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TWO-DIMENSIONAL SHAPE OF SAND MADE BY
CRUSHING ILLINOIS LIMESTONES OF DIFFERENT TEXTURES

Paul C. Heigold and J. E. Lamar

ABSTRACT

As grain shapes of sand used in making concrete can influence the characteristics of the concrete, a method has been devised for measuring and visualizing the two-dimensional shape of sand made by crushing limestone. The technique employs grain silhouettes and their intercepts on a 16-rayed radial grid. The data obtained are compared by computer with similar data for 111 shapes that were selected from samples of limestone sand and set up as standards.

One sample of calcite and 17 samples of 10-mesh by 14-mesh limestone sand, each consisting of 25 grains, were measured and the results grouped according to the lithology of the parent rock.

The results thus far show no consistent relations between lithology and grain shape, but certain shapes occurred frequently, including flatiron, narrow bulging oval, rectangular trapezoid with rounded ends, and truncated right triangle, in that order.

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INTRODUCTION

The shape of particles composing natural sand and sand that is made by crushing limestone is known to have an important bearing on the character of the concrete in which such sand is used (Gray and Bell, 1964; Willis, 1967). Various means of measuring the shape of limestone sand grains have been proposed (Gray and Bell, 1964; and Bibliography), including sizing with a sieve that has rectangular openings, measurement under a microscope, measurement of the radius of curvature of photographs of grains, measurement of the percentage of voids of unpacked sand, and noting the length of time required for the pieces to pass through a small orifice. Additional data on shape measurements of pieces of both natural sand and sands and coarser products made by crushing stone are given in papers listed in the Bibliography.

The procedure described in this paper presents what we believe is a different approach to grain shape. It was developed as a means of visualizing the two-dimensional shape of limestone sand grains and of determining the shape characteristics of particles produced by crushing limestones of different lithologic character. Briefly, it involves making photographic silhouettes of limestone particles, measurement of the intercepts of the outline of each particle on a grid of 16 equally spaced rays that have a common center, and comparing by computer the data so obtained with similar data for a number of "standard shapes" selected to cover a wide range of grain shapes. The standard shape that each grain most nearly resembles is determined, as is the shape that the total sample most nearly resembles. The end result is a visual image of the shape of the particles and of the mean particle shape of each sample.

The specific purposes of the investigation were (1) to test, and if possible perfect, the procedure described; (2) to develop a comprehensive and workable group of standard shapes; (3) to determine which shapes are most

common among crushed limestone particles in the 10- by 14-mesh size range; and (4) to ascertain whether lithologic differences in limestones are reflected in the shape of sand-sized particles made by crushing the limestones. The study is preliminary, but publication of its results to date may enable others to test the procedure and help solve some of the problems involved in its use, especially the problem posed by the amount of time required to measure the radial intercepts of the grain silhouettes.

PROCEDURES

Samples

The samples studied came from Illinois deposits and included 15 limestones, 2 dolomites, and 1 coarsely crystalline calcite. They have been grouped on the basis of their lithologic character in table 1.

All samples were taken from the faces of quarries except sample H, which consisted of coarse lumps of calcite from a vein in the now abandoned Hillside fluorspar mine at Rosiclare. The field samples were broken manually in the laboratory to about 1½-inch pieces, and these were fed into a laboratory jaw crusher set at one-quarter of an inch. The throughput of the crusher was sieved into several sizes, and the 10- by 14-mesh fraction was selected for this investigation. This size was chosen in the hope that it would reflect the breakage characteristics of the finer grained limestones and the effect, if any, of the cleavage of the calcite in coarsely crystalline and coarsely particulate limestones.

Selection of Standard Shapes

The standard shapes chosen for use in this study are shown in figure 1. They were selected after inspection of many grain silhouettes and are stylized models of the various grain shapes present. Each group of shapes, with one exception, consists of three sizes of the same form. At least one, and often all, of the shapes in each group was found during selection of the standard shapes from the various samples. In general, the individual silhouettes in each shape group are three-fourths, 1½ and 2 inches wide, although the 180 shape group includes four shapes. The shapes are numbered as shown in figure 1 and are subsequently referred to by these numbers. A group of shapes as a unit is referred to numerically with a zero ending; for instance, shapes 21, 22, and 23 form the 20 group and 371, 372, and 373 make up the 370 group.

Making Silhouettes

To make the silhouettes, a quantity of the 10- by 14-mesh stone sand was spread on a piece of glass, distributed by tapping, and placed in a photographic enlarger that was set to make silhouettes of average particles about 1½ inches in longest dimension. An area including about 60 grains was

TABLE 1—SOURCE AND CHARACTER OF SAMPLES

Sample	Source	Geologic unit	Lithology	
			Thin section	Naked eye
R7	Alton	St. Louis	Very finely crystalline	Lithographic
G	Thebes	Girardeau	Very finely crystalline	Lithographic
NF*	Chicago	Niagaran	Finely crystalline	Fine grained
McC	Marblehead	McCraney	Finely crystalline	Fine grained
T	Thebes	Kimmswick	Medium crystalline	Medium grained
NM*	Chicago	Niagaran	Medium crystalline	Medium grained
V	Valmeyer	Kimmswick	Coarsely crystalline	Coarse grained
H†	Rosiclare	—	Coarsely crystalline	Coarse grained
D	Dongola	Harrodsburg	Coarse particles in a fossil-hash matrix	Coarse grained
MC	Quincy	Burlington	Coarse particles in a fossil-hash matrix	Coarse grained
AC	Anna	Ste. Genevieve	Oolites and other coarse particles in a crystalline matrix	Coarse-grained oolite
PdR	Prairie du Rocher	Rocher	Small and medium detrital particles and/or oolites in a crystalline matrix	Medium grained, oolitic
SH	Valmeyer	Rocher		Medium to coarse grained, oolitic
AF	Anna	Ste. Genevieve		Medium grained, oolitic
R2	Alton	St. Louis	Principally small detritus, calcite matrix	Fine grained
R5	Alton	St. Louis		Fine grained
R8	Alton	St. Louis		Fine grained
MF	Marblehead	Burlington		Fine grained

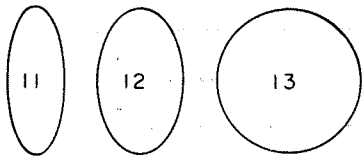
* Dolomite

† Vein calcite from abandoned Hillside fluorspar mine

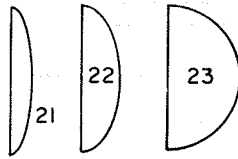
selected at random and enlarged. The larger sides of the grains are probably recorded by the silhouette. Several typical silhouettes are shown in figure 2. Twenty-five silhouettes, selected at random, were studied for each sample.

