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MATHEMATICAL METHODS FOR USE IN PLANNING REGIONAL WASTEWATER TREATMENT SYSTEMS

By

Masahisa Nakamura and E. Downey Brill, Jr.

Department of Civil Engineering and
the Institute for Environmental Studies
University of Illinois at Urbana-Champaign

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UNIVERSITY OF ILLINOIS
WATER RESOURCES CENTER
2535 Hydrosystems Laboratory
Urbana, Illinois 61801

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ABSTRACT

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A mathematical method presented here deals with regionalization of wastewater systems, a complex public sector planning problem. The method proposed focuses on generating alternative physical plans efficiently and systematically so that planning issues other than economic efficiency may be meaningfully integrated into the process of comparing alternative plans. Such a method, although simple in concept, can aid analysts in developing insights.

Two types of alternative plans can be generated by the method, single-time period plans and simplified multiperiod plans. In generating alternative plans, the method takes advantage of the structure of a branch-and-bound algorithm. A branch-and-bound tree may be transformed into a matrix called the imputed value incidence matrix which displays the incidence relationship between each of the alternative plans and the state of variables (regional facilities) associated with it. Once the matrix is constructed the imputed value of a given variable or a given set of variables can be obtained from the matrix.

An application of the method to a realistic example problem is presented and the interpretation of imputed values is discussed.

Nakamura, Masahisa
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PREFACE

The premise of this study has been that planning regional wastewater treatment systems is a complex public-sector problem and therefore the role of mathematical methods is limited. The first step in the research was to develop a branch-and-bound method which is simple to use in a single-time period problem; the method was specifically designed for generating alternative solutions efficiently. The second step was an extension of the method to include some features of the multiperiod planning problem where wastewater loads increase over time. The approach was to retain simplicity in generating alternatives at the expense of precision in obtaining a least-cost solution. The third step was to develop a method for examining the imputed values of individual facilities or groups of them; the procedure was designed for use in synthesizing a final plan.

An earlier Water Resources Center report, *The Japanese Regional Wastewater Treatment Systems, Research Report No. 129*, describes the complexity of such planning problems and sets the stage for the mathematical methods described herein. Although parts of the mathematical underpinnings are somewhat complex, the tools are easy to employ using a simple Fortran IV computer program. The design of these tools reflects the fact that they were specifically developed for use within a larger planning process; they are significantly different than methods designed for obtaining a "least cost" or "best compromise" solution. The approach described can also be applied to gain insights about other planning problems with economies of scale in potential facilities.

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1. INTRODUCTION

1.1 Preliminaries

"The purpose of computing is insight, not numbers," stated R. W. Hamming (1962) in his book on numerical methods of analysis. The statement reflects clearly the feeling of many researchers who apply mathematical methods to the analysis of various planning problems in the private and public sectors. The truth of this statement, however, applies more strongly in the public sector because of the multiplicity of the planning issues involved. A comprehensive set of numbers that adequately reflect all of the issues is often very hard to come by.

Planning regional wastewater systems is an exceedingly difficult public sector problem. The issues involved are usually very complex, diverse and interdependent. The resolution of some of the conflicting objectives is often beyond the scope of numbers obtained through mathematical analysis. The role of mathematical models for regionalization, therefore, seems to be limited to providing insight. The question posed here, then, is "How can a mathematical model be made to provide useful information on widely different alternatives most efficiently and most meaningfully?" This dissertation presents one mathematical model specifically designed for generating and comparing physical alternative plans for regional wastewater systems.

1.2 Regionalization and Planning Issues

During the last few decades regional management of wastewater collection and treatment has become a matter of concern in many metropolitan areas and in many cities and townships. In particular, the United States Congress passed the Federal Water Pollution Control Act Amendment of 1972 (PL 92-500) which requires stringent water quality management practices by municipalities and industries in the near future. Section 208 of the law requires regional facility planning.

This trend is not limited to the United States. Many of the developed nations have been and are today under severe pressure to cope with environmental pollution problems caused by rapid urbanization and industrialization (e.g., see Kelley, *et al.*, 1976). Regional wastewater management has been considered one of the most effective means of coping with water pollution as well as water resources management problems (e.g., Canham, *et al.*, 1971, and Lyon, 1967). Regional management in these nations where there are existing facilities may or may not lead to regionalization, the process of utilizing joint, or central, plants to serve several communities. Many of the developing parts of the world also face similar pollution problems because of extreme population densities now and in the future and because pollution abatement is virtually nonexistent. In these nations where there are limited human and economic resources, however, regionalization of wastewater facilities seems to be particularly attractive from the economic and managerial points of view (Thomas, 1972).

The mode of regionalization, therefore, is likely to depend much on the socio-economic and cultural background of a nation, state and locality. It also depends on the size and the physical state of the region under consideration. The planning issues raised in the process of regionalization are, thus, very diverse and difficult to generalize. The existing studies are generally based on a specific set of experiences. For example, Metzler, *et al.* (1971) have discussed past history and the current regional wastewater planning experiences in the state of New York. A discussion of institutional patterns in evolving regional water supply systems in some of the major metropolitan areas in the United States as well as in Massachusetts communities has been presented by Howards and Keynor (1971). Recent experiences with regionalization in Britain are discussed by Ardill (1974), Buckley (1975) and Okun (1975). In addition, Brill and Nakamura (1977-b) have provided a review of issues raised in the process of regionalization in Japan.

There exist some studies, however, which review comprehensively the issues involved in regionalization. For example, the United States Advisory Commission on Intergovernmental Relations (1962) has pointed out some of the major advantages of regional management of water supply and sewage disposal. It has also discussed the difficulties encountered during the early experiences with regionalization in the United States. A summary discussion of issues to consider in planning regional wastewater systems appears in Butrico and Coulter (1972).

With respect to the current federal policy of area-wide planning (208 planning), critical reviews and discussions of planning issues and planning processes have been presented by many (e.g., United States Environmental Protection Agency, 1973; National Science Foundation, 1976; Texas Advisory Commission on Intergovernmental Relations, 1974).

Some of the major issues related to planning regional wastewater systems are:

(1) Economies of Scale

Regional wastewater systems usually involve joint facilities for treating wastewater piped from several sources. The major advantages of regionalization are the potential economies of scale in capital and operation and maintenance costs associated with such joint facilities (e.g., see Linzing, 1972, and Classen, *et al.*, 1970). Of course, there are trade-offs. A system of large plants may require interceptor pipes, which also exhibit the economies of scale (Paintal, 1975), from individual waste sources to the central plants. Their cost may exceed the cost savings accruing from the economies of scale associated with large plants.

(2) Plant Performance

Large plants are considered more reliable than small plants because of the highly efficient management. Some of the contributing factors are simplified administration, concentration of skilled personnel, automation of auxiliary equipment, and reduction in the variability of wastewater quality and quantity. On the other hand, large effluent flows from a small number of plants may pose serious threats to the natural purification capacities of the receiving streams (Adams and Gemmell, 1973).

(3) Long Term Planning Flexibility

While regionalization of wastewater systems may provide technical flexibility in meeting the comprehensive goals of a region, it can also be argued that a larger number of smaller individual systems is more flexible in coping with the unforeseen future developments in socio-economic, political and technical affairs.

(4) Compatibility with Existing Systems

A regional wastewater system involving a small number of large facilities may significantly alter the existing condition of the region. For example, drainage patterns and water supply patterns are most likely to be significantly altered, since a large quantity of water is transported from many sources to large central treatment plants where it is discharged after treatment. For the same reason water reuse and recycle possibilities may be altered. Many existing land use patterns may be also affected. The size of a regional treatment plant may disturb the local living environments, and large interceptor pipes, once constructed, may promote unplanned growth of their immediate neighborhood and surrounding areas (Binkley, *et al.*, 1975). Such physical alterations invariably lead to political involvements of the communities concerned, and often lengthy and expensive transactions are required to settle the jurisdictional conflicts.

There are a number of other issues which are vital to the planning of regionalization. For example, the institutional and financial arrangements, which include the ownership and administration of the system as well as the cost allocation among participating municipalities and industries, are very important. Also legal constraints such as treatment regulations and water quality standards need to be carefully examined in planning a regional system.

Planning regional wastewater systems, therefore, is an exceedingly difficult problem which involves more than just solving a mathematical problem. In fact, the process of reaching decisions about any large-scale technological projects with social consequences involves a highly complex human interaction (Manheim, 1974). Nevertheless, in the case of planning regional wastewater systems, many of the issues are closely related to the number, size and location of regional plants and interceptors, and location analysis by means of mathematical techniques is an important part of the planning process, particularly when there are many possible alternative physical plans. In other words, different physical alternatives imply, to a great extent, different issues and different degrees of attractiveness.

In the following section existing mathematical methods for evaluating regional wastewater systems are reviewed briefly, and the current trend in dealing mathematically with complex public sector planning problems is discussed.

1.3 Review of Mathematical Methods of Analysis

In the body of literature dealing with analytical methods pertaining to water resources management, water pollution control and other public sector planning problems, there are some methods which deal specifically with regional water quality management and regional wastewater facility planning. For example, since the early 1960's many attempts have been made to construct mathematical models which deal with the water quality of river basins. Various techniques have been applied to several versions of this problem; a review of the literature in this area can be found in Pentland, *et al.* (1972) and Dracup (1970). A review of literature exclusively dealing with mathematical models of regional water quality management is given by Bundgaad-Nielsen and Hwang (1976).

One group of these models emphasizes the water quality aspect of the regionalization problem. The significance of this problem was first recognized in the late 1960's and in the early 1970's. The principal aim of the mathematical modeling approaches is to find the least-costly layout for regional wastewater treatment plants and the associated interceptors while satisfying the water quality constraints.

For example, Graves, *et al.* (1970) suggest a nonlinear formulation that allows at-source treatment, joint treatment at candidate sites, and bypass piping of water in order to meet explicit water quality constraints. Whitlatch (1975) has suggested a heuristic method, and Rossman (1974) used nonlinear programming and dynamic programming methods for solving this problem. Klemetson and Grenney (1976) have developed a dynamic programming model which is capable of analyzing the staging of regional facilities. Each of those models, however, deals basically with regions where waste-water sources are located along a river.

Mathematical methods for a network rather than linear configuration have also been examined in the past several years. Most of the formulations are modifications of general facility location problems involving concave cost functions and a single time period (Efroymson and Ray, 1966; Feldman, *et al.*, 1966, Sá, 1969). Because of the complexity of formulation, water quality constraints are generally excluded from these models. This separation is reasonable if high levels of waste treatment are assumed. For example, Meier (1971) has presented a branch-and-bound procedure to solve for the least-costly regional system. Deininger (1972) described an extreme point ranking algorithm for the same problem. A dynamic programming method for solving this problem has been suggested by Converse (1972). Wanielista and Bauer (1972), Joeres (1974) and Lauria (1975) all suggested mixed integer programming approaches. Also Jarvis, *et al.* (1975) suggested a network formulation and a group theoretic solution approach. A heuristic procedure offered by McConagha and Converse (1973) includes an evaluation of cost savings and cost allocation among participating municipalities. Weeter and Belardi (1975) improved the heuristic algorithm developed by McConagha and Converse and performed some sensitivity analysis on cost functions.

Some attempts have been made also to consider several planning periods. A heuristic method developed for general facility location problems has been proposed for application to wastewater regionalization problems by Bhalla and Rikkers (1971). Lauria (1975) showed that a mixed integer programming can be successfully applied to multiperiod analysis. Rossman (1977) applied the Weeter and Belardi algorithm and dynamic programming method to find an approximate multiperiod solution; his method was shown to be more efficient than the mixed integer programming approach proposed by Lauria.

The primary emphasis of all of the works cited above with the possible exception of that of McConagha and Converse has been to achieve computational efficiency and/or mathematical optimality in solving for the economically most favorable solution. The analysis of the overall desirability of planning alternatives is extremely difficult. There has been, however, some attempt to examine analytically planning issues other than economic efficiency and water quality. McAvoy (1973), for example, has proposed an affinity coefficient matrix for analyzing the potential for regionalizing separate political entities. The matrix is defined by several quantitative attributes associated with each of the neighboring communities. Giglio and Wrightington (1972) have analyzed the cost-sharing aspects of regional wastewater systems using several methods including game theoretic approaches. Heaney (1975) has suggested a game theoretic method for analyzing equity issues for a similar problem, managing urban storm water.

A review of some of the literature which presents methods to deal with the multiplicity of planning issues in the general public sector planning problems is in order at this point. One of the simplest and most widely practiced methods is to rely heavily on the judgment of an individual or a group of individuals with experience to reduce the number of alternatives at the outset to a handful of good alternatives. These alternatives are then examined in more detail. An example of such an approach to the regionalization problem is presented by Palm (1972). The comparison of several selected alternatives may be based on some systematic evaluation criteria. Benefit-cost analysis (e.g., Mishan, 1971) and other similar methods such as the Goal Achievement Matrix Method (Hill, 1973) may be used.

Although such an approach may be practical, it is often very difficult to preclude prejudicial judgment in selecting the candidate solutions. More rigorous methods of analysis involve, in general, mathematical optimization techniques. The simplest method is to formulate the model with a single objective and multiple constraint sets, each of which represents one of the planning issues. However, the number of issues which can be accommodated by an optimization method is generally very limited, partly because of the limitation in mathematical algorithms in dealing with a large number of variables and constraints, and partly because some issues cannot be represented by mathematical logic.

On the other hand, it is quite common to make an analysis of some of the selected issues on an individual basis using some simple optimization models. For example, in the facility location literature, there have been attempts to analyze some qualitative planning factors by introducing surrogate objectives for social utility. A review of such methods is found in ReVelle, *et al.* (1970). The main thrust of these attempts is to avoid the explicit quantifaction of qualitative factors involved. Similar attempts can be found in some water resources management problems (Cohon, 1973).

Mathematical methods which deal with problems involving more than one explicitly defined objective have been gaining considerable attention in recent years. Such methods are collectively called multiobjective optimization methods (Cohon, 1975). According to Cohon, the methods may be classified into three categories; generating techniques, techniques which rely on prior articulation of preferences, and techniques which rely on progressive articulation of preferences.

The methods which belong to the first category may be considered basically as extensions of the single-objective optimization methods with multiple sets of constraints. Based on the relationships between the constraints and objective function, one can generate a set of noninferior solutions among which a final choice can be made. The methods which belong to the second category are designed to take advantage of the explicitly expressed preference of a decision maker prior to the mathematical analysis. These methods, therefore, place a significant burden on the decision maker. They also hinder the process of gradually developing insights into the problem, unless the analysis can be repeated easily and efficiently. The third category includes the methods which are designed to moderate this difficulty.

Although some attempts have been made (e.g., Cohon and Marks, 1973, and Haimes, *et al.*, 1977), multiobjective methods, in general, are in the early development stage, and practical applications to public sector planning problems are yet limited. As for possible application to facility location problems such as the ones for regionalization of wastewater systems, difficulties stem from the demanding mathematical structure as well as from the large number of issues involved.

1.4 Research Orientation and Thesis Organization

The orientation of this research is based on the premise that it is very difficult to define, much less to find by mathematical means the "optimal" solution to such a complex public sector problem as planning regional wastewater systems (Rittel and Webber, 1973). Difficulties arise because many planning issues are involved and they are all closely interrelated. The resolution of some of the conflicting issues is simply beyond the scope of mathematical analysis. Recognizing this, a mathematical method for generating and comparing alternative plans has been proposed as an alternative to contemporary multiobjective optimization approaches.

Generation of alternatives is important for two reasons. First, the "optimal" solution to a mathematical model is most likely not the "optimal" solution to the real problem. Further, it is quite possible that some alternatives which are considerably different from the mathematically optimal solution may turn out to be very attractive. Second, generating alternative plans is a learning process, whereby important planning issues may be identified and the associated economic trade-offs can be examined. Also the display of a range of alternatives and of the economic trade-offs between them provides a basis for developing insights into the nature of a given regionalization problem.

The ability to generate alternatives mathematically depends on the properties of the particular modeling technique and on the problem to be solved. However, it is highly desirable, in general, for a mathematical model to satisfy the following criteria in order to be a useful tool for generating and comparing alternative plans; such a model should be capable of:

(1) generating many alternatives efficiently,

(2) generating alternatives systematically based on some quantitative measure, such as cost,

(3) generating alternatives in such a way that they may be most meaningfully related to various planning issues, and

(4) generating alternatives with prespecified characteristics.

Although many mathematical methods may satisfy one or more of these criteria, they may not be applicable to the regionalization problem because of its demanding mathematical structure. The branch-and-bound method presented here seems to adequately satisfy the above four criteria.

Branch-and-bound algorithms have been extensively used in the past for solving a wide variety of combinatorial problems. For example, Efroymson and Ray (1965) suggested the use of a branch-and-bound algorithm in solving plant location problems using integer programming. Liebman (1967) presented a branch-and-bound algorithm to minimize the cost of wastewater treatment under equity constraints. Sá (1968) treated the capacitated plant location problem using an approximation method and a branch-and-bound algorithm. A combination of a network algorithm and a branch-and-bound technique has been suggested by Marks and Liebman (1970) for solving a problem of locating solid waste management facilities. Also a branch-and-bound algorithm different from the one presented here has been proposed by Meier (1971) for obtaining the least-cost solution to the problem involving regionalization of wastewater systems.

A description of the basic concept and general properties of the algorithm is given by Agin (1966), Lawler and Wood (1966) and Mitten (1970). In short, such an algorithm works as follows. First, the entire set of alternatives may be partitioned into mutually exclusive subsets. Using an appropriate mathematical technique, lower and upper bounds on the least-cost alternative plan in each of the subsets are determined. If the lower bound found in one subset is greater than the upper bound in one of the other subsets, the least-cost solution to the entire set of alternatives does not belong to the former subset. Excluding such subsets, each of the remaining subsets may be partitioned further into mutually exclusive but smaller subsets, and a new bound is found on each of them. The process is continued until an alternative is found such that the lower bounds on all of the remaining alternatives (or the remaining subsets of alternatives) are found to exceed it. The algorithm is designed, therefore, to avoid complete enumeration of the feasible solutions. For a discussion of the planning process as a branch-and-bound process, the reader is referred to Harris (1970).

The evaluation of alternatives proceeds as follows. The most fundamental dichotomy of alternatives in the regionalization problem (and other location problems) is the grouping of those alternatives which contain a given facility and those which do not. From a planning point of view this is quite an attractive dichotomy, since, as noted earlier, many of the issues to be considered in planning regional systems are directly related to the physical configuration of the network of regional facilities. If there is only one facility, x, which is of special interest, then $C(x)$, the cost of the least-cost alternative with facility x, can be compared to $C(\overline{x})$, the cost of the least-cost alternative without it. If the economic efficiency is the only criterion for decision making, the alternative with the lowest cost would be selected. However, if there are other issues to be considered, then the cost difference, $C(\overline{x}) - C(x)$, can be evaluated. This cost difference is defined as the *imputed value* associated with facility x, and it is the basis for the imputed value analysis described later.

Based on the research orientation described above a mathematical model has been developed in three phases. The first phase has been devoted to developing a basic mathematical method for solving the regionalization problem involving static (single-period) waste flows. The objective here has been to make the model capable of identifying systematically many attractive solutions while maintaining computational efficiency and simplicity. The basic model and analysis procedure are presented in Chapter 2. The second phase has been devoted to modifying the model to generate alternative plans while taking into account a simplified form of multiperiod costs. The approach has been to take full advantage of the attractive features of the method developed for the static case. The multiperiod case is discussed in Chapter 3. The third phase involves the imputed value analysis. It is based on the transformation of information from the branch-and-bound tree into a matrix called the imputed value incidence matrix. Chapter 4 discusses the method in detail.

The model has been tested with a small hypothetical problem for each phase of development. A realistic example case has also been studied, as described in Chapter 5. The research findings are summarized, and additional discussion is provided in the concluding chapter.

2. GENERATING ALTERNATIVE PLANS: A SINGLE-PERIOD CASE

2.1 Introduction

As described in the previous chapter, many mathematical methods have been proposed in the past to solve for the least-cost solution to the regionalization problem with single-period (static) waste flows. They are generally based on the assumption of uniform secondary treatment at each regional plant. This type of problem, designated as the single-period regionalization problem, is considered significant primarily for two reasons. First, if the entire regional system is to be constructed for the entire design period within a short span of time, the least-cost solution to the single-period regionalization problem should provide a reasonably attractive solution. Second, the least-cost solution to the single-period problem may be regarded as an initial estimate of the least-cost solution to the multiperiod (dynamic) problem. In other words, the dynamic cost of the regional plan identified by a single-period cost analysis may be regarded as an upper bound on the multi-period least-cost solution. Operations and maintenance (O&M) costs in the static problem are sometimes treated as functions of treatment capacity just as costs of construction. In such cases a formulation like the one given in this chapter involving construction costs can be directly modified to include O&M costs. If any portion of the O&M costs is regarded as a function of waste flows rather than capacity, then a modification of the model becomes necessary. One such modification is proposed in Section 3.3-A.

Although the principal role of the mathematical model presented here is to generate alternative plans, the objective function of the formulation is to minimize cost. The method can, therefore, provide the least-cost solution to the single-period regionalization problem under given cost approximations. A special feature of the solution procedure, a branch-and-bound algorithm, is that it also identifies a number of alternative solutions. The method is also characterized by other features which improve computational efficiency.

The mathematical model and an example application are presented in the following sections. The branch-and-bound method is also compared to mixed integer programming. The reader is referred to Brill and Nakamura (1977-a) for an earlier discussion of the model presented here.

2.2 Basic Model

A mathematical formulation of the single-period, uniform-treatment regionalization problem is presented in this section. The concave cost functions are approximated using linear segments, and nonlinear constraints are associated with these segments. The solution method is a branch-and-bound algorithm that uses a network algorithm to solve subproblems. Some of the subproblems, however, can be readily solved by inspection steps.

A. General Formulation of the Basic Model

The mathematical model takes into account two types of regional facilities, treatment plants and interceptors. The mathematical objective of the model is to find the least-costly regional plan which specifies a configuration of plants and interceptors and their sizes. The formulation consists of an objective function and four types of constraints.

The objective function is the minimization of the sum of costs of plants and interceptors. Since the cost functions exhibit economies of scale, they are represented by piecewise linear segments as shown in Figure 2.1. In this example each facility cost is approximated by a fixed charge and by unit costs associated with the two piecewise linear segments.

In mathematical terms, the objective function is expressed as follows:

Minimize:

$$z = \sum_{ijk}\sum\sum C^{P}_{ijk} \cdot f_{ijk} + \sum_{ij}\sum x_{ij} + \sum_{jk}\sum C^{T}_{jk} \cdot q_{jk} + \sum_{j} y_{j} \qquad (2.1)$$

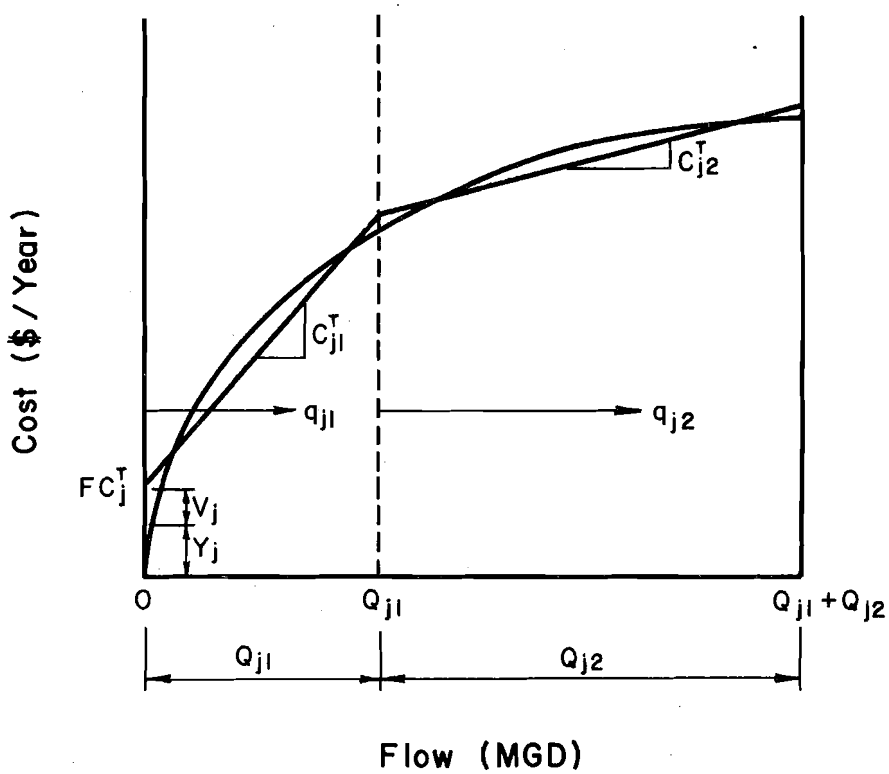

Figure 2.1 Piecewise Approximation of a Treatment-Plant Cost Function with a Fixed-Charge Component

Where the constants (upper case) and the variables (lower case) are:

C^P_{ijk} = unit cost of the kth segment of the piecewise-linear cost function for constructing the interceptor from location i to location j (dollars/year/million gallons per day (MGD)),

C^T_{jk} = unit cost of the kth segment of the piecewise-linear cost function for constructing a plant at site j (dollars/year/MGD),

f_{ijk} = kth piecewise variable for interceptor capacity from location i to location j (measured from break point) (MGD),

q_{jk} = kth piecewise variable for plant capacity at site j (measured from break point) (MGD),

x_{ij} = fixed cost variable for constructing an interceptor from location i to location j (either 0 or FC^P_{ij}) (dollars/year),

y_j = fixed cost variable for constructing a plant at site j (either 0 or FC^T_j) (dollars/year),

FC^P_{ij} = fixed cost associated with constructing an interceptor from location i to location j (dollars/year), and

FC^T_j = fixed cost associated with constructing a plant at site j (dollars/year).

The constraints given below ensure that the capacity variables and design flows maintain continuity at each waste source and at additional candidate plant sites:

$$\sum_{i}\sum_{k} f_{jik} - \sum_{i}\sum_{k} f_{ijk} + \sum_{k} q_{jk} = L_j \qquad \forall\ j \qquad (2.2)$$

where:

L_j = waste flow generated at location j (MGD).

If no waste flow is generated at location j, then L_j is zero. If location j is not a candidate site for a treatment plant, then the term $\sum_k q_{jk}$ is omitted.

The second type of constraint introduces the slack variable corresponding to each activity variable:

$$f_{ijk} + s_{ijk} = F_{ijk} \qquad \forall\ i, j, k \qquad (2.3)$$

$$q_{jk} + t_{jk} = Q_{jk} \qquad \forall\ j, k \qquad (2.4)$$

$$x_{ij} + u_{ij} + FC^P_{ij} \qquad \forall\ i, j \qquad (2.5)$$

$$y_j + v_j = FC^T_j \qquad \forall\ j \qquad (2.6)$$

where:

F_{ijk} = upper limit of the variable f_{ijk} (MGD),

Q_{jk} = upper limit of q_{jk} (MGD),

s_{ijk} = slack variable associated with f_{ijk} (MGD),

t_{jk} = slack variable associated with q_{jk} (MGD),

u_{ij} = slack variable associated with x_{ij} (dollars/year), and

v_j = slack variable associated with y_j (dollars/year).

The third type of constraint is the set of nonlinear constraints

$$f_{ijk} \cdot s_{ijk-1} = 0 \qquad \forall\ i, j \quad k = 2, \ldots, K^P_{ij} \qquad (2.7)$$

$$q_{jk} \cdot t_{jk-1} = 0 \qquad \forall\ j \quad k = 2, \ldots, K^T_j \qquad (2.8)$$

$$f_{ij1} \cdot u_{ij} = 0 \qquad \forall\ i, j \qquad (2.9)$$

$$q_{j1} \cdot v_j = 0 \qquad \forall\ j \qquad (2.10)$$

where:

K^P_{ij} = number of piecewise variables used for the capacity of an interceptor from location i to location j, and

K^T_j = number of piecewise variables used for plant capacity at site j.

The above constraints, together with those of the second type, ensure that the piecewise variables associated with each of the concave cost functions assume nonzero values in a proper sequence. For example, consider Figure 2.1 which illustrates the fixed-cost component for the approximation of a treatment plant cost function. Constraint set (2.10) requires that $v_j = 0$ (and, therefore, $y_j = FC^T_j$) before q_{j1} can be nonzero (treatment provided). Note that the s_{ijk} and t_{ijk} slack variables for the last segment of any approximation are not needed in constraint sets (2.7) and (2.8).

The last set of constraints are the nonnegativity requirements:

$$f_{ijk},\ q_{jk},\ s_{ijk},\ t_{jk},\ u_{ij},\ v_j,\ x_{ij},\ y_j \geq 0 \qquad (2.11)$$

It should be noted that several additional types of constraints might be useful in improving the computational efficiency of the branch-and-bound process. Often it is considered undesirable to split the waste flows such that two or more interceptor pipes originate at one site or such that a portion of a waste flow is treated and discharged at one site and the rest is piped elsewhere for treatment. If there is no capacity limit on any of the treatment plants and interceptors, split flows are unattractive economically because of the economies of scale. Split flows can be prevented in the mathematical method as follows. When an f_{ij1} variable is set equal to F_{ij1} in the branch-and-bound process (i.e., when branching in the constraint, $f_{ij2} \cdot s_{ij1} = 0$), then the f_{ijk} variables for the other values of the index j would be set to zero. This requirement can be expressed using the following nonlinear constraint set:

$$f_{ijl} \cdot f_{ij'k} = 0 \qquad \forall\ i, j, k \qquad \forall\ j, \neq j \tag{2.12}$$

Similarly, if a q_{jk} variable is set equal to Q_{jk}, then all f_{jik} variables can be set equal to zero. The corresponding mathematical constraints are:

$$q_{jk} \cdot f_{jik} = 0 \qquad \forall\ j, i, k \tag{2.13}$$

Additionally, it is obviously impractical and uneconomical to send some waste flows from i to j and some other flows from j to i. Therefore, the following constraint set can be added:

$$f_{ijl} \cdot f_{jik} = 0 \qquad \forall\ i, j, k \tag{2.14}$$

These constraints were used in some of the example problems, as described in Section 2.5 and in Chapter 5.

B. Branch-and-Bound Method for Nonlinear Binary Constraints

The objective function (2.1) and constraint sets, (2.2) through (2.6), form a linear programming formulation. If this portion of the problem is solved alone, however, it is very likely that some of the nonlinear constraints, (2.7) through (2.10), would be violated. If so, this solution is mathematically infeasible to the original formulation of the problem. However, these nonlinear constraints have a binary characteristic which suggests the following solution procedure.

Referring to Figure 2.1, consider a nonlinear constraint of the form, $q_{j2} \cdot t_{j1} = 0$. If such a constraint is violated, then its binary characteristic can be used as a basis for a "branching" in a branch-and-bound algorithm. On one branch q_{j2} would be set to zero, allowing t_{j1} to be nonzero and, as a result, q_{j1} to take on different values. Or, equivalently, if the piecewise variable associated with the second segment of the cost function for plant j is set to zero, then the piecewise variable associated with the first segment can take on any value. On the other branch, t_{j1} would be set to zero (in practice q_{j1} would simply be set to its upper bound, Q_{j1}), allowing q_{j2} to be nonzero. That is, if the piecewise variable associated with the first segment is constrained to its upper bound, then the piecewise variable associated with the second segment can take on any value.

These two conditions exhaust the possibilities for satisfying the nonlinear constraint, and linearity is maintained in the constraint set after each branching. Each of the two new linear problems is solved, and the branch-and-bound algorithm continues, producing a standard branch-and-bound tree like the one in Figure 2.2. When there is no violated nonlinear constraint in the solution to a subproblem, then that solution provides a feasible alternative to the original formulation of the problem. When there is no possibility of finding a feasible alternative costing less than a feasible solution already obtained, then the procedure terminates. Similar nonlinear constraints have also been suggested for formulating one type of water quality management problem for a river basin, although the solution procedure in that case is different (Brill, *et al.*, 1976).

C. Solving Subproblems

The linear portion of the formulation can be solved using any version of the simplex algorithm for linear programming. The branching can be performed either by giving a sufficiently high cost penalty to the variables to be set to zero or by setting the activity variables to their lower or upper bounds using constraints. Since many constraints (Sets (2.3), (2.4), (2.5), and (2.6)) simply place bounds on the variables, it would be desirable to use a linear programming code designed to handle bounds efficiently.

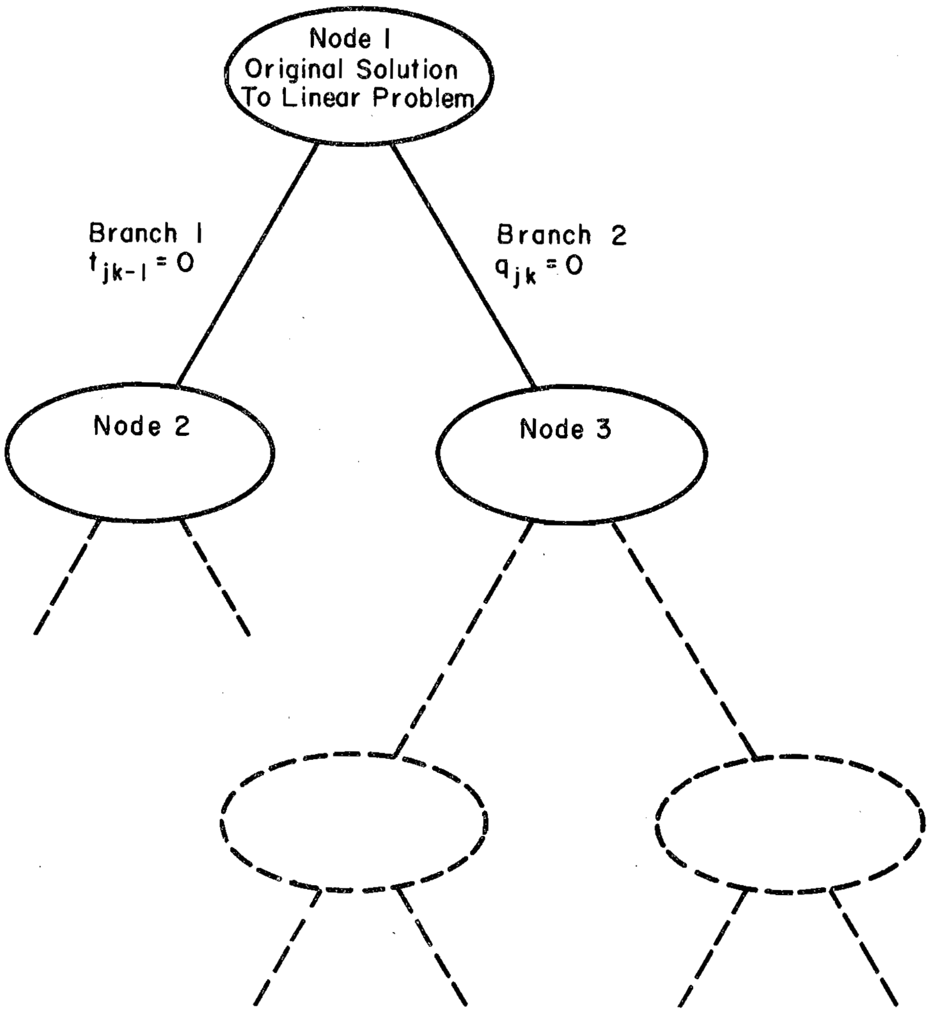

Figure 2.2 Starting Nodes and Branches of a Branch-and-Bound Tree

The linear programming problem considered here, however, can be viewed as a network flow problem. The network flow representation of the linear programming formulation consists of a set of nodes and a set of arcs. For example, Figure 2.3 illustrates a network flow representation of a region with two sources, each of which can also be a plant site. Node 1 and 2 represent waste sources and potential regional plants 1 and 2; nodes s and t are dummy nodes. The flow from s to node 1 has a required flow of L_1 (the waste originating at node 1), as indicated by the lower and upper bounds. The arcs connecting nodes 1 and 2 represent the piecewise capacity variables associated with potential interceptors between the two sites. For example, the first piecewise variable representing flow from site 1 to site 2 has a lower bound of zero, an upper bound of F_{122}, and a unit cost of C^P_{122}. The arcs from nodes 1 and 2 to node t represent the piecewise capacity variables associated with potential plants at sites 1 and 2. For example, the first arc from node 1 has a lower bound of zero, an upper bound of Q_{12}, and a unit cost of C^T_{12} for the plant capacity located at site 1. The entire network maintains a circulation of flow totaling $L_1 + L_2$ as indicated by the lower and upper bounds on the arc from t to s.

