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MATHEMATICAL MODELS AND OPTIMIZATION TECHNIQUES FOR USE IN ANALYSIS AND DESIGN OF WASTEWATER TREATMENT SYSTEMS

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ABSTRACT

A mathematical framework is developed for use in the design of a secondary wastewater treatment system. Mathematical models predicting the performance of various unit processes are used to construct a comprehensive system model. Three efficient optimization approaches to generate cost effective system designs are studied. The first approach transcribes the comprehensive system model into a nonlinear program that includes 64 variables and 58 constraints. A generalized reduced gradient algorithm is applied to solve this model. The second approach uses an existing algorithm for solving generalized geometric programs. Partitioning of model variables into two sets is necessary. A number of geometric programming subproblems resulting from the partitioning are solved. The third approach decomposes the wastewater system into a liquid and a sludge subsystem. The liquid subsystem is optimally designed, while the sludge subsystem design includes embedded optimization steps. The overall optimal design is obtained from coordination between the two subsystem designs. The comprehensive system model can be used as a tool for the analysis of process performance. Important insights about process design, modeling, and integration can be gained by exercising the model. Potentially fruitful areas for research can also be identified. This is illustrated through the use of an example problem.

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CHAPTER 1

INTRODUCTION

1.1. Preliminaries

The objective of present wastewater treatment plant design is to provide a cost effective processing system for a given wastewater. Such a system is relatively complex, containing a series of unit processes. Generally each of the unit processes is designed to achieve a specific goal, and only limited consideration is given during the design procedure to interactions among the unit processes. It would be desirable, of course, for engineers to have design procedures that take into full account the tradeoffs that are possible among the unit processes. For instance, minor modifications in the design of the liquid waste treatment portion of a conventional plant may produce significant cost savings in the solids handling portion of the plant.

Design engineers, however, are generally limited to using their past experiences and trial and error in considering these tradeoffs for a small number of options. One reason is that many of the unit processes are not well understood, and therefore a complete and compatible set of unit process models is not available for use in comprehensive, systematic design procedures. A second reason is that only limited progress has been made in combining the existing knowledge of individual unit processes to form comprehensive design procedures.

One approach that researchers have identified is to connect various unit process models within an overall system model and to apply a mathematical or enumerative optimization technique. The literature review in Section 1.3 provides an overview of the considerable progress that has been made since the first work in this area was reported by Lynn *et al.* in 1962.

The goals of this research are to extend the current capabilities in combining unit process models within an overall optimization framework as an analysis and design tool, and to highlight research needs that will improve the usefulness of unit process models in comprehensive system design. It is important to stress that wastewater treatment plant design is a complex process and that good designs generally cannot be achieved using only a mathematical, computerized model. The best system models are designed for use as tools by designers, who ultimately have the responsibility for taking into account factors not considered in the model. System models can be very useful, however, for obtaining an optimal solution for given input data and effluent requirements based on specified assumptions. By varying these conditions, the designer can use the model to facilitate the evaluation of options and tradeoffs.

Research in developing comprehensive design procedures is important because the need for wastewater treatment will clearly continue to require the commitment of significant resources at the national and international level. It is also important to improve the understanding of complete wastewater treatment systems so that innovative regulatory approaches to water quality management can be better evaluated. Examples of such approaches are time varying effluent requirements that change with receiving body conditions (see Reheis *et al.*, 1982, for an illustration) and basin wide management of a particular pollutant using transferable discharge permits (see Joeres and David, 1983, for a discussion of the program recently implemented by the Wisconsin Department of Natural Resources for the Fox River). In general, as more cost effective regulatory approaches are developed it will be even more important to understand better the options and tradeoffs in wastewater treatment. Perfect understanding (e.g., of costs) cannot be expected, but relative performances, costs, trends, etc. provide fundamental insights.

In the remainder of this chapter, research objectives and procedure are outlined in Section 1.2. Section 1.3 provides a thorough literature review of past research efforts on the optimization of wastewater treatment system design. Several guidelines to improve this research over previous studies are summarized. Section 1.4 describes the organization of the thesis.

1.2. Research Objectives

Progress in developing comprehensive system models can be roughly divided into two branches: 1) efforts to develop models that consider a wide range of unit processes and emphasize the selection from among them (e.g., an activated sludge process or a trickling filter) to form a treatment train, and 2) efforts that focus on a particular process train (perhaps with some options) and that emphasize the selection of design parameters (e.g., basin volumes). Models of each type can be used jointly since they emphasize different stages of the design process. Models of the first type could be used in selecting a general plant layout, and models of the second type could be used in refining recycle flows and in selecting design parameters for the given layout.

The overall objective of this thesis is to extend research along the second branch described above by developing a comprehensive system model of a conventional activated sludge secondary wastewater treatment system; several variations of the base treatment system are also considered. There are two major tasks under the objective: one is to develop efficient optimization techniques for solving the comprehensive system design model, and the other is to illustrate the use of the system model for the analysis of process performance and design. The specific steps taken to achieve this objective are the following:

1) Evaluate current unit process models to determine their suitability for use in a comprehensive system model and design procedure, and construct an overall wastewater system model which can be used to describe the performance of the system with given influent and design conditions.

2) Develop and apply optimization approaches for the design of the wastewater treatment system. Several approaches are examined for their applicability to optimizing the comprehensive system model.

3) Illustrate the use of the comprehensive system model as a tool for the analysis of performances, integration, and limitations of unit processes considered in the study. Several variations of the base treatment system are modeled to verify the insights obtained from the design optimization of the base system.

1.3. Literature Review

Past studies on the use of optimization models in the design and planning of wastewater treatment and sludge disposal systems can be roughly divided into two general categories: Optimal process synthesis and optimal process design. Process synthesis studies deal with the selection of the combination of unit processes that composes the least cost treatment system. Lynn, *et al.* (1962) pioneered the study of the optimal wastewater treatment plant synthesis. A network linear programming model was formulated to represent the BOD removal in a treatment plant that consists only of liquid waste treatment. Many assumptions had to be made in order to render the optimization model a linear program. The model was solved for the combination of unit processes that would remove a given amount of BOD at the least treatment cost.

Evenson *et al.* (1969) applied dynamic programming to select the unit processes that would result in the least cost design of a plant treating cannery processing wastes. Both liquid and sludge treatments were included in their system, with the sludge treatment train being a diverging branch in the dynamic programming framework. The removal of BOD was considered to be the only function of the plant. The structure of the waste treatment plant, with each unit processes represented as a "stage" and with the absence of recycle streams, made the application of dynamic programming possible. However, the design of unit processes in this study was very simplistic.

Shih and Krishnan (1969, 1973) also applied dynamic programming for the optimization of industrial waste treatment plant design. The problem was formulated as an initial-final state problem since the characteristics of the raw waste and the requirement of the treated effluent quality represent the boundary conditions. The performance of a unit process was considered to be its ability for removing BOD. The Decision Inversion Method proposed by Aris *et al.* (1964) was used to identify the least expensive liquid treatment system. The same methodology of process optimization was again demonstrated on a simplified problem by Shih and DeFilippi (1970). Lack of confidence in the performances of individual unit processes was considered by these authors a major handicap of the study.

The study of Shih and Krishnan (1969) appears to have attracted attention from other researchers. Ecker and McNamara (1971) formulated a geometric program for each of the process trains considered by Shih and Krishnan. The primal-dual relationship was used for solving these programs. The flowchart that has the lowest treatment cost was then identified by comparing the optimal cost of each process flowchart. Computational simplicity and the ease of performing sensitivity analysis for variations in effluent quality are features of the geometric programming approach for this problem.

Adam and Panagiotakopoulos (1977) discussed the weakness of using linear programming, dynamic programming, and geometric programming for wastewater treatment process design optimization. They proposed a network approach as an alternative solution technique for the problem studied by Shih and Krishnan (1969). Advantages of the network approach as claimed by the authors included its capability of handling multiple wastewater parameters (other than BOD), its indifference to the types of the cost functions and performance relationships, and its flexibility and efficiency. Unfortunately, with a simple example problem, none of these advantages were demonstrated by the proposed approach.

The fact that various optimization approaches have been applied to solve the same process optimization problem is indicative of the many special characteristics contained in this problem. The special arrangement of the unit processes in the system or the unique characteristics of the process performance relationships or cost functions may warrant the application of a specific optimization technique or the development of an innovative optimization procedure.

Sterling (1976) conducted a similar study to those discussed above on the optimal process selection and design using dynamic programming. Only BOD was included in the analysis of process performance, and the treatment included only liquid waste.

Patterson (1977) also developed a dynamic programming model for the optimal process selection and design of a liquid waste treatment system. An effort was specially made to identify those flowcharts that are good with respect to the total system cost, but different in the units being included. This allows the designer to examine different flowcharts and tradeoffs among these systems in more detail.

Mishra *et al.* (1973) considered optimization of both the structure and the design of a biological wastewater treatment system that included only liquid waste processing. Structural parameters, or stream splitting factors, were introduced into the model formulation to specify the arrangement of the unit processes. These structural parameters were continuous variables varying between zero and one. Both BOD and total suspended solids concentrations were modeled. The objective function was not complete because only the construction cost of the system was included. The simplex pattern search technique was employed to optimize this nonlinear programming model. Because the operation and maintenance costs were not included in the objective function, the optimal system selected by the technique was an activated sludge system, not a trickling filter system.

Bush and Silveston (1978) considered the optimal synthesis of the liquid processing portion of a complete waste treatment system. The structural parameter method used by

Mishra *et al.* was adopted. The constraints on the decision and state variables were expressed in terms of penalty functions. Five wastewater parameters were modeled. The complex method by Box (1965) was selected as the optimization algorithm.

While most efforts in optimal process synthesis focused on the liquid treatment system, Hasit *et al.* (1981) studied the optimization of a sludge management system using a mixed integer model. The design of the sludge treatment and disposal units were based on empirical loading factors to avoid nonlinearity in the model, and to make the model amenable to efficient optimization. Since the process performances were not modeled, the tradeoffs between performance and costs could not be evaluated. This model can be used to minimize overall sludge handling, transportation, and disposal costs both for a single plant and for a group of plants with or without centralized treatment.

The U. S. Army Corps of Engineers (1978) developed a computer program (CAPDET) in an effort to aid in the design of wastewater treatment facilities. The design procedures for a wide range of physical, chemical, and biological unit processes were programmed (the 1980 version of CAPDET contains 79 liquid stream processes and 14 sludge stream processes). Once the user specifies the unit processes to be considered for the design, CAPDET synthesizes and designs all possible treatment flowcharts that can be constructed from these unit processes using user-provided or default design criteria. Among all designs examined, the more cost-effective process trains and their detailed designs are given to the user as outputs. The effectiveness of CAPDET as a screening device and design aid was demonstrated by McGhee *et al.* (1983). Some problems encountered in the application of CAPDET were also noted by these authors, among them the most noticeable being the high computer user costs because of the enumerative nature of the program.

Rossman (1979, 1980) also developed a computer-aided procedure for the synthesis and design of wastewater treatment and sludge disposal systems. Information requirements from the user are similar to that for CAPDET. The computational procedure uses implicit enumeration coupled with a heuristic penalty method that accounts for the impact of return sidestreams from sludge processing. A unique feature of this work is that planning objectives other than system cost can be optimized in the program. Alternative designs that are energy efficient, or low in the initial construction cost, etc. can be identified and evaluated. The optimal design of the system is approximate in the sense that discrete values for the decision variables are supplied by the user.

To summarize, optimal process synthesis studies often deal with a variety of wastewater treatment unit processes. The mathematical models are basically used as screening devices for planning and design of wastewater treatment systems. They are used as design aids to specify good process trains; but the system design and performance in general cannot be predicted at a detailed level. If the tradeoffs among unit process designs or the applicability of unit process models for design are to be further explored, a process design optimization model will have to be employed.

Process design optimization models usually employ fairly detailed mathematical statements to describe the performance of a specified configuration (or possibly a few variations) of unit processes. They do not deal with the breadth of the options considered by the synthesis models. To use process synthesis models and process design models conjunctively, a process design model could be used in evaluating more thoroughly a process train selected using a synthesis model.

Naito *et al.* (1969) and Fan *et al.* (1970) studied the optimal design of an activated sludge subsystem consisting of aeration and final sedimentation. Various flow regimes in the aeration tank were considered. The simplex method of Nelder and Mead (1965) was employed to minimize the total capital cost of the system. The objective function was not complete since it left out the operation and maintenance costs which often play an important role in the design of wastewater treatment systems.

Berthouex and Polkowski (1970) investigated wastewater treatment plant design under uncertainty. Uncertainty in performance of system components was considered by applying the concept of propagation of variance. Only the liquid treatment train was optimized, sludge train design and cost estimation were based on typical design criteria. Thickening of activated sludge in the final clarifier was modeled by the limiting flux theory. The pattern search technique of Hooke and Jeeves (1961), with modifications to handle inequality constraints, was applied to solve this problem. Only a single local minimum was reported for the problem.

Scherfig *et al.* (1970) attempted to optimize the design of an activated sludge system using geometric programming. The primal problem of their model had a high degree of difficulty and was not amenable to the classic geometric programming solution approach. As a result, the system was decomposed into a sludge disposal system and a liquid waste treatment system. The sludge subsystem was optimally designed using a search algorithm to solve the dual problem. The liquid treatment train was designed by experience. These authors did not coordinate the designs of the two subsystems to identify the overall optimal system design. The capability of the classic geometric programming for solving the entire waste treatment plant design was shown to be limited by the high degree of difficulty and the lack of an efficient nonlinear programming technique.

Parkin and Dague (1972) indicated that an overall waste treatment system made up of individually optimized unit processes was seldom optimal. They assembled a design model for a treatment system that included both liquid and sludge processing. Six decision variables were identified and 720 alternative designs formed by different combinations of the values of the six decision variables were evaluated. This complete enumeration approach indicated that more than 60% of the treatment alternatives investigated were at least 20% more expensive than the least cost design. The importance of the cost-effective design of a waste treatment system was clearly demonstrated.

Middleton and Lawrence (1974) presented a unique technique for optimization of the activated sludge system. By adopting the concept of sludge age and the set of design equations proposed by Lawrence and McCarty (1970), they observed that the liquid and sludge process trains could be optimized independently for a fixed sludge age. An enumerative graphical search technique was developed based on the fact that each subsystem had only two decision variables. This optimization technique was specially designed to solve this formulation of the problem. It would become more complicated and inefficient if recycle streams generated in sludge processing are recycled to the liquid treatment train.

Middleton and Lawrence (1976) applied the same optimization technique to the design of a similar system where anaerobic digestion was substituted for aerobic digestion. Primary settling and sludge dewatering by vacuum filtration were also included. Simplifying assumptions were made such that the number of decision variables in this problem remained the same as in the previous problem even though more units were included. The assumptions that the primary settling tank removes suspended solids at a constant efficiency and that the final settling tank performs perfect clarification are unrealistic. However, they are essential for the solution technique to work. This is clearly a drawback of this approach. Only a single local minimum was found for this problem. The overall system cost was found to be quite insensitive to the sludge age.

Craig *et al.* (1978) used the complex algorithm (Box, 1965) to design the system studied by Middleton and Lawrence (1976). It was shown that this nonlinear programming algorithm was much more efficient than the graphical enumeration technique previously used. Multiple starting points were used in solving the nonlinear programming model, but only one local minimum was identified in this problem. Since the formulation had been purposely restricted by Middleton and Lawrence to include only five decision variables, the complex algorithm worked satisfactorily, outperforming the graphical enumeration significantly as far as computing time was concerned. The same algorithm was also successfully applied to an optimal activated sludge operation problem by the same group of researchers (Hughey *et al.*, 1982).

Bowden *et al.* (1976, 1978) reported another effort to develop a computerized procedure for wastewater treatment system design. Their model included liquid waste and sludge treatment units and recycle streams generated from sludge processing. Because of the presence of the recycle streams in the model, an iterative approach was used to determine a steady state solution. The objective function value corresponding to a set of decision variables could not be determined until a steady-state design was obtained. The search method by Powell (1964) was selected as the optimization algorithm. Although the computational experience was not explicitly reported, it is expected that the overall optimization procedure would not be very efficient because of the time requirement for obtaining the steady-state design by iteration.

In his study of sensitivity of the optimal wastewater treatment plant design with respect to state variables and technological parameters, Voelkel (1978) assembled an optimization model for a complete wastewater treatment system that contains recycle streams from sludge processing. Nine degrees of freedom were identified in his model. A modified complex algorithm was selected as the optimization technique. Voelkel applied the equation ordering algorithm of Rudd and Watson (1968) to select the decision variables in his model. Fixing the values of these selected decision variables permits more efficient solution for the steady-state design than the iterative approach. Voelkel did not report any computational experience with his optimization approach, nor did he discuss the quality of the solutions obtained from using this search technique.

Based on the above studies that used search techniques for optimization of wastewater treatment system design, it appears that these methods are not computationally efficient because of the nature of these methods and the need for obtaining a feasible solution by iteration. Although these methods are straightforward, they are likely to be very slow in obtaining the optimal system design for a complex arrangement of unit processes.

An optimization procedure that incorporates embedded optimization steps may serve well for the purpose of process design optimization. Tarrer *et al.* (1976) studied the activated sludge design under uncertainty. In developing their solution strategy, Tarrer *et al.* assumed that either the effluent BOD or total suspended solids constraint would be limiting if a least cost design is to be achieved. They subsequently developed a solution procedure with embedded nonlinear programming steps for the optimal design of their system. The major shortcoming of this work, however, is that it optimized only the liquid treatment process train, although the costs of sludge treatment were estimated using typical design criteria (and were included in the overall objective function).

Grady (1977) outlined the steps for using discrete dynamic programming for optimization of the activated sludge system. The problem formulation was similar to that of Tarrer *et al.*, i.e., only the liquid treatment train was considered. Grady observed that the problem could be formulated as three stages in series, each having one decision variable, provided that the sludge age was fixed. To implement the solution procedure, the sludge age was first calculated from an assumed effluent soluble BOD requirement. Designs were then made based on this sludge age using dynamic programming. It should be noted that if the complete treatment plant design is to be optimized, dynamic programming may not be an attractive technique because of the recycle streams, branches, and additional state variables that would be required in the system model.

Lauria *et al.* (1977) considered optimization of an activated sludge subsystem that included aeration and final settling. Through substitutions they reduced the objective to a function with only two variables. They solved the problem by using the classical calculus technique with Newton's method for solving systems of nonlinear equations. This approach would become impractical for a more complete treatment system because of the extensive computing requirements.

Hughes (1978) employed the same design equations used by Lauria *et al.* and optimized the system design using geometric programming. The problem had ten degrees of difficulty and a concave objective function. A problem of this type was considered unsolvable by Scherfig *et al.* in 1970. Advances made by Avriel *et al.* (1975), however, on the development of a solution technique for generalized geometric programs made the problem amenable to very efficient solution. With only two degrees of freedom in the problem, Hughes was able to verify that his solution was indeed the global minimum by mapping the response surface.

Although their main objective was to identify the most cost-effective sludge treatment and management scheme, Dick *et al.* (1976, 1978, 1979, 1981) considered both the liquid and sludge treatment trains and performed a sequence of very comprehensive studies on treatment process selection and design optimization. The interactions between the liquid and the sludge subsystems were considered in more detail than in previous studies. Side streams generated throughout the sludge processing train were assumed to be recycled to the liquid treatment train. The authors called for the use of fundamental design equations instead of empirical observations grounded purely on experience. Process models were complete except that the authors assumed a constant effluent solids concentration from the secondary clarifier regardless of the design condition. This assumption is unrealistic since the performance of the secondary clarifier varies significantly with the design and operation of the activated sludge subsystem. Based on their modeling work, the authors indicated that the physical properties of sludge influenced the optimal design to such an extent that more research on this aspect would be needed (Dick *et al.*, 1978). Predictive models for sludge characteristics as functions of basic design and operational variables were subsequently developed (Dick *et al.*, 1979, 1981).

Dick *et al.* developed a computer program for the selection of the least cost configuration of unit processes among alternative sludge management schemes. Because of the presence of recycle streams in the system, this program calculates the design parameters iteratively until a steady state design is achieved. This information is used interactively with a nonlinear programming code. This code uses the penalty function approach with the Davidon-Fletcher-Powell method (Davidon, 1959, and Fletcher and Powell, 1963) for the minimization of the resulting unconstrained problem. Dick *et al.* recognized that the number of potential decision variables can be very great when complex systems are being optimized. They discussed the factors that limit the number of design parameters actually needed as decision variables for the purposes of their study. In the demonstration runs presented, the design of the liquid train was fixed, i.e., the optimization was carried out for the sludge treatment system only. The computational requirements of their approach would be expected to increase considerably if it is applied to the entire wastewater treatment system. Dick *et al.* did not report an attempt to verify that the local optimum resulting from their solution strategy was indeed the global optimum, nor did they discuss the general issue of local optimality.

Tyteca *et al.* (1977) presented a thorough review of mathematical models developed for or used in wastewater treatment process design and optimization. Based on this work, Tyteca formulated an optimization model for a complete activated sludge system (Tyteca, 1981). His model included quite detailed models for unit processes except that he assumed perfect clarification in the secondary clarifier. The model had eight degrees of freedom and was unique in that dimensionless variables were used. The model was formulated as a geometric program which allows efficient computation of the analytical derivatives of the objective function and the constraints and systematic input of model data when implementing the optimizing code (Tyteca and Smeers, 1981, Smeers and Tyteca, 1984). The authors discussed one potential problem with their approach: the use of inequality constraints to replace equations as required for the standard geometric program formulation. In view of the size of this problem, Tyteca and Smeers decided to employ a well-tested nonlinear programming algorithm based on the generalized reduced gradient (GRG) method, rather than a special-purpose geometric programming (GP) code. A more general study of the use of a GRG algorithm to solve geometric programs was carried out by Ratner *et al.* (1978). They reported that for many test problems GRG compared well with special-purpose GP codes. An interesting conclusion of the Tyteca and Smeers study was that only a single local minimum was found for their highly nonlinear model.

Koelling (1982) used a quasi-enumerative search procedure for optimization of sewage treatment plant design. His study concentrated on the design of the activated sludge subsystem. Sludge processing units were then sized accordingly. The model has two degrees of freedom. An interesting feature of this work was that three objective functions were considered: the total system cost, the costs incurred by the federal government, and the costs incurred by the local municipality. It was observed that the "least cost" design changes with different objective functions. Koelling concluded that a design reached as a compromise of different interest groups seems to be more realistic than that obtained based on a single objective.

Suidan *et al.* (1983) formulated an optimization model for a simplified activated sludge system. Waste sludge was assumed to be dewatered by vacuum filtration and incinerated. Separate sludge thickening was not considered and recycle streams from the sludge processing system were ignored. Consequently, it was possible to simplify the model sufficiently to have only two decision variables. The univariate search technique was selected to solve this problem. Fibonacci search was employed for minimization in one dimension. It was reported that the response surface was unimodal and very flat in the vicinity of the optimum. These researchers applied the limiting flux theory to thickening in the primary settling tank and obtained an unrealistically high underflow solids concentration. As a consequence, they assumed that only a fraction of the primary clarifier area was effective for sludge thickening.

As shown in the literature review, the state of the art has evolved considerably over the last twenty years in the application of optimization concepts to wastewater treatment system analysis and design. There still, however, are areas where additional improvements can be made. The following summary of guidelines for future work is from the above discussion. These guidelines serve as a basis for the development of the comprehensive system model described in Chapter 2.

1) Construction, operation and maintenance costs: Both categories of costs should be included since different unit processes have different relative costs for construction, operation and maintenance.

2) Complete treatment system: The sludge treatment and disposal systems should be optimized together. Since the costs of these systems comprise a large portion of the total system cost, designs based on optimizing only the liquid treatment train are not likely to be optimal for an entire treatment system consisting of both liquid and sludge treatment units and sludge disposal.

3) Descriptive process models: Mathematical models describing the performance of units and interactions among units should be taken into account. For example, recycle streams from the sludge treatment system to the liquid treatment system have often been neglected but should be considered. Ultimate sludge disposal costs also must be considered.

4) Realistic assumptions: For many unit processes, a predictive model for process performance is not available due to the complexity of the process. Assumptions about process performance are necessary for a complete design of the treatment system. Limitations of the state-of-the-art in this area are probably best exemplified by an assumption made by a number of researchers that 100% of the solids are captured in the final settling tank. Because a substantial portion of effluent BOD results from the suspended solids, it is essential that a model relating the design and operational

parameters to the final effluent suspended solids concentration be included in the optimization model.

5) Meaningful constraints: Several previous studies have formulated the optimization problem with constraints on various design parameters or state variables. These constraints are based on past experience rather than on scientific fundamentals. Such constraints have often been used to force the model to produce "reasonable" results. This limits the usefulness of an optimization model by forcing it to work only in the range of variables found in conventionally designed systems. Important insights on process research may be lost with such a restricted optimization model. However, empirical models should only be used in the ranges within which they are developed when they are used to construct the overall system model.

6) Efficient optimization technique: Many optimization methods used previously can be applied only to a special and limited process scheme or only when simplifying assumptions about process designs are made. Few studies developed and presented methods that are specially tailored to take advantage of the unique structure of a complete wastewater treatment system.

One major purpose of this thesis is to make additional progress toward developing an efficient optimization method for use in designing a complete activated sludge treatment system. Significant cost savings in water pollution control efforts may ultimately be made possible with the aid of such a method for treatment process design. Attempts are made to incorporate many of the interactions that were omitted in previous studies into an optimization model to provide the designer with realistic insights about system design. Three optimization approaches that can be used efficiently to solve a complete mathematical model for the waste treatment system design are also presented.

A treatment plant design optimization model has been perceived by a number of researchers as a means to obtain the least cost system design. This role of an optimization model is suggested by the very nature of the optimization concept. The planning and design of a wastewater treatment system, however, is a complex problem. Many important issues such as energy requirements and system reliability may not be captured in a cost-optimization model. As a result, the optimal design obtained from solving such a model may only be meaningful mathematically. Another view suggests that the most appropriate role of this type of optimization model is as a decision-making aid. This role is more appropriate because of the importance of unmodeled issues and the uncertainties associated with planning a waste treatment system. The other major purpose of this thesis is to illustrate the use of such an optimization model as a tool for the analysis of process performance. An optimization model can lead to the examination of the validity of process models from the cost-effectiveness point of view. Useful insights about process performance, integration, or limitations are gained as valuable by-products from exercising an optimization model.

1.4. Thesis Outline

A comprehensive system design model of a wastewater treatment system is prerequisite for this research. Chapter 2 defines the base treatment system selected for this study, and provides a review of the representative process design models that describe the performances of those unit processes included in the base treatment system. Design equations and cost information used for the construction of the comprehensive system model are also described in this chapter. Several solution techniques examined for solving the comprehensive system model are described in Chapter 3, together with discussions of the performances of these techniques. Chapter 4 emphasizes the use of the optimization model as a tool for system and process analysis. Insights obtained from optimizing the system design are used as examples to illustrate the role of an optimization model. A summary, conclusions, and future research directions are presented in Chapter 5.

CHAPTER 2

DEVELOPMENT OF THE COMPREHENSIVE SYSTEM MODEL

2.1. Introduction

Design of unit processes in a wastewater treatment system follows two general approaches in current practice. One approach is simplistic, and involves the use of empirically determined design parameters. It has been observed qualitatively that these design parameters affect the performances of unit processes. However, quantitative measures of process performance cannot be obtained. As a result, designs based on past experience deny the engineers the opportunity to analyze the interactions among unit processes in a wastewater treatment system, which are essential to achieving a cost effective design.

The other approach for designing unit processes is to employ mathematical models which predict the process performance under given input and design conditions. Interrelationships among unit processes can be studied in detail to strive for cost-efficiency; performances of unit processes can be predicted to insure satisfactory effluent water quality. These process performance models may be developed from physical, chemical or biological principles, or from empirical data fitting; they may be time-dependent or time-independent; deterministic or stochastic. A thorough review of process performance models for unit processes typically employed in secondary wastewater treatment was given by Tyteca *et al.* (1977). The review of process performance models in this chapter is intended to highlight and update that effort. Time-dependent or stochastic models are not considered in this study. Unit process performance models were selected based on this review, and serve as building blocks for the comprehensive system model.

Ideally, a process model suitable for design should be able to describe the unit process performance over a wide range of operating and influent conditions. It should reflect realistic process performance, and include all relevant process variables that affect the process performance. Based on these guidelines, models developed from fundamental principles were given first priority for use in constructing the comprehensive system model because such models are valid regardless of the external conditions. Models developed from plant-scale studies were then considered, followed by models developed in laboratories. For complicated processes for which only empirical models are currently possible, those empirical models that predict process performance consistent with fundamental knowledge of treatment processes or with observed process responses were preferred. Since the development of an empirical model is generally specific to the system studied, the limitations of such models should be recognized.

Assuming that cost efficiency is a primary objective in the design of a wastewater treatment system, it is appropriate to write the comprehensive system model in the form of an optimization model. The objective function is to minimize the total system cost, which includes capital, operation and maintenance costs. The constraint set in the model is a collection of the independent design equations for all unit processes in the system and the mass balance relationships among the interconnected units. Restrictions on effluent water quality are also imposed on the design of the system as constraints.

The base wastewater treatment system selected for this study and the definition of the variables in the model are described in Section 2.2. Section 2.3 presents process performance models for units included in the base treatment system. Representative models for designing each unit process are reviewed, followed by the mathematical expression of a particular model that is incorporated into the overall system model. Section 2.4 deals with the formulation of the objective function in the cost minimization model, and includes discussions about available cost information. Section 2.5 illustrates the complete design procedure using the comprehensive system model with a numerical example.

2.2. System Description

2.2.1. Flowchart

A typical secondary wastewater treatment plant was selected as the base system for evaluation in this study. The flowsheet of the plant includes primary sedimentation of raw wastewater, organic material stabilization by the activated sludge process, gravity thickening of combined primary and waste activated sludge, two-stage anaerobic digestion of the thickened sludge, and sludge dewatering by vacuum filter. Final sludge disposal by sanitary landfill was assumed. Figure 2.1 depicts this study system. Supernatants generated in sludge processing were assumed to be recirculated to the head end of the plant for BOD and suspended solids removal.

2.2.2. Definition of System Variables

The complete design of the wastewater treatment system requires the specification of three groups of variables:

1) The parameters are those quantities that remain constant in the design; examples are the biological coefficients in the activated sludge process, pumping efficiency, cost of energy, etc.. A complete list of the parameters used in the system design and economic analysis is provided in Section 2.5.

2) The decision variables specify the dimensions or the design condition of a unit process. More specifically, the decision variables selected in this study are: overflow rate of the primary settling tank (L_p), mean cell residence time (θ_c), hydraulic retention time (θ), and sludge recycle ratio (r) in the activated sludge process, solids loading on the gravity thickener (L_g), digestion temperature (T_d) and solids retention time (θ_d) of the primary anaerobic digester, solids loading on the secondary digester (L_d), and filter yield from the vacuum filter (L_f).

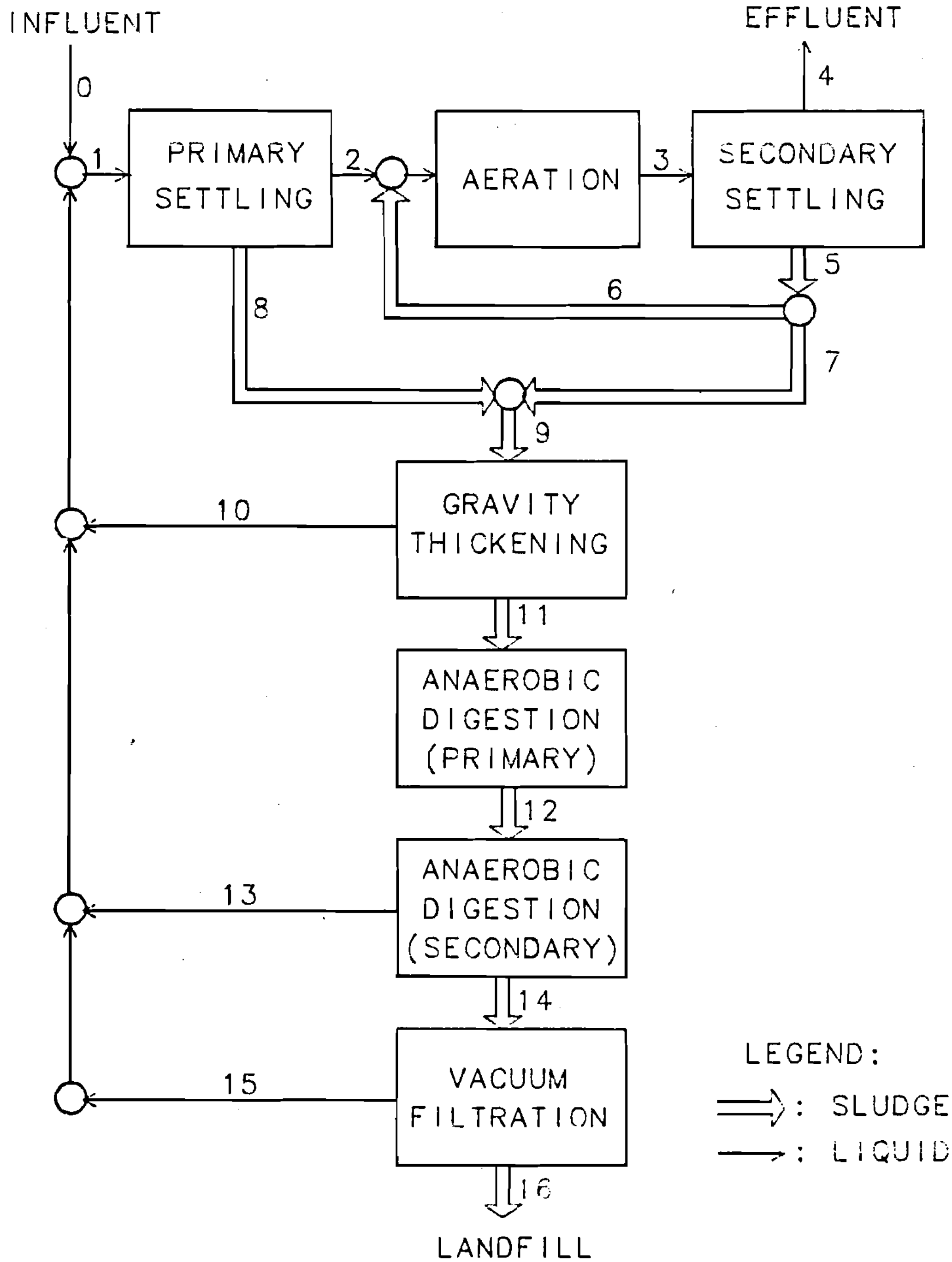


Figure 2.1 - Activated Sludge Treatment System

3) The state variables represent the wastewater characteristics at a particular stage during the treatment processes, and are defined at the seventeen control points shown in Figure 2.1. These state variables include flowrate, concentrations of soluble BOD_5, active biomass, biodegradable and inert (with respect to aerobic stabilization) volatile suspended solids, fixed suspended solids, and total suspended solids. The following notation and units for the state variables are used in the development of the comprehensive system model:

Q_j = Flowrate at control point j, m^3/hr

S_j = Soluble BOD_5 concentration at control point j, g/m^3

M_{aj} = Active biomass concentration at control point j in kg/m^3 unless noted otherwise

M_{dj} = Biodegradable volatile solids concentration at control point j in kg/m^3 unless noted otherwise

M_{ij} = Inert volatile solids concentration at control point j in kg/m^3 unless noted otherwise

M_{fj} = Fixed, or inorganic, solids concentration at control point j in kg/m^3 unless noted otherwise

M_{tj} = Total suspended solids concentration at control point j in kg/m^3 unless noted otherwise

j = Index of the control point, $j = 0,1, \ldots ,16$.

2.3. Problem Formulation

The constraints in the comprehensive system model are described in this section. In general, the design of a unit process can be considered using Figure 2.2. Vectors $\mathbf{Z}_i$ and $\mathbf{Y}_i$ represent the input states to unit i and output states from unit i, respectively. Vector $\mathbf{d}_i$ denotes the decisions made at unit i. The input and output states are related by a transformation function, or a technological function,

$$\mathbf{Y}_i = \mathbf{T}_i(\mathbf{Z}_i,\mathbf{d}_i) \tag{2.1}$$

where $\mathbf{T}_i$ is a vector function that defines the performance of the unit process.

The total cost of unit i, c_i, can be expressed as

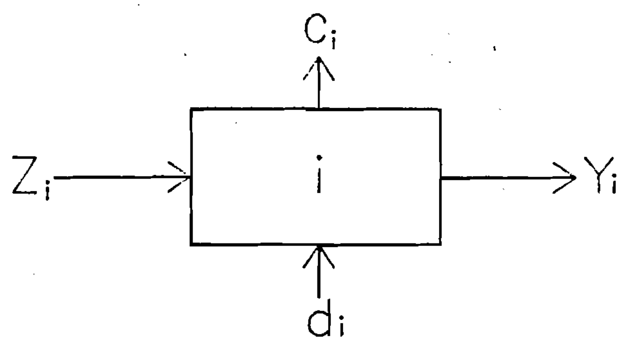


Figure 2.2 - Functional Diagram for Design of Unit Processes

$$c_i = c_i(\mathbf{Z}_i, \mathbf{d}_i) \tag{2.2}$$

The $\mathbf{T}_i$ functions are described in this section, while the c_i functions are described in the next.

2.3.1. Primary Sedimentation

Primary sedimentation is provided mainly for the removal of influent settleable solids. Organic matter in the form of suspended matter and semi-colloidal solids may also be removed from the wastewater. Fundamental understanding of the solids removal mechanism is limited to the ideal conditions of discrete spherical particles settling in laminar flow. The overflow rate was shown to be the single most important parameter controlling the solids removal efficiency according to the theory (Hazen, 1904, Camp, 1946). In practice, however, because of the flocculant nature of wastewater and the disturbance in the settling tank caused by hydraulic turbulence, density currents, scour and wind action, it is not possible to apply this basic knowledge to design.

Empirical relationships developed from plant operating data to describe the suspended solids and organic matter removal efficiencies in the primary settling tank are abundant in the literature. Smith (1968) proposed that solids removal efficiency is a function of the surface overflow rate. He developed a model using data from the WPCF Manual of Practice (1959). Berthouex and Polkowski (1970) developed a linear model with respect to the overflow rate based on the same data. This model is mathematically simple, but it is not an adequate representation of the observed data.

Other researchers have found that the influent suspended solids concentration is also important in predicting the solids removal efficiency. This observation seems reasonable considering that sewage contains a large portion of flocculant particles. Voshel and Sak (1968) developed two models relating the solids removal efficiency to both the influent solids concentration and the overflow rate based on their plant-scale study performed in Michigan. In England, two models have evolved over the past decade for the solids removal efficiency of primary sedimentation. The model of Tebbutt and Christoulas (1975) was developed from a pilot-scale study and was shown to describe plant operating data adequately. The CIRIA (1973) model used detention time instead of overflow rate to represent the hydraulic features of the settling tank. This model was based on data observed at sewage works in the London area. Dick *et al.* (1978) fitted the WPCF (1959) data to a model of the form proposed by Tebbutt and Christoulas. These models are summarized in Table 2.1.

It is noted that all models indicate that the solids removal efficiency increases with decreasing overflow rate and with increasing influent solids concentration when it is considered. Parameters in the models represent the degree of dependence of the solids removal on influent solids concentration and overflow rate. These parameters are related to the characteristics of the influent to the primary settling tank.

Figure 2.3 is used to illustrate the design of the primary settling tank. The overflow rate is the decision variable of this unit. The model of Voshel and Sak is selected to describe the removal of total suspended solids in the primary settling tank. The fraction of influent suspended solids remaining in the primary effluent is calculated as

$$\frac{M_{t2}}{M_{t1}} = 1 - v_1 M_{t1}^{v_2} L_p^{-v_3} \tag{2.3}$$

where v_1, v_2 and v_3 are positive parameters, and L_p is the overflow rate defined as

Table 2.1 - Models for Suspended Solids Removal Efficiency in the Primary Settling Tank

Models	Suspended Solids Removal Efficiency	Source of Data	Domain of Experiment
Smith (1968)	$.82 \exp(-.2112L_p)$	WPCF (1959)	$.42<L_p<3.75$
Voshel and Sak (1968)	1. $.139M_{t1}^{.27}L_p^{-.22}$ 2. $.340M_{t1}^{.17}L_p^{-.13}$ (Polymer addition)	Voshel and Sak (1968)	$70<M_{t1}<160$ $1.71<L_p<1.88$
Berthouex and Polkowski (1970)	$.82 - .142L_p$	WPCF (1959)	$.42<L_p<3.75$
CIRIA (1973)	$(.00043M_{t1}+.51)[1-\exp(-.7t)]$	CIRIA (1973)	--
Tebbutt and Christoulas (1975)	$.955 \exp(-\frac{265}{M_{t1}} - .0504L_p)$	Tebbutt and Christoulas (1975)	$100<M_{t1}<1000$ $1.04<L_p<6.25$
Dick *et al.* (1976)	$.84 \exp(-\frac{40}{M_{t1}} - .177L_p)$	WPCF (1959)	$.42<L_p<3.75$ $M_{t1}\simeq 230$

Note -- L_p : overflow rate ($m^3/m^2/hr$)
M_{t1} : influent suspended solids concentration (g/m^3)
t : detention time (hours)

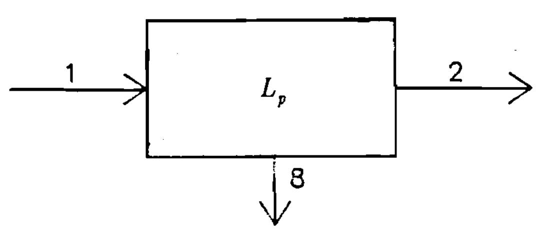


Figure 2.3 - Design of the Primary Settling Tank

$$L_p = \frac{Q_2}{A_p} \tag{2.4}$$

Note that M_{t1} and M_{t2} are in g/m^3, and the surface area of the primary clarifier, A_p, is in m^2.

Primary sludge concentration has been modeled by two approaches. The first approach assumes that this concentration is controlled by the hydraulic limitations of the sludge withdrawal mechanisms. As a result, a constant concentration is assigned to the primary sludge (see, for example, Voelkel, 1978). The second approach uses the differential thickening technique (see, for example, Dick and Suidan, 1975) which is based on the limiting flux theory (Dick, 1972) to calculate the primary sludge concentration. Thickening constants for primary sludge can be obtained from batch settling tests.

Many models have been proposed to define the batch sludge settling velocity as a function of the initial solids concentration (Vesilind, 1979). Vesilind (1968) proposed an exponential relationship,

$$u_i = a' \exp(-b' C_i) \tag{2.5}$$

where u_i is the batch settling velocity,

C_i is the initial solids concentration,

and a' and b' are empirically determined constants for the sludge.

Berthouex and Polkowski (1970) used equation (2.5) to develop a mathematical expression of the limiting flux.

$$G_L = a' b' C_u^2 \exp(-b' C_u) \tag{2.6}$$

where G_L is the limiting flux, and C_u is the underflow solids concentration.

Dick and Suidan (1975) also derived an expression, equation (2.8), for calculating the limiting flux based on the following batch settling velocity model proposed by Duncan and Kawata (1968),

$$u_i = a C_i^{-n} \tag{2.7}$$

$$G_L = [a(n-1)]^{\frac{1}{n}} \left(\frac{n}{n-1}\right) \left(\frac{Q_u}{A}\right)^{\frac{n-1}{n}} \tag{2.8}$$

where Q_u is the underflow flowrate from a thickener,

A is the surface area of a thickener,

and a and n are empirically determined constants.

The underflow solids concentration from a thickener can be calculated as

$$C_u = \frac{G_L A}{Q_u}$$

$$= [a(n-1)]^{\frac{1}{n}}(\frac{n}{n-1})(\frac{A}{Q_u})^{\frac{1}{n}} \tag{2.9}$$

Dick and Young (1972) have shown that equation (2.9) provides adequate prediction of pilot plant thickening data. This equation is used to describe the sludge thickening in the overall system model.

The thickening function of the primary settling tank is modeled, i.e., the primary sludge concentration is calculated, from equation (2.9) as

$$M_{t8} = [a_p(n_p-1)]^{\frac{1}{n_p}}(\frac{n_p}{n_p-1})(\frac{A_p}{Q_8})^{\frac{1}{n_p}} \tag{2.10}$$

where a_p and n_p are settling constants of the primary sludge obtained when the batch settling velocity is expressed in meters/hr and the sludge solids concentration in kg/m^3.

The flow and mass balance relationships around the primary settling tank are

$$Q_1 = Q_2 + Q_8 \tag{2.11}$$

$$Q_1 M_{t1} = Q_2 M_{t2} + 10^3 Q_8 M_{t8} \tag{2.12}$$

A unit conversion factor, 10^3, is inserted in equation (2.12) since M_{t1} and M_{t2} are in g/m^3 while M_{t8} is in kg/m^3.

Empirical models predicting the removal of organic matter in the primary settling tank also exist in the literature. Table 2.2 provides a sample of these models. Most of the models were developed from actual plant data except the one by Tebbutt and Christoulas (1975) which was developed from a pilot-plant study. A common feature of these models is the lack of fit of the data to the proposed model, generally with R^2 less than 0.6. Therefore none of these models is used in this study. Instead, the total BOD in the primary effluent is modeled by considering the soluble and suspended portions respectively.

The soluble BOD_5 concentration is assumed unaffected by primary sedimentation, i.e.,

$$S_2 = S_1 \tag{2.13}$$

$$S_8 = S_1$$

The concentrations of individual solids components are calculated based on the assumption that the settleable portion of each solids component is the same:

$$M_{a2} = M_{a1}\frac{M_{t2}}{M_{t1}}$$

$$M_{d2} = M_{d1}\frac{M_{t2}}{M_{t1}}$$

$$M_{i2} = M_{i1}\frac{M_{t2}}{M_{t1}}$$

$$M_{f2} = M_{f1}\frac{M_{t2}}{M_{t1}} \tag{2.14}$$

Table 2.2 - Models for Organic Matter Removal Efficiency in the Primary Settling Tank

Models	BOD Removal Efficiency	Source of Data	Domain of Experiment
Berthouex and Polkowski (1970)	$0.4 - .0588L_p$	WPCF (1959)	$.42 < L_p < 3.75$
CIRIA (1973)	$0.86e^2 - 0.029e$	CIRIA (1973)	--
Tebbutt and * Christoulas (1975).	$\dfrac{0.311 + 0.779e}{(\frac{411}{M_{t1}}) + 1.09}$	Tebbutt and Christoulas (1975)	$200 < M_{t1} < 800$ $0.26 < e < 0.63$
Tebbutt (1979)	$0.08 + 0.508e$	WPCF (1959)	$0.2 < e < 0.8$
Tebbutt (1979)	$-0.31 + 1.211e$	Tebbutt (1979)	$0.6 < e < 0.8$

* This model is for COD removal efficiency in the primary settling tank.

Note -- L_p : overflow rate ($m^3/m^2/hr$)

M_{t1} : influent suspended solids concentration (g/m^3)

e : suspended solids removal efficiency

$$M_{a8} = M_{a1}\frac{M_{t8}}{M_{t1}}$$

$$M_{d8} = M_{d1}\frac{M_{t8}}{M_{t1}}$$

$$M_{i8} = M_{i1}\frac{M_{t8}}{M_{t1}}$$

$$M_{f8} = M_{f1}\frac{M_{t8}}{M_{t1}}$$

where M_{a1}, M_{d1}, M_{i1} and M_{f1} and M_{a2}, M_{d2}, M_{i2}, M_{f2} are in g/m^3.

2.3.2. Activated Sludge

The activated sludge process consists of aerobic waste stabilization in the aeration tank, clarification of the aeration tank effluent and sludge concentration in the secondary clarifier, and recycle of the thickened sludge to the aeration tank to maintain the microbial population (Figure 2.4).

Tyteca *et al.* (1977) have reviewed various kinetic models proposed for the design of biological wastewater treatment processes. Among the models proposed, the first order models by McKinney (1962) and Eckenfelder (1966) and the Monod model by Lawrence and McCarty (1970) are the most widely accepted design models in practice. The design equations developed by Lawrence and McCarty are chosen as the basis for design of the activated sludge process. The aeration tank is assumed to be completely mixed. All

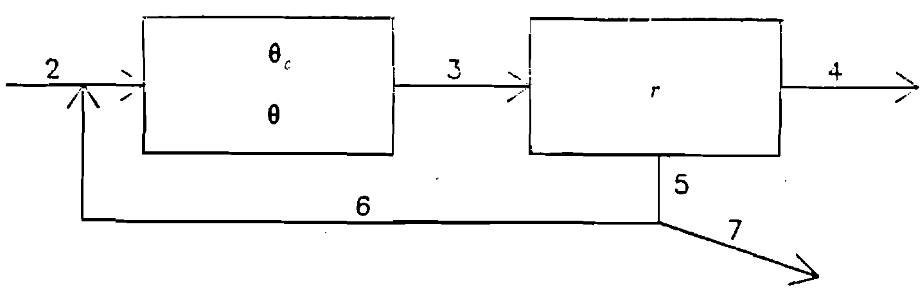


Figure 2.4 - Design of the Activated Sludge Process

biological activities are assumed to occur in the aeration tank, and the biodegradable volatile solids are assumed to be completely consumed in the tank, i.e., $M_{d3} = 0$. The substrate utilized in the process, S, is then

$$S = S_2 + (\frac{1.42 \text{ g } BOD_L}{\text{g } VSS})(\frac{\text{g } BOD_5}{1.5 \text{ g } BOD_L})M_{d2} - S_3 \tag{2.15}$$

where M_{d2} is the volatile biodegradable solids concentration in the primary effluent, S_3 is the soluble BOD_5 in the aeration tank effluent, and can be calculated as

$$S_3 = \frac{K_s(1 + b\theta_c)}{\theta_c(yk - b) - 1} \tag{2.16}$$

where K_s is the half-velocity constant, g BOD_5/m^3,

k is the maximum specific utilization coefficient, day^{-1},

y is the growth yield coefficient, g cell/g BOD_5,

b is the endogeneous decay coefficient, day^{-1}

and θ_c is the mean cell residence time, days.

The mean cell residence time, by definition, is

$$\begin{aligned}\theta_c &= \frac{10^3 M_{a3} V}{Q_4 M_{a4} + 10^3 Q_7 M_{a7} - Q_2 M_{a2}} \\ &= \frac{10^3 M_{a3}\theta}{(1-w)M_{a4} + 10^3 w M_{a7} - M_{a2}}\end{aligned} \tag{2.17}$$

where V is the volume of the aeration tank, m^3,

M_{a4} is the biomass concentration in the treated effluent, g/m^3,

M_{a7} is the biomass concentration in the underflow from the secondary clarifier,

θ is the hydraulic retention time in days, which is defined as

$$\theta = \frac{V}{24 Q_2} \quad , \tag{2.18}$$

w is the sludge wasting ratio defined as

$$w = \frac{Q_7}{Q_2} \quad , \tag{2.19}$$

and 10^3 is a unit conversion factor,

The biomass concentration in the aeration tank, M_{a3}, can be derived from the mass balance relationship of the substrate as

$$M_{a3} = 10^{-3}\frac{y}{1 + b\theta_c}\frac{\theta_c}{\theta}S \tag{2.20}$$

where 10^{-3} is a unit conversion factor.

The volatile inert suspended solids concentration in the mixed liquor is derived from the mass balance relationship and the assumption that the solid compositions remain unchanged through secondary sedimentation,

$$M_{i3} = \frac{1}{1 + (10^{-3})(\frac{\theta_c}{\theta})(\frac{M_{a2}}{M_{a3}})}[(10^{-3})M_{i2}(\frac{\theta_c}{\theta}) + (1 - f_d)bM_{a3}\theta_c]$$

or

$$\frac{M_{i3}}{M_{a3}} = \frac{1}{M_{a3}+(10^{-3})(\frac{\theta_c}{\theta})M_{a2}}[(10^{-3})M_{i2}(\frac{\theta_c}{\theta}) + (1-f_d)bM_{a3}\theta_c] \tag{2.21}$$

where f_d is the fraction of microbial cells that is degradable, and 10^{-3} is a unit conversion factor.

Similarly, the concentrations of the fixed suspended solids can be calculated and a ratio defined,

$$\frac{M_{f3}}{M_{a3}} = \frac{1}{M_{a3} + (10^{-3})(\frac{\theta_c}{\theta})M_{a2}}(10^{-3})M_{f2}(\frac{\theta_c}{\theta}) \tag{2.22}$$

Mass balance of biomass around the aeration tank yields

$$M_{a5} = (1 + \frac{1}{r} - \frac{\theta}{r\theta_c})M_{a3} - \frac{10^{-3}}{r}M_{a2} \tag{2.23}$$

where $r = \frac{Q_6}{Q_2}$ is the sludge recycle ratio, and 10^{-3} is a unit conversion factor.

The oxygen requirement for aeration is estimated using the Lawrence-McCarty Model as

$$O_2 = 24\times10^{-3} Q_2 S(1.5\frac{\text{g } BOD_L}{\text{g } BOD_5} - 1.42\frac{\text{g } BOD_L}{\text{g } cell}\frac{y}{1 + b\theta_c}) \tag{2.24}$$

where O_2 is the oxygen requirement in kg/day, and 24×10^{-3} is a unit conversion factor.

The air flow rate is calculated as

$$Q_a = \frac{1}{1440}\frac{C_s O_2}{\gamma\alpha(\beta C_s - DO)(1.024)^{T_L - 20}(OTE)\rho_{air}} \tag{2.25}$$

where Q_a is the air flow rate in m^3 air/min,

α and β are correction factors,

γ is the weight fraction of oxygen in air,

C_s is the dissolved oxygen saturation concentration at $20^o C$, g/m^3,

DO is the dissolved oxygen concentration maintained in the aeration tank, g/m^3,

OTE is the oxygen transfer efficiency,

T_L is the temperature of the aeration tank content, $^o C$,

ρ_{air} is the density of air, kg/m^3,

and (1/1440) is a unit conversion factor.

A minimum requirement for mixing of the aeration tank content is imposed on the model to maintain the complete-mix flow required in the tank. This constraint is transcribed as :

$$\frac{Q_a}{V} \geq \eta \tag{2.26}$$

where η is the minimum mixing requirement in m^3/m^3/min, whose value is assumed to be 0.02.

The dissolved oxygen concentration maintained in the aeration tank is assumed to be 1.5 g/m^3 so that the biological activity of a non-nitrifying activated sludge system will not be inhibited.

2.3.3. Secondary Sedimentation

A secondary clarifier performs two functions: clarification and thickening. Mixed liquor suspended solids (MLSS) from the aeration basin must be removed from the plant effluent to meet the water quality standards, while the settled solids should be concentrated for biomass recycle and further sludge processing. The surface area of the clarifier is determined from either the clarification or the thickening requirement (Dick, 1970).

Clarification efficiency of the secondary clarifier is a critical factor in determining the efficiency of the entire waste treatment system for both BOD and suspended solids removal. The effluent BOD from a secondary treatment plant consists of both soluble organics remaining or produced from the activated sludge process and the biodegradable suspended solids in the effluent. Depending on the operating conditions of the activated sludge process, suspended solids may account for more than half of the effluent total BOD. Thus the degree of uncertainty inherent in a model of the clarifier is very important. Influent solids concentration to a clarifier is usually in the range of 1500 to 3000 g/m^3, or possibly even greater. Since the desired effluent concentration is in the 10 to 20 g/m^3 range, removal efficiencies in excess of 99 percent are required. A slight deviation in this efficiency can have a pronounced impact on the quality of the effluent from the system.

The design conditions of both the aeration tank and the secondary clarifier affect the clarification efficiency. Parker (1983) provided an excellent review of how these design conditions influence the solids removal efficiency in the secondary clarifier. Because of the complexity involved in modeling the performance of this unit, a predictive model describing the clarification efficiency based on fundamental mechanisms is not currently available.

There are, however, a number of empirical models in the literature that predict the clarification performance of the secondary sedimentation tank (Table 2.3). Villiar (1967) developed a regression model based on results from bench scale experiments. Takamatsu

Table 2.3 - Empirical Models Predicting Total Suspended Solids Concentration in Secondary Clarifier Effluent

Models	Effluent TSS Concentration (mg/l)	Source of Data
Villier (1967)	$45.0(Q_4/A_f)^{.49} M_{t3}^{-.82} t_s^{-.439}$	Villier (1967)
Takamatsu and Naito (1967)	$63.2\ M_{t3}^{.5} \exp(-.74 t_s)$	Takamatsu and Naito (1967)
Agnew (1972)	1) $18.2 + 8.01(Q_4/A_f) - 3.3 M_{t3}$ 2) $73.2(Q_4/A_f)^{.12}\ FM^{.27} M_{t3}^{-.35} t_s^{-1.03}$	Agnew (1972)
Lech (1973)	$1.4(17.6 - .739T)(Q_4/A_f) M_{t3}$	Pflanz (1969)
Busby and Andrew (1975)	$10.88(Q_3/A_f) M_{t3}$	Pflanz (1969)
Keinath *et al.* (1977)	$4.5 + 7.48(Q_4/A_f) M_{t3}$	Pflanz (1969)
Tuntoolavest *et al.* (1980)	$-7.83 + 468 Q_a r - 70 r^2 + 14.59 M_{t3} + 13 r M_{t3}$ $-82.8 Q_a M_{t3} - 2.48 t_s M_{t3} + .162 M_{t3}(Q_4/A_f)$	Tuntoolavest *et al.* (1980)
Dietz and Keinath (1982)	$5.341 + .506 M_d - 1.406 t_l$	Dietz and Keinath (1982)
Chapman (1983)	$-180.6 + 4.03 M_{t3} + 133.24(Q_3/A_f)$ $+[90.16 - 62.54(Q_3/A_f)]H$	Chapman (1983)
Cashion and Keinath (1983)	$48.2 - 4.33\theta_c + 3.98\theta - .352\theta_c^2$ $-248\theta^2 + 28.6\theta_c\theta$	Cashion and Keinath (1983)

Note : A_f = surface area of secondary clarifier (m^2)
FM = food to microorganism ratio in the activated sludge system (g BOD/g MLSS/day)
H = side water depth (m)
Q_a = air flow rate to aeration tank (m^3/min)
Q_4 = effluent flowrate from secondary clarifier (m^3/hr)
Q_3 = influent flowrate to secondary clarifier (m^3/hr)
r = sludge recycle ratio to aeration tank
T = temperature of mixed liquor ($^\circ C$)
t_s = hydraulic detention time in secondary clarifier (hours)
t_l = detention time in clear zone (hours)
θ = hydraulic retention time in aeration basin (days)
θ_c = sludge age of the activated sludge system (days)
M_{t3} = mixed liquor suspended solids (MLSS) concentration (kg/m^3)
M_d = dilute blanket solids concentration (kg/m^3)

and Naito (1967) considered the effects of flow conditions on clarification efficiency using a calcium carbonate suspension. Pflanz (1969) reported results from a series of in-plant studies carried out in Germany. These experiments were carefully controlled to simulate steady state operation. The effluent solids concentration was shown to be proportional to the feed flow rate and solids concentration. Sludge settleability, temperature, and wind were also shown to affect clarifier performance. Lech (1973), Busby and Andrews (1975) and Keinath *et al.* (1977) have developed regression models from Pflanz's data.

Agnew (1972) proposed two models based on in-plant operating data. One of the models provided a satisfactory fit for short-term observations of effluent suspended solids concentrations. However, this model did not adequately predict the clarifier performance under varying operating conditions over a long period of time. A second model was then developed from data representing a wide range of operating conditions and sludges with different properties. This model included design parameters for the biological treatment unit as well as parameters representing the hydraulic efficiency of the clarifier. Both of Agnew's models predict that the effluent solids concentration decreases as MLSS concentration increases, which contradicts Pflanz's observations.

Tuntoolavest *et al.* (1980) used a laboratory-scale pilot plant facility supplied with synthetic wastewater in an attempt to resolve the issue over the effect of MLSS on clarification efficiency and to determine other design parameters that are important in influencing the clarification efficiency. Their results supported the trend predicted by Pflanz, i.e., that the effluent solids concentration increases with higher MLSS concentrations. They also observed that the turbulence level in the aeration tank, as measured by the air flow rate in their study, affected the clarification efficiency. This observation was consistent with the conclusion reached by Parker *et al.* (1971) that the floc-destructing environment of the aeration tank has a direct impact on sludge settling characteristics and the clarification efficiency. The thickening characteristics of the sludge were not found to be significantly related to changes of the design parameters they studied.

Dietz and Keinath (1982) presented a model based on a laboratory-scale study using calcium carbonate as settling particles. It was shown that the steady-state clarifier performance was most sensitive to the clear zone detention time in the clarifier. No consideration was given though to the issue of upstream operating conditions in an actual treatment plant.

Chapman (1983) studied the effects on clarification efficiency caused by a number of design variables. Among them, the side water depth of the clarifier, MLSS concentration, clarifier feed flow and underflow rates were found to have significant impacts on clarification efficiency. The air flow rate, however, was not an important factor. Chapman's results were also in agreement with Pflanz's observation that the effluent solids concentration increases with the MLSS concentration.

Cashion and Keinath (1983) studied the effects of solids retention time (SRT), hydraulic retention time (HRT), and clarifier overflow rate on the final clarifier solids removal efficiency in a laboratory-scale unit treating real wastewater. The SRT values in their study ranged from two to eight days, and the HRT values ranged from four to 12 hours. The effluent solids concentration was found to be insensitive to the overflow rate. High solids removal was attained in the regions defined by low SRT values and high HRT values or high SRT values and low HRT values. No apparent correlation was observed between the solids concentration of the influent to the clarifier and the effluent solids concentration.

Sludge settling characteristics were reported by Bisogni and Lawrence (1971) to be a function of sludge age. In their study with synthetic feed, activated sludge flocculated and settled better with increasing sludge age for sludge ages beyond three days. Dick *et al.* (1979) conducted similar experiments using real wastewater. They found that the correlation between sludge settling properties and sludge age was not significant. They also observed that influent suspended solids concentration exerted an effect on sludge settling behavior. It appears that currently there is no satisfactory model for predicting activated sludge settling characteristics as a function of operating parameters in the aeration tank.

The soluble BOD_5 concentration is assumed to be unchanged through sedimentation and sludge separation, i.e.,

$$S_3 = S_4 = S_5 = S_6 = S_7 \tag{2.27}$$

The total effluent BOD concentration includes both the soluble and the suspended portions. The effluent total suspended solids concentration is assumed to follow the model developed by Chapman for the secondary clarifier. The side water depth in Chapman's model is assumed to be a constant of 1.94 meters because the side water depth in the original pilot study was varied over only a small range (1.48 to 1.94 meters) and the effluent solids concentration is not very sensitive to this depth. The resulting model for secondary clarification becomes

$$M_{t4} = - c_1 + c_2 M_{t3} + c_3 \frac{Q_3}{A_f} \tag{2.28}$$

where M_{t3} and M_{t4} are both in g/m^3,

A_f is the surface area of the secondary clarifier, m^2,

and c_1, c_2 and c_3 are model parameters.

The effluent water quality requirements can be formulated as

$$S_3 + \left(\frac{1.42 \text{ g } BOD_L}{\text{g } cell}\right)\left(\frac{\text{g } BOD_5}{1.5 \text{ g } BOD_L}\right) f_d M_{a4} \leq S_{BOD} \tag{2.29}$$

$$M_{t4} \leq S_{TSS} \tag{2.30}$$

where S_{BOD} and S_{TSS} represent BOD_5 and total suspended solids restrictions, respectively, in the effluent, and are in g/m^3.

Since the volatile biodegradable suspended solids are assumed to be completely consumed during the activated sludge process (Section 2.3.2),

$$M_{d3} = M_{d4} = M_{d5} = M_{d6} = M_{d7} = 0$$

The ratios between the volatile inerts and the biomass and the inorganic solids and the biomass are assumed to be unaffected by secondary sedimentation or sludge separation. In other words,

$$\frac{M_{i3}}{M_{a3}} = \frac{M_{ij}}{M_{aj}} \quad ,j=4,5,6,7 \tag{2.31}$$

$$\frac{M_{f3}}{M_{a3}} = \frac{M_{fj}}{M_{aj}} \quad ,j=4,5,6,7 \tag{2.32}$$

Dick (1970) discussed the importance of including sludge thickening as an integral part of the design of a secondary clarifier. The underflow solids concentration from the clarifier is governed by the thickening model (equation (2.9)),

$$M_{t5} = [a_w(n_w - 1)]^{\frac{1}{n_w}}\left(\frac{n_w}{n_w - 1}\right)\left(\frac{A_f}{Q_5}\right)^{\frac{1}{n_w}} \tag{2.33}$$

where a_w and n_w are constants representing thickening properties of the waste activated sludge, and

$$Q_5 = (r + w)Q_2 \tag{2.34}$$

Decision variables selected for the design of the activated sludge process are the mean cell residence time, hydraulic retention time, and sludge recycle ratio.

2.3.4. Sludge Blending

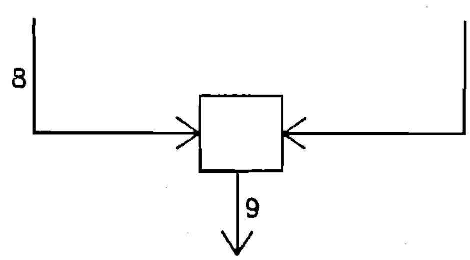


Figure 2.5 - Blending of Primary Sludge and Waste Activated Sludge

Since the primary and the waste activated sludges are combined before thickening (see Figure 2.5), a set of mass balance relationships is needed to calculate the characteristics of the influent to the thickener :

$$Q_9 = Q_7 + Q_8 \tag{2.35}$$

$$Q_9 M_{t9} = Q_7 M_{t7} + Q_8 M_{t8} \tag{2.36}$$

$$Q_9 S_9 = Q_7 S_7 + Q_8 S_8 \tag{2.37}$$

The settling characteristics of combined primary and waste activated sludge have been studied by Dick *et al.* (1978) and Suidan (1982) using plant operating data. Regression models were developed in both studies based on limited experimental data to relate the settling constants in equation (2.7) to the mass fraction of either the primary or the waste activated sludge.

The empirical relationships developed by Suidan are used to determine the thickening constants of the combined sludge:

$$a_c = a_w + a_1 f_p^{a_2} \tag{2.38}$$

$$n_c = n_w e^{n_1 f_p} \tag{2.39}$$

where f_p is the mass fraction of the primary sludge defined as

$$f_p = \frac{Q_8 M_{t8}}{Q_7 M_{t7} + Q_8 M_{t8}} \quad , \tag{2.40}$$

a_1, a_2, and n_1 are constants and a_c, n_c are constants characterizing the thickening of the combined primary and activated sludge.

2.3.5. Gravity Thickening

The design of the gravity thickener is illustrated by Figure 2.6. The underflow solids concentration is again calculated from equation (2.9),

$$M_{t11} = [a_c(n_c\text{-}1)]^{\frac{1}{n_c}} (\frac{n_c}{n_c - 1})(\frac{A_7}{Q_{11}})^{\frac{1}{n_c}} \tag{2.41}$$

where A_g is the surface area of the thickener in m^2.

The solids loading on the thickener is the decision variable. By definition, it is

$$L_g = \frac{Q_{11}M_{t11}}{A_g} \tag{2.42}$$

Combining equations (2.41) and (2.42),

$$M_{t11} = [a_c(n_c - 1)]^{\frac{1}{n_c-1}}\left(\frac{n_c}{n_c - 1}\right)^{\frac{n_c}{n_c-1}} L_g^{\frac{-1}{n_c-1}} \tag{2.43}$$

The flow and mass balance equations are

$$Q_{10} + Q_{11} = Q_9 \tag{2.44}$$

$$Q_{10}M_{t10} + Q_{11}M_{t11} = Q_9M_{t9} \tag{2.45}$$

There is no model available to predict the overflow solids concentration, M_{t10}. As a result, this concentration is treated as a parameter in the model, and is given a value of 0.2 kg/m^3.

The solids compositions in the thickener overflow and underflow are calculated from mass balance relationships based on the assumption that thickening does not affect the solids composition. For example,

$$M_{a10} = M_{a9}\frac{M_{t10}}{M_{t9}} = \frac{Q_7M_{a7} + Q_8M_{a8}}{Q_9}\frac{M_{t10}}{M_{t9}}$$

$$M_{a11} = M_{a9}\frac{M_{t11}}{M_{t9}} = \frac{Q_7M_{a7} + Q_8M_{a8}}{Q_9}\frac{M_{t11}}{M_{t9}}$$

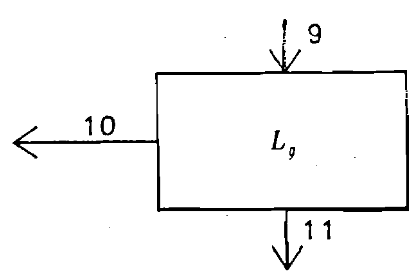


Figure 2.6 - Design of the Gravity Thickener

Similarly,

$$
\begin{aligned}
M_{d10} &= \frac{Q_8 M_{d8}}{Q_9} \frac{M_{t10}}{M_{t9}} \\
M_{d11} &= \frac{Q_8 M_{d8}}{Q_9} \frac{M_{t11}}{M_{t9}} \\
M_{i10} &= \frac{Q_7 M_{i7} + Q_8 M_{i8}}{Q_9} \frac{M_{t10}}{M_{t9}} \\
M_{i11} &= \frac{Q_7 M_{i7} + Q_8 M_{i8}}{Q_9} \frac{M_{t11}}{M_{t9}} \\
M_{f10} &= \frac{Q_7 M_{f7} + Q_8 M_{f8}}{Q_9} \frac{M_{t10}}{M_{t9}} \\
M_{f11} &= \frac{Q_7 M_{f7} + Q_8 M_{f8}}{Q_9} \frac{M_{t11}}{M_{t9}}
\end{aligned}
\tag{2.46}
$$

The soluble *BOD* is assumed not affected by gravity thickening, i.e.,

$$S_9 = S_{10} = S_{11} \tag{2.47}$$

2.3.6. Anaerobic Digester : Primary Tank

Conventional designs of an anaerobic digester use two-stage systems. The primary digester is generally mixed and heated to the fermentation temperature. Most sludge stabilization occurs in this unit. The secondary digester is not mixed and is primarily used to thicken the digested sludge.

The design of the primary digester depends on the kinetic model assumed for waste stabilization. There are several modeling approaches for the design of the primary digester. Lawrence and McCarty (1969) developed design equations based on Monod kinetics of substrate utilization. The underlying assumption for the Monod kinetics is that methane fermentation is the limiting step. Sewage sludge is a mix of complex organic solids, however, and it has been reported (Pfeffer, 1968) that except for very high loading rates, hydrolysis of the organic solids is the rate limiting step. A second modeling approach assumes that the stabilization rate is first order with respect to the biodegradable (under an anaerobic environment) volatile solids. The percent volatile solids that is degradable as well as the first order rate coefficient were found to be functions of digestion temperature for temperatures ranging from 25 to 35 ^{o}C (Pfeffer, 1981).

Chen and Hashimoto (1979, 1980) also proposed a set of equations for predicting digestion performance. To use this model for design, the biodegradable volatile solids concentration must be determined as a function of fermentation temperature.

Gossett and Belser (1982) studied the effect of sludge retention time in the activated sludge system on the performance of the anaerobic digester. A first order reaction was postulated for the conversion of active biomass in the digester influent into available substrate in the digester. The effect of temperature on digestion rate was not studied.

Wise (1980) summarized experimental results from studies involving stabilization of various organic residues at different temperatures. A first order kinetic model was assumed for total volatile solids destruction. Figure 2.7 depicts the correlation between the digestion rate coefficient and the fermentation temperature. The mathematical expression describing this relationship is

$$K_1 = 0.632 \exp[7.675(3.003 - \frac{1000}{T_d + 273})] \tag{2.48}$$

where K_1 is the first-order rate coefficient in day^{-1}, and T_d is the fermentation temperature in ^{o}C. This model is selected for primary digester design in this study because it covers a wide range of digestion temperatures. Consequently one decision variable for designing the primary digester is the fermentation temperature.

The primary digester is modeled as a complete-mix reactor where all sludge stabilization is assumed to take place (Figure 2.8). The solids compositions in the digester effluent are calculated based on the following assumptions: the volatile solids in the digester effluent are assumed to be nondegradable in the aerobic environment and to consist of no microorganisms that are capable of aerobic degradation of organic material. These assumptions are

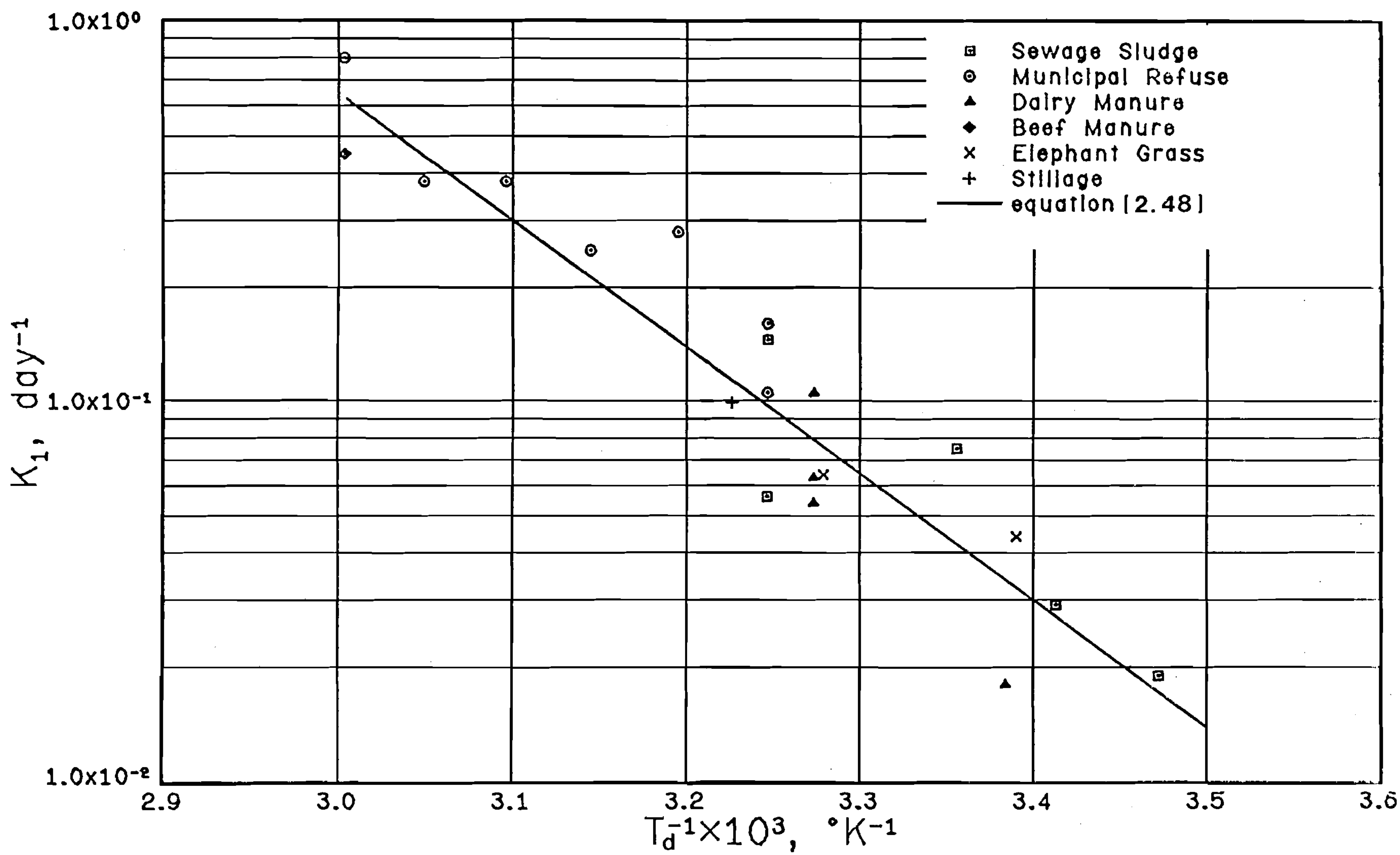


Figure 2.7 - Digestion Rate Coefficient as a Function of Fermentation Temperature

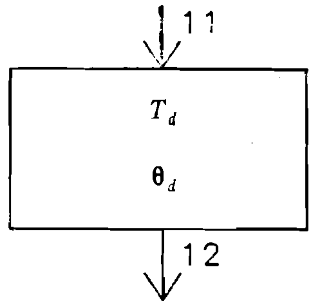


Figure 2.8 - Design of the Primary Digester

necessary in order to calculate the solids compositions of the digester supernatant which is recycled to the liquid processing train. The inorganic solids are assumed to be unaffected by anaerobic digestion. With these assumptions, the solids compositions of the primary digester effluent can be calculated as

$$M_{a12} = 0$$

$$M_{d12} = 0$$

$$M_{i12} = \frac{M_{a11} + M_{d11} + M_{i11}}{1 + K_1\theta_d} \tag{2.49}$$

$$M_{f12} = M_{f11}$$

where $$\theta_d = \frac{V_d}{24Q_{11}} \tag{2.50}$$

is the sludge age in days, which is equivalent to the hydraulic retention time for this digestion system without solids recycle, and V_d is the volume of the primary digester in m^3. The sludge age is the other decision variable for the design of this unit. No model is available for prediction of the soluble BOD_5 concentration of the digested sludge (S_{12}) when first order kinetics is used to describe the performance of the primary digester. Therefore, it is assumed to be a constant, 500 g/m^3, in this study.

