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An Adaptive Memory Programming framework for the Resource Constrained Project Scheduling Problem

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The Resource Constrained Project Scheduling Problem (RCPSP) is one of the most intractable combinatorial optimisation problems that combines a set of constraints and objectives met in a vast variety of applications and industries. Its solution raises major theoretical challenges due to its complexity, yet presenting numerous practical dimensions. Adaptive Memory Programming (AMP) is one of the most successful frameworks for solving hard combinatorial optimisation problems (e.g., vehicle routing and scheduling). Its success stems from the use of learning mechanisms that capture favourable solution elements found in high quality solutions. This paper challenges the efficiency of AMP for solving the RCPSP, to our knowledge, for the first time in the up-to-date literature. Computational experiments on well known benchmark RCPSP instances show that the proposed AMP consistently produces high-quality solutions in reasonable computational times.

Keywords: Adaptive Memory Programming, Project Scheduling, Resource Constraints

1. Introduction

Scheduling activities under the consideration of particular resource capacity and precedence constraints is a major complex scheduling problem faced in several different industries. Hartmann and Briskorn (2010) discuss on scheduling applications that can be modelled as Resource Constrained Project Scheduling Problem variants and extensions. Shop scheduling environments, sports league scheduling, scheduling of port operations, scheduling of multi-skilled workforce and school and university timetabling is only a sample of the RCPSP's vast range of applications (Hartmann and Briskorn 2010). The RCPSP model is hence not restricted only in its original field, that is project scheduling, but also addresses major scheduling problems faced in different industries.

The RCPSP can be described as follows. Let a single project consist of a set of activities J that must be executed to complete the project. Precedence constraints do not allow an activity i to start before all its immediate predecessors have finished. There exists a set of resources K with limited capacity, used for the execution of the activities. Each activity i has its own predefined duration d_i and spends a specific amount of resource units for its execution r_{ik}, for each resource $k \in K$. The goal is to find the resource and precedence feasible finish times of the activities, such that the finish time l_{n+1} of the last activity is minimised.

According to Garey and Johnson (1979), the RCPSP is one of the most intractable combinatorial optimisation problems and belongs to the *NP-hard* class of problems (Blazewicz *et al.* 1983). Due to its complexity, as well as its practical interest, the RCPSP has attracted much attention by

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researchers in the latest decades (Hartmann and Briskorn 2010; Kolisch and Hartmann 2006). Although there are significant achievements in the field of exact methods (see Zhu *et al.* 2006 for the multi-mode RCPSP and Brucker *et al.* 1998, Zamani 2001 and Demassey *et al* 2005 for the single mode RCPSP) the literature still favours metaheuristics for solving realistic scale problem instances. One of the most successful metaheuristic frameworks for solving hard combinatorial optimisation problems is Adaptive Memory Programming. AMP has produced the best-known results for major vehicle routing problems (Repoussis *et al.* 2010; Tarantilis *et al.* 2014; Gounaris *et al.* 2014) and binary quadratic programs (Glover *et al.* 1998), and has been applied for solving, among others, supply chain design problems (Cardona-Valdes *et al.* 2014; Velarde and Marti 2008).

Inspired and motivated by this stream of research, this paper contributes to the existing body of work by introducing for the first time in the RCPSP literature an AMP framework for solving the RCPSP. The proposed AMP combines memory structures to capture favourable solution elements and in this way produce high quality solutions. Specifically, the adaptive memory monitors the scheduling habits of the activities and the AMP framework uses this information to guide the search towards promising and unexplored regions of the solution space. To achieve this goal, our AMP consorts with a Greedy Randomised Reconstruction (GRR) method and a powerful local search algorithm in the spirit of Variable Neighbourhood Search (Paraskevopoulos *et al.* 2008). The resulting AMP framework was tested on large scale and well-known benchmark instances and outperformed the state-of-the-art of the literature under 1000 and 5000 schedules examined, while remaining highly competitive when 50000 schedules are examined.

The remainder of this paper is organised as follows. Section 2 discusses the relevant literature mainly focusing on the AMP and the RCPSP domains. Section 3 presents the solution methodology and describes in detail all components. Computational experience is presented in Section 4. Finally, conclusions and further research directions are discussed in Section 5.

2. Literature Review

In this section, we first present the relevant Adaptive Memory Programming literature and then talk about the recent developments on metaheuristics for solving the RCPSP.

Adaptive Memory Programming is a general-purpose solution framework that focuses on the exploitation of strategic memory structures (Glover 1997). The main goal is to capture common solution characteristics met in the search history and use this information to produce high quality solutions. This non problem-specific rationale enabled the successful application of AMP frameworks for solving a large variety of hard combinatorial optimisation problems.

Glover *et al.* (1998) developed an adaptive memory tabu search (AMP-TS) algorithm for solving binary quadratic programs that model various combinatorial optimisation problems. The proposed AMP-based TS collects information from the local optima throughout the search trajectory, i.e., captures each time a local move causes the objective function to increase or remain unchanged. Recency and frequency information are combined to guide effectively the search to new directions into the solution space. Computational experiments indicate the effectiveness and efficiency of the proposed solution methodology.

In a similar fashion, Fleurent and Glover (1999) demonstrated how constructive multi-start strategies for the Quadratic Assignment Problem (QAP) can be improved by using adaptive memory. Comparisons are made with Random Restart and Greedy Randomised Adaptive Search Procedure (GRASP) (Kontoravdis and Bard 1995), which both do not consider previously met solutions. The authors propose a framework that identifies “strong” solution elements, i.e., solution elements that are dominant in the majority of solutions, and adjust the preferences of the construction mechanism to apply intensification and diversification. The resulting multi-start method performs significantly better than other multi-start approaches for the QAP. Similarly, in our paper we demonstrate that the solution quality produced by our local search method significantly improves when the latter method is integrated within the proposed AMP framework.

Cardona-Valdes *et al.* (2014) consider the design of a two-echelon production distribution network with multiple manufacturing plants, distribution centres and a set of candidate warehouses. Their contribution is threefold a) the consideration of the demand uncertainty regarding the warehouse location problem, b) the focus on both economical and service quality objectives and an AMP-TS to address the underlying bi-objective stochastic optimisation problem. Similar to the supply chain design problem, Velarde and Marti (2008) applied an AMP framework for solving the supplier selection problem in an international setting under uncertainty in the macroeconomic conditions of the associated countries. The proposed AMP uses memory structures to discourage the selection of frequently selected suppliers and preserve a balance between intensification and diversification.

Vehicle routing problems have attracted much attention on the development of efficient AMP frameworks. The latter have produced the best-known results when tested on well-known benchmark problems of classical and rich vehicle routing variants (Repoussis *et al.* 2010; Tarantilis *et al.* 2014). Gounaris *et al.* (2014) presented an AMP framework to address the robust capacitated vehicle routing problem. Subroutes of the elite set of solutions represent the favourable solution elements and constitute the information stored in the adaptive memory. These subroutes are of varying size and are combined to create new provisional solutions.

To our knowledge, there is not any metaheuristic framework in the RCPSP literature that explicitly utilises adaptive memory structures within a solution framework. Nevertheless, there is a wide variety of metaheuristics for solving the RCPSP, which we discuss in the following.

Debels and Vanhoucke (2007) presented a decomposition-based Genetic Algorithm (GA) for solving the RCPSP. The proposed algorithm selects a time window of the RCPSP schedule and extracts the associated activities to this time window. A sub-problem is constructed and solved via an efficient GA. The improved subproblem solution is embedded into the main solution in a way to improve the makespan. More recently, Zamani (2013) proposed a GA that used a a specialised crossover operator that combines two coherent parts from one parent and one part from the other. The order of these parts change but they still remain contiguous. The author assumes that contiguity of the parts matters to preserve the fitness of their both parents. Computational experiments indicate the effectiveness of the method.

Swarm Intelligent (SI) population based algorithms have been also developed for solving the RCPSP. The latter are nature inspired procedures that emulate the behaviour of several species such as ants, bees and birds. Merkle *et al.* (2002) presented an Ant Colony Optimisation (ACO) algorithm for solving the RCPSP. Their algorithm uses an Activity List (AL) representation, while a Schedule Generation Scheme (SGS) is employed both in parallel and serial fashion to produce feasible schedules. Two pheromone strategies for the ant's decisions are integrated.

Another algorithm that falls into the SI category of population based algorithms is the Particle Swarm Optimisation (PSO). PSO is an evolutionary algorithm that emulates the behaviours of birds flocking. Zhang *et al.* (2006) applied a PSO solution framework for addresing the RCPSP. The proposed PSO algorithm operates on a priority-based particle representation which resembles the Random Key (RK) representation and ensures that feasible activity sequences, in terms of precedence constraints, are considered. The proposed PSO utilizes a parallel SGS to transform the particles information to feasible schedules. Lately, Chen (2011) proposed a "justification" PSO (jPSO) that enables a technique to adjust the starting times of each activity in an effort to reduce the makespan. The resulting jPSO operates on both forward and backward planning schemes.

