
  330 CONTINUE
      CALL PARM  (TLMT,MT,NINF,QMT,T,QOUT,DUR,DDT)
 1050 FORMAT(/,5X,'USING DIAMETER =',F6.2,1X,'RESULTED IN A FLOW DEPTH
     *GREATER THAN THE DIAMETER')
 1060 FORMAT(/,5X,'DIAMETER IS INCREASED TO THE NEXT HIGHER COMMERCIAL
     *PIPE SIZE =',F6.2)
 2100 FORMAT(//,5X,'PIPE DIAMETER =',F5.2,'FT',
     *            /,5X,'PIPE LENGTH =',F6.1,'FT',
     *            /,5X,'PIPE SLOPE =',F7.5,'FT/FT',
     *            /,5X,'MANNINGS ROUGHNESS N =',F6.4,
     *            /,5X,'NUMBER OF DX SECTIONS =',I3)
 2400 FORMAT(//,30X,'T(K)',5X,'QI(K)',5X,'QOUT(K)')
 2500 FORMAT(25X,3F10.2)
      RETURN
      END
```

```
      SUBROUTINE COMPDT(QI,QX,DIAM,DX,SO,AN,DDT,NINF,TLMT,DT,QB,DUR,MT,
     *                  TBASE)
C*********************************************************************
C*                                                                   *
C*     THIS SUBROUTINE COMPUTES THE DT TIME INCREMENT SO THAT        *
C*      K=DX/CEL IS GREATER THAN DT.  THE MAXIMUM CELERITY OCCURS    *
C*       AT A DEPTH/DIAMETER RATIO OF 0.6 SO KNOWING THE DEPTH FROM  *
C*     THIS RELATION THE MAXIMUM CELERITY IS COMPUTED USING DQDY/DAC*
C*                                                                   *
C*********************************************************************
C*                                                                   *
C*        QI( ) = TOTAL MANHOLE HYDROGRAPH FROM MAIN PROGRAM, EVERY DDT*
C*        QX( ) = TOTAL MANHOLE HYDROGRAPH AT DT TIME INCREMENTS WHICH *
C*                IS COMPUTED IN THIS SUBROUTINE                     *
C*        QDUMB( ) = DUMMY STORAGE ARRAY FOR QI( )                   *
C*        TREAD( ) = DUMMY TIME ARRAY FOR QI( )                      *
C*        NINF = NUMBER OF DT TIME INCREMENTS FOR MUSKINGUM METHOD, QX *
C*        DT = TIME INCREMENT BASED UPON K FOR CUNGE MUSKINGUM METHOD *
C*        DDT = TIME INCREMENT OF MANHOLE FLOW QI( ) WHICH READ INTO *
C*              COMPUTER PROGRAM ON THE MAIN PROGRAM                 *
C*                                                                   *
C*********************************************************************
      DIMENSION TREAD(300),QI(60),QX(300),QDUMB(300)
      TREAD(1) = 0.0
      TBASE = 0.0
      DO 50 KK=2,MT
      IF(KK. GE .5) GO TO 50
      IF(QI(KK-1).EQ.QI(1).AND.TREAD(KK-1).LT.DUR) TBASE=TREAD(KK-1
   50 TREAD(KK) = TREAD(KK-1) + DDT
      EF2 = 1.486*SQRT(SO)/AN
      YMCEL = 0.6*DIAM
      TH = 2.*ARCOS(1. - 2.*YMCEL/DIAM)
      FF4 = SIN(TH/2.)
      AC = .125*(TH-SIN(TH))*DIAM*DIAM
      DAC = DIAM*(1-COS(TH))/(2.*FF4)
      RC = (DIAM/4.)*(1.-SIN(TH)/TH)
      FF1 = RC**(-.33333)
      FF3 = RC**(.66667)
      DRC = (SIN(TH) - TH*COS(TH))/(TH*TH*FF4)
      DQDY = EF2*(0.66667*AC*FF1*DRC + FF3*DAC)
      CEL = DQDY/DAC
      XKMCEL = DX/CEL
      DT = XKMCEL - 1.0
      XT = TLMT/DT
      MHYD = 1
      NINF = XT + 1.0
      TTT = 0.0
      QB = QI(1)
      QX(1) = QB
      MINFLO = 0
      DO 53 KK = 1,MT
   53 QDUMB(KK) = QI(KK)
      DO 70 KK=2,NINF
      TTT = TTT + DT
      IF(TTT. LE .TBASE) GO TO 55
      IF(TTT. GT .DUR) GO TO 55
      CALL PAR(TLMT,TTT,MT,C,TREAD,QDUMB,QB,DUR,MHYD,MINFLO)
      IF(Q. LT .QB) Q = QB
      MINFLO = 1
      GO TO 60
   55 Q = QB
   60 QX(KK) = Q
   70 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE COEFF(DIAM,NDT,QF,SO,DX,CEL,XK,C1,C2,C3,QFULL,AN,
     *                 DMAX,VMXR,NDIACK)
C*********************************************************************
C*                                                                   *
C*    THIS SUBROUTINE COMPUTES THE COEFFICIENTS FOR THE CUNGE-       *
C*       MUSKINGUM ROUTING METHOD.  FIRST THE DEPTH OF FLOW IS COM-  *
C*       PUTED.  USING THE FLOW DEPTH THE CELERITY, DQ/DA, IS FOUND  *
C*       WHICH IS USED TO COMPUTE XK=ALNTH/CEL.  KNOWING  XK  ALFA IS *
C*       COMPUTED TO FIND THE MUSKINGUM COEFFICIENTS.                *
C*                                                                   *
C*                                                                   *
C*       QFULL = FULL FLOW CAPACITY OF PIPE, CFS                     *
C*       TH =  FLOW ANGLE                                            *
C*       YC = DEPTH OF FLOW, FT                                      *
C*       AC = AREA OF FLOW,FT**2                                     *
C*       DAC = DERIVATIVE OF AC W.R.T. YC                            *
C*       RC = HYDRAULIC RADIUS                                       *
C*       DRC = DERIVATIVE OF RC W.R.T. YC                            *
C*       TW = TOPWIDTH OF FLOW AT SECTION OF YC                      *
C*       DQDY = DERIVATIVE OF THE DISCHARGE W.R.T. YC                *
C*       CEL = CELERITY DQDY/DAC, FT/SEC                             *
C*       C1,C2,C3 = MUSKINGUM COEFFICIENTS                           *
C*                                                                   *
C*********************************************************************
      IF(NDT. EQ .1) YC = 0.3*DIAM
      IF(NDT. EQ .1) EF2 = 1.486*SQRT(SO)/AN
      IF(NDT. EQ .1) QFULL = EF2*DIAM**2.667
      EF1 = QF/EF2
C*********************************************************************
C*                                                                   *
C*    COMPUTE DEPTH OF FLOW FOR QF                                   *
C*                                                                   *
C*********************************************************************
      NTIME = 0
   20 CONTINUE
      NTIME = NTIME + 1
      TH = 2.*ARCOS(1. - 2.*YC/DIAM)
      FF4 = SIN(TH/2.)