The flowrate of the digester effluent is

$$Q_{12} = Q_{11} \tag{2.51}$$

The methane gas produced during digestion is calculated as

$$G = (1.42\frac{\text{kg } BOD_L}{\text{kg } VS})(0.35\frac{\text{m}^3\ CH_4}{\text{kg } BOD_L})Q_{11}(M_{a11} + M_{d11} + M_{i11})\frac{K_1\theta_d}{1 + K_1\theta_d}$$

$$+ (1.5\frac{\text{g } BOD_L}{\text{g } BOD_5})(0.35\frac{\text{m}^3\ CH_4}{\text{kg } BOD_L})(10^{-3}\frac{\text{kg}}{\text{g}})Q_{11}S_{11} \tag{2.52}$$

where G is the methane production rate in m^3/hr. The first term is the methane produced from stabilization of the volatile suspended solids, while the second term represents that from the soluble organics.

The energy value of the methane gas, E in $kWhr/yr$, is estimated to be

$$E = (35800\frac{kJ}{\text{m}^3\ CH_4})(\frac{1\ kWhr}{3600\ kJ})(8760\frac{hr}{yr})G$$

$$= 87113.3\ G \tag{2.53}$$

The heat requirements for raising the influent sludge to the digestion temperature, q_R in $kWhr/yr$, is

$$q_R = Q_{11}(10^3\frac{\text{kg}}{\text{m}^3})(4.2\frac{kJ}{\text{kg-}^oC})(T_d - T_0)(8760\frac{hr}{yr})(\frac{1\ kWhr}{3600\ kJ})$$

$$= 10.22 \times 10^3\ Q_{11}(T_d - T_0) \tag{2.54}$$

where T_0 is the influent sludge temperature in ^oC.

Assuming that the digester is approximately cylindrical, and all digester units are uniform in size, then the heat loss of the digester to the environment, q_L in $kWhr/yr$, can be estimated as

$$q_L = (8.76\frac{kWhr}{Watt\text{-}yr})UV_d a(T_d - T_a) \tag{2.55}$$

where U is the average heat conduction coefficient of the digester outside surfaces, $Watt/\text{m}^2\text{-}^oC$,

a is the ratio of the digester outside surface area to its volume, m^2/m^3,

and T_a is the average ambient temperature in ^oC.

The total heat requirement for the digester to maintain its operating temperature is

$$q = \frac{q_R + q_L}{\epsilon} \tag{2.56}$$

where ϵ is the heat transfer efficiency of the heat exchanger.

The net energy value of the digester gas is the energy produced by methane less the total heat requirement, or

$$N = E - q \tag{2.57}$$

where N is the net energy value in $kWhr/yr$. This net energy production from the digestion system is given a cost credit of 2.37 dollars/$10^6 kJ$ (0.25 dollars/therm) (Rimkus *et al.*, 1980) in the overall system economic analysis assuming the methane is used on site.

2.3.7. Anaerobic Digestion : Secondary Tank

The secondary digester is assumed to be unmixed and unheated, and is modeled as a gravity thickener with no methane fermentation taking place (Figure 2.9). The underflow solids concentration, from equation (2.43), is

$$M_{t14} = [\delta a_d(n_d - 1)]^{\frac{1}{n_d - 1}} \left(\frac{n_d}{n_d - 1}\right)^{\frac{n_d}{n_d - 1}} L_d^{\frac{-1}{n_d - 1}} \tag{2.58}$$

where $$L_d = \frac{Q_{14} M_{t14}}{A_d} \tag{2.59}$$

is the solids loading, and the decision variable for this unit,

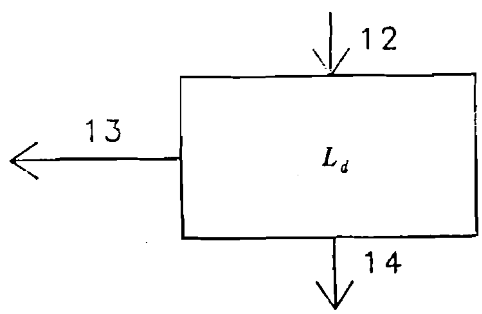


Figure 2.9 - Design of the Secondary Digester

A_d is the surface area of the secondary digester in m^2,

a_d and n_d are settling properties of a fully digested sludge,

and δ is a factor to discount the settling velocity of the digested sludge. In practice, the gas production in the secondary digester may be sufficiently high to cause some turbulence in the digester. The rising gas will reduce the settling velocity of the digested sludge, preventing the sludge from thickening to the degree expected from thickening theory alone. The use of the factor δ is intended to account for this observation. Initially, the value of δ is assumed to be 0.25. The sensitivity of the overall system design to this value is examined in Chapter 4.

There is no model available to predict the suspended solids concentration in the digester supernatant, M_{t13}. Therefore it is treated as a parameter in the model and is assumed to be a typical value of 4 kg/m^3.

The mass and flow balances around the secondary digester are

$$Q_{12} = Q_{13} + Q_{14} \tag{2.60}$$

$$Q_{12}M_{t12} = Q_{13}M_{t13} + Q_{14}M_{t14} \tag{2.61}$$

The soluble BOD_5 concentration is unaffected by this unit; therefore

$$S_{12} = S_{13} = S_{14} \tag{2.62}$$

The solids compositions are assumed to remain the same, or

$$\begin{aligned} M_{i13} &= M_{i12}\frac{M_{t13}}{M_{t12}} \\ M_{f13} &= M_{f12}\frac{M_{t13}}{M_{t12}} \\ M_{i14} &= M_{i12}\frac{M_{t14}}{M_{t12}} \\ M_{f14} &= M_{f12}\frac{M_{t14}}{M_{t12}} \end{aligned} \tag{2.63}$$

2.3.8. Vacuum Filtration

The design of the vacuum filter is shown schematically in Figure 2.10. Coackley and Jones (1956) compared several filtration theories and concluded that the model proposed by Carman (1933) fits experimental data most adequately. They developed the following equation for calculating the filter yield from Carman's analysis for given operating conditions and a sludge with known specific resistance,

$$L_f = 657.3\left(\frac{\chi P}{\mu r_s t_c}\right)^{\frac{1}{2}} W^{\frac{1}{2}} \tag{2.64}$$

where L_f is the filter yield in kg/m^2/hr,

χ is form time per cycle time,

P is the vacuum pressure applied in Newtons/m^2,

μ is the viscosity of filtrate in Newton-sec/m^2,

r_s is the specific resistance in m/kg,

t_c is the cycle time in minutes,

and $$W = \frac{Q_{16} M_{t16}}{Q_{15}} \tag{2.65}$$

is the mass of solids filtered per unit volume of filtrate in kg/m^3. Christensen (1983) has summarized the values of specific resistance for various sludges to be dewatered. The filter yield is the decision variable of this unit.

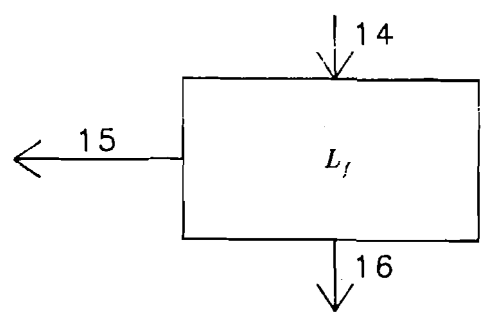


Figure 2.10 - Design of the Vacuum Filter

The size of the filter is

$$A_v = \frac{Q_{16} M_{t16}}{L_f} \tag{2.66}$$

where A_v is the filter area in m^2.

The mass and flow balance relationships around the unit give

$$Q_{14} = Q_{15} + Q_{16} \tag{2.67}$$

$$Q_{14} M_{t14} = Q_{15} M_{t15} + Q_{16} M_{t16} \tag{2.68}$$

The suspended solids concentration in the filtrate (M_{t15}) is assumed to be a constant of 2 kg/m^3 due to the lack of a predictive model.

The soluble BOD_5 concentration is the same throughout the process:

$$S_{14} = S_{15} = S_{16} \tag{2.69}$$

The solids components are:

$$\begin{aligned} M_{i15} &= M_{i14} \frac{M_{t15}}{M_{t14}} \\ M_{f15} &= M_{f14} \frac{M_{t15}}{M_{t14}} \\ M_{i16} &= M_{i14} \frac{M_{t16}}{M_{t14}} \\ M_{f16} &= M_{f14} \frac{M_{t16}}{M_{t14}} \end{aligned} \tag{2.70}$$

2.3.9. Recycle Streams

The side streams generated in sludge treatment are recycled back to the head end of the plant for the removal of the organics and the suspended solids (Figure 2.11). To arrive at a steady-state design of the system, flow and mass balances must be met where the recycle streams join the influent stream to the plant:

$$Q_1 = Q_0 + Q_{10} + Q_{13} + Q_{15} \tag{2.71}$$

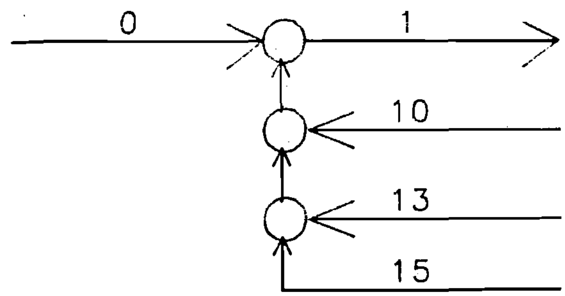


Figure 2.11 - Recirculation of Streams Generated in Sludge Treatment Back to the Plant Influent

$$Q_1S_1 = Q_0S_0 + Q_{10}S_{10} + Q_{13}S_{13} + Q_{15}S_{15} \tag{2.72}$$

$$10^{-3}Q_1M_{a1} = 10^{-3}Q_0M_{a0} + Q_{10}M_{a10} \tag{2.73}$$

$$10^{-3}Q_1M_{d1} = 10^{-3}Q_0M_{d0} + Q_{10}M_{d10} \tag{2.74}$$

$$10^{-3}Q_1M_{i1} = 10^{-3}Q_0M_{i0} + Q_{10}M_{i10} + Q_{13}M_{i13} + Q_{15}M_{i15} \tag{2.75}$$

$$10^{-3}Q_1M_{f1} = 10^{-3}Q_0M_{f0} + Q_{10}M_{f10} + Q_{13}M_{f13} + Q_{15}M_{f15} \tag{2.76}$$

$$M_{t1} = M_{a1} + M_{d1} + M_{i1} + M_{f1} \tag{2.77}$$

where M_{a0}, M_{d0}, M_{i0}, and M_{f0} are in g/m^3, and 10^{-3} is a unit conversion factor.

2.3.10. Sludge Disposal

Ultimate disposal of the sludge cake is an integral part of wastewater treatment systems. Multiple options are available, and they have been studied extensively by Dick *et al.* (1978, 1981) in their development of optimal sludge management strategies. Disposal by sanitary landfill is assumed in the base system.

The land area requirement is estimated using the following equation developed by Dick *et al.* (1978),

$$A_L = 3.62 \times 10^{-2} Q_{16}M_{t16} \tag{2.78}$$

where A_L is the land requirement in acres.

The wet tons of sludge landfilled per day, W_s, can be calculated, assuming a specific gravity of 1.04 for the dewatered sludge, as

$$W_s = (Q_{16}\frac{\mathrm{m}^3}{hr})(1.04\frac{\mathrm{g}}{\mathrm{cm}^3})(10^6\ \frac{\mathrm{cm}^3}{\mathrm{m}^3})(24\ \frac{hr}{day})(\frac{1\quad ton}{9.072\times 10^5\ \mathrm{g}})$$

$$= 27.513 Q_{16} \tag{2.79}$$

2.4. Cost Information

The total cost of the wastewater treatment system is the sum of the costs of all unit processes. Although cost data are abundant in the literature, only those data that relate costs to the capacities of the units are useful for this study. Smith (1968) developed cost functions from cost data collected by Logan *et al.* (1962) and Swanson (1966). Patterson and Banker (1971) presented the capital, operation and maintenance costs in graphical forms with respect to the sizes of the unit processes. Cost functions have been developed from this information by Middleton and Lawrence (1975), the U. S. Army Corps of Engineers (1978), and Rossman (1979). Dick *et al.* (1978) also developed a set of cost functions based on data presented by Patterson and Banker, Metcalf and Eddy, Inc. (1975), and Ettlich (1977).

These cost functions were compared for unit processes considered in this study using constant year (1971) dollars. The results of this comparison are summarized in Appendix A. Considerable variations in unit process costs were observed among different sources of data. Costs of wastewater treatment systems vary locally and depend on many factors. Therefore the cost functions considered in this study are only meaningful in the sense that they represent typical relative costs among unit processes.

Cost functions selected for use in this study are summarized in Table 2.4. They are based primarily on the data collected by Patterson and Banker. The firm pumping capacity is assumed to be two and a half times the average daily flow.

Costs for final sludge disposal by sanitary landfill are not listed in Table 2.4. These costs include capital and operation costs. The capital cost is calculated according to the equation presented by Rossman,

Table 2.4 - Summary of Cost Functions

	Capital (1971\$)	Operation (manhours/yr)	Maintenance (manhours/yr)	Material and Supply (1971\$/yr)	Power ($kWhr/yr$)
Primary Clarifier	$824A_p^{.77}$	$17.15A_p^{.6}$ ($A_p \geq 279$) $92.45A_p^{.3}$ ($A_p < 279$)	$9.23A_p^{.6}$ ($A_p \geq 279$) $106A_p^{.14}$ ($A_p < 279$)	$8.62A_p^{.76}$	–
Primary Sludge Pumping	$16042Q_8^{.53}$	$374Q_8^{.41}$	$166Q_8^{.43}$	$385Q_8^{.64}$	$23.85Q_8H/\epsilon_p$ †
Aeration Tank	$461V^{.71}$	–	–	–	–
Diffused Aeration	$8533Q_a^{.66}$	$187Q_a^{.48}$	$74.4Q_a^{.55}$	–	–
Secondary Clarifier	$824A_f^{.77}$	$17.15A_f^{.6}$ ($A_f \geq 279$) $92.45A_f^{.3}$ ($A_f < 279$)	$9.23A_f^{.6}$ ($A_f \geq 279$) $106A_f^{.14}$ ($A_f < 279$)	$8.62A_f^{.76}$	–
Return & Waste Sludge Pumping	$2779Q_5^{.53}$	$.333Q_5+390$	$.2375Q_5+370$	300 ($Q_5 < 63.2$) $40.57Q_5^{.52}$ ($Q_5 < 252$) $5.97Q_5^{.87}$ ($Q_5 < 632$) $2.54Q_5$ ($Q_5 > 632$)	$23.85Q_8H/\epsilon_p$
Gravity Thicker	$824A_g^{.77}$	$17.15A_g^{.6}$ ($A_g \geq 279$) $92.45A_g^{.3}$ ($A_g < 279$)	$9.23A_g^{.6}$ ($A_g \geq 279$) $106A_g^{.14}$ ($A_g < 279$)	$8.62A_g^{.76}$	–
Anaerobic Digester	$2323V_d^{.59}$	$1.29V_d^{.83}$ ($V_d \geq 5678$) $14V_d^{.55}$ ($V_d \geq 1968$) $192V_d^{.2}$ ($V_d < 1968$)	$0.83V_d^{.82}$ ($V_d \geq 5678$) $8.5V_d^{.55}$ ($V_d \geq 1968$) $113V_d^{.21}$ ($V_d < 1968$)	$14.4V_d^{.66}$ ($V_d \geq 2839$) $142V_d^{.37}$ ($V_d < 2839$)	–
Vacuum Filter	$29180A_v^{.71}$	$197.55Q_{16}^{.58}M_{t16}^{.58}$	$5.57Q_{16}^{.84}M_{t16}^{.84}$ ($Q_{16}M_{t16} \geq 519$) $20Q_{16}^{.63}M_{t16}^{.63}$ ($Q_{16}M_{t16} \geq 103$) $41.5Q_{16}^{.48}M_{t16}^{.48}$ ($Q_{16}M_{t16} < 103$)	$230Q_{16}^{.71}M_{t16}^{.71}+$ $182Q_{16}^{.86}M_{t16}^{.86}$	–
Recirculation Pumping	$2779Q_r^{.53}$ ‡	$0.333Q_r+390$	$0.2375Q_r+370$	300 ($Q_r \geq 63.2$) $40.57Q_r^{.52}$ ($Q_r < 252$) $5.97Q_r^{.87}$ ($Q_r < 632$) $2.54Q_r$ ($Q_r > 632$)	$23.85Q_rH/\epsilon_p$

† H is the pumping head in meters, and ϵ_p is the pumping efficiency.

‡ $Q_r = Q_{10} + Q_{13} + Q_{15}$

$$CC = A_L C_L + 6200 F_u W_s^{0.74} \tag{2.80}$$

where CC is the capital cost in present value (P.V.) dollars,

C_L is the unit cost of land, P.V. dollars/acre,

and F_u is a factor updating the cost from 1971 dollar to the present value.

Equation (2.80) can be rewritten in terms of Q_{16} and M_{t16} by substituting (2.78) and (2.79) for A_L and W_s, respectively,

$$CC = 3.62 \times 10^{-2} C_L Q_{16} M_{t16} + 72053 F_u Q_{16}^{0.74} \tag{2.81}$$

The annual manhours for the landfill operation is estimated using data from the U. S. Environmental Protection Agency (USEPA) Process Design Manual (1974),

$$OHRS = 8024 Q_{16}^{0.667} \tag{2.82}$$

where $OHRS$ is the annual operation manhour requirement for a landfill. The development of equation (2.82) is described in Appendix B.

The total annual cost in 1980 dollars is used to express the total system cost. A twenty-year design life and a $7\frac{3}{8}\%$ discount rate are assumed to amortize the capital costs. The USEPA National Average Wastewater Treatment Plant Index is used to update the capital costs and the costs for material and supply. Annual operation and maintenance costs are calculated by multiplying the manhour requirement by the hourly wage rates. The cost for pumping is the product of the power requirement and the unit power cost.

2.5. System Design

A complete set of equations for designing a secondary wastewater treatment system is presented in Section 2.3. The design of the overall treatment system for specified influent conditions and decision variables using these equations is illustrated in this section. In Chapter 3, a comprehensive model assembled based on a subset of the design equations described in Section 2.3 is presented. This model is optimized using a nonlinear programming algorithm to generate cost-effective designs for the studied wastewater treatment system. Description of this model is provided in Section 3.2.1.

There are nine degrees of freedom in the wastewater treatment system model. Therefore a complete system design requires specification of nine decision variables. The selected decision variables in the model are summarized in Table 2.5.

Bounds are imposed on the decision variables in the comprehensive system model. Table 2.6 summarizes these bounds. Most of the bounds cover typical range observed in practice for the conventional activated sludge process. The values of these bounds are set arbitrarily, but are relatively reasonable to avoid the possible lack of efficiency of an

Table 2.5 - Summary of Decision Variables in the Model

Unit	Decision Variables
Primary Sedimentation	Overflow Rate (L_p)
Activated Sludge (Aeration + Final Sedimentation)	Mean Cell Residence Time (θ_c) Hydraulic Retention Time (θ) Sludge Recycle Ratio (r)
Gravity Thickening	Solids Loading (L_g)
Anaerobic Digestion	
-Primary	Digestion Temperature (T_d) Solids Residence Time (θ_d)
-Secondary	Solids Loading (L_d)
Vacuum Filtration	Filter Yield (L_f)

Table 2.6 - Bounds on the Decision Variables

Variables	Lower Bound	Upper Bound
Overflow Rate, Primary Clarifier (m/hr)	0.5	6.0
Mean Cell Residence Time (days)	2.0	6.0
Hydraulic Retention Time (days)	0.1	0.5
Sludge Recycle Ratio	0.1	1.0
Solids Loading, Gravity Thickener (kg/m^2-hr)	0.5	2.0
Digestion Temperature ($^\circ$C)	20	60
Residence Time, Primary Digester (days)	5	30
Solids Loading, Secondary Digester (kg/m^2-hr)	0.5	2.0
Filter Yield (kg/m^2-hr)	5	50

optimization algorithm. Exceptions are the bounds on the activated sludge mean cell residence time and on the digestion temperature. Bounds imposed on the mean cell residence time are to prevent the process from failure. In addition to insure against process failure, the bounds on the digestion temperature define the domain on which the empirical model (equation (2.48)) is based. The solids concentration of the filtered cake is also constrained to be less than 150 kg/m^3 because the process model used for vacuum filter design does not predict a maximum cake concentration that can be obtained in practice. If the final solution obtained from optimizing the system design model suggests that some of the decision variables are at their imposed bounds, then the roles of these bounds are examined in detail. This is carried out in Chapter 4.

Design of the overall system may be carried out using several approaches once the decision variables are specified. A straightforward approach was employed in this study: unit processes are designed sequentially according to the system flowchart. Since only a few eqúations are solved in the design of each unit process, the computation required for one iteration of design is not excessive. However, a steady state design cannot be obtained in one iteration because of the presence of the recycle streams in the system. Characteristics of the recycle streams, however, are determined at the end of each iteration. A new set of influent conditions to the plant is calculated by mass balance relationships between the design plant influent and the recycle streams. A new iteration is then initiated using the newly calculated influent conditions. This direct substitution process is continued until the fractional changes of all influent state variable values become less than 10^{-6}.

An analysis computer program was written to carry out the calculations. Figure 2.12 shows the logic on which the design of the analysis program is based. The listing of the program and the instructions for using the program are given in Appendix C. More efficient calculation schemes than direct substitution for updating the initial design conditions are available (Westerberg *et al.*, 1979). However, since a typical steady-state design can be

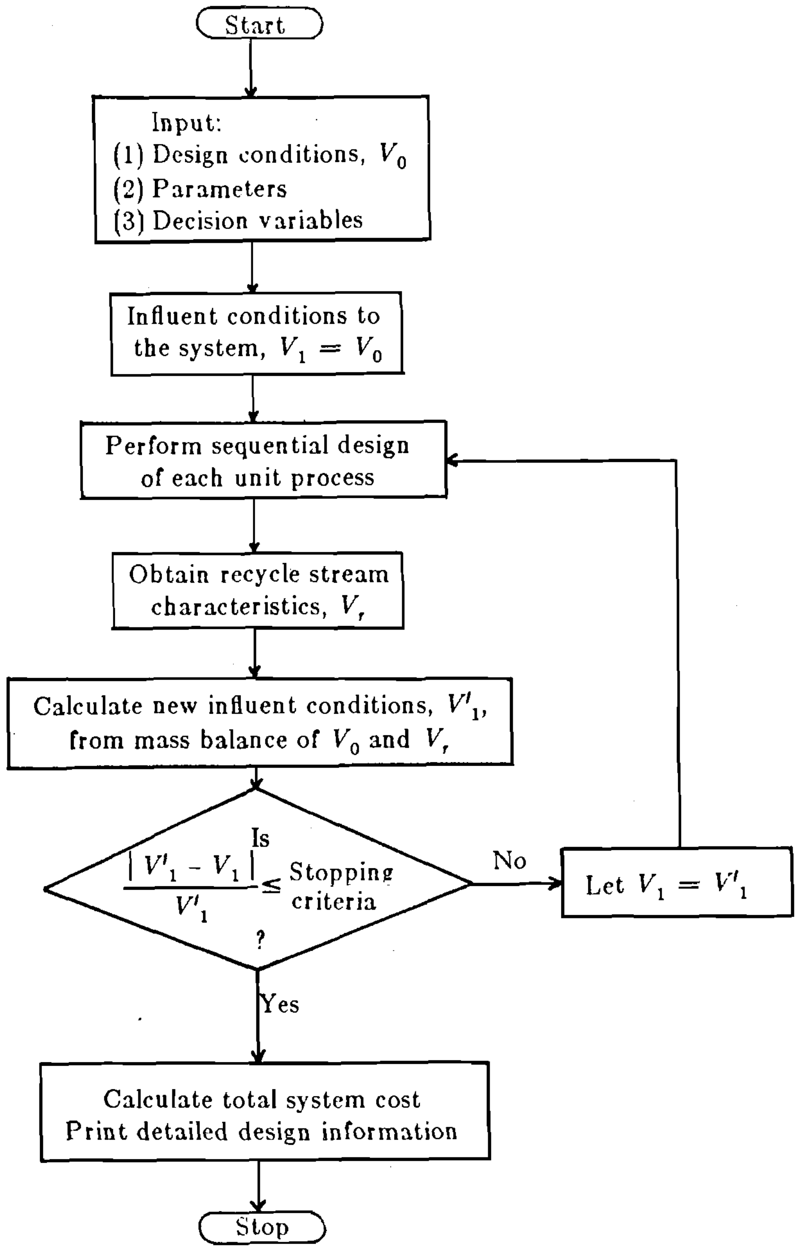


Figure 2.12 - Flow Diagram of the Analysis Program

achieved in less than ten iterations with computer time less than 0.3 seconds on the CDC Cyber 175 computer, the direct substitution strategy was considered adequate for this study.

An example system design obtained from using the analysis program is presented below. The wastewater treatment system was assumed to receive a typical domestic sewage with characteristics listed in Table 2.7. The parameters in the model are tabulated in Table 2.8. The values of the nine decision variables used for the system design are summarized in Table 2.9. Figure 2.13 (refer to Section 2.2.2 for the notation) describes the complete system design obtained from the analysis program for the conditions listed in Tables 2.7 to 2.9. It is noted that any arbitrarily selected values for the decision variables may lead to a design that does not meet the effluent requirements or may result in a filtered cake more concentrated than 150 kg/m^3. Such a design is called an infeasible design.

The analysis program is useful for examining the responses from the system model for given influent and design conditions and for generating system designs that can be used as starting solutions in various optimization procedures. This is illustrated in more detail in Chapter 3.

Table 2.7 - Base Design Conditions

Flowrate (m^3/hr)	1500
Soluble BOD$_5$ (g/m^3)	100
Active Biomass Conc. (g/m^3)	5
Volatile Biodegradable Suspended Solids Conc. (g/m^3)	100
Volatile Inert Suspended Solids Conc. (g/m^3)	45
Fixed Suspended Solids Conc. (g/m^3)	50
Total Suspended Solids Conc. (g/m^3)	200

Table 2.8 - Summary of Parameters in the System Model

Names (Units)	Value
Economic Data:	
Capital Recovery Factor	0.09716
Base (1971) Cost Index	150.6
Cost Index for 1980	362.0
Operating/Maintenance Wages (dollars/hr)	8.9
Land Cost, C_L (dollars/acre)	5000
Electricity Cost (dollars/ $kWhr$)	0.05
Pumping Head, H (meters)	10.0
Pumping Efficiency, ϵ_p	0.6
Primary Sedimentation:	
Constant in Voshel-Sak Model, v_1	0.139
Constant in Voshel-Sak Model, v_2	0.27
Constant in Voshel-Sak Model, v_3	0.22
Sludge Settling Characteristics:	
Thickening Constant, a_w	24.24
Thickening Constant, a_1	174.77
Thickening Constant, a_2	2.5
Thickening Constant, n_w	2.3747
Thickening Constant, n_1	0.1659
Activated Sludge Kinetics:	
Growth Yield Coefficient, y (g cell/g BOD_5)	0.4
Half-Velocity Constant, K_s (g BOD_5/m^3)	60
Maximum Specific Utilization Coeff., k (day^{-1})	5.0
Endogeneous Decay Coefficient, b (day^{-1})	0.04
Fraction of cells Degradable, f_d	0.77
Conversion (g BOD_L/g cell)	1.42
Conversion (g BOD_L/g BOD_5)	1.5
Secondary Sedimentation:	
Constant in Chapman Model, c_1	5.69
Constant in Chapman Model, c_2	0.00403
Constant in Chapman Model, c_3	11.91
Aeration:	
Alpha Factor in Aeration	0.8
Beta Factor in Aeration	0.95
DO Concentration in Aeraton Tank, DO (g/m^3)	1.5

Table 2.8 (continued)

Names (Units)	Value
DO Saturation Concentration, C_s (g/m^3)	9.17
Temperature of Mixed Liquor, T_L ($^\circ$C)	20.0
Oxygen Transfer Efficiency, OTE	0.08
Density of Air, ρ_{air} (kg/m^3)	1.2
Weight Fraction of Oxygen in Air, γ	0.232
Mixing Requirement, η (m^3 air/m^3/min)	0.02
Gravity Thickening:	
TSS of Thickener Supernatant, M_{t10} (kg/m^3)	0.2
Anaerobic Digestion:	
Temperature of Digester Influent, T_0 ($^\circ$C)	20.0
Methane Production (m^3/kg BOD$_L$)	0.35
Average Ambient Temperature, T_a ($^\circ$C)	10.0
Efficiency of Heat Exchanger, ϵ	0.85
Heat Conduction Coefficient, U (W/m^2-$^\circ$C)	1.0
Outside Surface Area and Volume Ratio for Digester, a	0.4
Worth of Digester Gas (dollars/10^6 kJ)	2.37
Soluble BOD$_5$ in Digester Supernatant, S_{12} (g/m^3)	500
Factor Accounting For Effect of Rising Gas on Thickening in Secondary Digester, δ	0.25
Thickening Constant for Digested Sludge, a_d	292.6
Thickening Constant for Digested Sludge, n_d	2.9
TSS of Digester Supernatant, M_{t13} (kg/m^3)	4.0
Height of Digester (m)	10.0
Vacuum Filtration:	
Form Time per Cycle Time, χ	0.33
Pressure Applied on Vacuum Filter, P (Nt/m^2)	83300
Viscosity of Filtrate, μ (Nt-sec/m^2)	0.00089
Cycle Time, t_c (min)	6.0
Specific Resistance of Sludge, r_s (m/kg)	10^{12}
TSS of Filtrate, M_{t15} (kg/m^3)	2.0
Effluent Standards:	
BOD$_5$ Concentration (mg/l)	30
TSS Concentration (mg/l)	30

Table 2.9 - Decision Variables for Example Treatment System Design

Decision Variables (Unit)	Value
Primary Clarifier Overflow Rate (m/hr)	3.0
Mean Cell Residence Time (days)	3.0
Hydaulic Retention Time (days)	0.15
Sludge Recycle Ratio	0.15
Solids Loading on Thickener ($kg/m^2/hr$)	1.0
Digestion Temperature (°C)	35
Retention Time in Digester (days)	15
Solids Loading on Digester ($kg/m^2/hr$)	1.0
Filter Yield ($kg/m^2/hr$)	8.0

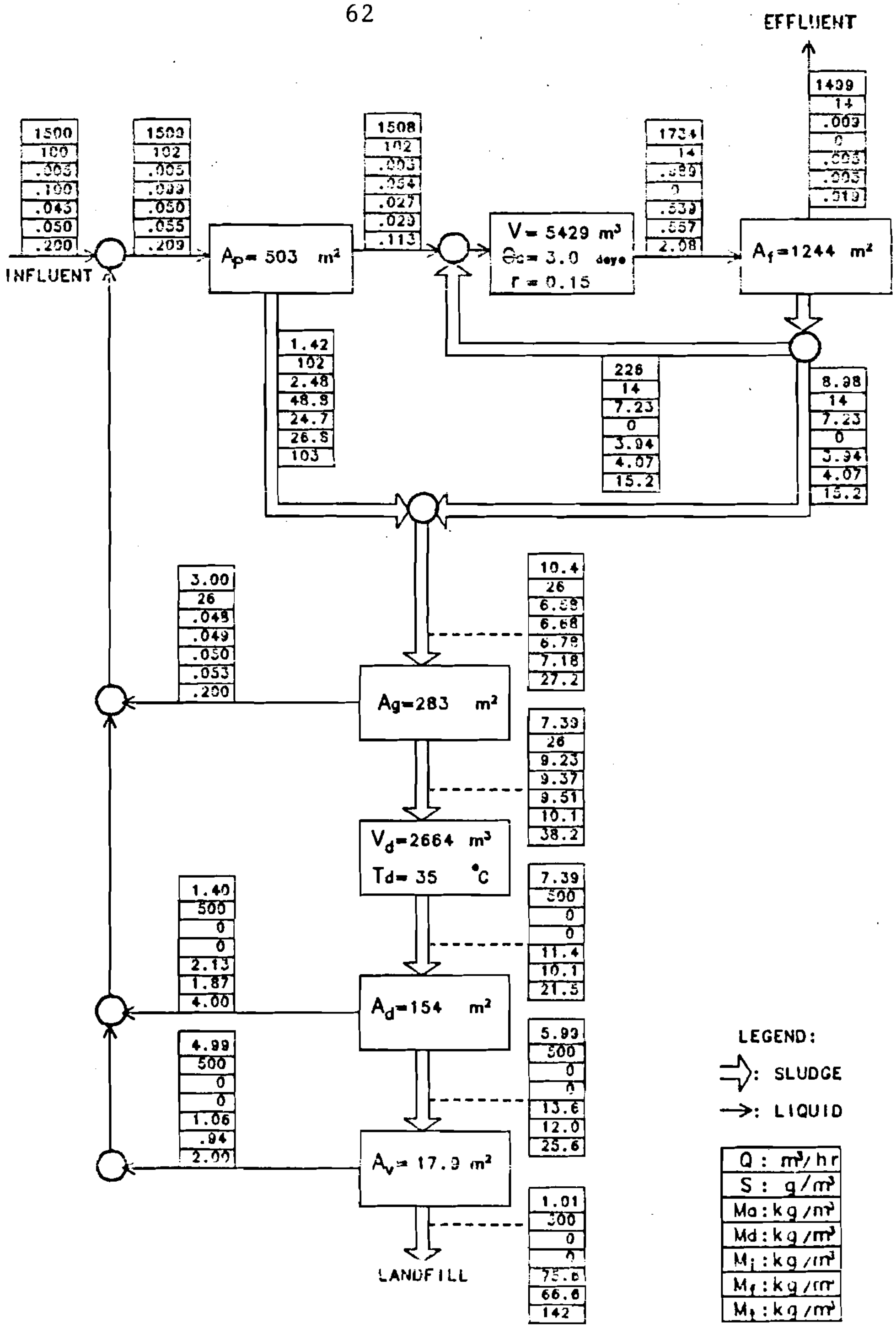


Figure 2.13 - An Example System Design Generated Using the Analysis Program

CHAPTER 3

OPTIMIZATION OF THE COMPREHENSIVE SYSTEM MODEL

3.1. Introduction

As described in Chapter 2, the design of a wastewater treatment system is formulated as an optimization model in which the total system cost is to be minimized subject to the unit process performance models and the effluent water quality requirements. This chapter discusses the techniques that were used in this study for solving the comprehensive system model. Illustrations of the use of these solution techniques are presented, and performances of these techniques are discussed.

The comprehensive system model is highly nonlinear; the objective function and the majority of the constraints are nonlinear. Most constraints are equations; exceptions are the ones specifying effluent water quality and the mixing requirement in the aeration tank. The problem is poorly scaled, usually with overflow and underflow rates (expressed in the same unit) from a separation unit differing in magnitude by several orders of ten. The complex arrangement of the units in the system appears to make it impractical to apply dynamic programming as the solution technique even though stages and states are clearly defined by the model. One approach to optimization examined in this study is to apply a well-tested nonlinear programming algorithm to solve the comprehensive system model directly. The generalized reduced gradient (GRG) algorithm developed by Lasdon *et al.* (1978), named GRG2, has been applied to many highly nonlinear programs with success. Studies of the computational experience with various constrained nonlinear programming methods have shown that the GRG algorithm is among the most efficient ones (Warren and Lasdon, 1979). GRG2 is well designed so that it competes favorably with more advanced algorithms such as sequential quadratic programming in terms of robustness and reliability (Schittkowski, 1982). Section 3.2 describes the use of GRG2 to optimize the comprehensive system model.

Special-purpose optimization algorithms developed for efficient solution of models with special characteristics may also be used to solve the comprehensive system model. The Interactive Generalized Geometric Programming (IGGP) code designed by Burns and Ramamurthy (1982) is an efficient algorithm with the capability of solving large-scale geometric programs. This algorithm is based on the primal condensation method proposed by Avriel *et al.* (1975). Burns and Ramamurthy extend this algorithm to solve problems with equality constraints. This extension allows the use of IGGP for solving the comprehensive system model. This is illustrated in Section 3.3.

A unique optimization procedure designed to take advantage of the special structure of the wastewater treatment system model was developed and is evaluated in Section 3.4. To solve the comprehensive system model by nonlinear programming directly, all equations have to be solved simultaneously. This mathematical operation is very costly with respect to computing requirements. A wastewater treatment system is generally composed of a liquid processing train and a sludge processing train, each consisting of individual unit processes provided to perform various treatment functions. By decomposing the entire treatment system, a series of subproblems with lower dimensionality can be solved instead of a large problem. Optimization techniques can be applied more effectively for solving these smaller problems, but coordination of the solutions is also required.

3.2. Generalized Reduced Gradient Algorithm for Optimization

The generalized reduced gradient algorithm is an extension of the reduced gradient algorithm by Wolfe (1963, 1967) to allow the solution of problems with nonlinear constraints. The earliest development of the algorithm was by Abadie and Carpentier (1969). Later improvements of the algorithm have incorporated many strategies for solving subproblems during the overall optimization procedure (see, for example, Himmelblau, 1972). GRG2 was used in this study.

GRG2 solves the following general nonlinear program:

$$\begin{aligned} \text{Minimize} \quad & c(\mathbf{X}) \\ \text{subject to} \quad & \mathbf{g}(\mathbf{X}) \leq 0 \\ & \mathbf{h}(\mathbf{X}) = 0 \\ & \mathbf{X}_L \leq \mathbf{X} \leq \mathbf{X}_u \end{aligned} \tag{3.1}$$

where c is a scalar objective function, and is the total cost of the wastewater treatment system in the comprehensive system model,

$\mathbf{X}$ is the vector of the variables in the model,

$\mathbf{g}$ is the vector of the inequality constraints,

$\mathbf{h}$ is the vector of the equality constraints,

and $\mathbf{X}_L$ and $\mathbf{X}_u$ are vectors representing the lower and the upper bounds of the variables, respectively.

The underlying concepts in developing GRG2 are described in detail by Lasdon *et al.* (1978).

3.2.1. Optimization Procedure

To make an optimization run, the user is asked to provide two files: one containing the program control parameter, initial solution to the problem, and bounds on the variables, and another specifying the model objective function and constraints. Instructions on using the program on the CDC Cyber computer can be found in the GRG User's Guide prepared by the Computing Services Office at the University of Illinois (1982).

The optimization model solved by GRG2 includes 64 variables and 55 design equations and three inequality constraints. The model is constructed based on the design equations discussed in Section 2.3. Detailed descriptions about the variables and the constraints in this optimization model are provided in Appendix D.

The control parameters in GRG2 are critical to the likelihood of obtaining convergence of the optimization procedure as well as to the quality of the final solution. The derivatives of the functions were approximated by the central differencing method. An equality constraint, $g(\mathbf{X}) = 0$, is considered to be satisfied when its value is in the ζ-neighborhood of zero, i.e., $|g(\mathbf{X})| \leq \zeta$. The value of this tolerance, ζ, was initially set to be 10^{-2}. The objective function generally improved significantly as the algorithm proceeded with this tolerance level. When the fractional change in the objective function became less than 10^{-4} for three consecutive iterations, the value of ζ was tightened to 10^{-4}. Then a phase-I optimization, which minimizes the sum of the constraint infeasibilities, was initiated until all constraints were satisfied to this final tolerance level and a feasible solution was found. Optimization of the true objective function was then begun until the termination criteria were met. The final solution obtained with this strategy was generally found to be superior to that obtained using a tight (10^{-4}) tolerance level throughout the optimization.

The basic variables were estimated using quadratic extrapolation. The one step version of the Broyden-Fletcher-Shanno variable metric method (see, for example, Avriel, 1976) was selected for generating search directions in the GRG2 runs.

Scaling of the variables as well as the constraints in the model has a direct effect on whether the optimization will be successful or not. No general rules are available; scaling nonlinear programs, as described by Lasdon and Beck (1981), is a "black art". Most variables in the model were scaled to have numerical values between 0.1 and 100 as suggested by the authors of GRG2. Some constraints were also scaled by trial-and-error in an attempt to achieving a balance among all constraints. Scaling factors in the optimization model solved by GRG2 are discussed in Appendix D.

3.2.2. Performance of GRG2

The efficiency of GRG2, the quality of the solutions obtained, and the effects on the solution of the imposed bounds on the selected variables are discussed in this subsection.

The computing time required for an optimization run varies with the starting solution and is highly dependent on the quality of the final solution. For all the GRG runs made in this study, the computing time never exceeded two minutes of central processing (CP) time on a CDC Cyber 175 computer when the program was run in batch mode with the control parameter values specified in Section 3.2.1. A FORTRAN V compiler was used to compile the program that contains the objective function and the constraints.

Based on the results from a number of test runs, it was noticed that varying some of the control parameters may result in a slightly better solution or a slightly faster optimization process for a particular starting solution and set of design conditions. However, in order for the results to be consistent and comparable, the control parameters used for running GRG2 were kept the same for all runs.

Computing experiences of some previous studies involving wastewater treatment system design models are listed in Table 3.1 for comparison. Although a straight comparison of the computing time requirements is not meaningful, this table does seem to indicate that the computing time using GRG2 for the comprehensive system model is at least comparable since the model solved is more complex than the others listed.

Because the model is highly nonlinear, multiple local optima are expected to be present. Different starting solutions were used to examine this issue. Table 3.2 summarizes the results of using five different starting solutions. The final solutions have objective function values that vary from 502,000 to 584,700 dollars/year, representing improvements in the objective function from the initial solutions from 17 (starting point No. 1) to 33% (starting point No. 5). All solutions call for designs that produce effluents exactly meeting the

Table 3.1 - Computing Experience in Optimizing Wastewater Treatment System Design

	Optimization Method	Execution Time (seconds)	Machine	Comments
Tang	GRG2	51-105*	CDC Cyber 175	9 degrees of freedom, 58 constraints, 64 variables.
Other Investigators :				
Middleton & Lawrence (1976)	Graphical Enumeration	96	IBM 360/65	5 degrees of freedom, No recycle.
Craig *et al.* (1978)	Box-Complex	1.65-2.82	CDC Cyber 173	5 degrees of freedom, No recycle.
Tyteca & Smeers (1981)	GRG for a geometric program	124-262	IBM 370/158	8 degrees of freedom, 35 constraints, 33 variables.

* For the base treatment system shown in Figure 2.1.

BOD_5 and total suspended solids standards. Among the five starting solutions, No. 4 and No. 5 differ only in the primary clarifier overflow rate, but the optimization results are very different. This is due to the fact that the initial solutions are quite different in the values of variables other than the decision variables. Figures 3.1 and 3.2 illustrate these two designs (notation is defined in Section 2.2.2). The design obtained with the higher overflow rate (starting point No. 5, Figure 3.2) has a higher mixed liquor suspended solids concentration in the aeration tank and has to waste more activated sludge for the same hydraulic retention time, sludge age, and sludge recycle ratio. Therefore the combined primary and waste activated sludges in the two designs exhibit quite different characteristics which result in very different values of the state variables when the sludge processing train is designed using the same design criteria. The importance of the choice of starting solution when using GRG2 to solve the comprehensive system model is obvious from this example.

The solution obtained by GRG is directly related to the bounds on the variables. This is best illustrated by an example. Two optimization runs starting from the same solution

Table 3.2 - Summary of Wastewater Treatment System Designs Obtained Using Different Starting Points

Variables (Units)	Starting Point				
	1	2	3	4	5
Primary Clarifier Overflow Rate (m/day)					
initial	72.0	36.0	36.0	24.0	72.0
final	115.9	80.0	78.2	17.4	144.0
Mean Cell Residence Time (days)					
initial	3.0	2.0	5.0	3.0	3.0
final	2.20	2.22	2.22	2.36	2.19
Hydraulic Retention Time (hr)					
initial	3.6	2.4	3.6	3.6	3.6
final	3.5	3.7	3.6	3.4	3.7
Sludge Recycle Ratio (%)					
initial	15.0	15.0	30.0	30.0	30.0
final	13.7	11.6	12.2	10.0	12.6
Solids Loading on Thickener (kg/m^2/day)					
initial	24.0	24.0	24.0	36.0	36.0
final	13.4	12.5	13.9	13.2	12.6
Digestion Temperature ($^\circ$C)					
initial	35.0	30.0	50.0	25.0	25.0
final	44.9	60.0	60.0	60.0	60.0
Retention Time in Digester (days)					
initial	15.0	15.0	25.0	20.0	20.0
final	17.7	14.7	12.9	16.3	13.9
Solids Loading on Digester (kg/m^2/day)					
initial	24.0	12.0	12.0	24.0	24.0
final	29.3	38.4	41.6	36.6	40.4
Filter Yield (kg/m^2/hr)					
initial	8.00	10.0	10.0	10.0	10.0
final	7.47	6.79	6.62	7.48	6.70
Cake Solids Concentration (kg/m^3)					
initial	142.2.	164.1†	164.1†	53.86	53.86
final	142.9	150.0	150.0	77.89	143.7
Effluent BOD$_5$ (mg/l)					
initial	20.5	30.8†	15.1	36.5†	27.5
final	30.0	30.0	30.0	30.0	30.0
Effluent TSS (mg/l)					
initial	19.3	23.9	20.0	59.5†	39.6†
final	30.0	30.0	30.0	30.0	30.0
Total System Cost (10^3 \$/yr)					
initial	656.0	678.0	654.7	738.1	748.0
final	542.9	506.1	506.7	584.7	502.0
Computer Time (CP seconds)	51.59	50.79	53.36	53.57	99.75

† : infeasible

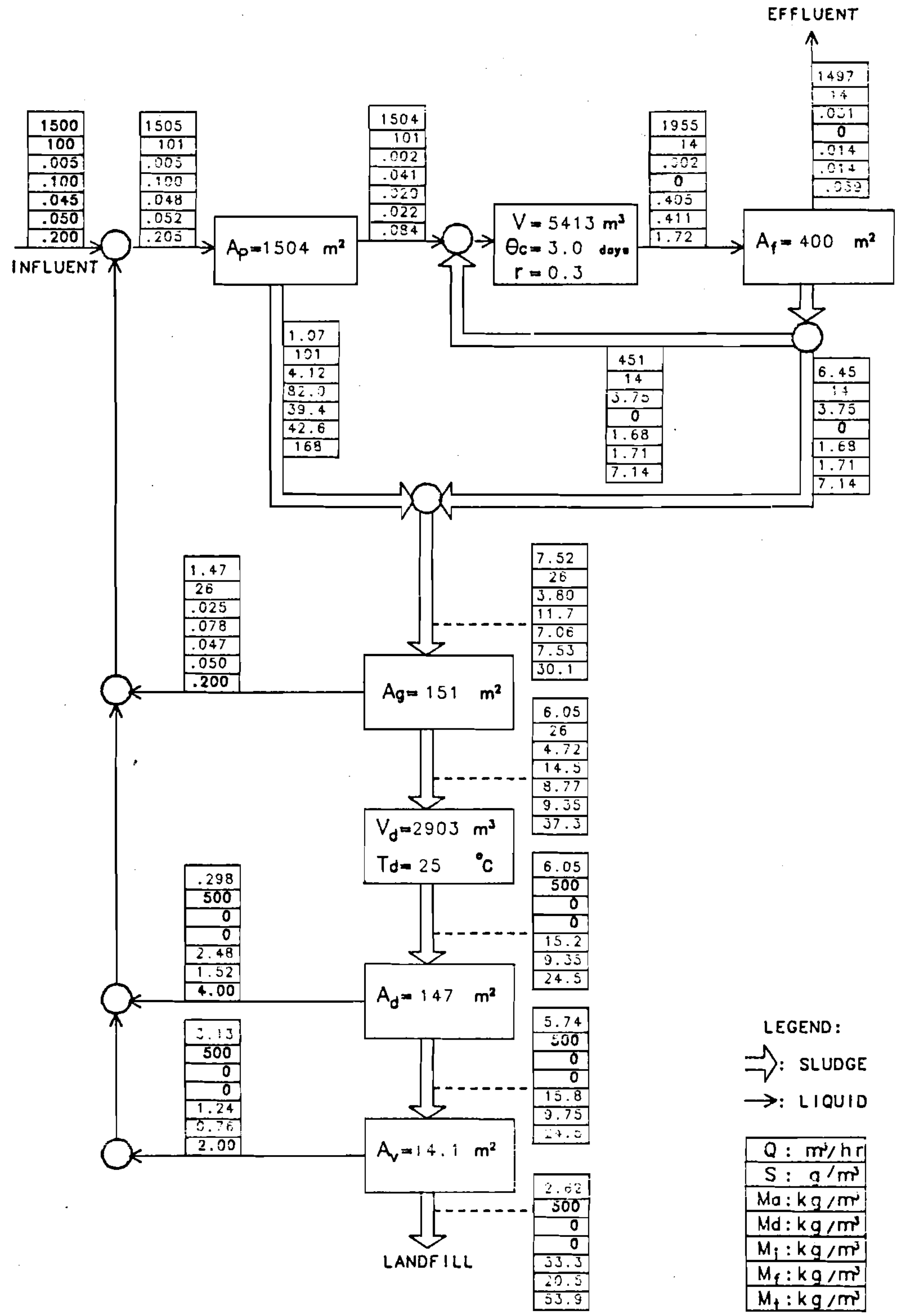


Figure 3.1- Wastewater Treatment System Design Using Starting Point No. 4

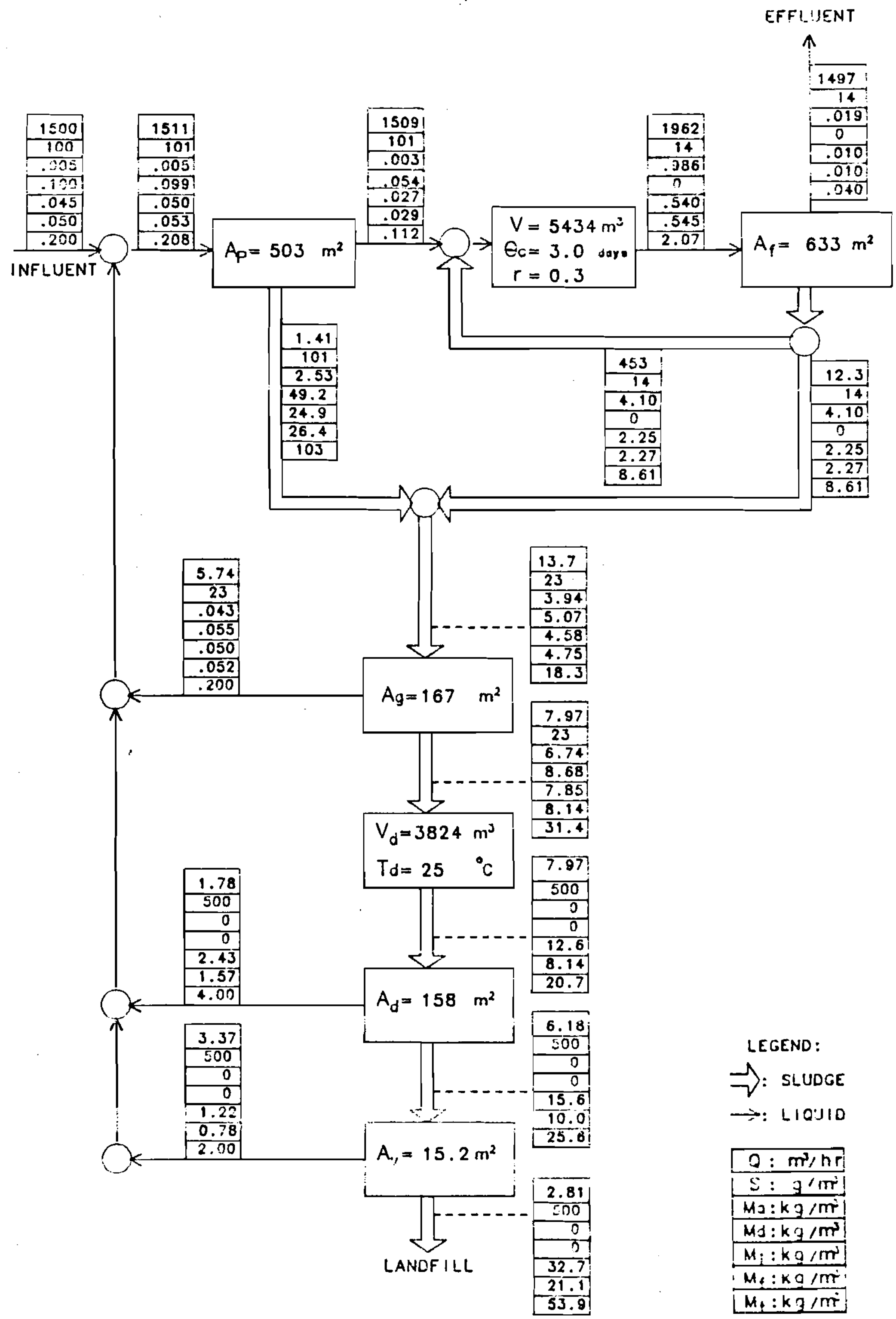


Figure 3.2- Wastewater Treatment System Design Using Starting Point No. 5

Table 3.3 - Solution Obtained Using GRG with Different Bounds on Selected Variables

Variables (Units)	Starting Point	Solution With Default Bounds	Solution With Modified Bounds
Primary Clarifier Overflow Rate (m/day)	36.0	80.0	43.8
Mean Cell Residence Time (days)	2.0	2.22	2.26
Hydraulic Retention Time (hr)	2.4	3.7	3.5
Sludge Recycle Ratio (%)	15.0	11.6	11.5
Solids Loading on Thickener ($kg/m^2/day$)	24.0	12.5	12.6
Digestion Temperature (°C)	30.0	60.0*	60.0*
Retention Time in Digester (days)	15.0	14.7	14.7
Solids Loading on Digester ($kg/m^2/day$)	12.0	38.4	36.3
Filter Yield ($kg/m^2/hr$)	10.0	6.79	6.92
Cake Solids Concentration (kg/m^3)	164.1	150.0*	150.0*
Effluent BOD_5 (mg/l)	30.8	30.0	30.0
Effluent TSS (mg/l)	23.9	30.0	30.0
Total System Cost (10^3 $/yr)	678.0	506.1	517.5
Computer Time (CP seconds)	–	50.79	39.53

* These values are at their specified bounds.

(starting point No. 2 in Table 3.2) were made with slightly different bounds on the decision variables. The solution shown in the second column of Table 3.3 was obtained using the default bound set summarized in Table 2.6. In the second optimization run, the upper bound on the primary clarifier overflow rate was changed from the default value of 144 to 240 meters/day, and the lower bound of the solids loadings on both the gravity thickener and the secondary digester were changed from 12 to 2.4 $kg/m^2/day$. These numbers have little physical significance and were used only for this experiment. The results of this run are summarized in the last column of Table 3.3. It is observed that the final objective function values are different by 2.3%. Note that none of the three decision variables for which the bounds were modified is at its bound in the final solution. The overflow rate for the primary clarifier in the two final solutions is the variable that showed the most significant difference in the two designs. This appears to be a weakness of GRG2 since most bounds on the decision variables, as described in Section 2.5, were arbitrarily selected and have little fundamental significance. Different nonbasic variables could be selected in GRG2 if different bounds are specified on the variables, resulting in different optimization processes and different solutions. Ideally, the optimal solution should not depend heavily on the bounds specified for the variables (which are not limiting the solution).

The solution process by GRG2 may terminate due to several reasons : a local optimum may be found, a feasible solution may be unavailable in the phase-I optimization, or some numerical difficulties such as scaling may cause the solution process to stop prematurely. Most of the optimization runs presented in this study terminated because the fractional change in the objective function was less than the specified tolerance for a specified number of iterations. The characteristics of the final solutions of this type are uncertain since they may or may not be local optima.

In summary, the solution obtained by GRG2 is observed to be affected by the starting point, the bounds on the variables, the tolerance levels of the equality constraints, the stopping criteria, and the various optimization strategies that are employed within the GRG2 optimization procedure. These difficulties associated with using GRG2 to optimize the comprehensive system model prompted the development of a strategy to evaluate the quality of the solutions obtained and to generate alternative good solutions that may be examined further from a practical perspective. The following subsection examines a strategy that is designed for this purpose.

Developing alternative procedures for optimization of the comprehensive system model is also suggested by the difficulties of using GRG2. Sections 3.3 and 3.4 describe two alternative solution procedures.

3.2.3. Exploration of the Feasible Design Space

Brill (1979) proposed that when using an optimization model of a complex planning problem with important unmodeled issues it may be desirable to use the model to explore alternative solutions. These alternatives can then be evaluated with respect to the unmodeled issues. The first step in his Hop-Skip-Jump (HSJ) method is to obtain an initial design using a single or multiple objective procedure. The next step then is to solve the following optimization problem :

$$\begin{aligned} &\text{Minimize} && \sum_{k \in K} x_k \\ &\text{subject to} && f_j(\mathbf{X}) \geq T_j \ , \quad \forall j && (3.2) \\ & && \mathbf{X} \in F_d \end{aligned}$$

where K is the index set of those variables which are nonzero in the initial design,

$f_j(\mathbf{X})$ is the jth objective function, and is a function of the solution vector, $\mathbf{X}$,

T_j is the target specified for the jth objective,

and F_d is the feasible solution space.

This formulation is designed to generate a maximally different solution from the initial solution. The objective function space can be explored by solving a sequence of problems in the form of program (3.2), and alternative designs can be generated and examined.

Extending this idea by a slight modification of the objective function in program (3.2), we can explore the feasible design space of the wastewater treatment system model by solving

$$\begin{aligned} &\text{Optimize} && \mathrm{F}(\mathbf{X}) \\ &\text{subject to} && \mathrm{c}(\mathbf{X}) \leq T && (3.3) \\ & && \mathbf{X} \in F_d \end{aligned}$$

where the objective function, F, is a function of the variables, and may be minimized or maximized. This function may be formed at random or using knowledge or engineering judgment of the problem. The total system cost, c($\mathbf{X}$), which is the objective of the original optimization problem (program (3.1)), is constrained to be less than or equal to a target, T, which may be arbitrarily determined, or which may be the same as the cost of the solution obtained from GRG2.

If a feasible solution can be obtained from solving the constrained formulation (3.3) with T set to the current best value of the objective function, then the new solution will be at least as good. The new solution may meet the target exactly, but it may represent a design that is different from the current solution.

Table 3.2 reveals characteristics in the decision variable values that result in cost effective designs of the base wastewater treatment system. While all five final designs have similar values for the mean cell residence time, hydraulic retention time, sludge recycle ratio, and solids loading rate on the gravity thickener, it is noted that "good" designs exhibit some special characteristics. Design No. 5 has its overflow rate on the primary clarifier at its upper bound (144 meters/day); designs No. 2, 3, and 5 all have the digestion temperature at the specified upper bound (60 °C); and designs No. 2 and 3 have the cake solids concentration at the upper bound of 150 kg/m^3. If these characteristics indeed lead to a more cost effective design than other feasible designs, then it may be possible to improve further the solution obtained from GRG by using program (3.3) to examine it with respect to these characteristics.

Program (3.3) was constructed for each of the five designs examined in Table 3.2. The objective functions and target values used to form program (3.3) as well as the results of solving program (3.3) are summarized in Tables 3.4 through 3.8. The solution obtained from GRG2 using starting point No. 1 was used as the starting point in Table 3.4 with the cake solids concentration being the objective function to be maximized. A different solution was obtained, but the total system cost remained the same. The difference between the two designs is primarily in the sludge processing train because the objective function chosen is related directly to the design of sludge treatment units. This solution was then used as the starting point for the next optimization where the overflow rate of the primary clarifier was maximized. This run produced another different design with the same total system cost. However, the major difference between this and the two previous designs is on the liquid

Table 3.4 - Exploring Design Space : Design No.1

Variable (Unit)	Objective Function, F		
	Maximize Cake Solids Conc.	Maximize Primary Clarifier Overflow Rate	Maximize Digestion Temp.
Primary Clarifier Overflow Rate (m/day)			
initial	115.9	115.9	144.0
final	115.9	144.0	144.0
Mean Cell Residence Time (days)			
initial	2.20	2.20	2.19
final	2.20	2.19	2.19
Hydaulic Retention Time (hr)			
initial	3.5	3.5	4.2
final	3.5	4.2	4.2
Sludge Recycle Ratio (%)			
initial	13.7	13.7	10.4
final	13.7	10.4	10.4
Solids Loading on Thickener ($kg/m^2/day$)			
initial	13.4	13.5	12.0
final	13.5	12.0	12.0
Digestion Temperature ($^\circ$C)			
initial	44.9	44.2	43.8
final	44.2	43.8	60.0
Retention Time in Digester (days)			
initial	17.7	18.7	18.7
final	18.7	18.7	18.5
Solids Loading on Digester ($kg/m^2/day$)			
initial	29.3	29.3	25.9
final	29.3	25.9	41.7
Filter Yield ($kg/m^2/hr$)			
initial	7.47	7.43	7.75
final	7.43	7.75	6.61
Cake Solids Concentration (kg/m^3)			
initial	142.9	150.0	150.0
final	150.0	150.0	150.0
Effluent BOD_5 (mg/l)			
initial	30.0	30.0	30.0
final	30.0	30.0	30.0
Effluent TSS (mg/l)			
initial	30.0	30.0	30.0
final	30.0	30.0	30.0
Total System Cost (10^3 \$/yr)			
initial	542.9	542.9	542.9
final	542.9	542.9	502.0
Computer Time (CP seconds)	7.58	16.34	13.33

processing train. This solution was then used for a third optimization run, and the digestion temperature was maximized. The total system cost improved significantly from 542,900 to 502,000 dollars/year. This cost reduction is the result of different designs in the sludge treatment system. Thus, in this case, the modified HSJ approach led to an improved solution in comparison to the first solution obtained using GRG2. The objective function value of the improved solution is the same as the best solution obtained using GRG2 and listed in Table 3.2.

Table 3.5 lists two optimization runs that started from the two final solutions given in Table 3.3, the solution obtained using starting point No. 2 and another solution obtained using different bounds on selected decision variables. The overflow rate of the primary clarifier was maximized in solving program (3.3). Two designs with very similar characteristics in sludge processing were obtained. The total system costs differ only slightly due to the difference in the activated sludge process design; both designs have slightly better objective function values than those obtained so far. This example illustrates that the effect of the bounds on the GRG2 solution can become less critical if an HSJ type approach is followed (i.e., by solving the constrained formulation of (3.3)). Different bounds on the variables or different control parameters used in runing GRG2 affect the solution in a complex problem. Solving the constrained formulation provides confidence to the solution quality, and generates different good designs.

Final solution No. 3 in Table 3.2 was used as the starting solution in Table 3.6. The primary clarifier overflow rate was first maximized. With the primary clarifier overflow rate at its specified upper bound, there was one equality constraint not satisfied to the specified tolerance level. To continue the optimization, the decision variables in this infeasible solution were used as input to the analysis program which generated a slightly different solution. This solution satisfied all constraints in the model, but the cake solids concentration violated its upper bound of 150 kg/m^3. Program (3.2) was then solved using this new starting point;

Table 3.5 - Exploring Design Space : Design No. 2

Variable (Unit)	Objective Function, F	
	Maximize Primary Clarifier Overflow Rate	Maximize Primary Clarifier Overflow Rate
Primary Clarifier Overflow Rate (m/day)		
initial	80.0	43.8
final	144.0	144.0
Mean Cell Residence Time (days)		
initial	2.22	2.26
final	2.19	2.19
Hydaulic Retention Time (hr)		
initial	3.7	3.5
final	3.8	4.2
Sludge Recycle Ratio (%)		
initial	11.6	11.5
final	12.5	10.0
Solids Loading on Thickener (kg/m^2/day)		
initial	12.5	12.6
final	12.3	12.6
Digestion Temperature ($^\circ$C)		
initial	60.0	60.0
final	60.0	60.0
Retention Time in Digester (days)		
initial	14.7	14.7
final	14.6	14.7
Solids Loading on Digester (kg/m^2/day)		
initial	38.4	36.3
final	40.0	38.9
Filter Yield (kg/m^2/hr)		
initial	6.79	6.92
final	6.70	6.76
Cake Solids Concentration (kg/m^3)		
initial	150.0	150.0
final	150.0	150.0
Effluent BOD$_5$ (mg/l)		
initial	30.0	30.0
final	30.0	30.0
Effluent TSS (mg/l)		
initial	30.0	30.0
final	30.0	30.0
Total System Cost (10^3 \$/yr)		
initial	506.1	517.5
final	500.4	501.0
Computer Time (CP seconds)	30.97	23.31

Table 3.6 - Exploring Design Space : Design No. 3

Variable (Unit)	Objective Function, F: Maximize Primary Clarifier Overflow Rate	Objective Function, F: Final GRG2 Solution
Primary Clarifier Overflow Rate (m/day)		
initial	78.2	144.0
final	144.0	144.0
Mean Cell Residence Time (days)		
initial	2.22	2.19
final	2.19	2.19
Hydraulic Retention Time (hr)		
initial	3.6	4.2
final	4.2	3.8
Sludge Recycle Ratio (%)		
initial	12.2	10.2
final	10.2	12.5
Solids Loading on Thickener ($kg/m^2/day$)		
initial	13.9	12.0
final	12.0	12.0
Digestion Temperature (°C)		
initial	60.0	60.0
final	60.0	60.0
Retention Time in Digester (days)		
initial	12.9	13.0
final	13.0	16.2
Solids Loading on Digester ($kg/m^2/day$)		
initial	41.6	37.2
final	37.2	40.2
Filter Yield ($kg/m^2/hr$)		
initial	6.62	6.86
final	6.86	6.69
Cake Solids Concentration (kg/m^3)		
initial	150.0	152.4
final	150.0	150.0
Effluent BOD_5 (mg/l)		
initial	30.0	30.0
final	30.0	30.0
Effluent TSS (mg/l)		
initial	30.0	30.0
final	30.0	30.0
Total System Cost (10^3 \$/yr)		
initial	506.7	500.6
final	500.6*	500.6
Computer Time (CP seconds)	21.94	19.85

* One constraint is violated in this solution.

the final GRG2 solution is given in Table 3.6. The total system cost of this solution is comparable to the best solution obtained so far (first solution in Table 3.5). Using the analysis program in this case helped to restart an optimization in which GRG2 failed to find a feasible solution by solving simultaneous design equations.

Table 3.7 provides another example of using the analysis program to restart the GRG optimization. Final solution No. 4 in Table 3.2 was used for the first optimization run in Table 3.7 in which the cake solids concentration was maximized. The primary clarifier overflow rate in this solution was then maximized. This resulted in an infeasible design with all constraints satisfied to 10^{-3}, but not the specified tolerance level of 10^{-4}. The decision variables in this final solution, with a minor modification of the value of the solids loading value on the secondary digester, were used as input to the analysis program. This modification is necessary for the analysis program to produce a feasible design of the secondary digester (i.e., the underflow solids concentration is higher than or equal to the influent solids concentration). The resulting design was used as the new starting point, and primary clarifier overflow rate was again maximized using program (3.3). A very different solution was obtained with an improved total system cost (from 551,500 to 523,700 dollars/year); it is the third solution listed in Table 3.7. Finally, the digestion temperature was maximized. The solution obtained, the last in Table 3.1, has a total system cost of 501,200 dollars/year, which represents a 10% savings of the total system cost from the GRG2 solution.

Final solution No. 5 has the best objective function value among the five designs in Table 3.2. Marginal reduction of the total system cost, however, was observed when the cake solids concentration was maximized (see Table 3.8). An alternative design with a nearly identical total system cost was obtained using a different objective function. This objective function minimizes the solids loading on the gravity thickener. Consequently it has a larger thickener which provides a digester influent with higher solids concentration. This allows the primary digester to be smaller, yet to achieve the same solids retention time.

Table 3.7 - Exploring Design Space : Design No. 4

Variable (Unit)	Objective Function, F: Maximize Cake Solids Concentration	Maximize Primary Clarifier Overflow Rate	Maximize Primary Clarifier Overflow Rate	Maximize Digestion Temperature
Primary Clarifier Overflow Rate (m/day)				
initial	17.4	16.0	18.7	144.0
final	16.0	18.7	144.0	144.0
Mean Cell Residence Time (days)				
initial	2.36	2.37	2.36	2.19
final	2.37	2.36	2.19	2.19
Hydaulic Retention Time (hr)				
initial	3.4	3.3	3.5	4.2
final	3.3	3.5	4.2	4.2
Sludge Recycle Ratio (%)				
initial	10.0	10.0	10.0	10.0
final	10.0	10.0	10.0	10.0
Solids Loading on Thickener ($kg/m^2/day$)				
initial	13.2	13.3	13.1	12.0
final	13.3	13.1	12.0	12.0
Digestion Temperature ($^\circ C$)				
initial	60.0	60.0	59.8	50.0
final	60.0	59.8	50.0	60.0
Retention Time in Digester (days)				
initial	16.3	16.3	16.3	16.2
final	16.3	16.3	16.2	16.1
Solids Loading on Digester ($kg/m^2/day$)				
initial	36.6	36.2	35.3	30.0
final	36.2	36.5	30.0	39.4
Filter Yield ($kg/m^2/hr$)				
initial	7.48	6.93	6.93	7.38
final	6.93	6.93	7.38	6.73
Cake Solids Concentration (kg/m^3)				
initial	77.89	150.0	167.9	150.0
final	150.0	150.0	150.0	150.0
Effluent BOD_5 (mg/l)				
initial	30.0	30.0	30.6	30.0
final	30.0	30.0	30.0	30.0
Effluent TSS (mg/l)				
initial	30.0	30.0	31.9	30.0
final	30.0	30.0	30.0	30.0
Total System Cost (10^3 \$/yr)				
initial	584.7	560.0	551.5	523.7
final	560.0	551.5*	523.7	501.2
Computer Time (CP seconds)	5.074	12.93	34.422	8.977

* : Solution infeasible with respect to the contraint tolerance of 10^{-4}, but all satisfied to 10^{-3}.

Table 3.8 - Exploring Design Space : Design No. 5

Variable (Unit)	Objective Function, F: Maximize Cake Solids Conc.	Objective Function, F: Maximize Cake Solids Conc.-100L_g *
Primary Clarifier Overflow Rate (m/day)		
initial	144.0	144.0
final	144.0	144.0
Mean Cell Residence Time (days)		
initial	2.19	2.19
final	2.19	2.19
Hydaulic Retention Time (hr)		
initial	3.7	3.7
final	3.7	3.7
Sludge Recycle Ratio (%)		
initial	12.6	12.6
final	12.8	12.7
Solids Loading on Thickener (kg/m^2/day)		
initial	12.6	12.6
final	12.6	12.0
Digestion Temperature (°C)		
initial	60.0	60.0
final	60.0	60.0
Retention Time in Digester (days)		
initial	13.9	13.9
final	13.9	13.9
Solids Loading on Digester (kg/m^2/day)		
initial	40.4	40.4
final	40.6	38.2
Filter Yield (kg/m^2/hr)		
initial	6.70	6.70
final	6.67	6.80
Cake Solids Concentration (kg/m^3)		
initial	143.7	143.7
final	150.0	150.0
Effluent BOD_5 (mg/l)		
initial	30.0	30.0
final	30.0	30.0
Effluent TSS (mg/l)		
initial	30.0	30.0
final	30.0	30.0
Total System Cost (10^3 \$/yr)		
initial	502.0	502.0
final	500.4	500.5
Computer Time (CP seconds)	6.052	12.386

* L_g is the solids loading on the gravity thickener as defined in Chapter 2.

These differences are given in Figures 3.3 and 3.4, which show the details of these two designs. Since the design of the liquid treatment train and the total system cost of these two designs are almost the same, the difference in the design of the thickener and the digester implies that there may be many possible combinations of the sizes of thickener and digester that would result in practically the same cost for sludge treatment.

These illustrations show that program (3.3) is potentially useful for generating altenative good designs for the wastewater treatment system considered. By relaxing the target values and forming different objective functions, many alternative designs can be produced which can then be evaluated for other important issues not present in a cost minimization model. Table 3.9 summarizes the final designs obtained from solving program (3.3) using the five solutions listed in Table 3.2. The total system cost ranges from 500,384 to 501,963 dollars/year; the differences are practically insignificant. These designs are similar because the objective functions used in obtaining them are similar. The size of the primary digester represents the most significant difference in the sludge processing train design, while the sizes of the aeration tank and the final clarifier are the major differences in the liquid train design. These solutions are discussed in more detail in Chapter 4.

Finally, it was observed that objective function values obtained using multiple starting points varied considerably and the best value is 502,000 dollars/year (Table 3.2). All solutions obtained by solving program (3.3) achieved a better objective value than this. Although the improvement may be small from a practical point of view for the particular wastewater treatment system model considered here, the difference could be greater in other cases. It suggests that this strategy may serve as a useful fine-tuning step for solving such problems using GRG2.

