Back to Parmenides[*]

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Abstract

After a brief introduction to issues that plague the realization of a theory of quantum gravity, I suggest that the main one concerns defining superpositions of causal structures. This leads me to a distinction between time and space, to a further degree than that present in the canonical approach to general relativity. With this distinction, one can make sense of superpositions as interference between alternative paths in the relational configuration space of the entire Universe. But the full use of relationalism brings us to a timeless picture of Nature, as it does in the canonical approach (which culminates in the Wheeler-DeWitt equation). After a discussion of Parmenides and the Eleatics' rejection of time, I show that there is middle ground between their view of absolute timelessness and a view of physics taking place in timeless configuration space. In this middle ground, even though change does not fundamentally exist, the illusion of change can be recovered in a way not permitted by Parmenides. It is recovered through a particular density distribution over configuration space which gives rise to 'records'. Incidentally, this distribution seems to have the potential to dissolve further aspects of the measurement problem that can still be argued to haunt the application of decoherence to Many-Worlds quantum mechanics. I end with a discussion indicating that the conflict between the conclusions of this paper and our view of the continuity of the self may still intuitively bother us. Nonetheless, those conclusions should be no more challenging to our intuition than Derek Parfit's thought experiments on the subject.

1 Quantum gravity implications for spacetime.

The non-technical summary of my argument

Quantum mechanics arose in the 1920's. General relativity has been around since the 1910's. And yet, we still have no quantum theory of gravity. What is taking us so long? I believe that the most challenging obstacle in our way is understanding what it means to superpose causal structures. I also believe that we can overcome this obstacle only if we accept a fundamental distinction between time and space. The distinction is timid in general relativity – even in its ADM form [1] – and here we want to push it further. In this spirit, here I will consider space to be fundamental and time to be a derived concept – a concept at which we arrive from change (a loose quote Ernst Mach).

Is this different perspective in conflict with what we have learned from general relativity? Not necessarily. For instance, Lorentz invariance can be a property recovered for *classical* solutions of the field equations [2]. From this point of view Einstein's unification of space and time would be a property of solutions of the equations of motion rather than a foundational principle of the theory. I don't want to flout the laws of relativity, only delegate their enforcement.

In fact, in the Hamiltonian framework of general relativity [1], relativity of simultaneity – refoliation invariance – is only recovered on-shell. I.e. it only has a Hamiltonian representation if we restrict our attention to space-times satisfying the equations of motion [3]. This is the first hint that by imposing space-time covariance quantum mechanically we might be over-stepping the principle's jurisdiction.

Following the ideas of Mach further, we find that general relativity falls short of his relational demands on Nature. According to Mach, the properties of objects can have no intrinsic meaning – it is only by comparison that we obtain objective knowledge. For example, in Newtonian mechanics, only relative positions of particles should enter our description of the Universe. There is no objective meaning in the

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absolute location of a particle, only in its relation to all the other constituents of the Universe [**?**]. In general relativity, even in vacuum the proper time of spacetime curves sets a scale, irrespective of any ruler to gauge their lengths. In more mathematical jargon, the spacetime length of (non-null) curves are not generally invariant under conformal transformations of the spacetime metric.[1]

If we accept the ontological split of spacetime into space and time, we gain access to a different kind of symmetry: the ones that act on each spatial field configuration. On the other hand, symmetries that had previously acted naturally on spacetime lose their simplicity and privileged status.

Focusing on space and its symmetries rather than spacetime, we can demand complete relationalism, to the satisfaction of Mach's demands. Then we find that there are two types of symmetries – of scale and of position – implied by spatial relationalism. Thus both the local scale and the spatial position are to be defined relationally – that is without ever referring to any absolute frame of reference.

These two types of transformation act 'intrinsically' on each spatial metric configuration. By 'intrinsic', I mean that their action on a configuration should only depend on the configuration itself, not on other configurations or on tangent vectors, which is why I call it a 'law of the instant'. Symmetries compatible with a timeless transition amplitude should be 'laws of the instant'. Fortuitously, the two relational symmetries mentioned above are shown to be the most general 'laws of the instant'. Thus, physical configuration space of a timeless theory becomes the quotient of configuration space wrt relational symmetries. In the case of gravity, this quotient is called conformal superspace. A curve in conformal superspace describes changes of scale-free geometry – or shapes – as it goes along.

On the other hand, symmetries that are inherited from a spacetime picture need not be laws of the instant. For example, the usual symmetry of refoliations is a remnant of spacetime diffeomorphisms. It is not a 'law of the instant' and is thus disallowed by spatial relationalism as I define it here.[2]

Once spacetime is nothing but the relational change of space, superposing geometries becomes a more straightforward problem to tackle. Quantum mechanical superpositions of three-geometries are realized through interfering paths in physical configuration space – the space of all relationally equivalent three-geometries.

This is a counter-intuitive view on superpositions. We instinctively believe quantum subsystems exist *within* a spacetime, together with ourselves as observers of these subsystems. We naturally expect therefore to see superpositions of macroscopic objects. But if instead we experience only instantaneous configurations, we should expect something different.

Interference, not superposition. Let me explain what is it that we should expect in the relational context of a closed Universe. Each *possible* 'spacetime' (not necessarily of the generally-covariant sort) is identified with a curve in physical configuration space (configuration space modulo relational symmetries). In this context, the timeless path integral formulation of gravity resembles the usual timeless path integral of quantum mechanics.

This analogy allows us to borrow from consistent histories [5] their notion of decoherence, or lack thereof – namely, interference. In the path integral formulation of consistent histories, one measures superposition through the use of an operator called the 'decoherence functional'. Given an 'in' and an 'out' configuration, q_1, q_2, and a partitioning of paths between q_1 and q_2 (called a *coarse-graining of the paths*), the decoherence functional gives a measure of interference between the elements of the partition.

With a preferred notion of space, and the accompanying preferred choice of relational field-configuration space, there is no need to dig so deep into the meaning of 'a superposition of spatial metrics'. Instantaneous configurations are experienced more, or less, frequently, according to the measure of the region around them. It is these frequencies which signal underlying interference patterns.

Timelessness The ideas developed here have ancient roots. They are a modern version of the classical debate between Parmenides and Heraclitus. Heraclitus held that nothing in this world is constant except change and becoming. Parmenides on the other hand believed that the ontic changes we experience with our senses are deceptive; behind the veil of our perceptions lays the changeless true nature of the World.

A much more recent incarnation of the Parmenidean view is found in the work of Julian Barbour (see [6]), which I use here as a nearer harbor from which to sail forth my arguments. Barbour observes that

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[1] The lengths of *null* curves are indeed invariant under conformal transformations. But even without matter the Einstein-Hilbert action is not explicitly conformally invariant. Upon a conformal transformation the action gains a kinetic term for the conformal factor.

[2] It is not enough to note that refoliations act differently on phase space than spatial diffeomorphisms (e.g. act as a groupoid vs as a group). Both transformations have infinitesimal actions on phase space points after all, and, apart from matters having to do with chaos [4], it is not clear why the action of one would permit a quotienting procedure, but not the other. Here the difference is made explicit.

timeless configuration space should be seen as the realm containing every possible 'now', or instantaneous configuration of the universe. In [7], Barbour attempts to accommodate timelessness more intuitively:

> An alternative is that our direct experience, including that of seeing motion, is correlated with only *configuration* in our brains: the correlate of the conscious instant is part of a point of configuration space [...] Our seeing motion at some instant is correlated with a single configuration of our brain that contains, so to speak, several stills of a movie that we are aware of at once and interpret as motion. [...] Time is not a framework in which the configurations of the world evolve. Time exists only so far as concrete configurations express it in their structure. The instant is not in time; time is in the instant.

Timelessness and the relational transition amplitude When there is some underlying notion of the passage of time, we think about configuration q_1 as 'now'; it is a configuration which has evolved from earlier initial conditions (e.g.: the Big Bang). We think of q_2 as some configuration in the future, and we want to predict the probability that our Universe will somehow become q_2.

In standard quantum mechanics, the amplitude for this transition is given by the usual propagator $G((t_1, q_1), (t_2, q_2))$. Conversely, we can think of this amplitude as saying that, having measured q_1 at a previous time, and some certain time having elapsed after that, we now want to calculate the probability density of q_2. Both alternatives represent the probability of q_2 'conditional' on q_1.

In the timeless context, the transition amplitude $W(q_1, q_2)$ is the substitute of $G((t_1, q_1), (t_2, q_2))$. It is construed as 'the amplitude related to finding a given relational configuration q_2, given that you 'now' observe q_1'. Finding when in the future? Any 'future time', as it will involve an integration over all time parameters (see eq. (25)). But this does not tell us how the present will become the future, even if the future is left unspecified. The alternative view of q_1 and q_2 presents similar problems. Loosely quoting Chiou [8] (page 16), $W(q_1, q_2)$ represents 'the amplitude related to measuring q_2, given that q_1 was measured at a *previous instance*'.

Although $W(q_1, q_2)$ appears to be mathematically self-consistent, we are still left with a host of questions unanswered. What 'previous instance'? How do we know that the previous configuration really 'happened', and how did it 'become' the present one? When Time is to be a derived concept at which we arrive from change, what is it that promotes change itself? What is it that drives the present to become the future?

Going back to a simple case where we can identify a time variable might provide some guidance. For a region in configuration space in which a variable that has 'time-like' properties exists, the configurations have the form $q^a = (t, q^i)$. It is true that in this case we obtain $W(q_1, q_2) = G((t_1, q_1^i), (t_2, q_2^i))$ (see eq. (28)), which is, at least in form, the amplitude we know exists for an absolute notion of time. Nonetheless, in previous equation, time is merely a part of the configuration, not an absolute clock external to the system.[3] The partial configuration t is on the same footing as the other configuration variables, and that footing is too slippery to support change itself. The main question – how do we exit the 'now'?– remains.

My answer will be that the past doesn't become the present – it is only embedded in the present. Every present exists, every present is unique, and some presents may be entangled with other presents.

Records and the illusion of a past Again quoting Barbour [7] as our proxy for Parmenides

> By analogy with Descartes's Cogito ergo sum, we know that the present instant is actualized. However, because we can never step out of the present instant, we can never know if any other instant is actually experienced. For we shall never know whether other possible instants, including what we take to be our own past, are actual or whether the present instant is unique.

Indeed, it seems undeniable that the present –each and every present – has a preferred ontological status than either its past or its future. Perhaps the difficulty of assigning ontological meaning to $W(q_1, q_2)$ is a reflection of this fact.

Everything we experience at any instant, including memories, must be etched into our instantaneous brain configuration. Records of apparent past events should exist in the present configuration. This is what Barbour alludes to by saying that 'time is in the instant'. But why do records give a consistent picture of the past? At any moment we are in the possession of a host of redundant records of the same event, and they better be in mutual accord, fitting into a larger explanatory framework we call "science".

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[3] And it is monotonic along flow curves *in phase space*. But there are no unique flow curves passing through the projection onto configuration q_1. See section 4.1 below.

I want to confront these questions about timelessness head on in a quantum gravity theory. In doing so, the main questions I need to answer are: why am I sure that I had a donut this morning? And why is that particular memory corroborated by the jelly stain on my shirt? Or perhaps more generally, how have we even come upon the concept of a concrete past which has evolved to the present according to some law of nature?

After a description of timeless quantum mechanics and its path integral formulation in section 4, I will introduce a volume form – or probability density – in configuration space, and a notion of semi-classical records – the key constructions to answer these questions.

Since certain spatial field configurations contain also an observer's conscious instant, the volume element around such a configuration expresses the amount of observers which all possess similar experiences. The volume form introduced will tightly depend on the given Lagrangian action, and we can determine paths that extremize this action between two configurations (if any such path exists).

The kernel of the argument lies in accepting timelessness by interpreting q_2 as configuration 'now', but having q_2 contain *records* of q_1.[4] In other words, even if there is no concrete sense in which q_1 'previously happened', in the presence of records of q_1, the volume of configurations in the region around q_2 become tightly correlated with the volume of configurations around q_1. The connection between the two infinitesimal volume forms implies that if there aren't many copies of an observer close to q_1, there will be even less around q_2. This not only guarantees a certain notion of unitarity, but, as a conditional probability, can be interpreted as an inference of the existence of the past: q_2 *infers* q_1.

Furthermore I can show that If q_2 has a record of q_1 *and* there is a unique extremal path between the two configurations – the classical limit– then the entire path has an ordering of records. Namely, parametrizing the path, $\gamma(t)$, such that $\gamma(0) = q_1$, $\gamma(t^* > 0) = q_2$, then $\gamma(t)$ is a record of $\gamma(t')$ iff $t < t'$.

In the classical limit, we are led to the idea of a complete past history, culminating always in the present. Different lines of evidence for the objective existence of the record q_1 seem to concur. There are redundant records of the breakfast donut – both my memory of it and the jelly stain on my shirt. After all, in this limit every aspect of our present configuration q_2 is connected to q_1 by a classical history of the fields. This nesting of records is also what allows us to effectively use "repeatability" of experiments. Even though we only have access to 'now', each previous run of an experiment can be encoded in a future run, in a very redundant manner subject to checks for consistency [9]. It allows frequentist approaches to probabilities.