      AC = .125*(TH-SIN(TH))*DIAM*DIAM
      DAC = DIAM*(1-COS(TH))/(2.*FF4)
      RC = (DIAM/4.)*(1.-SIN(TH)/TH)
      FF1 = RC**(-.33333)
      FF3 = RC**(.66667)
      DRC = (SIN(TH) - TH*COS(TH))/(TH*TH*FF4)
      FQO = EF1 - AC*FF3
      FPQO = -0.66667*AC*FF1*DRC - FF3*DAC
      FDD = FQO/FPQO
      YC = YC - FDD
      IF(NTIME. GE .45) GO TO 100
      IF(YC. LT .0.) WRITE(6,4000) YC
 4000 FORMAT(/,5X,'DEPTH =',F6.2)
      IF(YC. LT .0.) YC = 0.2*DIAM
      IF(YC. GE .DIAM) NDIACK = 100
      IF(YC. GE .DIAM) GO TO 100
      IF(ABS(FDD). GE .0.0005) GO TO 20
```

```
C*****************************************************************
C*                                                               *
C*     COMPUTE XK, CELERITY, ALFA                                *
C*                                                               *
C*****************************************************************
      V = QF/DAC
      IF(V. GT .VMXR) VMXR = V
      IF(YC. GT .DMAX) DMAX = YC
      TW = DIAM*FF4
      DQDY = FF2*(0.66667*AC*FF1*DRC + FF3*DAC)
      CEL = DQDY/DAC
      XK = DX/CEL
      ALFA = XK*QF/(SO*DX*DX*TW)
      IF(ALFA. GT .1.0) ALFA = 1.0
C*****************************************************************
C*                                                               *
C*     COMPUTE  MUSKINGUM COEFFICIENTS                           *
C*                                                               *
C*****************************************************************
      C1 = (2.-ALFA)/(2.+ALFA)
      C2 = ALFA/(2.+ALFA)
      C3 = C2
  100 CONTINUE
      RETURN
      END


      SUBROUTINE PAR(TLMT,TT,JT,QU,T,Q,QB,DUR,MHYD,MINFLO)
      DIMENSION T(300),Q(300)
C*******************************************************************
C*    THIS SUBROUTINE USES METHOD OF PARABOLIC INTERPOLATION TO OBTAIN *
C*    DISCHARGES FOR INFLOW AND OUTFLOW HYDROGRAPHS EVERY DT TIME      *
C*    INCREMENT FROM A GIVEN HYDROGRAPH                                *
C*******************************************************************
      IF(MINFLO. EQ.0) KI = 1
      JTT = JT - 2
      IF(MHYD. EQ .0 .AND. TT. GE .DUR) GO TO 250
         DO 200 JI=KI,JTT
         JJI = JI
         IF(T(JI). LE .TT. AND .TT. LT .T(JI+1)) GO TO 40
         GO TO 200
   40    CONTINUE
         IF(Q(JI). EQ .Q(JI+1)) GO TO 250
         IF(Q(JI+2). GT .Q(JI+1)) GO TO 50
         IF(Q(JI+1). GT .Q(JI)) GO TO 75
   50    CONTINUE
         J = JI
         T1 = T(J)
         T2 = T(J+1)
         T3 = T(J+2)
         Q1 = Q(J)
         Q2 = Q(J+1)
         Q3 = Q(J+2)
         GO TO 220
C
   75    CONTINUE
         J = JI
         T1 = T(J-1)
         T2 = T(J)
         T3 = T(J+1)
         Q1 = Q(J-1)
         Q2 = Q(J)
         Q3 = Q(J+1)
         GO TO 220
  200    CONTINUE
      GO TO 250
  220 CONTINUE
         QT = Q2 - (Q1-Q3)*(TT-T2)/(T3-T1) + 2.*(Q1+Q3-2.*Q2)*(TT-T2)*
     1   (TT-T2)/((T3-T1)*(T3-T1))
         QU = QT
      KI = JJI
      GO TO 300
  250 CONTINUE
      QU = QB
  300 CONTINUE
      RETURN
      END


      SUBROUTINE LININT(TT,CT,T,Q,QB,NKI,JTT)
      DIMENSION Q(300),T(300)
      IF(NKI. EQ .0) KI = 1
      DO 100 JI=KI,JTT
      JJI = JI
      IF(T(JI). LE .TT. AND .TT. LT .T(JI+1)) GO TO 110
  100 CONTINUE
  110 CONTINUE
      IF(Q(JI). EQ .Q(JI+1)) QT = QB
      IF(Q(JI). LT .Q(JI+1)) QT = Q(JI)+(Q(JI+1)-Q(JI))*(TT-T(JI))/
     *                            (T(JI+1)-T(JI))
      IF(Q(JI). GT .Q(JI+1)) CT =Q(JI+1) + (Q(JI)-Q(JI+1))*(TT-T(JI+1))/
     *                            (T(JI)-T(JI+1))
      KI = JJI - 2
      IF(KI. LT .1) KI = 1
      RETURN
      END
```

```
      SUBROUTINE PARM  (TLMT,MT,JT,QTTT,T,Q,DUR,DT)
      DIMENSION QTTT(60),T(300),Q(300)
C*********************************************************************
C*    THIS SUBROUTINE USES METHOD OF PARABOLIC INTERPOLATION TO OBTAIN *
C*    DISCHARGES FOR INFLOW AND OUTFLOW HYDROGRAPHS EVERY DT TIME      *
C*    INCREMENT FROM A GIVEN HYDROGRAPH                                *
C*********************************************************************
      KI = 1
      TT = 0.