The Radial Grid

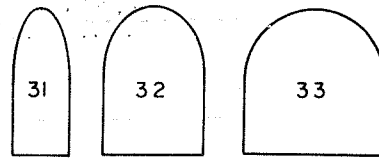
The radial grid used in this work is shown in figure 3 and is drawn on a standard piece of polar coordinate graph paper. The rays are lettered and the intercepts of the silhouettes and the standard shapes are



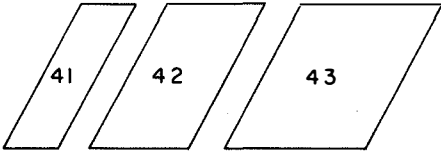
Group 10



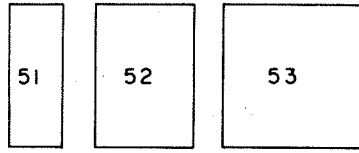
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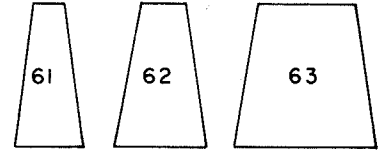
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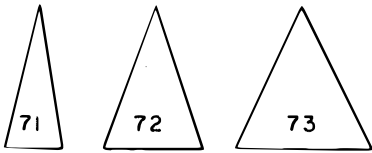
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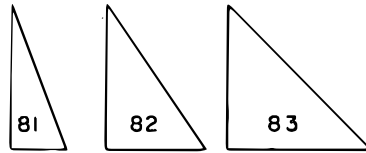
Group 50



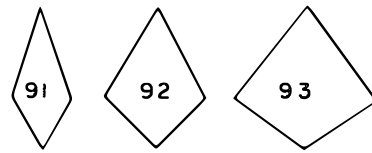
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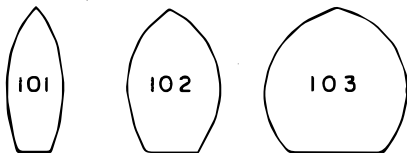
Group 70



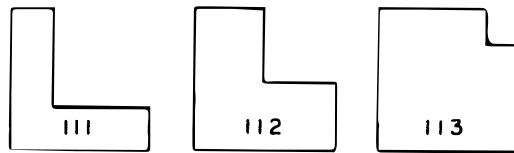
Group 80



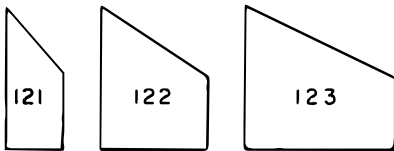
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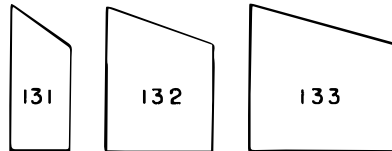
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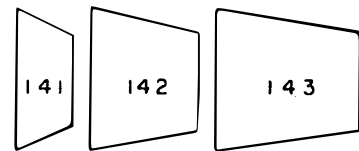
Group 110



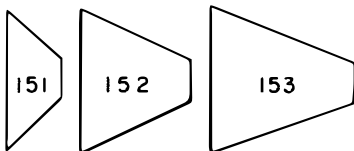
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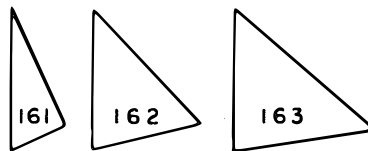
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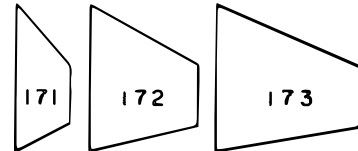
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Group 150

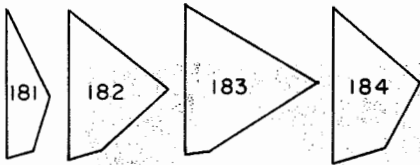


Group 160

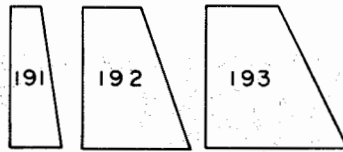


Group 170

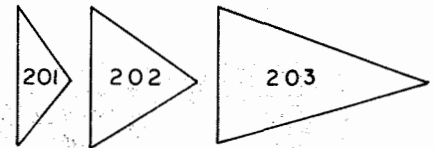
Fig. 1 - Standard shapes used in investigation. For easier visual comprehension These shapes are reduced from



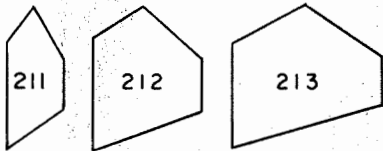
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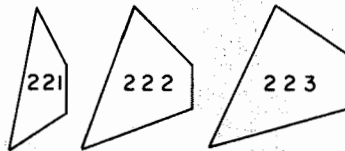
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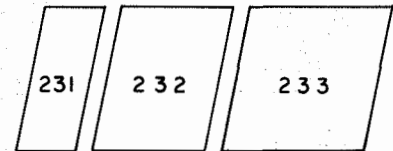
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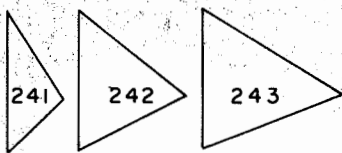
Group 210



Group 220



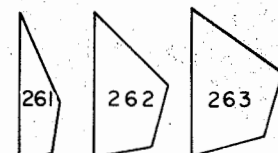
Group 230



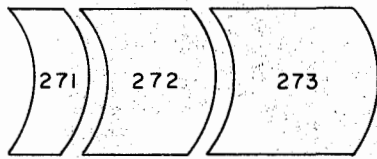
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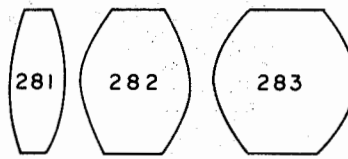
Group 250



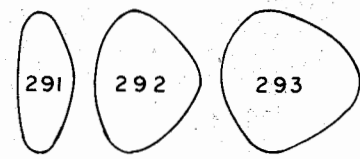
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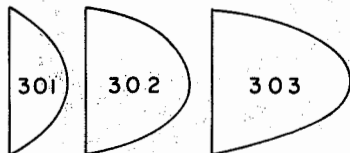
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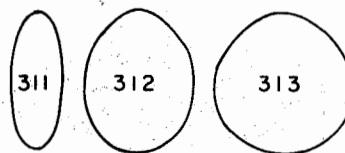
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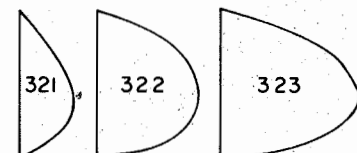
Group 290