From a Bayesian perspective, we are in fact completely justified in assuming that *configuration q_2 evolved from q_1*, realizing every configuration in between. The reconstruction of the past is complete.

The technical summary (tl;dr) This paper consists in a heuristic explanation of the following sequence of underlying technical arguments:

1. In nonrelativisitic quantum mechanics, to find the amplitude for a given initial state, one evolves a future outcome backwards in time to find the overlap with the current wavefunction using the Heisenberg picture. However, in the absence of a fixed causal structure, this procedure becomes ill-defined. The key requisite feature of quantum mechanics, unitarity, becomes muddied in the presence of alternate causal structures, as would happen in quantum gravity. The path integral transition amplitude between two fixed time-slices suffers from similar maladies. This leads me to consider a theory in which space is fundamental, and duration is derived from the change of space. A fundamentally timeless theory [6].

2. In this timeless theory, configuration space is singled out as the space of 'beables', or ontological entities. The primary object encoding physics is the timeless path integral transition amplitude between two configurations, $W(q_1, q_2)$ [8].

3. Following the principle that spatial configurations are primary, I will require symmetries to act solely as functions of the configuration variables, i.e. I demand that symmetries act as 'laws of the instant'. Taking the conservative point of view that the spatial metric represents the fundamental physical degrees of freedom, the only two types of symmetries satisfying this demand turn out to be spatial scale and diffeomorphism transformations, acting as $\delta_\epsilon g = \epsilon g$ and $\delta_\epsilon g = \mathcal{L}_\epsilon g$ [10].

4. To activate the relational (timeless) transition amplitude, $W(q_1, q_2)$, we require a way to define another configuration besides the present one.

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[4]It is not clear to me if this construction is or isn't a realization of what Barbour has called 'time capsules', of which I only learned recently. More work is required to connect the ideas.

5. The physical configuration space here is conformal superspace – the quotient of the space of metrics wrt diffeomorphisms and scale transformations. Conformal superspace for gravitational fields over the 3-sphere is a stratified manifold for which there is always a preferred stratum – the one corresponding to the spherical metric [11, 12]. I call a representative of this configuration, q^*.

6. Configuration q^* serves as a sort of 'vacuum state', yielding a preferred complex distribution on physical configuration space, $W(q) := W(q^*, q)$, given by the transition from this 'vacuum' to the given configuration. Requiring certain semi-classical properties of a probability density based on this amplitude, it was argued elsewhere [13] (but not proven) that $P(q) = |W(q)|^2$.[5]

7. But in our daily experiments we don't usually hark back to q^*. When a precise structure – called a record – exists, we can effectively infer a history between a recorded configuration, q_r, and its record-holding configuration, q. The existence of records implies that $W(q^*, q) = W(q^*, q_r)W(q_r, q)$. With this equation, the properties of $P(q)$ imply the equation for the probability of q becomes that of a conditional probability, conditional on the occurrence of q_r: $P(q) = P(q_r)P(q|q_r)$. I take this conditional probability property to represent the inference of the existence of a particular event in the past, namely, q_r, from: i) the present configuration, ii) the q^* hypothesis, and iii) the path integral. In a classical limit, we regain a complete continuous history in this manner. We test scientific hypotheses by comparing multiple and redundant records.

8. Without time, and with the role of measurements taken over by records, I argue that i) there is no measurement problem of quantum mechanics in this context, and ii) there is no superposition of metrics, but we can still infer interference of alternative paths in configuration space from the very structure of $P(q)$.

This new theory brings with it new questions: about locality, Bayesian probabilities, Born rule, etc, which I tried to answer as fully as I could in [15, 13, 10] and which I will only touch on here. Let me now lay down the argument contained in this paper, and the problems it aims to solve, at length.

2 The problems with quantizing gravity

2.1 A tale of two theories

General relativity is one of the pillars of our modern understanding of the Universe, deserving a certain degree of familiarity from all those who purport to study Nature, whether from a philosophical or mathematical point of view. The theory has such pristine logical purity that it can be comprehensively summarized by John A. Wheeler's famous quip:

$$\text{"Matter tells spacetime how to curve, and spacetime tells matter how to move."} \tag{1}$$

We should not forget however, that ensconced within Wheeler's sentence is our conception of spacetime as a dynamical geometrical arena of reality: no longer a fixed stage where physics unfolds, it is part and parcel of the play of existence.

In mathematical terms, we have:

$$\underbrace{R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}}_{\text{spacetime curving}} \quad \propto \quad \underbrace{T_{\mu\nu}}_{\text{sources for curving}} \tag{2}$$

Given the sources, one will determine a geometry given by the spacetime metric $g_{\mu\nu}$ – the ' matter tells spacetime how to curve' bit. Conversely, it can be shown that very light, very small particles will roughly follow geodesics defined by the geometry of the lhs of the equation – the 'spacetime tells matter how to move' part.[6]

A mere decade after the birth of GR, along came quantum mechanics, the new radical kid on the block. It was a framework that provided unprecedented accuracy in experimental confirmation, predictions of

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[5] A very similar argument was used by Page in a series of papers on "Sensible Quantum Mechanics". Page considers a similar measure (volume) on expectation values of certain preferred operators ('awareness operators,' one for each possible conscious perception). See [14] and references therein.

[6]This distinction is not entirely accurate, as the rhs of equation (2) usually also contains the metric, and thus it should be seen as a constraint on which kind of space-times with which kind of matter distributions one can obtain, "simultaneously" .

new physical effects and a reliable compass for the construction of new theories. And yet, it has resisted the intuitive understanding that was quickly achieved with general relativity. A much less accurate characterization than Wheeler's quip for general relativity has been borrowed from the pessimistic adage "everything that can happen, does happen".[7] The sentence is meant to raise the principle of superposition to the status of core concept of quantum mechanics (whether it expresses this clearly or not is very much debatable).

In mathematical terms, the superposition principle can be seen in the Schroedinger equation:

$$\hat{H}\psi = -i\hbar\frac{d}{dt}\psi \qquad (3)$$

whose linearity implies that two solutions ψ_1 and ψ_2 add up to a solution $\psi_1 + \psi_2$. In the path integral representation it is built-in. The very formulation of the generating function is a sum over all possible field configurations ϕ,

$$\mathcal{Z}[j] = \int \mathcal{D}\phi \exp\left[i \int (\mathcal{L} + \phi \cdot j)/\hbar\right] \qquad (4)$$

where $\mathcal{L}$ is the Lagrangian density.

Unfortunately, for the past 90 years, general relativity and quantum mechanics have not really gotten along. Quantum mechanics, the 'new kid on the block', soon claimed a large chunk of territory in the theoretical physics landscape, leaving a small sliver of no-man's land also outside the domain of general relativity. In most regimes, the theories will stay out of each other's way - domains of physics where both effects need to be taken into account for an accurate phenomelogical description of Nature are hard to come by. Nonetheless, such a reconciliation might be necessary even for the self-consistency of general relativity: by predicting the formation of singularities, general relativity "predicts its own demise", to borrow again the words of John Wheeler. Unless, that is, quantum effects can be suitably incorporated to save the day at such high curvature regimes.

2.2   The problems of quantum gravity

At an abstract level, the question we need to face when trying to quantize general relativity is: how to write down a theory that includes all possible superpositions and yet yields something like equation (2) in appropriate classical regimes? I for one am willing to consider a theory that incorporates these two principles as a successful candidate for a theory of quantum gravity.

Indeed, although the incompatibility between general relativity and quantum mechanics can be of technical character, it is widely accepted that it has more conceptual roots.

Is non-renormalizability the only problem?   The main technical obstacle cited in the literature is the issue of perturbative renormalizabilty. Gravity is a non-linear theory, which means that geometrical disturbances around a flat background can act as sources for the geometry itself. The problem is that unlike what is the case in other non-linear theories, the 'charges' carried by the non-linear terms in linearized general relativity become too 'heavy', generating a cascade of ever increasing types of interactions once one goes to high enough energies.

There are theories, such as Horava-Lifschitz gravity [17], which seem to be naively perturbatively renormalizable. The source of renormalizability here is the greater number of spatial derivatives as compared to that of time derivatives. This imbalance violates fundamental Lorentz invariance, breaking up spacetime into space and time. Unfortunately, the theory introduces new degrees of freedom that appear to be problematic (i.e. their influence does not disappear at observable scales).

And perhaps perturbative non-renormalizability is not the only problem. Indeed, for some time we have known that a certain theory of gravity called 'conformal gravity' (or 'Weyl squared') is also perturbatively renormalizable. The problem is that the theory is sick. Conformal gravity is not a unitary theory, which roughly means that probabilities will not be conserved in time. But, which time? And is there a way to have better control over unitarity?[8]

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[7]Recently made the title of a popular book on quantum mechanics [16].

[8]Another approach to quantum gravity called Asymptotic Safety [18] also suffers from such a lack of control of unitarity. This approach also explores the possible existence of gravitational theories whose renormalization will only generate dependence on a finite number of coupling constants, thus avoiding the loss of predictibility explained above.

A problem of Time At a more formal level, to combine (2) with our principle of superposition one should keep in mind that space-times define causal structures, and it is far from clear how one should think about these in a state of superposition. For instance, which causal structure should one use in an algebraic quantum field theory approach when declaring that space-like separated operators commute? [9]

Quantum field theory is formulated in a fixed spacetime geometry, while in general relativity spacetime is dynamical. Without a fixed definition of time or an a priori distinction between past and future, it is hard to impose causality or interpret probabilities in quantum mechanics.

For instance, in the S-matrix approach to quantum gravity [19], one can define the transition amplitude $\langle u|S|v \rangle$ with S being the S-matrix whose generating functional is given by (4), with $\phi = g_{\mu\nu}$.[10] Indeed, in such cases one can find (pseudo)unitarity and diffeomorphism invariance (the action is still non-renormalizable of course).

But there is another problem. One can only perform this construction if u and v are asymptotic states, at $t \to \pm\infty$. In these regions, diffeomorphisms are quite restricted, generating only rigid transformations.[11] No such general relativistic construction can be applied for the more realistic case in which u, v represent boundary states of a finite time interval. One could try to define the kernel of the transition amplitude (or transfer matrix) equivalent to (4) for a finite time, schematically (for pure gravity):

$$W(h_1, h_2) = \int_{h_1}^{h_2} \mathcal{D}g \, \exp\left[i \int \mathcal{L}/\hbar\right] \tag{5}$$

where h_i are initial and final geometries, i.e. for Σ_i Cauchy surfaces, $\partial M = \Sigma_1 \dot{\cup} \Sigma_2$, for the embedding $\imath_i : \Sigma_i \hookrightarrow M$, $h_{\mu\nu} = \imath^* g_{\mu\nu}$. The issue is that re-definitions of simultaneity surfaces – refoliations – shift h_i (in a g-dependent manner), and irreparably change (5).

A dynamical approach Can we formulate quantum gravity in a way which reflects the fundamental distinction between causal and non-causal? One way of approaching this question is to first use a more dynamical account of the theory. We don't need to reinvent such an account – it is already standard in the study of gravity, going by the acronym of ADM (Arnowitt-Deser-Misner) [1]. The main idea behind a dynamical point of view is to set up initial conditions on a spatial manifold M and construct the spacetime geometry by evolving in a given auxiliary definition of time.[12] Indeed most of the work in numerical general relativity requires the use of the dynamical approach. Such formulations allow us to use the tools of the Hamiltonian formalism of quantum mechanics to bear on the problem. With these tools, matters regarding unitarity are much easier to formulate, because there is a time with respect to which probabilities are to be conserved.

However, since the slicing of spacetime is merely an auxiliary structure, the theory comes with a constraint – called the Hamiltonian constraint – which implies a freedom in the choice of such artificial time slicings. The metric associated to each equal time slice, g_{ab}, and its associated momenta, π^{ab}, must be related by the following relation at each spatial point:

$$H := R - \frac{1}{g}\left(\pi^{ab}\pi_{ab} - \frac{1}{2}\pi^2\right) = 0 \tag{6}$$

where g stands for the determinant of the metric. The constraints (8) are commonly thought to guarantee that observables of the theory should not depend on the auxiliary 'foliation' of spacetime. However, as first shown in [3], redefinitions of surfaces of simultaneity in the canonical theory can only be translates as a proper Hamiltonian flow if one restricts attention to those space-times that satisfy the Einstein equations of motion. In other words, the Hamiltonian formalism embodies this sacred principle of relativity only on-shell.

And we should also be wary that the Hamiltonian formalism might be throwing the baby out with the bathwater. The freedom to refoliate spacetime implied by (8) also contains the generator of time

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[9]I am here assuming that metrics are indeed the fundamental variables that will still describe reality at a most fundamental level. It could be the case of course that they represent only emergent degrees of freedom from more fundamental ones.

[10]And I have simplified the expression by assuming that the momenta can be integrated out – giving the Lagrangian and defining $\mathcal{D}g$ from a projection of the Liouville measure – and also omitting the required gauge-fixings and Fadeev-Popov determinant present in the case of gauge-symmetries.

[11]I.e. no general refoliations are allowed, which is what allows one to speak of objectively of $t \to \pm$ in the first place.

[12] In this constructed space-time, the initial surface must be Cauchy, implying that one can only perform this analysis for space-times that are time-orientable.

evolution. In other words, time evolution becomes inextricably mixed with a certain type of gauge-freedom, leading some to conclude that in GR evolution is "pure gauge". This is one facet of what people have called "the problem of time" (see e.g. [20]).

