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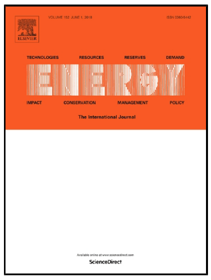

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A novel multilevel network slacks-based measure with an application in electric utility companies

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Abstract

In this paper, we developed an alternative Network Slacks-Based Data Envelopment Analysis Measure (NSBM) wherein the overall efficiency is expressed as a weighted average of the efficiencies of the individual processes. The advantage of this new model is that both overall efficiency and multi-divisional efficiencies have been calculated with a unified framework. The major merits of the proposed model are its ability to provide appropriate measure of efficiency, obtaining weight of processes from model, simultaneous assessment of intermediate variables considering them as both input and output. Finally, an application in electric power companies shows the practicality of the proposed model.

Keywords: Network DEA; Network SBM; Overall Efficiency; Divisional Efficiency.

1. Introduction

Data envelopment analysis (DEA) is a non-parametric technique for measuring the relative efficiency of peer decision making units (DMUs) that initially was proposed by Charnes, et al. (1978). DEA examine the efficiency of each DMU relative to an estimated production possibility frontier that determined by all DMUs and its advantage is that it does not require any assumption on the shape of the frontier surface. DEA goal is to classify the DMUs into two classes of efficient or inefficient ones. DEA has been recognized as a very powerful tool for measuring efficiency and productivity of Decision Making Units (Emrouznejad and Yang; 2018). The mathematical models for many applications of efficiency measurement should be formulated as multiple levels, such as network structures. Conventional Data Envelopment Analysis (DEA)

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models generally treats the decision making units (DMUs) in the network structures as a black box when measuring efficiency, and ignoring its internal structure. To overcome this problem, several network DEA models have been introduced in the literature for measuring the efficiency of DMUs according to internal structures.

DEA has been used in many areas including electricity power generation (Arabi et al, 2016; Özkara and Atak, 2015 and Lin et al, 2012) which is the case study of this paper but the conventional DEA models generally treats a DMU as black box when measuring its efficiency, and ignors its internal structure. Hence. in the traditional DEA models only inputs and outputs of the black box are considered in measuring the efficiency while intermediate products or linking activities are neglected (Bai et al., 2015, Emrouznejad and Thanassoulis, 2015).

In recent years, a number of DEA studies have focused production system with internal network structures, hence network DEA models been developed to measure the efficiency of DMUs considering their internal structures. Network DEA is an extension to traditional DEA, where in, the DMU is considered as a network of interconnected sub-stages with the connections indicating the flow of intermediate products (Despotis et al, 2016).

Liang et al. (2008), proposed a network DEA where global efficiency levels are obtained as the product of the individual efficiencies of each stage and differently from traditional DEA models, Wanke and Barros (2014) used this model for assessment of cost efficiency and productive efficiency of major Brazilian banks. Kao and Hwang (2008) applied a relational model to measure the efficiency of 24 non-life insurance companies in Taiwan, where the system efficiency have been decomposed as the product of the two process. Following this, Chen et al. (2009) proposed a network DEA model where it is capable to decompose the system efficiency into a weighted average of the two process efficiencies. Later, Wang et al. (2014) developed an additive two-stage DEA model in the presence of undesirable outputs and applied it to measuring the efficiency of the Chinese commercial banking system. Li et al. (2015) used a new DEA network model based on Virtual Frontier Network SBM that is divided into three stages to evaluate the efficiency of 22 international airlines from 2008 to 2012. Chen et al. (2010) developed a system distance function to measure the system efficiency of a basic two-stage system. One important feature of Chen et al. (2010) is its ability to find the projection of the intermediate product.

Shermeh et al. (2016) suggested a novel fuzzy network SBM model and applied to evaluate efficiency of the Iran regional power companies. Guan and Chen (2010) proposed a network DEA for measuring the innovation production process (IPP) that it provides systematic and simultaneous efficiency measures for the overall process and internal sub-processes. Tone and Tsutsui (2009) proposed a slacks-based network DEA model by identifying the production possibility sets (PPS). The main drawback of this model is that the weight of each division (sub-process) has been assigned using expert opinion, hence, these weights are assumed to be fixed and to be equal for all DMUs. However one of the main advantage of using DEA is to assign the weights based using the optimum value of the linear programming.

Many companies and organizations such as banking industry (Ebrahimnejad et al, 2014), airports (Maghbouli et al, 2014), hotels (Hsieh & Lin, 2010), insurance companies (Kao & Hwang, 2008) and etc. are formed of several divisions or processes that are connected together having division-specific inputs and outputs, that would require several interlinks between divisions or processes.

In this paper, we developed a new Network Slacks-Based Measure (NSBM) wherein the overall efficiency is expressed as a weighted average of the efficiencies of the individual stages. This approach can be applied under both constant returns to scale (CRS) and variable returns to scale (VRS) assumptions. The proposed model has two advantages. First, it has ability to provide overall measure of efficiency as well as efficiency score for each sub-process (stage). Secondly, the proposed model can calculate weight of each stages even if we have intermediate variables, e.g. variables that are output in one stage and input to the sub-sequent stage.

The rest of the paper is organized as follows. DEA model and network DEA model are briefly been presented in section 2 and 3, respectively. Section 4 discuss the proposed model. Section 5 applies the new approach to the evaluation of Iranian electric power companies. Finally, conclusion and direction for future research are given in sections 6.

2. Data Envelopment Analysis

Identifying efficient decision-making units (DMUs) plays a critical and vital role for achieving sustainable development in a competitive environment (Amado, 2012). Similarly, finding performance level of inefficient DMUs can help policy makers and regulators for providing better management strategies (Lin & Chiu, 2013). After the introduction of DEA by Charnes et al. (1978), DEA has become a popular empirical method for evaluating relative efficiency of a set of homogeneous DMUs, which utilize the same inputs to produce the same outputs.

DEA is recognized as a non-parametric frontier approaches and an excellent and robust efficiency analysis tool with a broad range of applications which practically efficient frontier was formed as the piecewise linear combination that connects the set of the best practice observations. DEA provides efficiency scores in format of the classical ratios of a weighted sum of outputs to a weighted sum of inputs (Ruiz & Sirvent, 2016). A DEA efficient frontier is not determined by some specific functional form (e.g. Cobb-Douglas production function), but formed by the actual data from the evaluated production units referred to as Decision Making Units (DMUs) (Paradi et al., 2011). The capability of dealing with multi-input and multi-output problems without requiring explicit specifications of the relationships between the inputs and outputs variable provides DEA more valuable than many other analytical tools.

3. Network Data Envelopment Analysis

Traditional DEA models does not consider internal structure, and so treat each DMU as black box, hence inputs for each DMU are converted into outputs without considering the internal procedures (Halkos et al., 2014). Network Data Envelopment Analysis (NDEA) was first introduced by Färe (1991), concerns using the DEA technique to measure the relative efficiency of a system, taking into account its internal structure (Kao, 2014). In the NDEA models DMUs regarded as a network structure, and the entire production system is divided into several sub-units (commonly called stages, divisions, sub-processes) in which intermediate products are considered as outputs of one sub-process which are then treated as inputs to the subsequent sub-processes. Hence, the results of network models become more meaningful, informative and reliable than those obtained from the conventional DEA models (Wang et al., 2014). In short, Network DEA is a nonparametric, sophisticated approach and useful technique to relative performance modeling that takes advantage of interrelated production frontiers, and captures the underlying performance information found in a firm's interacting sub-processes that would otherwise remain unknown to management (Avkiran & McCrystal, 2012).

