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Topological exciton Fermi surfaces in two-component fractional quantized Hall insulators

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A wide variety of two-dimensional electron systems (2DES) allow for independent control of the total and relative charge density of two-component fractional quantum Hall (FQH) states. In particular, a recent experiment on bilayer graphene (BLG) observed a continuous transition between a compressible and incompressible phase at total filling $\nu_{\mathrm{T}} = \frac{1}{2}$ as charge is transferred between the layers, with the remarkable property that the incompressible phase has a *finite interlayer polarizability*. We argue that this occurs because the topological order of $\nu_{\mathrm{T}} = \frac{1}{2}$ systems supports a novel type of interlayer exciton that carries Fermi statistics. If the fermionic excitons are lower in energy than the conventional bosonic excitons (i.e., electron-hole pairs), they can form an emergent neutral Fermi surface, providing a possible explanation of an incompressible yet polarizable state at $\nu_{\mathrm{T}} = \frac{1}{2}$. We perform exact diagonalization studies which demonstrate that fermionic excitons are indeed lower in energy than bosonic excitons. This suggests that a "topological exciton metal" hidden inside a FQH insulator may have been realized experimentally in BLG. We discuss several detection schemes by which the topological exciton metal can be experimentally probed.

Two-component quantum Hall systems have long been known to host rich phase diagrams, exhibiting intrinsically two-component fractional quantum Hall (FQH) states, broken symmetry states, and quantum phase transitions at fixed total filling fraction ν_{T} [1–3]. When tunneling between the components is effectively zero, the system acquires an enhanced total and relative $U(1)_{\mathrm{T}} \times U(1)_{\mathrm{r}}$ symmetry due to the independently conserved charges of the two components. This situation is most easily realized when the two components are related by spin or valley symmetry [4–8], or in double layer systems in which a barrier suppresses interlayer tunneling [9, 10].

A number of experimental platforms have been used to study the resulting phase diagram of two component FQH phases at $\nu_{\mathrm{T}} = \frac{1}{2}$, including wide quantum wells [11–14], ZnO heterostructures [8, 15], and, most recently, bilayer graphene (BLG), where the two components correspond to layer [16]. In many of these systems the relative filling $\nu_+ - \nu_-$ of the two components can be tuned *in situ*. In particular, Ref. 16 has reported the remarkable experimental observation of a BLG state at $\nu_{\mathrm{T}} = \frac{1}{2}$ that is incompressible, yet possesses a finite interlayer polarizability. This insulating state persists over the range of interlayer polarization $\nu_+ - \nu_- \approx 0 - 0.18$, which is strikingly large when compared with the typical width of FQH plateaux. In the presence of $U(1)_{\mathrm{r}}$, finite polarizability indicates a vanishing neutral gap, and hence hints at the discovery of a new phase of matter distinct from a fully gapped quantized Hall state.

In this Letter we address this experimental finding by analyzing the possible phases which can occur in a two component system at $\nu_{\mathrm{T}} = \frac{1}{2}$ as density is transferred between the two components. The $U(1)_{\mathrm{r}}$ symmetry ensures that intercomponent excitons can exist as long-lived excitations.We argue that the experimental observations of Ref. 16 could be explained by a FQH insulator whose interlayer excitons have delocalized into a degenerate quantum liquid.

The problem is particularly rich at $\nu_{\mathrm{T}} = \frac{1}{2}$ because the fractionalized nature of even-denominator FQH states guarantees that in addition to the familiar bosonic exciton (b-Exc), the system also hosts a topologically non-trivial fermionic exciton (f-Exc). If the f-Exc is lower in energy, the system naturally forms a "topological exciton metal" at finite f-Exc density. This new phase of matter would exhibit insulating charge transport but *metallic* counterflow resistance.