Upon a naive quantization of (8), one gets the infamous Wheeler-DeWitt equation:

$$\hat{H}\psi[g] = 0 \tag{7}$$

where $\psi[g]$ is a wave-functional over the space of three-geometries. And thus the classical 'problem of time' gets transported into the quantum regime. One could look at equation (7) as a *time-independent* Schroedinger equation, which brings us again to the notion of "frozen time", from (3). A solution of the equation will not be subject to time evolution; it will give a frozen probability wave-function on the space of three-geometries.[13]

Can we formulate quantum gravity in a way which reflects the fundamental distinction between causal and non-causal? One way of approaching this question is to first use a more dynamical account of the theory. We don't need to reinvent such an account – it is already standard in the study of gravity, going by the acronym of ADM (Arnowitt-Deser-Misner) [1]. The main idea behind a dynamical point of view is to set up initial conditions on a spatial manifold M and construct the spacetime geometry by evolving in a given auxiliary definition of time.[14] Indeed most of the work in numerical general relativity requires the use of the dynamical approach. Such formulations allow us to use the tools of the Hamiltonian formalism of quantum mechanics to bear on the problem. With these tools, matters regarding unitarity are much easier to formulate, because there is a time with respect to which probabilities are to be conserved.

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And we should also be wary that the Hamiltonian formalism might be throwing the baby out with the bathwater. The freedom to refoliate spacetime implied by (8) also contains the generator of time evolution. In other words, time evolution becomes inextricably mixed with a certain type of gauge-freedom, leading some to conclude that in GR evolution is "pure gauge". This is one facet of what people have called "the problem of time" (see e.g. [20]).

Other canonical approaches have so far similarly found insurmountable problems with the quantization of this constraint. I contend that this is because local time reparametrization represents an effective, but not fundamental symmetry. I.e. I contend that *invariance under refoliation is not present at a quantum mechanical level, but should be recovered dynamically for states that are nearly classical*. In fact, it is *already the case* that the Hamiltonian formalism represents changes of surfaces of simultaneity as a symmetry only on-shell. All we want is to extend this property to other possible formulations of gravity.

Nonetheless, apart from all of the technical difficulties, the property of timelessness represented by equation (7) should remain in any theory that is completely relational, in that it would not contain an explicit time variable. Briefly summarized, relationalism is the belief that all relevant physical information, including Time, should be deducible by the relation between physical objects. Thus the presence of an "external time" is odd from the point of view of relationalism, and should be extractable from

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[13] But equation (7) has some further problems of its own. It has operator ordering ambiguities, functional derivatives of the metric acting at a singular point, no suitable inner product on the respective Hilbert space with respectable invariance properties, etc. One could also attempt to interpret (7) as a Klein-Gordon equation with mass term proportional to the spatial Ricci scalar, but unlike Klein-Gordon, it is already supposed to be a quantized equation. Furthermore, the problem in defining suitable inner products are an obstacle in separating out a positive and negative spectrum of the Klein-Gordon operator.

[14] In this constructed space-time, the initial surface must be Cauchy, implying that one can only perform this analysis for space-times that are time-orientable.

internal properties of curves in configuration space.[15]

As a last remark in this section, I would like to point out that the presence of an auxiliary global time parameter parametrizing a curve in conformal superspace does *not* imply that one could detect a preferred surface of simultaneity. Duration is extracted from the local change in the physical degrees of freedom of the field, and should be thus completely relational, with no reference to background structures (see however the end of appendix B.1.1, for differences that could arise to the general relativistic notion of refoliation invariance).

3   The unique existence of the present

Time is change - Parmenides and Zeno

It could be argued that we do not "experience" space-times. We experience 'one instant at a time', so to say. We of course still appear to experience the passage of time, or perhaps more accurately, we (indirectly) experience changes in the spatial configuration of the world around us, through changes of the spatial configuration of our brain states.

But if present experience is somehow distinguished, how does "change" come about? This is where Parmenides has something to say that is relevant for our discussion. Parmenides was part of a group called the Eleatics, whose most prominent members were himself and Zeno, and whose central belief was that all change is illusory. The reasoning that led them to this conclusion was the following: if the future (or past) is real, and the future is not existing now, it would have both properties of existing and not existing, a contradiction (or a 'turning back on itself'). Without past and future, the past cannot transmute itself into future, and thus there is also no possible change. Of course, the argument hinges on the distinction we perceive between present, past and future.

Aristotle was able to give convincing arguments rebuking Zeno's paradox, by conceding that there is no time without change, but maintaining that Time should not be identified with change. Instead, change was just a measure of Time, or, in Aristotle's own words: "change is the unit of Time". Like numbers, change would be something capturing an existing aspect of Nature, without itself being part of Nature. For Aristotle, Time is like the real line. However, he argues, the time it takes to get from one place to another is not actually composed of an infinite number of finite lengths of time. Unfortunately, neither parties of the argument had the tools of calculus at their disposal, with which all of these notions can be made precise. Without these tools the argument was not completely resolved, but was largely deemed to be answered by Aristotle.

But even Aristotle did not fully provide an answer to what Time is supposed to be when past and future cannot exist. Augustine of Hippo noticed the loophole in Aristotle's argument, and picked up the question. Similarly to Parmenides, he concluded that change was an illusion and yet,

> How can the past and future be, when the past no longer is, and the future is not yet? As for the present, if it were always present and never moved on to become the past, it would not be time, but eternity.[...] Nevertheless we do measure time. We cannot measure it if it is not yet into being, or if it is no longer in being, or if it has no duration, or if it has no beginning and no end. Therefore we measure neither the future nor the past nor the present nor time that is passing. Yet we do measure time.

According to Augustine, Time is a human invention: the difference between future and past is merely the one between anticipation and memory.

To the extent that future and past events are real, they are real now, i.e. they are somehow encoded in the present configuration of the Universe. Apart from that, they can be argued not to exist. My memory of the donut I had for breakfast is etched into patterns of electric and chemical configurations of my brain, *right now*. We infer the past existence of dinosaurs because it is encoded in the genes of present species and in fossils in the soil. In a timeless Universe, what we actually do is deduce from the present that there exists a continuous curve of configurations connecting 'now' to some event in the past. But if the ideas of past and future were all false, how would we have come to have such illusions

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[15]In the case of shape dynamics [21], for the theory to possess the full gamut of spatial relationalism would require the theory to be invariant under the complete group of Weyl transformations, and not just the ones that preserve the total volume of space. Such a formulation exists [22], but it requires a preferred time parameter. This is the time parameter which can be *defined* to drive the change that defines duration relationally. Nonetheless, it will be a time parameter that somehow exists outside the domain of observability, which is still uncomfortable for relationalist sensibilities. What we are saying here does not apply to the original shape dynamics theories, based on York time but not *fully* relational.

in the first place? They surely are not false, and other instants should exist. But then how to connect a snapshot of the dinosaur dying with the snapshot of the archaeologist finding its remains? Does the Eleatic argument bring about a 'solipsism of the instant'?[16]

A more mathematical posing of the question. Let's call configuration space $\mathcal{M}$, which we endow with a reasonable topology (see the relevant subsection in 5). Given an appropriate action functional over $\mathcal{M}$, one obtains continuous curves that extremize this functional. It makes sense to have configuration 'bite on donut' connected by one such continuous curve to configuration 'me, reminiscing about donut, six hours later'. But what is the meaning of these curves? In which sense can we think of ourselves as traversing them?

We now turn to the meaning of timeless configuration space, and how to construct a theory of quantum gravity there.

4 Timelessness quantum mechanics in configuration space

What does a timeless, relational theory, quantum or classical, look like? A long literature exists on this matter, and it is of course beyond the scope of this work to give any reasonably detailed account of the subject. Instead, I will give a very brief account of the results of [8], which are specially useful for my purposes. Chiou translates canonical timeless quantum mechanics (see e.g. [23], briefly summarized in appendix A) into the path integral formulation – which is the approach I believe carries the most useful conceptual baggage.

4.1 Timeless path integral in quantum mechanics

We start with a finite-dimensional system, whose configuration space, $\mathcal{M}$, is coordinitized by q^a, for $a = 1, \cdots, n$. An observation yields a complete set of q^a, which is called an event. Let us start by making it clear that no coordinate, or function of coordinates, need single itself out as a reference parameter of curves in $\mathcal{M}$. The systems we are considering are not necessarily 'deparametrizable' – they do not necessarily possess a suitable notion of time variable.

Now let $\Omega = T^*\mathcal{M}$ be the cotangent bundle to configuration space, with coordinates q^a and their momenta p_a. The classical dynamics of a reparametrization invariant system is fully determined once one fixes the Hamiltonian constraint surface in Ω, given by $H = 0$. A curve $\gamma \in \mathcal{M}$ is a classical history connecting the events q_1^a and q_2^a if there exists an *unparametrized* curve $\bar{\gamma}$ in $T^*\mathcal{M}$ such that the following action is extremized:

$$S[\bar{\gamma}] = \int_{\bar{\gamma}} p_a dq^a \tag{9}$$

for curves lying on the constraint surface $H(q^a, p_a) = 0$, and are such that $\bar{\gamma}$'s projection to $\mathcal{M}$ is γ, connecting q_1^a and q_2^a.

Feynman's original demonstration of the equivalence between the standard form of non-relativistic quantum mechanics and his own path integral formulation relied on refining time slicings, which gave a straightforward manner by which to partition paths into smaller and smaller segments. Without absolute time, one must employ new tools in seeking to show the equivalence. For instance, a parametrized curve $\bar{\gamma} : [0,1] \rightarrow \Omega$ need not be injective on its image (it may go back and forth). This requires one to use a Riemann-Stieltjes integral as opposed to a Riemann one in order to make sense of the limiting procedure to infinite sub-divisions of the parametrization. Furthermore, one must then sum over all parametrizations, at which point the integral over τ in (25) ends up indeed giving a functional $\delta[H]$, and the entire transition amplitude (24) becomes:

$$W(q_1, q_2) = \int \mathcal{D}q^a \int \mathcal{D}p_a \, \delta[H] \exp\left[\frac{i}{\hbar} \int_{\bar{\gamma}} p_a dq^a\right] \tag{10}$$

where the path integral sums over paths whose projection starts at q_1 and ends at q_2. In the presence of gauge symmetries, if it is the case that these symmetries form a closed Lie algebra, one can in principle use the group averaging procedure mentioned in appendix A, provided one uses a similarly translation invariant measure of integration.

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[16]Julian Barbour tells me that the expression was originated in discussions with Fay Dowker.

For a strictly deparametrizable system,[17] one obtains again:

$$W(t_1, q_1^i, t_2, q_2^i) \sim \int \mathcal{D}t \, G(t_1, q_1^i, t_2, q_2^i) \sim G(t_1, q_1^i, t_2, q_2^i)$$

up to an irrelevant overall factor. Further, if the Hamiltonian is quadratic in the momenta, one can integrate them out and obtain the configuration space path integral with the Lagrangian form of the action.

The absence of change. For a theory that contains some driver of change, an absolute Time of some sort, we would extend our configuration space with an independent time variable, t, making the system effectively deparametrizable. With this absolute notion of Time, and ontological deparametrization of the system, evolution from t_1 to t_2 would not require any further definition. At this point we could stop, claiming that we have expounded on what we expect a relational theory of space to look like. We would be able to define a Schrodinger equation much as in the usual time-dependent framework, and go about our business. Shape dynamics employing the complete relational symmetries is a theory of that sort.[18] However, the presence of Time there is still disturbing from a relational point of view: where is this Time if not in the relations between elements of the configurations? Therefore, to fully satisfy our relational fantasies, we must again tackle the question posed at the introduction: without a driver for change, what is the meaning of a transition amplitude?

In the path integral representation of the timeless transition amplitude, $W(q_1, q_2)$ depends only on the configuration variables (momenta are being integrated over), and it again is not clear what ontological meaning to give to $W(q_1, q_2)$ by itself. I will require symmetries to be 'laws of the instant' precisely so that they are compatible with a theory defined at its most fundamental level by $W(q_1, q_2)$.

4.2 Timeless quantum gravity

In the last section, I reviewed a timeless path integral formulation of quantum mechanics by Chiou [8], building on previous results on timeless quantum mechanics (see [23] for a review). In these formulations, configuration space is the 'space of all possible instants'. This section will consist of a (extremely) summarized account of my work in [15, 13, 10].

Configuration space for timeless field theories, which I will still denote by $\mathcal{M}$, is the set of all possible field configurations over a given (in our case finite-dimensional, closed) manifold M. Each point of configuration space $q \in \mathcal{M}$ is a "snapshot" of the whole Universe.[19] One nice thing about this infinite-dimensional space is that it is metrizable, which gives it the minimal topological properties we want from the requirements of section 5.

Symmetries, relationalism and 'laws of the instant'

Relationalism In Hamiltonian language, the most general symmetry transformation acts through the Poisson bracket on configurations as

$$\delta_\epsilon g_{ij}(x) = \left\{ \int d^3x \, F[g, \pi; x'] \epsilon(x') \, , \, g_{ij}(x) \right\} \tag{11}$$

where ϵ is the gauge parameter, which in this infinite dimensional context is a function on M (not necessarily scalar), and we are using DeWitt's mixed functional dependence, i.e. F depends functionally on g_{ij} (not just on its value at x'), as denoted by square brackets, but it yields a function with position dependence – the " $; x'$)" at the end.

