Investigating dyscalculia from the lab to the classroom: a science of learning perspective

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Introduction

This chapter sets out to map the interdisciplinary journey from (i) neuroscience findings that illuminate some form of conceptual understanding, to (ii) the analysis of how this might affect the learning process in formal education, to (iii) a pedagogical design for practical application in a classroom. We contextualize this with reference to the phenomenon of dyscalculia, partly because it is a fascinating and difficult problem for education, and partly because its effects on behavior, cognition and the brain can be quite precisely described. For education, psychology and neuroscience to be able to collaborate effectively, we need to select the problems that provide a clear and rigorous account of what it takes to learn, and how the relevant processes and products of learning can be identified and measured for both neural and behavioural markers.

The section on the research programme explains the nature of the neuroscience research program on dyscalculia, and the second outlines some of the main findings. We then analyse what these findings mean for education, and use pedagogic theory and practice to specify the kind of intervention needed. Finally, we describe a particular application, which takes the form of an adaptive game-like teaching program, and the pedagogy that would be built around it in the classroom teaching context.

What SEN teachers know

People with typical numerical processing find it hard to imagine having no ‘number sense’. They will often observe that some very poor learners of arithmetic can count, as they can recite the number sequence, and point to distinct objects as they do it. Around the same stage of development children are also learning to recite another sequence: the alphabet; and as they do so they point to distinct objects. There are obvious similarities. The crucial difference is that while the alphabet is an entirely arbitrary sequence and could be in any order, the number sequence is ordered and has an internal structure. In the number sequence it is meaningful to ask:

what number do you add to 5 to reach 9?

It makes no sense at all to ask:

what letter do you add to K to reach P?

If a child were to interpret the number sequence as being similar to the alphabet sequence, then arithmetic becomes a baffling series of arbitrary rules that has no logic to it. If the processing of numerosity, because of an abnormality in the IPS, does not include seeing 5 as contained within 7, then what could *7 minus 5* possibly mean? It is a little like asking what letter is equivalent to *P minus K* - we would start reciting the alphabet using our fingers as the only route to finding a possible answer.

Learners with *dyscalculia* can usually learn to count in the sense that they can recite the number sequence, and can often put it in one-to-one correspondence with objects, though may not be completely systematic about that. The problem for them, is that the number sequence is cognitively rather like the alphabet. With little intuitive sense of number magnitude, they try to solve all problems by their one reliable method: counting. For example:

- To select a 9 dot card from a pile of cards, they select from cards at random and count dots till they find 9.
- To say which is the larger of two playing cards showing 5 and 8, they count all the symbols on each card, and note that 8 comes after 5.
- To count down from 10: count 1 to 10, then 1 to 9, then 1 to 8, etc.
- To place a playing card of 8 in sequence between a 3 and a 9 they count up spaces between the two to identify where the 8 should be placed.

They do not rely on spatial representations of number to help them, except for finger counting. These procedures make it very hard work to do simple arithmetic. To work out 5 + 7 they have to count up to 5 on their fingers, then count a further 7 to see which finger then end up with, then go back to the start and count up to that finger again to find out what number it is. Remembering number facts can help, but these are essentially meaningless arbitrary facts, so it is like building on sand; they rely so heavily on counting because memory is not reliable and counting is. SEN teachers frequently find that ideas that appear to have been mastered in one lesson are forgotten by the next one, which may be several days away, with nothing in between to reinforce the learning they have achieved.

The research programme on dyscalculia

Dyscalculia: the official position

Poor numeracy is a serious disability. It leads to poor educational, employment, and health lifetime outcomes (Bynner & Parsons, 1997; Parsons & Bynner, 2005). One form of low numeracy is developmental dyscalculia. A major UK government report on *Mental Capital and Wellbeing* summarized the current situation: "Developmental dyscalculia is currently the poor relation of dyslexia, with a much lower public profile. But the consequences of dyscalculia are at least as severe as those for dyslexia." (Beddington et al., 2008).

So what is dyscalculia? The term 'dyscalculia' does not appear[1] in DSM-5 (the researchers' and clinicians' official resource to "diagnose and classify mental disorders" (Association, 2013). In this manual all 'learning disabilities' are

[1] Incidentally, the term 'dyslexia' does not appear either.

unhelpfully defined as 'A persistent difficulty learning *academic skills* for at least 6 months despite intervention targeting the area(s) of difficulty.' The definition comes with a 'Specifier' to make it specific to maths, i.e. a difficulty 'in number sense, fact and calculation, and in mathematical reasoning', and in three degrees of 'Severity', Mild, Moderate or Severe. No norms are offered, and exclusions include: intellectual disabilities, visual or hearing impairments, mental and disorders (e.g. depression, anxiety, etc.), psycho-social difficulty, neurological disorders, and lack of access to adequate instruction.

To identify an individual as having this disability the advice is:

> *'The specific learning disorder is diagnosed through a clinical review of the individual's developmental, medical, educational, and family history, reports of test scores and teacher observations, and responses to academic interventions. The diagnosis requires persistent difficulties in reading, writing, arithmetic, or mathematical reasoning skills during formal years of schooling.' DSM-5 315.1*

The exclusions mean that a child or an adult cannot be identified as dyscalculic if they are depressed, anxious, with a low IQ, or with a sensory impairment. These conditions can cause poor academic skills, but seem to take precedence over other possible causes. It also means that different clinicians can operate with quite different criteria about who should and who should not be classified, and therefore how an intervention strategy should be designed and implemented.

Moreover, the individual should have no 'specific cognitive impairment'. (Would this include a specific impairment in phonological processing for dyslexia?)

