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OPTIMAL PATROL ROUTING AND SCHEDULING FOR PARKING  
ENFORCEMENT CONSIDERING DRIVERS' PARKING BEHAVIOR

BY

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THESIS

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## ABSTRACT

Logistics costs constitute a considerable proportion of overall daily expenses for many public sectors, among which parking enforcement agencies are some of the most prominent examples. While currently there is little research about the planning of efficient parking enforcement patrol operations, this work presents several models to generate patrol schemes that help parking departments achieve low operational costs and effective enforcement. This thesis considers two levels of problems: i) parking behavior of drivers based on given patrol frequency (but not schedule), and ii) parking enforcement patrol routing and scheduling based on the parking behavior of drivers. Driver determines optimal payment based on the distribution of parking duration, parking prices, citation fines, and patrol frequencies via a newsvendor model. As the intensity of parking enforcement increases, illegal parking is expected to occur less frequently. However, improving parking enforcement sometimes requires more frequent patrols that lead to higher agency costs. In order to find the optimal trade-off point, the problem is further formulated into a Vehicle Routing Problem (VRP). Solving this bi-level optimization problem means that the cost is reduced while anticipated parking offenses are limited to a certain level. We present a traditional discrete mixed-integer programming model, and a continuous approximation model based on the method of continuum approximation. Numerical tests are performed in order to examine the performance of these two models using randomly-generated datasets. Sensitivity analyses show that as parking price or demand increases, or citation fine decreases, more frequent patrols are required to maintain the healthy operation of the parking lots. The results also validate that the method of continuum approximation can offer good estimation of the agency cost for the parking patrol problem with comparatively minimal runtime.

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# TABLE OF CONTENTS

CHAPTER 1 INTRODUCTION .....	1
CHAPTER 2 LITERATURE REVIEW .....	6
2.1 Parking behavior .....	6
2.2 Periodic vehicle routing problem.....	8
2.3 Patrol models .....	11
2.4 Continuum approximation .....	12
CHAPTER 3 METHODOLOGY .....	15
3.1 Parking violation models .....	15
3.2 A discrete agency model.....	20
3.3 A continuous agency model.....	24
CHAPTER 4 NUMERICAL EXPERIMENTS .....	30
4.1 Parameter setting.....	30
4.2 Numerical test .....	32
4.3 Sensitivity test.....	36
4.4 Conclusion .....	42
CHAPTER 5 CONCLUSION AND FUTURE RESEARCH .....	44
5.1 Conclusion .....	44
5.2 Future research options.....	46
REFERENCES .....	49
APPENDIX A NUMERICAL TEST OF VIOLATION PROBABILITY LIMIT .....	57

# CHAPTER 1

## INTRODUCTION

Public agencies continuously strive to optimize their operational efficiencies by reducing costs and/or improving service performance. In many circumstances, such public sector organizations are responsible for keeping communities and societies running in a healthy manner with patrol operations. Two of the most important functions of patrols are: 1) rule-breakers can potentially be penalized by means of monetary losses or other forms of punishment, and 2) rule-breakers may become discouraged by the risk of being subject to penalties, and such people become more likely to obey regulations in the future. For instance, traffic enforcement patrols are believed to be important for limiting reckless driving behavior by issuing tickets and suspending driving licenses; frequent police patrols are generally associated with safer communities because the presence of police officers constitutes *per se* an effective warning to potential criminals; and border patrols are a primary reasons why smuggling does not occur on an hourly basis. This thesis focuses on effective parking enforcement patrols, which helps guarantee the turnover rate of parking spaces and helps maintain a prosperous and dynamic community.

Most parking lots in urban areas use pay systems by which parking fees are paid either before drivers leave their vehicles, or after drivers come back to use their vehicles. Paying afterwards means that a driver pays for a fixed span of time for parking, and they need not estimate the parking fees based on their anticipated stays. Moreover, as regards municipal parking lot owners, the expense of management can be quite small since most of the work can be conducted using machines, and little parking enforcement is required. However, this is not always the best solution for urban parking problems. For example, on-street parking facilities cannot be operated with making payments afterwards because it is not realistic to build entrances and exit in open environments. Parking lots which experiences high demand do not allow drivers to park for as long as they want, and a parking time limit is often specified. Therefore, the prepaid system of parking, meaning specifically metered parking system, plays an important role in modern life.



**Figure 1.1. Traditional meters (Marlith, 2008)**



**Figure 1.2. Fully-electric multi-space meter (Burdette, 2010)**

A parking meter is a device that is widely used to enforce the policies of on-street or off-street parking in North America and Europe. Drivers feed parking fees into the meters and are then allowed to park the vehicle for a specified period of time. Parking fees can be paid using coins, cash-keys, credit cards or through the Internet, depending on the type of meters or pay

stations. There are several different types of electronic and electromechanical meter machines, and some merely work as coin-collectors while others are equipped with more advanced technologies. Traditional parking meters are simply timers that collect coins, and flash after expiration to notify patrol officers of parking violations, see Figure 1.1. Modern parking meters can be fully electrical, and may send signals to technicians when repairs are needed or be able to maintain a number of parking spaces in real time, see Figure 1.2. Some advanced parking meters can even clear the remaining time when a vehicle departs, so that there is no chance for the next vehicle to park for free, or pay less for the parking duration. In order to maintain an optimal occupancy rate and turnover rate of the parking lots, municipalities need to ensure that vehicles leave before their meters expire. This means that potential parking violations require supervision and proper penalties if violations occur. Otherwise, the lack of regulation and management would encourage people to park their vehicles for as long as they want.

The geographic and demographic differences between different urban districts mean that parking lots are usually associated with different parking rates and regulation policies. In densely-populated areas, the meter rates are also relatively high due to demand. This implies that in such areas parking agents may patrol more frequently and violation penalties may be more expensive. Moreover, there are situations in which maximum parking time limits are enforced for the public good. Drivers should be informed of certain types of information by having parking rates and time limit posted on meters or pay stations. In order to properly decide how much drivers should pay for legal stays, drivers require more information about certain topics: citation penalties and patrol frequencies or schedules, which are not posted on the meters. This study assumes that drivers are frequent parkers who are more or less aware of the violation penalties and patrol frequencies.

Information about parking prices, patrol schedules, and citation penalties can help drivers make their choices: pay a sufficient amount, pay less than they should or even not pay at all. Obviously it is waste for drivers to pay more than they need to do so, but it is even more painful to have violation tickets appear on the dashboard. A driver who has received several parking citations will probably endeavor to avoid receiving additional parking citations in the same parking lot. It is reasonable for drivers to pay a little bit more in order to avoid potential penalties. On the contrary, a driver who is fortunate enough to park the vehicle for extra free

minutes is likely test his/her luck a second time. In such instances as noted above, drivers tend to judge whether or not a parking lot is inspected regularly based on personal observations. For one thing, intuitively, it is less likely to get a parking citation in a parking lot that is patrolled less frequently than others. For another, driver may have some notion that it is more likely to be inspected by an officer when the parking duration is longer, even if they are unaware of the exact patrol schedule or frequency.

In order to restrict parking offenses, parking enforcement officers are dispatched to patrol metered streets and certain off-street parking lots so as to inspect the meters or pay box receipts on the dashboards. Parking departments want to watch the turnover rate of particular high-demand parking lots, and might take measures such as charging higher parking fees, issuing citations with higher fines, or inspecting certain parking lots more frequently. The problem of patrol officer assignment and patrol route design is a typical Vehicle Routing Problem. Related topics have been studied by numerous researchers and many studies have shown that more efficient routing arrangements can help save expenses and satisfy customer demand. The problems associated with the routing of parking enforcement patrol systems have not yet been addressed, however.

Most vehicle routing problems involve the minimum number of visits made by each customer being specified based on the demand, while this parking patrol problem involves customer behavior having an interdependent relationship with the routing arrangements. More specifically, while most vehicle routing problems involve only a unilateral effect between customer and routing schemes, there exists an interaction effect that must be addressed in this problem. That is, patrol frequency affects driver behavior, which in turn impacts the expected cost of patrol and further affects decision-making concerning patrol frequency and routing. Also, as an uncapacitated VRP, the planning of parking patrol routes is more flexible in terms of vehicle assignments, compared with capacitated VRPs. Without the limitations of vehicle capacity, patrol officers can inspect as many parking lots as possible for as long as the duration of the tour involves a time limit.

This thesis consists of five chapters in total, including the introductory chapter. Chapter 2 presents a literature review of related works. Chapter 3 introduces the driver behavior model that

optimizes parking payments and the agency model that seeks to minimize expected system costs. The solution methods and numerical tests of the agency models are illustrated in Chapter 4, along with sensitivity analysis. Finally, the conclusion and options for future research can be found in the final chapter.

## CHAPTER 2

### LITERATURE REVIEW

This chapter provides an overview of the studies of the parking behavior of drivers and related studies in the field of Periodic Vehicle Routing Problems. It also introduces some case studies involving patrolling deployment problems, and illustrates the method of continuum approximation.

#### **2.1 Parking behavior**

The influence of parking policies on driver behavior is reflected in the types of parking, parking locations, car occupancy, and even travel frequency and travel mode in the target area (Bates et al., 1997). On the one hand, poor parking management policies can result in low transportation efficiency. For example, Shoup (2005) stated that overly low parking prices can produce parking demand inflation, which can lead to increased traffic volume, a considerable portion of which are vehicles cruising for parking spots. It can further negatively impact business efficiency in a given area (D’Acierno et al., 2006). On the other hand, good parking policies can positively impact urban transportation systems. Many researchers are of the opinion that parking price is one of the best measures for reducing the number of car trips (Albert and Mahalel, 2006; Kelly and Clinch, 2006; Simićević et al., 2013).

Most research on parking behavior notes that the main factors that influence the drivers are policies about parking charges and time limitations. These two measures have been proven to impact the demand for on-street parking. For instance, a willingness-to-pay survey conducted in New York City has shown that as parking prices rise, residents are less willing to park their vehicles in such an area (Guo and McDonnell, 2013). A state preference survey conducted in Sydney (Hensher and King, 2001) suggests that curtailing the hours of operation at specific locations can lead to the relocation of parking and some switching over to public transportation. However, parking price increases can lead to significantly greater use of public transportation,

and a noticeable increase in parking relocation. Although the prices and hours of operation may not result in losses to the central business district, it can significantly impact the demand for parking at certain specific parking locations. Thus, it is justifiable to assume that on-street parking demand is mainly related to the location of the parking spaces, parking prices, hours of operation, and parking time limitations.

In addition to the policies regarding parking prices and time limitations, parking enforcement, which restricts violations of parking regulations, also exerts a significant influence on the parking behavior of drivers (Petiot, 2004; Cullinane and Polak, 1992). At present, there is little research that has examined the connection between parking enforcement and illegal parking in the case of on-street meter parking, except for studies conducted by Petiot and Saltzman. According to Petiot (2004), the answer to the question of the primary determinant of parking meter violations is generally the weakness of enforcement efforts, for example, overly low parking fines. No empirical study has offered any substantial evidence to support the notion that an increase in parking fines results in decreases in parking violations. Robert M. Saltzman (1997) simulated the parking behavior of drivers and found that higher degrees of enforcement result in lower violation rates and higher system capacity. A higher degree of enforcement can be put into effect by toughening penalties, including policies such as increasing fines or removing illegally-parked vehicles. The turnover rate and the probability of finding ideal metered parking space would probably rise as a result.

In order to model the decision-making processes of drivers in metered parking slots, the Newsvendor model can be applied to compute optimal parking payments based on the distribution of actual parking duration. The Newsvendor model, which is also known as the single-period problem, optimizes the order quantity (which is the parking payment in this study) of a certain product in a probabilistic demand setting, such that the expected system profit can be maximized. At the end of a certain period of time, any remaining inventory would either be disposed of, or sold at a reduced price. If the order quantity were smaller than the demand, the system would be punished (Silver et al., 1998). The Newsvendor model has been proven by a large number of researchers during the last century to be applicable in several industries (Khouja, 1999).



## **2.2 Periodic vehicle routing problem**

A standard Periodic Vehicle Routing Problem (PVRP) is to assign each customer to one of the service schedules in a manner such that the headway between two visits meets the preset requirements and the total profit of the system is maximized. The services are offered by dispatching vehicles to visit customers to pick up packages, deliver goods, or conduct on-site services. PVRP has been studied by a large number of researchers for about half a century. The problem is so versatile that it can be applied to various situations and can be solved using multiple solution methods. The patrol routing problem for parking enforcement addressed in this thesis is a type of PVRP with no constraints on vehicle capacity, which is an uncapacitated-PVRP.

### **2.2.1 Problem setting of PVRP**

One major difference within the papers that address PVRP lies in the definition of the delivery schedule. Some of the research literature presets the number of visiting days for each customer (Beltrami and Bodin, 1974; Christofides and Beasley, 1984), while some specified a visiting frequency or constraints on the headway between two visits for each customer (Chao, Golden and Wasil, 1995; Cordeau, Gendreau and Laporte, 1997; Gaudioso and Paletta, 1992; Las Fargeas et al., 2012). The required visiting frequency or number visiting days during a period of time depends upon the characteristics of the customers in the model. For example, in the PVRPs of visiting student or patients, the frequency for each student or patient to be visited depends upon the types of disabilities of the students (Maya, Sørensen and Goos, 2012) or seriousness of their illnesses (An, Kim, Jeong and Kim, 2012). In an oil extraction problem, the optimum headway between two visits at a well depends upon the time needed for a mobile bump to lift the available oils and the recovery time of the well (Gonçalves, Ochi and Martins, 2005). In certain more modern cases, such as the patrolling of mobile data collectors, there can be a vehicle equipped with a data transceiver (Almi'ani, Selvadurai and Viglas, 2008), and the visiting frequency is determined according to the data generation speed of the sensors. A sensor that generates more data per unit of time should be visited more frequently.