Regarding the presence of gauge symmetries in configuration space, we would like to implement the most general relational principles that are applicable to space (as opposed to spacetime). At face value, the strictly relational symmetries should be:

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[17]I.e. one for which $[\hat{H}(t_1), \hat{H}(t_2)] = 0$. If this is not the case, the equality will only hold semi-classically.

[18]The absolute time used in the original version of shape dynamics [21], is of the form $\langle \pi^{ab} g_{ab} \rangle$, where brackets denote the spatial average. This quantity is only invariant wrt Weyl transformations that preserve the total volume of space, and is thus not completely relational. One can extend the conformal transformation to the full group, acquiring an absolute time parametrization [24].

[19] For instance, it could be the space of sections on a tensor bundle, $\mathcal{M} = C^\infty(TM \otimes \cdots TM \otimes TM^* \cdots TM^*)$. In the case of gravity, these are sections of the positive symmetric tensor bundle: $\mathcal{M} = C^\infty_+(TM^* \otimes_S TM^*)$.

- **Relationalism of locations.** In Newtonian particle mechanics this would imply that to predict the future behaviour of a system one requires the initial relative positions and velocities of the particles, not their absolute position and motion. It only holds if the total angular momentum of the system vanishes (see [25, 26]). In the gravitational field theory case, this property is represented by the (spatial) diffeomorphism group of the manifold M, Diff(M). This symmetry is generated by $F[g, \pi; x'] = \nabla_i \pi^i{}_j(x)$, which yields on configuration space $\delta_{\epsilon^a} g_{ij}(x) = \mathcal{L}_{\epsilon^a} g_{ij}(x)$.

- **Relationalism of scale.** In Newtonian particle mechanics this would imply that to predict the future behaviour of a system one furthermore requires only the relative distance of the particles, not the absolute scale. It only holds if the total dilatational momentum of the system vanishes (see [25, 26]). In the gravitational field theory this property is represented by the group of scale transformations (also called the Weyl group), $\mathcal{C}(M)$. This symmetry is generated by $F[g, \pi; x'] = g_{ij}\pi^{ij}(x)$, which yields on configuration space $\delta_{\epsilon} g_{ij}(x) = \epsilon(x) g_{ij}(x)$. In this case the gauge parameter is a scalar function, as opposed to a vector field for the diffeomorphisms.

Unlike what is the case with the constraints emerging from the Hamiltonian ADM formalism of general relativity, these symmetries form a (infinite-dimensional) closed Lie algebra.

Laws of the instant Even disregarding considerations about relationalism, as we saw in section 2.2, the issue with a finite-time gravitational transition amplitude, equation (5), is that local reparametrizations, or refoliations, don't act as a group in spatial configuration space, and thus do not allow one to form a "gauge-invariant' quotient from its action. One of the aims of the paper will be to find a fomulation of quantum gravity for which we can have a well-posed transition amplitude for states that are not infinitely "far apart" – after all, even the notion of "far apart" should emerge from expectation values.

We would also like symmetries to act solely on configuration space, in a manner compatible with the demand that $W(g_{ij}^1, g_{ij}^2)$ give all the information we need about a theory. To be compatible with presentism, I thus require that $\delta_\epsilon g_{ij}^1(x) = G[g_{ij}^1, \epsilon; x]$ for some mixed functional G – which crucially *only* depends on g_{ij}^1.

In other words, the action of the symmetry transformations of 'now', only depend on the content of 'now'. The action of these relational symmetries on each configuration is self-determined, they do not depend on tangent vectors $v_1 \in T_{g^1}\mathcal{M}$ or on configurations g_{ij}^3 different than the ones the symmetries are acting upon. In appendix B, I sketch a proof that the relational symmetries of scale and position are indeed the only symmetries whose action in phase space projects down to an intrinsic action on configuration space.[20]

The conclusion of this argument is that spatial relationalism is singled out by demanding that symmetries have an intrinsic action on configuration space. This feature is not realized by the action of the ADM scalar constraint (8), since it is a symmetry generated by terms quadratic in the momenta. Thus the transformation it generates on the metric requires knowledge of the conjugate momentum (and vice-versa).[21]

Lastly (and also unlike what is the case with the scalar constraint (8)), barring the occurrence of metrics with non-trivial isometry group, the action of these symmetries endows configuration space $\mathcal{M}$ with a well-defined, neat principal fiber bundle structure (see [12]), which enables their quantum treatment [10].

Given these symmetries (and the principal fiber bundle structure they form), we take the analogous of (10), schematically projected down onto the space of conformal geometries[22]:

$$W([g_1], [g_2]) = \int \mathcal{D}[g] \int \mathcal{D}[\pi] \exp\left[\frac{i}{\hbar} \int_{\bar{\gamma}} [\pi^{ab}] \, \delta[g_{ab}]\right] \tag{12}$$

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[20]This is not entirely true. There is the possibility of a third constraint, $\pi^{ab} = 0$. This constraint would subsume the other two, and imposing it would imply that indeed we live in a static, Eleatic Universe.

[21]Similarly, the action of these relational symmetries also act independently on the momenta, i.e. their action splits, and finite gauge transformations can be written schematically, in the form:

$$(g_{ab}, \pi^{ab}) \to (\exp\left(\int d\tau \, \xi(\tau)\right) \cdot g_{ab}, \exp\left(\int d\tau \, \xi(\tau)\right) \cdot \pi^{ab})$$

where $\exp \int d\tau \, \xi(\tau)$ represents the flow of the vector field part of $\xi(t)$ and the usual exponential of an infinitesimal Weyl transformation, and the $\cdot$ denotes the respective group action, e.g.: for the metric, pull-back for the diffeomorphisms and pointwise multiplication for Weyl, [?]).

[22]The full treatment of the gauge conditions requires a gauge-fixed BRST formalism, which is a level of detail I don't need here. See [10] for a more precise definition, equation (28), where we use $K(g_1, [g_2])$ as opposed to $W([g_1], [g_2])$.

where I have (again, schematically) used square brackets to denote the conformal-diffeo equivalence classes of the metric and momenta. In appendix A, we show how probabilities can then be calculated using the analogue of (27).

Still, even in the classical limit, there is a fundamental difference between the notion of duration in a spatially relational theory – such as the ones defined in appendix B in equations (33) and (35), which are to be input into (12) – and duration in a spacetime theory such as GR in its ADM form. In a 3+1 description of GR, given an initial and a final Cauchy surfaces and a unique spacetime interpolating between the two, duration along a world-line is read off from the lapse associated to that foliation. In (33) and (35) there is no inkling of a lapse anywhere to be seen. For a given unique extremal field history between $[g_1]$ and $[g_2]$, one must then *define* duration as a local measure of change in the conformal geometry. In this case, duration along a worldline is completely relational, and does not set a scale by itself, unlike what is the case in GR.

Records and timelessness

In a true spatially relational theory, an instantaneous state of an observer is encoded in a partial field configuration. There are no subjective overtones attributed to an observer – it is merely a (partial) state of the fields. Of course, there are many regions of configuration space where no such thing as an observer will be represented.

Since each point is a possible 'now', and there is no evolution, each 'now' has an equal claim on existing. This establishes the plane of existence, every 'now' that can exist, does exist! We are at least partway towards the adage of quantum mechanics. If this was a discrete space, we could say that each element has the same weight. This is known as the principle of indifference and it implies that we count each copy of a similar observer once.[23]

But configuration space is a continuous space, like $\mathbb{R}^2$ (but infinite-dimensional). Unlike what is the case with discrete spaces, there is no preferred way of counting points of $\mathbb{R}^2$. We need to imprint $\mathcal{M}$ with a volume form; each volume form represents a different way of counting configurations.

Born rule and the preferred configuration.   Contrary to what occurs in standard time-dependent Many Worlds quantum mechanics, I will define a single, standard time-independent 'volume element' over configuration space $\mathcal{M}$. Integrated over a given region, this volume element will simply give the volume, or the amount, of configurations in that region.[24]

The volume form $P([g])D[g]$ is defined as a positive scalar function of the transition amplitude, $P([g]) := F(W([g^*], [g]))$, where $F : \mathbb{C} \to \mathbb{R}^+$ with the extra property that it preserves the multiplicative group structure,

$$F(z_1 z_2) = F(z_1)F(z_2) \tag{13}$$

an important property to recover locality and an empiric notion of records from the transition amplitude [15]. This measure, F, gives a way to "count" configurations, and it is assumed to act as a positive functional of the only non-trivial function we have defined on $\mathcal{M}$, namely, the transition amplitude $W(q^*, q)$. Together with certain locality properties of $W(q^*, q)$ discussed in [15], and the *factorization property* of records (17) below, my hope is that (13) will uniquely lead to a derivation of the Born rule in the future.

Now, for simplicity of notation, let us denote the equivalence classes $[g]$ by the former coordinate variable, q, and just assume that $F(W(q^*, q)) = |W(q^*, q)|^2$, i.e.

$$P(q) = |W(q^*, q)|^2 \tag{14}$$

Still, in the definition of $P(q)$ I have sneaked in a 'in' configuration, q^*, which defines once and for all the static volume form over (reduced) configuration space. I define q^* roughly as the simplest, most structureless configuration of the fields in question.

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[23]The intuition obtained for Many Worlds in the discrete configuration spaces can be misleading for our purposes. In that case, each 'branch' can be counted, and one needs a further explanation to count them according to the Born rule. Based on this principle, and on the Epistemic Principle of Separability, Carroll et al claim that the Born rule can be derived [27].

[24]Of course, these volume forms are divergent and technically difficult to define. Properties of locality of the volume form, discussed in [15] are essential to show that nonetheless their definition reduces to the usual Born rule for isolated finite-dimensional systems. Furthermore, only ratios of the volume form have any meaning, and only in a Bayesian interpretation.

The preferred configuration, q^*. This might sound subjective, but in fact it is not. Reduced configuration spaces may not form smooth manifolds, but only what are called stratified manifolds, in general. This is because the symmetry group in question – whose action forms the equivalence relation by which we are quotienting – may act *qualitatively* differently on different orbits. If there are subgroups of the symmetry group, called stabilizer subgroups, whose action leave a point fixed, the symmetry does not act "fully" on said point (or rather, on the orbit corresponding to the point). Thus the quotient of configuration space wrt to the symmetry may vary in dimensionality.

Taking the quotient by such wavering actions of the symmetry group creates a patchwork of manifolds, whose union is called a stratified manifold. It is a space that has nested "corners" – each stratum has as boundaries a lesser dimensional stratum, and is indexed by the stabilizer subgroup of the symmetry group in question (e.g. isometries as a subgroup of Diff(M)). The larger the stabilizer group, the lower the strata. A useful picture to have in mind for this structure is a cube (seen as a manifold with boundaries). The interior of the cube has boundaries which decomposes into faces, whose boundaries decompose into lines, whose boundaries decompose into points. The higher the dimension of the boundary component, the smaller the isometry group that its constituents have.[25] Thus the interior of the cube might have no stabilizer subgroups associated to it, the face of the cube could be associated to a lower dimensional stabilizer subgroup than the edges, and the edges a lower one than the corners.

Configurations with the highest possible dimension of the stabilizer subgroup are what I define as q^* – they are the pointiest corners of reduced configuration space! And it is these preferred singular points of configuration space that we define as an origin of the transition amplitude.

Thus, depending on the symmetries acting of configuration space, and on the topology of M, one can have different such preferred configurations. For the case at hand – in which we have both scale and diffeomorphism symmetry and $M = S^3$ – there exists a *unique* such preferred point! The preferred q^* of $\mathcal{M}/(\text{Diff}(M) \ltimes \mathcal{C})$ is the one corresponding to the round sphere.

Semi-classical records: recovering Time   But, the astute reader may ask, having already defined q^*, we can set it as q_1 and obtain a meaningful transition amplitude $W(q_1, q_2)$ to 'now', represented by q_2? Yes, we can. At a fundamental level, q^*, together with a definition of F and the action, completely specify the physical content of the theory by giving the volume of configurations in a given region of $\mathcal{M}$.

But, if another class of object which I call *records*, exists, then one can more realistically model our practice of the scientific method. For after all, in our everyday laboratory usage of quantum mechanics, we don't calculate the transition amplitude to the origin of the Universe. Instead, one calculates the transition amplitude with respect to some not-too-distant initial conditions, some nearby 'in' state, q_1. The existence of records, embedded in the present configuration q, formalizes this notion. It makes sense out of amplitudes between a record and a record-holding configuration, leaving the actual amplitude between the record and the 'origin', q^*, redundant.