Referring to the original formulation, constraint set (2.2) represents flow conservation at the nodes and sets (2.3) through (2.6) represent capacity limits on the arcs. The objective function (2.1) corresponds to the minimization of costs over the entire network. The branching constraints required throughout the branch-and-bound algorithm can be readily added by setting the appropriate variables (arc flows in the network) to their lower or upper limits, as appropriate.

D. Cost Approximations

The computational effort required by the branch-and-bound method is greatly affected by the choice of the

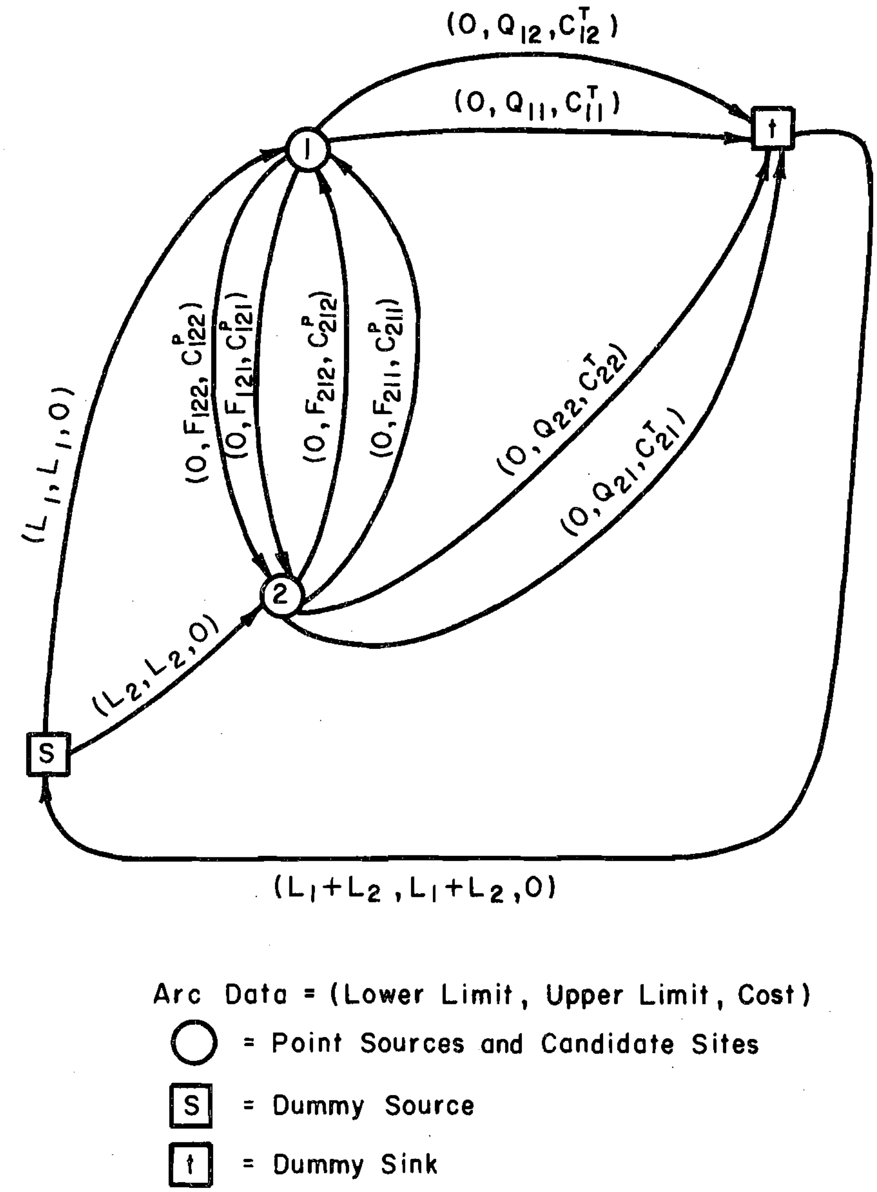

Figure 2.3 Network Representation of a Two-Source System Using Two-Piece Linearization

piecewise approximations of the concave cost functions. This issue is discussed using four alternative types of approximations:

(1) a fixed charge with one linear piece, FP;

(2) two linear pieces, PP;

(3) a fixed charge with two linear pieces, FPP; and

(4) three linear pieces, PPP.

The latter two cases offer better approximations of the original function at the expense of increased computational requirements. Also, as shown below, the FP and FPP approximations lead to computational advantages compared to the PP and PPP approximations, respectively.

Several factors can be considered in making the piecewise approximations. If a treatment plant is constructed at site j and split flows are not allowed, the capacity will be at least as large as the waste flows generated at site j. Similarly, if an interceptor is built from any site i to j, the capacity will also be at least as large as the waste flows generated at site i. Therefore, there is no advantage to making the upper limit, Q_{j1}, on the first piecewise variable associated with plant j less than L_j, the amount of waste flows generated at that site. Similarly, F_{ji1} should not be less than L_j.

In practice, however, the first linear segment in the FP approximations or in the FPP approximations may be placed such that the original cost function and the linear segment coincide at L_j. This is illustrated in Figure 2.4. The approximations of this particular kind are henceforth referred to as the FPI and FPPI to distinguish from the general fixed charge linear approximations. The letter I denotes the individual flows of L_j which differ in value from one waste source to another.

Similarly, if the Q_{j1} is placed exactly at L_j, then PPI and PPPI are used to distinguish these approximations from the general piecewise linear approximations. See, for example,

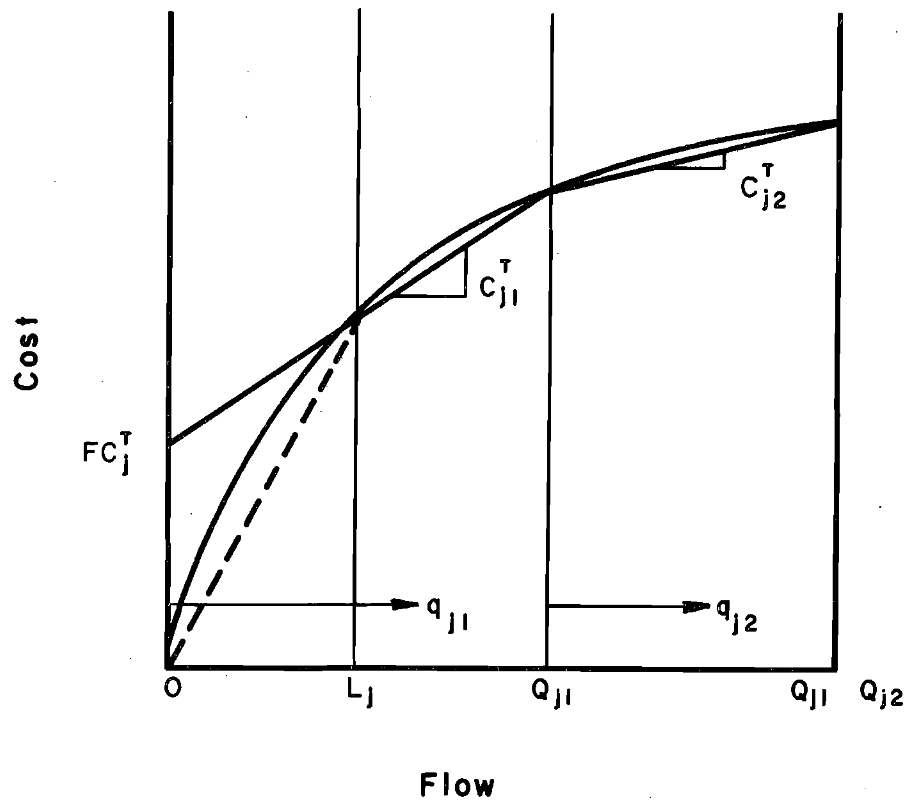

Figure 2.4 Piecewise Approximation of a Concave Cost Function When Split Flows Are Not Allowed

the dotted line in Figure 2.4. Such approximations, however, can be replaced by FPI and FPPI approximations, respectively, with no disadvantage.

As shown later, the general FP and FPP approximations are computationally more efficient than the PP and PPP approximations, respectively, since there are fewer variables that need to be considered in solving the network problems. Note that in practice the branchings performed using an FP and FPI approximation for a treatment plant would specify $q_{j1} = 0$ on one branch, and $q_{j1} \geq L_j$ and, in effect, $y_j = FC_j^T$ on the other branch. Similar branchings would be performed using the piping variables.

E. Converting an Infeasible Node Solution to a Feasible Alternative

As indicated above, it may be possible to reduce greatly the size of the branch-and-bound tree by finding a "good" feasible solution. Such solution could be found by using a heuristic algorithm similar to those developed by Kuehn and Hamburger (1963) and Feldman, *et al.* (1966) or by solving more refined problems as suggested by Lauria (1975). By using the method suggested here, however, a feasible upper bound can be obtained from each of the infeasible solutions (as they are determined) by the following simple conversion step. A solution is infeasible because one or more of the omitted, nonlinear constraints are violated, i.e., the piecewise variables assume values in an improper order. The waste flows, however, are physically meaningful since flow continuity is maintained and all wastes are treated. Only the cost calculations are in error because of the infeasible values of the piecewise variables. Thus, a feasible solution can be found simply by modifying the values of those piecewise variables that violate the omitted nonlinear constraints and by recalculating the objective function. For example, consider the PP case shown in Figure 2.5. When $q_{j1} = 0$ and $0 < q_{j2} \leq Q_{j2}$, the solution

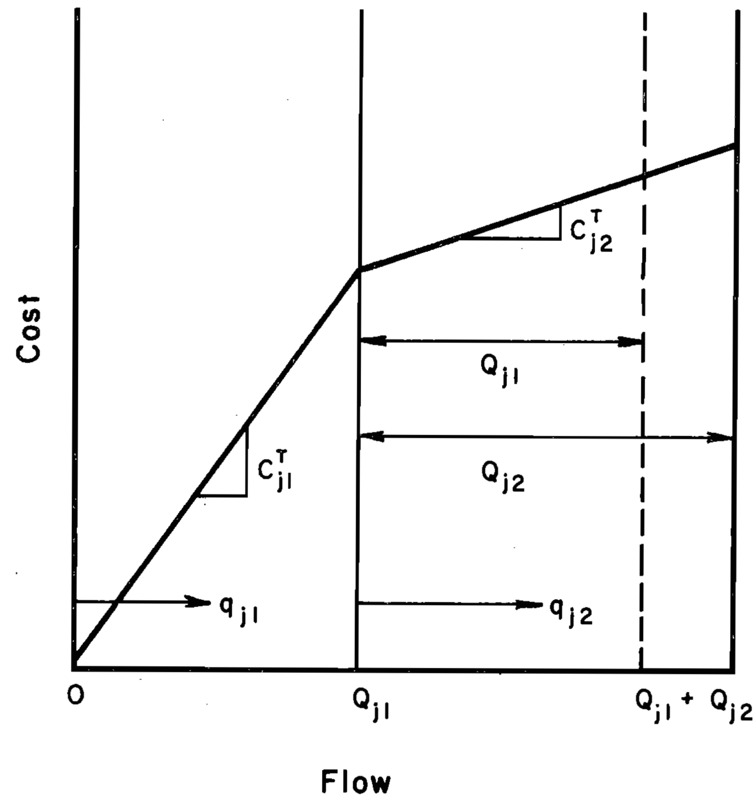

Figure 2.5 Two-Piece Linear Approximation and Relative Magnitudes of Q_{j1} and Q_{j2}

is infeasible. It can be made feasible by the following changes in the variable values:

(1) If $0 < q_{j2} \leq Q_{j1}$, then replace q_{j1} and q_{j2} with q'_{j1} and q'_{j2} such that $q'_{j1} = q_{j2}$ and $q'_{j2} = 0$.

(2) If $Q_{j1} < q_{j2} \leq Q_{j2}$, then set $q'_{j1} = Q_{j1}$ and $q'_{j2} = q_{j2} - Q_{j1}$.

In the case of an FP approximation a solution is infeasible when $q_{j1} > 0$ and $y_j < FC_j^T$. It can be converted to a feasible solution simply by letting $y_j = FC_j^T$. After correcting the ordering for all piecewise variables, the total cost can be computed accordingly, giving a feasible upper bound (which is an alternative regional plan) for each node in the branch-and-bound tree. A similar procedure can be applied in the PPP and FPP cases.

F. Obtaining Node Solution by Inspection

A very powerful step in the branch-and-bound method is based on an extension of the above discussion on finding feasible solutions; one may obtain the solution for some of the immediately following nodes by inspection. Consider the PP case shown in Figure 2.5. Assume that the current solution gives $q_{j1} = 0$ and $Q_{j1} < q_{j2} < Q_{j2}$. Since it is infeasible, a branching is performed in such a way that t_{j1} is set to zero (thus q_{j1} is set to Q_{j1}) on one branch (branch one), and q_{j2} is set to zero on the other (branch two). Then, the objective function value associated with the new node on branch one, C_n, is given by:

$$C_n = C_c + Q_{j1} \cdot (C_{j1}^T - C_{j2}^T) \qquad (2.15)$$

where C_c is the total cost for the current infeasible node. The subscripts "n" and "c" refer to "new" and "current", respectively. In other words, the only change is that q_{j1}

is increased by Q_{j1}, while q_{j2} is decreased by the same amount. Both before and after the modifications the value of q_{j2} is greater than zero and less than its upper limit. Since there are no advantages in increasing or decreasing the value of q_{j2} before the modifications are made, there are also none after they are made. It can be readily proven that all of the other variables would remain unchanged, and, as a result the next node solution is obtained by this inspection step. Similar methods of inspection can also be made to obtain the upper bound for some of the immediately following nodes also (see Brill and Nakamura (1977-a)).

When the current solution has many infeasibilities like the one described above, an additional node can be evaluated by inspection for each infeasibility. That is, a limb of the tree can be grown using a sequence of branches where only branch one is constructed for each successive node. This particular trait, which will be described in more detail for the FP case in Section 2.3-C, reduces considerably the number of subproblem computations required to find the least-cost solution.

In the FP case it is always possible to evaluate one of the two branches from each node by inspection. The only type of violation in the branch-and-bound is the entry of a q_{ji} (or an f_{ijl}) variable with a nonzero value when y_j (or x_{ij}) equals zero. Branch one will always yield $y_j = FC_j^T$ (or $x_{ij} = FC_{ij}^P$), and the value of q_{jl} (or f_{ijl}) will be unchanged. Thus, when all of the cost functions are approximated using the FP approach, it will be possible to determine one half of the node solutions by inspection.

The same basic principles apply to the FPP approximation. For example, in the FPP case, which is shown in Figure 2.4, if q_{j2} is constrained to be zero on branch two, then the remaining branch-and-bound process is exactly the same as in the FP case and the same inspection method applies. Similarly, the inspection method developed for the PP approximations also applies partially to the PPP case. Note, however, that a number of different possibilities need to be considered in the PPP case. For example, the occasion to carry out a branch-one inspection for the case $q_{j3} > 0$ and $q_{j1} = q_{j2} = 0$ depends on the relative magnitudes of q_{j3}, Q_{j1} and Q_{j2}.

The inspection methods for PPPI, FPPI, PPI and FPI are also more complex since the magnitude of L_j need to be taken into account. Also, the relative magnitudes of the upper limits of the piecewise variables (e.g., Q_{j1}, Q_{j2} and Q_{j3}) significantly affects the number of opportunities for using inspection steps.

2.3 Standardization of the Basic Model

The FP model is significant because any of the piecewise cost approximation methods presented in the preceding section can be reduced to a combination of FP approximations. The solution procedure and the structure of the branch-and-bound tree are uniform for all approximations. Any such modified model is called the FP model and the general mathematical formulation of the FP model is called the FP formulation. This uniformity leads to a straightforward way to form the imputed value incidence matrix which is discussed in Chapter 4.

A. FP Model

As described previously, the solution to the branch-one subproblem can always be obtained by inspection if the FP approximation is used. Also, this attractive property can be transferred to the other approximation methods by simply replacing a given set of piecewise segments with a set of equivalent FP approximations. For example, an FPP approximation can be represented by a combination of two FP approximations as shown in Figure 2.6. The first approximation is indicated by the line

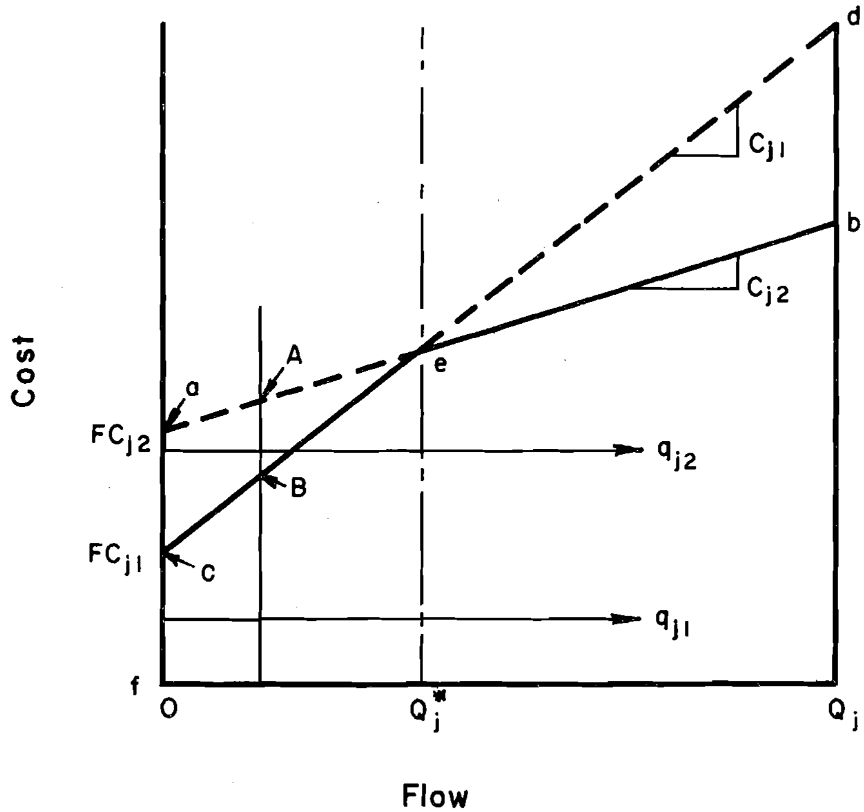

Figure 2.6 Replacing an FPP Approximation with Two FP Approximations

segments connecting points f-a-b, and the second approximation by points f-c-d. Q_j^* is the waste flow which corresponds to the intersection point e.

The new representation of the FPP approximation is called the *revised FPP approximation.* Cost approximations of this type have been applied in the past in solving the regionalization problem using mixed integer programming (Joeres, *et al.*, 1974, and Lauria, 1975).

Figure 2.7 illustrates the branch-and-bound process when the revised FPP approximation is used. Since Q is not limiting, q_{j2} becomes nonzero before q_{j1}. The branch-one inspection provides the solution to the subproblem associated with node n_1, as in the case of any FP approximation, by setting $y_j = FC_{j2}$. Also, q_{j1} will never become nonzero in the part of the tree under node n_1, since q_{j2} can be increased to any plant size at lower cost. The additional constraint, $q_{j2} = 0$ is introduced in solving the subproblem associated with node n_2. Since q_{j1} is not constrained, it may become nonzero in node solutions under n_2. A similar branching may become necessary at node m_0 to create two new nodes, m_1 and m_2, for variable q_{j1}. The optimal solution would never contain $q_{j2} < Q_j^*$ (e.g., point A in Figure 2.6), because a better solution (point B in Figure 2.6) can always be found.

The inspection method is applicable for finding branch-one solutions for both q_{j1} and q_{j2}, and the computational procedure is exactly the same as the case in which a single FP approximation is used. While all of the above discussion is based on the assumption that Q_j is not limiting, the same computational procedure can be applied if Q_j is limiting. As will be discussed later, however, the number of branchings increases, since q_{j1} and q_{j2} may assume nonzero values at the same time unless otherwise constrained.

The same basic approach can be taken in the PP and the PPP cases. For example, a PPP approximation can be represented by a linear segment through the origin and two FP

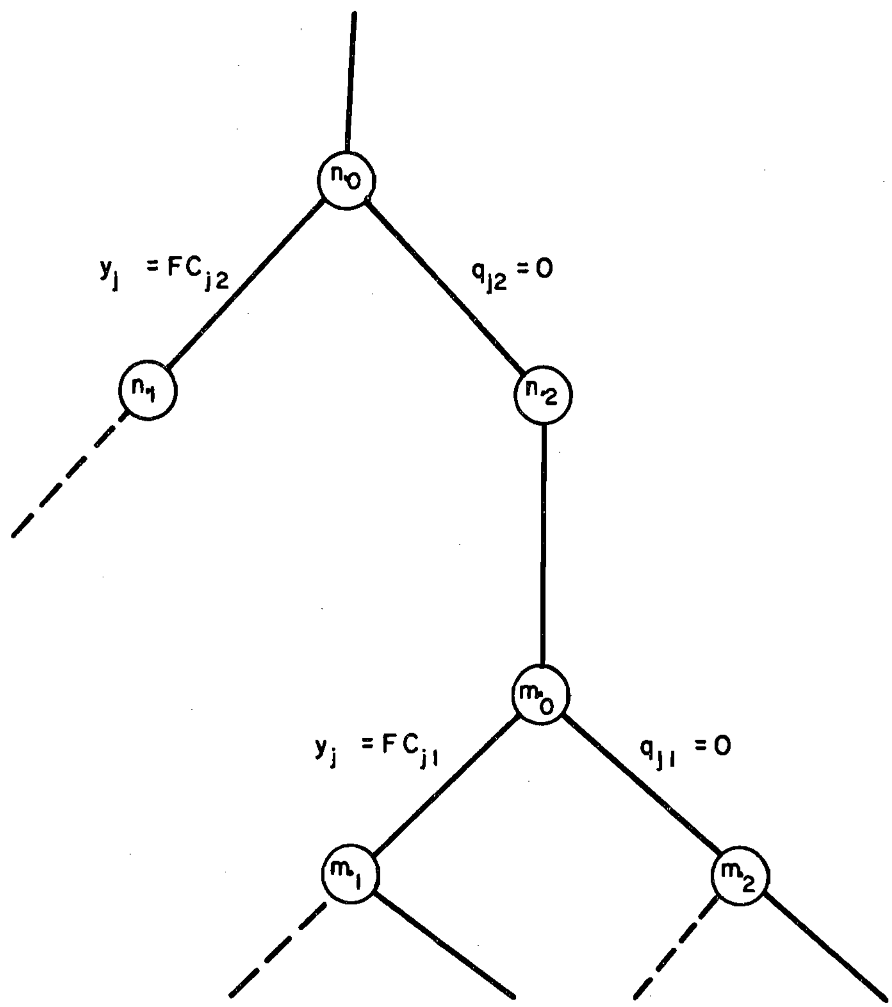

Figure 2.7 Example Branch-and-Bound Tree for the Revised FPP Approximation

approximations. Since there are two FP approximations, at most two pairs of branchings per function are required -- just as in the FPP case described in Figure 2.7. The PP case can easily be deduced from the PPP case. Note that the inspection method is applicable to one-half of the nodes on the tree. If the original PP approximation is used, an inspection step can be used only when the relationship, $Q_{j1} < q_{j2} < Q_{j2}$, holds.

In summary, the FP model is structurally simple, but it is extremely versatile in that its formulation and solution procedure apply to all of the cost approximations. The FP formulation of the regionalization problem is described in its entirety in the following subsection.

B. General Formulation of the FP Model

The FP formulation of the regionalization problem is analogous to the basic formulation described in Section 2.2-A. However, there are some differences. First, the capacity variables, f_{ijk} and q_{jk}, are defined differently. In the FP formulation they are defined as the capacity variables associated with the kth FP approximation rather than with the kth piecewise segment of a cost function (see Figures 2.1 and 2.6). Also, in the FP formulation, the upper bound is the same for all of the capacity variables associated with a given facility. Second, in the FP formulation, there are as many fixed charge segments as there are FP approximations used for a given cost function. Third, the nonlinear constraints are not needed for each pair of piecewise segments. Rather a nonlinear constraint must be defined for each FP component.

The objective function is expressed as follows:

Minimize

$$z = \sum_{ijk}\sum\sum C^{P}_{ijk} \cdot f_{ijk} + \sum_{ijk}\sum\sum x_{ijk} + \sum_{jk}\sum C^{T}_{jk} \cdot q_{jk} + \sum_{jk}\sum y_{jk} \tag{2.16}$$

where the constraints (upper case) and the variables (lower case) are:

C^P_{ijk} = unit cost of the kth FP approximation of the cost function for constructing the interceptor from location i to location j (dollars/year/MGD),

C^T_{jk} = unit cost of the kth approximation of the cost function for constructing a plant at site j (dollars/year/MGD),

f_{ijk} = capacity variable associated with the kth FP approximation of the interceptor cost from location i to location j (MGD),

q_{jk} = capacity variable associated with the kth FP variable of the plant cost at site j (MGD),

x_{ijk} = fixed cost variable associated with the kth FP approximation of the interceptor cost from location i to location j (dollars/year),

y_{jk} = fixed cost variable associated with the kth FP approximation of the plant cost at site j (either 0 or FC^P_{ijk})(dollars/year),

FC^P_{ijk} = fixed cost associated with the kth FP approximation of the interceptor cost function from location i to location j (dollars/year), and

FC^T_{jk} = fixed cost associated with the kth FP approximation of the plant cost function at site j (either 0 or FC^T_{ijk})(dollars/year)

The continuity constraint set is the same as Equation (2.2):

$$\sum_{i}\sum_{k} f_{jik} - \sum_{i}\sum_{k} f_{ijk} + \sum_{k} q_{jk} = L_j \qquad \forall\ j \qquad (2.17)$$

where L_j is the waste flow generated at source j (MGD). Equations (2.3) and (2.4) are replaced by the following simple upper bound constraint sets:

$$f_{ijk} \leq F_{ij} \qquad \forall\ i,\ j,\ k \qquad (2.18)$$

$$q_{jk} \leq Q_j \qquad \forall\ j,\ k \qquad (2.19)$$

where

F_{ij} = upper limit of the variables f_{ijk} for $k = 1, 2, \ldots, K_{ij}^P$,

Q_j = upper limit of the variables q_{jk} for $k = 1, 2, \ldots, K_j^T$,

K_{ij}^P = number of FP approximations associated with the interceptor cost function from location i to location j, and

K_j^T = number of FP approximations associated with the plant cost function at site j.

A slack variable is defined for each of the fixed charge variables:

$$x_{ijk} + u_{ijk} = F_{ijk}^P \qquad \forall\ i, j, k \tag{2.20}$$

$$y_{jk} + v_{jk} = F_{jk}^T \qquad \forall\ j, k \tag{2.21}$$

where

u_{ijk} = slack variable associated with x_{ijk} (MGD), and
v_{jk} = slack variable associated with y_{jk} (MGD).

The nonlinear constraints are defined as follows:

$$f_{ijk} \cdot u_{ijk} = 0 \qquad \forall\ i, j, k \tag{2.22}$$

$$q_{jk} \cdot v_{jk} = 0 \qquad \forall\ j, k \tag{2.23}$$

Last, the nonnegativity constraints are

$$f_{ijk}, q_{jk}, u_{ijk}, v_{jk}, x_{ijk}, y_{jk} \geq 0 \tag{2.24}$$

If all of the cost functions are approximated by a single FP approximation, then the subscript can be eliminated from the formulation. In the revised PP and PPP cases, only minor modifications in the formulation are needed.

Note again that for any treatment plant j, if Q_j is not limiting for q_{jk} for all k, then only one q_{jk} takes a nonzero value and the rest will remain at zero at any one time in the process of the branch-and-bound computation. The same is true with the interceptors. Therefore, the solution to the subproblem at any node on the branch-and-bound tree will always satisfy the constraint sets, (2.17) through (2.19). If on the other hand, $Q_j(f_{ij})$ is limiting for $q_{jk}(f_{ijk})$, then the following constraints must be introduced to ensure the feasibility of the solution:

$$q_{jk} \cdot y_{jk'} = 0 \qquad \forall\, j, k \quad \forall\, k' \neq k \tag{2.25}$$

$$f_{ijk} \cdot x_{ijk'} = 0 \qquad \forall\, i, j, k \quad \forall\, k' \neq k \tag{2.26}$$

In practice these constraints are enforced in such a way that when $y_{jk'}$, $(x_{ijk'})$ is set to $FC^T_{jk'}$, $(FC^P_{ijk'})$ in the branch-and-bound process, the q_{jk} (f_{ijk}) for $k \neq k'$ are automatically set to zero.

C. Solution Procedure for the FP Problems

The FP branch-and-bound method can be used to generate alternative solutions and, if desired, to identify the "least-cost" solution (in terms of a given set of cost approximations). A flow chart describing the procedure is shown in Figure 2.8. Also, the general structure of a branch-and-bound tree is shown in Figure 2.9. The structure of the tree is such that information associated with each alternative plan is readily retrievable.

The branch-and-bound process starts when the initial linear subproblem is solved using linear programming or a network flow algorithm. The initial subproblem consists only of the objective function (2.16) and constraint sets (2.17) through (2.19) along with (2.24). No branching constraints are added yet. The solution to this subproblem provides the objective function value z_1, a lower bound on the least-cost solution to the complete FP formulation. As shown in Figure 2.9, the entire string of nodes, 2, 3, ..., L, can then be generated by inspection along the limb of the tree originating from the branch-one side of node 1. This limb indicates that

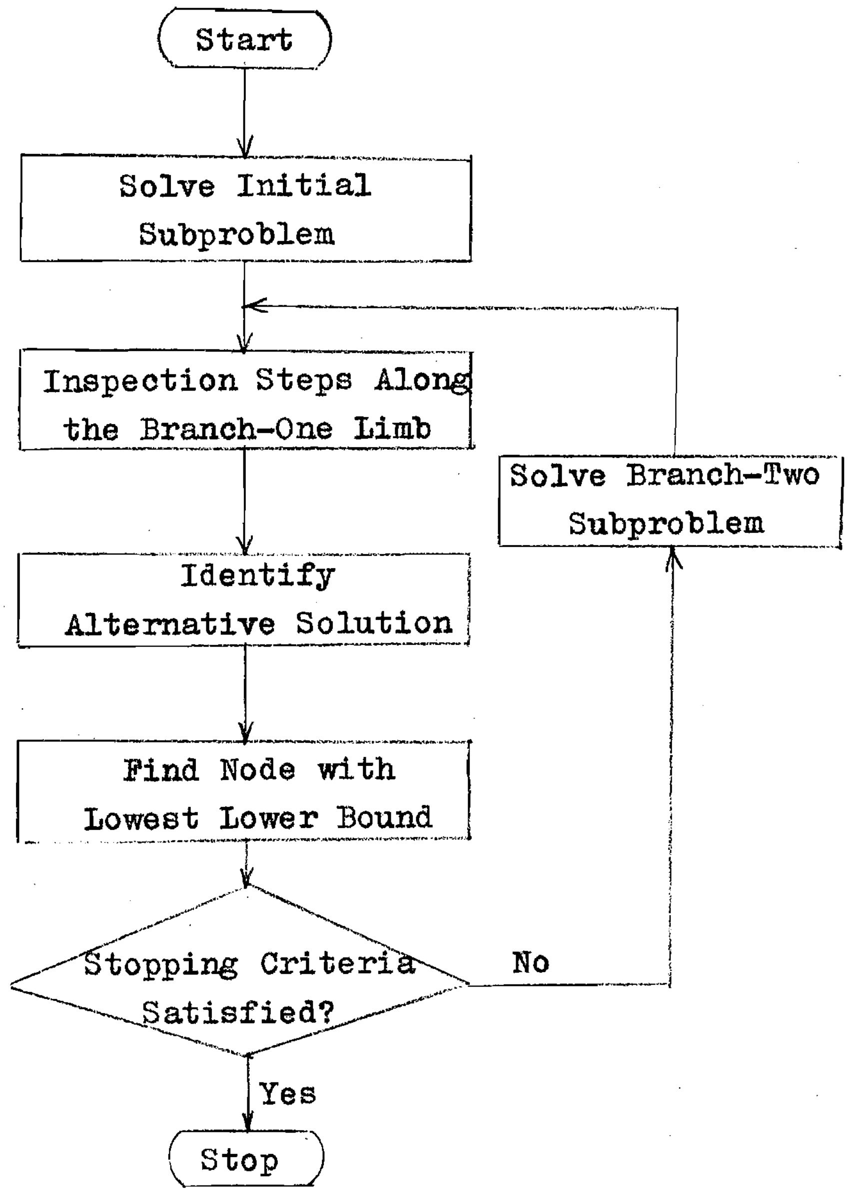

Figure 2.8 Flow Chart for the FP Branch-and-Bound Method

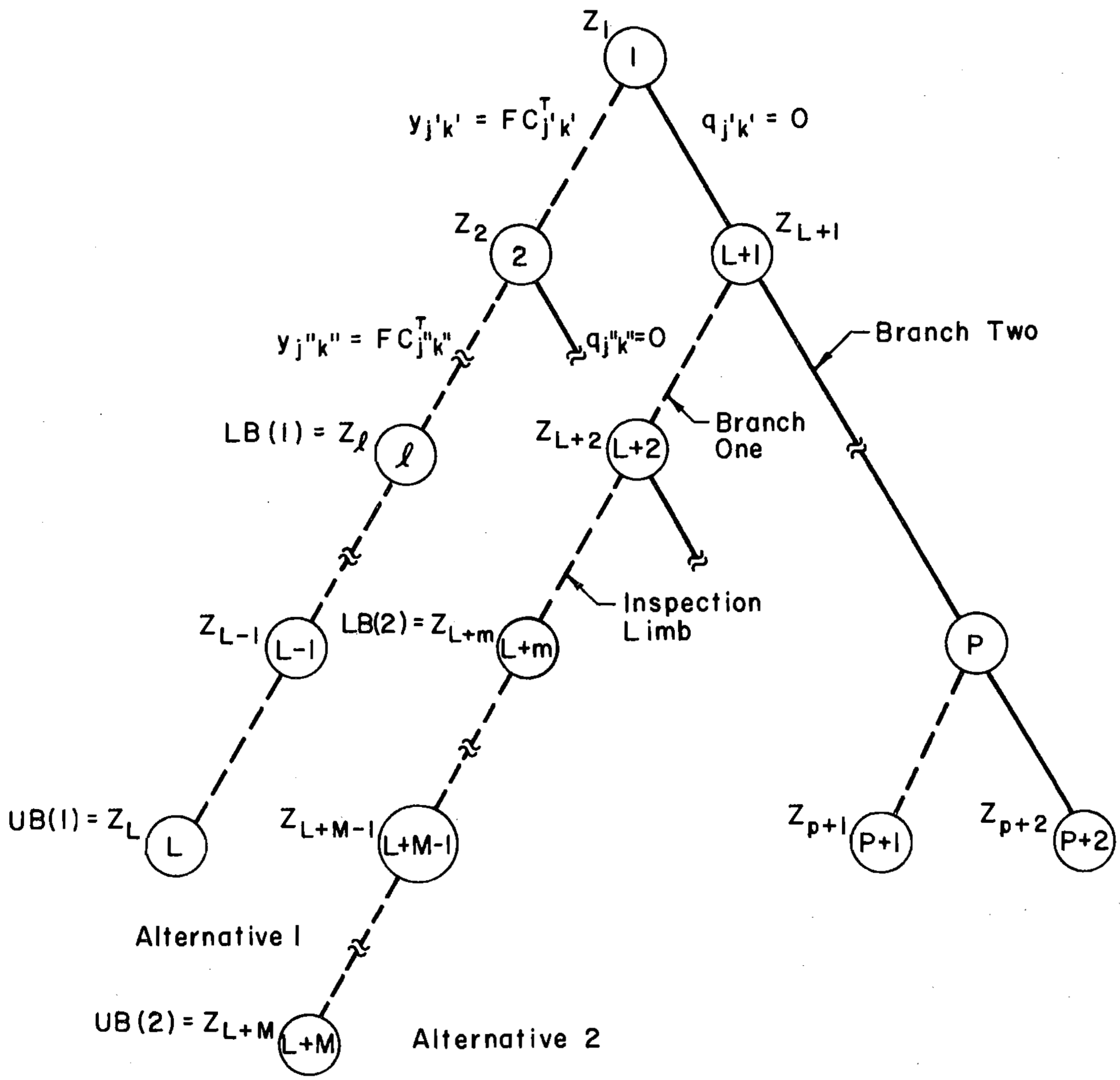

Figure 2.9 Schematic Representation of the FP Branch-and-Bound Tree

the solution to the initial subproblem contains L-1 violations of the nonlinear constraints (2.22 and 2.23). The fixed charge components corresponding to each of the violated constraints are added to z_1 one at a time to determine the objective function values, z_2, z_3, ..., z_L. The terminal node, L, provides a feasible alternative plan, as long as none of the Q_j's or F_{ij}'s are limiting. Flow conservation is satisfied, and the fixed charge associated with each of the FP variables in the solution is added to the objective function. If some of the Q_j's or F_{ij}'s are limiting, then the introduction of constraint sets (2.25) and (2.26) in the formulation provides alternatives with correct costs. These additional constraints do not alter the structure of the branch-and-bound tree, since they are added to the subproblems simultaneously with the ordinary branching constraints.