Due to the diversity of PVRP, the customers in some problems may also have particular special requirements regarding the service vehicle, although in many cases the vehicles are assumed to be identical. For instance, in the research conducted by Jang et al. (2006), thirty-nine representatives visited the retailers, and each retailer required a particular visiting frequency. Given that a representative tends to build relationships with the retailers visited previously, this problem can be further decomposed into multiple TSP problems. The TSP (travelling salesman problem) is a special case of an on-site service routing problem in which only one vehicle is dispatched. This is a very special case in which each vehicle is unique and each customer needs to be served by only one of the vehicles. In another study by Blakeley et al. (2003), the customers of Schindler, an elevator company, were visited by a number of maintenance technicians periodically and the technicians' skills may be different. Given that each specific type of maintenance work must be done periodically, the skills of the technicians and the maintenance cycles of the customers must be taken into consideration when assigning jobs to technicians.

In most of the cases mentioned above, the objective function is the cost of the entire system during a specified time period or the cost per unit of volume of the goods that are delivered. In some instances, the objective is to minimize the fleet size or the amount of labor required, when the required fleet size or number of staff members is closely correlated with the agency cost (Campbell and Hardin, 2005; Delgado, Laguna and Pacheco, 2005). The cost function can also be substituted for using the profit function, when the agency also earns some income from the dispatch service (Baptista, Oliveira and Zúquete 2002), particularly for private organizations. According to Francis et al. (2006a), when benefits are considered in the PVRP, the model will generate schemes in which some customers are visited with a greater frequency, such that the total cost can be further reduced. In the oil extraction transportation problem, it is less profitable to visit wells with extraction times and lift times longer than the recovery time, so these wells would probably be visited less frequently (Gonçalves, Ochi and Martins, 2005).

### 2.2.2 Solution methods of PVRP

Years ago, the solution methods of the VRPs were quite straightforward heuristics (Beltrami and Bodin, 1974; Russell and Igo, 1979) or exact approaches (Christofides and Beasley, 1984),

which were suitable for small cases. Particularly, in cases involving the PVRPs, the solution methods often have two stages: developing routes and assigning visiting days (Beltrami and Bodin, 1974; Baptista, Oliveira and Zúquete, 2002; Tan and Beasley, 1984; Christofides and Beasley, 1984). One could either develop the routes, and later assign the routes to days within a time period (a week or month), or first assign visiting days for each customer and then develop routes for each day in a time period (Coene, Arnout and Spieksma, 2010).

The most widely-known classical heuristics for the VRPs in the past is the Savings Algorithms devised by Clarke and Wright (1964). This algorithm can solve the problems in which the numbers of vehicles is a decision variable. Sequential improvement methods can also be applied in such problems, but it is not a competitive algorithm when compared with other available methods (Mole and Jameson, 1976; Christofides, 1976). The Sweep Algorithm, the Petal Algorithm, and the Cluster-First-Route-Second Algorithm involve similar procedures—they first cluster the nodes for one vehicle, and then generate the routes by solving the TSP (Gillett and Miller, 1974; Fisher and Jaikumar, 1981).

As computation speed increases in the contemporary era, larger instances can be solved using either meta-heuristics or mathematics-based approaches. Thus, the two stages used to solve PVRP which are mentioned above, developing routes and assigning visiting days, can be integrated and solved simultaneously by setting a large matrix of decision variables. The matrix can be three-dimensional or greater. For instance, Francis et al. developed and solved a PVRP (2006b) in which service schedule choices of the customers and vehicle routes were generated by solving one integrated model. Some state-of-art commercial solvers can solve such problems by using either exact methods or meta-heuristics. One of the most influential meta-heuristic methods is Tabu Search, which starts with a number of randomly-selected customers being assigned to certain service schedules. Tabu Search is similar to Simulated Annealing that they can both guide local search to extend beyond local optimality (Osman, 1993). Tabu search uses a forbidding strategy to prevent non-improving movements, which is conducted by identifying the attributes of such movements (Osman, 1991). The Variables Neighborhood Search algorithm is inspired by Simulated Annealing and has some advantages in terms of runtime. In large instances, Variables Neighborhood Search can outperform many other state-of-art techniques and generate better solutions (Hemmelmayr et al., 2009).

## 2.3 Patrol models

Patrol allocation problems have been discussed since the late 1970s (Chaiken et al., 1978; Chelst, 1978). There is a large body of literature that addresses the optimization problem of patrolling, and these studies can be classified into several categories.

Maximum coverage security patrol problem has been studied for decades. As the name implies, the objective underlying such problems is to design effective routes and schedule plans that cover as large a number as possible of *hot spots* during a given time period. In these patrol models, a *hot spot* is defined as an area where accidents occurred more frequently than in other places during a certain time period. For instance, highway patrol hotspots involve a certain combination of stretches of highway and a time of a day when crashes occur frequently (Çapar, Keskin and Rubin, 2015; Keskin and Li, 2012; Keskin, Li, Steil and Spiller, 2012). While on marine patrol, fleets of boats are sent to patrol regions in a manner such that each region will have at least one boat during a given time (Chircop et al., 2013). In many cases, these problems can be formulated into a Mixed Integer Linear Programming problem and can thus be solved using some state-of-art solvers using exact methods or meta-heuristics.

Patrolling upon request is common in property management or in police patrolling systems. One primary duty of such security patrols is to provide an immediate response to requests made by the residents. The objective of the problem is thus to minimize the response time, maximize the probability of successful responses (Lau et al., 2010) or minimize the system cost while control the average reaction time (Traffic Institute, 1993). The response time is defined as the lead time from the detection of an event to the arrival of a patrol officer on the scene. As a result of such a research objective, the blackspots, the locations with higher complaint rates, get patrolled more frequently. For example, Chelst (1978) allocated a fixed number of patrols to certain areas that had higher complaint rates in order to maximize the probability of intercepting complaints. Deploying patrol tasks using this method can help increase complaint interception probability by 30%, although it might lead to seriously uneven patrol distribution. More recently, Di Tella and Schargrotsky (2004) estimated and evaluated the deployment of police forces to patrol the area under random terrorist attack. Lou et al. (2011) investigated the freeway service patrol problem in order to minimize the total incident response

time in both deterministic and stochastic incidence occurrence settings. In a special case where all of the blocks or units in the patrol region have equal complaint rates, the solution would simply be the shortest Hamiltonian path (Chevaleyre, Sempe and Ramalho, 2004).

Meanwhile, the research objective and solution methods of patrolling problems in a dynamic environment can be quite different from the problems above. In a dynamic environment, there are no static hotspots or constant crime rates for the different blocks or units in the service region, but opponents that can change attack plans at any time. Under these settings, the patrol routes must be subject to flexible change so that the region can be defended efficiently or the opponents can be confused (Irvan et al., 2011). The patrol routing plan of the police force can be generated dynamically using machine learning models or through the re-computation of the models mentioned above (Melo et al., 2006). In order to meet this real-time computation requirement, the algorithms used must be sufficiently fast such that new routes can be generated as the environment changes (Chen and Yum, 2010; Chen, 2012). The objective of such a dynamic patrolling problem might be minimizing the amount of time that each block remained unpatrolled or optimizing system efficiency.

## **2.4 Continuum approximation**

In many cases people are interesting in optimizing detailed large-scale distribution systems by minimizing the expected cost of the entire system, such as the delivery service routing problem or the warehouse location problem. For VRP models with a considerable number of decision variables and constraints, the continuum approximation (CA) technique can be applied to reduce the model to an idealized continuous system in which smooth functions are used to describe the input data or even describe decisions (Smilowitz and Daganzo, 2007). Using this method, the locations of the customers are approximated into a density function. The method of CA is particularly applicable in complex and large-scale systems when an accurate expected cost must be estimated (Erera, 2000). It has been proven that the larger the instance, the more accurate the result it will achieve (Daganzo, 1999)

In some cases, solving the problem of multiple decision functions involves further simplification by partitioning the entire region into multiple delivery zones in which the

customers in a single zone are serviced by one vehicle (Ouyang, 2007). The reduced models can help identify the efficient strategies and greatly reduce the number of decision variables and computation time (Daganzo and Erera, 1999). According to Daganzo and Erera (1999), for vehicle routing problems in which the demand of customers remains static, an efficient strategy is to divide the service region  $\mathcal{R}$  into a number of delivery zones, each of which contains a certain amount of demand, see Figure 2.1. One vehicle is assigned to one zone and can meet all of the demand in the zone. The travel distance of the delivery system per unit area (which can be regarded as distance density) at any location  $\mathbf{x}$  should be a function of customer density  $\delta$ , location  $\mathbf{x}$ , and the average number of stops  $C$  made by one vehicle, see Equation 2.1.

$$\frac{2r(\mathbf{x})\delta}{C} + 0.57\delta^{1/2} \quad (2.1)$$

The average number of stops  $C$  can be approximated by the vehicle capacity divided by the average demand volume of one customer. The detour distance also depends on the metric used in the system. Here 0.57 is the Euclidean metric according to Daganzo (1999).

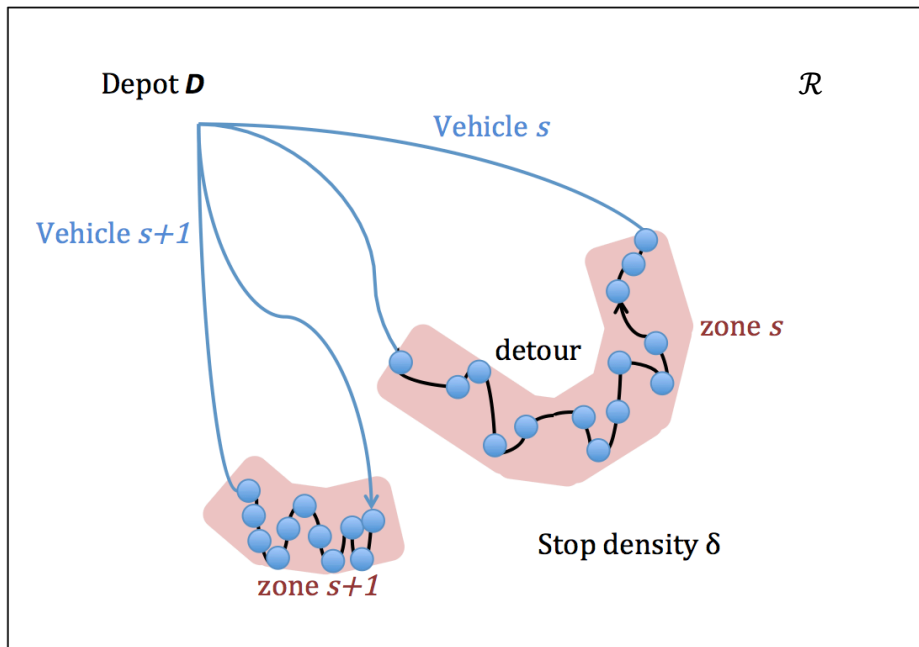


Figure 2.1. The zoning technique in the CA method (adapted from Daganzo et al., 1999)

The total travel distance can be approximated by integrating the distance per unit area over the entire service region. After the customer locations in each zone become known, the operational-level route planning of the systems can be conducted based on some cluster-first-route-second algorithms or meta-heuristics methods (Ouyang, 2007; Robust and Daganzo, 1990).

The continuum approximation method can be applied broadly to distribution problems and is found to be very helpful in assisting decision-making, in many studies (Ouyang and Daganzo, 2006; Smilowitz and Daganzo, 2007).

# CHAPTER 3

## METHODOLOGY

In this chapter, the driver behavior model and parking violation model are initially presented, followed by the formulation of the mathematical models of agency decision under both discrete and continuous settings.

### 3.1 Parking violation models

#### 3.1.1 Optimal parking payment of driver

Drivers make decisions regarding parking payments based on their estimates of the amount of time the vehicle is anticipated to be parked in a parking slot. It is assumed that drivers are blocked from the information about the schedule for parking lot patrols, but are aware of the time gap between two inspections. Therefore, the drivers can roughly assess the amount of potential penalties—longer illegal parking time or a higher patrol frequency can mean greater amounts of expected penalties.

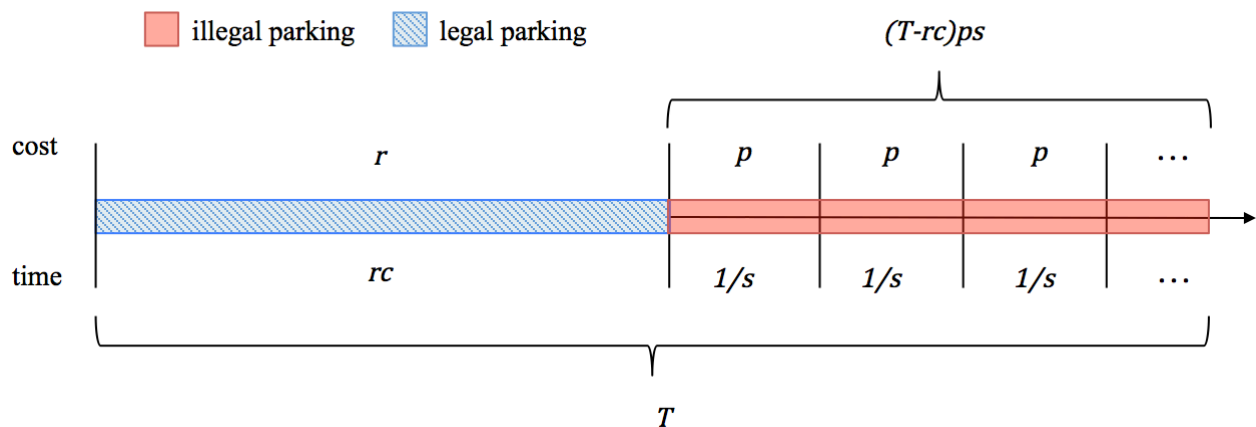


Figure 3.1. The overall parking cost of one vehicle



Several more assumptions are made in this model: i) the time limitation of the metered parking lots is not taken into consideration; ii) the actual parking time  $T$  at a certain parking lot follows a distribution with Probability Density Function of  $P(T)$  and a Cumulative Distribution Function  $F(T)$ ; iii) drivers want to minimize their total costs, see Figure 3.1, which is the sum of payments for parking and potential penalties; iv) one unit of money can be exchanged for  $c$  units of legal parking time; and finally, v) the expected penalty is the product of perceived patrol frequency  $s$ , the amount of fine for one parking ticket  $p$ , and the potential illegal parking time  $\max(0, T - rc)$ .