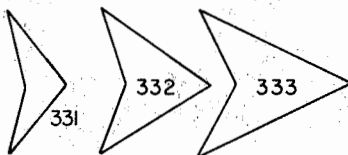
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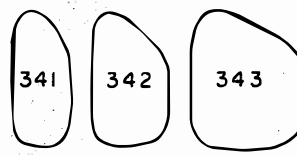
Group 310



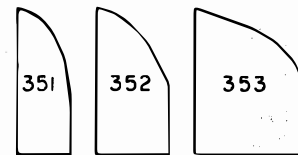
Group 320



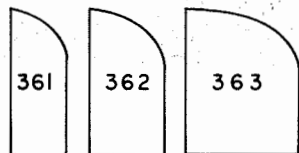
Group 330



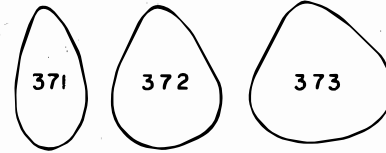
Group 340



Group 350



Group 360



Group 370

they are oriented in the position in which geometric forms are commonly viewed.
2-inch high working models.

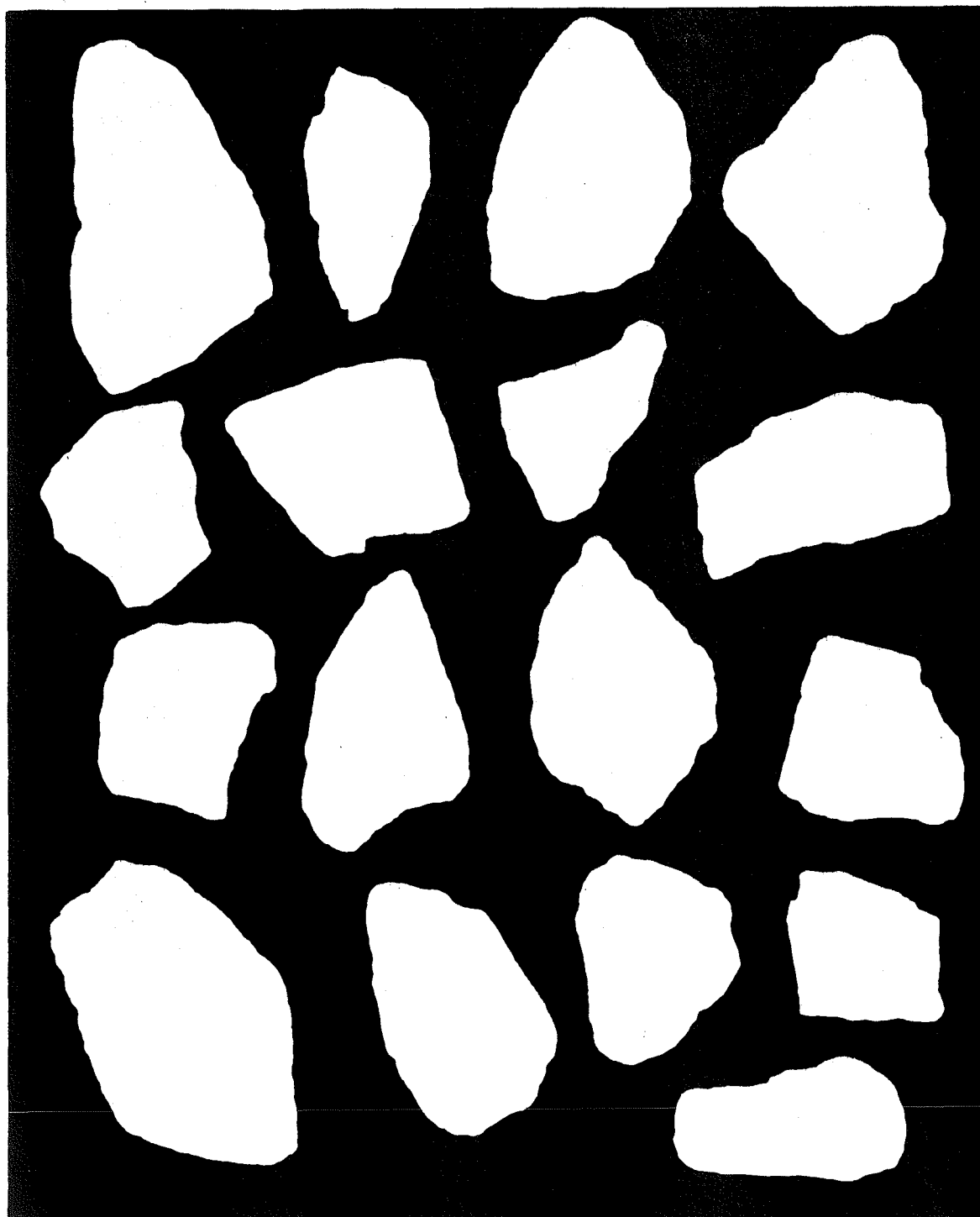


Fig. 2 - Silhouettes of limestone sand grains.