The sequence order of the branching variables along a limb of the tree can be based on the magnitude of the fixed charge. Adding fixed charges in descending order may help in pruning the branches closer to node 1 (vertex), since fewer branches may be needed before an intermediate node cost exceeds any cost limit which is used in the branch-and-bound process. However, the generation of nodes beyond those necessary for the completion of branch-and-bound process does not increase the computational burden significantly; the necessary fixed charges are simply added in the inspection steps.

The branch-two computation from the lowest-cost infeasible node, in this case node 1, is the next step. Again the subproblem, consisting of the objective function (2.16) and constraint sets (2.17) through (2.19) along with (2.24), is solved. One branching constraint (e.g., $q_{j'k'} = 0$ as shown in Figure 2.9) is added. Again a string of nodes, L+1, L+2, ..., L+M, is generated along the limb of the tree originating from the branch-one side of node L+1. A feasible alternative is identified at the terminal node, L+M. The branching procedure then follows the rule *branch from the lowest bound*

(Lawler and Wood, 1966, p. 712), and the branching continues as shown in Figure 2.9. The series of solving one subproblem by an optimization algorithm, carrying out a string of inspection steps, and identifying one feasible alternative can be repeated until a given stopping rule is satisfied.

The appropriate stopping rule depends on the purpose of application of the method. If the objective is to obtain z*, the least-cost solution in terms of a given approximation method, then the branch-and-bound process may be terminated when all of the infeasible node costs exceed the cost of a feasible alternative. Note that any feasible solution to the original problem will be found only at the bottom of the tree. If the objective is to generate alternatives within a given cost limit, z**, then the process can be continued until all of the infeasible-node costs exceed the cost limit.

Each limb of the tree grown by a string of inspections is referred to as an *inspection limb*; for example, the limb from node 1 to node L in Figure 2.9 is an inspection limb. There are two important node costs associated with each such limb. One is the feasible node cost (an upper bound) at the bottom of the limb, and the other is the infeasible node cost (a lower bound) associated with the first intermediate node from which no branch two has been extended. For example, in Figure 2.9 node L provides an upper bound, and node ℓ provides an infeasible lower bound. These two node costs are important for comparing economic trade-offs between different sets of alternative plans. A more detailed discussion on the use of lower and upper bounds is provided in Chapter 4.

The lowest of the inspection-limb lower bounds is the current lower bound, $\underline{z}$, of the branch-and-bound tree, and the current lowest of the inspection-limb upper bounds is defined as the upper bound, $\overline{z}$. Defining the lower bound on inspection-limb n as LB(n) and the upper bound as UB(n), the following relationships hold:

$$\underline{z} \leq z^* \leq \overline{z} \leq UB(n) \qquad \forall\, n \qquad (2.27)$$

$$\underline{z} \leq z^{**} \qquad \forall\, n \qquad (2.28)$$

If the objective is to find the least-cost solution, the branch-and-bound process terminates when:

$$z^* = \underline{z} = \overline{z} \leq LB(n) \leq UB(n) \qquad \forall\, n \qquad (2.29)$$

If alternative solutions are generated, the process terminates when:

$$z^{**} \leq \underline{z} \leq LB(n) \qquad \forall\, n \qquad (2.30)$$

The FP branch-and-bound trees contain, in general, many inspection limbs like the ones in Figure 2.9. At any given stage of procedure, many of the nodes, e.g., nodes $\ell+1$ through L in the figure, have not become candidates for branching since complete branching has not yet been performed on the preceding node, i.e., node ℓ. Since those nodes are not actively involved in the branch-and-bound process, they are called *inactive nodes*. Also the corresponding part of the inspection limb is called an *inactive portion* of the tree, and the extra inspection steps are called *inactive inspection steps*. Other nodes obtained by inspection, such as node 2, are in the *active portion* of the tree. The importance of inactive nodes lies in their potential for becoming active and leading to additional growth of the tree to generate more alternative solutions.

The following relationships hold among these different parts of the tree.

$$I + C = N \qquad (2.31)$$

$$N_a + N_i = N$$

$$I_a + I_i = I$$

$$C = I_a + 1 \qquad (2.34)$$

$$N_a = 2 \cdot I_a + 1 \qquad (2.35)$$

where

I = total number of inspection steps,
C = number of branch-two computations,
N = total number of nodes,
N_a = number of active nodes,
N_i = number of inactive nodes,
I_a = number of active inspection steps, and
I_i = number of inactive inspection steps.

Some of these indices shown above are compared for the example problems presented later.

2.4 Comparison of the Nonlinear Branch-and-Bound Method and the Mixed Integer Method

The regionalization problem can also be formulated as a mixed integer programming (MIP) problem (see, e.g., Joeres, *et al.* (1974) and Lauria (1975)). For example, if the cost of treatment plant j is represented by the revised FPP approximation described in Section 2.3-A and illustrated in Figure 2.7, one MIP formulation includes the following constraints:

$$q_{j1} \leq \delta_{j1} \cdot Q_{j1} \qquad (2.36)$$

$$q_{j2} \leq \delta_{j2} \cdot Q_{j2} \qquad (2.37)$$

$$\delta_{j1} + \delta_{j2} \leq 1 \qquad (2.38)$$

$$q_{jk} \geq 0 \qquad k = 1 \text{ and } 2 \qquad (2.39)$$

$$\delta_{jk} = 0, 1 \qquad k = 1 \text{ and } 2 \qquad (2.40)$$

where q_{jk} and Q_{jk} are described under Equation (2.23).

The objective function includes terms such as $\Sigma\, FC^T_{jk} \cdot \gamma_{jk}$, where FC^T_{jk} is the fixed cost associated with the kth FP component of the cost function for constructing the plant at site j.

The integer variables accomplish what the nonlinear binary constraints do in the formulation presented in this chapter. Note that the number of variables and constraints involved in the MIP formulation is greater. The MIP formulation can be solved directly using an MIP algorithm such as the one proposed by Gomory (1960). Also branch-and-bound algorithms, which are tailored to solve MIP formulations, are known to be very efficient (Garfinkel and Nemhauser, 1972). Such solution approaches, however, do not adequately fulfill the objective of this study -- to generate alternative plans. The primary objective of such methods is computational efficiency in finding the "optimal" solution. Thus, it requires some modifications, in general, to identify solutions which may be converted to physically meaningful alternative plans. In particular, when a fractional approach is used, the integer variables are treated like continuous variables and the branch-and-bound tree would not be as useful for comparing alternatives as in the case of the nonlinear branch-and-bound method.

Computationally, each method has its advantages and disadvantages. While the nonlinear branch-and-bound method can take advantage of inspection methods and a network flow algorithm to enhance computational efficiency, many of the MIP codes with the embedded branch-and-bound method contain various schemes to estimate the objective function values associated with unexplored nodes. Many of the general purpose MIP codes available today achieve high computational efficiencies. Some comparisons are made on the computational aspects of the two methods in the following section.

2.5 Illustrative Example

A small example problem is used to illustrate the branch-and-bound method for generating static alternative plans. While the structure of the example problem is simple, a number of interesting observations are made and outlined below; an earlier discussion is presented in Brill and Nakamura (1977-a). Computational requirement of a mixed integer programming approach are presented along with the requirements of the branch-and-bound method. A comparison of the FPP approximation and the revised FPP approximation is also presented.

A. Description of Hypothetical Example Problem

The relatively small hypothetical problem is shown in Figure 2.10; it consists of seven communities (sources of point waste) and eleven potential interconnecting routes. Of the seven point sources, two (sites 2 and 6) are not allowed to be candidate sites for regional treatment plants and six of the eleven links allow flow in only one direction. The amount of wastewater generated at each source is shown under column $L_j(10)$ in Table 2.1. Since the same example problem is used for the multitime period analysis, the wastewater production at two other years are also shown in the table.

This rather simple example has 640 feasible combinations of treatment plants and piping. For a problem of this size it is practical to find the least-cost plan and to examine many planning alternatives by enumeration. However, as the number of candidate sites becomes larger, the number of combinations grows exponentially, and total enumeration becomes impractical. If, for example, site 2 is allowed to be a candidate site for a regional treatment plant, the number of feasible combinations grows to 1152, or approximately twice the original number.

The cost functions used in this example problem are based on Deininger and Su (1971). They are:

$$TC^K = (0.560 \cdot q^{0.78}) \cdot 0.07095 \qquad (2.41)$$

$$TC^M = 0.067 \cdot q^{0.78} \qquad (2.42)$$

$$TC^P = (0.040 \cdot f^{0.50}) \cdot 0.05480 \cdot D \qquad (2.43)$$

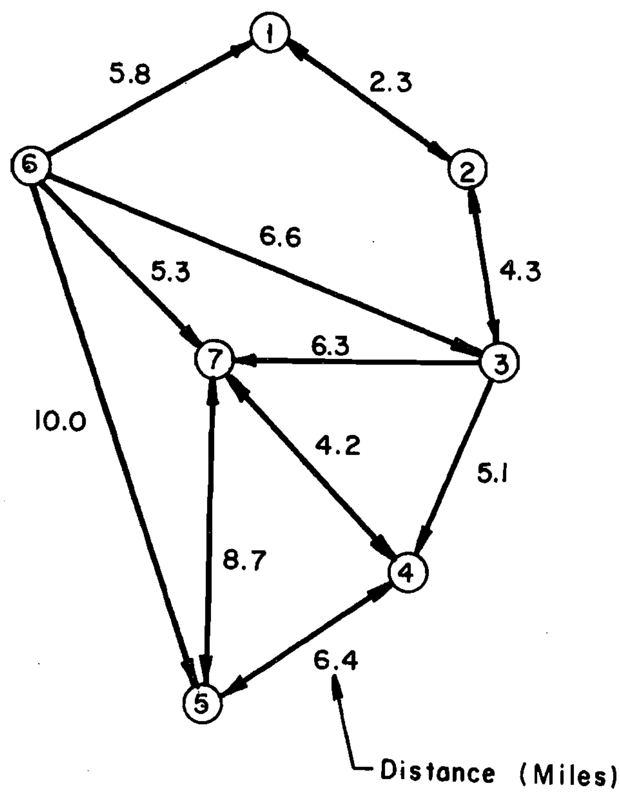

Figure 2.10 Interceptor Network and Waste Sources for the Hypothetical Example Problem

Table 2.1 Waste Flows Generated at Each Source for the Hypothetical Example Problem

Source	Waste Flows (Million Gallons per Day) $L_j(0)$	$L_j(10)$	$L_j(25)$
1	0.20	0.39	1.00
2	0.00	0.08	0.40
3	0.10	0.45	1.00
4	0.50	1.30	2.40
5	1.40	2.30	2.50
6	0.00	0.08	0.20
7	0.00	0.40	1.80

$L_j(t)$: Waste flows generated at the t-th year at site j.

where

TC^K = amortized construction cost of treatment plants (million dollars/year),

TC^M = annual operations and maintenance costs of treatment plants (million dollars/year),

TC^P = amortized construction cost of interceptors (million dollars/year),

q = design flow for treatment plants (MGD),

f = design flow for interceptors (MGD), and

D = distance (miles).

Construction costs of treatment plants and interceptors are amortized using 25 and 50 year design lives, respectively, and using a discount rate of 0.05. Operation and maintenance costs are not included in the single-period analysis. Constraint sets (2.12), (2.13) and (2.14), which prevent split flows and two-way flows, are included in the formulation.

B. Computational Results

Branch-and-bound trees have been grown using various cost approximation methods. Also three solution methods, the simplex method, the Out-of-Kilter Algorithm (OKA) and mixed integer programming, were used. An earlier experience with these approaches is described in Brill and Nakamura (1977-a). In summary, the following observations were presented.

(1) For a given method, as the number of piecewise variables increases, the size of the tree generally increases. The number of subproblems computed also increases approximately in proportion to the total number of nodes in the tree.

(2) The number of feasible (physically meaningful) alternatives tends to be proportional to the total number of nodes generated. In particular, in cases which involve the FPI or the FPPI approximations,

each subproblem computation resulted in one feasible alternative. If the first breakpoint of the piecewise segment is placed beyond the individual waste flows generated, however, split flows tend to occur in many alternatives.

(3) The inspection method has been proven to be very powerful, particularly in cases which involve the FP type approximations. One-half of the total number of nodes are generated by inspection. The numbers of subproblems actually computed in those cases, therefore, were less than the ones in the corresponding cases involving the PP or the PPP approximations.

(4) The least-cost configuration found depends to a great extent on the method used to approximate costs. Also different solutions were found when small changes were made in the locations of the piecewise segments, although the same type of approximations are used. This observation reinforces the importance of generating alternative solutions.

(5) The nonlinear branch-and-bound method seems to be a very efficient method for generating alternative regional plans. Furthermore, the OKA computer code was found to be much more efficient than the simplex computer code used to solve subproblems.

Some additional observations can be made by examining the information shown in Table 2.2 for seven different cases. Each case involves a different set of cost approximations. The table contains information pertaining to the type of approximations, the statistical data associated with the tree, the cost information, and the solution methods. The term "least cost" refers to the solution based on the given set of cost approximations, while the term "actual cost" refers

Table 2.2 Computational Statistics for Single-Period Example Problem

Case	A	B	C	D	E	F	G
No. of Piecewise Variables	5	5	5	5	7	7	7
Using PPI	5	5	0	0	0	0	0
FPI	0	0	5	5	7	5	5
FPPI	0	0	0	0	0	2	0
Revised FPPI	0	0	0	0	0	0	2
Total No. of Nodes	25	19	23	21	76	89	110
No. of Active Nodes	25	19	21	21	65	77	81
No. of Active Inspections	7	15*	10	17*	32	38	40
No. of Subproblems Computed	18	4	11	4	33	39	41
No. of Feasible Alternatives	11	2	11	3	31	38	30
Least Cost (\$ x 10^3/year)	181.5	181.5	181.5	181.5	184.6	184.6	184.6
Actual Cost (\$ x 10^3/year)	191.0	191.0	191.0	191.0	191.5	191.5	191.5
Solution Method	OKA	MIP	OKA	MIP	OKA	OKA	OKA

* No. of times the imbedded node value estimation is performed.

+ Excludes the solutions with improper FP approximations (see 3.3-A).

to the cost of this solution based on the original cost functions given in Equations (2.41) and (2.43).

When the approximate least-cost solution was found on the tree, the branch-and-bound process was terminated in each case. The computer program for the OKA was written in Fortran IV and was available at the University of Illinois. The OKA code developed by Clasen (1968) and other advanced codes recently developed (e.g., Barr, *et al.*, 1974, Glover, *et al.*, 1974, Bradley and Brown, 1975, and Hultz, *et al.*, 1976) may easily be substituted. The MIP code used is a part of the IBM MPSX and available at the University of Illinois.

Note from the table that one of the major differences between the nonlinear branch-and-bound and the MIP methods is in the number of feasible alternatives. While the nonlinear branch-and-bound tree provides at least eleven alternatives prior to the termination in each case, the MIP branch-and-bound tree provides only two and three feasible (physically meaningful) alternatives in cases B and D, respectively. The MIP method, on the other hand, has special capability for estimating the next node solution which may reduce the computational time considerably. For example, fifteen of the nineteen node values were computed by such routine in case B, and seventeen of the twenty-one node values were in case D. The actual number of subproblems computed, therefore, was only four in each case.

The CPU time required for cases A, C and E were about 0.6 seconds per subproblem on the University of Illinois DEC 10 system, while the MIP code on the IBM 360/75 system at the University of Illinois required 5.84 and 4.80 seconds for cases B and D, respectively. When the branch-and-bound tree was constructed for cases C and E, using a short Fortran program (about 350 steps including OKA) on the DEC 10 system, the total CPU time was 5.6 seconds and 12.9 seconds, respectively. The computational speed of the DEC 10 system is much lower than that of the IBM 360/75 system. However, no generalization is possible on the computational efficiency because of the limited experience.

The differences in the computational requirements of the FPPI and the revised FPPI approximations are shown in cases F and G, respectively. Case F resulted in a smaller tree than case G, and yet the number of feasible alternatives was greater than for case G. Most of the alternative configurations generated in case G were also generated in case F. Note, however, that the computer program for case F must include the provisions for all the possibilities for inspection of node solutions, while the program used for the FPI case is directly applicable to case G. The CPU times for the two problems were nearly identical at 12.7 for case F and 12.9 for case G.

An illustration of the structure of the FP tree for case E is given in Section 4.3. It will be shown that the information on the branch-and-bound tree can be used for obtaining the economic trade-offs among different alternatives.

3. GENERATING ALTERNATIVE PLANS: A SIMPLIFIED MULTIPERIOD CASE

3.1 Introduction

The branch-and-bound method developed in the previous chapter has a number of unique features which make it an attractive tool for planning analysis. The method presented so far, however, considers only a single time period; capacity expansion over different time periods is not taken into account. Thus, the cost differences between alternatives reflect only the single-period (static) trade-off values.

In this chapter a multiperiod case is considered. The analysis of public sector location problems based on multiperiod (dynamic) cost is extremely difficult, since it involves many more unknown factors than the case with the analysis based on static cost. Also the number of possible planning alternatives becomes considerably larger, because construction phasing adds another dimension. Problems possessing such characteristics are collectively called multiperiod facility location problems, and interest in the mathematical analysis of these problems has been extensive during the past several years.

The objective of multiperiod facility location problems is to find the least-costly locations of central facilities which satisfy demands that change from one time period to another, including the determination of time of construction. It involves the economics of capacity expansion and economies of scale. A major portion of the related literature considers location analysis on a plane rather than on networks. For example, Wesolowski (1973), Forcina (1974), Erlenkotter (1974), and Sweeney and Tatham (1976) have all proposed formulations and solution methods for multiperiod location analysis on a plane using dynamic programming and modifications. Meier (1974) has proposed a mixed-integer programming approach, Wesolowski and Truscott (1975) have tested mixed-integer programming and dynamic programming, and Eschenback and

Carlson (1975) have used a branch-and-bound technique for the same problem.

Several techniques have been suggested for dealing with regional wastewater systems. The direct application of a heuristic programming approach has been proposed by Bhalla and Rikkers (1971). Lauria (1975) proposed that mixed integer programming can be successfully applied to the multiperiod analysis of regionalization. Rossman (1977) has presented an efficient solution approach to the same problem using a heuristic technique in conjunction with dynamic programming.

3.2 Basic Assumptions

Besides the assumption of uniform secondary treatment, several additional simplifying assumptions are made in this study. These assumptions, which make the problem a special case of the general multiperiod regionalization problem, are based on two fundamental considerations. First, the primary objective of the proposed method is not to find the "optimal" solution in the strictly mathematical sense, but to generate many different alternative plans and to compare the economic trade-offs between them. Thus, rigorous pursuit of mathematical optimality is sacrificed for the approximate but efficient analysis of alternatives. Second, since the method developed for the single-period regionalization problem has proven quite efficient, an effort has been made to maintain the basic features of the method.

There are three basic assumptions for the multiperiod regionalization problem considered here:

(1) the growth of wastewater production at each source is linear,

(2) the interim design periods are predetermined, and each of the regional plants must be constructed stagewise to accommodate only the incremental design flows of each period, and

(3) the assignment of the individual waste sources to regional treatment plants remains unchanged over the entire design period.

Although some slight modifications of these assumptions may be possible without loss of generality of the method, these assumptions reflect the basic approach to the problem. Some additional but less critical assumptions are:

(4) existing facilities are not included in the analysis,

(5) the operation and maintenance (O&M) costs for plants are a function of wastewater produced rather than the capacity of the plant constructed,

(6) the O&M costs for interceptors are negligible,

(7) the interceptors are constructed at the outset for the entire design period, and

(8) the same cost function applies both to initial construction and to expansion of a treatment plant.

These additional assumptions may be modified to a significant degree depending on the specific characteristics of the regionalization problem.

The next sections discuss how simplified multiperiod construction costs and O&M costs can be transformed into single-period forms under these assumptions. The analytical procedure is discussed in the later sections.

3.3 Multiperiod Cost Approximations

Given the assumptions stated in Section 3.2, the approximate multiperiod costs of a plant can be transformed into a simple form involving only the fixed charge, the cost associated with the initial design-year flow and the cost associated with the ultimate design-year flow. The transformation is carried out in two steps. The first step is a transformation of the O&M costs over the entire design period. The second step is a transformation of the stagewise construction costs. All of these costs are finally aggregated to a single function. While the cost of constructing interceptors is assumed to be based only on the ultimate design flow, a modification is made of the cost functions so that they will also conform to the form of plant cost functions. The aggregate cost functions based on those transformations will be used for identifying the approximate least-cost solution and other alternative solutions to the multiperiod regionalization problem.

A. Plant O&M Costs

Based on the assumptions that wastewater production is linear with time at any waste source and that the assignment of waste sources to regional plants remains unchanged over the entire design period, the following relationship gives the waste flow treated at a regional plant at year t.

$$q_j(t) = q_j(o) + s_j \cdot t \qquad \forall j \quad 0 \leq t \leq T \tag{3.1}$$

where

$q_j(t)$ = waste flow to be treated at plant j at year t (MGD),
$q_j(O)$ = waste flow to be treated at plant j initially (MGD),
s_j = rate of growth of waste flow at plant j over T years (MGD/year), and
T = design period (year).

The annual O&M costs at year t for plant j, $\hat{M}_j(t)$ (dollars/year) can be expressed by an exponential function of the following form.

$$\hat{M}_j(t) = \alpha_M \cdot (q_j(t))^{\beta_M} \tag{3.2}$$

where

α_M = O&M costs of a reference plant, constant (dollars/year), and
β_M = economies of scale factor, constant.

The average equivalent O&M costs, M_j (dollars/year), for plant j is defined as:

$$M_j = \sum_{t=1}^{T} (\hat{M}_j(t) \cdot D^t) / \sum_{t=1}^{T} D^t$$

$$= \alpha_M \cdot \sum_{t=1}^{T} ((q_j(0) + s_j \cdot t)^{\beta_M} \cdot D^t) / \sum_{t=1}^{T} D^t \qquad (3.3)$$

where D is the discount factor, $1/(1+i)$, using a discount rate i. M_j, therefore, is the O&M costs of some hypothetical year, t*, which can be considered a uniform annual series over all T years, and when summed over T years after discounting, it equals the actual cumulative discounted O&M costs over T years (see Figure 3.1).

Also,

$$\hat{M}_j(t^*) = M_j$$

or

$$\alpha_M \cdot (q_j(0) + s_j \cdot t^*)^{\beta_M} = M_j \qquad (3.4)$$

Equating (3.3) and (3.4),

$$t^* = (1/s_j)\left[\left[\sum_{t=1}^{T} \left[q_j(0) + s_j \cdot t)\right]^{\beta_M} \cdot D^t / \sum_{t=1}^{T} D^t\right]^{1/\beta_M} - q_j(0)\right]$$

$$= \left[\sum_{t=1}^{T} (q_j(0)/s_j + t)^{\beta_M} \cdot D^t / \sum_{t=1}^{T} D^t\right]^{1/\beta_M} - q_j(0)/s_j \qquad (3.5)$$

Thus, t* is a function of $q_j(0)/s_j$ and the constants, T, β_M, and D. The term $q_j(0)/s_j$ has a dimension of time and it characterizes the pattern of waste generation, not the absolute amount of waste flow.

Figure 3.2 shows the relationship between T and t* for a given set of i and $q_j(0)/s_j$ and for a constant economies-of-scale factor, β_M, of 0.78, a typical value for wastewater treatment plants. Note from the figure that t* is rather insensitive to $q_j(0)/s_j$ because the term appears at two

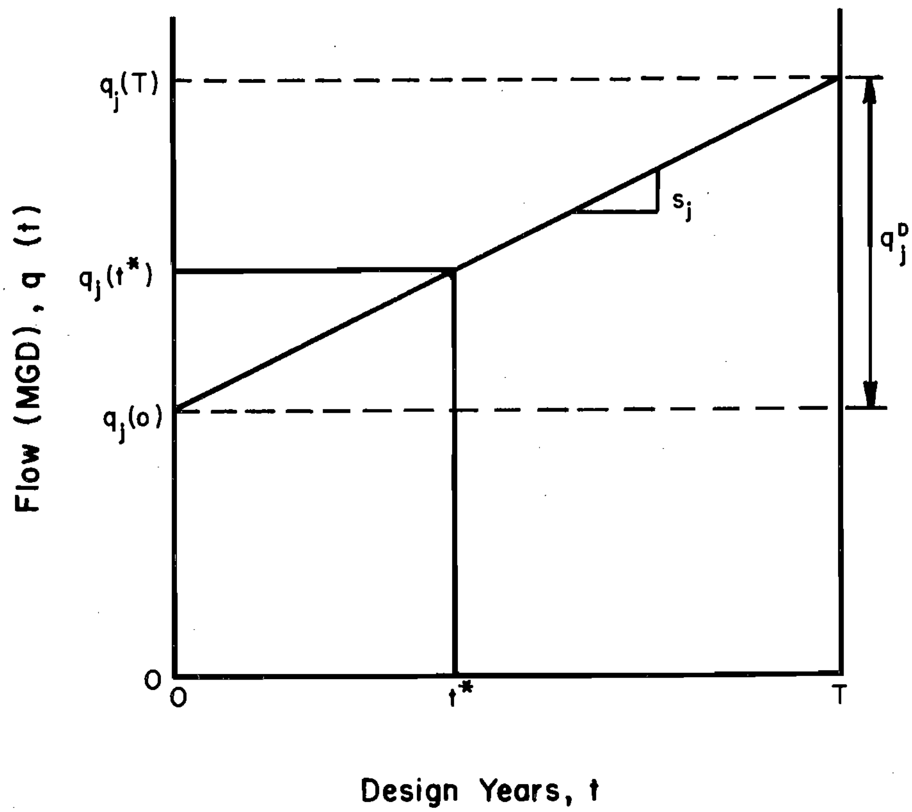

Figure 3.1 Growth of Waste Flows over T Years

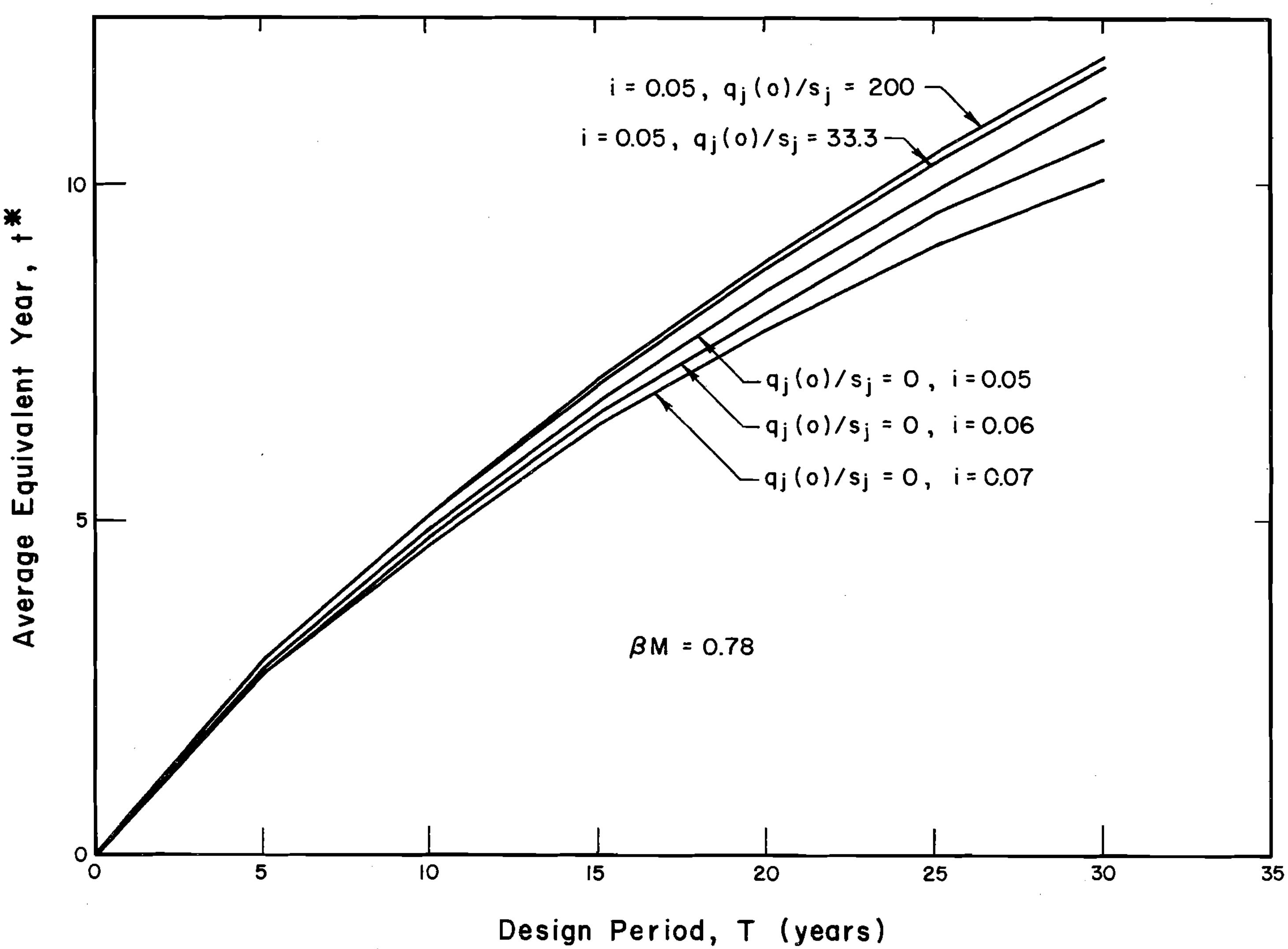

Figure 3.2 Average Equivalent Year, t*, as a Function of i, T and $(q_j(O)/s_j)$

separate locations in Equation (3.5) in such a way that its total contribution to t* is minimal. The values of t* for a wide range of nonzero values of $q_j(0)/s_j$ are only slightly larger than the t* corresponding to $q_j(0)/s_j = 0$ as shown for the case of $i = 0.05$. A similar trend has been shown to hold for other values of i and β_M. Thus, t* can be considered nearly independent of $q_j(0)$. This result is quite convenient since t* for $q_j(0) = 0$ can be identified *a priori* for a given set of T, β_M and i, and it can be used for nonzero values of $q_j(0)$. The error in the cost analysis caused by this underestimation of t* is minimal since it contributes only slightly to the O&M costs and even less to the sum of the O&M costs and construction costs.