The general approach

Our research-based approach is quite different from DSM-5. In the learning sciences, we want to understand *why* a learner is having difficulty in learning arithmetic, since this has to be the basis for an intervention strategy. We do not exclude learning disability (low IQ), or specific cognitive or neural differences, though we would of course, take into account the learning context at home and school (Butterworth, Varma, & Laurillard, 2011).

The general approach is to

- start with the observed behaviour of learners with low attainment in mathematics on a range of tests, from self-report, and from reports by their teachers and parents.
- develop a cognitive hypothesis based on the observations and reports, which may suggest a specific cognitive impairment.
- use cognitive theory about why some learners have difficulty or disability in learning arithmetic to develop a specific neural hypothesis based on the cognitive theory and other behavioural evidence
- plan an intervention strategy based on the cognitive and neural theories, guided by the principles of pedagogic design and the best practice of reflective practitioners
- implement the strategy in the form of guidance for teachers, parents and learners, and for relevant professionals, including educational and clinical psychologists.

- implement the strategy in the form of software that can help identify sufferers, and distinguish them from learners with low numeracy for other reasons than dyscalculia
- implement the strategy in the form of digital games that can support learners in both formal and informal settings, including individual learning away from teachers and parents
- evaluate the intervention against changes in behaviour, following intervention, and modified and retested against further changes in behaviour, and ultimately against predicted changes in neural activity and structure.

The general approach is illustrated in Figure 1, showing how the two disciplines are interwoven in an iterative series of research activities.

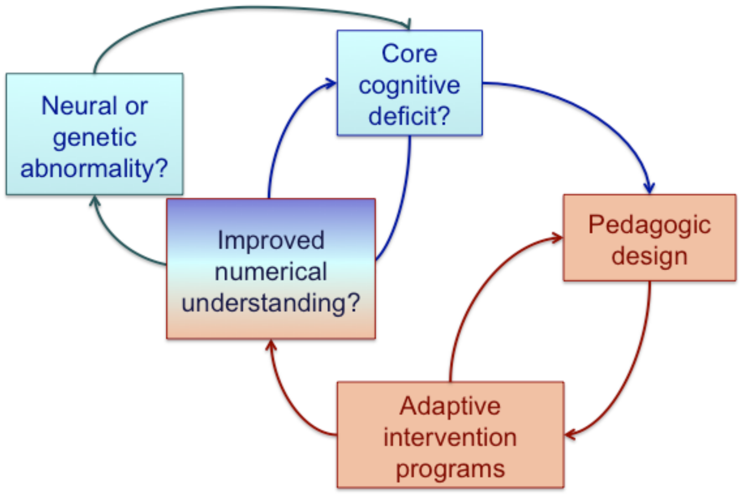


Figure 1. An iterative integration of neural and genetic factors with the development and testing of educational interventions

Investigating dyscalculia

In our first study of 9 year olds, we asked teachers to identify the children who, behaviourally, were really bad at arithmetic, but seemed to achieve normally in other subjects (Landerl, Bevan, & Butterworth, 2004). Those identified were formally tested with item-timed arithmetic, since response times to answer a question like "What is 3 plus 8?" can be diagnostic. For example, using a counting strategy to solve this problem is slow, especially if the child is 'counting all' ('one, two, three; one, two, three, four, five, six, seven, eight; one, two, three, four, five, six, seven, eight, nine, ten, eleven'). We have found in a series of studies that the development of arithmetical skills in children is reliably predicted by the time it takes to do very simple numerical tasks such as timed enumeration of dot displays up to 9 dots, and selecting the larger of two digits (Butterworth, 2003; Landerl et al., 2004; Reeve, Reynolds, Humberstone, & Butterworth, 2012; Reigosa-Crespo et al., 2012).

Of these children, we selected those who were 3SDs worse than matched controls. There was no special reason for this criterion, except to ensure that these children were really bad at arithmetic. These we provisionally termed 'dyscalculic'. We constructed four groups who were matched on language ability, IQ and short-term memory span[2]: (i) dyscalculics, (ii) dyslexics, (iii) a double deficit group who were both dyscalculic and dyslexic, and (iv) a control group matched for age, gender and classroom. We included learners who were dyslexic because many special needs teachers told us that dyslexics had trouble with maths, though for this group we selected only those who were not bad at arithmetic.

It turned out to be easy to construct these groups, which in itself showed that differences in IQ, language and short-term memory are not sufficient to cause dyscalculia. So what did make the difference? Our cognitive hypothesis was that the dyscalculics differed in something very simple, which must be domain-specific, and that would prove to be foundational for learning arithmetic. We already knew that maths abilities had a heritable component from twin studies (Alarcon, Defries, Gillis Light, & Pennington, 1997) and from studies of genetic anomalies, such as Turner Syndrome (Butterworth et al., 1999; Rovet, Szekely, & Hockenberry, 1994; Temple & Marriott, 1998), and that infants even in the first weeks of life could make numerical discriminations (Antell & Keating, 1983; Starkey & Cooper, 1980). Therefore we tested these four groups on tasks that seemed likely to capture differences in innate abilities, affected as little as possible by education. These tasks were: (i) timed enumeration of dot displays up to 9 dots, and (ii) magnitude comparison (selecting the larger of two digits). It turned out that the dyscalculic group and the double deficit group were significantly worse than controls on these measures, but the dyslexic group was not.