      JTT = JT - 2
      QTTT(1) = Q(1)
      DO 300 LL=2,MT
      TT = TT + DT
         DO 200 JI=KI,JTT
         JJI = JI
         IF(T(JI). LE .TT. AND .TT. LT .T(JI+1)) GO TO 40
         GO TO 200
   40    CONTINUE
         IF(Q(JI+2). GT .Q(JI+1)) GO TO 50
         IF(Q(JI+1). GT .Q(JI)) GO TO 75
   50    CONTINUE
         J = JI
         T1 = T(J)
         T2 = T(J+1)
         T3 = T(J+2)
         Q1 = Q(J)
         Q2 = Q(J+1)
         Q3 = Q(J+2)
         GO TO 220
C
   75    CONTINUE
         J = JI
         T1 = T(J-1)
         T2 = T(J)
         T3 = T(J+1)
         Q1 = Q(J-1)
         Q2 = Q(J)
         Q3 = Q(J+1)
         GO TO 220
  200    CONTINUE
  220 CONTINUE
         QT = Q2 - (Q1-Q3)*(TT-T2)/(T3-T1) + 2.*(Q1+Q3-2.*Q2)*(TT-T2)*
     1        (TT-T2)/((T3-T1)*(T3-T1))
         QTTT(LL) = QT
      KI = JJI
  300 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE WAVMD2(DIAM,ALNTH,AN,NN,SO,TLMT,    DUR,MT,DT,QMT,QI,
     *                  DMAX,VMXR,NROUT,NDSM,MRISK,LSPD,NCHECK)
C*********************************************************************
C*    KINEMATIC WAVE MODEL USING AN IMPLICIT SCHEME FOR FLOW IN      *
C*         CIRCULAR PIPES                                            *
C*********************************************************************
      DIMENSION QI(60),Y(11,2),G(11,2),T(11,2),V(11),Q(11),QMT(60),
     *          PDIA(24)
      DATA PDIA/8.,10.,12.,15.,16.,18.,21.,24.,27.,30.,36.,42.,48.,54.,
     $          60.,66.,72.,78.,84.,90.,96.,102.,114.,120./
      NROUT = 0
      NX = NN - 1
      IF(NDSM. EQ .0) WRITE(6,2100) DIAM,ALNTH,SO,AN,NX
      IF(NDSM. NE .0) GO TO 10
      WRITE(6,1000)
      WRITE(6,1010)
      TH = 0.0
      DO 15 I=1,MT
      WRITE(6,1020) TH,QI(I)
   15 TH = TH + DT
   10 CONTINUE
      DEPMIN = 0.0
      DEPMAX = 0.98*DIAM
      ANN = NN
      DX = ALNTH/(ANN-1.)
      DMAX = 0.
      VMXR = 0.
      EF2 = 1.486*SQRT(SO)/AN
C****************************
C*    INITIAL CONDITIONS    *
C****************************
      QB = QI(1)
      TI = 0.0
      X = 0.
      EF1 = QB/EF2
      YO = 0.3*DIAM
      MSEC = 0
   20 CONTINUE
      CALL GEOM(MSEC,YO,DIAM,TH,AO,CAO,DTO,RO,DRO,DDRO,GO,DGO,FF1,FF3,
     *          SO,AN,EF2)
      FOO = EF1 - AO*FF3
      FPQO = -0.66667*AO*FF1*DRO - FF3*DAO
      YO = YO - FOO/FPQO
      IF(ABS(FOO/FPQO). GE .0.0005) GO TO 20
      VO = EF2*FF3
      DO 25 M = 1,NN
      Y(M,1) = YO
      G(M,1) = GO
      T(M,1) = DAO
      V(M) = VO
      Q(M) = AO*VO
      X = X + DX
   25 CONTINUE
      IF(NDSM. NE .0) GO TO 28
      WRITE(6,1030)
      WRITE(6,1040) QB,V(NN),Y(NN,1),SO
   28 CONTINUE
      QMT(1) = Q(NN)
      J = 1
```

```
   30 TI = TI + DT
      J = J+ 1
      IF(TI. LT .DUR) QU = QI(J)
      IF (TI. GE .DUR) QU = QB
C***************************************
C*                                     *
C*    UPSTREAM BOUNDARY CONDITION      *
C*                                     *
C***************************************
      X = 0.