It is possible to incorporate t* directly into the piecewise cost approximation of the O&M costs over the design period. Let the exponential cost function for O&M costs be approximated by the FP method as shown in Figure 3.3. The FP approximation, consisting of $\hat{FC}_j^M$, the fixed charge (dollar/year), and $\hat{C}_j^M$, the unit cost for a linear segment (dollars/year/MGD), is:

$$\hat{TC}_j^M = \hat{FC}_j^M + \hat{C}_j^M \cdot q_j(t) \tag{3.6}$$

where $\hat{TC}_j^M(t)$ is an approximate annual O&M cost for plant j at year t (dollars/year). The superscript M denotes O&M costs (superscript K is introduced later for construction costs). Then, TC_j^M, the average equivalent annual O&M costs (dollars/years), is:

$$\begin{aligned} TC_j^M &= \sum_{t=1}^{T} \left[(\hat{FC}_j^M + \hat{C}_j^M \cdot q_j(t)) \cdot D^t \right] / \sum_{t=1}^{T} D^t \\ &= \left(\hat{FC}_j^M + \hat{C}_j^M \cdot q_j(t^*) \right) \cdot \sum_{t=1}^{T} D^t / \sum_{t=1}^{T} D^t \\ &= \hat{FC}_j^M + \hat{C}_j^M \cdot q_j(t^*) \end{aligned} \tag{3.7}$$

From Equation (3.1), $q_j(t^*)$ is given as:

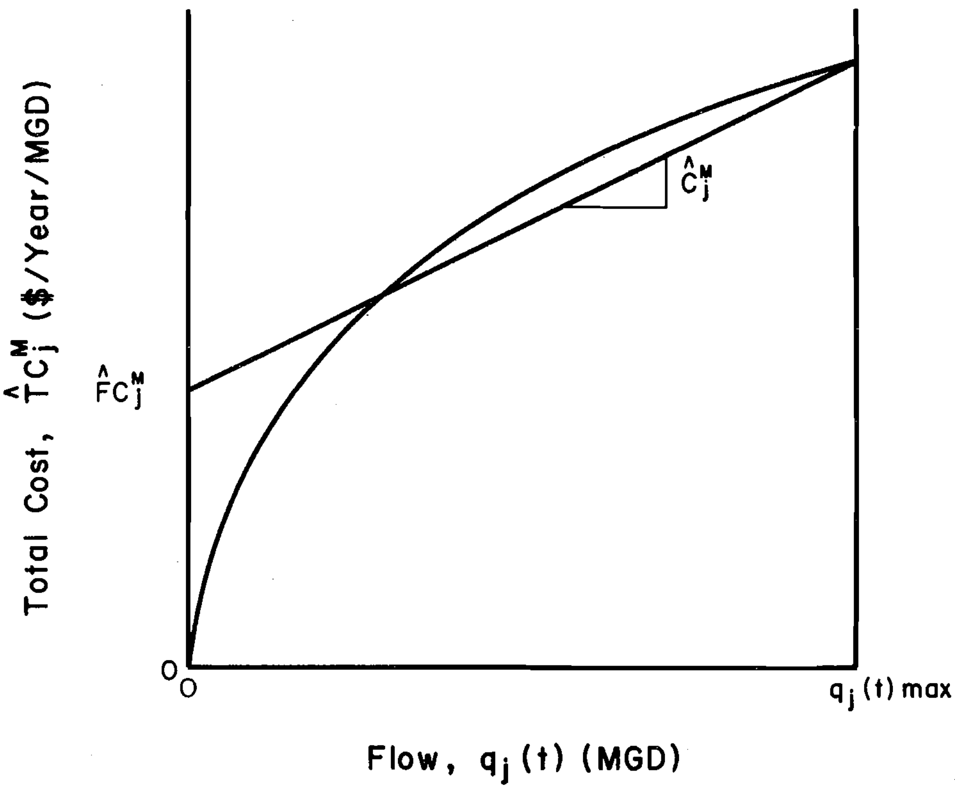

Figure 3.3 FP Approximation of O&M Cost

$$q_j(t^*) = q_j(0) + s_j \cdot t^*$$

$$= q_j(0) + (t^*/T) \cdot \left(q_j(T) - q_j(0)\right)$$

$$= (1 + t^*/T) \cdot q_j(0) + (t^*/T) \cdot q_j(T) \qquad (3.8)$$

Therefore, Equation (3.7) becomes:

$$TC_j^M = \hat{FC}_j^M + \hat{C}_j^M \cdot (1 - t^*/T) \cdot q_j(0)$$

$$+ \hat{C}_j^M \cdot (t^*/T) \cdot q_j(T)$$

$$= FC_j^M + C_j^M(0) \cdot q_j(0) + C_j^M(T) \cdot q_j(T) \qquad (3.9)$$

where

$$C_j^M(0) = \hat{C}_j^M \cdot (1 - t^*/T)$$

$$C_j^M(T) = \hat{C}_j^M \cdot (t^*/T)$$

and for consistence of notation $\hat{FC}_j^M$ is replaced with FC_j^M. Both $C_j^M(0)$ and $C_j^M(T)$ are constants and are associated with $q_j(0)$ and $q_j(T)$, respectively.

In summary, the average equivalent annual O&M costs can be expressed by a fixed charge and a unit cost modified by t^*/T. Note that when $q_j(0)$ is zero, Equation (3.9) reduces to an ordinary FP form and becomes a function only of $q_j(T)$. The cost approximation for O&M costs is the same for any stagewise construction program, as long as O&M costs are considered to be a function of waste flow rather than of capacity. Any portion of O&M costs which is a function of capacity can be included as part of the construction cost, which is a function of capacity.

B. Construction Cost of a Plant with Stagewise Expansions

Regional treatment plant capacities are assumed to be increased stagewise over T years. The number of interim design periods is given *a priori*, and every plant in the regional system must be constructed only for the required incremental capacity. This section describes how such stagewise construction costs can be expressed in terms of $q_j(0)$ and $q_j(T)$ just as with the O&M costs. The discussion is based on the two-stage construction case, for simplicity.

The amortized construction cost, K_j, of plant j for a design capacity of r_j can be expressed by an exponential cost function:

$$K_j = \alpha_K \cdot r_j^{\beta_K} \tag{3.10}$$

where

α_K = amortized construction cost of a reference plant, constant (dollars/year), and

β_K = economies of scale factor, constant.

Using the FP approximation method, the construction cost can be approximated as follows:

$$\hat{TC}_j^K = \hat{FC}_j^K + \hat{C}_j^K \cdot r_j \tag{3.11}$$

where

$\hat{TC}_j^K$ = approximate amortized construction cost for plant j at year t (dollars/year),

$\hat{FC}^K$ = fixed charge component of $\hat{TC}_j^K$ (dollars/year), and

$\hat{C}_j^K$ = piecewise cost component of $\hat{TC}_j^K$ (dollars/year/MGD).

Superscript K denotes construction costs.

As illustrated in Figure 3.4, for the two-stage case the plant will be constructed to a capacity, $q_j(0) + (q_j(T) - q_j(0)) \cdot (T_1/T)$, at year zero and an expansion of $(q_j(T) - q_j(0)) \cdot (T_2/T)$ will be added at year T_1. Equation (3.11) can be rewritten as follows:

$$\begin{aligned} TC_j^K &= \hat{FC}_j^K + \hat{C}_j^K \cdot \left[q_j(0) + (q_j(T) - g_j(0)) \cdot (T_1/T) \right] \\ &\quad + \left[\hat{FC}_j^K + \hat{C}_j^K \cdot (q_j(T) - q_j(0)) \cdot (T_2/T) \right] \cdot D^{T_1} \\ &= FC_j^K + C_j^K(0) \cdot q_j(0) + C_j^K(T) \cdot q_j(T) \end{aligned} \tag{3.12}$$

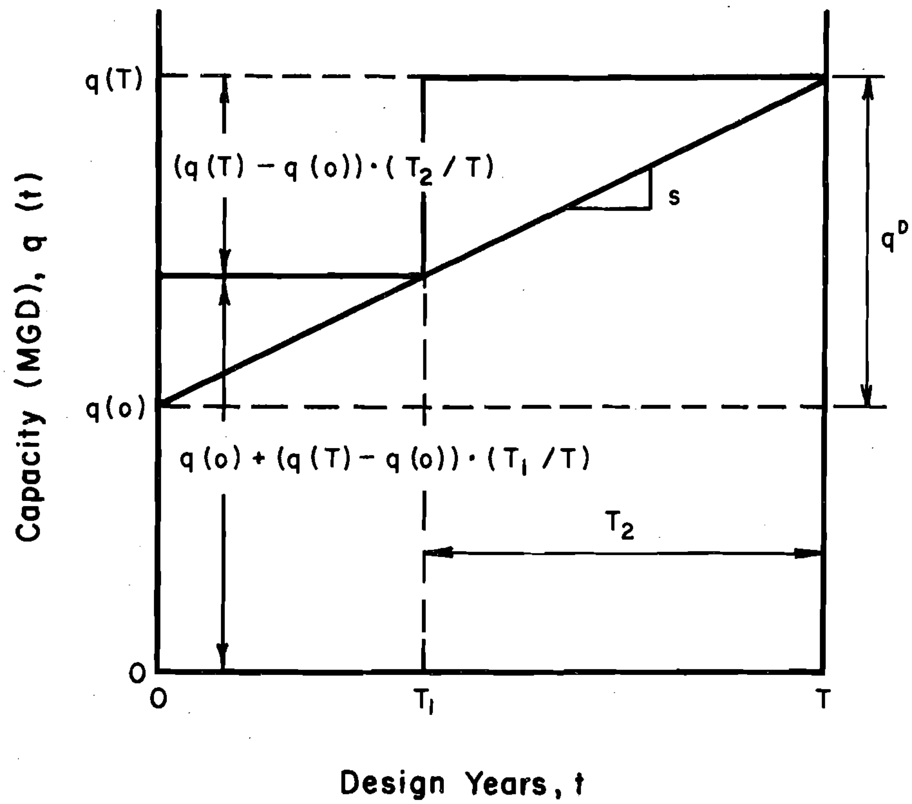

Figure 3.4 Two-Stage Construction of a Plant

where

$$FC_j^K = \hat{FC}_j^K(1 + D^{T_1})$$

$$C_j^K(0) = \hat{C}_j^K \cdot (1 - (T_1/T) - (T_2/T) \cdot D^{T_1})$$

$$C_j^K(T) = \hat{C}_j^K \cdot \left((T_1/T) + (T_2/T) \cdot D^{T_1}\right)$$

Note that both $C_j^K(0)$ and $C_j^K(T)$ are constants and are associated with $q_j(0)$ and $q_j(T)$, respectively. When $q_j(0)$ is zero, Equation (3.12) reduces to a simple FP form and becomes a function only of $q_j(T)$.

The two-stage construction case can be expanded to a general N-stage construction case without loss of generality. Only the fixed charge and two cost coefficients associated with $q_j(0)$ and $q_j(T)$ will change their forms as the number of stages increases, as follows:

$$FC_j^K = \hat{FC}_j^K \left[\left(\sum_{n=1}^{N} \prod_{m=1}^{n} D^{T_{m-1}}\right)\right] \quad (3.13)$$

$$C_j^K(0) = \hat{C}_j^K \cdot \left[1 - \sum_{n=1}^{N} \left[(T_n/T) \cdot \prod_{m=1}^{n} D^{T_{m-1}}\right]\right] \quad (3.14)$$

$$C_j^K(T) = \hat{C}_j^K \cdot \left[\sum_{n=1}^{N} \left[(T_n/T) \cdot \prod_{m=1}^{n} D^{T_{m-1}}\right]\right] \quad (3.15)$$

where T_o is defined as zero, and, thus, D^{T_o} is unity.

C. Aggregation of O&M Costs and Capital Costs of a Plant

The total plant cost is given by the sum of the O&M costs (Equation 3.9) and construction costs (Equation 3.12) as follows:

$$\begin{aligned} TC_j^T &= FC_j^M + FC_j^K + \left(C_j^M(0) + C_j^K(0)\right) \cdot q_j(0) \\ &\quad + \left(C_j^M(T) + C_j^K(T)\right) \cdot q_j(T) \\ &= FC_j^T + C_j^T(0) \cdot q_j(0) + C_j^T(T) \cdot q_j(T) \end{aligned} \quad (3.16)$$

where

$$TC_j^T = \text{total annual cost of plant j (dollars/year)},$$

$$FC_j^T = FC_j^M + FC_j^K ,$$

$$C_j^T(0) = C_j^M(0) + C_j^K(0), \text{ and}$$

$$C_j^T(0) = C_j^M(T) + C_j^K(T).$$

When $q_j(0) = 0$, Equation (3.16) contains only one variable $q_j(T)$, and it is of a simple FP form.

Equation (3.16) can be rewritten using q_j^D, the difference between $q_j(T)$ and $q_j(0)$ (see Figure 3.1), instead of $q_j(T)$ as follows:

$$\begin{aligned} TC_j^T &= FC_j^T + C_j^T(0) \cdot q_j(0) + C_j^T(T) \cdot q_j(T) \\ &= FC_j^T + \overline{C}_j^T \cdot q_j(0) + C_j^T(T) \cdot q_j^D \end{aligned} \quad (3.17)$$

where

$$\overline{C}_j^T = C_j^T(0) + C_j^T(T)$$

Equation (3.17) is used here to solve multiperiod problems which involve both $q_j(0)$ and $q_j(T)$.

D. Construction Cost of an Interceptor

It is assumed that the interceptors are constructed for the entire design period at the initial design year. Thus, the cost of constructing interceptors is based solely on the ultimate design flow.

$$TC_{ij}^P = FC_{ij}^P + C_{ij}^P(T) \cdot f_{ij}(T) \quad (3.18)$$

where

T_{ij}^P = construction cost of interceptor from location i to location j (dollars/year),

FC_{ij}^P = fixed charge component of TC_{ij}^P (dollars/year),

$C_{ij}^P(T)$ = piecewise cost component of TC_{ij}^P associated with capacity variable $f_{ij}(T)$ (dollars/year/MGD), and

$f_{ij}(T)$ = capacity of interceptor from location i to location j to be constructed for the ultimate design period T (MGD). It is equivalent to the ultimate design flow through the interceptor at year T.

Equation (3.18) is rewritten in the following form to be consistent with Equation (3.17) for solving the multiperiod problem.

$$TC^P_{ij} = FC^P_{ij} + C^P_{ij}(T) \cdot f_{ij}(0) + C^P_{ij}(T) \cdot f^D_{ij} \qquad (3.19)$$

where $f_{ij}(0)$ is that portion of the total capacity $f_{ij}(T)$, which corresponds to the initial design flows at year zero, and f^D_{ij} is defined as the difference between $f_{ij}(T)$ and $f_{ij}(0)$.

In the next section a solution method for the multiperiod regionalization problem, which involves cost approximations of the form represented by Equations (3.17) and (3.19), will be discussed.

3.4 Multiperiod Solution Method

The multiperiod solution method is essentially an extension of the branch-and-bound method presented in Chapter 2. It involves constructing two trees which are coupled by a set of constraints.

One special case of the multiperiod problem, however, can be reduced to the single-period formulation, and the branch-and-bound method of the previous chapter can be applied. This result is reached if the ratio of the initial design flow to the ultimate design flow is the same for every waste source. If this ratio is γ, the following relationship holds:

$$q_j(0) = q_j(T) \cdot \gamma \qquad \forall j \qquad (3.20)$$

Equation (3.16), then, reduces to the following form:

$$TC^T_j = FC^T_j + \left(\gamma \cdot C^T_j(0) + C^T_j(T)\right) \cdot q_j(T) \qquad (3.21)$$

The above equation along with Equation (3.18) leads to the same formulation as the single-period problem and the nonlinear branch-and-bound method can be applied.

If the above assumption does not hold, both the initial flows and ultimate flows must be taken into account in solving the multiperiod problem. The mathematical formulation and solution procedures are described below.

A. Multiperiod Formulation

The mathematical formulation of the multiperiod problem uses the cost approximations represented by Equations (3.17) and (3.19) for plants and interceptors, respectively. These equations are repeated below:

$$TC_j^T = FC_j^T + \bar{C}_j^T \cdot q_j(0) + C_j^T(T) \cdot q_j^D \qquad \forall\ j \qquad (3.17)$$

$$TC_{ij}^P = FC_{ij}^P + C_{ij}^P(T) \cdot f_{ij}(0) + C_{ij}^P(T) \cdot f_{ij}^D \qquad \forall\ i,\ j \qquad (3.19)$$

Two sets of variables are used in the above equations. The first set, $q_j(0)$ and $f_{ij}(0)$, represents the initial design flows, and the second set, q_j^D and f_{ij}^D, represents the incremental flows in between the initial and the terminal design years. The capacities to be constructed are represented by a combination of the two types of variables (see, e.g., Figure 3.4). Recall also that the fixed charge, FC_j^T, and cost coefficients, $\bar{C}_j^T$ and $C_j^T(T)$, in Equation (3.17) contain aggregate information about the O&M costs and stagewise construction costs for a plant at site j. As long as the assumption that growth of wastewater production is linear at each source holds, those two sets of variables are the only variables involved for any number of construction stages.

The two sets of variables are not independent, since it is also assumed that the assignment of waste sources to plants and interceptors remains unchanged over the planning period. In other words, if $q_j(0)$ consists of waste flows initially generated by a set of waste sources, then q_j^D must also consist of incremental flows generated by the same set of sources. Similarly, in the case of interceptors, if a set of waste sources is assigned to the interceptor from location i to location j, then both the initial flow, $f_{ij}(0)$, and the incremental flow, f_{ij}^D, consist of waste flows generated by the sources in the assigned set. This requirement is defined as the *compatibility requirement* of FP variables. If the above two sets of variables are introduced in one mathematical formulation, then, a set of constraints which impose the compatibility requirement becomes necessary. It is clearly quite cumbersome, since the number of constraints would generally be very large. It is not likely, further, that such a formulation can be solved efficiently.

The method proposed here uses a form of decomposition. If q_j^D and f_{ij}^D are disregarded from Equations (3.17) and (3.19), then the remaining cost components would form an FP approximation and the single-period branch-and-bound formulation becomes applicable. Similarly, if $q_j(0)$ and $f_{ij}(0)$ are disregarded, the remaining cost components would form a different FP approximation. Taking the first case, the nonlinear branch-and-bound formulation involves the following FP approximations:

$$TC_j^T(T) = FC_j^T + C_j^T(T) \cdot q_j^D \qquad \forall\ j \qquad (3.22)$$

$$TC_{ij}^P(T) = FC_{ij}^P + C_{ij}^P(T) \cdot f_{ij}^D \qquad \forall\ i,\ j \qquad (3.23)$$

The disregarded variables, $q_j(0)$ and $f_{ij}(0)$ along with their respective cost coefficients, form the remaining cost components.

$$TC_j^T(0) = \bar{C}_j^T(0) \cdot q_j(0) \qquad \forall\ j \qquad (3.24)$$

$$TC_{ij}^P(0) = C_{ij}^P(T) \cdot f_{ij}(0) \qquad \forall\ i,\ j \qquad (3.25)$$

If the compatibility requirement is imposed fully, then the following relationships hold:

$$TC_j^T = TC_j^T(0) + TC_j^T(T) \qquad \forall\ j \qquad (3.26)$$

$$TC_{ij}^P = TC_{ij}^P(0) + TC_{ij}^P(T) \qquad \forall\ i,\ j \qquad (3.27)$$

The above observation leads to a method for solving the multiperiod problem. Two decomposed problems are defined; one consisting of the FP approximations represented by Equations (3.22) and (3.23), and the other consisting of the linear approximations represented by Equations (3.24) and (3.25). The first problem is called the decomposed-problem 1 (DCP-1), and the second is called the decomposed-problem 2 (DCP-2). If the compatibility requirement is met between a solution to the first problem and a solution to the second problem, then, the two solutions can be joined to give an upper bound on the least-cost solution to the original problem represented by Equations (3.17) and (3.19). If, on the other hand, the compatibility requirement is satisfied only partially, the two solutions can be combined to give a candidate for a lower bound on the cost of the least-cost solution to the original problem. The method proposed here takes the latter approach.

The mathematical formulation of DCP-1 conforms exactly to the standard FP formulation given by Equations (2.16) through (2.24). Since only one FP approximation is involved for DCP-1, subscript k is dropped from the variables and constants, and thus no summation over index k is necessary. Also, the following changes are made:

f_{ij}^D and q_j^D replace f_{ijk} and q_{jk},

$C_{ij}^P(T)$ and $C_j^T(T)$ replace C_{ijk}^P and C_{jk}^T,

L_j^D replaces L_j,

F_{ij}^D replaces F_{ij}, and

Q_j^D replaces Q_j,

where L_j^D is the ultimate design flow less the initial design flow generated at site j, F^D_{ij} is the upper limit of variable f^D_{ij}, and Q^D_j is the upper limit of variable q^D_j.

The formulation of DCP-2 is a standard linear programming formulation consisting of an objective function, a set of flow conservation constraints and nonnegativity constraints. The formulation involves only Equations (2.16) through (2.18), again without subscript k and the summation over index k. The notation for variables and constants is changed as follows:

$f_{ij}(0)$ and $q_j(0)$ replace f_{ijk} and q_{jk},

$C^P_{ij}(T)$ and $\overline{C}^T_j(0)$ replaces C^P_{ijk} and C^T_{jk},

$L_j(0)$ replaces L_j,

$F_{ij}(0)$ replaces F_{ij}, and

$Q_j(0)$ replaces Q_j,

where $L_j(0)$ is the initial design flow generated at site j, $F_{ij}(0)$ is the upper limit of variable $f_{ij}(0)$, and $Q_j(0)$ is the upper limit of variable $q_j(0)$.

DCP-1 can be solved by the nonlinear branch-and-bound method proposed in the previous chapter, and DCP-2 can be solved by any linear programming method or a network flow algorithms.

The following section describes a method to couple the solutions to the two decomposed problems.

B. Coupled Branch-and-Bound Method

If the compatibility requirement is disregarded, then DCP-1 and DCP-2 are independent. The objective function of DCP-2 can be added to the cost of the least-cost solution of DCP-1 to provide a lower bound on the least-cost solution of the original problem with the compatibility requirement.

This lower bound can be improved by introducing the following *coupling constraint* sets to the formulation of DCP-2.

$$f_{ij}(0) \cdot u_{ij} = 0 \qquad \forall\, i, j \qquad (3.28)$$

$$q_j(0) \cdot v_j = 0 \qquad \forall\, j \qquad (3.29)$$

Note that variables $f_{ij}(0)$ and $q_j(0)$ are only in DCP-2, while slack variables u_{ij} and v_j are associated with fixed charge variables x_{ij} and y_j, and they appear in the nonlinear constraint sets in DCP-1 as follows:

$$f^D_{ij} \cdot u_{ij} = 0 \qquad \forall\, i, j \qquad (3.30)$$

$$q^D_j \cdot v_j = 0 \qquad \forall\, j \qquad (3.31)$$

The following discussion illustrates how those coupling constraints work. Consider, first, Equation (3.30). At some point of the DCP-1 branch-and-bound process, u_{ij} is set to zero on a branch one and f^D_{ij} is set to zero on the corresponding branch two. Then, according to Equation (3.28), $f_{ij}(0)$ is allowed to take on any value (when u_{ij} is set to zero), or $f_{ij}(0)$ is set to zero (when f^D_{ij} is set to zero). The latter holds because as f^D_{ij} is set to zero, u_{ij} would automatically assume the value FC^P_{ij} (since x_{ij} remains at zero). In other words, a tree structures similarly to the branch-and-bound tree associated with DCP-1 would be constructed in the process of solving each of the "coupled" subproblems for DCP-2. This tree is called the *constraint tree*. The branch-and-bound tree and the constraint tree are coupled to form a *coupled branch-and-bound tree*. The variables u_{ij} and v_j are now defined as the *coupling variables* between the two problems. This coupling step partially fulfills the compatibility requirement, as described later.

In practice, however, the coupling process is not so simple, because the branch-and-bound tree for DCP-1 is grown by inspection from each node where a subproblem is solved, while the corresponding constraint tree must be grown without such inspection steps. Consider the following example shown in Figure 3.5. Suppose a branching is to be performed from one of the infeasible nodes on the coupled branch-and-bound tree. The analysis procedure goes as follows. First, the node subproblem is solved for DCP-1. The objective function value z_1^1 is identified. A string of K-1 node costs is then obtained by inspection, and the node costs z_2^1, z_3^1, ..., z_K^1 are determined. Note that the solution identified at the bottom of the tree is a feasible alternative to DCP-1, since all the nonlinear constraints are satisfied. Now DCP-2 must be solved. A set of branching variables, which correspond to the set selected previously for DCP-1, must be constrained in exactly the same fashion based on the coupling constraints. Then the objective function value z_1^2 is identified. Now the coupling of node 1 of DCP-1 and node 1 of DCP-2 is completed. However, there are K-1 additional couplings to be performed before the entire set of nodes along the inspection limb of DCP-1 tree is coupled with the corresponding set of nodes on the DCP-2 tree. After each of the K-1 additional subproblem computations of DCP-2, the node costs z_2^2, z_3^2, ..., z_K^2, are determined, respectively. Now the coupling of the two limbs is completed, as shown in Figures 3.5-(1) and 3.5-(2). The node costs of the corresponding limb of the coupled tree are $z_1^0 = z_1^1 + z_1^2$, $z_2^0 = z_2^1 + z_2^2$, ..., $z_K^0 = z_K^1 + z_K^2$, as shown in Figure 3.5-(3). The number of subproblem computations required for the complete coupling of the two limbs is, therefore, K+1.

As the coupling procedure proceeds, the compatibility requirement becomes satisfied to a greater extent. Note, however, the coupling constraints for the FP variables do not specify the amount of waste flows to be assigned to a plant or to an interceptor, but they ensure that two coupled

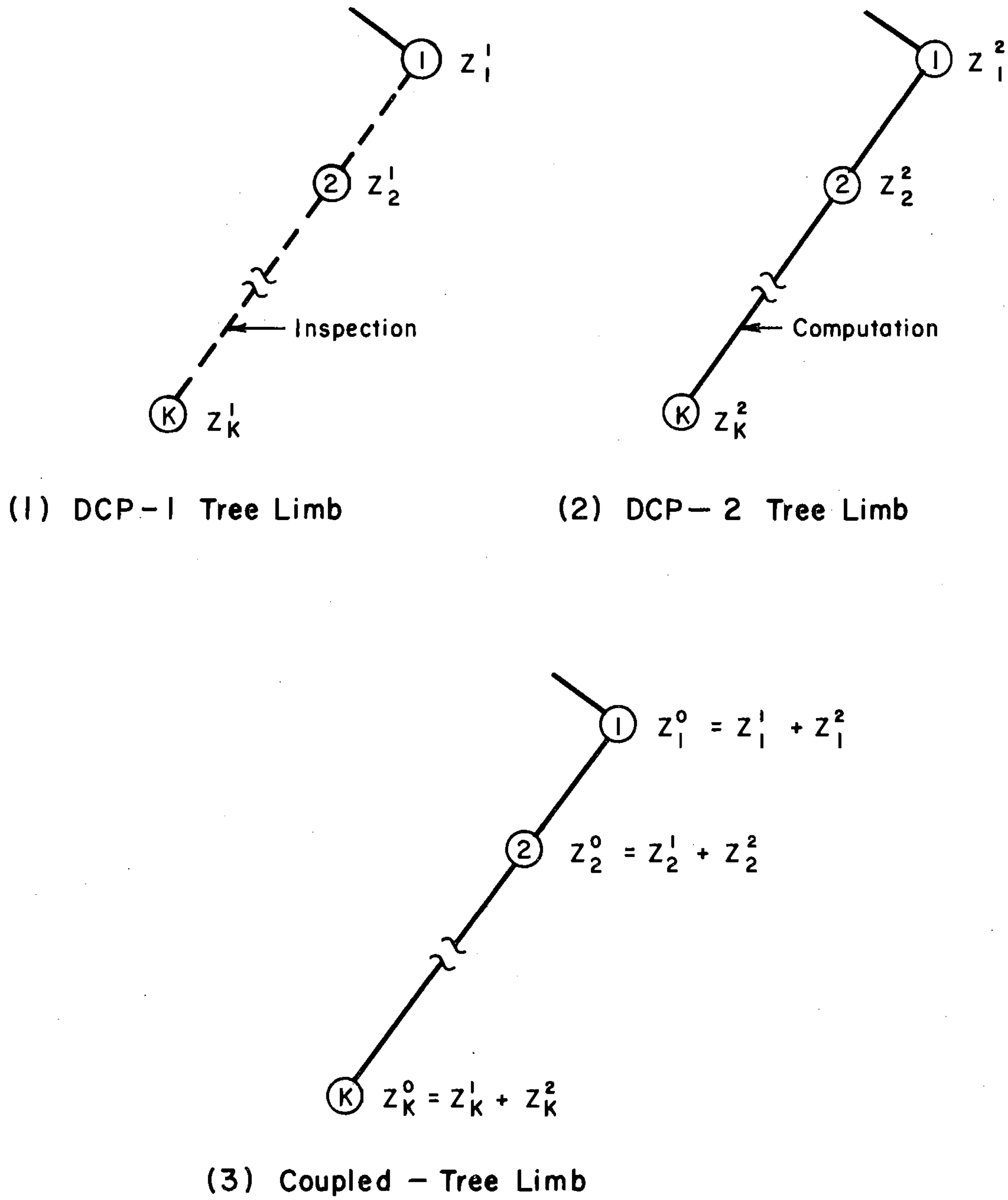

Figure 3.5 Coupling of Branch-and-Bound Tree and Constraint Tree

variables are simultaneously zero or simultaneously greater than zero. Therefore, the compatibility requirement may not be fully satisfied even at the bottom of the coupled branch-and-bound tree. Further, if there are some linear approximations without fixed charges in DCP-1 and DCP-2, then it would not be fully satisfied, since the variables associated with linear approximations do not engage in the coupling procedure. On the other hand, if FPI rather than FP approximations are used, it would be satisfied to a much greater extent, since each coupling constraint set specifies the minimum amount of waste flows to be assigned, for example, $q_j^D \geq L_j^D$ in DCP-1 and $q_1(0) \geq L_j(0)$ in DCP-2.

If the compatibility requirement is fully satisfied at the bottom of the limb of the coupled branch-and-bound tree, then the node solution is an alternative to the original multiperiod regionalization plan. If it is only partially satisfied, then the node cost gives a lower bound to the cost of that alternative. Therefore, when the least-cost multiperiod plan is to be found, the coupling process may be terminated along the limb of the coupled branch-and-bound tree whenever a node cost exceeds the current upper bound on cost. When, on the other hand, it is desired to identify alternative plans within a given cost range, then the coupling procedure may be continued until all of the infeasible node costs exceed that cost range. Note that any DCP-1 alternative solution can be converted to an alternative multiperiod solution by simply computing the multiperiod cost based on its flow assignment.

C. Modifications of the Coupled Branch-and-Bound Method

The coupled branch-and-bound method presented in Section 3.4-B provides many alternative multiperiod plans including the approximate least-cost solution. Computational efficiency of the method, however, depends on the number of subproblem computations required, particularly on the constraint tree for DCP-2. The large number of computations for DCP-2 offsets the computational efficiency attained by the inspection method for DCP-1.

For the purpose of obtaining the information on approximate costs of alternatives, which will be used for the trade-off analysis, the coupled branch-and-bound method may be modified as follows to increase the computational efficiency. Since $z_2^1 \leq z_3^1 \leq \ldots \leq z_K^1$, and $z_2^2 \leq z_3^2 \leq \ldots \leq z_K^2$, then, $z'^0_2 = z_2^1 + z_1^2$, $z'^0_3 = z_3^1 + z_1^2$, ..., and $z'^0_K = z_K^1 + z_1^2$ are less than or equal to z_2^0, z_3^0, ..., z_K^0, respectively. In other words, z_1^2 instead of z_k^2 (k=1, ..., K) can be added directly to each of the node costs on the DCP-1 branch-and-bound tree. This approach is equivalent to the relaxation of the compatibility requirement by omitting some coupling constraints in DCP-2. Therefore, the lower bounds in the coupled branch-and-bound tree becomes less tight. This modification reduces the number of subproblem computations in DCP-2 to only one for each DCP-1 subproblem computation. There is, however, a trade-off. Because of the looser lower bounds, the branch-and-bound process may have to be continued longer than in the previous case.

Note that when the values of the initial flow variables, $f_{ij}(0)$ and $q_j(0)$, are relatively small compared with the values of the incremental flow variables, f_{ij}^D and q_j^D, the relative contribution of the DCP-2 cost to the total cost of each alternative is even smaller since DCP-1 includes the fixed charge associated with both. If the costs of DCP-1 do dominate, the coupled tree would not be significantly different than a tree grown using the coupled branch-and-bound method without the modification. If, on the other hand, the values of the incremental flow variables are relatively small compared with the values of initial flow variables, the fixed charges can be combined with the initial flow variables rather than incremental flow variables. Then the DCP-2 can be solved by the nonlinear branch-and-bound method and the DCP-1 can be solved by a linear programming method. Thus the above modification of the coupled branch-and-bound method applies equally well to this case.

In summary, the modified coupled branch-and-bound method is computationally efficient and the information on the tree is just as useful as in the unmodified case.

3.5 Additional Considerations

The method of analysis for multiperiod regionalization problems can be modified further. For example, the FPP rather than the FP approximations can be used to approximate construction and O&M costs. Also, the coupled branch-and-bound method may be modified to deal with arbitrary growth of waste flows, using multiple branch-and-bound trees and constraint trees. Constraints to prevent split flows can be easily introduced to the current formulation of the problem. As the formulation is made more sophisticated, however, the solution procedure becomes more complex and time-consuming, and may defeat the purpose of generating alternatives efficiently.

On the other hand, the method proposed here is based on a number of assumptions. The assumptions may limit the practicality of this approach in determining the precise phasing schedule of the regional system. Such a capability is beyond the scope of this analytical method. The approach here is most useful for identifying many alternative plans systematically based on cost. The alternatives identified can be evaluated based on the economic trade-offs, using the imputed value method proposed in the next chapter. The illustrative example introduced in the following section focuses on generating alternatives in the multiperiod case.

3.6 Illustrative Example

The methods proposed for generating multiperiod alternative plans have been tested using the same hypothetical example problem described in Section 2.5. The waste-flow data over the 25 year period are shown in Table 2.1, and the regional facility network is shown in Figure 2.10. The cost functions used are given in Equations (2.41), (2.42) and (2.43) for plant construction, plant O&M costs, and interceptor construction, respectively. A discount rate of 0.05 and design lives of 25 years for plants and 50 years for interceptors are used. Waste flows increase at each site as shown in Table 2.1 and in Figure 3.6. The figure also indicates the lines connecting the initial and terminal year flows for each site; those lines are used later in the application of coupled branch-and-bound method. The examples presented are based on two-stage construction; the first design year is assumed to be the tenth year, and the second design year is assumed to be the terminal year.

Consider first the application of the single-period branch-and-bound method based on the assumption that the growth of waste flow at each source may be reduced to a simple linear form represented by Equation (3.20). Two values of γ were tested. In case A, γ was simply assumed to be zero, and in case B, γ was assumed to be 0.3, an approximate ratio of the sum of initial design year flows and the sum of terminal design year flows. These two approximations are shown in Figure 3.7. The computational results for the two cases are illustrated in Figure 3.8. Two cost relationships are indicated for each alternative solution represented by its approximate two-stage cost. The first is the one-stage (single-period) cost of that alternative based on the actual cost functions. The second is the two-stage cost of the same alternative based on the actual cost functions.

