Instant Preheating in Quintessential Inflation with $\alpha-$Attractors

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We investigate a compelling model of quintessential inflation in the context of α-attractors, which naturally result in a scalar potential featuring two flat regions; the inflationary plateau and the quintessential tail. The "asymptotic freedom" of α-attractors, near the kinetic poles, suppresses radiative corrections and interactions, which would otherwise threaten to lift the flatness of the quintessential tail and cause a 5th-force problem respectively. Since this is a non-oscillatory inflation model, we reheat the Universe through instant preheating. The parameter space is constrained by both inflation and dark energy requirements. We find an excellent correlation between the inflationary observables and model predictions, in agreement with the α-attractors set-up. We also obtain successful quintessence for natural values of the parameters. Our model predicts potentially sizeable tensor perturbations (at the level of 1%) and a slightly varying equation of state for dark energy, to be probed in the near future.

INTRODUCTION

One of the greatest discoveries in cosmology was that the Universe is currently undergoing accelerated expansion [1, 2]. To account for this, Einstein's General Relativity demands that the dominant component of the Universe content violates the strong energy condition. Assuming that it is a barotropic fluid, its pressure must be negative enough $p < -\frac{1}{3}\rho$. Such a mysterious substance is dubbed 'dark energy'.

By far, the simplest choice of dark energy is vacuum density, due to a non-zero cosmological constant, for which $p = -\rho$. The main problem with this idea is that the required value of the cosmological constant is staggeringly small such that the vacuum density is about 10^{120} times smaller than the Planck density, which corresponds to the cutoff scale of the theory. This has been called "the worst fine-tuning in Physics".[1] To overcome this, but at the expense of introducing new Physics, there have been alternative proposals put forward.

A substance which can exhibit pressure negative enough is a potentially dominated homogeneous scalar field. Thus, it is possible for a dynamical scalar field to drive the accelerated expansion of the Universe, therefore being a type of dark energy. The idea has long been in use when modelling cosmic inflation in the early Universe (inflationary paradigm). Mirroring the mechanism used to explain inflation, this idea can also be applied to address the current accelerated expansion. A scalar field responsible for late inflation is called quintessence; the fifth element after baryons, CDM, photons and neutrinos [3–5].

Since they are both based on the same idea, it is natural to attempt to unify cosmic inflation with quintessence. Indeed, the mechanism in which a scalar field is both driving primordial inflation and causing the current accelerated expansion, is called quintessential inflation [6]. Quintessential inflation is economical in that it models both inflation and quintessence in a common theoretical framework and employs a single degree of freedom. It also features some practical advantages, for example the initial conditions of quintessence are determined by the inflationary attractor. As such the infamous coincidence problem (which corresponds to late inflation occurring at present) is reduced to a constraint on the model parameters and not on initial conditions.

Quintessential inflation models require the inflaton potential energy density to survive until the present day to act as dark energy. Amongst other things, this necessitates a reheating mechanism alternative to the standard assumption, in which, after the end of inflation, the inflaton field decays into the thermal bath of the hot big bang. If inflaton decay is not considered, then reheating must occur by other means. Different reheating alternatives to inflaton decay include instant preheating [7, 8], curvaton reheating [9–11] and gravitational reheating [12, 13], amongst others [14], which may or may not happen exclusively. For example, gravitational reheating is *always* present, but because it is a very inefficient mechanism it is overwhelmed if another reheating mechanism is present. The outcome of any of these reheating mechanisms must complete and lead to radiation domination well before the time of Big Bang Nucleosynthesis (BBN). The temperature of the Universe when radiation domination takes over is called the reheating temperature $T_{\rm reh}$. Thus we need $T_{\rm reh} \gg 1\,{\rm MeV}$ to avoid disturbing BBN.

In quintessential inflation models, if reheating is not prompt, a period of kination exists, where the kinetic density of the inflaton is the dominant energy density in the Universe. During this period, the non-decaying mode for gravitational waves is not suppressed and this produces a spike in the gravitational wave spectrum at high frequencies. The energy density of these gravitational waves may be large enough to disturb the BBN process. As such it is constrained and provides an upper limit on the duration of

[1] by Laurence Krauss.

kination. This, in turn, is a lower bound on the reheating temperature.

The scalar potential in quintessential inflation typically features two flat regions, which when traversed by the scalar field, result in accelerated expansion, provided that the scalar field dominates the Universe. These are called the inflationary plateau and the quintessential tail and can lead to the primordial and current accelerated expansion respectively. Thus, the required potential is of runaway type, with the global minimum displaced at infinity, where the vacuum density is zero. The form of the quintessential potential is non trivial, especially since the two plateaus differ by more than a factor of 10^{100} in energy density. Consequently, one needs to use a theoretical framework that is valid at both these extreme energy scales.

A compelling way to naturally generate a scalar potential with the desired two plateaus is the idea of α-attractors, which is heavily used in inflationary model building [15–37]. Recently, we have presented a new quintessential inflation model along these lines [38]. Our model is in excellent agreement for inflationary observables with the CMB observations [39]. In our paper in Ref. [38] we utilised the mechanism of gravitational reheating to reheat the Universe after inflation, in alignment with the economy of the quintessential inflation idea. However, gravitational reheating is notoriously inefficient, with a very low reheating temperature of approximately $T_{\rm reh} \sim 10^4\,{\rm GeV}$. As a result, the spike of gravitational waves due to kination is large enough to challenge the BBN process. This is the price to pay for economy. In this paper we generalise our approach and employ the instant preheating mechanism to reheat the Universe. Thus, we envisage a coupling between our scalar field with some other degree of freedom such that, after the end of inflation, the rapid variation of the inflaton's expectation value leads to non-perturbative particle production, which generates the radiation bath of the hot big bang. The process is modulated by the coupling constant g. If g is small enough, instant preheating becomes comparable to gravitational reheating. Thus, in general we consider $T_{\rm reh} > 10^4\,{\rm GeV}$.

The setup of α-attractors also has another beneficial consequence, apart from generating the potential plateaus. It has to do with the suppression of radiative corrections near the poles (so along the plateaus), which otherwise threaten to lift the flatness of the quintessential tail, as well as the suppression of interactions, which would otherwise generate a 5th-force problem for quintessence. Both these problems plague models of quintessence and quintessential inflation alike. In Ref. [38], we had not fully realised the beneficial effect of the α-attractors setup in this respect. Thus, we aimed to avoid excessive interactions by keeping the non-canonical field sub-Planckian (the interactions are Planck-suppressed). However, as demonstrated in Refs. [40, 41], the suppression of loop corrections and interactions along the plateaus (near the poles) is such that even a super-Planckian excursion of the non-canonical inflaton is admissible. We investigate this issue in detail, but conservatively choose to avoid super-Planckian values for our non-canonical inflation field in our treatment.

We start with an overview of the model before noting how a change of reheating mechanism affects the number of remaining inflationary e-folds since observable scales left the horizon during primordial inflation. We calculate the inflationary observables before investigating how the quintessence requirements determine the parameter space in a model with instant preheating.

We use natural units, where $c = \hbar = 1$ and Newton's gravitational constant is $8\pi G = m_{\rm P}^{-2}$, with $m_{\rm P} = 2.43 \times 10^{18}\,{\rm GeV}$ being the reduced Planck mass.

THE MODEL

The Scalar Potential

We consider the following model:

$$\mathcal{L} = \mathcal{L}_{\rm kin} + \mathcal{L}_V + \Lambda\,, \tag{1}$$

where $\mathcal{L}_{\rm kin}$ is the kinetic Lagrangian density, $\mathcal{L}_V$ is the potential Lagrangian density and Λ is a cosmological constant.

The kinetic Lagrangian density is

$$\mathcal{L}_{\rm kin} = \frac{\frac{1}{2}(\partial\phi)^2}{(1-\frac{\phi^2}{6\alpha m_{\rm P}^2})^2}\,, \tag{2}$$

where $\alpha > 0$ is a parameter. This is the standard, non-canonical form in the context of α-attractors [15–18]. It can be realised in supergravity theories, when the Kähler manifold is not trivial, such that $\mathcal{L}_{\rm kin}$ features poles, characterised by the α parameter.

For the potential Lagrangian density, we consider a simple exponential function (possibly due to gaugino condensation [42–44]). Thus, we have

$$-\mathcal{L}_V = V(\phi) = V_0 e^{-\kappa\phi/m_{\rm P}}\,. \tag{3}$$

where κ is a parameter (without loss of generality, we consider $\kappa > 0$) and V_0 is a constant density scale.

In an effort to minimise its potential density, the expectation value of the field ϕ grows in time. However, it cannot cross the poles at $\pm\sqrt{6\alpha}\,m_{\rm P}$ [15–18]. Thus, starting in-between the poles, we expect that it finally approaches the value $\phi \to +\sqrt{6\alpha}\,m_{\rm P}$, which corresponds to non-zero potential density $V(\sqrt{6\alpha}\,m_{\rm P}) = V_0 e^{-\kappa\sqrt{6\alpha}}$.

We assume that, due to an *unknown* symmetry, the vacuum density is zero. This was the standard assumption before the discovery of dark energy. If a non-zero vacuum density is assumed then we have the usual ΛCDM cosmology. For motivating quintessence as the explanation of the dark energy observations, the vacuum density has to be zero. This fixes the cosmological constant in our model to the value

$$\Lambda = V(\sqrt{6\alpha}\,m_{\rm P}) = V_0 e^{-\kappa\sqrt{6\alpha}}. \tag{4}$$

Defining $n \equiv \kappa\sqrt{6\alpha}$ and incorporating Λ, the scalar potential can now be expressed as

$$V(\phi) = V_0 e^{-n}\left[e^{n\left(1-\frac{\phi}{\sqrt{6\alpha}m_{\rm P}}\right)} - 1\right] . \quad (5)$$

To assist our intuition, it is useful to consider a canonically normalised inflaton field φ. The form of the kinetic Lagrangian density in Eq. (2) suggests the field redefinition is obtained when $\frac{\partial\phi}{\partial\varphi} = 1 - \frac{\phi^2}{6\alpha m_{\rm P}^2}$, which gives

$$\phi = \sqrt{6\alpha}\, m_{\rm P}\ \tanh\left(\frac{\varphi}{\sqrt{6\alpha}m_{\rm P}}\right) . \quad (6)$$

Then, the scalar potential, in terms of the canonical scalar field becomes

$$V(\varphi) = e^{-2n}M^4\Big\{\exp\left[n\Big(1-\tanh\frac{\varphi}{\sqrt{6\alpha}m_P}\Big)\right]-1\Big\} \quad , \quad (7)$$

where we have defined $M^4 \equiv e^n V_0$, which stands for the inflation energy scale. Note, also, that $\Lambda = e^{-2n}M^4$.

Whereas the range of the non-canonical inflaton field ϕ is bounded by the poles in $\mathcal{L}_{\rm kin}$: $-\sqrt{6\alpha} < \phi/m_{\rm P} < \sqrt{6\alpha}$, the range of the canonical inflaton field φ is unbounded: $-\infty < \varphi < +\infty$. This is because the poles are transposed to infinity when we switch from ϕ to φ. In effect, the scalar potential $V(\varphi)$ becomes "stretched" as ϕ approaches the poles [15–18]. Therefore, the potential $V(\varphi)$ features two plateaus experienced by the field, one at early and one at late times.

At early times ($\varphi \to -\infty, \phi \to -\sqrt{6\alpha}\, m_{\rm P}$), the potential in Eq. (7) can be simplified to

$$V(\varphi) \simeq M^4 \exp\Big(-2ne^{\frac{2\varphi}{\sqrt{6\alpha}m_{\rm P}}}\Big) , \quad (8)$$

which gives rise to the inflationary plateau. In the opposite limit, towards late times ($\varphi \to +\infty, \phi \to +\sqrt{6\alpha}\, m_{\rm P}$), the potential in Eq. (7) becomes

$$V = 2ne^{-2n}M^4 \exp(-2\varphi/\sqrt{6\alpha}\, m_{\rm P}) . \quad (9)$$

This corresponds to the quintessential tail. It is evident that the potential density asymptotes to zero as $\varphi \to +\infty$.

The evolution of the quintessential inflaton field goes as follows. The field slow-rolls along the early-time plateau, obeying the slow-roll constraints and inflating the Universe. Inflation ends when the potential becomes steep and curved. Afterwards, the inflaton field falls down the steep slope of the potential. A period of kination ensues, when the Universe is dominated by the kinetic density of the scalar field. Kination ends when the Universe is reheated and radiation takes over. As such, the duration of kination is inversely proportional to the reheating temperature $T_{\rm reh}$, which defines the moment when radiation domination begins and reheating completes. The field continues to roll until it runs out of kinetic energy and freezes at a particular value φ_F. It remains dormant at φ_F until late times, when it becomes quintessence and its residual potential density drives the Universe expansion into acceleration again.