      EF1 = QU/EF2
      YU = Y(1,1)
      MSEC = 0
      NCHECK = 0
   40 CONTINUE
      IF(YU. LE .0.0) GO TO 190
      IF(YU. GE .DEPMAX) GO TO 190
      CALL GEOM(MSEC,YU,DIAM,TH,AU,TU,DTU,RU,DRU,DDRU,GU,DGU,FF1,FF3,SO,
     *          AN,EF2)
      FQU = EF1 - AU*FF3
      FPQU = -.66667*AU*FF1*DRU - FF3*TU
      YU = YU - FQU/FPQU
      NCHECK = NCHECK + 1
      IF(NCHECK. GE .15) GO TO 220
      IF(ABS(FQU/FPQU). GE .0.0005) GO TO 40
      VU = EF2*FF3
      Y(1,2) = YU
      G(1,2) = GU
      T(1,2) = TU
      V(1) = VU
      Q(1) = QU
C*************************
C*    INTERIOR POINTS    *
C*************************
      FDX = 1./(2.*DX)
      FDT = 1./(4.*DT)
      DO 150 I = 2,NN
         X = X + DX
         YC = Y(I-1,2)
         GD = G(I-1,2)
         TD = T(I-1,2)
         YD = Y(I-1,2)
         YA = Y(I-1,1)
         YB = Y(I,1)
         MSEC = 1
      NCHECK = 0
  100    CONTINUE
         IF(YC. LE .0.0) GO TO 190
         IF(YC. GE .DEPMAX) GO TO 190
         CALL GEOM(MSEC,YC,DIAM,TH,AC,TC,DTC,RC,DRC,DDRC,GC,DGC,FF1,FF3,
     *          SO,AN,EF2)
         FKW = FDX*(GD+GC)*(YC-YD) + FDT*(TD+TC)*(YD+YC-YA-YB)
         FPKW = FDX*((GD+GC) + (YC-YD)*DGC) + FDT*((TD+TC) + (YD+YC-YA-
     *          YB)*DTC)
         YC = YC - FKW/FPKW
      NCHECK = NCHECK + 1
      IF(NCHECK. GE .15) GO TO 220
         IF(ABS(FKW/FPKW). GE .0.0005) GO TO 100
         V(I) = EF2*FF3
         Q(I) = AC*V(I)
         Y(I,2) = YC
         G(I,2) = GC
         T(I,2) = TC
         IF(DMAX. LE .YC) DMAX = YC
         IF(VMXR. LE .V(I)) VMXR = V(I)
  150    CONTINUE
      X = 0.
      DO 160 M = 1,NN
      Y(M,1) = Y(M,2)
      G(M,1) = G(M,2)
      T(M,1) = T(M,2)
      X = X + DX
  160 CONTINUE
      CMT(J) = Q(NN)
      IF(TI. LE .TLMT) GO TO 30
      IF(NROUT. EQ .2) GO TO 220
      IF(DMAX. GT .DEPMIN) GO TO 220
      IF(LSPD. EQ .0) GO TO 220
      DO 170 MM=1,24
      MD = MM
      DIAT = PDIA(MM)/12.
      IF(DIAM. EQ .DIAT) GO TO 180
  170 CONTINUE
  180 DIAM = PDIA(MD-1)/12.
      IF(NDSM. EQ .0) WRITE(6,1070) DIAM
      GO TO 10
  190 CONTINUE
      IF(NDSM. EQ .0) WRITE(6,1050) DIAM
      DO 200 MM=1,24
      MD = MM
      DIAT = PDIA(MM)/12.
      IF(DIAM. EQ .DIAT) GO TO 210
  200 CONTINUE
  210 DIAM = PDIA(MD+1)/12.
      IF(NDSM. EQ .0) WRITE(6,1060) DIAM
      NROUT = 2
      GO TO 10
  220 RETURN
 1000 FORMAT (/,24X,'INFLOW HYDROGRAPH')
 1010 FORMAT(/,19X,'TIME, SEC',4X,'DISCHARGE, CFS')
 1020 FORMAT(20X,F7.1,10X,F5.1)
 1030 FORMAT(/,3X,'INITIAL CONDITIONS')
 1040 FORMAT(5X,'BASE FLOW=',F8.4,5X,'VELOCITY=',F8.4,5X,'DEPTH=',F8.4,
     *5X,'SO=',F8.4)
 1050 FORMAT(/,5X,'USING DIAMETER =',F6.2,1X,'RESULTED IN A FLOW DEPTH
     *GREATER THAN THE DIAMTER')
 1060 FORMAT(/,5X,'DIAMETER IS INCREASED TO THE NEXT HIGHER COMERCIAL PI
     *PE SIZE =',F6.2)
 1070 FORMAT(/,5X,'DIAMETER IS DECREASED TO NEXT SMALLER COMMERCIAL PIPE
     * SIZE =',F6.2)
 2100 FORMAT(//,5X,'PIPE DIAMETER =',F5.2,'FT',
     *          /,5X,'PIPE LENGTH =',F6.1,'FT',
     *          /,5X,'PIPE SLOPE =',F7.5,'FT/FT',
     *          /,5X,'MANNINGS ROUGHNESS N =',F6.4,
     *          /,5X,'NUMBER OF DX SECTIONS =',I3)
      END
```

```
      SUBROUTINE GEOM(MSEC,YC,DIAM,TH,AC,DAC,DDAC,RC,DRC,DDRC,GC,DGC,
     *                FF1,FF3,SO,AN,FF2)
C*****************************************************************************
C*    THIS SUBROUTINE COMPUTES THE FLOW CHARACTERISTICS FOR THE            *
C*       POINTS ON THE X-T SOLUTION PLANE OF THE IMPLICIT TECHNIQUE.       *
C*       GIVEN ARE THE DEPTH OF FLOW, YC, AND THE DIAMETER, DIAM.          *
C*                                                                         *
C*       AC = AREA OF FLOW,CFS                                             *
C*       DAC = DERIVATIVE OF AC W.R.T YC                                   *
C*       RC = HYDRAULIC RADIUS                                             *
C*       DRC = DERIVATIVE OF RC W.R.T. YC                                  *
C*       GC = DERIVATIVE OF THE DISCHARGE W.R.T. YC                        *
C*       DDAC = SECOD DERIVATIVE OF AC W.R.T. YC                           *
C*       DDRC = SECOND DERIVATIVE OF THE DISCHARGE W.R.T. YC               *
C*       DGC = SECOND DERIVATIVE OF DISCHARGE W.R.T. YC                    *
C*****************************************************************************
      TH = 2.*ARCOS(1. - 2.*YC/DIAM)
      FF4 = SIN(TH/2.)
