A Virtual Restoration System for Broken Pottery

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Abstract

Aiming to assist the archaeologist to restore excavated sherds, a virtual restoration system for pottery shape relics is developed based on their two characteristic features: symmetry axis and profile. An effective method of estimating the axis of the surface of revolution is presented first; then an approach for computing the complete profile is proposed. Because some pottery contains holes or is constituted of small fragments, it is difficult to compute the complete profile. An effective method is proposed to compute the complete profile by assembling a number of sherds. Meanwhile, in order to ensure a valid match, some human/computer interactions are applied to remove erroneous matches. Finally, texture mapping technology is applied to virtual restored pottery in order to show the pottery more realistically. The proposed methods are applied to the recovery of Yao Zhou's famous ancient porcelains.

Keywords: *surface of revolution, restoration of pottery shape relics, symmetric axis, texture mapping*

1 INTRODUCTION

With a long history and a profound civilization, China is a country where a large number of cultural artifacts are found every year. Most of these excavated remains are pot-shaped or bowl-shaped and were made on a potter's wheel. But they are often damaged or broken. In addition, there are frequently missing parts of the vessels, which are never found. When sherds are excavated, they must be measured, classified, archived and restored in the traditional ways. Archaeologists do all the work by hand, which is a time-consuming and repetitive task. Therefore, we have developed a computer-aided restoration system in order to protect cultural heritage through modern technology and to avoid secondary damage inadvertently arising through the work of archaeologists.

The symmetric axis is defined as the central line of a pot. The profile of a pot is defined as the intersection plane with the pot along the symmetric axis. These are the two characteristics of a pot through which the vessel can be restored. Andrews used a variety of features and feature comparisons automatically to reconstruct pottery vessels from sherds.[1] Afterwards, a framework to pair-wise match different sherds was developed; however, this paper aims at reassembling the fractures. Shape Lab assembled pots through three steps: matching sherds based on the break-curve, estimating the axis, and estimating the profile curve.[2] A Bayesian approach was used to represent the geometric parameters. Mara estimated the rotational axis based on the Hough method and profile, and then classified the sherds by comparison of the extreme points and primitives.[3] Kampel proposed an approach for matching the rotationally symmetric fragments, first aligning the axis of rotation and then calculating the translation matrix along the z-axis and the rotation matrix around the z-axis.[4] Halir proposed a method to estimate the profile of pottery, but the orientation of the sherds was estimated by archaeologists.[5] Lai proposed a surface-fitting algorithm combined with a B-spline surface and surface

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[1]Stuart Andrews and David H. Laidlaw, "Toward a Framework for Assembling Broken Pottery Vessels," in *Proceedings of the Eighteenth National Conference on Artificial Intelligence, July 28–August 1, 2002, Edmonton, Alberta, Canada* (Menlo Park, California: AAAI Press, 2002) 945–946.

[2]D. B. Cooper et al., "Bayesian Pot-Assembly from Fragments as Problems in Perceptual-Grouping and Geometric-Learning," in *Proceedings of the 16th International Conference on Pattern Recognition, 11–15 August 2002, Quebec City, Canada*, vol. 3 (Los Alamitos, California: IEEE Computer Society, 2002) 297–303.

[3]H. Mara and M. Kampel, "Profile Extraction of Archaeological Fragments for Classification," in *Proceedings of the 8th Computer Vision Winter Workshop, February 3–6, 2003, Valtice, Czech Republic* (Prague: Czech Pattern Recognition Society, 2003) 67–72.

[4]Martin Kampel and Robert Sablatnig, "3D Puzzling of Archaeological Fragments," in *Proceedings of the 9th Computer Vision Winter Workshop, February 4–6, 2004, Piran, Slovenia* (Ljubljana: Slovenian Pattern Recognition Society, 2004) 31–40.