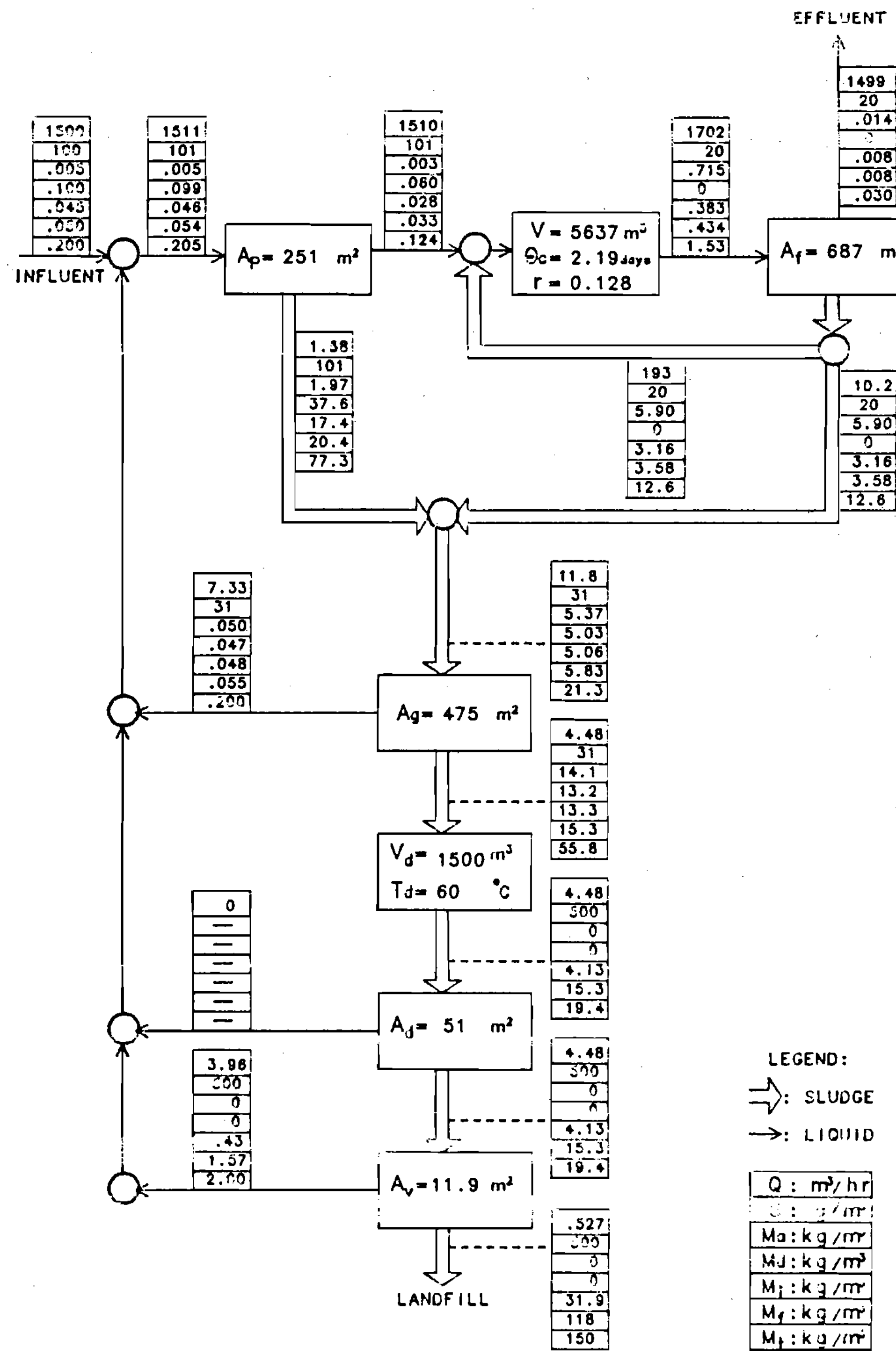


Figure 3.3- Final Design Obtained From Solving Program (3.3): Maximizing Cake Solids Concentration

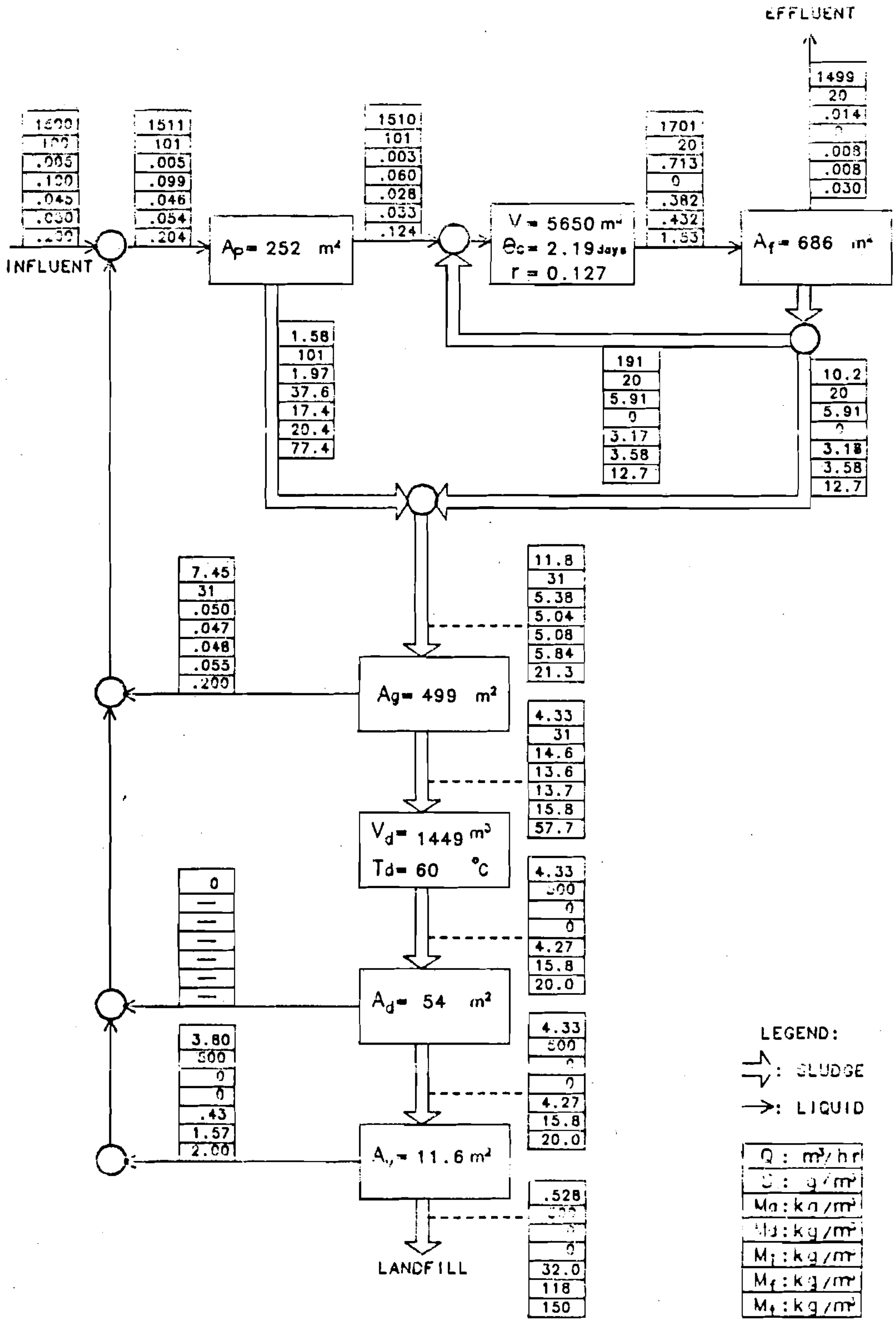


Figure 3.4- Final Design Obtained From Solving Program (3.3): Maximizing (Cake Solids Concentration - 100 x Solids Loading on Thickener)

Table 3.9 - Summary of Final Solutions Obtained From Solving Program (3.3)

Design No. :	1	2	3	4	5	6	7
Starting Point No. :	1	2	2	3	4	5	5
Liquid Processing:							
Primary Clarifier (m^2)	252	252	251	252	251	251	252
Aeration Tank (m^3)	6273	5687	6407	5686	6408	5637	5650
Final Clarifier (m^2)	658	684	653	684	653	687	686
Air Flow Rate (m^3/min)	242	242	242	242	242	242	242
Effluent BOD_5 (g/m^3)	30	30	30	30	30	30	30
Effluent TSS (g/m^3)	30	30	30	30	30	30	30
Sludge Processing:							
Mass Fraction of Primary Sludge	.486	.486	.486	.485	.486	.486	.487
Thickener (m^2)	500	486	500	499	500	475	499
Thickener Supernatant (m^3/hr)	6.82	7.34	6.70	7.41	6.70	7.33	7.45
Primary Digester (m^3)	1923	1540	1528	1685	1672	1500	1449
Secondary Digester* (m^3)	430	520	530	510	510	510	510
Digester Supernatant (m^3/hr)	0	0	0	0	.045	0	0
Vacuum Filter (m^2)	11.4	11.7	11.6	11.5	11.4	11.9	11.6
Filtrate (m^3/hr)	3.83	3.89	3.81	3.82	3.78	3.96	3.80
Cake Flowrate (m^3/hr)	.501	.523	.522	.513	.513	.527	.528
Cake Concentration (kg/m^3)	150	150	150	150	150	150	150
Total System Cost (10^3\$/yr)	501.963	500.384	500.954	500.627	501.228	500.422	500.467

* Height of the digester is assumed to be 10 m.

3.3. IGGP Algorithm for Optimization

3.3.1. Introduction

The Generalized Geometric Programming (GGP) algorithm for solving geometric programs was developed by Avriel *et al.* (1975). The algorithm condenses polynomials to monomials (a posynomial is a polynomial with only positive coefficients, and a monomial is a posynomial with only a single term) at a given point and then linearizes the monomials by logarithmic transformation. A linear program is then solved in each iteration. There are a number of computer codes that implement this basic idea (Dembo, 1980). Burns and Ramamurthy (1982) have developed a code that can be used interactively on the CDC Cyber computers at the University of Illinois. The original algorithm developed by Avriel *et* *al.* deals exclusively with inequality constraints. Equality constraints have to be converted to inequalities in order for the optimization to proceed. Strategies for this conversion have been proposed (See, for example, Blau and Wilde, 1969). Burns and Ramamurthy (1983) discussed the deficiencies of these strategies and extended the idea of condensation of polynomials to the treatment of equality constraints. Favorable results were obtained from their algorithm when it was applied to solve generalized geometric programs with equality constraints. This algorithm, named Interactive Generalized Geometric Programming (IGGP), was used to solve the comprehensive system model described in Chapter 2.

IGGP solves the following geometric program:

$$\begin{aligned} \text{Minimize} \quad & P_0^+(\mathbf{X}') - P_0^-(\mathbf{X}') \\ \text{subject to} \quad & P_k^+(\mathbf{X}') - P_k^-(\mathbf{X}') \leq 0 \ , \quad k = 1,\dots,K \\ & P_j^+(\mathbf{X}') - P_j^-(\mathbf{X}') = 0 \ , \quad j = 1,\dots,J \\ & 0 < \mathbf{X}'_L \leq \mathbf{X}' \end{aligned} \tag{3.4}$$

where P_0^+, P_0^-, P_k^+, P_k^-, P_j^+ and P_j^- are posynomials,

$\mathbf{X}'_L$ is the vector of the lower bounds on the model variables, N x 1,

$\mathbf{X}' = [\ x_{1,\dots,}\ x_N\ ]^T$, N x 1,

and N is the number of variables in the model.

Two restrictions are noted in program (3.4). The objective function and the constraints in the model have to be polynomials in order to apply the algorithm. The variables in the model have to be strictly positive.

Program (3.4) can be restated as

$$\begin{aligned} \text{Minimize} \quad & x_0 \\ \text{subject to} \quad & \frac{P_0^+(\mathbf{X})}{P_0^-(\mathbf{X}) + x_0} \leq 1 \end{aligned} \tag{3.5}$$

$$\frac{P_k^+(\mathbf{X})}{P_k^-(\mathbf{X})} \leq 1 \ , \quad k = 1,\ldots,K$$

$$\frac{P_j^+(\mathbf{X})}{P_j^-(\mathbf{X})} = 1 \ , \quad j = 1,\ldots,J$$

$$0 < \mathbf{X}_L \leq \mathbf{X}$$

where $\mathbf{X} = [\ x_0, x_1, \ldots, x_N\ ]^T$ is the (N+1) x 1 solution vector, and $\mathbf{X'}_L$ is the (N+1) x 1 vector of lower bound. The denominator of each inequality constraint in program (3.5) is condensed to a monomial at a point $\mathbf{X} = \overline{\mathbf{X}}$, while both the denominator and the numerators are condensed to monomials for each equality constraint at $\overline{\mathbf{X}}$. The resulting program becomes

$$\begin{aligned} \text{Minimize} \quad & x_0 \\ \text{subject to} \quad & P_k^-(\mathbf{X},\overline{\mathbf{X}}) \leq 1 \ , \quad k = 0,1,\ldots,K \\ & M_j^-(\mathbf{X},\overline{\mathbf{X}}) = 1 \ , \quad j = 1,\ldots,J \\ & \mathbf{X}_L \leq \mathbf{X} \end{aligned} \qquad (3.6)$$

where P_k^- is a posynomial and M_j^- is a monomial resulting from the condensation at point $\overline{\mathbf{X}}$.

Program (3.6) is linearized by logarithmic transformation. A linear program (LP) is solved, and the most violated inequality polynomial is linearized at the LP solution and is appended to the LP tableau as a cutting plane. Additional cutting planes are added until all of the inequality polynomials are satisfied within a specified tolerance. Cutting planes are added only for the inequality constraints, the equality constraints are simply log-linearized once in each iteration. The detailed development of this method is documented by Burns and Ramamurthy (1983).

Convergence to a Kuhn-Tucker solution of the GGP without equality constraints was shown by Avriel and Williams (1970). Burns and Ramamurthy did not prove their method will converge to a Kuhn-Tucker solution. Nevertheless, it is an attractive approach to test because it can solve large-scale problems efficiently by transforming the nonlinear program to a linear program. Also it is interesting to test the proposed strategy of Burns and Ramamurthy for handling equality constraints in GGP using the comprehensive system model which includes primarily equality constraints. These tests and their results are provided in the next two subsections.

3.3.2. Optimization Procedure

As mentioned in the previous subsection, the optimization model has to be transformed to a GGP and the variables have to be strictly positive to apply the IGGP. Most of the design equations in the comprehensive system model can be transformed to polynomials with the exceptions of equations (2.39), (2.41), and (2.48). The requirement for the variables to be strictly positive is not a practical problem. Although one variable became zero in final solutions obtained using GRG2, most variables are strictly positive because of what they represent in the system. Where necessary, however, a small positive number can be imposed as the lower bound for those variables that otherwise may turn out to be zero.

Modifications of equations (2.39) and (2.41) are necessary in order to use IGGP. If the mass fraction of the primary sludge, f_p, is fixed in the model, then the thickening constants of the combined primary and waste activated sludge can be calculated immediately from equations (2.38) and (2.39). When these constants become known, the thickening equation (2.41) for the combined sludge can be transformed into a polynomial. Thus, by fixing f_p two equations were dropped from the model, and equation (2.41) was simplified to form a polynomial.

Equation (2.48) calculates the first-order digestion rate coefficient as a function of the fermentation temperature. Because this model is empirical, alternative modeling of the experimental data used to develop equation (2.48) is possible. Polynomial models that satisfy the standard GGP format were used to fit the experimental data. It was found that a third degree polynomial fits the data reasonably well,

$$K_1 = 0.06457 - 5.1358\times10^{-3}T_d + 1.2061\times10^{-4}T_d^2 + 1.918\times10^{-6}T_d^3 \quad (3.7)$$

where K_1 is the rate coefficient in day^{-1}, and T_d is the fermentation temperature in °C. Figure 3.5 presents equation (3.7) in graphical form.

With the above modifications, the comprehensive system model can be transcribed to a GGP which has 62 variables and 57 constraints; 54 of the constraints are equalities. The design of IGGP allows the objective function to be specified only interactively. Since many cost functions describing the costs of unit processes are composed of several piecewise segments (see Table 2.4), it is necessary to guess the capacities of these units in advance to determine the segment of the function in which the final solution falls. Ideally, if the final solution specifies a size of a particular unit that is not in the range assumed, the cost function for that unit should be revised in IGGP. However, knowing that the cost functions used in this study are only approximations of the cost data and involve uncertainty, and that the differences are small (see Table 3.11 for a comparison of the total system costs calculated by the complete and the simplified cost functions), this trial-and-error approach was not performed. Consequently the objective function value obtained from the IGGP solution may be slightly different from that obtained from the GRG2. The cost functions used in the GGP model are summarized in Table 3.10. A listing of the GGP model is attached in Appendix E.

The solution process proceeds by searching over a range of values of f_p for the best solution. A starting point can be obtained from the analysis program. The value of f_p is then fixed at a given value, and an optimal design is obtained by IGGP. Theoretically, the initial solution does not have to be feasible since IGGP can start from an infeasible solution and perform Phase-I optimization. Any one-dimensional search technique can be used to obtain the optimal value of f_p which results in the least cost design of the system. The design found in this manner will be a locally optimal design for the overall system. This solution strategy is sometimes referred to as partitioning, or projection in the operations

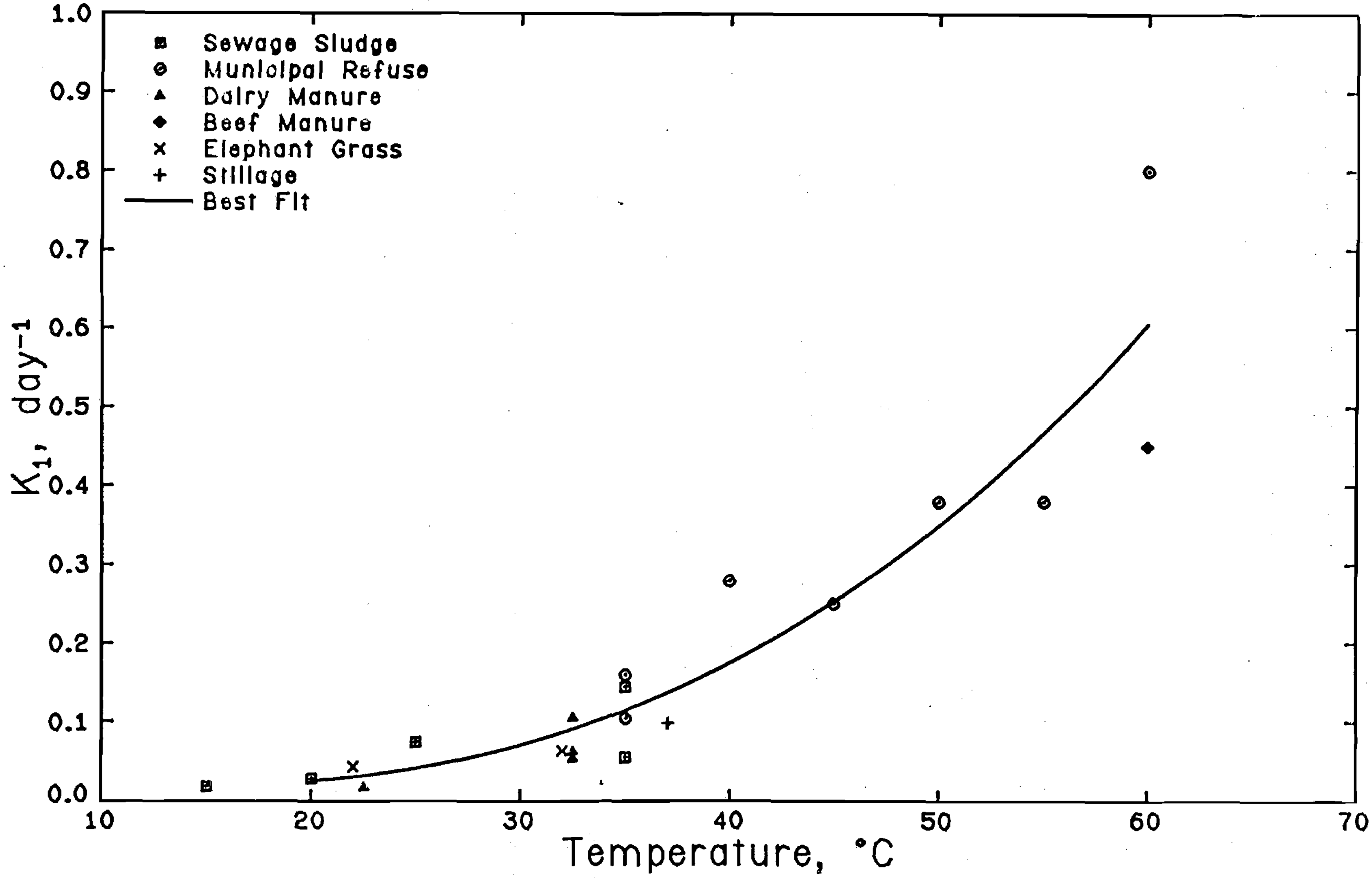


Figure 3.5- Modeling Digestion Rate Coefficient as a Polynomial With Respect to Fermentation Temperature

Table 3.10 - Summary of Cost Functions Used in IGGP

	Capital (1971$)	Operation (manhours/yr)	Maintenance (manhours/yr)	Material and Supply (1971$/yr)	Power ($kWhr/yr$)
Primary Clarifier	$824A_p^{.77}$	$92.45A_p^{.3}$	$106A_p^{.14}$	$8.62A_p^{.76}$	--
Primary Sludge Pumping	$16042\,Q_8^{.53}$	$374\,Q_8^{.41}$	$166\,Q_8^{.43}$	$385\,Q_8^{.64}$	$23.85\,Q_8H/\epsilon_p$ †
Aeration Tank	$461\,V^{.71}$	--	--	--	--
Diffused Aeration	$8533\,Q_a^{.66}$	$187\,Q_a^{.48}$	$74.4\,Q_a^{.55}$	--	--
Secondary Clarifier	$824A_f^{.77}$	$17.15A_f^{.6}$	$9.23A_f^{.6}$	$8.62A_f^{.76}$	--
Return & Waste Sludge Pumping	$2779\,Q_5^{.53}$	$.333\,Q_5+390$	$.2375\,Q_5+370$	$40.57\,Q_5^{.52}$	$23.85\,Q_8H/\epsilon_p$
Gravity Thicker	$824A_g^{.77}$	$17.15A_g^{.6}$	$9.23A_g^{.6}$	$8.62A_g^{.76}$	--
Anaerobic Digester	$2323\,V_d^{.59}$	$192\,V_d^{.2}$	$113\,V_d^{.21}$	$142\,V_d^{.37}$	--
Vacuum Filter	$29180A_v^{.71}$	$197.55\,Q_{16}^{.58}M_{t16}^{.58}$	$20\,Q_{16}^{.63}M_{t16}^{.63}$	$230\,Q_{16}^{.71}M_{t16}^{.71}$ + $182\,Q_{16}^{.86}M_{t16}^{.86}$	--
Recirculation Pumping	$2779\,Q_r^{.53}$‡	$0.333\,Q_r+390$	$0.2375\,Q_r+370$	300	$23.85\,Q_rH/\epsilon_p$

† H is the pumping head in meters, and ϵ_p is the pumping efficiency.

‡ $Q_r = Q_{10} + Q_{13} + Q_{15}$

research literature (Geoffrion, 1971).

3.3.3. Performance of IGGP

Solutions were obtained for the conditions listed in Tables 2.6 to 2.8. Because the majority of the constraints in the model are equalities, IGGP essentially solves the linear program resulting from the log-linearization of the condensed equality constraints. If the operating point is near the final solution, the condensation and the linearization are more accurate than if the operating point is far away. It was noticed during the test runs that when f_p is specified to be very different from the value in the initial solution provided by the analysis program, i.e., the starting solution for optimization is infeasible, IGGP may not be able to find a feasible starting solution using its Phase-I optimization routine. As a result, feasible starting solutions were used. The initial designs were generated by the analysis program. These designs corresponded to different values of f_p, and the optimal solutions corresponding to these f_p's were obtained by IGGP. In this approach, the values of f_p cannot be controlled directly, and an efficient one-dimensional search method could not be used to locate the optimal f_p. For the eleven initial designs specified, the values of f_p ranged from 0.44 to 0.61. Figure 3.6 depicts the total system cost versus f_p. The computing time for individual IGGP runs varied from 2.5 to 5.7 seconds. The results are summarized in Table 3.11.

It is observed from Figure 3.6 that the system cost is very sensitive to f_p when f_p is less than about 0.47, and is relatively insensitive to f_p otherwise. The solutions obtained with f_p less than 0.47 are characterized by a high primary clarifier overflow rate (at its upper bound of 144 meters/day), and by effluent BOD_5 and suspended solids values that are below the assumed standards. The solutions obtained with f_p greater than 0.49 exhibit the opposite characteristics. This observation reveals the two extremes of the system design when f_p is fixed. When the mass fraction of the primary sludge is relatively small, the pri-

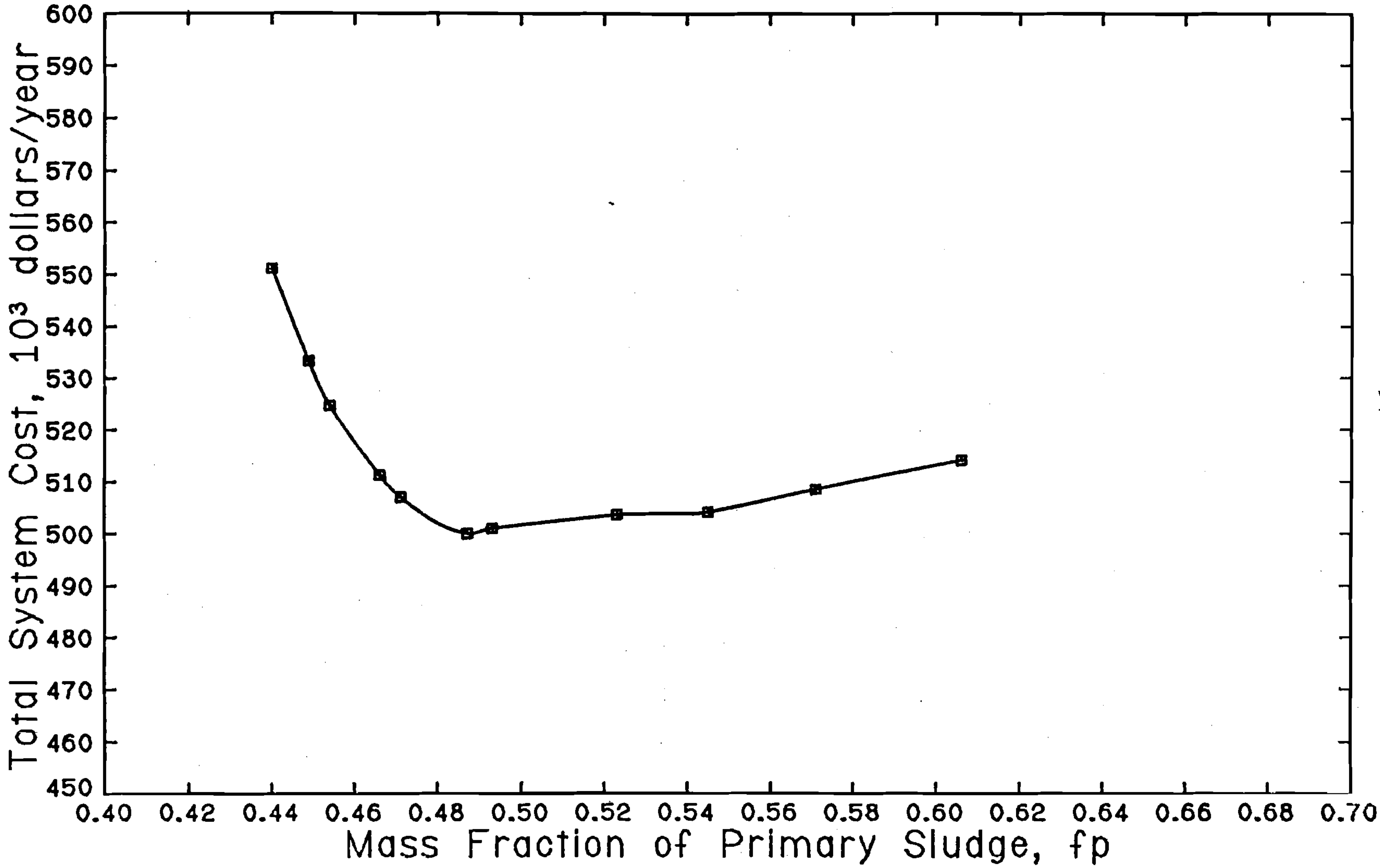


Figure 3.6- Using IGGP for Optimal System Design: Total System Cost vs. Mass Fraction of Primary Sludge

Table 3.11 - Wastewater Treatment System Designs Obtained Using IGGP

	Mass Fraction of Primary Sludge, f_p										
	0.440	0.449	0.454	0.466	0.471	0.487	0.493	0.523	0.545	0.571	0.606
Primary Clarifier Overflow Rate (m/day)	144.0	144.0	144.0	144.0	144.0	141.8	132.7	95.9	77.7	59.6	41.8
Mean Cell Residence Time (days)	2.00	2.00	2.00	2.00	2.03	2.19	2.20	2.21	2.22	2.24	2.26
Hydaulic Retention Time (hr)	2.5	2.8	3.0	3.4	3.5	3.8	3.7	3.7	3.6	3.6	3.5
Sludge Recycle Ratio (%)	10.0	10.0	10.0	10.0	10.4	12.4	13.1	12.5	12.6	11.8	11.8
Solids Loading on Thickener (kg/m^2/day)	12.0	12.0	12.0	12.0	13.5	12.0	12.0	12.9	12.0	12.3	12.0
Digestion Temperature (°C)	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0
Retention Time in Digester (days)	17.0	16.1	15.7	16.5	11.3	19.3	18.3	14.4	19.7	16.6	18.6
Solids Loading on Digester (kg/m^2/day)	42.8	41.2	40.6	40.8	41.2	42.8	41.4	39.7	42.4	37.6	36.9
Filter Yield (kg/m^2/hr)	6.56	6.64	6.67	6.66	6.63	6.56	6.63	6.72	6.57	6.84	6.88
Cake Solids Concentration (kg/m^3)	150.0	150.0	150.0	150.0	150.0	150.0	150.0	150.0	150.0	150.0	150.0
Effluent BOD_5 (mg/l)	26.2	27.4	28.1	29.7	30.0	30.0	30.0	30.0	30.0	30.0	30.0
Effluent TSS (mg/l)	12.8	15.5	17.6	22.1	24.3	30.0	30.0	30.0	30.0	30.0	30.0
Total System Cost (10^3 \$/yr)	551.2	533.4	524.8	511.3	507.1	500.1	501.0	503.7	504.2	508.6	514.2
Total System Cost* (10^3\$/yr)	551.1	533.6	525.0	511.8	507.2	500.5	501.2	505.1	507.1	511.8	519.1
Computer Time (CP seconds)	4.57	4.20	3.45	5.05	4.33	5.50	4.44	4.68	2.90	5.39	5.69

* These values are the total system costs calculated using the complete cost functions listed in Table 2.4.

Total Computer Time : 50.20 CP seconds.

mary settling tank is small. A large secondary clarifier is needed to produce a highly concentrated waste activated sludge for recycle to the aeration tank. This thickening requirement causes the plant to produce a high quality (low suspended solids) effluent. The thickening characteristics of the combined primary and secondary sludge are not as good as when f_p is large. Therefore a large thickener is needed. On the other hand, when f_p is large, the primary clarifier is large, the waste activated sludge is thickened to a smaller degree, and the clarification requirement of the secondary clarifier dominates the system design. Consequently the effluent water quality standards are binding. It appears that an optimal f_p value exists between 0.471 and 0.487 (see Figure 3.6) where the primary clarifier overflow rate is high and both the effluent BOD_5 and total suspended solids standards are binding.

More points may be used as starting points to run IGGP and to refine the curve shown in Figure 3.6 if it is desired to know the optimal value of f_p more accurately. This was not carried out in this study because: 1) the cost is relatively insensitive near the optimal f_p, and 2) the trend for optimal design conditions has become obvious through the analysis. If the Phase-I optimization in the IGGP performed more reliably for the system model, then locating the optimal system design could be done effectively by using a one-dimensional search technique such as Fibonacci search.

The total computer time for running IGGP and generating the points in Figure 3.6 was about 50 seconds for the test problem. Therefore the total time required in this optimization approach is comparable to that of GRG2. The solution obtained with f_p equal to 0.487 is shown in Figure 3.7. This design is similar to design No. 4 in Table 3.9 except for the digestion system. The total system cost calculated using the complete cost functions (Table 2.4) is 500,500 dollars/year which compares well to the solutions obtained using GRG2 (see Table 3.9). It is noted that the solutions in Table 3.9 have f_p values that range from 0.485 to 0.487 and that are within the final interval for f_p (0.471 to 0.487) determined by the IGGP solution process.

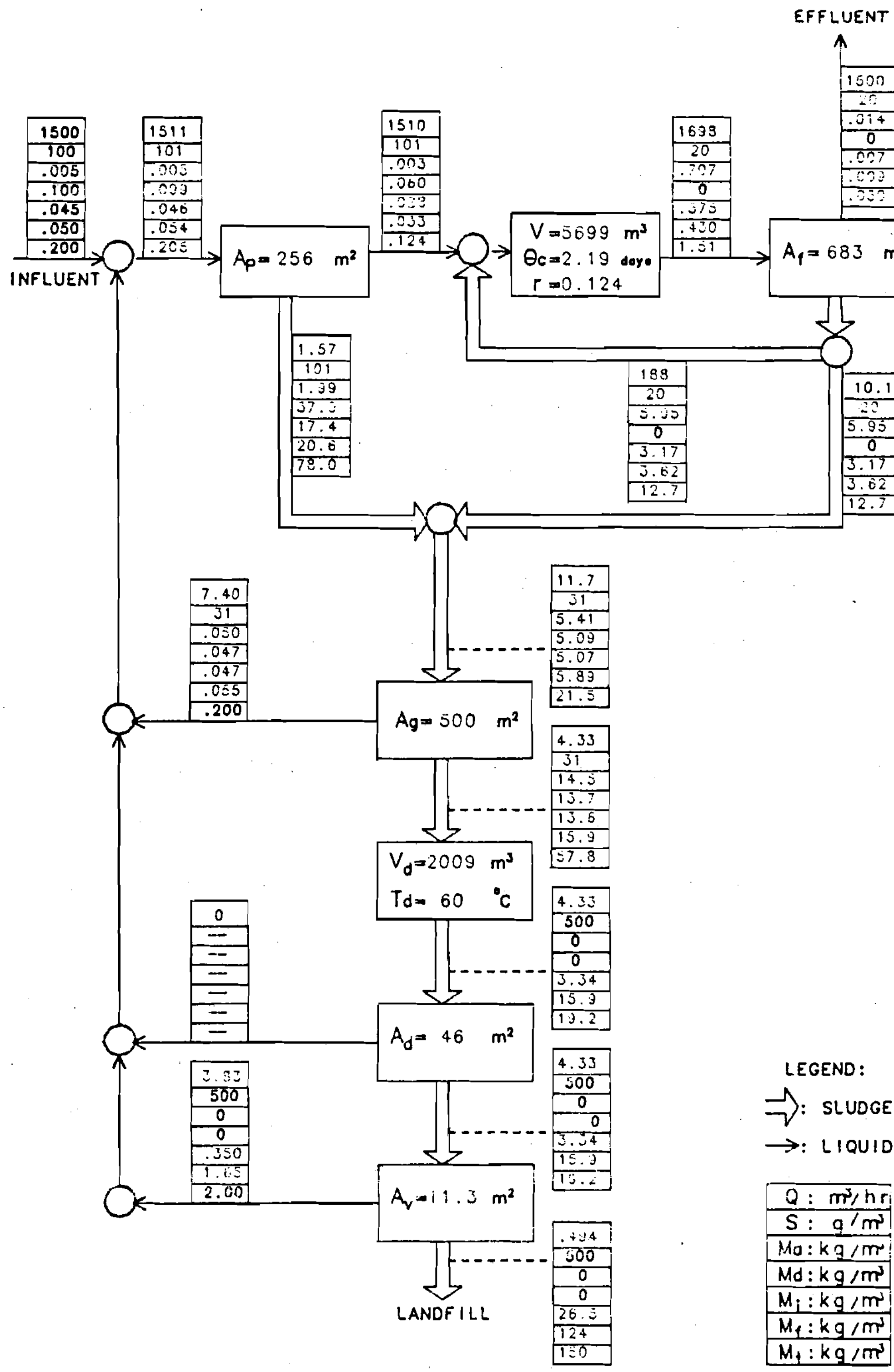


Figure 3.7- Design Obtained From IGGP: $f_p = 0.487$

3.4. Decomposition Approach for Optimization

3.4.1. Introduction

A wastewater treatment system is very complex in nature — not only because the design of individual unit processes may be complicated, but also because various interactions among the unit processes are complicated. In general, however, a wastewater treatment system can be considered to consist of a liquid treatment portion and a sludge treatment and disposal portion. For the base system (Figure 2.1), the liquid subsystem includes the primary settling tank and the activated sludge process, while the sludge subsystem contains the other units in the system. The inputs to the liquid subsystem are the influent wastewater and the recycle streams generated in the sludge treatment. The liquid subsystem produces primary and secondary sludges which are inputs to the sludge subsystem.

This section presents a specially tailored approach for solving the comprehensive system model. The overall system is decomposed into a liquid subsystem and a sludge subsystem. The design of the liquid subsystem is optimized. The optimal design of the liquid subsystem has been studied by many researchers (Section 1.3) and many alternative optimization techniques have been shown to be applicable to this problem. The solution obtained from optimizing the liquid subsystem design is then treated as input to the sludge subsystem. Embedded optimization steps are used in the sludge subsystem design. The optimal solution for the entire system is then obtained by coordinating the designs of the liquid and sludge subsystems. This approach may be especially useful for design engineers since alternative designs of each system are explicitly examined and tradeoffs between the two subsystems can be readily evaluated.

Formal decomposition techniques for nonlinear programs were first developed by researchers in the mid 1960's (for example, the feasible decomposition method by Brosilow *et al.* (1965) and the dual-feasible method by Brosilow and Lasdon (1965)). A large complex system is decomposed into a number of small subsystems each with its goals and constraints. Each subsystem is optimized separately, and results from the subsystem optimization are coordinated so that an optimal solution for the overall system can be obtained. Mathematical programming basis for nonlinear decomposition is well documented in Schoeffler (1970) and Lasdon (1970).

Although the decomposition approaches have numerous advantages for solving complex, interconnecting large-scale system models as discussed by Haimes (1977), the efficiency and robustness of these methods depend strongly on the characteristics of the problem. Westerberg (1972) discussed the use of decomposition techniques for steady-state chemical process synthesis and design problems. Limitations of the decomposition approaches were identified, and some computational experiences were reported.

While general decomposition approaches were not used to solve the comprehensive system model, the idea of decomposing the model into smaller problems was adopted for developing an optimization procedure that is unique for this particular problem. The procedure preserves such advantages of the decomposition approaches as conceptual simplification of a complex system, reduction in dimensionality, and flexibility in using different techniques for optimizing different subsystems.

3.4.2. Optimization Procedure

The overall wastewater treatment system was divided into two subsystems, one represents liquid processing and the other sludge processing. This conceptual simplification of the system and the interactions between the two subsystems are shown in Figure 3.8. The input to the liquid subsystem is the combination of the plant influent and the recycle streams generated from sludge processing, i.e., the output from the sludge subsystem. The output from the liquid subsystem (i.e., the combined primary and waste activated sludge) serves as input to the sludge subsystem.

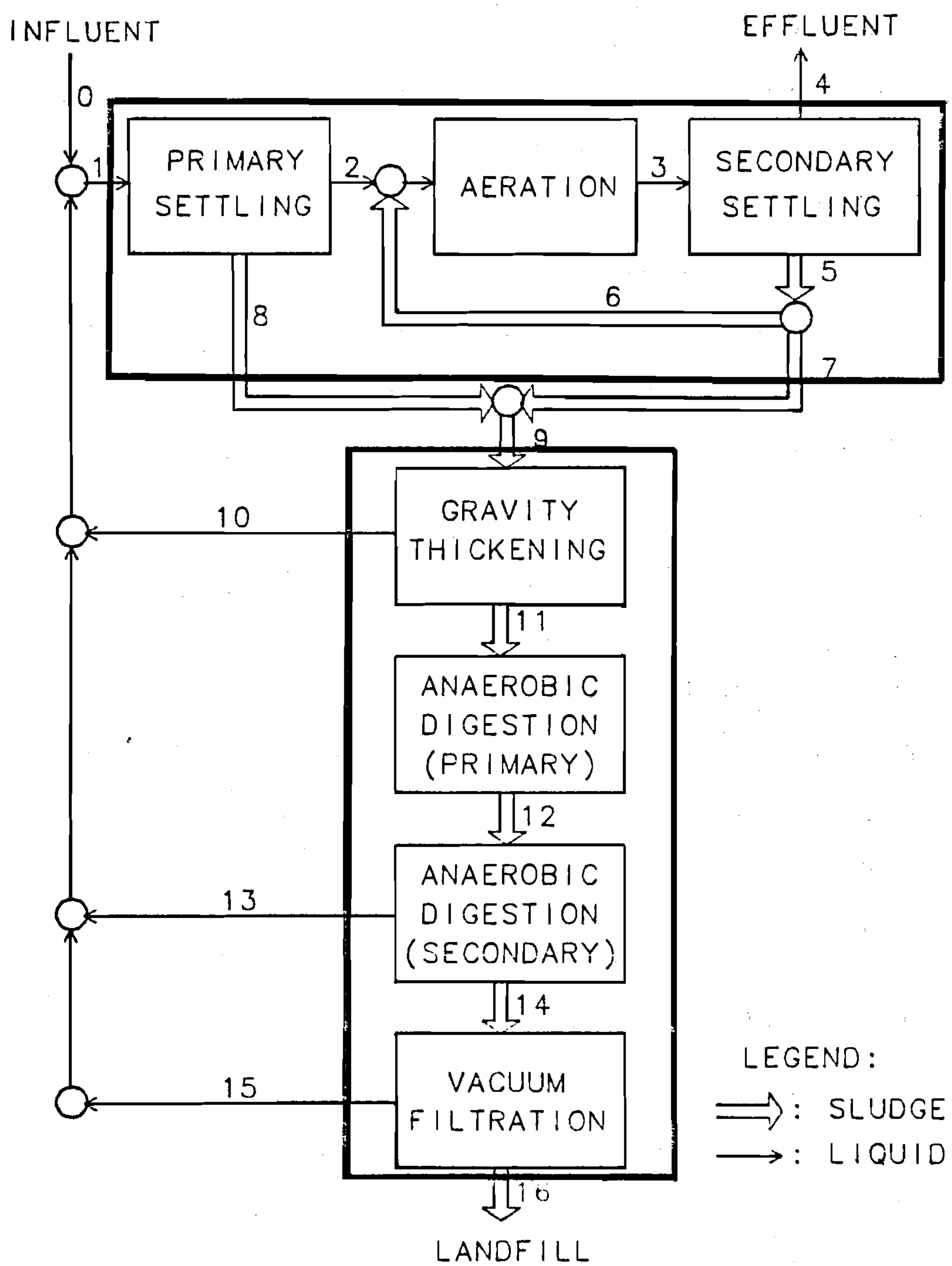


Figure 3.8- Subsystem Formed By Tearing the Interactions Between Liquid and Sludge Processing Trains

The design of the liquid system cannot be determined unless the characteristics of the recycle streams, i.e., the state variables at control points 10, 13 and 15, are known. There are twelve unknown state variables at these three control points that connect the liquid and sludge subsystems. These interacting variables are Q_{10}, S_{10}, M_{a10}, M_{d10}, M_{i10}, M_{f10}, Q_{13}, M_{i13}, M_{f13}, Q_{15}, M_{i15} and M_{f15}. The soluble BOD_5 of the digester supernatant (S_{13}) and the filtrate (S_{15}) have been assumed to be a constant (Section 2.3). Because of the lack of process models for predicting the total suspended solids concentrations of the thickener supernatant (M_{t10}), digester supernatant (M_{t13}), and filtrate (M_{t15}), these concentrations have been assumed to be constants, or

$$M_{a10} + M_{d10} + M_{i10} + M_{f10} = M_{t10} = \text{constant} \tag{3.8}$$

$$M_{i13} + M_{f13} = M_{t13} = \text{constant} \tag{3.9}$$

$$M_{i15} + M_{f15} = M_{t15} = \text{constant} \tag{3.10}$$

It is desirable to eliminate as many of the interacting variables as possible in order to efficiently coordinate the designs of the two subsystems. The solids concentration in the thickener supernatant is usually much less than the solids concentration in the digester supernatant or in the filtrate for a well-operated gravity thickener with high solids recovery efficiency, i.e.,

$$M_{t10} << M_{t13} \tag{3.11}$$

$$M_{t10} << M_{t15} \tag{3.12}$$

Consequently the contribution of the suspended solids from the thickener supernatant to the plant influent is small compared to that of the solids from the digester supernatant and filtrate if the thickener decant, digester supernatant, and filtrate have flowrates in the same order of magnitude. It is assumed that the suspended solids mass in the thickener supernatant can be neglected in the recycle mass balances. This additional assumption is made only for the decomposition solution approach. This assumption allows the variables M_{a10}, M_{d10}, M_{i10}, and M_{f10} to be eliminated from the group of interacting variables. It is also assumed for the decomposition approach that the soluble BOD_5 concentration of the thickener decant (S_{10}) is much less than that of the digester supernatant or of the filtrate in the calculation of recycle BOD mass balance. This assumption allows the interacting variable S_{10} to be eliminated.

It has also been assumed that the total suspended solids in the digester supernatant consist of only the volatile and aerobically nondegradable solids (M_{i13}) and the inert solids (M_{f13}). Since the secondary digester is modeled as a thickener and the vacuum filter is a physical separation unit, the solids species in the filtrate are expected to be in the same proportion as in the digester supernatant, i.e.,

$$\frac{M_{i13}}{M_{f13}} = \frac{M_{i15}}{M_{f15}} = z \tag{3.13}$$

Once the ratio, z, is determined, the solids compositions in the digester supernatant and in the filtrate can be calculated from equations (3.9), (3.10), and (3.13).

With the above assumptions, the recycle stream characteristics can be determined with the specification of only four interacting variables: the flowrates of thickener decant (Q_{10}), digester supernatant (Q_{13}), and filtrate (Q_{15}), and the ratio between the volatile inert solids and the inorganic solids concentrations in the digested sludge (z). The liquid subsystem can be readily designed for known characteristics of the recycle stream.

The complete decomposition procedure is now stated as follows :

1) Assume values for Q_{10}, Q_{13}, Q_{15} and z. Calculate from mass balance relationships (equations (3.14) to (3.20) below) the influent characteristics to the liquid subsystem.

$$Q_1 = Q_0 + Q_{10} + Q_{13} + Q_{15} \tag{3.14}$$

$$Q_1 S_1 = Q_0 S_0 + Q_{13} S_{13} + Q_{15} S_{15} \tag{3.15}$$

$$Q_1 M_{a1} = Q_0 M_{a0} \tag{3.16}$$

$$Q_1 M_{d1} = Q_0 M_{d0} \tag{3.17}$$

$$10^{-3}Q_1M_{i1} = 10^{-3}Q_0M_{i0} + Q_{13}\frac{z}{1+z}M_{t13} + Q_{15}\frac{z}{1+z}M_{t15} \tag{3.18}$$

$$10^{-3}Q_1M_{f1} = 10^{-3}Q_0M_{f0} + Q_{13}\frac{1}{1+z}M_{t13} + Q_{15}\frac{1}{1+z}M_{t15} \tag{3.19}$$

$$M_{t1} = M_{a1} + M_{d1} + M_{i1} + M_{f1} \tag{3.20}$$

where M_{a0}, M_{d0}, M_{i0} and M_{f0} are in g/m^3, and 10^{-3} is a unit conversion factor. The magnitudes for Q_{10}, Q_{13}, Q_{15}, and z can be roughly decided from running the analysis program using several different starting points. The assumed value for z is not critical in this approach. This is explained in more detail in step (4).

2) Optimize the liquid subsystem design using any efficient optimization technique. GRG2 was used in this study. The model has 21 variables in 17 equations and three inequality constraints. Therefore it has four degrees of freedom. A listing of the GRG program describing this model is attached in Appendix F.

3) Calculate the mass and flow characteristics of the combined primary and waste activated sludge based on the optimal design from the liquid subsystem optimization. The combined sludge is the input to the sludge processing train.

4) Determine the most cost effective sludge subsystem design for the assumed values of Q_{10}, Q_{13}, and Q_{15}. This is an optimization problem with one degree of freedom in the ratio z. Except for the solids compositions of the waste activated sludge, the liquid subsystem design is not affected by the value of z specified in step (1) because neither the volatile inert solids nor the inorganic solids is removed in the activated sludge process. This is illustrated by a numerical example in the next subsection where the liquid subsystem design is optimized for different influent conditions. Starting from the optimal design for the liquid subsystem, the solids compositions of the waste activated sludge can be readily calculated for a given value of z using equations (2.21) and (2.22):

$$\frac{M_{i3}}{M_{a3}} = \frac{1}{M_{a3}+(10^{-3})(\frac{\theta_c}{\theta})M_{a1}(\frac{M_{t2}}{M_{t1}})}[(10^{-3})M_{i1}(\frac{M_{t2}}{M_{t1}})(\frac{\theta_c}{\theta}) + (1-f_d)bM_{a3}\theta_c] \tag{3.21}$$

$$\frac{M_{f3}}{M_{a3}} = \frac{1}{M_{a3}+(10^{-3})(\frac{\theta_c}{\theta})M_{a1}(\frac{M_{t2}}{M_{t1}})}(10^{-3})M_{f1}(\frac{M_{t2}}{M_{t1}})(\frac{\theta_c}{\theta}) \tag{3.22}$$

In the above equations, M_{i1} and M_{f1} are determined by z, and all other variables in the right-hand-side are known from the optimal design of the liquid subsystem, which is obtained in step(2).

The influent characteristics of the combined primary and waste activated sludge can be determined once the solids compositions of the waste activated sludge are calculated (see equations (2.31) and (2.32)). The sludge subsystem design then proceeds as follows:

4.1) For the gravity thickener, there is one degree of freedom in the design for given influent conditions, i.e., complete design of this unit requires one design variable to be specified. The supernatant flowrate, Q_{10}, is treated as that variable in this approach since its value is specified in step (1).

4.2) For the primary digester, there are two decision variables, digestion temperature and solids retention time. Since the characteristics of the digester influent are known from the thickener design, and the digester effluent is characterized by the ratio between the two effluent solids concentrations, z, the primary digester design can be formulated as another optimization problem. The net cost of the primary digestion system is minimized subject to the effluent characteristics as specified by z. Recall from Section 2.3.6 that

$$M_{i12} = \frac{M_{a11} + M_{d11} + M_{i11}}{1 + K_1\theta_d} \tag{3.23}$$

$$M_{f12} = M_{f11} \quad , \tag{3.24}$$

the solids ratio z can be written from equations (3.23) and (3.24) as

$$z = \frac{M_{i13}}{M_{f13}} = \frac{M_{i12}}{M_{f12}} = \frac{M_{a11} + M_{d11} + M_{i11}}{M_{f11}(1 + K_1\theta_d)} \tag{3.25}$$

since the solids compositions are assumed to be unaffected by the secondary digester.

Specification of the digestion temperature results in the determination of the digestion rate coefficient, K_1. The solids retention time, θ_d, can then be calculated from equation (3.25), and the primary digester design is completely defined. This is a one-dimensional optimization problem with respect to the digestion temperature. Fibonacci search was employed to find the optimal digestion temperature that is accurate to within 1 °C.

4.3) The design of the secondary digester is similar to that of the gravity thickener. The decision variable is chosen to be the digester supernatant flowrate, Q_{13}, whose value is specified in step (1).

4.4) The design of the vacuum filter requires the specification of one design variable which is chosen as the filtrate flowrate, Q_{15}. Its value is specified in step (1).

Repeat steps (4.1) to (4.4) for different values of z. Golden section search was used to identify the optimal value of z for the sludge subsystem design. The computer program designed to carry out the calculations in step (4) is attached in Appendix G.

5) Sum the costs for the liquid subsystem obtained in step (2) and for the best sludge subsystem obtained in step (4) and obtain the total cost for the entire system. This cost is for an assumed set of interacting variables Q_{10}, Q_{13}, and Q_{15}. A complete flowchart describing steps (1) to (5) is shown in Figure 3.9.

6) Different combinations of values for the interacting variables can be selected. The total system cost can be calculated for each combination following steps (1) through (5), and the trend for a cost-effective design can be identified.

This proposed procedure transforms the original problem which has nine decision variables into two subproblems. The liquid subsystem design has four decision variables; and

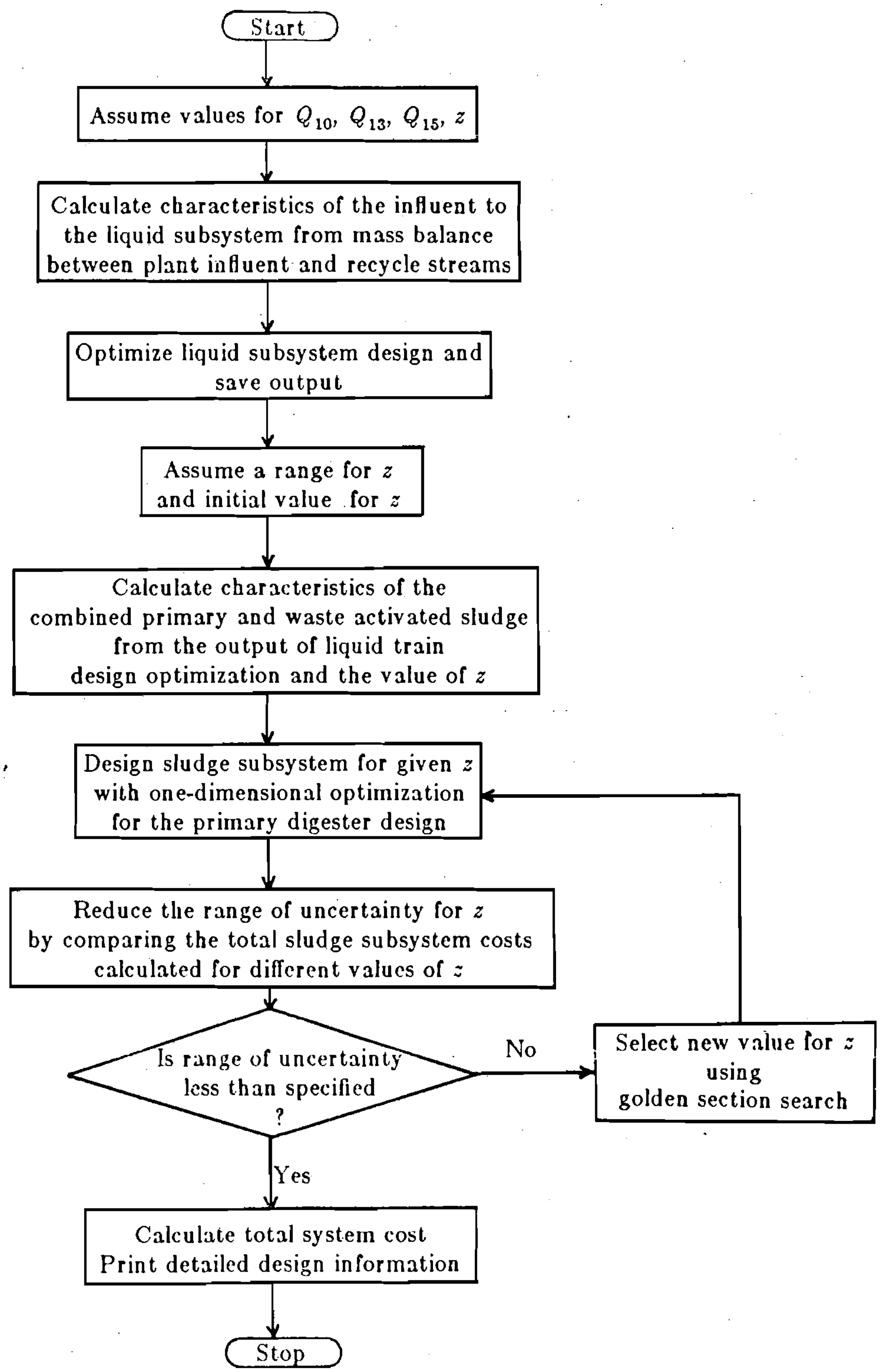


Figure 3.9 - Flowchart of the Decomposition Approach

the sludge subsystem design has two decision variables (z and T_d), each can be determined optimally using embedded one-dimensional optimization. The search for the overall optimal system design is a problem with three decision variables (Q_{10}, Q_{13} and Q_{16}). This concept is illustrated by Figure 3.10. The solutions obtained using the decomposition approach are only approximations to the comprehensive system design model described in Chapter 2 because of the additional assumptions made in developing this approach. These assumptions neglect the soluble BOD and suspended solids concentrations in the thickener supernatant. The validity of these assumptions are examined in the next subsection, so are the performance of the decomposition procedure for optimizing the complete wastewater treatment system design and the performance of the embedded techniques for optimizing the subsystem designs are also discussed.

3.4.3. Performance of the Optimization Approach

Step (2) in the above decomposition approach is essential to the overall optimization procedure. To examine the objective function surface of the liquid subsystem, different design conditions and multiple starting points were investigated. Table 3.12 summarizes solutions obtained when the base design conditions (see Section 2.5) are treated as the

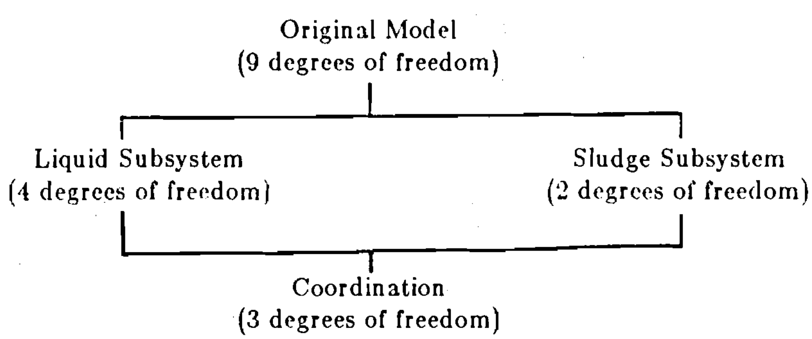


Figure 3.10 - Concept of the Decomposition Approach

Table 3.12 - Optimization of the Liquid Treatment Subsystem

Influent Conditions:

Flowrate	= 1500 m^3/hr
Soluble BOD_5	= 100 g/m^3
Active Biomass	= 5 g/m^3
Volatile Biodegradable Solids	= 100 g/m^3
Volatile Inert Solids	= 45 g/m^3
Inorganic Solids	= 50 g/m^3

Variables (Units)	Solution Obtained Using GRG2							
	1	2	3	4	5	6	7	8
Primary Clarifier Overflow Rate (m/day)								
initial	36.0	24.0	48.0	72.0	96.0	120.0	12.0	24.0
final	144.0	144.0	144.0	144.0	144.0	144.0	144.0	144.0
Mean Cell Residence Time (days)								
initial	5.0	3.0	6.0	4.0	2.0	3.0	5.0	6.0
final	2.18	2.18	2.18	2.18	2.18	2.18	2.18	2.18
Hydraulic Retention Time (hr)								
initial	3.6	4.8	6.0	2.4	2.4	2.4	4.8	6.0
final	3.6	3.6	3.6	3.6	3.6	3.6	3.6	3.6
Sludge Recycle Ratio (%)								
initial	30	20	15	25	25	40	25	35
final	12.3	12.3	12.3	12.3	12.3	12.4	12.3	12.4
Effluent BOD_5 (mg/l)								
initial	16.0	35.6	13.1	16.1	34.4	22.4	26.1	25.5
final	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0
Effluent TSS (mg/l)								
initial	21.3	56.2	17.3	16.5	34.7	24.65	43.5	49.5
final	28.8	28.8	28.8	28.8	28.8	28.8	28.8	28.8
Liquid System Cost (10^3 \$/yr)								
initial	339.9	297.0	338.8	371.1	254.9	298.1	367.0	338.9
final	253.9	253.9	253.9	253.9	253.9	253.9	253.9	253.9
Computer Time (CP seconds)	2.878	4.409	2.471	3.816	2.574	2.308	4.377	3.936

influent to the liquid train. All GRG runs were made interactively with the same control parameter values specified in Section 3.2.1. The computing time requirement is much less than that for the complete model which includes 64 variables and 58 constraints (as opposed to 21 and 20, respectively). The solution process also appears to be robust; widely different initial solutions converge to essentially the same solution. These observations are encouraging for the approach of decomposing the overall system model into smaller subsystems whose mathematical expressions are amenable to efficient and robust solution techniques.

Table 3.13 summarizes the liquid subsystem design optimization for a different set of influent conditions which has a higher flowrate and suspended solids concentration than the base conditions. Five starting points were tested, and four of them converged to the same optimal solution. The optimization runs with starting point No. 4 stopped short of the optimum, but the objective function value and the design are almost the same as the optimal solution. This indicates the flatness of the objective function surface of this subproblem.

The influent conditions examined in Table 3.13 were varied one at a time to observe the effect of each condition on the liquid system design. The results are tabulated in Table 3.14. Case 1 is the original solution from the first column of Table 3.13. A change in the flowrate (Case 2) affects the liquid system cost, but has little effect on the system design. An increase in the influent soluble BOD_5 (Case 3) increases the cost of the subsystem. A higher biomass concentration is maintained in the aeration tank when the size of the tank remains at the minimum level. A large secondary clarifier is included for thickening purposes. Thus the effluent suspended solids concentration decreases. The effect of the increased volatile suspended solids in the influent (Case 4) is similar to that caused by an increased soluble BOD_5 concentration.

Increasing the influent volatile inert solids (Case 5) or the inorganic solids (Case 6) by the same amount (5 mg/l) results in two almost identical designs with the only difference

Table 3.13 - Optimization of the Liquid Treatment Subsystem

Influent Conditions :

Flowrate = 1515 m^3/hr

Soluble BOD_5 = 100 g/m^3

Active Biomass = 5 g/m^3

Volatile Biodegradable Solids = 100 g/m^3

Volatile Inert Solids = 50 g/m^3

Inorganic Solids = 55 g/m^3

Variables (Units)	Solution Obtained Using GRG2				
	1	2	3	4	5
Primary Clarifier Overflow Rate (m/day)					
initial	36.0	24.0	24.0	72.0	120.0
final	144.0	144.0	144.0	144.0	144.0
Mean Cell Residence Time (days)					
initial	5.0	3.0	6.0	4.0	3.0
final	2.15	2.15	2.15	2.17	2.15
Hydraulic Retention Time (hr)					
initial	3.6	4.8	6.0	2.4	2.4
final	3.7	3.7	3.7	3.7	3.7
Sludge Recycle Ratio (%)					
initial	30	20	35	25	40
final	12.8	12.7	12.7	13.0	12.7
Effluent BOD_5 (mg/l)					
initial	15.4	41.5	23.5	15.9	21.6
final	30.0	30.0	30.0	30.0	30.0
Effluent TSS (mg/l)					
initial	20.5	74.2	45.9	16.6	23.4
final	29.4	29.4	29.4	30.0	29.4
Liquid System Cost (10^3 \$/yr)					
initial	348.0	300.9	343.6	385.4	306.6
final	256.4	256.4	256.4	256.4	256.4
Computer Time (CP seconds)	2.389	4.866	3.168	3.028	2.526

Table 3.14 - Liquid Subsystem Design Optimization for Different Influent Conditions

	Case					
	1	2	3	4	5	6
Influent Conditions :						
Flowrate (m^3/hr)	1515	1510	1515	1515	1515	1515
Soluble BOD_5 (mg/l)	100	100	105	100	100	100
Active Biomass (mg/l)	5	5	5	5	5	5
Volatile Degradable Solids (mg/l)	100	100	100	105	100	100
Volatile Inert Solids (mg/l)	50	50	50	50	55	50
Inorganic Solids (mg/l)	55	55	55	55	55	60
Final Solutions :						
Primary Clarifier Overflow Rate (m/day)	144	144	144	144	144	144
Mean Cell Residence Time (days)	2.15	2.15	2.16	2.15	2.14	2.14
Hydraulic Retention Time (hr)	3.7	3.7	3.8	3.7	3.7	3.7
Sludge Recycle Ratio (%)	12.8	12.8	12.8	12.7	13.0	13.0
Effluent BOD_5 (mg/l)	30	30	30	30	30	30
Effluent TSS (mg/l)	29.4	29.4	29.1	29.2	29.7	29.7
Liquid System Cost (10^3 \$/yr)	256.4	255.8	259.8	258.4	256.8	256.8
Computer Time (CP seconds)	2.389	2.474	3.328	4.094	2.817	2.889

Note : Starting point No. 1 in Table 3.13 was used in all runs.

being the composition of the sludge produced. The volatile inert and inorganic solids are not treated in the activated sludge process, and they do not contribute to the effluent BOD. To avoid excessive build-up of these solids in the system, which would require a larger aeration tank and a larger final clarifier, more solids have to be wasted either in the overflow or to the sludge processing train. A low sludge age and high solids concentration in the effluent are direct consequences of this increased solids concentration in the influent. The fact that the liquid system cost is not affected by the ratio between the volatile inert and inorganic solids has important implication in the analysis of the sludge treatment subsystem design (step (4) of the decomposition procedure). It allows the optimization of value of the ratio of the volatile inert and the inorganic solids concentrations (z) in the sludge sybsystem based on only one optimization run for the liquid subsystem design.

As mentioned above, golden section search was used in the sludge subsystem design optimization of the value of the ratio z. A typical cost curve resulting from this search is shown in Figure 3.11. The cost curves exhibited this general shape for all runs made in this study. This shape results in fast convergence of the sludge subsystem design.

The search for the cost-effective overall system design was carried out by examining various combinations of Q_{10}, Q_{13}, and Q_{15}. During the liquid subsystem design, the solution obtained from each GRG run was saved and used as the starting solution for the next run. It was observed that this strategy saves computing time by about 50% when compared to the strategy of starting from an arbitrarily chosen solution. This is because the starting solution is closer to the final optimal solution. As was shown in the test runs for liquid subsystem design optimization (Tables 3.12 through 3.14), the cost surface of this problem is flat, and convergence to a unique local optimum was often observed. These observations support the use of a previously determined optimal solution as the starting point for a new optimization run.

Tables 3.15 to 3.17 present results obtained from the proposed optimization approach. A coarse grid enumeration was performed for various combinations of values of Q_{13} and Q_{15} for Q_{10} equal to 1.0, 4.0, and 7.0 m^3/hr, respectively. The computing time required to solve the liquid subsystem problem ranged from 1.58 to 2.91 seconds when GRG2 was used interactively on a CDC Cyber 175 computer. The computing time for sludge subsystem design averaged about 0.08 seconds. Fifty-three runs altogether were made to explore any trends exhibited by the cost-effective designs.

The following observations can be made from the results in Tables 3.15 to 3.17. For fixed values of Q_{10} and Q_{13}, the total system cost decreases as Q_{15} increases, which implies an increasingly efficient vacuum filter for sludge dewatering. The total system cost keeps decreasing until the cake concentration equals the assumed upper bound of 15%. For fixed Q_{10} and Q_{15}, an increase in Q_{13} implies a larger secondary digester which produces a more concentrated sludge for dewatering and final disposal. Therefore the total system cost decreases. For fixed values of Q_{13} and Q_{15}, increasing Q_{10} produces decreasing system costs.

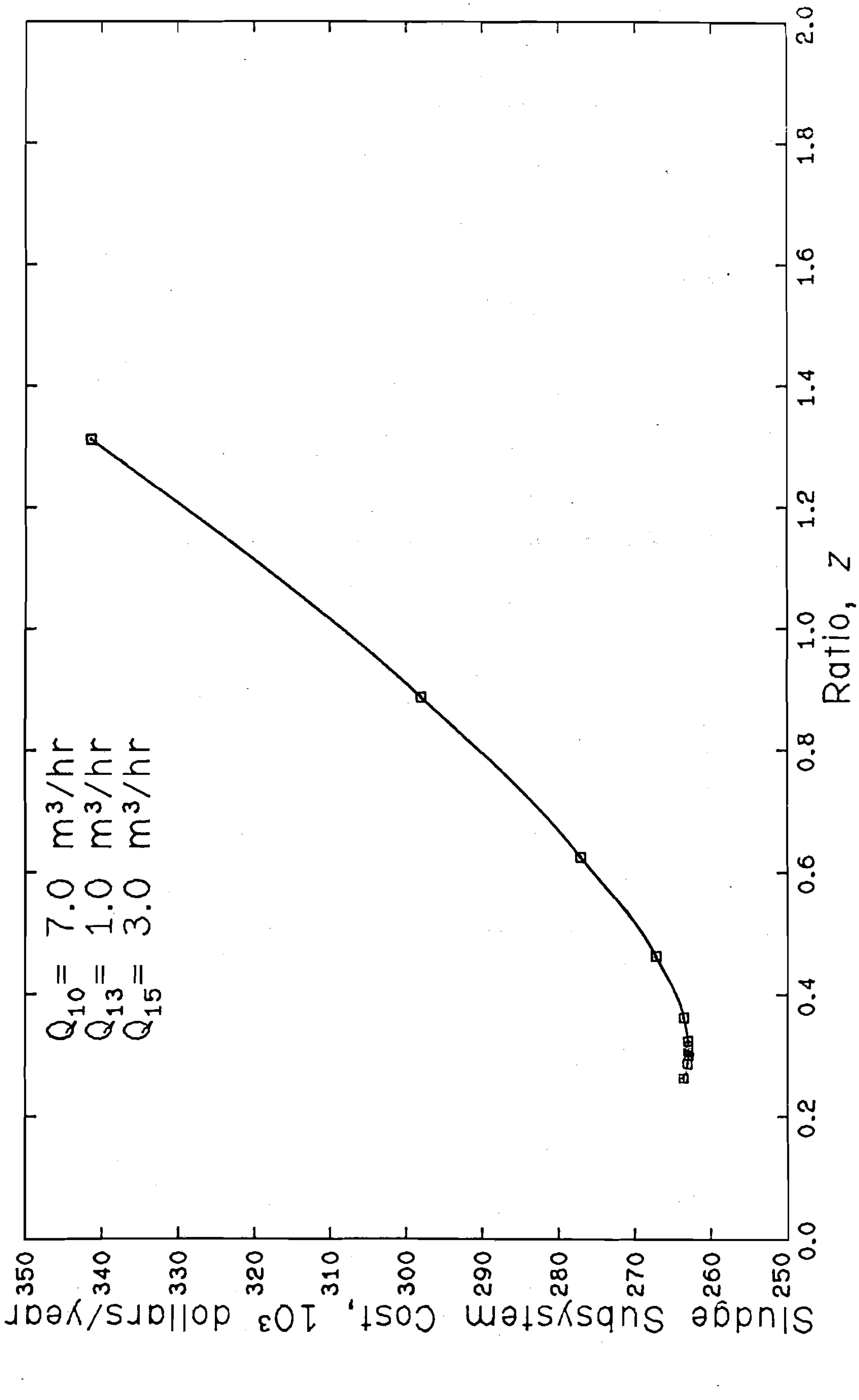


Figure 3.11- Golden Section Search For the Optimal *z*

Table 3.15 - Approximate Designs of Wastewater Treatment System : $Q_{10} = 1.0\ m^3/hr$

Q_{13} (m^3/hr)	0.0						1.0					
Q_{15} (m^3/hr)	1.0	3.0	5.0	7.0	9.0	11.0	1.0	3.0	5.0	7.0	9.0	10.14
Liquid Subsystem :												
Primary Clarifier Overflow Rate (m/day)	144	144	144	144	144	144	144	144	144	144	144	144
Mean Cell Residence Time (days)	2.17	2.17	2.16	2.15	2.15	2.14	2.18	2.16	2.15	2.15	2.14	2.14
Hydraulic Retention Time (hr)	3.63	3.66	3.69	3.71	3.74	3.75	3.62	3.69	3.71	3.74	3.75	3.76
Sludge Recycle Ratio (%)	12.4	12.5	12.6	12.7	12.8	13.0	12.3	12.6	12.7	12.8	13.0	13.0
Cost (10^3 $/yr)	254.3	255.0	255.8	256.5	257.3	258.0	253.9	255.5	256.3	257.2	257.8	258.2
Computer Time (CP seconds)	2.315	1.875	1.875	1.830	1.845	1.782	1.716	1.821	1.734	1.891	1.724	1.674
Sludge Subsystem :												
Solids Loading on Thickener (kg/m^2/day)	47.9	48.5*	49.0*	49.7*	50.2*	50.8*	48.7*	49.1*	49.6*	50.2*	50.8*	51.1*
Digestion Temperature (°C)	59.3	59.3	59.3	59.3	59.3	59.3	59.3	59.3	59.3	59.3	59.3	59.3
Retention Time in Digester (days)	6.95	7.02	7.05	7.08	7.03	6.92	7.02	6.72	7.11	7.17	6.92	6.81
Solids Loading on Digester (kg/m^2/day)	152*	152*	152*	152*	151*	149*	134*	130*	133*	132*	134*	133*
Filter Yield (kg/m^2/hr)	14.7	8.43	6.48	5.44	4.77*	4.30*	14.7	8.48	6.49	5.45	4.75*	4.47*
Cake Solids Concentration (kg/m^3)	11	13	16	23	40	151*	11	14	19	28	58	151*
Cost (10^3 $/yr)	583.9	544.3	459.9	440.0	373.2	284.5	563.9	523.5	473.2	414.2	339.2	283.9
Computer Time (CP seconds)	.078	.083	.081	.080	.085	.082	.080	.082	.084	.083	.079	.078
Total System Cost (10^3 $/yr)	838.2	799.3	751.7	696.6	630.4	542.6	817.9	779.1	729.5	671.3	597.0	542.1

* Infeasible if the bounds on the decision variables (see Table 2.6) are considered.

Table 3.15 (continued)

Q_{13} (m^3/hr)	3.0					5.0				7.0		
Q_{15} (m^3/hr)	1.0	3.0	5.0	7.0	8.42	1.0	3.0	5.0	6.70	1.0	3.0	4.97
Liquid Subsystem :												
Primary Clarifier Overflow Rate (m/day)	144	144	144	144	144	144	144	144	144	144	144	144
Mean Cell Residence Time (days)	2.15	2.15	2.14	2.14	2.13	2.14	2.13	2.13	2.13	2.13	2.12	2.12
Hydraulic Retention Time (hr)	3.71	3.71	3.75	3.78	3.80	3.74	3.78	3.80	3.83	3.81	3.83	3.86
Sludge Recycle Ratio (%)	12.7	12.9	13.0	13.0	13.1	13.0	13.0	13.2	13.2	13.1	13.2	13.3
Cost (10^3 \$/yr)	255.8	256.5	257.3	258.0	258.5	256.7	257.5	258.2	258.9	257.7	258.5	259.2
Computer Time (CP seconds)	2.107	1.581	1.917	1.764	1.773	1.789	1.699	1.586	1.695	1.665	1.695	1.887
Sludge Subsystem :												
Solids Loading on Thickener (kg/m^2/day)	49.6*	50.3*	50.8*	51.3*	51.7*	50.8*	51.3*	51.8*	52.3*	51.9*	52.4*	52.8*
Digestion Temperature (°C)	59.3	59.3	59.3	59.3	59.3	59.3	59.3	59.3	59.3	59.3	59.3	59.3
Retention Time in Digester (days)	7.27	7.16	7.05	6.79	6.74	7.04	6.89	6.82	6.62	6.81	7.17	6.42
Solids Loading on Digester (kg/m^2/day)	102*	101*	100*	98*	103*	70.3*	70.1*	70.5*	74.5*	43.3	46.2	48.0
Filter Yield (kg/m^2/hr)	14.5	8.33	6.44	5.45	4.88*	14.4	8.30	6.42	5.45	14.3	8.18	6.31
Cake Solids Concentration (kg/m^3)	13	18	26	51	152*	17	24	45	151*	23	39	150
Cost (10^3 \$/yr)	522.0	478.3	422.3	353.0	282.3	478.9	428.6	363.4	281.8	432.7	373.9	283.0
Computer Time (CP seconds)	.078	.081	.085	.077	.085	.078	.082	.086	.079	.080	.079	.083
Total System Cost (10^3 \$/yr)	777.8	734.8	679.5	611.0	540.8	735.7	686.1	621.7	540.7	690.4	632.4	542.2

* Infeasible if the bounds on the decision variables (see Table 2.6) are considered.

Total Computer Time : 45.188 CP seconds.

Table 3.16 - Approximate Designs of Wastewater Treatment System : $Q_{10} = 4.0\ m^3/hr$

Q_{13} (m^3/hr)	0.0					1.0			
Q_{15} (m^3/hr)	1.0	3.0	5.0	7.0	7.53	1.0	3.0	5.0	6.68
Liquid Subsystem :									
Primary Clarifier Overflow Rate (m/day)	144	144	144	144	144	144	144	144	144
Mean Cell Residence Time (days)	2.17	2.17	2.16	2.16	2.15	2.17	2.16	2.15	2.15
Hydraulic Retention Time (hr)	3.62	3.65	3.68	3.71	3.72	3.65	3.68	3.71	3.73
Sludge Recycle Ratio (%)	12.4	12.5	12.6	12.7	12.7	12.5	12.6	12.7	12.7
Cost (10^3 $/yr)	254.4	255.1	255.9	256.6	256.8	254.9	255.6	256.4	257.0
Computer Time (CP seconds)	1.729	1.868	1.935	1.846	2.075	2.185	2.048	1.904	1.904
Sludge Subsystem :									
Solids Loading on Thickener ($kg/m^2/day$)	27.9	28.7	29.5	30.3	30.4	28.6	29.5	30.2	30.9
Digestion Temperature (°C)	59.3	59.3	59.3	59.3	59.3	59.3	59.3	59.3	59.3
Retention Time in Digester (days)	8.77	8.81	8.74	8.60	8.53	8.88	9.02	9.15	8.58
Solids Loading on Digester ($kg/m^2/day$)	89.2*	90.4*	91.1*	91.3*	91.0*	73.9*	75.3*	76.6*	77.2*
Filter Yield ($kg/m^2/hr$)	14.2	8.16	6.28	5.29	5.10	14.2	8.12	6.24	5.38
Cake Solids Concentration (kg/m^3)	15	20	32	84	151*	16	24	43	151*
Cost (10^3 $/yr)	478.0	430.0	369.3	290.3	261.5	455.8	405.4	339.6	261.2
Computer Time (CP seconds)	.080	.084	.079	.083	.077	.084	.084	.085	.081
Total System Cost (10^3 $/yr)	732.4	685.2	625.2	546.3	518.3	710.7	661.0	595.9	518.2

* Infeasible if the bounds on the decision variables (see Table 2.6) are considered.