Hence, in order to minimize the expected costs, the optimal payment should be formulated as follows:

$$r^* = \underset{r}{\operatorname{argmin}} E(r + ps \cdot \max(0, T - rc)) \quad (3.1)$$

where  $E(\max(0, T - rc)) = \int_{rc}^{\infty} (T - rc)P(T)dT$ . The optimal solution  $r^*$  to the equation is formulated as:

$$r^* = \frac{1}{c} F^{-1}\left(1 - \frac{1}{pcs}\right) \quad (3.2)$$

The domain of  $F^{-1}$  is  $[0,1]$ . Thus,  $r^*$  exists only when  $1 \leq pcs$ . This implies that the penalty  $p$  should exceed the parking payment for the parking duration of one patrol headway. Otherwise, the driver would not pay the parking fee but instead pay the ticket. Therefore, the optimal payment is:

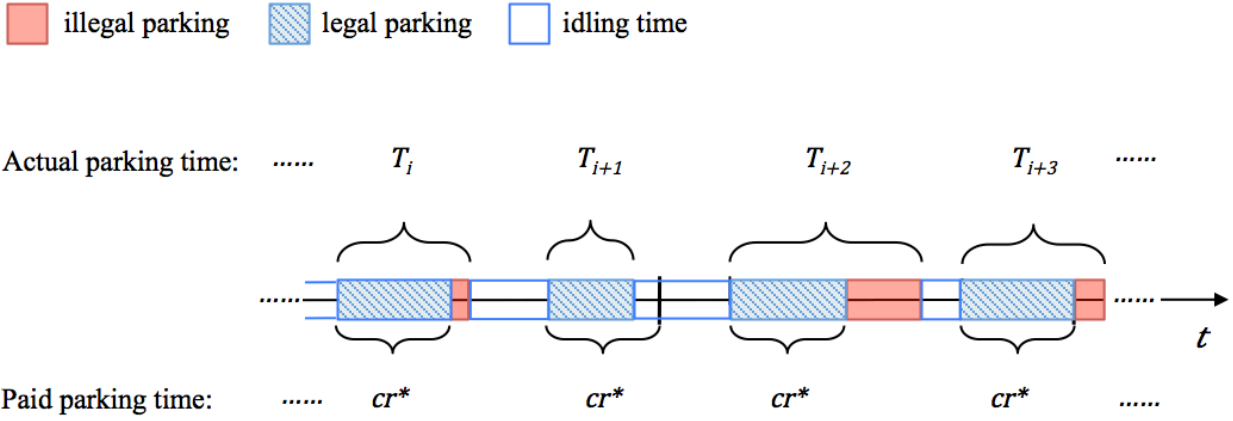
$$r^* = \begin{cases} \frac{1}{c} F^{-1}\left(1 - \frac{1}{pcs}\right), & \text{if } 1 \leq pcs \\ 0, & \text{o. w.} \end{cases} \quad (3.3)$$

For the sake of simplicity, it is assumed that  $1 \leq pcs$  and thus  $r^* = \frac{1}{c} F^{-1}\left(1 - \frac{1}{pcs}\right)$  in the remainder of the thesis.

### 3.1.2 Expected illegal parking duration

The inspection cost consists of the labor and gasoline costs that can be attributed to checking meters, which is assumed to be the product of the total number of meters inspected and the cost to inspect on single meter. This cost is highly correlated with driver behavior. For one thing, if the inspection frequency remains unchanged, while parking violations occur more frequently, this situation is probably due to high parking fees or low violation penalties. The officers would thus expect to spend more time issuing citations. Another issue is that in seeking to restrict illegal parking, patrol frequency could be increased at certain parking lots in order to persuade the drivers to obey parking regulations. Hence, it is important to analyze the influence of patrol frequency and driver behavior on illegal parking time per hour at metered parking lots.

A metered parking slot is a queuing system with one service station and no space for queues. The arrival of the customers can be assumed to follow the Poisson process with average arrival rate  $\lambda$ . Moreover, we assume that parking time follows a same distribution for all drivers at a certain parking lot. They thus have the same optimal payment. The service time  $T$  (which is the actually parking time in this problem) for each customer follows a general distribution as mentioned above, while paid parking time is  $cr^*$ . The operation of such a service system is a M/G/1/1 queue with an arrival rate  $\lambda$  (-/hr) and a service rate  $\mu$  (-/hr),  $\mu = \frac{1}{E(T)}$ . The system contains only one parking slot and the overall capacity is one. A customer is thus lost permanently if the customer arrives when the parking slot is occupied by another vehicle. See Figure 3.2,  $T_i$  is the actual parking time of the  $i^{th}$  driver, and  $cr^*$  is the optimal legal parking time for all drivers.



**Figure 3.2. The operation of a metered parking slot over an infinite timeline**

The system has only two types of status: empty or full. Let  $P_0$  be the probability of system being empty and  $P_1$  be the probability of system being full.  $P_0$  should also represent the blocking probability, which is the probability of customers being lost. Intuitively we have:

$$P_1 + P_0 = 1 \quad (3.4)$$

The “traffic intensity”  $\rho_c$  is actually the occupied time of the parking slot in one unit of time, and is determined by  $\lambda_c$ , the rate at which customers actually enter the parking slot, and the expected service time  $E(T)$ .  $\lambda_c$  is equal to the arrival rate of the system, and the loss rate due to the system blocked is subtracted from this, see Equation 3.6.

$$\rho_c = \lambda_c \cdot E(T) = \lambda_c / \mu \quad (3.5)$$

$$\lambda_c = \lambda(1 - P_1) \quad (3.6)$$

Note that  $\rho_c$  is essentially the fraction of time that the parking slot is occupied, it should be equal to  $P_1$ . Then we have:

$$P_1 = \rho_c = \frac{\lambda(1 - P_1)}{\mu} \quad (3.7)$$

Solving Equation 3.7, we can write:

$$P_1 = \frac{\lambda}{\lambda + \mu} \quad (3.8)$$

Assuming that the parking space is occupied, the expected ratio of expired time over total occupied time is:

$$P_E = \int_{cr^*}^{\infty} \frac{(T - cr^*)}{T} P(T) dT \quad (3.9)$$

Note that the  $P_E$  is not the probability of parking expiration conditional upon parking space being occupied, but rather the expected ratio of illegal parking time to total occupied time. Therefore, the probability  $P_{vio}$  of one metered parking space being illegally occupied by a vehicle should be:

$$P_{vio} = P_1 P_E = \frac{\lambda}{\lambda + \mu} \int_{cr^*}^{\infty} \frac{(T - cr^*)}{T} P(T) dT \quad (3.10)$$

When  $T$  follows a normal distribution with a mean  $\mu^{-1}$  and variance  $\sigma^2$ ,  $P_{vio}$  can be written as:

$$P_{vio} = \frac{\lambda}{\lambda + \mu} \left( \frac{1}{pcs} - \int_{cr^*}^{\infty} \frac{cr^*}{T} \frac{1}{\sigma} \phi \left( \frac{T - \mu^{-1}}{\sigma} \right) dT \right) \quad (3.11)$$

in which  $cr^* = \sigma \cdot \Phi^{-1} \left( 1 - \frac{1}{pcs} \right) + \mu^{-1}$ , and  $\Phi$  and  $\phi$  are, respectively, the CDF and PDF of Standard Normal Distribution. As  $s$  grows larger,  $\frac{1}{pcs}$  will certainly decrease, while  $cr^*$  will increase.  $P_E$ , see Equation 3.9, decreases as  $cr^*$  increases. Thus,  $P_{vio}$  is monotonically decreasing when  $s$  is increasing. When every variable in Equation 3.11 is fixed except for  $s$ , we can write  $P_{vio}$  as:

$$P_{vio} = g(s) \quad (3.12)$$

where  $g(\cdot)$  is a function of frequency. Thus,  $s$  can be written as:

$$s = g^{-1}(P_{vio}) \quad (3.13)$$

For the sake of simplicity, the expected violation duration in unit time  $P_{vio}$  is referred to as violation probability in the remainder of this thesis. Assuming that each parking meter is independent of the others in the sense that all of the events which occur at this meter do not depend on the other meters, the expected number of violations encountered during one patrol of an parking lot should be the product of the total number of meters and the violation probability  $P_{vio}$  in this parking lot.

### 3.2 A discrete agency model

In the region  $\mathcal{R}$ , a number of parking lots need to be patrolled by enforcement officers. A patrol tour is defined as a trip that starts and ends at the depot. And during each tour, the parking enforcement officer must travel along one of the specified routes. Each parking lot is characterized with a pre-set number of meters and deemed to be a node, and the officers are assigned to visit the parking lots during each patrol tour, as shown in Figure 3.3. The travelling distance between each couple of nodes, and the number of routes are also fixed as input parameters. The routes and the patrol frequencies of the routes are the decisions in this model.

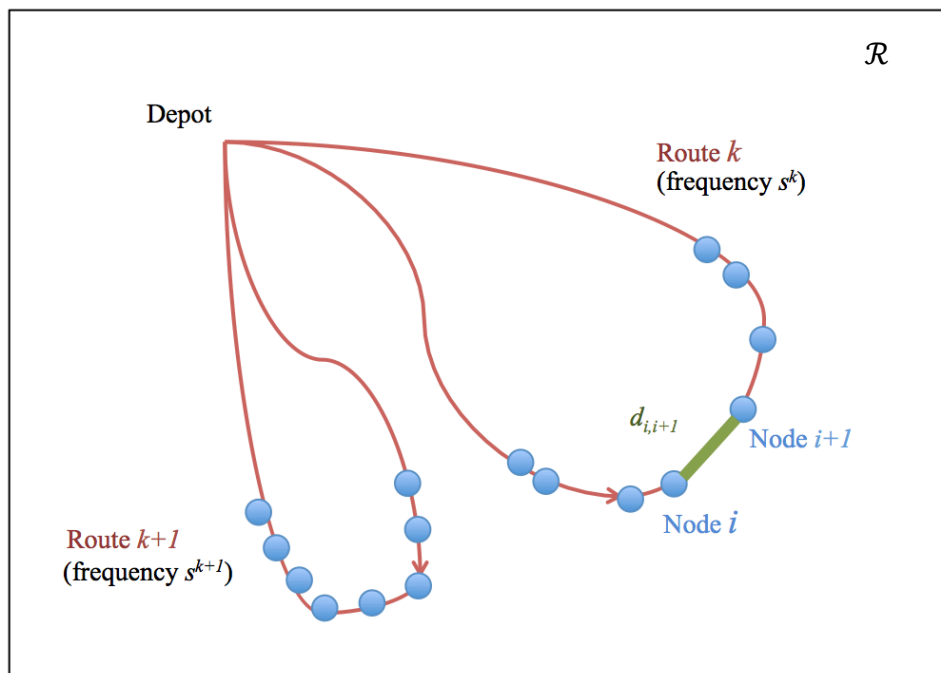


Figure 3.3. Patrol routes in the service region  $\mathcal{R}$

The following notations are used to formulate the discrete patrol routing model:

$N$	total number of nodes
$I$	set of nodes, $I = \{1, 2, \dots, N\}$ , and Node 0 is the depot
$c_i$	the parking charge rate at parking lot $i$ (hr/\$)
$\lambda_i$	arrival rate at parking lot $i$ (-/hr)
$\mu_i$	service rate at parking lot $i$ with $\mu_i = 1/E(T_i)$ (-/hr)
$r_i^*$	optimum parking payment at node $i$ (\$)
$P_{vio,i}$	expected violation duration in unit time at node $i$ , see details in Section 3.1.2
$K$	set of routes, $k \in K = \{0, 1, 2, \dots,  K  - 1\}$ represents a patrol route
$D$	set of network arcs, $D = \{(i, j) : i, j \in I \cup \{0\}\}$
$d_{ij}$	distance of arc $(i, j)$ (mile)
$u$	variable cost associated with the distance travelled (\$/mile)
$\beta_i$	fixed cost of inspection at node $i \in I$ (\$)
$\gamma_i$	cost of issuing citations at node $i$ assuming that all parking slots are illegally parked (\$)
$P_{max}$	violation probability limit (-)
$C_{max}$	travel distance limit of one vehicle tour (mile)

The decision variables are as follows:

$$X_{ij}^k = \begin{cases} 1, & \text{if } arc(i, j) \in D \text{ is in route } k \\ 0, & \text{otherwise} \end{cases}$$

$$s^k \quad \text{patrol frequency of route } k, k \in K \text{ (-/hr)}$$

The expected travelling cost per unit time  $z^k$  of route  $k$  is linear to the summation of the costs traveling through the arcs:

$$z^k = \sum_{i, j \in I \cup \{0\}} X_{ij}^k d_{ij} u \quad (3.14)$$

The expected inspection cost per unit time  $z_i$  at parking lot  $i$  is:

$$z_i = \sum_{k \in K} \sum_{j \in I \cup \{0\}} X_{ij}^k \cdot s^k \cdot (\beta_i + \gamma_i \cdot P_{vio,i}) \quad (3.15)$$

The objective is to minimize the expected system cost per unit time with constraint on violations at each parking lot:

$$\min \quad Z = \sum_{k \in K} s^k z^k + \sum_{i \in I} z_i \quad (3.16a)$$

subject to

$$P_{vio,i} = \frac{\lambda_i}{\lambda_i + \mu_i} \int_{c_i r_i^*}^{\infty} \frac{(T_i - c_i r_i^*)}{T_i} P(T_i) dT_i \leq P_{max} \quad \forall i \in I \quad (3.16b)$$

$$c_i r_i^* = F_i^{-1} \left( 1 - \frac{1}{p c_i \sum_{k \in K} \sum_{j \in I \cup \{0\}} X_{ij}^k \cdot s^k} \right) \quad \forall i \in I \quad (3.16c)$$

$$0 \leq s^k \quad \forall k \in K \quad (3.16d)$$

$$\sum_{k \in K} \sum_{j \in I \cup \{0\}} X_{ij}^k = 1 \quad \forall i \in I \quad (3.16e)$$

$$\sum_{j \in I \cup \{0\}} X_{ij}^k = \sum_{j \in I \cup \{0\}} X_{ji}^k \quad \forall i \in I \cup \{0\}, k \in K \quad (3.16f)$$

$$\sum_{i,j \in I \cup \{0\}} X_{ij}^k \leq |Q| - 1 \quad \forall Q \subseteq I, k \in K \quad (3.16g)$$

$$\sum_{i \in I \cup \{0\}} \sum_{j \in I \cup \{0\}} X_{ij}^k \cdot d_{ij} \leq C_{max} \quad \forall k \in K \quad (3.16h)$$

$$X_{ij}^k \in \{0,1\} \quad \forall (i,j) \in D, k \in K \quad (3.16i)$$

Constraints 3.16b and 3.16c are set for the purpose of enforcing the probabilities of violation occurring at all of the parking lots under a limit, in which  $\sum_{j \in I \cup \{0\}} X_{ij}^k$  indicates whether or not node  $i$  is covered by route  $k$ , see the detailed formulation in Section 3.2. Constraints 3.16d are non-negativity constraints of the decision variables  $s^k$ . Constraints 3.16e make sure that each parking lot must be visited along one and only on route. Constraints 3.16f ensure the equality of inflow and outflow at each node. Constraints 3.16g are to eliminate the subtours. Constraints

3.16h restrict the travel distance of each patrol tour. Constraints 3.16i specify the space of decision variables  $X_{ij}^k$ .