These findings supported our cognitive hypothesis that the dyscalculics at 9 years had a ‘core deficit’ in an inherited domain-specific mechanism that could be identified by simple tests of timed enumeration of sets of objects and by timed magnitude comparison (Butterworth, 2005). It also guided the development of a software product that a teacher or other professional could use to help identify the dyscalculics and differentiate them from other children equally bad on a standardized test of timed simple arithmetic (Butterworth, 2003). Other studies used a somewhat different characterization of the domain-specific mechanism, but also used a very simple test that depended relatively little on education: ‘numerical acuity’, a two-alternative forced-choice task to select the display with more dots. It turned out that children who were bad at this were also bad at learning arithmetic (Piazza et al., 2010). However, training (Dewind & Brannon, 2012) and education do seem to play a role even in this simple task (Piazza, Pica, Izard, Spelke, & Dehaene, 2013).

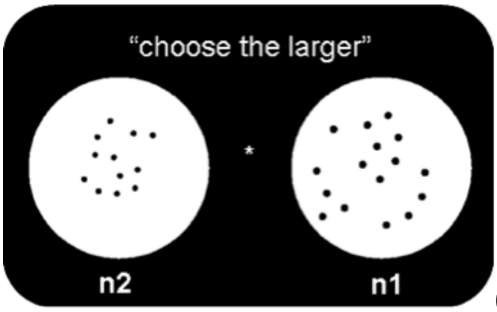
The early results gave us confidence to carry out a longitudinal study based on the cognitive hypothesis of a core deficit in this mechanism (Reeve et al., 2012). Here we tested timed Dot Enumeration (DE) along with other numerical and cognitive tests on seven occasions between kindergarten and 11 years. DE is a very simple test: the learner says the number of dots in a visual array as quickly as possible. Using cluster analysis based on four parameters of the number of dots against RT (slope of the subitizing[3] range, slope of the counting range, the point of discontinuity where the slope changes, and the overall average RT), we identified three clusters at each testing. Children tended to stay in the same cluster throughout the testing, suggesting

[2] Apart from the dyslexic groups who had a span of about one item fewer than the controls

[3] ‘Subitizing’ refers to the rapid, accurate, and confident judgments of number performed for small numbers of items. It is claimed that up to about 4 items (there are individual differences, including age differences) counting is not necessary to for accuracy, while for larger numbers of items – the ‘countng range’ - counting is needed for accuracy.

that this is a stable measure of individual differences. The slowest cluster at kindergarten, about 7% of the total of 159 children, also were way behind their peers in the other two clusters at each testing in their accuracy of age-appropriate arithmetic, from single-digit addition at 6 years, to three-digit subtraction, multiplication and division at 10 years.

At the same time, several other teams had identified very simple cognitive markers of dyscalculia using our method or similar methods. For example, (Piazza et al., 2010) found children and adults differed in how well they could tell which of two clouds of dots had more dots. See Figure 2.



(a)

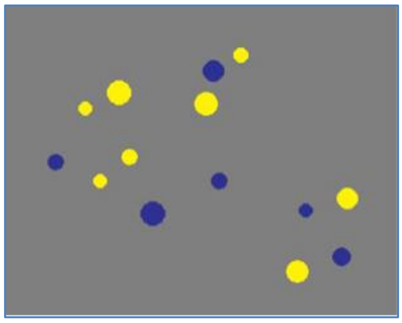

(b)

Figure 2. Two examples of the 'clouds of dots' task. 2a shows the method used by (Piazza et al., 2010). 2b shows the method used by (Halberda, Mazzocco, & Feigenson, 2008), where the task is to say whether there are more blue or more yellow dots.

Accuracy on this task depends on the difference between the numerosities of the clouds: that is, the more different they are the higher the accuracy, and also the faster the response. This is called the 'distance effect'. On this task, 10 year old dyscalculics performed at the level of typically developing 5 year olds. That is, the distance effect was different for the dyscalculics: they need a bigger difference to be reliably accurate. More generally, individual differences on this task correlated with performance on arithmetic tasks (Halberda et al., 2008; Mazzocco, Feigenson, & Halberda, 2011). Several other studies pointed to differences on very simple numerical tasks, such as deciding whether two squares match the digit 2, or placing the number 7 on a line with the ends marked 0 and 10 (e.g. Geary et al., 2009). In fact a tool for screening for dyscalculia and differentiating from non-dyscalculic causes of poor arithmetic was based on measures of this kind (Butterworth, 2003), and a large-scale prevalence study used Dot Enumeration as the criterion from distinguishing dyscalculia from 'calculation dysfluency' (Reigosa-Crespo et al., 2012).

These studies suggested that it may be possible to find differences in brain activity when dyscalculics carry out these simple tasks. Several studies have established small regions in the parietal lobes, the left and right 'intraparietal sulci', (IPS) are reliably activated when children or adults compare the numerosities of sets. In fact, activity in these regions show the distance effect. That is, the more similar the numerosities of the sets to be compared, the greater the activation (Castelli, Glaser, & Butterworth, 2006; Pinel, Dehaene, Rivière, & Le Bihan, 2001). A study of dyscalculic 12 year olds revealed that these children did not show the distance effect in the right parietal lobe, while matched controls did (Price, Holloway, Räsänen, Vesterinen, & Ansari, 2007). It is worth noting that although these tasks activate both left and right parietal lobes in adults, the balance of activation changes with age so that children tend to show more activation in the right, while adult show more activation in the left (Cantlon, Brannon, Carter, & Pelphrey, 2006).

The structure of the dyscalculic brain in these regions is also different from typically developing individuals. In adolescents, there is lower grey-matter density in the left IPS of dyscalculics compared with controls (Isaacs, Edmonds, Lucas, & Gadian, 2001), while in younger children there is lower grey matter density in the right IPS (Rotzer et al., 2008). White matter, the tracts that connect different grey matter regions, is also different in children with low numeracy (Rykhlevskaia, Uddin, Kondos, & Menon, 2009). From the ages of 8 to 14 years in typically developing children, white matter volume in several tracts increases with age, showing that the distant regions are becoming better connected. However, in dyscalculics this does not seem to happen (though it may happen later: Ranpura et al., 2013). There also seem to be differences in the white matter connections between the frontal lobes – the region that supports reasoning – and the hippocampus – the structure that supports long-term memory (see Moeller, Willmes, & Klein, 2015 for a recent review).