It is apparent from the figure that the approximate costs in Case B represent the actual two-stage costs better than those in Case A. Also note that the actual two-stage costs and actual one-stage costs are very close in both cases. The close fit between the approximate costs and actual costs in Case B does not imply that this method of analysis is justified

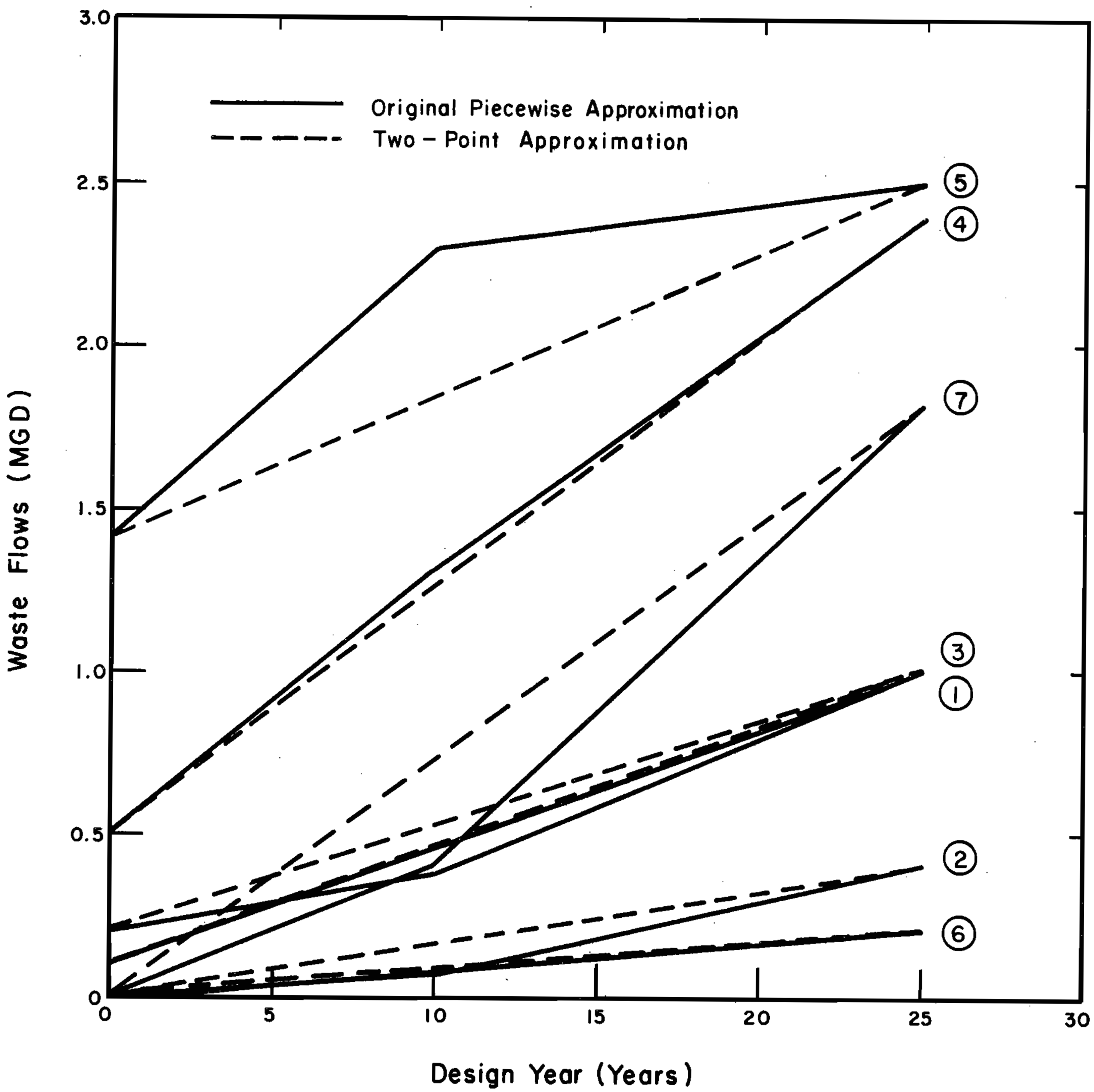

Figure 3.6 Original Piecewise Approximation and Two-Point Approximation of Waste Flows for the Hypothetical Example Problem

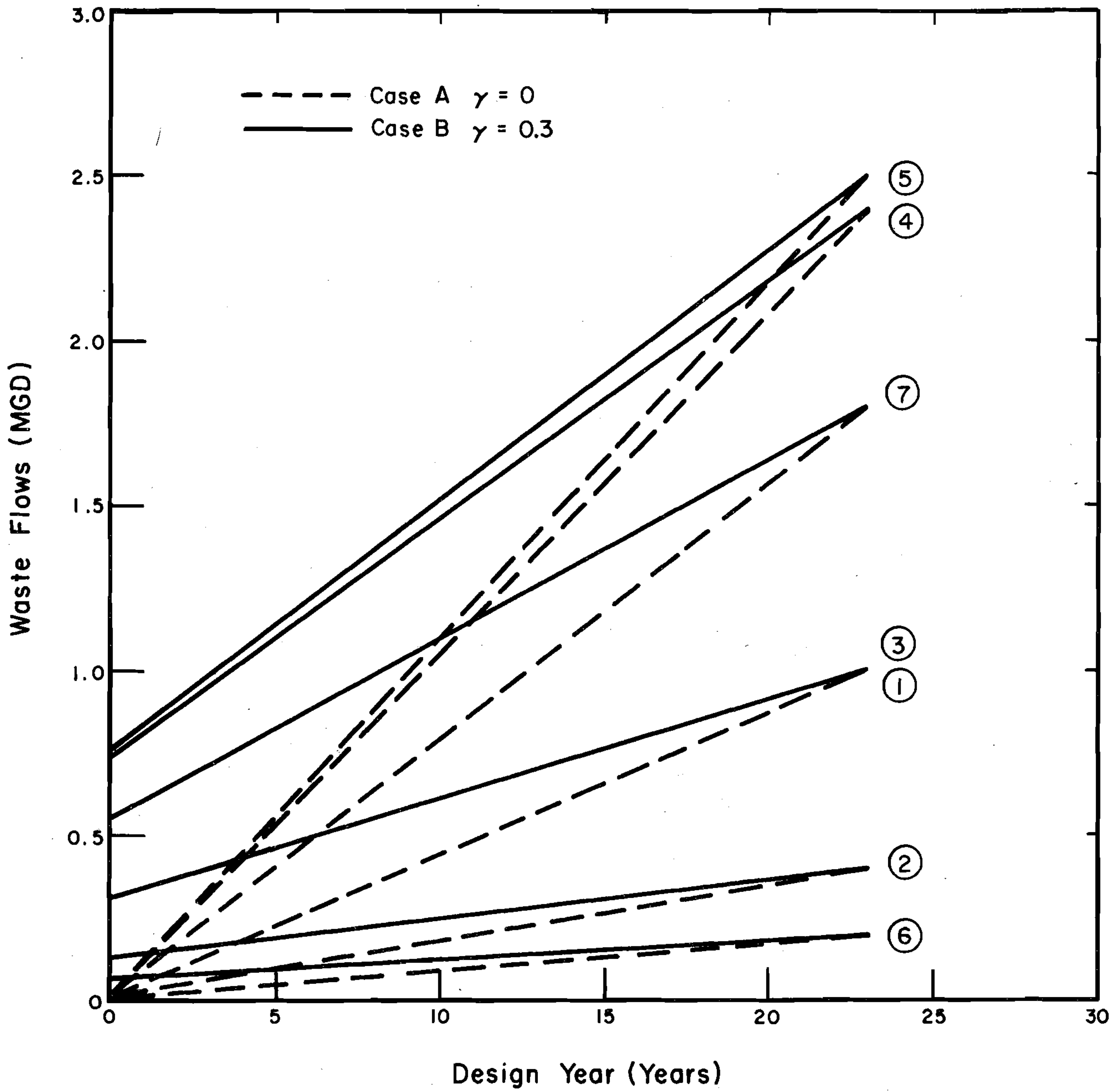

Figure 3.7 Fixed Ratio Linear Approximations of Waste Flows for the Hypothetical Example Problem

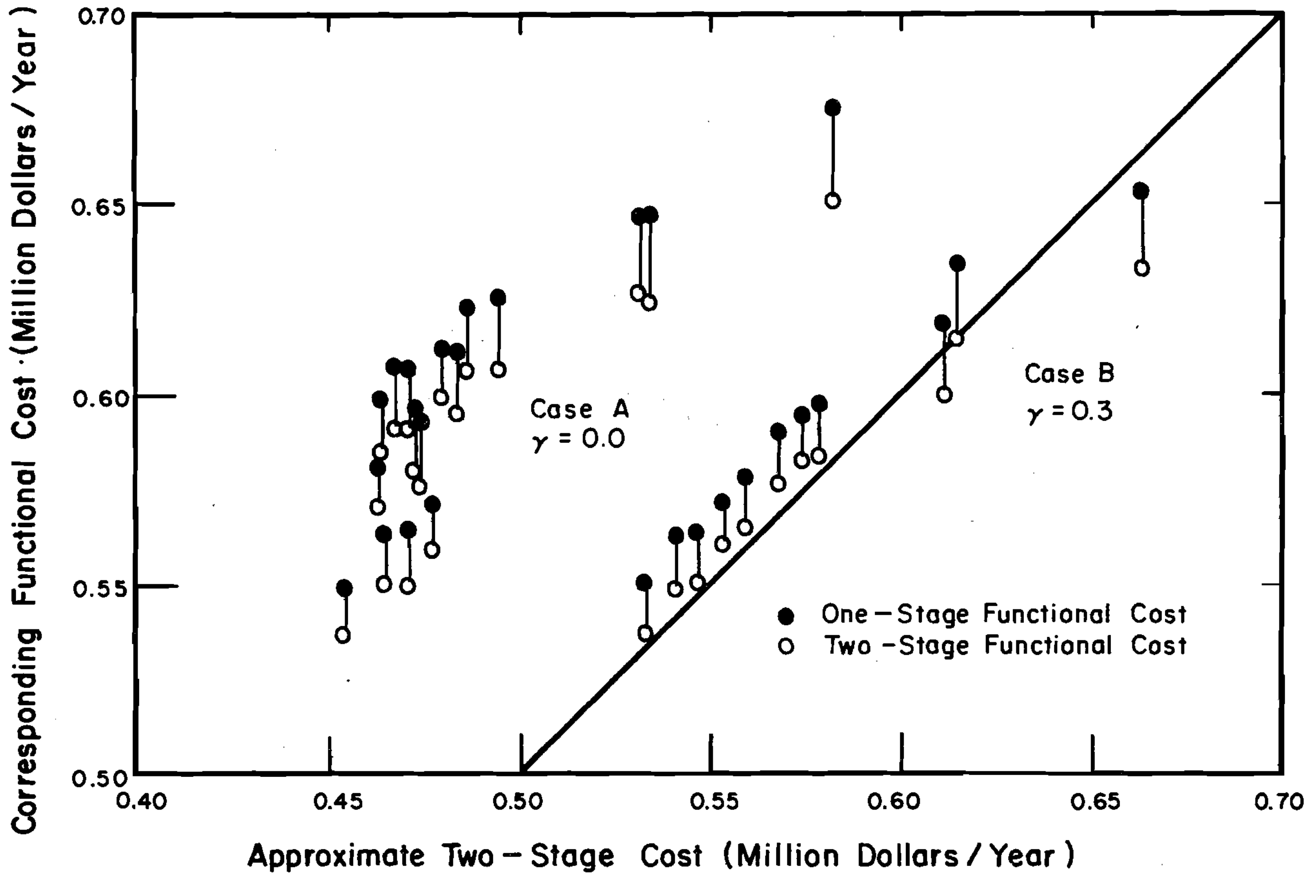

Figure 3.8 Alternatives Generated by the Static Two-Stage Branch-and-Bound Method

for any growth patterns of waste flow, since the assumption given by Equation (3.20) is not generally applicable. However, it is interesting to note that the approximation did show a close fit in this particular example.

The relatively small difference between the one-stage and two-stage costs based on actual cost functions is not accidental. Recall the assumption that the interceptor costs occur only at the initial year. Now that the one-stage and two-stage costs are computed for the same facility locations, the interceptor costs are exactly the same for both cases. Further, since the O&M costs are based solely on the annual flows treated by the plant, they are the same for both cases also. The only cost difference incurred is due to the economies of scale for plant construction and to the discounting of the second stage plant construction costs. This difference seems to be relatively small for this example problem.

The computational results for the coupled branch-and-bound method, Case C, based on the two-point flow approximation given in Figure 3.6, are shown in Figure 3.9. Although the approximate two-stage cost for each alternative gives a slight underestimation of the actual cost, the relative fit is quite close. Again the one-stage and two-stage costs based on the cost functions show relatively small difference. Although the results of Cases B and C turned out to be quite similar with respect to their close representations of actual costs, the additional mathematical flexibility of the latter outweighs the simplicity of the former. The coupled branch-and-bound method can be applied to any set of two-point approximations of waste flows over the design period, and what is more, the infeasible lower bounds on the branch-and-bound tree provide useful information on the alternatives yet to be generated.

As in the case with the single-period branch-and-bound method, the tree can be grown to generate many additional alternative plans by simply increasing the cut-off value z**. For example, if the tree is grown to the point where the lowest

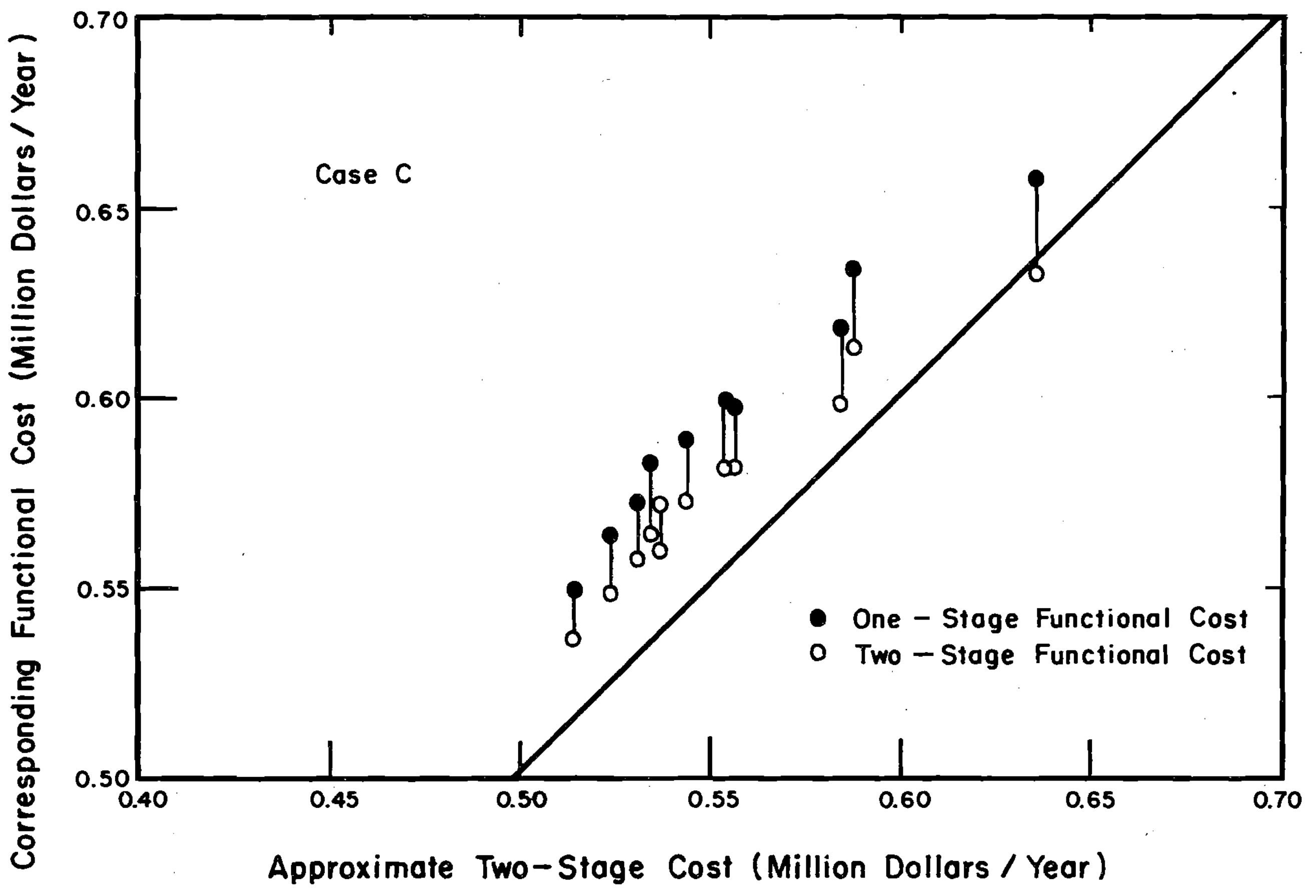

Figure 3.9 Alternatives Generated by the Coupled Branch-and-Bound Method for Two-Stage Construction

of the infeasible lower bounds exceeds z** of $550,000/year rather than terminating the computation when the approximate least-cost solution z* of $516,000/year is found, the number of generated alternatives increases from 12 to 34. As z** is increased further to $570,000/year, then the number of alternatives also increases further to 45. The computational statistics for the three runs are shown in Table 3.1. The analysis of the alternatives generated in each of the three runs will be discussed in Section 4.4 of the following chapter.

Table 3.1 Computational Statistics for Generating Additional Alternatives for Case C

	Run 1	Run 2	Run 3
Cut-off Cost† (z^{**})	516.0+	550.0	570.0
Total No. of Nodes	46	97	118
No. of Active Nodes	27	83	107
No. of Active Inspections	13	41	53
No. of Subproblems Computed	28	84	108
No. of Feasible Alternatives	12	34	45

† Thousand dollars per year.

+ Also the least cost (z^*).

4. COMPARING ALTERNATIVE PLANS: IMPUTED VALUE ANALYSIS

4.1 FP Branch-and-Bound Tree and Imputed Values

Given the four criteria presented in Chapter 1 for measuring the performance of a mathematical model as a tool to generate and compare alternative plans, the branch-and-bound method appears to perform quite well. First, the method takes advantage of a network flow algorithm and inspection steps to make it computationally efficient. Second, it can identify the lower and upper bounds on the cost of alternatives which are systematically generated. Third, economic trade-offs among different sets of alternatives can be related to other planning issues for gaining insights. And last, the branch-and-bound tree may be grown at will to generate alternatives with pre-specified physical characteristics.

In particular, the FP branch-and-bound method appears quite attractive because of its flexibility in adapting to many different cost approximations, because of its versatility in handling both single-period and multiperiod formulations, and because of its mathematical simplicity which results in high computational efficiency. One of its most attractive features, however, is that it allows efficient comparisons of economic trade-offs associated with regional facilities and boundaries in terms of the "*imputed values*". An imputed value is defined as the cost difference between the least-cost solution with a facility (a set of facilities) and the one without it.

Imputed value analysis is based on the binary grouping of alternatives. There are those alternative plans which contain a given facility and those which do not. The fundamental mathematical approach of the FP branch-and-bound process is also a binary grouping of alternatives into mutually exclusive subsets. The mathematical problem is constrained to generate two kinds of alternatives: one allows a facility to be constructed, and the other does not. This dichotomy is automatically ensured as long as the facility cost is approximated by any combination of the FP approximation. When a cost approximation involves a piecewise linear segment through the origin, as in the case of the revised PP or the revised PPP approximations, the linear segment can be artificially constrained in adapting to a branch-and-bound process. Consider, for example, the case illustrated in Figure 2.7. The figure shows the branch-and-bound process for the revised FPP approximation. The alternatives generated under node m_o are forced to exclude the treatment plant (plant j in this case), and those under nodes n_1 and m_1 are forced to include it. The imputed value of plant j is simply the cost difference between the least-cost alternative which belongs to the former group and the one which belongs to the latter.

An imputed value of facility x, as described in Chapter 1, is the implicit economic gain (if $C(\bar{x}) - C(x) \geq 0$) or implicit economic loss (if $C(\bar{x}) - C(x) \leq 0$) of including that facility in the regional plan, as opposed to excluding it from the plan. In other wrods, it is a measure of the economic trade-off between two mutually exclusive sets of alternative plans. Since it is an implicit economic value, it is relatively easy to gain a substantive "feel" for the significance of such a facility or such a set of facilities in the regional plan. For example, one may wish to examine issues other than cost and to compare them with the imputed values. Such an exercise provides an opportunity to gain insights into the problem of planning wastewater facilities in the given region.

Conceptually, the comparison of alternatives may be carried out directly using the branch-and-bound tree as described in Chaper 1. For example, a least-cost solution is found to require an interceptor, say interceptor A, which is relatively undesirable for noneconomic reasons (e.g., the crossing of a political boundary). The existing tree can be evaluated to explore this issue. All of the nodes which are at the end of tree branches may be reevaluated to see if they contain interceptor A in the corresponding alternative plans. If necessary, the tree can be extended to find the least-cost solution without it. It may be possible, however, to examine the already existing feasible solutions and to choose one which appears attractive with respect to all of the planning objectives. Since lower bounds are available for each node solution, it may not be necessary to examine any new branches.

The economic savings incurred by crossing the boundary is the difference between the cost of the least-cost alternative which excludes interceptor A and the one which includes it. It becomes attractive to cross over the political boundary only when there are economic savings which exceed the implicit costs of political transactions associated with the boundary. Therefore, the difference between the two costs as defined above can be regarded as the imputed value of crossing the political boundary.

The set of imputed values provides valuable information. First, it provides an estimate of the trade-offs between cost and the political issue related to crossing the boundary. Second, it suggests the relative importance of the boundary in comparison to the other boundaries. Such information may be important in arriving at a relatively small set of alternative regional plans for more detailed evaluation.

While the conceptual application of the branch-and-bound tree described above for analyzing alternative plans may be used for small problems, it may not provie very practical for large ones, because the size of the tree becomes too large for display, and because the retrieval of information becomes too cumbersome. The following section deals with the transformation of the branch-and-bound tree into a form of matrix which is designed to be more practical for analyzing the imputed values in large problems.

4.2 The Imputed Value Incidence Matrix

A. Structure of the Matrix

The fundamental structure of the FP branch-and-bound tree has been described in Section 2.3-C and is illustrated in Figure 2.9. This structure is common to both the single-period branch-and-bound tree and the coupled branch-and-bound tree in the multiperiod problems. While the FP tree is very well structured, it cannot be used directly for an imputed value analysis since it is extremely difficult to retrieve information systematically. It is possible, however, to transform some of the information on the branch-and-bound tree into matrix form. The matrix displays the incidence relationship between the state of the branching variables and the alternative physical plans identified at the extreme ends of the inspection limbs. Since a simple search through the matrix can provide imputed values, it is defined as the *imputed value incidence matrix*. Also, for convenience, it will henceforth be referred to simply as the *incidence matrix*.

In any FP problem each alternative plan is described by the state of the FP variables. When an FP (FPI) variable is constrained to be greater than zero (greater than L_j) on branch one, then the facility represented by the variable would be forced to exist in the alternative plan generated. When it is constrained to be zero on branch two, then the facility would be prevented from existing. Such incidence relationship can be expressed by introducing the incidence index, a_{mn}. The index describes the state of the nth FP variable in the mth alternative plan. The index may take the value of 1 (representing "branch one") or 2 (representing "branch two"). The incidence matrix contains M rows for the alternatives and N columns for the FP variables.

Additional index values are also used. First, some FP variables may not be constrained in the process of generating an alternative, and yet they may assume the value zero in the solution to a subproblem. Since these variables assume the value zero but are not so constrained, the index value associated with them is defined to be -2. In this way it is possible to distinguish between such variables and those constrained to be zero with an index value of 2.

Second, while all of the variables associated with the inspection limb from a parent node would be assigned the index value of 1, some of these nodes belong to the active portion and others are in the inactive portion of the limb. Since the variables in the active portion define the lower bound on the cost of all the possible alternatives which could be generated by further branching of the tree, they are distinguished from the ones in the inactive portion. The variables which belong to the inactive portion are assigned the index value of -1 and the variables which belong to the active portion retain the index value of 1.

The four index values described above are illustrated using the example shown in Figure 4.1. The figure displays the inspection limb m' which is associated with alternative m' and the inspection limb associated with alternative m. The following notation is used to represent various nodes and variables. There are K nodes, or K-1 branch-one inspection steps, on the inspection limb m. These nodes are denoted 1_m, 2_m, ..., k_m, ..., K_m. Associated with each of these nodes is its cost, $z(1_m)$, $z(2_m)$, ..., $z(k_m)$, ..., $z(K_m)$. The notation for inspection limb m' is similar. Node 1_m is the parent node for inspection limb m, and node $1_{m'}$ is the parent node for inspecton limb m'. In the entire tree there are N branching variables, of which L are shown. They are n_1, n_2, ..., n_ℓ, ..., n_L; n_1 and n_2 are associated with inspection limb m', and n_3 through n_L are associated with inspection limb m.

The incidence index a_{mn} can be described in the context of branch-and-bound process. Assume first that inspection limb m' has been constructed, but branch two has not been extended from node $2_{m'}$. Since the inspection limb m is not generated yet, a_{mn} for all n is null. At some point of the branch-and-bound process, branch two from node $2_{m'}$ provides node 1_m, which leads to inspection limb m and the feasible alternative at node K_m. At this point the mth row is added to the incidence matrix. The elements of this row correspond to variables n_1, n_2, ..., n_L and contain the following incidence index values:

n_1	n_2	n_3	n_4	.	.	n_ℓ	$n_{\ell+1}$	.	.	n_L
1	2	-1	-1			-1	-1			-1

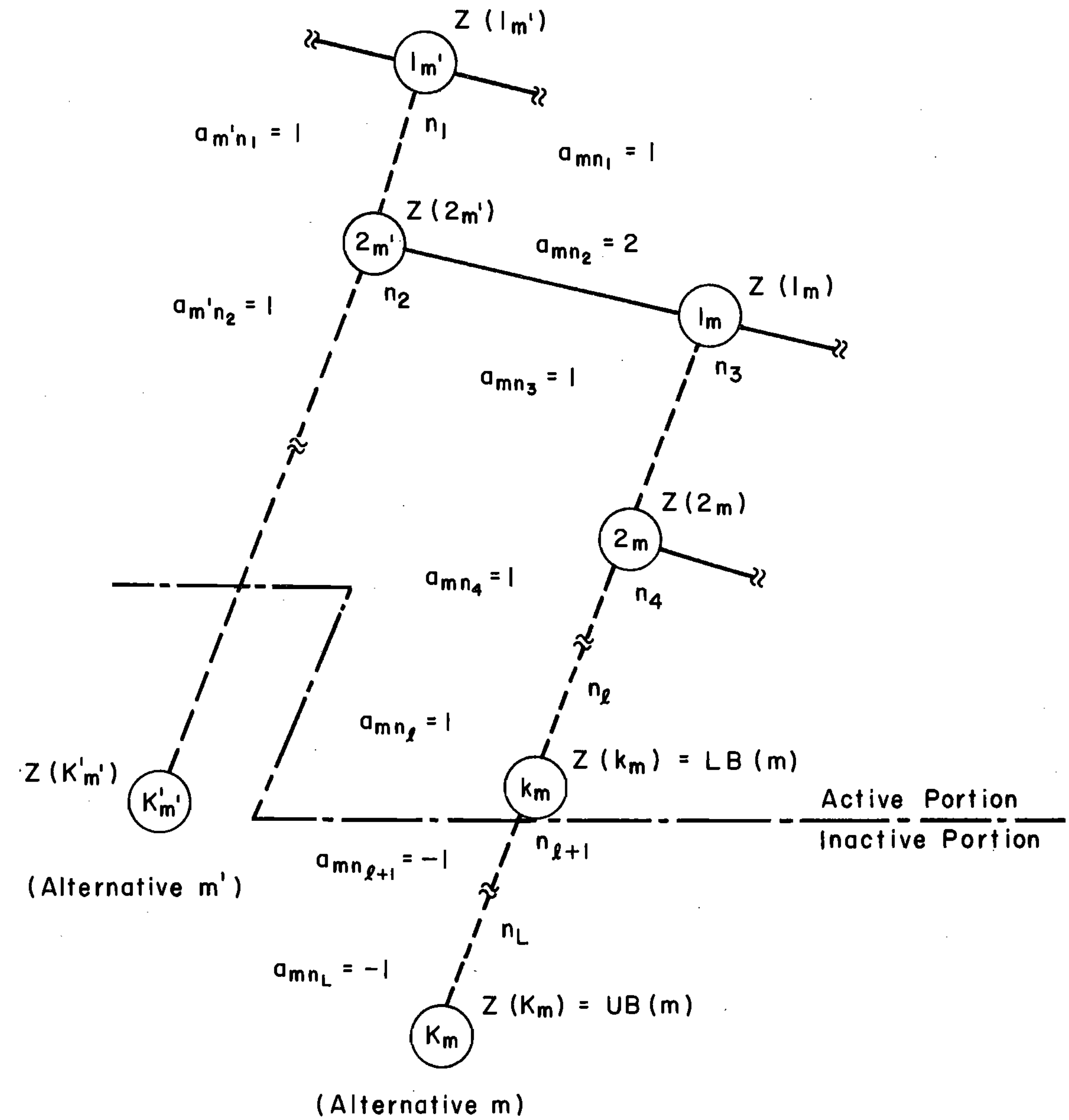

Figure 4.1 An Example Tree for Illustration of Incidence Index, a_{mn}

The value of a_{mn_1} is 1 because the branch-one inspection from node 1_m, involves variable n_1 and leads to an active node. The value of a_{mn_2} is 2 because the branch-two computation constrains variable n_2 to be zero. The values of a_{mn_3}, ..., a_{mn_L} are all -1, because branch-one inspections are performed and, at this point in the branch-and-bound process, they are inactive nodes. If, for example, node 2_m is selected for branching at a later point in the algorithm, then a_{mn_3} would change from -1 to 1.

A similar process continues until the branch-and-bound process terminates. Suppose at the time of termination of the branch-and-bound process the active portion of the tree is extended to node k_m. Then $z(k_m)$ is a lower bound for any possible alternatives yet to be generated under node k_m. Also, $z(K_m)$ at the end of the limb is an upper bound for the least-cost solution among such alternatives since it represents the cost of one of the feasible alternatives. In other words, while the mth row of the matrix represents the mth alternative identified at the bottom of the tree through the mth inspection limb, it may also be considered to represent the mth subset of alternatives of which only one is explicitly specified. At this point the column elements of the row m in the matrix are:

$$\begin{array}{cccccccccc} n_1 & n_2 & n_3 & n_4 & \cdot & \cdot & n_\ell & n_{\ell+1} & \cdot\ \cdot & n_L \\ 1 & 2 & 1 & 1 & & & 1 & -1 & & -1 \end{array}$$

Note here that the lower bound on the cost of mth subset of alternatives is defined only by variables with an index value of 1 or 2. The branching variables associated with a path from the vertex of the tree to node 1_m, are not shown in Figure 4.1, but the column elements corresponding to such variables are assigned the incidence index value of 1 or 2. Suppose there are variables which are observed to assume the value zero in the solution to the subproblem associated with node 1_m. The column elements of row m corresponding to these variables are assigned the incidence index value of -2.

The schematic description of the incidence matrix is shown in Figure 4.2. As described previously, there are M rows for M alternatives and N columns for variables. Therefore, there are M•N matrix elements, each of which contains one of the four incidence index values. The lower and upper bounds associated with each row and the index values of each row provide the information required for the imputed value analysis. The incidence matrix is constructed simultaneously as the branch-and-bound process proceeds. As the process continues, the number of alternatives being generated increases, and the number of rows of the matrix also increases (the number of columns remains the same). However, as noted above, as alternatives are generated, some of the incidence index values may change. Another important characteristic of the matrix is the fact that all of the necessary information for an imputed value analysis is available and can be used repeatedly to calculate imputed values. Further, the incidence information is sufficient to reconstruct the original branch-and-bound tree, if it is desired to add branches from any of the inactive nodes.

In summary, an incidence matrix contains the following information:

(1) M feasible alternative plans generated on the branch-and-bound tree,

(2) lower and upper bounds on the cost of the least-cost alternative in each of M sets of possible alternatives, and

(3) the structure of the original branch-and-bound tree and the location of inactive nodes from which additional branches can be added.

B. Obtaining Imputed Values from the Matrix

The procedure for obtaining imputed values can be illustrated using a hypothetical incidence matrix which contains all of the

	Cost		Variables							
ALT	UB	LB	1	2	.	.	n	.	.	N
1	U_1	L_1	a_{11}	a_{12}	.	.	a_{1n}	.	.	a_{1N}
2	U_2	L_2	a_{21}	a_{22}			a_{2n}			a_{2N}
.	.	.	.	.			.			.
.	.	.	.	.			.			.
m	U_m	L_m	a_{m1}	a_{m2}	.	.	a_{mn}	.	.	a_{mN}
.	.		.	.			.			.
.	.		.	.			.			.
M	U_M	L_M	a_{M1}	a_{M2}	.	.	a_{Mn}	.	.	a_{MN}

ALT: Numerical index associated with each of M sets of potential alternatives.

UB : Upper bound on the cost of the least-cost alternative in each of M sets of potential alternatives.

LB : Lower bound on the cost of the least-cost alternative in each of M sets of potential alternatives.

a_{mn}: Incidence index of n th variable in the m th set of alternatives.

Figure 4.2 Imputed Value Incidence Matrix possible alternative plans (actually infinite in number) for a given regionalization problem. One can identify the least-cost alternative among those which include facility x (i.e., have an incidence index value of 1 under the column representing facility x), and the least-cost alternative among the remaining alternatives which exclude it (i.e., have an incidence index value of 2). The exact imputed value of facility x is obtained by subtracting the cost of the former from the cost of the latter. The imputed values of any facility or any combinations of facilities can be obtained in just the same way.

Although there are an infinite number of solutions for any given regionalization problem, the branch-and-bound process would provide, at its termination, M explicitly specified alternative plans. Each plan belongs to one of the M subsets of potential solutions. Further, the cost range of the least-cost solution in each subset of solutions is specified by lower and upper bounds. The upper bound is given by the cost of the alternative explicitly specified. The lower bound is given by the cost of the subproblem solution which is only partially constrained and has a lower cost than the least-cost solution in that subset.