We consider two main scenarios. First we discuss systems in which the two components arise from a crossing between an $N = 0$ and $N = 1$ LL in the limit where the distance d between them is small compared to the magnetic length ℓ_B, as occurs in ZnO [8], wide quantum wells [12], and BLG [16]. Our exact diagonalization calculations show that the f-Exc is indeed lower in energy than the b-Exc. In the second scenario, relevant to a bilayer with $d/\ell_B \gtrsim 1$, we consider a crossing of two $N = 0$ levels, where we also argue that the f-Exc will determine the nature of the intermediate phase.

$(N_+, N_-) = (1, 0)$*: the Pfaffian exciton metal.* In the experiment of Ref. 16, an electric field perpendicular to the bilayer causes the first excited $N_+ = 1$ LL in the top layer to cross in energy with the lowest $N_- = 0$ LL in the bottom layer. The filling fraction of the two layers is $\nu_+ = 1/2 - \delta$ and $\nu_- = \delta$. Because $N_+ = 1$, when $\delta = 0$ the system is observed to form an incompressible FQH state in the top layer, roughly analogous to the $5/2$-plateau of GaAs [17]. Based on numerical evidence [16, 18], as well as the recent experimental observation of a half-integer thermal Hall effect [19], we will assume that the system forms a Moore-Read Pfaffian FQH state.[20] However all even-denominator FQH states must contain a charge $-e$ boson, so will lead to essentially the same conclusions. When $\delta = 1/2$, the particles reside in an $N_- = 0$ level, so the system forms a compressible CFL [21–23].

What is the fate of the system at intermediate δ? As δ increases from zero, the top layer loses charge to the bottom layer. Due to the strong Coulomb interaction between layers, excitons will form, with $-e$ charge in the top layer and e charge in the bottom layer, with a binding energy on the order of the interlayer Coulomb interaction. Crucially, at $\nu_{\mathrm{T}} = \frac{1}{2}$ this system supports two topologically distinct types of excitons. The conventional bosonic exciton (b-Exc) is formed when an electron is transferred from the Pfaffian state in the top layer to the bottom layer. On the other hand, the Pfaffian state also has a charge $-e$ bosonic excitation, which can be thought of as a Laughlin quasiparticle associated with inserting two flux quanta into the system. A bound state of the charge $-e$ boson in the top Pfaffian layer and an electron in the bottom is a *fermionic* exciton (f-Exc). In contrast to the b-Exc, the f-Exc is a topologically non-trivial quasiparticle; it can also be thought of as a bound state of the b-Exc and the anyonic "neutral fermion" ψ_{NF} of the Pfaffian phase. As we will demonstrate within the long wavelength effective field theory, this f-Exc is coupled to an emergent Z_2 gauge field. A pair of f-Exc's is topologically equivalent to a pair of b-Exc's.

Because excitons are neutral particles, they have some nonzero dispersion $\epsilon(k)$ and can delocalize. If the excitons attract, there may be an instability and the transition will be discontinuous, but otherwise we can consider three types of ground states for the excitons: density-wave, condensate, and metal.[24] First, depending on the interactions between the excitons, it may be preferable for the excitons to form a density-wave state, for example stripes or a Wigner crystal. In the presence of weak disorder that pins the density wave, this state can be viewed as a localized state of excitons, e.g. a Bose glass or Anderson insulator for the b-Exc, f-Exc respectively.[25, 26]

As the density of excitons increases with δ, the b-Exc can potentially undergo a quantum phase transition to a superfluid, spontaneously breaking U$(1)_r$. Analogous to the $\nu_{\mathrm{T}} = 1$ exciton condensate [2, 3], the condensation of the b-Exc leads to an interlayer coherent Moore-Read Pfaffian state. Alternatively, if the f-Exc are more stable, increasing their density leads to a Fermi surface whose volume is set by δ. In this case, the Pfaffian state coexists with a Fermi surface of f-Exc's, leading to insulating charge transport but metallic counterflow. There is no sharp transition between the Anderson insulator state and the "metallic' state of excitons, because in two dimensions all states are localized by disorder. At finite temperature there is a crossover from the localized to delocalized regime as the temperature is increased, with a crossover temperature $T^* \sim e^{-\epsilon_F/W}$, where ϵ_F is the Fermi energy and W is the disorder strength.[26]