Regarding Scatter Search (SS) implementations, a hybrid methodology was proposed by Debels *et al.* (2006). The authors developed an SS algorithm which uses the electromagnetism principles for the solution selection. The proposed method emulates the forces in a electromagnetic field to depict the attractiveness of a new solution to the solutions of the reference set. A Standardized RK solution representation scheme is presented, while particular intensification procedures were developed. Paraskevopoulos *et al.* (2012) proposed a Hybrid Evolutionary Algorithm (HEA), that is based on SS, operates on an event-list representation and employs an Iterated Local Search algorithm as a post-improvement method. It is worth mentioning that the AMP algorithm, presented herein, is developed on the basins of the event-list (EL) representation and adopts the local search operators proposed by Paraskevopoulos *et al.* (2012). Our proposed AMP contributes to the existing body of work by introducing an adaptive memory-based solution reconstruction procedure, able to produce high quality solutions even in early stages of the search process; note that the proposed AMP outperforms the state-of-the-art of the literature for large scale RCPSP instances when 1000 and 5000 schedules are examined. Furthermore, an innovative local search method is presented that performs a thorough exploration of the solution neighbourhoods.

3. Solution Methodology

In this section, we first present the solution representation used by all the components of the algorithm and then describe in detail the components of the main algorithm.

3.1 *The event-list representation*

The most popular representation schemes of the literature is the activity list and the random key representation (Kolisch and Hartmann 2006). The main problem that typically occurs when these representations are used, is that multiple representation lists (either ALs or RKs) can be associated with the same solution (Debels and Vanhoucke 2007). The latter is expected to reduce the efficiency of metaheuristic methodologies, since useless replicas of the same solution exist in the solution space and therefore the search is typically delayed and misguided. Debels and Vanhoucke (2007) used several transformation mechanisms and a Standardized RK representation scheme that addressed the aforementioned problem. The Event List representation used in this paper (Paraskevopoulos *et al.* 2012) addresses the inefficiencies of the AL and RK without the use of transformation mechanisms. The structure of the EL involves time information and efficiently depicts the RCPSP solution. Two different ELs cannot be associated with the same schedule/solution as they are composed by sets of activities that start at the same time, i.e., the events, while these events are ordered according to their starting times. If either the events or the order of these events, or both are different in two particular event lists, different solutions are considered. This property of the EL representation enables both local search to move effectively into the solution space, as well as it equips evolutionary algorithms with an efficient inheritance process. In this paper, we took advantage of the particular structure of the EL, and we introduce compound local search moves that consider "event relocations" within the schedule.

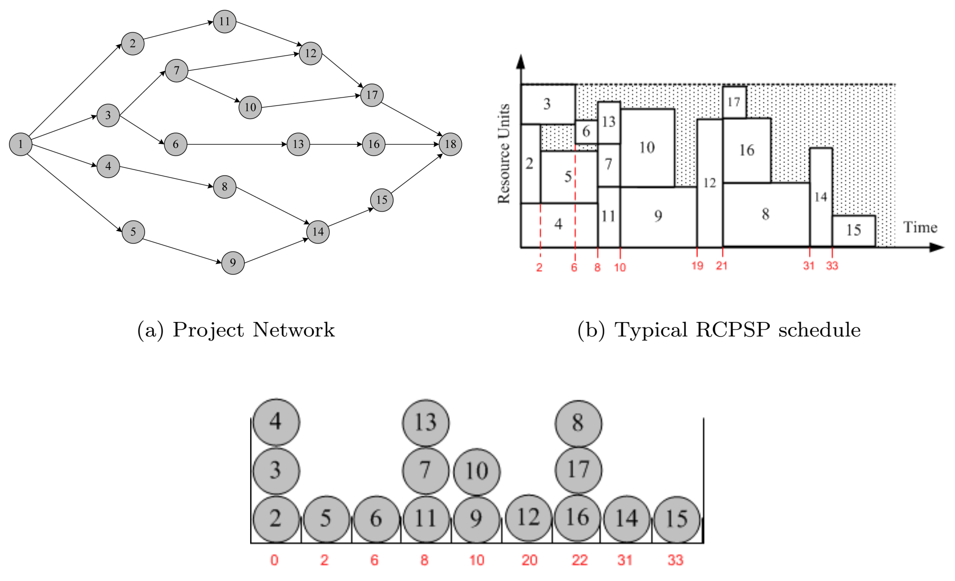
Figure 1 illustrates a project of 16 activities and a capacity-limited resource. The numbers, that appear on the x axis in Figure 1b and in the event list, indicate the starting times, i.e., the dates of the events. The resource consumption and duration of the activities are shown in Figure 1b. The latter shows a feasible solution of the problem, obtained by using a serial SGS.

3.2 *The objective function*

A hierarchical objective function is used, inspired by Paraskevopoulos *et al.* (2012). The primary objective is the makespan, and the secondary objective is a measure that mainly represents the deviation of the finishing times l_{sj} of the activities j in a solution s from their Earliest Finishing Times (EFT_j) lower bounds calculated by the Critical Path Method (CPM). Typically, the largest this deviation is, the less is the quality of a solution, when comparing two solutions with the same makespan. The equation below gives the hierarchical objective function used in this paper:

$$f(s) = min(l_{s(n+1)}, \frac{\sum_{j\in J} \frac{l_{sj}-EFT_j}{EFT_j+1}}{|J|}) \tag{1}$$

where $l_{s(n+1)}$ is the finishing time of the last activity, i.e., the makespan, $|J|$ the cardinality of



(a) Project Network

(b) Typical RCPSP schedule

(c) Event list representation

Figure 1. A typical Resource Constrained Project Scheduling Problem

set J, i.e., the number of activities.

This is a common practice in the relevant literature (Mendes *et al.* 2009), as the solution space is largely composed by solutions with the same makespan (Czogalla and Finik 2009), and this hierarchical objectives typically help the search to move within the space more effectively.

3.3 *The Adaptive Memory Programming Framework*

The proposed AMP consists of two phases; the initialisation and the exploitation phase. In the initialisation phase a construction heuristic is applied for λ iterations, throughout which μ different and good quality solutions are selected to comprise the set M. This pool of solutions M informs the adaptive memory α, which stores the scheduling history for each activity. In the exploitation phase, The AMP framework manipulates M through an exploration of search trajectories initiated from new provisional solutions. In particular, a GRR method (Kontoravdis and Bard 1995) uses the information stored in the adaptive memory and produces a provisional solution s. Given s, a Reactive Variable Neighbourhood Search (ReVNS) algorithm is applied for improving the solution quality. If the provisional solution s is “sufficiently” improved through the application of the ReVNS, i.e., the provisional solution satisfies some elitist selection criteria, the pool of solutions is updated with the new, improved by ReVNS, solution s' and the worst cost solution is removed from the pool. At this point, the adaptive memory is updated as well. The AMP continues with another provisional solution (whether the previous solution was improved or not) until the termination criteria are met. The AMP framework is described by Algorithm 1. The algorithm terminates after a number of schedules $maxSc$ has been examined.

The function *InitialiseAdaptiveMemory* uses the pool of solutions and informs the adaptive memory (see Section 3.4.1 for details). Similarly, *UpdateAdaptiveMemory* updates the adaptive memory α with new information, each time a new solution s' is inserted into the pool of solutions. *Selection-Criteria* is a boolean function that returns “true” if the solution s' meets the criteria to be inserted into the pool of solutions, and “false” otherwise. *UpdatePool* removes the worst cost solution from the pool, and inserts the new s' one. *GRR* is the reconstruction method introduced in Section 3.5.1, and produces a provisional solution s. $ReVNS^{cost}$ and $ReVNS^{dist}$ are two different versions of the ReVNS proposed in this paper, compliant with two different objectives, respectively.