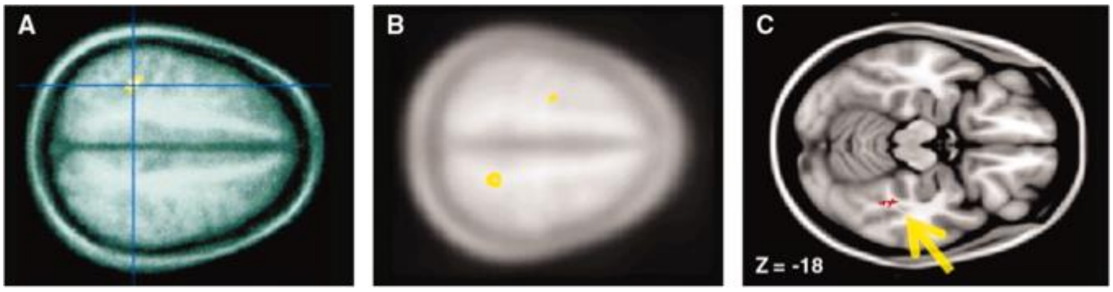


Figure 3 Structural abnormalities in young dyscalculic brains suggesting the critical role for the IPS. (Reproduced from (Butterworth et al., 2011))

Figure 3 shows regions where the dyscalculic brain is different from that of typically developing controls. Both left and right IPS are implicated, possibly with a greater impairment for left IPS in older learners. (A) There is a small region of reduced gray-matter density in left IPS in adolescent dyscalculics (Isaacs et al., 2001) . (B) There is right IPS reduced gray-matter density (yellow area) in 9-year-olds dyscalculics (Rotzer et al., 2008). (C) There is reduced probability of connections from right fusiform gyrus to other parts of the brain, including the parietal lobes (Rykhlevskaia et al., 2009).

What this means for learning

We know that training on novel maths tasks changes the activity of brains in adults, but there has to date been very little work on the effects of training on the brains of dyscalculics (but see Kucian et al., 2011). One of the aims of our work is indeed to test the effects of training on dyscalculic brains. Training could and should improve performance on the trained task, and also on transfer tasks, but this in itself does not clarify whether the training makes the pattern of activity more normal (as it does in dyslexia training: Eden et al., 2004), or whether alternative networks are created to carry out the task. What is primary for the teacher and the learner is whether performance improves, but for the science of learning it is important to determine whether improvement comes about by making the learner better at using typical strategies or by recruiting compensatory strategies.

As we mentioned above, one important factor in dyscalculia is genetics. This is not to say that all dyscalculics have inherited the condition. Neural abnormalities can be due to many other causes, such as prematurity, perinatal trauma, fetal alcohol syndrome, and so on. In fact, twin studies show that the effects of non-shared environment – the experiences of one of the twins but not the other – can be as important as genetics (see Butterworth & Kovas, 2013 for a review).

In Figure 4, we summarise what is currently known about the causal basis of dyscalculia: the cognitive activities affected by the biology of regions of the brain, the arithmetic activities affected by cognition, and the educational contexts in which these are addressed.

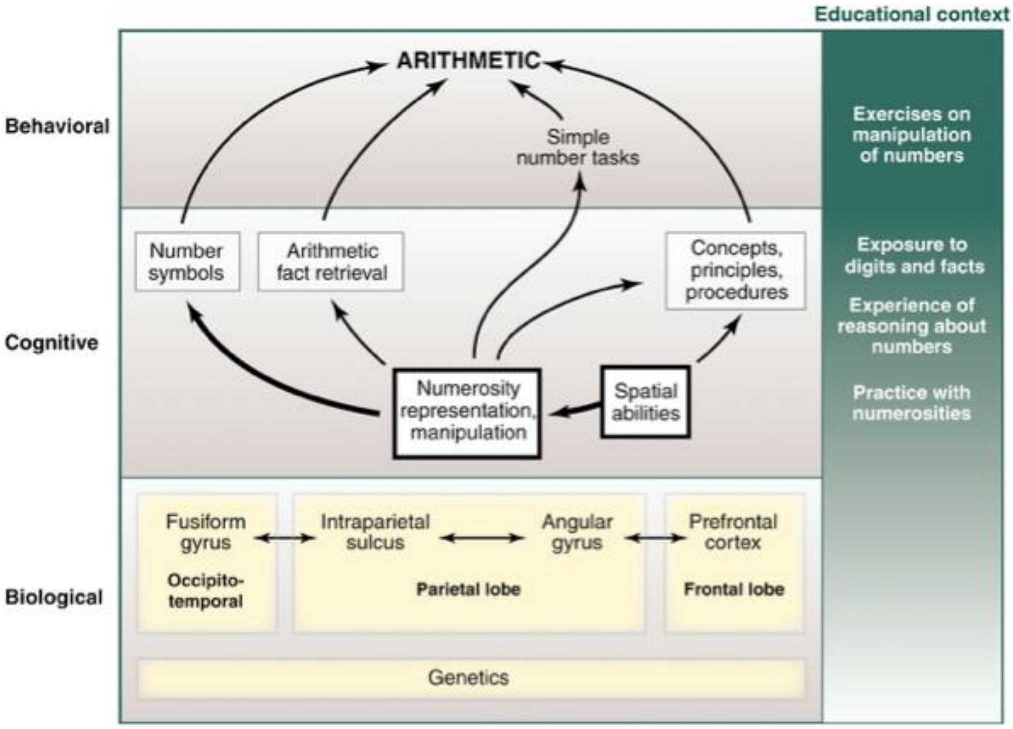


Figure 4. Causal model of possible inter-relations between educational context with biological, cognitive, and behavioral functions . (Reproduced from Butterworth et al., 2011)

If parietal areas, especially the IPS, fail to develop normally, there may be an impairment at the cognitive level in numerosity representation and consequential impairments for other relevant cognitive systems revealed in behavioral abnormalities. The link between the occipitotemporal and parietal cortex is required for mapping number symbols (digits and number words) to numerosity representations. Prefrontal cortex supports learning new facts and procedures. The multiple levels of the theory suggest the instructional interventions on which educational scientists should focus.