Table 3.16 (continued)

Q_{13} (m³/hr)	3.0			5.0			7.0	
Q_{15} (m³/hr)	1.0	3.0	5.0	1.0	3.0	3.28	1.0	1.55
Liquid Subsystem :								
Primary Clarifier Overflow Rate (m/day)	144	144	144	144	144	144	144	144
Mean Cell Residence Time (days)	2.15	2.15	2.14	2.14	2.14	2.13	2.13	2.13
Hydraulic Retention Time (hr)	3.71	3.71	3.74	3.75	3.77	3.78	3.80	3.81
Sludge Recycle Ratio (%)	12.7	12.9	13.0	12.9	13.0	13.0	13.1	13.1
Cost (10^3 \$/yr)	255.9	256.6	257.4	256.8	257.6	257.7	257.8	258.0
Computer Time (CP seconds)	1.602	1.735	1.834	1.723	1.686	1.782	1.741	1.781
Sludge Subsystem :								
Solids Loading on Thickener (kg/m²/day)	30.2	31.0	31.8	31.6	32.4	32.5	33.1	33.3
Digestion Temperature (°C)	59.3	59.3	59.3	59.3	59.3	59.3	59.3	59.3
Retention Time in Digester (days)	9.29	9.47	9.29	9.57	9.23	9.16	11.2	9.70
Solids Loading on Digester (kg/m²/day)	47.9	50.1*	51.0*	25.6	27.6	28.4	10.2*	10.4*
Filter Yield (kg/m²/hr)	13.9	7.99	6.17	13.7	7.93	7.51	13.3	10.8
Cake Solids Concentration (kg/m³)	22	37	154*	34	107	151*	77	146
Cost (10^3 \$/yr)	409.5	350.9	263.5	360.4	283.3	264.8	314.2	283.0
Computer Time (CP seconds)	.081	.084	.083	.085	.086	.082	.088	.080
Total System Cost (10^3 \$/yr)	665.3	607.5	520.8	617.2	540.9	522.5	572.0	541.0

* Infeasible if the bounds on the decision variables (see Table 2.6) are considered.

Total Computer Time : 32.784 CP seconds.

Table 3.17 - Approximate Designs of Wastewater Treatment System : $Q_{10} = 7.0\ m^3/hr$

Q_{13} (m^3/hr)	0.0					1.0				3.0		
Q_{15} (m^3/hr)	1.0	2.0	3.0	4.0	4.1	1.0	2.0	3.0	3.25	1.0	1.5	1.55
Liquid Subsystem :												
Primary Clarifier Overflow Rate (m/day)	144	144	144	144	144	144	144	144	144	144	144	144
Mean Cell Residence Time (days)	2.17	2.17	2.17	2.16	2.16	2.17	2.16	2.16	2.16	2.15	2.15	2.15
Hydraulic Retention Time (hr)	3.63	3.64	3.65	3.66	3.66	3.65	3.66	3.68	3.68	3.70	3.71	3.71
Sludge Recycle Ratio (%)	12.4	12.4	12.5	12.5	12.5	12.5	12.5	12.6	12.6	12.7	12.7	12.7
Cost (10^3 \$/yr)	254.5	254.9	255.2	255.6	255.6	255.0	255.3	255.7	255.8	255.9	256.1	256.1
Computer Time (CP seconds)	1.781	2.912	1.950	2.420	2.038	1.893	1.747	1.991	1.796	1.734	1.920	1.888
Sludge Subsystem :												
Solids Loading on Thickener ($kg/m^2/day$)	12.0	12.5	12.9	13.3	13.3	12.8	13.3	13.7	13.8	14.4	14.6	14.7
Digestion Temperature (°C)	59.3	59.3	59.3	59.3	59.3	59.3	59.3	59.3	59.3	59.3	59.3	59.3
Retention Time in Digester (days)	12.7	12.5	12.3	12.1	12.0	12.9	12.7	12.5	12.4	14.4	14.3	14.3
Solids Loading on Digester ($kg/m^2/day$)	38.1	39.0	39.7	40.3	40.4	27.3	28.2	29.0	29.1	10.3*	10.8*	10.8*
Filter Yield ($kg/m^2/hr$)	13.6	9.60	7.82	6.77	6.68	13.5	9.51	7.76	7.45	13.1	10.7	10.6
Cake Solids Concentration (kg/m^3)	26	35	55	129	149	33	50	106	151*	78	138	148
Cost (10^3 \$/yr)	368.0	337.6	300.3	252.6	246.9	342.3	307.7	263.0	248.4	292.3	266.8	264.0
Computer Time (CP seconds)	.081	.078	.077	.079	.086	.083	.078	.085	.082	.083	.082	.084
Total System Cost (10^3 \$/yr)	622.5	592.5	555.5	508.2	502.5	597.3	563.0	518.7	504.2	548.2	522.9	520.1

* Infeasible if the bounds on the decision variables (see Table 2.6) are considered.

Total Computer Time : 25.048 CP seconds.

This is attributed to a larger gravity thickener which reduces the volume of sludge to be processed in the subsequent unit processes.

Thus, the trend that indicates a cost-effective design is obvious from this analysis: for this example problem, the cost of the liquid subsystem is not very sensitive to the recycle flowrates, and it is the design of the sludge subsystem that determines the most cost-effective overall system design. To make the sludge subsystem design cost efficient, the volume of the sludge to be processed should be minimized. The above analysis indicates that the gravity thickener is most cost effective for achieving this goal. Although an increased level of sludge concentration produces higher *BOD* and suspended solids mass in the recycle streams to the liquid train, the marginal increase in liquid subsystem cost is much less than the reduced cost for sludge treatment and disposal. The best design obtained from the coarse grid enumeration has $Q_{10} = 7.0$, $Q_{13} = 0.0$, and $Q_{15} = 4.1$ m^3/hr (Table 3.17).

Figures 3.12 to 3.14 depict the cost surfaces for the different combinations of supernatant flowrates. These Figures are graphical representations of the results in Tables 3.15 to 3.17. It is obvious from these plots that the total system cost decreases as Q_{13} or Q_{15} increases for a fixed Q_{10}; the total system cost decreases more rapidly for a unit increase of Q_{15} than a unit increase of Q_{13}. The boundary of the feasible region outside which the cake concentration exceeds its upper bound is also shown approximately in each case by the hashed line. It is noted that the boundary is very flat, meaning that many alternative designs are available at approximately the same total system cost. These alternative designs are different mainly in their designs of the sludge subsystem, although some of them may violate other constraints set on the decision variables. For example, the design with $Q_{10} =$ 4.0 m^3/hr, $Q_{13} = 1.0$ and $Q_{15} = 6.68$ m^3/hr has a total system cost of 518,200 dollars/year; another design with $Q_{10} = 4.0$, $Q_{13} = 5.0$ and $Q_{15} = 3.28$ m^3/hr has a total system cost of 522,500 dollars/year (see Table 3.16); and the third design with $Q_{10} = 7.0$, $Q_{13} = 1.0$ and

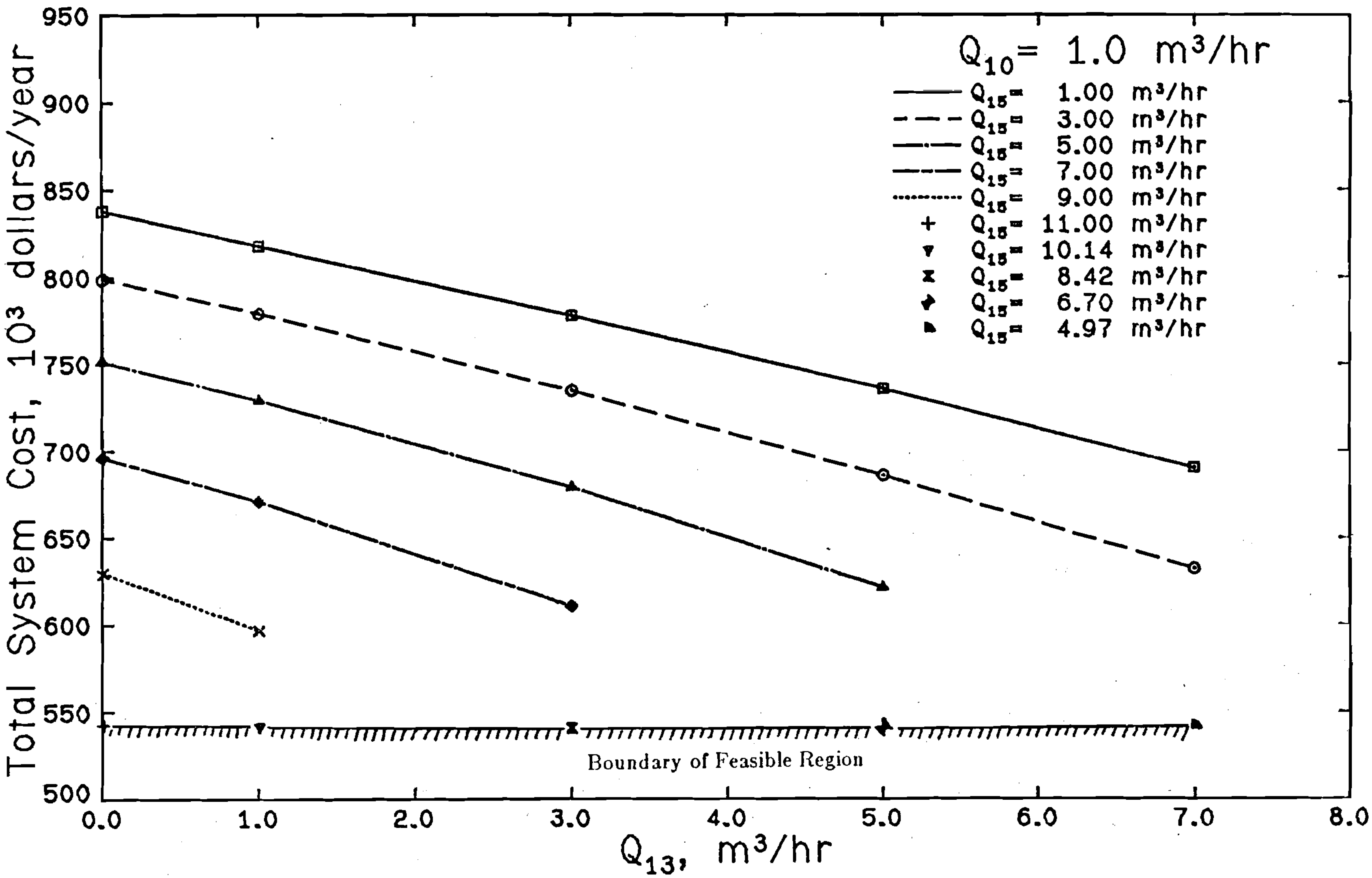


Figure 3.12- Total System Cost vs. Digester Supernatant Flowrate- $Q_{10} = 1.0$ m^3/hr

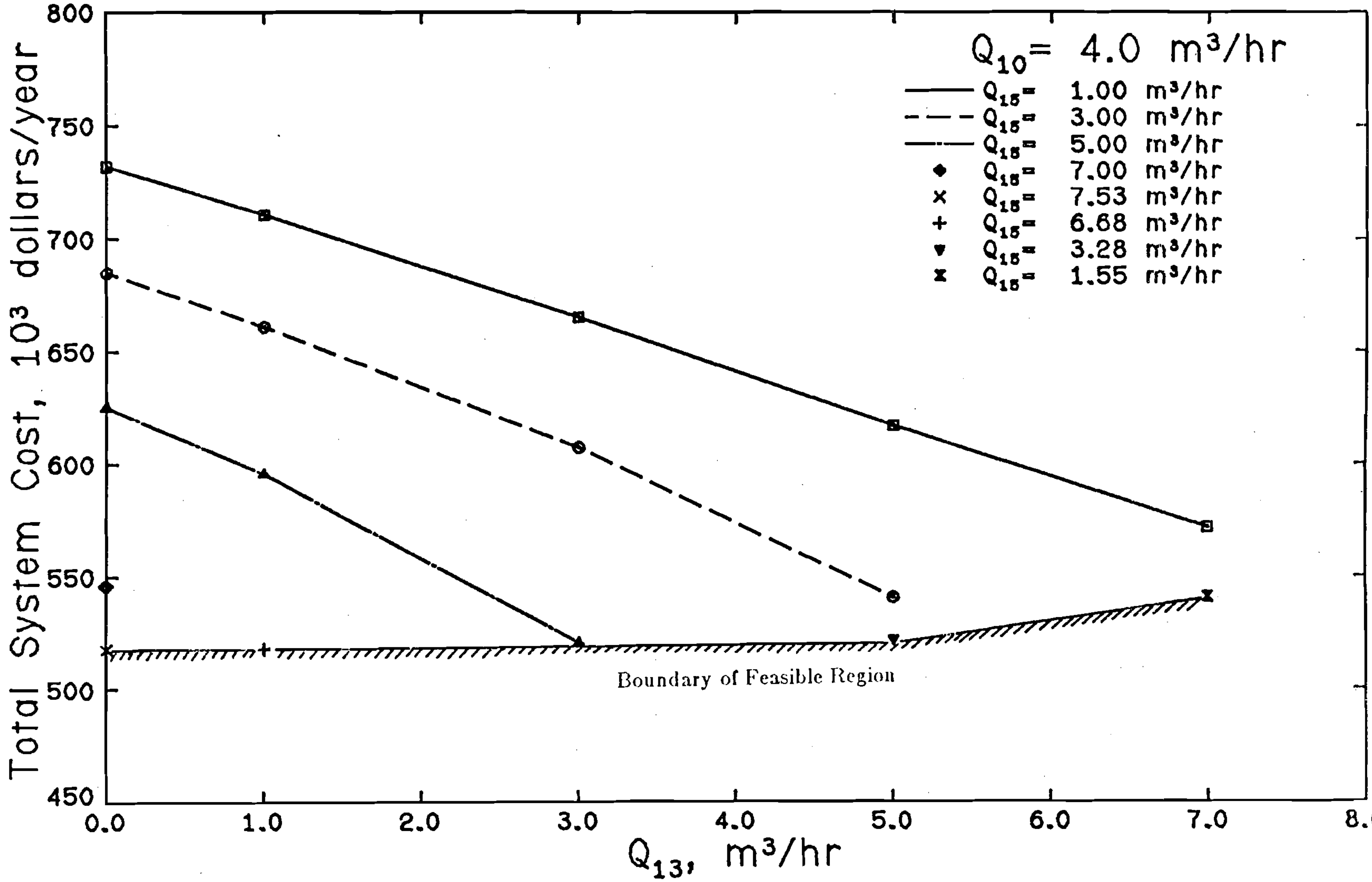


Figure 3.13- Total System Cost vs. Digester Supernatant Flowrate- $Q_{10} = 4.0\ m^3/hr$

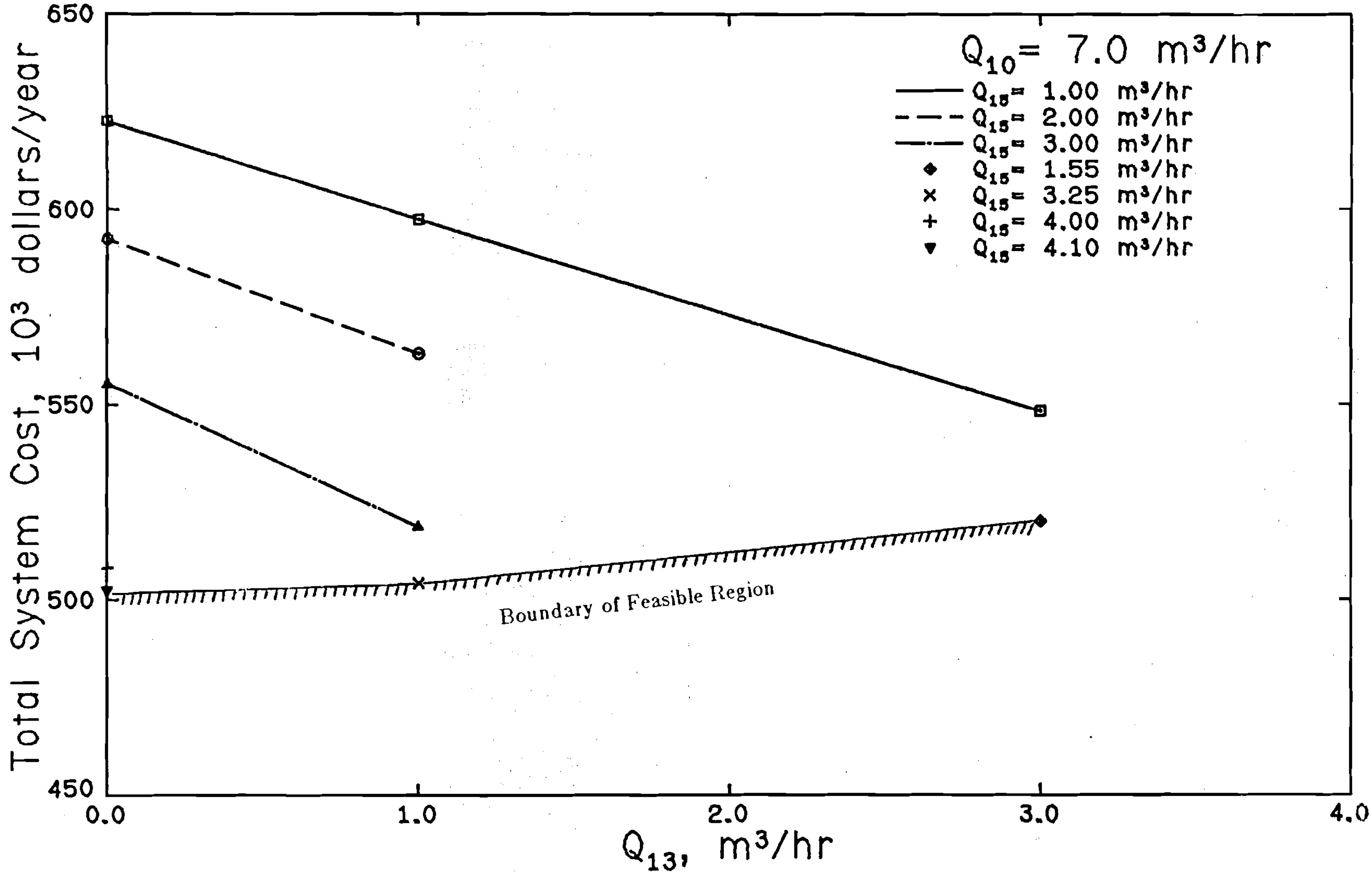


Figure 3.14- Total System Cost vs. Digester Supernatant Flowrate- $Q_{10} = 7.0\ m^3/hr$

Q_{15} = 3.0 m^3/hr has a total system cost of 518,700 dollars/year (see Table 3.17). These three designs are very different in their design of the gravity thickener, the secondary digester and the vacuum filter. All three designs have approximately the same total system cost. However, the solids loading on the secondary digester in the first design is 77.2 kg/m^2/day which is infeasible if the bounds on the decision variables (see Table 2.6) are considered.

If more accurate identification of the most cost efficient design is desired, a fine-tuning step can be employed. As an example, the neighborhood around the best solution described above (given in Table 3.17) was explored based on the trend observed in the coarse grid enumeration. Five runs were made, and the results are summarized in Table 3.18. Although the second design in Table 3.18 with Q_{10} =7.2, Q_{13} = 0 and Q_{15} = 3.9 m^3/hr has the lowest total system cost among the five designs, the extent of violation of its cake solids concentration is also the greatest. Therefore, the fine-tuning process was continued. The final design Q_{10} = 7.27, Q_{13} = 0 and Q_{15} = 3.80 m^3/hr has a total system cost about 501,700 dollars/year. This design is shown in detail in Figure 3.15. Compared with the designs obtained by GRG2 (Tabel 3.9), this design is most similar to the one shown in Figure 3.4 in terms of the state variables in the model. However, this design suggests a smaller aeration tank, a smaller primary digester, and larger final settling tank, secondary digester and vacuum filter. Also, the cake solids concentration is slightly above the upper bound used in the original model solved by GRG2 (see Table 2.6). It is noted that the maximum digestion temperature that can be obtained in the decomposition approach is 59.3 °C because of the stopping criterion specified in the Fibonacci search. The actual upper bound for this variable in the model is 60 °C.

As mentioned above, the solutions obtained using this approach are only approximations to the comprehensive system model because the soluble BOD and the solids concentrations in the thickener supernatant are neglected. The approximation is better when the

Table 3.18 - Fine-tuning Solutions in the Decomposition Approach

Q_{10} (m^3/hr)	7.1	7.2	7.3	7.26	7.27
Q_{13} (m^3/hr)	0	0	0	0	0
Q_{15} (m^3/hr)	4.0	3.9	3.75	3.82	3.80
Liquid Subsystem :					
Primary Clarifier Overflow Rate (m/day)	144	144	144	144	144
Mean Cell Residence Time (days)	2.16	2.16	2.16	2.16	2.16
Hydraulic Retention Time (hr)	3.66	3.66	3.66	3.66	3.66
Sludge Recycle Ratio (%)	12.5	12.5	12.5	12.5	12.5
Cost (10^3 \$/yr)	255.6	255.6	255.5	255.5	255.5
Computer Time (CP seconds)	1.873	1.656	1.670	2.006	1.651
Sludge Subsystem :					
Solids Loading on Thickener (kg/m^2/day)	12.9	12.4	11.9*	12.1	12.0
Digestion Temperature (°C)	59.3	59.3	59.3	59.3	59.3
Retention Time in Digester (days)	12.3	12.3	12.8	12.7	12.7
Solids Loading on Digester (kg/m^2/day)	38.9	37.3	36.0	36.8	36.5
Filter Yield (kg/m^2/hr)	6.75	6.84	6.95	6.89	6.91
Cake Solids Concentration (kg/m^3)	152*	156*	147	152*	151*
Cost (10^3 \$/yr)	246.0	245.2	247.4	246.0	246.2
Computer Time (CP seconds)	.082	.080	.082	.082	.083
Total System Cost (10^3 \$/yr)	501.6	500.8	502.9	501.5	501.7

* Infeasible in the optimization model solved by GRG.

Total Computer Time : 9.265 CP seconds.

thickener supernatant flowrate is small compared to that of the digester and filter supernatants. It is interesting to examine the errors associated with the designs with high thickener supernatant flowrates. Tables 3.19 to 3.21 summarize three designs that have high thickener supernatant flowrates. The values of the decision variables obtained from the decomposition approach were used as inputs to the analysis program (Section 2.5) which calculates the exact values of the state variables in the model. Important design variables calculated from the decomposition approach as well as using the analysis program are compared with each other. The errors in Tables 3.19 to 3.21 for these variables are all less than 1%. These values offer an indication of the maximum possible errors in the decomposition approach; the errors are expected to be smaller when the thickener flowrate is smaller. For the

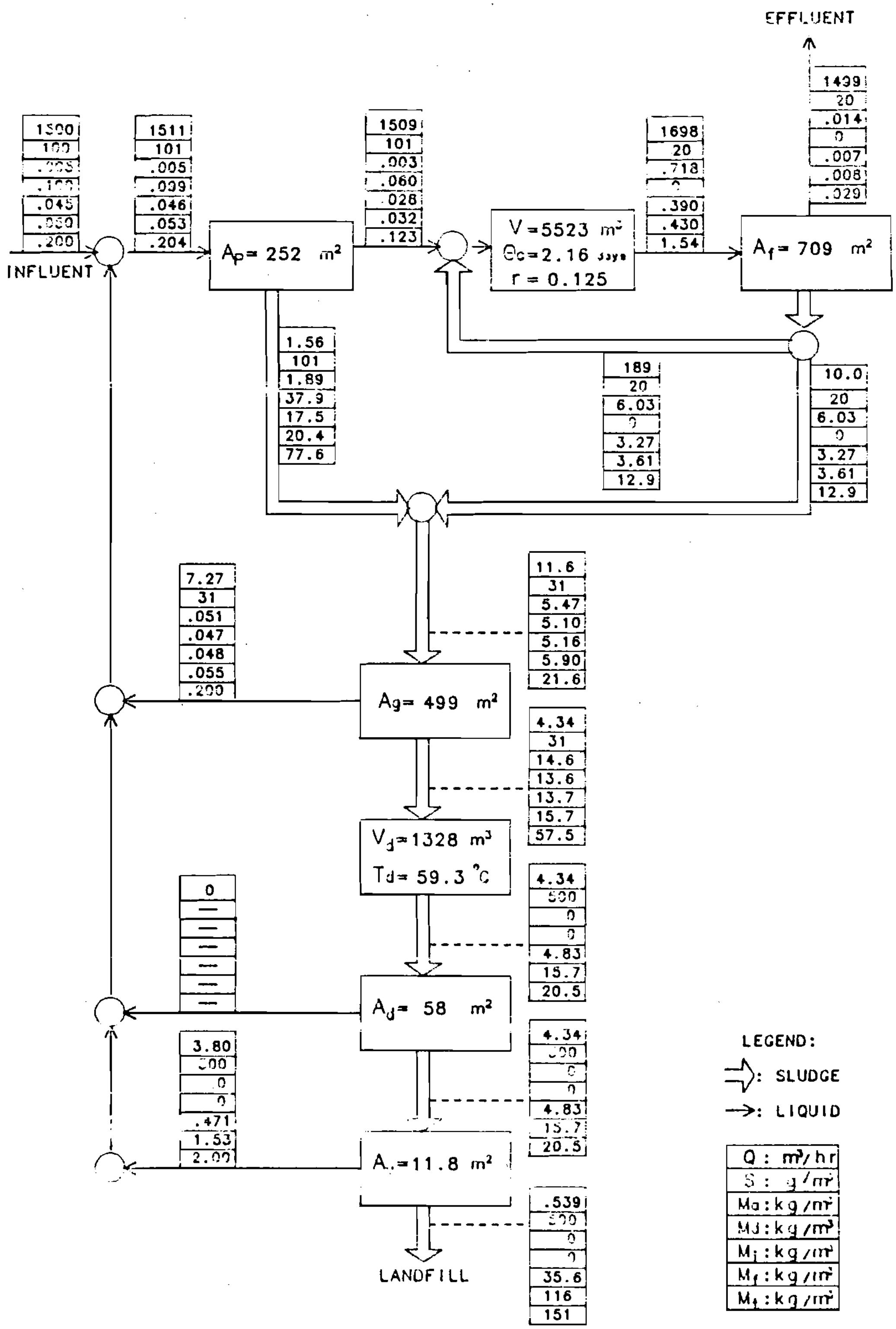


Figure 3.15- Best Design Obtained From The Decomposition Approach

Table 3.19 - Examination of Assumptions in the Decomposition Approach

Primary Clarifier Overflow Rate	= 144 m/day
Mean Cell Residence Time	= 2.17 days
Hydraulic Retention Time	= 3.65 hr
Sludge Recycle Ratio	= 12.5 %
Solids Loading on Thickener	= 12.8 kg/m^2/day
Digestion Temperature	= 59.3 °C
Retention Time in Digester	= 12.9 days
Solids Loading on Digester	= 27.3 kg/m^2/day
Filter Yield	= 13.5 kg/m^2/hr

	Approximated Design	Exact Design	Error* (%)
Primary Clarifier -			
Surface Area (m^2)	251.24	251.25	.0040
Solids Removal (%)	39.330	39.379	.12
Underflow Solids (%)	7.7842	7.7585	.33
Aeration Tank -			
Volume (m^3)	5501.6	5501.8	.0036
Biomass (mg/l)	718.65	719.80	.16
MLSS (mg/l)	1534.6	1537.1	.16
Final Clarifier -			
Surface Area (m^2)	707.60	710.20	.37
Effluent BOD_5 (mg/l)	30.000	29.968	.11
Effluent TSS (mg/l)	29.028	28.935	.32
Gravity Thickener -			
Surface Area (m^2)	463.15	466.11	.64
Influent Solids (%)	2.1679	2.1691	.55
Underflow Solids (%)	5.5135	5.5127	.015
Supernatant (m^3/hr)	7.0000	7.0379	.54
Primary Digester -			
Volume (m^3)	1394.2	1403.3	.65
Effluent Solids (%)	1.9502	1.9503	.0051
Secondary Digester -			
Surface Area (m^2)	73.653	74.138	.66
Supernatant (m^3/hr)	1.0000	1.0063	.63
Vacuum Filter -			
Surface Area (m^2)	6.0589	6.0988	.66
Cake Solids (%)	3.2734	3.2734	.00
Supernatant (m^3/hr)	1.0000	1.0066	.66
Total System Cost (10^3 \$/yr)	597.23	598.87	.27

$$* \text{ Error } (\%) = \frac{|\text{Approximated design value - Exact design value}|}{\text{Exact design value}} \times 100$$

Table 3.20 - Examination of Assumptions in the Decomposition Approach

Primary Clarifier Overflow Rate	= 144 m/day
Mean Cell Residence Time	= 2.16 days
Hydraulic Retention Time	= 3.68 hr
Sludge Recycle Ratio	= 12.6 %
Solids Loading on Thickener	= 13.7 kg/m^2/day
Digestion Temperature	= 59.3 °C
Retention Time in Digester	= 12.5 days
Solids Loading on Digester	= 29.0 kg/m^2/day
Filter Yield	= 7.76 kg/m^2/hr

	Approximated Design	Exact Design	Error*(%)
Primary Clarifier -			
Surface Area (m^2)	251.57	251.58	.0040
Solids Removal (%)	39.454	39.503	.12
Underflow Solids (%)	7.7203	7.6947	.33
Aeration Tank -			
Volume (m^3)	5550.2	5550.4	.036
Biomass (mg/l)	713.26	714.40	.16
MLSS (mg/l)	1540.2	1542.8	.17
Final Clarifier -			
Surface Area (m^2)	705.91	708.65	.39
Effluent BOD_5 (mg/l)	30.000	29.966	.11
Effluent TSS (mg/l)	29.190	29.090	.34
Gravity Thickener -			
Surface Area (m^2)	441.01	443.84	.64
Influent Solids (%)	2.1534	2.1547	.060
Underflow Solids (%)	5.2872	5.2864	.015
Supernatant (m^3/hr)	7.0000	7.0372	.53
Primary Digester -			
Volume (m^3)	1426.5	1435.9	.65
Effluent Solids (%)	1.9134	1.9137	.016
Secondary Digester -			
Surface Area (m^2)	72.058	72.544	.67
Supernatant (m^3/hr)	1.0000	1.0060	.60
Vacuum Filter -			
Surface Area (m^2)	10.465	10.535	.66
Cake Solids (%)	10.606	10.606	.00
Supernatant (m^3/hr)	3.0000	3.0202	.67
Total System Cost (10^3 \$/yr)	518.67	520.00	.25

$$* \text{ Error } (\%) = \frac{|\text{Approximated design value - Exact design value}|}{\text{Exact design value}} \times 100$$

Table 3.21 - Examination of Assumptions in the Decomposition Approach

Primary Clarifier Overflow Rate	= 144 m/day
Mean Cell Residence Time	= 2.16 days
Hydraulic Retention Time	= 3.66 hr
Sludge Recycle Ratio	= 12.5 %
Solids Loading on Thickener	= 12.0 kg/m^2/day
Digestion Temperature	= 59.3 ℃
Retention Time in Digester	= 12.7 days
Solids Loading on Digester	= 36.5 kg/m^2/day
Filter Yield	= 6.91 kg/m^2/hr

	Approximated Design	Exact Design	Error*(%)
Primary Clarifier -			
Surface Area (m^2)	251.58	251.59	.0040
Solids Removal (%)	39.371	39.422	.13
Underflow Solids (%)	7.7631	7.7364	.35
Aeration Tank -			
Volume (m^3)	5522.6	5522.8	.0036
Biomass (mg/l)	717.67	718.87	.17
MLSS (mg/l)	1537.8	1540.4	.17
Final Clarifier -			
Surface Area (m^2)	708.99	711.77	.39
Effluent BOD$_5$ (mg/l)	30.000	29.966	.013
Effluent TSS (mg/l)	29.035	28.935	.35
Gravity Thickener -			
Surface Area (m^2)	499.21	502.54	.66
Influent Solids (%)	2.1628	2.1640	.055
Underflow Solids (%)	5.7529	5.7520	.016
Supernatant (m^3/hr)	7.2700	7.3108	.56
Primary Digester -			
Volume (m^3)	1327.7	1336.8	.68
Effluent Solids (%)	2.0529	2.0531	.0097
Secondary Digester -			
Surface Area (m^2)	58.575	58.995	.71
Supernatant (m^3/hr)	.0000	.0000	--
Vacuum Filter -			
Surface Area (m^2)	11.799	11.881	.69
Cake Solids (%)	15.114	15.130	.11
Supernatant (m^3/hr)	3.8000	3.8264	.69
Total System Cost (10^3 \$/yr)	501.77	503.02	.25

$$* \text{ Error } (\%) = \frac{|\text{Approximated design value - Exact design value}|}{\text{Exact design value}} \times 100$$

parameters and design conditions considered in this example, the simplifying assumptions appear to be very reasonable. It is also noted that the objective function value calculated in the decomposition approach is slightly lower than that calculated from the analysis program in all three cases. Because the suspended solids in the thickener supernatant are ignored in the decomposition approach, the cost for liquid treatment is underestimated, but this error appears insignificant from a practical point of view.

There may be many modifications of the basic decomposition approach outlined in this section. Alternative optimization techniques may be used to optimize the liquid subsystem design. For example, IGGP can be applied to solve this subsystem design. Dynamic programming or any other nonlinear programming techniques are also possible candidates. As for the coordination of the subsystem designs, it may be possible to employ more efficient optimization technique than the coarse grid enumeration to find the combination of recycle flowrates (Q_{10}, Q_{13}, and Q_{15}) that results in the least total system cost. These modifications are potentially capable of refining and improving the proposed basic approach.

3.5. Summary

The comprehensive system model described in Chapter 2 can be optimized using three optimization techniques. The first approach solves the nonlinear programming model, which contains 64 variables, 55 equality constraints, and three inequality constraints, directly using the generalized reduced gradient algorithm developed by Lasdon *et al.* (GRG2). The solution obtained from applying GRG2 depends on the various control parameters assigned, the initial solution, bounds on model variables, and constraint and variable scaling. Computational experience with a particular problem is helpful for obtaining "good quality" solutions. Multiple starting points are necessary to ascertain the quality of the solution obtained. An approach derived from the Hop-Skip-Jump method can be used as a tool to improve and fine-tune the solution obtained by solving the base nonlinear programming wastewater treatment system model. Good but different solutions can also be obtained using this approach. The computing time requirements for GRG2 are comparable to those reported in the literature for solving wastewater treatment system models using other optimization techniques.

The comprehensive system model can also be formulated as a geometric program by modifying the constraint set and by assigning a value to one variable in the model. An efficient package for solving geometric programs (IGGP) can be employed for solving the subproblems resulting from the partitioning process. A one-dimensional enumeration can be used to search for the optimal value of the fixed variable. This second level search could be more efficient if IGGP would be able to start from an infeasible starting point and to proceed with the optimization efficiently. This is prevented by the large number of equality constraints in the model. The computing time for solving the geometric programming subproblems is usually less than five seconds. Therefore IGGP would be more attractive for wastewater treatment systems that can be described completely as a geometric program.

Because of the unique structure of the wastewater treatment system under study, an approach that decomposes the wastewater treatment system into two interacting subsystems was developed for optimization of the overall system design. The liquid subsystem design can be optimized using GRG2 for specified recycle characteristics from the sludge subsystem. This problem contains 21 variables, 17 equality constraints, and three inequality constraints, and it can be solved very efficiently by GRG2. The solution obtained from the liquid subsystem optimization provides input to the sludge subsystem. The design of the sludge subsystem is carried out sequentially for each unit process. Two one-dimensional optimization searches are embedded in the sludge subsystem design. The computing time requirement for the sludge subsystem design is trivial. A coarse grid enumeration is employed for the second level optimization that searches for the combination of the interacting variables that produces the lowest total system cost. Trends for cost-effective system designs can be identified in this approach with confidence. The total computing time for one set of design conditions is comparable to that required when using GRG2 for the entire model. Improvement in the computing time may be possible if an another optimization technique is substituted for enumeration in the second level problem. Several simplifying assumptions are necessary in using the decomposition approach. These assumptions appear very reasonable for the example problem. It is noted that if the same assumptions are applied to the original model evaluated using GRG2, three variables and three constraints can be omitted. However, the model is still of considerable size, and the same difficulties discussed above in using GRG2 for solving the entire system model are expected to occur.

Using the GRG2 algorithm to solve the comprehensive system model is the most straightforward approach for optimization. Once formulated, the model can be used repetitively to examine various influent and design conditions with only minor adjustments of the input data files. However, if the flowchart is modified, the system model needs to be revised and most variables and constraints in the model need to be relabeled which may involve extensive effort. If the size of the problem increases, however, the efficiency of the algorithm decreases drastically. Therefore although it is useful as a tool for process analysis because it can be applied directly, it may not be the best strategy for optimizing a complex wastewater treatment system. The use of this algorithm for the analysis of wastewater treatment processes is illustrated in more detail in Chapter 4.

IGGP is an efficient program for solving geometric programs. However, for the wastewater treatment system model that contains a large number of equality constraints, the optimization performs better with feasible starting solution. Therefore the second level problem of finding the optimal value of the partitioned variable cannot be solved by efficient optimization technique. In addition, the model has to be formulated as a geometric program before IGGP can be applied, which may not always be possible because process design equations may be of any mathematically complicated forms.

The decomposition approach is specially developed to solve the comprehensive system model by taking advantage of the unique structure of the waste treatment system and reducing the dimensionality of the problem. By decomposing the overall system into interacting subsystems, different optimization algorithms can be applied to solve different subsystem designs. Nonlinear programming algorithms are also more efficient for solving problems of smaller size. This approach is also quite flexible, since design of some unit processes is done on a modular basis. Consequently, modifications of the process flowchart will not cause extensive revision of the system model in terms of human effort. The identification of any trend related to cost-efficient design is especially useful since it suggests design guidelines. Also, many solutions with good total system costs are identified in this approach. These solutions can then be evaluated with respect to other planning issues that are not captured in the cost minimization model.

CHAPTER 4

AN ILLUSTRATION OF THE USE OF THE OPTIMIZATION MODEL FOR PROCESS ANALYSIS AND DESIGN

4.1. Introduction

An optimization model can be used to obtain cost effective designs of the wastewater treatment system defined by the selected process performance models and parameters. Using an optimization model also enables the designer to analyze process performances systematically and effectively. Detailed design of the entire wastewater treatment system can then be performed following the guidelines or trends suggested from the modeling study.

In this chapter the role of an optimization model is explored, and it is shown that such a model may be used for more than just identifying a least-cost system design. Specifically, such a model can be used as a tool for the analysis of treatment process performance and of alternative treatment plant configurations. Potentially important research areas or design guidelines can also be identified from these insights.

The hypothetical wastewater treatment system described in Figure 2.1 was designed using various optimization approaches described in Chapter 3 for the design conditions summarized in Tables 2.6 to 2.8. The final designs obtained from using GRG2 are summarized in Table 3.9. These designs provide the basis for the following discussion. They have several common characteristics; the overflow rate of the primary settling tank, the digester operating temperature, and the solids concentration of the cake from the vacuum filter are at their upper bounds. The implications associated with a variable being at its specified bound in the final solution may provide useful insights. Relaxing such a bound may imply that the total system cost could be reduced. It may be necessary, however, to extrapolate process models. Additional research may be needed to justify such extensions if bounds imposed on the decision variables represent ranges recommended for design or limits within which the process model is developed. On the other hand, if the bounds represent the limits outside which process failure will occur, then extrapolation of a process model is inappropriate. Modification of the process flowchart may also be suggested when a variable is at its bound. For instance if an unusually high upper bound on a loading rate is approached in the optimization solutions, then it may be desirable to eliminate that unit process.

Design of wastewater treatment systems is subject to uncertainties. Uncertainties arise from parameter estimation, cost information, the prediction of influent characteristics, possible changes in the water quality regulations, and the lack of knowledge about the performance of some unit processes. While design is usually carried out by assuming steady-state conditions, an operating wastewater treatment plant is more likely to receive sewage varying with time in quantity as well as in strength. There may also be other important planning issues that are specific for each plant; examples are energy requirements, effluent limitation on a specific pollutant, and system reliability concerns. In light of these realistic considerations, the design obtained from the mathematical optimization of a comprehensive system design model needs to be examined carefully or modified so that the final plant being constructed will meet the design goals.

This chapter presents observations and discussions drawn from an examination of the solutions obtained from the optimization of the example wastewater treatment system. The discussion is on a unit-by-unit basis. Finally the design of wastewater treatment plant is considered as a two-objective problem to illustrate a simplistic approach for design under uncertainty. The tradeoff between economic efficiency and a flow safety factor is studied. This design approach allows the use of an optimization model as a useful preliminary design aid.

4.2. Primary Sedimentation

Typical design guidelines for a primary settling tank generally call for the overflow rate to be less than or equal to 40 meters/day under the average flow conditions (see, for example, Great Lakes- Upper Mississippi River Board of State Sanitary Engineers, 1978). In their pilot scale studies, Tebbutt and Christoulas (1975) investigated the performance of primary settling tanks for overflow rates up to 150 meters/day. Their results implied that the current practice is too conservative. As a result, an upper bound of 144 meters/day was imposed on the overflow rate in the comprehensive system model. The final design showed that the overflow rate is at this upper bound.

This solution suggests that the total system cost may be further reduced by relaxing the upper bound on the overflow rate because of a negative reduced gradient associated with this variable in the final solution. Two major questions arise:

1) Is the Voshel-Sak model a valid representation of the primary clarifier performance when the overflow rate is as high as that assumed in the comprehensive system model?

2) Is the primary clarifier a cost-effective unit in the assumed wastewater treatment system?

Extrapolating the Voshel-Sak model to high overflow rates shows that solids removal efficiency decreases only marginally as the loading increases substantially. This is depicted in Figure 4.1. It is expected that the solids removal efficiency will decrease sharply when the overflow rate reaches a critical value. Therefore the behavior of the primary settling tank at high overflow rates should be an area of further investigation.

To address the second question, the primary settling tank was eliminated from the base system. The modified system is shown in Figure 4.2. The GRG2 was used to determine an optimal design under the base conditions listed in Tables 2.6 through 2.8. The GRG2 model describing the system design has 51 variables, 43 equality constraints, and

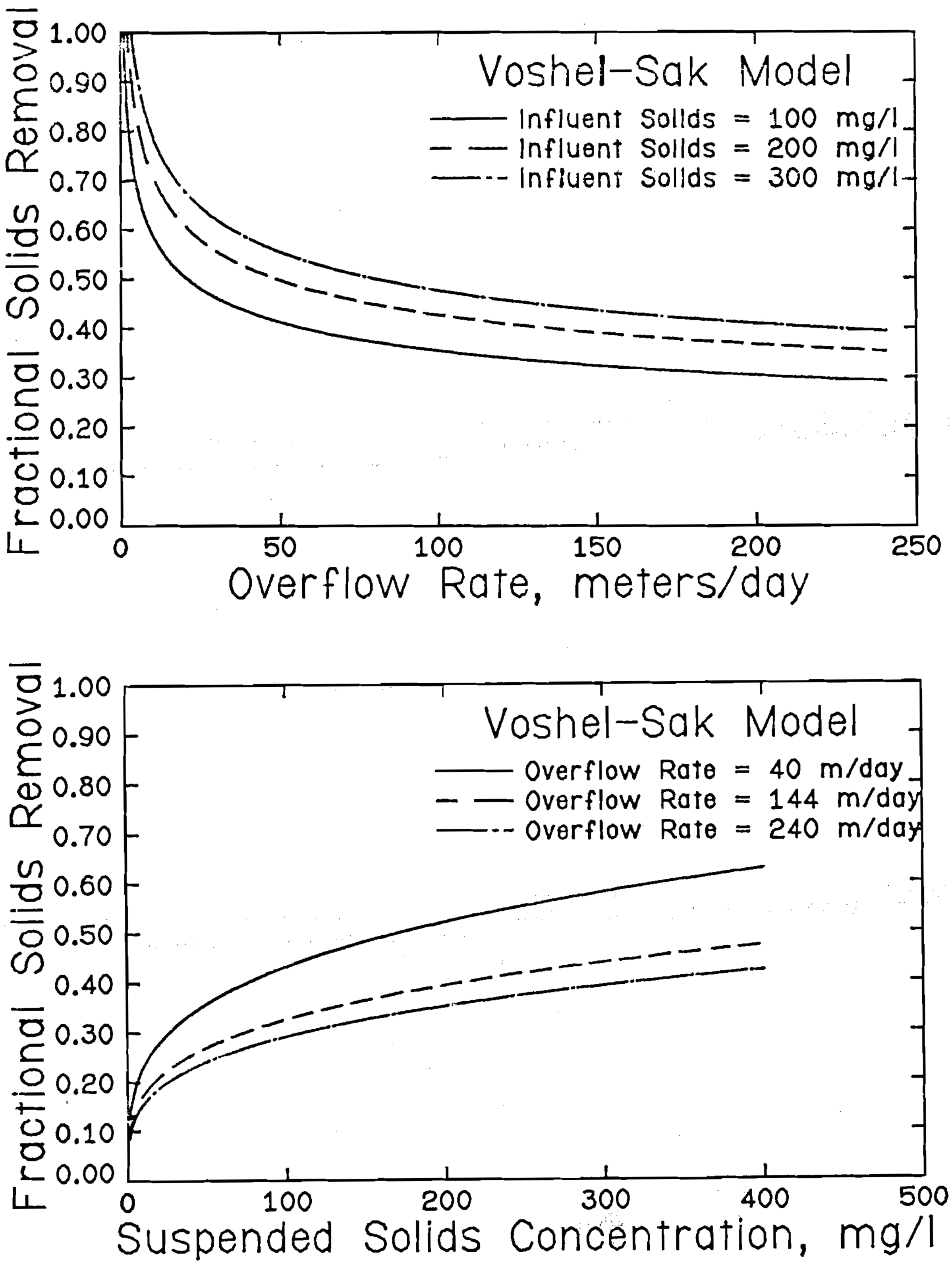


Figure 4.1- Performance of Primary Clarifier as Predicted by the Voshel-Sak Model

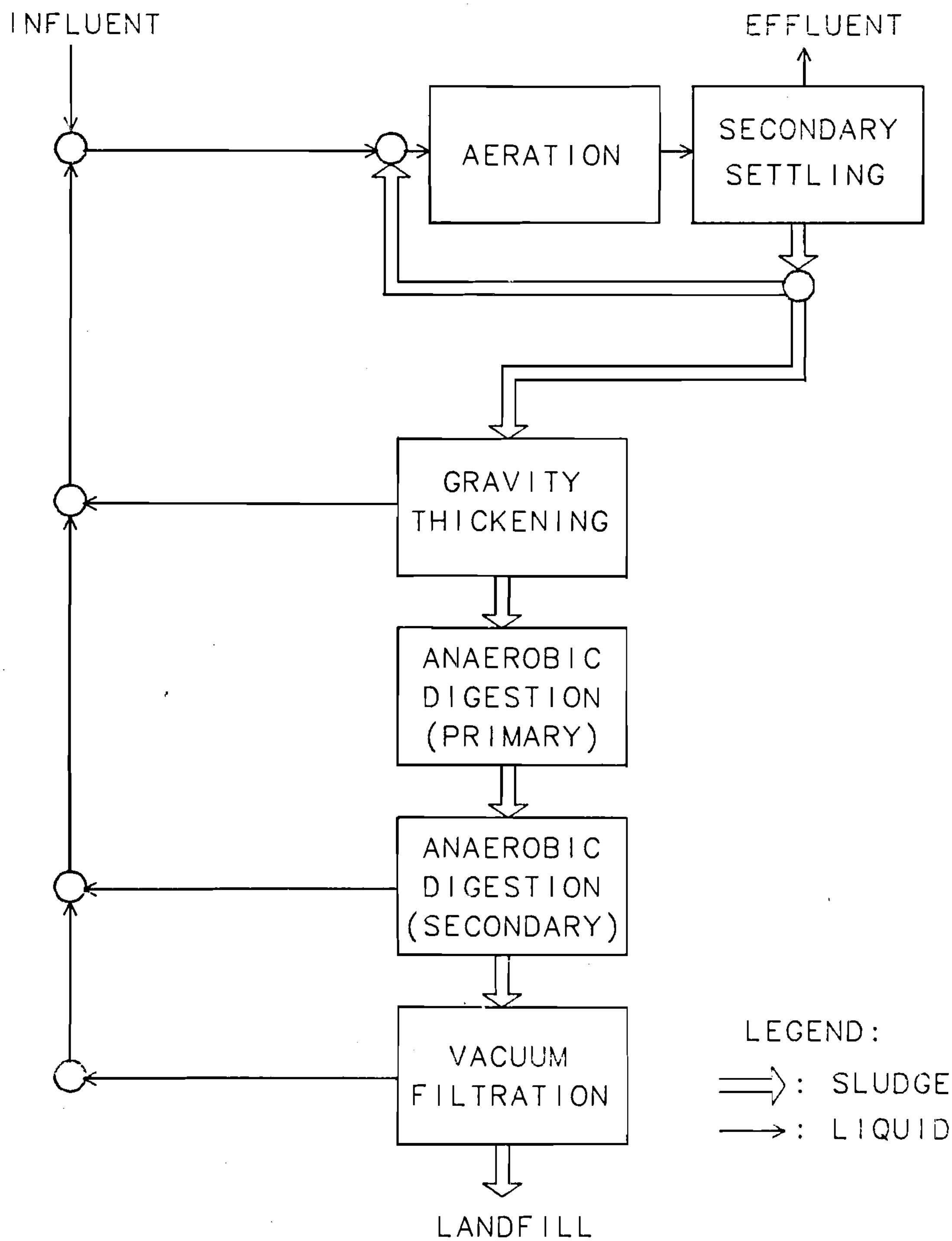


Figure 4.2- Wastewater Treatment System Without a Primary Clarifier

three inequality constraints. The computer program listing of this model is included in Appendix H. Table 4.1 summarizes the results of the optimization.

Three different starting points were used for the GRG2 runs. The final designs are very similar, and the total system cost without the primary clarifier is about 492,500 dollars/year, or 1.6% less than the final design with the primary clarifier. The final design obtained from using starting point No. 1 is shown in Figure 4.3 (refer to Section 2.2.2 for the notation). A comparison between this design and the one with the primary clarifier in the system (design No. 6 in Table 3.9) is shown by Table 4.2. Without the primary clarifier in the system, a larger aeration tank and final clarifier are needed to achieve the same effluent water quality. However, the total sludge production is less because of the absence of primary sludge. Therefore the costs for sludge treatment and disposal are less. However, the biological parameters used for design of the system without the primary clarifier are likely to be different from those of the system with the primary clarifier. This is a weakness of this analysis and further research is necessary to determine how the biological parameters are affected by the absence of the primary clarifier. For the base design conditions, with the assumption that biological parameters are constant, provision of the primary clarifier appears to be unjustified as far as the economic efficiency of the system is concerned.

To explore further the role of the primary clarifier, the influent volatile biodegradable suspended solids concentration was increased to 200 mg/l while the other parameters in the model remained unchanged. Five different starting points were used for the GRG2 optimization runs, and the results are tabulated in Table 4.3. In contrast to the results when the base design conditions were evaluated, the primary clarifier overflow rate is not at the upper bound of 144 meters/day in any of the final solutions. This suggests that the presence of this unit is cost-effective for these design conditions. The final design obtained from starting point No. 5 is shown in Figure 4.4; this design has the lowest total system cost (545,000 dollars/year) among the five final designs.

Table 4.1 - Treatment Plant Design Optimization : Base System Without a Primary Clarifier

Variable (Unit)	Solution Obtained Using GRG2		
	1	2	3
Mean Cell Residence Time (days)			
initial	3.0	5.5	2.5
final	2.08	2.08	2.08
Hydraulic Retention Time (hr)			
initial	6.0	8.0	4.0
final	4.6	4.7	4.7
Sludge Recycle Ratio (%)			
initial	25.0	30.0	15.0
final	14.7	14.6	14.5
Solids Loading on Thickener ($kg/m^2/day$)			
initial	24.0	36.0	18.0
final	12.3	12.0	12.1
Digestion Temperature ($^\circ C$)			
initial	35	35	35
final	60	60	60
Retention Time in Digester (days)			
initial	15.0	20.0	10.0
final	12.4	13.4	12.9
Solids Loading on Digester ($kg/m^2/day$)			
initial	24.0	36.0	18.0
final	34.9	34.6	34.7
Filter Yield ($kg/m^2/hr$)			
initial	8.0	6.9	9.0
final	7.02	7.03	7.03
Cake Solids Concentration (kg/m^3)			
initial	142.2	162.2	125.5
final	150.0	150.0	150.0
Effluent BOD_5 (mg/l)			
initial	25.1	14.0	21.7
final	30.0	30.0	30.0
Effluent TSS (mg/l)			
initial	39.3	24.3	16.1
final	30.0	30.0	30.0
Total System Cost (10^3 \$/yr)			
initial	585.1	661.7	655.5
final	492.6	492.5	492.5
Computer Time (CP seconds)	38.073	85.200	26.636

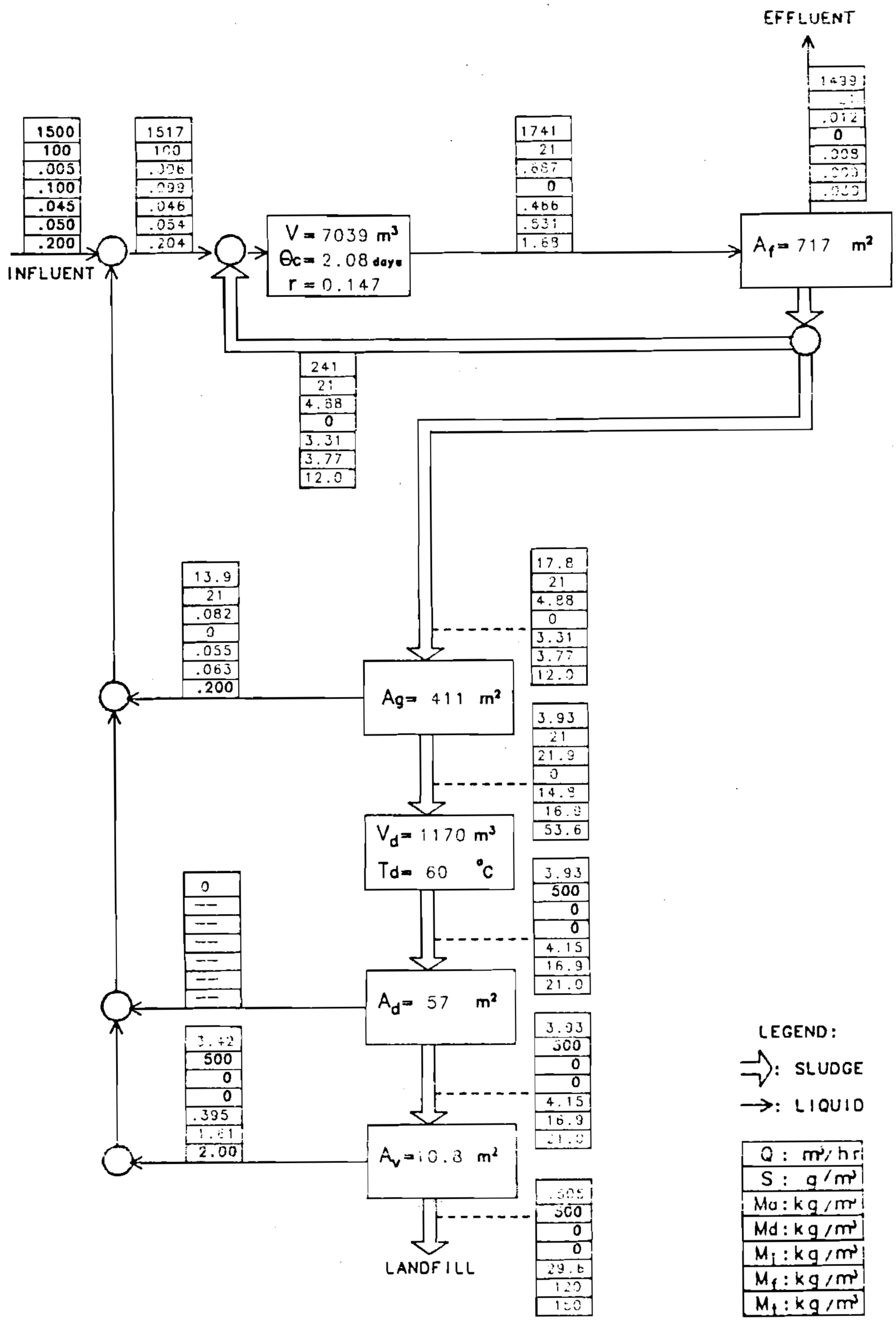


Figure 4.3 - Final Design for the Wastewater Treatment System Without a Primary Clarifier

Table 4.2 - Final Designs With and Without a Primary Clarifier in the System

Variables (Units)	With Primary Clarifier	Without Primary Clarifier
Primary Clarifier Surface Area (m^2)	251	--
Mean Cell Residence Time (days)	2.19	2.08
Aeration Tank Volume (m^3)	5637	7038
Sludge Recycle Ratio (%)	12.8	14.7
Final Clarifier Surface Area (m^2)	687	717
Thickener Influent Flowrate (m^3/hr)	11.8	17.8
Thickener Influent Solids Concentration (kg/m^3)	21.3	12.0
Thickener Surface Area (m^2)	475	411
Digestion Temperature ($^\circ$C)	60	60
Primary Digester Volume (m^3)	1500	1170
Retention Time in Digester (days)	14.0	12.4
Vacuum Filter Surface Area (m^2)	6.7	10.8
Cake Solids Concentration (kg/m^3)	150	150
Effluent BOD_5 (mg/l)	30.0	30.0
Effluent TSS (mg/l)	30.0	30.0
Total System Cost (10^3 \$/yr)	500.4	492.5

The same design conditions were then examined for a system without a primary clarifier. Four starting points were tested, and the final designs were very similar (Table 4.4). The total system would cost 556,300 dollars/year, which is slightly (2%) higher than that for the base system designed for the same conditions. This design is shown in Figure 4.5. A comparison of the two designs is shown by Table 4.5. It is not surprising to observe that the primary clarifier is cost-effective when the influent wastewater contains high concentration of suspended organic materials. This trend would be expected to apply to even higher, or lower, influent suspended solids levels than those considered here. In general, depending on the design conditions, the observations that can be drawn from a wastewater treatment system optimization study may be very different.

4.3. Activated Sludge

The final designs for the base system are characterized by an effluent that just meets the assumed water quality standards. However, it is possible that only one of the two con-

Table 4.3 - Summary of Wastewater Treatment System Design : Influent Volatile Biodegradable Solids = 200 mg/l

Variables (Units)	Solution Obtained Using GRG2				
	1	2	3	4	5
Primary Clarifier Overflow Rate (m/day)					
initial	36.0	24.0	32.0	36.0	24.0
final	130.0	79.9	116.6	69.2	113.3
Mean Cell Residence Time (days)					
initial	2.0	3.0	4.0	5.0	6.0
final	2.38	2.41	2.39	2.42	2.39
Hydraulic Retention Time (hr)					
initial	2.4	3.6	6.0	4.8	10.8
final	4.3	4.2	4.3	4.1	4.3
Sludge Recycle Ratio (%)					
initial	15.0	30.0	25.0	25.0	10.0
final	14.1	13.6	14.0	13.5	13.9
Solids Loading on Thickener ($kg/m^2/day$)					
initial	12.0	36.0	40.0	24.0	12.0
final	12.0	12.0	12.0	12.0	12.0
Digestion Temperature ($^\circ$C)					
initial	35.0	25.0	35.0	35.0	35.0
final	60.0	60.0	60.0	60.0	60.0
Retention Time in Digester (days)					
initial	15.0	20.0	15.0	15.0	15.0
final	14.7	17.1	14.4	16.0	15.1
Solids Loading on Digester ($kg/m^2/day$)					
initial	12.0	24.0	18.0	24.0	12.0
final	48.0	48.0	48.0	48.0	48.0
Filter Yield ($kg/m^2/hr$)					
initial	12.0	7.8	12.0	8.0	10.0
final	6.41	6.31	6.37	6.31	6.31
Cake Solids Concentration (kg/m^3)					
initial	79.9	186.3	52.2	142.2	164.2
final	121.0	150.0	132.3	148.3	150.0
Effluent BOD_5 (mg/l)					
initial	31.5	35.7	34.9	17.8	24.7
final	30.0	30.0	30.0	30.0	30.0
Effluent TSS (mg/l)					
initial	21.7	50.7	59.5	23.4	41.1
final	30.0	30.0	30.0	30.0	30.0
Total System Cost (10^3 \$/yr)					
initial	788.4	774.3	799.9	768.7	779.4
final	554.6	547.0	550.9	548.1	545.0
Computer Time (CP seconds)*	536	88.273	666	570	525

* Except for starting point No. 2, all computer times reported on this table are recorded when the optimization model and GRG2 are run on a Harris computer. A subroutine calculating the analytical derivatives for all functions in the model is incorporated in these runs.

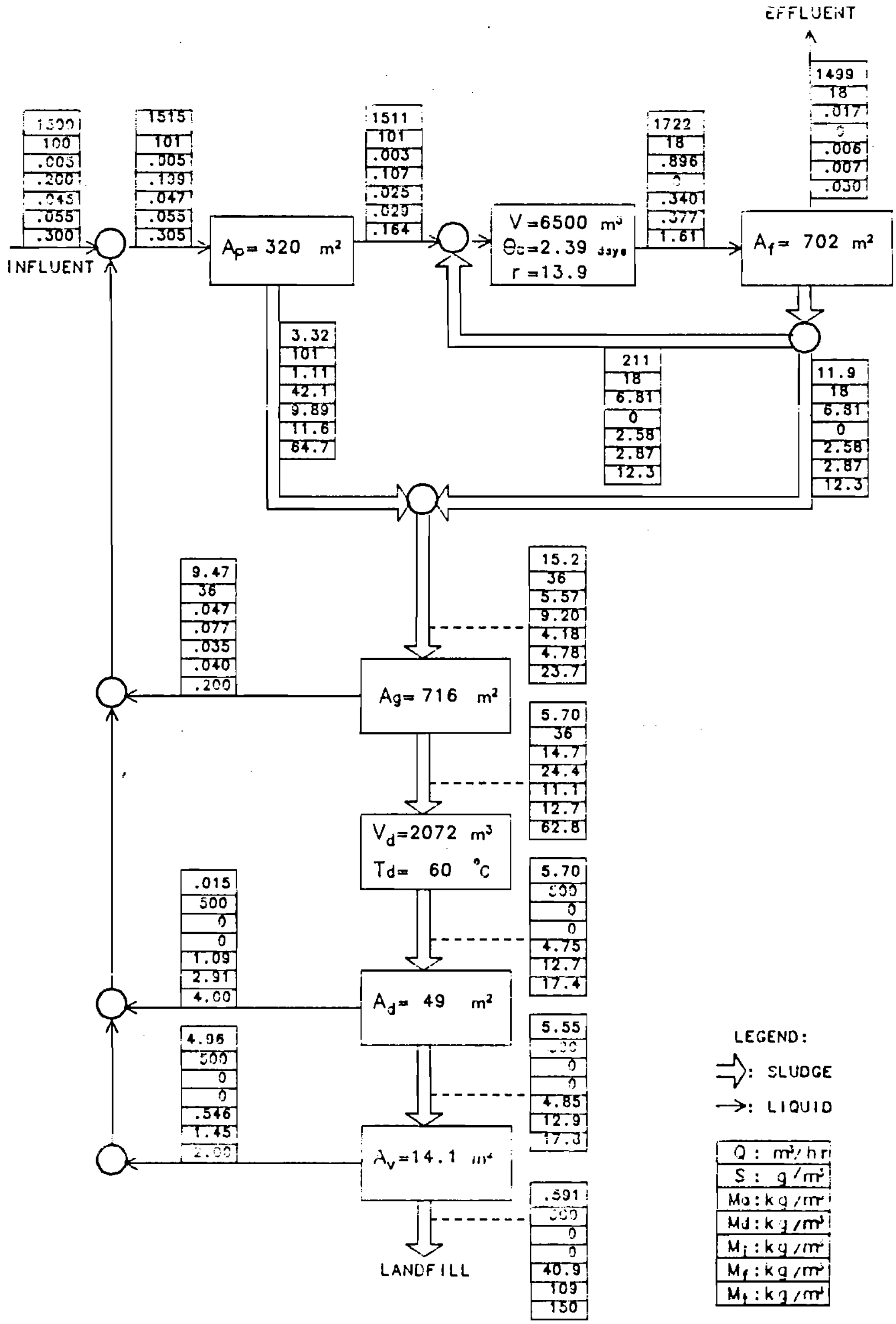


Figure 4.4- Best Design for the Base System: Influent Volatile Biodegradable Suspended Solids Concentration = 200 mg/l

Table 4.4 - Treatment Plant Design Optimization : Base System Without a Primary Clarifier, Influent Volatile Biodegradable Suspended Solids Concentration = 200 mg/l

Variables (Units)	Solution Obtained Using GRG2			
	1	2	3	4
Mean Cell Residence Time (days)				
initial	4.0	3.0	5.0	6.0
final	2.27	2.27	2.27	2.27
Hydraulic Retention Time (hr)				
initial	4.8	6.0	4.8	12.0
final	5.8	5.8	5.8	5.8
Sludge Recycle Ratio (%)				
initial	15.0	10.0	50.0	30.0
final	17.1	17.1	17.1	17.1
Solids Loading on Thickener ($kg/m^2/day$)				
initial	12.0	24.0	18.0	36.0
final	12.0	12.0	12.0	12.0
Digestion Temperature ($^\circ C$)				
initial	35	35	35	35
final	60	60	60	60
Retention Time in Digester (days)				
initial	15.0	20.0	25.0	10.0
final	14.6	14.0	15.1	14.4
Solids Loading on Digester ($kg/m^2/day$)				
initial	12.0	24.0	18.0	30.0
final	45.5	44.9	46.0	45.4
Filter Yield ($kg/m^2/hr$)				
initial	10.0	7.8	8.7	7.4
final	6.42	6.45	6.40	6.43
Cake Solids Concentration (kg/m^3)				
initial	164.1	186.3	162.1	145.1
final	150.0	150.0	150.0	150.0
Effluent BOD_5 (mg/l)				
initial	15.1	17.7	15.7	16.8
final	30.0	30.0	30.0	30.0
Effluent TSS (mg/l)				
initial	13.8	11.3	21.8	30.4
final	30.0	30.0	30.0	30.0
Total System Cost (10^3 \$/yr)				
initial	837.2	743.7	779.4	757.6
final	556.3	556.3	556.3	556.3
Computer Time (CP seconds)	26.105	37.745	45.062	30.451

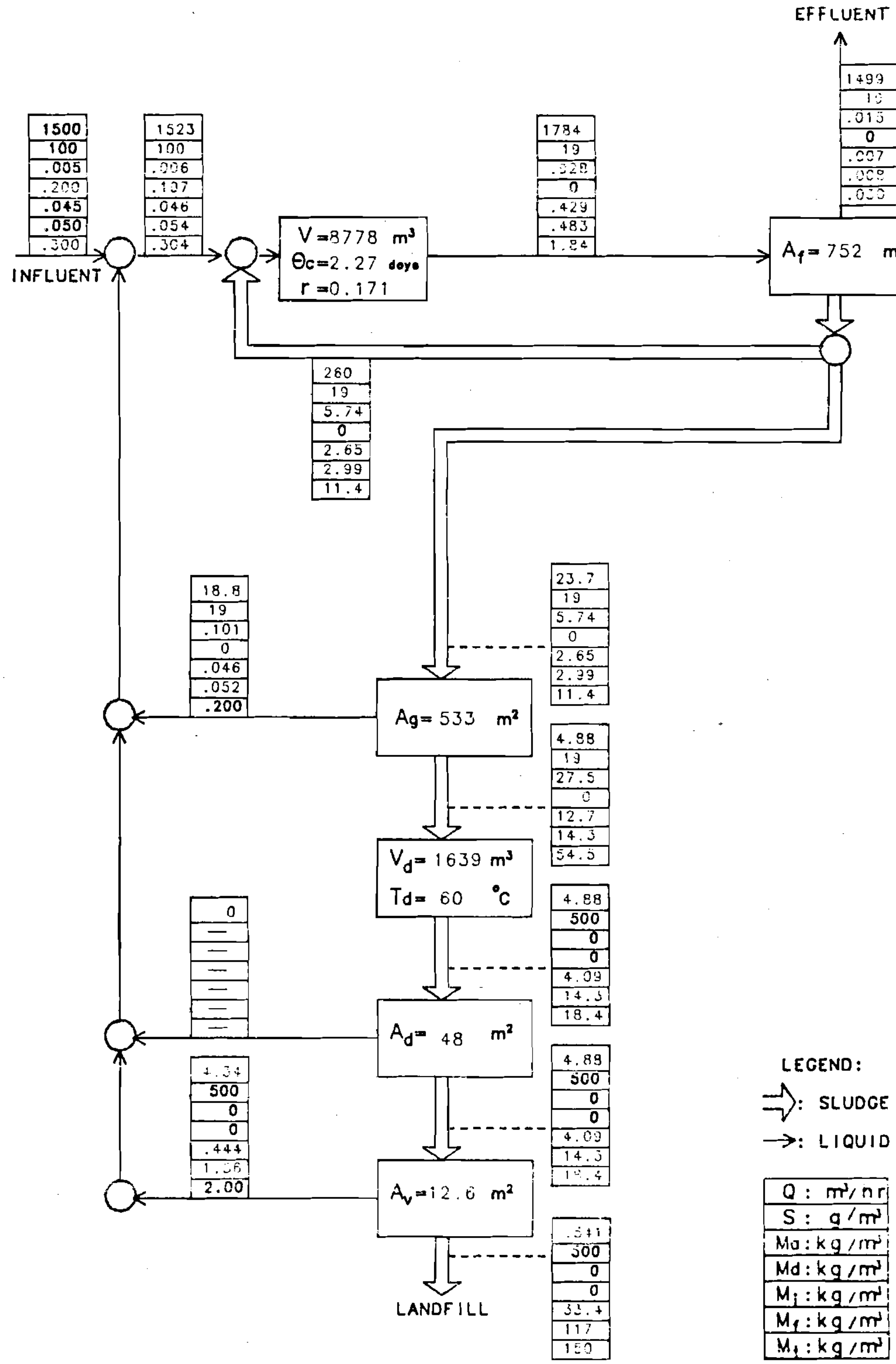


Figure 4.5- Best Design for the System Without a Primary Clarifier:
Influent Volatile Biodegradable Solids Concentration = 200 mg/l

Table 4.5 - Final Designs With and Without a Primary Clarifier in the System : Influent Volatile Biodegradable Suspended Solids Concentration = 200 mg/l

Variables (Units)	With Primary Clarifier	Without Primary Clarifier
Primary Clarifier Surface Area (m^2)	320	--
Mean Cell Residence Time (days)	2.39	2.27
Aeration Tank Volume (m^3)	6500	8778
Sludge Recycle Ratio (%)	13.9	17.1
Final clarifier Surface Area (m^2)	702	752
Thickener Influent Flowrate (m^3/hr)	15.2	23.7
Thickener Influent Solids Concentration (kg/m^3)	23.7	11.4
Thickener Surface Area (m^2)	716	533
Digestion Temperature ($^\circ$C)	60	60
Primary Digester Volume (m^3)	2072	1639
Retention Time in Digester (days)	15.1	14.0
Vacuum Filter Surface Area (m^2)	14.1	12.6
Cake Solids Concentration (kg/m^3)	150	150
Effluent BOD_5 (mg/l)	30.0	30.0
Effluent TSS (mg/l)	30.0	30.0
Total System Cost (10^3 \$/yr)	545.0	556.3

straints would be binding in the final solution if a different set of design conditions are considered. In the final solutions listed in Table 3.9, the sludge ages are about 2.2 days for the design conditions assumed in Tables 2.6 to 2.8 since no provision for nitrification is considered in the model. The sludge recycle ratios (10~13%) are lower than what is usually experienced in practice because the effluent suspended solids concentration increases with the recycle ratio according to Chapman's model. Good sludge thickening in the final settling tank is also suggested at this low value of the sludge recycle ratio.

Sludge settling characteristics could be affected by the sludge age. Bisogni and Lawrence (1971) showed that sludge flocculated and settled better with longer sludge ages. This observation was questioned by Dick and Hasit (1981). Currently there is no consensus on how sludge age affects the activated sludge settling properties. If longer sludge ages do enhance sludge thickening, then the design sludge age should perhaps be longer than that obtained for the base system design.

Increased organic loading to the wastewater treatment plant would be expected to have a direct effect on the design of the activated sludge process. The influent soluble BOD_5 concentration was increased to 200 mg/l and the model was optimized with GRG2 using five different starting points. The results are summarized in Table 4.6. Although the initial designs are quite different, with sludge ages ranging from two to six days, the final solutions obtained by GRG2 are very similar. The system design obtained with starting point No. 1 is shown in Figure 4.6. A comparison of this design with the final design (No. 6 in Table 3.9) obtained for the base design conditions (influent soluble $BOD_5 = 100$ mg/l) is shown by Table 4.7. It is observed that the design of the primary clarifier is not affected by changing the influent soluble BOD_5. This is consistent with the assumption made in the primary clarifier design that the soluble BOD is unaffected by this unit. The design determined for the high influent soluble BOD_5 condition has a slightly higher sludge age in order to meet the same effluent water quality requirements. The aeration tank is bigger, and the MLSS concentration is higher because of the higher organic loading. The sludge production rate is high, resulting in higher costs for sludge treatment and disposal.

4.4. Secondary Sedimentation

The clarification model describing the solids removal of the final settling tank in the activated sludge process plays a critical role in the design of wastewater treatment plants. Most previous researchers (see, for example, Middleton and Lawrence, 1976, Tyteca, 1981) assumed that the final clarifier is 100% efficient in the removal of suspended solids. If the effluent is assumed to be free of suspended solids, then the system design model is subject only to a restriction on the BOD_5 concentration.