The cost attributed to the processing of citations is relatively small compared to the transportation and inspection cost (see details in Section 4.1 and the numerical test in Appendix A). Assuming this part of cost being omitted, the objective function, Equation 3.16a, can be transformed into Equation 3.17a. Furthermore, considering that in reality the headways in logistics services such as transit are usually choices that are made among particular numbers such as 5 minutes or 1 hour, instead of any randomly-chosen number, and for the purpose of simplifying the problem, constraint 3.16d is modified into 3.17c.

The following notation is added to the discrete model:

$S$  set of feasible patrol frequencies,  $S = \{\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, 1, \frac{4}{3}, 2\}$  (-/hr)

The modified model could be written as:

$$\min \quad Z = \sum_{k \in K} \sum_{i, j \in I \cup \{0\}} s^k X_{ij}^k d_{ij} + \sum_{i=1}^N \sum_{k \in K} \sum_{j \in I \cup \{0\}} X_{ij}^k s^k \beta_i \quad (3.17a)$$

subject to

$$s^k \geq g_i^{-1}(P_{max}) \cdot \sum_{j \in I \cup \{0\}} X_{ij}^k \quad \forall k \in K, i \in I \quad (3.17b)$$

$$s^k \in S \quad \forall k \in K \quad (3.17c)$$

$$\sum_{k \in K} \sum_{j \in I \cup \{0\}} X_{ij}^k = 1 \quad \forall i \in I \quad (3.17d)$$

$$\sum_{j \in I \cup \{0\}} X_{ij}^k = \sum_{j \in I \cup \{0\}} X_{ji}^k \quad \forall i \in I \cup \{0\}, k \in K \quad (3.17e)$$

$$\sum_{i, j \in Q} X_{ij}^k \leq |Q| - 1 \quad \forall Q \subseteq I, k \in K \quad (3.17f)$$



$$\sum_{i \in I \cup \{0\}} \sum_{j \in I \cup \{0\}} X_{ij}^k \cdot d_{ij} \leq C_{max} \quad \forall k \in K \quad (3.17g)$$

$$X_{ij}^k \in \{0,1\} \quad \forall (i,j) \in D, k \in K \quad (3.17h)$$

Therefore, the original nonlinear non-convex mixed integer problem has been modified into a Mixed Integer Linear Programming problem.

### 3.3 A continuous agency model

In the continuous model, the discrete parameters and variables are transformed into continuous functions. These functions are assumed to be smooth, continuous and vary slowly over the entire service region  $\mathcal{R}$ . It is assumed that the parking lots are distributed across the entire service region according to a density function, and each parking lot should be patrolled according to a frequency.

The notations in this part follow previous models, but now are mostly functions of  $\mathbf{x}$ . The following new notations are introduced to formulate the continuous model:

- $\omega(\mathbf{x})$  spatial density of nodes (number of parking lots per unit area) at point  $\mathbf{x}$
- $d(\mathbf{x})$  distance between the depot from point  $\mathbf{x}$  (mile)

The decision functions and variables are introduced as follows:

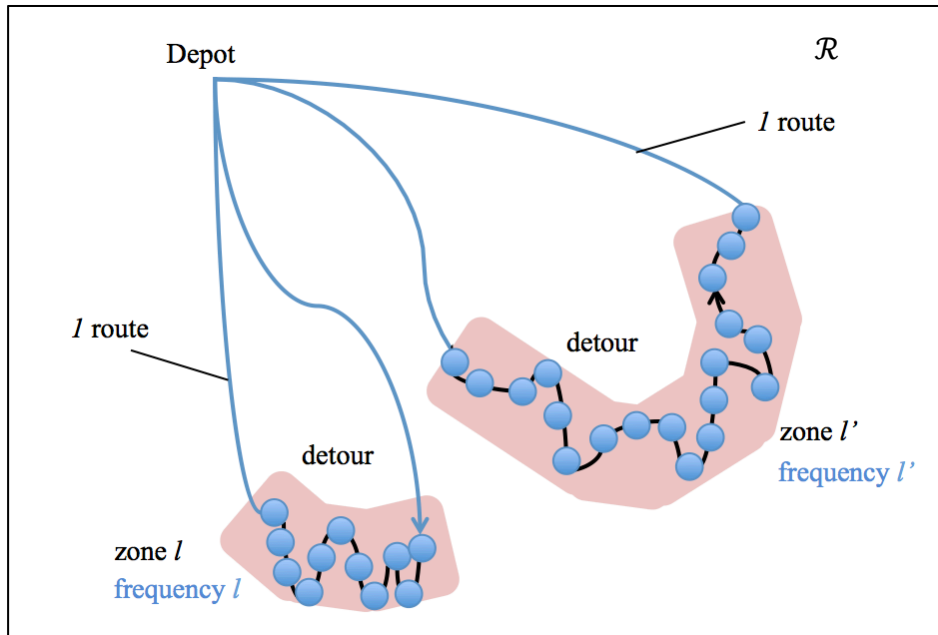
- $f^l(\mathbf{x})$  fraction of nodes at  $\mathbf{x}$  being visited according to frequency  $l \in S$
- $Y^l$  number of routes patrolled with frequency  $l \in S$

The travelling cost of each patrol vehicle can be distributed to each node and be computed by integrating the cost of the nodes over the whole region. The travel distance for patrolling the nodes at  $\mathbf{x}$  with a frequency  $l$  consists of the line-haul between the depot and the vicinity of point  $\mathbf{x}$  and detours between nodes, see Equation 3.18. The first term of the equation is the line-haul per unit area for the nodes patrolled according frequency  $l$ , in which  $\omega(\mathbf{x})f^l(\mathbf{x})$  is the number of nodes per unit area patrolled with frequency  $l$ ,  $\int_{\mathcal{R}} \omega(\mathbf{x})f^l(\mathbf{x})d\mathbf{x}$  is the total number of nodes patrolled with frequency  $l$ , and  $2d(\mathbf{x})$  is the distance between  $\mathbf{x}$  and the depot.

The average distance between two nodes near point  $\mathbf{x}$  can be approximated by the product of  $[f^l(\mathbf{x}) \cdot \omega(\mathbf{x})]^{-\frac{1}{2}}$  and a metric constant. Assuming that the distance is computed in terms of Euclidean distance, the detour cost per unit of area which is the second term in Equation 3.18 should be the product of  $[f^l(\mathbf{x}) \cdot \omega(\mathbf{x})]^{-\frac{1}{2}}$ , node density  $f^l(\mathbf{x}) \cdot \omega(\mathbf{x})$ , and a Euclidean metric constant 0.57 (Daganzo, 1999).

$$\frac{2d(\mathbf{x})\omega(\mathbf{x})f^l(\mathbf{x})}{\int_{\mathcal{R}} \omega(\mathbf{x})f^l(\mathbf{x})d\mathbf{x}} + 0.57[f^l(\mathbf{x}) \cdot \omega(\mathbf{x})]^{\frac{1}{2}} \quad (3.18)$$

When there is no constraint on the travel distance of or the number of stops made by each vehicle, an optimal method will allow a vehicle to visit as many nodes as possible, see Figure 3.4. However, the duration of one vehicle tour should be restricted according to certain labor regulations (Daganzo, 1999). Thus, the nodes that are determined to be patrolled with frequency  $l$  should be assigned to a number of separate vehicle routes, instead of one single vehicle route, see Figure 3.5. While the length of a detour remains the same as the second term in Equation 3.18, the original line-haul distance term should be multiplied by  $Y^l$ , which is the number of routes associated with frequency  $l$ .



**Figure 3.4. Patrol zoning without travel distance constraints**

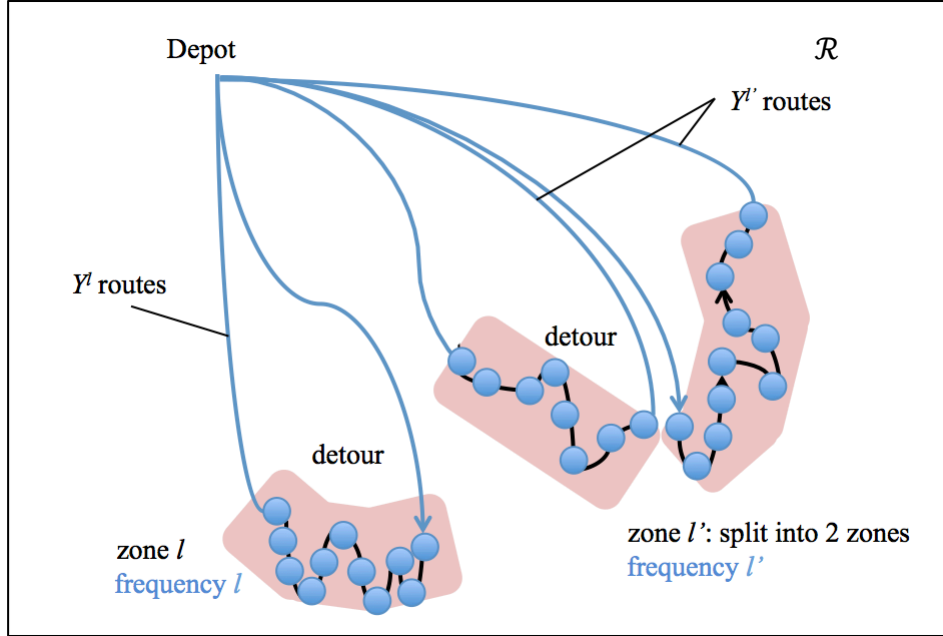


Figure 3.5. Patrol zoning with travel distance constraints

Let  $\Psi^l(\mathbf{x})$  denote the expected cost at  $\mathbf{x}$  of a dispatch associated with frequency  $l$ .  $\Psi^l(\mathbf{x})$  can be divided into the traveling cost and the meter inspection cost in each parking lot. The first term in Equation 3.19 is the travel cost, which is derived from Equation 3.18, while the second term is the cost of inspection. The cost for processing citations is again omitted.

$$\Psi^l(\mathbf{x}) = \left\{ \frac{2d(\mathbf{x})\omega(\mathbf{x})f^l(\mathbf{x})Y^l}{\int_{\mathcal{R}} \omega(\mathbf{x})f^l(\mathbf{x})d\mathbf{x}} + 0.57[f^l(\mathbf{x}) \cdot \omega(\mathbf{x})]^{\frac{1}{2}} \right\} \cdot u + [f^l(\mathbf{x}) \cdot \omega(\mathbf{x})] \cdot \beta(\mathbf{x}) \quad (3.19)$$

The objective of this continuous vehicle routing problem is to minimize the expected system cost per unit time while controlling the violation probability of each parking lot, see Equations 3.20.

$$\min \quad \bar{Z} = \int_{\mathbf{x} \in \mathcal{R}} \left( \sum_{l \in S} \Psi^l(\mathbf{x}) \cdot l \right) d\mathbf{x} \quad (3.20a)$$

subject to

$$\frac{\lambda(\mathbf{x})}{\lambda(\mathbf{x}) + \mu(\mathbf{x})} \int_{c(\mathbf{x})r^{*l}(\mathbf{x})}^{\infty} \frac{(T - c(\mathbf{x})r^{*l}(\mathbf{x}))}{T} P(\mathbf{x}, T) dT \leq \frac{P_{max}}{[f^l(\mathbf{x})]} \quad \forall l \in S, \mathbf{x} \in \mathcal{R} \quad (3.20b)$$

$$r^{*l}(\mathbf{x}) = \frac{1}{c(\mathbf{x})} F^{-1}\left(\mathbf{x}, 1 - \frac{1}{pc(\mathbf{x})l}\right) \quad \forall l \in S, \mathbf{x} \in \mathcal{R} \quad (3.20c)$$

$$\int_{\mathbf{x} \in \mathcal{R}} \left( \frac{2d(\mathbf{x})\omega(\mathbf{x})f^l(\mathbf{x})Y^l}{\int_{\mathcal{R}} \omega(\mathbf{x})f^l(\mathbf{x})d\mathbf{x}} + 0.57[f^l(\mathbf{x}) \cdot \omega(\mathbf{x})]^{\frac{1}{2}} \right) d\mathbf{x} \leq Y^l \cdot C_{max} \quad \forall l \in S \quad (3.20d)$$

$$\sum_{l \in S} f^l(\mathbf{x}) \geq 1 \quad \forall \mathbf{x} \in \mathcal{R} \quad (3.20e)$$

$$0 \leq f^l(\mathbf{x}) \leq 1 \quad \forall l \in S, \mathbf{x} \in \mathcal{R} \quad (3.20f)$$

$$f^l(\mathbf{x}) \leq Y^l \quad \forall \mathbf{x} \in \mathcal{R}, l \in S \quad (3.20g)$$

$$\sum_{l \in S} Y^l \leq |K| \quad (3.20h)$$

$$Y^l \in \{0, 1, 2 \dots |K|\} \quad \forall l \in S \quad (3.20i)$$

Constraints 3.20b and 3.20c are set in order to enforce the probability of violation in all parking lots being under a limit, see the detailed formulation in Section 3.2.1. Constraints 3.20d restrict the travel distance of each patrol vehicle. Constraints 3.20e ensure that each parking lot must be covered by at least one patrol route. Constraints 3.20f and 3.20i specify the space of decision variables  $f^l(\mathbf{x})$  and  $Y^l$ . Constraints 3.20g enforce that when some nodes near  $\mathbf{x}$  are to be patrolled with frequency  $l$ , at least one route associated with frequency  $l$  must be constructed. Constraint 3.20h ensures that the total number of routes is below the limit. Constraints 3.20i specify the space of  $Y^l$ .

