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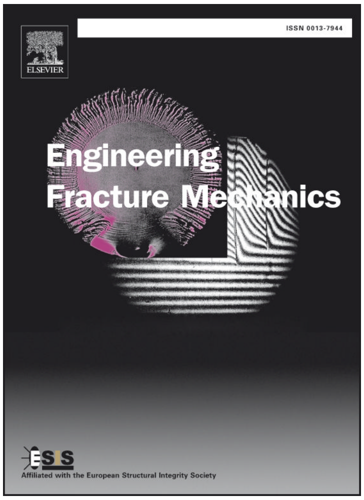

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A NOVEL APPROACH TO ESTIMATE THE EVOLUTION OF FRACTURE ENERGY AND TENSILE SOFTENING CURVE OF CONCRETE FROM VERY EARLY AGE

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Abstract:

Concrete fracture properties and their evolution over time are critical inputs for numerous engineering aspects. Despite substantial efforts invested, there exists a crucial need to establish a comprehensive model for reliable estimation of such evolution. In this paper, combining reliable experimental data and in-depth analyses, a novel approach for estimating of the evolution of fracture energy and tensile softening curve of concrete from early age is proposed. Fundamentally, the approach relies on three criteria, namely (i) Tensile strength, (ii) Tensile strength-fracture energy correlation and especially, (iii) Centroid coordinates of the area under actual stress-crack opening curve. Through detailed assessment of all mentioned criteria and with provided examples of direct applications, the capability and reliability of the approach are clearly demonstrated.

Keywords: Fracture energy; tensile softening curve; centroid coordinate; bi-linear approximation; power approximation.

1. Introduction

Since its first introduction by Hillerborg [1], the cohesive model has become widespread with *fracture energy and tensile softening behaviour* being two of the most important factors, when it comes to the study of concrete fracture properties. Along with extensive effort invested so far for mature concrete [2-10], there exists a crucial necessity to establish an appropriate model, to estimate the evolution of both indicated factors at different age, spanning from concrete's infancy to its maturity. The obtained knowledge is *not only* critical for numerous direct applications such as pre-casting, pre-stressing and practical design against premature cracking, *but also* important for further research by simulation/modelling.

Despite its significance, very limited available studies can be found, and most of which unfortunately suffers at least one of the following drawbacks:

- First of all, it is *the lack of a reliable-, systematic- and logical basis* to be able to confirm each model's applicability to other cases. For example, Dao et al. [11] in his paper introduced an empirical formula to estimate fracture energy, based solely on its linear correlation with tensile strength observed in their studies. The applicability of their empirical factors for other concretes unfortunately was left unconfirmed. Similar situation was also observed in Østergaard's study [12], where several formulae were presented with empirical constants determined for each concrete individually;
- Secondly, *virtually none of the available methods can comprehensively estimate the evolution of fracture energy and its tensile softening curve altogether*. Apart from Dao [11], Østergaard [12] and Elices-Planas [13], Kang et al. [14] investigated the variation of the areas under the first- and second linear trend line of his bi-linear approximation model separately. Considering 4 different self-consolidating concretes (SCC), he presented some empirical formulae to estimate such areas. Although with a logical approach, the proposed model was shown to have large variation with almost all his experimental data;
- Lastly yet importantly, it is the difficulty of limited concrete age restrained in each study. Dao et al. [11] only considered concrete at 2.5-10 hours after mixing, while other groups totally disregarded that period and concentrated on later age. *No successful effort of combining these two periods together into a comprehensive model can be found*.
- In the standardization field, CEB-FIP 1990 [15] appears to be the first to suggest a model to characterize tensile softening curve, based on known G_F. The model accepts one important assumption – the ratio of f_1/f_t equals to 0.15, where f_t is tensile strength and f_1 is the coordinate of the kink point in a bi-linear approximation model (point B – Figure 2). However, as indicated by Kang et al. [14] and also implied by Østergaard [12], the assumed factor is mostly lower than actually recorded in real tests, not to mention the ratio tends to greatly vary over time, especially during the first 1-3 days of age.

In this paper, a distinct approach for the stated purpose for concrete from as early as 2 hours of age is presented, forming a unified- and comprehensive alternative for previous models. The approach originates from (i) A convincing estimation of early-age tensile strength, (iii) The confirmed multi-linear correlation between fracture energy and tensile strength and especially, (iii) The respected *centroid coordinates* of the area under stress-crack opening curve. Adopting experimental data from 6 different studies with a total of 15 different concretes, combined with in-depth analytical assessment, the proposed approach was shown satisfactorily applicable for most considered cases.