Let us now describe the above scenario more concretely in terms of a long wavelength effective field theory. c_+ and c_- denote the electrons in the two layers. To describe the system at $\nu_{\mathrm{T}} = 1/2$, we attach two flux quanta to each electron, to obtain composite fermions (CFs) ψ_+ and ψ_-. It is convenient to describe this in terms of a parton construction (see e.g. [27]) $c_\pm = b\psi_\pm$, where b is a charge-e boson and ψ_+, ψ_- are the neutral CFs. b and $\psi_\pm$ carry charge 1 and -1, respectively, under an internal emergent gauge field a, associated with the phase rotations $b \to e^{i\theta} b$, $\psi_\pm \to e^{-i\theta}\psi_\pm$ which keep the physical electron operator invariant. Introducing $A_{\mathrm{T}} = A_+ + A_-$ as an external probe gauge field for $U(1)_{\mathrm{T}}$, and $A_r = (A_+ - A_-)/2$ as a probe gauge-field for the $U(1)_{\mathrm{r}}$, the $\psi_\pm$ carry charge $\pm 1/2$ under A_r.

Next, we assume a mean-field ansatz where b forms a bosonic $\nu = 1/2$ Laughlin state, and $\langle a \rangle = 0$. The resulting field theory can be written as

$$\mathcal{L} = -\frac{2}{4\pi}\tilde{a}\partial\tilde{a} + \frac{1}{2\pi}(a + A_{\mathrm{T}})\partial\tilde{a} + \mathcal{L}_\psi(\psi_\pm, a, A_r). \quad (1)$$

Here $a\partial a \equiv \epsilon^{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda$, $\frac{1}{2\pi}\epsilon^{\mu\nu\lambda}\partial_\nu \tilde{a}_\lambda$ is the conserved current for the b particles, and the first term on the RHS above is the effective action for a bosonic $1/2$ Laughlin FQH state [28]:

$$\begin{aligned}\mathcal{L}_\psi = \sum_{\alpha=\pm} [&\psi_\alpha^\dagger (i\partial_t + a_t + \alpha A_{r;t}/2)\psi_\alpha \\ &+ \frac{1}{2m_\alpha}\psi_\alpha^\dagger (i\partial_i + a_i + \alpha A_{r;i}/2)^2 \psi_\alpha + \cdots], \end{aligned} \quad (2)$$

where $\cdots$ indicates higher order interactions among the CFs. We can now consider a variety of possible mean-field states for the CFs $\psi_\pm$.

(1) *Two-component composite Fermi liquid*. Here, $\psi_\pm$ both form a composite Fermi sea. This describes a CFL state with two Fermi surfaces, with Fermi wave vectors $k_{F\pm} = \ell_B^{-1}\sqrt{2\nu_\pm}$, where $\nu_\pm$ is the electron filling in the two layers. This phase is most natural when $\nu_- \sim 1/2$.

(2) Z_2 *fractionalized exciton metal*. We consider a state where species ψ_+ forms a paired state, $\langle \psi_+ \psi_+ \rangle \neq 0$, while ψ_- continues to form a Fermi surface with $k_{F-} = \ell_B^{-1}\sqrt{2\nu_-}$. This breaks the $U(1)$ gauge symmetry down to Z_2, and the Higgs mechanism sets $a + A_r/2 = 0$. In the limit $\nu_- = 0$, we expect ψ_+ forms a $p_x + ip_y$ state since the system is described by a Moore-Read Pfaffian state in the top layer [36] [29].

As ν_- is increased, the system is described by a Pfaffian state in ψ_+ together with a Fermi sea of ψ_-. Since we have locked $a = -A_r/2$, Eq. (2) implies that ψ_- effectively becomes coupled only to A_r, with *unit* charge. Physically, this implies that ψ_- is a fermion which carries a unit dipole moment perpendicular to the layers, and can thus be identified with the f-Exc.