Here we should highlight the importance of the parameter $n = \kappa\sqrt{6\alpha}$. The value of n modulates both the steepness of the potential and the inflaton value where the potential drops from the early-time to the late-time plateau. As such, this controls φ_F, the value the field freezes at, when it runs out of kinetic energy after reheating.

The Range of α

At late times, there are two attractor solutions to the Klein-Gordon equation depending on whether the quintessence field is eventually dominant or not over the background matter. It has been shown that, when the background density becomes comparable to the field's residual potential density $V(\varphi_F)$, the field unfreezes and briefly oscillates about the attractor before settling on the attractor solution [45]. The question of which attractor solution the field eventually follows is controlled by the value of α, which determines the slope of the quintessential tail.

The latest Planck observations suggest that the density parameter of dark energy is $\Omega_\Lambda = 1 - \Omega_{\rm K} - \Omega_m$, where $\Omega_{\rm K} = 0.000 \pm 0.005$ is the curvature density parameter and $\Omega_m = 0.308 \pm 0.012$ is the density parameter of matter. This results in $\Omega_\Lambda = 0.692 \pm 0.017$. Planck also demands that the effective barotropic parameter of dark energy is $w_{\rm DE} = -1.023^{+0.091}_{-0.096}$ (*Planck* TT+lowP+ext)[2] at 2-σ. We investigate this in the appendix and we find that demanding that our model satisfies these observational requirements results in the bound $\alpha \gtrsim 1.5$ (i.e. $\sqrt{6\alpha} \gtrsim 3$). In all cases, the scalar field has unfrozen but is yet to settle on the attractor solution. This results in $\dot{w}_{\rm DE} \neq 0$, which lies within current Planck bounds (see appendix) but can be potentially observable in the near future, where the dot denotes time derivative.

We can obtain an upper bound on α by avoiding super-Planckian values for the non-canonical field ϕ. The motivation for this is to suppress radiative corrections and the 5th-force problem, which plague quintessence models [38]. However, the bound is soft, as both loop corrections and interactions are suppressed near the poles [40, 41] as we discuss in the penultimate section of this paper. Still, being conservative, we choose to avoid a super-Planckian non-canonical inflation field. Therefore, the relevant range for α is the following:

$$3 \lesssim \sqrt{6\alpha} \lesssim 5 \quad \Leftrightarrow \quad 1.5 \leq \alpha \leq 4.2 . \quad (10)$$

For the above range, there is a theoretical prejudice in view of maximal supergravity, string theory, and M-theory, for particular values of α satisfying $3\alpha = 5, 6, 7$ [33, 46, 47].

[2] "ext" includes the *Planck* lensing, BAO, JLA and H_0 data sets.

Inflationary Observables

The inflationary observables predicted by this model are [38]:

$$r = 16\epsilon = 12\alpha \left(N_* + \frac{\sqrt{3\alpha}}{2}\right)^{-2}, \tag{11}$$

$$n_s = 1 - \frac{2}{\left(N_* + \frac{\sqrt{3\alpha}}{2}\right)} - \frac{3\alpha}{2\left(N_* + \frac{\sqrt{3\alpha}}{2}\right)^2} \simeq 1 - \frac{2}{N_*}, \tag{12}$$

and

$$\begin{aligned} n_s' &\equiv \frac{\mathrm{d}\ln n_s}{\mathrm{d}\ln k} \\ &= -\frac{1}{\left(N_* + \frac{\sqrt{3\alpha}}{2}\right)} \frac{2\left(N_* + \frac{\sqrt{3\alpha}}{2}\right) + 3\alpha}{\left(N_* + \frac{\sqrt{3\alpha}}{2}\right)^2 - 2\left(N_* + \frac{\sqrt{3\alpha}}{2}\right) - \frac{3}{2}\alpha} \\ &\simeq -\frac{2}{N_*^2 - 2N_*} \end{aligned} \tag{13}$$

where r is the tensor to scalar ratio, n_s is the spectral index of the scalar perturbations and n_s' its running. In the above, N_* is the number of remaining inflationary e-folds when the cosmological scales left the horizon during inflation and the last equations in Eqs. (12) and (13) correspond to $\alpha \ll N_*^2$ and are the standard α-attractors results.

The value of N_* is dependent on $T_{\rm reh}$ via the equation[3]

$$N_* \simeq 61.93 + \ln\left(\frac{V_{\rm end}^{1/4}}{m_P}\right) + \frac{1}{3}\ln\left(\frac{V_{\rm end}^{1/4}}{T_{\rm reh}}\right). \tag{14}$$

As will be discussed in the following sections, $T_{\rm reh}$ depends on n and α as well as the efficiency of the reheating mechanism. As such, the value of N_* is determined iteratively. However, using $N_* = 62$ (in view of Eq. (14)) and $\sqrt{6\alpha} = 4$ (the middle point of the range in Eq. (10)) in Eqs. (12) and (13) gives approximate results:

$$n_s = 0.968 \quad \text{and} \quad n_s' = -5.46 \times 10^{-4}. \tag{15}$$

Assuming $N_* = 62$ and considering the range in Eq. (10), Eq. (11) gives

$$0.005 \leq r \leq 0.012. \tag{16}$$

These inflationary observables are in excellent agreement with the latest Planck observations [39]. As shown later, the resulting values of n_s, n_s' and r using the actual values of N_*, obtained from considering the reheating mechanism and the quintessence requirements, remain in excellent agreement with the observations and are very close to the above.

[3] In Ref. [38], N_* was calculated exactly to be 63.49, due to a cancellation of all dependence on $T_{\rm reh}$ in the case of gravitational reheating.

Finally, for the energy scale of inflation we find [38],

$$\left(\frac{M}{m_P}\right)^2 = \frac{3\pi\sqrt{2\alpha\mathscr{P}_\zeta}}{\left(N_* + \frac{\sqrt{3\alpha}}{2}\right)} \exp\left[\frac{3\alpha}{4}\left(N_* + \frac{\sqrt{3\alpha}}{2}\right)^{-1}\right], \tag{17}$$

where $\mathscr{P}_\zeta = (2.199 \pm 0.066) \times 10^{-9}$, is the spectrum of the scalar curvature perturbation [39]. With $N_* = 62$ and α in the range in Eq. (10), we find $M \simeq 10^{16}\,\text{GeV}$, which is at the scale of grand unification. The actual values of M can be seen in Fig. 8.

REHEATING AND QUINTESSENCE

Inflaton Freezing

As mentioned earlier, n affects the freezing value of the field, φ_F. During kination the field is oblivious of the potential and the Klein-Gordon equation reduces to $\ddot{\varphi}+3H\dot{\varphi} \simeq 0$. Consequently, following the treatment in Ref. [38], it is easy to show that during kination, the scalar field grows as

$$\varphi = \varphi_{\rm IP} + \sqrt{\frac{2}{3}}\, m_{\rm P} \ln\left(\frac{t}{t_{\rm IP}}\right), \tag{18}$$

where the subscript 'IP' denotes the moment of instant preheating, when radiation is generated (discussed in the following subsection), and we consider that kination continues for a while after instant preheating occurs. Because radiation density scales as $\rho_r \propto a^{-4}$, once created, radiation eventually takes over, since for a kinetically dominated scalar field we have $\rho_{\rm kin} \equiv \frac{\dot{\varphi}^2}{2} \propto a^{-6}$. Thus, for the density parameter of radiation during kination we have $\Omega_r = \rho_r/\rho_\phi \propto a^{-2}$. Denoting as 'reh' the moment of reheating, i.e. the moment when the radiation bath comes to dominate the Universe, we have $\Omega_r^{\rm reh} = 1$ by definition. Therefore, the radiation density parameter at instant preheating is

$$\Omega_r^{\rm IP} = \Omega_r^{\rm reh}\left(\frac{a_{\rm IP}}{a_{\rm reh}}\right)^2 = \left(\frac{t_{\rm IP}}{t_{\rm reh}}\right)^{\frac{2}{3}}, \tag{19}$$

where $\Omega_r^{\rm IP} \equiv (\rho_r/\rho)_{\rm IP}$ is the radiation density parameter at instant preheating and we considered that during kination $a \propto t^{1/3}$. Inserting the above into Eq. (18) we find

$$\varphi_{\rm reh} = \varphi_{\rm IP} - \sqrt{\frac{3}{2}}\, m_{\rm P} \ln(\Omega_r^{\rm IP}). \tag{20}$$

Now, as shown in Ref. [38], during radiation domination, the field continues to roll for a while as

$$\varphi = \varphi_{\rm reh} + \sqrt{\frac{2}{3}}\, m_{\rm P}\left(1 - \sqrt{\frac{t_{\rm reh}}{t}}\right). \tag{21}$$

The above suggests that the field freezes at a value φ_F, given by

$$\varphi_F = \varphi_{\rm IP} + \sqrt{\frac{2}{3}}\left(1 - \frac{3}{2}\ln\Omega_r^{\rm IP}\right) m_{\rm P}, \tag{22}$$

where we used Eq. (20). Here, we assume that the generation of radiation is almost instantaneous, as is discussed later. A relationship between n and φ_F can be obtained from the final energy density requirements for dark energy. Starting from the requirement that the density of quintessence must be comparable to the density of the Universe today:

$$\frac{\rho_{\rm inf}}{\rho_0} \simeq \frac{M^4}{V(\varphi_F)} \simeq \frac{e^{2\varphi_F/\sqrt{6\alpha}m_{\rm P}}}{2ne^{-2n}} \simeq 10^{108}\,, \tag{23}$$

where $\rho_{\rm inf}$ is the energy density during inflation, we find

$$2n - \ln(2n) = 108\ln 10 - \frac{2}{\sqrt{6\alpha}}\frac{\varphi_F}{m_{\rm P}}\,, \tag{24}$$

and combining Eqs. (22) and (24) gives

$$2n - \ln(2n) = 108\,\ln 10 - \frac{2}{\sqrt{6\alpha}}\sqrt{\frac{2}{3}}\Big(1 - \frac{3}{2}\ln\Omega_r^{\rm IP}\Big)\,, \tag{25}$$

where we assumed that the right-hand-side of Eq. (22) is dominated by the last term. This is so when $\Omega_r^{\rm IP} \ll 1$, which can be challenged only for very high reheating efficiency. However, as we show later, such efficiency is excluded because of backreaction constraints. Also, high reheating efficiency would mean that radiation domination begins almost right after instant preheating. This would result in a high reheating temperature, incompatible with gravitino over-production considerations.

The parameter space for n is related to the density of produced radiation at the end of inflation. Hence, changing the reheating efficiency affects the parameter space for n. $\Omega_r^{\rm IP}$ is larger the more efficient instant preheating is, meaning the scalar field rolls less far in field space before it freezes. So, maintaining the same final energy density (comparable to the density at present), requires a higher n value. To find bounds on n, we derive bounds on $\Omega_r^{\rm IP}$ and evolve the equations of motion numerically, in order to determine exactly when instant preheating occurs and how this affects the variables we need to constrain.

The equations of motion used are:

$$3m_{\rm P}^2H^2 = \frac{1}{2}\dot\varphi^2 + V(\varphi)\,, \tag{26}$$

$$-2\dot H m_{\rm P}^2 = \dot\varphi^2\,, \tag{27}$$

$$\ddot\varphi = -3H\dot\varphi - V'(\varphi)\,, \tag{28}$$

where the prime denotes differentiation with respect to φ and dots denote differentiation with respect to time.

Instant Preheating

For instant preheating we presume the inflaton ϕ is coupled to some other scalar field χ. In particular, we consider an interaction at an enhanced symmetry point (ESP) at $\phi = \phi_0$. The Lagrangian density near the ESP is

$$\mathcal{L} = \mathcal{L}(\phi_0) + \mathcal{L}_{\rm int}\,, \tag{29}$$

where $\mathcal{L}(\phi_0)$ is determined by Eq. (1) evaluated at ϕ_0. The interaction Lagrangian density near the ESP is

$$\mathcal{L}_{\rm int} = -\frac{1}{2}g^2(\phi-\phi_0)^2\chi^2 - h\chi\psi\bar\psi\,, \tag{30}$$

where g and h are perturbative coupling constants, and ψ denotes a fermion field, coupled to χ. The fermion is taken to be light, such that the χ-particles decay into a radiation bath. We consider $h \sim 1$, which means that the decay of χ is immediate.