This assumption can be expected to have significant impact on the entire treatment plant design. The comprehensive system model was modified to examine this issue; the water quality constraints are reduced to

Table 4.6 - Treatment Plant Design Optimization : Influent Soluble BOD_5 = 200 mg/l

Variables (Units)	Solution Obtained Using GRG2				
	1	2	3	4	5
Primary Clarifier Overflow Rate (m/day)					
initial	36.0	24.0	32.0	36.0	24.0
final	144.0	144.0	144.0	144.0	144.0
Mean Cell Residence Time (days)					
initial	2.0	3.0	4.0	5.0	6.0
final	2.47	2.47	2.47	2.47	2.47
Hydraulic Retention Time (hr)					
initial	2.4	3.6	6.0	4.8	10.8
final	5.1	5.1	5.1	5.1	5.1
Sludge Recycle Ratio (%)					
initial	15.0	30.0	25.0	25.0	10.0
final	15.8	15.7	15.8	15.8	15.8
Solids Loading on Thickener ($kg/m^2/day$)					
initial	12.0	36.0	40.0	24.0	12.0
final	12.0	12.0	12.0	12.0	12.0
Digestion Temperature ($^{\circ}C$)					
initial	30.0	25.0	35.0	30.0	35.0
final	60.0	60.0	60.0	60.0	60.0
Retention Time in Digester (days)					
initial	15.0	20.0	15.0	15.0	15.0
final	14.2	14.2	13.7	14.2	13.6
Solids Loading on Digester ($kg/m^2/day$)					
initial	12.0	24.0	18.0	24.0	12.0
final	48.0	48.0	48.0	48.0	48.0
Filter Yield ($kg/m^2/hr$)					
initial	10.0	10.0	12.0	8.0	12.0
final	6.31	6.31	6.31	6.32	6.31
Cake Solids Concentration (kg/m^3)					
initial	164.1	53.9	52.2	142.2	79.9
final	150.0	150.0	150.0	147.0	150.0
Effluent BOD_5 (mg/l)					
initial	29.0	26.6	24.3	15.7	16.4
final	30.0	30.0	30.0	30.0	30.0
Effluent TSS (mg/l)					
initial	14.6	27.5	31.9	17.6	20.7
final	30.0	30.0	30.0	30.0	30.0
Total System Cost (10^3 \$/yr)					
initial	822.4	937.2	858.0	855.5	876.7
final	577.1	577.1	577.2	577.9	577.3
Computer Time (CP seconds)	64.4	92.9	71.5	78.0	105.2

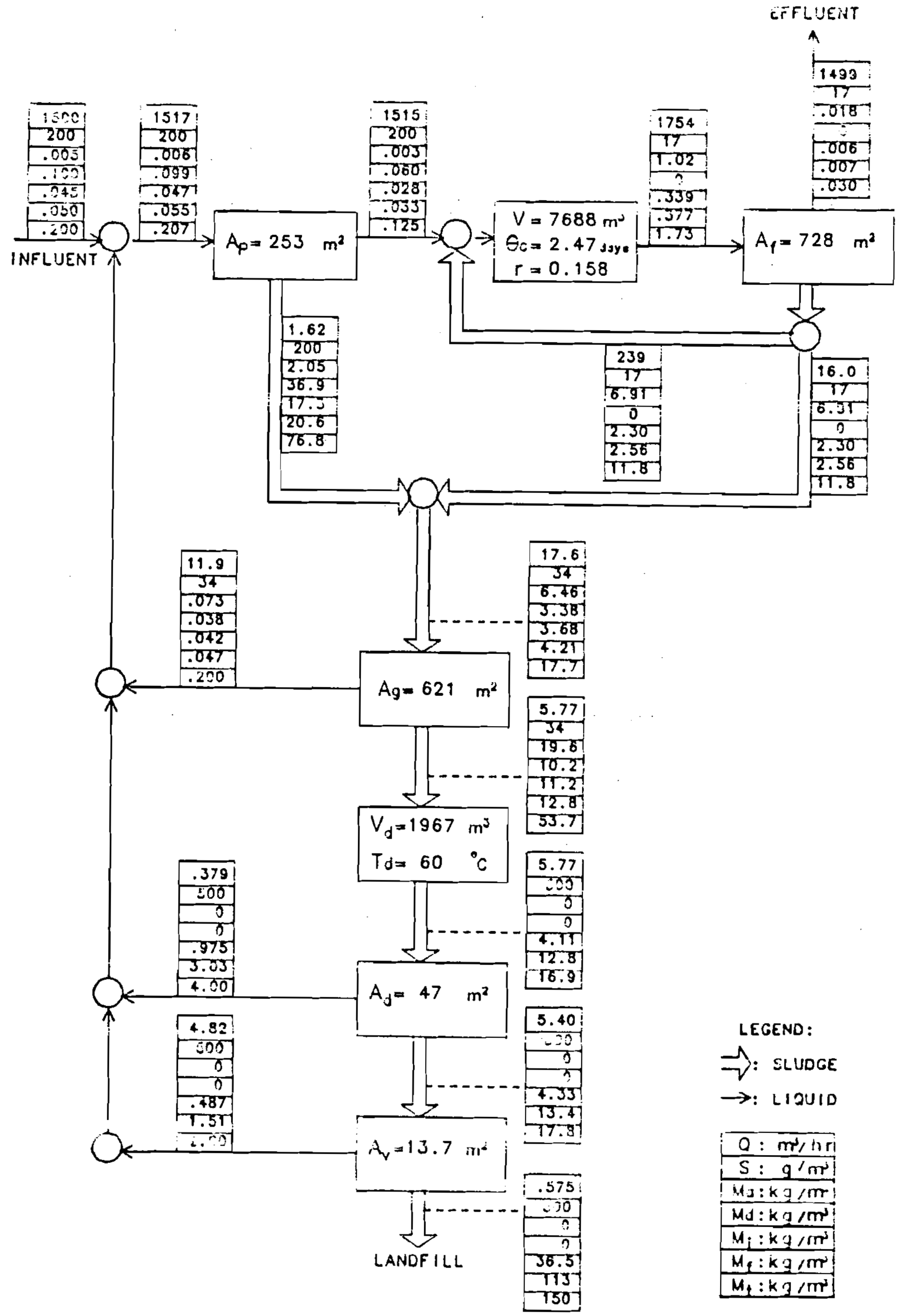


Figure 4.6- Best Design for the Base System: Influent Soluble $BOD_5 = 200$ mg/l

Table 4.7 - Final Designs With Different Influent Soluble BOD_5 Concentration

Variables (Units)	Influent Soluble BOD_5 100 mg/l	200 mg/l
Primary Clarifier Overflow Rate (m/day)	144.0	144.0
Mean Cell Residence Time (days)	2.19	2.47
Hydraulic Retention Time (hr)	3.7	5.1
Aeration Tank Volume (m^3)	5637	7688
MLSS Concentration (mg/l)	1530	1730
Sludge Recycle Ratio (%)	12.8	15.8
Final Clarifier Surface Area (m^2)	687	728
Thickener Influent Flowrate (m^3/hr)	11.8	17.6
Thickener Influent Solids Concentration (kg/m^3)	21.3	17.7
Mass of Sludge Processed (kg/hr)	251	312
Solids Loading on Thickener (kg/m^2/day)	12.6	12.0
Thickener Surface Area (m^2)	475	621
Digestion Temperature (^{o}C)	60.0	60.0
Retention Time in Digester (days)	13.9	14.2
Primary Digester Volume (m^3)	1500	1967
Solids Loading on Digester (kg/m^2/day)	40.6	48.0
Filter Yield (kg/m^2/hr)	6.67	6.31
Vacuum Filter Surface Area (m^2)	6.7	13.7
Cake Solids Concentration (kg/m^3)	150.0	150.0
Effluent BOD_5 (mg/l)	30.0	30.0
Effluent TSS (mg/l)	30.0	30.0
Total System Cost (10^3 \$/yr)	500.4	577.1

$$S_3 \leq BOD_5 \text{ standard} \tag{4.1}$$

where S_3 is the soluble BOD_5 concentration in the plant effluent as defined in Chapter 2. Since the total suspended solids concentration of the effluent is assumed to be zero, no constraint is needed for suspended solids.

Optimization runs were made with a total (soluble) BOD_5 standard of 15 and 10 mg/l for the base design conditions except the influent soluble BOD_5 concentration was changed from 100 to 200 mg/l. The results of these two runs are summarized in Table 4.8. These final designs show that the total system costs are much less than that obtained originally (577,100 dollars/year in Table 4.6) even though the BOD_5 standards are much more stringent (30 mg/l initially). If Chapman's model for clarification correctly calculates the

Table 4.8 - Summary of Optimal Wastewater Treatment System Design Obtained Assuming Complete Clarification

Variable (Unit)	$BOD_5 \leq 15$ mg/l	$BOD_5 \leq 10$ mg/l
Primary Clarifier Overflow Rate (m/day)		
initial	32.0	32.0
final	144.0	144.0
Mean Cell Residence Time (days)		
initial	4.0	4.0
final	2.78	4.07
Hydraulic Retention Time (hr)		
initial	6.0	6.0
final	7.5	10.9
Sludge Recycle Ratio (%)		
initial	25.0	25.0
final	32.8	28.8
Solids Loading on Thickener ($kg/m^2/day$)		
initial	40.0	40.0
final	19.2	16.4
Digestion Temperature ($^\circ C$)		
initial	35.0	35.0
final	60.0	60.0
Retention Time in Digester (days)		
initial	15.0	15.0
final	15.6	15.6
Solids Loading on Digester ($kg/m^2/day$)		
initial	18.0	18.0
final	48.0	48.0
Filter Yield ($kg/m^2/hr$)		
initial	12.0	12.0
final	6.56	6.41
Cake Solids Concentration (kg/m^3)		
initial	52.2	52.2
final	96.7	120.7
Effluent BOD_5 (mg/l)		
initial	24.3	24.3
final	75.3	51.0
Effluent TSS (mg/l)		
initial	31.9	31.9
final	139.1	120.0
Total System Cost (10^3 \$/yr)		
initial	858.0	858.0
final	496.7	535.8
Computer Time (CP seconds)	98.7	78.1

effluent suspended solids concentration, then the designs shown in Table 4.8 are in fact unacceptable because the actual BOD_5 concentration would be greater than 50 mg/l and the actual total suspended solids concentration greater than 120 mg/l; these concentrations are well beyond the water quality restrictions.

This example illustrates that it is important, of course, for a comprehensive system model to include complete performance relationships for all unit processes in the system. Performance relationships for some unit processes, however, may not be available or not be reliable. In such cases, making simplifying assumptions are crucial since an "optimal design" obtained is not likely to be optimal or even feasible when the process mechanisms are taken into account. This example also supports the view that in general it is more important to use such a system model as a tool to identify the limitations of current process models and future research areas, and to analyze the trends for cost-effective process synthesis or design, rather than to use such a model to obtain the "optimal system design."

4.5. Sludge Thickening

Sludge thickening in a wastewater treatment plant is provided to reduce the volume of sludges for processing and final disposal. Very large thickeners are specified by the solutions to the base system model, and the digester influents have concentrations higher than 5% in all designs in Table 3.9. With these high solids concentrations, the costs of heating the digester influent become outweighed by the benefits that can be derived from the methane production in the digester. Therefore an efficiently designed thickener is the key to a cost-effective sludge treatment train. However, there should be a practical limit on thickener design beyond which the limiting flux theory is no longer applicable for predicting underflow solids concentration. The lower bound for the solids loading of the gravity thickener in the model is 12 kg/m^2/day which is lower than values usually observed in practice. The modeling study suggests that the limitations of the limiting flux theory be investigated. A long detention time in the thickener may cause sludge degradation in the thickener and problems in sludge transport.

There are a number of possible schemes for sludge thickening other than that assumed in the base flowchart. One such scheme has been analyzed using the system model. If the limiting flux theory is valid for primary sludge thickening as assumed, then recirculation of the waste activated sludge to the primary clarifier would appear to be very attractive. The separate thickener could then be eliminated from the system. and yet a very concentrated sludge could be obtained from the primary settling tank and pumped directly to the digester. This scheme is depicted in Figure 4.7. The thick sludge would have a significant impact in reducing the cost of sludge treatment and disposal.

The GRG optimization model was modified to represent the flowchart shown in Figure 4.7. The revised optimization model has 51 variables, 43 equations, and three inequality constraints. A listing of the optimization model is in Appendix I.

Results of optimizing the treatment system design using GRG2 are listed in Table 4.9. Five starting points were used in this exercise. The final solutions have objective function values ranging from 466,200 to 469,200 dollars/year, representing cost reductions of 6.2 to 6.8% from the cost of the base system designed for the same conditions (which has total system cost of about 500,400 dollars/year). The final design obtained from starting point No. 1 in Table 4.9 is shown in Figure 4.8.

Because of the use of the primary clarifier as a thickener, the final solutions specify that the size of this unit be from 400 to 750 m^2, which are significantly larger values than the 250 m^2 obtained for the base system. In the design shown in Figure 4.8, the primary sludge is about 7.5% (75.5 kg/m^3) with a flowrate of 3.45 m^3/hr. This sludge is a highly concentrated digester influent with a high organic content, which helps to produce more methane gas at a moderate digester retention time (17.6 days). A comparison of the pri-

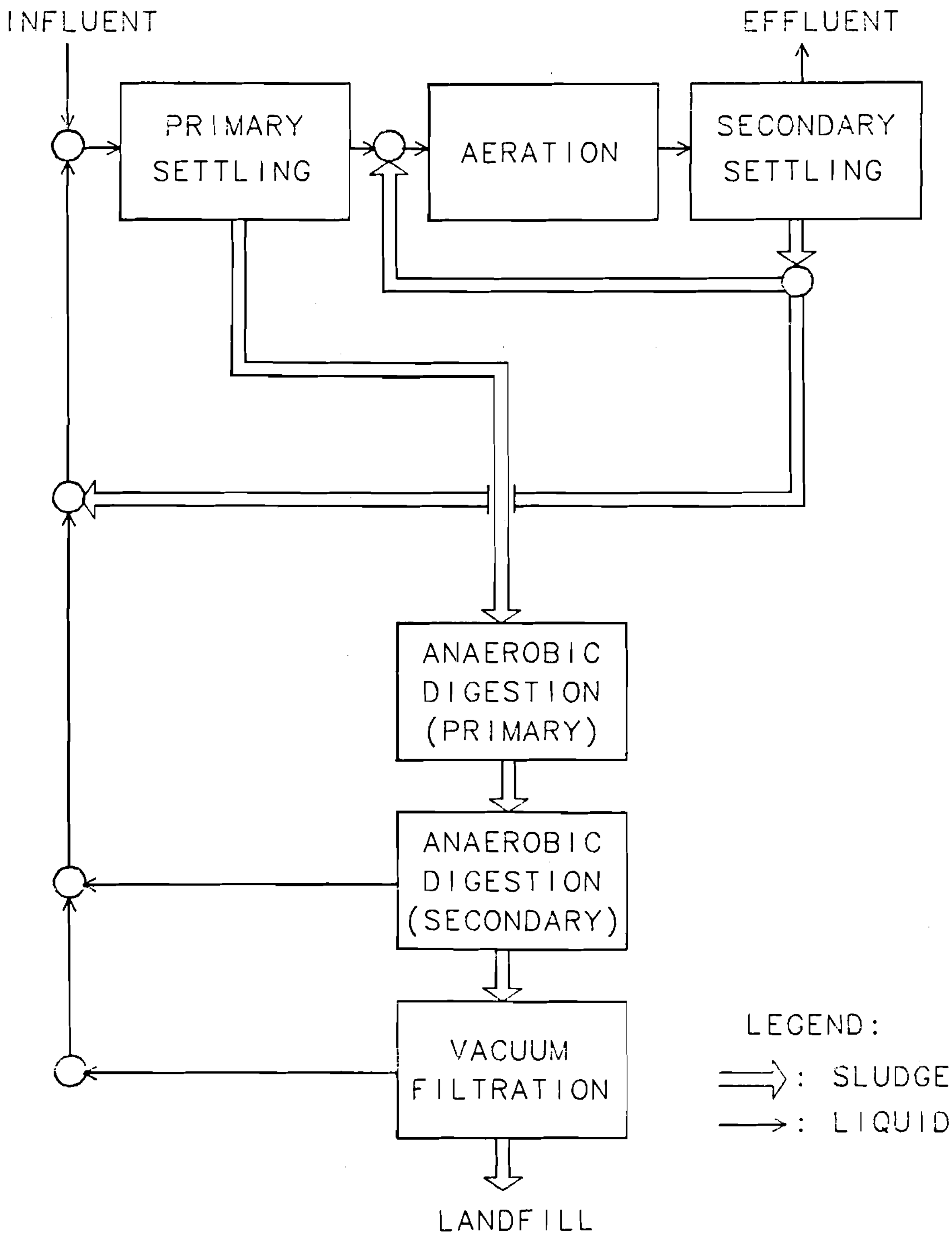


Figure 4.7- Recirculation of Waste Activated Sludge to Primary Clarifier

Table 4.9 - Summary of System Design Optimization : Waste Activated Sludge Returned to Primary Settling Tank

Variables (Units)	Solution Obtained Using GRG2				
	1	2	3	4	5
Primary Clarifier Overflow Rate (m/day)					
initial	72.0	36.0	48.0	60.0	120.0
final	70.7	58.1	48.1	57.3	90.8
Mean Cell Residence Time (days)					
initial	3.0	2.5	5.0	5.5	4.5
final	2.27	2.29	2.30	2.29	2.25
Hydraulic Retention Time (hr)					
initial	3.6	2.9	6.0	4.8	3.6
final	3.4	3.2	3.3	3.5	3.4
Sludge Recycle Ratio (%)					
initial	20.0	15.0	30.0	20.0	30.0
final	11.5	12.4	11.3	10.6	11.5
Digestion Temperature ($^{\circ}$C)					
initial	50	35	35	35	35
final	60	60	60	60	60
Retention Time in Digester (days)					
initial	25.0	20.0	10.0	25.0	20.0
final	17.6	23.1	19.1	20.8	15.3
Solids Loading on Digester (kg/m^2/day)					
initial	12.0	2.4	2.4	4.8	7.2
final	26.5	23.8	18.8	22.8	32.2
Filter Yield (kg/m^2/hr)					
initial	10.0	20.0	20.0	14.5	12.0
final	7.69	7.98	8.64	8.10	7.20
Cake Solids Concentration (kg/m^3)					
initial	164.1	161.4	161.4	153.3	168.7
final	150.0	150.0	150.0	150.0	150.0
Effluent BOD_5 (mg/l)					
initial	26.0	27.8	28.8	14.8	16.7
final	30.0	30.0	30.0	30.0	30.0
Effluent TSS (mg/l)					
initial	33.6	28.1	56.1	20.2	22.0
final	30.0	30.0	30.0	30.0	30.0
Total System Cost (10^3 $/yr)					
initial	505.1	692.3	724.2	670.4	653.7
final	466.2	468.0	469.2	467.4	467.0
Computer Time (CP seconds)	23.929	29.507	46.465	27.029	33.950

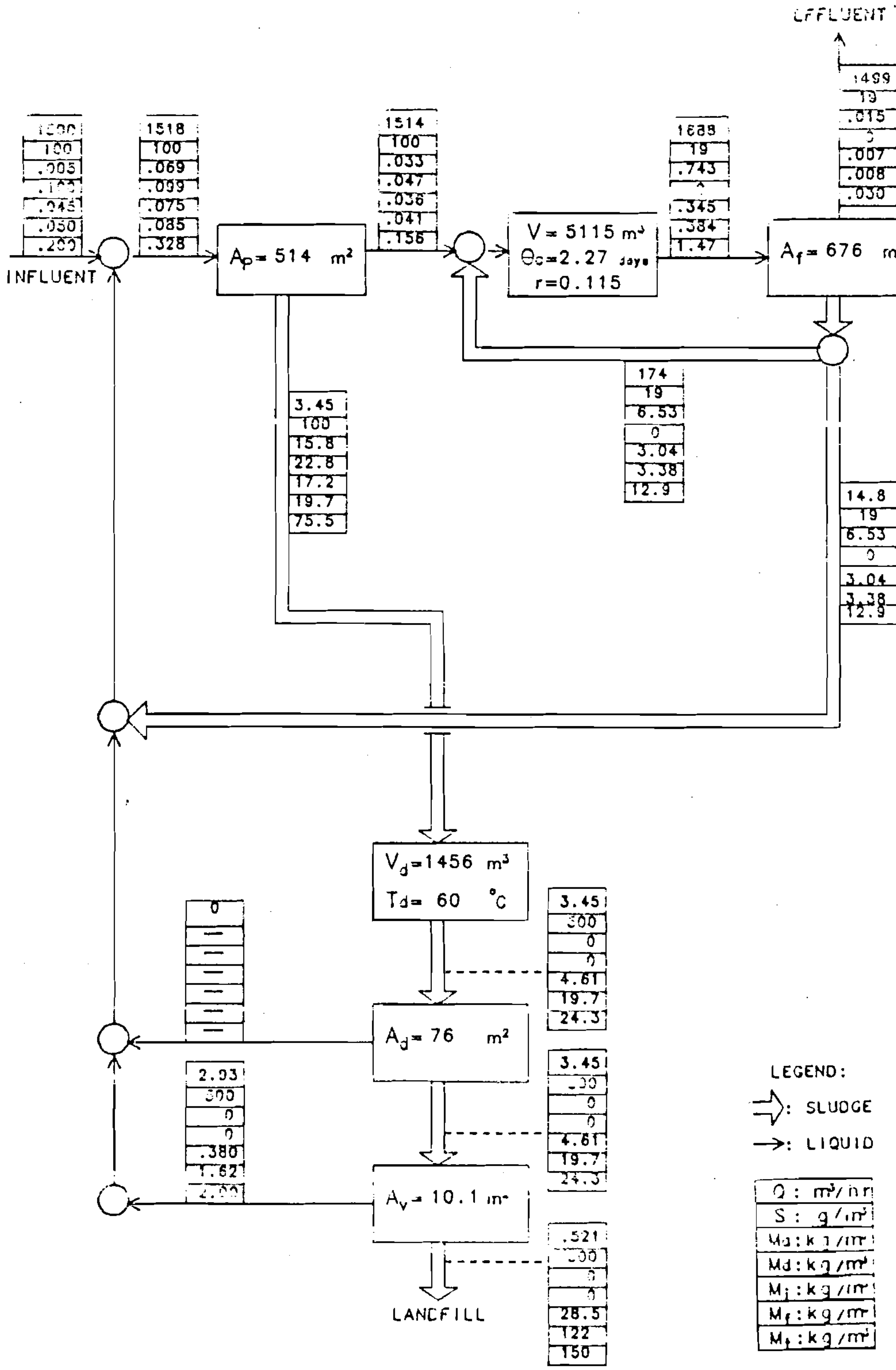


Figure 4.8- Best Design for the System Where Waste Activated Sludge is Returned to Primary Clarifier

mary digester designs for the base system (design No. 6 in Table 3.9) and the modified system is shown by Table 4.10.

It is recognized that thickening characteristics of the combined waste activated sludge and the raw wastewater may be different from those of the raw influent alone. Also because the waste activated sludge contains a high concentration of microbial mass, biological stabilization of soluble organics is possible in the primary clarifier. Experimental work on the use of the primary clarifier as a thickener is necessary to verify the results from the modeling study.

Primary sludge concentration has been modeled by many researchers as a constant. This modeling approach was examined by fixing the primary sludge concentration to 4% in the optimization models. Table 4.11 summarizes the designs obtained from this approach and from using the differential thickening technique for the base system. The influent soluble BOD_5 is 200 mg/l in these runs. In general, the two solutions show the same characteristics for a cost-effective design. The liquid subsystem designs appear to be similar regardless of the approach selected to model the primary sludge concentration. The mass fractions of the primary sludge are about the same in the two designs, which results in very similar

Table 4.10 - Comparison of Primary Digester Designs for the Base System and the System in Figure 4.7

Design Conditions	Base System	Wasted Activated Sludge Returned to Primary Clarifier
Influent Flowrate (m^3/hr)	4.48	3.45
Influent Volatile Solids (kg/m^3)	40.6	53.8
Influent Total Solids (kg/m^3)	55.8	75.5
Digester Volume (m^3)	1500	1456
Digestion Temperature (°C)	60	60
Solids Retention Time (days)	13.9	17.6
Methane Production (m^3/day)	1947	2114
Heating Requirement (10^6 $kWhr/yr$)	2.47	1.96
Net Value from Digester Gas (10^6 $kWhr/yr$)	4.60	5.71

Table 4.11 - Comparison of Optimal Designs by Different Models to Determine Primary Sludge Concentration

Variables (Units)	Limiting Flux	Constant (4%)
Primary Clarifier Overflow Rate (m/day)	144	144
Primary Sludge Concentration (%)	7.7	4.0
Mean Cell Residence Time (days)	2.47	2.47
Hydraulic Retention Time (hr)	5.1	5.1
Sludge Recycle Ratio (%)	15.8	15.7
Solids Loading on Thickener ($kg/m^2/day$)	12.0	12.0
Thickener Surface Area (m^2)	620	621
Digestion Temperature ($^\circ C$)	60.0	60.0
Primary Digester Volume (m^3)	1970	1920
Retention Time in Digester (days)	14.2	13.8
Solids Loading on Digester ($kg/m^2/day$)	48.0	48.0
Filter Yield ($kg/m^2/hr$)	6.31	6.31
Cake Solids Concentration (kg/m^3)	150.0	150.0
Effluent BOD_5 (mg/l)	30.0	30.0
Effluent TSS (mg/l)	30.0	30.0
Total System Cost (10^3 \$/yr)	577.1	581.8
Computer Time (CP seconds)	64.4	54.4

Notes: 1) Starting point No. 1 in Table 4.6 is used for these runs.
2) Influent Soluble $BOD_5 = 200$ mg/l

thickener sizing. However, because of the difference in the digester influent flowrate and solids concentration due to the different modeling approaches for the primary sludge concentration, the primary digesters are designed differently in the two solutions in Table 4.11. Therefore the total system costs in the two designs are slightly different.

A similar modification of the primary sludge concentration was also made in the model describing the wastewater treatment system with recirculation of the waste activated sludge to the primary clarifier (see Figure 4.7). Solutions were obtained for the base conditions in which the influent soluble BOD_5 concentration is 100 mg/l. Three different starting points were used to run GRG2. Results are tabulated in Table 4.12.

It is interesting to note that starting points No. 1 and No. 3, although very different, converge to exactly the same point in the optimization. This solution is displayed in Figure 4.9. A comparison between this design and that obtained by modeling the primary sludge

Table 4.12 - Summary of System Design Optimization : Waste Activated Sludge Returned to Primary Settling Tank, Primary Sludge @ 4%

Variables (Units)	Solution Obtained Using GRG2		
	1	2	3
Primary Clarifier Overflow Rate (m/day)			
initial	72.0	36.0	24.0
final	144.0	144.0	144.0
Mean Cell Residence Time (days)			
initial	2.5	3.0	5.0
final	2.21	2.22	2.21
Hydraulic Retention Time (hr)			
initial	3.6	2.4	4.8
final	3.6	3.6	3.6
Sludge Recycle Ratio (%)			
initial	15.0	25.0	15.0
final	11.7	11.7	11.7
Digestion Temperature ($^\circ$C)			
initial	35.0	35.0	35.0
final	60.0	59.9	60.0
Retention Time in Digester (days)			
initial	20.0	15.0	10.0
final	11.7	5.0	11.7
Solids Loading on Digester (kg/m^2/day)			
initial	2.4	12.0	24.0
final	48.0	48.0	48.0
Filter Yield (kg/m^2/hr)			
initial	20.0	10.0	7.9
final	6.31	6.31	6.31
Cake Solids Concentration (kg/m^3)			
initial	161.4	164.1	160.1
final	150.0	150.0	150.0
Effluent BOD$_5$ (mg/l)			
initial	28.8	22.3	16.6
final	30.0	30.0	30.0
Effluent TSS (mg/l)			
initial	33.8	22.6	21.6
final	30.0	30.0	30.0
Total System Cost (10^3 \$/yr)			
initial	704.0	668.7	706.4
final	489.4	507.3	489.4
Computer Time (CP seconds)	39.398	36.433	29.373

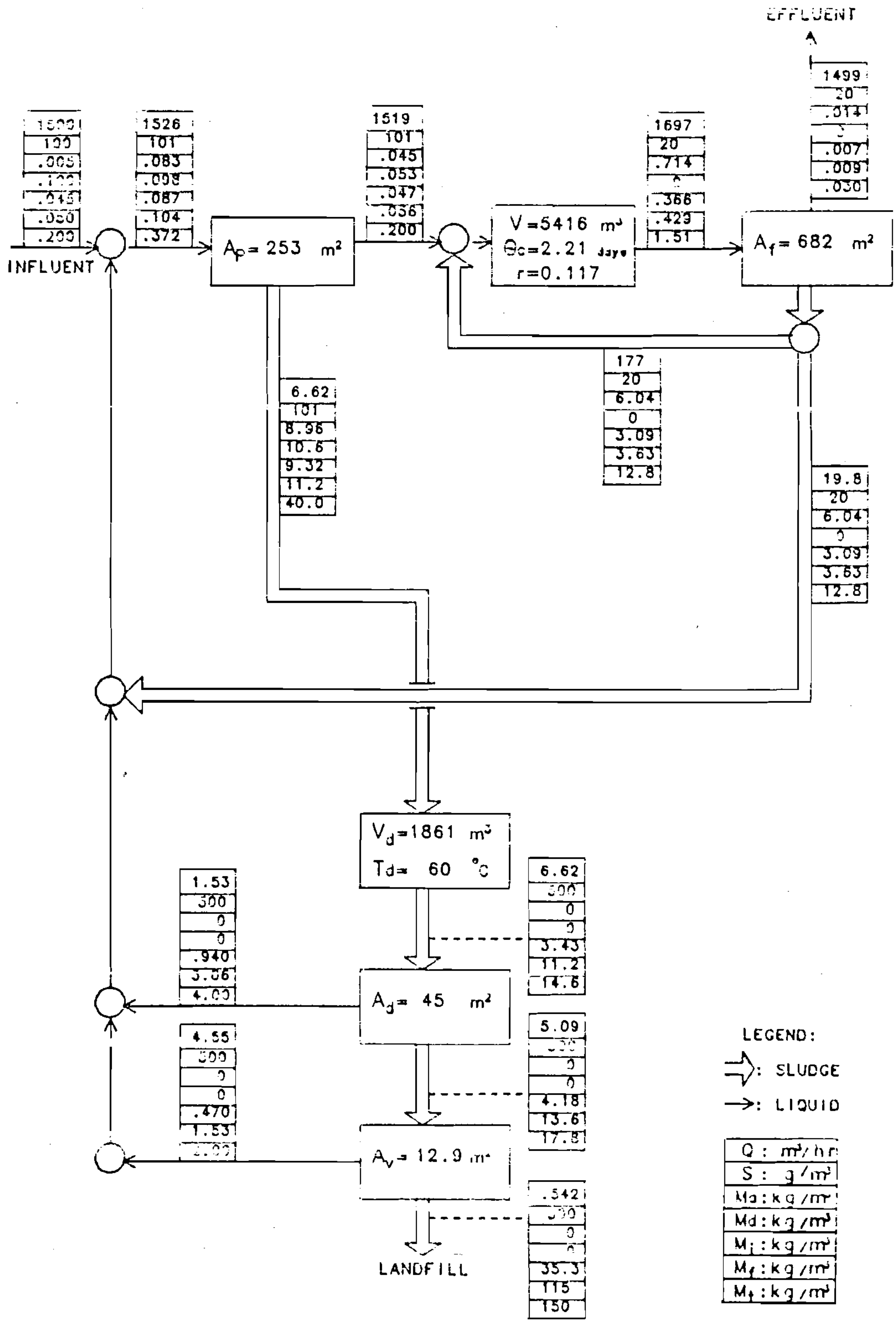


Figure 4.9- Best Design for the System Where Waste Activated Sludge is Returned to Primary Clarifier: Primary Sludge @ 4%

concentration using the limiting flux theory (Solution No. 1 in Table 4.9) is shown in Table 4.13. The difference in total system cost is more pronounced in this case than in the previous example, with the limiting flux approach costing about 5% less than the approach which assumes that the primary sludge concentration is independent of the primary clarifier surface area. The major difference in the system design is, as expected, in the primary clarifier. When the limiting flux theory is used to calculate the primary sludge concentration, the underlying assumption is that the primary clarifier serves as a thickener as well. In this case, this use is necessary in the most cost-efficient design since it reduces the volume of the sludge to be processed. This is also the reason why this thickening scheme, i.e., returning the waste activated sludge to the primary clarifier, is potentially attractive. On the other hand, if the primary sludge is fixed at 4%, then the thickening function of the primary clarifier is neglected. No matter how small the primary clarifier is, the thickened sludge from the clarifier is always at the same concentration of 4%. This causes the optimization program to select the size of the primary clarifier that is as small as possible.

In summary, modeling the primary sludge concentration as a constant has little effect on the solution obtained for a cost-efficient wastewater treatment system design for the base system; for this system, however, the thickening potential of the primary clarifier is limited because of the sludge thickening scheme specified. In contrast, when a system flowchart is designed specifically to take advantage of the thickening capability of the primary settling tank, then this capability may be more important. It is also noted that if the primary sludge concentration were modeled initially as a constant in the base system design, the final solutions obtained may have suggested designs with a good total system cost, but these solutions would not have suggested the alternative sludge thickening scheme of returning the waste activated sludge to the primary clarifier. This insight was directly provided by the model, however, when the primary sludge concentration was modeled using the limiting flux theory.

Table 4.13 - Comparison of Designs by Different Modeling Approaches on Primary Sludge Concentration : Waste Activated Sludge Returned to Primary Clarifier

Variables (Units)	Limiting Flux	Constant (4%)
Primary Clarifier Surface Area (m^2)	514	253
Primary Sludge Concentration (%)	7.6	4.0
Mean Cell Residence Time (days)	2.27	2.21
Aeration Tank Volume (m^3)	5115	5416
Hydraulic Retention Time (hr)	3.4	3.6
Sludge Recycle Ratio (%)	11.5	11.7
Digester Influent Flowrate (m^3/hr)	3.45	6.62
Digestion Temperature ($^\circ$C)	60	60
Primary Digester Volume (m^3)	1456	1861
Retention Time in Digester (days)	17.6	11.7
Secondary Digester Volume (m^3)	760	450
Filter Yield (kg/m^2/hr)	7.69	6.31
Vacuum Filter Surface Area (m^2)	10.1	12.9
Cake Solids Concentration (kg/m^3)	150.0	150.0
Effluent BOD_5 (mg/l)	30.0	30.0
Effluent TSS (mg/l)	30.0	30.0
Total System Cost (10^3 \$/yr)	466.2	489.4

This example has illustrated that the comprehensive system model can be used to analyze cost-efficient process integration. Results presented here are dependent on the settling properties of the primary, the activated, and the combined primary and activated sludges, as well as the limitations of all of the unit process models. Consequently it is the methodology of the analysis and the philosophy of using the optimization model for process analysis that are important. An optimization model enables the design engineer to investigate alternative flowcharts efficiently. Insights about the impact on the entire plant due to design modification of a single unit process can be obtained. Such information should be viewed as supplementing the traditional knowledge used by the design engineer (not as replacing any of it).

4.6. Anaerobic Digestion

The final solutions obtained for the base system and all design conditions considered suggest thermophilic digestion because the digestion temperature is at its specified upper bound of 60 °C. This upper bound cannot be relaxed because the activities of the anaerobic microorganisms will decrease drastically and finally stop completely when the digestion temperature goes higher than this temperature. Thermophilic digestion results in a high degree of organics stabilization and high methane production, which is given a cost credit in the model. The solids concentrations of the digester influent in the final solutions are all higher than 5%. This high concentration results in low energy requirements for heating the influent.

It is recognized that the unit process model used in this analysis (equation (2.48)) is based on a number of assumptions and is developed from limited experimental data. It appears that fine-tuning of that model would be worthwhile to verify the benefits associated with a thermophilic digestion system.

The final solutions in Table 3.9 also call for the elimination of the secondary digester since the influent solids concentration to this unit is almost identical to the underflow solids concentration at the design loading rate. Both the secondary digester and the vacuum filter are provided to achieve the same purpose, sludge concentration. Because of the poor settling characteristics of the digested sludge, it is more economical to concentrate the sludge by vacuum filter than by the secondary digester. It is interesting that this insight, which was obtained using the model for the example problem, is consistent with the observations by Lawler and Singer (1984) who suggested the elimination of the secondary digester in a treatment plant based on their survey of the performance of the secondary digester as a thickener at a number of existing plants.

Eliminating the secondary digester from the system layout may not be desirable in practice, however, since the secondary digester provides reliability to the system. Because of the reliability problems associated with operating an anaerobic digester, it may be desirable to have the secondary digester in the system. Cleaning the primary cell is also possible without the necessity of operational modifications if secondary digesters are present. In addition, methane production is generally observed in the secondary digester, which contributes to the net energy production. Since these considerations are not captured in the mathematical model, it would be desirable to explore the role of the secondary digester in more detail in an actual design exercise.

This example also brings up the general question of the role of an optimization model. Planning and design of a wastewater treatment system in general is very complicated. Using mathematical models for design may not include all important considerations in a realistic treatment system design situation. For example, the system that has the least system cost may not satisfy other design criteria such as ease of operation or high degree of system reliability. Mathematical models should be used to generate alternative system designs that are good with respect to thèse important design criteria. Traditional engineering design concepts can then be exercised to determine the most adequate system design.

Because of the consideration given to the rising digester gas on the digested sludge settling characteristics, the digested sludge settling velocity was assumed to be only one-fourth of that of a fully digested sludge in the calculation of the digested sludge solids concentration (see Section 2.3.7). This factor discounts the digested sludge settling velocity from what is predicted by the limiting flux theory alone. For a thermophilic digestion system, the percent of organics stabilization is very high in the primary digester. Therefore the effect of digester gas on sludge settling in the secondary digester becomes less significant, and a larger factor is more appropriate. A factor of 0.90 was substituted for 0.25 in the secondary digester design, and one optimization run was made for the base system and design conditions. The results are summarized in Table 4.14 and depicted in Figure 4.10. Also listed in Table 4.14 for comparison is the solution obtained for the base design conditions (design No. 6 in

Table 4.14 - Sensitivity of the System Design to the Digested Sludge Settling Characteristics

Variables (Units)	Initial	Final*	Base Design
Primary Clarifier Overflow Rate (m/day)	72.0	144.0	144.0
Mean Cell Residence Time (days)	3.0	2.19	2.19
Hydraulic Retention Time (hr)	3.6	3.8	3.7
Sludge Recycle Ratio (%)	30.0	12.6	12.8
Solids Loading on Thickener ($kg/m^2/day$)	36.0	12.0	12.6
Digestion Temperature ($^\circ C$)	25.0	60.0	60.0
Retention Time in Digester (days)	20.0	14.0	13.9
Solids Loading on Digester ($kg/m^2/day$)	24.0	94.0**	40.6
Filter Yield ($kg/m^2/hr$)	12.0	7.73	6.67
Cake Solids Concentration (kg/m^3)	196.9	150.0	150.0
Effluent BOD_5 (mg/l)	26.1	30.0	30.0
Effluent TSS (mg/l)	36.8	30.0	30.0
Total System Cost (10^3 \$/yr)	644.6	484.9	500.4

* Computer time for optimization : 67.034 CP seconds.

** Upper bound of digester solids loading (48 $kg/m^2/day$) is relaxed in this run.

Table 3.9). When the digested sludge is assumed to have better settling properties, the total system cost is lowered to 484,900 dollars/year. The solution specifies a small secondary digester surface area (22 m^2) which would concentrate the digested sludge from 2.0 to 2.4% (20.2 to 24.5 kg/m^3 in Figure 4.10). The solids loading on the secondary digester, however, is extremely high at 94 $kg/m^2/hr$, and the secondary digester begins to play a role in the overall wastewater treatment system. This suggests that the settling properties of the digested sludge have a direct effect on the arrangement of the digestion system (i.e., should a secondary digester be included or not) if the limiting flux theory is valid at the high solids loading. Since data in this area are lacking, laboratory analysis of digested sludge settling characteristics under various fermentation conditions should be performed to identify the appropriate role of the secondary digester.

4.7. Vacuum Filter

The solids cake concentrations in the final solutions are at the specified upper bound of 15% for all conditions considered. As discussed in Section 2.5, this upper bound was arbi-

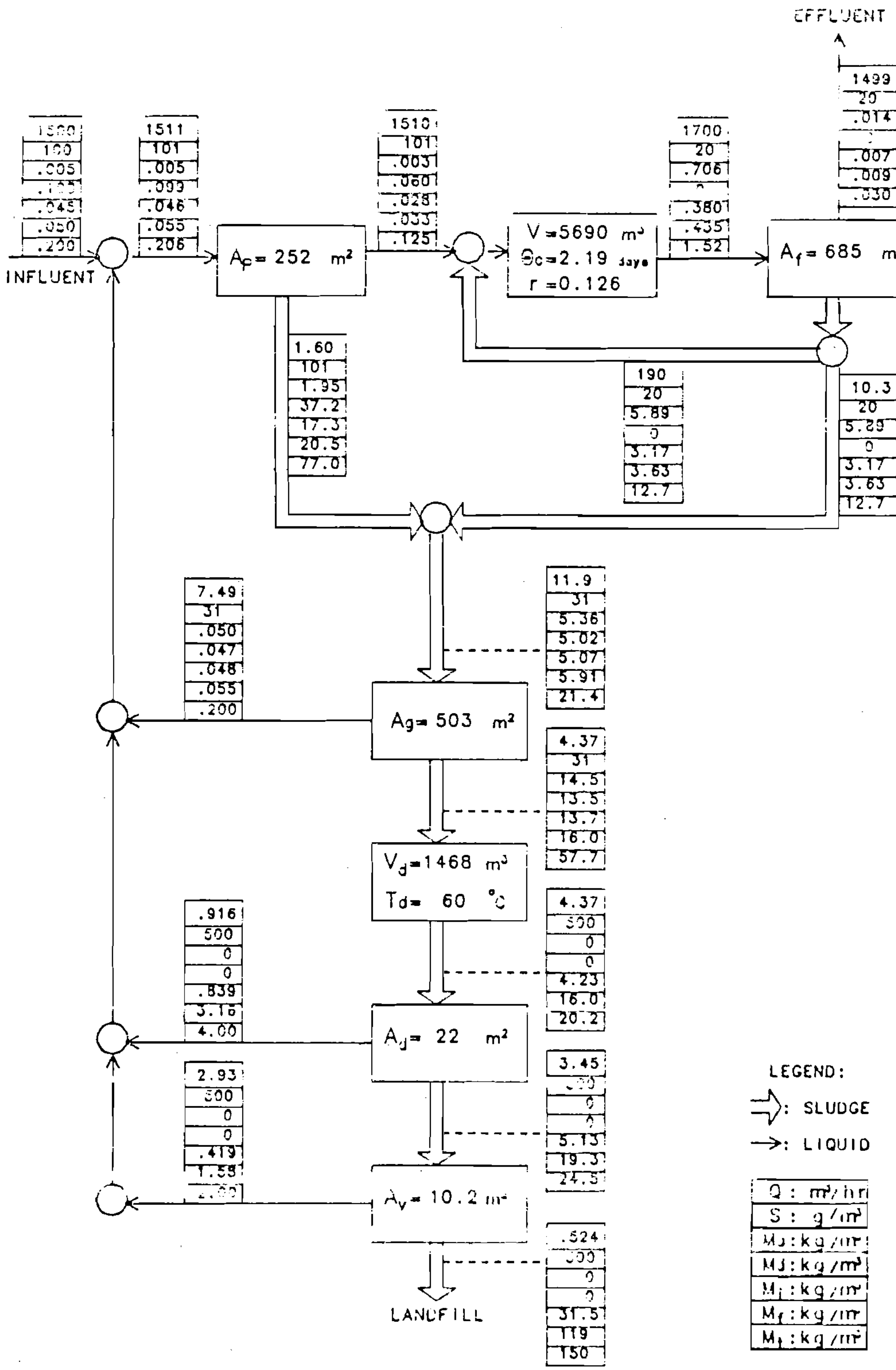


Figure 4.10- Best Design for the Base System: Improved Settling Properties of Digested Sludge

trarily set because the model used for the vacuum filter design does not predict a maximum cake concentration that can practically be attained. Since the final disposal of dewatered sludge is relatively expensive, and since the filter area requirement is insensitive to the filtered cake concentration at high concentration levels (see Figure 4.11), the cake concentration was driven to its upper bound in the solutions obtained.

The limitation of the vacuum filter design model appears to be that it is only applicable within a limited range of design conditions. For example, the air drying mechanism is not considered in the development of this design equation. This is an area where additional research is needed to refine the present model for vacuum filter design.

4.8. Design Under Uncertainty: A Multi-objective Approach

As discussed in the introduction of this chapter, the design of wastewater treatment plants involves many uncertainties. Parameter uncertainty in the design of wastewater treatment systems has been dealt with by Berthouex and Polkowski (1970), and Tarrer *et al.* (1976). Key parameters were assumed to follow a certain statistical distribution, and the means and the standard deviations were taken into account in mathematical models. There are three major difficulties with this approach: 1) The statistical distributions of the design parameters are usually unknown and have to be assumed, 2) the resulting mathematical model becomes very complicated, and 3) uncertainties on process performance models, cost information, and design conditions are not included.

An alternative approach to handling uncertainty in engineering design is to perform sensitivity analysis for model parameters. Voelkel (1978) performed sensitivity analysis of the parameters in his model and recorded the sensitivity of the overall system design to the unit changes of these parameters. The major drawback of this approach is that the optimization procedure may terminate at local optima because the model is nonlinear. A distinct trend for the system cost as a function of the perturbed parameter may not be attained.

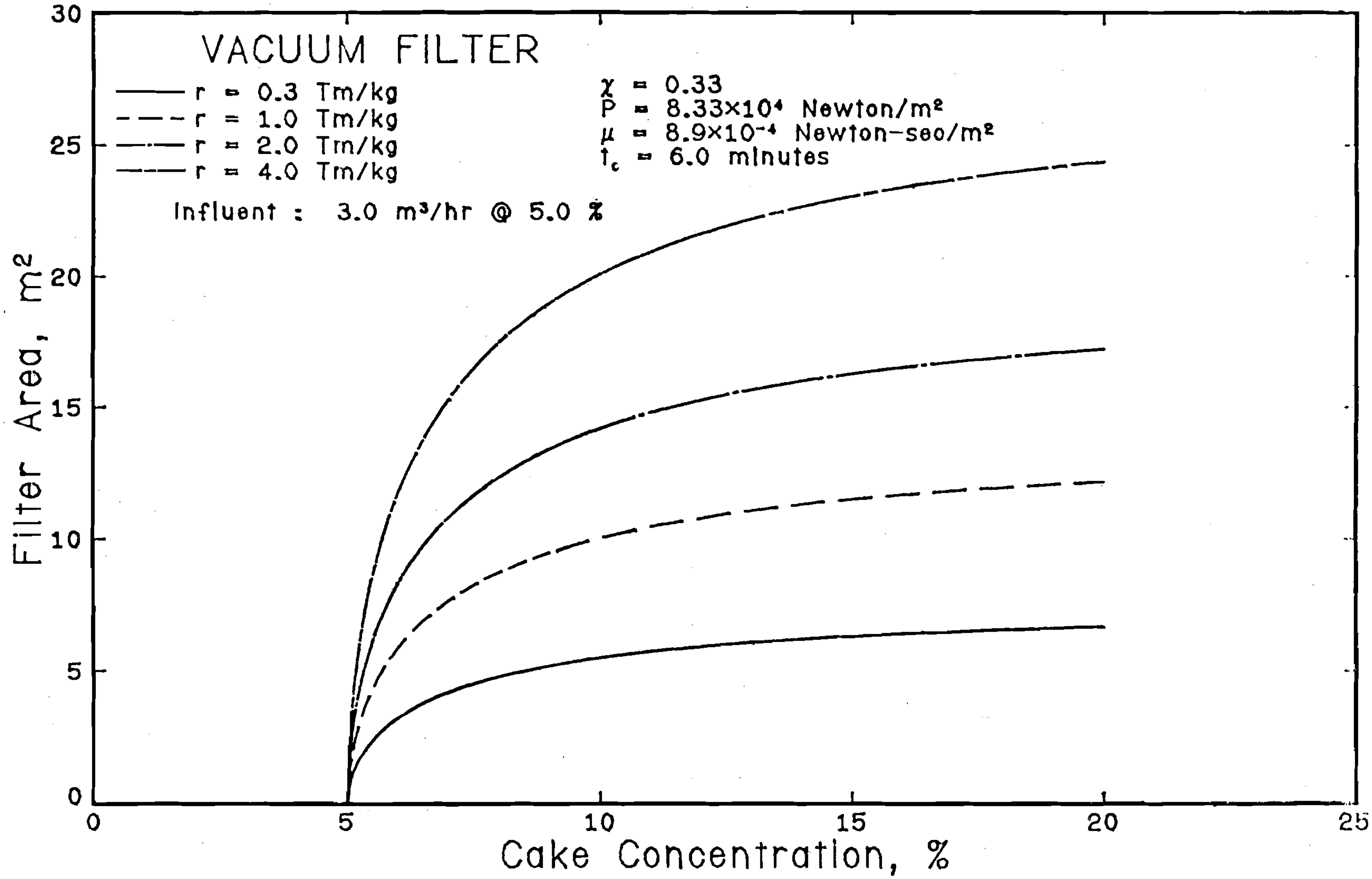


Figure 4.11- Vacuum Filter Surface Area Requirement vs. Solids Cake Concentration

A common strategy for dealing with design uncertainty is to apply a safety factor. This factor allows some flexibility and redundancy to be built into the system design. However, safety factors escalate the total system cost. In other words, there exists a tradeoff between the system's cost and reliability. In this respect, the design of a wastewater treatment system can be modeled as a multi-objective problem.

Use of safety factors also provides an allowance for other unmodeled but important design considerations. The most cost-effective design may be deficient with respect to other design criteria as discussed in the previous section on the role of the secondary digester. The optimization model can be used to generate many alternative designs that are noninferior when the system's cost and reliability are considered as two planning objectives.

As an illustrative example, cost and safety were assumed to be two objectives in the design of a wastewater treatment plant. To deal with the uncertainty issue, a flowrate safety factor was used to provide more capacity to the system so that it could be operated with more flexibility. This safety factor was assumed to be a multiple of the design flowrate. The base design conditions were assumed, and GRG2 was used for optimization. The results

Table 4.15 - Treatment Plant Design Optimization for Different Flowrate Safety Factor

Variables (Units)	Flowrate Safety Factor				
	1.0	1.5	2.0	2.5	3.0
Primary Clarifier Overflow Rate (m/day)	144.0	144.0	144.0	144.0	144.0
Mean Cell Residence Time (days)	2.19	2.19	2.19	2.19	2.19
Hydraulic Retention Time (hr)	3.8	3.8	3.8	3.8	3.8
Sludge Recycle Ratio (%)	12.5	12.4	12.3	12.2	12.1
Solids Loading on Thickener (kg/m^2/day)	12.5	12.0	12.0	12.0	12.0
Digestion Temperature ($^\circ$C)	60	60	60	60	60
Retention Time in Digester (days)	14.2	13.9	13.7	14.5	14.4
Solids Loading on Digester (kg/m^2/day)	40.3	38.2	38.0	38.9	38.7
Filter Yield (kg/m^2/hr)	6.69	6.81	6.82	6.76	6.78
Cake Solids Concentration (kg/m^3)	150.0	150.0	150.0	150.0	150.0
Effluent BOD_5 (mg/l)	30.0	30.0	30.0	30.0	30.0
Effluent TSS (mg/l)	30.0	30.0	30.0	30.0	30.0
Total System Cost (10^3 $/yr)	500.4	642.5	768.7	884.1	990.9

are summarized in Table 4.15. It is interesting to note that the final designs exhibit similar values for the decision variables. The tradeoff between the design safety factor and the system cost is depicted in Figure 4.12. This curve is slightly convex due to the economies of scale in the design of wastewater treatment systems.

To account for the design uncertainties mentioned above, the design engineer may select a safety factor greater than one based on the design flowrate or influent pollutant concentrations. This is similar to design based on the maximum daily flow except that the peaking factor becomes the second objective in the model. The design made according to this approach is more realistic since design flows may be exceeded, and because there are uncertainties in the model. With better knowledge about the design parameters or process performance models, a smaller safety factor may be used.

4.9. Summary

The role of the comprehensive system model developed in Chapter 2 as a tool for use in the analysis and design of secondary wastewater treatment systems is illustrated in this chapter. Recognizing the limitations of a cost-minimization system model, the intent of this work has not been to obtain the "least-cost design." Through the use of the model, potentially important research areas in treatment process design are identified from the cost-effectiveness viewpoint. For example, the solids removal behavior of the primary clarifier at overflow rates higher than usually recommended in design practice should be examined. The importance of a model describing clarification in the activated sludge final clarifier is also illustrated. Sludge thickening at low solids loading is critical to the design of the sludge processing train. Anaerobic digestion in the thermophilic range is another area that should be investigated. The settling characteristics of the digested sludge determine the role of the secondary digester in the overall treatment system; correlations between the digested sludge settling properties and the degree of organics stabilization should be studied. Refinements of

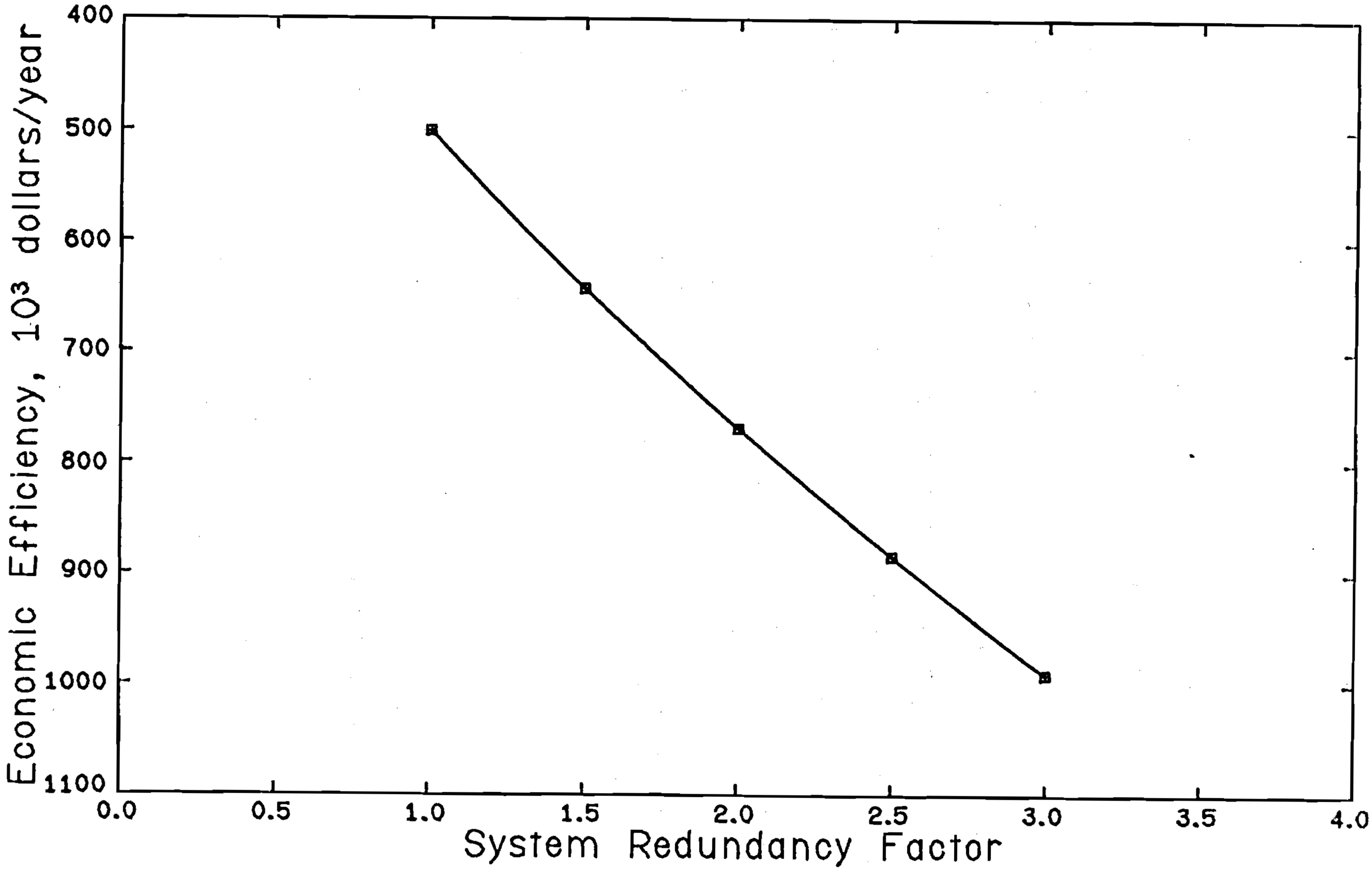


Figure 4.12- Approximation of the Noninferior Set With Flow Safety Factor and System Cost as Two Design Objectives

the modeling approach for vacuum filter design is also an area that warrants future research work.

The comprehensive system model also helps to identify innovative process flowsheet design. A primary clarifier may not be cost-effective when the influent volatile solids concentration is not very high. The secondary digester may not be cost-effective when the digested sludge has poor settling characteristics. If the primary sludge is allowed to settle to its full potential, then the use of the primary clarifier as a gravity thickener may be attractive economically. These insights are examples which show the usefulness of the comprehensive system model as a process synthesis aid. By taking into account the interactions among unit processes, the most meaningful system flowcharts can be analyzed. This represents potential savings in computations and design work which are nontrivial requirements of process synthesis. However, it should be recognized that when an optimization model is used for process synthesis, the parameters used in design may change according to the design of upstream treatment units. Experimental work may be necessary to further verify the conclusions obtained from the mathematical modeling study.

Realistic planning and design of wastewater treatment systems are generally multi-objective since objectives other than the system cost have to be considered. Uncertainty in process modeling and design is an important design consideration since there is a tradeoff between the system's cost and reliability. A simplistic approach for the design of a wastewater treatment system considering cost and reliability as two objectives is presented. It is expected that a design made based on this type of approach would be more realistic than "the optimal design" obtained by a simple application of the original model.

CHAPTER 5

SUMMARY AND FUTURE RESEARCH

5.1. Introduction

The design of wastewater treatment systems involves many tradeoffs because of the complex arrangement of the unit processes. With increasing understanding of the fundamentals of the wastewater treatment mechanisms, researchers have been developing mathematical models that can be used to describe the levels of performance of the various unit processes. Use of these mathematical models for design allows engineers to examine the tradeoffs in a wastewater treatment system in detail and to strive for cost-effective system designs.

There are other uses of a comprehensive design model for a wastewater treatment system in addition to obtaining cost-effective system designs. Limitations of process performance models and potential research areas can be identified. Important insights about process flowsheets can be gained from exercising such a model. Innovative water quality management strategies for a river basin can be better evaluated using a model for wastewater treatment plants as the basis. Also, issues that are important in planning and design of wastewater treatment systems but that are unmodeled can be evaluated.

Efficient mathematical programming techniques are essential if a comprehensive system model is to achieve extensive use. Because a system model is very complicated mathematically, research must be done to develop efficient optimization procedures.

In this thesis, a complete model for use in the design of a secondary wastewater treatment system is developed. This model includes state-of-the-art process design models to predict the performance of the treatment system. The construction of the model is described in detail in Chapter 2, and is briefly summarized in Section 5.2.

Chapter 3 discusses the use of two existing optimization algorithms and one new approach for solving the comprehensive system model developed in Chapter 2. Some key observations are summarized in Section 5.3.

Recognizing the unmodeled issues and uncertainties involved in the design of wastewater treatment systems, the use of the comprehensive system model as a tool for the analysis of process performance is illustrated in Chapter 4. Conclusions from using the model as an analysis tool are summarized in Section 5.4.

The comprehensive system model developed in this research may serve as basis for additional research in the area of environmental systems analysis. Several areas in treatment process design and modeling were identified in Chapter 4 as potentially fruitful for achieving more cost-effective system designs. Section 5.5 provides a summary of these possible future research directions.

5.2. Comprehensive System Design Model

A typical secondary wastewater treatment system was selected for initial evaluation in this study. This system includes primary sedimentation, aeration and secondary sedimentation (activated sludge), gravity thickening of combined primary and waste activated sludge, two-stage anaerobic digestion, vacuum filter dewatering, and final sludge disposal by sanitary landfill. Supernatants generated in sludge processing are recycled to the head end of the plant.

Wastewater parameters represent the state of the wastewater or sludge during different stages of the treatment process. These state variables include flowrate, soluble $BOD5$ concentration, and concentrations of active biomass, volatile biodegradable suspended solids, volatile inert suspended solids, fixed suspended solids, and total suspended solids. Nine decision variables need to be specified in order to define the system design completely.

The solids removal efficiency in the primary clarifier was modeled using the Voshel-Sak (1968) equation. The primary sludge concentration was calculated based on the limiting flux theory. The Lawrence-McCarty (1970) model was selected for the design of the activated sludge process. Clarification of the aeration tank effluent is critical in determining the efficiency of the overall wastewater treatment. This function of the final clarifier was modeled based on an equation proposed by Chapman (1983). Thickening in the final clarifier and in the gravity thickener was modeled using the differential thickening technique be Dick and Suidan (1975). Sludge stabilization in the primary anaerobic digester is a function of digestion temperature and solids retention time. A mathematical model based on limited experimental data summarized by Wise (1980) was developed to describe the performance of the primary digester. The secondary digester was modeled as a gravity thickener, and the differential thickening technique was employed for design. Vacuum filter design was based on the estimated filter yield.

To estimate the total system cost, cost information summarized by Patterson and Banker (1971) was used to calculate the costs of each unit process in the system. Sludge disposal costs were estimated based on models developed by Dick *et al.* (1978), Rossman (1979), and USEPA Process Design Manual (1974). An analysis computer program was written to design the wastewater treatment system for specified influent and design conditions. Unit processes were designed sequentially according to the system flowchart. The steady-state design of the overall system was obtained through iterations because of the presence of the recycle streams in the system. This program is useful for examining the system response corresponding to different input and design conditions and for generating system designs that can be used as initial solutions in an optimization procedure.

5.3. Optimization Techniques for Wastewater Treatment System Model

Three optimization approaches were taken in this study to solve the comprehensive wastewater treatment model. Because the model is more complicated than previously studied ones, efficient optimization techniques are essential.

The first technique examined was to formulate the system design problem as a nonlinear program and to solve it directly using a generalized reduced gradient algorithm (GRG2). The resulting nonlinear program has 64 variables, 55 equations, and three inequality constraints. Computing time for this model ranged from 51 to 105 central processing seconds on a CDC Cyber 175 computer. This performance can be considered to be at least comparable to previous studies that solved less complicated wastewater treatment system models using other optimization techniques. Once the GRG2 model is formulated, it can be used repetitively to examine different influent or design conditions with minor adjustments of the data file. This allows its use as a tool for process analysis. However, extensive revision of the model is necessary if an alternative treatment flowchart is to be examined.

The solutions obtained from using GRG2 depend on the various control parameters specified, the bounds on the variables, the initial solutions, and the numerical characteristics of the model. A modified HSJ (Brill, 1979) approach was used to examine the quality of these solutions. This strategy explores the feasible design space using objective functions that are formed based on the knowledge about the problem. Numerical examples have shown that this strategy helped to improve the total system cost of the solution obtained from solving the original model directly using GRG2. This strategy can also be used to identify designs that are similar in the total system cost, but are different with respect to the sizes of the unit processes in the wastewater treatment system. This is particularly useful if there are unmodeled issues in the design of the treatment system. For the example problems, seven different system designs were obtained using the proposed strategy (see Table 3.9). The differences among these designs were not significant, however, because similar objective functions were used to generate these designs.

The system design can also be formulated as a generalized geometric program (GGP) if one variable in the model is fixed and one equation is modified. An efficient algorithm (IGGP) for solving GGP was used to solve the subproblems resulting from the specification of the variable. These subproblems have 62 variables, 54 equality constraints, and three inequality constraints. The computing time for solving one subproblem ranged from 2.5 to 5.7 seconds. To obtain the optimal design for the original problem, different values of the fixed variable have to be examined; a subproblem has to be solved for each value assumed. For the example problem, eleven subproblems were solved for a total computer time of about 50 seconds on the Cyber 175 computer. The final solution obtained from this approach compared well with that obtained from GRG2 as far as the characteristics of the cost-effective designs.

A unique approach was also developed for the identification of cost-effective designs. This approach decomposes the overall system into a liquid subsystem and a sludge subsystem. The liquid subsystem design was optimized using GRG2 for a specified set of recycle stream characteristics. The output from the liquid subsystem, i.e., the combined primary and waste activated sludge, was treated as input to the sludge subsystem. The design of the sludge subsystem was carried out for the specified set of recycle stream characteristics. Two one-dimensional optimizations were embedded in the sludge subsystem design. One advantage of this approach is that the overall system which contains nine degrees of freedom can be reduced to two subproblems with four and two degrees of freedom, respectively. Optimization techniques can be applied to solve these smaller problems more efficiently and more reliably. The subsystem designs, however, must be coordinated to obtain the overall optimal design. This coordination involved determination of the values of three interacting variables. A coarse grid enumeration technique was employed to identify the set of interacting variables that results in the least system cost. Several assumptions were used in this approach to reduce the number of the interacting variables so that the coordination could be carried out more efficiently. These assumptions were shown to be adequate with three numerical examples. Total computer time of about 100 seconds was necessary for the example problem tested. This computing time is comparable to those required in the previous two approaches. Trends for cost-effective designs were clearly identified using this approach.

Another advantage of the decomposition approach is that many alternative solutions can be obtained during the optimization process. These solutions are very different in their designs, but the total system costs are similar. Therefore they can be evaluated with respect to unmodeled issues.

5.4. Use of Model for Process Analysis

A mathematically optimal solution is the result of optimizing the comprehensive system model. This mathematically least-cost design is not expected to be the best final plan to be implemented in a realistic design situation because the design of wastewater treatment system typically involves other important but unmodeled issues. However, the characteristics of this solution provide useful insights about process research and design. This use of a comprehensive system model as an analysis and design tool is illustrated in Chapter 4.

Several research areas in process modeling were identified by an examination of the solutions obtained from GRG2. The solids removal efficiency of the primary clarifier at high overflow rates, sludge thickening at low solids loadings, and sludge solids stabilization by thermophilic anaerobic digestion are examples of these potential research areas.

Information on process flowsheets was also obtained from the modeling study. The use of the primary clarifier as a thickener was cost-effective if primary sludge is allowed to thicken to its full potential. The role of the primary clarifier depends on the characteristics of the influent wastewater. The role of the secondary digester depends on the settling characteristics of the digested sludge. These results of course depend heavily on the parameter values used. It is recognized that parameter values used for design of some unit processes may be functions of the influent characteristics to these units. Without such information, results obtained on process synthesis from the use of the comprehensive system model should be examined carefully. Experimental evaluation may be necessary to confirm the modeling results.

Uncertainties in designing wastewater treatment systems have been dealt with by researchers using various approaches. Traditionally, engineers have employed peaking factors to design some units in a wastewater treatment system to provide a system with reliability. There is a tradeoff between the system's cost and reliability, i.e., the more reliable the system is, the more it costs. This problem can be considered as a two-objective problem, and noninferior designs can be generated. These designs can be evaluated based on other design criteria.

5.5. Future Research

Several potential research areas in process modeling and design have been suggested from the use of the comprehensive system model. There are other areas that deserve future investigation:

1) Optimization techniques: As mentioned above, one advantage of the decomposition approach is that different optimization techniques can be used to solve different subsystem designs. Alternative optimization techniques for optimizing the liquid subsystem and for coordination could be studied to improve the efficiency of this approach. There are alternative strategies for defining the subsystems and for implementation of the decomposition approach. The robustness of the decomposition approach, i.e., the performance of this approach under different influent conditions should be studied. The applicability of the approach to other system flowcharts can also be investigated. This proposed work is potentially capable of refining the decomposition approach and

making it a useful design and analysis tool.

2) Sensitivity analysis: Information on the effect of a particular parameter in the model on the overall system design is useful for system design and process analysis. Potential research areas can be identified where the system model is very sensitive to a particular parameter. The settling characteristics of the digested sludge serves as an example to illustrate the importance of the sensitivity analysis.

3) Reliability analysis: The reliability of the system designed based on the optimization of a comprehensive model can be evaluated. Design safety factors on particular unit processes can be determined more rationally when different influent conditions are imposed on the system designed based on the average flow and are subject to the constraint that the effluent water quality standards have to be met. This information helps to establish guidelines for practical wastewater treatment system design. Considering system cost and a flow safety factor as two objectives in wastewater treatment system design is an alternative approach of analyzing the system reliability. This has been illustrated using an example problem in Chapter 4.

4) Model verification: Realistic plant operating data may be used in a given situation to determine the most appropriate process performance relationships. These models can then be used in a realistic design condition. If the facility already exists, then this information can be used in the comprehensive system model to identify cost-effective operation of a wastewater treatment system.

5) Water quality management: The model can be used to generate information that relates the cost of a wastewater treatment system to its waste removal efficiency. Such information is useful in studies involving innovative water quality management strategies. Modifications of the comprehensive system model may be necessary, however, for specific situations (for example, if multiple pollutants are to be controlled).

APPENDIX A

COST FUNCTIONS OF UNIT PROCESSES

Five sets of cost functions representing three sources of cost information were compared in this study. Table A.1 summarizes the cost functions studied and the sources of the information (see Section 2.4 for a discussion of this cost information).

In Figures A.1 to A.9, capital costs are expressed in 1971 dollars. The USEPA National Average Wastewater Treatment Plant Index is used to convert costs to this common basis.

Middleton and Lawrence, CAPDET, and Rossman all developed their cost functions based on the information furnished by Patterson and Banker. However, these functions vary considerably in the degree of complexity. The function that is the simplest among the three was selected for use in the study if no significant difference is observed among the predictions of these three sets of functions. Cost functions incorporated into the comprehensive system model are summarized in Table 2.4.