[5]Radim Halir and Jan Flusser, "Estimation of Profiles of Sherds of Archaeological Pottery," in *Proceedings of the Czech Pattern Recognition Workshop, February 17–20, 1997, Milovy, Czech Republic* (Prague: Czech Pattern Recognition Society, 1997) 126–130.

blending algorithm for reconstructing surfaces.[1] Qian presented an algorithm to compute the axis and profile of a surface of revolution.[2]

In this paper, an effective virtual restoration method for ceramic pottery is proposed. First, the symmetric axis of surface of revolution is estimated. After aligning the axis of each sherd, the sherds can be reassembled quickly and easily, and then the complete profile of the pottery can be exactly calculated. The paper is organized as follows. In the next section, the method of data capture is introduced. The methods of calculating the symmetric axis and complete profile are discussed in Section 3 and Section 4. Texture mapping technology is proposed in Section 5. In Section 6, we review the whole workflow of the restoration system. In Section 7, we conclude the paper and point out the directions for future research. The processes of the whole restoration system are shown in figure 1.

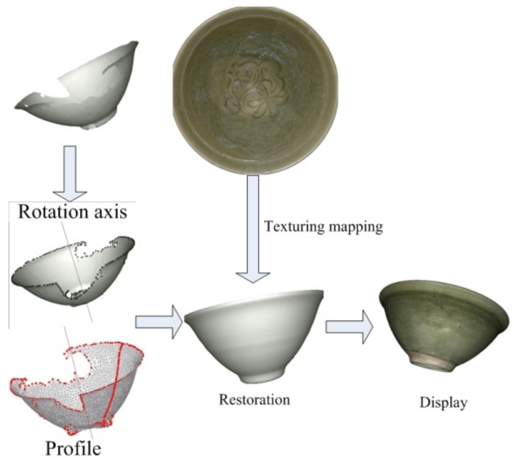


Figure 1. *The procedure of the restoration system of pottery.*

2 DATA CAPTURE

With the development of 3D data capture devices, information including the geometry, topology, and textures of models can be easily acquired using a 3D laser scanner. However, it is difficult for many scanners to capture the geometric information of ceramics. The ceramic surface reflects light strongly, so that the geometric information of the ceramic is often poorly acquired. Non-reflective material is laid on the ceramic surface in order to minimize the reflected light and to acquire better geometric information.

3 ESTIMATING THE AXIS OF THE SURFACE OF REVOLUTION

Pottery is regarded as a symmetric surface and the symmetric axis is the most important characteristic of the pottery, so the method of estimating the symmetric axis from scattered 3D points is a key technology for reconstruction. The curvature method, the Hough-inspired method, and the line geometry method are three methods to estimate the axis. For the curvature method, the intersection curve that is generated by the plane perpendicular with the rotation axis must be a circle, whose curvature is constant. Qian first reconstructs a piecewise linear surface Σ' by interpolating the sampled points S.[3] Based on the partial reconstruction surface Σ', the axis is determined and the whole surface of revolution is reconstructed. For the Hough-inspired method, one obtains the rotation axis based on transforming the normal vector of the sampled points of the model in the Hough space. Sablatnig exploits an approach to estimate the axis on the basis that the normal vector of rotational symmetric objects intersects their axis of the surface of revolution.[4] All intersections of the surface normal vectors n_i locate at the axis of symmetry R. First compute the normal n_i of all vertexes p_i based on minor component analysis,[5] then transform all the lines (p_i, n_i) into Hough space. Finally, the optimal axis is estimated by principal component analysis.[6] For the line geometry method, the axis is estimated based on normals of surfaces of revolution intersecting the axis of revolution. Pottmann takes the surface of the revolution as the helical motion and calculates an approximate normal, then the rotation axis is estimated by minimizing the deviation function.[7] However, this method has some problems because it does not always obtain eigenvectors.

We propose an effective method of estimating the rotation axis that avoids computing curvature and the disadvantage of line geometry. An optimal axis is estimated based on Pottmann's method, and then we apply a new optimization method to estimate the exact axis (fig. 2). Here v_i $(i=1,2,\cdots,k)$ is the vertex of the model and $n_i(i=1,2,\cdots,k)$ is its normal, i is the index of

[1] Jiing-Yih Lai and Wen-Der Ueng, "Reconstruction of Surfaces of Revolution from Measured Points," *Computers in Industry* 41 (2000): 147–161.