The paper is divided into two major parts. The first three sections discuss the approach, its fundamental criteria, and detailed assessment of each criterion. The last section presents two examples of actual application, from which the interpretation of the proposed approach into 2 typical approximation curves – bi-linear- and power function – is clearly illustrated.

2. Descriptions of the concept

2.1. Why centroid coordinates?

In this study, the selection of a contributing factor for the new approach complies with the following conditions:

- First of all, the factor should play a fundamental role in the determination of fracture energy and softening curve, also considering its sufficiency and simplification for practice; and
- Lastly yet importantly, such factor should have a smooth-, consistent- and predictable developing trend over time. This condition is critical for a reliable estimation.

To illustrate the selection process, a typical concrete mix of type I class N cement and the water-to-cementitious materials (w/c) ratio of 0.32 (Østergaard [12]) is examined. In his study, Østergaard employed a wedge-splitting test (WST) setup, with a bi-linear approximation model being applied to interpret his recorded data.

To have a comprehensive comparison, time-dependent variation of most key factors contributing to the characterization of the bi-linear model are collected and plotted in Figure 1. Beside tensile strength f_t and fracture energy G_F, other investigated factors include (Figure 2):

- Slopes of the first- and second linear trend line a_1 and a_2, respectively;
- Relative coordinates of the kink point under a stress-crack opening curve (point B), $\alpha_1 = \sigma_1/f_t$ and $\alpha_2 = \omega_1/\omega_c$;
- Centroid coordinates of the area under the tensile softening curve, σ_{cg} and ω_{cg};
- Ultimate crack opening at zero stress (point D), ω_c.

As shown in Figure 1, among these factors, the development of centroid coordinates appears to be (i) More consistent with obviously less variation of data points, (ii) Smoother – with no abnormality, and (iii) Totally predictable. Other factors, although with relatively clear trend, do not seem to promise more accurate estimation. More importantly, as clearly shown in the next section, both coordinates, together with other criteria, can form a fundamental approach for estimation which is sufficient- and applicable for most popular approximation curves.