Consider, for example, identifying the imputed value range of facility x, assuming that the facility cost is approximated with a single FP variable, n_x. The imputed value is defined as the difference between $C(\overline{n}_x)$, the cost of the least-cost alternative having variable n_x constrained to be zero, and $C(n_x)$, the cost of the least-cost alternative having variable n_x constrained to be greater than zero. The upper bound on $C(n_x)$ and $C(\overline{n}_x)$ can be readily obtained from the upper bounds of the M sets of alternatives. The lowest upper bound, $UB(n_x)$ is equivalent to the least-cost alternative among those which specifies the variable n_x to be positive (a_{mn_x} is 1 or -1). Similarly, the lowest upper bound, $UB(\overline{n}_x)$, on $C(\overline{n}_x)$ is equivalent to the least-cost alternative among those M alternatives which specify the variable n_x to be zero (a_{mn_x} is 2 or -2). Therefore,

$$UB(n_x) = \min[U_m \mid a_{mn_x} = 1 \text{ or } -1] \tag{4.1}$$

$$UB(\bar{n}_x) = \min[U_m \mid a_{mn_x} = 2 \text{ or } -2] \quad (4.2)$$

where U_m is the upper bound associated with the mth inspection limb and is shown in the mth row of the incidence matrix, as illustrated in Figure 4.2. Note here that one of the two sets of alternatives described above may be empty.

Finding a lower bound on $C(n_x)$ or $C(\bar{n}_x)$ is somewhat more involved. Consider finding the lower bound on $C(n_x)$ first. Among the M sets of possible alternative represented by the M rows, some have already been constrained to have facility X as a regional facility. The incidence index, a_{mn_x}, for such rows is 1. The lower bounds associated with such rows are candidates for the lower bound on $C(n_x)$. Note, however, that the lower bounds associated with rows whose incidence index under the n_x th column is either -1 or -2 are also candidates, since variable n_x may later be constrained to be positive and the incidence index value may be changed to 1. Therefore,

$$LB(n_x) = \min\ [L_m \mid a_{mn_x} = 1, -1 \text{ or } -2] \quad (4.3)$$

where L_m is the lower bound associated with the mth inspection limb. The lower bound on $C(\bar{n}_x)$ is identified in a similar fashion. The incidence index value of the candidate alternative sets must be either 2, -1 or -2. Therefore,

$$LB(\bar{n}_x) = \min\ [L_m \mid a_{mn_x} = 2, -1 \text{ or } -2] \quad (4.4)$$

Note, however, that the lower bound may be improved for $LB(n_x)$ when a_{mn_x} is -1. The incidence index -1 means that variable n_x is not constrained, or it is in the inactive portion of the inspection limb. Adding n_x to the active constraint set is equivalent to adding the fixed charge associated with n_x, $FC(n_x)$, to the current lower bound. Therefore,

$$LB(n_x) = \min_m \left\{[L_m \mid a_{mn_x} = 1, -2],\ [(L_m \mid a_{mn_x} = -1) + FC(n_x)]\right\} \quad (4.5)$$

The ranges for $C(n_x)$ and $C(\bar{n}_x)$ as well as for the imputed value associated with variable n_x, $IV(n_x)$, are given as follows:

$$LB(n_x) \leq C(n_x) \leq UB(n_x) \tag{4.6}$$

$$LB(\bar{n}_x) \leq C(\bar{n}_x) \leq UB(\bar{n}_x) \tag{4.7}$$

$$ILB(n_x) \leq IV(n_x) \leq IUB(n_x) \tag{4.8}$$

where

$$ILB(n_x) = LB(\bar{n}_x) - UB(n_x) \tag{4.9}$$

$$IUB(n_x) = UB(\bar{n}_x) - LB(n_x) \tag{4.10}$$

Note that the lower bound on the imputed value, $ILB(n_x)$, and the upper bound on the imputed value, $IUB(N_x)$, depend on the relative values of the lower and upper bonds on $C(n_x)$ and $C(\bar{n}_x)$, and can take negative values. A negative imputed value indicates that the least-cost solution without facility x (n_x assumes the value zero) has a lower cost than the least-cost solution with facility x (n_x assumes a positive value). Note that for the analysis of a single-facility imputed value either $C(n_x)$ or $C(\bar{n}_x)$ is equal to z*, the overall least-cost solution, if the branch-and-bound tree is extended to provide z* on one of its nodes.

The impact of including or excluding a set of facilities rather than a single facility is just as important, since many planning issues are related to a group of facilities. As examples, the water quality of a particular stream may be a critical issue related to the location of plants at any of the potential sites along its length, the water reuse policy of a region may be evaluated by placing a group of treatment plants in specific strategic locations, and jurisdictional boundaries which encompass several plants and interceptors may be studied for their political implications.

Depending on the analysis to be performed, the imputed value of a set of facilities may be defined in many different ways. For example, the imputed value associated with a pair of variables, n_x and n_y, may be defined by any of the following:

(1) $$IV(n_x, n_y/\bar{n}_x, \bar{n}_y) = C(\bar{n}_x, \bar{n}_y) - C(n_x, n_y) \tag{4.11}$$

(2) $$IV(n_x, n_y/\bar{n}_x, n_y) = C(\bar{n}_x, n_y) - C(n_x, n_y) \tag{4.12}$$

(3) $$IV(n_x, n_y/n_x, \bar{n}_y) = C(n_x, \bar{n}_y) - C(n_x, n_y) \tag{4.13}$$

(4) $$IV(n_x, \bar{n}_y/\bar{n}_x, \bar{n}_y) = C(\bar{n}_x, \bar{n}_y) - C(n_x, \bar{n}_y) \tag{4.14}$$

$$(5) \qquad IV(n_x, \bar{n}_y/\bar{n}_x, n_y) = C(\bar{n}_x, n_y) - C(n_x, \bar{n}_y) \qquad (4.15)$$

$$(6) \qquad IV(\bar{n}_x, n_y/\bar{n}_x, \bar{n}_y) = C(\bar{n}_x, \bar{n}_y) - C(\bar{n}_x, n_y) \qquad (4.16)$$

where, for example, $C(\bar{n}_x, \bar{n}_y)$ indicates the cost of the least-cost solution with both n_x and n_y constrained to be zero, $C(n_x, n_y)$ indicates the cost of the least-cost solution with both n_x and n_y constrained to be greater than zero, and $IV(n_x, n_y/\bar{n}_x, n_y)$ is the notation used to denote the imputed value as defined by the difference between the two. Note also that

$$IV(N_1/N_2) = -IV(N_2/N_1) \qquad (4.17)$$

$$IV(N_1/N_1) = 0 \qquad (4.18)$$

where N_1 and N_2 denotes a given set of indexed variables. These cases are omitted from the above list.

The same basic principles developed for one variable apply to the analysis of imputed values involving sets of variables. An application example of such an analysis is given in the following section.

4.3 Illustrative Examples

Two illustrative examples of an imputed value analysis are described here. The first example is based on the incidence matrix associated with Case E of the single-period example problem shown in Table 2.2. The second example is based on the incidence matrix associated with Case C of the multiperiod analysis given in Section 3.6. The imputed value analysis procedure and computational results are presented. All of the analyses were carried out by hand. A more detailed discussion on practical applications is presented in Chapter 5.

A. Imputed Value Analysis for a Single-Period Example Problem

Case E of the single-period example problem involves seven FPI variables; two are for interceptors, and five are for plants. The structure of the imputed value incidence matrix associated with the branch-and-bound tree is described, and a procedure for using this information is outlined.

A portion of the branch-and-bound tree for this example is shown in Figure 4.3. Three alternative plans are identified at the bottom of the inspection limbs. Also one infeasible solution is identified. The alternatives identified are designated as Alternative 1, Alternative 2 and Alternative 4 according to the order of generation. Alternative 3 is not shown in the figure. The incidence matrix corresponding to the entire branch-and-bound tree is shown in Figure 4.4. The rows 1, 2 and 4, of course, correspond to the paths from the vertex to the nodes 6, 11 and 20, respectively.

The structure of the tree is represented by the incidence matrix (see Figure 4.4). For example, consider the inspection limb associated with Alternative 1. The variables 1, 2, 4 and 6 are constrained to be positive and they are in the active portion of the tree. Thus, the incidence index values associated with columns, 1, 2, 4 and 6 are all 1 in row 1. Variable 7 is not constrained to be in the active portion of the tree. Thus the index value is -1. Although variables 3 and 5 are not explicitly constrained to be zero, the index values for them are 2, because they are implicitly constrained by constraints that prevent split flows. Similar relationships exist between each row of the matrix and each path on the tree.

Assuming that the FPI approximations used in this example case closely represent the actual cost of the facilities, the matrix provides abundant information on the imputed values of various facilities and various combinations of facilities. Some example results of an imputed value analysis are provided in Table 4.1. The table shows 9 cases; seven are for a single facility, and two are for a pair of facilities.

The computational procedure of the imputed value analysis, described in Section 4.3, is illustrated for Case 5 in which the imputed value of the fifth variable, which corresponds to the interceptor 7-5, is to be obtained. First, an upper bound on the cost of the alternatives which are constrained to include this interceptor is:

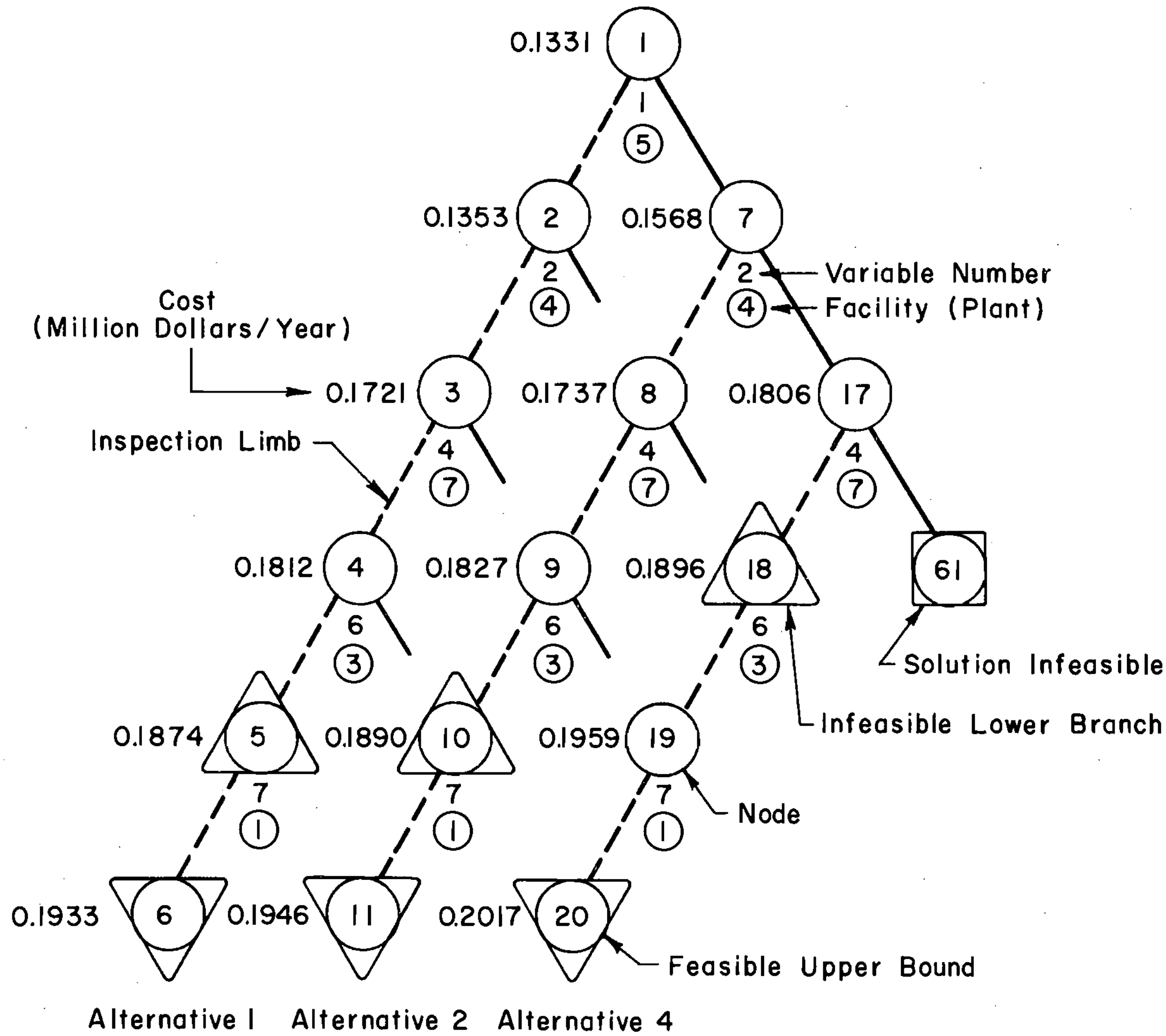

Figure 4.3 A Portion of the Branch-and-Bound Tree for Case E of the Single-Period Example Problem

ALT	COST UB	COST LB	Variables 1	2	3	4	5	6	7
1	193.3	187.4	1	1	2	1	2	1	-1
2	194.8	189.0	2	1	2	1	2	1	-1
3	191.0	185.1	1	2	1	1	2	1	-1
4	201.7	189.6	2	2	-2	1	2	-1	-1
5	188.9	188.9	1	2	2	1	2	1	1
6	198.9	186.8	1	2	2	2	1	-1	-1
7	193.5	187.6	1	2	1	2	1	1	-1
8	195.7	189.9	1	1	2	2	1	1	-1
9	189.2	189.2	1	2	1	2	2	1	1
10	191.4	185.6	2	1	2	2	-2	1	-1
11	189.9	189.9	1	1	2	2	2	1	1
12	185.5	185.5	1	2	2	1	2	2	1
13	185.7	185.7	1	2	1	2	2	2	1
14	184.6	184.6	1	1	2	2	2	2	1
15	188.0	188.0	1	1	2	2	2	2	2
16	187.5	187.5	1	2	1	1	2	2	1
17	186.2	186.2	2	1	2	2	-2	2	1
18	190.8	190.8	1	2	2	1	2	2	2
19	191.3	191.3	1	2	1	2	2	2	2
20	189.5	189.5	2	1	2	2	-2	2	2
21	188.0	188.0	1	1	2	1	2	2	1
22	190.0	190.0	1	2	1	2	1	2	1
23	192.7	192.7	1	2	1	1	2	2	2
24	191.3	191.3	1	1	2	1	2	2	2
25	189.6	189.6	2	1	2	1	2	2	1
26	191.2	191.2	1	2	2	1	2	1	2
27	191.8	191.8	1	2	1	2	2	1	2
28	190.5	184.6	1	1	2	2	1	2	1
29	192.9	192.9	2	1	2	1	2	2	2
30	192.2	192.2	1	1	2	2	2	1	2
31	195.2	195.2	1	2	1	2	1	2	2
		Facility	5	4	4-5	7	7-5	3	1

Figure 4.4 Imputed Value Incidence Matrix for Case E of the Single-Period Example Problem

Table 4.1 Imputed Value Analysis for Case E of the Single-Period Example Problem

<table>
<tr><th rowspan="2">Case</th><th rowspan="2">Variable</th><th rowspan="2">Facility</th><th colspan="2">Imputed Value*</th></tr>
<tr><th>LB</th><th>UB</th></tr>
<tr><td>1</td><td>1</td><td>5</td><td>1.0</td><td>1.6</td></tr>
<tr><td>2</td><td>2</td><td>4</td><td>1.5</td><td>0.9</td></tr>
<tr><td>3</td><td>3</td><td>4-5</td><td colspan="2">-1.1</td></tr>
<tr><td>4</td><td>4</td><td>7</td><td colspan="2">-0.9</td></tr>
<tr><td>5</td><td>5</td><td>7-5</td><td>-5.4</td><td>0.0</td></tr>
<tr><td>6</td><td>6</td><td>3</td><td>-4.3</td><td>-0.5</td></tr>
<tr><td>7</td><td>7</td><td>1</td><td colspan="2">3.3</td></tr>
<tr><td>8</td><td>1,2</td><td>4,5</td><td>5.0</td><td>7.1</td></tr>
<tr><td>9</td><td>3,5</td><td>4-5,7-5</td><td>-5.4</td><td>-3.0</td></tr>
</table>

* Thousand dollars/year.

LB: Lower Bound.

UB: Upper Bound.

$$UB(5) = \min_m \left[U_m \mid a_{m5} = 1, -1 \right] = 190.0 \qquad (4.19)$$

Second, an upper bound on the cost of alternatives which are constrained to have variable 5 equal to zero is:

$$UB(\overline{5}) = \min_m \left[U_m \mid a_{m5} = 2, -2 \right] = 184.6 \qquad (4.20)$$

Third, a lower bound on the cost of all potential alternatives which may include the interceptor is:

$$LB(5) = \min_m \left\{ \left[L_m \mid a_{m5} = 1, -2 \right], \left[(L_m \mid a_{m5} = -1) + FC(nx) \right] \right\}$$
$$= 184.6 \qquad (4.21)$$

Fourth, a lower bound on the cost of all potential alternatives which may not include the interceptor is:

$$LB(5) = \min_m L_m \{a_{m5} = 2, -1, -2\} = 184.6 \qquad (4.22)$$

Therefore,

$$184.6 \leq C(5) \leq 190.0$$

$$184.6 \leq C(\overline{5}) \leq 184.6$$

and

$$-5.4 \leq IV(5) \leq 0 \ .$$

In other words, it costs somewhere between zero and \$5,400/year more to have the interceptor in the regional plan than it does to exclude that interceptor. If it is the only facility that requires consideration of issues other than cost, then the maximum extra cost of \$5,400/year gives one quantitative measure of its economic trade-off. The cost may be assessed against the implicit values of the other issues associated with the interceptor.

The range of imputed value may be tightened by extending the branches from the tree. For example, four inspections and four subproblem computations are required to tighten the above range to the actual imputed value, \$5,400/year. In many cases, however, lower and upper bounds provide information that is as useful as the exact imputed value.

B. Imputed Value Analysis for a Multiperiod Example Problem

Case C of the multiperiod problem involves nine FPI variables, each of which represents a facility. The alternatives were generated using the coupled branch-and-bound method. The computational results have already been presented in Section 3.6. Recall that the coupled branch-and-bound tree was grown stagewise in three different runs. The imputed value ranges for five individual facilities and six sets of facilities have been analyzed from the incidence matrix associated with each of the three runs. The three matrices are shown in Figure A-1, A-2 and A-3 in Appendix A. A summary of the imputed value analysis for eleven different cases is shown in Table 4.2. The number of facilities directly involved in the analysis is one in Cases 1 through 7, two in Cases 8 through 10, and three in Case 11. In cases 3 and 10 the imputed value analysis was performed while keeping the variable 1 positive. In other words, those cases give the conditional imputed values. The same is true with Case 6, but in this case both variables 1 and 3 are kept positive.

The following observations can be made based on the information in the table:

(1) The most noticeable trend is that the ranges of imputed values become tighter as the tree is grown further to generate more alternatives. The total number of subproblems computed increased from 14 in the first run, when the approximate least-cost solution z* was identified as $516,100/year, to 42 and then to 54 as the cutoff value z** was increased to $550,000 and to $570,000/year, respectively. Note that as the cutoff value is increased, the matrix becomes larger, and some of

Table 4.2 Imputed Value Analysis for Case C of the Multiperiod Example Problem

Case	Variable*	Imputed Value Range ($ x 10^3/year) Run 1 LB	Run 1 UB	Run 2 LB	Run 2 UB	Run 3 LB	Run 3 UB
1	1	-15.0	-1.2	-15.0		-15.0	
2	3	-10.6	-2.1	-10.6		-10.6	
3+	(1),3	-72.6	-4.5	-35.8	-27.2	-35.8	
4	5	-40.4	0.0	-40.4	0.0	-40.4	0.0
5	7	-15.0	0.0	-15.0	0.0	-15.0	0.0
6+	(1),(3),7	-54.2	87.5	-13.7	3.4	5.1	
7	9	1.1	10.6	10.6		10.6	
8	1,3	-73.8	-19.5	-50.7	-42.1	-50.7	
9	3,7	-73.8	-2.1	-55.8	-10.6	-55.8	
10+	(1),3,7	-72.6	NI	-23.2	-14.7	-23.2	
11	1,3,7	-73.8	-33.1	-55.8	-47.3	-55.8	

* Variables 1, 3, 5, 7 and 9 correspond to plants 5, 7 and 3 and interceptors 4-5 and 3-4, respectively.

\+ The facilities represented by the variables in parenthesis are constrained to exist in all alternative plans involved in the analysis.

NI: Not identified(the tree is not grown sufficiently yet).

LB: Lower bound.

UB: Upper bound.

the negative incidence index values change to positive values in the subsequent run(s).

(2) The upper and lower bounds converged as the tree is grown larger. The upper and lower bounds converged in nine of the eleven cases in Run 3, while only three cases converged in Run 2.

(3) The imputed values become noticeably larger in general as the number of variables in the set being examined increases. However, since an imputed value represents the cost difference between two sets of alternatives, it is not always true. (The least-cost solution with both plants A and B constrained to exist would tend to have a greater cost than the least-cost solution with only plant A constrained to exist. Similarly, the least-cost solution with both plants A and B constrained out of the plan would tend to have a greater cost than the least-cost solution with only plant A constrained out. But the cost difference between the least-cost solutions containing or excluding two plants, A and B, may or may not be greater than the cost difference between the least-cost solutions containing or excluding a single plant, A.) For example, the absolute imputed values in Case 3 was greater than the corresponding absolute imputed value in Case 2, while the absolute imputed values in Case 10 was less than the corresponding absolute imputed values in Case 9. In other words, the relative significance of the existence of a facility or a combination of facilities depends on the existence of other facilities.

(4) Generally, the lower and upper bounds on an imputed value provide relatively strong indications of the magnitude of the actual imputed value. For example, all three converged imputed values in Run 2 correspond

> to the lower or upper bounds identified for the corresponding cases in Run 1. Similarly, eight of the nine converged imputed values in Run 3 correspond to the upper or lower bounds of the corresponding cases in Run 2.

The significance of the imputed value analysis for planning will be discussed using a large scale example problem in the following chapter.

5. EXAMPLE APPLICATIONS

5.1 Problem Description and Input Data

In this chapter a relatively large scale example case is studied, using the method developed in previous chapters for generating and comparing alternative regionalization plans. The area chosen for study is DuPage County, Illinois.

DuPage County is a growing area where regionalization of wastewater systems is considered beneficial. For example, the current proliferation of small and inefficient treatment facilities has been considered to be detrimental to the environment, and those plants may be replaced by larger and better-operated regional systems which can improve the quality of the environment at lower cost. Further, the topography of the region is such that wastewater can be transported through interceptors without great difficulty, and that there are potentially favorable locations for constructing regional treatment plants. In fact, extensive study of alternative regionalization plans has been performed by the Northern Illinois Planning Commission (1969) and by the Illinois Pollution Control Board (1974).

Although this case study makes use of the available information on DuPage County, the example problems are constructed using many simplifying assumptions. It is, therefore, not the intent of the study to examine or evaluate the proposed regionalization policy of the county, nor are the example problems in any way designed to influence the current and future regionalization efforts.

The primary objective of this study is to demonstrate that the branch-and-bound method described in previous chapters is applicable to problems of realistic size and is potentially useful for examining widely different plans. To fulfill the objective five example problems are constructed. The first two problems are designed to allow only large regional facilities, and they are called the large-scale problems. The third and fourth problems are designed so that only small and medium size regional facilities are allowed, and they are called the small-scale problems. Further, the first and third problems involve multiperiod (dynamic) costs. The fifth problem is the same as the first one except that the cost functions are modified to examine the effects on the branch-and-bound process and on the imputed values. These five problems are identified as the Static Large-Scale Problem (S-LSP), the Dynamic Large-Scale Problem (D-LSP), the Static Small Scale Problem (S-SSP), the Dynamic Small-Scale Problem (D-SSP) and the Modified Static Large-Scale Problem (MS-LSP).

Each of the static problems is solved for the approximate least-cost solution as described in Section 2.3-C and each of the dynamic problems is solved for the approximate least-cost solution as described in Section 3.4-C. Split flows are allowed only in the small-scale problems. The alternatives generated for each problem are examined, and a trade-off analysis is performed on each problem based on some hypothetical scenarios.

A. General Description of DuPage County

DuPage County is located immediately west of, and at its closest point only fifteen miles from downtown Chicago. It is nearly rectangular in shape as shown in Figure 5.1 and it covers an area of 331 square miles. Prior to World War II it was primarily agricultural farmland with a few rail commuter suburbs and farm trade centers. The postwar growth in the metropolitan economy has brought population and development pressures to the county, and today the eastern half is already densely inhabited and the western half is expected to sustain the growth in the next few decades.

The population of this county has been growing steadily at a high rate, and it is estimated to be 572,000 in 1975 and to grow to 1,200,000 by 2005. There is only a limited amount

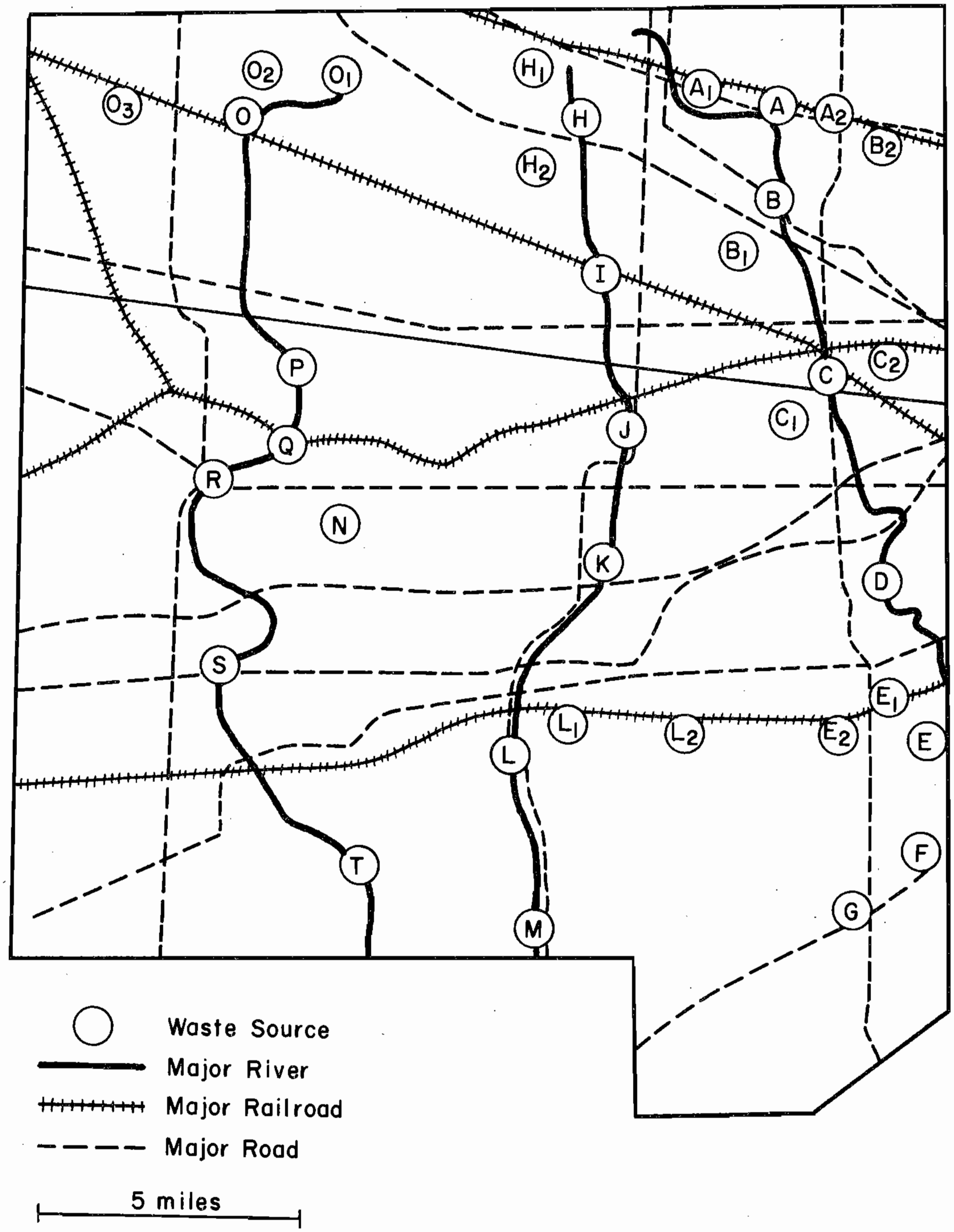

Figure 5.1 Waste Sources in DuPage County, Illinois

of industry in the county; industrial waste does not currently comprise a significant portion of the total wastewaters generated within the county, nor is it expected to do so in the future.

Topographically the county is highlighted by extensive glacial deposits which produce the rolling characteristic of the countryside. The general slope of the terrain is from the north to the south with a maximum difference in elevation of approximately 250 feet. The county is drained by three major streams nearly parallel to each other several miles apart.

B. Basic Assumptions

The same set of cost functions used for the illustrative examples (see Section 2.5-A) is used in this study. Similarly, the discount rate and design lives of treatment plants and interceptors are assumed, as before, to be 0.05, 25 years and 50 years, respectively. When the method is applied in an actual planning problem, however, the cost functions and design parameters should be evaluated for the particular conditions of the region under examination.

The estimated wasteflows generated by each of the 20 major aggregate waste sources in the next 30 years are based on population estimates for each of the 9 subregions identified by the Illinois Pollution Control Board (1974). The per capita wastewater production is assumed to be 100 gallons/day. The interim construction years for the multitime period problem are assumed to be the 10th (1985) and 20th (1995) years in addition to the initial year, 1975. No consideration is given to the existing facilities.

The FPI approximations are used for both static and dynamic problems. The modified coupled branch-and-bound method is used for solving the dynamic problems.

C. Original Regional Network

The schematic description of the original regional network consisting of wastewater sources and possible interceptor routes is shown in Figure 5.2. There are 20 wastewater sources and 22 interconnecting links for interceptors. The amount of wastewater generated at each source node and the distance between each pair of interconnected nodes are shown in Tables 5.1 and 5.2. The linear approximation of growth of waste flows for the dynamic problems is based on projections for the years 1985 and 2005, since the first design target year is 1985. The waste flow at the initial reference year, $L_j(0)$, is that of 1975 for every source and is given by extrapolation. The original network will be modified in each of the following problems; some of the wastewater sources are aggregated and represented as a single source and some of the interceptor routes are deleted. The maximum capacity for each interceptor and plant is determined for each new network.

D. Large-Scale Problems

The selection of candidate plant sites and interceptor routes for the large-scale problems is based on the consideration that each of the plants can accommodate flows from at least several wastewater sources. As shown in Figure 5.3, six potential large-scale plant sites are established in this problem. Three of the six sites are located about midway through each of the interceptor routes running vertically from the north to the south, and the remaining three are located at the extreme south end of these interceptor routes. There are nine wastewater sources of which six coincide with candidate plant sites. There are altogether eleven interconnecting links for interceptors. Three of the eleven interceptor links allow flow in either direction. Some of the nodes which appear in the original network shown in Figure 5.2 are collapsed into other nodes; Table 5.3 summarizes the relationship between the two networks. The S-LSP cost data for plants and for interceptors are given respectively in Tables B-1 and B-2 in Appendix B. The minimum capacity of a plant is given by the waste flow generated at that site, and the maximum capacity is given by the maximum possible flow which can be assigned to the plant. The same principle

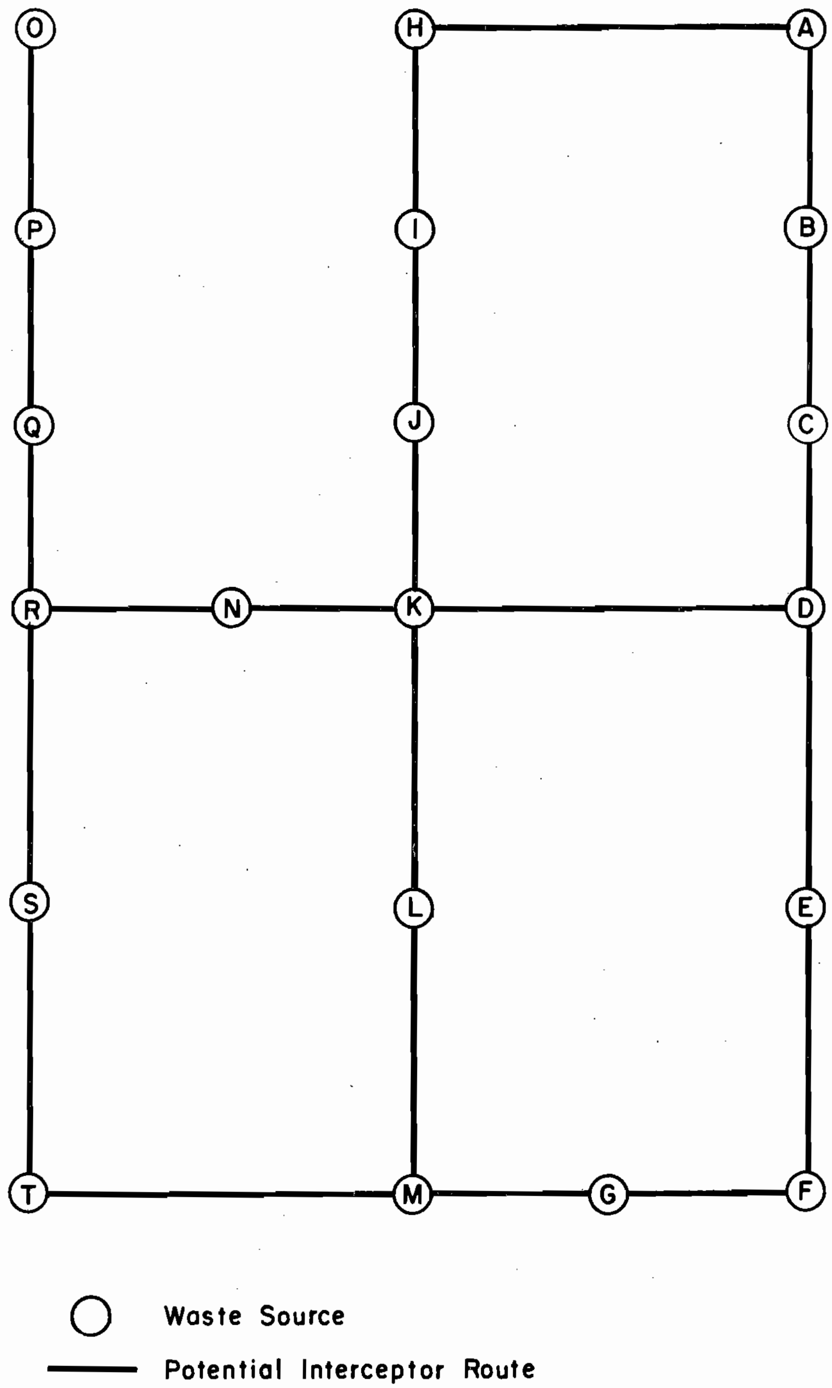

Figure 5.2 Original Network

Table 5.1 Projection of Wastewater Production

Source Index	Community Index	Community Name	1970	1975	1985	2005
A	A1	Itasca	1.4	5.4	5.6	5.8
	A2	Wooddale				
B	B1	Addison	4.8	9.3	9.3	12.0
	B2	Bensenville				
C	C	Villa Park	7.4	8.5	8.5	8.8
D	D	Oakbrook	0.6	1.2	1.2	1.2
E	E1	Hinsdale	3.3	6.9	7.1	7.5
	E2	Clarendon Hills				
F	F	(Dummy Source)	0.0	0.0	0.0	0.0
G	G	Darien	0.8	0.8	1.6	3.0
H	H1	Roselle	0.8	0.8	1.0	1.4
	H2	Bloomingdale				
I	I	Glendale	1.1	1.3	1.6	2.2
J	J	Lombard	3.7	4.4	5.3	7.0
K	K	Glen Ellyn	2.2	2.6	3.1	4.1
L	L1	Lisle	4.9	8.8	10.4	13.5
	L2	Downers Grove				
M	M	Woodridge	1.5	1.8	3.2	5.9
N	N	Wheaton	2.9	4.8	6.5	10.0
O	O1	Hanover Park	0.5	1.2	1.8	3.1
	O2	Bartlett				
	O3	Wayne				
P	P	Carol Stream	0.6	1.4	2.2	3.7
Q	Q	Winfield	0.4	0.9	1.5	2.6
R	R	West Chicago	1.0	2.3	3.6	6.1
S	S	Warrenville	0.3	0.4	0.7	1.0
T	T	Naperville	2.5	4.0	5.3	8.0

* Million gallons/day (MGD).