The scalar field, χ, can be expressed in terms of the creation and annihilation operators, and the Fourier modes of this expansion obey a wave equation with a frequency dependent on the effective mass of χ. Certain solutions to this wave equation are growing solutions and this translates into an exponential increase of the occupation number n_k for a particular mode, when particle production occurs [7, 8]. The adiabaticity condition

$$\frac{\dot\omega_k}{\omega_k^2} < 1\,, \tag{31}$$

where ω_k is the frequency of the Fourier expanded wave equation, must be violated for particle production to occur. For the interaction terms used here, this leads to:

$$|\dot m_\chi| \ll m_\chi^2\,, \tag{32}$$

with $m_\chi^2 = g^2(\phi-\phi_0)^2$. Thus, particle production takes place when:

$$|\dot\phi| > g(\phi-\phi_0)^2\,, \tag{33}$$

which gives the following range for ϕ

$$\phi_0 - \sqrt{\frac{|\dot\phi|}{g}} \le \phi \le \phi_0 + \sqrt{\frac{|\dot\phi|}{g}}\,. \tag{34}$$

The above is the window of ϕ in which particle production occurs.

The careful reader may have noticed that the interaction considered regards ϕ, the original non-canonically normalised field. Hence, we need to find ϕ and $\dot\phi$ to check the adiabaticity constraint. We can find ϕ using Eq. (6) from which we readily obtain

$$\dot\phi = \mathrm{sech}^2\Big(\frac{\varphi}{\sqrt{6\alpha}m_{\rm P}}\Big)\dot\varphi\,, \tag{35}$$

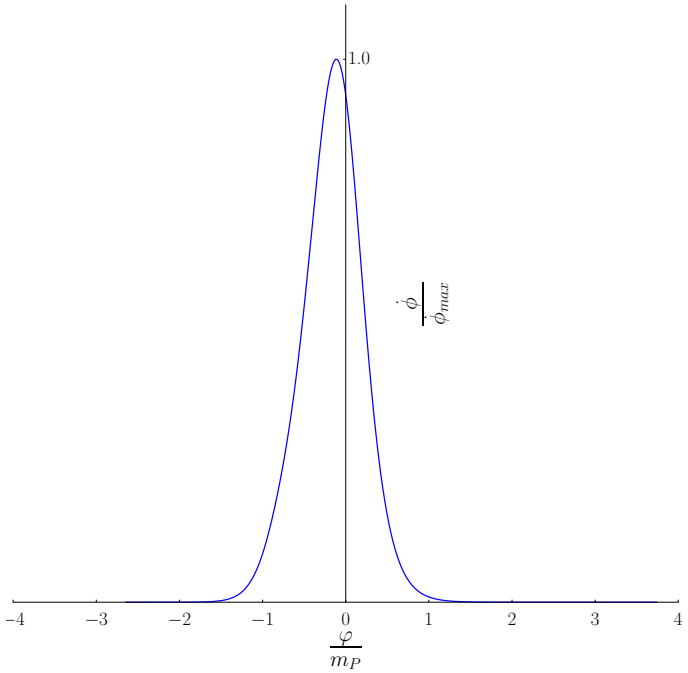

FIG. 1: This plot depicts where $\dot\phi$ is maximised in the φ direction.

where we obtain φ and $\dot\varphi$ from the computation, but for completeness:

$$\varphi = \sqrt{6\alpha} m_{\rm P} \tanh^{-1}\Big(\frac{\phi}{\sqrt{6\alpha} m_{\rm P}}\Big) \tag{36}$$

$$\text{and} \qquad \dot\varphi = \frac{\dot\phi}{1-\frac{\phi^2}{6\alpha m_{\rm P}^2}}\,, \tag{37}$$

which are analytically cyclic. It is clear from this computation that the region where $\dot\phi$ is maximised and particle production occurs is very close to $\phi = 0$, meaning $\phi \simeq \varphi$ (c.f. Eq. (6)) and ϕ is almost canonical. This can be seen clearly in Fig. 1. This is because, when the non-canonical ϕ is near the poles it hardly varies, even when the canonical φ changes substantially. Thus, it is not possible to violate the adiabaticity condition in Eq. (31) in this region. Therefore, there may be many ESPs along the φ direction, but only near $\phi \simeq \varphi \simeq 0$ can we have particle production.

The number density of produced χ particles [7, 8] is

$$n_\chi = \int \frac{{\rm d}^3k}{(2\pi)^3}\, n_k = \frac{1}{2\pi^2}\int_0^\infty k^2 n_k {\rm d}k\,, \tag{38}$$

where the occupation number

$$n_k = \exp\Big(-\frac{\pi m_\chi^2}{\dot m_\chi}\Big)\,, \tag{39}$$

is suppressed when $\dot m_\chi < m_\chi$, evidencing why the adiabaticity condition in Eq. (32), controls particle production.

Combining Eq. (38) with the χ particle effective mass provides the density of the produced χ particles [7, 8]

$$\rho_\chi^{\rm IP} = \frac{g^{5/2}|\dot\phi_{\rm IP}|^{3/2}\phi_{\rm IP}}{8\pi^3}\,. \tag{40}$$

The instant preheating efficiency is maximised when ϕ is near the final edge of the production window in Eq. (34) because, even though we expect a continuous contribution to n_χ whilst ϕ is in this region, the produced χ-particles are diluted by the expansion of the Universe. Therefore, we expect only the ones produced near the end of the particle production regime to contribute significantly to $\rho_\chi^{\rm IP}$. As such, from Eq. (34), taking $\phi_0 \simeq 0$, we set

$$\phi_{\rm IP} = \sqrt{\frac{\dot\phi_{\rm IP}}{g}}\,, \tag{41}$$

which simplifies Eq. (40) to

$$\rho_r^{\rm IP} = \rho_\chi^{\rm IP} = \frac{g^2\dot\phi_{\rm IP}^2}{8\pi^3}\,, \tag{42}$$

where we have considered that $\dot\phi > 0$ because the field is rolling towards larger values and we have assumed that the decay of the χ-particles to radiation is instantaneous.

For each choice of n, the quintessence requirements stipulate the required value of φ_F and hence $\Omega_r^{\rm IP}$. The value of $\Omega_r^{\rm IP}$ is

$$\Omega_r^{\rm IP} = \frac{\rho_\chi^{\rm IP}}{\rho_\chi^{\rm IP}+\rho_{\phi,a}^{\rm IP}} = \frac{\rho_r^{\rm IP}}{\rho_{\phi,b}^{\rm IP}}\,. \tag{43}$$

where $\rho_\chi^{\rm IP} = \rho_r^{\rm IP}$ is defined in Eq. (42) and the subscript 'a/b' refers to after/before instant preheating. Inserting the above in Eq. (42), a rearrangement quickly yields:

$$g = \sqrt{\frac{8\pi^3}{\dot\phi_{\rm IP}^2}\,\Omega_r^{\rm IP}\rho_\phi^{\rm IP}}\,, \tag{44}$$

where we have omitted subscript 'b' for simplicity. Note that $\rho_{\phi,a} \simeq \rho_{\phi,b}$ when $\Omega_r^{\rm IP} \ll 1$.

For each choice of n, we calculate φ_F from Eq. (23) and insert this into Eq. (22) to obtain $\Omega_r^{\rm IP}$ as a function of n. As the reheating variables are also functions of n, we now have g in terms of only n. However, as noted previously, $\phi_{\rm IP}$ and $\dot\phi_{\rm IP}$ are themselves dependent on g and so this requires iteration. This is the procedure to obtain a value of g for a given value of n.

CONSTRAINTS FROM REHEATING AND QUINTESSENCE

Immediate Constraints on n

An immediate sanity check arises: if $\varphi_F < \varphi_{\rm IP}$ then the combination of n and α is disallowed. This allows us at first glance to constrain n. For the complete range of allowed α values, $1.5 \leq \alpha \leq 4.2$, we find

$$n \leq 130\,. \tag{45}$$

The fact that this approach produces an upper limit on n makes sense because a larger n value makes the potential steeper and means lower V values will be reached earlier in field space. Hence, to equate $V(\varphi_F)$ with dark energy today will require a lower value for φ_F. As such, ensuring $\varphi_F > \varphi_{\rm IP}$ results in an upper bound on n.

Keeping g Perturbative and Ensuring Radiation Domination

The first constraint on g is found by requiring $g < 1$, for a perturbative coupling constant, which provides a tight upper bound on n:

$$\begin{aligned} \alpha = 1.5: &\qquad n \leq 124\,, \\ \alpha = 4.2: &\qquad n \leq 125\,. \end{aligned} \tag{46}$$

However, to obtain the correct Universe history, we also need to ensure we have a period of radiation domination after instant preheating, which might provide a tighter bound. In a quintessential inflation model with a period of kination after radiation generation, this is never a problem because the density of the produced radiation scales as $\rho_r \propto a^{-4}$ whilst the density of the kinetically dominated field scales as $\rho_\phi \propto a^{-6}$. Hence, to ensure radiation domination we need to ensure that the scalar field remains kinetically dominated after instant preheating. Note, that the transfer of energy to χ-particles during instant preheating comes from the kinetic energy density of the inflaton only, therefore $V(\phi_a) = V(\phi_b) \equiv V(\phi_{\rm IP})$. Were there not enough kinetic density left, the inflaton would become potentially dominated and would embark to a new bout of inflation. Thus, we need to ensure that the kinetic energy of the inflaton is greater than the potential energy after instant preheating. This leads to

$$\rho_{\phi,a} - V(\phi_{\rm IP}) > V(\phi_{\rm IP}) \quad \Rightarrow \quad \rho_\chi < \rho_{\phi,b} - 2V(\phi_{\rm IP})\,, \tag{47}$$

where the subscripts 'a' and 'b' refer to after and before instant preheating respectively and we have used that $\rho_{\phi,b} = \rho_{\phi,a} + \rho_\chi$. Eq. (47) gives us an upper limit on the allowed energy density of produced χ particles, which translates to an upper limit on the perturbative coupling g, from the equation for the energy density, Eq. (42). However, it turns out that this constraint is automatically satisfied for a perturbative coupling with $g < 1$.

Backreaction Constraint

We must also consider the back reaction of produced χ-particles on ϕ, which may further constrain the allowed value of g. The equation of motion for the scalar field, including back reaction, is given by [7, 8]

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = -g n_\chi \frac{\phi}{|\phi|}\,, \tag{48}$$

where

$$n_\chi = \frac{(g|\dot{\phi}|)^{3/2}}{8\pi^3} \exp\Big(-\frac{\pi m_\chi^2}{\dot{m}_\chi}\Big) \tag{49}$$

and we consider that, near the ESP, ϕ is canonically normalised ($\phi \simeq \varphi$), as discussed.

The exponential is suppressed during particle production and so the right hand side of Eq. (48) becomes

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = -\frac{g^{5/2}\dot{\phi}^{3/2}}{8\pi^3}\,, \tag{50}$$

where we have also considered $\dot{\phi} > 0$. As back reaction increases, the magnitude of the right-hand side of this equation grows to have more and more of an effect on the dynamics [8]. This is maximised at $\phi = \phi_{\rm IP}$ (i.e. for maximum n_χ). Computing this at that moment, we find that to avoid back-reaction effects requires roughly

$$g \lesssim 10^{-3}\,. \tag{51}$$

In detail, the above upper bound on g depends on the value of α as depicted in our results, see Figs. 2-10.

Gravitino Constraint

Finally, because this is a model rooted in supergravity, constraints from over-production of gravitinos have to be taken into account. The over-production of gravitinos needs to be controlled because they can either contribute to the mass of dark matter and overclose the Universe or they can decay and disrupt the production of nuclei during BBN. Gravitino production is strongly correlated with reheating temperature. In general, the bound $T_{\rm reh} < \mathcal{O}(10^9)$ GeV

g	Allowed n values	Allowed κ values
0.001	$119 \leq n \leq 122$	$24.3 \leq \kappa \leq 39.6$
0.01	$121 \leq n \leq 123$	$24.5 \leq \kappa \leq 40.3$
0.1	$123 \leq n \leq 124$	$24.7 \leq \kappa \leq 41.0$
1.0	$125 \leq n \leq 126$	$25.1 \leq \kappa \leq 41.7$

TABLE I: Allowed n and κ values for specific choices of g, within the allowed α range, before consideration of backreaction and gravitino constraints

α	n	N_*	n_s	$r/10^{-3}$	$n_s'/10^{-4}$
1.5	118	62.7	0.968	4.42	−5.25
1.5	124	59.1	0.966	4.97	−5.92
4.2	121	63.5	0.968	11.8	−5.11
4.2	125	59.4	0.966	13.9	−5.86

TABLE II: For the allowed range of n, prior to consideration of backreaction and gravitino constraints, the corresponding values of N_* and the inflationary observables are shown.