However, ψ_- is still coupled to an emergent Z_2 gauge field, corresponding to the remnant of a after the pairing of the ψ_+ fermions, reminiscent of the 'orthogonal metal' phase [30]. Importantly, the ψ_+ and ψ_- fermions are both coupled to this Z_2 gauge field, so are non-trivially entangled. In particular, the f-Exc will acquire a π-phase upon encircling the Pfaffian's non-Abelian charge $e/4$ quasiparticle; hence the f-Exc will see any localized $\pm e/4$ quasiparticles pinned to the disorder potential as sources of random π-flux.

A model wave function for this state can be written as follows: $\Psi_{\text{exFS}}(\{z_i, w_a\}) = \mathcal{P}_{\text{LLL}}\psi_{\text{f-Exc}}(\{\mathbf{r}_a\})\prod_{a<b}(w_a - w_b)^2\prod_{i,a}(z_i - w_a)^2\text{Pf}\left(\frac{1}{z_i - z_j}\right)\prod_{i<j}(z_i - z_j)^2$. Here z and w are the complex coordinates of the electrons in the top and bottom layers, respectively, with $w_a = r_{a;x} + ir_{a;y}$. $\psi_{\text{f-Exc}}(\{\mathbf{r}_a\})$ is the wave function for the excitons, which can be taken to be in a Fermi sea. $\mathcal{P}_{\text{LLL}}$ denotes projection to the lowest LL. While this wave function is written as if both layers are in the lowest LL, it should be transposed to the case where the Pf layer is in the first LL by acting with the LL raising operator on each z-electron, $\prod_i(\partial_{z_i} - \frac{\bar{z}_i}{4\ell_B^2})$.

(3) *Interlayer coherent FQH states: exciton condensates.* We can consider a state where both $\psi_\pm$ CFs form a paired state, $\langle\psi_+\psi_+\rangle \neq 0$, $\langle\psi_-\psi_-\rangle \neq 0$, which breaks $U(1)_{\text{r}}$ and gives interlayer coherence. As a result these phases have a Goldstone mode and superfluid-like counterflow. We can further distinguish two cases:

(a) $\langle\psi_+\psi_-\rangle \neq 0$. In this case, since we also have $\langle\psi_+\psi_+\rangle \neq 0$, we can treat $\langle\psi_+\psi_-\rangle$ and $\langle\psi_+^\dagger\psi_-\rangle$ as equivalent. Since $\psi_+^\dagger\psi_-$ carries unit $U(1)_{\text{r}}$ charge, its expectation value implies that the interlayer $U(1)_{\text{r}}$ is completely broken. This corresponds to the case where the b-Exc form a condensate.

(b) $\langle\psi_+\psi_-\rangle = 0$. In this case, pairs of the f-Exc have condensed, implying that the interlayer $U(1)_{\text{r}}$ is spontaneously broken down to Z_2. This leaves behind a mod-2 conservation law for the exciton number. Since pairs of f-Exc are topologically equivalent to pairs of b-Exc, this state can also be viewed as a state where *pairs* of b-Exc have condensed.

Note that in both case (a) and (b), we can further consider various types of paired states for the $\psi_\pm$ fermions, e.g., whether they are weak or strong pairing superconductors [29]. Wave functions for these interlayer coherent FQH states can be written as $\Psi(\{x_i, \sigma_i\}) = \text{Pf}\left[g_{\sigma_i\sigma_j}(\mathbf{r}_i - \mathbf{r}_j)\right]\prod_{i<j}(x_i - x_j)^2$, where x_i is now the complex coordinate of the ith electron including both layers and $\sigma_i = \pm$ is its layer index. $g_{\sigma_i\sigma_j}(\mathbf{r}_i - \mathbf{r}_j)$ is the pair wave function. For example, if we take $g_{\sigma_i\sigma_j}(\mathbf{r}_i - \mathbf{r}_j) = \frac{\Delta_{\sigma\sigma'}}{x_i - x_j}$, this would correspond to the case where $\langle\psi_\sigma(k)\psi_{\sigma'}(-k)\rangle = \Delta_{\sigma\sigma'}(k_x + ik_y)$.