\+ Wastewater flows of more than two communities are aggregated.

Table 5.2 Distance between Waste Sources

From	To	Distance (Miles)
A	B	4.6
A	H	3.6
B	C	3.1
C	D	5.2
D	E	4.4
D	K	6.0
E	F	2.1
F	G	3.0
G	M	6.1
H	I	3.5
I	J	6.1
J	K	3.0
K	L	4.2
K	N	5.7
L	M	3.3
M	T	6.5
N	R	3.2
O	P	4.7
P	Q	1.8
Q	R	1.8
R	S	4.7
S	T	5.7

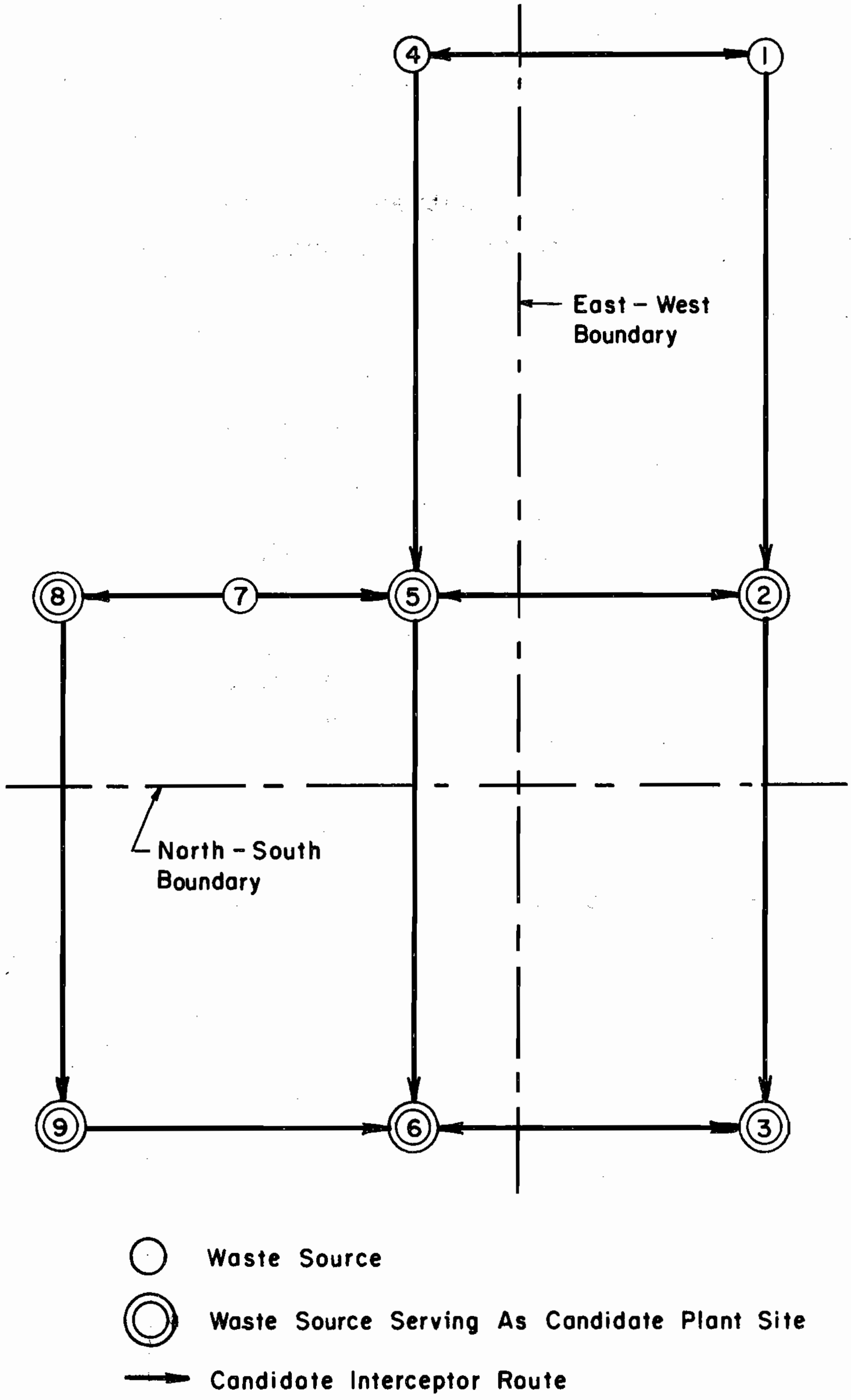

Figure 5.3 Large-Scale Problem Network

Table 5.3 Relationships between the Original Network and the Large-Scale Network

Node in LS-N*	Node in O-N+
1	A
2	B, C, D
3	E, F, G
4	H
5	I, J, K
6	L, M
7	N
8	O, P, Q, R
9	S, T

* Large-scale network.

+ Original network.

applies to interceptor capacities, except they are based on the flows at the origin of the interceptor links. The MS-LSP cost data are the same as the ones for the S-LSP except that: the costs of the plant at site 2 and of the interceptors from node 1 to node 2, node 2 to node 5, and node 3 to node 6 are assumed to be 20 percent more than the corresponding costs for the S-LSP; and the costs of the plants at nodes 6 and 8 are assumed to be 20 percent less. These assumptions are arbitrary, but the intent here is to examine how the size of the branch-and-bound tree and the costs of alternatives generated differ from the original S-LSP. The D-LSP cost data for plants are given in Table B-3 in Appendix B. The interceptor cost data for the D-LSP are the same as those for the S-LSP.

E. Small-Scale Problems

Flow directions and the capacity limits on plants and interceptors in the small-scale problems are specified *a priori* so that a plant would receive waste flows from at most three or four sources. Since these problems are highly constrained, many mathematically infeasible solutions emerge on the branch-and-bound tree. Also split flows are expected to occur in many of the alternatives generated. If split flows are allowed (as assumed here), many feasible alternatives can be generated. On the other hand, if they are not, the number of feasible alternatives would be quite limited.

The network associated with the small-scale problems consists of fifteen wastewater sources and fifteen interconnecting interceptor links (see Figure 5.4). Of the fifteen sources, eleven are also candidate plant sites. There is only one link, 8-9, which allows flow in either direction. Note that the links between nodes D and K and between nodes R and S in the original network shown in Figure 5.2 are omitted. The relationship between the original network and the network for the small-scale problems is given in Table 5.4. The S-SSP cost data are given in Tables B-4 and B-5 in Appendix B for

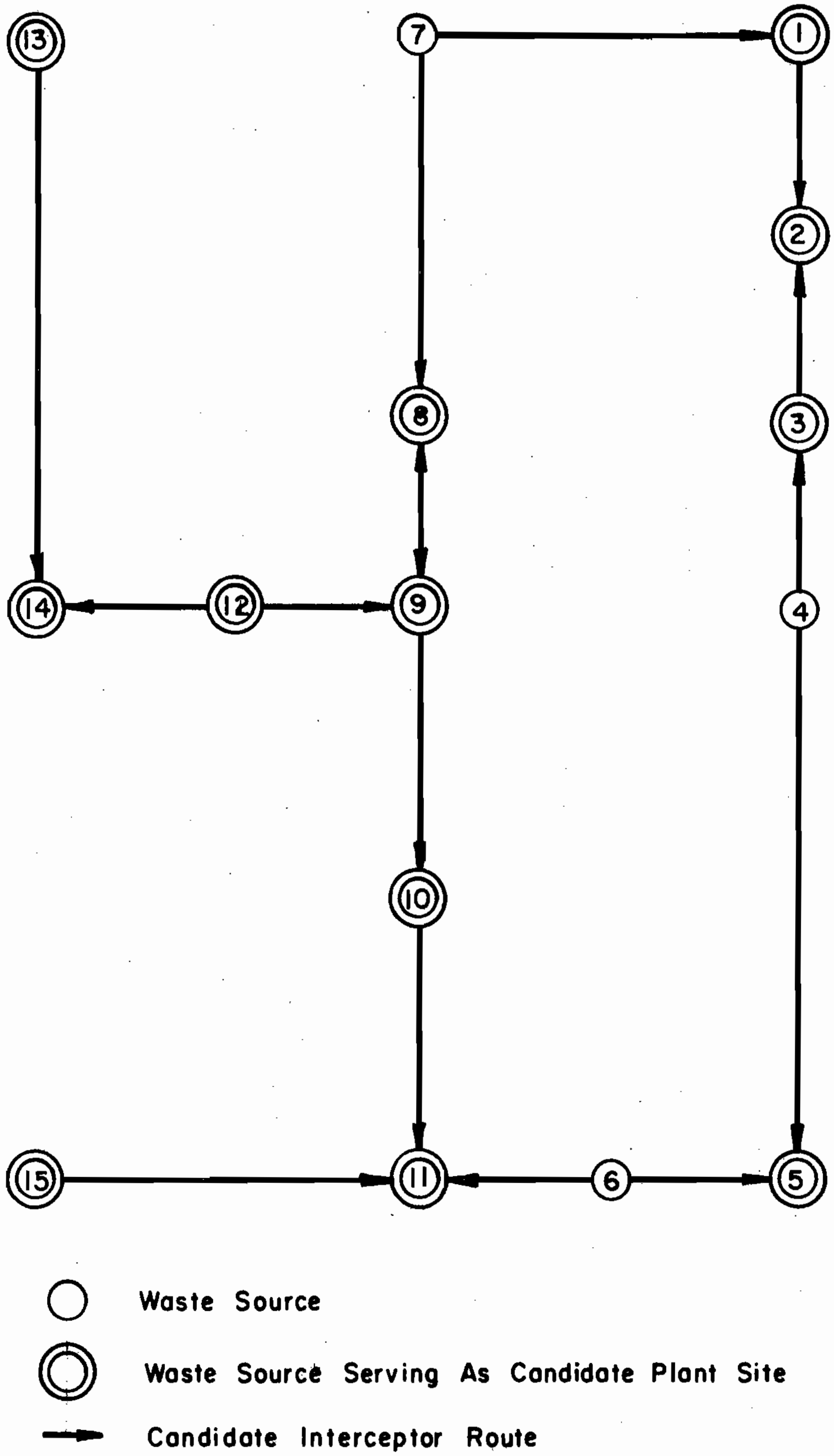

Figure 5.4 Small-Scale Problem Network

Table 5.4 Relationships between the Original Network and the Small-Scale Network

Node in SS-N*	Node in O-N+
1	A
2	B
3	C
4	D
5	E, F
6	G
7	H
8	I, J
9	K
10	L
11	M
12	N
13	O
14	P, Q, R
15	S, T

* Small-scale network.
+ Original network.

plants and interceptors, respectively, and the D-SSP data for plants are given in Table B-6 in Appendix B. The minimum and maximum capacities are determined as in the case of the large-scale problems. The interceptor cost data for the D-SSP are the same as those for the S-SSP.

The results of the branch-and-bound analysis for each of the five problems are presented in the next two sections.

5.2 Frequency Distribution of Alternative Plans

The statistics on the computational requirements for obtaining the least-cost solution are presented in Table 5.5 for each of the five problems. Only a moderate computation was required in each case, although the size of the tree and the number of alternatives generated varied from one problem to the other. Several observations about the information given in the table are presented below.

The total number of nodes and the number of alternatives generated on the branch-and-bound tree for the dynamic problems, D-LSP and D-SSP, was found to be about one-half of those for the corresponding static problems, S-LSP and S-SSP. On the other hand, the number of subproblem computations required for the former problems was about the same as for the latter problems. In other words, for the dynamic problems, it takes about twice as many computations to generate the same number of alternatives, but it requires half as many node evaluations to reach the least-cost solution. The small size trees for the dynamic problems are attributed to high fixed charges associated with the branch-and-bound variables. Twice as many computations were required because one is needed for the branch-and-bound tree, and another is needed for the constraint tree.

The number of subproblems computed, the number of alternatives generated, and the number of nodes on the branch-and-bound tree for the MS-LSP were found to be two-thirds of those for the S-LSP. The modification of costs resulted in a reduction of computational requirements for finding the least-

Table 5.5 Computational Results for Five Example Problems

	S-LSP	D-LSP	S-SSP	D-SSP	MS-LSP
No. of FP Variables	20	20	28	28	20
Total No. of Nodes	362	169	659	347	234
No. of Active Nodes	187	87	301	153	121
No. of Active Inspections	93	43	150	76	60
No. of Subproblems Computed	94	88*	151	154*	61
No. of Feasible Alternatives	52	25	65	28	36
Least Cost ($ million/year)	2.256	4.274	2.473	5.723	1.988
Maximum Cost+	107.5	109.8	104.7	103.4	114.5
CPU Time (seconds)	24.34	17.51	77.42	51.46	18.59

* Includes the coupled subproblems.

\+ Identified on the branch-and-bound tree grown to find the least-cost solution, and expressed as a percentage of the corresponding least cost.

cost solution, since the bounding of the nodes on the branch-and-bound tree becomes more efficient for asymmetric problems.

The range of feasible costs among the generated alternatives was greater for the large-scale problems than for the small-scale problems. This is due primarily to the fact that the incremental costs from one alternative to another is much smaller for the small-scale problems, since many of the potentially high cost alternatives become mathematically infeasible. Also, the cost range associated with the MS-LSP turned out to be much greater than the one associated with the S-LSP, because the former problem has a less symmetric cost structure. Of course, the cost ranges may widen as the branch-and-bound trees are grown further to generate more alternatives.

The frequency distribution of alternative plans for the large-scale and the small-scale problems with respect to the number of plants are shown in Figures 5.5 and 5.6, respectively. The difference in the pattern of frequency distribution between the static and dynamic problems are evident. While many of the alternatives generated in the S-LSP and MS-LSP have three or four plants, no alternative in the D-LSP has more than two plants. Similarly, while some of the S-SSP alternatives have seven plants, none of the D-SSP do.

Figures 5.7 and 5.8 show the frequency distribution of alternatives with respect to each of the candidate plants for the large-scale and small-scale problems, respectively. In Figure 5.7 one can note the trend expected from the design of the MS-LSP in comparison to the S-LSP. Since the plant located at site 6 (or plant 6) is 20 percent less costly, it appears more frequently than in the S-LSP. The same is true with plant 8. Similarly, plant 2 which is 20 percent more costly appears less frequently. The noticeably less-frequent occurrence of plants 3 and 9 resulted from waste shipments to plant 6. The D-LSP follows basically the similar pattern as the S-LSP, except that plant 8 occurs less frequently. Also the D-LSP shows a greater tendency toward centralization at plants 3 and 6.

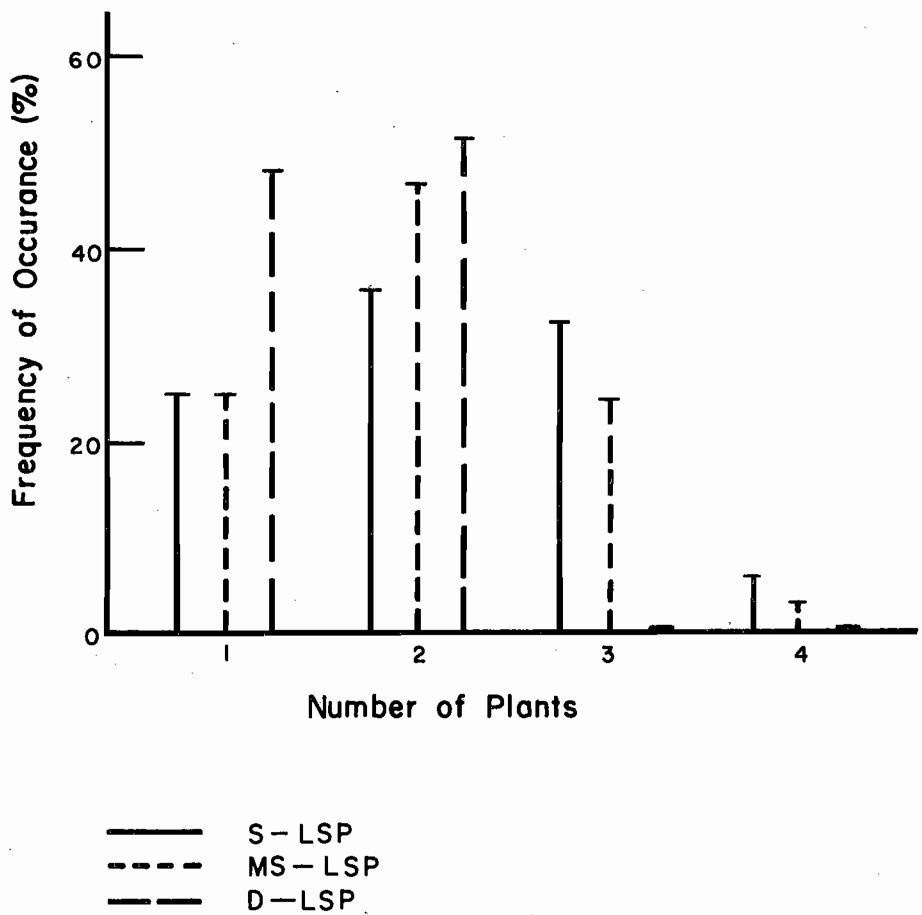

Figure 5.5 Frequency Distribution of Alternative Plans with Respect to Number of Plants for Large-Scale Problems

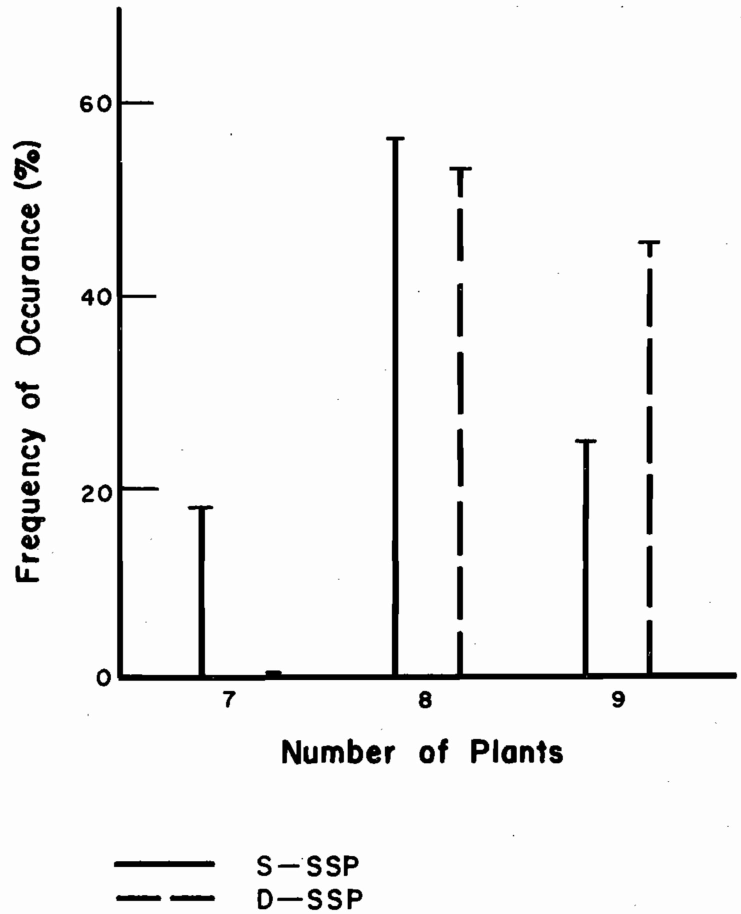

Figure 5.6 Frequency Distribution of Alternative Plans with Respect to Number of Plants for Small-Scale Problems

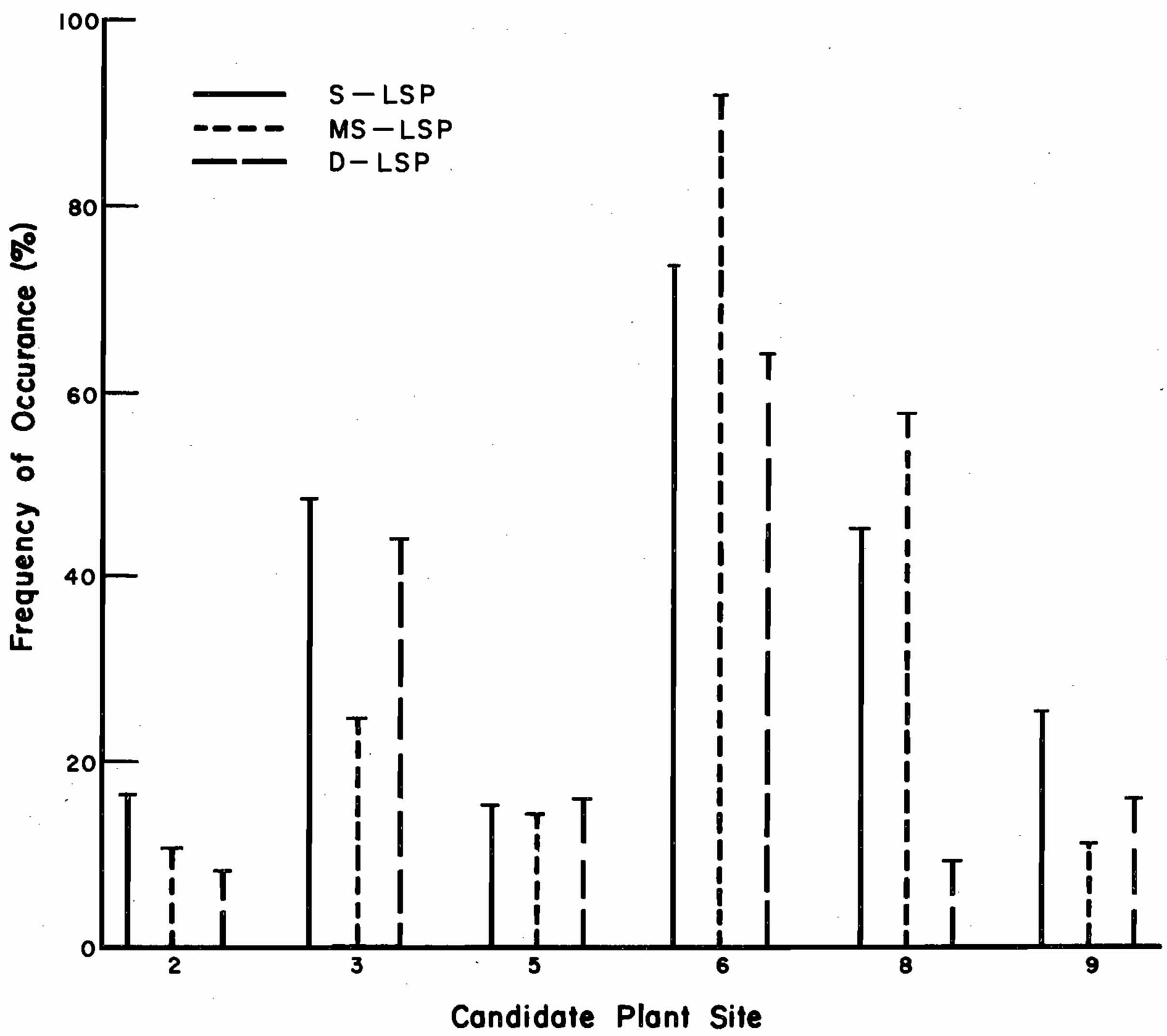

Figure 5.7 Frequency Distribution of Alternative Plans with Respect to Plant Sites for Large-Scale Problems

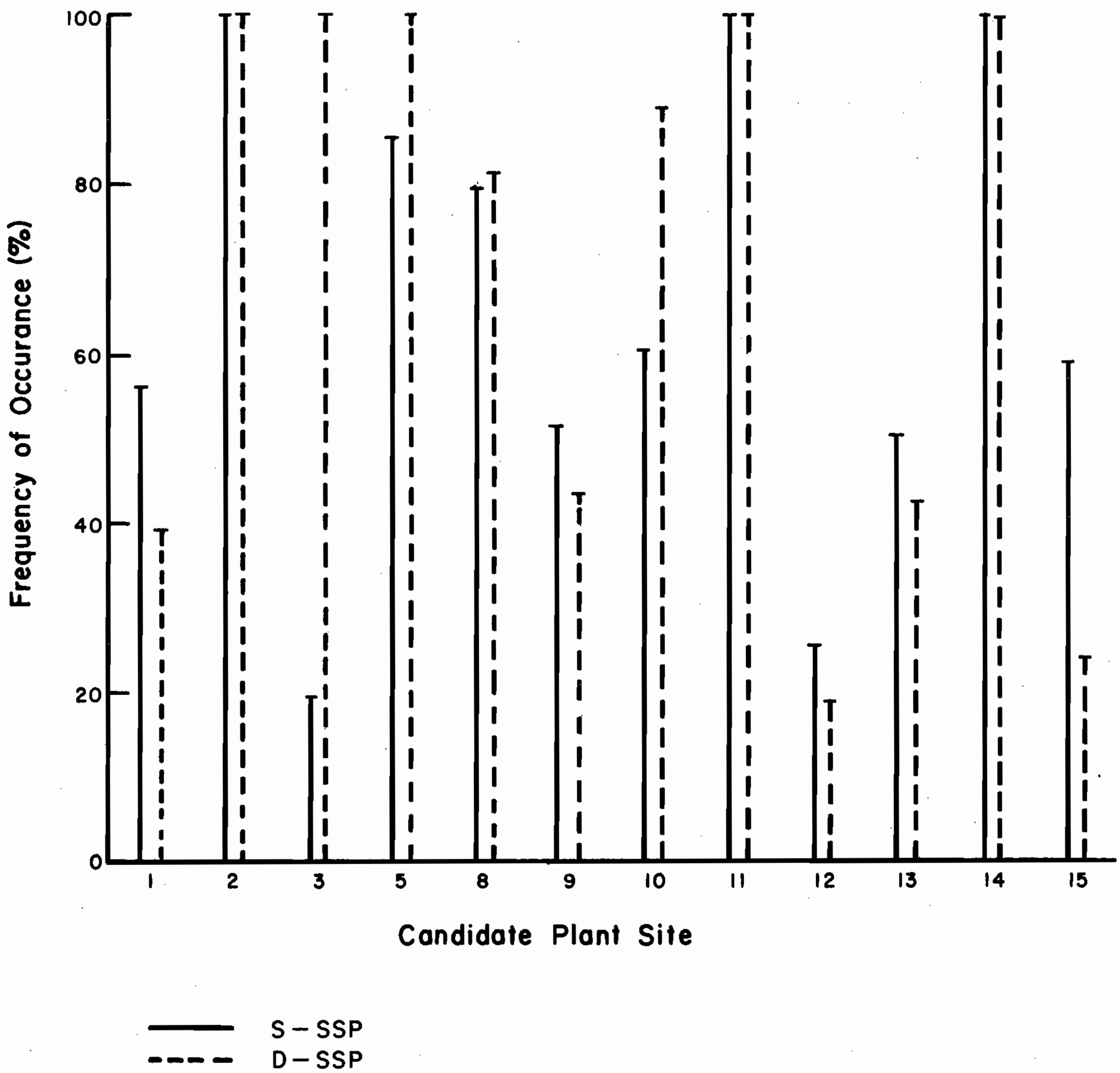

Figure 5.8 Frequency Distribution of Alternative Plans with Respect to Plant Sites for Small-Scale Problems

In the small-scale problems, shown in Figure 5.8, the most noticeable tendency is that many of the plants appear in all of the alternatives generated. This tendency is greatest in the D-SSP where five of the twelve plants appear in all of the alternatives generated. These plants are necessary to maintain mathematical feasibility because of the capacity limits on plants which are given *a priori*. Their sizes, however, are not known prior to solving the problem. Although omitted here, a similar analysis can be made regarding interceptors.

It is not possible from these example problems to make any conclusive statements as to the general patterns of alternatives generated in any given static and dynamic problems, since such results are clearly dependent on the individual regional network. However, it is important to recognize that the patterns do depend on the cost criteria used. In all, these preliminary analyses provide insights about the system behavior under different problem designs.

The following section presents the imputed values associated with some of the facilities in the alternative regionalization plans generated by the five example problems. It also provides the hypothetical scenarios which have been examined using the imputed value matrices associated with the branch-and-bound trees for the three static problems.

5.3 Trade-off Analysis Using Imputed Value Incidence Matrices

Various economic trade-off analyses were performed for each of the five static and dynamic problems, and the computational results are presented here in three parts. The first part presents the imputed value ranges for each of the five problems. They are based on the *original imputed value incidence matrices* which are associated with the *original branch-and-bound trees* grown to the point of identifying the respective least-cost solutions. The second part is devoted to analyses of alternative solutions, or regional plans, based on six hypothetical scenarios. These analyses are performed for three static problems by growing additional branches from the respective branch-and-bound trees to provide additional alternatives. The incidence matrix associated with each of the extended trees, or the *augmented branch-and-bound trees*, is called an *augmented imputed value incidence matrix*. The third part provides observations on the computational results given in the first two parts.

It was demonstrated that an imputed value incidence matrix provides valuable insights into the characteristics of the alternative plans generated and provides opportunities to make trade-off analyses by iterative use of the computer capability. However, since each of these problems is constructed based on many arbitrary simplifications, no attempts have been made to relate the imputed values obtained from one problem to those obtained from another.

A. Imputed Values from the Original Matrices

The imputed values obtained from the original matrix for some of the regional facilities as well as some combinations of them in each of the five problems are described in Tables 5.6 and 5.7. Note that ranges are given using the method described in Chapter 4, and the values give the economic trade-offs between placing a facility (or a set of facilities) in the regional plan and excluding that facility. They are expressed as a percentage of the cost of the corresponding least-cost solution, without signs, since no attempts are made here to evaluate the individual imputed values.

Table 5.6 shows eight cases for the small-scale problems. Case 1, for example, shows the imputed value range for plant 3 in the S-SSP as 0.0 - 2.2, indicating that the cost difference between the least-cost solution with plant 3 and the one without it is greater than or equal to 0.0 and less than or equal to 2.2 percent of the overall least-cost solution. The first six cases deal with a single plant, and the seventh and eighth cases deal with two and three plants, respectively.

The imputed value for all cases in both the S-SSP and D-SSP is less than 2.2 percent of the corresponding least-cost solution.

Table 5.6 Typical Imputed Values for Facilities in S-SSP and D-SSP

		Imputed Value (% of least cost)	
Case	Plant at site	S-SSP	D-SSP
1	3	0.0 - 2.2	*
2	8	0.0 - 1.1	0.2 - 1.2
3	10	0.0 - 0.6	0.0 - 1.3
4	12	0.0 - 1.2	0.3 - 1.3
5	13	0.0 - 0.9	0.0 - 1.2
6	15	0.0 - 0.6	0.0 - 1.2
7	9,13	0.6 - 1.2	1.3 - 1.8
8	9,13,15	0.6 - 1.2	1.3 - 1.8

* Not identified. All of the alternatives generated so far contain a plant at site 3.

Table 5.7 Typical Imputed Values for Facilities in S-LSP, D-LSP and MS-LSP

		Imputed Value (% of least cost)		
Case	Facility at Site	S-LSP	D-LSP	MS-LSP
1	2	0.1-2.7	5.7-7.8	2.7-10.1
2	3	0.1-2.0	0.0-0.7	0.7- 5.3
3	5	0.1-4.2	6.0-7.0	0.5- 4.6
4	6	0.1-2.1	0.0-0.5	0.7- 5.1
5	8	0.0-1.6	2.3-5.5	0.0
6	9	0.1-3.9	4.8-6.4	0.7- 5.1
7	2-3	0.0-0.3	0.0-0.1	0.0
8	2-3,5-6	0.2-5.5	0.1-9.3	0.6-10.3
9	2-3,5-6,8-9	2.3-5.5	1.3- *	1.0-10.3

* Not identified.

This result can also be inferred from the percent cost range between the lowest and the highest costs, 4.7 for the S-SSP and 3.4 for the D-SSP, as shown in Table 5.5. In other words, there is only a slight cost difference between the least-cost alternative with one of the specified facilities (or a combination of them) and the least-cost alternative without it. The contribution of each of these facilities to the total cost of the regional system is small in the small-scale problems; the placement of one or a few of them in or out of the system does not significantly alter the total cost.

On the other hand, as shown in Table 5.7, the imputed value ranges are larger in the S-LSP and the D-LSP, primarily because the facilities are much larger and the cost of one facility represents a larger portion of the costs of the entire system. This trend becomes even more noticeable when the cost structure of the problem is less symmetric. As indicated in the table, both the upper and lower bounds on the imputed values of the MS-LSP are much greater than those of the S-LSP, and their ranges are also much greater.

In either case, however, the imputed values of a small number of facilities do not seem to be large enough to be significant at the screening stage of a planning process. This result occurs because there are no pronounced physical features in this problem which make any specific set of alternatives particularly attractive. It is to be noted, however, that if there are pronounced physical features which lead to greater cost differences among alternatives, the branch-and-bound process for identifying the least-cost solution would require fewer node evaluations, there would be somewhat fewer alternatives generated, and the imputed values would most likely be greater.

B. Analysis of Hypothetical Scenarios

The design of a mathematical model usually involves many simplifications and implicit assumptions. It is, therefore, quite important that a mathematical model have an analytical capability to deal with as many as possible of the hidden aspects of the problem. In the case of the regionalization problem, important issues drastically affect the desirability of different combination of candidate plant sites and interceptor routes. It is very important that a mathematical method be capable of generating alternative plans and facilitating comparisons between them. Excessive computations, of course, would be required to deal with all of the possible permutations of candidate facilities in most realistic problems. The branch-and-bound tree and the associated imputed value incidence matrix offers promise in coping with this demanding problem, as is demonstrated next using different planning scenarios.