According to the assumptions regarding continuous and slow-varying parameters in the continuum approximation method, the experimental region can be decomposed into a set of geographic subregions. In each subregion  $m$ ,  $m \in M$ , the parameters of the points should vary very slowly and approach a constant such that all of the input functions of the subregion can be simplified into constant parameters. In addition, a subregion  $m$  must be large enough to contain

at least one route. The shape of a subregion can be square, triangular, rectangular or any arbitrary shape. Hence, the problem can be decomposed into several subproblems for each subregion. For subregion  $m$ , let  $A_m$  be the area of  $m$ , let  $d_m$  be the distance from the weighted centroid of  $m$  to the depot, let  $\omega_m$  be the average node density, and let  $\beta_m$  be the average inspection cost per node. The decision variables are  $f_m^l$  and  $Y^l$ , respectively, and these are the fraction of nodes in subregion  $m$  patrolled according to frequency  $l$  and number of routes associated with patrol frequency  $l$ . The expected cost per unit time can be written as:

$$A_m \sum_{l \in S} l \left\{ \left( \frac{2d_m \omega_m f_m^l Y^l}{\sum_{m \in M} \omega_m f_m^l \Omega_m} + 0.57(f_m^l \cdot \omega_m)^{\frac{1}{2}} \right) \cdot u + f_m^l \cdot \omega_m \cdot \beta_m \right\} \quad (3.21)$$

The CA problem can be written as:

$$\min \quad \bar{Z} = \sum_{m \in M} \sum_{l \in S} l \cdot A_m \left\{ \frac{2ud_m \omega_m f_m^l Y^l}{\sum_{m \in M} \omega_m f_m^l \Omega_m} + 0.57u(f_m^l \omega_m)^{\frac{1}{2}} + \beta_m f_m^l \omega_m \right\} \quad (3.22a)$$

subject to

$$f_m^l = 0 \quad \forall l < g_m^{-1}(P_{max}) \quad (3.22b)$$

$$\sum_{l \in S} f_m^l \geq 1 \quad \forall m \in M \quad (3.22c)$$

$$\sum_{m \in M} \left( \frac{2d_m \omega_m f_m^l Y^l}{\sum_{m \in M} \omega_m f_m^l \Omega_m} + 0.57(f_m^l \cdot \omega_m)^{\frac{1}{2}} \right) \cdot A_m \leq Y^l \cdot C_{max} \quad \forall l \in S \quad (3.22d)$$

$$0 \leq f^l(\mathbf{x}) \leq 1 \quad \forall l \in S, \mathbf{x} \in \mathcal{R} \quad (3.22e)$$

$$0 \leq f_m^l \leq 1 \quad \forall m \in M, l \in S \quad (3.22f)$$

$$f_m^l \leq Y^l \quad \forall m \in M, l \in S \quad (3.20g)$$

$$\sum_{l \in S} Y^l \leq |K| \quad (3.20h)$$

$$Y^l \in \{0,1,2 \dots |K|\} \quad \forall l \in S \quad (3.20i)$$

The problem now contains a reduced number of decision variables, and the non-linear objective function and constraints, and can thus be solved rapidly using commercially available solvers.

Based on the optimal values of decision variables, we can further elaborate upon the design of the vehicle routes for each patrol frequency  $l$ . In each subregion, the original parking lot locations replace the node density function, and then the vehicle routing zoning method can be applied to design the detailed routes for each vehicle tour. In this thesis we mainly focus on the optimum agency cost, while the design of routes in the continuous model is omitted.

## CHAPTER 4

### NUMERICAL EXPERIMENTS

Due to the absence of real-world parking enforcement data, for the purpose of numerical testing of the problem, several data sets are generated and the problems are solved using the data. In this chapter, the results of numerical testing of both the discrete model and the CA model are presented, followed by the sensitivity tests of some of the most seemingly important parameters.

#### 4.1 Parameter setting

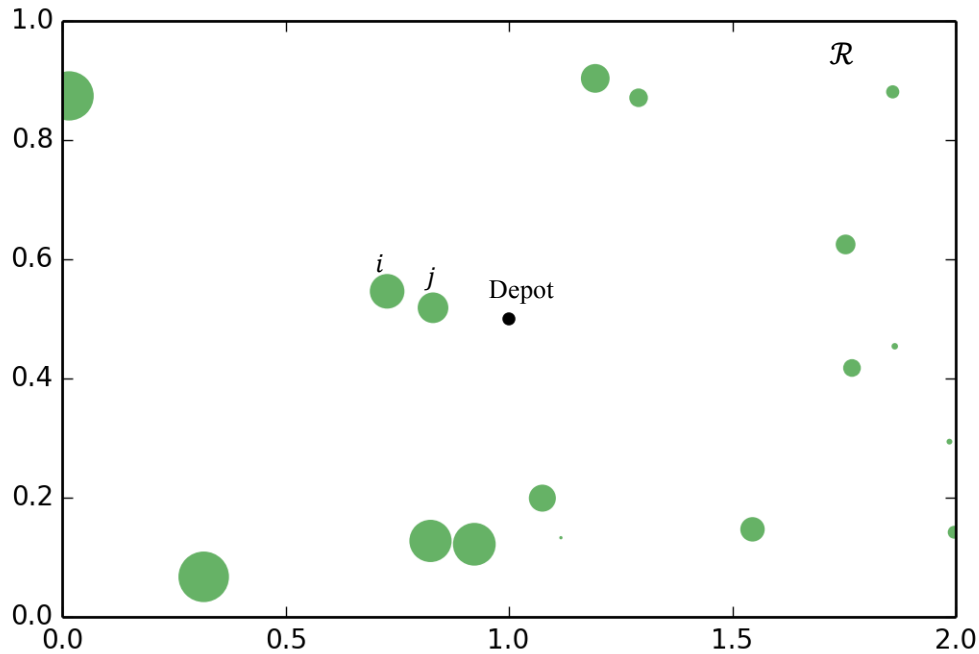
The data used in this experiment is randomly generated from some reasonable ranges according to a list of peer-reviewed papers, government statistics, and news articles. Parking lots are almost evenly distributed across a  $1 \times 2$  mile<sup>2</sup> service region, and the depot is assumed to be at the center of the rectangle. The values of the parameters of each parking lot, including the inspection cost, driver arrival rate, average parking duration and the parking rate, are randomly selected from a reasonable range, see Table 4.1.

According to some case studies of parking demand in the United States, the average arrival rate  $\lambda$  for one parking space is usually less than 1 per hour (Kimley-Horn and Associates, Inc., 2013a; Kimley-Horn and Associates, Inc., 2013b; Ottosson et al., 2013). The price of parking is assumed to vary from 0.25 to 1 (\$/hr), which covers the most likely parking rates in small towns and cities, so the parking rate  $c$  (hr/\$) should be within this range [1,4]. The average parking time  $\mu^{-1}$  in a metered parking space may vary from 1 hour to 2 hours (Ottosson et al., 2013), thus each  $\mu_i^{-1}$  is randomly selected from this range. The variable travel cost is \$1.55/mile, including a \$0.55/mile for gas and maintenance for law enforcement vehicles in United States (Vincentric, LLC, 2010). Also included is \$1/mile for officer wages (considered to be an hourly wage of \$19.92 (Bureau of Labor Statistics, 2014) for patrol officers in Illinois and a 20mph travel speed, which is based on field observations). The distances between nodes are calculated using the Euclidean metric. The fine amount for a single citation is assumed to be \$20 or more,

which is the case in many small and medium-sized cities. The cost of processing a parking citation is usually much lower than the amount of fine for one parking citation, such that the revenue from the tickets can be used to offset the cost in other municipal projects (McNerthney, 2011; Murray, 2014). We assume a \$2 processing cost for one parking citation herein. The number of on-street parking slots per block usually contains five to fifteen slots as an estimate based on field observations in the Champaign-Urbana area of Illinois. Without data source about the inspection cost per parking slot, we assume that the inspection cost per parking slot is \$0.5, including the labor cost, fuel and maintenance cost for a patrol vehicle with speed lower than 5 mph. Also, due to the absence of the literature about parking violation tolerance, we chose a reasonable value range of violation probability limit in this thesis based on the numerical test in Appendix A.

**Table 4.1. Value ranges of the parameters**

Parameter	Value	Unit	Parameter	Value	Unit	Parameter	Value	Unit
$\lambda_i$	[0,1]	-/hr	$\mu_i^{-1}$	[1,2]	-/hr	$\sigma_i$	$0.2\mu_i^{-1}$	hr
$c_i$	[1,4]	hr/\$	$\beta_i$	[2.5, 7.5]	\$	$ K $	$\mathbb{Z}^+$	-
$p$	20.0	\$	$P_{max}$	[0.0005,0.003]	-	$u$	1.55	\$/mile



**Figure 4.1. Demand/price distribution of Case I**



The west side of the service region is assumed to be subject to greater demand and higher parking prices, see Figure 4.1, the parking demand distribution of Case I. Each green circle represents a parking lot, and larger size of the circle means higher demand (-/hr) and parking price (\$).

In order to compute the parameters of the continuous model, the region had to be partitioned into two subregions, according to the distribution of required patrol frequency,  $g^{-1}(P_{vio})$ . Base on the method illustrated in Section 3.3, the parameters of each subregion should be set at the average of the nodes in it.

## 4.2 Numerical test

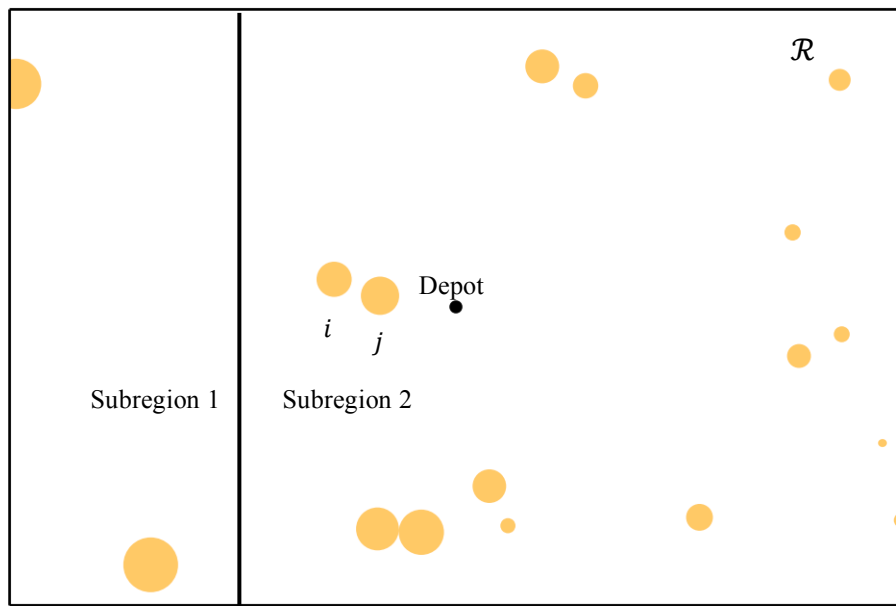
In this section, we solve the case with nodes distributed as seen in Figure 4.1 under both discrete and continuous settings using some state-of-art solvers. There are some difficulties in solving these models. For the discrete model, the number of constraints increases exponentially as the number of nodes increases, so does the computation time. Therefore, to reduce the computation time, some constraints like subtour elimination constraints (3.16g) and tour length constraints (3.16h) were relaxed initially. If subtours or overly long vehicle tours exist in the solution, the corresponding constraints should be re-added into the model formulation, then the model should be resolved (Francis et al., 2006b). Similarly, for the continuous model, the nonlinear constraints (3.22d) associated with tour length are relaxed initially and should be re-added into the model when the solution contains routes that are longer than the limit. These heuristic approaches are used to simplify the solution process.

The values, or value ranges, of some of the parameters in this case are exhibited in Table 4.2. Other parameters are randomly generated based on the value ranges in Table 4.1. There are in total seventeen parking lots in the service region. The required patrol frequency,  $g^{-1}(P_{vio})$ , of each parking lot is shown in Figure 4.3. A larger size of the yellow circle represents the fact that the parking lot requires a higher patrol frequency. In comparing Figure 4.1 and 4.2, one can see that, generally speaking, a parking lot with greater demand and a higher price requires more frequent inspection, but greater demand and higher price definitely do not mean that a higher patrol frequency is required, since the required patrol frequency remains dependent on other

parameters, including  $T_i$ ,  $p$ , and  $limit$ . For example, in Figure 4.1 and 4.2, the demand of parking lot  $i$  has a greater demand and higher price than  $j$ , but  $j$  still requires more frequent inspections.