To summarise, the basic science reveals a core deficit in numerosity processing specific to dyscalculia. That is to say, that dyscalculics have a deficit in their 'number sense'. In the next section, we show how this can be a target for intervention.

From research to education

To help SEN learners with mastery of basic number concepts, teachers use intensive practice with materials-based manipulation activities(Anning & Edwards, 1999), focused on the core concepts of number, as this can help to bring dyscalculic learners closer to the norm (Butterworth & Yeo, 2004). To help with retention they ask their learners to talk about what they are doing, and when they have done it successfully to describe what they did, and why it was right (Butterworth & Yeo, 2004) . These methods take time, and have no place in the classroom for typically developing learners. SEN teachers are very aware that there is no point in forcing the age-stage link to the curriculum. These learners must be allowed to take their time, and build up the concepts that typically developing learners have been building throughout the pre-school years (Yeo, 2003).

From diagnosis to intervention

The neuroscience identifies the origin of dyscalculia, and explains it as a congenital neural difference that amounts to 'a deficit in number sense', and from this we can infer that an educational intervention must attempt to build the automatized connections between digit, numerosity and length that a typically developing (TD) learner has. It does not yet tell us how those connections were built. All we know, from behavioural studies with infants, is that there is an innate basis to grasp of numerosity, and, from studies of dyscalculics, that it is heritable (Butterworth, 2010). We do not yet know how the perception of numerosity builds up to the number sense, i.e. that 5 is contained within 7, or that 3 + 2 = 2 + 3. To develop an intervention likely to assist dyscalculics , therefore, we turn to a combination of (i) building on what SEN teachers know and know to be successful (Bird, 2007; Butterworth & Yeo, 2004), and (ii) making use of techniques that provide the immense number of transactions it takes to build the neural connections to draw an efficient connection between the concepts of digit, numerosity, and length that make possible the representations and processing that arithmetic requires (Butterworth et al., 2011).

Our fundamental pedagogic aim, therefore, is to strengthen numerosity processing. We argue that the optimal approach is to create game-like digital environments that scaffold the learner's development of number sense.

The advantage of game-like digital environments is their highly engaging quality. Dyscalculic learners are able to focus and maintain time on task for a long time and hence practice far more examples than they could with a teacher, even one-to-one (Butterworth & Laurillard, 2010). They offer the best way of enabling a learner to process a very large number of numerical transactions over short enough periods of time that they are likely to learn something about them, and to remember what they learn.

A second advantage of using a digital environment is that it can support independent learning by providing the feedback and therefore supporting learning beyond the classroom. In the long gaps between one lesson and another at school it is very easy to forget the ideas mastered, but the personal mobile device, with an enticing game, that is satisfying to play, is an always-present personal tutor – if it is well designed.

The pedagogical principles

The spread of tablets and personal mobile devices for children has led to the development of a great many game-like apps for basic maths. They are certainly engaging because they do have the motivational qualities of games, but they lack any game-play design based on pedagogic principles. The predominant form of feedback is the right/wrong of multiple-choice questions, or their equivalent. This is the instructionist design that requires an external judge of what the learner has done.

The alternative pedagogy is ‘constructionist’, which enables ‘learning without being taught’ (Butterworth & Laurillard, 2010; diSessa, 2001; Healy & Kynigos, 2010; Noss, Healy, & Hoyles, 1997; Papert, 1980). The nature of the task, and therefore the learning process, is closer to learning in the world, because the digital format creates a task environment in which the goal is shared (in the same sense in which a game goal is shared), the actions are within the learner’s repertoire of possible actions, and the feedback is intrinsic, i.e. the environment changes according to their action. There is no extrinsic judgmental feedback. The learner is situated within an environment that affords learning because they are able to self–correct, having seen the effect of their action in relation to the goal. For example, a child learning to use a spoon aims for the yogurt pot, but using the wrong angle knocks it over: they are very aware of having to aim more carefully. They do not need to be told they missed, so the processing of that action is more likely to integrate the form of the action with its result than if they had been blindfolded and simply told ‘wrong, try again’. In this case there would be no informational feedback to supply the link between action, goal and improved action. The aim of a constructionist design is to emulate the nature of encounters with the world, which affords learning.

A second pedagogical principle is to adapt to the learner’s current level, where the ‘zone of proximal development’ (Vygotsky, 1978) is maintained, not by a more competent individual, but by an algorithm that tracks current performance and decides whether to increase the difficulty level or not. In this way the tasks remain challenging, the learner is motivated to move through different levels of the game, and so build the concept (Mariotti, 2009).

These two principles were used in the design of a game tested with typically developing learners, age 5-7 .

From successful SEN teachers and the basic principles of good pedagogy, therefore, we have the basis to specify the pedagogical features of the digital learning environment, i.e. that it must:

- provide intensive learning
- set familiar materials-based tasks
- sequence tasks and stages to build a concept of numerosity
- adapt difficulty to the performance of the learner
- ask the learner to construct answers to achieve a goal
- give meaningful feedback that enables the learner to self-correct.