Algorithm 1: Adaptive Memory Programming Framework

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Input: $\lambda$ (initial population size), $\mu$ (pool size), where $\lambda \geq \mu$, $\delta$ (number of local search
       iterations without an improvement), $\vartheta_{\max}$ (number of perturbations without an
       improvement), $\delta t$ (size of the time bucket)
Output: $M$, $s_{\text{best}} \in M$
***Initialization Phase***
M ←ConstructionHeuristic($\mu, \lambda$);
$\alpha \leftarrow$ InitialiseAdaptiveMemory($M$,$\delta t$) ; $RCLsize = 1$;
***Exploitation Phase***
while Number of schedules examined are less or equal to maxSc do
    $s \leftarrow \text{GRR}(\alpha, RCLsize)$;
    $s^* \leftarrow ReVNS^{dist}(s, \delta/2, \vartheta_{max})$;
    $(s', \vec{h}) \leftarrow ReVNS^{cost}(s^*, \delta, \vartheta_{max})$;
    if SelectionCriteria($M, s'$) then
        UpdatePool($M, s'$); $RCLsize = 1$;
        $\alpha \leftarrow$ UpdateAdaptiveMemory($M, s', \delta t$) ;
    else
        if $RCLsize < \mu$ then
            $RCLsize = RCLsize + 1$;
        else
            $RCLsize = 1$;
```

3.4 *Initialisation Phase*

The initialisation phase aims at producing an initial population of solutions that are both diversified and of good quality. A construction heuristic is proposed that combines a serial SGS with a backward, a forward and a bidirectional planning schemes. At each iteration of the initialisation phase, the planning scheme is selected at random with equal probability and is preserved until all activities are scheduled and a complete solution is constructed. The activity to be scheduled at a particular iteration is selected at random from all precedence feasible activities at that iteration. The construction heuristic is applied repeatedly until λ different solutions are created, among which μ are selected to build the pool M. Details about the creation of the pool of solutions are given in Section 3.4.2. The initial population is exploited in the exploitation phase.

3.4.1 The adaptive memory

A major element of the proposed algorithm is the adaptive memory α, which records the track of the search to identify favourable time buckets for each activity and guides the search towards high quality solution regions. The main intuition behind this strategy is that frequently visited solution elements (favourable time buckets in this instance) can be possibly found in optimum or higher quality solutions, especially if these elements belong to high quality solutions (Tarantilis 2005; Repoussis *et al.* 2010).

To capture the favourable solution elements appropriate memory structures are used. Taking advantage of the event-list representation structure the time information needed is easily extracted and stored in a two dimensional array α, i.e., the adaptive memory. To create the array α, the time horizon T is divided into $Tb = T/\delta t$ time buckets and the size of α is $|J| \times Tb$, where $|J|$ is the number of activities and δt the size of the time bucket in time units.

Initially, all the elements $\{i, t\}$ of α are set equal to 0. In the initialisation phase, α is initialised with the information included in the pool of solutions M. In particular, each element $\{i, t\}$ of α is incremented by b_s, if activity i is scheduled at time bucket t and belongs to solution $s \in M$. The value of $b_s = 1/f(s)$ reflects the importance of the solution quality into the information stored in α, where $f(s)$ is the primary objective as defined by equation (1). The higher the cost of a solution s, the lower is the b_s and thus the lower is the impact of the solution elements of s into the formation of the adaptive memory. The latter is calculated according to the equation given below:

$$\alpha(i,t) = \sum_{s\in M} b_s I^s_{it}, \forall i \in J, \forall t \in \{1,...,Tb\} \tag{2}$$

where i represents an activity in solution s and I^s_{it} is a binary variable equal to 1 if activity i is scheduled within the time bucket t, i.e., $(t-1)\delta t \leq ST_{si} \leq t\delta t$, where ST_{si} is the starting time of i, and 0 otherwise. Every time, a new solution is inserted into the pool of solutions, α is updated by using the procedure described above. Note that only the activity's starting time ST_{si} needs to be (or equally the finish time l_{si} for the backward scheduling scheme) stored within the respective time bucket, and not the whole execution time interval of the activity.

3.4.2 The selection criteria

The pool of solutions and the adaptive memory α are updated if certain criteria are met. The goal is to maintain a balance between quality and diversity and to avert premature convergence. Our diversity measure uses the distance measure of Chen *et al.* (2010b) between pairs of solutions, $D(s,s^*) = \sum_{j\in J} |l_{sj} - l_{s^*j}|$, for two solutions s and s^*, where l_{sj} and l_{s^*j} are the finish times of activity j in solutions s and s^*, respectively, J is the set of activities and M is the pool of solutions. The total dissimilarity for pool M is then given by:

$$\text{TD}(M) = \sum_{s,s^*\in M} D(s,s^*), \tag{3}$$

where the sum is over all $\mu(\mu-1)/2$ pairs of solutions in set M.

The creation of the initial pool of solutions proceeds as follows. The first μ solutions among the λ generated within the Initialisation phase are inserted into M. The remaining $\lambda-\mu$ solutions are then considered sequentially for replacing a solution in M. Specifically, if such a solution s satisfies the condition $f(s) < f(s_{\text{best}})$, where s_{best} is a least cost solution in M, or if $f(s) < f(s_{\text{worst}})$ and $\text{TD}(M) < \text{TD}(M \setminus \{s_{\text{worst}}\} \cup \{s\})$, then s is inserted into M. Otherwise, s is not included in M. When s is inserted, solution $s_{\text{worst}} \in M$, where s_{worst} has the largest cost among the solutions in M, is removed from M. Note that, as soon as the pool of solutions is updated with a new solution, the adaptive memory is updated as well. The same rationale is followed in the *Exploitation Phase.*

3.5 *Exploitation Phase*

The initial population is exploited in the second phase of AMP, the exploitation phase. In the following, the components of the exploitation phase are presented.

3.5.1 The greedy randomised reconstruction method

The way the information is extracted from the adaptive memory, is crucial and affects the efficiency and effectiveness of the proposed solution methodology. The information stored in the adaptive memory, is extracted by using a greedy randomised rationale (Kontoravdis and Bard 1995), which adaptively selects the information according to the track record of the search history.

For this purpose, a list, namely the Random Candidate List -RCL_i, is used to store the highest score elements $\alpha\{i,t\}$ of activity i, $\forall t \in \{1,...,Tb\}$. Each activity i is assigned with its own list RCL_i. The size of the list is dynamic and starts from 1 and increases as no improvement in the solution quality is observed, until the maximum size μ is reached. If a better solution than $s_{best} \in M$

is found the RCL size is reinitialised to 1, for intensification purposes. Details on how the RCL size is controlled are given in Algorithm 1.

The provisional solution is built step by step, by scheduling one activity at a time. Starting from an empty schedule, a set $\mathcal{A}$ of precedence feasible activities is created. The first activity to be scheduled is the activity that has the highest score $\alpha^*\{i,t\}$, i.e., $\max_{i\in\mathcal{A}}\{\alpha^*\{i,t\}\}$, among all the precedence feasible activities $i \in \mathcal{A}$. The score $\alpha^*\{i,t\}$ is derived by selecting at random one of the elements of the list RCL_i. Also, the element $\alpha^*\{i,t\}$ provides the favourable time bucket t_i^* that activity i should be scheduled.

A scheduling procedure tries then to schedule the activity i as close as possible to the desirable t_i^* without violating any resource capacity constraints. Note that, it is not always possible to schedule all the activities to their favourable time bucket t_i^* due to the resource constraints and due to the serial nature of the scheduling procedure and they are thus scheduled as close as possible to their t_i^*. As soon as activity i is scheduled, it is removed from the set $\mathcal{A}$, set $\mathcal{A}$ is populated with more precedence feasible activities and the procedure iterates until $\mathcal{A}$ is empty, i.e., all activities $i \in J$ have been scheduled.

To illustrate the mechanics of the adaptive memory and the reconstruction procedure we provide the following example. Assume the problem instance of Figure 1 and a pool of solutions with size $\mu = 9$. The time horizon T is set equal to 40 and divided in time buckets of size $\delta t = 5$, thus the number of time buckets is $Tb = T/\delta t = 8$. The adaptive memory α has a size of 18x8, compliant with the 18 activities and the 8 time buckets available. Assume also that, after some iterations of the reconstruction mechanism, some of the activities have already been scheduled. Then, on a particular iteration, set $\mathcal{A}$ contains activities 16, 10 and 9 and the size of RCL is equal to 2. Assume also that the adaptive memory records of these three activities are $\alpha\{9,t\} = \{0,0,0.345,0.546,0.034,0,0\}$, $\alpha\{10,t\} = \{0,0,0.897,0.098,0,0.023,0\}$, $\alpha\{16,t\} = \{0,0,0.054,0.345,0.245,0,0\}$, with $t \in \{1,...,8\}$. Given all this information, the RCLs are created. Because the RCL size is equal to two, the two highest score elements of each $\alpha\{i,t\}$ are selected. Therefore the following RCLs are created: $RCL_9 = \{0.546, 0.345\}$, $RCL_{10} = \{0.897, 0.098\}$ and $RCL_{16} = \{0.345, 0.245\}$. Next step, an element of RCL_i for each i is selected at random. Let the following elements be the selected ones: $\alpha^*\{9,t\} = 0.345$, $\alpha^*\{10,t\} = 0.897$ and $\alpha^*\{16,t\} = 0.345$, and the respective t_i^*s are, $t_9^* = 3$, $t_{10}^* = 3$, and $t_{16}^* = 4$.

Among the three activities, the activity that has the highest score $\alpha^*\{i,t\}$ is activity 10, and will be scheduled next. Specifically, activity 10 will be scheduled as close as possible to time bucket 4, i.e., the time interval from 15 to 20 (from (4-1)$\times\delta t$ to 4$\times\delta t$, δt=5) within the time horizon, without violating any resource capacity constraints. Then activity 10 is removed from set $\mathcal{A}$, more precedence feasible activities are inserted into $\mathcal{A}$ and the procedure continues by comparing the adaptive memory records of the activities in the updated set $\mathcal{A}$, until all activities $i \in J$ have been scheduled.

3.6 *The reactive variable neighbourhood search*

The provisional solution produced by the reconstruction method is subject for improvement by a ReVNS. The basic idea of the variable neighbourhood local search is the systematic change of neighbourhoods within local search (Paraskevopoulos *et al.* 2008). The systematic change of the neighbourhood is following an increasing cardinality scheme, such that small neighbourhoods are chosen initially and as an improvement in the solution quality is not observed, the neighbourhood cardinality is increased by selecting different neighbourhood types. In our implementation, y_{max} neighbourhood types are introduced, as many as the size of the largest event in the solution s. Note that, the events are groups of activities that start at the same time, thus y_{max} is defined by the maximum number of activities that start together at the same time in the solution s. Therefore, for example if the largest event of a solution is composed by 5 activities, the neighbourhood types considered are y_{max} =5. The neighbourhoods are created by rescheduling events of the size y, $y \in [1,...,y_{max}]$, to all possible positions in the event list (see 3.6.2 for details). If an improvement in the total cost of the solution, in terms of equation (1), is observed, the neighbourhood index is reinitialised to y_{max}. Note that, y_{max} fully depends on the structure of the current solution. Consequently, despite the fact that at an iteration the neighbourhood cardinality is y_{max}, after performing a specific local move, y_{max} must be re-calculated according to the largest size of the event in the new solution.

The *Shaking* step, the *Local Search* phase and the *Perturbation* compose the ReVNS framework. In the Shaking step, a local move is applied and a neighbouring solution s' is produced. The local search preserves the neighbourhood type y and performs only local moves of this neighbourhood type y. Note that, after δ iterations that the local search has not found any better solution, the procedure terminates and the best solution is returned. To escape from local optima, the perturbation mechanism is applied to guide the search towards unexplored regions of the solution space and apply diversification. The pseudocode of the proposed ReVNS is given by Algorithm (2).

Algorithm 2: Reactive Variable Neighbourhood Search