Table A.1 - Summary of Cost Information

Cost Functions	Source
Smith (1968)	Logan *et al.* (1962) Swanson (1966)
Middleton & Lawrence (1975)	Patterson & Banker (1971)
Dick *et al.* (1978)	Patterson & Banker (1971) Metcalf & Eddy, Inc. (1975) Ettlich (1977)
CAPDET (1978)	Patterson & Banker (1971)
Rossman (1979)	Patterson & Banker (1971) Ettlich (1977)

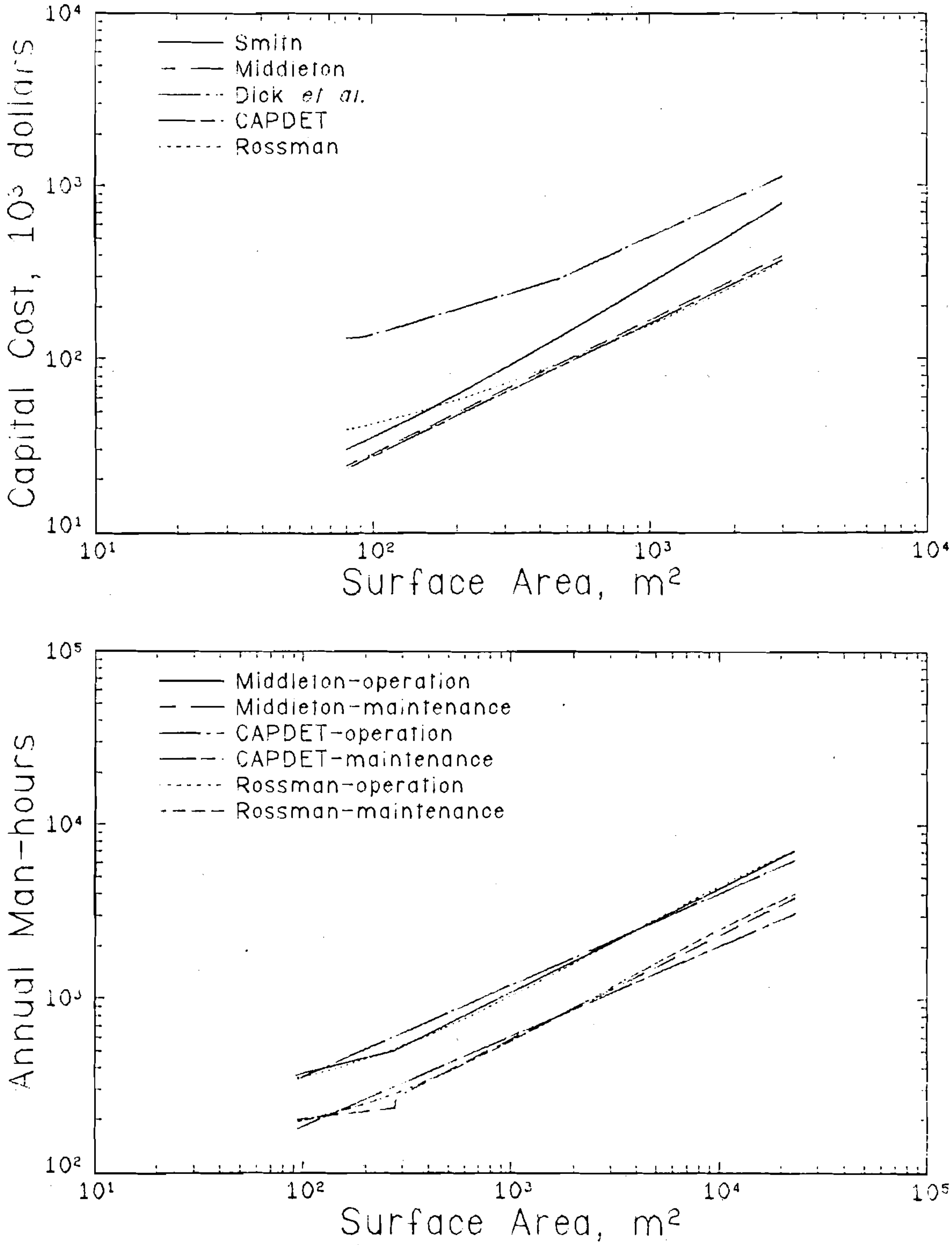
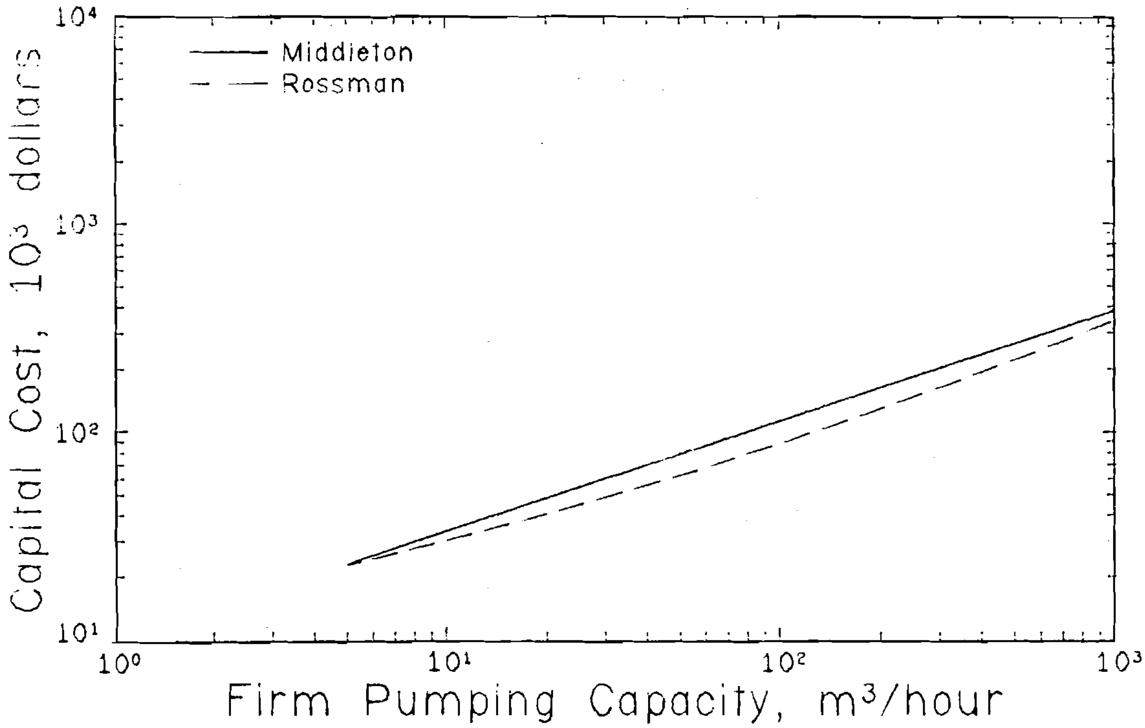


Figure A.1- Cost Functions for Primary Clarifier



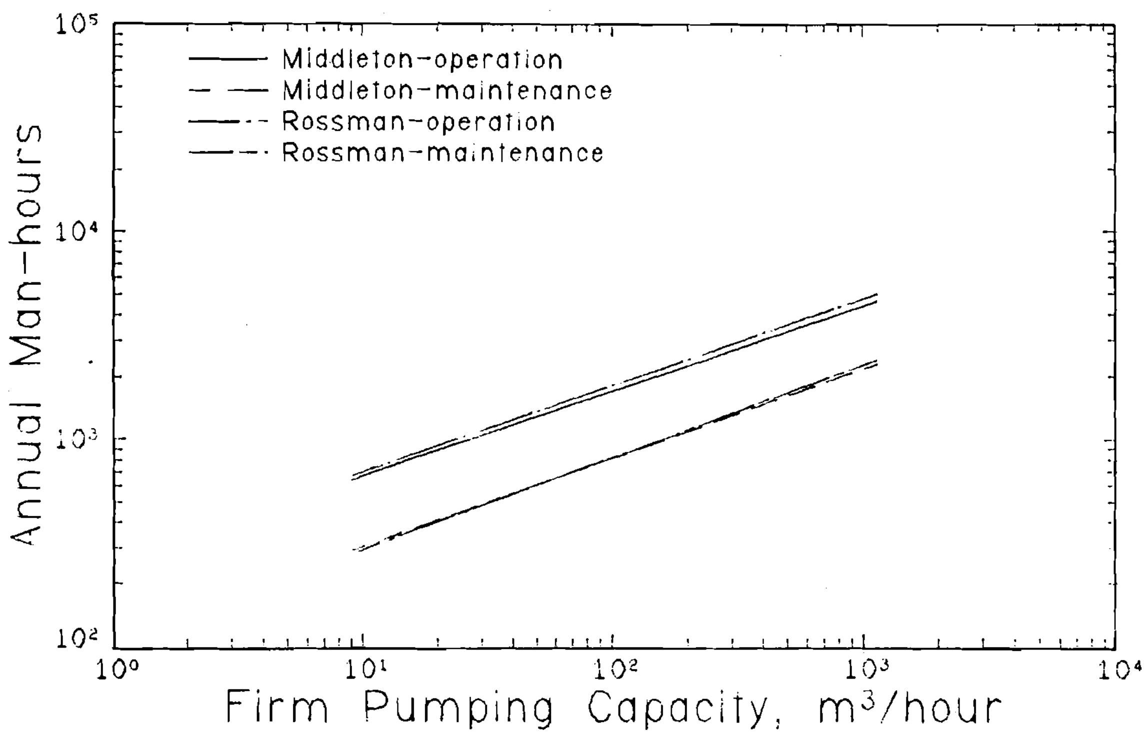


Figure A.2- Cost Functions for Primary Sludge Pumping

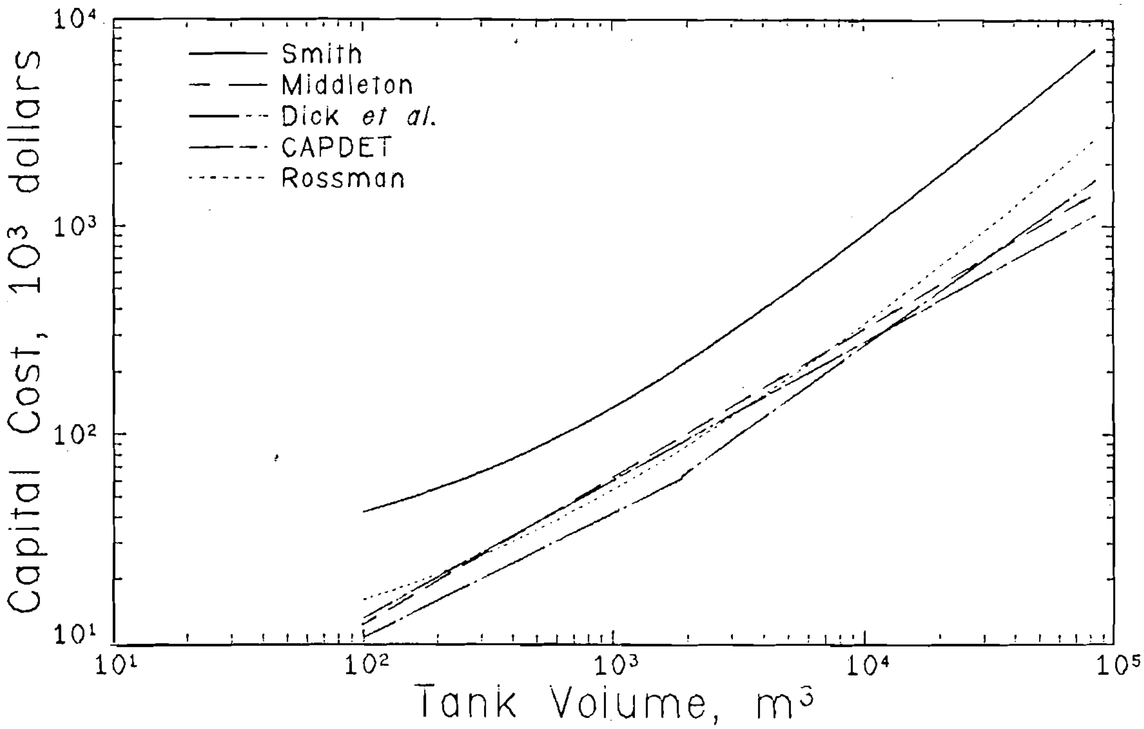
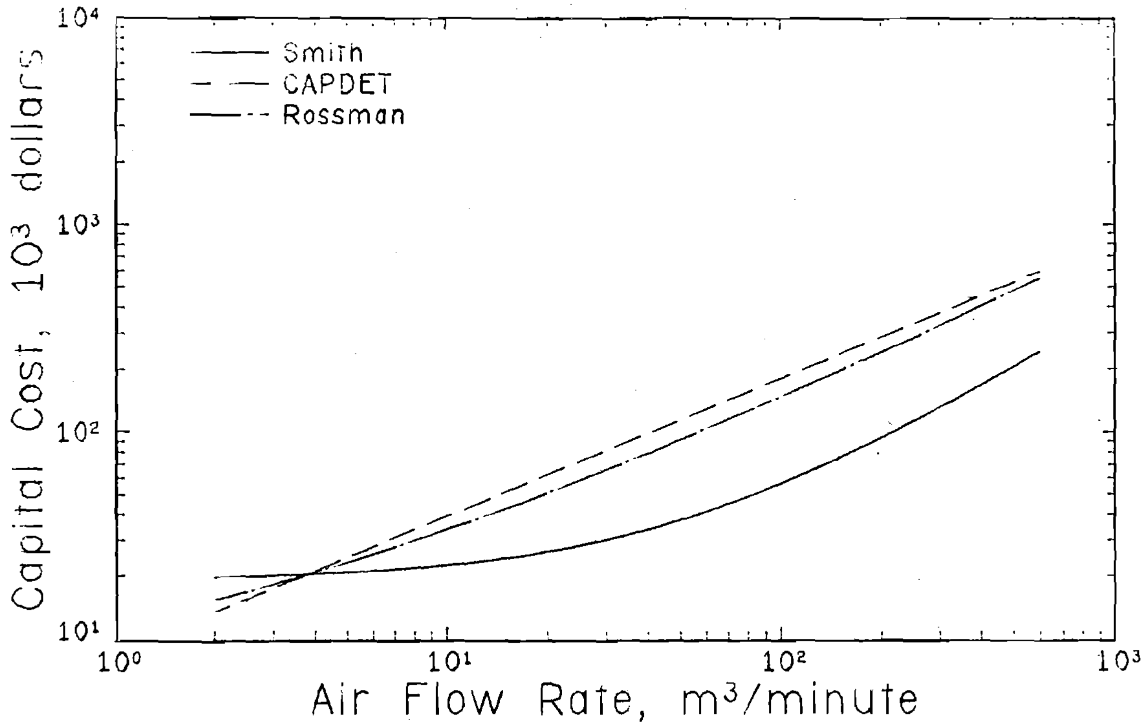


Figure A.3- Cost Functions for Aeration Tank



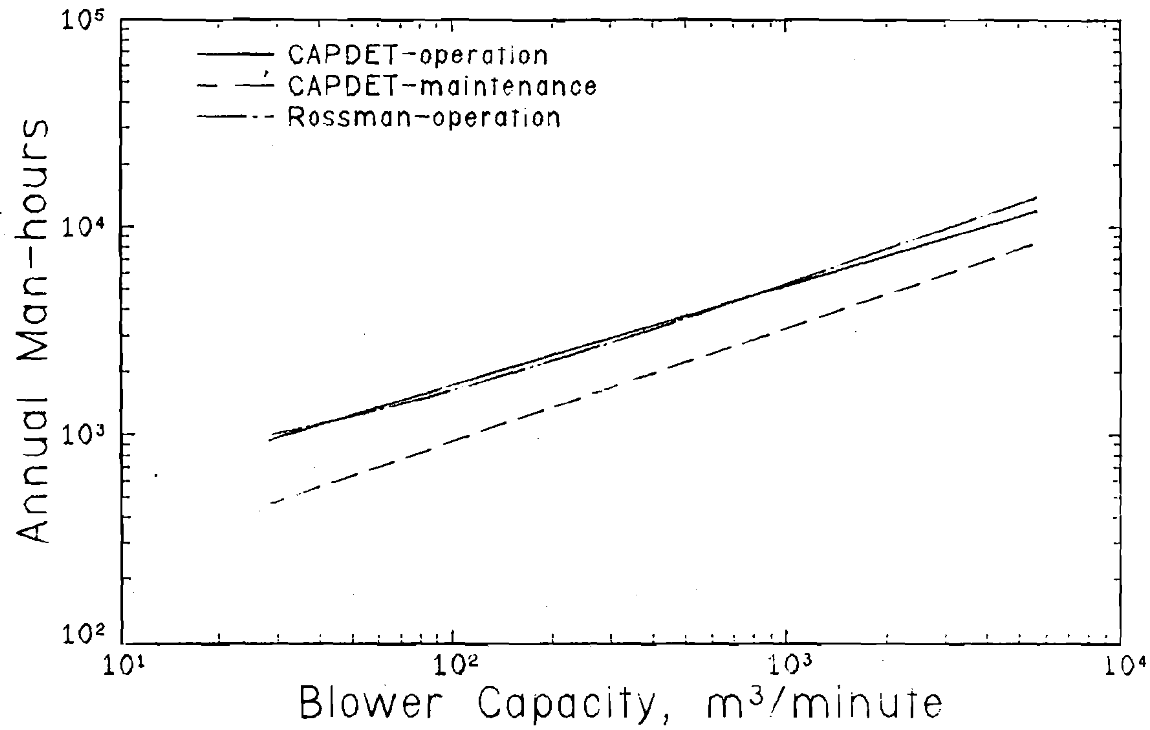


Figure A.4- Cost Functions for Activated Sludge Aeration

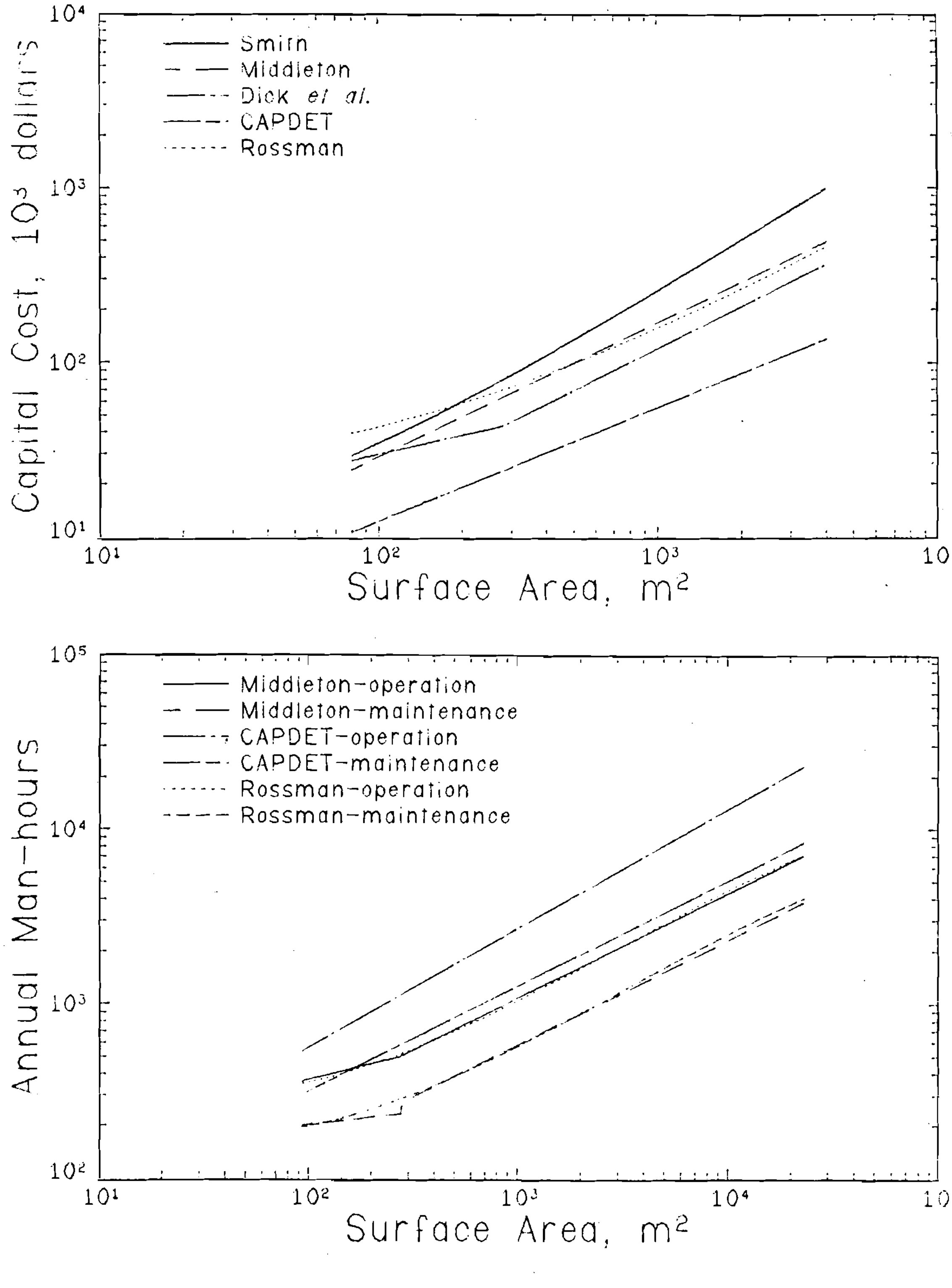
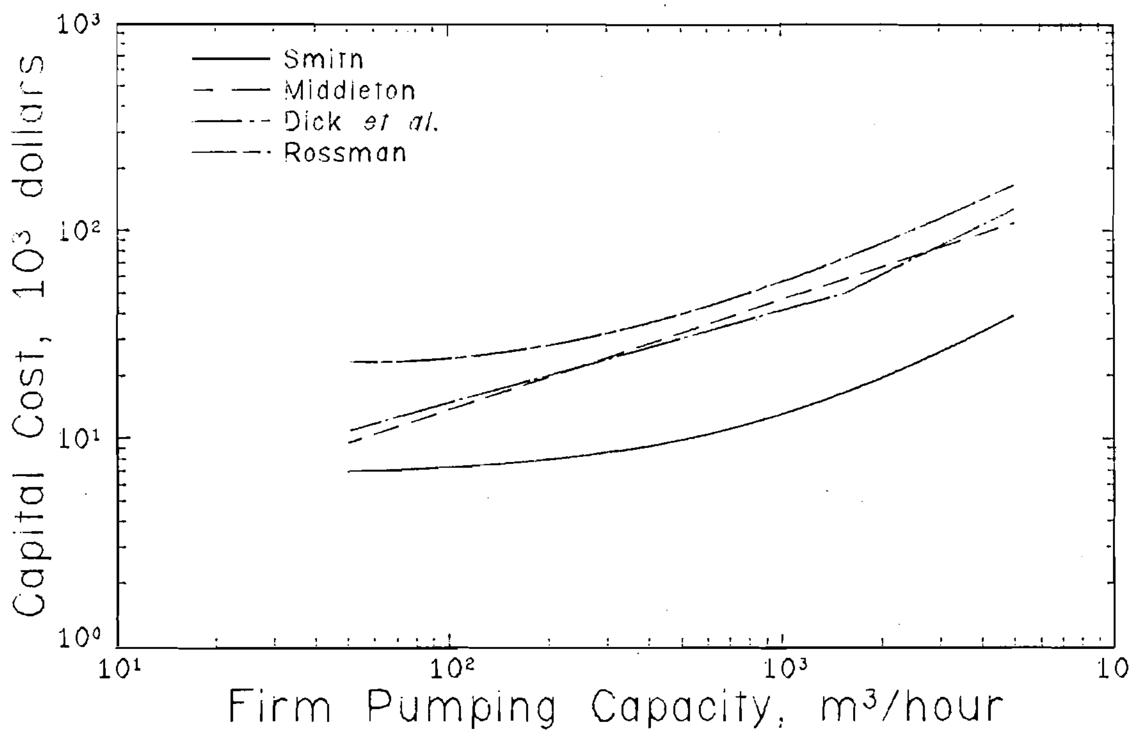


Figure A.5- Cost Functions for Secondary Clarifier



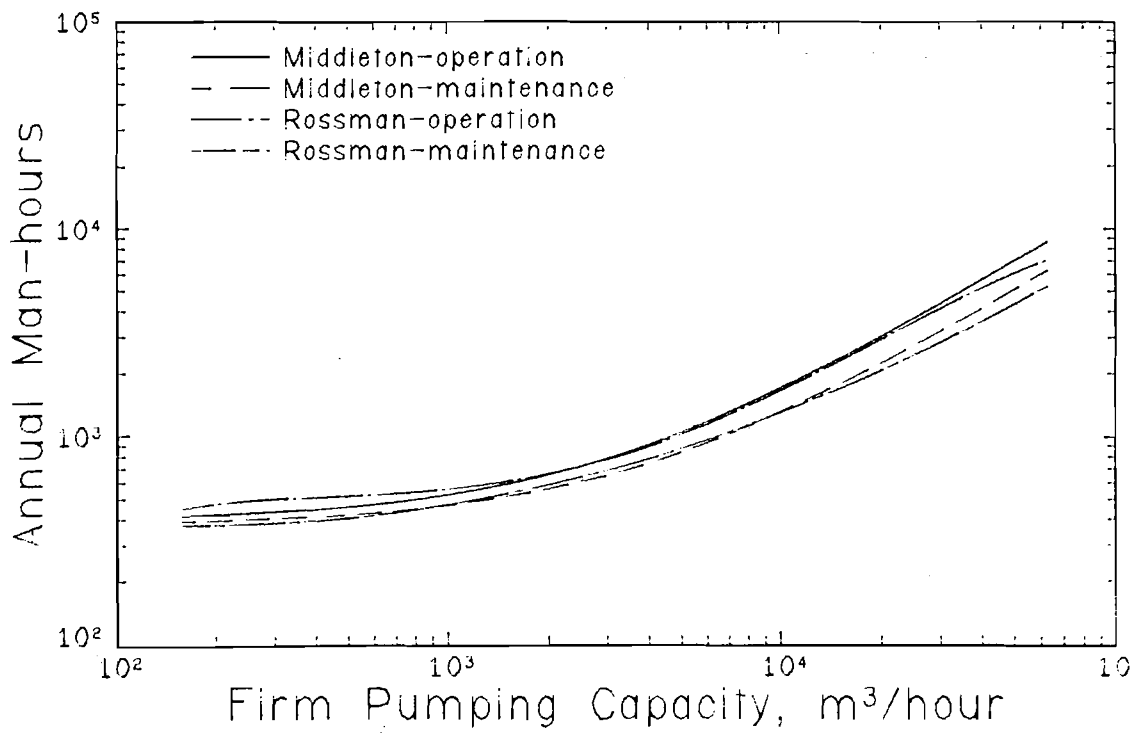


Figure A.6- Cost Functions for Recirculation Pumping

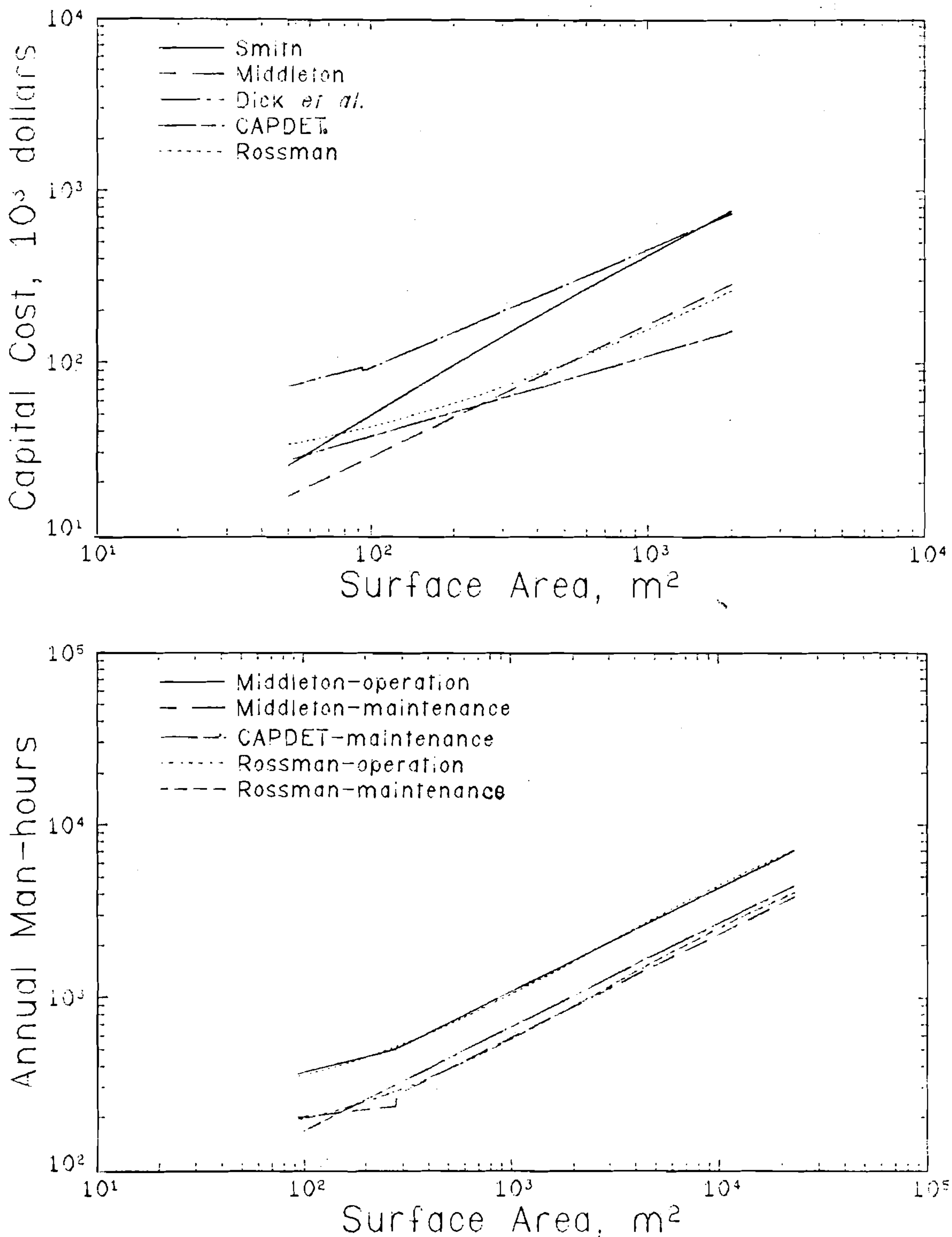


Figure A.7- Cost Functions for Gravity Thickener

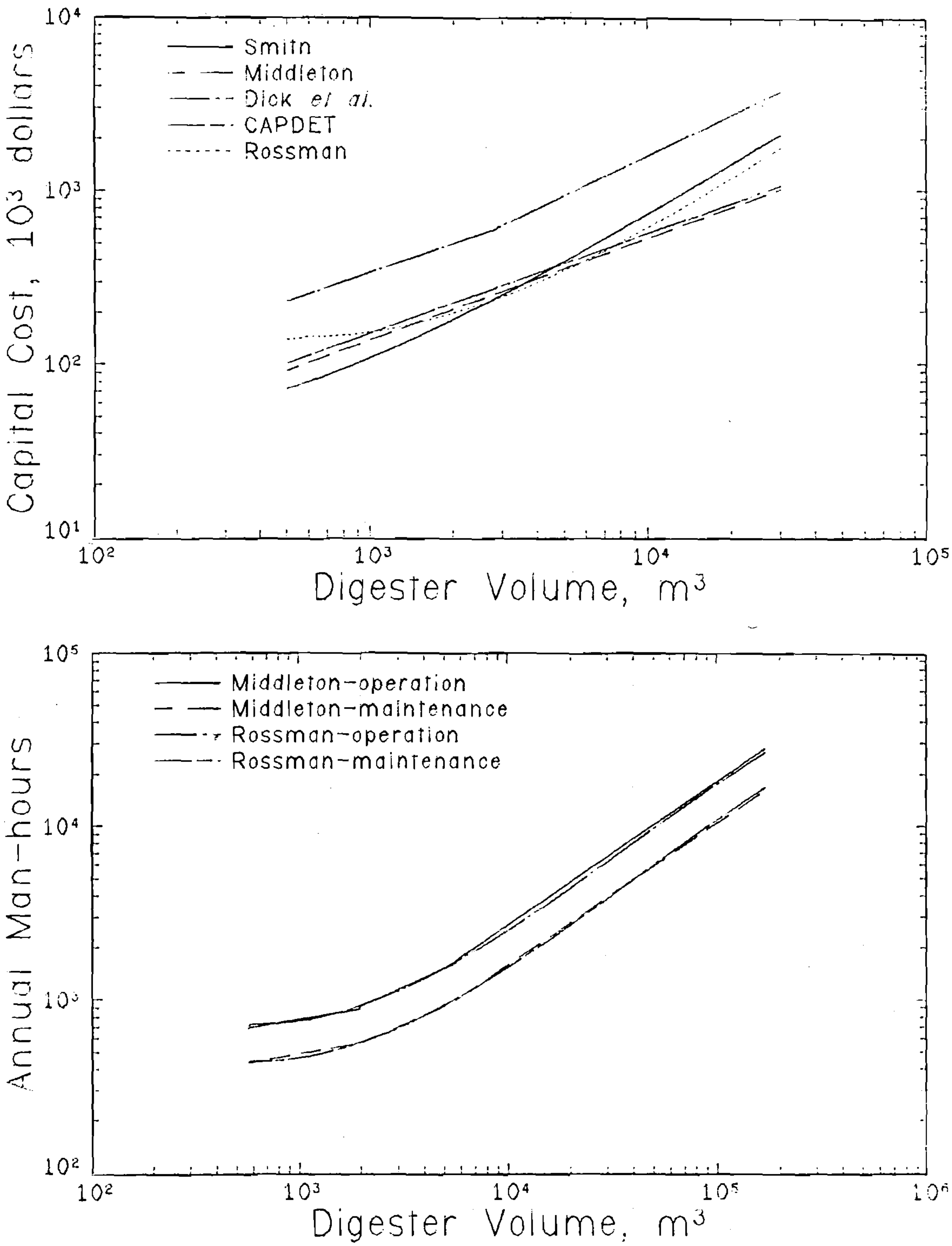
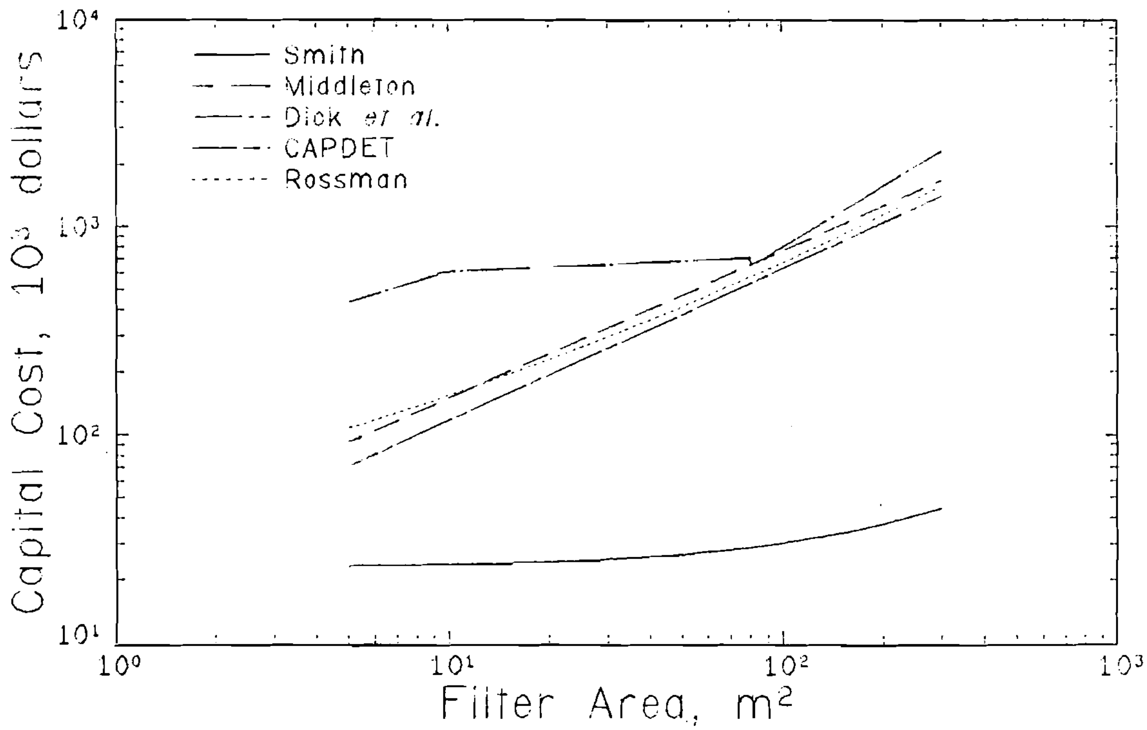


Figure A.8- Cost Functions for Anaerobic Digester



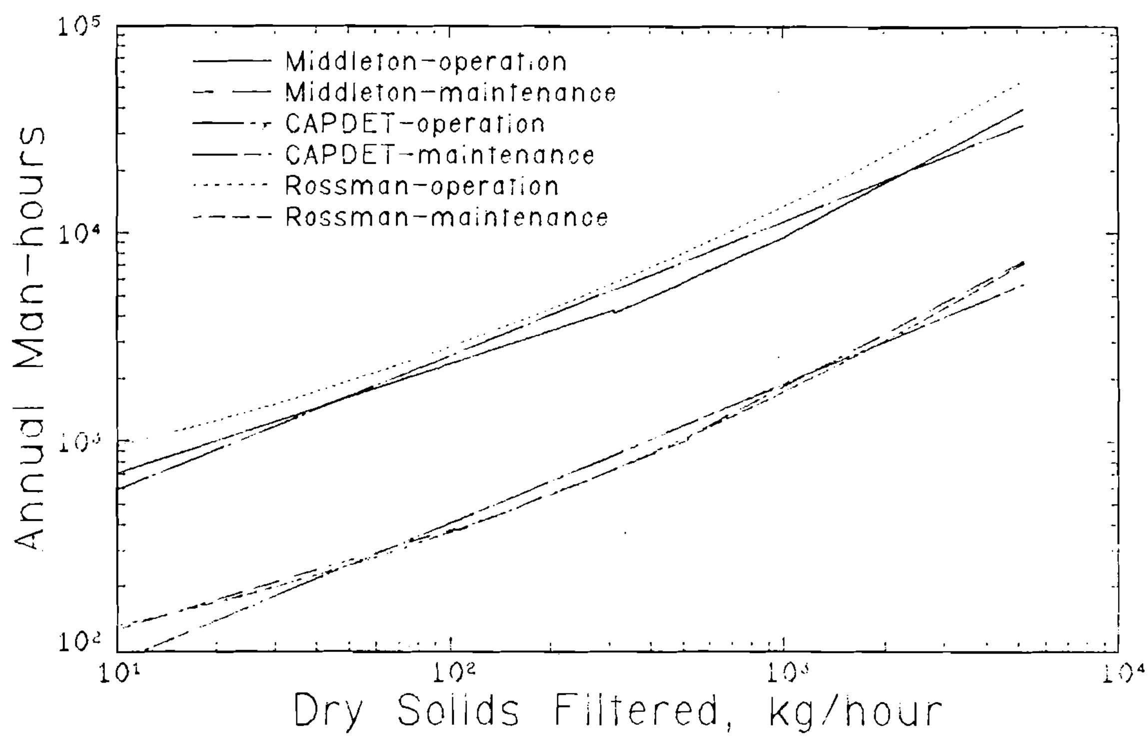


Figure A.9- Cost Functions for Vacuum Filter

APPENDIX B

ESTIMATING OPERATION COST FOR SLUDGE LANDFILL

The following development is based on Figure 9-1 on Page 9-4 in the USEPA Process Design Manual - Sludge Treatment and Disposal.

Let W_s = wet tons of sludge landfilled per day,

OMC = annual operation cost for sludge landfill.

According to Figure 9-1,

$$\frac{OMC}{365 W_s} = 15.081 \; W_s^{-0.333}$$

or

$$OMC = 5504.6 W_s^{0.667}$$

Since the labor rate is 6.25 dollars/hr in Figure 9-1, the annual manhours, $OHRS$, can be calculated as

$$OHRS = \frac{OMC}{6.25} = 880.7 \; W_s^{0.667}$$

Using equation (2.77),

$$\begin{aligned} OHRS &= 880.7 \; W_s^{0.667} \\ &= 880.7(27.513 Q_{16})^{0.667} \\ &= 8024 \; Q_{16}^{0.667} \end{aligned}$$

APPENDIX C

ANALYSIS PROGRAM FOR DESIGN OF WASTEWATER TREATMENT SYSTEMS

The analysis program can be used to determine a complete system design for the base wastewater treatment system (Figure 2.1) and two variations of the base system (Figures 4.2 and 4.7). Two input data files are necessary to run this program. The first file contains the parameters in the model, and the second the decision variables. Specifications of these two files are described below.

Model parameters include the influent wastewater characteristics, effluent water quality standards, and parameters for process design and economic analysis. The input order of these parameters in the data file is shown in Table C.1.

Table C.1 - Input Data to the Analysis Program : Model Parameters

Card No.	Comment
1	Influent Characteristics
2	Effluent Water Quality Standards
≥3	Parameters

The influent characteristics are the design flowrate (m^3/hr), the soluble BOD_5 concentration (mg/l), and the concentrations of active biomass, volatile biodegradable solids, volatile inert solids, and fixed solids (all in mg/l). The second card specifies, in order, the effluent BOD_5 and the total suspended solids standards (both in mg/l). Table C.2 lists the parameters used for process design and cost calculations according to their input order. Free format input is used. An example input file looks like this:

```
1    1500 100 5 100 45 50
2    30 30
3    0.09716   150.6    362.0    8.9     5000.0
4    0.05      10.0     0.6      0.0     0.0
5    .139      0.27     0.22     0.0     0.0
6    1.0       24.24    174.44   2.5     2.3747
7    0.1659    0.0      0.4      60.0    5.0
```

8	0.04	0.77	1.42	1.5	5.69
9	0.00403	11.91	0.0	0.0	0.0
10	0.8	0.95	1.5	9.17	20.0
11	0.08	1.2	0.232	1.024	0.02
12	0.0	0.0	0.2	0.0	0.0
13	0.0	0.00	0.632	3.003	20.0
14	0.35	10.0	0.85	1.0	0.4
15	2.5	0.0	0.0	0.0	0.0
16	500.	0.25	292.6	2.90	4.0
17	10.0	0.0	657.3	0.33	83300.0
18	8.9E-4	6.0	1.E12	2.00	1.E-3

The decision variables selected in this study were summarized in Table 2.5. The values of these variables are specified in the second input data file to run the analysis program. Table C.3 summarizes the information requirements of this file.

An example input file containing the decision variables is shown below:

```
1    INITIAL DESIGN FOR BASE SYSTEM
2    0
3    0 1 0
4    1.50000
5    2.00000 .150000
6    0.100000
7    1.000000
8    30.0000 15.0000
9    0.5000000
10   10.00000
```

Three files are produced from running the analysis program. The first file contains the detailed design information for the specified flowchart and the values of the decision variables. The itemized costs for the unit processes included in the flowchart are summarized in a second output file. The third file has the values of the variables that are in the GRG optimization model.

The analysis program is listed on the next few pages.

Table C.2 - Summary of Parameters in the System Model

Names (Units)	Value	Index
Economic Data:		
Capital Recovery Factor	0.09716	1
Base (1971) Cost Index	150.6	2
Cost Index for 1980	362.0	3
Operating/Maintenance Wages (dollars/hr)	8.9	4
Land Cost, C_L (dollars/acre)	5000	5
Electricity Cost (dollars/$kWhr$)	0.05	6
Pumping Head, H (meters)	10.0	7
Pumping Efficiency, ϵ_p	0.6	8
Primary Sedimentation:		
Constant in Voshel-Sak Model, v_1	0.139	11
Constant in Voshel-Sak Model, v_2	0.27	12
Constant in Voshel-Sak Model, v_3	0.22	13
Sludge Settling Characteristics:		
Thickening Constant, a_w	24.24	17
Thickening Constant, a_1	174.77	18
Thickening Constant, a_2	2.5	19
Thickening Constant, n_w	2.3747	20
Thickening Constant, n_1	0.1659	21
Activated Sludge Kinetics:		
Growth Yield Coefficient, y (g cell/g BOD_5)	0.4	23
Half-Velocity Constant, K_s (g BOD_5/m^3)	60	24
Maximum Specific Utilization Coeff., k (day^{-1})	5.0	25
Endogeneous Decay Coefficient, b (day^{-1})	0.04	26
Fraction of cells Degradable, f_d	0.77	27
Conversion (g BOD_L/g cell)	1.42	28
Conversion (g BOD_L/g BOD_5)	1.5	29
Secondary Sedimentation:		
Constant in Chapman Model, c_1	5.69	30
Constant in Chapman Model, c_2	0.00103	31
Constant in Chapman Model, c_3	11.91	32
Aeration:		
Alpha Factor in Aeration	0.8	36
Beta Factor in Aeration	0.95	37
DO Concentration in Aeraton Tank, DO (g/m^3)	1.5	38

Table C.2 (continued)

Names (Units)	Value	Index
DO Saturation Concentration, C_s (g/m^3)	9.17	39
Temperature of Mixed Liquor, T_L (°C)	20.0	40
Oxygen Transfer Efficiency, OTE	0.08	41
Density of Air, ρ_{air} (kg/m^3)	1.2	42
Weight Fraction of Oxygen in Air, γ	0.232	43
Temperature Coefficient	1.024	44
Mixing Requirement, η (m^3 air/m^3/min)	0.02	45
Gravity Thickening:		
TSS of Thickener Supernatant, M_{t10} (kg/m^3)	0.2	48
Anaerobic Digestion:		
Coeff. for Digestion Rate Model	0.632	53
Coeff. for Digestion Rate Model	3.003	54
Temperature of Digester Influent, T_0 (°C)	20.0	55
Methane Production (m^3/kg BOD_L)	0.35	56
Average Ambient Temperature, T_a (°C)	10.0	57
Efficiency of Heat Exchanger, ϵ	0.85	58
Heat Conduction Coefficient, U (W/m^2-°C)	1.0	59
Outside Surface Area and Volume Ratio for Digester, a	0.4	60
Worth of Digester Gas (dollars/therm)	2.5	61
Soluble BOD_5 in Digester Supernatant, S_{12} (g/m^3)	500	66
Factor Accounting For Effect of Rising Gas on Thickening in Secondary Digester, δ	0.25	67
Thickening Constant for Digested Sludge, a_d	292.6	68
Thickening Constant for Digested Sludge, n_d	2.9	69
TSS of Digester Supernatant, M_{t13} (kg/m^3)	4.0	70
Height of Digester (m)	10.0	71
Vacuum Filtration:		
Coeff. for Calculating Filter Yield	657.3	73
Form Time per Cycle Time, χ	0.33	74
Pressure Applied on Vacuum Filter, P (Nt/m^2)	83300	75
Viscosity of Filtrate, μ (Nt-sec/m^2)	0.00089	76
Cycle Time, t_c (min)	6.0	77
Specific Resistance of Sludge, r_s (m/kg)	10^{12}	78
TSS of Filtrate, M_{t15} (kg/m^3)	2.0	79
Unit Conversion Factor	0.001	80

Table C.3 - Input Data to the Analysis Program : Decision Variables

Card No.	Comments
1	Title
2	Output print level, =0 : only the final results are printed =1 : design of every iteration is printed
3	Process flowchart, 0. 1. 0. : Figure 2.1 1. 1. 0. : Figure 4.2 0. 0. 1. : Figure 4.7
4	Primary clarifier overflow rate (m/hr), delete this card if primary clarifier is not in the system (Figure 4.2)
5	Sludge age, θ_c (days), Activated sludge recycle ratio
6	Hydraulic retention time (days)
7	Gravity thickener solids loading ($kg/m^2/hr$), delete this card if waste activated sludge is recycled to primary settling tank (Figure 4.7)
8	Digestion temperature (°C), Solids retention time, θ_d (days)
9	Secondary digester solids loading ($kg/m^2/hr$)
10	Vacuum filter filter yield ($kg/m^2/hr$)

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```
      PROGRAM MAIN(PAR,DECVAR,OUTPUT,DETAIL,COST,GRGDATA,RCYCLE,
     +             TAPE4=PAR,TAPE5=DECVAR,TAPE6=DETAIL,
     +             TAPE8=GRGDATA,TAPE9=COST,TAPE7=RCYCLE)
      COMMON/STATE/VIN,VOUT,VSIDE,ARY1,ARY2,ITE
      COMMON/PARVAR/U,V,X
      COMMON/CONTRL/LPRINT,B1,B2,B3
      COMMON/DPST/OR,IUNIT1
      COMMON/DAS/SRT,HRT,ASRR,IUNIT2,BODSTD,TSSSTD
      COMMON/BLEND/AC,NC
      COMMON/DGT/SLGT,IUNIT3
      COMMON/DPAND/SRTD,TEMP
      COMMON/DSAND/SLSAND,IUNIT4
      COMMON/DVF/YIELD
      REAL VIN(7),VOUT(7),VSIDE(7),U(100),V(20),X(100)
      REAL ARY1(7),ARY2(7)
      REAL VOLD(7),TEST(7),INFLOW(7)
      INTEGER LPRINT
      REAL NC
      CHARACTER* 1 TITLE(80)
C
      READ(4,*)  (INFLOW(I),I=1,6)
      INFLOW(7)=INFLOW(3)+INFLOW(4)+INFLOW(5)+INFLOW(6)
      READ(4,*)  BODSTD,TSSSTD
      READ(4,*)  (U(I),I=1,80)
C
      READ(5,90)TITLE
90    FORMAT(80A1)
      READ(5,*)LPRINT
      READ(5,*)B1,B2,B3
      IF(B1.EQ.1.0) GOTO 11
      READ(5,*)OR
11    READ(5,*)SRT,ASRR
      READ(5,*)HRT
      IF(B3.EQ.1.0) GOTO 13
      READ(5,*)SLGT
13    READ(5,*)TEMP,SRTD
      READ(5,*)SLSAND
12    READ(5,*)YIELD
      IUNIT1=1
      IUNIT2=1
      IUNIT3=1
      IUNIT4=1
C
      V(1)=U(1)*U(3)/U(2)
      V(2)=U(3)/U(2)
      V(3)=23.85*U(6)*U(7)/U(8)
      V(4)=U(17)+U(18)
      V(5)=U(20)*EXP(U(21))
      V(6)=(V(4)*(V(5)-1.0))**(1./V(5))*V(5)/(V(5)-1.0)
      V(7)=(U(17)*(U(20)-1.0))**(1./U(20))*U(20)/(U(20)-1.0)
      V(8)=U(28)/U(29)
      V(9)=U(39)/1440./U(36)/(U(37)*U(39)-U(38))/U(41)
     +     /U(42)/U(43)/U(44)**(U(40)-20.0)
      V(10)=U(28)*U(56)
      V(11)=(U(67)*U(68)*(U(69)-1.0))**(1./U(69))*U(69)/(U(69)-1.0)
      V(12)=U(73)*(U(74)*U(75)/(U(76)*U(77)*U(78)))**0.5
      V(13)=U(69)-1.
C
      WRITE(6,99)
99    FORMAT(2X,104('*'))
```

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```
      WRITE(6,100)TITLE
100   FORMAT(/,15X,80A1,/)
      WRITE(6,99)
C
      ITE=0
      DO 20 I=1,7
      VIN(I)=0.0
      VOLD(I)=INFLOW(I)
20    CONTINUE
C
32    CALL MB(VIN,INFLOW)
      IF(ITE.EQ.0) GOTO 26
        DO 21 I=1,7
        TEST(I)=(VIN(I)-VOLD(I))/VIN(I)
        IF(ABS(TEST(I)).GT.1.E-6) THEN
          IF(ITE.EQ.40) THEN
             PRINT *,'MAXIMUM NUMBER OF ITERATION (40)
     +       REACHED FOR RECYCLE STREAM MASS BALANCE'
             STOP
          ELSE
            ITE=ITE+1
            CALL COPY(VIN,VOLD)
            GO TO 27
          ENDIF
        ENDIF
21    CONTINUE
      LPRINT=LPRINT+1
      IF(LPRINT.EQ.1) THEN
        ITE=ITE+1
        GO TO 28
      ELSE
        GO TO 40
      ENDIF
C
26    ITE=ITE+1
27    IF(LPRINT.EQ.0) GO TO 30
      WRITE(7,77) (VIN(I),I=1,6)
77    FORMAT(E14.6)
28    WRITE(6,110) ITE
110   FORMAT(///,20X,'ITERATION',I5,///)
30    CALL BRANCH(B1,VIN,ARY1)
C
      IF(B1.EQ.1.0) THEN
        CALL AS
        DO 51 I=1,7
        ARY1(I)=0.0
        VSIDE(I)=0.0
51      CONTINUE
        VOUT(1)=0.0
        GOTO 52
      ENDIF
C
      CALL PST
      CALL MB(ARY1,VOUT)
      CALL AS
      IF(B3.EQ.1.0 .AND. B2.EQ.0.0) THEN
        CALL COPY(VIN,ARY1)
        CALL COPY(VSIDE,VIN)
        DO 61 I=3,7
        VIN(I)=VIN(I)*1.E-3
```

```
61       CONTINUE
         GOTO 33
        ENDIF
        CALL BRANCH(B2,VSIDE,VOUT)
        CALL BRANCH(B3,VIN,ARY1)
52      CALL SLGMIX
        CALL GT
        CALL MB(ARY1,ARY2)
        CALL MB(VOUT,VSIDE)
        CALL COPY(VOUT,VIN)
33      CALL PAND
        CALL COPY(VOUT,VIN)
        CALL SAND
        CALL MB(ARY1,VSIDE)
        CALL COPY(VOUT,VIN)
        CALL VF
        CALL MB(ARY1,VSIDE)
        CALL COPY(ARY1,VIN)
C
        DO 35 I=3,7
        VIN(I)=VIN(I)*1.E3
35      CONTINUE
C
        GO TO 32
C
   40 CALL OBJ
C
        IF(B1.EQ.1.0.OR.B3.EQ.1.0)THEN
        NVAR=51
        ELSE
        NVAR=64
        ENDIF
        WRITE(8,120) (K,X(K),K=1,NVAR)
  120 FORMAT(3X,I3,4X,E20.10)
C
        STOP
        END
C
C
C
        SUBROUTINE PST
        COMMON/STATE/VIN,VOUT,VSIDE,ARY1,ARY2,ITE
        COMMON/PARVAR/U,V,X
        COMMON/CONTRL/LPRINT,B1,B2,B3
        COMMON/DPST/OR,IUNIT1
        REAL VIN(7),VOUT(7),VSIDE(7),ARY1(7),ARY2(7),U(100),V(20),X(100)
        REAL KP1
C
        IF(VIN(1).EQ.0.0) RETURN
C
        IF(IUNIT1.EQ.0.AND.ITE.EQ.1) OR=OR/24.
        R=U(11)*VIN(7)**U(12)/OR**U(13)
        R1=1.0-R
C
        KP1=1.E3*V(6)/VIN(7)
C
C USE NEWTON'S METHOD TO SOLVE FOR PRIMARY SLUDGE FLOWRATE:
C
        N=1
        VSIDE(1)=1.E-3*VIN(1)
```



```
21      VOUT(1)=VIN(1)-VSIDE(1)
        F=VIN(1)-VOUT(1)*R1-KP1*VSIDE(1)**((V(5)-1.)/V(5))
       +  *(VOUT(1)*U(16)/OR)**(1./V(5))
        IF(ABS(F).LE.1.E-6) GOTO 22
        FPRIME=R1-KP1*(U(16)*VOUT(1)/OR/VSIDE(1))**(1./V(5))
       +         *(1.-VIN(1)/V(5)/VOUT(1))
        VSIDE(1)=VSIDE(1)-F/FPRIME
        N=N+1
        IF(N.GE.50) THEN
        PRINT *,'MAXIMUM NUMBER OF ITERATION REACHED IN PRIMARY
       + SETTLING TANK DESIGN'
        STOP
        ENDIF
        GOTO 21
C
22      IF(VSIDE(1).LT.0.) THEN
        PRINT *,'FAILED TO FIND A FEASIBLE SOLUTION IN
       +PRIMARY SETTLING TANK DESIGN'
        STOP
        ENDIF
C
        VOUT(7)=VIN(7)*R1
        APST=VOUT(1)/OR
        VSIDE(7)=V(6)/(VSIDE(1)/U(16)/APST)**(1./V(5))*1.E3
        VOUT(2)=VIN(2)
        VSIDE(2)=VIN(2)
C
        RATIO=VSIDE(7)/VIN(7)
C
        DO 1 I=3,6
          VOUT(I)=VIN(I)*R1
          VSIDE(I)=VIN(I)*RATIO
1       CONTINUE
C
        IF(LPRINT.LT.1)GOTO 100
        WRITE(6,13)OR*24.,APST,R*100.
13      FORMAT(2X,'**PRIMARY SETTLING TANK DESIGN--',//,
       +'      OVERFLOW  RATE   =',F12.5,'  M/DAY,',/,
       +'      SURFACE   AREA   =',F12.5,'  SQ M',/,
       +'      SOLIDS REMOVAL   =',F12.5,'  PERCENT.',/)
        WRITE(6,14)
14      FORMAT(15X,'Q(CU M/HR)',4X,'S(G/CU M)',3X,'MA(G/CU M)',3X,
       +'MD(G/CU M)',3X,'MI(G/CU M)',3X,'MF(G/CU M)',5X,'MT(G/CU M)')
        WRITE(6,10) (VIN(I),I=1,7)
        WRITE(6,11) (VOUT(I),I=1,7)
        WRITE(6,12) (VSIDE(I),I=1,7)
10      FORMAT(2X,'INFLUENT',3X,F12.5,5(1X,F12.5),1X,F13.5)
11      FORMAT(2X,'EFFLUENT',3X,F12.5,5(1X,F12.5),1X,F13.5)
12      FORMAT(2X,'UNDERFLOW',2X,F12.5,5(1X,F12.5),1X,F13.5)
C
        X(10)=R1
        X(8)=OR
        X(1)=VIN(1)/60.0
        X(11)=VOUT(1)/60.
        X(12)=VSIDE(1)
        X(7)=VIN(7)
        X(9)=APST*1.E-2
        X(3)=VIN(3)
        X(4)=VIN(4)
        X(5)=VIN(5)
```

```
      X(6)=VIN(6)
      X(2)=VIN(2)
      IF(B3.NE.1.)THEN
      X(64)=VSIDE(7)*1.E-5
      ELSE
      X(50)=VSIDE(7)*1.E-5
      X(51)=VSIDE(6)*1.E-3
      ENDIF
C
100   RETURN
      END
C
C
C
      SUBROUTINE AS
      COMMON/STATE/VIN,VOUT,VSIDE,ARY1,ARY2,ITE
      COMMON/PARVAR/U,V,X
      COMMON/CONTRL/LPRINT,B1,B2,B3
      COMMON/DAS/SRT,HRT,ASRR,IUNIT2,BODSTD,TSSSTD
      REAL VIN(7),VOUT(7),VSIDE(7),ARY1(7),ARY2(7),U(100)
      REAL V(20),X(100),SN(3),F(3)
      REAL NW
C
C
C AERATION:
C
      IF(IUNIT2.EQ.0.AND.ITE.EQ.1) HRT=HRT/24.
C
      C=SRT/HRT
      VIN(1)=ARY1(1)*(1.0+ASRR)
      VIN(2)=U(24)*(1.0+U(26)*SRT)/(SRT*(U(23)*U(25)-U(26))-1.0)
      FOOD=ARY1(2)+V(8)*ARY1(4)-VIN(2)
      VIN(3)=U(23)/(1.0+U(26)*SRT)*C*FOOD
      VIN(4)=0.0
      VIN(5)=(ARY1(5)*C+(1.0-U(27))*U(26)*VIN(3)*SRT)/
     +          (1.0+C*(ARY1(3)/VIN(3)))
      VIN(6)=ARY1(6)*C/(1.0+C*(ARY1(3)/VIN(3)))
      VIN(7)=VIN(3)+VIN(4)+VIN(5)+VIN(6)
C
      VAT=ARY1(1)*HRT*24.0
C
      ORMT=ARY1(1)*FOOD*(U(29)-U(28)*U(23)/(1.+U(26)*SRT))*1.E-3*24.
      AFR=ORMT*V(9)
      AFUV=AFR/VAT
      IF(AFUV.LT.U(45)) THEN
        PRINT *,'MIXING REQUIREMENT CONTROLS OXYGEN DEMAND'
      ENDIF
C
      IF(LPRINT.LT.1) GOTO 150
      WRITE(6,100) SRT,ASRR*1.E2,VAT,HRT*24.,ORMT,AFR
100   FORMAT(//,2X,'**ACTIVATED SLUDGE SYSTEM DESIGN--',//,
     +'        SLUDGE  AGE   =',F12.5,' DAYS,',/,
     +'      SLUDGE RECYCLE =',F12.5,' PERCENT,',//,
     +2X,'(A). AERATION TANK--',//,
     +'          VOLUME       =',F12.5,' CU M,',/,
     +'      RETENTION TIME =',F12.5,' HOURS,',/,
     +'   OXYGEN REQUIREMENT=',F12.5,' KG/D,',/,
     +'      AIR  FLOWRATE  =',F12.5,' CU M/MIN.',/)
      WRITE(6,110)
110   FORMAT(15X,'Q(CU M/HR)',4X,'S(G/CU M)',3X,'MA(G/CU M)',3X,
```



```
     +'MD(G/CU M)',3X,'MI(G/CU M)',3X,'MF(G/CU M)',5X,'MT(G/CU M)')
      WRITE(6,120) (ARY1(I),I=1,7)
      WRITE(6,130) (VIN(I),I=1,7)
120   FORMAT(2X,'INFLUENT',3X,F12.5,5(1X,F12.5),1X,F13.5)
130   FORMAT(2X,'EFFLUENT',3X,F12.5,5(1X,F12.5),1X,F13.5)
C
150   IF(B1.NE.1.0) GOTO 160
      X(1)=ARY1(1)/60.0
      DO 155 I=2,7
      X(I)=ARY1(I)
155   CONTINUE
C
C SECONDARY SETTLING:
C
160   YY=VIN(5)/VIN(3)
      ZZ=VIN(6)/VIN(3)
      XX=VIN(7)/VIN(3)
C
      Q2=ARY1(1)
      XA2=ARY1(3)
C
      ARY1(3)=((ASRR+1.0-1.0/C)*VIN(3)-XA2)/ASRR
      ARY1(7)=ARY1(3)*XX
C
C SETTING UP THE COEFFICIENTS FOR SIMULTANEOUS EQUATIONS:
      NW=U(20)
      A11=ARY1(3)*1.0E-3
      A12=-1.0/XX*1.0E-3
      A13=VIN(1)/XX*1.0E-3
      RHS1=(VIN(3)*VAT/SRT/24.0+Q2*XA2+ASRR*Q2*ARY1(3))*1.E-3
      A21=V(7)
      RHS2=ARY1(7)*1.0E-3
      A31=U(32)*Q2*(1.0+ASRR)*(-1.0)
      A32=1.0
      RHS3=-U(30)+U(31)*VIN(7)
C
C PROVIDING STARTING VALUES FOR VARIABLES:
        IAREA=1
        SN(1)=ASRR*Q2
        SN(2)=15.0
180     SN(3)=500.0*(2.*IAREA-1.0)
C
      N=3
C
      CALL FUNC(N,SN,F,NW,A11,A12,A13,A21,A31,A32,RHS1,RHS2,RHS3)
C
      CALL QUASI(N,SN,F,NW,A11,A12,A13,A21,A31,A32,RHS1,
     +RHS2,RHS3,ITEQN)
C
      IF(ITEQN.GE.50) THEN
      IF(IAREA.GT.20) THEN
      PRINT *,'MAXIMUM NUMBER OF ITERATIONS REACHED IN
     + ACTIVATED SLUDGE DESIGN'
      STOP
      ELSE
        IAREA=IAREA+1
        GOTO 180
        ENDIF
        ENDIF
C
```

```
      IF(SN(1).LE.0.0.OR.SN(2).LE.0.0.OR.SN(3).LE.0.0) THEN
      PRINT *,'FAILED TO FIND A FEASIBLE SOLUTION IN
     +ACTIVATED SLUDGE DESIGN'
      STOP
      ENDIF
C
      ARY1(1)=SN(1)
      VOUT(7)=SN(2)
      AF=SN(3)
C
      VOUT(1)=VIN(1)-ARY1(1)
      VOUT(2)=VIN(2)
      VOUT(3)=VOUT(7)/XX
      VOUT(4)=0.0
      VOUT(5)=VOUT(3)*YY
      VOUT(6)=VOUT(3)*ZZ
C
      ARY1(2)=VOUT(2)
      ARY1(4)=0.0
      ARY1(5)=ARY1(3)*YY
      ARY1(6)=ARY1(3)*ZZ
C
      SLFST=ARY1(1)*ARY1(7)/AF*1.E-3
      OFRATE=VOUT(1)/AF*24.0
C
      EFFBOD=VOUT(2)+V(8)*U(27)*VOUT(3)
      EFFTSS=VOUT(7)
C
      IF(LPRINT.LT.1) GOTO 700
C
      IF(EFFBOD.GT.BODSTD) THEN
        PRINT *,'WARNING-BOD5 STANDARD VIOLATED'
      ENDIF
      IF(EFFTSS.GT.TSSSTD) THEN
        PRINT *,'WARNING-TSS STANDARD VIOLATED'
      ENDIF
      WRITE(6,200)AF,OFRATE,SLFST,EFFBOD,EFFTSS
200   FORMAT(/,2X,'(B). SECONDARY SETTLING TANK--',//,
     +'       SURFACE  AREA   =',F12.5,' SQ M,',/,
     +'       OVERFLOW RATE   =',F12.5,' M/DAY,',/,
     +'       SOLIDS LOADING  =',F12.5,' KG/HR/SQ M,',/,
     +'       EFFLUENT BOD5   =',F12.5,' GM/CU M,',/,
     +'       EFFLUENT TSS    =',F12.5,' GM/CU M.'/)
      WRITE(6,300)
300   FORMAT(15X,'Q(CU M/HR)',4X,'S(G/CU M)',3X,'MA(G/CU M)',3X,
     +'MD(G/CU M)',3X,'MI(G/CU M)',3X,'MF(G/CU M)',5X,'MT(G/CU M)')
      WRITE(6,400) (VIN(I),I=1,7)
      WRITE(6,500) (VOUT(I),I=1,7)
      WRITE(6,600) (ARY1(I),I=1,7)
400   FORMAT(2X,'INFLUENT',3X,F12.5,5(1X,F12.5),1X,F13.5)
500   FORMAT(2X,'EFFLUENT',3X,F12.5,5(1X,F12.5),1X,F13.5)
600   FORMAT(2X,'UNDERFLOW',2X,F12.5,5(1X,F12.5),1X,F13.5)
C
      IF(B1.NE.1.0) THEN
      X(23)=VOUT(3)
      X(24)=ARY1(3)*1.E-3
      X(17)=VOUT(2)
      X(13)=SRT
      X(14)=HRT
      X(20)=ASRR
```



```
      X(15)=VAT*1.E-3
      X(16)=VIN(3)*1.E-3
      X(26)=FOOD
      X(25)=AF*1.E-2
      X(18)=YY
      X(19)=ZZ
      X(27)=AFR/60.
      ELSE
      X(18)=VOUT(3)
      X(19)=ARY1(3)*1.E-3
      X(12)=VOUT(2)
      X(8)=SRT
      X(9)=HRT
      X(15)=ASRR
      X(10)=VAT*1.E-3
      X(11)=VIN(3)*1.E-3
      X(21)=FOOD
      X(20)=AF*1.E-2
      X(13)=YY
      X(14)=ZZ
      X(22)=AFR/60.
      ENDIF
C
C ACTIVATED SLUDGE SEPARATION:
C
C
700   VOUT(1)=ASRR*Q2
      VIN(1)=ARY1(1)-VOUT(1)
C
      DO 60 I=3,7
      ARY1(I)=ARY1(I)*1.0E-3
      VOUT(I)=ARY1(I)
      VIN(I)=ARY1(I)
60    CONTINUE
      VOUT(2)=ARY1(2)
      VIN(2)=ARY1(2)
C
      IF(LPRINT.LT.1) GO TO 20000
      WRITE(6,800)
800   FORMAT(/,2X,'(C). ACTIVATED SLUDGE SEPARATION--',/)
      WRITE(6,900)
900   FORMAT(15X,'Q(CU M/HR)',4X,'S(G/CU M)',2X,'MA(KG/CU M)',2X,
     +'MD(KG/CU M)',2X,'MI(KG/CU M)',2X,'MF(KG/CU M)',
     +4X,'MT(KG/CU M)')
      WRITE(6,910) (ARY1(I),I=1,7)
      WRITE(6,920) (VOUT(I),I=1,7)
      WRITE(6,930) (VIN(I),I=1,7)
910   FORMAT(2X,'UNDERFLOW',2X,F12.5,5(1X,F12.5),1X,F13.5)
920   FORMAT(2X,'RECYCLE',4X,F12.5,5(1X,F12.5),1X,F13.5)
930   FORMAT(2X,'WASTED',5X,F12.5,5(1X,F12.5),1X,F13.5)
C
      IF(B1.NE.1.0) THEN
      X(21)=VIN(1)/Q2*1.E2
      X(22)=ASRR+X(21)*1.E-2
      X(28)=VIN(1)
      X(29)=VIN(7)
      GOTO 20000
      ELSE
      X(16)=VIN(1)/Q2*1.E2
      X(17)=ASRR+X(16)*1.E-2
```

```
      X(23)=VIN(1)
      ENDIF
20000 RETURN
      END
C
      SUBROUTINE QUASI(N,X,F,NC,A11,A12,A13,A21,A31,
     +                 A32,RHS1,RHS2,RHS3,ITE)
      DIMENSION F(3),FNEW(3),H(3,3),HY(3),S(3),SH(3)
      DIMENSION U(3,3),X(3),Y(3)
      REAL NC
C
      ITE=1
C
C INITIATE H(I,J):
      DO 10 I=1,N
      DO 10 J=1,N
      IF(I.EQ.J)THEN
        H(I,J)=1.0
        ELSE
        H(I,J)=0.0
      ENDIF
10    CONTINUE
C
C COMPUTE X'(I)  (THE NEW SOLUTION) :
15    CALL MATVEC(N,H,F,S)
      DO 16 I=1,N
      S(I)=-S(I)
16    CONTINUE
C
C   TEST FOR CONVERGENCE:
        NSTOP=0
        DO 20 I=1,N
      IF(I.EQ.2)THEN
        IF(ABS(S(I)).LE.1.E-5)THEN
        NSTOP=NSTOP+1
        ENDIF
      ELSE
        IF(ABS(S(I)).LE.1.E-3)  THEN
          NSTOP=NSTOP+1
        ENDIF
      ENDIF
20      CONTINUE
C
        IF(NSTOP.EQ.N.OR.ITE.GE.200)  GO TO 100
C
C   UPDATE X(I):
        DO 25 I=1,N
        X(I)=X(I)+S(I)
25      CONTINUE
C
C COMPUTE Y(I):
      CALL FUNC(N,X,FNEW,NC,A11,A12,A13,A21,A31,
     +A32,RHS1,RHS2,RHS3)
      DO 30 I=1,N
      Y(I)=FNEW(I)-F(I)
      F(I)=FNEW(I)
30    CONTINUE
C
C UPDATE H(I,J):
C    COMPUTE H(I,J)*Y(J):
```



```
        CALL MATVEC(N,H,Y,HY)
C     COMPUTE TRANSPOSE(S(1*N)) * H(N*N)  * Y(N*1)  :
        SHY=0.0
        DO 35 I=1,N
        SHY=SHY+S(I)*HY(I)
35      CONTINUE
C     COMPUTE TRANSPOSE(S(1*N)) * H(N*N)  :
         CALL VECMAT(N,S,H,SH)
C     UPDATE:
        DO 40 I=1,N
        DO 40 J=1,N
        U(I,J)=(S(I)-HY(I))*SH(J)/SHY
        H(I,J)=H(I,J)+U(I,J)
40      CONTINUE
C
C  CONTINUE ITERATIONS:
      ITE=ITE+1
      GO TO 15
C
100   RETURN
      END
C
C SUBROUTINE 'MATVEC' PERFORMS THE POST-MULTIPLICATION OF
C    A MATRIX (N*N) BY A VECTOR (N*1)
C
      SUBROUTINE MATVEC(N,A,B,C)
      DIMENSION A(3,3),B(3),C(3)
C
      DO 1 I=1,N
      C(I)=0.0
      DO 2 J=1,N
      C(I)=C(I)+A(I,J)*B(J)
2     CONTINUE
1     CONTINUE
      RETURN
      END
C
C SUBROUTINE VECMAT PERFORMS THE PRE-MULTIPLICATION OF
C    A MATRIX (N*N) BY A VECTOR (1*N)
C
      SUBROUTINE VECMAT(N,X,Y,Z)
      DIMENSION X(3),Y(3,3),Z(3)
C
      DO 1 J=1,N
      Z(J)=0.0
      DO 2 I=1,N
      Z(J)=Z(J)+X(I)*Y(I,J)
2     CONTINUE
1     CONTINUE
      RETURN
      END
C
      SUBROUTINE FUNC(N,X,F,NC,A11,A12,A13,A21,A31,A32,
     +RHS1,RHS2,RHS3)
      REAL NC
      DIMENSION X(3),F(3)
C
      F(1)=A11*X(1)+A12*X(1)*X(2)+A13*X(2)-RHS1
      F(2)=A21**NC*(X(3)/X(1))-RHS2**NC
      F(3)=A31/X(3)+A32*X(2)-RHS3
```

```
      RETURN
      END
C
      SUBROUTINE SLGMIX
      COMMON/STATE/VIN,VOUT,VSIDE,ARY1,ARY2,ITE
      COMMON/PARVAR/U,V,X
      COMMON/CONTRL/LPRINT,B1,B2,B3
      COMMON/BLEND/AC,NC
      REAL VIN(7),VOUT(7),VSIDE(7),ARY1(7),ARY2(7),
     +U(100),V(20),X(100)
      REAL TEMP(7)
      REAL NC
C
      DO 1 I=3,7
      VOUT(I)=VOUT(I)*1.E-3
      VSIDE(I)=VSIDE(I)*1.E-3
1     CONTINUE
C
      IF(VOUT(1).EQ.0.0)  THEN
        DO 11 I=2,7
        VOUT(I)=0.0
11      CONTINUE
        GOTO 13
      ENDIF
C
      IF(VIN(1).EQ.0.0)  THEN
        DO 12 I=2,7
        VIN(I)=0.0
12      CONTINUE
      ENDIF
C
13    DO 2 I=1,7
      TEMP(I)=VIN(I)
2     CONTINUE
C
C THICKENING CHARACTERISTICS OF COMBINED SLUDGE:
      PRISLG=VOUT(1)*VOUT(7)
      FP=PRISLG/(PRISLG+TEMP(1)*TEMP(7))
      AC=U(17)+U(18)*FP**U(19)
      NC=U(20)*EXP(U(21)*FP)
C
      X(61)=FP
      X(62)=AC
      X(63)=NC
C
      VIN(1)=VOUT(1)+TEMP(1)
      DO 10 I=2,7
      VIN(I)=(TEMP(I)*TEMP(1)+VOUT(I)*VOUT(1))/VIN(1)
10    CONTINUE
C
      IF(LPRINT.LT.1) GOTO 1000
      WRITE(6,100)
100   FORMAT(//,2X,'**SLUDGE BLENDING--',/)
      WRITE(6,110)
110   FORMAT(15X,'Q(CU M/HR)',4X,'S(G/CU M)',2X,'MA(KG/CU M)',2X,
     +'MD(KG/CU M)',2X,'MI(KG/CU M)',2X,'MF(KG/CU M)',4X,
     +'MT(KG/CU M)')
      WRITE(6,120) (VOUT(I),I=1,7)
      WRITE(6,130) (TEMP(I),I=1,7)
      WRITE(6,140) (VIN(I),I=1,7)
```



```
120   FORMAT(2X,'PRIMARY',4X,F12.5,5(1X,F12.5),1X,F13.5)
130   FORMAT(2X,'ACTIVATED',2X,F12.5,5(1X,F12.5),1X,F13.5)
140   FORMAT(2X,'COMBINED ',2X,F12.5,5(1X,F12.5),1X,F13.5)
C
      IF(B1.NE.1.0)  THEN
      X(30)=VIN(1)
      X(31)=VIN(2)
      X(32)=VIN(7)
      GOTO 1000
      ELSE
      X(24)=VIN(7)
      ENDIF
1000  RETURN
      END
C
      SUBROUTINE GT
      COMMON/STATE/VIN,VOUT,VSIDE,ARY1,ARY2,ITE
      COMMON/PARVAR/U,V,X
      COMMON/CONTRL/LPRINT,B1,B2,B3
      COMMON/DGT/SLGT,IUNIT3
      COMMON/BLEND/AC,NC
      REAL VIN(7),VOUT(7),VSIDE(7),ARY1(7),ARY2(7),U(100),
     +V(20),X(100)
      REAL NC,KC
C
      IF(IUNIT3.EQ.0.AND.ITE.EQ.1)  SLGT=SLGT/24.
C
      KC=(AC*(NC-1.0))**(1.0/NC)*NC/(NC-1.0)
      VOUT(7)=SLGT*(KC/SLGT)**(NC/(NC-1.0))
C
      IF(VOUT(7).LT.VIN(7))  THEN
        SLGT=KC**NC/VIN(7)**(NC-1.0)
        PRINT *,'THICKENER DESIGN INFEASIBLE--INFLUENT
     +SOLIDS CONCENTRATION GREATER THAN UNDERFLOW SOLIDS
     + CONCENTRATION FOR THE SOLIDS LOADING SPECIFIED. ----
     + CHANGE SLUDGE THICKENING SCHEME, OR USE SLUDG
     +E LOADING LESS THAN ',SLGT,'  KG/SQ M/HR'
        STOP
      ENDIF
C
      ARY2(7)=U(48)
      VOUT(1)=(VIN(1)*VIN(7)-VIN(1)*ARY2(7))/(VOUT(7)-ARY2(7))
      ARY2(1)=VIN(1)-VOUT(1)
      VOUT(2)=VIN(2)
      ARY2(2)=VIN(2)
      R=VOUT(7)/VIN(7)
      ZSTAR=ARY2(7)/VIN(7)
      DO 10 I=3,6
      VOUT(I)=VIN(I)*R
      ARY2(I)=VIN(I)*ZSTAR
10    CONTINUE
C
      AG=VOUT(1)*VOUT(7)/SLGT
C
      PSR=VOUT(1)*VOUT(7)/(VIN(1)*VIN(7))
C
      IF(LPRINT.LT.1) GOTO 1000
      WRITE(6,100) SLGT*24.,AG,PSR*1.E2
100   FORMAT(//,2X,'**GRAVITY THICKENER DESIGN--',//,
     +'      SOLIDS LOADING  =',F12.5,' KG/DAY/SQ M,',/,
```

```
     +'      SURFACE AREA     =',F12.5,' SQ M,',/,
     +'      SOLIDS RECOVERY =',F12.5,' PERCENT.',/)
      WRITE(6,110)
110   FORMAT(15X,'Q(CU M/HR)',4X,'S(G/CU M)',2X,'MA(KG/CU M)',2X,
     +'MD(KG/CU M)',2X,'MI(KG/CU M)',2X,'MF(KG/CU M)',
     +4X,'MT(KG/CU M)')
      WRITE(6,120) (VIN(I),I=1,7)
      WRITE(6,130) (VOUT(I),I=1,7)
      WRITE(6,140) (ARY2(I),I=1,7)
120   FORMAT(2X,'INFLUENT',3X,F12.5,5(1X,F12.5),1X,F13.5)
130   FORMAT(2X,'UNDERFLOW',2X,F12.5,5(1X,F12.5),1X,F13.5)
140   FORMAT(2X,'SUPERNATANT',F12.5,5(1X,F12.5),1X,F13.5)
C
      IF(B1.NE.1.0) THEN
      X(35)=ARY2(1)
      X(36)=VOUT(1)
      X(37)=VOUT(7)
      X(34)=AG*1.E-2
      X(38)=VOUT(6)
      X(39)=X(28)*(1.E3*X(24))*U(48)/X(30)/X(32)
      X(40)=1.E3*X(12)*(1.E2*X(64))*U(48)/X(30)/X(32)/X(7)
      X(33)=SLGT
      GOTO 1000
      ELSE
      X(27)=ARY2(1)
      X(28)=VOUT(1)
      X(29)=VOUT(7)
      X(26)=AG*1.E-2
      X(30)=VOUT(6)
      X(31)=1.E3*X(19)*U(48)/X(24)
      X(25)=SLGT
      ENDIF
1000  RETURN
      END
C
      SUBROUTINE PAND
      COMMON/STATE/VIN,VOUT,VSIDE,ARY1,ARY2,ITE
      COMMON/PARVAR/U,V,X
      COMMON/CONTRL/LPRINT,B1,B2,B3
      COMMON/DPAND/SRTD,TEMP
      REAL VIN(7),VOUT(7),VSIDE(7),ARY1(7),ARY2(7),
     +U(100),V(20),X(100)
C
      TVS=VIN(3)+VIN(4)+VIN(5)
      RATE=U(53)*EXP(10.*ALOG(10.)/3.*(U(54)-1.E3/(TEMP+273.)))
C
      VOUT(1)=VIN(1)
      VOUT(2)=U(66)
      VOUT(3)=0.0
      VOUT(4)=0.0
      VOUT(5)=TVS/(1.+RATE*SRTD)
      VOUT(6)=VIN(6)
      VOUT(7)=VOUT(3)+VOUT(4)+VOUT(5)+VOUT(6)
C
      VSSOUT=VOUT(3)+VOUT(4)+VOUT(5)
      VSDEST=(TVS-VSSOUT)/TVS
      CH4=V(10)*VIN(1)*(TVS-VSSOUT)+U(29)*U(56)*1.E-
     +3*VIN(1)*VIN(2)
      EVCH4=CH4*3.58E4*8.76/3.6
      Q1=10.22E3*VIN(1)*(TEMP-U(55))
```



```
      Q2=U(59)*VIN(1)*SRTD*U(60)*(TEMP-U(57))*210.24
      Q=(Q1+Q2)/U(58)
      EVNET=(EVCH4-Q)*1.E-6
C
      VDIG=SRTD*VIN(1)*24.0
C
      IF(LPRINT.LT.1) GOTO 1000
      WRITE(6,100)TEMP,SRTD,VDIG,VSDEST*1.E2,CH4*24.,EVNET
100   FORMAT(//,2X,'**ANAEROBIC DIGESTER DESIGN--',//,
     +2X,'(A). PRIMARY TANK--',/,
     +'      TEMPERATURE      =',F12.5,' DEG. C.,',/,
     +'      SLUDGE  AGE      =',F12.5,' DAYS,',/,
     +'          VOLUME       =',F12.5,' CU M,',/,
     +'    VOLATILE SOLIDS',/,
     +'         DESTRUCTION   =',F12.5,' PERCENT.',/,
     +'  METHANE PRODUCTION   =',F12.5,' CU M/DAY',/,
     +'    NET ENERGY VALUE   =',F12.5,' MILLION KWH/YR',/)
      WRITE(6,110)
110   FORMAT(15X,'Q(CU M/HR)',4X,'S(G/CU M)',2X,'MA(KG/CU M)',2X,
     +'MD(KG/CU M)',2X,'MI(KG/CU M)',2X,'MF(KG/CU M)',
     +4X,'MT(KG/CU M)')
      WRITE(6,120) (VIN(I),I=1,7)
      WRITE(6,130) (VOUT(I),I=1,7)
120   FORMAT(2X,'INFLUENT',3X,F12.5,5(1X,F12.5),1X,F13.5)
130   FORMAT(2X,'EFFLUENT',3X,F12.5,5(1X,F12.5),1X,F13.5)
C
      IF(B1.NE.1.0) THEN
      IF(B3.NE.1.) THEN
      X(44)=RATE
      X(45)=Q*1.E-6
       X(46)=TVS
       X(47)=VOUT(5)
       X(41)=TEMP
       X(42)=SRTD
       X(43)=VDIG*1.E-3
       X(48)=CH4
       X(55)=VOUT(5)/VOUT(7)
       X(49)=EVNET
      GOTO 1000
      ELSE
      X(33)=RATE
      X(34)=Q*1.E-6
       X(35)=TVS
       X(36)=VOUT(5)
       X(30)=TEMP
       X(31)=SRTD
       X(32)=VDIG*1.E-3
       X(37)=CH4
       X(44)=VOUT(5)/VOUT(7)
       X(38)=EVNET
      GOTO 1000
      ENDIF
      ELSE
      X(35)=RATE
      X(36)=Q*1.E-6
       X(37)=TVS
       X(38)=VOUT(5)
       X(32)=TEMP
       X(33)=SRTD
       X(34)=VDIG*1.E-3
```

```
        X(39)=CH4
        X(46)=VOUT(5)/VOUT(7)
        X(40)=EVNET
       ENDIF
1000   RETURN
       END
C
       SUBROUTINE SAND
       COMMON/STATE/VIN,VOUT,VSIDE,ARY1,ARY2,ITE
       COMMON/PARVAR/U,V,X
       COMMON/CONTRL/LPRINT,B1,B2,B3
       COMMON/DSAND/SLSAND,IUNIT4
       REAL VIN(7),VOUT(7),VSIDE(7),ARY1(7),ARY2(7),
      +U(100),V(20),X(100)
C
       IF(IUNIT4.EQ.0.AND.ITE.EQ.1) SLSAND=SLSAND/24.0
       VOUT(7)=SLSAND*(((U(67)*U(68)*V(13))**(1.0/
      +U(69))*U(69)/V(13))
      +         /SLSAND)**(U(69)/V(13))
C
       IF(VOUT(7).LT.VIN(7)) THEN
         SLSAND=V(11)**U(69)/VIN(7)**V(13)
         PRINT *,'SECONDARY DIGESTER DESIGN INFEASIBLE --
      + INFLUENT SOLIDS CONCENTRATION GREATER THAN
      + UNDERFLOW SOLIDS CONCENTRATION FOR THE
      +SOLIDS LOADING SPECIFIED. ---- CHANGE DEWATERING SCHEME,
      +OR USE LOADING LESS THAN ',SLSAND,'  KG/SQ M/HR'
         STOP
       ENDIF
C
       VSIDE(7)=U(70)
       VOUT(1)=(VIN(1)*VIN(7)-VIN(1)*VSIDE(7))/(VOUT(7)-VSIDE(7))
       VSIDE(1)=VIN(1)-VOUT(1)
       VOUT(2)=VIN(2)
       VSIDE(2)=VIN(2)
       DO 20 I=3,6
       VOUT(I)=VIN(I)*VOUT(7)/VIN(7)
       VSIDE(I)=VIN(I)*VSIDE(7)/VIN(7)
20     CONTINUE
C
       ADIG=VOUT(1)*VOUT(7)/SLSAND
C
       PSSR=VOUT(1)*VOUT(7)/(VIN(1)*VIN(7))
C
       IF(LPRINT.LT.1) GOTO 1000
       WRITE(6,100)SLSAND*24.,ADIG,PSSR*1.E2
100    FORMAT(/,2X,'(B). SECONDARY TANK--',//,
      +'       SOLIDS LOADING =',F12.5,' KG/DAY/SQ M,',/,
      +'       SURFACE  AREA  =',F12.5,' SQ M,',/,
      +'       SOLIDS CAPTURE =',F12.5,' PERCENT.',/)
       WRITE(6,110)
110    FORMAT(15X,'Q(CU M/HR)',4X,'S(G/CU M)',2X,'MA(KG/CU M)',2X,
      +'MD(KG/CU M)',2X,'MI(KG/CU M)',2X,
      +'MF(KG/CU M)',4X,'MT(KG/CU M)')
       WRITE(6,120) (VIN(I),I=1,7)
       WRITE(6,130) (VOUT(I),I=1,7)
       WRITE(6,140) (VSIDE(I),I=1,7)
120    FORMAT(2X,'INFLUENT',3X,F12.5,5(1X,F12.5),1X,F13.5)
130    FORMAT(2X,'UNDERFLOW',2X,F12.5,5(1X,F12.5),1X,F13.5)
140    FORMAT(2X,'SUPERNATANT',F12.5,5(1X,F12.5),1X,F13.5)
```



```
C
       IF(B1.NE.1.0)THEN
       IF(B3.NE.1.) THEN
       X(52)=VSIDE(1)
       X(53)=VOUT(1)
       X(51)=ADIG*1.E-2
       X(50)=SLSAND
       X(54)=VOUT(7)
       GOTO 1000
       ELSE
       X(41)=VSIDE(1)
       X(42)=VOUT(1)
       X(40)=ADIG*1.E-2
       X(39)=SLSAND
       X(43)=VOUT(7)
       GOTO 1000
       ENDIF
       ELSE
       X(43)=VSIDE(1)
       X(44)=VOUT(1)
       X(42)=ADIG*1.E-2
       X(41)=SLSAND
       X(45)=VOUT(7)
       ENDIF
1000   RETURN
       END
C
       SUBROUTINE VF
       COMMON/STATE/VIN,VOUT,VSIDE,ARY1,ARY2,ITE
       COMMON/PARVAR/U,V,X
       COMMON/CONTRL/LPRINT,B1,B2,B3
       COMMON/DVF/YIELD
       REAL VIN(7),VOUT(7),VSIDE(7),ARY1(7),ARY2(7),
      +U(100),V(20),X(100)
C
C
       W=YIELD*YIELD/U(73)/U(73)/(U(74)*U(75)/
      +(U(76)*U(78)*U(77)))
C
       VSIDE(7)=U(79)
       VSIDE(1)=VIN(1)*VIN(7)/(W+U(79))
       VOUT(1)=VIN(1)-VSIDE(1)
       IF(VOUT(1).LE.0.0) THEN
       WW=VIN(7)-U(79)
         YIELD=SQRT(WW*U(73)*U(73)*U(74)*U(75)/U(76)/U(77)/U(78))
         PRINT *,'VACUUM FILTER DESIGN INFEASIBLE--
      +INFLUENT FLOW RATE LESS THAN UNDERFLOW FLOW RATE FOR
      + THE FILTER YIELD SPECIFIED ----USE
      +FILTER YIELD GREATER THAN ',YIELD,' KG/SQ M/HR'
         STOP
       ENDIF
C
       VOUT(7)=(VIN(1)*VIN(7)-VSIDE(1)*VSIDE(7))/VOUT(1)
       PSR=VOUT(1)*VOUT(7)/(VIN(1)*VIN(7))
       AV=VOUT(1)*VOUT(7)/YIELD
       VOUT(2)=VIN(2)
       VSIDE(2)=VIN(2)
       R=VOUT(7)/VIN(7)
       RATIO=VSIDE(7)/VIN(7)
       DO 10 I=3,6
```

```
      VOUT(I)=VIN(I)*R
      VSIDE(I)=VIN(I)*RATIO
10     CONTINUE

      IF(LPRINT.LT.1) GOTO 1000
      WRITE(6,100) YIELD,AV,PSR*1.E2
100   FORMAT(//,2X,'**VACUUM FILTER DESIGN--',//,
     +'      FILTER YIELD     =',F12.5,' KG/SQ M/HR,',/,
     +'      SURFACE AREA     =',F12.5,' SQ M,',/,
     +'      SOLIDS RECOVERY =',F12.5,' PERCENT.',/)
      WRITE(6,110)
110   FORMAT(15X,'Q(CU M/HR)',4X,'S(G/CU M)',2X,'MA(KG/CU M)',2X,
     +'MD(KG/CU M)',2X,'MI(KG/CU M)',2X,
     +'MF(KG/CU M)',4X,'MT(KG/CU M)')
      WRITE(6,120) (VIN(I),I=1,7)
      WRITE(6,130) (VOUT(I),I=1,7)
      WRITE(6,140) (VSIDE(I),I=1,7)
120   FORMAT(2X,'INFLUENT',3X,F12.5,5(1X,F12.5),1X,F13.5)
130   FORMAT(2X,'UNDERFLOW',2X,F12.5,5(1X,F12.5),1X,F13.5)
140   FORMAT(2X,'SUPERNATANT',F12.5,5(1X,F12.5),1X,F13.5,/////)
C
      IF(B1.NE.1.0)THEN
      IF(B3.NE.1.) THEN
      X(58)=VSIDE(1)
      X(59)=VOUT(1)
      X(60)=VOUT(7)
      X(57)=AV
      X(56)=YIELD
      GOTO 1000
      ELSE
      X(47)=VSIDE(1)
      X(48)=VOUT(1)
      X(49)=VOUT(7)
      X(46)=AV
      X(45)=YIELD
      GOTO 1000
      ENDIF
      ELSE
      X(49)=VSIDE(1)
      X(50)=VOUT(1)
      X(51)=VOUT(7)
      X(48)=AV
      X(47)=YIELD
      ENDIF
1000  RETURN
      END
C
      SUBROUTINE BRANCH(F,A,B)
      REAL A(7),B(7)
C
      B(1)=F*A(1)
      A(1)=A(1)-B(1)
      DO 1 I=2,7
      B(I)=A(I)
1     CONTINUE
C
      RETURN
      END
C
```



```
C
C
      SUBROUTINE MB(A,B)
      REAL A(7),B(7)
C
      D=A(1)+B(1)
      DO 1 I=2,6
      A(I)=(A(1)*A(I)+B(1)*B(I))/D
1     CONTINUE
      A(7)=A(3)+A(4)+A(5)+A(6)
      A(1)=D
C
      RETURN
      END
C
C
C
      SUBROUTINE COPY(A,B)
      REAL A(7),B(7)
C
      DO 1 I=1,7
      B(I)=A(I)
1     CONTINUE
C
      RETURN
      END
C
      SUBROUTINE OBJ
      COMMON/PARVAR/U,V,X
      COMMON/CONTRL/LPRINT,B1,B2,B3
      REAL U(100),V(20),X(100)
      REAL NETBEN
C
      WRITE(9,81)
81    FORMAT(/////,2X,'SUMMARY OF COST ESTIMATES:',///)
      WRITE(9,98)
98    FORMAT(126('*'),//)
      WRITE(9,10000)
10000 FORMAT(37X,'CAPITAL',10X,'OPERATION',8X,'MAINTENANCE',
     1       9X,'MATERIAL',13X,'POWER')
      WRITE(9,10001)
10001 FORMAT(34X,'DOLLARS/YEAR',5X,4('DOLLARS/YEAR',7X),//)
      WRITE(9,98)
C
      IF(B1.NE.1.0) THEN
      APST=1.E2*X(9)
      PSF=X(12)
      VAT=1.E3*X(15)
      AFR=60.*X(27)
      AFST=1.E2*X(25)
      Q5=60.*X(11)*X(22)
      IF(B3.NE.1.) THEN
      AGT=1.E2*X(34)
      VPAND=1.E3*X(43)
      VSAND=1.E2*U(71)*X(51)
      Q16X16=X(59)*X(60)
      AVF=X(57)
      SFR=X(59)
      QCYCLE=X(35)+X(52)+X(58)
      X49=X(49)
```

```
      ELSE
      AGT=0.0
      VPAND=1.E3*X(32)
      VSAND=1.E2*U(71)*X(40)
      Q16X16=X(48)*X(49)
      AVF=X(46)
      SFR=X(48)
      QCYCLE=X(41)+X(47)
      X49=X(38)
      ENDIF
      ELSE
      APST=0.0
      PSF=0.0
      VAT=1.E3*X(10)
      AFR=60.*X(22)
      AFST=1.E2*X(20)
      Q5=60.*X(1)*X(17)
      AGT=1.E2*X(26)
      VPAND=1.E3*X(34)
      VSAND=1.E2*U(71)*X(42)
      Q16X16=X(50)*X(51)
      AVF=X(48)
      SFR=X(50)
      QCYCLE=X(27)+X(43)+X(49)
      X49=X(40)
      ENDIF
C
C PRIMARY SETTLING TANK:
C
      CCPST=824.*APST**.77
      IF(APST.GE.279.) THEN
      COPST=17.15*APST**0.6
      CMPST=9.23*APST**0.6
      ELSE
      COPST=92.45*APST**.3
      CMPST=106.*APST**.14
      ENDIF
      CSPST=8.62*APST**.76
C
C PRIMARY SLUDGE PUMPING:
C
      CCPSP=16042.*PSF**.53
      COPSP=374.*PSF**.41
      CMPSP=166.*PSF**.43
      CSPSP=385.*PSF**.64
      CPPSP=PSF
C
C AERATION TANK:
C
      CCAT=461.*VAT**.71
      CCDAA=8533.*AFR**.66
      CODAA=187.*AFR**.48
      CMDAA=74.4*AFR**.55
C
C FINAL SETTLING TANK:
C
      CCFST=824.*AFST**.77
      IF(AFST.GE.279.) THEN
      COFST=17.15*AFST**0.6
      CMFST=9.23*AFST**0.6
```



```
      ELSE
      COFST=92.45*AFST**.3
      CMFST=106.*AFST**.14
      ENDIF
      CSFST=8.62*AFST**.76
C
C RETURN SLUDGE PUMPING:
C
      CCRSP=2779.*Q5**.53
      CORSP=.333*Q5
      CMRSP=.2375*Q5
      IF(Q5.LT.158.) THEN
      CSRSP=300.
      ELSE
      IF(Q5.LT.631.) THEN
      CSRSP=40.57*Q5**.52
      ELSE
      IF(Q5.LT.1580.) THEN
      CSRSP=5.97*Q5**.87
      ELSE
      CSRSP=2.540*Q5
      ENDIF
      ENDIF
      ENDIF
      CPRSP=Q5
C
C GRAVITY THICKENER:
C
      CCGT=824.*AGT**.77
      IF(AGT.GE.279.) THEN
      COGT=17.15*AGT**0.6
      CMGT=9.23*AGT**0.6
      ELSE
      COGT=92.45*AGT**.3
      CMGT=106.*AGT**.14
      ENDIF
      CSGT=8.62*AGT**.76
C
C PRIMARY DIGESTER:
C
      CCPAND=2323.*VPAND**.59
      IF(VPAND.GE.5678.) THEN
      COPAND=1.29*VPAND**.83
      CMPAND=.83*VPAND**.82
      ELSE
      IF(VPAND.GE.1968.) THEN
      COPAND=14.*VPAND**.55
      CMPAND=8.5*VPAND**.55
      ELSE
      COPAND=192.*VPAND**.2
      CMPAND=113.*VPAND**.21
      ENDIF
      ENDIF
      IF(VPAND.GE.2839.) THEN
      CSPAND=14.4*VPAND**.66
      ELSE
      CSPAND=142.*VPAND**.37
      ENDIF
C
C SECONDARY DIGESTER:
```

```
C
      CCSAND=2323.*VSAND**.59
      IF(VSAND.GE.5678.) THEN
      COSAND=1.29*VSAND**.83
      CMSAND=.83*VSAND**.82
      ELSE
      IF(VSAND.GE.1968.) THEN
      COSAND=14.*VSAND**.55
      CMSAND=8.5*VSAND**.55
      ELSE
      COSAND=192.*VSAND**.2
      CMSAND=113.*VSAND**.21
      ENDIF
      ENDIF
      IF(VSAND.GE.2839.) THEN
      CSSAND=14.4*VSAND**.66
      ELSE
      CSSAND=142.*VSAND**.37
      ENDIF
C
C VACUUM FILTER:
C
      CCVF=29180.*AVF**.71
      CSVF=230.*Q16X16**.71+182.*Q16X16**.86
      COVF=197.55*Q16X16**.58
      IF(Q16X16.GE.519.) THEN
      CMVF=5.57*Q16X16**.84
      ELSE
      IF(Q16X16.GE.103.) THEN
      CMVF=20.*Q16X16**.63
      ELSE
      CMVF=41.5*Q16X16**.48
      ENDIF
      ENDIF
C
C RECIRCULATION PUMPING:
C
      CCRP=2779.*QCYCLE**.53
      CORP=.333*QCYCLE
      CMRP=.2375*QCYCLE
      IF(QCYCLE.LT.158.) THEN
      CSRP=300.
      ELSE
      IF(QCYCLE.LT.631.) THEN
      CSRP=40.57*QCYCLE**.52
      ELSE
      IF(QCYCLE.LT.1580.) THEN
      CSRP=5.97*QCYCLE**.87
      ELSE
      CSRP=2.540*QCYCLE
      ENDIF
      ENDIF
      ENDIF
      CPRP=QCYCLE
C
      CCPST=CCPST*V(1)
      CCPSP=CCPSP*V(1)
      CCAT=CCAT*V(1)
      CCDAA=CCDAA*V(1)
      CCFST=CCFST*V(1)
```



```
      CCRSP=CCRSP*V(1)
      CCRP=CCRP*V(1)
      CCGT=CCGT*V(1)
      CCPAND=CCPAND*V(1)
      CCSAND=CCSAND*V(1)
      CCVF=CCVF*V(1)
      CCSD=V(1)*72053.*SFR**.74+U(5)*3.62E-2*Q16X16
     +     *U(1)
      COPST=COPST*U(4)
      CMPST=CMPST*U(4)
      COPSP=COPSP*U(4)
      CMPSP=CMPSP*U(4)
      CODAA=CODAA*U(4)
      CMDAA=CMDAA*U(4)
      COFST=COFST*U(4)
      CMFST=CMFST*U(4)
      CORSP=CORSP*U(4)
      CMRSP=CMRSP*U(4)
      CORP=CORP*U(4)
      CMRP=CMRP*U(4)
      COGT=COGT*U(4)
      CMGT=CMGT*U(4)
      COPAND=COPAND*U(4)
      CMPAND=CMPAND*U(4)
      COSAND=COSAND*U(4)
      CMSAND=CMSAND*U(4)
      COVF=COVF*U(4)
      CMVF=CMVF*U(4)
      COMSD=U(4)*8024.*SFR**.667
      CSPST=CSPST*V(2)
      CSPSP=CSPSP*V(2)
      CSFST=CSFST*V(2)
      CSRSP=CSRSP*V(2)
      CSRP=CSRP*V(2)
      CSGT=CSGT*V(2)
      CSPAND=CSPAND*V(2)
      CSSAND=CSSAND*V(2)
      CSVF=CSVF*V(2)
      CPPSP=CPPSP*V(3)
      CPRSP=CPRSP*V(3)
      CPRP=CPRP*V(3)
      NETBEN=U(61)*3.6E3/1.055*X49
C
      TOTAL=CCPST+CCPSP+CCAT+CCDAA+CCFST+CCRSP+CCRP
     +     +CCGT+CCPAND+CCSAND+CCVF+CCSD
     +     +COPST+CMPST+COPSP+CMPSP+CODAA+CMDAA+COFST
     +     +CMFST+CORSP+CMRSP+COGT+CMGT
     +     +COPAND+CMPAND+COSAND+CMSAND
     +     +COVF+CMVF+COMSD+CORP+CMRP
     +     +CSPST+CSPSP+CSFST+CSRSP+CSGT+CSPAND+CSSAND+CSRP
     +     +CSVF
     +     +CPPSP+CPRSP+CPRP
     +     -NETBEN
C
      WRITE(9,1000)CCPST,COPST,CMPST,CSPST
      WRITE(9,1001)CCPSP,COPSP,CMPSP,CSPSP,CPPSP
      WRITE(9,1002)CCAT
      WRITE(9,1003)CCDAA,CODAA,CMDAA
      WRITE(9,1004)CCFST,COFST,CMFST,CSFST
      WRITE(9,1005)CCRSP,CORSP,CMRSP,CSRSP,CPRSP
```

```
      WRITE(9,1006)CCGT,COGT,CMGT,CSGT
      WRITE(9,1007)CCPAND,COPAND,CMPAND,CSPAND
      WRITE(9,1008)CCSAND,COSAND,CMSAND,CSSAND
      WRITE(9,1009)CCVF,COVF,CMVF,CSVF
      WRITE(9,1013)CCRP,CORP,CMRP,CSRP,CPRP
      WRITE(9,1010)CCSD,COMSD
      WRITE(9,1011)NETBEN
      WRITE(9,98)
      WRITE(9,1012)TOTAL
C
1000  FORMAT(5X,'PRIMARY SETTLING TANK',10X,F8.0,3(10X,F8.0),/)
1001  FORMAT(5X,'PRIMARY SLUDGE PUMPING',9X,F8.0,4(10X,F8.0),/)
1002  FORMAT(5X,'AERATION TANK',18X,F8.0,/)
1003  FORMAT(5X,'DIFFUSED AIR AERATION',10X,F8.0,2(10X,F8.0),/)
1004  FORMAT(5X,'SECONDARY SETTLING TANK',8X,F8.0,3(10X,F8.0),/)
1005  FORMAT(5X,'RECYCLE SLUDGE PUMPING',9X,F8.0,4(10X,F8.0),/)
1006  FORMAT(5X,'GRAVITY THICKENER',14X,F8.0,3(10X,F8.0),/)
1007  FORMAT(5X,'PRIMARY ANAEROBIC DIGESTER',5X,F8.0,
     +3(10X,F8.0),/)
1008  FORMAT(5X,'SECONDARY ANAEROBIC DIGESTER',
     +3X,F8.0,3(10X,F8.0),/)
1009  FORMAT(5X,'VACUUM FILTER',18X,F8.0,3(10X,F8.0),/)
1013  FORMAT(5X,'RECYCLE STREAM PUMPING',9X,
     +F8.0,4(10X,F8.0),/)
1010  FORMAT(5X,'FINAL SLUDGE DISPOSAL',10X,F8.0,10X,F8.0,/)
1011  FORMAT(//,5X,'WORTH OF NET ENERGY FROM METHANE
     += ',F8.0,' DOLLAR
     +S/YEAR.',//)
1012  FORMAT(//,20X,'TOTAL SYSTEM COST
     += ',F8.0,' DOLLARS/YEAR.')
C
      RETURN
      END
```