[2] Xiaoyuan Qian and Xuegang Huang, "Reconstruction of Surfaces of Revolution with Partial Sampling," *Journal of Computational and Applied Mathematics*, 163 (1) (2004): 211–217.

[3] Ibid.

[4] Robert Sablatnig and Martin Kampel, "Model-Based Registration of Front- and Backviews of Rotationally Symmetric Objects," *Computer Vision and Image Understanding* 87 (2002): 90–103.

[5] L. Xu, E. Oja, and C.Y. Suen, "Modified Hebian Learning for for Curve and Surface Fitting," *Neural Networks* 5 (1992): 441–457.

[6] I. T. Jolliffe, *Principal Component Analysis*, 2nd ed. (New York: Springer, 2002).

[7] H. Pottmann, M. Peternell, and B. Ravani, "An Introduction to Line Geometry with Applications," *Computer-Aided Design* 31 (1999): 3–16.

the vertex. The line (v_i, n_i) and the plane $oo'v_i$ should be in the same plane, because the normal of every vertex on the surface of rotation interacts with every other at the rotation axis, which is represented as $[(v_i - o), (v_i - o'), n_i)] = 0$.

The error function is $\delta(o,o') = \sum_{i=1}[norm(A \times B) \bullet n_i]^2$, where $norm()$ denotes vector normalization, vector A denotes $(v_i - o)$ and vector B denotes $(v_i - o')$. Then in order to estimate the axis, $\delta(o,o')$ should be minimized based on line searching. The result is shown in Table 1.

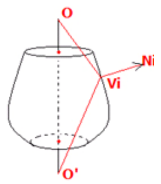


Figure 2. *Estimating the symmetric axis.*

Index	Orientation of axis(x,y,z)	
	Pottmann	Our method
1	-0.8526 0.4012 -0.3349	-0.8867 0.3298 -0.3239
2	0.5761 -0.2860 0.7657	0.5800 -0.2863 0.7626
3	0.0467 0.0161 0.9988	0.0531 0.0189 0.9984

Table 1. *Results of different methods to estimate rotational axis.*

4 Computing the Complete Profile of the Pottery

Excavated pottery is often broken up into sherds, so it is difficult to compute the complete profile. However, this profile is very important for reconstructing the pottery. We need to reassemble some fragments to generate a bigger sherd in order to obtain the complete profile. The following is the detailed algorithm:

- Step 1: Reassemble the neighboring fragments based on matching features of different sherds.
- Step 2: After reassembling different sherds, compute the complete profile of the pottery.

4.1 Matching the Fragments of the Pottery

Assuming two sherds located in different coordinates respectively, p_1 represents the feature curve of one sherd which locates in $I_A(O'_A, e'_{A1}, e'_{A2}, e'_{A3})$ and p_2 represents the feature curve of another sherd, which locates in $I_B(O'_B, e'_{B1}, e'_{B2}, e'_{B3})$. If two sherds match, they must be reassembled. Then transformation matrix M should be computed based on the shape matching. One model is transformed from its local coordinates to the world coordinates by the translation matrix and the rotation matrix.

$$P = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = A_A \begin{pmatrix} x'_A \\ y'_A \\ z'_A \end{pmatrix} + \begin{pmatrix} x_{A0} \\ y_{A0} \\ z_{A0} \end{pmatrix} = A_B \begin{pmatrix} x'_B \\ y'_B \\ z'_B \end{pmatrix} + \begin{pmatrix} x_{B0} \\ y_{B0} \\ z_{B0} \end{pmatrix} \quad \textbf{(1)}$$

Here (x,y,z) represents the position in the world coordinates, (x_{A0}, y_{A0}, z_{A0}) represents the position of the origin of I_A in the world coordinates, and (x'_A, y'_A, z'_A) represents the position in the local coordinates I_A. (x_{B0}, y_{B0}, z_{B0}) represents the position of the origin of I_B in the world coordinate frame, and (x'_B, y'_B, z'_B) represents the position in the local coordinates I_B. A_A, A_B represent separately the rotation matrix.