α	n	κ	$T_{\rm reh}$ (GeV)	M (GeV)	$V_0^{1/4}$ (GeV)
1.5	118	39.3	3.84×10^{6}	8.50×10^{15}	1.31×10^{3}
1.5	124	41.3	2.22×10^{11}	8.76×10^{15}	3.01×10^{2}
4.2	121	24.1	2.35×10^{5}	1.10×10^{16}	8.04×10^{2}
4.2	125	24.9	6.07×10^{10}	1.15×10^{16}	3.06×10^{2}

TABLE III: For the allowed range of n, prior to consideration of backreaction and gravitino constraints, the corresponding values of $T_{\rm reh}$, M and $V_0^{1/4}$ are shown.

[48–50] is adequate.[4]

We can derive $T_{\rm reh}$ in this model from the relationship

$$T_{\rm reh}=\Big(\frac{30}{\pi^2 g_*}\rho_r^{\rm reh}\Big)^{1/4}\,, \tag{52}$$

where 'reh' denotes the moment of reheating, which is the onset of the radiation era. Employing Eq. (19) we readily find

$$\rho_r^{\rm reh}=\rho_\phi^{\rm reh}=\rho_\phi^{\rm IP}(\Omega_r^{\rm IP})^3\,, \tag{53}$$

where we used that $\rho_\phi\propto a^{-6}$ during kination. Inserting this into Eq.(52) we find

$$T_{\rm reh}=\Big[\frac{30}{\pi^2 g_*}\rho_\phi^{\rm IP}(\Omega_r^{\rm IP})^3\Big]^{1/4}=\Big[\frac{30}{\pi^2 g_*}\rho_r^{\rm IP}(\Omega_r^{\rm IP})^2\Big]^{1/4}\,, \tag{54}$$

where we also considered that $\rho_r=\Omega_r\rho_\phi$. We find $\Omega_r^{\rm IP}$ as follows

$$\Omega_r^{\rm IP}=\frac{\rho_r^{\rm IP}}{\rho_\phi^{\rm IP}}=\frac{g^2\dot\phi_{\rm IP}^2}{8\pi^3}\frac{2}{\phi_{\rm IP}^2}=\frac{g^2}{4\pi^3}\,, \tag{55}$$

where we considered Eq. (42) and that $\rho_\phi^{\rm IP}=\frac{1}{2}\dot\phi_{\rm IP}^2$ during kination. Thus, $\Omega_r^{\rm IP}\sim10^{-2}g^2$, which means that, since $g<1$, $\Omega_r^{\rm IP}$ is very small. Given that the dependence of $(\rho_\phi^{\rm IP})^{1/4}$ on g is weak, Eq. (54) suggests $T_{\rm reh}\propto g^{3/2}$. This is easy to understand by considering that a large value of g means that more radiation is generated at instant preheating. Consequently, reheating happens earlier and therefore $T_{\rm reh}$ is large. To limit $T_{\rm reh}$ to small enough values we need to avoid a large g.

In our model, the bound $T_{\rm reh}<\mathcal{O}(10^9)\,$GeV translates to an upper bound on g of roughly

$$g\lesssim10^{-2}\,. \tag{56}$$

As in the previous subsection, in detail, the above upper bound on g depends on the value of α as depicted in our results, see Figs. 2-10.

A Lower Bound on g

The first constraint on a lower g value is to ensure that radiation domination occurs before BBN, but this constraint is not a worry for we find $T_{\rm reh}\gg1\,$MeV in all cases.

We may obtain a lower bound on $\rho_\chi^{\rm IP}$, and hence g, from the nucleosynthesis constraint on the energy density of produced gravitational waves during kination. We follow the treatment in Ref. [52] to find the lower bound on g. The BBN constraint demands

$$\Big(\frac{\rho_g}{\rho_r}\Big)_{\rm reh}\lesssim10^{-2}\,, \tag{57}$$

where

$$\Big(\frac{\rho_g}{\rho_r}\Big)_{\rm reh}=\frac{64}{3\pi}h_{\rm GW}^2\Big(\frac{\rho_\phi}{\rho_r}\Big)_{\rm IP}\,. \tag{58}$$

Using the relations

$$h_{\rm GW}^2=\frac{H_{\rm end}^2}{8m_{\rm P}^2}\quad\text{and}\quad H_{\rm end}^2\simeq\frac{V_{\rm end}}{3m_{\rm P}^2}\,, \tag{59}$$

where the subscript 'end' signifies the end of inflation, we can re-express this as

$$\Big(\frac{\rho_g}{\rho_\gamma}\Big)_{\rm reh}=\frac{8}{9\pi}\frac{V_{\rm end}}{m_{\rm P}^4}\frac{1}{\Omega_r^{\rm IP}}\lesssim10^{-2}\,. \tag{60}$$

Substituting the above in Eq. (55) we get

$$g\geq20\pi\sqrt{\frac{8}{9}}\frac{V_{\rm end}^{1/2}}{m_{\rm P}^2}\simeq10\Big(\frac{M}{m_{\rm P}}\Big)^2\sim10^{-4} \tag{61}$$

where we considered that $V_{\rm end}=M^4e^{-\sqrt{3\alpha}}$ [38]. For the last equation we considered $M\simeq10^{16}\,$GeV as suggested by Fig. 8.

[4] However, in some cases, the bound can be much a tighter: $T_{\rm reh}<\mathcal{O}(10^6)\,$GeV [51].

α	n	N_*	n_s	$r/10^{-3}$	$n_s'/10^{-4}$
1.5	118	62.74	0.968	4.42	-5.25
1.5	119	62.14	0.968	4.51	-5.35
4.2	121	63.54	0.968	11.8	-5.11
4.2	122	62.53	0.967	12.2	-5.28

TABLE IV: Final values for the parameters when considering the tightest constraints on g, from the backreaction bound.

α	n	κ	$T_{\rm reh}$ (GeV)	M (GeV)	$V_0^{1/4}$ (GeV)
1.5	118	39.3	3.84×10^{6}	8.5×10^{15}	1.31×10^{3}
1.5	119	39.7	2.39×10^{7}	8.5×10^{15}	1.03×10^{3}
4.2	121	24.1	2.35×10^{5}	1.1×10^{16}	8.04×10^{2}
4.2	122	24.3	5.07×10^{6}	1.1×10^{16}	6.31×10^{2}

TABLE V: Final values for the parameters when considering the tightest constraints on g, from the backreaction bound.

RESULTS AND DISCUSSION

The two unavoidable constraints are the upper bound on n, ensuring $g < 1$ because of perturbativity, and the lower limit on g ensuring a period of kination that does not disturb BBN through overproduction of gravitational waves. This bound is $g \gtrsim 10^{-4}$. These bounds result in the parameter space

$$\alpha = 1.5: \qquad 118 \leq n \leq 124\,, \tag{62}$$

$$\alpha = 4.2: \qquad 121 \leq n \leq 125\,. \tag{63}$$

The upper constraint on g arising from the avoidance of backreaction in the instant preheating mechanism results in a bound on g of approximately $g \lesssim 10^{-3}$. However, this bound can be sidestepped if the decay $\chi \to \psi\bar{\psi}$ is rapid, as is often assumed. All that is required is a large enough h value for this coupling. The upper bound on g arising from gravitino over-production constraints is important in a model rooted in supergravity. This bound is roughly $g \lesssim 10^{-2}$. Because of this bound, the parameter space is reduced to

$$\alpha = 1.5: \qquad 118 \leq n \leq 122\,, \tag{64}$$

$$\alpha = 4.2: \qquad 121 \leq n \leq 124\,. \tag{65}$$

The allowed values of n and κ for a selection of g values are shown in Table I, without consideration of the backreaction and gravitino bounds. For the extremal values of

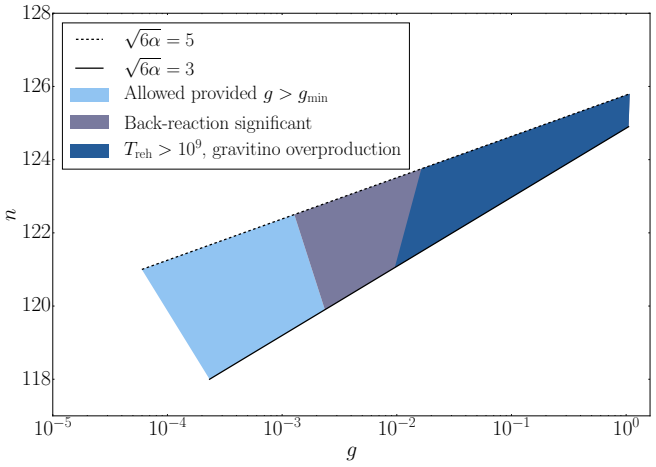

FIG. 2: Allowed parameter space for n, for the range of allowed g values and allowed α values between 1.5 and 4.2. The bounds arising from backreaction and gravitino constraints are indicated.

n the corresponding values of N_*, n_s, r, n_s', $T_{\rm reh}$, M and $V_0^{1/4}$ are shown in Tables II and III. Figs. 2, 3, 4, 5, 7, 8 9 and 10 document how the parameter space is altered when the backreaction and gravitino bounds are included. Values of N_*, n_s r, n_s', $T_{\rm reh}$, M and $V_0^{1/4}$ for the most constricted final parameter space are shown in Tables IV and V.

With a lower value of $\Omega_r^{\rm IR}$, the inflaton rolls to larger distances before it freezes. To fulfil dark energy requirements, this requires a lower n value. The results found here for n demonstrate this. The two different α values result in different n requirements because α controls the slope of the quintessential tail (c.f. Eq. (9)). A smaller/larger α-value means a steeper/gentler quintessential tail. Thus, for a given value of φ_F, we require smaller/larger n-values for a smaller/larger-α value.

SUPPRESSED INTERACTIONS

In general, quintessence models require an extremely flat potential over super-Planckian distances. This gives rise to two problems. First, the flatness of such a potential can be lifted by sizeable radiative corrections. Second, because the mass of the quintessence field is extremely small $\sim H_0 \sim 10^{-33}\,$eV, the corresponding wavelength is very large (Horizon sized), which can give rise to the infamous 5th force problem, that amounts to sizable violations of the Equivalence Principle.

However, in the context of α-attractors both the above dangers are averted. Indeed, as discussed in Ref. [40, 41], when near the kinetic poles ($\phi/m_{\rm P} \approx \pm\sqrt{6\alpha}$, equivalently $|\varphi|/m_{\rm P} \gg \sqrt{6\alpha}$), the inflaton interactions are exponentially suppressed and the field becomes "asymptotically free". The same is true for the loop corrections to the potential. We now briefly demonstrate this regarding the interactions.

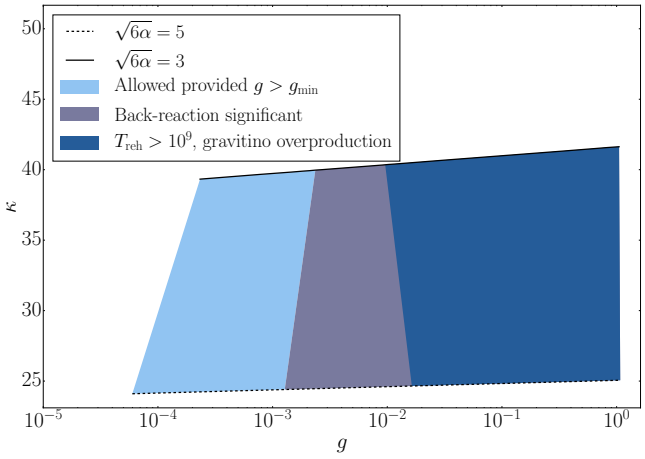

FIG. 3: Allowed parameter space for κ, for the range of allowed g values and allowed α values between 1.5 and 4.2. The bounds arising from backreaction and gravitino constraints are indicated.

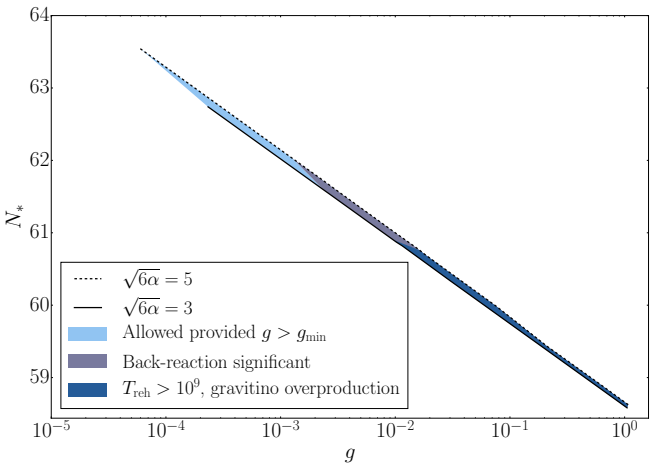

FIG. 4: N_* values for the range of g values and allowed α values between 1.5 and 4.2. The bounds arising from backreaction and gravitino constraints are indicated.