4. A new NDEA model based on SBM

The Network DEA models that are based on the radial measure of efficiency, (e.g. the CCR (Charnes et al., 1978) or the BCC (Banker et al., 1984)) follow the assumption that inputs reduction or outputs enhancement undergo the same proportional changes. In this paper, we have developed a Slacks-Based Model (SBM) for measuring efficiency of network system as well as measuring efficiency of sub-processes of the network. The SBM is a non-radial method in which it is suggested for measuring the efficiencies when inputs and outputs may change non-proportionally. The SBM deals directly with output shortage and input surplus. Suppose *n* DMUs $(DMU_j;\ j=1,\ldots,n)$ consisting of K processes $(k=1,2,\ldots,K)$ and m_k be the numbers of inputs to process k and r_k be the numbers of output from process k. Also (k, h) denote the link leading from process k to process h. $X_{ij}^{(k)}$ denote the input resources to $\mathrm{DMU_j}$ at process k, $Y_{rj}^{(k)}$ denote the output products from $\mathrm{DMU_j}$ at process k and $Z_j^{(k,h)}$ denote the linking intermediate products from process k to process h (output from k and input to h).

In this case, the production possibility set with the constant returns-to-scale (CRS) assumptions is defined by:

$$
\begin{aligned}
&\sum_j A_j^{(k)} X_{ij}^{(k)} \le X_{i0}^{(k)}, && (k=1,\ldots,K),\\
&\sum_j A_j^{(k)} Y_{rj}^{(k)} \ge Y_{r0}^{(k)}, && (k=1,\ldots,K),\\
&\sum_j A_j^{(h)} Z_j^{(k,h)} = \sum_j A_j^{(k)} Z_j^{(k,h)}, && (\forall (k,h)),\\
&\begin{cases} \sum_j A_j^{(h)} Z_j^{(k,h)} \le Z_0^{(k,h)}, & (\forall (k,h))\ \ (as\ inputs\ to\ h),\\ \quad or & \\ \sum_j A_j^{(k)} Z_j^{(k,h)} \ge Z_0^{(k,h)}, & (\forall (k,h))\ \ (as\ outputs\ \ from\ k),\end{cases} \\
&A_j^{(k)} \ge 0, && (\forall j,k).
\end{aligned}
\tag{1}
$$

Where $A^{(k)}$ is the intensity vector corresponding to process k $(k=1,2,\ldots,K)$.

In the above model, if the linking activities are considered as inputs, then the constraints $\sum_j A_j^{(h)} Z_j^{(k,h)} \le Z_0^{(k,h)}, (\forall (k,h))$ are considered and if the linking activities are considered as outputs, then constraints of $\sum_j A_j^{(k)} Z_j^{(k,h)} \ge Z_0^{(k,h)}, (\forall (k,h))$ are included in the model. However if the links are considered either input type (the less the better) or output type (the more the better), we include the both above constraints into the non-oriented models. Hence, the above relations for DMU_0 can be written as follows:

$$
\begin{aligned}
&\sum_j A_j^{(k)} X_{ij}^{(k)} + s_i^{-(k)} = X_{i0}^{(k)}, && (k=1,\dots,k),\\
&\sum_j A_j^{(k)} Y_{rj}^{(k)} - s_r^{+(k)} = Y_{r0}^{(k)}, && (k=1,\dots,k),\\
&\sum_j A_j^{(h)} Z_j^{(k,h)} = \sum_j A_j^{(k)} Z_j^{(k,h)}, && (\forall (k,h)),\\
&\begin{cases} \sum_j A_j^{(h)} Z_j^{(k,h)} + s_h^{-(k,h)} = Z_0^{(k,h)}, & (\forall (k,h)),\\ \quad or & \\ \sum_j A_j^{(k)} Z_j^{(k,h)} - s_k^{+(k,h)} = Z_0^{(k,h)}, & (\forall (k,h)),\end{cases}\\
&A_j^{(k)},\, s_i^{-(k)},\, s_r^{+(k)},\, s_h^{-(k,h)},\, s_k^{+(k,h)} \ge 0, && (\forall j,\, i,\, r,\, k, (k,h)).
\end{aligned} \tag{2}
$$

It should be noted that under variable returns-to-scale (VRS) assumption the convexity constraint $\sum_{j=1}^{n} A_j^{(k)} = 1, (\forall k)$ should be added to the model, where $s_i^{-(k)}$ and $s_r^{+(k)}$ are the input and output slack vectors of process k, respectively. Also $s_h^{-(k,h)}$ and $s_k^{+(k,h)}$ are the slacks of the link (k, h) as input to process h and output from process k, respectively. If $t_h^{(k,h)}$ and $t_k^{(k,h)}$, respectively, be the numbers of linking activities as inputs to process h and output from process k; according to SBM model, efficiency of any process in the network system is defined as (the mean reduction rate of inputs divided by the mean expansion rate of outputs):

$$
\theta_0^k = \frac{\left[1 - \dfrac{1}{m_k + \sum_{h=2}^{K} t_h^{(k,h)}} \left(\sum_{i=1}^{m_k} \dfrac{s_i^{-(k)}}{X_{i0}^{(k)}} + \sum_{h=2}^{K} \dfrac{s_h^{-(k,h)}}{Z_0^{(k,h)}} \right)\right]}{\left[1 + \dfrac{1}{r_k + \sum_{k=1}^{K-1} t_k^{(k,h)}} \left(\sum_{r=1}^{r_k} \dfrac{s_r^{+(k)}}{Y_{r0}^{(k)}} + \sum_{k=1}^{K-1} \dfrac{s_k^{+(k,h)}}{Z_0^{(k,h)}} \right)\right]}; \tag{3}
$$

According to the proposed model in this paper, the overall efficiency in network system is expressed as a weighted average of the efficiencies of the individual processes as:

$$
\theta_0^{Global} = \sum_{k=1}^{k} w_k \theta_0^k = \sum_{k=1}^{k} w_k \cdot \frac{\left[1 - \dfrac{1}{m_k + \sum_{h=2}^{K} t_h^{(k,h)}} \left(\sum_{i=1}^{m_k} \dfrac{s_i^{-(k)}}{X_{i0}^{(k)}} + \sum_{h=2}^{K} \dfrac{s_h^{-(k,h)}}{Z_0^{(k,h)}} \right)\right]}{\left[1 + \dfrac{1}{r_k + \sum_{k=1}^{K-1} t_k^{(k,h)}} \left(\sum_{r=1}^{r_k} \dfrac{s_r^{+(k)}}{Y_{r0}^{(k)}} + \sum_{k=1}^{K-1} \dfrac{s_k^{+(k,h)}}{Z_0^{(k,h)}} \right)\right]}, \tag{4}
$$