APPENDIX D

GRG MODEL FOR BASE SYSTEM DESIGN OPTIMIZATION

The GRG optimization model for the base treatment system (Figure 2.1) has 64 variables and 59 functions (constraints plus objective function). The file containing all functions in the model is listed as GCOMP8 on the following pages. A list of the variables in the model is provided in Table D.1. The reader is refered to Chapter 2 for the notation used in this table.

Table D.1 - Summary of Model Variables : Base System

Variable Index	Unit	Meaning
1	m^3/min	$Q_1/60$
2	g/m^3	S_1
3	g/m^3	M_{a1}
4	g/m^3	M_{d1}
5	g/m^3	M_{i1}
6	g/m^3	M_{f1}
7	g/m^3	M_{t1}
8	m^3/hr	L_p
9	$100m^2$	$A_p/100$
10	--	M_{t2}/M_{t1}
11	m^3/min	$Q_2/60$
12	m^3/hr	Q_8
13	days	θ_c
14	days	θ
15	$1000m^3$	$V/1000$
16	kg/m^3	M_{a3}
17	g/m^3	S_3
18	--	M_{i3}/M_{a3}
19	--	M_{f3}/M_{a3}
20	--	r
21	--	$100w$
22	--	$r+w$
23	g/m^3	M_{a4}
24	kg/m^3	M_{a5}
25	$100m^2$	$A_f/100$
26	g/m^3	S
27	m^3/sec	$Q_a/60$

Table D.1 (continued)

Variable Index	Unit	Meaning
28	m^3/hr	Q_7
29	kg/m^3	M_{t7}
30	m^3/hr	Q_9
31	g/m^3	S_9
32	kg/m^3	M_{t9}
33	$kg/m^2/hr$	L_g
34	$100m^2$	$A_g/100$
35	m^3/hr	Q_{10}
36	m^3/hr	Q_{11}
37	kg/m^3	M_{t11}
38	kg/m^3	M_{f11}
39	kg/m^3	$Q_7 M_{a5} M_{t10}/Q_9/M_{t9}$
40	--	$1000\, Q_8 M_{t8} M_{t10}/Q_9/M_{t9}/M_{t1}$
41	°C	T_d
42	days	θ_d
43	$1000m^3$	$V_d/1000$
44	day^{-1}	K_1
45	$10^6 kWhr/year$	q
46	kg/m^3	$M_{a11} + M_{d11} + M_{i11}$
47	kg/m^3	M_{t12}
48	m^3/hr	G
49	$10^6 kWhr/year$	N
50	$kg/m^2/hr$	L_d
51	$100m^2$	$A_d/100$
52	m^3/hr	Q_{13}
53	m^3/hr	Q_{14}
54	kg/m^3	M_{t14}
55	--	M_{i12}/M_{t12}
56	$kg/m^2/hr$	L_f
57	m^2	A_v
58	m^3/hr	Q_{15}
59	m^3/hr	Q_{16}
60	kg/m^3	M_{t16}
61	--	f_p
62	--	a_c
63	--	n_c
64	$100kg/m^3$	$M_{t8}/100$

The constraints in the GRG model and their corresponding equation numbers (see Chapter 2) are summarized in Table D.2. It is convenient to define some "secondary variables" in constructing the GRG model to avoid repetitive computation. As shown in Table D.2, several secondary variables (variable No.29, 38, 39, 40, 46, and 55) are defined by the constraints which are not described but are derived from design equations presented in Chapter 2.

Table D.2 - Description of the Constraints in the GRG Model

Constraint No. in GRG Model	Corresponding Equation No. from Chapter 2
1	2.3
2	2.4
3	2.11
4	2.12
5	2.17
6	2.18
7	2.16
8	2.20
9	2.21
10	2.22
11	2.23
12	2.33
13	2.34
14	2.28
15	2.24 & 2.25
16	2.15
17	2.19
18	2.35
19	2.36
20	2.37
21	definition of X(29)*
22	2.42
23	2.44
24	2.45
25	2.41
26	definition of X(39)*
27	definition of X(40)*
28	definition of X(38)*
29	2.48
30	definition of X(46)*
31	2.58
32	2.49
33	2.52
34	2.54 - 2.56
35	2.53 & 2.57
36	2.60
37	2.59
38	2.58
39	2.61

Table D.2 (continued)

Constraint No. in GRG Model	Corresponding Equation No. from Chapter 2
40	definition of X(55)*
41	2.67
42	2.68
43	2.64 & 2.65
44	2.66
45	2.71
46	2.72
47	2.73
48	2.74
49	2.75
50	2.76
51	2.77
52	2.40
53	2.38
54	2.39
55	2.10
56	2.29
57	2.30
58	2.26

* : X denotes the variables in the GRG model. See Table D.1.

The initial solution to the model and the control parameters for the optimization (see Section 3.2.1) are specified in another input data file. An example is of this file is also listed in this Appendix under the name GRGDATA. The user's manual for GRG2 should be consulted for the details.

```
      PROGRAM MAIN(INPUT,PAR,OUTPUT,LASTVAR,TAPE7,TAPE8,
     +             TAPE5=INPUT,TAPE6=OUTPUT,TAPE4=LASTVAR,
     +             TAPE9=PAR)
      DIMENSION Z(20000)
      COMMON Z
      DATA NCORE/20000/
      CALL GRG(Z,NCORE)
      END
C
C SUBROUTINE GCOMP:
C
      SUBROUTINE GCOMP(G,X)
      COMMON/INITBK/INIT
      DIMENSION G(59),X(64)
      REAL INFLOW(6),STD(2),U(80),V(20)
      IF(INIT.EQ.0) GOTO 50
      READ(9,*)  (INFLOW(I),I=1,6)
      READ(9,*)  (STD(I),I=1,2)
      READ(9,*)  (U(I),I=1,80)
      V(1)=U(1)*U(3)/U(2)
      V(2)=U(3)/U(2)
      V(3)=23.85*U(6)*U(7)/U(8)
      V(4)=U(17)+U(18)
      V(5)=U(20)*EXP(U(21))
      V(6)=(V(4)*(V(5)-1.0))**(1./V(5))*V(5)/(V(5)-1.0)
      V(7)=(U(17)*(U(20)-1.0))**(1./U(20))*U(20)/(U(20)-1.0)
      V(8)=U(28)/U(29)
      V(9)=U(39)/1440./U(36)/(U(37)*U(39)-U(38))/U(41)
     +     /U(42)/U(43)/U(44)**(U(40)-20.0)
      V(10)=U(28)*U(56)
      V(11)=(U(67)*U(68)*(U(69)-1.0))**(1./U(69))*U(69
     +      )/(U(69)-1.0)
      V(12)=U(73)*(U(74)*U(75)/(U(76)*U(77)*U(78)))**0.5
      V(13)=U(69)-1.
C
50    EPBND=1.E-6
      DO 100 I=1,64
      IF(X(I).LT.EPBND)  THEN
        DO 200 J=1,59
        G(J)=1.E30
200     CONTINUE
        RETURN
      ENDIF
100   CONTINUE
C
C PRIMARY SETTLING TANK DESIGN:
C
      G(1)=1.0-U(11)*X(7)**U(12)/X(8)**U(13)-X(10)
      G(2)=(60.*X(11))/(1.E2*X(9))-X(8)
      G(3)=(60.*X(11))+X(12)-(60.*X(1))
      G(4)=1.E-5*X(7)*X(10)*60.*X(11)+X(64)*X(12)-1.E
     +     -5*60.*X(1)*X(7)
C
C ACTIVATED SLUDGE DESIGN:
C
      G(5)=X(13)-(1.E3*X(16))*X(14)/((1.-(1.E-2*X(21)))*X(23)+
     +     (1.E-2*X(21))*(1.E3*X(2444))-X(3)*
     +     X(10))
```



```
      G(6)=X(14)-(1.E3*X(15))/(60.*X(11))/24.
      G(7)=X(17)-U(24)*(1.+U(26)*X(13))/(X(13)*(U(23)*U(25)
     +      -U(26))-1.)
      G(8)=X(14)*(1.E3*X(16))/X(13)-U(23)*X(26)/(1.+U(26)*
     +      X(13))
      G(9)=(1.E3*X(16))*X(18)+X(18)*X(13)*X(3)*X(10)/X(14)-
     +      X(5)*X(10)
     +    *X(13)/X(14)-(1.-U(27))*U(26)*(1.E3*X(16))*X(13)
      G(10)=X(19)*(1.E3*X(16))+X(19)*X(13)*X(3)*X(10)/X(14)
     +      -X(6)*X(10)*X(13)/X(14)
      G(11)=X(24)+U(80)*X(3)*X(10)/X(20)-(1.+1./X(20)-X(14)
     +     /X(20)/X(13))*X(16)
      G(12)=(1.+X(18)+X(19))*X(24)-V(7)/
     +      ((60.*X(11))/(1.E2*X(25))*X(22))**(1./U(20))
      G(13)=X(22)-X(20)-(1.E-2*X(21))
      G(14)=(1.+X(18)+X(19))*X(23)+U(30)-U(31)*(1.+X(18)+X(19))
     +    *(1.E3*X(16))-U(32)*(60.*X(11))/(1.E2*X(25))*(1.+X(20))
      G(15)=60.*X(27)-24.E-3*U(29)*V(9)*60.*X(11)*X(26)
     +      +U(80)*U(28)*V(9)*
     +     (1.E3*X(15))*(1.E3*X(16))/X(13)
      G(16)=X(26)+X(17)-X(2)-V(8)*X(4)*X(10)
C
C SLUDGE MIXING:
C
      G(17)=X(28)-(60.*X(11))*(1.E-2*X(21))
      G(18)=X(30)-X(12)-X(28)
      G(19)=X(28)*X(29)+(1.E2*X(64))*X(12)-X(30)*X(32)
      G(20)=X(30)*X(31)-X(17)*X(28)-X(2)*X(12)
      G(21)=U(80)*(1.+X(18)+X(19))*(1.E3*X(24))-X(29)
C
C GRAVITY THICKENER DESIGN:
C
      G(22)=X(33)-X(36)*X(37)/(1.E2*X(34))
      G(23)=X(35)+X(36)-X(30)
      G(24)=U(48)*X(35)+X(36)*X(37)-X(30)*X(32)
      G(25)=X(37)-(X(62)*(X(63)-1.))**(1./X(63))*X(63)/(X(63)-1.)/
     +     (X(36)/(1.E2*X(34)))**(1./X(63))
      G(26)=U(48)*(1.E3*X(24))*X(28)/X(30)/X(32)-X(39)
      G(27)=1.E3*(1.E2*X(64))*U(48)*X(12)/X(7)/X(30)/X(32)-X(40)
      G(28)=U(80)/U(48)*X(19)*X(37)*X(39)+U(80)/U(48)*X(6)*
     +      X(37)*X(40)-X(38)
C
C PRIMARY DIGESTER DESIGN:
C
      G(29)=X(44)-U(53)*EXP(ALOG(10.)*10./3.*(U(54)-
     +       1.E3/(X(41)+273.)))
      G(30)=X(38)+X(46)-X(37)
      G(31)=X(42)-(1.E3*X(43))/X(36)/24.
      G(32)=X(47)-X(46)/(1.+X(44)*X(42))
      G(33)=X(48)-U(56)*X(36)*(U(28)*(X(46)-X(47))
     +      +U(29)*1.E-3*X(31))
      G(34)=X(45)-(10.22*1.E-3*X(36)*(X(41)-U(55))
     +      +U(59)*(1.E3*X(43))*U(60)
     +      *8.76E-6*(X(41)-U(57)))/U(58)
      G(35)=X(49)-(3.58E4*8.76/3.6*X(48)*1.E-6-X(45))
C
C SECONDARY DIGESTER DESIGN:
C
```

```
      G(36)=X(36)-X(52)-X(53)
      G(37)=X(53)*X(54)/(1.E2*X(51))-X(50)
      G(38)=X(54)-V(11)**(U(69)/V(13))/X(50)**(1./V(13))
      G(39)=X(36)*(X(38)+X(47))-U(70)*X(52)-X(53)*X(54)
      G(40)=X(55)-X(47)/(X(47)+X(38))
C
C VACUUM FILTER DESIGN:
C
      G(41)=X(53)-X(58)-X(59)
      G(42)=X(53)*X(54)-X(58)*U(79)-X(59)*X(60)
      G(43)=X(56)-V(12)*SQRT(X(59)*X(60)/X(58))
      G(44)=X(57)-X(59)*X(60)/X(56)
C
C MASS BALANCE OF RECYCLE STREAMS:
C
      G(45)=(60.*X(1))-INFLOW(1)-X(35)-X(52)-X(58)
      G(46)=U(80)*(60.*X(1))*X(2)-U(80)*INFLOW(1)*INFLOW(2)
     +      -U(80)*X(35)*X(31)
     +     -U(80)*X(52)*U(66)-U(80)*U(66)*X(58)
      G(47)=U(80)*60.*X(1)*X(3)-U(80)*INFLOW(1)*INFLOW(3)
     +      -U(80)*X(35)*X(39)
     +      -U(80)*X(3)*X(35)*X(40)
      G(48)=U(80)*60.*X(1)*X(4)-U(80)*INFLOW(1)*INFLOW(4)
     +      -U(80)*X(4)*X(35)*X(40)
      G(49)=U(80)*60.*X(1)*X(5)-U(80)*INFLOW(1)*INFLOW(5)
     +      -U(80)*X(18)*X(35)*X(39)
     +      -U(80)*X(5)*X(35)*X(40)-U(70)*X(52)*X(55)-X(55)*X(58)
     +     *U(79)
      G(50)=U(80)*60.*X((1)*X(6)-U(80)*INFLOW(1)*INFLOW(6)
     +      -U(80)*X(19)*X(35)*X(39)
     +      -U(80)*X(6)*X(35)*X(40)-U(70)*X(52)*(1.-X(55))
     +      -X(58)*U(79)*(1.-X(55))
      G(51)=X(3)+X(4)+X(5)+X(6)-X(7)
C
C SETTLING CHARACTERISTICS OF COMBINED PRIMARY &
     +  WASTE ACTIVATED SLUDGES:
C
      G(52)=X(61)-(1.E2*X(64))*X(12)/((1.E2*X(64))*X(12)+
     +      X(28)*X(29))
      G(53)=X(62)-U(17)-U(18)*X(61)**U(19)
      G(54)=X(63)-U(20)*EXP(U(21)*X(61))
C
C THICKENING MODEL FOR PRIMARY SEDIMENTATION:
C
      G(55)=X(64)-V(6)*((1.E2*X(9))*U(16)/X(12))**(1./V(5))*1.E-2
C
C EFFLUENT WATER QUALITY STANDARDS:
C
      G(56)=X(17)+U(27)*V(8)*X(23)-STD(1)
      G(57)=X(23)*(1.+X(18)+X(19))-STD(2)
C
C MIXING REQUIREMENT IN AERATION TANK:
C
      G(58)=(U(45)-(60.*X(27))/(1.E3*X(15)))*1.E2
C
C OBJECTIVE FUNCTION:
C
C PRIMARY SETTLING TANK:
```



```
C
      CCPST=824.*(1.E2*X(9))**.77
      IF((1.E2*X(9)).GE.279.) THEN
      COPST=17.15*(1.E2*X(9))**0.6
      CMPST=9.23*(1.E2*X(9))**0.6
      ELSE
      COPST=92.45*(1.E2*X(9))**.3
      CMPST=106.*(1.E2*X(9))**.14
      ENDIF
      CSPST=8.62*(1.E2*X(9))**.76
C
C PRIMARY SLUDGE PUMPING:
C
      CCPSP=16042.*X(12)**.53
      COPSP=374.*X(12)**.41
      CMPSP=166.*X(12)**.43
      CSPSP=385.*X(12)**.64
      CPPSP=X(12)
C
C AERATION:
C
      CCAT=461.*(1.E3*X(15))**.71
      CCDAA=8533.*(60.*X(27))**.66
      CODAA=187.*(60.*X(27))**.48
      CMDAA=74.4*(60.*X(27))**.55
C
C FINAL SETTLING TANK:
C
      CCFST=824.*(1.E2*X(25))**.77
      IF((1.E2*X(25)).GE.279.) THEN
      COFST=17.15*(1.E2*X(25))**0.6
      CMFST=9.23*(1.E2*X(25))**0.6
      ELSE
      COFST=92.45*(1.E2*X(25))**.3
      CMFST=106.*(1.E2*X(25))**.14
      ENDIF
      CSFST=8.62*(1.E2*X(25))**.76
C
C RETURN SLUDGE PUMPING:
C
      Q5=(60.*X(11))*X(22)
      CCRSP=2779.*Q5**.53
      CORSP=.333*Q5
      CMRSP=.2375*Q5
      IF(Q5.LT.158.) THEN
      CSRSP=300.
      ELSE
      IF(Q5.LT.631.) THEN
      CSRSP=40.57*Q5**.52
      ELSE
      IF(Q5.LT.1580.) THEN
      CSRSP=5.97*Q5**.87
      ELSE
      CSRSP=2.54*Q5
      ENDIF
      ENDIF
      ENDIF
      CPRSP=Q5
```

```
C
C RECIRCULATION PUMPING:
C
      QCYCLE=X(35)+X(52)+X(58)
      CCRP=2779.*QCYCLE**.53
      CORP=.333*QCYCLE
      CMRP=.2375*QCYCLE
      IF(QCYCLE.LT.158.) THEN
      CSRP=300.
      ELSE
      IF(QCYCLE.LT.631.) THEN
      CSRP=40.57*QCYCLE**.52
      ELSE
      IF(QCYCLE.LT.1580.) THEN
      CSRP=5.97*QCYCLE**.87
      ELSE
      CSRP=2.540*QCYCLE
      ENDIF
      ENDIF
      ENDIF
      CPRP=QCYCLE
C
C GRAVITY THICKENER:
C
      CCGT=824.*(1.E2*X(34))**.77
      IF((1.E2*X(34)).GE.279.) THEN
      COGT=17.15*(1.E2*X(34))**0.6
      CMGT=9.23*(1.E2*X(34))**0.6
      ELSE
      COGT=92.45*(1.E2*X(34))**.3
      CMGT=106.*(1.E2*X(34))**.14
      ENDIF
      CSGT=8.62*(1.E2*X(34))**.76
C
C PRIMARY DIGESTER:
C
      CCPAND=2323.*(1.E3*X(43))**.59
      IF((1.E3*X(43)).GE.5678.) THEN
      COPAND=1.29*(1.E3*X(43))**.83
      CMPAND=.83*(1.E3*X(43))**.82
      ELSE
      IF((1.E3*X(43)).GE.1968.) THEN
      COPAND=14.*(1.E3*X(43))**.55
      CMPAND=8.5*(1.E3*X(43))**.55
      ELSE
      COPAND=192.*(1.E3*X(43))**.2
      CMPAND=113.*(1.E3*X(43))**.21
      ENDIF
      ENDIF
      IF((1.E3*X(43)).GE.2839.) THEN
      CSPAND=14.4*(1.E3*X(43))**.66
      ELSE
      CSPAND=111142.*(1.E3*X(43))**.37
      ENDIF
C
C SECONDARY DIGESTER:
C
      VSAND=1.E2*U(71)*X(51)
```



```
      CCSAND=2323.*VSAND**.59
      IF(VSAND.GE.5678.) THEN
      COSAND=1.29*VSAND**.83
      CMSAND=.83*VSAND**.82
      ELSE
      IF(VSAND.GE.1968.) THEN
      COSAND=14.*VSAND**.55
      CMSAND=8.5*VSAND**.55
      ELSE
      COSAND=192.*VSAND**.2
      CMSAND=113.*VSAND**.21
      ENDIF
      ENDIF
      IF(VSAND.GE.2839.) THEN
      CSSAND=14.4*VSAND**.66
      ELSE
      CSSAND=142.*VSAND**.37
      ENDIF
C
C VACUUM FILTER:
C
      Q16X16=X(59)*X(60)
      CCVF=29180.*X(57)**.71
      CSVF=230.*Q16X16**.71+182.*Q16X16**.86
      COVF=197.55*Q16X16**.58
      IF(Q16X16.GE.519.) THEN
      CMVF=5.57*Q16X16**.84
      ELSE
      IF(Q16X16.GE.103.) THEN
      CMVF=20.*Q16X16**.63
      ELSE
      CMVF=41.5*Q16X16**.48
      ENDIF
      ENDIF
C
      G(59)=V(1)*(CCPST+CCPSP+CCAT+CCDAA+CCFST+CCRSP+CCRP
     +       +CCGT+CCPAND+CCSAND+CCVF+72053.*X(59)**.74)
     +       +U(4)*(COPST+CMPST+COPSP+CMPSP+CODAA+CMDAA+COFST
     +       +CMFST+CORSP+CMRSP+COGT+CMGT+COPAND+CMPAND+COSAND+CMSAND
     +       +COVF+CMVF+CORP+CMRP+8024.*X(59)**.667)
     +       +V(2)*(CSPST+CSPSP+CSFST+CSRSP+CSGT+CSPAND+CSSAND+CSRP
     +       +CSVF)
     +       +V(3)*(CPPSP+CPRSP+CPRP)
     +       +U(1)*U(5)*(3.62E-2)*X(59)*X(60)-U(61)*
     +       3.6E3/1.055*X(49)
      RETURN
      END
C
C SUBROUTINE REPORT
C
      SUBROUTINE REPORT(G,X,M,N,CON,VAR,XO)
      DIMENSION X(N),G(M),CON(M),VAR(N),XO(N)
      COMMON/INITBK/INIT
      IF(INIT.EQ.1) RETURN
      WRITE(4,10) (I,X(I),I=1,N)
10    FORMAT(3X,I3,4X,E20.10)
      RETURN
      END
```

```
   64   59
NAME EXAMPLE DATA FILE FOR GRG2
BOU
 R   1   7          1.0000E-5        1.0000E+6
 R   8              5.0000E-1        6.0000E+0
 R   9  12          1.0000E-5        1.0000E+6
 R  13              2.0000E+0        6.0000E+0
 R  14              1.0000E-1        5.0000E-1
 R  15  16          1.0000E-5        1.0000E+6
 R  17              1.0000E-5        3.0000E+1
 R  18  19          1.0000E-5        1.0000E+6
 R  20              1.0000E-1        1.0000E+0
 R  21  22          1.0000E-5        1.0000E+6
 R  23              1.0000E-5        3.0000E+1
 R  24  32          1.0000E-5        1.0000E+6
 R  33              5.0000E-1        2.0000E+0
 R  34  40          1.0000E-5        1.0000E+6
 R  41              2.0000E+1        6.0000E+1
 R  42              5.0000E+0        3.0000E+1
 R  43              1.0000E-5        1.0000E+6
 R  44              2.7100E-2        6.3200E-1
 R  45  49          1.0000E-5        1.0000E+6
 R  50              5.0000E-1        2.0000E+0
 R  51  54          1.0000E-5        1.0000E+6
 R  55              1.0000E-5        1.0000E+0
 R  56              5.0000E+0        5.0000E+1
 R  57  59          1.0000E-5        1.0000E+6
 R  60              5.0000E+1        1.5000E+2
 R  61              1.0000E-5        1.0000E+0
 R  62              2.4240E+1        1.9901E+2
 R  63              2.3747E+0        2.8100E+0
 R  64              4.0000E-1        2.0000E+0
END
INI
SEPARATE
     1          .2517629410E+02
     2          .1006992953E+03
     3          .5191348902E+01
     4          .9951173846E+02
     5          .4603023294E+02
     6          .5380996602E+02
     7          .2045432863E+03
     8          .6000000000E+01
     9          .2515001293E+01
    10          .6057846444E+00
    11          .2515001293E+02
    12          .1576870399E+01
    13          .2192800000E+01
    14          .1747300000E+00
    15          .6328024933E+01
    16          .6367755458E+00
    17          .1978924694E+02
    18          .5364729982E+00
    19          .6049371720E+00
    20          .1024300000E+00
    21          .6313827548E+00
    22          .1087438275E+00
    23          .1402380995E+02
    24          .6327395442E+01
    25          .6555106441E+01
    26          .1379776550E+03
    27          .4035911060E+01
```



```
    28          .9527570668E+01
    29          .1354954895E+02
    30          .1110444107E+02
    31          .3127876356E+02
    32          .2261201153E+02
    33          .5000000000E+00
    34          .4994782394E+01
    35          .6773148575E+01
    36          .4331292492E+01
    37          .5765925995E+02
    38          .1574435845E+02
    39          .4801768850E+02
    40          .4750800442E+00
    41          .6000000000E+02
    42          .1299000000E+02
    43          .1350323747E+01
    44          .6319854333E+00
    45          .2361422460E+01
    46          .4191490150E+02
    47          .4551272433E+01
    48          .8050203029E+02
    49          .4651377738E+01
    50          .1550000000E+01
    51          .5669453550E+00
    52          .7445906924E-02
    53          .4323846585E+01
    54          .2032369287E+02
    55          .2242488770E+00
    56          .6857400000E+01
    57          .1170741494E+02
    58          .3797051406E+01
    59          .5267951787E+00
    60          .1523978018E+03
    61          .4858721675E+00
    62          .5294457576E+02
    63          .2574041692E+01
    64          .7736809844E+00
END
ROW
 E   1  55             0.0000E+0
 L  56  58             0.0000E+0
 O  59
END
PRI
IPR      1
END
LIM
NST      3
END
EPS
EPN        1.0000E-4
EPI        1.0000E-2
EPT        1.0000E-4
END
MET
FDC
QUA
CG
CGS
MIN
END
GO
STO
```

APPENDIX E

IGGP MODEL FOR BASE SYSTEM DESIGN

The IGGP model contains 62 variables, 57 constraints, and 90 parameters. Variables No. 1 to 60 are defined the same as in the GRG model (Table D.1 in Appendix D). Variable No. 61 is the total recycle stream flowrate in m^3/hr, or

$$X(61) = Q_{10} + Q_{13} + Q_{15}$$

Variable No. 62 is the primary sludge concentration in 100 kg/m^3 ($M_{t8}/100$).

Parameters in the GGP model are the same as that in the GRG model. Exceptions are listed in Table E.1.

Table E.1 - Parameters that are Unique in the IGGP Model

Parameter Index	Meaning in GGP Model
15	$[a_p(n_p-1)]^{1/n_p}(\frac{n_p}{n_p-1})$
22	$[a_w(n_w-1)]^{1/n_w}(\frac{n_w}{n_w-1})$
35	$\frac{1}{1440}\ \frac{C_s}{\gamma\alpha(\beta C_s-DO)(OTE)\rho_{air}}$ (See equation (2.25))
46	$[a_c(n_c-1)]^{1/n_c}(\frac{n_c}{n_c-1})$
51 - 54	Parameters in equation (3.7)
72	$[\delta a_d(n_d-1)]^{1/n_d}(\frac{n_d}{n_d-1})$
81	$657.3(\frac{\chi P}{\mu r_s t_c})^{1/2}$ (See equation (2.62))
82	Influent flowrate to plant, m^3/hr
83	Influent soluble BOD_5, g/m^3
84	Influent active biomass, g/m^3
85	Influent volatile degradable solids, g/m^3
86	Influent volatile inert solids, g/m^3
87	Influent fixed solids, g/m^3
88	BOD_5 standard, g/m^3
89	Total suspended solids standard, g/m^3
90	Mass fraction of the primary sludge, f_p

If the value of f_p is changed, then parameter No. 46 in the GGP model, which corresponds to this f_p, needs to be calculated using equations (2.38) and (2.39) for a_c and n_c. Exponents in constraint No. 25 which represents thickening of the combined primary and activated sludge also have to be modified since this equation is:

$$L_g M_{t11}^{(n_c-1)} - \left\{ [a_c(n_c-1)]^{\frac{1}{n_c}} \left(\frac{n_c}{n_c-1} \right) \right\}^{n_c} = 0$$

The listing of the IGGP model is on the next few pages.

```
                    IGGP
   INTERACTIVE GENERALIZED GEOMETRIC PROGRAMMING
             84/08/04.  10.33.08.

SESSION TITLE : IGGP MODEL FOR WASTEWATER TREATMENT PLANT DESIGN

 -COMMAND-->  READ
  READING DATA FILE:
 UPDATED MODEL

 -COMMAND-->  CHE OBJ
 -COMMAND-->  L ALL

 OBJECTIVE:0
    +    28571.P1P2^-1.P3X9^ .77   +    368.P4X9^ .3
    +    202.P4X9^ .14    + 285.4P2^-1.P3X9^ .76
    + 16042.P1P2^-1.P3X12^ .53   +    374.P4X12^ .41
    +    166.P4X12^ .43   +    385.P2^-1.P3X12^ .64
    +    23.85P6P7P8^-1.X12   + 62187.P1P2^-1.P3X15^ .71
    +    127257.P1P2^-1.P3X27^ .66   +    1335.P4X2
 7^ .48    +    707.P4X27^ .55   +    28571.P1P2^-1.P3X25^ .77
    +    271.8P4X25 ^ .6    +    146.3P4X25^ .6    +
    285.4P2^-1.P3X25^ .76    +    24339.P1P2^-1.P
 3X11^ .53X22^ .53    +    19.98P4X11X22    +    14.25P4X11X22
    +    341.P2^-1.P 3X11^ .52X22^ .52    +
    1431.P6P7P8^-1.X11X22    +    28571.P1P2^-1.P3X34^ .77
    +    271.8P4X34^ .6    +    146.3P4X34^ .6    +
    285.4P2^-1.P3X34^ .76    + 136788.P1P2^-1.P3X43^ .59
    +    764.P4X43^ .2    +    482.P4X43^ .21    +
  1829.P2^-1.P3X43^ .37    +    35160.P1P2^-1.P3P71^ .59X51^ .59
    +    482.3P4P 71^ .2X51^ .2    +    297.2P4P71^ .21X51^ .21
    +    780.3P2^-1.P3P71^ .37X51^ .37    +
    2779.P1P2^-1.P3X61^ .53    +    29180.P1P2^-1.P3X57^ .71
    +    230.
 P2^-1.P3X59^ .71X60^ .71    +    182.P2^-1.P3X59^ .86X60^ .86
    +    .333P4X61 +    197.55P4X59^ .58X60^ .58    +
    20.P4X59^ .63X60^ .63    +    .362E-01P1
 P5X59X60    +    72053.P1P2^-1.P3X59^ .74    +
    8024.P4X59^ .667    +    .2375P
 4X61    +    23.85P6P7P8^-1.X61    +    722.    -    3412.3P61X49

 -COMMAND-->  CHE ALL

 G(   1): EQUALITY--PST1
    +    1.    -    1.P11X7^ .27X8^-.22    -    1.X10

 G(   2): EQUALITY--PST2
    +    1.    -    .6X8^-1.X9^-1.X11

 G(   3): EQUALITY--PST3
    +    60.X11    +    1.X12    -    60.X1

 G(   4): EQUALITY--PST4
    +    .6E-03X7X10X11    +    1.X12X62    -    .6E-03X1X7

 G(   5): EQUALITY--AS1
```



```
    +    1.    +    10.X21X23^-1.X24    -    .1E-01X21    -
    1.X3X10X23^-1.    - 1000.X13^-1.X14X16X23^-1.

 G(   6): EQUALITY--AS2
    +    1.    -    1.44X11X14X15^-1.

 G(   7): EQUALITY--AS3
    +    1.P23P24^-1.P25X13X17    -    1.P24^-1.P26X13X17    -
         1.P24^-1.X17 -    1.    -    1.P26X13

 G(   8): EQUALITY--AS4
    +    1.    +    1.P26X13    -    .1E-02P23X13X14^-1.X16^-1.X26

 G(   9): EQUALITY--AS5
    +    1.P27    +    1.P26^-1.X13^-1.X18    +    .1E-02P26^-
         1.X3X10X14^-1.X16^- 1.X18    -    1.    -
         .1E-02P26^-1.X5X10X14^-1.X16^-1.

 G(  10): EQUALITY--AS6
    +    1.    -    1.X3X6^-1.X19    -
         1000.X6^-1.X10^-1.X13^-1.X14X16X19

 G(  11): EQUALITY--AS7
    +    1.    +    1.X20    -    1.X13^-1.X14    -
         .1E-02X3X10X16^-1.    -    1 .X16^-1.X20X24

 G(  12): EQUALITY--AS8
    +    1.    +    1.X18    +    1.X19    -    1.24P22X11^-
         .421106X22^-.421106X24 ^-1.X25^ .421106

 G(  13): EQUALITY--AS9
    +    1.    -    1.X20X22^-1.    -    .1E-01X21X22^-1.

 G(  14): EQUALITY--AS10
    +    1.    +    1.X18    +    1.X19    +    1.P30X23^-1.    -
         1000.P31X16X23 ^-1.    -    1000.P31X16X18X23^-1.    -
         1000.P31X16X19X23^-1.    -    .6P32X11 X23^-1.X25^-1.
    -    .6P32X11X20X23^-1.X25^-1.

 G(  15): EQUALITY--AS11
    +    1.    -    41.6667P29^-1.P35^-1.X11^-1.X26^-1.X27    -
         694.444P28P29^ -1.X11^-1.X13^-1.X15X16X26^-1.

 G(  16): EQUALITY--AS12
    +    1.X2^-1.X26    +    1.X2^-1.X17    -    1.    -
         1.P28P29^-1.X2^-1.X4X1 0

 G(  17): EQUALITY--MIX1
    +    1.    -    .6X11X21X28^-1.

 G(  18): EQUALITY--MIX2
    +    1.    -    1.X12X30^-1.    -    1.X28X30^-1.

 G(  19): EQUALITY--MIX3
    +    1.    -    1.X28X29X30^-1.X32^-1.    -    100.X12X30^-
         1.X32^-1.X62
 G(  20): EQUALITY--MIX4
    +    1.    -    1.X17X28X30^-1.X31^-1.    -
         1.X2X12X30^-1.X31^-1.
```

```
G(  21): EQUALITY--MIX5
   +    1.   +    1.X18   +    1.X19   -    1.X24^-1.X29

G(  22): EQUALITY--GT1
   +    1.X33X34   -    .1E-01X36X37

G(  23): EQUALITY--GT2
   +    1.X35   +    1.X36   -    1.X30

G(  24): EQUALITY--GT3
   +    1.P48X35   +    1.X36X37   -    1.X30X32

G(  25): EQUALITY--GT4
   +    1.X33X37^ 1.57459   -    1.P46^ 2.57459

G(  26): EQUALITY--GT5
   +    1.P48X24X28   -    .1E-02X30X32X39

G(  27): EQUALITY--GT6
   +    1.P48X12X62   -    .1E-04X7X30X32X40

G(  28): EQUALITY--GT7
   +    .1E-02X19X37X39   +    .1E-02X6X37X40   -    1.P48X38

G(  29): EQUALITY--PAND1
   +    1.X44   +    1.P52X41   -    1.P51   -
     1.P53X41^ 2.   -    1.P54X41 ^ 3.

G(  30): EQUALITY--PAND2
   +    1.   -    1.X37^-1.X38   -    1.X37^-1.X46

G(  31): EQUALITY--PAND3
   +    1.   -    .24E-01X36X42X43^-1.

G(  32): EQUALITY--PAND4
   +    1.   -    1.X42X44X46^-1.X47   -    1.X46^-1.X47

G(  33): EQUALITY--PAND5
   +    1.   +    1.P28^-1.P56^-1.X36^-1.X47^-1.X48   -
        1.X46X47^-1.   - .1E-02P28^-1.P29X31X47^-1.

G(  34): EQUALITY--PAND6
   +    1.   +    1.X45X49^-1.   -    .871133E-01X48X49^-1.

G(  35): EQUALITY--PAND7
   +    1.P58X45   +    .1022E 01P55X36   +
   .876E-02P57P59P60X43 -.10 22E-01X36X41 -
   .876E-02P59P60X41X43

G(  36): EQUALITY--SAND1
   +    1.X52   +    1.X53   -    1.X36

G(  37): EQUALITY--SAND2
   +    1.X50X51   -    .1E-01X53X54

G(  38): EQUALITY--SAND3
   +    1.X50X54^ 1.9   -    1.P72^ 2.9

G(  39): EQUALITY--SAND4
   +    1.X36X38   +    1.X36X47 -  1.P70X52 -  1.X53X54
```



```
G(  40): EQUALITY--SAND5
   +    1.X38X55   +    1.X47X55   -    1.X47

G(  41): EQUALITY--VF1
   +    1.   -    1.X53^-1.X58   -    1.X53^-1.X59

G(  42): EQUALITY--VF2
   +    1.   -    1.P79X53^-1.X54^-1.X58   -
     1.X53^-1.X54^-1.X59X60

G(  43): EQUALITY--VF3
   +    1.   -    1.P81X56^-1.X58^-.5X59^ .5X60^ .5

G(  44): EQUALITY--VF4
   +    1.   -    1.X56^-1.X57^-1.X59X60

G(  45): EQUALITY--RECYCLE
   +    1.   -    1.X35X61^-1.   -    1.X52X61^-1.
   -    1.X58X61^-1.

G(  46): EQUALITY--MB1
   +    60.X1   -    1.P82   -    1.X61

G(  47): EQUALITY--MB2
   +    1.   -    .166667E-01P82P83X1^-1.X2^-1.   -
     .166667E-01X1^-1.X2^-1.  X31X35   -
    .166667E-01P66X1^-1.X2^-1.X52   -
    .166667E-01P66X1^-1.X2^-1.X 58

G(  48): EQUALITY--MB3
   +    1.   -    .166667E-01P82P84X1^-1.X3^-1.   -
  .166667E-01X1^-1.X3^-1. X35X39 - .166667E-01X1^-1.X35X40

G(  49): EQUALITY--MB4
   +    1.   -    .166667E-01P82P85X1^-1.X4^-1.
   -    .166667E-01X1^-1.X35X40

G(  50): EQUALITY--MB5
   +    1.   -    .166667E-01P82P86X1^-1.X5^-1.   -
     .166667E-01X1^-1.X5^-1.  X18X35X39   -
    .166667E-01X1^-1.X35X40   -    16.6667P70X1^-1.X5^-1.X52X55
   -    16.6667P79X1^-1.X5^-1.X55X58

G(  51): EQUALITY--MB6
   +    1.   +    16.6667P70X1^-1.X6^-1.X52X55   +
    16.6667P79X1^-1.X6^-1.X5 5X58   -
    .166667E-01P82P87X1^-1.X6^-1.   -
    .166667E-01X1^-1.X6^-1.X19X35 X39   -
    .166667E-01X1^-1.X35X40   -    16.6667P70X1^-
    1.X6^-1.X52   -    16.  6667P79X1^-1.X6^-1.X58

G(  52): EQUALITY--MB7
   +    1.   -    1.X3X7^-1.   -    1.X4X7^-1.   -
    1.X5X7^-1.   -    1.X6X7 ^-1.

G(  53): EQUALITY--PS CONC
   +    1.X62   -    .516968E-01P15P16^ .356732X9^
      .356732X12^-.356732
```

```
G( 54): EQUALITY--DEFN OF FP
  +   1.   -   1.P90   -   .1E-01P90X12^-1.X28X29X62^-1.

G( 55): BOD STANDARD
  +   1.X17   +   1.P27P28P29^-1.X23   -   1.P88

G( 56): TSS STANDARD
  +   1.X23   +   1.X18X23   +   1.X19X23   -   1.P89

G( 57): MIXING REQUIREMENT
  +   1.P45   -   .6E-01X15^-1.X27
```

OBJECTIVE VALUE: 675263. =

VARIABLE	LOWER BOUND	OPERATING PT.	PREV. OP. PT.	UPPER BOUND	NAME
X(1)	25.0000	25.1589	25.1508	30.0000	X(1)
X(2)	50.0000	101.500	101.445	200.000	X(2)
X(3)	.100000E-03	5.07200	5.05517	100.000	X(3)
X(4)	.100000E-03	99.4600	99.5166	200.000	X(4)
X(5)	.100000E-03	50.3900	47.1788	100.000	X(5)
X(6)	.100000E-03	53.9000	57.2102	100.000	X(6)
X(7)	.100000E-03	208.800	208.961	300.000	X(7)
X(8)	.500000	4.00000	1.86938	6.00000	X(8)
X(9)	1.00000	3.77000	8.06559	100.000	X(9)
X(10)	.100000	.566600	.487542	1.00000	X(10)
X(11)	20.0000	25.1338	25.1295	30.0000	X(11)
X(12)	.100000E-03	1.50500	1.27950	10.0000	X(12)
X(13)	2.00000	3.00000	2.23616	6.00000	X(13)
X(14)	.100000	.150000	.146565	.500000	X(14)
X(15)	.500000	5.42890	5.30366	50.0000	X(15)
X(16)	.100000	1.00800	.717329	10.0000	X(16)
X(17)	.100000E-03	13.7700	19.3229	30.0000	X(17)
X(18)	.100000E-03	.562000	.484410	2.00000	X(18)
X(19)	.100000E-03	.573200	.563703	2.00000	X(19)
X(20)	.100000	.150000	.119031	1.00000	X(20)
X(21)	.100000E-03	.606600	.551731	5.00000	X(21)
X(22)	.100000E-03	.156100	.124548	1.50000	X(22)
X(23)	.100000E-03	8.60200	14.6476	30.0000	X(23)
X(24)	.100000E-03	7.37300	6.32803	20.0000	X(24)
X(25)	.100000E-03	13.4300	6.75027	100.000	X(25)
X(26)	.100000E-03	141.100	128.053	200.000	X(26)
X(27)	.100000E-03	4.18900	3.74575	10.0000	X(27)
X(28)	.100000E-03	9.14800	8.31883	100.000	X(28)
X(29)	.100000E-03	15.7400	12.9605	100.000	X(29)
X(30)	.100000E-03	10.6500	9.59833	100.000	X(30)
X(31)	.100000E-03	26.1700	30.2702	100.000	X(31)
X(32)	.100000E-03	26.3600	28.0821	100.000	X(32)
X(33)	.500000	1.00000	.500000	2.00000	X(33)
X(34)	.100000E-03	2.80200	2.68929	100.000	X(34)
X(35)	.100000E-03	3.11400	3.06167	100.000	X(35)
X(36)	.100000E-03	7.53900	6.53647	100.000	X(36)
X(37)	.100000E-03	37.1700	41.1428	100.000	X(37)
X(38)	.100000E-03	9.79000	11.2581	100.000	X(38)
X(39)	.100000E-03	48.0400	39.0604	100.000	X(39)
X(40)	.100000E-03	.466600	.574271	100.000	X(40)
X(41)	20.0000	30.0000	60.0000	60.0000	X(41)
X(42)	5.00000	15.0000	12.0862	30.0000	X(42)

X(43)	.100000E-03	2.71400	1.89602	100.000	X(43)
X(44)	.250000E-01	.070830	.604906	.605000	X(44)
X(45)	.100000E-03	1.13000	3.53426	100000.	X(45)
X(46)	.100000E-03	27.3800	29.8547	100.000	X(46)
X(47)	.100000E-03	13.2700	3.59241	100.000	X(47)
X(48)	.100000E-03	52.9400	85.4262	200.000	X(48)
X(49)	.100000E-03	3.48200	3.90750	100.000	X(49)
X(50)	.500000	1.00000	2.00000	2.00000	X(50)
X(51)	.100000E-03	1.70300	.458835	100.000	X(51)
X(52)	.100000E-03	.884300	1.37558	100.000	X(52)
X(53)	.100000E-03	6.65500	5.16350	100.000	X(53)
X(54)	.100000E-03	25.6000	17.7723	100.000	X(54)
X(55)	.100000E-03	.575500	.241440	100.000	X(55)
X(56)	5.00000	8.00000	6.30820	50.0000	X(56)
X(57)	.100000E-03	19.9100	13.0847	100.000	X(57)
X(58)	.100000E-03	5.53500	4.61322	100.000	X(58)
X(59)	.100000E-03	1.12000	.550271	100.000	X(59)
X(60)	50.0000	142.200	150.000	150.000	X(60)
X(61)	.100000E-03	9.53300	9.04815	100.000	X(61)
X(62)	.400000	.908800	1.26397	2.00000	X(62)

PARAMETER	VALUE	NAME
P(1) =	.971600E-01	
P(2) =	150.600	
P(3) =	362.000	
P(4) =	8.90000	
P(5) =	5000.00	
P(6) =	.500000E-01	
P(7) =	10.0000	
P(8) =	.600000	
P(9) =	.100000E+11	
P(10) =	.100000E+11	
P(11) =	.139000	
P(12) =	.270000	
P(13) =	.220000	
P(14) =	.100000E+11	
P(15) =	12.6774	
P(16) =	1.00000	
P(17) =	24.2400	
P(18) =	174.770	
P(19) =	2.50000	
P(20) =	2.37470	
P(21) =	.165900	
P(22) =	7.56198	
P(23) =	.400000	
P(24) =	60.0000	
P(25) =	5.00000	
P(26) =	.400000E-01	
P(27) =	.770000	
P(28) =	1.42000	
P(29) =	1.50000	
P(30) =	5.69000	
P(31) =	.403000E-02	
P(32) =	11.9100	
P(33) =	.100000E+11	
P(34) =	.100000E+11	
P(35) =	.495601E-01	
P(36) =	.800000	
P(37) =	.950000	

```
P(  38)=  1.50000
P(  39)=  9.17000
P(  40)=  20.0000
P(  41)=  .800000E-01
P(  42)=  1.20000
P(  43)=  .232000
P(  44)=  1.02400
P(  45)=  .200000E-01
P(  46)=  9.12960
P(  47)=  .100000E+11
P(  48)=  .200000
P(  49)=  .100000E+11
P(  50)=  .100000E+11
P(  51)=  .645700E-01
P(  52)=  .513580E-02
P(  53)=  .120610E-03
P(  54)=  .191800E-05
P(  55)=  20.0000
P(  56)=  .350000
P(  57)=  10.0000
P(  58)=  .850000
P(  59)=  1.00000
P(  60)=  .400000
P(  61)=  2.50000
P(  62)=  .100000E+11
P(  63)=  .100000E+11
P(  64)=  .100000E+11
P(  65)=  .100000E+11
P(  66)=  500.000
P(  67)=  .250000
P(  68)=  292.600
P(  69)=  2.90000
P(  70)=  4.00000
P(  71)=  10.0000
P(  72)=  8.36756
P(  73)=  657.300
P(  74)=  .330000
P(  75)=  83300.0
P(  76)=  .890000E-03
P(  77)=  6.00000
P(  78)=  .100000E+13
P(  79)=  2.00000
P(  80)=  .100000E-02
P(  81)=  1.49133
P(  82)=  1500.00
P(  83)=  100.000
P(  84)=  5.00000
P(  85)=  100.000
P(  86)=  45.0000
P(  87)=  50.0000
P(  88)=  30.0000
P(  89)=  30.0000
P(  90)=  .487155

 STATUS   CONSTR.      VALUE          NAME
---------------------------------------------------
   ON     G(    1)=  .712057E-07 EQUALITY--PST1
   ON     G(    2)= -.185676E-04 EQUALITY--PST2
   ON     G(    3)= -.100000E-02 EQUALITY--PST3
   ON     G(    4)= -.741799E-04 EQUALITY--PST4
```



```
   ON     G(    5)=  .747580E-04 EQUALITY--AS1
   ON     G(    6)= -.147360E-06 EQUALITY--AS2
   ON     G(    7)= -.400000E-04 EQUALITY--AS3
   ON     G(    8)=  .158730E-03 EQUALITY--AS4
   ON     G(    9)= -.354017E-03 EQUALITY--AS5
   ON     G(   10)=  .104801E-03 EQUALITY--AS6
   ON     G(   11)= -.236063E-04 EQUALITY--AS7
   ON     G(   12)= -.982754E-04 EQUALITY--AS8
   ON     G(   13)=  .217809E-03 EQUALITY--AS9
   ON     G(   14)=  .438038E-03 EQUALITY--AS10
   ON     G(   15)= -.193008E-03 EQUALITY--AS11
   ON     G(   16)=  .211947E-03 EQUALITY--AS12
   ON     G(   17)=  .330293E-04 EQUALITY--MIX1
   ON     G(   18)= -.281690E-03 EQUALITY--MIX2
   ON     G(   19)= -.106578E-03 EQUALITY--MIX3
   ON     G(   20)= -.536758E-04 EQUALITY--MIX4
   ON     G(   21)=  .383779E-03 EQUALITY--MIX5
   ON     G(   22)= -.246300E-03 EQUALITY--GT1
   ON     G(   23)=  .300000E-02 EQUALITY--GT2
   ON     G(   24)=  .113430      EQUALITY--GT3
   ON     G(   25)= -.246283      EQUALITY--GT4
   ON     G(   26)=  .317944E-02 EQUALITY--GT5
   ON     G(   27)=  .406686E-04 EQUALITY--GT6
   ON     G(   28)=  .348582E-03 EQUALITY--GT7
   ON     G(   29)= -.100000E-05 EQUALITY--PAND1
   ON     G(   30)=  .710543E-14 EQUALITY--PAND2
   ON     G(   31)= -.147384E-04 EQUALITY--PAND3
   ON     G(   32)=  .412290E-03 EQUALITY--PAND4
   ON     G(   33)= -.645587E-03 EQUALITY--PAND5
   ON     G(   34)=  .637272E-04 EQUALITY--PAND6
   ON     G(   35)= -.182920E-03 EQUALITY--PAND7
   ON     G(   36)=  .300000E-03 EQUALITY--SAND1
   ON     G(   37)= -.680000E-03 EQUALITY--SAND2
   ON     G(   38)=  .130166      EQUALITY--SAND3
   ON     G(   39)= -.558600E-01 EQUALITY--SAND4
   ON     G(   40)=  .103000E-02 EQUALITY--SAND5
   ON     G(   41)=  .710543E-14 EQUALITY--VF1
   ON     G(   42)=  .199568E-03 EQUALITY--VF2
   ON     G(   43)=  .373835E-04 EQUALITY--VF3
   ON     G(   44)=  .100452E-03 EQUALITY--VF4
   ON     G(   45)= -.314696E-04 EQUALITY--RECYCLE
   ON     G(   46)=  .100000E-02 EQUALITY--MB1
   ON     G(   47)= -.481334E-03 EQUALITY--MB2
   ON     G(   48)= -.816320E-04 EQUALITY--MB3
   ON     G(   49)= -.437149E-04 EQUALITY--MB4
   ON     G(   50)=  .201534E-04 EQUALITY--MB5
   ON     G(   51)= -.135388E-04 EQUALITY--MB6
   ON     G(   52)= -.105364E-03 EQUALITY--MB7
   ON     G(   53)= -.612652E-03 EQUALITY--PS CONC
   ON     G(   54)= -.828671E-05 EQUALITY--DEFN OF FP
   ON     G(   55)= -9.95972      BOD STANDARD
   ON     G(   56)= -11.6330      TSS STANDARD
   ON     G(   57)= -.262967E-01 MIXING REQUIREMENT

KEYWORD     VALUE
-------     -----
EPSCON      .10E-02   CONSTRAINT TOLERANCE (RHS)
EPSX        .10E-02   DISTANCE FROM CONSTR. SURFACE TOL.
EPSMIN      .10E-07   MINIMUM TOLERANCE.
XCHAR        "^"      EXPONENTIATION CHARACTER.
```

```
MAXTRM       70     MAXIMUM NO. OF CHARACTERS PER TERM.
P             5     NUMBER OF LP COLUMNS PRICED.
FACTOR      4.0     FACTORIZATION FREQUENCY/NVAR
DNCP         25     DEFAULT MAX. NO. OF CUTTING PLANES.
INCP         25     CUTTING PLANE INCREMENT.
TRACE         0     TO TERMINAL
GRAD         NO     EXTRA FEASIBILITY CHECK.
 SYSTEM      0    CURRENT SUBSYSTEM
 LIST        1     1:BOTH 0:SUM ONLY
 BIG= 100.   SMALL= -100.
METHOD = SAFE
 AUTOBOUND =    1     1=TRUE,0=FALSE

-COMMAND--> EXIT
 CP TIME = 1.169 SEC.
```

APPENDIX F

GRG MODEL FOR LIQUID SUBSYSTEM OPTIMIZATION

The liquid subsystem to be optimized is shown in Figure 3.8. The GRG model describing the design of this subsystem has 21 variables and 21 functions. The wastewater parameters at control point No.1 (see Figure 3.8) are input to the program. The variables in this model is given in Table F.1, followed by the listing of the program that defined the model.

Another program is used to generate the initial solution needed for the GRG run. This program is listed under the name DESIGN11. A file specifying the decision variables is needed to run DESIGN11. Input requirements of this file are summarized in Table F.2.