We expect the inflaton to have Planck-suppressed interactions with other fields. Following Ref. [40, 41], lets sketch this by considering another scalar field σ with which the inflaton is coupled as

$$\delta V = \frac{1}{2} h \left(\frac{\phi}{m_{\rm P}}\right)^q \phi^2 \sigma^2, \tag{66}$$

where $q \geq 0$ and $h = \mathcal{O}(1)$. Then, the strength of the interaction is estimated by $\mathcal{G} = \partial_\varphi^2 \partial_\sigma^2 \delta V$. It is straightforward to find

$$\mathcal{G} = \left(\frac{\partial\phi}{\partial\varphi}\right)^2 (q+1)(q+2) h \left(\frac{\phi}{m_{\rm P}}\right)^q. \tag{67}$$

Now, near the pole (down the quintessential tail) we have

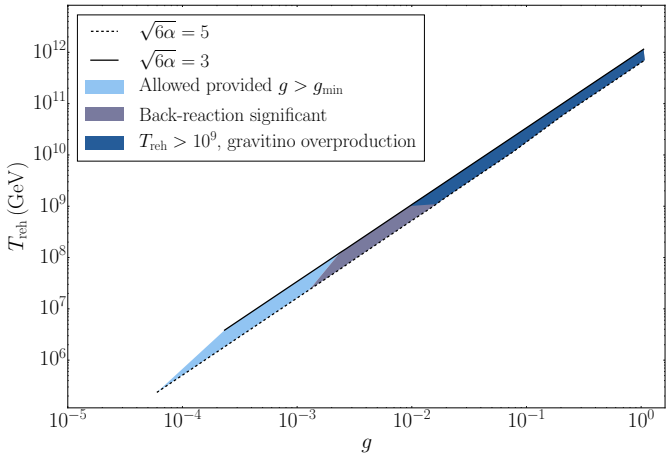

FIG. 5: $T_{\rm reh}$ values for the range of g values and allowed α values between 1.5 and 4.2. The bounds arising from backreaction and gravitino constraints are indicated.

$\phi/m_{\rm P} = \sqrt{6\alpha}$. Using this and in view of Eq. (6), we find

$$\mathcal{G} = \frac{(q+1)(q+2)\, h\, (6\alpha)^{q/2}}{\cosh^4 \frac{\varphi_F}{\sqrt{6\alpha}\, m_{\rm P}}}. \tag{68}$$

Taking $q \sim h \sim \alpha \sim 1$ and $\varphi_F \gg m_{\rm P}$ we obtain that the strength of the interaction is suppressed as

$$\mathcal{G} \sim \exp\left(-\frac{4\varphi_F}{\sqrt{6\alpha}\, m_{\rm P}}\right). \tag{69}$$

It should be noted here that this suppression is not due to assuming a Planck-suppressed interaction, as can be readily seen by taking $q = 0$ in Eq. (68).

It is straightforward to obtain an estimate of the above value of $\mathcal{G}$. Indeed, ignoring $\varphi_{\rm IP}$ and using Eq. (55) we have

$$\varphi_F/m_{\rm P} \simeq \sqrt{\frac{2}{3}} \left[1 - 3 \ln\left(g/2\pi^{3/2}\right)\right]. \tag{70}$$

Inserting the above into Eq. (69), we obtain

$$\mathcal{G} \sim e^{-4/3\sqrt{\alpha}} \left(\frac{g}{2\pi^{3/2}}\right)^{4/\sqrt{\alpha}} \tag{71}$$

Using this we obtain the values shown in Fig 6, which demonstrates that the interaction strength is drastically diminished. The above argument can be generalised to non-perturbative interactions, which are expected to be of the form $\sim \exp(-\beta_i \phi/m_{\rm P})\mathcal{L}_i$, where $\mathcal{L}_i$ is any 4-dimensional Lorentz-invariant operator. Considering the interaction strength, we always obtain a factor $\left(\frac{\partial\phi}{\partial\varphi}\right)^2 \sim \exp\left(-\frac{4\varphi_F}{\sqrt{6\alpha}\, m_{\rm P}}\right)$ in the limit $\phi/m_{\rm P} \to \sqrt{6\alpha}$. As shown in Fig. 6, the interaction strength is exponentially suppressed, which overcomes the 5th force problem.

In a similar manner, loop corrections are also suppressed so the flatness of the quintessential tail is safely protected from radiative corrections [40, 41].

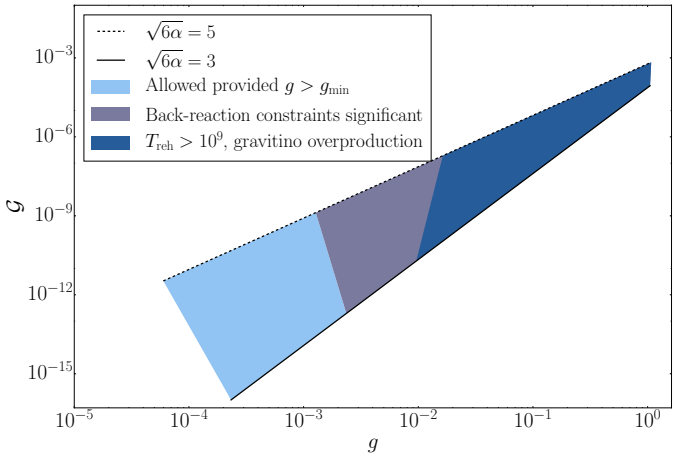

FIG. 6: The interaction strength, $\mathcal{G}$ for the range of allowed g values and allowed α values between 1.5 and 4.2. The bounds arising from backreaction and gravitino constraints are indicated.

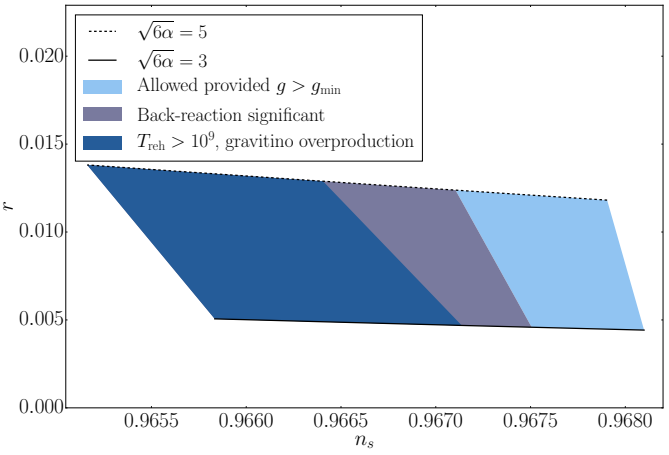

FIG. 7: n_s and r values for the range of n values indicated in Eq.(62), for allowed α values between 1.5 and 4.2. The bounds arising from backreaction and gravitino constraints are indicated.

CONCLUSIONS

We have investigated a model of quintessential inflation in the context of α-attractors in supergravity. We considered a simple exponential potential $V(\phi) = V_0 e^{-\kappa\phi/m_P}$ (c.f. Eq. (3)) and the standard α-attractors kinetic term, which features two poles at $\phi = \pm\sqrt{6\alpha}\, m_{\rm P}$ (c.f. Eq. (2)). Switching to a canonically normalised inflaton, the scalar potential gets "stretched" as the poles are transposed to infinity [15–18], thereby generating the inflationary plateau and the quintessential tail. After inflation, the field becomes kinetically dominated as it "jumps off the cliff" of the inflationary plateau. A period of kination ensues. This necessarily ends when the Universe becomes dominated by radiation and the hot big bang begins. This radiation is generated through the mechanism of instant preheating. For this we assume that the inflaton ϕ is coupled with some other scalar field χ such that, after the end of inflation when the inflaton's variation peaks, the effective mass of the χ-particles is varying non-adiabatically. This adiabaticity breaking results in particle production of χ-particles, which soon decay into a newly formed radiation bath. The strength of the interaction between ϕ and χ is parametrised by the coupling g (c.f. Eq. (30)).

We have investigated the parameter space available, when the observational constraints on the abundance and barotropic (equation of state) parameter of dark energy are considered. We were conservative in avoiding a super-Planckian inflaton field ϕ, even though the suppression of loop corrections and interactions of the inflaton near the poles in α-attractors [40, 41] would mean that, even if the inflaton were super-Planckian, the flatness of the quintessential runaway potential would be preserved and there would not be a fifth-force problem. Moreover, we have taken into account backreaction constraints, which

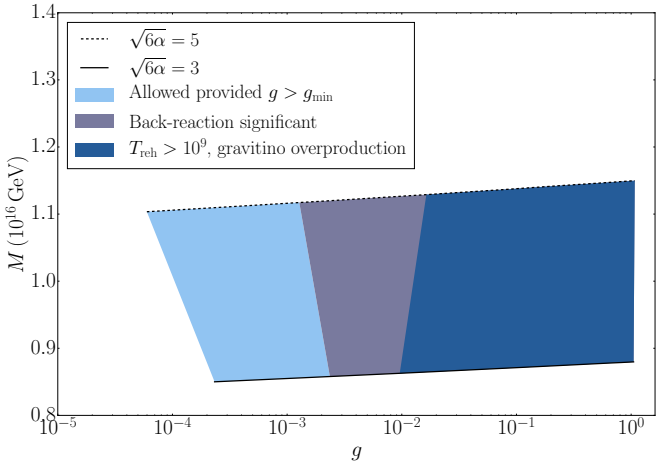

FIG. 8: M values for the range of g values and allowed α values between 1.5 and 4.2. The bounds arising from backreaction and gravitino constraints are indicated.

threaten to shut down χ-particle production and gravitino constraints on the reheating temperature.

When all the constraints are applied we find that our model is successful for natural values of the model parameters. In particular, for the coupling we find $g \sim 10^{-4} - 10^{-2}$, while we also have $V_0^{1/4} \sim 1\,{\rm TeV}$, which is the electroweak energy scale (Fig. 9). The inflationary scale is $M \simeq 10^{16}\,{\rm GeV}$, which is at the energy scale of grand unification (Fig. 8). For the slope of the exponential potential we find $\kappa \simeq 24 - 40$ (Fig 3), i.e. $\kappa \sim 0.1 m_{\rm P}/M$, meaning that in the potential the inflaton is suppressed by the scale $\sim 10^{17}\,{\rm GeV}$ (string scale?).

We also find that the cosmological scales exit the horizon about $N_* \simeq 62 - 63$ e-folds before the end of inflation (Fig. 4) and that the reheating temperature is

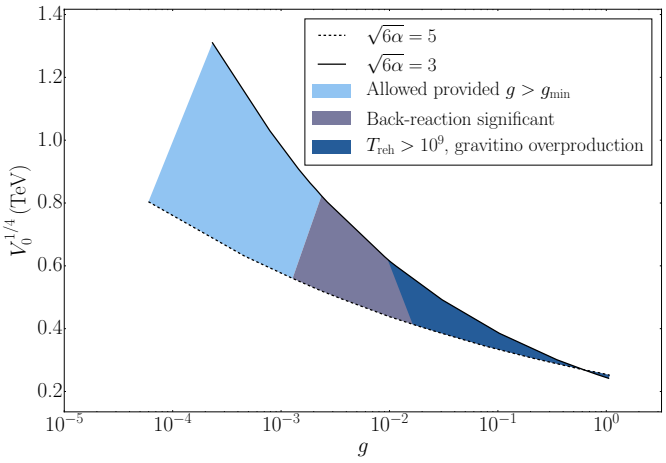

FIG. 9: $V_0^{1/4}$ values for the range of g values and allowed α values between 1.5 and 4.2. The bounds arising from backreaction and gravitino constraints are indicated.

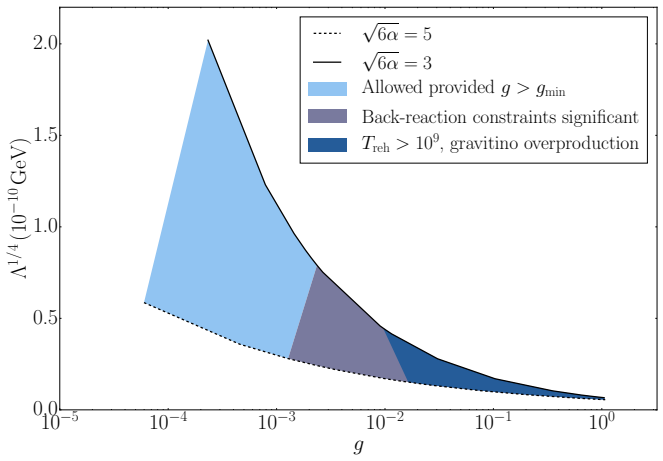

FIG. 10: $\Lambda^{1/4}$ values for the range of g values and allowed α values between 1.5 and 4.2. The bounds arising from backreaction and gravitino constraints are indicated.