      AC = .125*(TH-SIN(TH))*DIAM*DIAM
      DAC = DIAM*(1-COS(TH))/(2.*FF4)
      RC = (DIAM/4.)*(1.-SIN(TH)/TH)
      FF1 = RC**(-.33333)
      FF2 = FF1**4
      FF3 = RC**(.66667)
      DRC = (SIN(TH) - TH*COS(TH))/(TH*TH*FF4)
      GC = FF2*(.66667*AC*FF1*DRC + FF3*DAC)
      IF(MSEC. EQ .0) GO TO 10
      FF5 = FF4**3
      DDAC = (2.*SIN(TH/2.)*SIN(TH) - COS(TH)*(1.-COS(TH)))/FF5
      DDRC=(4./DIAM)*(TH*TH*TH*FF4*SIN(TH) - (SIN(TH)-TH*COS(TH))*
     *      (TH*TH*COS(TH/2.)/2. + 2.*TH*FF4))/(FF5*TH**4)
      DGC = FF2*(.66667*(AC*FF1*DDRC - 0.33333*AC*FF2*DRC*DRC + FF1*DRC*
     *      DAC) + FF3*DDAC + 0.66667*FF1*DAC*DRC)
   10 CONTINUE
      RETURN
      END
```

APPENDIX F

PROJECT PUBLICATIONS

A list of publications wholly or partially supported by the research project (excluding this report) is given as follows.

(1) Tang, W. H. and B. C. Yen, "Hydrologic and Hydraulic Design Under Uncertainties," Proceedings, International Symposium on Uncertainties in Hydrologic and Water Resource Systems, Vol. 2, pp. 868-882, Tucson, Ariz., Dec. 1972.

(2) Mays, L. W. and B. C. Yen, "Optimal Cost Design of Branched Sewer Systems," Water Resources Research, Vol. 11, No. 1, pp. 37-47, Feb. 1975.

(3) Yen, B. C., W. H. Tang, and L. W. Mays, "Designing Storm Sewers Using the Rational Method," Water and Sewage Works, Part I in Vol. 121, No. 10, pp. 92-95, Oct. 1974, Part II in Vol. 121, No. 11, pp. 84-85, Nov. 1974.

(4) Tang, W. H., L. W. Mays and B. C. Yen, "Optimal Risk-Based Design of Storm Sewer Networks," Jour. Env. Eng. Div., ASCE, Vol. 101, No. EE3, pp. 381-398, June 1975.

(5) Mays, L. W., B. C. Yen and W. H. Tang, "Worth of Data for Optimal Design of Storm Sewers," Proceedings, 16th Congress of the International Association for Hydraulic Research, Vol. 4, pp. 34-42, Sao Paulo, Brazil, July 1975.

(6) Yen, B. C. and A. S. Sevuk, "Design of Storm Sewer Networks," Jour. Env. Eng. Div., ASCE, Vol. 101, No. EE4, pp. 535-553, Aug. 1975.

(7) Yen, B. C. and W. H. Tang, "Risk-Safety Factor Relation for Storm Sewer Design," Jour. Env. Eng. Div., ASCE, Vol. 102, No. EE2, pp. 509-516, April 1976.

(8) Mays, L. W., "Optimal Layout and Design of Storm Sewer Systems," Ph.D. Thesis, Dept. of Civil Eng., University of Illinois at Urbana-Champaign, Ill., 1976.

(9) Mays, L. W. and H. G. Wenzel, Jr., "A Serial DDDP Approach for Optimal Design of Multi-level Branching Storm Sewer Systems," to be published in Water Resources Research, April 1977.

(10) Yen, B. C., H. G. Wenzel, Jr., L. W. Mays and W. H. Tang, "New Models for Optimal Sewer Systems Design," to appear in the Proceedings of the U. S. EPA Conf. on Modeling and Simulation, Cincinatti, Ohio, April 1976.

(11) Mays, L. W., H. G. Wenzel, Jr. and J. C. Liebman, "A Heuristic Model for Layout and Design of Sewer Systems," Accepted for publication in Jour. Water Resources Plan. and Manage., ASCE, 1977.

The abstracts of the publications listed above are reproduced in the following pages.