(4) *Pfaffian FQH states with localized excitons.* Finally, we can consider a state where ψ_+ is paired, while the ψ_- fermions form a density wave state, or, in the presence of disorder, are localized. This is the state which, in the language of excitons used earlier, corresponds to a Pfaffian FQH state in one layer with some density of localized excitons. As explained above, this disordered state is not a sharply distinct phase from the Z_2 fractionalized exciton metal, but rather a different regime of the same phase. The topological order of such a state is simply that of the Pfaffian FQH state, regardless of whether the b-Exc or f-Exc are lower in energy.

Exact diagonalization study of the Pfaffian's exciton energies. While we have enumerated a variety of consistent possibilities, it is a matter of microscopic energetics which will actually occur. A comprehensive numerical investigation is

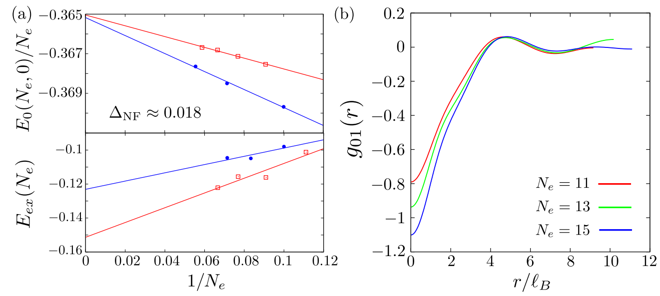

FIG. 1: (**a**) Top panel shows the energy per electron $E(N_1 = N_e, N_0 = 0)/N_e \to e_0$ when all electrons are in the $N = 1$ layer. The odd-even effect [31] confirms that the Pfaffian ground state occurs for N_e-even, while N_e-odd corresponds to the ψ_{NF} excited state. Extrapolating the energy difference in $1/N_e$ we obtain the neutral fermion gap $\Delta_{\text{NF}} \sim 0.018$. In the bottom panel, we transfer one electron from the $N = 1$ to the $N = 0$ layer and measure the energy relative to the vacuum, $E_{ex}(N_e) = E(N_e - 1, 1) - e_0 N_e$. There is again an odd-even effect, but reversed: the bosonic exciton (blue, $N_e = 2m$) is considerably higher in energy than the fermionic exciton (red, $N_e = 2m + 1$). (**b**) The f-Exc pair correlation function between the $N = 0, 1$ layers, $g_{01}(r)$, shows that the electron and hole bind together into an exciton of size $\sim 4\ell_B$.

presented elsewhere,[PRB] but here we address the most important question: does the b-Exc, or f-Exc, have lower energy? We answer this question using exact diagonalization of the Coulomb Hamiltonian on a sphere, keeping both an $N = 0$ and $N = 1$ LL. [37]

To explain the results in Fig. 1 we must recall some facts about the Pfaffian state on a sphere. The Pfaffian ground state occurs when the number of electrons N_e and the number of flux quanta N_ϕ satisfies $N_\phi = 2N_e - 5$. When N_e is even, the sphere has a unique, gapped ground state. In the top panel of Fig. 1(a), we show the energy per electron $E(N_1 = N_e, N_0 = 0)/N_e$ when all electrons are in the $N = 1$ layer. Calculations are done for the Coulomb interaction with energies expressed in units of $e^2/\epsilon\ell_B$. Using standard finite-size corrections [18, 32] and linear extrapolation in $1/N_e$ for N_e-even, we find the thermodynamic vacuum energy per particle of the Pfaffian to be $e_0 \approx -0.365$. However, when N_e is odd, there is a dispersing band of low energy states [33, 34]. This can be understood by appealing to the "superconducting" nature of Pfaffian phase [29]: when the number of CFs is odd, one CF must remain as an unpaired BdG quasiparticle, which is precisely the neutral fermion ψ_{NF} excitation. By measuring the ground state energy differences $E(N_e) - e_0 N_e$, where N_e is odd and e_0 is the energy per electron in the thermodynamic limit (top panel in Fig. 1a), we estimate neutral fermion gap $\Delta_{\text{NF}} \sim 0.018$, in line with earlier studies [33, 34].