$T_{\rm reh} \sim 10^5 - 10^8\,{\rm GeV}$ (Fig. 5), which satisfies gravitino constraints as required. For the inflationary observables we obtain the values $n_s = 0.968$ for the spectral index and $n_s' = -(5-6)\times 10^{-4}$ for its running. For the tensor to scalar ratio we obtain $r \simeq 0.004 - 0.012$, which may well be observable (Fig. 7). These values are within the 1-σ contour of the Planck results [39].

The α-attractors setup may also be realised without relying on supergravity [15–18]. In this case, the gravitino constraints may not be necessary. Also, backreaction effects can be dispensed with when the χ-particles decay rapidly into radiation, such that they don't backreact and close the resonance. If we remove these constraints, our parameter space is substantially enlarged. In particular, g can approach unity, while N_* can be as low as $N_* \simeq 59$ and the reheating temperature can be as large as $T_{\rm reh} \sim 10^{11}\,{\rm GeV}$. Regarding the inflationary observables, the spectral index can become as low as $n_s = 0.966$, but r is not changed much.

The required cosmological constant is $\Lambda^{1/4} \sim 10^{-10}\,{\rm GeV}$ (Fig. 10), which is somewhat larger that the value $\sim 10^{-3}\,{\rm eV}$ required in ΛCDM, but the improvement is not much. One may worry that, if one is prepared to accept the scale 0.1 eV, why not stay with ΛCDM in the first place. The answer is two-fold. Firstly, in contrast to ΛCDM, our required value for Λ is not imposed ad hoc to satisfy the observations. Instead, it is generated by the requirement that the vacuum energy asymptotes to zero (cf. Eq. (4)). In other words, the (unknown) mechanism which demands zero vacuum density is the one which imposes our value of Λ. The second reason has to do with the future horizon problem in string theories [53–56]. In a nutshell, in ΛCDM, there is a future event horizon, which makes the asymptotic future states not well defined because they are not causally connected. As a result, the formulation of the S-matrix is problematic [57, 58]. Our model may overcome this problem as follows: Since the eventual value of the vacuum density is zero, this means that the size of the future event horizon increases to infinity. Thus, future states are well defined and the future horizon problem is overcome. Finally, it is important to point out that our model considers a varying barotropic parameter of dark energy, which will be tested in the near future.

In summary, we have shown that our model of quintessential inflation with α-attractors, first introduced in Ref. [38], works well with instant preheating, improving the robustness of the model.

Note: After our paper originally appeared in the arXiv, Ref. [59] came out, which considers quintessential inflation with α-attractors beyond the single-field exponential potential.

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REFERENCES

[1] Adam G. Riess *et al.* (Supernova Search Team), "Observational evidence from supernovae for an accelerating universe and a cosmological constant," Astron. J. **116**, 1009–1038 (1998), arXiv:astro-ph/9805201 [astro-ph].

[2] S. Perlmutter *et al.* (Supernova Cosmology Project), "Measurements of Omega and Lambda from 42 high redshift supernovae," Astrophys. J. **517**, 565–586 (1999), arXiv:astro-ph/9812133 [astro-ph].

[3] P. J. E. Peebles and Bharat Ratra, "The Cosmological constant and dark energy," Rev. Mod. Phys. **75**, 559–606 (2003), arXiv:astro-ph/0207347 [astro-ph].

[4] Bharat Ratra and P. J. E. Peebles, "Cosmological Consequences of a Rolling Homogeneous Scalar Field," Phys. Rev. **D37**, 3406 (1988).

[5] R. R. Caldwell, Rahul Dave, and Paul J. Steinhardt, "Cosmological imprint of an energy component with general equation of state," Phys. Rev. Lett. **80**, 1582–1585 (1998), arXiv:astro-ph/9708069 [astro-ph].

[6] P. J. E. Peebles and A. Vilenkin, "Quintessential inflation," Phys. Rev. **D59**, 063505 (1999), arXiv:astro-ph/9810509 [astro-ph].

[7] Gary N. Felder, Lev Kofman, and Andrei D. Linde, "Instant preheating," Phys. Rev. **D59**, 123523 (1999), arXiv:hep-ph/9812289 [hep-ph].

[8] A. H. Campos, H. C. Reis, and R. Rosenfeld, "Preheating in quintessential inflation," Phys. Lett. **B575**, 151–156 (2003), arXiv:hep-ph/0210152 [hep-ph].

[9] Bo Feng and Ming-zhe Li, "Curvaton reheating in nonoscillatory inflationary models," Phys. Lett. **B564**, 169–174 (2003), arXiv:hep-ph/0212213 [hep-ph].

[10] J. C. Bueno Sanchez and Konstantinos Dimopoulos, "Curvaton reheating allows TeV Hubble scale in NO inflation," JCAP **0711**, 007 (2007), arXiv:0707.3967 [hep-ph].

[11] Tomohiro Matsuda, "NO Curvatons or Hybrid Quintessential Inflation," JCAP **0708**, 003 (2007), arXiv:0707.1948 [hep-ph].

[12] L. H. Ford, "Gravitational Particle Creation and Inflation," Phys. Rev. **D35**, 2955 (1987).

[13] E. J. Chun, S. Scopel, and I. Zaballa, "Gravitational reheating in quintessential inflation," JCAP **0907**, 022 (2009), arXiv:0904.0675 [hep-ph].

[14] Edmund J. Copeland, M. Sami, and Shinji Tsujikawa, "Dynamics of dark energy," Int. J. Mod. Phys. **D15**, 1753–1936 (2006), arXiv:hep-th/0603057 [hep-th].

[15] Renata Kallosh, Andrei Linde, and Diederik Roest, "Superconformal Inflationary α-Attractors," JHEP **11**, 198 (2013), arXiv:1311.0472 [hep-th].

[16] Renata Kallosh, Andrei Linde, and Diederik Roest, "Universal Attractor for Inflation at Strong Coupling," Phys. Rev. Lett. **112**, 011303 (2014), arXiv:1310.3950 [hep-th].

[17] Sergio Ferrara, Renata Kallosh, Andrei Linde, and Massimo Porrati, "Minimal Supergravity Models of Inflation," Phys. Rev. **D88**, 085038 (2013), arXiv:1307.7696 [hep-th].

[18] Sergio Ferrara, Renata Kallosh, Andrei Linde, and Massimo Porrati, "Higher Order Corrections in Minimal Supergravity Models of Inflation," JCAP **1311**, 046 (2013), arXiv:1309.1085 [hep-th].

[19] Renata Kallosh, Andrei Linde, and Diederik Roest, "Large field inflation and double α-attractors," JHEP **08**, 052 (2014), arXiv:1405.3646 [hep-th].

[20] Renata Kallosh and Andrei Linde, "Planck, LHC, and α-attractors," Phys. Rev. **D91**, 083528 (2015), arXiv:1502.07733 [astro-ph.CO].

[21] Diederik Roest and Marco Scalisi, "Cosmological attractors from alpha-scale supergravity," Phys. Rev. **D92**, 043525 (2015), arXiv:1503.07909 [hep-th].

[22] Andrei Linde, "Single-field α-attractors," JCAP **1505**, 003 (2015), arXiv:1504.00663 [hep-th].

[23] Marco Scalisi, "Cosmological α-attractors and de Sitter landscape," JHEP **12**, 134 (2015), arXiv:1506.01368 [hep-th].

[24] John Joseph M. Carrasco, Renata Kallosh, and Andrei Linde, "α-Attractors: Planck, LHC and Dark Energy," JHEP **10**, 147 (2015), arXiv:1506.01708 [hep-th].

[25] Mario Galante, Renata Kallosh, Andrei Linde, and Diederik Roest, "Unity of Cosmological Inflation Attractors," Phys. Rev. Lett. **114**, 141302 (2015), arXiv:1412.3797 [hep-th].

[26] John Joseph M. Carrasco, Renata Kallosh, and Andrei Linde, "Cosmological Attractors and Initial Conditions for Inflation," Phys. Rev. **D92**, 063519 (2015), arXiv:1506.00936 [hep-th].

[27] Takahiro Terada, "Generalized Pole Inflation: Hilltop, Natural, and Chaotic Inflationary Attractors," Phys. Lett. **B760**, 674–680 (2016), arXiv:1602.07867 [hep-th].

[28] K. Sravan Kumar, J. Marto, P. Vargas Moniz, and Suratna Das, "Non-slow-roll dynamics in $\alpha-$attractors," JCAP **1604**, 005 (2016), arXiv:1506.05366 [gr-qc].

[29] Yoshiki Ueno and Kazuhiro Yamamoto, "Constraints on α-attractor inflation and reheating," Phys. Rev. **D93**, 083524 (2016), arXiv:1602.07427 [astro-ph.CO].

[30] Mehdi Eshaghi, Moslem Zarei, Nematollah Riazi, and Ahmad Kiasatpour, "CMB and reheating constraints to α-attractor inflationary models," Phys. Rev. **D93**, 123517 (2016), arXiv:1602.07914 [astro-ph.CO].

[31] Michal Artymowski and Javier Rubio, "Endlessly flat scalar potentials and α-attractors," Phys. Lett. **B761**, 111–114 (2016), arXiv:1607.00398 [astro-ph.CO].

[32] Renata Kallosh, Andrei Linde, Diederik Roest, and Timm Wrase, "Sneutrino inflation with α-attractors," JCAP **1611**, 046 (2016), arXiv:1607.08854 [hep-th].

[33] Sergio Ferrara and Renata Kallosh, "Seven-disk manifold, α-attractors, and B modes," Phys. Rev. **D94**, 126015 (2016), arXiv:1610.04163 [hep-th].

[34] S. D. Odintsov and V. K. Oikonomou, "Inflationary α-attractors from $F(R)$ gravity," Phys. Rev. **D94**, 124026 (2016), arXiv:1612.01126 [gr-qc].

[35] Alessandro Di Marco, Paolo Cabella, and Nicola Vittorio, "Reconstruction of α-attractor supergravity models of inflation," Phys. Rev. **D95**, 023516 (2017), arXiv:1703.06472 [astro-ph.CO].

[36] Artur Alho and Claes Uggla, "Inflationary α-attractor cosmology: A global dynamical systems perspective," Phys. Rev. **D95**, 083517 (2017), arXiv:1702.00306 [gr-qc].

[37] Konstantinos Dimopoulos and Michal Artymowski, "Initial conditions for inflation," Astropart. Phys. **94**, 11–16 (2017), arXiv:1610.06192 [astro-ph.CO].

[38] Konstantinos Dimopoulos and Charlotte Owen, "Quintessential Inflation with α-attractors," JCAP **1706**, 027 (2017), arXiv:1703.00305 [gr-qc].

[39] P. A. R. Ade *et al.* (Planck), "Planck 2015 results. XX. Constraints on inflation," Astron. Astrophys. **594**, A20 (2016), arXiv:1502.02114 [astro-ph.CO].

[40] Andrei Linde, "On the problem of initial conditions for inflation," (2017) arXiv:1710.04278 [hep-th].

[41] Renata Kallosh and Andrei Linde, "Cosmological Attractors and Asymptotic Freedom of the Inflaton Field," JCAP **1606**, 047 (2016), arXiv:1604.00444 [hep-th].

[42] Lars Gorlich, Shamit Kachru, Prasanta K. Tripathy, and Sandip P. Trivedi, "Gaugino condensation and nonperturbative superpotentials in flux compactifications," JHEP **12**, 074 (2004), arXiv:hep-th/0407130 [hep-th].

[43] Michael Haack, Daniel Krefl, Dieter Lust, Antoine Van Proeyen, and Marco Zagermann, "Gaugino Condensates and D-terms from D7-branes," JHEP **01**, 078 (2007), arXiv:hep-th/0609211 [hep-th].

[44] Zygmunt Lalak, Graham G. Ross, and Subir Sarkar, "Racetrack inflation and assisted moduli stabilisation," Nucl.