(1)

Proceedings, Vol. 2, pp. 868-882

Tucson, Arizona, December 1972

INTERNATIONAL SYMPOSIUM ON UNCERTAINTIES IN HYDROLOGIC AND WATER RESOURCE SYSTEMS

HYDROLOGIC AND HYDRAULIC DESIGN UNDER UNCERTAINTIES

by Wilson H. Tang
Assistant Professor
and Ben Chie Yen
Associate Professor
Department of Civil Engineering
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Illinois, 61801 USA

ABSTRACT

Uncertainties arise at various aspects and phases in hydrologic and hydraulic design of engineering projects. In the past, only hydrologic uncertainties such as basic randomness in flood or rainfall frequencies has been considered. This paper proposes a practical model whereby all sources of uncertainties associated with a hydraulic design, including design model reliability, insufficient data, material variability, as well as hydrologic randomness etc., can be systematically analyzed, combined, and incorporated in the evaluation of the overall risk of alternative designs. Formulation leading to the selection of a design for a specified risk level is developed. An example of storm sewer design is presented to illustrate in detail the proposed formulation. Based on the assessment of uncertainties, the precipitation rate and the errors in the simplified design model are the major sources of uncertainties. A risk-based sewer pipe design curve is developed for practical purpose. For a specified risk level of 1%, the required safety factor in the design flow is shown to be about 2.0.

(2)

VOL. 11, NO. 1 WATER RESOURCES RESEARCH FEBRUARY 1975

Optimal Cost Design of Branched Sewer Systems

LARRY W. MAYS AND BEN CHIE YEN

Department of Civil Engineering, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801

Techniques using dynamic programing (DP) and discrete differential dynamic programing (DDDP) to achieve optimal cost design of pipe sizes and elevations of branched sewer systems have been developed and demonstrated by an example. The branched system is decomposed into equivalent serial subsystems, which are then solved in sequence. DDDP requires less computer time than DP, although it cannot guarantee global optimization. Major factors affecting the efficiency in using DDDP are the location and width of the initial trial trajectory corridor, the number of states (lattice points) used, and the reduction rate of the state increment during iterations.

(3)

Sewage Collection

Reprinted with permission from Water & Sewage Works,
434 S. Wabash Ave., Chicago, Illinois 60605.
Part 1 in pp. 92-95, Vol. 121, No. 10, October 1974
Part 2 in pp. 84-85, Vol. 121, No. 11, November 1974.

Designing Storm Sewers Using the Rational Method

By Ben Chie Yen, Wilson H. Tang, and Larry W. Mays
Civil Engineering Department, University of Illinois

Three techniques for design of storm sewers using the rational formula are discussed in this article. They are the standard rational method, risk-based rational method, and risk-based optimal-cost rational method. Two different versions are introduced for each of the latter two methods.

The rational method[1] is probably the most popular method used for design of storm sewers. Although criticisms have been raised on the adequacy of the method,[2,3] and several other more occurate methods have recently been proposed[4,5,6] the rational method, because of its simplicity, will probably be in continued use for sewer designs when high accuracy of runoff rate is not required.

Furthermore, engineers have long realized the uncertainties involved in the data and other information needed for design of sewers using the rational method. Yet such uncertainties and the related benefit-cost-damage relationship have not been taken explicitly into consideration in design. Also, even without considering the uncertainties and the related costs and damages, the rational method has been used incorrectly by a surprisingly large number of engineers in design of storm sewers. This is probably the consequence of the fact that among the standard design manuals and textbooks, only one, an ASCE design manual[1] presented the method for a sewer system correctly however without adequate explanation.

The purpose of this paper is to present the computational procedure of design of storm sewers using the rational method with and without considering the risks and costs. The theory of the rational method can be found in standard textbooks and manuals[1,7] and the theory of the risk-based cost optimization can be referred to a recent paper by the authors.[8]

(4)

11360 OPTIMAL RISK-BASED DESIGN OF STORM SEWER

KEY WORDS: Costs; Drainage; **Dynamic programming; Economic analysis; Environmental engineering;** Mathematical models; **Optimization;** Probability theory; **Risk;** Sanitary engineering; **Storm sewers;** Uncertainty principle; Urban development

ABSTRACT: Optimal design of sewer network has conventionally been pursued within a deterministic framework. However, such design may be appropriate because of existence of uncertainties affecting the performance of a storm sewer system, e.g., precipitation rate, insufficient data, errors in design equations, and other factors. These uncertainties are systematically evaluated and incorporated in the proposed design of a storm sewer system. The optimal choice of slopes and diameters of the entire sewer network is based on a tradeoff between risk due to potential flood losses and the cost of installation of the sewers. Discrete differential dynamic programming technique is used in the risk-based design.

REFERENCE: Tang, Wilson H., Mays, Larry W., and Yen, Ben Chie, "Optimal Risk-Based Design of Storm Sewer Networks," *Journal of the Environmental Engineering Division*, ASCE, Vol. 101, No. EE3, **Proc. Paper 11360**, June, 1975, pp. 381-398

(5)

Proceedings of the 16th Congress
(Sao Paulo, Brazil, 1975)
Vol. 4, pp. 34-42

INTERNATIONAL ASSOCIATION FOR HYDRAULIC RESEARCH

WATER RESOURCES SYSTEMS (Subject D.a.)

WORTH OF DATA FOR OPTIMAL DESIGN OF STORM SEWERS

By Larry W. MAYS, Graduate Research Assistant,
Ben Chie YEN, Associate Professor
and Wilson H. TANG, Associate Professor

Department of Civil Engineering
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Urbana, Illinois 61801 USA

SYNOPSIS

Design of water resources engineering projects is always under uncertainties. This fact bears important consequences when the optimal design is sought. The uncertainties often complicate the determination of the optimal design, and probabilistic consideration is generally a necessity. To illustrate this point, optimal design of a branched storm sewer systems is used as an example. The result demonstrates the relative importance of and sensitivity to the accuracy of the data on rainfall, transformation of rainfall to runoff, sewer size, change in roughness, accuracy of design equations, and design duration and return period of rainfall. The method can be applied to other water resources systems and the result can identify the most critically needed data as well as the worth of the data.