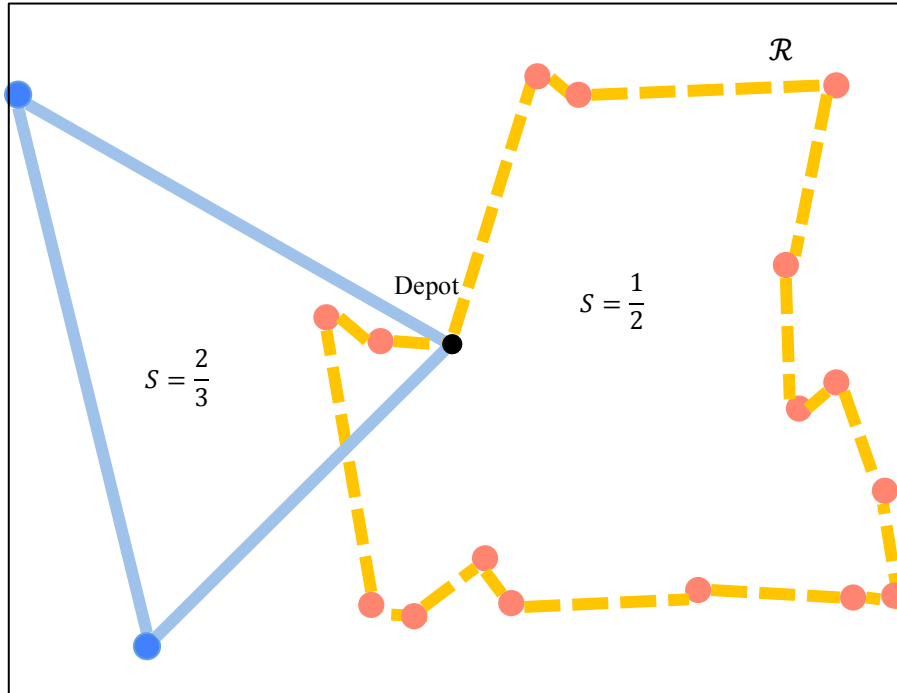
**Table 4.2. Value or value range of the parameters of Case I and II**

Parameter	Value	Unit	Parameter	Value	Unit	Parameter	Value	Unit
$\lambda_i$	[0.31,0.62]	-/hr	$T_i$	[1,1.38]	hr	$p$	20.0	\$
$P_{max}$	0.002	-	$ K $	2	-	$c_i$	[1,2]	hr/\$



**Figure 4.2. Required patrol frequency distribution of Case I**

The discrete model is solved using Gurobi (Gurobi Optimization, I., 2014) based on Python using an i7 processor and 8 GB of memory. The subtour constraints are added back into the problems, while the tour length constraints are relaxed. The computation time is 2 hour 55 minutes. Figure 4.3 shows that in Case I, Route 1 (the solid blue lines) covers the high-demand nodes, with the patrol frequency being  $\frac{2}{3}$ , while Route 2 (the dotted yellow lines) covers the rest of the nodes with the patrol frequency being  $\frac{1}{2}$ . The optimum cost is found to be \$47.8508 with a 0.0% optimality gap.



**Figure 4.3. Routes generated in the discrete model of Case I**

To solve the case with the method of continuum approximation, we first partition the service region  $\mathcal{R}$  into two subregions based on the distribution of the required patrol frequency, see the detailed description of the partitioning method in Section 3.3. The two subregions are shown in Figure 4.2. Then the values of the parameters of the continuous model can be computed by taking the average over each subregion, see Table 4.3.

The continuous model is solved using the Knitro solver (Ziena Optimization LLC, 2011) based on AMPL (Fourer et al. 2003) with an i7 processor and 8 GB of memory. Given that Knitro only provides a local optimum, we conducted the computation several times with different initial values in order to obtain a global optimum by enumerating a number of feasible solutions that are as great as possible. The computation time is 0.41 second with the tour length constraints being relaxed. The optimum cost is \$46.5645 with 0 feasibility error, and the values of the decision variables can be found in Table 4.4.

**Table 4.3. Value of the parameters of the continuous model in Case I**

Parameter	Unit	Subregion 1	Subregion 2
$\lambda_m$	-/hr	0.4498	0.3451
$\mu_m^{-1}$	hr	1.1635	1.1888
$c_m$	hr/\$	0.9992	1.3132
$\sigma_m$	hr	0.2327	0.2377
$r_m$	mile	0.75	0.25
$A_m$	mile <sup>2</sup>	0.5	1.5
$\omega_m$	-/mile <sup>2</sup>	4.0	10.0
$\beta_m$	\$	4.8120	4.7361
$g_m^{-1}(P_{max})$	hr	0.5585	0.3725

**Table 4.4. Solution of the continuous model in Case I**

Frequency ( $l$ )	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	1	$\frac{4}{3}$	2
Decision variables						
$f_1^l$	0	0	1	0	0	0
$f_2^l$	0	1	0	0	0	0
$Y^l$	0	1	1	0	0	0

In both the discrete and continuous model, routes with patrol frequencies of  $\frac{1}{2}$  and  $\frac{2}{3}$  are generated to serve the parking lots in region  $\mathcal{R}$ . Although the design of vehicle routes is omitted in the continuous model, it still provides an accurate estimation of the optimal agency cost with a percentage difference that is less than 3%.

The continuum approximation model yields an optimum cost that is slightly smaller than the result of the discrete model. One possible reason is that, although the set of feasible frequencies and maximum number of patrol routes remain the same, the partitioning and approximation steps in the CA method are actually making the customers more evenly distributed than that of the discrete model. For example, in Subregion 2, which is the low demand area, most of the nodes in it are located near the four sides of the subregion but are away from the depot. This may lead to the solution value of the discrete model being larger than in the

case of the continuous model because the line-haul cost is potentially higher. This conclusion will be further supported in the following section of sensitivity analysis.

### 4.3 Sensitivity test

In this section, sensitivity tests of both the discrete and the continuous models are presented in order to illustrate the impact of parking demand  $\lambda$ , parking rate  $c$ , parking citation fine  $p$ , and violation probability limit  $P_{max}$ . We generate a new instance with fewer nodes, Case II, as the benchmark case, since the computation time of the discrete model in Case I is overly long. The percentage differences between the discrete models and continuous models are also presented to validate that continuum approximation method is accurate to some extent for the purpose of estimating the agency cost.

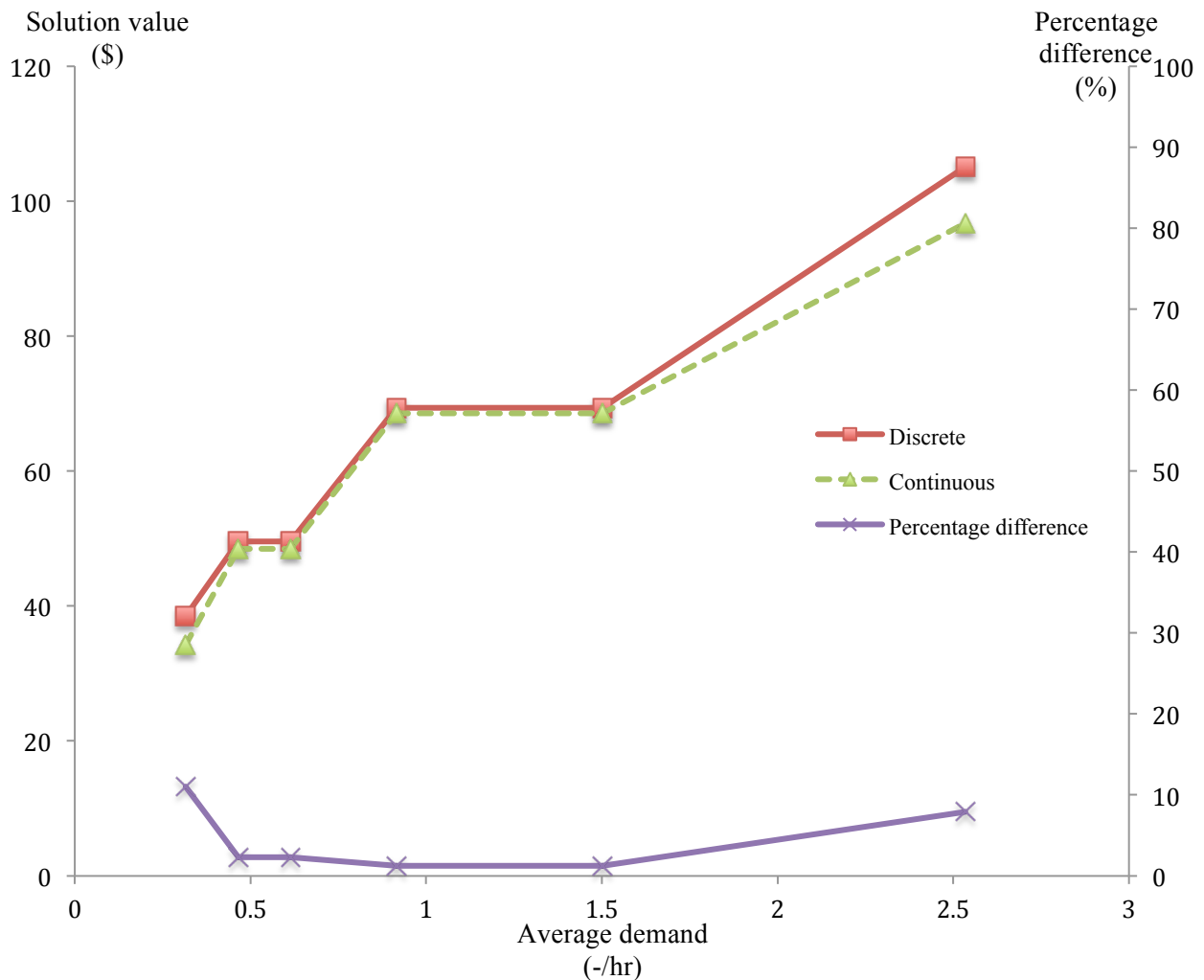
In Case II, only fifteen nodes are generated while the values and value ranges of the parameters remain the same as those in Table 4.1 and 4.2. Using the same method in Section 4.2, we obtain the optimum cost of \$49.5823 with a 0.00% optimality gap by solving the discrete model. The computation time is around 20 minutes, which is much less than that of Case I. This is due to the reduction in the number of decision variables and a much greater reduction in the number of constraints. The optimum cost of the continuum approximation model turns out to be \$48.4533 with 0 feasibility error. The computation time is 0.31 seconds. Both of the models generate routes with patrol frequencies of  $\frac{1}{2}$  and  $\frac{2}{3}$ , and the percentage difference between the solution values of the two models is less than 3%.

#### 4.3.1 Impact of demand

While the distribution of nodes and other parameters remain unchanged, the demand  $\lambda_i$  of each parking lot in the service region are multiplied by a scalar  $a$ . Considering the range of the parking demand in real life,  $a$  should satisfy  $0 \leq a \leq 10$ . For  $a \in \{0.677, 1, 1.32, 1.97, 3.23, 5.45\}$ , the optimum agency cost is computed using the same method explained in Section 4.2 and is plotted in Figure 4.6. The horizontal axle is the average demand of the parking lots in the service region. The purple line with cross marks gives the

percentage difference between the solution values of the discrete model and the continuous model.

Figure 4.4 shows that as the demand increases, the required patrol frequency and optimum agency cost rise stepwise. It rises stepwise because that the frequency variable  $l$  does not take continuous values. This also occurs in the sensitivity analysis of other parameters. Hence, there exist a number of demand thresholds, see the inflection points in Figure 4.4. When the demand varies between two thresholds, the optimal agency cost and the patrol scheme will remain unchanged. Otherwise, the agency cost and the patrol scheme will definitely change.



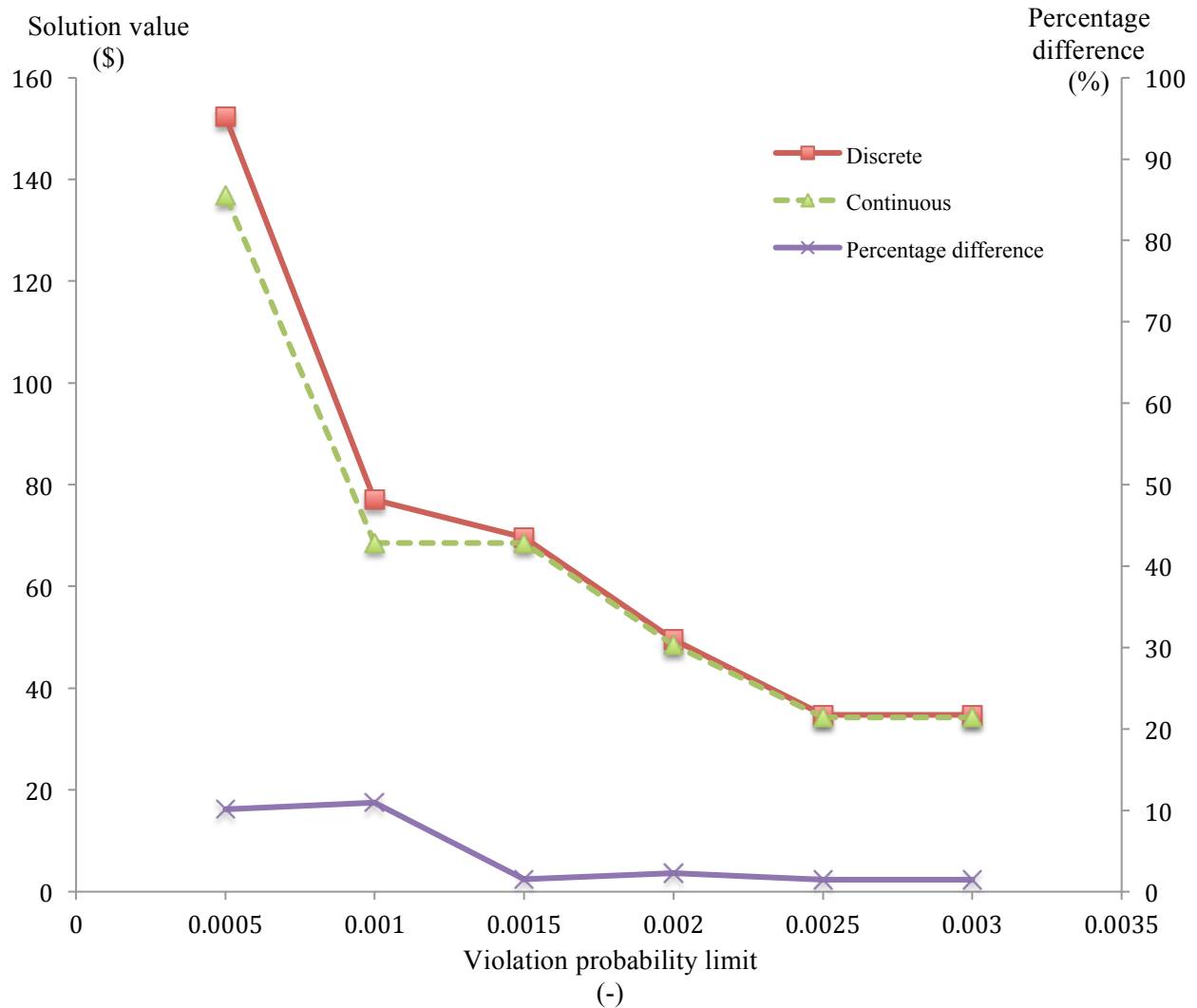
**Figure 4.4. Impact of parking demand on solution value**

Note that the solution values of the continuous model move in the same steps as those of the discrete model, and the percentage difference between the two models is generally below 10%.