$$\begin{pmatrix} x'_A \\ y'_A \\ z'_A \end{pmatrix} = A_A^{-1} A_B \begin{pmatrix} x'_B \\ y'_B \\ z'_B \end{pmatrix} + A_A^{-1} \begin{pmatrix} x_{B0} - x_{A0} \\ y_{B0} - y_{A0} \\ z_{B0} - z_{A0} \end{pmatrix} \quad \textbf{(2)}$$

If R is defined as rotation matrix, here $R = A_A^{-1} A_B$ and T is defined as translation matrix, here $\mathbf{T} = A_A^{-1} \begin{pmatrix} x_{B0} - x_{A0} \\ y_{B0} - y_{A0} \\ z_{B0} - z_{A0} \end{pmatrix}$, then the equation is:

$$\begin{pmatrix} x'_A \\ y'_A \\ z'_A \end{pmatrix} = R \begin{pmatrix} x'_B \\ y'_B \\ z'_B \end{pmatrix} + T \quad \textbf{(3)}$$

On the basis of matching two sherds, the relationship of different fractures can be determined. If they are neighbors, i.e. the contours of fractures match, the rotation matrix R and translation matrix T can be computed. However, if the pottery is broken up into many fractured sherds, it is very difficult to calculate the transformation matrix quickly. At least six variables should be calculated, i.e. three rotation variables and three translation variables, so it is very difficult to compute the matrices quickly. Meanwhile, more error may have occurred. In this paper, we present an effective approach that only needs to compute two variables according to the characteristic of surface of revolution. One is the angle around the rotation axis, and the other is the translation distance along the rotation axis. It is easy and quick to reassemble the different sherds.

The transformation matrix is computed by two steps:

- Step 1: Aligning the rotation axis of different sherds. Assuming the pots are composed of many sherds, here R_i represents the i index sherd, then the rotation axis l_i of every sherd can be computed based on section 2. Then l_i is represented:

$$\begin{cases} x = x_0 + t\cos\alpha \\ y = y_0 + t\cos\beta \\ z = z_0 + t\cos\gamma \end{cases} \quad \textbf{(4)}$$

Assuming all the sherds belong to the same object, then they have the same rotation axis. Firstly, calculate the transformation matrix M in order to align the axis in the same coordinates based on matching the axis l_i of each sherd, $M = R_{(\alpha,\beta,\gamma)}T_{(x,y,z)}$. Here R is rotation matrix and T is translation matrix.

- Step 2: Computing the transformation matrix. Then the transformation matrix M_1 is computed in order to reassemble the two sherds based on matching the curve, $M_1 = R_{(z)}T_{(z)}$. Here $R_{(z)}$ means the rotation matrix around the axis and $T_{(z)}$ means the translation matrix goes through the rotation axis (fig. 3).

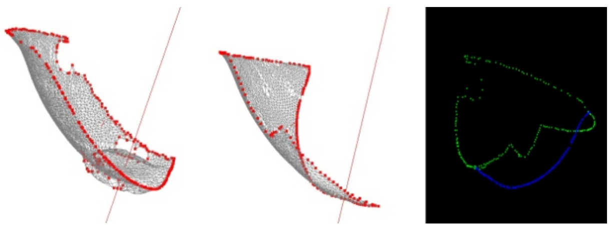


Figure 3. *Aligning axis of pottery.*

4.2 Computing the Complete Profile

The profile of the sherd is defined as the intersection of the sherd with the plane along the rotation axis of the pot to which the sherd belongs. However, sometimes the newly generated sherd has some holes after reassembly. In order to obtain the complete profile of the pottery, the longest profile is taken as the profile of the pottery.

Assuming O represents the center of the bounding box on the symmetric axis, the symmetric axis is denoted as $I(O,d)$. Here is a way to choose the intersecting plane:

- First, the contour line of the sherd provides important information for us to compute the profile, which has been sampled and represented as $S(s_1, s_2, \cdots, s_r)$.
- Second, a cross section plane through rotation axis $\pi_t(o,n_t)$ is created, which goes through point s_b; n_t represents the normal of the plane, $n_t = norm(d \times s_t)$. The cross section planes can be recreated from the beginning points to the ending points of the contours by orders around the orientation ψ_1 (see fig. 4).
- Third, calculate the position of all vertexes of the surface of the pottery which are intersected with section plane $\pi_t(o,n_t)$. The details include three steps:

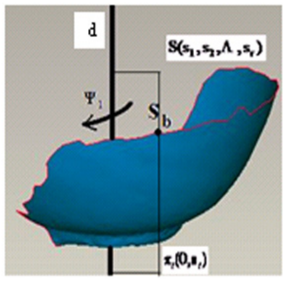


Figure 4. *Generating the new plane of a sherd.*

- Step 1: The local coordinate $\chi_t(o, I_t, J_t, K_t)$ is built, and then the orientation of J is the same orientation with the rotation axis, $J = d$. Additionally, the vector K is the same orientation as the normal of the plane, which is composed by $d \times (s_t - o)$, $K = norm(d \times (s_t - o))$. As illustrated above, the vector I has the same orientation with the plane which is composed by $J \times K$, $I = J \times K$.
- Step 2: The transformation matrix from the local coordinate $\chi_t(o, I_t, J_t, K_t)$ to the original one is obtained, which is denoted by Tr_t, $Tr_t = [I_t, J_t, K_t]$. It is easy to understand that the transformation matrix from the original coordinates to local coordinates $\chi_t(o, I_t, J_t, K_t)$ is denoted by Tr_t', $Tr_t' = Inv(Tr_t)$. Here Inv represents the inverse matrix.

Each cluster of plane $\pi_t(o,n_t)$ is used to intersect with the sherds, and the intersection points' set of current intersections is denoted by U_t. Each point in set U_t is represented in local coordinates. Then $\chi_t(o, I_t, J_t, K_t)$ is sorted by x-component of the coordinates. After deleting the points whose x value is less than 0, the point sets $U = \bigcup_{i=1}^{k} U_t$ are obtained, which compose the profile of the sherds completely.

- Step 3: After obtaining the points' set U, we can fit the complete profile curve based on B-spline. The result is shown in table 2:

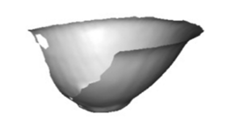
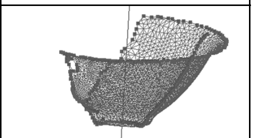
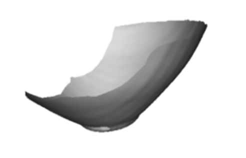
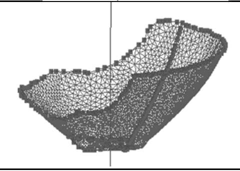
Index	Sherds	Axis and Profile
1		
2		

Table 2. *The profiles of the different models.*

Only the external surface is usually acquired by the 3D laser scanner device; however, what is needed here is both the inner part and the outer part. The surface of revolution is reconstructed by the rotation axis (A_{in} and A_{out}) and profile (Pr_{in} and Pr_{out}), which are fitted by B-spline curves. So the reconstructed result is composed of two surfaces; one is inside and the other is outside. The profile of the outside has been generated (fig. 5). Meanwhile, the ends of the profiles do not connect to each other.

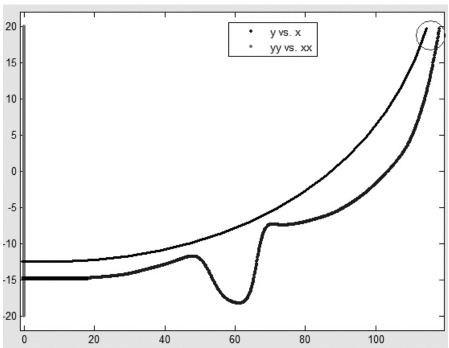


Figure 5. *Inner profile and outer profile.*

p_{inner} and p_{outer} represent the profile of the inner surface and of the external surface separately. *P1* and *P2* represent the end points of the profile. New points are generated, which are defined as *P3* and *P4* separately, along the tangent orientation of end points of the profile (fig. 6). We could easily fit a B-spline curve by control points *P1*, *P2*, *P3* and *P4*. Finally, by connecting the B-spline curves p_{inner} and p_{outer}, we could get the complete profile of the artifacts, which is defined as p_r . The inner and outer surfaces of the model can be reconstructed by revolving profile p_r around the rotation axis.