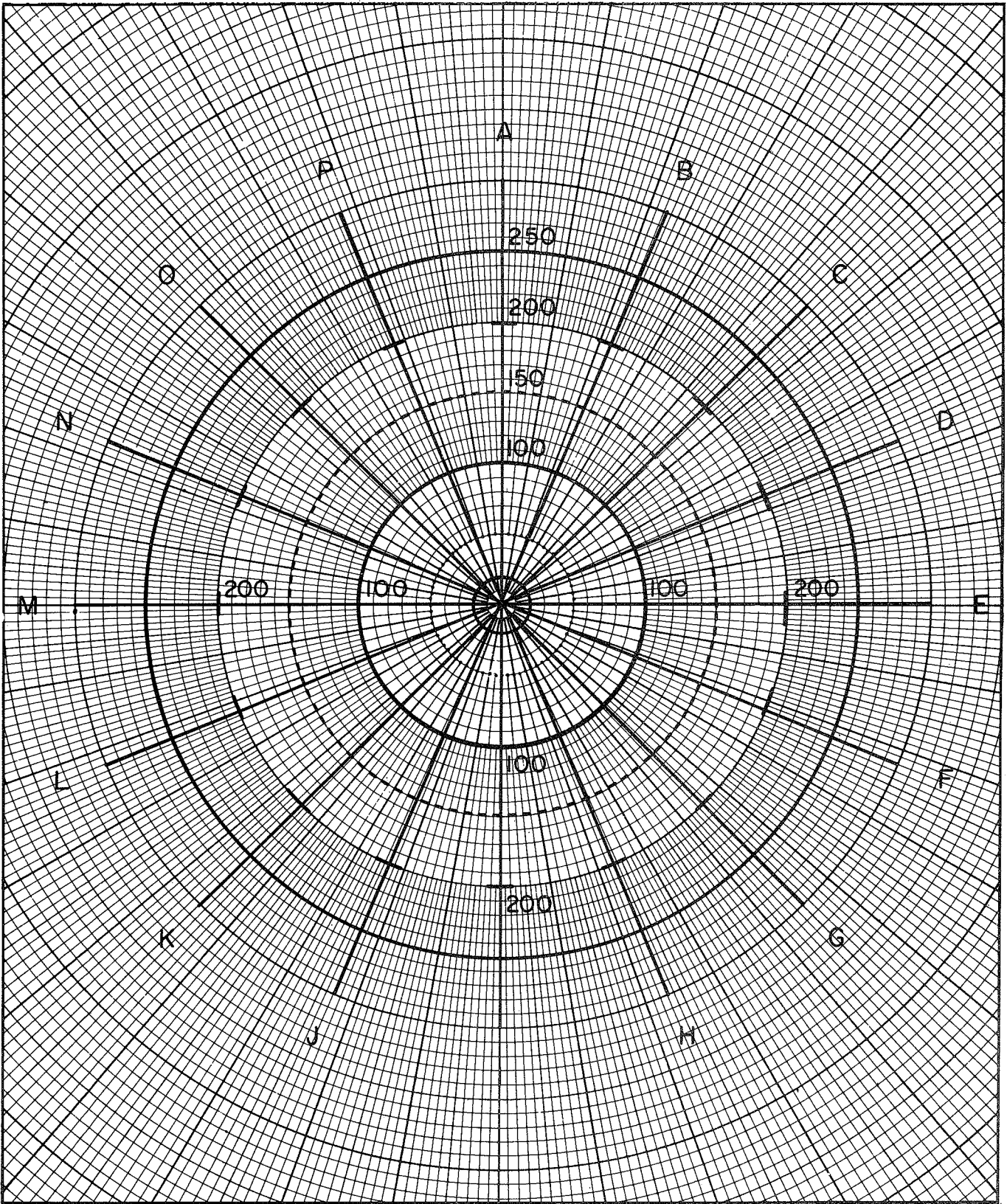


Fig. 3 - Radial grid on which silhouettes are measured.

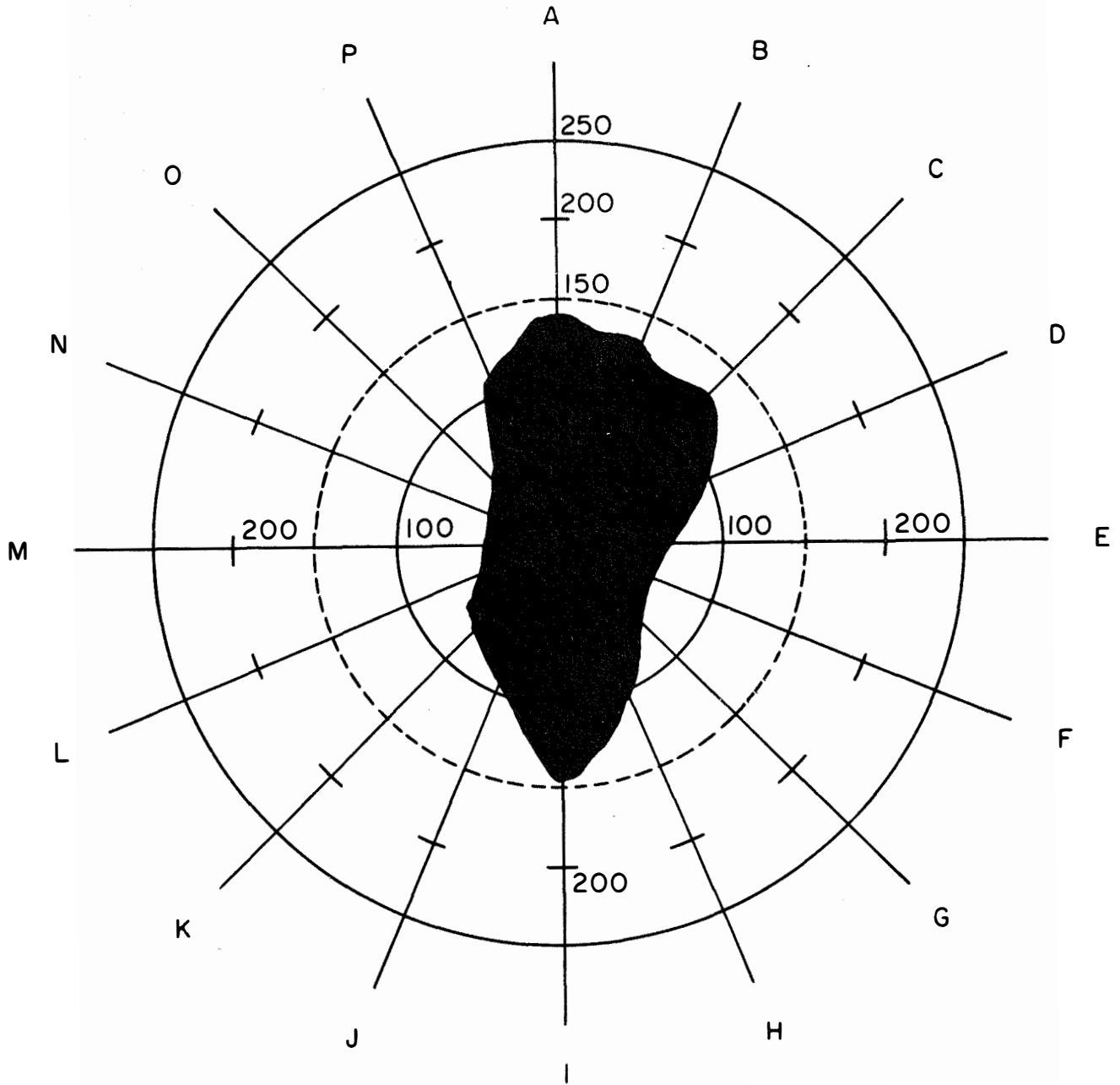


Fig. 4 - Positioning of grain silhouettes on radial grid. The silhouette is shown in black but it is normally white.

read on the rays beginning with ray A and progressing clockwise. Intercepts are measured in tenths and estimated hundredths of an inch.