```
Input: Provisional solution $s$, $\delta$ (number of local search iterations without an improvement),
       $\vartheta_{\max}$ (number of perturbations without an improvement), $\delta t$ (size of the time bucket)
Output: $s_{ReVNSbest}$
$\vartheta = 1$; $\vec{g} \leftarrow 0$; $\vec{h} \leftarrow 1$;
$y_{max}$ =ReturnSizeofEvents(s);$y = y_{max}$;
while $\vartheta < \vartheta_{max}$ do
    $s' \leftarrow$Shaking$(s, y)$;
    $y_{max}$ =ReturnSizeofEvents$(s')$;
    $(s^*, \vec{g}, \vec{h}, y_{max}) \leftarrow$ LocalSearch$(y, y_{max}, \delta, s', \vec{g}, \vec{h})$;
    if $f(s^*) > f(s) \& y > 1$ then
        $y = y - 1$;
    else if $f(s^*) > f(s) \& y = 1 \& \vartheta < \vartheta_{max}$ then
        $s \leftarrow$ Perturbation$(s^*, \vec{h}, \vartheta)$; $\vartheta = \vartheta + 1$;
        $y_{max}$ =ReturnSizeofEvents(s); $y = y_{max}$;
    else if $f(s^*) < f(s)$ then
        $s \leftarrow s^*$; $y_{max}$ =ReturnSizeofEvents(s);$y = y_{max}$;
    else if $f(s^*) < f(s_{ReVNSbest})$ then
        $s_{ReVNSbest} \leftarrow s^*$;$\vartheta = 1$;$\vec{g} \leftarrow 0$;
```

The function *ReturnSizeofEvent* scans the current solution, finds the maximum number of activities that start (or finish if backward scheduling scheme is considered) at the same time, and returns this value. *Shaking* picks at random one particular neighbour, of the neighbourhood type y. Details on *LocalSearch* and *Perturbation* are explained in the following sections. Prior to this, however, we describe the two versions of the proposed ReVNS, according to two different objective functions.

3.6.1 The two versions of the ReVNS

There are two versions of the ReVNS: the cost minimisation $ReVNS^{cost}$ and the distance minimisation $ReVNS^{dist}$. The first version of ReVNS is using the equation (1) as the objective function. The second version of ReVNS, can be described as follows. Due to precedence and resource constraints the procedure described in Section 3.5.1 cannot accurately transfer the adaptive memory information to a solution's schedule. In other words, there are deviations between the favourable time-bucket t_i^* and the time bucket an activity i is actually scheduled. The latter fact is inevitable since the schedules are built in a sequential fashion. To achieve a more efficient inheritance of the adaptive memory information, $ReVNS^{dist}$ is applied using a different objective than this of makespan minimisation, just before the cost minimisation takes place (see Algorithm 1 for details).

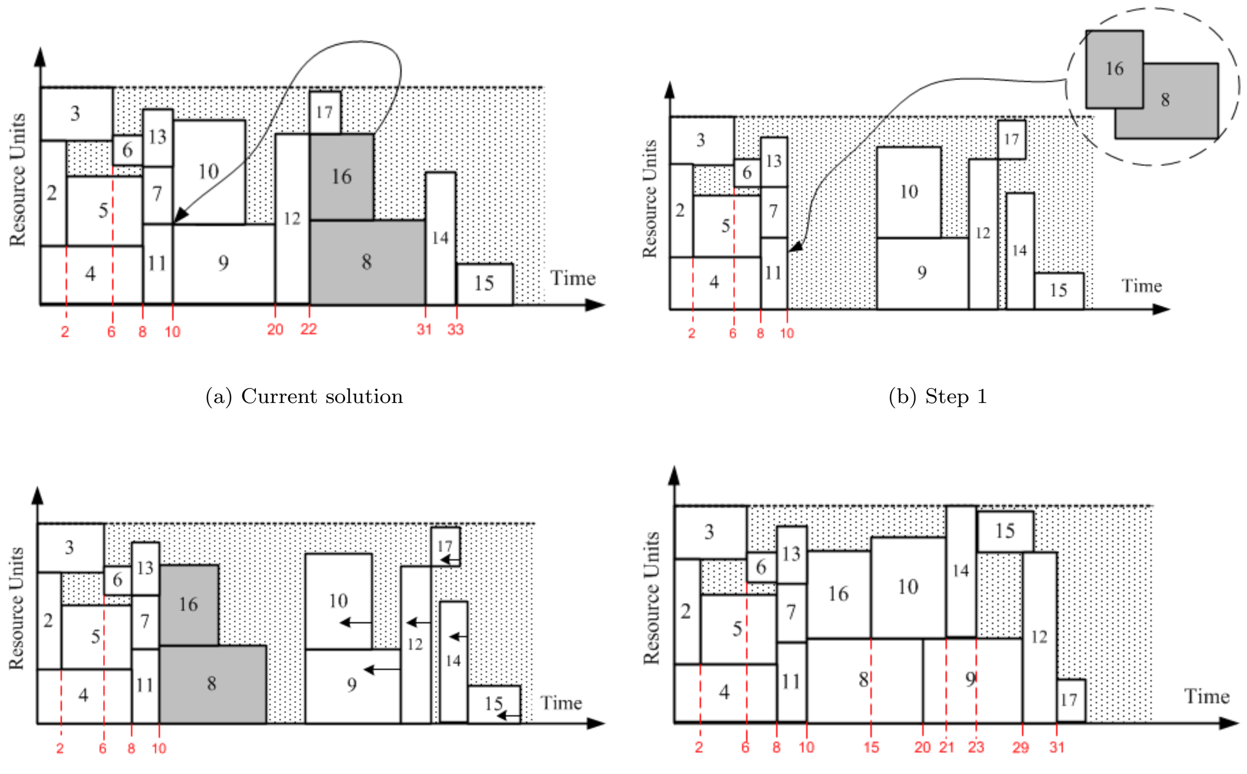
The new objective is related to the total distance between the desired (extracted by the adaptive memory) and the actual starting times for the activities of the provisional solution s and is given below:

$$f'(s) = \sum_{i=1}^{N} |ST_{si} - t_i^* \delta t| \tag{4}$$

where ST_{si} are the starting times of activity i in solution s and t_i^* is the favourable time-bucket for activity i. Therefore, Algorithm (2) can be broken down to two versions, Algorithm (2a) where $f(s)$ is equation (1) and Algorithm (2b) where $f(s)$ is equation (4).

3.6.2 The neighbourhood operators

ReVNS operates compound moves and uses a frequency based memory (represented by memory structure $\vec{g}$) to escape from local optima by penalising neighbours met before. An array $\vec{g}$ of size $|J|$xTb is used to store the number of times a particular activity i has participated into a local move and has been scheduled in the time bucked t. After an improvement in the solution quality is observed, $\vec{g}$ is reinitialised to the zero vector. Note that the role of $\vec{g}$ is totally different than this of the adaptive memory α and they contain different types of information; $\vec{g}$ stores information regarding the local search history to help penalising frequently selected moves and guide effectively the trajectory local search, and on the contrary adaptive memory α consorts with the pool of solutions to capture favourable solution elements, which are combined to produce high quality provisional solutions.



(a) Current solution

(b) Step 1

(c) Step 2

(d) Final neighbouring solution

Figure 2. Local search operator-event relocation

Figure 2 illustrates the process of evaluating a neighbouring solution using the event relocation operator, based on the problem instance shown in Figure 1. Given the current solution, assume that we examine relocating the event that is composed by activities 16 and 8, as the arrow shows. All events up to that position remain as they are in the schedule and from that point and onwards everything is rescheduled (Figure 2b). First the relocated event is scheduled (Figure 2c) and then all the rest events are scheduled as early as possible. Note that, even though we consider event relocations, the activities are scheduled independently as early as possible. In the neighbouring solution (Figure 2d), new events have emerged and thus a new Event List is created. Note that, this is only a single neighbour created by the event relocation operator.

The following equation defines the local move cost from solution s to a trial solution s' as

$$\Delta f_{\text{move}}(s, s') = f(s') - f(s) + \beta \sum_{i \in J} \sum_{t=0}^{T_b} z_{it} g_{it}, \tag{5}$$

where β is a scaling parameter, defined as the fraction of the $f(s_{best})/|J|$, and z_{it} has a value equal to 1 if the activity i is scheduled in time bucket t in the current local move from s to s', and a value 0 otherwise. The component $\beta \sum_{(i \in J} \sum_{t=0}^{T_b} z_{it} g_{it}$, is added to the cost of the local move to penalise moves that involve frequently selected activities.

Trial solutions with smaller values of Δf_{move} are generally preferred. However, it may be that this number is large enough to prevent the search from selecting a high-quality neighbour s'. To avert such cases, an aspiration criterion is used: if $f(s') < f(s_{\text{ReVNSbest}})$, the penalty component is ignored so that $\Delta f_{\text{move}} = f(s') - f(s)$.

It is worth denoting that as soon as, a local move is applied the structure of the solution changes and *ReturnSizeofEvents* is called again and a new value for y_{max} is calculated, the y'_{max}. If the current neighbourhood type $y > y'_{max}$ then $y = y'_{max}$, otherwise no action is required.

3.6.3 The perturbation strategy

A major component of the proposed ReVNS is its perturbation strategy. The goal is to partially rebuild the current local optimum solution, such that the new diversified solution preserves some information from the local optimum. The proposed perturbation strategy performs local moves of neglected activities during local search and tries to reschedule them to the least favourable time buckets. For this purpose, a long term memory $\vec{h}$, similar to $\vec{g}$, is introduced that stores the number of times the activity i has participated in a local move and has been scheduled into a particular time bucket t. Initialization sets $h_{it} = 1$ for all $i \in J$, and $t = 1, ..., T_b$. The h_{it} values have a similar purpose to the g_{it} values of Section 3.6.2 except that $\vec{h}$ is never re-initialised, instead a scaling process is applied for the h_{it}. Specifically, to avoid h_{it} become very large for some (i, t), we periodically divide h_{it} by $h_{i\,\min}$ for all $i \in J$, where $h_{i\,\min} = \min_{t=1,...,T_b} h_{it}$,with $h_{it} > 1$.

The perturbation normally “ruins and recreates” a part of the current solution in order to move to unexplored regions of the solution space. The part of the solution we ruin and recreate in this proposed perturbation is defined by a subset $\mathcal{F} \subseteq J$ of activities that are subject for rescheduling during the perturbation process. Specifically, $\vartheta/\vartheta_{max}$ of the total number of activities are selected and stored in list $\mathcal{F}$ such that $|\mathcal{F}| = \frac{\vartheta|J|}{\vartheta_{max}}$. The activities that comprise the list $\mathcal{F}$ are selected as follows. The set of activities J are sorted in an ascending order of their average score of $\overline{h}_{it} = (\sum_{t=0, h_{it}>1}^{T_b} h_{it})/T_b$. The function *SortActivities* will thus sort the more neglected during the search activities in the top of the list R and the less neglected towards the end. Next, *FillCandidateSet* considers the fraction $\vartheta/\vartheta_{max}$ and starts from the top of the list, inserts activities to the list $\mathcal{F}$ and it stops until the index $i = \frac{\vartheta|J|}{\vartheta_{max}}$ is met. Then, an iterative process relocates-reschedules all activities $i \in \mathcal{F}$. In the following we explain how rescheduling is done.

Each activity i has its own allowable range of relocation positions $[w_i, v_i]$ in the event list, where w_i, v_i the earliest and latest positions in the event list that the particular activity can be relocated-

Algorithm 3: Perturbation