Where w_k is the relative weight or importance of process k such that $\sum_{k=1}^{k} w_k = 1;\ w_k \geq 0\ (\forall k)$. These weights are not decision variables, but rather they are functions of the decision variables. We thus propose deriving the overall efficiency of the system by solving the following problem:

$$
\begin{aligned}
\theta_0^{Global} = Min & \left[\begin{array}{l} w_1 \cdot \dfrac{1 - \dfrac{1}{m_1}\displaystyle\sum_{i=1}^{m_1} \dfrac{s_i^{-(1)}}{X_{i0}^{(1)}}}{1 + \dfrac{1}{r_1 + \sum_{k=1}^{K-1} t_1^{(1,h)}} \left(\displaystyle\sum_{r=1}^{r_k} \dfrac{s_r^{+(1)}}{Y_{r0}^{(1)}} + \sum_{k=1}^{K-1} \dfrac{s_1^{+(1,h)}}{Z_0^{(1,h)}} \right)} + \cdots \\ + w_k \cdot \dfrac{\left[1 - \dfrac{1}{m_k + \sum_{h=2}^{K} t_h^{(k,h)}} \left(\displaystyle\sum_{i=1}^{m_k} \dfrac{s_i^{-(k)}}{X_{i0}^{(k)}} + \sum_{k=2}^{K} \dfrac{s_h^{-(k,h)}}{Z_0^{(k,h)}} \right) \right]}{\left[1 + \dfrac{1}{r_k} \left(\displaystyle\sum_{r=1}^{r_k} \dfrac{s_r^{+(k)}}{Y_{r0}^{(k)}} \right) \right]} \end{array} \right], \\
s.t. \quad & \sum_j A_j^{(k)} X_{ij}^{(k)} + s_i^{-(k)} = X_{i0}^{(k)}, \qquad (k = 1, \ldots, k), \\
& \sum_j A_j^{(k)} Y_{rj}^{(k)} - s_r^{+(k)} = Y_{r0}^{(k)}, \qquad (k = 1, \ldots, k), \\
& \sum_j A_j^{(\mathrm{h})} Z_j^{(k,h)} = \sum_j A_j^{(k)} Z_j^{(k,h)}, \qquad (\forall (k,h)), \\
& \left\{ \begin{array}{ll} \displaystyle\sum_j A_j^{(h)} Z_j^{(k,h)} + s_h^{-(k,h)} = Z_0^{(k,h)}, & (\forall (k,h)), \\ \qquad or & \\ \displaystyle\sum_j A_j^{(k)} Z_j^{(k,h)} - s_k^{+(k,h)} = Z_0^{(k,h)}, & (\forall (k,h)), \end{array} \right. \\
& A_j^{(k)},\ s_i^{-(k)},\ s_r^{+(k)}, s_h^{-(k,h)}, s_k^{+(k,h)} \geq 0, \qquad (\forall j, i,\ r,\ k).
\end{aligned} \tag{5}
$$

It is observed that model (5) cannot be turned into a linear program (LP) using the usual Charnes and Cooper (1962) theory of fractional programming. Therefore, we seek an alternative way to convert model (5) into a linear model, by appropriate choice of the weights (w_k).

Note that w_k is intended to represent the relative importance or contribution of the performances of any process, to the overall performance of the DMU. In this paper to transform the non-linear model (5) into linear model, we consider the contribution of each process from the mean expansion rate of outputs (or the contribution of each process in inefficiency of overall system) as weight of that process. Then weight of each process is defined as:

$$w_k = \frac{1 + \dfrac{1}{r_k + \sum_{k=1}^{K-1} t_k^{(k,h)}} \left(\sum_{r=1}^{r_k} \dfrac{s_r^{+(k)}}{Y_{r0}^{(k)}} + \sum_{k=1}^{K-1} \dfrac{s_k^{+(k,h)}}{Z_0^{(k,h)}} \right)}{\sum_{k=1}^{K} \left[1 + \dfrac{1}{r_k + \sum_{k=1}^{K-1} t_k^{(k,h)}} \left(\sum_{r=1}^{r_k} \dfrac{s_r^{+(k)}}{Y_{r0}^{(k)}} + \sum_{k=1}^{K-1} \dfrac{s_k^{+(k,h)}}{Z_0^{(k,h)}} \right) \right]}. \tag{6}$$

The advantage of this model is that the weights are not assigned by decision makers. The proposed model lets each DMU to choose the most desirable weights as long as the efficiency of the overall system is maximized. Hence, under CRS assumptions, non-linear model (5) is written as follows:

$$\begin{aligned}
\theta_0^{Global} = Min\ & \frac{\sum_{k=1}^{K} \left[1 - \dfrac{1}{m_k + \sum_{h=2}^{K} t_h^{(k,h)}} \left(\sum_{i=1}^{m_k} \dfrac{s_i^{-(k)}}{X_{i0}^{(k)}} + \sum_{h=2}^{K} \dfrac{s_h^{-(k,h)}}{Z_0^{(k,h)}} \right) \right]}{\sum_{k=1}^{K} \left[1 + \dfrac{1}{r_k + \sum_{k=1}^{K-1} t_k^{(k,h)}} \left(\sum_{r=1}^{r_k} \dfrac{s_r^{+(k)}}{Y_{r0}^{(k)}} + \sum_{k=1}^{K-1} \dfrac{s_k^{+(k,h)}}{Z_0^{(k,h)}} \right) \right]}, \\
s.t.\quad & \sum_j A_j^{(k)} X_{ij}^{(k)} + s_i^{-(k)} = X_{i0}^{(k)}, \qquad (k = 1,\ldots,k), \\
& \sum_j A_j^{(k)} Y_{rj}^{(k)} - s_r^{+(k)} = Y_{r0}^{(k)}, \qquad (k = 1,\ldots,k), \\
& \sum_j A_j^{(\mathrm{h})} Z_j^{(k,h)} = \sum_j A_j^{(k)} Z_j^{(k,h)}, \qquad (\forall (k,h)), \\
& \begin{cases} \sum_j A_j^{(h)} Z_j^{(k,h)} + s_h^{-(k,h)} = Z_0^{(k,h)}, & (\forall (k,h)), \\ \qquad or & \\ \sum_j A_j^{(k)} Z_j^{(k,h)} - s_k^{+(k,h)} = Z_0^{(k,h)}, & (\forall (k,h)), \end{cases} \\
& A_j^{(k)},\ s_i^{-(k)},\ s_r^{+(k)}, s_h^{-(k,h)}, s_k^{+(k,h)} \geq 0, \qquad (\forall j, i, r, k).
\end{aligned} \tag{7}$$

Applying the Charnes and Cooper (1962) transformation, assume $S_i^- = ts_i^-$, $S_r^+ = ts_r^+$, $S_h^{-(k,h)} = ts_h^{-(k,h)}, S_k^{+(k,h)} = ts_k^{+(k,h)}$, $\lambda_j = tA_j$, where $t = \left(\sum_{k=1}^{K} \left[1 + \dfrac{\left(\sum_{r=1}^{r_k} \dfrac{s_r^{+(k)}}{Y_{r0}^{(k)}} + \sum_{k=1}^{K-1} \dfrac{s_k^{+(k,h)}}{Z_0^{(k,h)}} \right)}{r_k + \sum_{k=1}^{K-1} t_k^{(k,h)}} \right] \right)^{-1}$

problem (7) can be converted to the following linear programming (LP) model:

$$
\begin{aligned}
\theta_0^{Global} = Min & \left[kt - \sum_{k=1}^{K} \left(\frac{1}{m_k + \sum_{h=2}^{K} t_h^{(k,h)}} \left(\sum_{i=1}^{m_k} \frac{S_i^{-(k)}}{X_{i0}^{(k)}} + \sum_{h=2}^{K} \frac{S_h^{-(k,h)}}{Z_0^{(k,h)}} \right) \right) \right], \\
s.t. \quad & kt + \sum_{k=1}^{K} \left[\frac{1}{r_k + \sum_{k=1}^{K-1} t_k^{(k,h)}} \left(\sum_{r=1}^{r_k} \frac{S_r^{+(k)}}{Y_{r0}^{(k)}} + \sum_{k=1}^{K-1} \frac{S_k^{+(k,h)}}{Z_0^{(k,h)}} \right) \right] = 1, \\
& \sum_j \lambda_j^{(k)} X_{ij}^{(k)} + S_i^{-(k)} = tX_{i0}^{(k)}, \qquad (k = 1,\ldots,k), \\
& \sum_j \lambda_j^{(k)} Y_{rj}^{(k)} - S_r^{+(k)} = tY_{r0}^{(k)}, \qquad (k = 1,\ldots,k), \\
& \sum_j \lambda_j^{(\mathrm{h})} Z_j^{(k,h)} = \sum_j \lambda_j^{(k)} Z_j^{(k,h)}, \qquad (\forall (k,h)), \\
& \sum_j \lambda_j^{(h)} Z_j^{(k,h)} + S_h^{-(k,h)} = tZ_0^{(k,h)} + S_k^{+(k,h)}, \qquad (\forall (k,h)), \\
& \sum_j \lambda_j^{(k)} Z_j^{(k,h)} - S_k^{+(k,h)} = tZ_0^{(k,h)} - S_h^{-(k,h)}, \qquad (\forall (k,h)), \\
& S_k^{+(k,h)} \le Md^{(k,h)}, \qquad (\forall (k,h)), \\
& S_h^{-(k,h)} \le M(1 - d^{(k,h)}), \qquad (\forall (k,h)), \\
& \lambda_j^{(k)},\ S_i^{-(k)},\ S_r^{+(k)}, S_h^{-(k,h)}, S_k^{+(k,h)} \ge 0,\ \ d^{(k,h)} \in \{0,1\}, (\forall j, i, r, k).
\end{aligned} \tag{8}
$$

In Model 8, if $S_k^{+(k,h)} = 0$, the constraints of the linking activity for outputs $(\sum_j \lambda_j^{(k)} Z_j^{(k,h)} - S_k^{+(k,h)} = tZ_0^{(k,h)} - S_h^{-(k,h)})$ are redundant, if $S_h^{-(k,h)} = 0$, the constraints of the linking activity for input $(\sum_j \lambda_j^{(h)} Z_j^{(k,h)} + S_h^{-(k,h)} = tZ_0^{(k,h)} + S_k^{+(k,h)})$ are redundant. As previously mentioned, under VRS assumption the convexity constraint of $\sum_{j=1}^{n} \lambda_j^{(k)} = t, (\forall k)$ should be added to the model.

After obtaining the overall efficiency of system (θ_0^{Global}), efficiency of each process can be calculated according to the following procedure. First, by giving the pre-emptive priority, the following model determines the efficiency (θ_0^1), while maintaining the overall efficiency score at (θ_0^{Global}) as calculated from model (8).

$$
\begin{aligned}
&\theta_0^1 = Min\left[\frac{1-\dfrac{1}{m_1}\displaystyle\sum_{i=1}^{m_1}\frac{s_i^{-(1)}}{X_{i0}^{(1)}}}{1+\dfrac{1}{r_1+\sum_{k=1}^{K-1}t_1^{(1,h)}}\left(\displaystyle\sum_{r=1}^{r_k}\frac{s_r^{+(1)}}{Y_{r0}^{(1)}}+\sum_{k=1}^{K-1}\frac{s_1^{+(1,h)}}{Z_0^{(1,h)}}\right)}\right]\\
&s.t \quad \sum_j A_j^{(k)}X_{ij}^{(k)}+s_i^{-(k)}=X_{i0}^{(k)}, \qquad (k=1,\ldots,k),\\
&\qquad \sum_j A_j^{(k)}Y_{rj}^{(k)}-s_r^{+(k)}=Y_{r0}^{(k)}, \qquad (k=1,\ldots,k),\\
&\qquad \sum_j A_j^{(\mathrm{h})}Z_j^{(k,h)}=\sum_j A_j^{(k)}Z_j^{(k,h)}, \qquad (\forall(k,h)),\\
&\qquad \begin{cases}\displaystyle\sum_j A_j^{(h)}Z_j^{(k,h)}+s_h^{-(k,h)}=Z_0^{(k,h)}, \qquad (\forall(k,h)),\\ \qquad\qquad or\\ \displaystyle\sum_j A_j^{(k)}Z_j^{(k,h)}-s_k^{+(k,h)}=Z_0^{(k,h)}, \qquad (\forall(k,h)),\end{cases}\\
&\qquad \frac{\displaystyle\sum_{k=1}^{K}\left[1-\frac{1}{m_k+\sum_{h=2}^{K}t_h^{(k,h)}}\left(\sum_{i=1}^{m_k}\frac{s_i^{-(k)}}{X_{i0}^{(k)}}+\sum_{h=2}^{K}\frac{s_h^{-(k,h)}}{Z_0^{(k,h)}}\right)\right]}{\displaystyle\sum_{k=1}^{K}\left[1+\frac{1}{r_k+\sum_{k=1}^{K-1}t_k^{(k,h)}}\left(\sum_{r=1}^{r_k}\frac{s_r^{+(k)}}{Y_{r0}^{(k)}}+\sum_{k=1}^{K-1}\frac{s_k^{+(k,h)}}{Z_0^{(k,h)}}\right)\right]}=\theta_0^{Global},\\
&\qquad A_j^{(k)},\, s_i^{-(k)},\, s_r^{+(k)}, s_h^{-(k,h)}, s_k^{+(k,h)}\geq 0, \qquad (\forall j,i,r,k).
\end{aligned}
\tag{9}
$$

Or equivalently (by the same methods),

$$\theta_0^1 = Min\left[t - \frac{1}{m_1}\sum_{i=1}^{m_1}\frac{S_i^{-(1)}}{X_{i0}^{(1)}}\right]$$

