Inverse design of a suspended Kirchhoff rod: From theory to practice

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Keywords: inverse design, Kirchhoff rod, experimental and theoretical curve framing

Estimating the natural shape of a given hanging deformed rod, made of a known material, is a relevant problem in many industrial, graphic design, or even biological applications [1, 2, 3]. To tackle this problem we examine the static Kirchhoff equations for thin elastic rods [4] in the reverse direction, that is, when the shape at equilibrium is known and the natural shape is unknown. Our study is focussed on the case of an isotropic rod clamped at one end and free at the other, subjected to gravity. The input shape is described as a mere geometric curve that we subsequently frame to compute a material curvature field and feed our inverse problem. One interesting property we prove is that the natural shape of the rod satisfying equilibrium exists and is unique, regardless of the infinity of framings compatible with the input curve. The natural shape is computed efficiently by solving in sequence three linear initial value problems, starting from any framing of the input curve. We illustrate our theoretical results trough numerical examples of well-known curves to which we apply our inverse procedure. We validate, by direct simulation [2], that indeed those natural shapes fall, under the effect of gravity, onto the expected equilibrium. We stress on the fact that the obtained rest shapes are complex and far from intuitive, making the simplicity of this procedure very useful. For example, we show that the intricate rest shape depicted in Figure 1 (left) will fall into a perfect helix of radius 15 mm and pitch 20 mm (see fig. 1 rigth) if the rod is made of a material with elastic modulus of 5 Mpa and a 3mm cross section diameter.

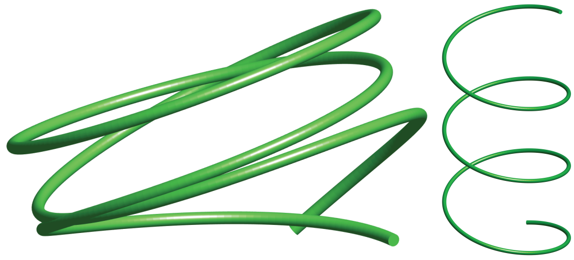

Figure 1: **Left.-** Inverse natural shape for a perfect helix under gravity. **Right.-** Equilibrium shape.

Finally, we complement this study with experimental corroborations. By means of a standard array of cameras, we spatially reconstruct real elastic hanging rods with well-defined geometrical features. We find some good agreements with model prediction despite the experimental limitations on the estimation of the curvature fields of the rod's center lines.

Acknowledgment: This work was supported by the European Research Council grant GEM (StG-2014-639139).

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