The contrast between instructionist and constructionist approaches is captured in the Conversational Framework account of learning in the context of formal education (Laurillard, 2012). It models learning in terms of two kinds of interaction, communication, and practice. The teacher communicates ideas through language and representation, enabling the learner to learn through acquisition of concepts by listening or reading and through discussion (Frith, 2007), as in Figure 5(a). The teacher also sets up a learning practice modelling environment in which the learner puts their developing concepts into practice in order to achieve some defined goal, and through which those concepts are modified as a result of feedback on actions in relation to the goal. The feedback may be extrinsic in the practice cycle, from the teacher or from a computer program evaluating the learner's actions, as in Figure 5(b). Or the feedback may be intrinsic in the modelling cycle, from the world or from a computer model giving informational feedback on the learner's actions, as in Figure 5(c).

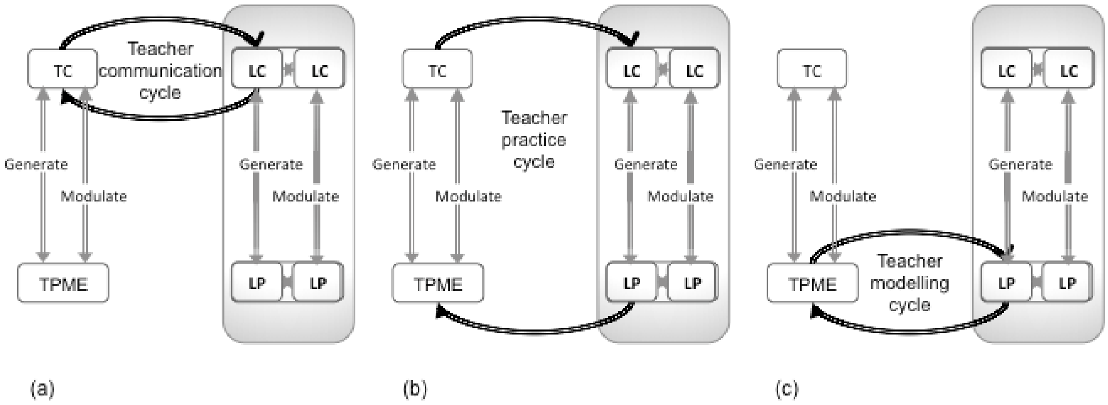


Figure 5: The learner learning concepts (LC) and practice (LP) through interaction in (a) the teacher communication cycle, (b) the teacher practice cycle, and (c) the teacher modelling cycle, from the teacher's conceptual organisation (TC), and the practice modelling environment set up by the teacher (TPME).

The Conversational Framework represents the contrast between instructionist and constructionist pedagogies in terms of the contrast between extrinsic feedback in the practice cycle and intrinsic feedback in the modelling cycle.

Figure 6(a) shows the *instructionist* version of the goal-action-feedback-revised action sequence, where the teacher's evaluation or guidance can be put into practice without necessarily engaging the learner's conceptual processing, inevitably so if the feedback does not make explicit the connection between the goal, the action, and what is needed to improve the action, resulting in a trial-and-error response.

Figure 6(b) shows the *constructionist* version of the sequence, where the feedback on action comes from the world, or from the program representing the result of the action. In this case, there is an explicit relationship between the goal, the action and what the action achieved in comparison with the goal. The learner may make a random trial-and-error attempt at revising their action, but also has the information they need to inform their own decision on how to revise it, without recourse to teacher guidance.

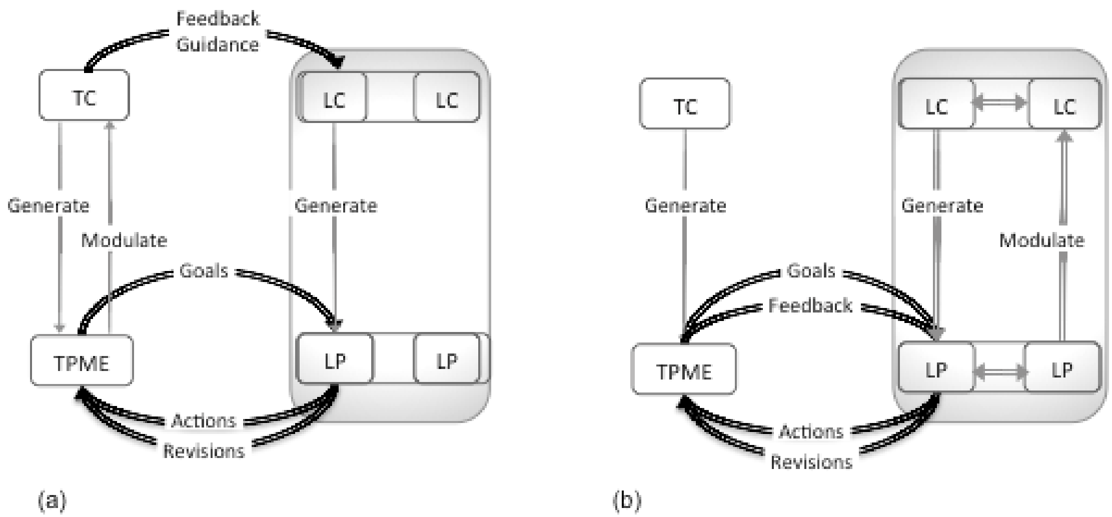


Figure 6: (a) The teacher reflects on learner's action to provide extrinsic feedback, (b) The practice modelling environment provides intrinsic feedback, prompting learner reflection

Engaging the learner in making a connection between their practice and their concepts is essential if they are to build a meaningful relationship between the two, and so develop a full conceptual understanding of the task set. This is what we aim for in designing a game-like digital environment for learning number concepts.