$$\begin{aligned}
s.t.\quad & t + \frac{1}{r_1 + \sum_{k=1}^{K-1} t_1^{(1,h)}}\left(\sum_{r=1}^{r_k}\frac{S_r^{+(1)}}{Y_{r0}^{(1)}} + \sum_{k=1}^{K-1}\frac{S_1^{+(1,h)}}{Z_0^{(1,h)}}\right) = 1, \\
& \sum_j \lambda_j^{(k)} X_{ij}^{(k)} + S_i^{-(k)} = tX_{i0}^{(k)}, \qquad (k = 1,\ldots,k), \\
& \sum_j \lambda_j^{(k)} Y_{rj}^{(k)} - S_r^{+(k)} = tY_{r0}^{(k)}, \qquad (k = 1,\ldots,k), \\
& \sum_j \lambda_j^{(\mathrm{h})} Z_j^{(k,h)} = \sum_j \lambda_j^{(k)} Z_j^{(k,h)}, \qquad (\forall (k,h)), \\
& \sum_j \lambda_j^{(h)} Z_j^{(k,h)} + S_h^{-(k,h)} = tZ_0^{(k,h)} + S_k^{+(k,h)}, \qquad (\forall (k,h)), \\
& \sum_j \lambda_j^{(k)} Z_j^{(k,h)} - S_k^{+(k,h)} = tZ_0^{(k,h)} - S_h^{-(k,h)}, \qquad (\forall (k,h)), \\
& S_k^{+(k,h)} \le Md^{(k,h)}, \qquad (\forall (k,h)), \\
& S_h^{-(k,h)} \le M(1 - d^{(k,h)}), \qquad (\forall (k,h)), \\
& \left(kt - \sum_{k=1}^{K}\left[\frac{1}{m_k + \sum_{h=2}^{K} t_h^{(k,h)}}\left(\sum_{i=1}^{m_k}\frac{S_i^{-(k)}}{X_{i0}^{(k)}} + \sum_{h=2}^{K}\frac{S_h^{-(k,h)}}{Z_0^{(k,h)}}\right)\right]\right) - \\
& \theta_0^{Global}\left((k-1)t + \sum_{k=2}^{K}\left[\frac{1}{r_k + \sum_{k=2}^{K-1} t_k^{(k,h)}}\left(\sum_{r=1}^{r_k}\frac{S_r^{+(k)}}{Y_{r0}^{(k)}} + \sum_{k=2}^{K-1}\frac{S_k^{+(k,h)}}{Z_0^{(k,h)}}\right)\right]\right) = \theta_0^{Global}, \\
& \lambda_j^{(k)}, S_i^{-(k)}, S_r^{+(k)}, S_h^{-(k,h)}, S_k^{+(k,h)} \ge 0,\; d^{(k,h)} \in \{0,1\}, \qquad (\forall j, i, r, k).
\end{aligned} \tag{10}$$

Using the same procedure we can calculate the efficiency of other processes.

Assume optimal solution to (8) is $(\theta^*, t^*, \lambda_j^{(k)*}, S_i^{-(k)*}, S_r^{+(k)*}, S_h^{-(k,h)*}, S_k^{+(k,h)*})$. Then, we have an optimal solution of NSBM as defined by:

$$E^* = \theta^*;\quad A_j^{(k)*} = \frac{\lambda_j^{(k)*}}{t^*};\quad s_i^{-(k)*} = \frac{S_i^{-(k)*}}{t^*};\quad s_r^{+(k)*} = \frac{S_r^{+(k)*}}{t^*};\quad s_h^{-(k,h)*} = \frac{S_h^{-(k,h)*}}{t^*};\quad s_k^{+(k,h)*} = \frac{S_k^{+(k,h)*}}{t^*}. \tag{11}$$

Hence, for each inefficient DMU, the efficient target (projected on the frontier) can be calculated from the following equations:

$$\begin{aligned}
& X_{i0}^{k*} \leftarrow X_{i0}^{k} - s_i^{-(k)*}\;;\; (k = 1, 2, \ldots, K), \\
& Y_{r0}^{k*} \leftarrow Y_{r0}^{k} + s_r^{+(k)*}\;;\; (k = 1, 2, \ldots, K), \\
& \mathrm{Z}_{j0}^{(k,h)*} \leftarrow \mathrm{Z}_{j0}^{(k,h)}\, A_{j0}^{(k)*} = \mathrm{Z}_{j0}^{(k,h)}\, A_{j0}^{(h)*}\;;\; (\forall (k,h)).
\end{aligned} \tag{12}$$

According to the proposed NSBM model, a DMU is overall efficient if and only if it is efficient in all processes; i.e. $\theta_k^* = 1$ & $E_0^* = 1$. This condition is equivalent to no input excesses and no output shortfalls in any optimal solution.

5. An application in electric power companies

In this section we apply the proposed NSBM model to a dataset comprised of 16 Iranian electric power company for describing Network SBM (we first ran the model on all DMUs and we found that two DMUs are outliers, therefor we removed those two DMUs).

Fig. 1 exhibits typical vertically integrated electric utility companies consisting of generation, transmission and distribution processes.

The generation process (Process 1) uses several inputs such as labor 1 and fuel and produces electric power. Then it becomes an intermediate input for the transmission process. In the transmission process (Process 2), companies utilize labor 2 and electric power generated inputs. Electricity through transmission lines is sent to distribution process as intermediate output or sales to large customers that do not utilize distribution line. The distribution process (Process 3) uses electric power distributed and labor 3 inputs and provides electricity to small customers.

------- [Figure 1 about here] -------

Table 1 exhibits inputs, outputs and links data of the all companies. The data in this table is normalized by sum in each column (i.e. $(a_{ij} \big/ \sum_{j=1}^{n} a_{ij})$), hence we do not denote the units. This change has no effect on the efficiency scores, since the proposed NSBM models employed are units-invariant.

------- [Table 1 about here] -------

5. 1. Results of proposed NSBM model

Whenever the operations of the component stages are taken into account, we are facing with a network system, and the conventional model must then be extended to suitable network structure. Many researchers have pointed that obtaining efficiency of decision making units using the conventional black-box DEA model may produce misleading results, one should to use a suitable network model. The overall efficiency as well as efficiency of each stage and their associated weights could help policy makers to identify the most influential factors in the performance of the production unit, Hence, improving each inefficient stage will effectively enhance the performance of the production unit. In this respect, the results of the NSBM model (model 8) under both CRS and VRS assumptions have been presented in Table 2 and 3, respectively. By applying proposed procedure, the efficiencies of the three processes have been calculated, for example divisional efficiency of first process, obtain from model (10). The number in parentheses below each divisional efficiency is the weight associated with that process calculated from Eq. (6) based on the optimal solution of model (8).

As expected, the overall score or system efficiency is a weighted average of the three process efficiencies. Taking DMU A as an example, one has $0.5855 = (0.2566 \times 0.6367) + (0.3218 \times 0.5143) + (0.4216 \times 0.6086)$. The numbers in parentheses below reference set column (Lambda) are the intensity vector corresponding in each division.

According to Table 2, we can conclude that there is no efficient DMU in process 1, while we can see that the other two processes have at least one efficient DMU; this demonstrates that all DMUs in process 1 need improvement. On the other hand, lack of divisionally efficient DMUs in some division and inefficient reference units are characteristics of network DEA models which cannot be expected by conventional DEA models.

------- [Table 2 about here] -------

As can be seen in Table 3, the overall scores of NSBM model with VRS assumptions tend to be higher than those of with CRS assumptions. However, this is quite natural and as it should be we can see that the sum of the lambdas in each process (VRS assumptions) equals to one.

------- [Table 3 about here] -------

According to Fig. 2 efficiency of generation process is less than the system efficiency (overall score) in most companies (except A, B, and H), while the efficiency of distribution process is higher than the system efficiency in most companies (except B, C, and G). The average of efficiency under CRS assumptions are 0.5185 in the generation process, and 0.6325 in the transmission process, while the average efficiency in the distribution process is 0.6544. This concludes that the generation is the weakest process in the electric power companies. The sum of weights assigned to companies in the three processes (generation, transmission, and distribution) are 4.4057, 4.5177, and 5.0748, respectively, which represents the contribution of each process in the total efficiency of the system. As results, the contribution of the generation, transmission and distribution process in the efficiency of the whole system are 31.48%, 32.27%, and 36.25%, respectively.

------- [Figure 2 about here] -------

It should be noted that in under the standard DEA approach, the scores under the VRS assumption are not less than the ones under CRS assumption. Table 4 shows that this is true for the overall efficiency scores in the power electric companies. However, we note that this is not the case for Company "A" for the first division and, Companies "D" and "H" for the second division. This may be attributed to the fact that the production possibility set using the constraints for the overall efficiency model (8) and divisional efficiencies are not the same, hence the intermediate scores may not obey the conventional principles (see also Chen et al 2009).