RÉSUMÉ

Le calcul des projets de ressources en eau est toujours incertain. Ce fait est encore plus important quand il s'agit de retrouver la solution optima. Pour parer aux incertitudes qui compliquent souvent la détermination de la solution optima, des considérations probabilistiques sont en général nécéssaires. Pour illustrer ce fait, le calcul optimum d'un système d'égouts branché est utilisé. Le résultat démontre l'importance relative et la sensitivité de la précision des donneés sur la précipitation, la transformation de la pluie en ruissellement, la dimension des égouts, les variations de la rugosité, la précision des équations de base, la durée des calculs et la fréquence de la précipitation. La méthode peut être appliquée à d'autres systèmes de ressources en eau et les résultats peuvent identifier les données les plus nécessaires aussi bien que leur mérite.

(6)

11503 DESIGN OF STORM SEWER NETWORKS

KEY WORDS: **Computer applications; Drainage; Environmental engineering;** Floods; Hydraulic design; Mathematical models; Retention dams; Runoff; **Sewers; Storm drains;** Storm runoff; Storm water; **Urbanization**

ABSTRACT: Five methods for design of storm and combined sewers are cor ıared by determining the size of the sewers of an example network. The Illinois Sto n Sewer Systems Simulation (ISS) Model appears to be the most accurate method sı able for large expensive networks, and can also be used to study on-line retention stc ıge. The conventionally used steady-flow routing method with shifting of hydrograp due to sewer flow time is simple and does not require use of a computer; thus it ca be used for small networks and when high accuracy is not required. The version of sı ıdy-flow routing method without time shifting of hydrographs is not recommended ! :cause it consistently gives overdesign. The Chicago method provides no improvement :n design and yet is more complicated than the time-shifting version of steady-flow routing method. The EPA Storm Water Management Model and the kinematic-wave model are improvements over the steady-flow routing method. They can be used for large networks but presumably are less accurate than the ISS Model.

REFERENCE: Yen, Ben Chie, and Sevuk, Ahmet Suha, "Design of Storm Sewer Networks," *Journal of the Environmental Engineering Division*, ASCE, Vol. 101, No. EE4, **Proc. Paper 11503**, August, 1975, pp. 535-553

(7)

Risk-Safety Factor Relation for Storm Sewer Design

By Ben Chie Yen,[1] M. ASCE and Wilson H. Tang,[2] A. M. ASCE

Introduction

Recently, the writers (7) proposed a new method for optimal design of diameters and slopes of pipes of storm sewer networks. Uncertainties from various sources are considered and a tradeoff is made betwen potential rainstrom flood damages and the cost of installation of the sewers. One key step in the proposed method is the establishment of the risk-safety factor relationship. Although the underlying theory has been presented elsewhere (8) and summarized in Ref. 7 for storm sewer design, questions requiring clarification of the details have been raised. Through a numerical example, this paper presents the details of the procedure to establish the risk-safety factor relationship. The usefulness of the relationship so established is neither restricted to the optimal design method proposed in Ref. 7 nor to only sewer design (8).

Note.—This paper is part of the copyrighted Journal of the Environmental Engineering Division, Proceedings of the American Society of Civil Engineers, Vol. 102, No. EE2, April, 1976. Manuscript was submitted for review for possibible publication on June 26, 1975.

[1]Assoc. Prof. of Civ. Engrg., Univ. of Illinois at Urbana-Champaign, Urbana, Ill.
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(8)

OPTIMAL LAYOUT AND DESIGN OF STORM SEWER SYSTEMS

Larry Wesley Mays, Ph.D.
Department of Civil Engineering
University of Illinois at Urbana-Champaign, 1976

The objective of this research is to develop an optimization model for minimum cost layout and design of storm sewer systems. This model can be used to determine the optimal layout, size, and crown elevations of the sewer pipes and depths of the manholes. The required input for the model are the locations of the manholes and outlets, design inflows at manholes, arbitrary surface topography, and design constraints.

The optimization is achieved by using a screening model which consists of two conjunctive phases: a model for the combined layout and design and a supplementary design model for prescribed layouts. The combined layout and design model is an optimization procedure based upon discrete differential dynamic programming (DDDP) which simultaneously varies both the system layout and pipe design. Drainage lines, which are imaginary lines separating manholes for investigation of possible pipe connections, are used to divide the system into stages for optimization. At each stage of the DDDP procedure, a connectivity model, which is formulated as a set-partitioning problem, is solved to determine the minimum cost layout for that stage. The combined layout and design model is an iterative stage-by-stage procedure resulting in a complete system layout and design after each iteration. When changes

in the system layout occur for successive iterations of the combined model, the design model for given layouts is used to compute the optimal design of the previously generated layout by the combined layout and design model. The design model is also an iterative stage-by-stage optimization procedure based upon DDDP. The hydraulics of flow are accounted for using Manning's equation and peak flows at various manholes are added to compute flowrates for the connecting downstream pipes.