Phys. **B766**, 1–20 (2007), arXiv:hep-th/0503178 [hep-th].
[45] Edmund J. Copeland, Andrew R Liddle, and David Wands, "Exponential potentials and cosmological scaling solutions," Phys. Rev. **D57**, 4686–4690 (1998), arXiv:gr-qc/9711068 [gr-qc].
[46] Renata Kallosh, Andrei Linde, Diederik Roest, and Yusuke Yamada, "$\overline{D3}$ induced geometric inflation," JHEP **07**, 057 (2017), arXiv:1705.09247 [hep-th].
[47] Renata Kallosh, Andrei Linde, Timm Wrase, and Yusuke Yamada, "Maximal Supersymmetry and B-Mode Targets," JHEP **04**, 144 (2017), arXiv:1704.04829 [hep-th].
[48] John R. Ellis, Andrei D. Linde, and Dimitri V. Nanopoulos, "Inflation Can Save the Gravitino," Phys. Lett. **118B**, 59–64 (1982).
[49] Masahiro Kawasaki, Fuminobu Takahashi, and T. T. Yanagida, "The Gravitino-overproduction problem in inflationary universe," Phys. Rev. **D74**, 043519 (2006), arXiv:hep-ph/0605297 [hep-ph].
[50] Masahiro Kawasaki, Fuminobu Takahashi, and T. T. Yanagida, "Gravitino overproduction in inflaton decay," Phys. Lett. **B638**, 8–12 (2006), arXiv:hep-ph/0603265 [hep-ph].
[51] Kazunori Kohri, Takeo Moroi, and Akira Yotsuyanagi, "Big-bang nucleosynthesis with unstable gravitino and upper bound on the reheating temperature," Phys. Rev. **D73**, 123511 (2006), arXiv:hep-ph/0507245 [hep-ph].
[52] Abhineet Agarwal, R. Myrzakulov, M. Sami, and Naveen K. Singh, "Quintessential inflation in a thawing realization," Phys. Lett. **B770**, 200–208 (2017), arXiv:1708.00156 [gr-qc].
[53] Simeon Hellerman, Nemanja Kaloper, and Leonard Susskind, "String theory and quintessence," JHEP **06**, 003 (2001), arXiv:hep-th/0104180 [hep-th].
[54] W. Fischler, A. Kashani-Poor, R. McNees, and S. Paban, "The Acceleration of the universe, a challenge for string theory," JHEP **07**, 003 (2001), arXiv:hep-th/0104181 [hep-th].
[55] Tom Banks, "Cosmological breaking of supersymmetry?" *Superstrings. Proceedings, International Conference, Strings 2000, Ann Arbor, USA, July 10-15, 2000*, Int. J. Mod. Phys. **A16**, 910–921 (2001), [,270(2000)], arXiv:hep-th/0007146 [hep-th].
[56] Tom Banks and W. Fischler, "M theory observables for cosmological space-times," (2001), arXiv:hep-th/0102077 [hep-th].
[57] Lisa Dyson, Matthew Kleban, and Leonard Susskind, "Disturbing implications of a cosmological constant," JHEP **10**, 011 (2002), arXiv:hep-th/0208013 [hep-th].
[58] Naureen Goheer, Matthew Kleban, and Leonard Susskind, "The Trouble with de Sitter space," JHEP **07**, 056 (2003), arXiv:hep-th/0212209 [hep-th].
[59] Yashar Akrami, Renata Kallosh, Andrei Linde, and Valeri Vardanyan, "Dark energy, α-attractors, and large-scale structure surveys," (2017), arXiv:1712.09693 [hep-th].
[60] P. A. R. Ade *et al.* (Planck), "Planck 2015 results. XIII. Cosmological parameters," Astron. Astrophys. **594**, A13 (2016), arXiv:1502.01589 [astro-ph.CO].
[61] Konstantinos Dimopoulos, "The Curvaton hypothesis and the eta-problem of quintessential inflation, with and without branes," Phys. Rev. **D68**, 123506 (2003), arXiv:astro-ph/0212264 [astro-ph].
[62] Urbano França and Rogerio Rosenfeld, "Fine tuning in quintessence models with exponential potentials," JHEP **10**, 015 (2002), arXiv:astro-ph/0206194 [astro-ph].
[63] James M. Cline, "Quintessence, cosmological horizons, and self-tuning," JHEP **08**, 035 (2001), arXiv:hep-ph/0105251 [hep-ph].
[64] Christopher F. Kolda and William Lahneman, "Exponential quintessence and the end of acceleration," (2001), arXiv:hep-ph/0105300 [hep-ph].
[65] Alex Kehagias and Georgios Kofinas, "Cosmology with exponential potentials," Class. Quant. Grav. **21**, 3871–3886 (2004), arXiv:gr-qc/0402059 [gr-qc].

APPENDIX: THE RANGE OF α

In this Appendix we calculate the appropriate range for α, upon which our results are based. To do this, we first investigate exponential quintessence.

We consider single field quintessence with a canonical scalar field φ and a scalar potential $V(\varphi)$, which drives the currently observed accelerated expansion. This simple model assumes only a minimal coupling between gravity and φ, and is thus described by the action

$$S = \int \mathrm{d}^4x\sqrt{-g}\mathcal{L} + S_m(g_{\mu\nu};\Psi_n)\,, \tag{72}$$

where the scalar tensor Lagrangian density is

$$\mathcal{L} = \frac{1}{2}m_{\rm P}^2 R - \frac{1}{2}(\partial\varphi)^2 - V(\varphi)\,, \tag{73}$$

and S_m is the action for any matter fields present, Ψ_n, coupled to gravity.

To explore the dynamics of any late universe quintessence model we assume a wCDM cosmology. We have an FRW metric and assume the effects of ρ_r are negligible and that $\rho_\Lambda = 0$. As such, the content of the Universe is modelled as two perfect fluid components; our scalar field φ, and a non-relativistic background matter fluid, denoted by subscript 'm', with equations of state $p_i = w_i\rho_i, i \in \{\varphi, m\}$ where $w_\varphi = w_\varphi(t)$ and $p_m = 0 \Rightarrow w_m = 0$. Ignoring perturbations and any spatial curvature, because $\Omega_{\rm K} \simeq 0$ [60], we have

$$g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu = -\mathrm{d}t^2 + a^2(t)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j\,, \tag{74}$$

$$\rho_\varphi = \frac{1}{2}\dot\varphi^2 + V(\varphi), \quad p_\varphi = \frac{1}{2}\dot\varphi^2 - V(\varphi)\,, \tag{75}$$

$$w_\varphi = \frac{p_\varphi}{\rho_\varphi} = \frac{\frac{1}{2}\dot\varphi^2 - V(\varphi)}{\frac{1}{2}\dot\varphi^2 + V(\varphi)}, \qquad w = \frac{\Sigma_i p_i}{\Sigma_i\rho_i} = \frac{p_\varphi}{\rho}\,, \tag{76}$$

where $\rho = \Sigma_i\rho_i = \rho_m + \rho_\varphi$. The evolution equations are

$$-2\dot H m_{\rm P}^2 = \dot\varphi^2 + \rho_m\,, \tag{77}$$

$$\dot\rho_m = -3H\rho_m\,, \tag{78}$$

$$\frac{\delta S}{\delta\varphi} = \ddot\varphi + 3H\dot\varphi + V'(\varphi) = 0\,, \tag{79}$$

conditional on the Friedman equation

$$3m_{\rm P}^2H^2 = \frac{1}{2}\dot\varphi^2 + V(\varphi) + \rho_m\,. \tag{80}$$

Specific quintessence models are distinguished by considering suitable forms of $V(\varphi)$ (typically of runaway type) which are flat enough to lead to the current accelerated expansion at late times. In quintessential inflation models, this region is called the quintessential tail. Quintessential inflation considers so-called "thawing" quintessence, where, until recently, φ was frozen, but near the present it unfreezes and begins to evolve into a possible slow-roll regime as it dominates the Universe and drives the current accelerated expansion.

Here we consider the specific case of exponential quintessence where

$$V(\varphi) = V_Q \exp(-\lambda\varphi/m_{\rm P})\,, \tag{81}$$

where V_Q is a constant density scale and λ is a constant parameter.

The Klein-Gordon equation, Eq. (79) admits two attractor solutions which depend on the eventual dominance vs. sub-dominance of φ with regard to the background matter:

$$\text{Dominant } \left(\text{for } \lambda < \sqrt{3(1+w_b)}\right):$$
$$V = \frac{2(6-\lambda^2)}{\lambda^4}\left(\frac{m_{\rm P}}{t}\right)^2 \;\&\; \rho_{\rm kin} = \frac{2}{\lambda^2}\left(\frac{m_{\rm P}}{t}\right)^2$$
$$\Rightarrow\; \rho_\varphi = \frac{12}{\lambda^4}\left(\frac{m_{\rm P}}{t}\right)^2. \tag{82}$$

$$\text{Subdominant } \left(\text{for } \lambda > \sqrt{3(1+w_b)}\right):$$
$$V = \frac{2}{\lambda^2}\left(\frac{1-w_b}{1+w_b}\right)\left(\frac{m_{\rm P}}{t}\right)^2 \;\&\; \rho_{\rm kin} = \frac{2}{\lambda^2}\left(\frac{m_{\rm P}}{t}\right)^2$$
$$\Rightarrow\; \rho_\varphi = \frac{4}{\lambda^2(1+w_b)}\left(\frac{m_{\rm P}}{t}\right)^2, \tag{83}$$

where $\rho_{\rm kin} \equiv \frac{1}{2}\dot\varphi^2$ and w_b is the barotropic parameter of the background; being $w_b = 0$ for matter.

The solutions differ with regard to the evolution of ρ_φ in comparison to that of ρ_m, which is fixed at $\rho_m \propto a^{-3}$. For dominant quintessence Eq. (82), $\rho_\varphi \propto a^{-\lambda^2}$, and for subdominant quintessence Eq. (83), $\rho_\varphi \propto a^{-3}$. The subdominant quintessence attractor solution is called a scaling solution because ρ_φ/ρ_m stays constant. The value of λ determines both the slope of the quintessential tail, and which attractor solution the field eventually follows. The value of $\lambda = \sqrt{3}$ (since $w_b = 0$) represents the boundary between the two attractor solutions, i.e. as λ increases toward $\sqrt{3}$, the evolution of ρ_φ increasingly moves towards that of $\rho_\varphi \propto a^{-3}$.

Copeland et al [45] used a phase-plane analysis and found that, for $\lambda < \sqrt{3}$ the dominant quintessence attractor solution (Eq. (82)) is a stable node. For $\sqrt{3} < \lambda < \sqrt{6}$ and $\lambda > \sqrt{6}$ the subdominant quintessence attractor solution (Eq. (83)) is a stable node/spiral and a stable spiral respectively. After unfreezing, the field briefly oscillates about the attractor before settling on the attractor solution. For $\lambda < \sqrt{3}$, it is easy to show that $w = -1 + \lambda^2/3$ on the attractor [38]. This means that $\lambda < \sqrt{2}$ results in $w < -1/3$, which leads to eternal accelerated expansion. For $\sqrt{2} \lesssim \lambda < \sqrt{3}$ ($\sqrt{3} \lesssim \lambda < 2\sqrt{6}$), the brief oscillation of the field about the dominant quintessence (subdominant quintessence) attractor, may result in a bout of transient accelerated expansion [61–65].

We numerically explore the cosmological dynamics of this single field quintessence model, to a confirmed accuracy of 10^{-4}(4 d.p) for all cosmological parameters. We use the latest Planck observations to constrain the range of λ for which any current eternal or transient accelerated expansion is present. The latest Planck observations [60] suggest that the density parameter of dark energy is $\Omega_\Lambda = 1 - \Omega_{\rm K} - \Omega_m$, where $\Omega_{\rm K} = 0.000 \pm 0.005$, and $\Omega_m = 0.308 \pm 0.012$. This results in $\Omega_\Lambda = 0.692 \pm 0.017$.

As we have a time-varying w_φ, we model a Taylor expansion of w_φ to first order

$$w_\varphi = w_{\rm DE} + \left(1 - \frac{a}{a_0}\right) w_a\,, \tag{84}$$

where $w_a = -(\mathrm{d}w_\varphi/\mathrm{d}a)_0 = -\dot w_{\rm DE}$, the subscript '0' denotes values today, when $a = a_0$ and $w_\varphi(a_0) = w_{\rm DE}$. We use the Planck bounds [60] of $w_{\rm DE} = -1.023^{+0.091}_{-0.096}$ at 2-σ in our constraint on possible ranges of values for λ. This translates to $w_0 = -0.7112 \pm 0.0821$, where w_0 is the barotropic parameter of the Universe at present, $w_0 = (p_\varphi/\rho)_0$ (cf. Eq. (76)).

Demanding that our model satisfies these observational requirements, the Universe today has to lie within the range $(\rho_\varphi/\rho_m)_0 = \Omega_\Lambda/\Omega_m = 2.2523 \pm 0.1429$, within which we can investigate any current eternal or transient accelerated expansion found. We start with the frozen field, where $\dot\varphi_F = 0$ and $(\rho_\varphi/\rho_m)_F \ll 1$, where the subscript '$F$' denotes frozen values.