A similar method can be used to measure the energy difference between the f-Exc and b-Exc, see bottom panel of Fig. 1(a). Let $E(N_1, N_0)$ be the ground state energy for $N_e = N_1 + N_0$ electrons in the $N = 1, 0$ levels respectively, keeping fixed the number of flux $N_\phi = 2N_e - 5$. The b-Exc occurs when $N_0 = 1$ and N_e is even; in contrast, the f-Exc occurs for $N_0 = 1$ and N_e is odd. We define the exciton energies by subtracting off the Pfaffian's extrapolated vacuum energy e_0 [18, 32],

$$E_{ex}(N_e) = E(N_e - 1, 1) - e_0 N_e. \quad (3)$$

The exciton energy $E_{ex}(N_e)$ also shows an odd-even effect, Fig. 1(a), but in contrast to the vacuum, *odd* N_e (the f-Exc) is now lower in energy by about $\Delta_{\text{b-Exc}} - \Delta_{\text{f-Exc}} \gtrsim 0.02$. Note that since the b-Exc can decay into an f-Exc and a ψ_{NF}, we do not expect to see a difference much greater than $\Delta_{\text{NF}} \approx 0.018$, though the energy of the *metastable* b-Exc may be much larger.

To verify that the electron and hole are forming a tightly bound exciton, we examine the inter-layer pair correlation function in the f-Exc sector, $g_{01}(r) \equiv A\langle\langle \hat{n}_0(r)(\hat{n}_1(0) - \bar{n}_1)\rangle\rangle$, where $\bar{n}_1 = N_e/A$ is the average density in the Pfaffian ground state and A is the area of the sphere. The f-Exc carries angular momentum $L = 3/2$, so the double brackets denote an average over the L-multiplet. We subtract $\bar{n}_1$ so that $-\int d^2r g_{01}(r) = 1$ can interpreted as the probability for the electron and hole to be at distance r. As we see in Fig. 1(b), they indeed bind together into an exciton of size $\sim 4\ell_B$.

In summary, exact diagonalization of the Coulomb Hamiltonian shows that as charge is transferred between layers the electrons and holes form tightly bound excitons, and the non-trivial f-Exc is the lowest energy exciton. At dilute exciton densities this "single particle" energy will dominate over interactions, indicating that a fermionic exciton metal is more likely than a bosonic condensate.

A number of experimental signatures could be used to distinguish these scenarios:

Counterflow– Counterflow transport is a clear way to distinguish between localized Bose/Fermi excitons, interlayer coherent FQH states, and the exciton metal. Assuming the ability to independently contact the two layers, one can measure the counterflow conductivity: $j_{\text{r}} = \sigma_{\text{r}} E_{\text{r}}$, where $j_{\text{r}} = j_+ - j_-$ is the relative current and $E_{\text{r}} = E_+ - E_-$ is the difference in electric field between the two layers. When $\langle\psi_+\psi_+\rangle \neq 0$, j_{r} is simply the current of the ψ_- fermions. The DC "counterflow conductivity" σ_{r} will thus be zero, finite, or infinite, depending on whether the b-Exc have Bose condensed, the f-Exc have formed a Fermi sea (with temperature T greater than the localization cross-over scale), or the excitons have localized. A *dissipative* counterflow conductivity, in an incompressible FQH insulator, is a striking property of the exciton metal state.

