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An Experimental Study of Eddy Diffusion Coefficients,  
Evapotranspiration and Water Use Efficiency

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## ABSTRACT

### AN EXPERIMENTAL STUDY OF EDDY DIFFUSION COEFFICIENTS, EVAPOTRANSPIRATION AND WATER USE EFFICIENCY

Measurements of mass transfer coefficients were made directly by the use of point and line sources. The gas used was propane. The studies revealed that the form of the profile of mass transfer coefficients was markedly dependent on crop geometry, with a local maximum and minimum within the crop canopy.

Further studies demonstrated the anisotropy of mass transfer coefficients above the crop canopy wherein the downwind component was greater than the crosswind component. Crosswind and vertical components were nearly equal.

Studies were also made on energy balances within crops and with individual plant leaves.

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KEYWORDS--\*energy balances/ \*evapotranspiration/ \*mass transfer  
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## 1. INTRODUCTION

Use and reuse of water by plants is a prime purification process in the hydrologic cycle.

The rate of use, or evapotranspiration rate, when integrated over time permits evaluation of the potential intake of soil water and runoff. Rainfall is relatively easily measured and runoff in stream flow can be estimated with precision. However, there are still two unknowns remaining in the water balance--the deep percolation and evapotranspiration.

Our objectives included development and examination of a novel method of measuring evapotranspiration. Since deep percolation and lateral seepage result in movement of soluble salts, it is of considerable importance to assess this component of the water balance. One can propose the concept of a solutograph having characteristics analogous to the hydrograph.

Mineral-free rain water picks up soluble material on contact with the land surface. It carries this load of dissolved salts as it moves as soil water or as runoff. The soluble salt balance both in soils and in bodies of fresh water is thus a function of the rates of water flow and salt concentrations in these separate subsystems.

Since plants contain say 10 percent mineral matter and with an annual accretion of dry matter of about 4 tons per acre per annum, they remove about 800 lbs. per acre of mineral matter. In the process some 1500 tons of water are transpired. Assuming seepage and runoff are 300 tons per acre then the plants removal could constitute a saving of 3000 ppm dissolved salts in the runoff water. However, unless insoluble plant residues are accumulating in the soil, the rate of decomposition of plant residues will match the rate of accretion of living plant material with consequent release of a similar amount of mineral matter.

The plant or crop cover then plays its most important role in not only reducing the phase and amplitude of fluctuations in runoff water itself, but in reducing the phase and amplitude of variations in its mineral content.

Both these ameliorating effects are dependent not only on evapotranspiration rates but also on growth rates. These rates, on a ground area basis, reflect the integrated behavior of the plant community-environment system. However, it is not only analysis of this macrosystem but extension of the analysis to single plant and single leaf performance which will provide understanding and improved utilization of these plant community-environment systems.

Ability to measure plant growth in terms of carbon fixation and evapotranspiration provides the means of accounting whereby water balance and potential nutrient fixation rates can be assessed. Further, the water use efficiency of natural and crop plant communities is directly derived from such measurements and can be stated in terms of photosynthetic rate in relation to water use.

## 2. THEORETICAL BACKGROUND

### 2.1 - One-Dimensional Flow in the Soil-Plant-Atmosphere System

In plant communities, in the absence of horizontal gradients in environmental properties, the fluxes can be taken as one-dimensional, vertical fluxes. In the process of photosynthesis carbon dioxide is consumed and in the process of transpiration, water vapor is produced.

Thus, within the canopy of the crop or plant community a flow equation can be written

$$\frac{\partial e}{\partial t} = \frac{\partial}{\partial z} (F) + S(z,t)$$

where we are concerned with the flow  $F$  of some property of concentration  $C$  which is both time ( $t$ ) and space ( $z$ ) dependent and which is further produced at a rate  $S$  which, also, is time and space dependent.

$S$ , the source or sink term, might be regarded as the rate of consumption or rate of production of  $\text{CO}_2$  or  $\text{H}_2\text{O}$  respectively by say individual leaf or unit leaf area. Above the crop

$$S = 0$$

and

$$\int_z^{z_1} S \cdot dz = F$$

for  $z_1 > h$  where  $h$  is the crop height. The flow  $F$  occurs by eddy diffusion which is orders of magnitude greater than molecular diffusion and, for  $\frac{\partial c}{\partial t} = 0$ , the steady state condition,

$$F = K \frac{dc}{dz}$$

which defines  $K$ , the eddy diffusion coefficient.

Thus in the steady state, the flux of water (evapotranspiration rate) or carbon dioxide (net photosynthesis) can be determined from measures of the appropriate gradient and the eddy diffusion coefficient above the crop.

When the same data are available within the crop, estimates of the distribution of source strength can be obtained.

## 2.2 - Methods of Arriving at Fluxes

To estimate  $K$ , the eddy diffusion coefficient, various methods have been employed<sup>(1,2,3)\*</sup>. All methods make assumptions on the equality of the eddy transfer coefficients for mass, momentum and/or sensible heat. Since mass transfer was the object of study here, it was decided to introduce artificial sources and, from concentration distributions about these sources, to compute the eddy diffusion coefficient. Ideally, with an extensive plane source of known strength, the proportionality constant between flux (invariant with height in the steady state) and gradient is, unambiguously, the eddy diffusion coefficient. However, the mechanics of introducing such a source proved too great to be a practical undertaking.