------- [Table 4 about here] -------

One of the significant results obtained from the proposed NSBM model is that the best company in terms of overall efficiency is the one that acts in the acceptable level in all processes. For example, overall efficiency of DMU D (0.7247) is greater than overall efficiency of DMU N (0.6988), however none of the processes are efficient in DMU D. The efficiency scores for the generation, the transmission and the distribution processes are 0.5188, 0.8734 and 0.7699, respectively, while the distribution process is fully efficient (100%) for this DMU N.

According to Fig. 3, efficiency of generation process is less than the system efficiency (overall score) in most companies (except C, H, and I), while the efficiency of distribution process is higher than the system efficiency in most companies (except B, and C). The efficiency scores for the generation, the transmission and the distribution processes are 0.6787, 0.7612 and 0.8189, respectively. Further, the sum of weights assigned to electric power companies in the three processes (generation, transmission, and distribution) are 4.4774, 4.7315, and 5.7911, respectively.

------- [Figure 3 about here] -------

5. 2. Analyzing inefficient DMUs

Based on formulas (12) we can calculate the target for each inefficient DMU_0 by projecting it to the best practice frontier. It should be noted that the projected DMU is the target of inefficient DMU on the frontier and hence it is overall efficient. For example, using NSBM model (model 8) and formula (12), the projection of inefficient DMU A (under CRS assumptions) is calculated as follows:

$\theta_0^{Global} = 0.5855027$,

$s_1^{-(1)*} = 0.036208834$, $s_2^{-(1)*} = 0.017592635$, $s_1^{-(2)*} = 0.022218597$,

$s_1^{-(3)*} = 0.00000000$, $s_1^{+(2)*} = 0.000000$, $s_1^{+(3)*} = 0.06031114$,

$s_2^{-(1,2)*} = 0.031850098$, $s_3^{-(2,3)*} = 0.0000000$, $s_1^{+(1,2)*} = 0.000000$,

$s_2^{+(2,3)*} = 0.034641277$, $A_E^{(1)} = 0.376284832$, $A_B^{(2)} = 0.239784764$,

$A_G^{(2)} = 1.943836846$, $A_N^{(3)} = 3.015873424$

For this DMU A, the target, on the frontier, for inputs, outputs and intermediate variables are reported in Table 5. As expected, projection of intermediate variables as input or output are exactly the same.

------- [Table 5 about here] -------

According to the results the policy makers should pay more attention to the generation process compared to the transmission and distribution processes, since averages efficiency of the generation process under both CRS and VRS assumptions are much lower than the other two processes. For illustration purpose Table 5 provides details of adjustments required for company “A” (as an example) to achieve the overall efficiency of 100%. We have run non-oriented models and so Fig. 4 (a & b) illustrates how electric power company “A” could save Labor and Fuel in order to be efficient.

------- [Figure 4a & 4b about here] -------

6. Conclusions and direction for future research

In this paper, we have developed a network DEA model based on the slacks-based measures approach. The main advantage of SBM is that they express the efficiency of weakly efficient DMUs more appropriately than the traditional radial models. In the proposed NSBM model, the overall or system efficiency is expressed as a weighted average of the divisional efficiencies, so that the contribution of each process from the mean expansion rate of outputs (or the contribution of each process in inefficiency of overall system) accounts for the importance or weight of each division. Therefore, we can calculate the overall efficiency and multi-divisional efficiencies in a unified framework. The major merits of the proposed model are its ability to provide appropriate measure of efficiency, obtaining weight of stages from model and without the intervention of the human factor (decision maker) ensuring that those are the most desirable weights in overall efficiency calculation. This paper also applies the proposed model to evaluate 14 Iranian electric power. For this purpose, the process of electric power companies have been divided to three stages (the generation, the transmission and the distribution processes) and it is been shown the policy makers should pay more attention the first stage (the generation process) in order to improve the system (overall) efficiency). Researcher interested may apply the proposed models in other applications, especially by improving the model to handle undesirable factors such as CO2 emissions.

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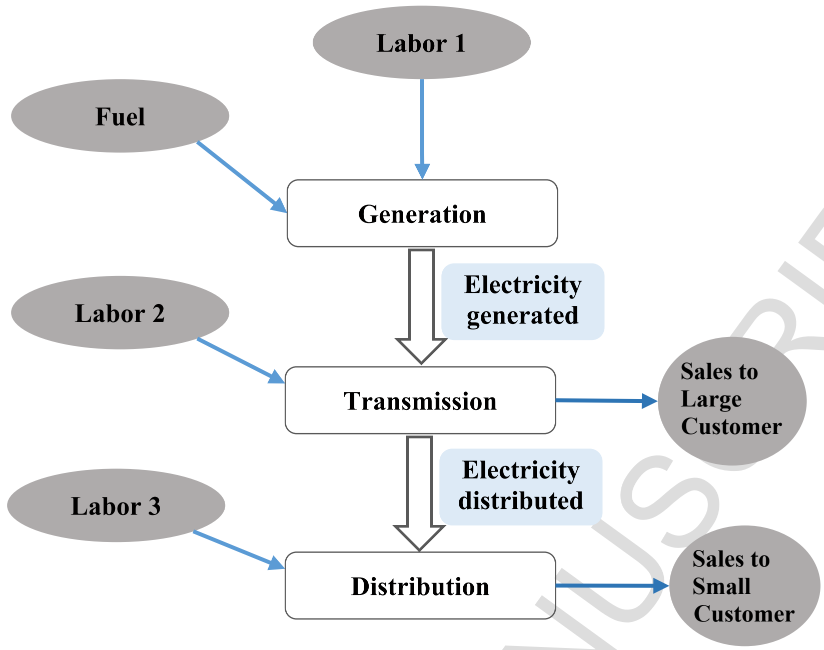

Fig. 1. Vertically integrated electric utility company

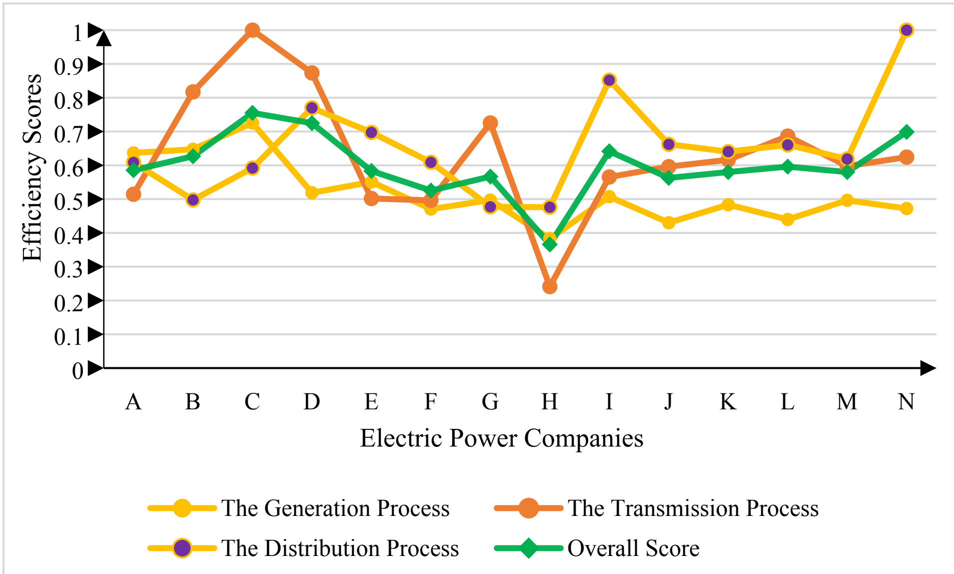

Fig 2. Comparisons of scores between the overall and process efficiencies (CRS assumptions)