The system one should have in mind as an example of such a structure is the Mott bubble chamber [28]. In it, emitted particles from α-decay in a cloud chamber condense bubbles along their trajectories. A quantum mechanical treatment involving a timeless Schroedinger equation finds that the wave-function peaks on configurations for which bubbles are formed collinearly with the source of the α-decay. In this analogy, a 'record holding configuration' would be any configuration with n collinear condensed bubbles, and any configuration with $n' \leq n$ condensed bubbles along the same direction would be the respective 'record configuration'. In other words, the $n + 1$-collinear bubbles configuration holds a record of the n-bubbles one. For example, to leading order, the probability amplitude for n bubbles along the θ direction obeys

$$P[(n, \theta), \cdots, (1, \theta)] \simeq P[(n', \theta), \cdots, (1, \theta)]P[(n', \theta), \cdots, (1, \theta)|(n, \theta), \cdots, (1, \theta)] \tag{15}$$

where $n' < n$, and $P[A|B]$ is the conditional probability for B given A.

Let us sketch how this comes about in the present context. When semi-classical approximations may be made for the transition amplitude between q^* and a given configuration, we have

$$W_{\text{cl}}(q^*, q) = \sum_{\gamma_j} \Delta_j^{\frac{1}{2}} \exp\left((i/\hbar)S_{\text{cl}}[\gamma_j]\right) \tag{16}$$

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[25]E.g. let $\mathcal{M}_o$ be the set of metrics without isometries. This is a dense and open subset of $\mathcal{M}$, the space of smooth metrics over M. Let I_n be the isometry group of the metrics g_n, such that the dimension of I_n is d_n. Then the quotient space of metrics with isometry group I_n forms a manifold with boundaries, $\mathcal{M}_n/\text{Diff}(M) = \mathcal{S}_n$. The boundary of $\mathcal{S}_n$ decomposes into the union of $\mathcal{S}_{n'}$ for $n' > n$ (see [11]).

where the γ_j are curves that extremize the action and Δ are certain weights for each one. Roughly speaking, when all of γ_j go through a configuration $q_r \neq q$, I will define q as *possessing a semi-classical record* of q_r. Note that this is a statement about q, i.e. it is q that contains the record.[26]

Indeed, for records, it can be shown that the amplitude suffers a decomposition (this is shown in [13], and works also for strings of records)

$$W(q^*, q) \simeq W(q^*, q_r)W(q_r, q) \tag{17}$$

To show this, one uses the same techniques as to show that the usual semi-classical transition amplitude has the semi-group property:

$$W_{\text{cl}}((q_1, t_1), (q_3, t_3)) = \int dq_2 W_{\text{cl}}((q_1, t_1), (q_2, t_2))W_{\text{cl}}((q_2, t_2), (q_3, t_3)) \tag{18}$$

In a simplified case of a system deparametrizable around the components t_2^r of the record configurations, we immediately recover

$$W_{\text{cl}}(q^*, q) \simeq \sum_{q_r} W_{\text{cl}}(q^*, q_r)W_{\text{cl}}(q_r, q)$$

and for a single record we recover (17).

Calculating the probability of q from equation (17), we get an equation of conditional probability, of q on q_r,

$$P(q_r) = P(q|q_r)P(q_r) \tag{19}$$

Equation (19) thus reproduces the Mott bubble equation, (15).

If records are present, it would make absolute sense for 'observers' in q to attribute some of its properties to the 'previous existence' of q_r. It is as if configuration q_r had to 'happen' in order that q came into existence. If q has some notion of history, q_r participated in it.

When comparing relative amplitudes between possibly finding yourself in configurations q_1 or q_2, both possessing the same records, the amplitude $W(q^*, q_r)$ factors out, becoming irrelevant. We don't need to remember what the origin of the Universe was, when doing experiments in the lab.[27]

Records and conservation of probability Now, one of the main questions that started our exploration of theories that are characterized by the timeless transition amplitude, was the difficulty in defining concepts such as conservation of probability for theories such as quantum gravity, which have no fixed causal structure. Are we in a better position now? Yes.

Redefining $W(\phi^*, \phi) = \int_{\gamma \in \Gamma(\phi^*, \phi)} \mathcal{D}\gamma \exp\left[iS[\gamma(\lambda)]/\hbar\right]$ (without A), and taking $\psi(\phi) := A W(\phi^*, \phi)$ the static wave-function over configuration space, we have:

$$\int \mathcal{D}\phi\, \psi(\phi)\overline{\psi(\phi)} = A^2 \int \mathcal{D}\phi \left(W(\phi^*, \phi)\overline{W(\phi^*, \phi)} \right)$$

$$= A^2 \int \mathcal{D}\phi \left(W(\phi^*, \phi)W(\phi, \phi^*) \right) = A^2 \int \mathcal{D}\phi \underbrace{W(\phi^*, \phi^*)}_{1} = A^2 V(\mathcal{M}) \tag{20}$$

where $V(\mathcal{M})$ is the volume of configuration space according to the projection of the Liouville measure and we used the composition property of the path integral in the third equality. Thus, as long as we normalize $A = \frac{1}{V(\mathcal{M})^{1/2}}$, i.e. $\psi(\phi) = \frac{1}{V(\mathcal{M})^{1/2}}W(\phi^*, \phi)$, we get:

$$\int_{\mathcal{M}} \mathcal{D}\phi\, \psi(\phi)\overline{\psi(\phi)} = 1$$

and thus, for $\mathcal{M}_{(r)}$ the space of configurations with records of ϕ_r, for which $\psi(\phi) = \frac{1}{A}\psi(\phi_r)W(\phi_r, \phi)$ (reinstating the normalization factor A), the total quantum volume (probability) of the region is:

$$P(\mathcal{M}_{(r)}) = \int_{\mathcal{M}_{(r)}} \mathcal{D}\phi\, \psi(\phi)\overline{\psi(\phi)} \simeq \frac{|\psi(\phi_r)|^2}{A^2} \int_{\mathcal{M}_{(r)}} \mathcal{D}\phi\, |W(\phi_r, \phi)|^2 \leq |\psi(\phi_r)|^2$$

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[26] A more precise definition of records using semi-classical coarse-grainings is left for the appendix C, definition 2.

[27] But note that whenever a record exists, the preferred configuration q^* is also a record. In fact, one could have defined it as *the* record, of all of configuration space. Indeed, it does have the properties of being as unstructured as possible, which we would not be amiss in taking to characterize an origin of the Universe.

since

$$\int_{\mathcal{M}_{(r)}} \mathcal{D}\phi\, |W(\phi_r, \phi)|^2 \leq \int_{\mathcal{M}} \mathcal{D}\phi\, |W(\phi_r, \phi)|^2 = A^2 \tag{21}$$

In other words, probabilities of future events (i.e. of configurations which have a record) cannot exceed the probability of the past events (of the records).

Also, in the limit in which the amplitude $W(\phi_r, \phi)$ completely concentrates on $\mathcal{M}_{(r)}$ we obtain the equality in (21), which thus just says that the probability is conserved. If one were to restrict oneself to the semi-classical regime, this limit indicates that there there is very little volume around extremal curves from ϕ_r to ϕ for $\phi \notin \mathcal{M}_{(r)}$, relative to those curves that end in $\phi \in \mathcal{M}_{(r)}$. The weights of each extremal path are given by the Van-Vleck determinant, $\Delta_i = \frac{\delta \pi_r^i}{\delta \phi}$, where π_r^i is the initial momentum required to reach that final ϕ. Having small Van-Vleck determinant means that slight variations of the initial momentum give rise to large deviations in the final position.

Without getting into the intricacies of coarse-grainings of configuration space, I would like to speculate on a parallel with a well-known heuristic example in the study of entropy: suppose that ϕ_r contains a broken egg. If ϕ represents a configuration with that same egg[28] unbroken (still connected to ϕ_r by an extremal curve), small deviations in initial velocity of configurational change at ϕ_r will result in a a final configuration very much different (very far from) ϕ. In other words, the harder it is to "un-brake" an egg, the more the volume inequality (21) will saturate and the more accurate the conservation of probability will become. In this spirit, I would thus suggest a connection between the notion of records proposed here and a notion of an entropic arrow of Time.

The recovery of Time   I believe that indeed, it is difficult to assign meaning to some future configuration q in the timeless context. Instead, what we do, is *to compare expectations 'now', with retrodictions, which are embedded in our records, or memories.* When we have a record at q, then q_r itself acquires meaning. Accordingly, records imbue $W(q_r, q)$ with stronger epistemological status.

Furthermore, it is easy to show that when q_1 is a record of q_2 and there is a unique classical path between the two configurations, then the entire path has an ordering of records. Namely, parametrizing the path, $\gamma(t)$, such that $\gamma(0) = q_1, \gamma(t^*) = q_2$, then $\gamma(t)$ is a record of $\gamma(t')$ iff $t < t'$. This finally gives us back a complete notion of history, which is recovered only in the complete classical limit! This retrodiction aspect begs for a Bayesian treatment of probabilities, which works well (but we leave this analysis for the appendix D).

If there is nothing to empirically distinguish between our normal view of history – as having actually happened – on one hand, and the tight correlation between the present and the embedded past on the other, why should we give more credence to the former interpretation? Bayesian analysis can pinpoint no pragmatic distinction, and I see no reasons for preferring one over the other, except psychological ones.

5   What are we afraid of?

What usually unsettles people – including me – about this view is the damage it does to the idea of a continuous conscious self. The egalitarian status of each and all instantaneous configurations of the Universe – carrying on their backs our own present conscious states – raises alarms in our heads. Could it be that each instant exists only unto itself, that all our myriad instantaneous states of mind *exist separately*? This proposal appears to conflict with the narrative we have construed of our selves – of having a continuously evolving and self-determining conscious experience.

But perhaps, upon reflection, it shouldn't bother us as much as it does. First of all, the so-called Block Time view of the self does not leave us in much better shape in certain respects of this problem. After all, the general relativistic worldline does not imply an 'evolving now' – it implies a collection of them, corresponding to the entire worldline. For the (idealized) worldline of a conscious being, each element of this collection will have its own, unique, instantaneous experience.

Nonetheless, in at least one respect, the worldline view still seems to have one advantage over the one presented in this paper. Even in the classical limit – for which aspects of a one-parameter family of configurations becomes embedded in a 'now' – the presented view still appears more fractured, more disconnected, less linear than the worldline view. We start off with a one-parameter family of individual

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[28]Same for all practical purposes – its relations to the rest of the configuration consist of what we would identify as the same egg. This is supposed to be an entirely heuristic discussion, for a true study of entropy we would have to formalize the corresponding coarse-grainings in phase space.

conscious experiences, and, like Zeno, we imagine that an inverse limiting procedure focusing on the 'now' will eventually tear one configuration from the 'next', leaving us stranded in the 'now', separated from the rest of configuration space by an infinitesimal chasm. This is what I mean here by 'solipsism of the instant'. In the first subsection of this section, I will explain how this intuitive understanding can only find footing in a particular choice of (non-metric) topology for configuration space. That topology is not compatible with our starting point of applying differential geometry to configuration space.

The entirety of our intuitive dumbfoundness concerning our individual experiences however, is predicated on the belief that we know what a continuous self really means. In fact, it is not easy to put our finger on the meaning of this assumption. Moreover, there absolutely are meanings which are compatible with the notion arrived at here. This is what I discuss in the second subsection.

Zeno's paradox and solipsism of the instant: a matter of topology

It might not seem like it, but the discussion about whether we have a 'a collection of individual instants' as opposed to 'a continuous curve of instants' hinges, albeit disguisedly, on the topology we assume for configuration space. Our modern dismissal of Zeno's paradox relies on the calculus concept of a limit. But in fact, a *limit point* in a topological space first requires the notion of topology: a limit point of a set C in a topological space X is a point $p \in X$ (not necessarily in C) that can be "approximated" by points of C in the sense that every neighborhood of p with respect to the topology on X also contains a point of C other than p itself.

In the finest topology – the discrete topology – each subset is declared to be open. On the real line, this would imply that every point is an open set. Let us call an abstract pre-curve in X the image of an injective mapping from $\mathbb{R}$ (endowed with the usual metric topology) to the set X. Thus no pre-curve on X can be continuous if X is endowed with the finest topology. Because the mapping is injective, the inverse of each point of its image (which is an open set in the topology of X) is a single point in $\mathbb{R}$, which is not an open set in the standard metric topology of $\mathbb{R}$. Likewise, with the finest topology, Zeno's argument becomes inescapable – when every point is an open set, there are no limit points and one indeed cannot hop continuously from one point to the next. We are forever stuck 'here', wherever here is.

In my opinion, the idea that Zeno and Parmenides were inductively aiming at was precisely that of a discrete topology, where there is a void between any two given points in the real line. If X is taken to be configuration space, this absolute "solipsism of the instant" would indeed incur on the conclusions of the Eleatics, and frozen time would necessarily follow. However, *the finest topology cannot be obtained by inductively refining metric topologies.*

With a more appropriate, e.g. metric, topology, we can only iteratively get to open neighborhoods of a point, neighborhoods which include a continuous number of other configurations. That means for example that smooth functions on configuration space, like $P(q)$, are too blunt an instrument – in practice its values cannot be used to distinguish individual points. No matter how accurately we measure things, there will always be open sets whose elements we cannot parse.

The point being that with an appropriate topology we can have timelessness in a brander version than the Eleatics, even assuming that reality is entirely contained in configuration space without any absolute time. With an appropriate coarser (e.g. metric) topology on configuration space, we do not have to worry about a radical "solipsism of the instant": in the classical limit there are continuous curves interpolating between a record and a record-holding configuration. I can safely assume that there is a *continuous sequence* of configurations connecting me eating that donut this morning to this present moment of reminiscence. This is all that we can ask for to forbid 'gaps' in our experience.