Instead, line and point sources were used and, from diffusion theory and continuity considerations, analysis of concentration distributions provided estimates of the eddy diffusion coefficient.

## 2.3 - Use of Controlled Sources

Propane gas was released from both point and "infinite" line sources and concentrations at varying distances from the source were measured. Propane was released at points in and above the crop and air samples were pumped from sampling points about the points of release

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\*Raised numbers in parentheses refer to reference list.

into an instrument trailer, for analysis using a gas chromatograph. An automated system allowed continuous sampling and duplicate analyses were made of the mean concentration averaged over half hour sampling periods. The analysis system made 24 analyses every half hour, duplicate analyses at 12 sampling stations.

From these concentration distribution data and the known source strength (ml of propane gas per sec. or ml of propane gas per cm per sec.) estimates of the eddy diffusion coefficient were obtained.



### 3. MAIN FINDINGS

#### 3.1 - Profile Studies

Studies were made in soybean crops at various stages of maturity and height profiles of the eddy diffusion coefficient were obtained.

The form of these profiles was markedly dependent on crop geometry. While the rows were distinguishable, the profile showed a local maximum and minimum. The former occurred in the lower part of the crop canopy where leaf area per unit volume of canopy was low. The local minimum occurred in the zone of highest leaf area density, say, at  $2/3 h$  where  $h$  was the crop height.

These were the first direct measures which unequivocally demonstrated that the form of the relation between  $K$  (the eddy diffusion coefficient) and height could not be inferred from the inside one-dimensional aerodynamic methods of analysis<sup>(4,5)</sup>.

#### 3.2 - The Downwind, Crosswind and Vertical Components of $K$

Further studies, using horizontal and vertical line sources as well as point sources simultaneously, showed the anisotropy of mass transfer coefficients in and above the crop canopy. Our results showed that the downwind component,  $K_x$ , was greater than the crosswind component,  $K_y$ , which was of similar magnitude to the vertical component,  $K_z$ .

Again, these data on variation in mass transfer coefficients in natural environments were the first of their kind and were not based on assumptions of equality of mass and heat transfer coefficients.

### 3.3 - Energy Balances Within the Crop

In conjunction with the mass transfer studies, partitioning of net radiation was undertaken, as well as the attenuation of short wave radiation within the crop.

These studies again highlighted the effects of anisotropy of the crop canopy on the microclimate. The most significant finding was the clear demonstration of the need to develop crop production and evapotranspiration models based on two- and three-dimensional fluxes.

### 3.4 - Leaf Energy Balances

The source and sink terms within the canopy are generated by the exchange between the bulk air-plant medium and the plant itself.

In general terms

$$S = HAC$$

where H is an exchange coefficient and  $\Delta C$  a concentration difference between the leaf and the surrounding air.

Extensive studies were made of radiant energy, sensible heat and latent heat exchange between single leaves and their environment. These showed clearly that the exchange coefficients for sensible heat and latent heat (water vapor) were not the same for simple, freely evaporating surfaces.

In these studies equipment was developed which resulted in control, for the first time, of the overall leaf energy balance. All energy exchange processes were controlled, and measurable, including the long wave reradiation from the leaf.

Considerable insight was gained into the significance of long wave reradiation in both laboratory and field situations.

#### 4. APPLICATION OF FINDINGS

The water and solute balances as well as energy balances in the biosphere result from flow and exchange of these components of the system. It is necessary to define the physical nature of the flow or exchange process and to be able to measure the rate of flow or exchange before the interlocking of component subsystems can be undertaken.

Where direct measurement of the rate of flow is not feasible, indirect methods based on knowledge of the flow process must be undertaken.

Our findings have immediate application in both controlled experimentation and field surveys of mass exchange relevant to water and salt (nutrient) balances as well as energy balances.

Concerning the latter, Dr. E. R. Perrier has undertaken extension and development of methods of analysis of momentum or energy exchange in two-dimensional fields. His work confirms the "partial isolation" of flow within the crop canopy. This result is implicit in the form of the profiles of eddy diffusion coefficients found in this study. The further application of these findings will add considerably to improved understanding of fluid flow over irregular surfaces.

Our methods provide means of estimating evapotranspiration and net photosynthesis over periods of days. Already we have found that water stress in soybeans limits their photosynthetic activity, even under relatively low radiation loads. Further, in 24-hour studies, we have found abundant evidence of the role of stomatal control not only in limiting water use and, regrettably, photosynthesis, but also in the maintenance of high respiratory rates at night.

The study carried out has provided published accounts of a number of facets of crop evapotranspiration and crop growth. Further, the work has generated a continuing program in crop aerodynamics, crop energy balance and the nature of plant physiological processes limiting exchange between the plant or crop and its environment. In terms of water use and quality, plant growth and cover not only modify the form of short-term and long-term hydrographs but also the pattern of soluble salt fluxes in the overland and seepage flows.

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