#### 4.3.2 Impact of violation probability limit

The tolerance level for parking violations may vary upon occasion. In many situations, e.g., during the Thanksgiving shopping season, patrol officers may have to perform their duties more strictly while the parking prices and fines remain constant. We tested the sensitivity of the models to the tolerance level for violations by incrementally changing the parking violation limit,  $P_{max} \in \{0.0005, 0.001, 0.0015, 0.002, 0.0025, 0.003\}$ , and computing the corresponding solutions, see Figure 4.5.

As the violation probability limit rises, the system provides greater tolerance for parking violations, and the patrol cost per unit time decreases. When the violation probability is set to zero, the patrol frequency approached a value greater than ten billion and the corresponding agency cost can be seen as infinite. Note that the method of CA provides a particularly good estimation of the agency cost when the violation limit is larger than 0.0015, and the percentage differences are generally smaller than 10%.



**Figure 4.5. Impact of violation probability limit on solution value**

### 4.3.3 Impact of parking price

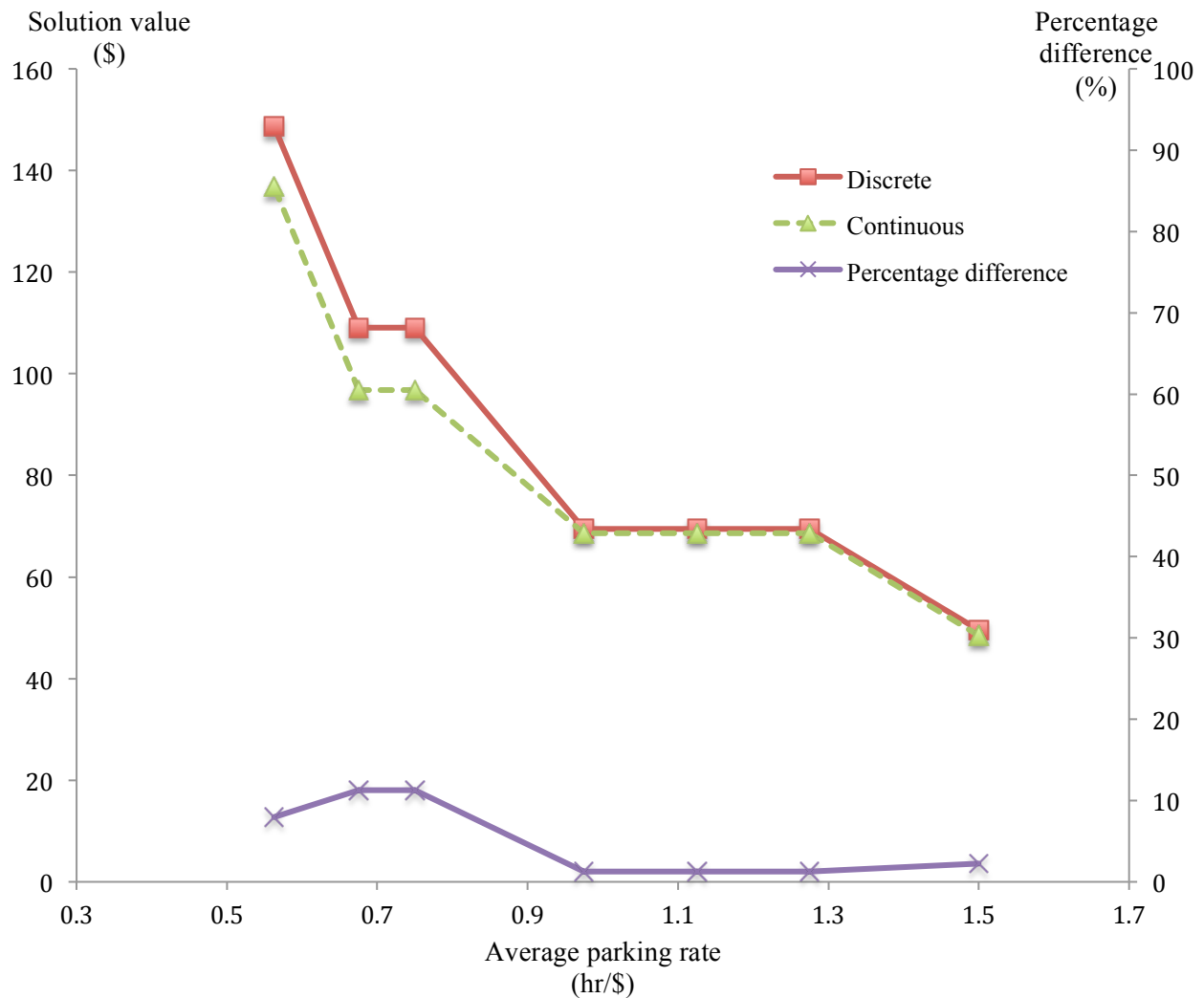
We examined the impact of parking price by substituting the parking rate  $c_i$  for  $\cdot c_i$ ,  $a \in \{0.375, 0.45, 0.5, 0.65, 0.75, 0.85, 1\}$ , and computing the corresponding solution values while keeping other parameters constant. Note that parking rate  $c$  is the inverse of the parking price. The solution values and the percentage differences are plotted in Figure 4.6.

In a similar manner, the models are sensitive to changes in parking price. As the price decreases, which means  $c$  goes up, drivers are expected to be more willing to pay parking fees and less likely to risk receiving parking citations. On the contrary, when parking price rises to a point such that the gap between the fine and the parking fee is relatively small, it becomes more



economical for drivers to pay less than they should have paid. The consequence is that parking enforcement officers had to inspect the parking lots more frequently in order to keep the violation probability under the limit.

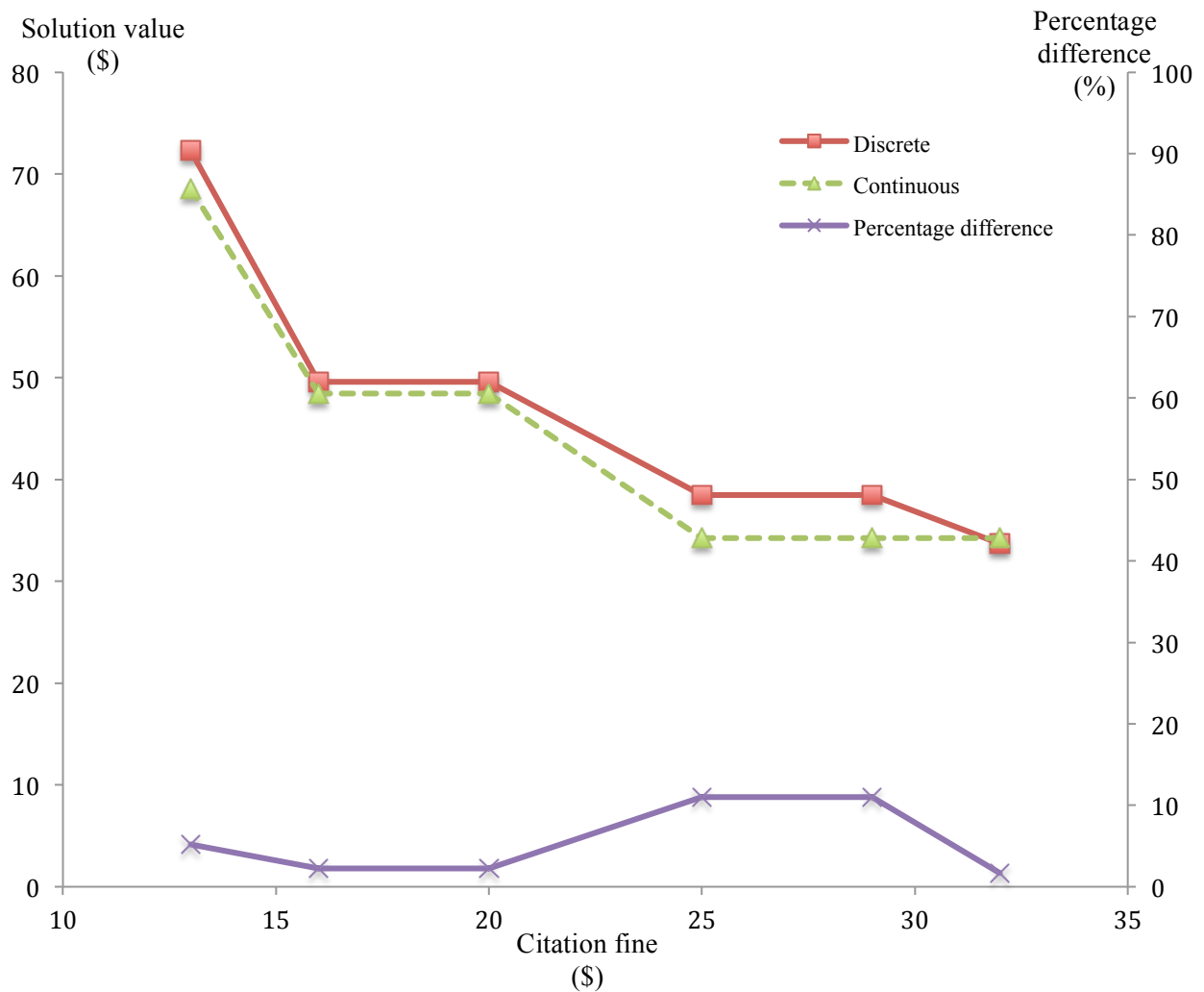
When the parking rate is less than one hour per dollar which means price is greater than one dollar per hour, the estimation of agency cost using the CA method becomes less accurate, with the percentage difference being around 10%, but it still moves in the same steps with the solution value of the discrete model.



**Figure 4.6. Impact of parking price on solution value**

#### 4.3.4 Impact of parking fine

Online forums and local newspapers often have featured discussions involving advocates of lowering or increasing local parking fines. Intuitively, a higher penalty can lead to reduced number of illegal behaviors because less effort is required to inspect the parking lots. In order to explore the sensitivity of the model to the value of parking fine, we raised the value of  $p$  stepwise and monitored the changes in the solution,  $p \in \{13, 16, 20, 23, 25, 29, 32\}$ .

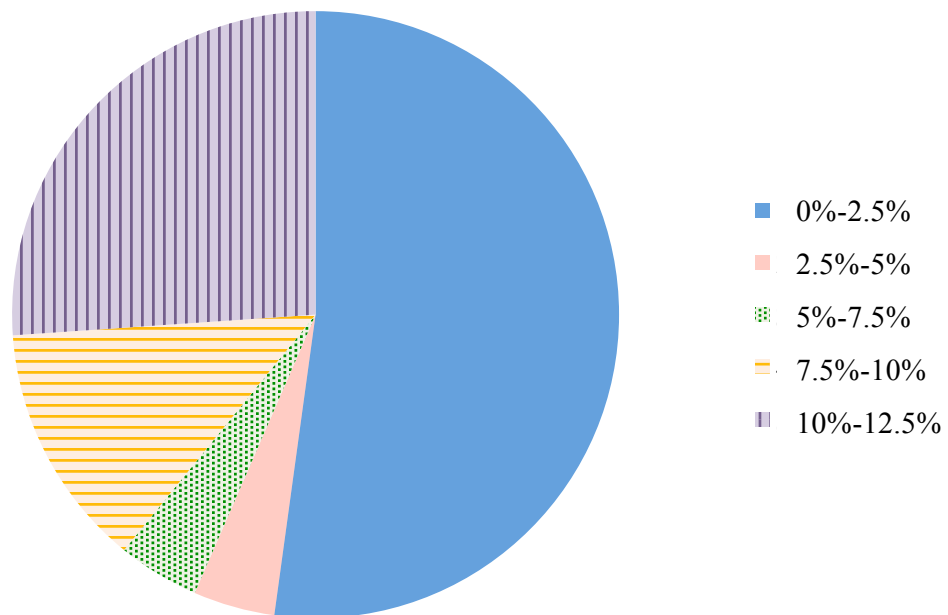


**Figure 4.7. Impact of parking fine on solution value**

Figure 4.7 shows that the solution value would decrease as  $p$  increases, which proves the assumption made earlier that the parking offence rate decreases as parking enforcement becomes stricter. The estimation of the CA method is not as accurate as what we find in the sensitivity tests of the other parameters. Generally the percentage differences are below 11%.

## 4.4 Conclusion

In this chapter, the parking enforcement patrol models in Section 3.2 and 3.3 are tested with the assumed data. The discrete agency model, a Mixed Integer Linear Problem, is solved with the commercial solver Gurobi (Gurobi Optimization, I., 2014) using an exact method, while the continuous agency model is solved with KNITRO (Ziena Optimization LLC, 2011) using a multi-start algorithm. With the method of continuum approximation, the numbers of decision variables and constraints are greatly reduced, and as a result the runtime of this idealized model is relatively minimal, compared with the original discrete model. Generally, the solution values of the CA models are slightly smaller than those of the discrete model. Overall, 23 different instances are generated and tested in this chapter. And about 61% of these instances yield the percentage differences between the discrete and continuous models that are less than 5.5%, see details in Figure 4.8.