Several grids with fewer rays were tried but failed to yield a sufficient number of readings to give a reasonable replica of the silhouette being studied when its intercepts were plotted and progressively connected by straight lines. The intercepts of 16 rays produced replicas that were deemed adequate for this study. Increasing the number of rays would improve accuracy but would also impose a prohibitive amount of work. Any procedure that could read the intercepts on the rays mechanically, either with a radial or some other type of grid, would greatly facilitate making the measurements involved and, if more than 16 rays could be read rapidly, increase accuracy.

Orientation of Standard Shapes and Silhouettes

Obviously, the grain silhouettes and the standard shapes must be oriented in some standard position to be comparable. The shapes in figure 1 are arranged in what might be described as a normal viewing position, but their placement is not based on any firm common property of the silhouette shapes. One characteristic common to both silhouettes and shapes is their longest dimension or axis, and this was chosen as the primary basis for orientation. The rules for orienting both shapes and silhouettes (fig. 4) are as follows:

1. The longest dimension or axis of each silhouette or shape is placed along the vertical axis of the grid (A—I).
2. The midpoint of the longest axis is placed so that it coincides with the center point of the grid.
3. The larger area on either side of the vertical axis is placed to the right of the vertical axis.
4. The larger of the areas in the two quadrants to the right is placed at the top.

For most silhouettes or standard shapes, no difficulty is encountered in determining which half is larger. If serious difficulty arises, a planimeter may be used to determine the area of the sides. However, if differences cannot be recognized by eye they are not usually sufficient to affect shape determination greatly. The same applies to which quadrant is placed upward.

Measuring Silhouettes of Standard Shapes

Two procedures are available for measuring silhouettes and shapes. The first involves cutting out the silhouettes and shapes, marking their longest axis, its midpoint, and the axis at right angles at the midpoint. The cutouts are placed on the radial grid, suitably oriented, and their intercepts read.

With the second method, a transparent photographic copy can be made of the grid and placed on silhouettes or standard shapes so that the intercepts

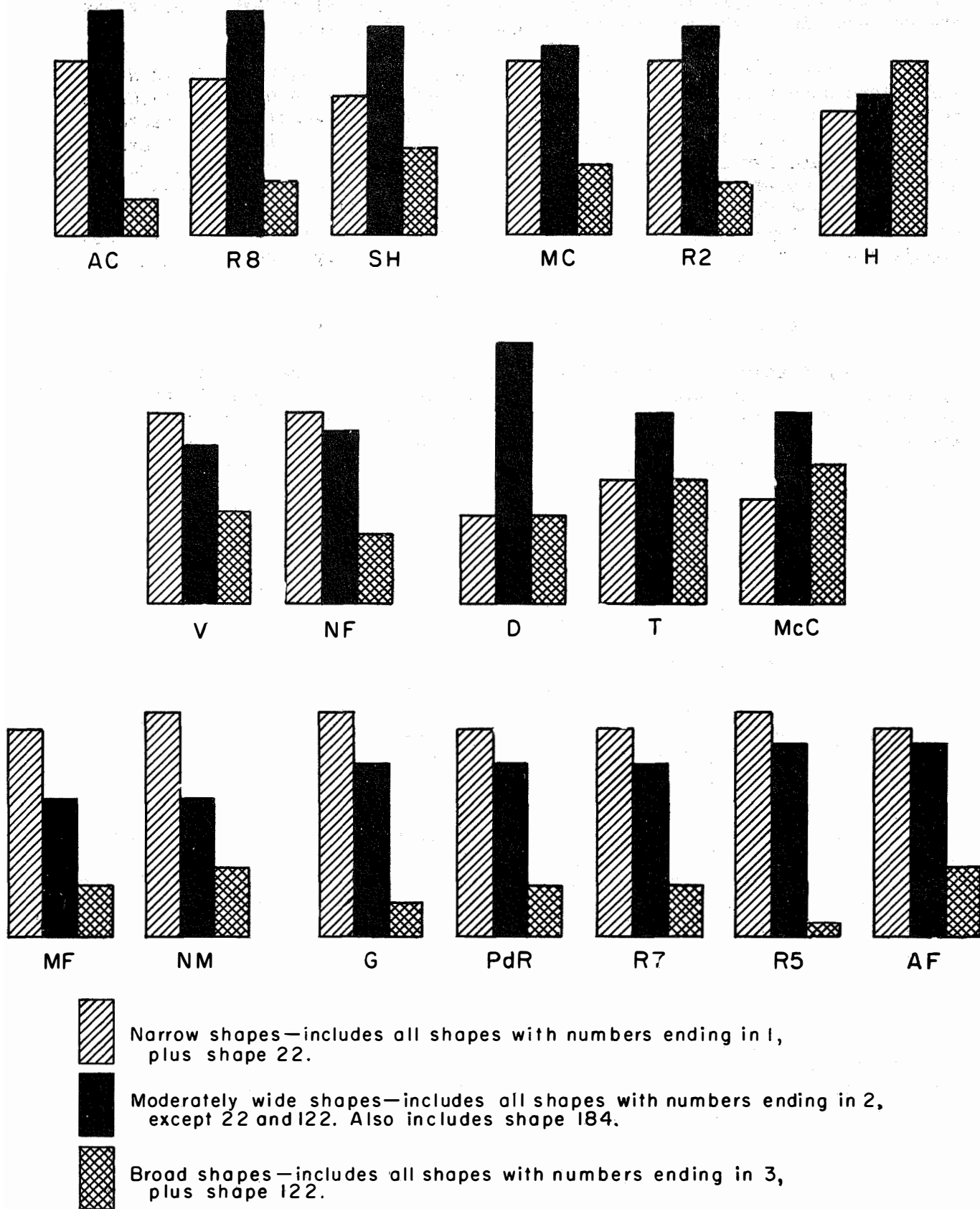


Fig. 5 - Histograms showing number of narrow, moderately wide, and broad pieces in samples. Histograms are grouped according to similarity of shape.

can be read. The procedure may require the use of a light table because some of the shapes and silhouettes will need to be turned over to allow proper placement.

RESULTS

Grain Width and Lithology

Most groups of standard shapes include three widths, narrow, moderately wide, and broad. A few exceptions are noted at the bottom of table 2, which tabulates the data for the 18 samples studied in relation to their lithology. The distribution of pieces is shown graphically in figure 5. Only a few correlations between lithology and grain width were found. Samples R7 and G are both lithographic and have comparable histograms, as do fine-grained samples R2 and R8. Sample H has a distinctive histogram, as might be expected because it is the only crystalline calcite in the study. No other significant relations between grain width and lithology were observed.