The continuity of the self - Locke, Hume and Parfit

John Locke considered personal identity (or the self) to be founded on memory, much like my own view here. He says in "Of Ideas of Identity and Diversity":

> "This may show us wherein personal identity consists: not in the identity of substance, but [...] in the identity of consciousness. [...] This personality extends itself beyond present existence to what is past, only by consciousness"

David Hume, wrote in " A Treatise of Human Nature" that when we start introspecting, "we are never intimately conscious of anything but a particular perception; man is a bundle or collection of different perceptions which succeed one another with an inconceivable rapidity and are in perpetual flux and movement.".

Indeed, the notion of self, and continuity of the self, are elusive upon introspection. I believe, following Locke, that our self is determined biologically by patterns in our neural connections. Like any other physical structure, under normal time evolution these patterns are subject to change. What we consider to be a 'self' or a 'personality', is inextricably woven with the notion of continuity of such patterns in (what we perceive as) time. Yes, these patterns may change, but they do so continuously. It is this continuity which allows us to recognize a coherent identity.

In Reasons and Persons, Derek Parfit puts these intuitions to the test. He asks the reader to imagine entering a "teletransporter" a machine that puts you to sleep, then destroys you, copying the information of your molecular structure and then relaying it to Mars at the speed of light. On Mars, another machine re-creates you, each atom in exactly the same relative position to all the other ones. Parfit poses the question of whether or not the teletransporter is a method of travel – is the person on Mars the same person as the person who entered the teletransporter on Earth? Certainly, when waking up on Mars, you would feel like being you, you would remember entering the teletransporter in order to travel to Mars, you would also remember eating that donut this morning.

Following this initial operation, the teletransporter on Earth is modified to not destroy the person who enters it. Each replica left on Earth would claim to be you, and also remember entering the teletransporter, and then getting out again, still on Earth. Using thought experiments such as these, Parfit argues that any criteria we attempt to use to determine sameness of personal identity will be lacking. What matters, to Parfit, is simply what he calls "Relation R": psychological connectedness, including memory, personality, and so on.

This is also my view, at least intellectually if not intuitively. And it applies to configuration space and the general relativistic worldline in the same way as it does in Parfit's description. In our case there exists a past configuration, *represented* (but not contained) in configuration 'now' in the form of a record. This past configuration has in it neural patterns that bear a strong resemblance to neural patterns contained in configuration 'now'. Crucially, these two configurations are connected by *continuous extremal paths in configuration space*, ensuring that indeed we can act as if they are psychologically connected. We can, and should, act as if one classically evolved from the other; our brain states are consistent with the evolved relations between all subsystems out there in the world that we can access. Furthermore, I would have a stronger Relation R with what I associate with future configurations of my (present) neural networks, than to other brain configurations (e.g. associated to other people). There seems to be no further reason for this conclusion to upset us, beyond those reasons that already make us uncomfortable with Parfit's thought experiment.

6   Conclusions

The idea of timelessness is certainly counter-intuitive.

But our own personal histories can indeed be pieced together from the static landscape of configuration space. Such histories are indiscernible from, but still somehow feel less real than our usual picture of our pasts. Even more than the worldline view of the self, the individual existence of *every* instant still seems to leave holes in the integrity of our life histories. This feeling is due to our faulty intuitions about the topology of configuration space.

Nonetheless, even after ensuring mathematical continuity of our notion of history, the idea of time-lessness and of *all* possible states of being threatens the ingrained feeling that we are self-determining – since all these alternatives exist timelessly, how do we determine our future? But this is a hollow threat. Forget about timelessness; free will and personal identity are troublesome concepts all on their own, we should not fear doing them damage. I like to compare these concepts to mythical animals: Nessie, Bigfoot, unicorns and the like. They are constructs of our minds, and – apart from blurry pictures – shall always elude close enough inspection. Crypto-zoologists notwithstanding, Unicorns are not an endangered species. We need not be overly concerned about encroaching on their natural habitat.

Crippling Time

"Time does not exist. There is just the furniture of the world that we call instants of time. Something as final as this should not be seen as unexpected. I see it as the only simple and plausible outcome of the epic struggle between the basic principles of quantum mechanics and general relativity. For the one – on its standard form at least – needs a definite time, but the other denies it. How can theories with such diametrically opposed claims coexist

peacefully? They are like children squabbling over a toy called time. Isn't the most effective way to resolve such squabbles to remove the toy?"[6]

Loosely following the Eleatic view of the special ontological status of the present, here we have carved Time away from spacetime, being left with timeless configuration space as a result. If Time is the legs which carries space forward, we might seem to have emerged from this operation with a severely handicapped Universe.

The criticism is to the point. Even if Time does not exist as a separate entity in the Universe, our conception of it needs to be recovered somehow. If there is no specific variable devoted to measuring time, it needs to be recovered from relational properties of configurations. This essay showed that this can be done.

Rehabilitating Time The plan was to recover Time by using a semi-classical approximation of a fundamentally timeless quantum mechanics theory in configuration space. Mathematically, before all else we need to find a way to breath life into the configuration space propagator, $W(q_1, q_2)$, by defining a second configuration other than any present configuration, 'now'.

This first step was accomplished by defining a preferred configuration q^*, playing a role similar to the quantum mechanical 'vacuum'. It is preferred in that it is the most structureless point of reduced configuration space, and it is also the configuration representing the " pointiest corner" of configuration space. The precise choice of q^* depends both on the topology of the spatial manifold M and on the relational symmetry group at hand. For the case of $M = S^3$ and the symmetry group $\text{Diff}(M) \ltimes \mathcal{C}$, q^* is represented by (the orbit of) the round metric on S^3.

With the introduction of the preferred 'in' configuration, we defined a positive scalar density (a volume form, or probability density) on configuration space, $P(q)\mathcal{D}q$, given by the transition probability from the vacuum to the given configuration. Restrictions of locality (see [15]) and factorization properties (required to make $V(q)$ compatible with the later introduction of records), limit our choices, leading us to conjecture that we can recover the Born form, $P(q) = |W(q^*, q)|^2$.

But in any case, this state of affairs is still not completely satisfactory. The number $|W(q^*, q)|^2$, which makes reference to some quantum mechanical vacuum state, or " preferred initial configuration", is too removed from everyday practice of physics, and it seems to say nothing about why I believe I really had a donut this morning. To get around this, we need the emergence of records which are more local (in configuration space). In the semi-classical path integral representation, there indeed exists such a candidate object to play this role. We have named this object a *semi-classical record* (or just a record here).

If q_r is the record possessed by the configuration 'now', q, then $W(q_r, q)$ can encode our immediate pasts, through a correlation of amplitudes. *The probability of q becomes the conditional probability of q given q_r, $P(q) = P(q|q_r)P(q_r)$.* Time emerges as something that we infer from the present. Its emergence requires a shift from the notion of "state" (or spacetime) to that of "process", between a record and the present. Since all subsystems of the configuration evolve to the same tune along an extremal path, relations between these subsystems may be intelligible and consistent.

This timeless picture requires a Bayesian approach to science, which I briefly discuss in the appendix D. In certain circumstances, all of the configurations between q_r and q will themselves define an ordering of records, completely reproducing our notion of continuous history. In such cases, each previous run of an experiment is encoded in each posterior run. This feature gives us access to frequentist approaches to probabilities, even though we only have access to configuration 'now'. The equations emerging for a Bayesian treatment for the fitness of a given theory are identical to the usual, timeful ones, as was shown in [13].

The more records a certain configuration has, the more data one has to test their theories. Consistency of multiple records within a given configuration increases our level of confidence in a theory, and inconsistency decreases it, as expected. The more such types of consistent structure a configuration has, the more we will have perceived time to have passed. In other words, an arrow of time points from q^* – the most symmetric configuration – to ones that have consistent records.

Regarding duration, unlike GR – where proper time is intrinsic to the worldline itself – duration needs to be defined as a measure of local change in conformal geometry. [29]

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[29]Not only is this possible, but it has been done for one theory that is not intrinsically formulated in spacetime. In [29, 24], it was shown that weak matter perturbations evolving through standard unitary Hamiltonian evolution in shape dynamics, will perceive conformal geometry change – or duration – in the exact proportion to rebuild an Einstein spacetime (in a particular foliation, called CMC).

We have gone from the picture of a Universe that limps, to one that lilts, conducted by the complex structures present in configuration space.

Relationalism and Laws of the Instant Looking for a theory that will allow a more natural description of unitarity, probability, and superposition, we are led to require of it no external input other than what is contained in configuration space itself. Interestingly, this demand puts severe restrictions on the types of symmetries that can exist. Namely, we only allow those symmetries whose action depends solely on the configuration on which it is acting. The ones that obey these restrictions we call 'Laws of the Instant".

Had we allowed symmetries which are not 'Laws of the Instant', the transition amplitude would not have been a well-defined object by itself, and records would not be invariant under such symmetries, thus losing objective meaning. Fortuitously, we find that the most general Laws of the Instant are those that indeed embody the full gamut of relational symmetries.

I want to repeat once more that the theory of general relativity does not accommodate all manifestations of spacetime relationalism. In particular, it does not incorporate relationalism of scale: the theory is not conformally invariant (unlike its unitarily-challenged cousin, conformal gravity). In a 3+1 formulation, general relativity still does not respect the Laws of the Instant; refoliations are not intrinsic to configuration space.

Spatially relational theories also have better control over questions of unitarity (unlike conformal gravity), and have the correct number of degrees of freedom (unlike Horava gravity), although we have said little in these directions here. Not to mention that configuration space only has a principal fiber bundle structure for symmetry groups which are 'Laws of the Instant', and this is a useful structure to have for explicitly writing the path integral [10].

Actions which represent completely spatially relational theories would be the ones suggested by the principles expounded on in this paper. The challenge is to find ones which are also in accord with experiment. From these principles, the derivation of shape dynamics, with its non-local Hamiltonian and preferred Time, is without a doubt overly contrived, and requires a detour through general relativistic territory. In the future, we need to investigate the phenomenology of the more natural relational model, given by (35) in the appendix. That is the classical theory suggested by the principles expounded on in this paper.[30]

What gives, Wheeler's quip or superpositions? Neither, really.

Perhaps our shortcomings in the discovery of a viable theory of quantum gravity are telling us that *spacetime* is the obstacle. Though at first sight we are indeed mutilating the beautiful unity of space and time, this split should not be seen as a step back from Einstein's insights. I believe the main insight of general relativity, contained in Wheeler's sentence (1), is about the dynamism of space and time themselves. There is no violence being done to this insight here.

Spatial geometry appears dynamic – it warps and bends throughout evolution whenever we are in the classical regime. Regarding the dynamism of Time, the notion of 'duration' *is* emergent from relational properties of space. Thus duration too, is dynamic and space-dependent.

Nonetheless, all relational properties are encoded in the *static* landscape of configuration space. The point is that this landscape is full of hills and valleys, dictated by the preferred volume form that sits on top of it. From the way that the volume form distributes itself on configuration space, certain classical field histories – special curves in configuration space – can give a thorough illusion of change. I have argued that this illusion is indistinguishable from how we perceive motion, history, and time.

Moreover, with regards to the quantum mechanics adage, the processes $W(q_r, q)$ straightforwardly embody "everything that can happen, does happen". The concept of superposition of causal structures (or even that of superposition of geometries), is to be replaced by interference between paths in configuration space. Those same hills and valleys in configuration space that encode classical field histories reveal the valleys and troughs of interference patterns. A very shallow valley around a point – for example representing an experimental apparatus *and* a fluorescent dot on a given point on a screen – indicates the scarcity of observers sharing that observation. By looking at the processes between records and record-holding configurations, we can straightforwardly make sense of interference, or lack thereof, between (coarse-grained) histories of the Universe.

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[30] The most general local model obeying the Laws of the Instant which are second order in time derivatives also can admit a potential term which is a function of the Chern-Simons functional. Thus (35) is the simplest action without a potential, i.e. that can be written as a geodesic in conformal superspace [10]. Unfortunately for the program, it seems likely that it will yield different experimental predictions than general relativity.

In all honesty, I don't know if formulating a theory in which space and time appear dynamical, and in which we can give precise meaning to superpositions of alternative histories, is enough to quantize gravity. Although the foundations seem solid, the proof is in the pudding, and we must further investigate tests for these ideas.

But I also don't believe that dropping Time from the picture is abdicating hard-won knowledge about spacetime. Indeed, we can recover a notion of history, we can implement strict relationalism, we transfigure the 'measurement problem', and we can make sense of a union of the principles of quantum mechanics and geometrodynamics.[31]

It seems to me that there are many emotions against this resolution, but very few arguments; as I said at the beginning of these conclusions, accepting timelessness is deeply counter-intuitive. But such a resolution would necessarily change only how we view reality, while still being capable of fully accounting for how we experience it. The consequences for quantum gravity still need to be unraveled. Even at the classical level, from our search for a natural action embodying timelessness we were led to (35), a theory that now needs to be phenomenologically investigated. But this whole approach should be seen as a framework, not as a particular theory. And indeed, in the non-relativistic regime of quantum mechanics, we are not looking for new experiences of reality, but rather for new ways of viewing the ones we can already predict, a new framework to interpret them with. This is the hallmark of a philosophical insight, albeit in the present case one heavily couched on physics. As Wittgenstein once said: "Once the new way of thinking has been established, the old problems vanish; indeed they become hard to recapture. For they go with our way of expressing ourselves and, if we clothe ourselves in a new form of expression, the old problems are discarded along with the old garment."