**Figure 4.8. Values of percentage difference between the two models**

The parking enforcement patrol models are very sensitive to the changes in some of the parameters, especially the violation tolerance level. The system cost increases exponentially as the violation probability limit  $P_{max}$  decreases. When the tolerance level of parking violations remains unchanged, the parking agency can reduce the patrol cost by increasing the citation fine

or cutting the parking price, both of which are effective methods. Another important finding is that parking demand changes may influence the optimum patrol scheme. When parking demand fluctuates between some thresholds, the efficiency of patrol system will not be affected. But if the parking demand changes go beyond some limits, the agency will have to re-compute the patrol scheme in order to maintain the healthy operation of the parking system. Thus, the patrol department should periodically check the parking demand and make new patrol plans.

## CHAPTER 5

### CONCLUSION AND FUTURE RESEARCH

#### 5.1 Conclusion

This thesis applied the method of mathematical modeling and the approach of continuum approximation in order to design a more efficient patrol system for parking enforcement.

Chapter 2 reviews the research literature related to i) the characteristics of driver behavior when parking, in the peer-reviewed papers, ii) the problem settings and solution methods of multiple periodic vehicle routing problems, iii) model formulation for some popular patrol problems, and iv) the method of continuum approximation.

Chapter 3 presents a driver model in which stricter enforcement results in higher parking payments from drivers, and the two agency models that have the objective of minimizing expected system cost per unit time. The perspective of drivers is that the parking payment that minimizes the expected overall cost of the parking duration should be determined based on whatever limited information the drivers can obtain. The optimal parking payment is formulated as a function of parking prices, violation fines, parking time distribution and patrol frequency. The perspective of agency is that the patrol cost should be minimized with the parking violation under control. The violation probability for one parking slot is formulated based on the Queueing Theory. The probability of a parking slot being illegally occupied is formulated as function of parking demand, the distribution of parking duration, and drivers' optimal payment.

In order to formulate the models that aim at minimizing agency cost, several important assumptions are made: i) under the time-invariant demand setting, the arrival rate at each parking lot is a constant, so the headway between two inspections is also assumed to be time-invariant; ii) the violation limits at all parking lots are of the same value; iii) the events at one parking slot are independent of the events at any other parking slot; iv) the number of vehicle routes is specified as a preset parameter; v) the citation processing cost is omitted. Given the above

results, two agency models are built under discrete and continuous settings, respectively. In the discrete model, the parking lots are assigned to several routes and each route is patrolled according to a certain frequency, which is one of the decisions of the model. Two routes can share the same patrol frequency, while each parking lot can only be assigned to one route. In the continuous model, the parking demand, parking time, and parking rate are all assumed to be continuous functions of location. The agency costs are computed by, first, formulating the cost per unit area, and, second, by integrating the cost over the entire region. The problem is further simplified by partitioning the service region into multiple subregions in a manner such that, within each subregion, the parameters can be approximated into the average over the subregion. Note that the construction of the vehicles routes is skipped in this continuous model.

In Chapter 4, the agency models are tested using assumed data, and the results of sensitivity analysis provide some significant insights into the efficient parking enforcement patrol scheme. Note that the tour length constraints are relaxed when solving these small cases. Compared with that of the discrete model, the computation time of the continuous model is minimal. The computation time for the discrete model, would increase exponentially as the number of nodes increased. As regards the continuous model, the number of the instances has little impact on the number of decision variables or constraints, so the runtime can remain relatively small even for extremely large problems. The method of CA can be used as an efficient tool for estimating system cost and assisting decision-makings of new policies, a conclusion that has been arrived at by multiple researchers.

In conclusion, as regards parking patrol routing problems, it is valid to apply the method of continuum approximation in system cost estimations and parametric analyses, because it can rapidly provide important decision-making information for new measures or policies. As regards parking prices, violation fines and patrol frequencies, all of these play significant roles in controlling the violation rate of drivers. Regular surveys should be conducted to verify the parking demand, parking purposes and other customer information involving the parking lots, so that patrol schemes can be updated to ensure parking enforcement efficiency as the situation changes.

## 5.2 Future research options

The application of the models and solution methods above are restricted to problems with time-invariant customer demands and fixed number of patrol routes. Potential extensions of the parking patrol routing problems include: i) relaxing the assumptions regarding parking demand, violation limit and so on mentioned in Section 5.1; ii) modifying the objective functions by adding benefit terms; iii) constructing vehicle routes in the continuous model; iv) testing larger instances with the nonlinear tour length constraints.

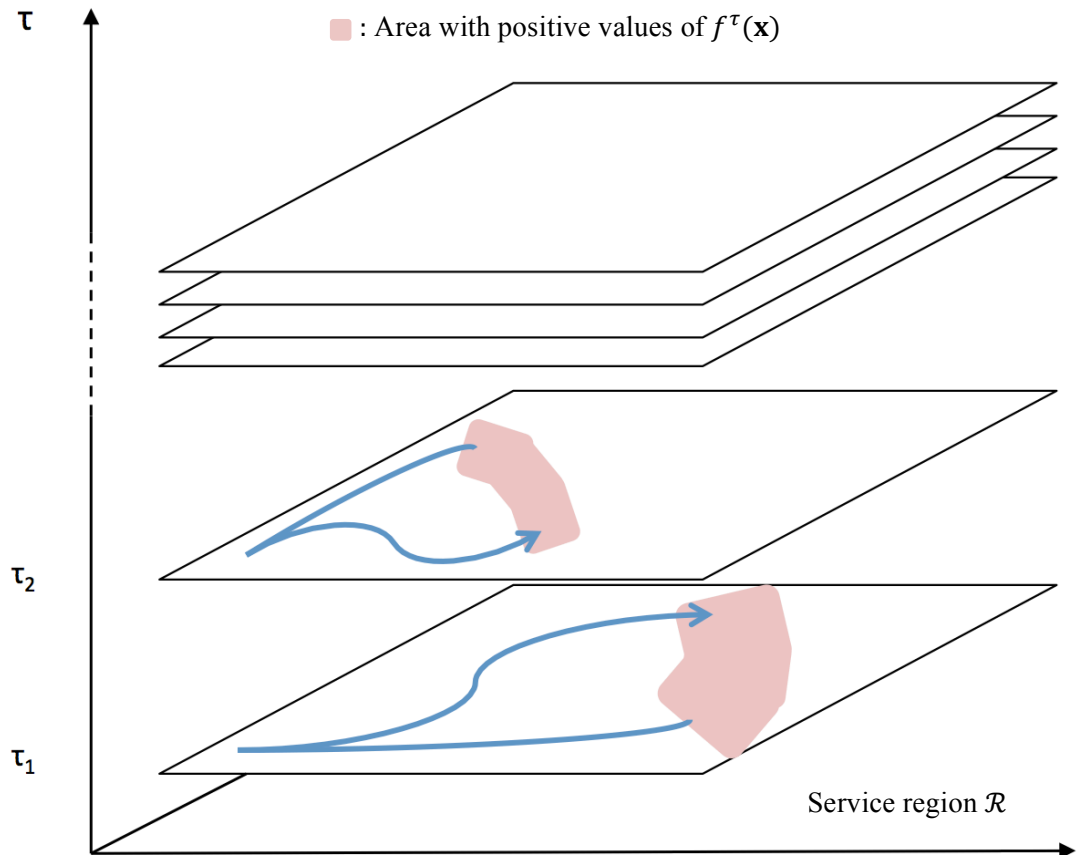
Due to the limits of testing data, the computational study herein is restricted to small instances with assumed data. In small instances, for example, Case I and Case II generated in Chapter 4, the violation probability can be assumed to be constant for all parking lots, and the tour length can hardly extend beyond the limit  $C_{max}$ . Thus the tour length constraints are relaxed when solving the models in this work. However, when the size of the instances grows larger, the violation probability limit should vary upon location to reflect the functional differences between locations. And the tour length constraints in the continuous model, which are nonlinear constraints, will further extend the computation time of the models. Thus, large instances involving real data should be tested to examine the performance of the continuous model of parking enforcement patrol problem formulated in this thesis.

Given the time-invariant setting of parking demand and patrol frequency settings in this paper, it is not permissible for parking lots that require different levels of inspections be patrolled by the same vehicle. While under the time-variant setting, the feasible dispatch time set  $S_c$  can be a substitute for  $S$ , the set of feasible patrol frequencies. Thus, the parking lots associated with different patrol levels are allowed to be visited by one vehicle that departs at a scheduled time. The agency cost can be further reduced, and the model can be more applicable in practical use. Under the continuous setting, the problem basically involves decision-makings regarding both time and space, see Figure 6.1

And the decision variables would be:

$n^\tau$  numbers of vehicles dispatched at time  $\tau$ ,  $n^\tau \in \{0,1,2,3 \dots\}$ ,  $\tau \in S_c$

$f^\tau(\mathbf{x})$  fraction of nodes at point  $\mathbf{x}$  visited by vehicles dispatched at time  $\tau$



**Figure 5.1. Time-variant model of parking patrol problem**

In many cities, the revenues from parking charges and violation tickets are not completely given to the local parking department. Revenues can be shared and used to offset the costs of education, public safety, and advocacy projects unrelated to parking, and other programs (McNerthney, 2011; Murray, 2014). It is the case that a small portion of the revenue goes to the city or county to offset the costs of parking, patrolling and the processing of the tickets. Thus, there are some benefits to be derived from effective patrolling operations. The objective function could be changed from cost to net cost by adding the benefit parameters. When the net income from one parking ticket is a positive value, the model would probably generate patrol plans that inspect the parking lots more frequently. Again, this will be a tradeoff between the patrol costs and benefits. Sensitivity analysis should be conducted to examine the effect of the changes in these benefit parameters.

Last, but not least,, analyzing the design of vehicle routing zones can be conducted to further narrow the gap between the agency costs generated by mathematical modeling with



discrete inputs and the method of continuum approximation. First, a number of vehicles routing zones should be created such that one patrol vehicle is assigned to visit all the nodes in one and only one zone. The near-optimum shape of such zones has been proved to be a narrow wedge elongated towards the depot (Newell and Daganzo, 1986). Second, after replacing the continuous density function with the original customer locations, the exact routes for each patrol vehicle can be generated (Ouyang, 2007). A comparative analysis between the two modeling approaches should also be conducted to further analyze the method of CA in operational-level patrol routing problems.

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## APPENDIX A

### NUMERICAL TEST OF VIOLATION PROBABILITY LIMIT

To find a reasonable value range of the violation probability limit  $P_{max}$ , a numerical test is conducted. First, a sample of 500 nodes is generated with different values of  $\lambda_i$ ,  $c_i$ , and  $T$  randomly generated respectively, from range  $[0.31,4]$ ,  $[0.375,2]$  and  $[1,1.38]$ , see detailed description of these parameters in Section 4.1. For each combination of difference values of  $p$  and  $s^k$ , we calculated the value of  $P_{vio}$  of all nodes, then we obtained the minimum, average and maximum values of  $P_{vio}$  from the sample. Second, we generated a sample of 1000 nodes and repeat the first step. As shown in Table A, in each row, the statistics of the 1000-size sample and the 500-size sample are very close, thus it is reasonable to assume that the sample large enough to provide accurate estimation of the population.

The value of violation probability limit should be high enough to ensure a healthily functioning parking system and meanwhile below a reasonable limit such that the patrol cost is affordable for the parking department. Hence, the value range of  $P_{max}$  is set to be  $[0.0005,0.003]$ , according to the average values of the two samples. With the citation processing cost being \$2, number of parking slot per street block being 10, total number of parking lots being 15, and the patrol frequency being 1 (-/hr), the expected citation processing cost per hour should be in range  $[0.15,0.9]$ , which is much smaller than the transportation and inspection cost, see Section 4.1.

**Table A.1. Value of  $P_{vio}$  under different parameter settings**

$p$	$s^k$	$P_{vio}$					
		500 nodes generated			1000 nodes generated		
		Min.	Ave.	Max.	Min.	Ave.	Max.
20	0.5	0.00085	0.00496	0.01988	0.00080	0.00511	0.02052
	0.67	0.00043	0.00342	0.01334	0.00056	0.00353	0.01377
	1	0.00031	0.00206	0.00780	0.00035	0.00212	0.00805
	1.33	0.00027	0.00145	0.00542	0.00025	0.00150	0.00559
	2	0.00017	0.00089	0.00325	0.00015	0.00091	0.00336

**Table A.1. (Cont.)**

$p$	$s^k$	$P_{vio}$					
		500 nodes generated			1000 nodes generated		
		Min.	Ave.	Max.	Min.	Ave.	Max.
25	0.5	0.00065	0.00372	0.01457	0.00061	0.00383	0.01504
	0.67	0.00043	0.00258	0.00990	0.00043	0.00266	0.01022
	1	0.00028	0.00157	0.00586	0.00027	0.00161	0.00605
	1.33	0.00020	0.00111	0.00409	0.00019	0.00114	0.00422
	2	0.00013	0.000686	0.00248	0.00012	0.00070	0.00255
30	0.5	0.00052	0.00295	0.01139	0.00049	0.00304	0.01176
	0.67	0.00037	0.00206	0.00780	0.00035	0.00212	0.00805
	1	0.00023	0.00126	0.00645	0.00021	0.00129	0.00480
	1.33	0.00016	0.00089	0.00326	0.00015	0.00092	0.00337
	2	0.00010	0.00055	0.00198	0.00009	0.00337	0.00205
35	0.5	0.00043	0.00243	0.00929	0.00041	0.00251	0.00959
	0.67	0.00031	0.00170	0.00639	0.00029	0.00176	0.00660
	1	0.00019	0.00104	0.00384	0.00018	0.00107	0.00396
	1.33	0.00013	0.00074	0.00270	0.00013	0.00076	0.00279
	2	0.00008	0.00046	0.00165	0.00008	0.00047	0.00170