The screening model has been programmed in Fortran IV for computer application. Approximately 268 K bytes of the computer storage are required for a sewer system of 200 manholes. The screening model is applied to a hypothetical storm sewer system having relatively flat ground slopes and to an actual storm sewer system having rather steep ground slopes. Through the use of these examples various factors affecting the efficient use of the screening model are identified and discussed. Experience gained from several examples indicates that the required computation time for solution is small. Guidelines are presented for determining various model input parameters. Several conclusions are made concerning the design constraints, pipe designs, system layouts, outlet location, and drainage line construction including how their variation and cost interactions affect the final optimal layout and design of the storm sewer system.

(9)

A SERIAL DDDP APPROACH FOR OPTIMAL DESIGN OF MULTI-LEVEL BRANCHING STORM SEWER SYSTEMS

LARRY W. MAYS AND HARRY G. WENZEL, JR.

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ABSTRACT

7310 Economics
A SERIAL DDDP APPROACH FOR OPTIMAL DESIGN OF MULTI-LEVEL BRANCHING STORM SEWER SYSTEMS
Larry W. Mays and Harry G. Wenzel (Department of Civil Engineering, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801)

The problem of determining a minimum cost design of a multi-level branching storm sewer system is formulated using a serial approach to describing the system and proceeding through the computational algorithm. Discrete differential dynamic programming (DDDP) is the basic search technique employed. Results for an example using the serial approach are compared to those achieved using an earlier nonserial DDDP approach. The several advantages of the serial approach becomes increasingly important as the size and level of branching of the system increases. These advantages are outlined and discussed, demonstrating the superiority of the serial over the nonserial approach. (Operations research, storm runoff, storm sewers, water resources)

Mays, L. W. and H. G. Wenzel, Jr., "A Serial DDDP Approach for Optimal Design of Multi-level Branching Storm Sewer Systems," to be published in Water Resources Research, April 1977.

(10)

NEW MODELS FOR OPTIMAL SEWER SYSTEM DESIGN

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Wilson H. Tang
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Urbana, Illinois 61801

SUMMARY

Three new models have been developed for the least-cost design of storm sewers. All three models consider the sewers as a system. The basic model designs the crown elevations, slopes, and diameters of the sewers. The sewer system layout is predetermined. Routing is accomplished by lagging the hydrographs by a travel time. Optimization is achieved through a discrete differential dynamic programming technique to produce the least-cost design of the system based on specified cost functions for installation of the sewers and manholes. The second model is an expansion of the basic model incorporating risk-based damage costs in the design procedure, and the risks for each sewer associated with the least-cost design are also given as part of the design results. The third model is similar to the basic model except that the least-cost sewer layout is also a part of the design result instead of being predetermined.

Yen, B. C., H. G. Wenzel, Jr., L. W. Mays and W. H. Tang, "New Models for Optimal Sewer Systems Design," to appear in the Proceedings of the U. S. EPA Conf. on Modeling and Simulation, Cincinatti, Ohio, April 1976.

(11)

A HEURISTIC MODEL FOR LAYOUT AND DESIGN OF SEWER SYSTEMS

By

Larry W. Mays, A.M. ASCE, Harry G. Wenzel, Jr.
and Jon C. Liebman, Members, ASCE

ABSTRACT

An optimization model has been developed for computing the minimum cost layout and design of sewer systems for arbitrary topographic, physiographic, and hydrologic conditions. The heuristic procedure has been developed as a screening model consisting of two conjunctive phases: a combined layout and design model and a design model for given layouts. The combined layout and design model is an optimization procedure based upon discrete differential dynamic programming (DDDP) and a connectivity model formulated as a set-partitioning problem. This iterative stage-by-stage procedure results in a complete system layout and design after each iteration. The design model for given layouts is also based upon DDDP and is used to compute the minimum cost designs for layouts generated by the combined model. Through the use of a hypothetical example several factors affecting efficient use of the layout and design model are identified and discussed.

Key Words: Sewer systems, Urban drainage, Economic analysis, Optimization, Storm sewers, Sanitary sewers, Dynamic programming, Integer programming, Urban development, Computer

Mays, L. W., H. G. Wenzel, Jr. and J. C. Liebman, "A Heuristic Model for Layout and Design of Sewer Systems," Accepted for publication in Jour. Water Resources Plan. and Manage., ASCE, 1977.

APPENDIX G

PROJECT PERSONNEL

Many technical and non-technical persons have involved in the project at various times and capacities, including four faculty members, two professional staff, and four graduate research assistants. Throughout the project period no one person has been supported financially by the project on a full-time basis during the regular academic years. The names of the technical personnel together with their responsibilities and periods of engagement are listed as follows:

Name	Title	Starting Date	Termination Date
B. C. Yen	Co-Principal Investigator, Associate Professor of Civil Engineering	10-1-72	3-1-76
H. G. Wenzel, Jr.	Co-Principal Investigator, Associate Professor of Civil Engineering	10-1-72	3-1-76
W. H. Tang	Co-Investigator Associate Professor of Civil Engineering	10-1-72	[illegible]-1-76
D. D. Meredith	Co-Investigator Assistant Professor of Civil Engineering	10-1-72	8-20-73
A. S. Sevuk	Visiting Research Associate	10-1-72	1-20-74
L. W. Mays	Visiting Research Assistant Professor	1-21-76	3-1-76
N. N. Adelmeyer	Graduate Research Assistant	2-1-73	5-20-74
A. O. Akan	Graduate Research Assistant	10-1-72	3-1-76

A. Haldar	Graduate Research Assistant	8-21-74	8-20-75
L. W. Mays	Graduate Research Assistant	6-16-73	1-20-76

In addition, drafting and secretary-clerk staff was provided by the Department of Civil Engineering of the University and accounting service was provided by the Water Resources Center of the University of Illinois.