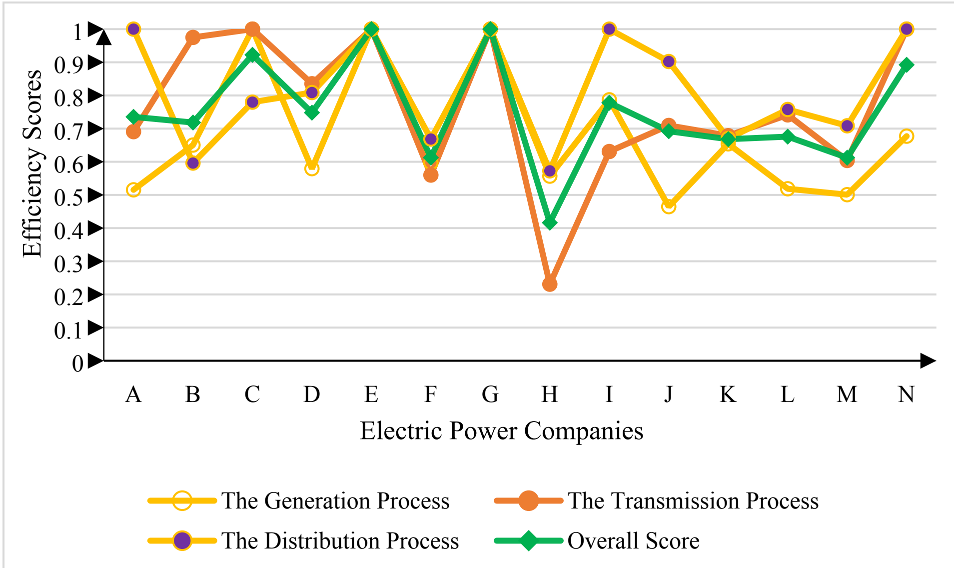

Fig 3. Comparisons of scores between the overall and process efficiencies (VRS assumptions)

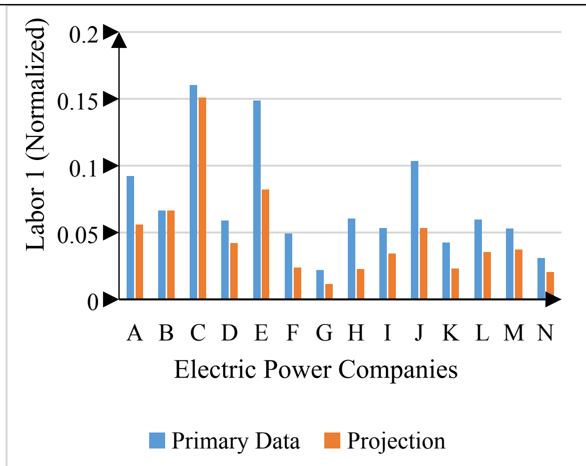

Fig4a. Projected value for Labor (for all DMU)

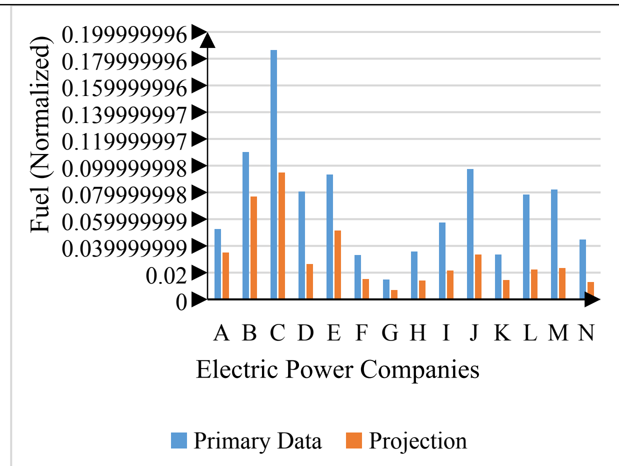

Fig4b. Projected value for Fuel (for all DMU)

Table 1. Data for 14 electric power companies

DMU	Process 1		Process 2		Process 3		Link	
	Labor 1	Fuel	Labor 2	Sales to Large Customers	Labor 3	Sales to Small Customers	Electricity generated	Electricity distributed
A	0.0922	0.0527	0.0621	0.0643	0.0950	0.0938	0.0904	0.0682
B	0.0664	0.1103	0.0658	0.1117	0.0709	0.0562	0.0634	0.0722
C	0.1601	0.1864	0.1068	0.1971	0.1436	0.0989	0.1578	0.0869
D	0.0590	0.0806	0.0472	0.0773	0.0468	0.0591	0.0561	0.0518
E	0.1488	0.0933	0.1888	0.1445	0.1580	0.1553	0.1556	0.1314
F	0.0490	0.0331	0.0497	0.0376	0.0515	0.0517	0.0463	0.0576
G	0.0219	0.0147	0.0124	0.0193	0.0220	0.0221	0.0223	0.0440
H	0.0603	0.0357	0.1540	0.0195	0.0692	0.0694	0.0576	0.1387
I	0.0532	0.0574	0.0683	0.0484	0.0509	0.0811	0.0508	0.0750
J	0.1033	0.0975	0.0882	0.0933	0.1021	0.1024	0.0987	0.0968
K	0.0425	0.0334	0.0360	0.0391	0.0449	0.0450	0.0406	0.0453
L	0.0595	0.0783	0.0534	0.0653	0.0611	0.0615	0.0568	0.0586
M	0.0528	0.0819	0.0360	0.0556	0.0524	0.0526	0.0741	0.0395
N	0.0309	0.0447	0.0311	0.0269	0.0315	0.0511	0.0295	0.0341

Table 2. Results of NSBM (CRS assumptions)