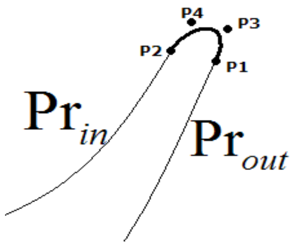


Figure 6. *Connection of inner profile and outer profile.*

5 TEXTURE MAPPING

The texture is the most important characteristic of pottery, since more information can be perceived through the texture of the pottery. In order to display a more realistic pottery model, the texture of every excavated pottery piece is captured by the digital camera. Then the texture is mapped onto the restoration models based on Equation 5:

$$\begin{cases} u = (0.5 - 0.5 * i / height) * \cos(\theta * pi / 180) + 0.5 \\ v = (0.5 - 0.5 * i / height) * \sin(\theta * pi / 180) + 0.5 \end{cases} \quad \textbf{(5)}$$

Here i is the index of the vertex set, θ is the angle around the rotation axis, and *height* represents the length of the line that the profile projects on the rotation axis. The restoration results are shown in figure 7.

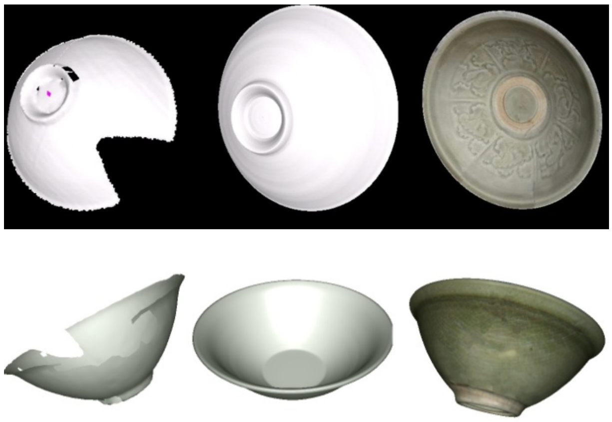


Figure 7. *Virtual restoration results of fractured pottery.*

6 FUNCTION OF THE RESTORATION SYSTEM

The restoration system for broken pottery is constituted of five modules: the Reading module, for reading different kinds of digital models; the Math module, which contains much of the mathematical library, especially the matrix library and vector library; the Display module, the model that should be visualized on the screen; the Reconstruction module, which performs restoration of the sherds on the basis of rotation axis and complete profile; and the Output module, which saves and outputs the restored model to standard file formats. The main functions of the system are shown in figure 8.

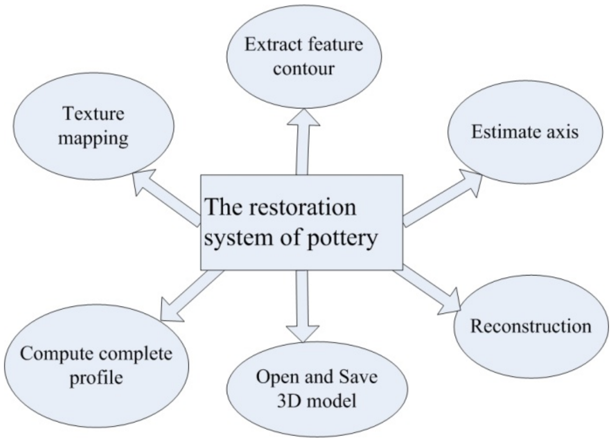


Figure 8. *Functions of the restoration system.*

7 CONCLUSION AND FUTURE WORK

In this paper, we present how to restore pottery based on the rotation axis and profile. We use the laser scanner and structured light scanner to capture the information of the model and we solve some problems occurring in the procedure of data capture. Then an efficient method to estimate the rotation axis is proposed based on the Pottmann and optimization methods. Next, aiming at calculating the profile of sherds, we propose a method to obtain the complete profile. Finally, the texture is mapped onto the model, which can be displayed in a much more realistic way. In China, however, many artifacts, such as bronze objects, have symmetry but not complete axial symmetry. We plan to study how to restore the shapes of these artifacts in the future.

ACKNOWLEDGEMENTS

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