Grain Shape and Lithology

Table 3 shows the numbers of those shapes peculiar to each lithologic group of samples; that is, they occur only in that group. Of the eight lithologic groups, six have distinctive shapes, two do not. More samples must be tested, however, to determine whether these data are apparent or real.

Frequency of Shape Occurrence

Figure 6 indicates the frequency of occurrence of the various standard shapes among 450 grains studied. Figure 7 shows a larger view of the 10 most common shapes. The most common shape was a moderately wide flat-iron; 36 grains had this shape. Next in abundance was a narrow, distorted oval, which occurred 31 times. Of the 33 standard shapes noted in figure 6, 13 are narrow, 14 moderately wide, and 6 broad. Of the 349 limestone particles involved in figure 6, 148 (42 percent) were narrow, 162 (46 percent) moderately wide, and 39 (12 percent) broad. Eleven of the 33 shapes shown in figure 6 are symmetrical with respect to their vertical axes; 22 are asymmetrical.

Correlation Coefficient

The measurement of shape by computer gives results as a number, the correlation coefficient, which indicates how nearly the intercepts of any given grain on the radial grid coincide with the intercepts of the standard shapes on the grid.

Regarding the coefficients themselves, consider a set of standard shapes, $\{B_j\}$, $j = 1, \dots, J$, and a set of samples, $\{S_i\}$, $i = 1, \dots, I$. Each sample S_i is itself a set, $\{A_{ik}\}$, $k = 1, \dots, K$, composed of individual grains.

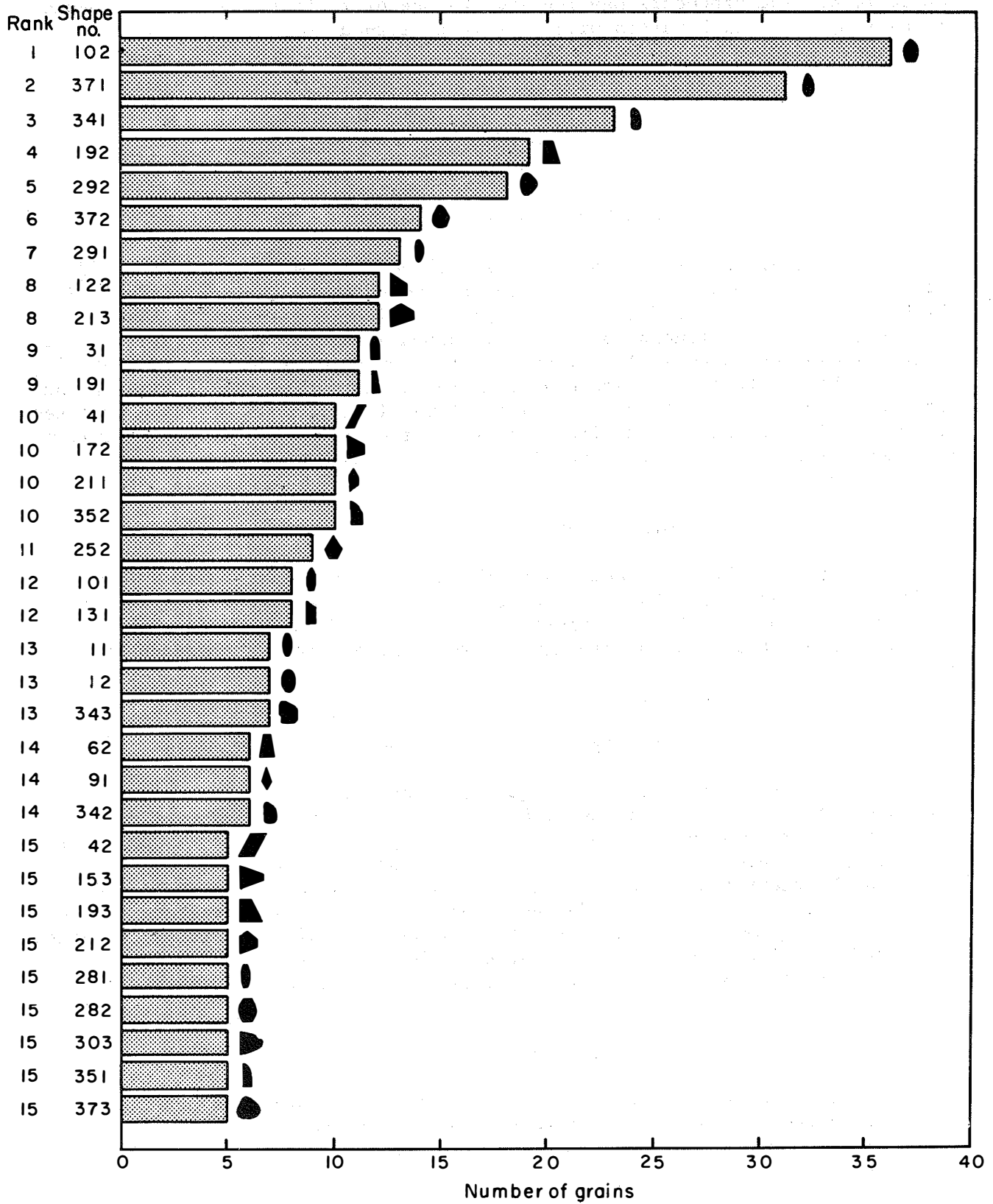


Fig. 6 - Frequency of occurrence of standard shapes based on more than five occurrences.














TABLE 2--GRAIN WIDTH IN RELATION TO LITHOLOGY

Sample	No. of narrow pieces*	No. of moderately wide pieces*	No. of broad pieces*	Lithology
R7	12	10	3	Very fine lithographic
G	13	10	2	Very fine lithographic
NF	11	10	4	Finely crystalline
McC	6	11	8	Finely crystalline
T	7	11	7	Medium crystalline
NM	13	8	4	Medium crystalline
V	11	9	5	Coarsely crystalline
H	7	8	10	Coarsely crystalline (calcite)
D	5	15	5	Coarse particles in hash matrix
MC	10	11	4	Coarse particles in hash matrix
AC	10	13	2	Coarse particles in crystalline matrix
PdR	12	10	3	Small and medium particles in crystal- line matrix
SH	8	12	5	
AF	12	11	2	
R2	10	12	3	Small detritus
R5	13	11	1	Small detritus
R8	9	13	3	Small detritus
MP	12	8	3	Small detritus

* Narrow - all shape numbers ending in 1, plus 22; moderately wide - all shape numbers ending in 2, except 22 and 122, and including 184; broad - all shape numbers ending in 3, plus 122.