Design for a number concepts game

Several digital games, based on these pedagogic principles and aiming to develop aspects of number, have been tested with small numbers of learners (Butterworth & Laurillard, 2010; Butterworth et al., 2011). The new Science of Learning Research Centre[4] provides an opportunity to test this approach with larger learner groups, comparing typically developing with dyscalculic learners. With its focus on the relationship between education, cognition and neuroscience it also enables the investigation of both behavioural and neural responses to this constructionist type of game design, in comparison with the typical educational app taking an instructionist or simple testing approach.

To integrate number concepts with the basics of arithmetic manipulation, the aim of a new game is to enable dyscalculic learners to develop a sense of the meaning of numbers, not as an arbitrary sequence, but as a structure, which can be combined and split to make other numbers, and therefore represent a meaningfully ordered sequence. With this conceptual basis, the manipulations of addition and subtraction become meaningful.

The design of the 'Sets Game' is as follows

- The screen displays sets of discrete objects, clustered in sets of different numerosities
- The objects are initially colour-coded according to the set size
- Two tools are always available, one for combining, one for splitting sets

[4] Based at the University of Queensland and the University of Melbourne, and funded by the Australian Research Council

- The goal is to combine or split the sets to match the target set displayed at the top of the screen, until all sets are matched
- Later levels of difficulty use larger sets, digits on the right-hand object to denote the cardinality of its set, no colours, and finally remove objects to leave just digits on the screen
- When sets are combined or split the resulting set changes colour, or changes rightmost digit to denote the new numerosity

Figures 7.1 to 7.3 show how the game play looks to the learner.

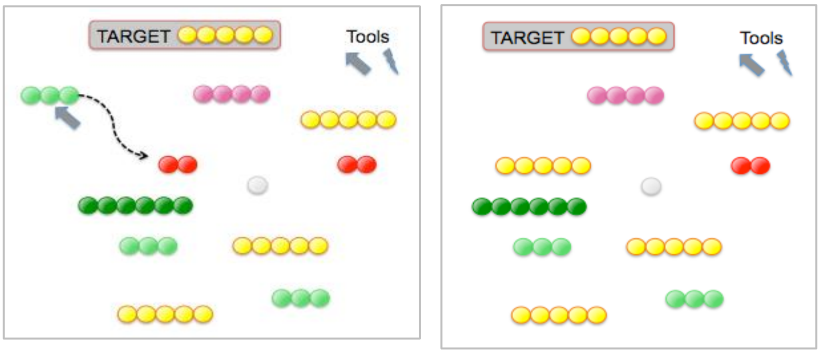


Figure 7.1: the learner moves the 3 group to combine with the 2 group, which makes a target 5 group and changes colour to match its size

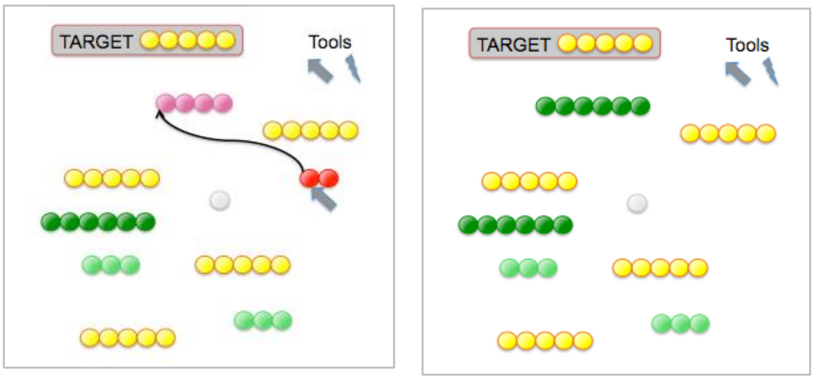


Figure 7.2: the learner moves the 2 group to combine with the 4 group, which makes a 6 group and changes colour to match its size

Figure 7.3 the learner now has to split the 6 group at the right place to make a target 5 group with an extra 1, both of which change colour to match their size

The total number of objects on the screen is programmed to be a multiple of the target number, to ensure that the task can be completed.

At the final level the learner is combining and splitting a screen full of just digits, and is then ready to move on to the formal representation of these manipulations of addition and subtraction.

The game has the concreteness of materials-based manipulations, but the significant advantage that it can represent the change of numerosity by the change in colour, as well as other rewards, such as a small animated movement of the set, and sounds. There is nothing else visual, to avoid distraction; other visual rewards come at the end of each game, along with the score and, with sufficient score, a new level.

There is never any negative evaluation of the learner. They can tell for themselves when they have matched the target, and can repeat the manipulations as often as they wish – even playing with irrelevant manipulations without penalty. Games of this kind appear to be sufficiently rewarding that the learner remains on task for long continuous periods, and that does help to establish the connections they need to make between the nature of the goal and their action to achieve it.

Using feedback

The further experimentation on games of this type should replicate established findings on feedback, that:

- Feedback is effective if it directs information to more effective self-regulation, so that students invest more effort or commitment in the task (Kluger & DeNisi, 1996)
- Teachers can help by creating a learning environment which emphasises self-monitoring and self-regulation to enhance learning (Hattie, Biggs, & Purdie, 1996).
- Feedback about the task is powerful when the task information can be used for improving strategy processing (Hattie & Timperley, 2007).