```
Input: $Q$ a large positive number, Local optima solution $s^*$, $\vartheta$ and $\vartheta_{\max}$ (number of
        perturbations without an improvement), long term memory $\vec{h}$
Output: $s$
$R \leftarrow$ SortActivities($\vec{h}$);
$\mathcal{F} \leftarrow$ FillCandidateSet($\vartheta, \vartheta_{\max}, R$);
while $\mathcal{F} \neq \emptyset$ do
    $i \leftarrow$ SelectFirstActivity($\mathcal{F}$);
    min=Q;pos=0;time=0;
    for all possible relocation positions pos in the event-list for activity $i$ do
        $time$=CheckMove($pos, i$);
        if $h_{i,time} < min$ then
            min=$h_{i,time}$; $pos^* = pos$;
    if $min \neq Q$ then
        $s \leftarrow$ ApplyMove($s^*, i, pos^*$);$s^* \leftarrow s$;
        $\mathcal{F} \leftarrow$ RemoveFirstElement($\mathcal{F}$);
    else
        $\mathcal{F} \leftarrow$ RemoveFirstElement($\mathcal{F}$);
```

rescheduled, respectively, according to the precedence constraints. A function *CheckMove* is used to return the time the activity i will be scheduled after the local move is applied on the position *pos*. The goal is to select the position for relocation-rescheduling that corresponds to the minimum value $h_{i,time}$ for all times *time* that are derived by scheduling activity i at all the possible positions w_i and v_i in the event list. Given the best position *pos** a local move is applied to relocate activity i from its original position to the position *pos** in the event list. Then, a serial SGS is called to calculate the new timings for all activities $i \in J$ and produce a solution s. Finally, the procedure removes the first element from the list $\mathcal{F}$ and iterates with the next activity in the list, until the list $\mathcal{F}$ is empty.

4. Numerical experiments

Computational experimentation was conducted on the benchmark problem instances of Kolisch *et al.* (1995) for the evaluation of the proposed AMP. The problem instances come in the sets J30, J60, and J120 which include problems with 30, 60, and 120 activities, respectively. In total, J30 contains 480 instances, J60 contains 480 instances and J120 contains 600 instances. The proposed algorithm was developed in Visual C++ 2010, and run on a PIV 2,8 GHz PC.

The basis of comparisons among heuristic methodologies for the RCPSP is the number of schedules examined and is used as the termination criterion. The main goal is to assess on the design of an algorithm and not on the programming skills of the developers (Kolisch *et al.* 1995). Thus, the termination criterion followed in this paper, as well as in the literature, is the 1000, 5000 and 50000 schedules examined.

4.1 *Parameter settings-ReVNS*

To investigate on the ReVNS' suitability to serve as an improvement method integrated into AMP, ReVNS was tested as an autonomous multi-start solution method.

ReVNS introduces two parameters; a parameter δ to define the maximum number of local search oscillations that an improvement is not observed, and the maximum number of perturbations performed ϑ_{max}, without an improvement into the current solution. A balance between the number of the restarts and the ReVNS iterations on each initial solution, is preserved.

4.1.1 Maximum number of perturbations ϑ_{max}

The purpose of perturbation is to perturb a solution, such that to produce diversified solution structures while keeping some of the elements of the local optima. We have conducted computational experiments to define θ_{max}. The latter indicates a) (explicitly) the maximum number of perturbations that will be applied on a single solution without observing any cost improvement before ReVNS restarts from new solution and b) (implicitly) the number of activities that will be removed and rescheduled at each perturbation. For example, if θ_{max}=3, 1/3 of the total number of activities will be removed and rescheduled at the first perturbation, and 2/3 of activities will be removed and rescheduled at second etc. We tried different values for θ_{max}, i.e., 2, 3, 4, 5 and 6. Higher numbers than 3 created weak perturbations as less number of activities were being removed and rescheduled, whereas 2 was creating low quality solutions. Based on this observation, the number of perturbations ϑ_{max} is set equal to 3.

4.1.2 Maximum number δ of local search iterations without improvement

The size of the largest event into various solutions, was found to vary between 3 and 6 according to the specifications of the incumbent problem instances. Local search will thus perform at the worst case $3 \times \delta$ iterations without an improvement, as the number of neighbourhood types is equal to the size of the largest event in the current solution. If the latter quantity is multiplied by the number of perturbations ϑ_{max}, we can roughly estimate the number of ReVNS iterations without an improvement. The goal is to preserve a balance between multiple restarts and ReVNS, under the limitations of the total number of schedules examined. Therefore, δ was set equal to 10, which means $3 \times 10 \times 3 = 90$ local search iterations without an improvement.

4.2 *Parameter settings-AMP*

The proposed AMP introduces five parameters; the pool size $|M| = \mu$, the number of initial solutions λ considered to build the pool of solutions, the maximum local search oscillations that an improvement has not been observed δ, the number of perturbations ϑ_{max}, and the size of the timebucket used by adaptive memory δt. The parameter λ was set according to the termination criterion, i.e., equal to 100 if the maximum number of schedules $maxSc$ examined is 1000, λ=200 if $maxSc$=5000, and λ=500 if $maxSc$=50000. Since δ and ϑ_{max} are common parameters for the ReVNS, and are discussed previously, in the following the sensitivity of parameters μ and δt is discussed.

4.2.1 Size of the time bucket δt

Parameter δt is the size of the time bucket used to store information about the starting times (or finishing times when a backward SGS is considered) of the activities. Giving small values to δt leads to keeping more precise information in the adaptive memory. To fully exploit this feature a large pool of solutions should be employed to provide to the detailed adaptive memory different scheduling positions of the activities in the time horizon. As δt increases the different starting times of an activity in different solutions in the pool, tend to be associated to the same time bucket. At this case, there is no need for a large pool and a smaller one shall be employed. Small values for δt=2 or δt=3 combined with large pool sizes led to a delayed convergence, exceeding the limited number of schedules. To the other end, large values, e.g., δt=10 caused a premature convergence. Finally, according to several numerical experiments, δt was set equal to 5.

Table 1. Calibration of the parameter μ

Instances	Avg Total Cost for different μ			
	8	10	12	14
6-1	152.20	152.80	152.30	151.80
7-1	105.00	105.10	105.80	105.50
11-1	179.20	179.40	179.90	180.50
12-1	142.80	143.60	143.10	143.40
16-1	202.30	202.60	202.10	202.10
17-1	143.90	144.00	143.80	143.90
26-1	178.10	180.30	179.50	179.30
27-1	111.60	111.80	111.80	112.10
31-1	204.20	205.30	204.80	205.50
32-1	150.90	151.10	150.90	151.10
36-1	216.00	216.20	216.40	216.30
37-1	148.80	149.10	148.90	148.90
46-1	196.20	196.40	196.70	198.10
47-1	142.70	144.30	144.80	144.50
51-1	213.00	214.90	213.90	214.50
52-1	182.90	184.10	183.80	183.80
56-1	242.80	242.80	242.70	243.20
57-1	190.30	190.00	190.00	190.10
Avg	172.38	172.99	172.84	173.03