In order to measure the degree of relation between each sample and the standard shapes, the IBM 360/75 computer on the campus of the University of Illinois at Urbana was programmed to calculate a set of correlation coefficients, $\{r_{ijk}\}$, $i = 1, \dots, I$; $j = 1, \dots, J$; $k = 1, \dots, K$. The 16 intercepts were used for each individual grain and each standard shape. The standard shape that yielded the highest correlation coefficient for each grain was thought to be the representative shape for that grain. Exact coincidence of the intercepts of a standard shape and an individual grain would give a correlation coefficient of 1.000. In most cases the highest correlation coefficient between an individual grain in a sample and a standard shape was more than 0.900. Coefficients between 0.700 and 0.800 occurred, but were in the minority. The range of correlation coefficients generally was large enough to make the "best" fit clearly discernible.

TABLE 3—SHAPES PECULIAR TO EACH LITHOLOGIC GROUP OF SAMPLES

Shape group	Samples							
	R7, G	NF, McC	T, NM	V, H	D, MC	AC	PdR SH, AF	R2, R5, R8, MF
41								
62								
63								
71								
101								
153								
211								
213								
233								
252								
342								
343								
352								

No shapes peculiar to these groups

Mean Correlation Coefficient

The most representative standard shape for a given sample (S_i) of K grains was determined by calculating the following set of means:

$$\bar{r}_{ij} = \frac{\sum_{k=1}^K r_{ijk}}{K} \quad j = 1, \dots, J$$

The maximum mean correlation coefficient, \bar{r}_{ij} , for a particular sample (S_i) was used as the criterion for the standard shape most representative of that sample.

Figure 8 shows graphically the three highest mean correlation coefficients for 18 samples. The degree of correlation is shown in table 4. The fact that the highest mean correlation coefficients were rather large—that is, greater than 0.800—for so many samples shows that the standard shapes indicated by the coefficients are a good approximation of the average shape of the entire sample.

TABLE 4—RANGE IN CORRELATION COEFFICIENTS

Range of mean correlation coefficients	Mean correlation coefficients		
	1st correlation	2nd correlation	3rd correlation
0.850-0.875	1	1	0
0.825-0.849	5	2	1
0.800-0.824	5	6	6
0.775-0.799	5	6	6
0.750-0.774	2	2	4
0.700-0.749	0	1	1
Coefficients above 800	11	9	7

When the texture of the limestone samples was compared with the shapes indicated by the correlation coefficients (fig. 8), all the fine- and very fine-grained samples (R7, G, NF, McC, P2, R5, and R8) had narrow shapes in their three highest mean correlation coefficients. PdR and AF also were narrow, but they are medium grained. Among the other medium-grained and coarse-grained rocks (table 1), sample D had a moderately wide average grain width in all three coefficients, and samples T and MC had two moderately wide average widths and one narrow width. Broad grains were restricted to samples H and NM; these samples have no obvious lithologic kinship.

The frequency of the standard shapes for the three highest mean coefficients is indicated in table 5. The first mean correlation coefficient

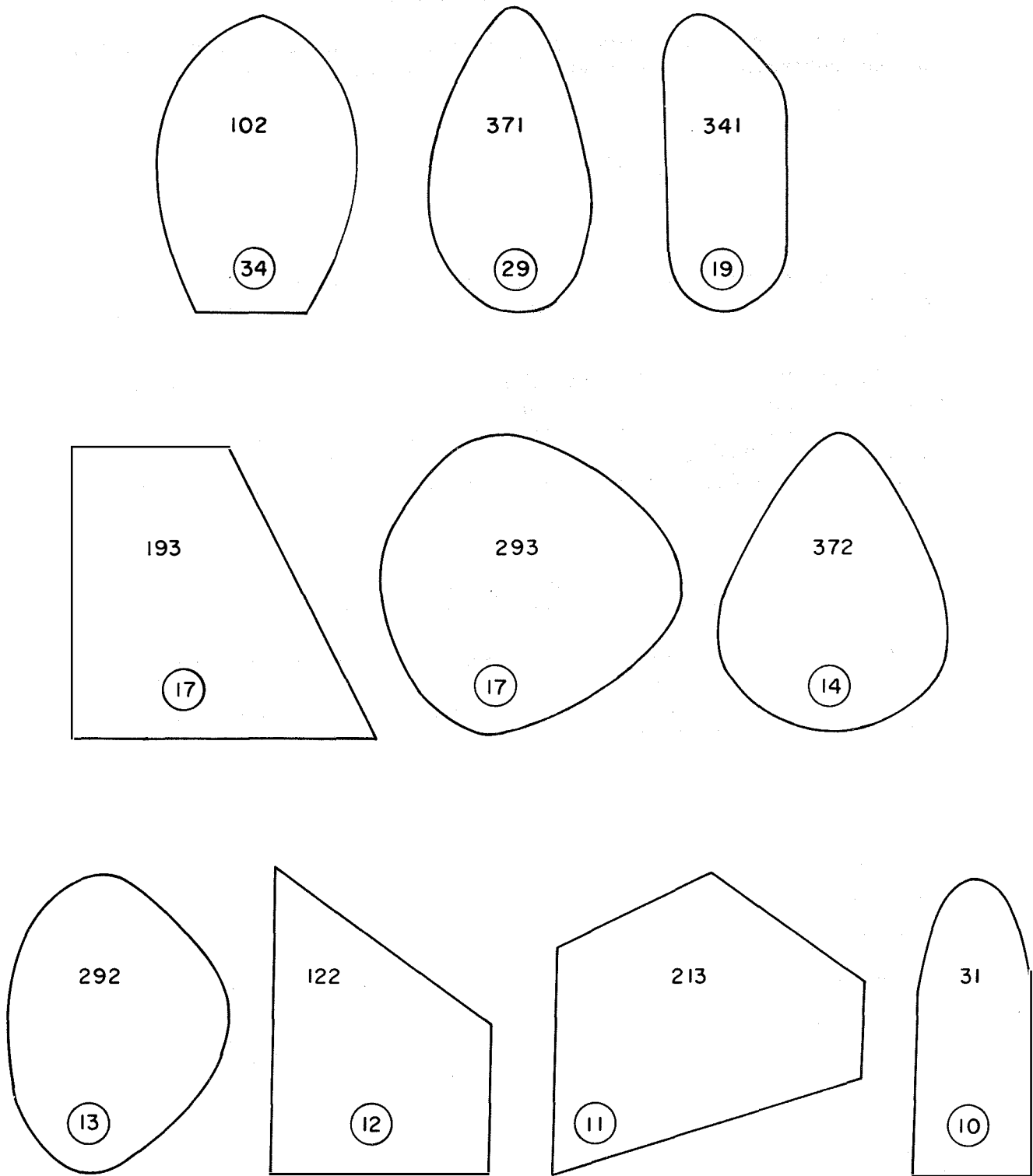














Fig. 7 - Ten most common shapes of grains. The upper number is the shape number; the circled number indicates shape frequency among the 450 grains studied. The shapes shown make up 39 percent of the grains measured.