DMU	Overall Score	Divisional Score			Reference Set		
		P1(W1)	P2(W2)	P3(W3)	P1(Lambda)	P2(Lambda)	P3(Lambda)
A	0.5855	0.6367 (0.2566)	0.5143 (0.3218)	0.6086 (0.4216)	E1(0.3763)	B2(0.2398), G2(1.9438)	N3(3.0159)
B	0.6268	0.6474 (0.2882)	0.8173 (0.2693)	0.4975 (0.4425)	E1(0.1912), M1(0.7188)	B2(0.5606), C2(0.1322), G2(1.1934)	I3(1.3929)
C	0.7551	0.7251 (0.3015)	1.0000 (0.3015)	0.5920 (0.3970)	E1(1.0141)	C2(1.0000)	N3(2.5484)
D	0.7247	0.5188 (0.3061)	0.8734 (0.3061)	0.7699 (0.3878)	E1(0.2820)	B2(0.6920)	N3(1.4652)
E	0.5833	0.5508 (0.3333)	0.5017 (0.3333)	0.6973 (0.3334)	E1(0.5508)	B2(1.2375), G2(0.3246)	N3(3.0391)
F	0.5253	0.4706 (0.3333)	0.4963 (0.3333)	0.6089 (0.3334)	E1(0.1607)	B2(0.2807), G2(0.3234)	N3(1.0117)
G	0.5666	0.4971 (0.3333)	0.7254 (0.3333)	0.4772 (0.3334)	E1(0.0757)	B2(0.1603), G2(0.0721)	N3(0.4325)
H	0.3659	0.3832 (0.3310)	0.2411 (0.3380)	0.4761 (0.3310)	E1(0.1508)	G2(1.0525)	N3(1.3581)
I	0.6415	0.5072 (0.3333)	0.5655 (0.3333)	0.8519 (0.3334)	E1(0.2294)	B2(0.3081), G2(0.7243)	N3(1.5871)
J	0.5627	0.4302 (0.3333)	0.5960 (0.3333)	0.6621 (0.3334)	E1(0.3589)	B2(0.7913), G2(0.2546)	N3(2.0039)
K	0.5801	0.4835 (0.3333)	0.6164 (0.3333)	0.6404 (0.3334)	E1(0.1536)	B2(0.3240), G2(0.1509)	N3(0.8806)
L	0.5961	0.4398 (0.3302)	0.6864 (0.3302)	0.6604 (0.3396)	E1(0.2382)	B2(0.5846)	N3(1.2378)
M	0.5801	0.4964 (0.2608)	0.5975 (0.3177)	0.6188 (0.4215)	E1(0.2509)	B2(0.3838), G2(0.6594)	N3(1.6635)
N	0.6988	0.4721 (0.3333)	0.6243 (0.3333)	1.0000 (0.3334)	E1(0.1368)	B2(0.1492), G2(0.5301)	N3(1.0000)

Table 3. Results of NSBM (VRS assumptions)

DMU	Overall Score	Divisional Score			Reference Set		
		P1(W1)	P2(W2)	P3(W3)	P1(Lambda)	P2(Lambda)	P3(Lambda)
A	0.7354	0.5154 (0.3333)	0.6908 (0.3334)	1.0000 (0.3333)	E1(0.1794), G1(0.8206)	B2(0.4868), G2(0.4026), H2(0.1106)	A3(1.0000)
B	0.7183	0.6504 (0.3652)	0.9750 (0.2704)	0.5961 (0.3644)	E1(0.1417), M1(0.8583)	B2(0.2412), C2(0.3943), G2(0.3645)	I3(0.8220), N3(0.1780)
C	0.9224	1.0000 (0.3238)	1.0000 (0.3238)	0.7798 (0.3524)	C1(1.0000)	C2(1.0000)	E3(0.5427), N3(0.4573)
D	0.7481	0.5793 (0.3028)	0.8353 (0.3317)	0.8088 (0.3655)	E1(0.1935), G1(0.8065)	B2(0.6277), G2(0.3723)	I3(0.6748), N3(0.3252)
E	1.0000	1.0000 (0.3333)	1.0000 (0.3333)	1.0000 (0.3333)	E1(1.0000)	E2(1.0000)	E3(1.0000)
F	0.6124	0.5971 (0.3117)	0.5599 (0.3117)	0.6685 (0.3766)	E1(0.0611), G1(0.9389)	B2(0.1981), G2(0.8019)	I3(0.3786), N3(0.6214)
G	1.0000	1.0000 (0.3333)	1.0000 (0.3333)	1.0000 (0.3333)	G1(1.0000)	G2(1.0000)	G3(1.0000)
H	0.4163	0.5567 (0.2779)	0.2308 (0.4442)	0.5726 (0.2779)	E1(0.0786), G1(0.9214)	B2(0.2549), G2(0.7451)	E3(0.1756), N3(0.8244)
I	0.7780	0.7869 (0.3296)	0.6309 (0.4111)	1.0000 (0.2593)	E1(0.1285), G1(0.3864), M1(0.4851)	B2(0.8701), G2(0.0298), H2(0.0280), J2(0.0721)	I3(1.0000)
J	0.6922	0.4647 (0.3333)	0.7095 (0.3333)	0.9023 (0.3334)	E1(0.2785), G1(0.7215)	B2(0.8006), G2(0.0800), H2(0.1194)	A3(0.7136), E3(0.1999), N3(0.0865)
K	0.6681	0.6541 (0.2901)	0.6787 (0.3052)	0.6701 (0.4047)	E1(0.0661), G1(0.9339)	B2(0.2143), G2(0.7857)	I3(0.3898), N3(0.6102)
L	0.6758	0.5186 (0.3209)	0.7414 (0.3209)	0.7577 (0.3582)	E1(0.1535), G1(0.8465)	B2(0.4978), G2(0.5022)	I3(0.5853), N3(0.4147)
M	0.6123	0.5008 (0.2889)	0.6039 (0.3459)	0.7085 (0.3652)	E1(0.1211), G1(0.8789)	B2(0.3929), G2(0.6071)	I3(0.5129), N3(0.4871)
N	0.8924	0.6772 (0.3333)	1.0000 (0.3333)	1.0000 (0.3334)	E1(0.0540), G1(0.9460)	N2(1.0000)	N3(1.0000)

Table 4. Scale efficiency of NSBM

DMU	Overall Score	Divisional Score		
		P1(W1)	P2(W2)	P3(W3)
A	0.796	1.235	0.744	0.609
B	0.873	0.995	0.838	0.835
C	0.819	0.725	1	0.759
D	0.969	0.896	1.046	0.952
E	0.583	0.551	0.502	0.697
F	0.858	0.788	0.886	0.911
G	0.567	0.497	0.725	0.477
H	0.879	0.688	1.045	0.831
I	0.825	0.645	0.896	0.852
J	0.813	0.926	0.84	0.734
K	0.868	0.739	0.908	0.956
L	0.882	0.848	0.926	0.872
M	0.947	0.991	0.989	0.873
N	0.783	0.697	0.624	1

Table 5. Projection of DMU A to efficient frontiers

Variables	DMU A	Projection	Projection (Actual Value)
Labor 1	0.0922	0.0922-0.036208834=0.0559912 0.376284832×0.1488=0.0559912	912
Fuel	0.0527	0.0527-0.017592635=0.0351073 0.376284832×0.0933=0.0351073	235061
Labor 2	0.0621	0.0621-0.022218597=0.0398814 (0.239784764×0.0658)+(1.943836846×0.0124)= 0.0398814	32
Sales to Large Customers	0.0643	0.0643+0.0000=0.0643 (0.239784764×0.1117)+(1.943836846×0.0193)= 0.0643000	16230
Labor 3	0.0950	0.0950-0.0000=0.0950 3.015873424×0.0315=0.0950000	743
Sales to Small Customers	0.0938	0.0938+0.06031114=0.1541111 3.015873424×0.0511=0.1541111	10269277
Electricity generated (as output P1)	0.0904	0.376284832×0.1556=0.0585499	17317789
Electricity generated (as input P2)	0.0904	0.0904-0.031850098=0.0585499 (0.239784764×0.0634)+(1.943836846×0.0223)= 0.0585499	17317789
Electricity distributed (as output P2)	0.0682	0.0682+0.034641277=0.10284128 (0.239784764×0.0722)+(1.943836846×0.0440)= 0.10284128	13560459
Electricity distributed (as input P3)	0.0682	3.015873424×0.0341=0.10284128	13560459

- Developing network DEA model to decompose the system efficiency.
- Pareto target estimation for intermediate products of electric power companies.
- The transmission process is more efficient than generation & distribution.
- To improve the efficiency, policy makers should make more attention to the generation process.