Table F.1 - Summary of Model Variables: Liquid Subsystem

Variable Index	Unit	Meaning
1	m^3/hr	L_p
2	$100m^2$	$A_p/100$
3	--	M_{t2}/M_{t1}
4	m^3/min	$Q_2/60$
5	m^3/hr	Q_8
6	days	θ_c
7	days	θ
8	$1000m^3$	$V/1000$
9	kg/m^3	M_{a3}
10	g/m^3	S_3
11	--	M_{i3}/M_{a3}
12	--	M_{f3}/M_{a3}
13	--	r
14	--	$100w$
15	--	$r+w$
16	g/m^3	M_{i4}
17	kg/m^3	M_{a5}
18	$100m^2$	$A_f/100$
19	g/m^3	S
20	m^3/sec	$Q_a/60$
21	$100kg/m^3$	$M_{t8}/100$

Table F.2 - Input Data to the Analysis Program : Liquid Subsystem

Card No.	Comments
1	Title
2	Primary clarifier overflow rate (m/hr)
3	Sludge age (days), Activated sludge recycle ratio
4	Hydraulic retention time (days)

```
      PROGRAM MAIN(INPUT,PAR1,OUTPUT,LAST11,TAPE7,TAPE8,
     +             TAPE5=INPUT,TAPE6=OUTPUT,TAPE9=PAR1,
     +             TAPE3=LAST11)
      DIMENSION Z(9000)
      COMMON Z
      DATA NCORE/9000/
      CALL GRG(Z,NCORE)
      END
C
C SUBROUTINE GCOMP:
C
      SUBROUTINE GCOMP(G,X)
      COMMON/INITBK/INIT
C     COMMON/BONDRY/Q1,S1,XA1,XD1,XI1,XF1,XT1
      DIMENSION G(21),X(21)
      REAL INF(6),STD(2),U(80),V(20)
      IF(INIT.EQ.0) GOTO 50
      READ(9,*) (INF(I),I=1,6)
      READ(9,*)  (STD(I),I=1,2)
      READ(9,*)  (U(I),I=1,80)
C
      Q1=INF(1)
      S1=INF(2)
      XA1=INF(3)
      XD1=INF(4)
      XI1=INF(5)
      XF1=INF(6)
      XT1=XA1+XD1+XI1+XF1
      Q1=Q1/60.
C
      V(1)=U(1)*U(3)/U(2)
      V(2)=U(3)/U(2)
      V(3)=23.85*U(6)*U(7)/U(8)
      V(4)=U(17)+U(18)
      V(5)=U(20)*EXP(U(21))
      V(6)=(V(4)*(V(5)-1.0))**(1./V(5))*V(5)/(V(5)-1.0)
      V(7)=(U(17)*(U(20)-1.0))**(1./U(20))*U(20)/(U(20)-1.0)
      V(8)=U(28)/U(29)
      V(9)=U(39)/1440./U(36)/(U(37)*U(39)-U(38))/U(41)
     +     /U(42)/U(43)/U(44)**(U(40)-20.0)
      V(10)=U(28)*U(56)
      V(11)=(U(67)*U(68)*(U(69)-1.0))**(1./U(69))*U(69)/
     +          (U(69)-1.0)
      V(12)=U(73)*(U(74)*U(75)/(U(76)*U(77)*U(78)))**0.5
      V(13)=U(69)-1.
C
50    EPBND=1.E-6
      DO 100 I=1,21
      IF(X(I).LT.EPBND) THEN
        DO 200 J=1,21
        G(J)=1.E30
200     CONTINUE
        RETURN
      ENDIF
100   CONTINUE
C
C PRIMARY SETTLING TANK DESIGN:
C
      G(1)=1.0-U(11)*XT1**U(12)/X(1)**U(13)-X(3)
      G(2)=(60.*X(4))/(1.E2*X(2))-X(1)
      G(3)=(60.*X(4))+X(5)-(60.*Q1)
      G(4)=1.E-5*XT1*X(3)*60.*X(4)+X(21)*X(5)-1.E-5*60.*Q1*XT1
```



```
C
C ACTIVATED SLUDGE DESIGN:
C
      G(5)=X(6)-(1.E3*X(9))*X(7)/((1.-(1.E-2*X(14)))*X(16)+
     +       (1.E-2*X(14))*(1.E3*X(17))-XA1*
     +       X(3))
      G(6)=X(7)-(1.E3*X(8))/(60.*X(4))/24.
      G(7)=X(10)-U(24)*(1.+U(26)*X(6))/((((X(6)*(U(23)*U(25)
     +         -U(26))-1.)
      G(8)=X(7)*(1.E3*X(9))/X(6)-U(23)*X(19)/(1.+U(26)*X(6))
      G(9)=1.E3*X(9)*X(11)+X(11)*X(6)*XA1*X(3)/X(7)-XI1*X(3)
     +      *X(6)/X(7)-(1.-U(27))*U(26)*(1.E3*X(9))*X(6)
      G(10)=X(12)*(1.E3*X(9))+X(12)*X(6)*XA1*X(3)/X(7)
     +        -XF1*X(3)*X(6)/X(7)
      G(11)=X(17)+U(80)*XA1*X(3)/X(13)-(1.+1./X(13)-X(7)
     +       /X(13)/X(6))*X(9)
      G(12)=(1.+X(11)+X(12))*X(17)-V(7)/
     +        ((60.*X(4))/(1.E2*X(18))*X(15))**(1./U(20))
      G(13)=X(15)-X(13)-(1.E-2*X(14))
      G(14)=(1.+X(11)+X(12))*X(16)+U(30)-U(31)*(1.+X(11)+X(12))
     +       *(1.E3*X(9))-U(32)*(60.*X(4))/(1.E2*X(18))*(1.+X(13))
      G(15)=60.*X(20)-24.E-3*U(29)*V(9)*60.*X(4)*X(19)
     +         +U(80)*U(28)*V(9)*
     +       (1.E3*X(8))*(1.E3*X(9))/X(6)
      G(16)=X(19)+X(10)-S1-V(8)*XD1*X(3)
C
C THICKENING MODEL FOR PRIMARY SEDIMENTATION:
C
      G(17)=X(21)-V(6)*((1.E2*X(2))*U(16)/X(5))**(1./V(5))*1.E-2
C
C
C EFFLUENT WATER QUALITY STANDARDS:
C
      G(18)=X(10)+U(27)*V(8)*X(16)-STD(1)
      G(19)=X(16)*(1.+X(11)+X(12))-STD(2)
C
C MIXING REQUIREMENT IN AERATION TANK:
C
      G(20)=(U(45)-(60.*X(20))/(1.E3*X(8)))*1.E2
C
C OBJECTIVE FUNCTION:
C
C PRIMARY SETTLING TANK:
C
      CCPST=824.*(1.E2*X(2))***.77
      IF((1.E2*X(2)).GE.279.) THEN
      COPST=17.15*(1.E2*X(2))**0.6
      CMPST=9.23*(1.E2*X(2))**0.6
      ELSE
      COPST=92.45*(1.E2*X(2))**.3
      CMPST=106.*(1.E2*X(2))**.14
      ENDIF
      CSPST=8.62*(1.E2*X(2))**.76
C
C PRIMARY SLUDGE PUMPING:
C
      CCPSP=16042.*X(5)**.53
      COPSP=374.*X(5)**.41
      CMPSP=166.*X(5)**.43
      CSPSP=385.*X(5)**.64
      CPPSP=X(5)
C
```

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```
C AERATION:
C
      CCAT=461.*(1.E3*X(8))**.71
      CCDAA=8533.*(60.*X(20))**.66
      CODAA=187.*(60.*X(20))**.48
      CMDAA=74.4*(60.*X(20))**.55
C
C FINAL SETTLING TANK:
C
      CCFST=824.*(1.E2*X(18))**.77
      IF((1.E2*X(18)).GE.279.) THEN
      COFST=17.15*(1.E2*X(18))**0.6
      CMFST=9.23*(1.E2*X(18))**0.6
      ELSE
      COFST=92.45*(1.E2*X(18))**.3
      CMFST=106.*(1.E2*X(18))**.14
      ENDIF
      CSFST=8.62*(1.E2*X(18))**.76
C
C RETURN SLUDGE PUMPING:
C
      Q5=(60.*X(4))*X(15)
      CCRSP=2779.*Q5**.53
      CORSP=.333*Q5
      CMRSP=.2375*Q5
      IF(Q5.LT.158.) THEN
      CSRSP=300.
      ELSE
      IF(Q5.LT.631.) THEN
      CSRSP=40.57*Q5**.52
      ELSE
      IF(Q5.LT.1580.) THEN
      CSRSP=5.97*Q5**.87
      ELSE
      CSRSP=2.540*Q5
      ENDIF
      ENDIF
      ENDIF
      CPRSP=Q5
C
      G(21)=(V(1)*(CCPST+CCPSP+CCAT+CCDAA+CCFST+CCRSP)
     +      +U(4)*(COPST+CMPST+COPSP+CMPSP+CODAA+CMDAA+COFST
     +      +CMFST+CORSP+CMRSP)
     +      +V(2)*(CSPST+CSPSP+CSFST+CSRSP)
     +      +V(3)*(CPPSP+CPRSP))*1.E-0
      RETURN
      END
      SUBROUTINE REPORT(G,X,M,N,CON,VAR,XO)
      DIMENSION X(N),G(M),CON(M),VAR(N),XO(N)
      COMMON/INITBK/INIT
      IF(INIT.EQ.1) RETURN
      WRITE(3,10) (I,X(I),I=1,N)
10    FORMAT(3X,I3,4X,E20.10)
      RETURN
      END
```

```
      PROGRAM MAIN(PAR1,DECVAR,OUTPUT,GRGD11,
     +            TAPE4=PAR1,TAPE5=DECVAR,
     +             TAPE8=GRGD11)
C
      COMMON/STATE/VIN,VOUT,VSIDE,ARY1
      COMMON/PARVAR/U,V,X
      COMMON/DPST/OR,IUNIT1
      COMMON/DAS/SRT,HRT,ASRR,IUNIT2,BODSTD,TSSSTD
      REAL VIN(7),VOUT(7),VSIDE(7),U(100),V(20),X(30)
      REAL ARY1(7),INFLOW(7)
      REAL NC
      CHARACTER* 1 TITLE(80)
C
      READ(4,*)(INFLOW(I),I=1,6)
      READ(4,*)BODSTD,TSSSTD
      READ(4,*)(U(I),I=1,80)
C
C
      READ(5,90)TITLE
90    FORMAT(80A1)
      READ(5,*)OR,IUNIT1
      READ(5,*)SRT,ASRR
      READ(5,*)HRT,IUNIT2
C
      V(1)=U(1)*U(3)/U(2)
      V(2)=U(3)/U(2)
      V(3)=23.85*U(6)*U(7)/U(8)
      V(4)=U(17)+U(18)
      V(5)=U(20)*EXP(U(21))
      V(6)=(V(4)*(V(5)-1.0))**(1./V(5))*V(5)/(V(5)-1.0)
      V(7)=(U(17)*(U(20)-1.0))**(1./U(20))*U(20)/(U(20)-1.0)
      V(8)=U(28)/U(29)
      V(9)=U(39)/1440./U(36)/(U(37)*U(39)-U(38))/U(41)
     +      /U(42)/U(43)/U(44)**(U(40)-20.0)
      V(10)=U(28)*U(56)
      V(11)=(U(67)*U(68)*(U(69)-1.0))**(1./U(69))*U(69)/(U(69)-1.0)
      V(12)=U(73)*(U(74)*U(75)/(U(76)*U(77)*U(78)))**0..5
      V(13)=U(69)-1.
C
      VIN(1)=INFLOW(1)
      VIN(2)=INFLOW(2)
      VIN(3)=INFLOW(3)
      VIN(4)=INFLOW(4)
      VIN(5)=INFLOW(5)
      VIN(6)=INFLOW(6)
      VIN(7)=VIN(3)+VIN(4)+VIN(5)+VIN(6)
C
      CALL PST
      DO 88 I=1,7
      ARY1(I)=VOUT(I)
88    CONTINUE
      CALL AS
C
      WRITE(8,120)(K,X(K),K=1,21)
  120 FORMAT(3X,I3,4X,E20.10)
C
      STOP
      END
C
C
C
      SUBROUTINE PST
```



```
      COMMON/STATE/VIN,VOUT,VSIDE,ARY1
      COMMON/PARVAR/U,V,X
      COMMON/DPST/OR,IUNIT1
      REAL VIN(7),VOUT(7),VSIDE(7),ARY1(7),U(100),V(20),X(30)
      REAL KP1
C
      IF(IUNIT1.EQ.0) OR=OR/24.
      R=U(11)*VIN(7)**U(12)/OR**U(13)
      R1=1.0-R
C
      KP1=1.E3*V(6)/VIN(7)
C
C USE NEWTON'S METHOD TO SOLVE FOR PRIMARY SLUDGE FLOWRATE:
C
      N=1
      VSIDE(1)=1.E-3*VIN(1)
21    VOUT(1)=VIN(1)-VSIDE(1)
      F=VIN(1)-VOUT(1)*R1-KP1*VSIDE(1)**((V(5)-1.)/V(5))
     +   *(VOUT(1)*U(16)/OR)**(1./V(5))
      IF(ABS(F).LE.1.E-6) GOTO 22
      FPRIME=R1-KP1*(U(16)*VOUT(1)/OR/VSIDE(1))**(1./V(5))
     +        *(1.-VIN(1)/V(5)/VOUT(1))
      VSIDE(1)=VSIDE(1)-F/FPRIME
      N=N+1
      IF(N.GE.50) THEN
      PRINT *,'MAXIMUM NUMBER OF ITERATION REACHED IN
     +PRIMARY SETTLING TANK DESIGN'
      STOP
      ENDIF
      GOTO 21
C
22    IF(VSIDE(1).LT.0.) THEN
      PRINT *,'FAILED TO FIND A FEASIBLE SOLUTION IN
     +PRIMARY SETTLING TANK DESIGN'
      STOP
      ENDIF
C
      VOUT(7)=VIN(7)*R1
      APST=VOUT(1)/OR
      VSIDE(7)=V(6)/(VSIDE(1)/U(16666)/APST)**(1./V(5))*1.E3
      VOUT(2)=VIN(2)
      VSIDE(2)=VIN(2)
C
      RATIO=VSIDE(7)/VIN(7)
C
      DO 1 I=3,6
        VOUT(I)=VIN(I)*R1
        VSIDE(I)=VIN(I)*RATIO
1     CONTINUE
C
      X(3)=R1
      X(1)=OR
      X(4)=VOUT(1)/60.
      X(5)=VSIDE(1)
      X(2)=APST*1.E-2
      X(21)=VSIDE(7)*1.E-5
C
      RETURN
      END
C
C
C
```

```
      SUBROUTINE AS
      COMMON/STATE/VIN,VOUT,VSIDE,ARY1
      COMMON/PARVAR/U,V,,,,,X
      COMMON/DAS/SRT,HRT,ASRR,IUNIT2,BODSTD,TSSSTD
      REAL VIN(7),VOUT(7),VSIDE(7),ARY1(7),U(100),V(20),X(30)
      REAL SN(3),F(3)
      REAL NW
C
C
C AERATION:
C
      IF(IUNIT2.EQ.0) HRT=HRT/24.
C
      C=SRT/HRT
      VIN(1)=ARY1(1)*(1.0+ASRR)
      VIN(2)=U(24)*(1.0+U(26)*SRT)/(SRT*(U(23)*U(25)-U(26))-1.0)
      FOOD=ARY1(2)+V(8)*ARY1(4)-VIN(2)
      VIN(3)=U(23)/(1.0+U(26)*SRT)*C*FOOD
      VIN(4)=0.0
      VIN(5)=(ARY1(5)*C+(1.0-U(27))*U(26)*VIN(3)*SRT)/
     +        (1.0+C*(ARY1(3)/VIN(3)))
      VIN(6)=ARY1(6)*C/(1.0+C*(ARY1(3)/VIN(3)))
      VIN(7)=VIN(3)+VIN(4)+VIN(5)+VIN(6)
C
      VAT=ARY1(1)*HRT*24.0
C
      ORMT=ARY1(1)*FOOD*(U(29)-U(28)*U(23)/(1.+U(26)*SRT))*1.E-3*24.
      AFR=ORMT*V(9)
      AFUV=AFR/VAT
      IF(AFUV.LT.U(45)) THEN
        PRINT *,'MIXING REQUIREMENT CONTROLS OXYGEN DEMAND'
      ENDIF
C
C SECONDARY SETTLING:
C
150   YY=VIN(5)/VIN(3)
      ZZ=VIN(6)/VIN(3)
      XX=VIN(7)/VIN(3)
C
      Q2=ARY1(1)
      XA2=ARY1(3)
C
      ARY1(3)=((ASRR+1.0-1.0/C)*VIN(3)-XA2)/ASRR
      ARY1(7)=ARY1(3)*XX
C
C SETTING UP THE COEFFICIENTS FOR SIMULTANEOUS EQUATIONS:
      NW=U(20)
      A11=ARY1(3)*1.0E-3
      A12=-1.0/XX*1.0E-3
      A13=VIN(1)/XX*1.0E-3
      RHS1=(VIN(3)*VAT/SRT/24.0+Q2*XA2+ASRR*Q2*ARY1(3))*1.E-3
      A21=V(7)
      RHS2=ARY1(7)*1.0E-3
      A31=U(32)*Q2*(1.0+ASRR)*(-1.0)
      A32=1.0
      RHS3=-U(30)+U(31)*VIN(7)
C
C PROVIDING STARTING VALUES FOR VARIABLES:
      IAREA=1
      SN(1)=ASRR*Q2
      SN(2)=15.0
180   SN(3)=500.0*(2.*IAREA-1.0)
```



```
C
      N=3
C
      CALL FUNC(N,SN,F,NW,A11,A12,A13,A21,A31,A32,
     +          RHS1,RHS2,RHS3)
C
      CALL QUASI(N,SN,F,NW,A11,A12,A13,A21,A31,A32,
     +           RHS1,RHS2,RHS3,ITEQN)
C
      IF(ITEQN.GE.50) THEN
      IF(IAREA.GT.5) THEN
      PRINT *,'MAXIMUM NUMBER OF ITERATIONS REACHED
     + IN ACTIVATED SLUDGE DESIGN'
      STOP
      ELSE
      IAREA=IAREA+1
      GOTO 180
      ENDIF
      ENDIF
C
      IF(SN(1).LE.0.0.OR.SN(2).LE.0.0.OR.SN(3).LE.0.0) THEN
      PRINT *,'FAILED TO FIND A FEASIBLE SOLUTION IN
     +ACTIVATED SLUDGE DESIGN'
      STOP
      ENDIF
C
      ARY1(1)=SN(1)
      VOUT(7)=SN(2)
      AF=SN(3)
C
      VOUT(1)=VIN(1)-ARY1(1)
      VOUT(2)=VIN(2)
      VOUT(3)=VOUT(7)/XX
      VOUT(4)=0.0
      VOUT(5)=VOUT(3)*YY
      VOUT(6)=VOUT(3)*ZZ
C
      ARY1(2)=VOUT(2)
      ARY1(4)=0.0
      ARY1(5)=ARY1(3)*YY
      ARY1(6)=ARY1(3)*ZZ
C
      X(16)=VOUT(3)
      X(17)=ARY1(3)*1.E-3
      X(10)=VOUT(2)
      X(6)=SRT
      X(7)=HRT
      X(13)=ASRR
      X(8)=VAT*1.E-3
      X(9)=VIN(3)*1.E-3
      X(19)=FOOD
      X(18)=AF*1.E-2
      X(11)=YY
      X(12)=ZZ
      X(20)=AFR/60.
C
C ACTIVATED SLUDGE SEPARATION:
C
      VOUT(1)=ASRR*Q2
      VIN(1)=ARY1(1)-VOUT(1)
C
      X(14)=VIN(1)/Q2*1.E2
```

```
      X(15)=ASRR+X(14)*1.E-2
      RETURN
      END
C
      SUBROUTINE QUASI(N,X,F,NC,A11,A12,A13,A21,A31,
     +                 A32,RHS1,RHS2,RHS3,ITE)
      DIMENSION F(3),FNEW(3),H(3,3),HY(3),S(3),SH(3)
      DIMENSION U(3,3),X(3),Y(3)
      REAL NC
C
      ITE=1
C
C INITIATE H(I,J):
      DO 10 I=1,N
      DO 10 J=1,N
      IF(I.EQ.J)THEN
        H(I,J)=1.0
        ELSE
        H(I,J)=0.0
      ENDIF
10    CONTINUE
C
C COMPUTE X'(I)  (THE NEW SOLUTION) :
15    CALL MATVEC(N,H,F,S)
      DO 16 I=1,N
      S(I)=-S(I)
16    CONTINUE
C
C   TEST FOR CONVERGENCE:
        NSTOP=0
        DO 20 I=1,N
      IF(I.EQ.2)THEN
        IF(ABS(S(I)).LE.1.E-5)THEN
        NSTOP=NSTOP+1
        ENDIF
      ELSE
        IF(ABS(S(I)).LE.1.E-3)  THEN
          NSTOP=NSTOP+1
        ENDIF
      ENDIF
20      CONTINUE
C
        IF(NSTOP.EQ.N.OR.ITE.GE.200) GO TO 100
C
C   UPDATE X(I):
        DO 25 I=1,N
        X(I)=X(I)+S(I)
25      CONTINUE
C
C COMPUTE Y(I):
      CALL FUNC(N,X,FNEW,NC,A11,A12,A13,A21,A31,A32,
     +          RHS1,RHS2,RHS3)
      DO 30 I=1,N
      Y(I)=FNEW(I)-F(I)
      F(I)=FNEW(I)
30    CONTINUE
C
C UPDATE H(I,J):
C     COMPUTE H(I,J)*Y(J):
        CALL MATVEC(N,H,Y,HY)
C     COMPUTE TRANSPOSE(S(1*N)) * H(N*N) * Y(N*1) :
        SHY=0.0
```



```
        DO 35 I=1,N
        SHY=SHY+S(I)*HY(I)
35      CONTINUE
C     COMPUTE TRANSPOSE(S(1*N)) * H(N*N) :
        CALL VECMAT(N,S,H,SH)
C     UPDATE:
        DO 40 I=1,N
        DO 40 J=1,N
        U(I,J)=(S(I)-HY(I))*SH(J)/SHY
        H(I,J)=H(I,J)+U(I,J)
40      CONTINUE
C
C  CONTINUE ITERATIONS:
      ITE=ITE+1
      GO TO 15
C
100   RETURN
      END
C
C SUBROUTINE 'MATVEC' PERFORMS THE POST-MULTIPLICATION OF
C     A MATRIX (N*N) BY A VECTOR (N*1)
C
      SUBROUTINE MATVEC(N,A,B,C)
      DIMENSION A(3,3),B(3),C(3)
C
      DO 1 I=1,N
      C(I)=0.0
      DO 2 J=1,N
      C(I)=C(I)+A(I,J)*B(J)
2     CONTINUE
1     CONTINUE
      RETURN
      END
C
C SUBROUTINE VECMAT PERFORMS THE PRE-MULTIPLICATION OF
C     A MATRIX (N*N) BY A VECTOR (1*N)
C
      SUBROUTINE VECMAT(N,X,Y,Z)
      DIMENSION X(3),Y(3,3),Z(3)
C
      DO 1 J=1,N
      Z(J)=0.0
      DO 2 I=1,N
      Z(J)=Z(J)+X(I)*Y(I,J)
2     CONTINUE
1     CONTINUE
      RETURN
      END
C
      SUBROUTINE FUNC(N,X,F,NC,A11,A12,A13,A21,A31,A32,
     +                RHS1,RHS2,RHS3)
      REAL NC
      DIMENSION X(3),F(3)
C
      F(1)=A11*X(1)+A12*X(1)*X(2)+A13*X(2)-RHS1
      F(2)=A21**NC*(X(3)/X(1))-RHS2**NC
      F(3)=A31/X(3)+A32*X(2)-RHS3
      RETURN
      END
```

APPENDIX G

SLUDGE SUBSYSTEM DESIGN

The inputs to this program are: [1] wastewater parameters at control point No. 1 (see Figure 3.8), [2] optimal solution for the liquid subsystem (see Appendix F), and [3] the recycle flowrates Q_{10}, Q_{13}, and Q_{15}. Only the final design of the sludge subsystem is printed out. The program listing is on the next few pages.

```
      PROGRAM MAIN(PARSLG,LAST11,INPUT,STATES,OUTPUT,
     +             TAPE4=PARSLG,TAPE5=LAST11,TAPE6=STATES)
      COMMON/STATE0/Q0,S0,XA0,XD0,XI0,XF0
      COMMON/STATE1/Q1,S1,XA1,XD1,XI1,XF1,XT1
      COMMON/PAR/U
      COMMON/VAR/X,XA2,Q7,RATIO
      COMMON/STATE9/Q9,S9,XA9,XD9,XI9,XF9,XT9,FP
      COMMON/FLOWS/Q10,Q13,Q15
      DIMENSION U(80),STD(2),X(21)
      DATA GOLD/0.618034/
      DATA Q0/1500./,S0/100./,XA0/5./,XD0/100./,XI0/45./,XF0/50./
      READ(4,*)Q1,S1,XA1,XD1,XI1,XF1
      READ(4,*)(STD(I),I=1,2)
      READ(4,*)(U(I),I=1,80)
      XT1=XA1+XD1+XI1+XF1
      READ(5,9)(J,X(I),I=1,21)
9     FORMAT(3X,I3,4X,E20.10)
      Q7=60.*X(4)*1.E-2*X(14)
      RATIO=1.E2*X(21)/XT1
      Q9=Q7+X(5)
      S9=(Q7*X(10)+X(5)*S1)/Q9
      XA9=(Q7*X(17)+X(5)*XA1*RATIO)/Q9
      XD9=X(5)*XD1*RATIO/Q9
      XI9=(Q7*X(17)*X(11)+X(5)*XI1*RATIO)/Q9
      XF9=(Q7*X(17)*X(12)+X(5)*XF1*RATIO)/Q9
      FP=X(5)*1.E2*X(21)/(Q7*X(17)*(1.+X(11)+X(12))+X(5)*1.E2*X(21))
      XT9=XA9+XD9+XI9+XF9
      XA2=XA1*X(3)
C
      PRINT *,'INPUT Q10, Q13, Q15 (CU M/HR)'
      READ *,Q10,Q13,Q15
      Q16=Q9-Q10-Q13-Q15
      IF(Q16.LE.0.)THEN
      PRINT *,'NEGATIVE SLUDGE FLOW'
      GOTO 1000
      ENDIF
      IC=0
      RIFL=.2
      RIFU=2.0
C
      BL=RIFL
      BU=RIFU
      RANGE=BU-BL
      B=GOLD*RANGE+BL
      A=(1.-GOLD)*RANGE+BL
      CALL SLUDGE(OBJA,A,IC)
      CALL SLUDGE(OBJB,B,IC)
20    IF(OBJA.LE.OBJB) THEN
      BU=B
      B=A
      OBJB=OBJA
      A=BL+BU-B
      CALL SLUDGE(OBJA,A,IC)
      GOTO 10
      ELSE
      BL=A
      A=B
      OBJA=OBJB
      B=BU+BL-A
      CALL SLUDGE(OBJB,B,IC)
```



```
      GOTO 10
      ENDIF
C
10    ROU=B-A
      IF(ROU.LE.0.005)THEN
      C=(A+B)/2.
      IC=1
      CALL SLUDGE(OPT,C,IC)
      GOTO 30
      ELSE
      GOTO 20
      ENDIF
C
30    PRINT *,'OPTIMAL XI/XF=',C
      PRINT *,'OPTIMAL COST FOR SLUDGE TREATMENT=',OPT
      GOTO 1001
1000  PRINT *,'DESIGN INFEASIBLE'
1001  STOP
      END
C
      SUBROUTINE SLUDGE(TCOST,RIF,IC)
      COMMON/STATE0/Q0,S0,XA0,XD0,XI0,XF0
      COMMON/STATE1/Q1,S1,XA1,XD1,XI1,XF1,XT1
      COMMON/PAR/U
      COMMON/VAR/X,XA2,Q7,RATIO
      COMMON/STATE9/Q9,S9,XA9,XD9,XI9,XF9,XT9,FP
      COMMON/FLOWS/Q10,Q13,Q15
      DIMENSION U(80),X(21),F(0:15),DX(16)
      REAL L(16)
      REAL NC,ND,KC,KD
      XI1=(Q0*XI0+1.E3*RIF/(1.+RIF)*(Q13*U(70)+Q15*U(79)))/Q1
      XF1=(Q0*XF0+1.E3/(1.+RIF)*(Q13*U(70)+Q15*U(79)))/Q1
      DENOM=X(9)+1.E-3*X(6)/X(7)*XA2
      DUMMY=X(3)*X(6)/X(7)
      X(11)=(1.E-3*XI1*DUMMY+(1.-U(27))*U(26)*X(9)*X(6))/DENOM
      X(12)=1.E-3*XF1*DUMMY/DENOM
      XI9=(Q7*X(17)*X(11)+X(5)*XI1*RATIO)/Q9
      XF9=(Q7*X(17)*X(12)+X(5)*XF1*RATIO)/Q9
      CHECK=XA9+XD9+XI9-RIF*XF9
      IF(CHECK.LE.0.)THEN
      PRINT *,'RATIIIIIO XI/XF TOO LARGE'
      STOP
      ENDIF
C
      IF(IC.EQ.0)GOTO 601
      WRITE(6,60)Q9,S9,XA9,XD9,XI9,XF9,XT9
60    FORMAT(5X,'INPUT TO SLUDGE TREATMENT TRAIN:',/,
     +       10X,'Q9=',F12.6,'CU M/HR',/,10X,'S9=',F12.6,
     +       'G/CU M',/,10X,'XA9=',F12.6,'KG/CU M',/,10X,'XD9=',
     +       F12.6,'KG/CU M',/,10X,'XI9=',F12.6,'KG/CU M',/,10X,
     +       'XF9=',F12.6,'KG/CU M',/,10X,'XT9=',F12.6,
     +       'KG/CU M',/)
601   Q11=Q9-Q10
      XT11=(Q9*XT9-Q10*U(48))/Q11
      AC=U(17)+U(18)*FP**U(19)
      NC=U(20)*EXP(U(21)*FP)
      KC=(AC*(NC-1.0))**(1.0/NC)*NC/(NC-1.0)
      AG=Q11*(XT11/KC)**NC
      SLGT=Q11*XT11/AG
      CCGT=824.*AG**.77
```

```
      IF(AG.GE.279.) THEN
      COGT=17.15*AG**0.6
      CMGT=9.23*AG**0.6
      ELSE
      COGT=92.45*AG**.3
      CMGT=106.*AG**.14
      ENDIF
      CSGT=8.62*AG**.76
      TCGT=CCGT*U(1)*U(3)/U(2)+U(4)*(COGT+CMGT)+CSGT*U(3)/U(2)
      XF11=XT11*XF9/XT9
      TVS=XT11-XF11
      IF(IC.EQ.0)GOTO 602
      WRITE(6,59) RIF
59    FORMAT(//,'RATIO XI/XF=',F10.5,/)
      WRITE(6,61)Q11,TVS,XF11,XT11
      WRITE(6,62)AG,SLGT
61    FORMAT(10X,'Q11=',F12.6,'CU M/HR',/,10X,'TVS=',
     + F12.6,'KG/CU M'
     +       ,/,10X,'XF11=',F12.6,'KG/CU M',/,10X,'XT11=',F12.6,
     +       'KG/CU M',/)
62    FORMAT(10X,'AREA OF THICKENER=',F12.6,'SQ M',/,10X,
     +       'SOLIDS LOADING ON THICKENER=',F12.6,'KG/SQ M/HR',/)
602   XF12=XF11
      XI12=RIF*XF12
      VSDEST=TVS-XI12
      CHEC=TVS/XI12-1.0
C USE FIBONACCI SEARCH TO FIND THE OPTIMAL DIGESTER TEMPERATURE
C FINAL INTERVAL OF UNCERTAINTY IS ONE DEGREE CENTIGRADE
C
      A=20.0
      B=60.0
      D=1.0
      L(1)=B-A
      T=L(1)/D
      F(0)=1.0000
      F(1)=1.0
      DO 10 N=2,100
      F(N)=F(N-1)+F(N-2)
      IF(T.LE.F(N))GOTO 11
10    CONTINUE
11    M=N-1
      DO 20 K=1,M
      DX(K+1)=F(N-K-1)/F(N-K+1)*L(K)
      L(K+1)=L(K)-DX(K+1)
20    CONTINUE
C
      J=2
      AX=A+DX(J)
      CALL COST(COSTA,AX,U,CHEC,Q11,S9,VSDEST)
30    XB=B-DX(J)
      CALL COST(COSTB,XB,U,CHEC,Q11,S9,VSDEST)
31    IF(COSTA.LE.COSTB) GOTO 32
      A=AX
      AX=XB
      COSTA=COSTB
      J=J+1
      IF(J.GT.N)GOTO 40
      GOTO 30
32    B=XB
      XB=AX
```



```
      COSTB=COSTA
      J=J+1
      IF(J.GT.N)GOTO 41
      AX=A+DX(J)
      CALL COST(COSTA,AX,U,CHEC,Q11,S9,VSDEST)
      GOTO 31
40    A=AX
      XMIN=XB
      FMIN=COSTB
      GOTO 50
41    B=XB
      XMIN=AX
      FMIN=COSTA
50    RATE=U(53)*EXP(10.*ALOG(10.)/3.*(U(54)-1.E3/(XMIN+273.)))
      SRTD=CHEC/RATE
      VDIG=SRTD*Q11*24.
      TCPAND=FMIN
      XT12=XI12+XF12
      IF(IC.EQ.0)GOTO 603
      WRITE(6,70)Q11,XI12,XF12,XT12
70    FORMAT(10X,'Q12=',F12.6,'CU M/HR',/,10X,'XI12=',F12.6
     + 'KG/CU M'
     +       ,/,10X,'XF12=',F12.6,'KG/CU M',/,10X,'XT12=',F12.6,
     +       'KG/CU M',/)
      WRITE(6,71)XMIN,SRTD,VDIG
71    FORMAT(10X,'DIGESTER TEMPERATURE=',F12.6,'DEG C',/,10X,
     +       'SLUDGE AGE=',F12.6,'DAYS',/,10X,'DIGESTER VOLUME=',
     +       F12.6,'CU M',/)
603   Q14=Q11-Q13
      XT14=(Q11*XT12-Q13*U(70))/Q14
      AD=U(68)
      ND=U(69)
      KD=(U(67)*U(68)*(U(69)-1.))**(1./U(69))*U(69)/(U(69)-1.)
      AD=Q14*(XT14/KD)**ND
      SLSAND=Q14*XT14/AD
      VSAND=U(71)*AD
      CCSAND=2323.*VSAND**.59
      IF(VSAND.GE.5678.) THEN
      COSAND=1.29*VSAND**.83
      CMSAND=.83*VSAND**.82
      ELSE
      IF(VSAND.GE.1968.) THEN
      COSAND=14.*VSAND**.55
      CMSAND=8.5*VSAND**.55
      ELSE
      COSAND=192.*VSAND**.2
      CMSAND=113.*VSAND**.21
      ENDIF
      ENDIF
      IF(VSAND.GE.2839.) THEN
      CSSAND=14.4*VSAND**.66
      ELSE
      CSSAND=142.*VSAND**.37
      ENDIF
      TCSAND=CCSAND*U(1)*U(3)/U(2)+U(4)*(COSAND+CMSAND)
     +       +U(3)/U(2)*CSSAND
      XF14=XT14*XF12/XT12
      XI14=XT14-XF14
C
      IF(IC.EQ.0)GOTO 604
```

```
      WRITE(6,80)Q14,XI14,XF14,XT14
      WRITE(6,81)AD,SLSAND
80    FORMAT(10X,'Q14=',F12.6,'CU M/HR',/,10X,'XI14=',F12.6
     + 'KG/CU M'
     +       ,/,10X,'XF14=',F12.6,'KG/CU M',/,10X,'XT14=',F12.6,
     +       'KG/CU M',/)
81    FORMAT(10X,'AREA OF SECONDARY DIGESTER=',F12.6,'SQ M',/,10X,
     +       'SOLIDS LOADING ON DIGESTER=',F12.6,'KG/SQ M/HR',/)
C
604   Q16=Q14-Q15
      XT16=(Q14*XT14-Q15*U(79))/Q16
      W=Q16*XT16/Q15
      YIELD=U(73)*SQRT(W*U(74)*U(75)/U(76)/U(77)/U(78))
      AV=Q16*XT16/YIELD
C
      Q16X16=Q16*XT16
      CCVF=29180.*AV**.71
      CSVF=230.*Q16X16**.71+182.*Q16X16**.86
      COVF=197.55*Q16X16**.58
      IF(Q16X16.GE.519.) THEN
      CMVF=5.57*Q16X16**.84
      ELSE
      IF(Q16X16.GE.103.) THEN
      CMVF=20.*Q16X16**.63
      ELSE
      CMVF=41.5*Q16X16**.48
      ENDIF
      ENDIF
C
      CCSD=U(1)*U(3)/U(2)*72053.*Q16**.74+U(5)*3.6662E-2*Q16*XT16
     +      *U(1)
      COMSD=U(4)*8024.*Q16**.667
C
      TCVFSD=U(1)*U(3)/U(2)*CCVF+U(4)*(COVF+CMVF)+U(3)/U(2)*CSVF
     +       +CCSD+COMSD
C
      XF16=XT16*XF14/XT14
      XI16=XT16-XF16
C
      IF(IC.EQ.0)GOTO 605
      WRITE(6,90)Q16,XI16,XF16,XT16
      WRITE(6,91)AV,YIELD
90    FORMAT(10X,'Q16=',F12.6,'CU M/HR',/,10X,'XI16=',F12.6,
     + 'KG/CU M'
     +       ,/,10X,'XF16=',F12.6,'KG/CU M',/,10X,'XT16=',F12.6,
     +       'KG/CU M',/)
91    FORMAT(10X,'AREA OF VACUUM FILTER=',F12.6,'SQ M',/,10X,
     +       'FILTER YIELD=',F12.6,'KG/SQ M/HR',/)
605   QCYCLE=Q10+Q13+Q15
      CCRP=2779.*QCYCLE**.53
      COSP=.333*QCYCLE
      CMRP=.2375*QCYCLE
      IF(QCYCLE.LT.158.) THEN
      CSRP=300.
      ELSE
      IF(QCYCLE.LT.631.) THENN
      CSRP=40.57*QCYCLE**.52
      ELSE
      IF(QCYCLE.LT.1580.) THEN
      CSRP=5.97*QCYCLE**.87
```



```
      ELSE
      CSRP=2.540*QCYCLE
      ENDIF
      ENDIF
      ENDIF
      CPRP=QCYCLE
      TCRP=U(1)*U(3)/U(2)*CCRP+U(4)*(CORP+CMRP)+CSRP*U(3)/U(2)
     +       +CPRP*23.85*U(6)*U(7)/U(8)
      TCOST=TCGT+TCPAND+TCSAND+TCVFSD+TCRP
      IF(IC.EQ.0)GOTO 606
      WRITE(6,92)TCOST
92    FORMAT(///,'TOTAL COST FOR SLUDGE TREATMENT AND DISPOSAL =',
     +  F10.0,'  DOLLARS/YEAR',//)
606   RETURN
      END
C
      SUBROUTINE COST(TC,TEMP,U,CHEC,Q11,S9,VSDEST)
      DIMENSION U(80)
C
      RATE=U(53)*EXP(10.*ALOG(10.)/3.*(U(54)-1.E3/(TEMP+273.)))
      SRTD=CHEC/RATE
      VDIG=SRTD*Q11*24.
      CH4=U(28)*U(56)*Q11*VSDEST+U(29)*U(56)*1.E-3*Q11*S9
      EVCH4=CH4*3.58E4*8.76/3.6
      Q1=10.22E3*Q11*(TEMP-U(55))
      Q2=U(59)*VDIG*U(60)*(TEMP-U(57))*8.76
      Q=(Q1+Q2)/U(58)
      EVNET=(EVCH4-Q)*1.E-6
      CCPAND=2323.*VDIG**.59
      IF(VDIG.GE.5678.) THEN
      COPAND=1.29*VDIG**.83
      CMPAND=.83*VDIG**.82
      ELSE
      IF(VDIG.GE.1968.) THEN
      COPAND=14.*VDIG**.55
      CMPAND=8.5*VDIG**.55
      ELSE
      COPAND=192.*VDIG**.2
      CMPAND=113.*VDIG**.21
      ENDIF
      ENDIF
      IF(VDIG.GE.2839.) THEN
      CSPAND=14.4*VDIG**.66
      ELSE
      CSPAND=142.*VDIG**.37
      ENDIF
      TC=U(1)*U(3)/U(2)*CCPAND+U(4)*(COPAND+CMPAND)
     +    +U(3)/U(2)*CSPAND-U(61)*3.6E3/1.055*EVNET
      RETURN
      END
```

APPENDIX H

GRG MODEL FOR SYSTEM WITHOUT A PRIMARY CLARIFIER

The flowchart of the system is shown in Figure H.1. There are 51 variables and 47 equations in the GRG model describing the design of this system. Table H.1 provides a list of the variables in the model.

Table H.1 - Summary of Model Variables : Base System Without a Primary Clarifier

Variable Index	Unit	Meaning
1	m^3/min	$Q_2/60$
2	g/m^3	S_2
3	g/m^3	M_{a2}
4	g/m^3	M_{d2}
5	g/m^3	M_{i2}
6	g/m^3	M_{f2}
7	g/m^3	M_{t2}
8	days	θ_c
9	days	θ
10	$1000m^3$	$V/1000$
11	kg/m^3	M_{a3}
12	g/m^3	S_3
13	--	M_{i3}/M_{a3}
14	--	M_{f3}/M_{a3}
15	--	r
16	--	$100w$
17	--	$r+w$
18	g/m^3	M_{a4}
19	kg/m^3	M_{a5}
20	$100m^2$	$A_f/100$
21	g/m^3	S
22	m^3/sec	$Q_s/60$
23	m^3/hr	Q_9
24	kg/m^3	M_{t9}
25	$kg/m^2/hr$	L_g
26	$100m^2$	$A_g/100$
27	m^3/hr	Q_{10}
28	m^3/hr	Q_{11}
29	kg/m^3	M_{t11}

Table H.1 (continued)

Variable Index	Unit	Meaning
30	kg/m^3	M_{f11}
31	kg/m^3	$M_{a5}M_{t10}/M_{t9}$
32	°C	T_d
33	days	θ_d
34	$1000m^3$	$V_d/1000$
35	day^{-1}	K_1
36	$10^6 kWhr$/year	q
37	kg/m^3	$M_{a11}+M_{d11}+M_{i11}$
38	kg/m^3	M_{t12}
39	m^3/hr	G
40	$10^6 kWhr$/year	N
41	$kg/m^2/hr$	L_d
42	$100m^2$	$A_d/100$
43	m^3/hr	Q_{13}
44	m^3/hr	Q_{14}
45	kg/m^3	M_{t14}
46	--	M_{i12}/M_{t12}
47	$kg/m^2/hr$	L_f
48	m^2	A_v
49	m^3/hr	Q_{15}
50	m^3/hr	Q_{16}
51	kg/m^3	M_{t16}

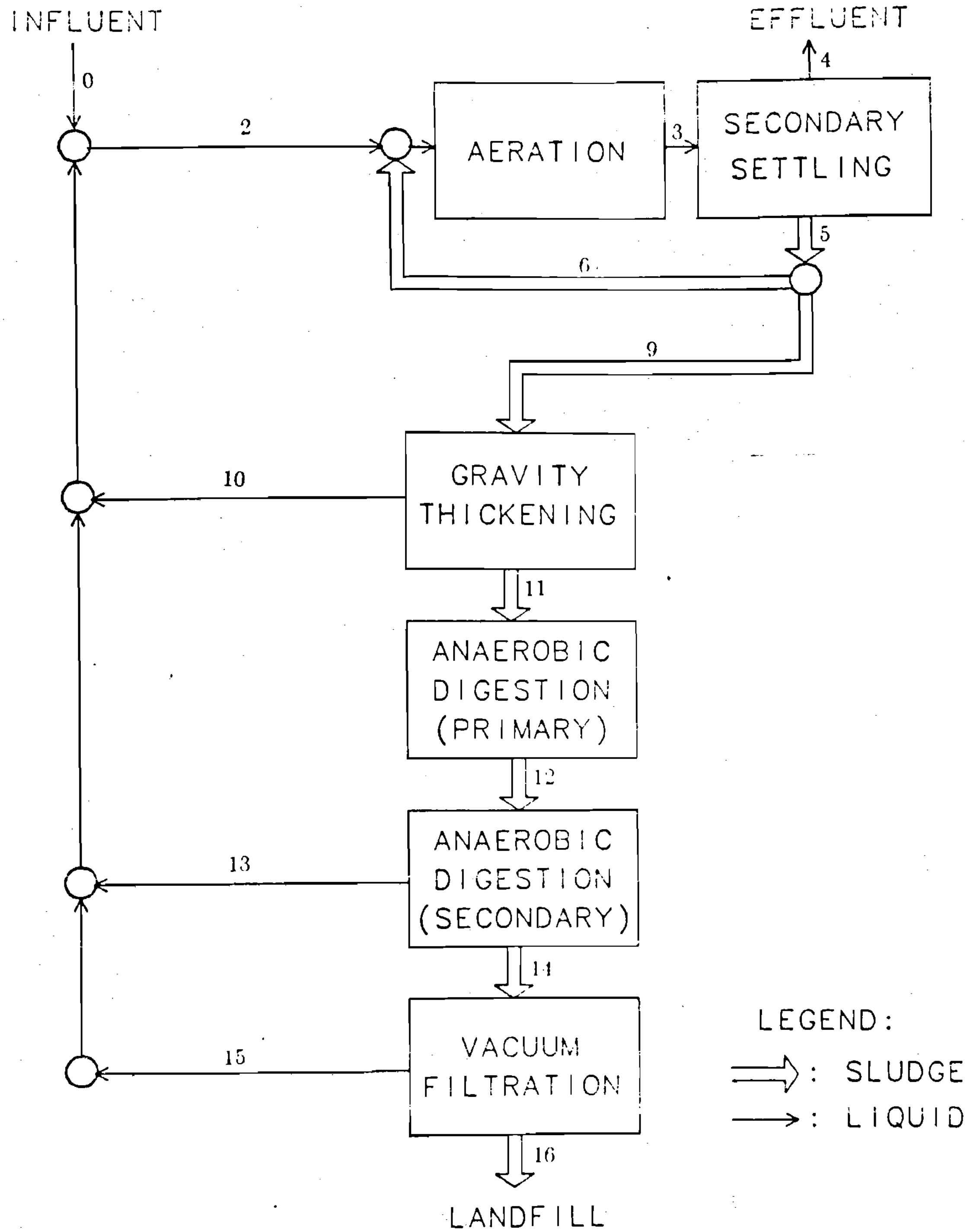

Figure H.1- Wastewater Treatment System Without a Primary Clarifier

```
      PROGRAM MAIN(INPUT,PAR,OUTPUT,LASTVAR,TAPE7,TAPE8,
     +             TAPE5=INPUT,TAPE6=OUTPUT,TAPE4=LASTVAR,
     +             TAPE9=PAR)
      DIMENSION Z(20000)
      COMMON Z
      DATA NCORE/20000/
      CALL GRG(Z,NCORE)
      END
C
C SUBROUTINE GCOMP:
C
      SUBROUTINE GCOMP(G,X)
      COMMON/INITBK/INIT
      DIMENSION G(47),X(51)
      REAL INFLOW(6),STD(2),U(80),V(20)
      IF(INIT.EQ.0) GOTO 50
      READ(9,*)  (INFLOW(I),I=1,6)
      READ(9,*)  (STD(I),I=1,2)
      READ(9,*)  (U(I),I=1,80)
      V(1)=U(1)*U(3)/U(2)
      V(2)=U(3)/U(2)
      V(3)=23.85*U(6)*U(7)/U(8)
      V(4)=U(17)+U(18)
      V(5)=U(20)*EXP(U(21))
      V(6)=(V(4)*(V(5)-1.0))**(1./V(5))*V(5)/(V(5)-1.0)
      V(7)=(U(17)*(U(20)-1.0))**(1./U(20))*U(20)/(U(20)-1.0)
      V(8)=U(28)/U(29)
      V(9)=U(39)/1440./U(36)/(U(37)*U(39)-U(38))/U(41)
     +    /U(42)/U(43)/U(44)**(U(40)-20.0)
      V(10)=U(28)*U(56)
      V(11)=(U(67)*U(68)*(U(69)-1.0))**(1./U(69))*U(69)/(U(69)-1.0)
      V(12)=U(73)*(U(74)*U(75)/(U(76)*U(77)*U(78)))**0.5
      V(13)-U(69)-1.
C
50    EPBND=1.E-6
      DO 100 I=1,51
      IF(X(I).LT.EPBND) THEN
        DO 200 J=1,47
        G(J)=1.E30
200     CONTINUE
        RETURN
      ENDIF
100   CONTINUE
C
C ACTIVATED SLUDGE DESIGN:
C
      G(1)=X(8)-(1.E3*X(11))*X(9)/((1.-(1.E-2*X(16)))*X(18)+
     +     (1.E-2*X(16))*(1.E3*X(19))-X(3))
      G(2)=X(9)-(1.E3*X(10))/(60.*X(1))/24.
      G(3)=X(12)-U(24)*(1.+U(26)*X(8))/(X(8)*(U(23)*U(25)-
     +      U(26))-1.)
      G(4)=X(9)*(1.E3*X(11))/X(8)-U(23)*X(21)/(1.+U(26)*X(8))
      G(5)=(1.E3*X(11))*X(13)+X(13)*X(8)*X(3)/X(9)-X(5)
     +    *X(8)/X(9)-(1.-U(27))*U(26)*(1.E3*X(11))*X(8)
      G(6)=X(14)*(1.E3*X(11))+X(14)*X(8)*X(3)/X(9)
     +     -X(6)*X(8)/X(9)
      G(7)=X(19)+U(80)*X(3)/X(15)-(1.+1./X(15)-X(9)
     +    /X(15)/X(8))*X(11)
      G(8)=(1.+X(13)+X(14))*X(19)-V(7)/
     +     ((60.*X(1))/(1.E2*X(20))*X(17))**(1./U(20))
      G(9)=X(17)-X(15)-(1.E-2*X(16))
      G(10)=(1.+X(13)+X(14))*X(18)+U(30)-U(31)*(1.+X(13)+X(14))
```



```
     +     *(1.E3*X(11))-U(32)*(60.*X(1))/(1.E2*X(20))*(1.+X(15))
      G(11)=60.*X(22)-24.E-3*U(29)*V(9)*60.*X(1)*X(21)
     +      +U(80)*U(28)*V(9)*
     +     (1.E3*X(10))*(1.E3*X(11))/X(8)
      G(12)=X(21)+X(12)-X(2)-V(8)*X(4)
C
C SLUDGE MIXING:
C
      G(13)=X(23)-(60.*X(1))*(1.E-2*X(16))
      G(14)=U(80)*(1.+X(13)+X(14))*(1.E3*X(19))-X(24)
C
C GRAVITY THICKENERR DESIGN:
C
      G(15)=X(25)-X(28)*X(29)/(1.E2*X(26))
      G(16)=X(27)+X(28)-X(23)
      G(17)=U(48)*X(27)+X(28)*X(29)-X(23)*X(24)
      G(18)=X(29)-(U(17)*(U(20)-1.))**(1./U(20))*U(20)/(U(20)-1.)/
     +     (X(28)/(1.E2*X(26)))**(1./U(20))
      G(19)=U(48)*(1.E3*X(19))/X(24)-X(31)
      G(20)=U(80)/U(48)*X(14)*X(29)*X(31)-X(30)
C
C PRIMARY DIGESTER DESIGN:
C
      G(21)=X(35)-U(53)*EXP(ALOG(10.)*10./3.*(U(54)-1.E3/
     +      (X(32)+273.)))
      G(22)=X(30)+X(37)-X(29)
      G(23)=X(33)-(1.E3*X(34))/X(28)/24.
      G(24)=X(38)-X(37)/(1.+X(35)*X(33))
      G(25)=X(39)-U(56)*X(28)*(U(28)*(X(37)-X(38))+U(29)*
     +      1.E-3*X(12))
      G(26)=X(36)-(10.22*1.E-3*X(28)*(X(32)-U(55))
     +      +U(59)*(1.E3*X(34))*U(60)
     +      *8.76E-6*(X(32)-U(57)))/U(58)
      G(27)=X(40)-(3.58E4*8.76/3.6*X(39)*1.E-6-X(36))
C
C SECONDARY DIGESTER DESIGN:
C
      G(28)=X(28)-X(43)-X(44)
      G(29)=X(44)*X(45)/(1.E2*X(42))-X(41)
      G(30)=X(45)-V(11)**(U(69)/V(13))/X(41)**(1./V(13))
      G(31)=X(28)*(X(30)+X(38))-U(70)*X(43)-X(44)*X(45)
      G(32)=X(46)-X(38)/(X(38)+X(30))
C
C VACUUM FILTER DESIGN:
C
      G(33)=X(44)-X(49)-X(50)
      G(34)=X(44)*X(45)-X(49)*U(79)-X(50)*X(51)
      G(35)=X(47)-V(12)*SQRT(X(50)*X(51)/X(49))
      G(36)=X(48)-X(50)*X(51)/X(47)
C
C MASS BALANCE OF RECYCLE STREAMS:
C
      G(37)=(60.*X(1))-INFLOW(1)-X(27)-X(43)-X(49)
      G(38)=U(80)*(60.*X(1))*X(2)-U(80)*INFLOW(1)*INFLOW(2)
     +       -U(80)*X(27)*X(12)
     +      -U(80)*X(43)*U(66)-U(80)*U(66)*X(49)
      G(39)=U(80)*60.*X(1)*X(3)-U(80)*INFLOW(1)*INFLOW(3)
     +       -U(80)*X(27)*X(31)
      G(40)=U(80)*60.*X(1)*X(4)-U(80)*INFLOW(1)*INFLOW(4)
      G(41)=U(80)*60.*X(1)*X(5)-U(80)*INFLOW(1)*INFLOW(5)
     +       -U(80)*X(13)*X(27)*X(31)
     +       -U(70)*X(43)*X(46)-X(46)*X(49)
```

```
     +      *U(79)
      G(42)=U(80)*60.*X(1)*X(6)-U(80)*INFLOW(1)*INFLOW(6)
     +        -U(80)*X(14)*X(27)*X(31)
     +        -U(70)*X(43)*(1.-X(46))
     +        -X(49)*U(79)*(1.-X(46))
      G(43)=X(3)+X(4)+X(5)+X(6)-X(7)
C
C EFFLUENT WATER QUALITY STANDARDS:
C
      G(44)=X(12)+U(27)*V(8)*X(18)-STD(1)
      G(45)=X(18)*(1.+X(13)+X(14))-STD(2)
C
C MIXING REQUIREMENT IN AERATION TANK:
C
      G(46)=(U(45)-(60.*X(22))/(1.E3*X(10)))*1.E2
C
C OBJECTIVE FUNCTION:
C
C AERATION:
C
      CCAT=461.*(1.E3*X(10))**.71
      CCDAA=8533.*(60.*X(22))**.66
      CODAA=187.*(60.*X(22))**.48
      CMDAA=74.4*(60.*X(22))**.55
C
C FINAL SETTLING TANK:
C
      CCFST=824.*(1.E2*X(20))**.77
      IF((1.E2*X(20)).GE.279.) THEN
      COFST=17.15*(1.E2*X(20))**0.6
      CMFST=9.23*(1.E2*X(20))**0.6
      ELSE
      COFST=92.45*(1.E2*X(20))**.3
      CMFST=106.*(1.E2*X(20))**.14
      ENDIF
      CSFST=8.62*(1.E2*X(20))**.76
C
C RETURN SLUDGE PUMPING:
C
      Q5=(60.*X(1))*X(17)
      CCRSP=2779.*Q5**.53
      CORSP=.333*Q5
      CMRSP=.2375*Q5
      IF(Q5.LT.158.) THEN
      CSRSP=300.
      ELSE
      IF(Q5.LT.631.) THEN
      CSRSP=40.57*Q5**.52
      ELSE
      IF(Q5.LT.1580.) THEN
      CSRSP=5.97*Q5**.87
      ELSE
      CSRSP=2.54*Q5
      ENDIF
      ENDIF
      ENDIF
      CPRSP=Q5
C
C RECIRCULATION PUMPING:
C
      QCYCLE=X(27)+X(43)+X(49)
      CCRP=2779.*QCYCLE**.53
```



```
      CORP=.333*QCYCLE
      CMRP=.2375*QCYCLE
      IF(QCYCLE.LT.158.) THEN
      CSRP=300.
      ELSE
      IF(QCYCLE.LT.631.) THEN
      CSRP=40.57*QCYCLE**.52
      ELSE
      IF(QCYCLE.LT.1580.) THEN
      CSRP=5.97*QCYCLE**.87
      ELSE
      CSRP=2.54*QCYCLE
      ENDIF
      ENDIF
      ENDIF
      CPRP=QCYCLE
C
C GRAVITY THICKENER:
C
      CCGT=824.*(1.E2*X(26))**.77
      IF((1.E2*X(26)).GE.279.) THEN
      COGT=17.15*(1.E2*X(26))**0.6
      CMGT=9.23*(1.E2*X(26))**0.6
      ELSE
      COGT=92.45*(1.E2*X(26))**.3
      CMGT=106.*(1.E2*X(26))**.14
      ENDIF
      CSGT=8.62*(1.E2*X(26))**.76
C
C PRIMARY DIGESTER:
C
      CCPAND=2323.*(1.E3*X(34))**.59
      IF((1.E3*X(34)).GE.5678.) THEN
      COPAND=1.29*(1.E3*X(34))**.83
      CMPAND=.83*(1.E3*X(34))**.82
      ELSE
      IF((1.E3*X(34)).GE.1968.) THEN
      COPAND=14.*(1.E3*X(34))**.55
      CMPAND=8.5*(1.E3*X(34))**.55
      ELSE
      COPAND=192.*(1.E3*X(34))**.2
      CMPAND=113.*(1.E3*X(34))**.21
      ENDIF
      ENDIF
      IF((1.E3*X(34)).GE.2839.) THEN
      CSPAND=14.4*(1.E3*X(34))**.66
      ELSE
      CSPAND=142.*(1.E3*X(34))**.37
      ENDIF
C
C SECONDARY DIGESTER:
C
      VSAND=1.E2*U(71)*X(42)
      CCSAND=2323.*VSAND**.59
      IF(VSAND.GE.5678.) THEN
      COSAND=1.29*VSAND**.83
      CMSAND=.83*VSAND**.82
      ELSE
      IF(VSAND.GE.1968.) THEN
      COSAND=14.*VSAND**.55
      CMSAND=8.5*VSAND**.55
      ELSE
```

```
      COSAND=192.*VSAND**.2
      CMSAND=113.*VSAND**.21
      ENDIF
      ENDIF
      IF(VSAND.GE.2839.) THEN
      CSSAND=14.4*VSAND**.66
      ELSE
      CSSAND=142.*VSAND**.37
      ENDIF
C
C VACUUM FILTER:
C
      Q16X16=X(50)*X(51)
      CCVF=29180.*X(48)**.71
      CSVF=230.*Q16X16**.71+182.*Q16X16**.86
      COVF=197.55*Q16X16**.58
      IF(Q16X16.GE.519.) THEN
      CMVF=5.55557*Q16X16**.84
      ELSE
      IF(Q16X16.GE.103.) THEN
      CMVF=20.*Q16X16**.63
      ELSE
      CMVF=41.5*Q16X16**.48
      ENDIF
      ENDIF
C
      G(47)=V(1)*(CCAT+CCDAA+CCFST+CCRSP+CCRP
     +      +CCGT+CCPAND+CCSAND+CCVF+72053.*X(50)**.74)
     +      +U(4)*(CODAA+CMDAA+COFST
     +      +CMFST+CORSP+CMRSP+COGT+CMGT+COPAND
     +      +CMPAND+COSAND+CMSAND
     +      +COVF+CMVF+CORP+CMRP+8024.*X(50)**.667)
     +      +V(2)*(CSFST+CSRSP+CSGT+CSPAND+CSSAND+CSRP
     +      +CSVF)
     +      +V(3)*(CPRSP+CPRP)
     +      +U(1)*U(5)*(3.62E-2)*X(50)*X(51)-U(61)*
     +      3.6E3/1.055*X(40)
      RETURN
      END
C
C SUBROUTINE REPORT
C
      SUBROUTINE REPORT(G,X,M,N,CON,VAR,XO)
      DIMENSION X(N),G(M),CON(M),VAR(N),XO(N)
      COMMON/INITBK/INIT
      IF(INIT.EQ.1) RETURN
      WRITE(4,10) (I,X(I),I=1,N)
10    FORMAT(3X,I3,4X,E20.10)
      RETURN
      END
```

APPENDIX I

GRG MODEL FOR THE SYSTEM WHERE WASTE ACTIVATED SLUDGE IS RECIRCULATED TO PRIMARY CLARIFIER

The system is shown in Figure I.1. There are 51 variables and 47 equations in the GRG model. A list of the model variables is provided in Table I.1.

Table I.1 - Summary of Model Variables : Waste Activated Sludge Recirculated to the Primary Clarifier

Variable Index	Unit	Meaning
1	m^3/min	$Q_1/60$
2	g/m^3	S_1
3	g/m^3	M_{a1}
4	g/m^3	M_{d1}
5	g/m^3	M_{i1}
6	g/m^3	M_{f1}
7	g/m^3	M_{t1}
8	m^3/hr	L_p
9	$100m^2$	$A_p/100$
10	–	M_{t2}/M_{t1}
11	m^3/min	$Q_2/60$
12	m^3/hr	Q_8
13	days	θ_c
14	days	θ
15	$1000m^3$	$V/1000$
16	kg/m^3	M_{a3}
17	g/m^3	S_3
18	–	M_{i3}/M_{a3}
19	–	M_{f3}/M_{a3}
20	–	r
21	–	$100w$
22	–	$r+w$
23	g/m^3	M_{a4}
24	kg/m^3	M_{a5}
25	$100m^2$	$A_f/100$
26	g/m^3	S
27	m^3/sec	$Q_a/60$
28	m^3/hr	Q_7
29	kg/m^3	M_{t7}
30	°C	T_d
31	days	θ_d

Table I.1 (continued)

Variable Index	Unit	Meaning
32	$1000 m^3$	$V_d/1000$
33	day^{-1}	K_1
34	$10^6 kWhr/year$	q
35	kg/m^3	$M_{a11}+M_{d11}+M_{t11}$
36	kg/m^3	M_{t12}
37	m^3/hr	G
38	$10^6 kWhr/year$	N
39	$kg/m^2/hr$	L_d
40	$100 m^2$	$A_d/100$
41	m^3/hr	Q_{13}
42	m^3/hr	Q_{14}
43	kg/m^3	M_{t14}
44	–	M_{t12}/M_{t12}
45	$kg/m^2/hr$	L_f
46	m^2	A_v
47	m^3/hr	Q_{15}
48	m^3/hr	Q_{16}
49	kg/m^3	M_{t16}
50	$100 kg/m^3$	$M_{t8}/100$
51	kg/m^3	M_{t8}

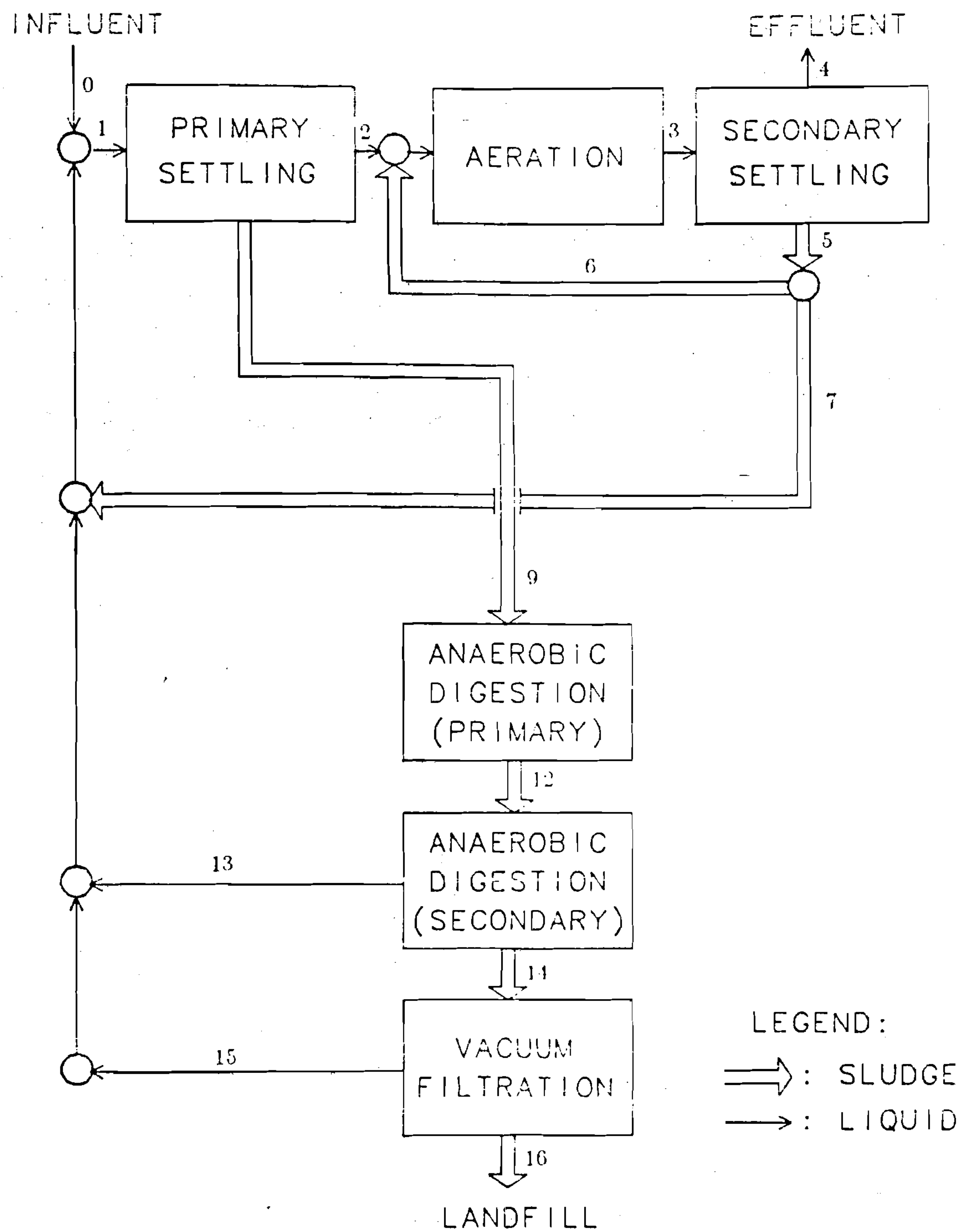

Figure I.1- Recirculation of Waste Activated Sludge to Primary Clarifier

```
      PROGRAM MAIN(INPUT,PAR,OUTPUT,LASTVAR,TAPE7,TAPE8,
     +                 TAPE5=INPUT,TAPE6=OUTPUT,TAPE4=LASTVAR,
     +                 TAPE9=PAR)
      DIMENSION Z(20000)
      COMMON Z
      DATA NCORE/20000/
      CALL GRG(Z,NCORE)
      END
C
C SUBROUTINE GCOMP:
C
      SUBROUTINE GCOMP(G,X)
      COMMON/INITBK/INIT
      DIMENSION G(47),X(51)
      REAL INFLOW(6),STD(2),U(80),V(20)
      IF(INIT.EQ.0) GOTO 50
      READ(9,*)  (INFLOW(I),I=1,6)
      READ(9,*)  (STD(I),I=1,2)
      READ(9,*)  (U(I),I=1,80)
      V(1)=U(1)*U(3)/U(2)
      V(2)=U(3)/U(2)
      V(3)=23.85*U(6)*U(7)/U(8)
      V(4)=U(17)+U(18)
      V(5)=U(20)*EXP(U(21))
      V(6)=(V(4)*(V(5)-1.0))**(1./V(5))*V(5)/(V(5)-1.0)
      V(7)=(U(17)*(U(20)-1.0))**(1./U(20))*U(20)/(U(20)-1.0)
      V(8)=U(28)/U(29)
      V(9)=U(39)/1440./U(36)/(U(37)*U(39)-U(38))/U(41)
     +        /U(42)/U(43)/U(44)**(U(40)-20.0)
      V(10)=U(28)*U(56)
      V(11)=(U(67)*U(68)*(U(69)-1.0))**(1./U(69))*U(69)/(U(69)-1.0)
      V(12)=U(73)*(U(74)*U(75)/(U(76)*U(77)*U(78)))**0.5
      V(13)=U(69)-1.
C
50    EPBND=1.E-6
      DO 100 I=1,51
      IF(X(I).LT.EPBND) THEN
        DO 200 J=1,47
        G(J)=1.E30
200     CONTINUE
        RETURN
      ENDIF
100   CONTINUE
C
C PRIMARY SETTLING TANK DESIGN:
C
      G(1)=1.0-U(11)*X(7)**U(12)/X(8)**U(13)-X(10)
      G(2)=(60.*X(11))/(1.E2*X(9))-X(8)
      G(3)=(60.*X(11))+X(12)-(60.*X(1))
      G(4)=1.E-5*X(7)*X(10)*60.*X(11)+X(50)*X(12)-1.E-5*60.
     +        *X(1)*X(7)
C
C ACTIVATED SLUDGE DESIGN:
C
      G(5)=X(13)-(1.E3*X(16))*X(14)/((1.-(1.E-2*X(21)))*X(23)+
     +        (1.E-2*X(21))*(1.E3*X(24))-X(3)*
     +        X(10))
      G(6)=X(14)-(1.E3*X(15))/(60.*X(11))/24.
      G(7)=X(17)-U(24)*(1.+U(26)*X(13))/(X(13)*(U(23)*U(25)-
     +        U(26))-1.)
      G(8)=X(14)*(1.E3*X(16))/X(13)-U(23)*X(26)/(1.+U(26)*X(13))
      G(9)=(1.E3*X(16))*X(18)+X(18)*X(13)*X(3)*X(10)/X(14)-
```



```
     +        X(5)*X(10)
     +     *X(13)/X(14)-(1.-U(27))*U(26)*(1.E3*X(16))*X(13)
      G(10)=X(19)*(1.E3*X(16))+X(19)*X(13)*X(3)*X(10)/X(14)
     +        -X(6)*X(10)*X(13)/X(14)
      G(11)=X(24)+U(80)*X(3)*X(10)/X(20)-(1.+1./X(20)-X(14)
     +       /X(20)/X(13))*X(16)
      G(12)=(1.+X(18)+X(19))*X(24)-V(7)/
     +        ((60.*X(11))/(1.E2*X(25))*X(22))**(1./U(20))
      G(13)=X(22)-X(20)-(1.E-2*X(21))
      G(14)=(1.+X(18)+X(19))*X(23)+U(30)-U(31)*(1.+X(18)+X(19))
     +        *(1.E3*X(16))-U(32)*(60.*X(11))/(1.E2*X(25))*(1.+X(20))
      G(15)=60.*X(27)-24.E-3*U(29)*V(9)*60.*X(11)*X(26)
     +        +U(80)*U(28)*V(9)*
     +       (1.E3*X(15))*(1.E3*X(16))/X(13)
      G(16)=X(26)+X(17)-X(2)-V(8)*X(4)*X(10)
C
C SLUDGE MIXING:
C
      G(17)=X(28)-(60.*X(11))*(1.E-2*X(21))
      G(18)=U(80)*(1.+X(18)+X(19))*(1.E3*X(24))-X(29)
C
C PRIMARY DIGESTER DESIGN:
C
      G(19)=X(51)*1.E-2-X(6)*X(50)/X(7)
      G(20)=X(33)-U(53)*EXP(ALOG(10.)*10./3.*(U(54)-1.E3/
     +         (X(30)+273.)))
      G(21)=X(51)+X(35)-X(50)*1.E2
      G(22)=X(31)-(1.E3*X(32))/X(12)/24.
      G(23)=X(36)-X(35)/(1.+X(33)*X(31))
      G(24)=X(37)-U(56)*X(12)*(U(28)*(X(35)-X(36))+U(29)*1.E-
     +         3*X(2))
      G(25)=X(34)-(10.22*1.E-3*X(12)*(X(30)-U(55))
     +        +U(59)*(1.E3*X(32))*U(60)
     +        *8.76E-6*(X(30)-U(57)))/U(58)
      G(26)=X(38)-(3.58E4*8.76/3.6*X(37)*1.E-6-X(34))
C
C SECONDARY DIGESTER DESIGN:
C
      G(27)=X(12)-X(41)-X(42)
      G(28)=X(42)*X(43)/(1.E2*X(40))-X(39)
      G(29)=X(43)-V(11)*(1.E2*X(40)/X(42))**(1./U(69))
      G(30)=X(12)*(X(51)+X(36))-U(70)*X(41)-X(42)*X(43)
      G(31)=X(44)-X(36)/(X(36)+X(51))
C
C VACUUM FILTER DESIGN:
C
      G(32)=X(42)-X(47)-X(48)
      G(33)=X(42)*X(43)-X(47)*U(79)-X(48)*X(49)
      G(34)=X(45)-V(12)*SQRT(X(48)*X(49)/X(47))
      G(35)=X(46)-X(48)*X(49)/X(45)
C
C MASS BALANCE OF RECYCLE STREAMS:
C
      G(36)=(60.*X(1))-INFLOW(1)-X(28)-X(41)-X(47)
      G(37)=U(80)*(60.*X(1))*X(2)-U(80)*INFLOW(1)*INFLOW(2)
     +        -U(80)*X(28)*X(17)
     +       -U(80)*X(41)*U(66)-U(80)*U(66)*X(47)
      G(38)=U(80)*60.*X(1)*X(3)-U(80)*INFLOW(1)*INFLOW(3)
     +        -X(28)*X(24)
      G(39)=U(80)*60.*X(1)*X(4)-U(80)*INFLOW(1)*INFLOW(4)
      G(40)=U(80)*60.*X(1)*X(5)-U(80)*INFLOW(1)*INFLOW(5)
     +        -X(18)*X(28)*X(24)-U(70)*X(41)*X(44)-X(44)*X(47)*U(79)
```

```
      G(41)=U(80)*60.*X(1)*X(6)-U(80)*INFLOW(1)*INFLOW(6)
     +        -X(19)*X(28)*X(24)-U(70)*X(41)*(1.-X(44))
     +        -X(47)*U(79)*(1.-X(44))
      G(42)=X(3)+X(4)+X(5)+X(6)-X(7)
C
C THICKENING MODEL FOR PRIMARY SEDIMENTATION:
C
      G(43)=X(50)-V(6)*((1.E2*X(9))*U(16)/X(12))**(1./V(5))*1.E-2
C
C EFFLUENT WATER QUALITY STANDARDS:
C
      G(44)=X(17)+U(27)*V(8)*X(23)-STD(1)
      G(45)=X(23)*(1.+X((18)+X(19))-STD(2)
C
C MIXING REQUIREMENT IN AERATION TANK:
C
      G(46)=(U(45)-(60.*X(27))/(1.E3*X(15)))*1.E2
C
C OBJECTIVE FUNCTION:
C
C PRIMARY SETTLING TANK:
C
      CCPST=824.*(1.E2*X(9))**.77
      IF((1.E2*X(9)).GE.279.) THEN
      COPST=17.15*(1.E2*X(9))**0.6
      CMPST=9.23*(1.E2*X(9))**0.6
      ELSE
      COPST=92.45*(1.E2*X(9))**.3
      CMPST=106.*(1.E2*X(9))**.14
      ENDIF
      CSPST=8.62*(1.E2*X(9))**.76
C
C PRIMARY SLUDGE PUMPING:
C
      CCPSP=16042.*X(12)**.53
      COPSP=374.*X(12)**.41
      CMPSP=166.*X(12)**.43
      CSPSP=385.*X(12)**.64
      CPPSP=X((12)
C
C AERATION:
C
      CCAT=461.*(1.E3*X(15))**.71
      CCDAA=8533.*(60.*X(27))**.66
      CODAA=187.*(60.*X(27))**.48
      CMDAA=74.4*(60.*X(27))**.55
C
C FINAL SETTLING TANK:
C
      CCFST=824.*(1.E2*X(25))**.77
      IF((1.E2*X(25)).GE.279.) THEN
      COFST=17.15*(1.E2*X(25))**0.6
      CMFST=9.23*(1.E2*X(25))**0.6
      ELSE
      COFST=92.45*(1.E2*X(25))**.3
      CMFST=106.*(1.E2*X(25))**.14
      ENDIF
      CSFST=8.62*(1.E2*X(25))**.76
C
C RETURN SLUDGE PUMPING:
C
      Q5=(60.*X(11))*X(22)
```



```
      CCRSP=2779.*Q5**.53
      CORSP=.333*Q5
      CMRSP=.2375*Q5
      IF(Q5.LT.158.) THEN
      CSRSP=300.
      ELSE
      IF(Q5.LT.631.) THEN
      CSRSP=40.57*Q5**.52
      ELSE
      IF(Q5.LT.1580.) THEN
      CSRSP=5..97*Q5**.87
      ELSE
      CSRSP=2.54*Q5
      ENDIF
      ENDIF
      ENDIF
      CPRSP=Q5
C
C RECIRCULATION PUMPING:
C
      QCYCLE=X(41)+X(47)
      CCRP=2779.*QCYCLE**.53
      CORP=.333*QCYCLE
      CMRP=.2375*QCYCLE
      IF(QCYCLE.LT.158.) THEN
      CSRP=300.
      ELSE
      IF(QCYCLE.LT.631.) THEN
      CSRP=40.57*QCYCLE**.52
      ELSE
      IF(QCYCLE.LT.1580.) THEN
      CSRP=5.97*QCYCLE**.87
      ELSE
      CSRP=2.54*QCYCLE
      ENDIF
      ENDIF
      ENDIF
      CPRP=QCYCLE
C
C PRIMARY DIGESTER:
C
      CCPAND=2323.*(1.E3*X(32))**.59
      IF((1.E3*X(32)).GE.5678.) THEN
      COPAND=1.29*(1.E3*X(32))**.83
      CMPAND=.83*(1.E3*X(32))**.82
      ELSE
      IF((1.E3*X(32)).GE.1968.) THEN
      COPAND=14.*(1.E3*X(32))**.55
      CMPAND=8.5*(1.E3*X(32))**.55
      ELSE
      COPAND=192.*(1.E3*X(32))**.2
      CMPAND=113.*(1.E3*X(32))**.21
      ENDIF
      ENDIF
      IF((1.E3*X(32)).GE.2839.) THEN
      CSPAND=14.4*(1.E3*X(32))**.66
      ELSE
      CSPAND=142.*(1.E3*X(32))**.37
      ENDIF
C
C SECONDARY DIGESTER:
C
```

```
      VSAND=1.E2*U(71)*X(40)
      CCSAND=2323.*VSAND**.59
      IF(VSAND.GE.5678.) THEN
      COSAND=1.29*VSAND**.83
      CMSAND=.83*VSAND**.82
      ELSE
      IF(VSAND.GE.1968.) THEN
      COSAND=14.*VSAND**.55
      CMSAND=8.5*VSAND**.55
      ELSE
      COSAND=192.*VSAND**.2
      CMSAND=113.*VSAND**.21
      ENDIF
      ENDIF
      IF(VSAND.GE.2839.) THEN
      CSSAND=14.4*VSAND**.66
      ELSE
      CSSAND=142.*VSAND**.37
      ENDIF
C
C VACUUM FILTER:
C
      Q16X16=X(48)*X(49)
      CCVF=29180.*X(46)**.71
      CSVF=230.*Q16X16**.71+182.*Q16X16**.86
      COVF=197.55*Q16X16**.58
      IF(Q16X16.GE.519.) THEN
      CMVF=5.57*Q16X16**.84
      ELSE
      IF(Q16X16.GE.103.) THEN
      CMVF=20.*Q16X16**.63
      ELSE
      CMVF=41.5*Q16X16**.48
      ENDIF
      ENDIF
C
      G(47)=V(1)*(CCPST+CCPSP+CCAT+CCDAA+CCFST+CCRSP+CCRP
     +     +CCPAND+CCSAND+CCVF+72053.*X(48)**.74)
     +     +U(4)*(COPST+CMPST+COPSP+CMPSP+CODAA+CMDAA+COFST
     +     +CMFST+CORSP+CMRSP+COPAND+CMPAND+COSAND+CMSAND
     +     +COVF+CMVF+CORP+CMRP+8024.*X(48)**.667)
     +     +V(2)*(CSPST+CSPSP+CSFST+CSRSP+CSPAND+CSSAND+CSRP
     +     +CSVF)
     +     +V(3)*(CPPSP+CPRSP+CPRP)
     +     +U(1)*U(5)*(3.62E-2)*X(48)*X(49)-U(61)*
     +     3.6E3/1.055*X(38)
      RETURN
      END
C
C SUBROUTINE REPORT
C
      SUBROUTINE REPORT(G,X,M,N,CON,VAR,XO)
      DIMENSION X(N),G(M),CON(M),VAR(N),XO(N)
      COMMON/INITBK/INIT
      IF(INIT.EQ.1) RETURN
      WRITE(4,10) (I,X(I),I=1,N)
10    FORMAT(3X,I3,4X,E20.10)
      RETURN
      END
```

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