TABLE 5—FREQUENCY OF SHAPES FOR
THREE HIGHEST MEAN CORRELATION COEFFICIENTS

Shape	Frequency		
	1st correlation coefficient	2nd correlation coefficient	3rd correlation coefficient
33 	2	1	1
102 	1		1
103 		6	1
122 	3	2	1
211 		1	4
251 		1	
252 	1	1	1
291 			3
341 	4	3	3
343 			1
353 		1	
371 	7	2	2

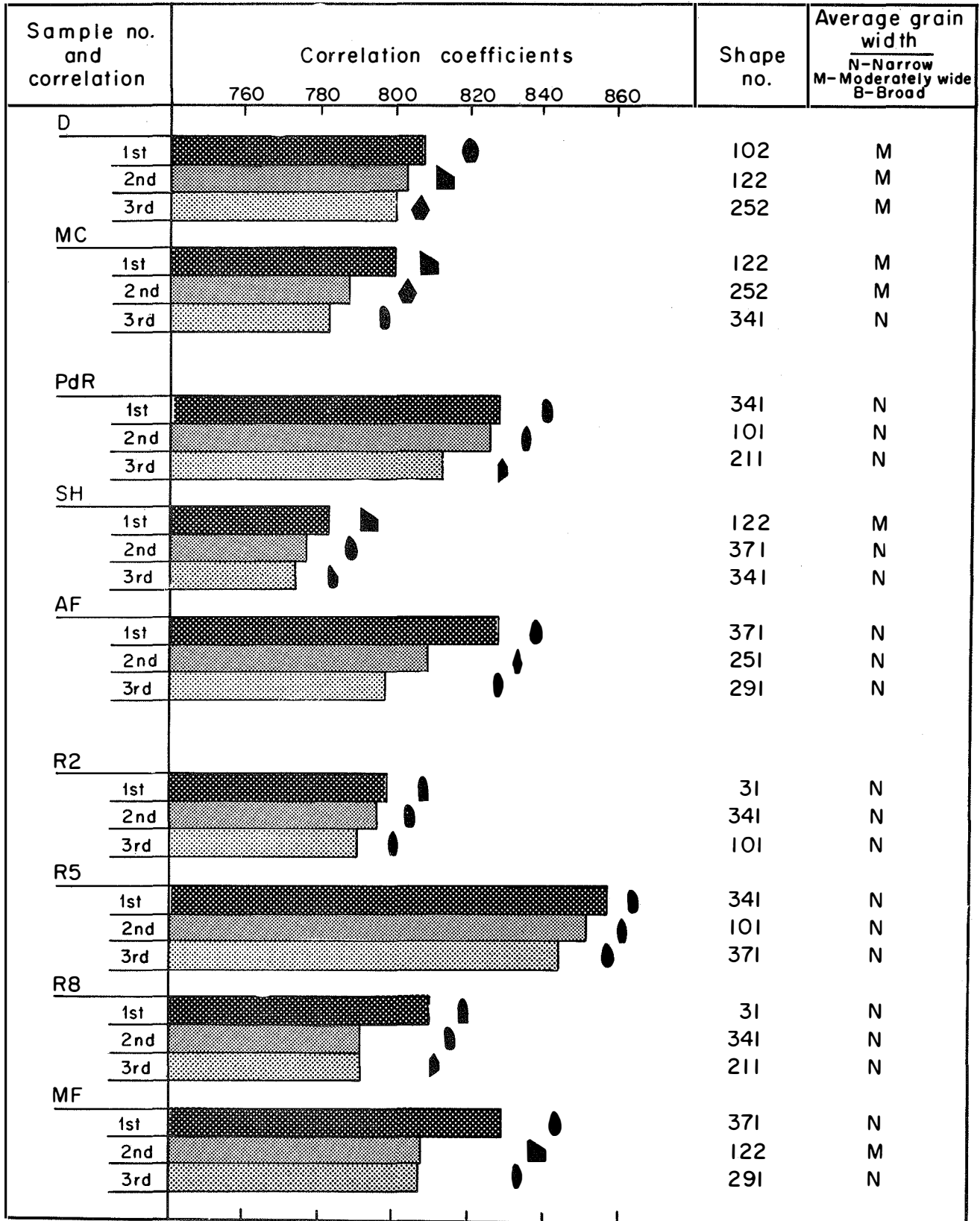


Fig. 8 - First, second, and third mean correlation

Sample no. and correlation	Correlation coefficients						Shape no.	Average grain width N-Narrow M-Moderately wide B-Broad
	760	780	800	820	840	860		
R7								
1st							341	N
2nd							101	N
3rd							371	N
G								
1st							371	N
2nd							101	N
3rd							31	N
NF								
1st							371	N
2nd							101	N
3rd							341	N
McC								
1st							371	N
2nd							341	N
3rd							291	N
T								
1st							252	M
2nd							31	N
3rd							122	M
NM								
1st							122	M
2nd							371	N
3rd							343	B
V								
1st							341	N
2nd							211	N
3rd							102	M
H								
1st							371	N
2nd							353	B
3rd							211	N
AC								
1st							371	N
2nd							101	N
3rd							122	M

coefficients arranged by lithologic groups.

determinations include only six shapes, whereas the second and third mean coefficients involve nine and ten shapes, respectively. The difference between first and second coefficients is expectable. The similarity in the number of shapes involved in the second and third coefficients suggests about an equal degree of "fit" in both cases.

CONCLUSIONS

A two-dimensional shape can be assigned to particles of limestone sand, or any other sizable particulate material, by comparing by computer certain parameters of silhouettes of the particles with those of standard shapes. A study of the 10- by 14-mesh fraction of the sands resulting from crushing 1 calcite and 17 limestones of various lithologies yielded some suggestive, but not definitive, evidence of the effect of limestone lithology on the shape of sand-sized particles produced by crushing. Some similarity in grain width was shown by two fine-grained samples and by two lithographic samples. The calcite sample had different shape characteristics than the limestones. Most common shapes were the flatiron, narrow distorted oval, rectangular trapezoid with rounded ends, and truncated right triangle.

The procedure involved in measuring the parameters of the silhouettes is time consuming; partial or complete automation of it would increase the rapidity of the procedure. Further studies of additional samples, consisting of a statistically significant number of grains and of grains of other sieve sizes, should be made.

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