4.2.2 Size of the pool of solutions μ

Aiming to achieve a high level of efficiency in limited number of schedules one must consider the speed of convergence into the limited number of schedules. To investigate on this, we run a sample of large scale problems of class J120 with different parameter settings for μ and the average result over ten runs for each parameter setting is shown in Table 1. The termination criterion was $maxSc$=50000 schedules, while the rest of the parameters were set fixed λ=500, $\delta t = 5$, $\delta = 10$ and $\vartheta_{max} = 3$. As Table 1 shows, the effect of the μ varies according to the different problem solved. On average, the best value for μ seems to be 8, and this is fixed for all the runs of AMP.

4.3 *Comparative Analysis*

Experimentation was conducted on both the ReVNS and the AMP framework to challenge their efficiency, compared to the-state-of-the-art of the RCPSP literature.

4.3.1 The performance of the ReVNS

To investigate on the appropriateness of the ReVNS as a post-improvement method integrated into the proposed AMP, numerical experiments were conducted considering ReVNS as an autonomous solution method, and local search algorithms of the literature was the basis of comparisons, presented in Table 2.

Table 2 provides the performance results of ReVNS compared to local search methods proposed in literature for the RCPSP. There are not many local search methods in literature for the RCPSP.

Table 2. Comparisons of the ReVNS to local search metaheuristics on benchmarks sets J30, J60, and J120.

Pr. Set	**Algorithm**	**RS**	**Reference**	**% Dev**		
				1000	5000	50000
J30	ReVNS	EL	This paper	0.03	0.01	0.00
	AILS	EL	Paraskevopoulos et al. (2012)	0.05	0.01	0.00
	SA	AL	Bouleimen and Lecocq (2003)	0.38	0.23	n/a
	TS	AL	Nonobe and Ibaraki (2002)	0.46	0.16	0.05
	TS-insertion rules	AL	Artiques *et al.* (2003)	n/a	n/a	n/a
	Hybrid LS techniques	AL	Pesek *et al.* (2007)	n/a	n/a	n/a
J60	ReVNS	EL	This paper	11.47	11.10	10.88
	AILS	EL	Paraskevopoulos et al (2012)	11.34	11.10	10.91
	SA	AL	Bouleimen and Lecocq (2003)	12.75	11.90	n/a
	TS	AL	Nonobe and Ibaraki (2002)	12.97	12.18	11.58
	Hybrid LS techniques	AL	Pesek *et al.* (2007)	n/a	n/a	11.10
	TS-insertion rules	AL	Artiques *et al.* (2003)	n/a	12.05	n/a
J120	ReVNS	EL	This paper	34.74	33.36	32.21
	AILS	EL	Paraskevopoulos et al (2012)	34.38	32.67	32.66
	TS-insertion rules	AL	Artiques *et al.* (2003)	n/a	36.74	n/a
	TS	AL	Nonobe and Ibaraki (2002)	40.86	37.88	35.85
	SA	AL	Bouleimen and Lecocq (2003)	42.81	37.68	n/a
	Hybrid LS techniques	AL	Pesek *et al.* (2007)	n/a	n/a	n/a

The results reported herein, as well as in other papers of literature (Tseng and Chen 2006), take into consideration one schedule at each iteration of the local search. Thomas and Salhi (1998) was not included in the comparisons as the authors have used different benchmarks for their evaluation tests. Lastly, the filter and fan method proposed by Ranjbar (2008) was not considered as the author does not report results with respect to the different schedule modes, i.e., 1000, 5000 and 50000 schedules.

The second column of Table 2 gives the abbreviations for the solution methodologies, i.e., SA for Simulated Annealing, VNS for Variable Neighbourhood Search, TS for Tabu Search, AILS for Adaptive Iterated Local Search and hybrid LS for hybrid Local Search. The third column reports the representation schemes, while the next column gives the references. Lastly, the multicolumn that follows reports the average deviation % from the optimum and from the CPM lower bounds. In particular, for J30 set the % deviation is calculated with regard to the optimum values and for J60 and J120 the % deviations are derived with the CPM lower bounds. The three different columns refer to the different termination criteria of 1000, 5000, and 50000 schedules examined. Table 2 shows that ReVNS performs better than the local search methods of literature for all termination criteria. In particular, ReVNS at 50000 schedules achieves 0,00 (%), 10,88 % and 32,21 % deviations for J30, J60 and J120 sets, respectively.

Table 3. Statistics on the results derived by AILS and ReVNS on the J30 benchmark set

Measure	AILS			ReVNS		
	1000	5000	50000	1000	5000	50000
Sum of all costs	28334	28319	28317	28326	28319	28317
Average cost	59.03	59.00	59.00	59.01	59.00	59.00
Geometric mean cost	57.52	57.50	57.50	57.51	57.50	57.50
Max % Dev from CPM LBs	2.94	1.92	1.18	2.94	1.92	1.18
No of Optimum solutions	463	476	479	470	476	479
Avg. % Dev from Optima	0.05	0.01	0.00	0.03	0.01	0.00

Table 4. Statistics on the results derived by AILS and ReVNS on the J60 benchmark set

Measure	AILS			ReVNS		
	1000	5000	50000	1000	5000	50000
Sum of all costs	38635	38521	38483	38679	38551	38477
Average cost	80.49	80.25	80.17	80.58	80.32	80.16
Geometric mean cost	78.65	78.47	78.41	78.72	78.53	78.42
Max % Dev from CPM LBs	110.39	105.20	103.90	106.49	105.20	102.60
No of Optimum solutions	295	295	295	295	295	295
Avg. % Dev from CPM LBs	11.35	11.10	10.91	11.47	11.10	10.88

Table 5. Statistics on the results derived by AILS and ReVNS on the J120 benchmark set

Measure	AILS			ReVNS		
	1000	5000	50000	1000	5000	50000
Sum of all costs	76256	75773	75279	76476	75686	75038
Average cost	127.09	126.29	125.47	127.46	126.14	125.06
Geometric mean cost	120.83	120.17	119.52	121.20	120.10	119.21
Max % Dev from CPM LBs	209.9	208.08	203.03	203.96	203.03	200.00
No of Optimum solutions	144	162	168	150	160	170
Avg. % Dev from CPM LBs	34.38	32.67	32.66	34.74	33.36	32.21

To compare AILS and ReVNS separately, we derived different statistical measures shown below in the Tables 3, 4 and 5. ReVNS is in general better than AILS in J30 and J60 benchmark sets, and this difference becomes larger when it comes to the large problem instances of the benchmark set J120, under the limit of 50000 schedules. More specifically, ReVNS achieves a better sum of all costs by 241 units, a better average cost by 0.41, a better geometric mean cost by 0.31, a better maximum % deviation from the CPM LBs by 3.03, it produces two more optimum solutions than AILS, and a better average % deviation from the CPM LBs by 0.45.

Note that, in Tables 4 and 5 (and in Tables 8 and 9), the row *No of Optimum solutions* indicate the number of solutions for which the total cost matches with the Lower Bound derived by the CPM. Therefore, there could be more optimum solutions found, but this information is not known. This is not the case for the J30 benchmark set, as for the latter all optimum solutions are known.

Table 6. Comparisons to the-state-of-the-art algorithms on benchmarks sets J30, J60, and J120.

Pr. Set	**Algorithm**	**RS**	**Reference**	**% Dev.**		
				1000	5000	50000
J30	AMP	EL	This paper	0.02	0.01	0.00
	ReVNS	EL	This paper	0.03	0.01	0.00
	HEA	EL	Paraskevopoulos *et al.* (2012)	0.03	0.01	0.00
	AILS	EL	Paraskevopoulos *et al.* (2012)	0.05	0.01	0.00
	GA-MBX	AL	Zamani (2013)	n/a	n/a	0.00
	SS-PR	AL	Mobini *et al.* (2009)	0.05	0.02	0.01
	b-GA	RK	Goncalves et al. (2011)	0.32	0.02	0.01
	GA. TS-PR	AL	Kochetov and Stolyar (2003)	0.10	0.04	0.00
	GAPS	RK	Mendes *et al.* (2009)	0.06	0.02	0.01
	ACOSS	AL	Chen *et al.* (2010b)	0.14	0.06	0.01
	SS-FBI	RK	Debels *et al.* (2006)	0.27	0.11	0.01
	GA	TO-RK	Debels and Vanhoucke (2007)	0.15	0.04	0.02
	GA-hybrid FBI	AL	Valls *et al.* (2008)	0.27	0.06	0.02
	NG (FBI)	AL	Agarwal *et al.* (2011)	0.13	0.10	n/a
	jPSO	AL	Chen (2011)	0.29	0.14	0.03
	TS	AL	Nonobe and Ibaraki (2002)	0.46	0.16	0.05
	GA	AL	Hartmann (2002)	0.38	0.22	0.08
	ANGEL	AL	Tseng and Chen (2006)	0.22	0.09	n/a