Only a change in the value of λ affects the evolution of our model once the field is unfrozen. A relative decrease (increase) in the value of $\rho^F_\varphi = V(\varphi_F)$, for a given ρ^F_m, only increases (decreases) the evolution time of the model until a_0 today, i.e. the model is extended backwards (forwards) to an earlier (later) time when φ is frozen. Similarly, any change in the value of φ_F can be expressed as a change in V_Q, and so for a given value of λ, also has no effect on the dynamics. Conversely, since Ω_Λ is fixed by the observations, changes in φ_F without a change in V_Q must instead be accompanied with corresponding changes in λ, such that the contribution of quintessence to the density budget at present remains fixed .

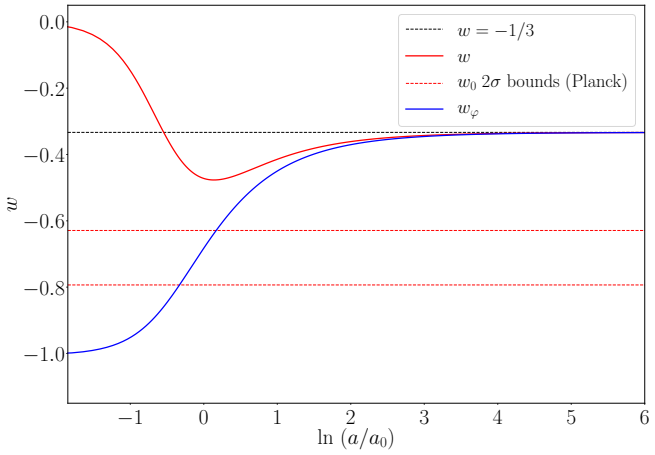

FIG. 11: Transient accelerated expansion for $\lambda = \sqrt{2}$. We find $w < -1/3$, but the minimum value of w is well outside of the Planck bounds.

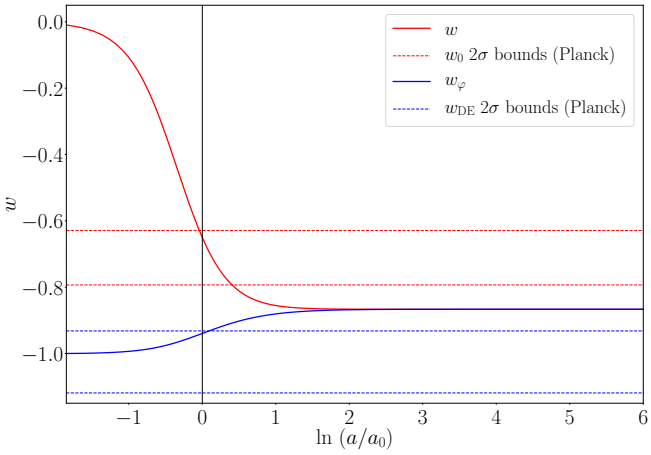

FIG. 12: Eternal accelerated expansion for $\lambda = \sqrt{0.4}$. The vertical line $\ln(a/a_0) = 0$ indicates present day values of w and w_φ (w_0 and $w_{\rm DE}$ respectively), which also fall within the required Planck bounds for w and w_φ today. The scalar field has unfrozen, but is yet to settle on the attractor solution.

Transient Accelerated Expansion

For brief periods of transient accelerated expansion with $w < -1/3$, we find a range of numerically valid λ values bridging the dominant and subdominant quintessence regimes

$$\sqrt{2} \lesssim \lambda < \sqrt{3.38} \tag{85}$$

However, the values for w that we find in this scenario are incompatible with the Planck constraints for the entire range of λ values above. As the minimum value of w reached during any period of evolution increases with increasing λ, we only need to look at $\lambda = \sqrt{2}$ to illustrate our findings. This is shown in Fig. 11, where we are using $\lambda = \sqrt{2}$. It can be clearly seen that the minimum value of w is not nearly small enough to match the Planck observational bounds, and so all higher values of λ are also ruled out.[5]

Eternal Accelerated Expansion

We know theoretically that $w < -1/3$ for $\lambda < \sqrt{2}$. When applying the Planck constraints we find that the cosmologically viable range is reduced to of $\lambda < \sqrt{0.46}$. We find that, in all cases, the scalar field at present has unfrozen but is yet to settle on the attractor solution. This is illustrated in Fig. 12 for $\lambda = \sqrt{0.4}$, where it can be clearly seen the field has yet to evolve to its attractor solution. It can also be clearly seen that the present day values at $\ln(a/a_0) = 0$ are within the Planck bounds.

As illustrated in Fig. 15, we find that it is the bound for $w_{\rm DE} = -1.023^{+0.091}_{-0.096}$ that constrains our possible range of values to $\lambda < \sqrt{0.46}$. This can also be seen in Fig. 12, where the value of $w_{\rm DE}$ is closer to the upper Planck bound for $w_{\rm DE}$ compared to the value for w_0, which is further within the upper Planck bound for w_0. When increasing λ, we find that $w_{\rm DE}$ exits the upper Planck bound for $w_{\rm DE}$ before w_0 exits the upper Planck bound for w_0. If we ignore this constraint and just demand that $w_0 = -0.7112 \pm 0.0821$ today, then our range of possible values for λ extends to $\lambda < \sqrt{0.68}$.

Using our Taylor expansion of w_φ to first order, (cf. Eq. (84)), we obtain a range of values for $|w_a|$ that are of $\mathcal{O}(10^{-2}) - \mathcal{O}(10^{-3})$. These values easily lie within current Planck bounds [60], but can be potentially observable in the near future, e.g. by EUCLID. This is illustrated in Figs. 13 and 14.

The above are valid in general for exponential quintessence. We now apply our findings to our quintessential inflation model with α-attractors We convert from λ to α, using $\alpha = 2/3\lambda^2$ (cf. Eq. (9)), and restate all our findings in terms of α. We find that only values of $\alpha \geq 1.5$ accord with all the required Planck constraints and set an upper bound of $\alpha = 4.2$ to avoid a super-Planckian ϕ. Figs. 13 and 15 are labelled both in terms of λ^2 and α.

[5] Figure 11 also highlights the validity of $w = -1 + \lambda^2/3$ requiring $\lambda < \sqrt{2}$ for eternal accelerated expansion, as we can clearly see $w = -1/3$ in the attractor limit where $\lambda = \sqrt{2}$. We can also see w_φ moving toward the same value because we are in the dominant quintessence regime.

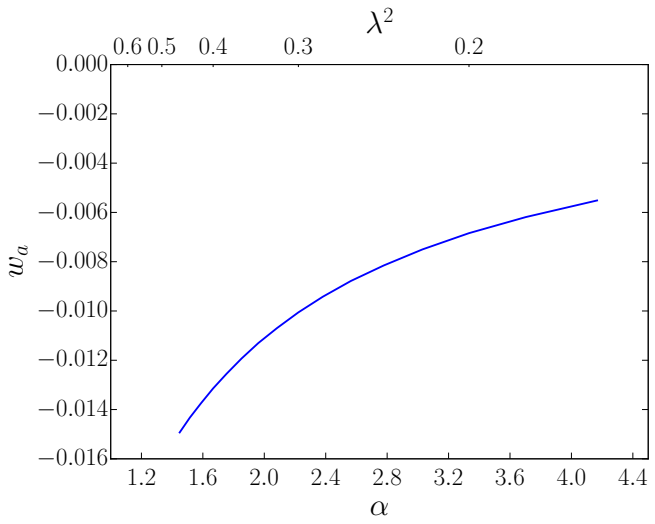

FIG. 13: w_a against α and λ^2 for $\lambda^2 < 0.46 \Leftrightarrow \alpha > 1.45$, values in the text are quoted to 2 s.f.

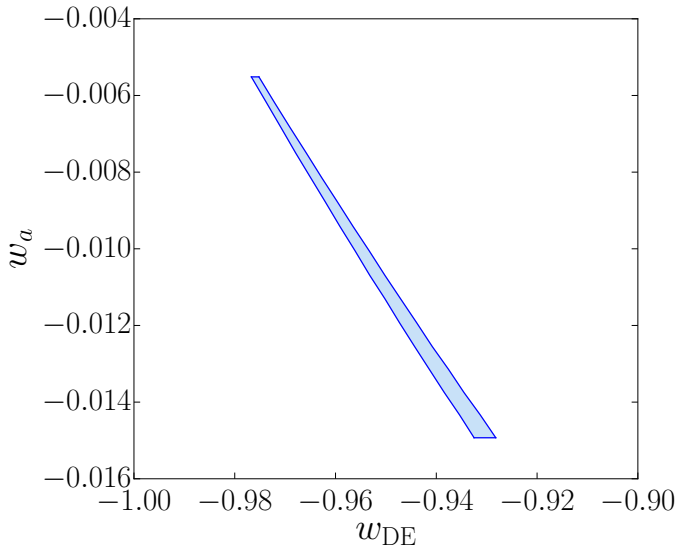

FIG. 14: w_a versus $w_{\rm DE}$. The allowed parameter space depicted lies well within the 1-σ Planck contour.

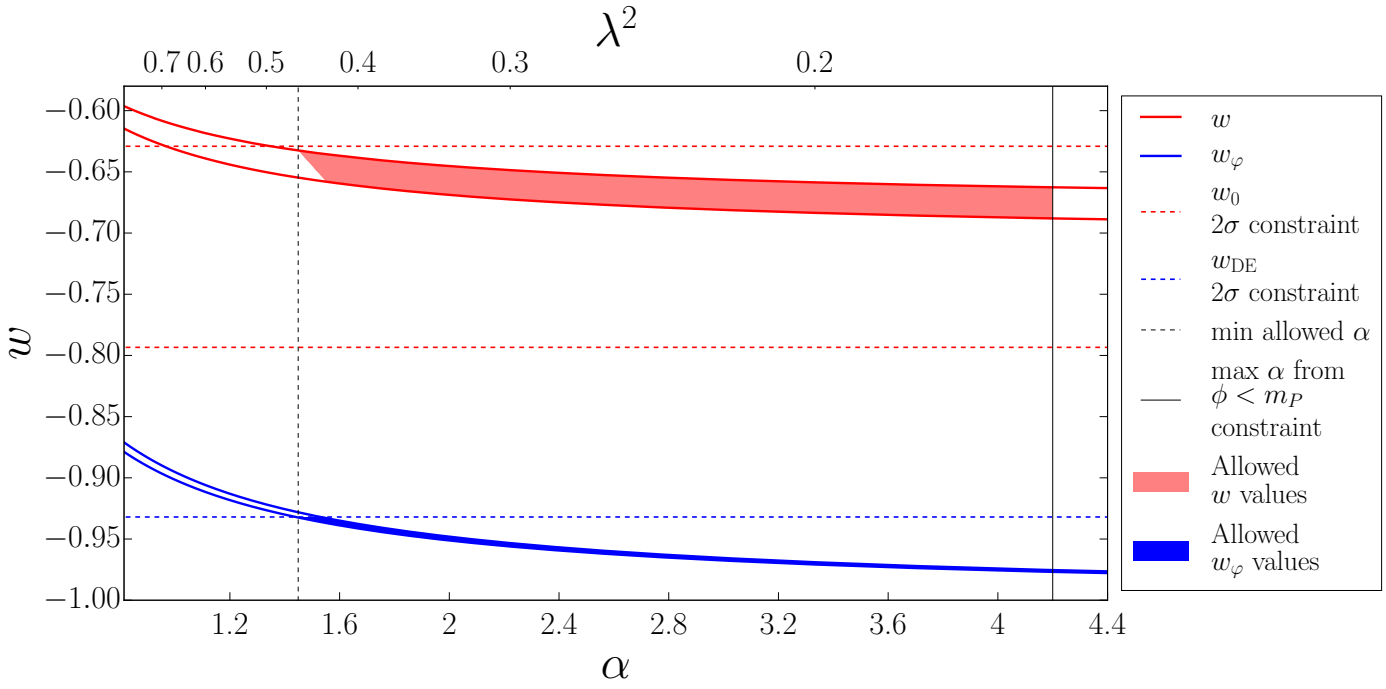

FIG. 15: Possible range of values for α and λ^2, from the Planck constraints on w. It is shown that the 2-σ upper bound on $w_{\rm DE}$ is satisfied only for $\lambda^2 < 0.46$ or equivalently $\alpha = 2/3\lambda^2 > 1.45$. The allowed ranges of w and w_φ reflect the observed range in Ω_Λ/Ω_m. Values in the text are quoted to 2 s.f.