Altogether six hypothetical scenarios are presented here. Scenarios 1 and 2 are associated with the S-SSP, Scenarios 3 and 4 with the MS-LSP, and Scenarios 5 and 6 with the S-LSP. Although each is associated with one of the three static problems, similar analyses may be performed using the dynamic models. These analyses are based on the original as well as the augmented incidence matrices.

As a first step of analysis the alternatives which satisfy or may satisfy the given conditions are identified on the original incidence matrix. Recall that each row of the matrix represents a set of alternatives, and each set of alternatives includes one alternative which is completely specified by the inspection limb (primary alternatives). The upper bound on the cost of the least-cost alternative in the set is given the primary alternative.

For example, consider the case where it is desired to find the least-cost solution which contains a given facility. From the column of the imputed value matrix corresponding to the given facility, one can find the rows (or sets of alternatives) which contain the incidence index values:

1 (The facility is already constrained to exist in all of the alternatives in the set),

-1 (The facility is not yet constrained to exist to be excluded in any alternatives in the set of alternatives, but the primary alternative contains the facility), and

-2 (The facility is not yet constrained to exist nor to be excluded in the set, but the primary alternative does not contain the facility).

The alternatives which belong to the first group already contain the facility, and, therefore, no additional constraints are required to include it in the analysis of the corresponding node of the branch-and-bound tree. The alternatives which belong to the latter two groups are constrained to contain the facility.

On the other hand, if it is desired to find the least-cost alternative which does not contain the facility, it is necessary to find the alternative which contain the index values 2, -2, and -1. The alternatives which belong to the first group already exclude the facility, and, therefore, no additional constraints are required, and the alternatives which belong to the latter two groups can be constrained to exclude it.

When there are multiple facilities which are constrained to be "in" or "out" of the solution, an appropriate combination of the above two situations must be considered. For example, consider the case where two facilities are to be considered. There are nine incidence index values to consider; whether both are constrained in, both are constrained out, or one is constrained in and the other out. In practice, however, the identification requires only a simple search through the matrix.

B.1 Scenarios 1 and 2

Scenarios 1 and 2 pertain to S-SSP (see Figure 5.4), and they deal with a situation where it is desired to modify the original set of candidate facilities after the imputed value analysis based on the original incidence matrix. For example, a set of candidate plant sites may be removed from further consideration and an additional set of alternative plans may be generated to examine such issues as municipal water supply, flood control, stream water quality, and, perhaps, the local autonomy of communities.

In Scenario 1, four sites 3, 8, 10 and 12 are removed from the set of candidate plant sites. Note that the removal of these sites not only limits the number of available plant sites but also breaks the interceptor links through these sites because of the capacity limits on the remaining plants. The overall effect would be to use the remaining eight sites which tend to be dispersed at the outer edge of the network except site 9 in the center.

In Scenario 2, site 15 is removed from the set of candidate plant sites, and interceptor links 3-2, 8-9 and 9-8 are also removed. In addition, a plant is constrained to be constructed at site 12. These additional constraints arbitrarily divide the region diagonally into two subregions; the northwestern portion and the southeastern portion. Regionalization would be carried out in each of the two subregions, but the plants would still be dispersed over each of them because of the capacity limits.

Table 5.8 summarizes the analysis procedure and results for the two scenarios. The table contains the following information:

(1) Set No. - The number of the particular set of alternatives in the original branch-and-bound tree. It is also the row number of the original incidence matrix.

(2) Cost Range - The range of the cost of the least-cost alternative in the corresponding alternative set.

(3) Least Cost - The least cost identified on the augmented branch-and-bound tree.

(4) No. Comp. - The number of subproblem computations required to identify the least-cost alternative on the augmented branch-and-bound tree.

Cost figures are presented using four significant digits for illustration purposes only. The table indicates that in each

Table 5.8 Analysis of Scenarios 1 and 2

Scenario	Set No.	From Original Matrix	From Augmented Matrix	
		Cost Range	Least Cost	No. Comp.
	5	2.479 - 2.504	2.504	4
	29	2.479 - 2.504	2.504	4
1	53	2.481 - 2.556	*	1
	54	2.481 - 2.556	*	1
	12	2.486 - 2.564	*	1
	28	2.493 - 2.549	2.597	28
2	43	2.525 - 2.559	2.559	50
	48	2.479 - 2.536	2.563	12

* Solution infeasible.

scenario four sets of alternatives were identified in the original branch-and-bound tree as the possible candidates for further analysis.

In Scenario 1, all of the alternatives in the four sets have already been constrained to exclude sites 3, 8 and 10 from the candidate plant sites in the original branch-and-bound tree. (Therefore, the incidence index values corresponding to the variables representing these sites are 2.) The variable representing site 12 has not been constrained in any set, but the primary alternatives in sets 5 and 29 do not contain any plant at that site. (Therefore, its incidence index value is -2 in these sets.) In sets 53 and 54 they include plants at site 12. (The index value is -1 in these sets.) These four sets provide candidates for the least cost alternative for Scenario 1.

Using the augmented branch-and-bound tree, it was found that in sets 5 and 29 the least-cost alternative was in fact the primary alternative (the alternative whose cost is the upper bound of the range), and that in sets 53 and 54 there was no feasible alternative which excludes the four plant sites. The number of subproblems computed for constructing the augmented branch-and-bound tree was four for sets 5 and 29, and only one for sets 53 and 54. Of the four computations only one resulted in a feasible alternative for sets 5 and 29. The data in Table 5.8 for sets 5 and 9 (and 53 and 54) are identical because of rounding off of costs obtained by nearly identical branching specifications throughout the tree.

A similar analytical procedure applies to Scenario 2. In this case, however, the number of computations required to find the least-cost alternatives in the augmented branch-and-bound tree were much greater than in Scenario 1. The computational requirements seem to depend on the additional constraints; additional branching was required for variables with minor fixed charges.

The results of the analysis for the two scenarios indicate the following. The cost of the least-cost alternative for Scenario 1, which achieves a more dispersed configuration, is identified as $2.504 million per year as compared with the original least-cost solution of $2.473 million per year. The difference of $31,000 per year is the incremental cost of such a decentralization. Note, however, that the incremental cost is only 1.3 percent of the original least-cost solution, and it is well within the range of computational error. Similarly, the cost of the least-cost alternative for Scenario 2, which divides the region diagonally in two subregions, is identified as $2.559 million per year. The difference of $85,000 per year is the incremental cost of such modification. This difference is only 3.5 percent of the original least-cost solution. These small imputed values result from the small capacity limits of regional facilities and also from the lack of pronounced physical features (see Section 5.3-A). If the imputed values are, in fact, so small, then the selection of a plan may be based mainly on issues other than costs.

B.2 Scenarios 3 and 4

Scenario 3 is an evaluation of a hypothetical boundary line which divides the region into northern and southern subregions. Scenario 4, on the other hand, is an evaluation of a hypothetical boundary line which divides the region into eastern and western subregions. The imputed value analyses of these boundary lines were performed for the large-scale regionalization using the MS-LSP (see Figure 5.3).

The imputed value analysis for Scenario 3 involves finding the least-cost alternative which contains all of the three network links, 2-3, 5-6 and 8-9, and the least-cost alternative which contains none of them. These three links cross the hypothetical north-south boundary line of the region. In Scenario 4, the imputed value analysis involves finding the least-cost alternative which contains the links 1-4 or 4-1, 2-5 or 5-2, and 3-6 or 6-3, and finding the least-cost alternative which contains none of the six interceptor routes. These interceptor links cross the hypothetical east-west boundary line.

From the original incidence matrix it is known that: 1) the least-cost solution which contains all of the interceptor links over the hypothetical north-south boundary line costs \$1.988 million per year, 2) the least-cost solution which contains one interceptor link from each of the three pairs over the hypothetical east-west boundary line is \$1.989 million per year, 3) the least-cost solution which does not contain any of the interceptor links over the north-south boundary line is known to be somewhere between \$2.008 and \$2.192 million per year, and 4) the least-cost solution which contains none of the links over the east-west boundary line is known to cost at least \$2.010 million per year but its upper bound is not known. Although the information provided by the original incidence matrix may be sufficient for planning analysis, these ranges can be tightened to find the actual imputed value.

A summary of analyses for the two scenarios is presented in Table 5.9. The table includes information similar to that presented in Table 5.8. Four significant digits are presented for cost figures for illustration. The least costs for some alternative sets are given in terms of a range, since for each scenario the new least-cost alternative was found prior to the complete branching of the augmented tree. In Scenario 3, alternative set 17 was already constrained not to include any interceptor links over the north-south boundary line; therefore its upper bound was used to prune the branches on the augmented branch-and-bound tree. In Scenario 4, the annual cost of the feasible alternative identified in set 23, \$2.117 million per year, was replaced by the cost of the new feasible alternative identified in set 24, \$2.116 million per year in pruning other branches.

The least-cost solution identified in the augmented branch-and-bound tree for Scenario 3 costs \$2.192 million per year, and for Scenario 4, it costs \$2.116 million per year. The imputed value of the hypothetical north-south boundary line, therefore, is the difference between \$2.192 and \$1.988 million, or \$204,000 per year. Similarly, the imputed value of the hypothetical east-

Table 5.9 Analysis of Scenarios 3 and 4

Scenario	Set No.	From Original Matrix Cost Range	From Augmented Matrix Cost Range	No. Comp.
	5	2.008 - 2.099	2.239 - 2.295	7
	9	2.105 - 2.196	2.231 - 2.323	9
3	13	2.019 - 2.081	2.249 - 2.277	18
	17	2.060 - 2.192	2.192	7
	36	2.049 - 2.299	2.270 - 2.431	19
	2	2.063 - 2.289	*	1
	21	2.043 - 2.226	*	1
	23	2.010 - 2.102	2.117	8
4	24	2.016 - 2.017	2.116	10
	26	2.094 - 2.326	*	1
	32	2.043 - 2.293	2.228 - 2.325	1
	36	2.049 - 2.299	*	1

* Solution infeasible.

west boundary line is $127,000 per year. In other words, it costs about 10 percent more for the north-south subregionalization and about 6 percent more for the east-west subregionalization in this particular problem. Although the imputed values turned out to be much greater in these cases, whether or not any planning significance can be associated with imputed values of this magnitude depend on individual cases. If the magnitude of the imputed values is considered significant, the trade-offs between the imputed values and noncost issues need to be carefully examined in a more refined and detailed fashion.

B.3 Scenarios 5 and 6

Many different cases can be evaluated using the same imputed value incidence matrix. This advantage is illustrated using two scenarios for the S-LSP. The scenarios presented here analyze the cost increase associated with an increase in the number of plants to be constructed in the region and the cost increase associated with the increase in capacity of a given plant. The same incidence matrix is used for both scenarios and for different cases within the same scenario.

Scenario 5 deals with a situation in which a number of plants is given *a priori*. Three cases were analyzed as shown in Table 5.10. Case 1 allows only two plants, and Cases 2 and 3 allow three and four plants respectively. The incremental cost over the overall least-cost solution, which contains only one plant, is determined for each of the three cases from the original matrix. The maximum incremental cost for the 2-plant, case is $36,000 per year, or 1.6 percent increase. The maximum incremental cost for the 3-plant case is $80,000 per year, or 3.5 percent increase. Since these maximum costs do not seem to be very significant, the analysis for identifying the exact incremental cost may simply be terminated in these cases. On the other hand, the range of the incremental cost for the 4-plant case was found to be much greater, from the minimum of $28,000 to a maximum of $141,000 per year. Thus the augmented branch-and-bound tree was constructed, and the exact incremental

Table 5.10 Analysis of Scenarios 5 and 6

Scenario	Case	Incremental Cost+	% Incremental Cost
	1: 2 plants	0. - 36.	0.0 - 1.6
5	2: 3 plants	28. - 80.	1.2 - 3.5
	3: 4 plants	28. - 141.	1.2 - 6.3
		141.*	6.3*
	1: 27.8 MGD	28. - 61.	1.2 - 2.7
		61.*	2.7*
6	2: 42.5 MGD	33. - 151.	1.5 - 6.7
		92.*	4.1*
	3: 52.5 MGD	33. - 151.	1.5 - 6.7
		95.*	4.2*

* From the augmented imputed value incidence matrix.

+ Thousand dollars per year.

cost was found to be equal to the maximum value of \$141,000 per year. In other words, cost must increase at least 6.6 percent in comparison to the least-cost solution if there are to be four plants in the region.

Scenario 6 analyzes the situation in which the capacity of a plant at site 2 is varied. Lower bounds on that plant's capacity are implied as branching constraints are added. For example, if a plant is located at site 2, its capacity must be at least 27.8 MGD ($q_2 \geq L_2$). If the interceptor between site 1 and site 2 is constructed ($f_{12} \geq L_1$), the capacity of the plant must be at least 42.5 MGD. As in Scenario 5, the incremental costs can be identified. The range of the incremental cost for case 1 which is obtained from the original imputed value matrix indicates that the maximum is \$61,000 per year, only 2.7 percent of the least cost. Although no further analysis may be needed since the maximum is so small, the exact incremental cost was identified using the augmented tree, as the maximum of the range. Similarly, the ranges of the incremental costs for cases 2 and 3 were determined from the original matrix, and the exact incremental costs were determined from the augmented matrix. The exact incremental costs for each of these cases turned out to be about the midpoint of the original range.

Because of the problem structure (see Section 5.3-A) the incremental costs in these two scenarios are also rather small. In an actual application such small differences would not probably be significant. The general applicability of the method to a planning process, however, seems to be adequately demonstrated by these example problems. A systematic analysis of the imputed value incidence matrix generates abundant information with a limited number of additional computations.

C. Summary of Example Applications

The imputed value incidence matrix is used for identifying economic trade-off values among different sets of alternative plans in the previous two sections. It was demonstrated that the analysis can be performed efficiently and that the matrix provides abundant information on economic trade-offs. Although it is not possible to make any generalization based on this particular example, some observations on the results of the analysis are summarized as follows:

(1) the cost ranges associated with the large-scale problems were much greater than those associated with the small-scale problems,

(2) the typical cost of alternatives in the large-scale problems was significantly less than the cost of alternatives in the small-scale problems,

(3) the cost ranges associated with the dynamic problems were much greater than those associated with the static problems, and

(4) the cost range within any given problem was relatively small. As a result, the economic trade-off values including the imputed values between alternative plans also turned out to be relatively small.

The first observation is quite intuitive. As discussed earlier, a slight change in the location pattern of large facilities would alter the total costs significantly, while such is not the case with small facilities.

The second observation implies that there is an imputed value between large-scale and small-scale solutions. For example, the cost difference ($217,000 per year) between the least-cost in the S-LSP and the least-cost in the S-SSP can be considered as an imputed value of scale. In other words, the implicit benefits associated with implementing a plan with small plants should exceed that value before such a plan becomes attractive. Implicit benefits of small-scale regionalization could include, for example: an improvement in water quality because of more dispersed effluents, the enhancement of water recycle possibilities at the fringe areas, and the coherent development of wastewater systems with regard to land use plans or to jurisdictional boundaries. Of course, the analysis of such benefits is an extremely difficult process in itself, and it was not included in this study.

The third observation stems from the assumption that each plant is to be expanded exactly three times at a set of fixed intervals. The fixed cost associated with each plant, and thus the cost ranges between alternative solutions, are relatively larger in the dynamic problems than in the static problems. The small cost range within any given problem, the fourth observation, results primarily from the relatively symmetric network of plant sites and interceptors and from the simplified problem design. The small cost ranges imply that in these particular example problems there are many solutions that are nearly as efficient economically as the least-cost solution.

The economic trade-off values obtained from the incidence matrix provide a guide to a systematic evaluation of alternative plans. For example, if the trade-off between two sets of alternatives is relatively small, then economic efficiency may not be a dominant factor in determining the overall desirability of one set over the other. On the other hand, if the cost trade-off is large, then the evaluation of the additional planning issues becomes important.

Although no attempts have been made in this study to integrate the evaluation process of issues other than costs, the easy access to economic trade-off values itself is a significant contribution to the formulation of a regionalization plan which is attractive in a realistic sense. The imputed value method provides an opportunity for the analyst to reflect on the trade-offs associated with planning issues which might affect the overall desirability of a plan.

As for the computational aspects, the augmented branch-and-bound analyses were performed using a FORTRAN IV program which has an interactive capability of generating alternatives from any nodes of the original tree (i.e., from any row of the incidence matrix). The number of subproblems computed vary, depending on the problem analyzed as indicated in Tables 5.8 and 5.9. The computation time required for each analysis also varies. For example, the total CPU time required for Scenario 1 was 8.3 seconds for 10 subproblem computations and the total CPU time required for Scenario 2 was 44.62 seconds for 91 subproblem computations, using the DEC 10 system at the University of Illinois. Once imputed value matrices were obtained, the trade-off analyses were carried out by hand. Of course, a simple utility program may be easily substituted for hand computations.

6. SUMMARY AND DISCUSSION

Public sector planning problems generally involve a number of complex quantitative and qualitative planning issues. Planning regional wastewater systems is no exception. There are many planning issues such as the cost of regional plants and interceptors, the quality and quantity of the receiving streams, land use planning, and political autonomy of the participating communities.

Because of this complexity, mathematical optimization methods offer the potential of being valuable tools for use in planning regional wastewater systems. The general emphasis to date in using mathematical models has been, however, on finding the mathematically optimum solution under a set of assumed conditions. Although such solutions are useful, it is not always possible to apply mathematical methods to find the truly optimal solution since it is quite likely that better plans might be drastically different from the theoretical optimum. As discussed in Chapter 1, the branch-and-bound method proposed here is an attempt to modify such an approach based on the premise that it is not always possible to define, much less to find, the optimal solution and that mathematical models should stimulate thoughts and provide an intuitive understanding of the behavior of the system under study. The emphasis in designing the method has been placed on generating many alternative plans in a systematic fashion so that many valuable insights can be obtained through the interaction between the analyst and the model.

Chapter 2 demonstrates that the branch-and-bound method developed for single-period problems is efficient in generating many alternative plans, including the approximate least-cost solution. It is also shown that the branch-and-bound method is so structured that the economic trade-offs between alternatives can be obtained in a very systematic fashion. The use of a network solution algorithm and an inspection method for identifying many node values on the branch-and-bound tree contribute to computational efficiency. Also, as shown in Chapter 3, the method can be extended to a special case of the multiperiod problem.

The alternatives that are generated can be displayed, and the trade-offs can be analyzed efficiently using the imputed value incidence matrix, which is a direct transformation of some of the information on the branch-and-bound tree. The matrix structure and the imputed value analysis procedure are described in Chapter 4. The proposed method was applied to a realistic problem in Chapter 5, and the computational results indicate that the method is potentially very useful, as it can provide many valuable insights in the process of generating and comparing alternative plans.

While there are many attractive features which make this method potentially very useful for use in planning regional wastewater systems, it has some shortcomings. They are:

(1) the method involves cost approximations which may introduce significant errors in the costs of alternatives and the imputed values between alternatives.

(2) it is relatively difficult to analyze the sensitivity of the alternatives which are generated with respect to cost parameters such as the discount rate and design lives, and

(3) many simplifying assumptions are introduced in the multiperiod analysis which may not necessarily suit a given problem under study.

The first two shortcomings are common to many mathematical methods which make use of piecewise approximations for the original cost functions. They are not critical, however, to the proposed method, since the primary objective is to generate alternative plans based on approximate costs, rather than to find the exact optimum. Further, each of the alternatives may be reevaluated with any desired cost functions with a minor computational burden. Similarly, the actual multiperiod costs based on the original waste generation patterns and any desired phasing schedule may be obtained relatively easily for each alternative generated. In general, simplifying approximations and assumptions are less critical for a method of generating alternatives than for a method designed to find the optimal solution.

The imputed value incidence matrix can be refined further by incorporating weights to the branching variables. Such weights can be selected in such a way that they represent the magnitude of an impact which is associated with a particular set of facilities. Once the matrix is constructed, the matrix structure remains unchanged, but the selected sets of weights may alter the alternatives selected for further analysis. Of course, such an approach faces the same difficulties which any other methods involving weights face, namely the difficulty in defining the weights.

In an actual planning process of facility location, mathematical analysis may have only a limited role. For example, the process may be dominated by political issues, which require a complex human interaction for their resolution. Further, the value of a plan changes over time as the economic, political and social climate changes. A plan which is considered the best today may no longer be so tomorrow, and the planning process must continually adjust to such changes. Still, mathematical analysis can make a significant contribution if appropriately designed and used. The branch-and-bound method proposed here for the analysis of location of regional wastewater facilities has been designed to provide an opportunity for the analyst to develop insights and intuitive understanding of the behavior of the system under study so that he may have a more active role in integrating the complex planning issues. The general methodology is potentially applicable to many public sector location problems that are similar to the problem described here.

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APPENDIX A

IMPUTED VALUE INCIDENCE MATRICES FOR CASE C OF THE MULTIPERIOD EXAMPLE PROBLEM

Table A-1 Imputed Value Incidence Matrix for Run 1

ALT	COST		Variables								
	UB	LB	1	2	3	4	5	6	7	8	9
1	636.7	536.3	1	1	-1	-1	-2	-2	-2	-2	-1
2	585.8	546.7	2	1	1	-1	-2	-2	-2	-2	-1
3	544.7	536.2	2	2	1	1	-2	-2	-2	-1	-2
4	589.9	535.6	1	2	1	-1	-2	-2	-1	-1	-2
5	553.7	533.2	1	2	2	1	-2	1	-1	-2	-1
6	537.8	517.3	1	2	2	2	-2	1	-1	-2	-1
7	536.1	529.2	2	1	2	1	-2	-2	-2	-2	-1
8	516.1	516.1	2	1	2	2	-2	-2	-2	-2	1
9	531.1	524.2	1	2	2	2	-2	2	1	-2	-1
10	526.7	518.2	2	2	1	2	-2	-2	-2	-1	-2
11	556.5	524.5	2	1	2	2	-1	-2	-2	-2	2
12	547.0	526.4	1	2	2	1	-2	2	-1	-2	-1
		Facility	5	4	7	1	3	7-5	4-5	3-7	3-4

Table A-2 Imputed Value Incidence Matrix for Run 2

ALT	COST UB	COST LB	Variables 1	2	3	4	5	6	7	8	9
1	636.7	597.7	1	1	1	-1	-2	-2	-2	-2	-1
2	585.8	578.9	2	1	1	1	-2	-2	-2	-2	-1
3	544.7	544.7	2	2	1	1	-2	-2	-2	1	-2
4	589.9	567.7	1	2	1	1	-2	-2	-1	-1	-2
5	553.7	553.7	1	2	2	1	-2	1	1	-2	1
6	537.8	537.8	1	2	2	2	-2	1	1	-2	1
7	536.1	536.1	2	1	2	1	-2	-2	-2	-2	1
8	516.1	516.1	2	1	2	2	-2	-2	-2	-2	1
9	531.1	531.1	1	2	2	2	-2	2	1	-2	1
10	526.7	526.7	2	2	1	2	-2	-2	-2	1	-2
11	556.5	556.5	2	1	2	2	1	-2	-2	-2	2
12	547.0	547.0	1	2	2	1	-2	2	1	-2	1
13	571.4	571.4	1	2	2	2	1	1	2	-2	-2
14	561.3	561.3	2	2	1	2	1	-2	-2	2	-2
15	563.5	563.5	1	2	2	2	1	2	1	-2	2
16	566.7	566.7	2	1	2	1	1	-2	-2	-2	2
17	567.7	567.7	1	2	2	2	1	1	1	-2	2
18	584.3	584.3	1	2	2	1	-1	1	2	-2	-2
19	553.6	553.6	2	1	2	1	2	-2	-2	1	2
20	571.9	563.4	1	2	1	2	-2	-2	1	-1	-2
21	545.0	545.0	1	2	2	2	2	1	1	1	2
22	571.5	571.5	2	2	1	1	1	-2	-2	2	-2
23	600.6	576.5	1	1	2	1	-2	-1	-2	-2	-1
24	548.7	548.7	1	2	2	2	2	1	2	1	-2
25	550.9	550.9	2	2	1	1	2	-2	-2	2	-2
26	573.8	573.8	1	2	2	1	1	2	1	-2	2
27	560.7	552.2	1	2	2	1	2	2	1	-1	2
28	580.5	556.5	1	1	2	2	-2	-1	-2	-2	-1
29	546.1	546.1	2	1	2	1	2	-2	-2	2	2
30	565.7	558.8	2	1	1	2	-2	-2	-2	-2	-1
31	580.6	580.6	1	2	2	1	1	1	1	-2	2
32	557.4	557.4	1	2	2	1	2	1	1	1	2
33	560.0	560.0	1	2	2	1	2	1	1	2	2
34	566.8	558.2	1	2	1	2	-2	-2	2	-1	-2
		Facility	5	4	7	1	3	7-5	4-5	3-7	3-4

Table A-3 Imputed Value Incidence Matrix for Run 3

ALT	COST UB	COST LB	Variables 1	2	3	4	5	6	7	8	9
1	636.7	597.7	1	1	1	-1	-2	-2	-2	-2	-1
2	585.8	578.9	2	1	1	1	-2	-2	-2	-2	-1
3	544.7	544.7	2	2	1	1	-2	-2	-2	1	-2
4	589.9	581.3	1	2	1	1	-2	-2	1	-1	-2
5	553.7	553.7	1	2	2	1	-2	1	1	-2	1
6	537.8	537.8	1	2	2	2	-2	1	1	-2	1
7	536.1	536.1	2	1	2	1	-2	-2	-2	-2	1
8	516.1	516.1	2	1	2	2	-2	-2	-2	-2	1
9	531.1	531.1	1	2	2	2	-2	2	1	-2	1
10	526.7	526.7	2	2	1	2	-2	-2	-2	1	-2
11	556.5	556.5	2	1	2	2	1	-2	-2	-2	2
12	547.0	547.0	1	2	2	1	-2	2	1	-2	1
13	571.4	571.4	1	2	2	2	1	1	2	-2	-2
14	561.3	561.3	2	2	1	2	1	-2	-2	2	-2
15	563.5	563.5	1	2	2	2	1	2	1	-2	2
16	566.7	566.7	2	1	2	1	1	-2	-2	-2	2
17	567.7	567.7	1	2	2	2	1	1	1	-2	2
18	584.3	584.3	1	2	2	1	1	1	2	-2	-2
19	553.6	553.6	2	1	2	1	2	-2	-2	1	2
20	571.9	571.9	1	2	1	2	-2	-2	1	1	-2
21	545.0	545.0	1	2	2	2	2	1	1	1	2
22	571.5	571.5	2	2	1	1	1	-2	-2	2	-2
23	600.6	576.5	1	1	2	1	-2	-1	-2	-2	-1
24	548.7	548.7	1	2	2	2	2	1	2	1	-2
25	550.9	550.9	2	2	1	1	2	-2	-2	2	-2
26	573.8	573.8	1	2	2	1	1	2	1	-2	2
27	560.7	560.7	1	2	2	1	2	2	1	1	2
28	580.5	573.6	1	1	2	2	-2	1	-2	-2	1
29	546.1	546.1	2	1	2	1	2	-2	-2	2	2
30	565.7	565.7	2	1	1	2	-2	-2	-2	-2	1
31	580.6	580.6	1	2	2	1	1	1	1	-2	2
32	557.4	557.4	1	2	2	1	2	1	1	1	2
33	559.9	559.9	1	2	2	1	2	1	1	2	2
34	566.7	566.7	1	2	1	2	-2	-2	2	1	-2
35	553.1	553.1	1	2	2	1	2	2	1	2	2
36	561.2	561.2	1	2	2	1	2	1	2	1	-2
37	563.6	563.6	1	2	2	1	2	1	2	2	-2
38	567.0	567.0	1	1	2	2	-2	2	-2	-2	1
39	601.3	601.3	1	2	1	2	1	-2	2	2	-2
40	571.6	571.6	2		1	2	-2	-2	-2	1	2
41	607.4	607.4	1	1	2	2	-1	2	-2	-2	2
42	606.1	574.1	2	1	1	2	-1	-2	-2	2	2
43	573.9	573.9	1	2	1	2	-2	-2	1	2	1
44	606.4	574.4	1	2	1	2	-1	-2	1	2	2
45	584.7	576.2	1	2	1	1	-2	-2	2	-1	-2
		Facility	5	4	7	1	3	7-5	4-5	3-7	3-4

APPENDIX B

COST APPROXIMATION DATA FOR THE DUPAGE COUNTY EXAMPLE

Table B-1 Piecewise Approximations of the Annual Costs of Plants in the S-LSP

Plant Site	Minimum Capacity	Maximum Capacity	Fixed Cost ($/yr.)	Unit Cost ($/MGD/yr.)
2	22.0	52.5	132,700	14,100
3*	10.5	106.9	153,200	12,900
5	13.3	52.5	104,400	14,600
6*	19.4	106.9	153.200	12,900
8	15.5	25.5	89,100	16,000
9	9.0	34.5	76,400	16,000

* Approximated by a piecewise linearization between 20 MGD and 100 MGD.

Table B-2 Piecewise Approximations of the Annual Costs of Interceptors in the S-LSP

Interceptor Route	Minimum Capacity (MGD)	Maximum Capacity (MGD)	Fixed Cost ($/year)	Added* F.C. ($/year)	Unit Cost ($/MGD)
1-2	5.8	7.2	13,800	75,500	3,860
2-3	22.0	52.5	30,600	45,600	1,150
1-4	5.8+	5.8	9,500	-	1,650
4-1	1.4+	1.4	4,600	-	3,340
2-5	22.0	29.2	33,000	-	1,300
5-2	13.3	30.5	28,900	-	1,430
3-6	10.5	63.0	45,900	-	1,790
6-3	19.4	96.5	60,700	-	1,400
4-5	1.4	7.2	7,100	33,500	3,770
5-6	13.3	52.5	26,000	52,500	1,390
7-5	10.0+	10.0	19,700	-	1,980
7-8	10.0+	10.0	11,100	-	1,110
9-6	9.0	34.5	28,300	-	1,600
8-9	15.5	25.5	40,000	52,500	2,500

* Fixed cost associated with collapsed nodes.

+ Piecewise linear approximation of the cost function is based on the assumption that the minimum capacity is 0.1 MGD less than the maximum capacity.

Table B-3 Piecewise Approximations of the Annual Costs of Plants in the D-LSP

Plant Site		Minimum Capacity (MGD)	Maximum Capacity (MGD)	Fixed Cost ($/yr.)	Unit Cost ($/MGD/yr.)
2	a)	17.6	36.9	357,600	38,500
	b)	22.0	52.5	-	22,800
3	a)	7.7	65.4	269,300	36,200
	b)	10.5	106.9	-	20,900
5	a)	8.3	36.9	280,200	40,100
	b)	13.3	52.5	-	23,300
6	a)	10.6	65.4	410,100	35,200
	b)	19.4	106.9	-	20,100
8	a)	5.8	10.6	235,200	44,900
	b)	15.5	25.5	-	24,700
9	a)	4.4	15.0	202,600	44,700
	b)	9.0	34.5	-	24,800

a): Data for the first portion of the coupled branch-and-bound problem dealing with the initial-year flows.

b): Data for the second portion of the coupled branch-and-bound problem dealing with the terminal-year flows.

Table B-4 Piecewise Approximations of the Annual Costs of Plants in the S-SSP

Plant Site	Minimum Capacity (MGD)	Maximum Capacity (MGD)	Fixed Cost ($/yr.)	Unit Cost ($/MGD/yr.)
1	5.8	7.2	37,400	20,500
2	12.0	26.6	80,800	16,300
3	8.8	10.0	50,100	18,900
5	7.5	11.7	49,700	18,900
8	9.2	13.3	56,700	18,200
9	4.1	14.1	40,100	19,400
10	13.5	17.6	73,600	17,000
11	5.9	19.4	52,500	18,000
12	10.0*	10.0	52,500	18,700
13	3.1*	3.1	20,900	24,200
14	12.4	22.4	77,400	16,600
15	9.0*	9.0	48,300	19,100

* Piecewise linear approximation of the cost function is based on the assumption that the minimum capacity is 0.1 MGD less than the maximum capacity.

Table B-5 Piecewise Approximations of the Annual Costs of Interceptors in the S-SSP

Interceptor Route	Minimum Capacity* (MGD)	Maximum Capacity (MGD)	Fixed Cost ($/year)	Added F.C.+ ($/year)	Unit Cost ($/MGD)
1-2	5.8	5.8	12,100	-	2,100
3-2	8.8	8.8	10,100	-	1,100
4-3	1.2	1.2	6,100	-	5,300
4-5	1.2	1.2	4,700	15,000	5,000
7-1	1.4	1.4	4,600	-	3,400
6-5	3.0	3.0	5,600	-	1,900
6-11	3.0	3.0	11,500	-	3,900
7-8	1.4	1.4	4,600	6,500	4,500
8-9	9.2	9.2	9,900	-	1,100
9-8	4.1	4.1	6,600	-	1,600
9-10	4.1	4.1	9,300	-	2,300
10-11	13.5	13.5	13,300	-	1,000
12-9	10.0	10.0	19,700	-	2,000
12-13	10.0	10.0	11,100	-	1,100
15-11	9.0	9.0	21,300	-	2,400
13-14	3.1	3.1	9,500	17,500	4,400

* Piecewise linear approximation of the cost function is based on the assumption that the minimum capacity is 0.1 MGD less than the maximum capacity.

\+ Fixed cost associated with collapsed nodes.

Table B-6 Piecewise Approximations of the Annual Costs of Plants in the D-SSP

Plant Site		Minimum Capacity (MGD)	Maximum Capacity (MGD)	Fixed Cost ($/yr.)	Unit Cost ($/MGD/yr.)
1	a)	5.4	6.2	100,900	55,500
	b)	5.8	7.2	-	35,600
2	a)	8.0	14.2	194,400	46,400
	b)	12.0	19.2	-	27,200
3	a)	8.4	9.6	135,200	51,000
	b)	8.8	10.0	-	35,200
5	a)	0.0	7.7	114,000	63,500
	b)	0.0	10.5	-	37,400
8	a)	4.4	7.0	127,000	52,600
	b)	7.0	11.1	-	30,200
9	a)	2.6	7.4	107,100	53,500
	b)	4.1	14.1	-	30,300
10	a)	8.8	11.4	197,600	46,600
	b)	13.5	17.6	-	26,900
11	a)	2.3	11.1	141,500	49,800
	b)	6.1	19.6	-	27,900
12	a)	4.8	4.8	140,500	51,900
	b)	10.0	10.0	-	29,000
13	a)	1.2	1.2	56,400	67,600
	b)	3.1	3.1	-	37,200
14	a)	4.6	9.4	20,410	46,600
	b)	12.4	22.4	-	25,700
15	a)	4.4	4.4	126,900	53,100
	b)	9.0	9.0	-	29,700

a): Data for the first portion of the coupled branch-and-bound problem dealing with the initial-year flows.

b): Data for the second portion of the coupled branch-and-bound problem dealing with the terminal-year flows.