J60	HEA	EL	Paraskevopoulos *et al.* (2012)	11.05	10.72	10.54
	b-GA	RK	Goncalves et al. (2011)	n/a	11.56	10.57
	SS-PR	AL	Mobini *et al.* (2009)	11.12	10.74	10.57
	AMP	EL	This paper	11.09	10.74	10.65
	GA-MBX	AL	Zamani (2013)	n/a	n/a	10.65
	GAPS	RK	Mendes *et al.* (2009)	11.72	11.04	10.67
	ACOSS	AL	Chen *et al.* (2010b)	11.72	11.04	10.67
	GA	TO-RK	Debels and Vanhoucke (2007)	11.45	10.95	10.68
	SS-FBI	RK	Debels *et al.* (2006)	11.73	11.10	10.71
	GA-hybrid FBI	AL	Valls *et al.* (2008)	11.56	11.10	10.73
	GA. TS-PR	AL	Kochetov and Stolyar (2003)	12.21	11.27	10.74
	ReVNS	EL	This paper	11.47	11.10	10.88
	AILS	EL	Paraskevopoulos et al (2012)	11.34	11.10	10.91
	NG (FBI)	AL	Agarwal *et al.* (2011)	11.51	11.29	n/a
	jPSO	AL	Chen (2011)	12.03	11.43	11.00
	GA	AL	Hartmann (2002)	12.21	11.70	11.21
	ANGEL	AL	Tseng and Chen (2006)	11.94	11.27	n/a
	TS	AL	Nonobe and Ibaraki (2002)	12.97	12.18	11.58
J120	ACOSS	AL	Chen *et al.* (2010b)	35.19	32.48	30.56
	HEA	EL	Paraskevopoulos *et al.* (2012)	33.32	32.12	30.78
	GA	TO-RK	Debels and Vanhoucke (2007)	34.19	32.34	30.82
	AMP	EL	This paper	**33.08**	**31.80**	30.88
	GA-hybrid FBI	AL	Valls *et al.* (2008)	34.07	32.54	31.24
	GA-MBX	AL	Zamani (2013)	n/a	n/a	31.30
	SS-PR	AL	Mobini *et al.* (2009)	34.49	32.61	31.37
	GAPS	RK	Mendes *et al.* (2009)	35.87	33.03	31.44
	SS-FBI	RK	Debels *et al.* (2006)	35.22	33.10	31.57
	GA. TS-PR	AL	Kochetov and Stolyar (2003)	34.74	33.36	32.06
	ReVNS	EL	This paper	34.74	33.36	32.21
	AILS	EL	Paraskevopoulos et al (2012)	34.38	32.67	32.66
	b-GA	RK	Goncalves et al. (2011)	n/a	35.94	32.76
	jPSO	AL	Chen (2011)	35.71	33.88	32.89
	NG (FBI)	AL	Agarwal *et al.* (2011)	34.65	34.15	n/a
	GA	AL	Hartmann (2002)	37.19	35.39	33.21
	ANGEL	AL	Tseng and Chen (2006)	36.39	34.49	n/a
	TS	AL	Nonobe and Ibaraki (2002)	40.86	37.88	35.85

4.3.2 Performance of the AMP

Having tested the proposed improvement method's performance, the comparative analysis proceeds by evaluating AMP's performance. Table 6 reports the results of metaheuristic methodologies of the literature on J30, J60 and J120 problem sets. The table has an identical structure to Table 2. The second column presents the abbreviations of the solution methodologies as originally named by the authors (ACOSS – Ant Colony Optimization Scatter Search, PR–Path Relinking, GAPS–Genetic Algorithm for Project Scheduling, GA-MBX–Genetic Algorithm Magnet-based Crossover, b-GA–biased Genetic Algorithm, FBI–Forward Backward Improvement, ANGEL–ANt colony optimization GEnetic algorithm Local search, NG–NeuroGenetic). The next column give the representation scheme, e.g., TO-RK for the Topological Order Random Key representation, while the next column gives the references. Lastly, the multicolumn presents the % deviations as described in Table 2. The order of the papers is in accordance to the algorithmic performances under 50000 schedules, for each different benchmark set.

In Table 6, the Decomposition-based GA (DBGA) of Debels and Vanhoucke (2007) is not considered, since the authors do not report the number of schedules, since the latter cannot be explicitly calculated due to the decomposition phase of the algorithm. Nevertheless, the authors report DGBA's computational times, thus included in the Tables 10 and 11.

AMP achieves 0, 02%, 0, 01% and 0, 00% average deviation from the optimum solutions of the J30 set at 1000, 5000 and 50000 schedules, respectively. Regarding the J60 set, AMP obtains 11,09 %, 10,74 and % 10,65 % average deviation from the CPM lower bounds. On the large scale benchmark set J120, our AMP outperforms all the solution approaches when 1000 and 5000 schedules are examined, presenting 33.08 % and 31.80 % deviations from the CPM lower bounds. The latter results improve the current best results presented by Paraskevopoulos *et al.* (2012) by 0.24% and 0.32%. When 50000 schedules are examined, AMP remains competitive presenting 30.88 % deviation from the CPM lower bounds. Lastly, it is worth mentioning that, ReVNS holds a good ranking in the list of Table 6, justifying his role as an improvement method. Nevertheless, the AMP framework is by far better than the ReVNS, improving by 1.66%, 1.56% and 1.33% the results produced by ReVNS for 1000, 5000 and 50000 schedules respectively, highlighting the effectiveness of AMP and its impact on the solution quality.

To compare HEA and AMP separately, we present different statistical measures in Tables 7, 8 and 9. One could observe that AMP's performance is very competitive compared to HEA's. AMP's main strength is to produce very good results in fewer schedules examined, as clearly shown in Table 6. More specifically, regarding the 5000 schedules examined, AMP achieves a better sum of all costs by 179 units, a better average cost by 0.3, a better geometric mean cost by 0.24, it produces 4 more optimum solutions than HEA and it lastly achieves a better average % deviation from the CPM LBs by 0.32. AMP performs also better than HEA under the limitation of 1000 schedules. Specifically, AMP achieves a better sum of all costs by 137 units, a better average cost by 0.23, a better geometric mean cost by 0.2, a better maximum % deviation from the CPM LBs by 4.04, it produces 6 more optimum solutions than HEA and it lastly achieves a better average % deviation from the CPM LBs by 0.24.

Table 7. Statistics on the results derived by HEA and AMP on the J30 benchmark set

Measure	HEA			AMP		
	1000	5000	50000	1000	5000	50000
Sum of all costs	28326	28319	28317	28323	28319	28317
Average cost	59.01	59.00	59.00	59.01	59.00	59.00
Geometric mean cost	57.51	57.50	57.50	57.51	57.50	57.50
Max % Dev from CPM LBs	2.94	1.92	1.18	1.92	1.92	1.18
No of Optimum solutions	470	476	479	472	476	479
Avg. % Dev from Optima	0.03	0.01	0.00	0.02	0.01	0.00

Table 8. Statistics on the results derived by HEA and AMP on the J60 benchmark set

Measure	HEA			AMP		
	1000	5000	50000	1000	5000	50000
Sum of all costs	38532	38425	38361	38539	38420	38389
Average cost	80.28	80.05	79.92	80.07	80.32	80.01
Geometric mean cost	78.48	78.31	78.22	78.51	78.32	78.27
Max % Dev from CPM LBs	106.49	103.9	101.3	105.2	101.3	101.3
No of Optimum solutions	297	297	297	297	297	297
Avg. % Dev from CPM LBs	11.05	10.72	10.54	11.09	10.74	10.65

Table 9. Statistics on the results derived by HEA and AMP on the J120 benchmark set

Measure	HEA			AMP		
	1000	5000	50000	1000	5000	50000
Sum of all costs	75663	74979	74096	75526	74800	74277
Average cost	126.11	124.97	123.49	125.88	124.67	123.8
Geometric mean cost	120.12	119.16	117.99	119.92	118.92	118.23
Max % Dev from CPM LBs	207.07	198.99	195.96	203.03	202.02	196.97
No of Optimum solutions	144	162	173	150	166	171
Avg. % Dev from CPM LBs	33.32	32.12	30.78	33.08	31.80	30.88

4.3.3 Comparisons on the computational times

In this section the computational times are reported. There are papers that report running times that are linked to 5000 (and lower) schedules, while other report more than 50000 schedules or they do not mention the number of schedules, all included in Tables 10 and 11.

Table 10. Computational times for up to 5000 schedules examined

Pr. Set	**Alg.**	**Reference**	Dev. %	Orig. CPU(s) Avg.	Orig. CPU(s) Max.	PCPUS	Nrm. CPU(s) Avg	Nrm. CPU(s) Max.	No Sch. (*1000)	CPU GHz
J30	AMP	This paper	0.01	0.1	1.5	1610	0.1	1.5	5.0	2.80
	HEA	Paraskevopoulos *et al.* (2012)	0.01	0.2	1.6	1610	0.2	1.6	5.0	2.80
	b-GA	Goncalves et al. (2011)	0.02	1.8	n/a	1565	1.8	n/a	5.0	2.40
	ACOSS	Chen *et al.* (2010b)	0.14	0.1	3.3	1121	0.1	2.3	5.0	1.86
	GA-MBX	Zamani (2013)	n/a	0.2	n/a	1121	0.1	n/a	5.0	1.86