Polarizability– The polarizability is defined as $\lim_{\omega\to 0, q\to 0}\langle p(q,\omega)p(-q,-\omega)\rangle$, where $p(x,t) = n_+(x,t) - n_-(x,t)$ is the difference in density between the two components. All states considered above have finite polarizability. When the excitons are localized by disorder in either the bosonic or fermionic case, the polarizability is set by the disorder strength; in the Bose exciton condensate state it is set by the superfluid density, and in the exciton Fermi sea it is set by the density of states at the Fermi surface. The latter can be understood within the field theory presented above: if $\langle\psi_+\psi_+\rangle \neq 0$, then ψ_- is a f-Exc, $p \sim \psi_-^\dagger\psi_- + const$, and polarizability is simply the compressibility of the f-Exc state. The exciton Fermi sea can be distinguished the temperature dependence of the polarizability or by the application of a periodic potential: when the wave vector of the periodic potential becomes commensurate with $2k_F$, Bragg scattering induces an exciton band gap and modulates the polarizability.

Specific heat and thermal conductivity – Another characteristic distinguishing feature of the different exciton states appears in the specific heat and the thermal conductivity. The thermal conductivity of the exciton metal will be linear in temperature: $\kappa \sim C_v v_F \ell \sim T$, where ℓ is the mean free path of the excitons, v_F is their Fermi velocity, and $C_v \sim T$ is the specific heat of the exciton Fermi surface. Since such a state has zero electrical conductivity at zero temperature, this would imply an infinite violation of the Wiedemann-Franz law. In contrast, the thermal conductivity of the exciton localized state $\kappa \to 0$ at zero temperature, although the specific heat is still expected to be linear in T in this phase.

$(N_+, N_-) = (0, 0)$*: (331) fractional exciton metal.* Finally, we briefly mention an alternative platform for an exciton metal. In QH bilayers with $d/l_B > 1$ at filling $(\nu_+, \nu_-) = (1/4, 1/4)$ the bilayer can form a 331 state. This state has been observed when both components partially fill the $N_\pm = 0$ LL,[3] though it may also happen more generally. On the other hand, when $(\nu_+, \nu_-) = (1/2, 0)$ or $(0, 1/2)$, the system will form a CFL state. What is the fate of the system in the intermediate regime $(\nu_+, \nu_-) = (1/4 + \delta, 1/4 - \delta)$? The 331 state also possesses an f-Exc, which contains charge $e/2$ and $-e/2$ in the two layers, respectively Note that this is quite distinct from the scenario considered earlier, where the f-Exc in the Pfaffian state contained charge e and $-e$ in the two layers. Since the b-Exc has charge e and $-e$ while the f-Exc has charge $e/2$ and $-e/2$, we expect that the Coulomb repulsion would cause the b-Exc to be unstable to decaying into two f-Exc's. As δ is tuned away from zero, the finite density of f-Exc's can form a Fermi sea. In terms of the effective theory, the 331 can be described by writing $c_\pm = b_\pm\psi$, and assuming a mean-field state where $b_\pm$ each form $\nu = 1/2$ bosonic Laughlin states in each layer, and ψ forms a $\nu = 1$ IQH state. The exciton in this picture can be thought of as a pair of $\pm e/2$ quasiparticles of the bosonic Laughlin states in each layer. As in the case of the Pfaffian exciton metal, the 331 state also possesses charge $\pm e/4$ quasiparticles, which the f-Exc's see as sources of an effective π-flux.

In summary, we have shown that bilayer QH systems at $\nu_{\text{T}} = \frac{1}{2}$ can support a variety of fractionalized exciton phases due to the interplay of $\text{U}(1)_{\text{r}}$ symmetry and even-denominator fractionalization. We find numerical evidence that the lowest-energy exciton in these systems is a fermion, suggesting that the Z_2-fractionalized "topological exciton metal" is a strong possibility. An even-denominator states at finite layer polarization has already been observed in BLG,[16] so we hope future experiments can look for the dramatic transport signatures of this phase.

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[36] More generally, when ψ_+ forms any BCS paired odd angular momentum state, then other variants of the Pfaffian state are realized.

[37] Note the two LLs have a different number of orbitals due to the "shift" [35]: the number of orbitals in $N=0$ LL is $N_{\rm orb,0} = N_\phi+1$, where N_ϕ is the number of flux quanta, while in $N=1$ LL the number of orbitals is $N_{\rm orb,1} = N_\phi + 3$. This difference is important for accurate finite-size calculations.