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APPENDIX

A   Timeless quantum mechanics, the canonical theory

Configuration space, $\mathcal{M}$, is coordinitized by q^a, for $a = 1, \cdots, n$. An observation yields a complete set of q^a, which is called an event.

$\Omega = T^*\mathcal{M}$ is the cotangent bundle to configuration space, with coordinates q^a and their momenta p_a. The classical dynamics is fully determined once one fixes the Hamiltonian constraint surface in Ω, given by $H = 0$, where $H : \Omega \to \mathbb{R}^k$ is the Hamiltonian of the system. If $k = 1$, then we have a single Hamiltonian constraint, whose action only generates reparametrizations of curves in phase space, if $k > 1$, then we have further gauge-invariance.

A curve $\gamma \in \mathcal{M}$ is a physical motion connecting the events q_1^a and q_2^a if there exists an *unparametrized* curve $\bar{\gamma}$ in $T^*\mathcal{M}$ such that the following action is extremized:

$$S[\bar{\gamma}] = \int_{\bar{\gamma}} p_a dq^a \tag{22}$$

for curves lying on the constraint surface $H(q^a, p_a) = 0$, and are such that $\bar{\gamma}$'s projection to $\mathcal{M}$ connects q_1^a and q_2^a. By parametrizing the curve with a parameter τ, we get the familiar form:

$$S[\bar{\gamma}] = \int d\tau \left(p_a \dot{q}^a - N_i(\tau) H^i(q^a, p_a) \right) \tag{23}$$

where the N^i are Lagrange multipliers.

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[31] Perturbative techniques of course still need to be employed, even in the semi-classical limit, to make sense of the weights Δ in (16). This, and other issues to do with renormalizability are left for future study.

One can now define the fundamental transition amplitudes between configuration eigenstates:[32]

$$W(q_1, q_2) := \langle q_1 | \hat{P} | q_2 \rangle \tag{24}$$

where $\hat{P}$ is the "evolution operator":

$$\hat{P} := \int d\tau \, e^{-i\tau \hat{H}} \tag{25}$$

where $\hat{H}$ is the canonically quantized (with Weyl ordering) Hamiltonian. Note that since one integrates over all τ, the projector is parametrization independent. To obtain physical states, one need still use the quantization of the constraints, as in (7):

$$\hat{H} |\psi\rangle = 0 \tag{26}$$

We will for now implicitly consider the case of a single constraint, $H : \Omega \to \mathbb{R}$.

Given two regions in configuration space, R_1, R_2, we have that the probability of an observation in R_2 given an observation in R_1 is:

$$P(R_1, R_2) = \left| \frac{W(R_1, R_2)}{\sqrt{W(R_1, R_1)} \sqrt{W(R_2, R_2)}} \right|^2 \tag{27}$$

where

$$W(R_1, R_2) = \int_{R_1} dq_1 \int_{R_2} dq_2 \, W(q_1, q_2)$$

Standard non-relativistic quantum mechanics through deparametrizable systems

If one can single out a degree of freedom to parametrize motion on a whole region of configuration space, we can write $q^a = (t, q^i)$, in which case one gets a momenta conjugate to time and writes $H(t, q^i, p_t, p_i) = p_t + H_o(t, q^i, p_i)$. In this case, by inserting a decomposition of the identity in terms of eigenstates p_t and E of $\hat{p}_t$ and $\hat{H}_o$, one obtains:

$$W(q_1^a, q_2^a) = W(t_1, q_2^i, t_2, q_2^i) = \int dE \, e^{-iE(t_1 - t_2)} \langle q_1^i | E \rangle \langle E | q_2^i \rangle = G(t_1, q_2^i, t_2, q_2^i) \tag{28}$$

where $G(t_1, q_2^i, t_2, q_2^i)$ is the usual transition amplitude in quantum mechanics.[33]

Gauge transformations

The presence of more than one constraint, i.e. $H : \Omega \to \mathbb{R}^k$, indicates further gauge symmetries of the system. In this case we must impose all of the respective equations (26) simultaneously, which generally is difficult. When the the commutation of the constraints form a true Lie Algebra:

$$[\hat{H}^i, \hat{H}^j] = f^{ij}{}_k \hat{H}^k, \tag{29}$$

i.e. when $f^{ij}{}_k$ have no phase space-dependence, different methods can be used to find the projection onto the physical states, the most straightforward of which is called 'group-averaging' [31]. This consists in integrating over the group:

$$|\Psi\rangle = \int_G d\mu(U) \hat{U} |\psi\rangle$$

where $d\mu$ is the Haar measure, which is translation invariant. In the more general case, this technique will in general incur in anomalies.[34]

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[32] As much as possible, I want to avoid technicalities which won't be required here. Having said this, formally one would have had to define the so-called kinematical Hilbert space $\mathcal{K}$ for the quantum states over $\mathcal{M}$ by using a Gelfand triple over $\mathcal{M}$ with measure $d^d q^a = dq^1 \cdots dq^d$, i.e. $\mathcal{S} \subset \mathcal{K} \subset \mathcal{S}'$. This is not necessary in my case, because we will not require a Hilbert space, as we will see.

[33] One should be careful to note however, that in standard non-relativistic quantum theory, time is not an operator, and thus $\Delta t = 0$, i.e. measurements are made at a specific instant. Thus, although at the level of transition amplitudes, equation (24) for deparametrizable systems reproduces $G(t_1, q_2^i, t_2, q_2^i)$, the probabilities for measurements performed with some inaccuracy Δt in the time variable need not match. The basic reason is that according to (27), one sums over the transition amplitudes first, and then one takes the squared norm. Thus they are summed interferentially. For standard quantum mechanics with time dependence, one takes the squared norm at each instant and then integrates over the time taken by the measurement. The temporal resolution Δt necessary for a good agreement between the two theories was studied in [30] for simple systems.

[34] In the more general case one should use BRST techniques, but in the Lie groupoid case – i.e. when the structure constants depend on the fields, $f^{ij}{}_k(q, p)$ – is still much more problematic, as one has a BRST charge that is not (usually) of rank 1 in ghost momenta [32].

B Relational symmetries and laws of the instant

In the case at hand, suppose that the transformations in phase space are given by a Hamiltonian vector field, associated to a smeared functional $F[g, \pi, \eta]$, polynomial in its variables. For this to have an action on configuration space that is independent of the momenta, $F[g, \pi, \eta]$ must be linear in the momenta. This already severely restricts the forms of the functional to

$$F[g, \pi, \eta] = \int F_1(g, \eta)_{ab} \pi^{ab}$$

A Poisson bracket here results in

$$\{F[g, \pi, \eta_1], F[g, \pi, \eta_2]\} = \int d^3x \left(\frac{\delta F_1(g, \eta_1)_{ab}}{\delta g_{cd}} \pi^{ab} F_1(g, \eta_2)_{cd} - \frac{\delta F_1(g, \eta_2)_{ab}}{\delta g_{cd}} \pi^{ab} F_1(g, \eta_1)_{cd} \right)$$

where $F_1(g, \eta)_{ab}$ must be a covariant tensor of rank two.

If F_1 has no derivatives of the metric, it will straightforwardly commute. But with no derivatives the only objects we can form are:

$$F_1(g, \eta)_{ab} = \eta g_{ab} \;\;, \text{ and } \;\; F_1(g, \eta)_{ab} = \eta^{ab} g_{ab}$$

In the first case, these are just conformal transformations, in the second, they would imply that $\pi^{ab} = 0$, a constraint killing any possibility of dynamics, which is still consistent (also consistent with the strictly Eleatic Universe).

The point now is to show that only with one derivative – which implies a Lie derivative for a covariant object – they still weakly commute. With more derivatives of the metric, the conjecture is that one does not close the algebra. For example,

$$F_1(g, \eta)_{ab} = \eta(\alpha R_{ab} + \beta R g_{ab})$$

it is straightforward but tedious to show that the algebra does not close for any values of α and β. If one instead chose a term of the form $\beta R \eta_{ab}$ one can show that the rank of this constraint is not constant along phase space. Furthermore, it implies that almost everywhere $\pi^{ab} = 0$, as before. The conjecture is that these conclusions hold order by order in number of derivatives of the metric.

One could also impose a more stringent definition of 'law of the instant' , as acting independently on both metric and momenta. Of course, the proof is much more straightforward then, yielding:

$$(g_{ab}, \pi^{ab}) \to \left(\exp\left(\int d\tau \, \xi(\tau) \right) \cdot g_{ab} \, , \exp\left(\int d\tau \, \xi(\tau) \right) \cdot \pi^{ab} \right) \tag{30}$$

where $\exp \int d\tau \, \xi(\tau)$ represents the flow of the vector field part of $\xi(t)$ and the usual exponential of an infinitesimal Weyl transformation, and the $\cdot$ denotes the respective group action. On the metric, this action is by pull-back for the diffeomorphisms and pointwise multiplication for Weyl, and for the momenta it is by inverse of the push-forward for the diffeomorphisms and the inverse scalar multiplication for Weyl [?].

The further issue with the refoliation constraint, is that it only forms a true equivalence relation on-shell. That is, suppose $g_{ab}^2 \sim g_{ab}^1$ and $g_{ab}^2 \sim g_{ab}^3$ according to some initial metric velocities $\dot{g}_{ab}^1$, and $\dot{g}_{ab}^3$, and lapses $\lambda_1(t)$ and $\lambda_2(t)$, respectively. Call the curve that solves the equations of motion for the metric (with zero shift) between g_{ab}^2 and g_{ab}^1 with these conditions, $\gamma_1(t)$, and $\gamma_2(t)$ for the curve obeying the analogous conditions between g_{ab}^2 and g_{ab}^3, respectively. Since $g_{ab}^1 \sim g_{ab}^3$ only if there exists a solution curve connecting the two, we would only have the transitive property if the opposite of the initial momenta of the solution curve from g_{ab}^2 to g_{ab}^1, at g_{ab}^2 is the same as the initial momenta for the curve connecting g_{ab}^2 and g_{ab}^3. I.e. if

$$-\frac{d}{dt}_{|t=0} \mathcal{T}_{\lambda_t^1} g_{ab}^2(x) = -\lambda_1'(x) \pi_{ab}^1(x) = \frac{d}{dt}_{|t=0} \mathcal{T}_{\lambda_t^2} g_{ab}^2(x) = \lambda_2'(x) \pi_{ab}^2(x)$$

This is what is meant by saying that the Hamiltonian constraint in ADM gravity [1] generates space-time refoliations only on-shell. It means we can only have an equivalence relation under very special circumstances – of equality between fields which do not depend exclusively on the configurations $g_{ab}^1, g_{ab}^2, g_{ab}^3$ themselves. This conclusion also has implications for the locality of the field theory, as discussed in [15].

B.1 Configuration space metrics

Superspace One nice thing about the space of metrics is that itself comes with a supermetric, defined, for $v, w \in T_g \mathcal{M}$, at the base point $g_{ab} \in \mathcal{M}$ by:[35]

$$\langle v, w \rangle_g = \int d^3x \sqrt{g}\, g^{ac} g^{bd} v_{ab} w_{cd} \tag{31}$$

This supermetric induces a metric topology on $\mathcal{M}$.[36] Furthermore, the inner product (31) is invariant wrt to diffeomorphisms acting through pull-back. That is, the directions along the diffeomorphism orbits in $\mathcal{M}$ are Killing wrt the metric (31) (see [33]).

Conformal Superspace However, the supermetric (31) is not invariant wrt conformal transformations, because of the presence of $\sqrt{g}$. Thus we construct:

$$\langle v, w \rangle_g = \int d^3x \sqrt{g}\, \sqrt{C^{ef} C_{ef}}\, g^{ac} g^{bd} v_{ab} w_{cd} \tag{32}$$

where C_{ab} is the Cotton-York tensor, which is both traceless and transverse, and has the correct conformal weight for the inner product to be conformally invariant.

B.1.1 Examples of relational conformal geoemetrodynamical theories

Let me exemplify the constructions above with two different actions. The first is that of shape dynamics, and is given in Hamiltonian form by

$$H_{SD} = \int \sqrt{g}\left((e^{6\phi_o} - \rho \pi^{ab} g_{ab} - \pi^{ab} \mathcal{L}_\xi g_{ab} \right) \tag{33}$$

where $\mathcal{L}_\xi$ denotes the Lie derivative, and ϕ_o is defined implicitly from the modified Lichnerowicz-York equation [34]:

$$e^{-6\phi} \frac{\pi_{ab}\pi^{ab}}{\sqrt{g}} + \sqrt{g}\left(e^{6\phi}(-(t^2/6) + 2\Lambda) + e^{2\phi}(-R + 8(\nabla_a \phi \nabla^a \phi + \nabla^2 \phi)) \right) = 0 \tag{34}$$

The problem with (33) is that it can be put in this form only when it is deparametrizable (see [26]). Otherwise, one must use not the full group of scale transformations as symmetries, but only those that preserve the *total* volume of space. I.e. instead of $\rho \pi^{ab} g_{ab}$ one must use $(\rho - \langle \rho \rangle)\pi^{ab} g_{ab}$ where

$$\langle \rho \rangle = \frac{\int \sqrt{g} \rho}{\int \sqrt{g}}$$

This is a non-local restriction, and is hard to make sense of from a purely relational manner.