	SS-PR	Mobini *et al.* (2009)	0.02	0.1	0.2	1763	0.1	0.2	5.0	3.00
	DBGA	Debels and Vanhoucke (2007)	0.04	0.1	n/a	998	0.1	n/a	5.0	1.80
	SS-FBI	Debels *et al.* (2006)	0.11	0.1	0.1	998	0.1	0.1	5.0	1.80
	ANGEL	Tseng and Chen (2006)	0.08	0.1	n/a	250	0.1	n/a	2.0	1.00
	LSSPER	Palpant *et al.* (2004)	0.00	10.3	123.0	1374	8.8	105.0	1.1	2.30
	TS	Nonobe and Ibaraki (2002)	0.05	9.1	n/a	145	0.8	n/a	5.0	0.30
J60	AMP	This paper	10.74	4.6	55.2	1610	4.6	55.2	5.0	2.80
	HEA	Paraskevopoulos *et al.* (2012)	10.72	5.4	47.6	1610	5.4	47.6	5.0	2.80
	b-GA	Goncalves et al. (2011)	11.56	0.5	n/a	1565	0.5	n/a	5.0	2.40
	ACOSS	Chen *et al.* (2010b)	10.98	0.7	13.8	1121	0.5	9.6	5.0	1.86
	GA-MBX	Zamani (2013)	n/a	0.4	n/a	1121	0.3	n/a	5.0	1.86
	SS-PR	Mobini *et al.* (2009)	10.74	0.2	0.4	1763	0.3	0.4	5.0	3.00
	DBGA	Debels and Vanhoucke (2007)	10.95	0.1	n/a	998	0.1	n/a	5.0	1.80
	SS-FBI	Debels *et al.* (2006)	11.10	0.2	0.3	998	0.1	0.2	5.0	1.80
	ANGEL	Tseng and Chen (2006)	11.27	0.8	n/a	250	0.1	n/a	2.0	1.00
	LSSPER	Palpant *et al.* (2004)	10.81	38.8	223.0	1374	33.1	190.3	2.2	2.30
	TS	Nonobe and Ibaraki (2002)	11.55	24.5	n/a	145	2.2	n/a	5.0	0.30
J120	AMP	This paper	31.80	45.6	167.5	1610	45.6	167.5	5.0	2.80
	HEA	Paraskevopoulos *et al.* (2012)	32.12	43.5	137.1	1610	43.5	3.7	5.0	2.80
	b-GA	Goncalves et al. (2011)	35.94	1.8	n/a	1565	1.8	n/a	5.0	2.40
	ACOSS	Chen *et al.* (2010b)	32.48	3.8	39.8	1121	2.6	0.1	5.0	1.86
	GA-MBX	Zamani (2013)	n/a	0.7	n/a	1121	0.5	n/a	5.0	1.86
	SS-PR	Mobini *et al.* (2009)	32.61	0.7	1.1	1763	0.8	0.0	5.0	3.00
	DBGA	Debels and Vanhoucke (2007)	32.18	0.3	n/a	998	0.2	n/a	5.0	1.80
	SS-FBI	Debels *et al.* (2006)	33.10	0.7	0.9	998	0.4	0.0	5.0	1.80
	ANGEL	Tseng and Chen (2006)	34.49	4.8	n/a	250	0.7	n/a	2.0	1.00
	LSSPER	Palpant *et al.* (2004)	32.41	207.9	501.0	1374	177.5	427.6	5.0	2.30
	TS	Nonobe and Ibaraki (2002)	34.99	645.3	n/a	145	58.1	n/a	5.0	0.30

To be able to normalise the computational times according to the different computational power used, we used the performance lists of http://www.cpubenchmark.net/. All the comparisons were made according to the Passmark CPU Score (PCPUS). As we were unable to find PCPUS for Sun and AMD 400 MHz systems on http://www.cpubenchmark.net/, we used an Intel equivalent. Every computational time was normalised by using AMP as the reference point, e.g., Nrm. CPU(Alg)= PCPUS(Alg)Orig.CPU(Alg)/PCPUS(AMP).

Tables 10 and 11 report the papers sorted according to the date of publication. The four first columns are common with the previous tables , while the fifth and the sixth column present the average and the maximum CPU time in seconds, respectively, as reported by the original authors. Note that the abbreviation LSSPER stands for Local Search with SubProblem Exact Resolution, PBA stands for Population based approach and FF stands for Filter and Fan. The seventh column reports the score of PCPUS, and using that, columns eight and nine report the normalised running times. The number of schedules and the processor's frequency is reported on the tenth and eleven columns, respectively. This is not sufficient though for determining the effectiveness of the solution approaches, since the computer architecture and operating systems must also be thoroughly investigated. Tables 10 and 11 indicate that the running times of AMP are reasonable, illustrating its effectiveness in solving large scale RCPSPs.

5. Conclusions and Further Research

This paper presented, for the first time in the relevant literature, an AMP framework for solving the RCPSP. The proposed AMP combined adaptive memory stratagies to effectively guide the search

Table 11. Computational times for 50000 (and more) schedules examined.

Pr. Set	**Alg.**	**Reference**	Dev.	Orig. CPU(s)		PCPUS	Nrm. CPU(s)		No Sch.	CPU
			%	Avg.	Max.		Avg	Max.	(*1000)	GHz.
J30	GA-MBX	Zamani (2013)	0.00	1.3	n/a	1121	0.9	n/a	50	1.86
	AMP	This paper	0.00	0.2	3.7	1610	0.2	3.7	50	2.80
	HEA	Paraskevopoulos *et al.* (2012)	0.00	0.2	2.9	1610	0.2	2.9	50	2.80
	b-GA	Goncalves et al. (2011)	0.01	18.0	n/a	1565	17.4	n/a	50	2.40
	SS-PR	Mobini *et al.* (2009)	0.01	0.8	1.4	1763	0.9	1.5	50	3.00
	SS-PR(fast)	Mobini *et al.* (2009)	0.02	0.7	1.4	1763	0.7	1.5	n/a	3.00
	GAPS	Mendes *et al.* (2009)	0.01	5.0	n/a	340	1.1	n/a	50	1.33
	FF	Ranjbar (2008)	0.00	5.0	5.0	1763	5.5	5.5	n/a	3.00
	DBGA	Debels and Vanhoucke (2007)	0.02	0.5	n/a	998	0.3	n/a	50	1.80
	SS-FBI	Debels *et al.* (2006)	0.01	0.7	1.3	998	0.4	0.8	50	1.80
	PBA	Valls *et al.* (2004)	0.10	1.2	5.5	145	0.1	0.5	n/a	0.40
	VNS	Fleszar and Hindi (2004)	0.01	0.6	5.9	250	0.1	0.9	n/a	1.00
	TS-FBI	Valls *et al.* (2003)	0.06	1.6	6.2	145	0.1	0.6	n/a	0.40
J60	GA-MBX	Zamani (2013)	10.65	2.8	n/a	1121	2.0	n/a	50	1.86
	AMP	This paper	10.65	15.3	93.8	1610	15.3	93.8	50	2.80
	HEA	Paraskevopoulos *et al.* (2012)	10.54	16.3	89.2	1610	16.3	89.2	50	2.80
	b-GA	Goncalves et al. (2011)	10.57	5.3	n/a	1565	5.2	n/a	50	2.40
	SS-PR	Mobini *et al.* (2009)	10.57	2.0	3.0	1763	2.2	3.3	50	3.00
	SS-PR (fast)	Mobini *et al.* (2009)	10.91	1.1	n/a	1763	1.2	n/a	n/a	3.00
	GAPS	Mendes *et al.* (2009)	10.67	20.1	n/a	340	4.2	n/a	50	1.33
	FF	Ranjbar (2008)	10.56	5.0	5.0	1763	5.5	5.5	n/a	3.00
	DBGA	Debels and Vanhoucke (2007)	10.68	1.1	n/a	998	0.7	n/a	50	1.80
	SS-FBI	Debels *et al.* (2006)	10.71	1.9	2.6	998	1.2	1.6	50	1.80
	PBA	Valls *et al.* (2004)	10.89	3.7	22.6	145	0.3	2.0	n/a	0.40
	VNS	Fleszar and Hindi (2004)	10.94	8.9	80.7	250	1.4	12.5	1653	1.00
	TS-FBI	Valls *et al.* (2003)	11.12	2.8	14.6	145	0.2	1.3	n/a	0.40
J120	GA-MBX	Zamani (2013)	31.30	5.8	n/a	1121	4.0	n/a	50	1.86
	AMP	This paper	30.88	118.5	601.0	1610	118.5	601.0	50	2.80
	HEA	Paraskevopoulos *et al.* (2012)	30.78	123.5	551.0	1610	123.5	551.0	50	2.80
	b-GA	Goncalves et al. (2011)	32.76	18.0	n/a	1565	17.4	n/a	50	2.40
	SS-PR	Mobini *et al.* (2009)	31.37	7.1	10.1	1763	7.7	11.1	50	3.00
	SS-PR (fast)	Mobini *et al.* (2009)	32.27	6.2	n/a	1763	6.7	n/a	n/a	3.00
	GAPS	Mendes *et al.* (2009)	31.44	112.5	n/a	340	23.7	n/a	50	1.33
	FF	Ranjbar (2008)	31.42	5.0	5.0	1763	5.5	5.5	n/a	3.00
	DBGA	Debels and Vanhoucke (2007)	30.69	3.0	n/a	998	1.9	n/a	50	1.80
	SS-FBI	Debels *et al.* (2006)	31.57	6.7	9.2	998	4.1	5.7	50	1.80
	PBA	Valls *et al.* (2004)	31.58	59.4	264.0	145	5.4	23.8	n/a	0.40
	VNS	Fleszar and Hindi (2004)	33.10	219.9	1127.0	250	34.1	175.0	10778	1.00
	TS-FBI	Valls *et al.* (2003)	34.53	17.0	43.9	145	1.5	4.0	n/a	0.40

into the solution space, while using a ReVNS as an improvement method. The proposed ReVNS conducts a thorough exploration of the solution neighbourhood and consorts with a perturbation strategy to escape from local optima.

The main goal of this paper was to prove whether the AMP framework, which is quite successful in solving hard combinatorial optimisation problems, is suitable for solving large scale RCPSPs. Computational experiments on well known RCPSP instances showed that AMP produces consistently high quality solutions for all data sets and performance criteria. In particular, AMP outperforms all solution methodologies for large scale RCPSPs under the termination criteria of 1000 and 5000 schedules, while remaining highly competitive when the 50000 schedules are used as a termination criterion. It is worth mentioning also that, ReVNS, a major AMP component, presents a good performance justifying its role as an improvement method.

Challenging the proposed adaptive memory strategies for capturing elite solution elements of other important variants of the classical RCPSP, e.g., multiple modes and/or multiple projects, is a worth pursuing research avenue.

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