However, as Hattie and Timperley have pointed out (p91):

> *"too much feedback at the task level can lead to trial-and-error strategies and less cognitive effort to develop informal hypotheses about the relationship between the instructions, the feedback and the intended learning" .*

In an adaptive digital game the feedback is entirely focused on the task level, so it is important to investigate this aspect. However, as the nature of the task is conceptual, and the goal can only be achieved by using cognitive effort to clarify the relationship between goal and feedback, it should be the case that, under these conditions, task-level feedback could achieve the intended learning.

Application

Dyscalculia has been found to occur due a specific and often severe deficit in the capacity to mentally represent the number of objects in a set – its 'numerosity'. This means that learning the connection between the representation of numerosities and the words and symbols for them is difficult for dyscalculic learners. Remembering facts based on representations of numerosity, as typically developing learners can – such as single digit additions and multiplication tables – is also difficult for them. Therefore, effective remediation must strengthen their representations of numerosities, link them to counting words and numerals, and help them make sense of the meaning of numbers in terms of their internal structures and the relations between them.

As outlined above, we created a digital game that addresses all these requirements.

The digital game outlined in this chapter can be used in the learner's own time, because the task adaptation and meaningful intrinsic feedback can support independent learning. However, this gives the learner no opportunity to discuss and articulate the mathematical concepts and relations. So it is important to integrate independent, technology-based learning with the classroom teaching, where learners manipulate concrete materials, work in pairs or groups, and have plenty of opportunity to articulate what they have done, as the expert SEN teachers do. A learning design for one such session is proposed here, based on the designs used by these teachers (Butterworth and Yeo, 2004).

The design has been implemented in the Learning Designer tool[5], and exported to a word document, as reproduced in Appendix 1. The design shows a plan for a 50-minute classroom session leading to individual learner use of the Sets Game described above, to be followed by a class discussion in the next session a few days later.

The design can be found on the Learning Designer website in the Browser screen, from where it can be loaded into the Designer screen for editing.

[5] http://learningdesigner.org - this online design tool is free and open to anyone who registers.

Appendix 1: Learning Design for: Developing number sense

Context

Topic: Set combinations

Total learning time: 180

Number of students: 10

Description: This is a design for a week's worth of teaching and learning for a student who attends class, and has access to a personal mobile device for learning beyond the classroom. It assumes learners have learned to count using 1-1 correspondence. Based on Butterworth and Yeo, *Dyscalculia Guidance* (2004), nferNelson, pages 55-59.

Aim

To develop a sense of the way numbers can combine and split to make other numbers

Outcome

Construct (Application): Able to construct a target number from combining or splitting different sets of numerosities.

Teaching-Learning activities

Build sets of 4 from sets of 1, 2 and 3

Read Watch Listen *2 minutes* *10 students* *Tutor is available*
Watch how I can make up a group of 4 counters. I have a group of 2 here, and another group of 2 here. Count each group. Now I bring them together.
How many are in this group?
From this one I'm adding 3, 4, so I now have a group of 4.
I added a group of 2 to a group of 2 to make a group of 4.

Practice *2 minutes* *1 students* *Tutor is available*
Now take counters from the pile to make a group of 2 and another group of 2. Put them together and tell me what you have made.
Use the counters to make groups of 2, then build a group of 4 from 2 groups of 2.

Read Watch Listen *2 minutes* *10 students* *Tutor is available*
Watch how I can do this with lots of groups, and make them all into groups of 4. [Use several sets of 1, 2, and 3] - I can combine a 1 and a 1 to make a 2, and now I can add another 2 group to make a 4 group.
Here’s a group of 3. I add a 1 to that to make 4, and so on till I have just groups of 4.

Practice *3 minutes* *2 students* *Tutor is available*
Work in pairs and take it in turns to combine these groups so that you make just groups of 4. Each time you make one, describe how you did it - "I added a group of 1 to a group of 3 to make a group of 4".

Produce 3 minutes 2 students Tutor is available
Explain what you did to your partner. Is your partner's explanation correct?

Combine and split sets of 1 to 6 to make sets of 3

Read Watch Listen 3 minutes 10 students Tutor is available
If I have a group of 6 and want to make a group of 3, what should I do? [Demonstrate splitting the group of 6 into two groups of].
How am I going to make this group of 5 into a group of 3? I can split it to make a group of 3 and now I have a group of 2 as well. "I took 2 away from 5 to make 3" How can I make that into a group of 3 - if I add it to this 1. Here you have several groups of different numbers.
Can you bring them together, or split them up until you have just groups of 3?

Practice 5 minutes 2 students Tutor is available
Work in pairs and take it in turns to combine and split these groups so that you make just groups of 3.
Each time you make one, describe how you did it - "I added a group of 2 to a group of 1 to make a group of 3", "I took 3 away from 6 to make 3", and so on.

Repeat the same design: combine and split sets of 1 to 10 to make sets of 2 to 9

Produce 5 minutes 1 students Tutor is available
Split the pile of counters into small groups of different size for their partner.

Practice 10 minutes 1 students Tutor is available
Each learner rolls a dice to decide which number they are aiming to make, take a pile of the counters and make them into groups of the target number by combining and splitting them.

Do the same exercise with the Sets game on a tablet

Read Watch Listen 5 minutes 10 students Tutor is available
In this game you have to make all the groups on the screen into the same as the one at the top. You can use the combine and split tools to combine and split the groups. When you've matched all the groups on the screen you move on to the next level.

Practice 10 minutes 1 students Tutor is available
Now work through Level 1 and see if you can get to Level 3 in 10 minutes.

Working through levels in the Sets Game

Practice 120 minutes 1 students Tutor is not available
Work individually to complete each successive Level in the game. [They should use it for 3 sessions of 20 minutes each day before the next classroom session.]

Discuss 10 minutes 10 students Tutor is available
In class, discuss which Levels in the Sets game were easy or difficult and why. Learners should each describe what they did in the last game they used.

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