The second action is given in its Lagrangian Jacobi form (see [34, 10]): We have the reparametrization and conformal-diffeomorphism invariant geodesic action:

$$S = \int dt \sqrt{\int_M \sqrt{g} d^3x \sqrt{C^{ab}C_{ab}}\, (\dot{g}^{cd} - (\mathcal{L}_\xi g)^{cd} - \rho g^{cd})(\dot{g}_{cd} - (\mathcal{L}_\xi g)_{cd} - \rho g_{cd}))} \tag{35}$$

where ξ^a and ρ are the Lagrange multipliers corresponding to diffeomorphisms and conformal transformations, respectively.

To stress, this is a fully conformal diffeomorphism invariant action with the same physical degrees of freedom as general relativity, but which does *not* have local refoliation invariance, only a global reparametrization one. Equation (35) is furthermore a purely geodesic-type action in Riem, with just one global lapse and thus one global notion of time, as such it also possesses inherent value in a relationalist setting. Classical solutions are one-parameter collections of conformal geometries, which extremize the

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[35] In fact, it comes with a one-parameter family of supermetrics, where we substitute $g^{ac}g^{bd} \to g^{ac}g^{bd} + \lambda g^{ab}g^{cd}$. This supermetric however is only positive for $\lambda > -1/3$. Note also that we are not symmetrizing the DeWitt supermetric because we are assuming that it is acting on tangent vectors of $\mathcal{M}$, which are already symmetric.

[36] More formally, one would work with what is called a weak Whitney topology, which roughly is a norm on the jet bundles of the sections. We will ignore these more formal aspects here.

total length according to a given supermetric (32). Although this theory is completely relational – it is not yet clear whether it will reproduce standard tests of general relativity, and more analysis is required.

In [10] we formulate the theory in a given parameter gauge given by arc-length (in superspace metric). In this gauge, one can ask: are the propagation equations for fields hyperbolic? I.e. do they contain the same number of space and time derivatives. If they don't, then the velocity of a such a field can depend on the velocity of the source, and the energy wrt the source. In the case of that exemple, the equations for electrogmanetism seem to be constrained to be hyperbolic, but not the gravitational ones, it seems, at least in this gauge. In any case, I think there are two distinct questions: can one somehow detect a preferred reference frame? To which I believe the answer is "no", if we have no time but only abstract duration from change. And a second question is: do the relative velocity of massless fields depend on their energy relative to source and sink? These are all matters that require further investigation.

As a last comment, I address a usual concern: we never know our metric configuration with infinite precision. That is true. To deal with this, one should use not the usual configuration space, but an effective one. For instance, one which only takes into account eigenmodes of the Laplacian at each point above a certain cut-off. Of course, one should then use the effective action to that scale, as opposed to the bare action above. These issues are briefly discussed in [15].

C Preferred coarse-grainings and records

The following definitions are taken from [13]. They assume that configuration space (or reduced configuration space) possesses a metric, with respect to which one can define tubular neighborhoods of given radii around non-intersecting paths. $\Gamma(\phi^*, \phi)$ is the space of unparametrized paths between ϕ^* and ϕ.

A *tubular neighborhood* of a given path γ in $\mathcal{M}$ is roughly a small enough tube around γ such that the tube doesn't self-intersect. More precisely, a tubular neighborhood of a submanifold L embedded in a Riemannian manifold N is a diffeomorphism between the normal bundle of L and an open set of N, for which the zero section reduces to the identity on L. One first defines the exponential map (that the exponential map is still well defined in the infinite-dimensional setting is shown in [33]), $\mathrm{Exp} : T\mathcal{M} \to \mathcal{M}$. Restricting the exponential map to the normal bundle of γ (i.e. to act only on vectors orthogonal to γ'), $\mathrm{Exp} : (T\gamma)^\perp \subset \mathcal{M} \to \mathcal{M}$, since γ is compact, one can always find a maximum radius $\rho_{\max}$ such that there are no self-intersections of the tubular neighborhood, i.e. Exp is a diffeomorphism between $(T\gamma)^\perp_{\rho_{\max}}$ (normal vectors with maximal length $\rho_{\max}$) and its image in $\mathcal{M}$.

Defining coarse–grained histories between two configurations ϕ^* and ϕ_f, requires us to first define a collection of subsets of $\Gamma(\phi^*, \phi_f)$, $\{C_\alpha, \alpha \in \Lambda\}$. The amplitude kernel for C_α is:

$$W_\alpha := \int_{C_\alpha} \mathcal{D}\gamma(t) e^{-iS[\gamma(t)]/\hbar} \tag{36}$$

Using the configuration space metric (which defines extremal paths as geodesics), we have the orthogonal plane $P := (T\gamma_{\mathrm{cl}})^\perp$. A basis for all the deviation vector fields from the classical path, $X \in \mathbb{X}$, can be formed by all the vectors in P.

Thus, for each deviation vector field $X \in P$ from γ_{cl}, we can form the one-parameter family of paths $\gamma_X(u)$ given by

$$\gamma_X(u, t) := \gamma_{\mathrm{cl}}(t) + \mathrm{Exp}_{\gamma_{\mathrm{cl}}(t)}(uX(t)) \tag{37}$$

where $X(t) \in P_{\gamma_{\mathrm{cl}}(t)}$. We define the set of coarse-grained paths seeded by $\gamma_{\mathrm{cl}}^\alpha$ with radius ρ as:

$$C_\alpha^\rho = \{\gamma_X(u) \mid u \leq \rho\} \tag{38}$$

this is a particular set of paths; not defined by "all those that don't enter the region". It thus avoids criticisms of Halliwell regarding the definition of coarse-grainings by "crossing properties of regions of space-time (here, configuration space)", [?].

We already have a $\rho_{\max}$ given above by the no-intersection criterion. Now, to define the minimum one, we consider the partial transition amplitude:

$$W_{\alpha,\rho}(\phi_i, \phi_f) = \int_{C_\alpha^\rho} \mathcal{D}\gamma \exp\left[iS[\gamma]/\hbar\right] \tag{39}$$

Now, by the results of [35], if there is a single extremal path γ_{cl} between ϕ_i, ϕ_f, there exists a small enough radius such that

$$W_{\alpha,\rho}(\phi_i, \phi_f) \approx \Delta_{\gamma_{\mathrm{cl}}} \exp\left[iS[\gamma_{\mathrm{cl}}]/\hbar\right] \approx W(\phi_i, \phi_f) \tag{40}$$

where the approximation works up to orders of $\hbar^2$. Now, of course, for a large enough ρ, the partial amplitude stops approximating the full amplitude. It is no longer a good sampling of all the paths, e.g. the exponential might even cease being a local diffeomorphism. It might not be able to detect a second extremal path away from $\gamma_{\rm cl}$ for example, since it does not comprise all possible variations there. This is represented by the fact that for higher order of approximations one requires higher order of derivatives of $S[\gamma_{\rm cl}]$.

Since in practice we don't have access to the full path integral, only to approximations, we should be content in having not an absolute minimum radius, but an optimal radius only up to some order of approximation:

$$\rho_{\min}^{\alpha}(\epsilon) = \arg\left(\sup_{\rho \in \mathbb{R}^+}(|W_{\alpha,\rho}(\phi_i, \phi_f) - W(\phi_i, \phi_f)| < \epsilon)\right) \tag{41}$$

roughly, assuming that at some point the error starts increasing with ρ (for instance, at some point the Riemann exponential map might even cease to be a diffeomorphism), this should be the largest radius, if it exists, such that $|W_{\alpha,\rho}(\phi_i, \phi_f) - W(\phi_i, \phi_f)| \sim \mathcal{O}(\epsilon^2)$. Now we define

Definition 1 (Extremal coarse-grainings (ECs)) *An ϵ-exhaustive extremal coarse-graining for the paths in $\Gamma(\phi^*, \phi)$ is a coarse-graining $\{C_\alpha^\rho, \alpha \in I\}$ seeded in the extremal paths $\{\gamma_\alpha^{cl}, \alpha \in I\}$ between ϕ^* and ϕ. Where each C_α^ρ is given by (38), with elements $\gamma_X(u)$ given by (37) and radii given by $\rho_{min}^\alpha(\epsilon)$ in (41).*

This definition works when the lengths of the extremal paths, $S[\gamma_\alpha^{cl}] \ll \epsilon$, otherwise there is usually no such ρ. If $\rho_{\min}(\epsilon) \geq \rho_{\max}$ the coarse-graining cannot be exhaustive to the given order (ϵ) without self-intersection. See figure 1 for an illustration of extremal coarse-grainings.

Heuristically, the width of the coarse grainings around an extremal path should be wide enough so that the total amplitude is approximated to the order of some formal parameter ϵ (usually taken to be $\hbar$). For $1 \ll S_{\gamma_{\rm cl}}/\hbar$ the amplitude is carried by a tight bundle of paths around each classical path, not posing a tight constraint on the width of the coarse-graining.

Records are then defined as:

Definition 2 (Semi-classical record (single segmented)) *Given an initial configuration ϕ^*, ϕ and $\{C_\alpha\}_{\alpha \in I}$ the ϵ-extremal coarse graining (def. 1) between ϕ^* and ϕ, of radii ρ_{min}^α given in (41), we will say ϕ holds a (single-segmented) semi-classical record of a field configuration ϕ_r to order ϵ, if ϕ_r is contained in every C_α. (see figure 1 below).*
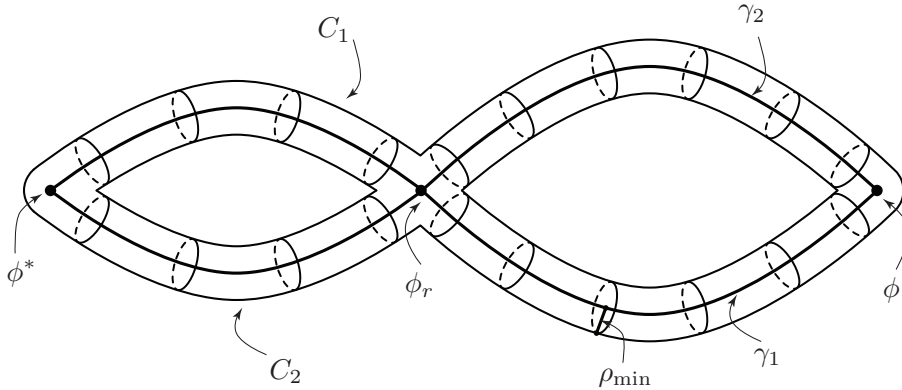

Figure 1: An extremal coarse-graining between ϕ^* and ϕ in configuration space $\mathcal{M}$, consisting of the elements C_1 and C_2, seeded by the extremal paths γ_1, γ_2, with radius $\rho_{\min}$. Here ϕ contains a single segmented semi-classical record of ϕ_r (see [13]).

With these definitions, it can be shown [13] that

Proposition 1 *For a (single-segmented) semi-classical record, we have:*

$$W(\phi^*, \phi) = W(\phi^*, \phi_r)W(\phi_r, \phi) \tag{42}$$

D   Bayesian analysis

Let us call 'an observation' E a property of correlations within a configuration. We call the manifold $\mathcal{M}_{(E_o)}$ the manifold which has records of observation E_o. Given a theory $\mathbb{T}_i$ (where i indexes the theory we are discussing), the probability of observation E_1 is given by the relative volume of observers:

$$P(E|\mathbb{T}_i) = \frac{P_i(E_1)}{P_i(\mathcal{M}_{(E_o)})} := \frac{\int_{E_1} F_i(\phi)\mathcal{D}\phi}{\int_{\mathcal{M}_{(E_o)}} F_i(\phi)\mathcal{D}\phi} \tag{43}$$

In Bayesian analysis we want to judge the ability of the theories to explain a given distribution of the observation given the theory This number is called the *'likelihood' of the theory* $\mathbb{T}_i$ given the observation of E. We want to compare the chances that we will find ourselves correlated with a "new" observation E_1, according to the two distinct theories above. Assuming that (42) holds for E_o and E_1, that F factorizes according to (13),

$$\int_{E_1} F_i(\phi)\mathcal{D}\phi = F_i(K(\phi^*, E_o)) \int_{E_1} F_i(K(E_o, \phi))\mathcal{D}\phi$$

and using Bayes rule, we can determine the posterior probability of the theory given the records E_o and observation E_1:

$$P(\mathbb{T}_i|E_1) \approx \frac{P(\mathbb{T}_i)P(E_1|\mathbb{T}_i, E_o)}{P(E_1)} \tag{44}$$

where $P(E_1|\mathbb{T}_i, E_o)$ is obtained from (43) by the replacement $F_i(\phi) \to F_i(K(E_o, \phi))$. It should be interpreted as the probability that you would find E_1 *if T_i is correct*, and you have already 'observed' E_o (it is a record). For configurations that have many records of properties of E we can – by just looking at these records – update the prior of the next (or present) such observation E_n. This is the standard way in which we test our theories, nothing needs to change.

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