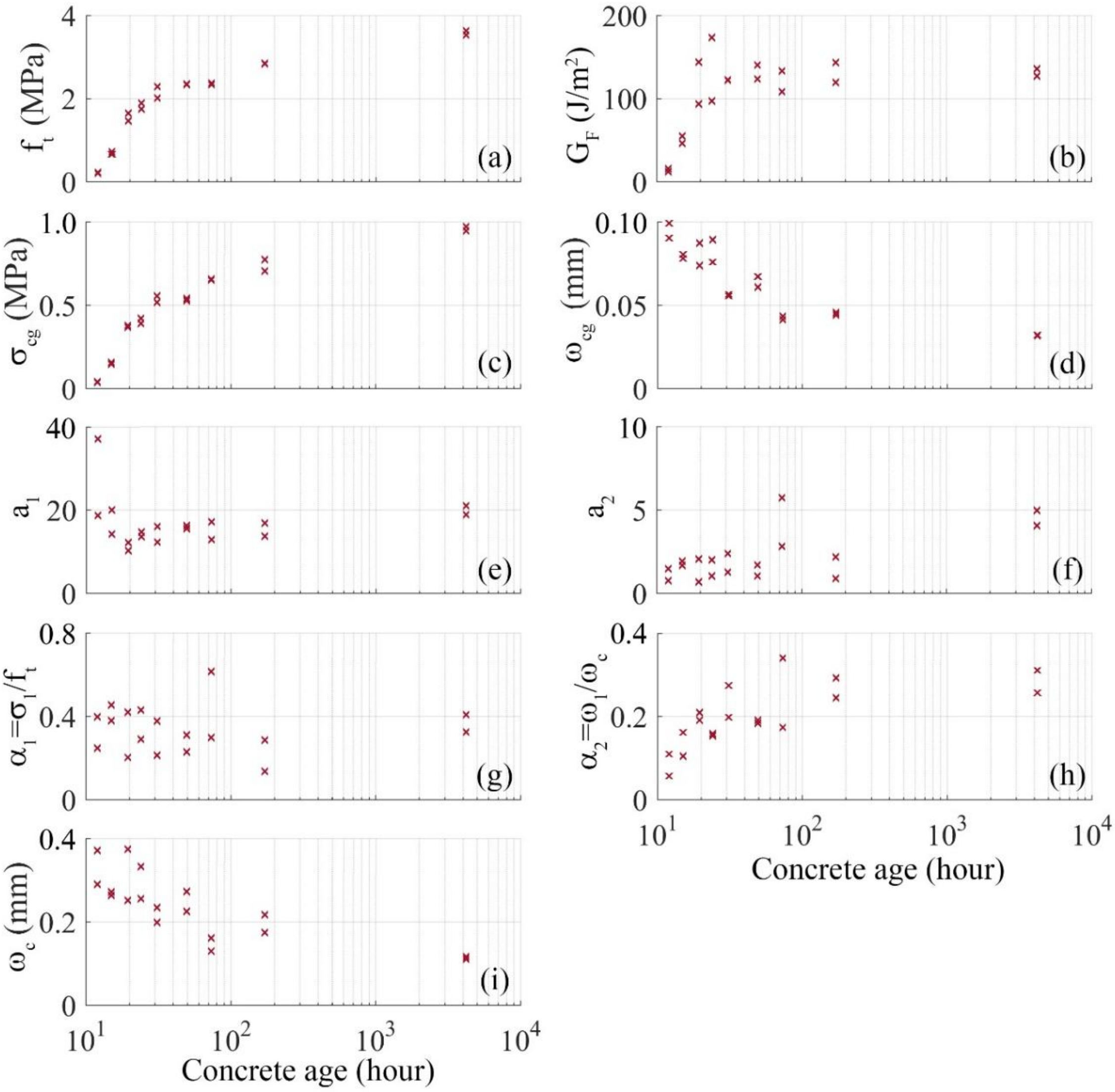
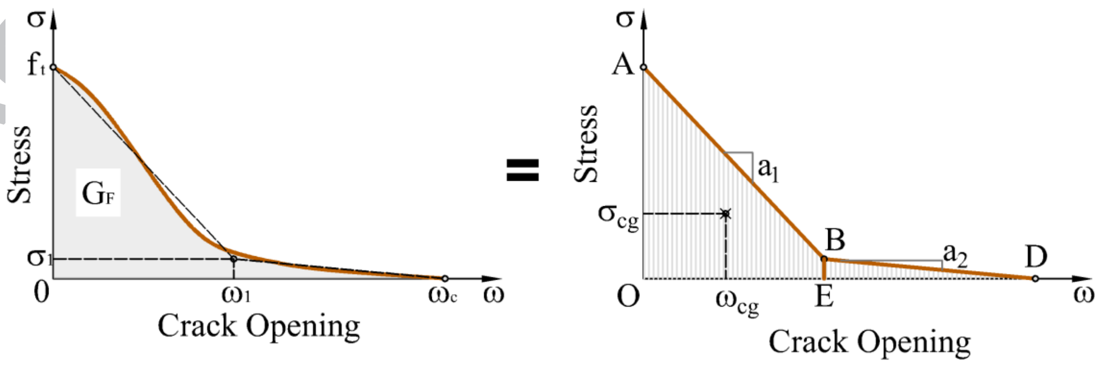


Figure 1. Experimental data adopted from Østergaard [12] and important factors of his bi-linear approximation model.

2.2. Fundamental criteria for estimation



(a) (b)

Figure 2. Bi-linear model to approximate tensile softening curve and fracture energy.

Examining a stress-crack opening curve of concrete at a certain age in Figure 2a, the area under that curve (G_F) as well as the shape of the curve itself can be considered appropriately estimated if the following criteria are satisfied:

1. Position of point A of the actual curve – *or tensile strength* $f_t(t)$ – can be accurately predicted;
2. A *correlation between tensile strength* $f_t(t)$ *and fracture energy* $G_F(t)$ can be established in advance; and
3. *The centroid coordinates* (σ_{cg}(t) and ω_{cg}(t)) of the actual stress-crack opening curve can be reliably estimated. Regardless of the approximation curve employed, these coordinates must be respected.

Among these 3, only criterion 1 and 2 combined are sufficient for an estimation of fracture energy; nevertheless, for tensile softening curve, criterion 3 must be incorporated. As shown in Table 1 and in Section 4, the above criteria are sufficient for both bi-linear- and power approximation curves, from which all their required unknowns can be conveniently determined.

Table 1. Formulation of two popular approximation models for concrete at a certain age.

	Bi-linear approximation	Power approximation
σ(w)	$\sigma(\omega)=\begin{cases} f_t-(f_t-\sigma_1)\dfrac{\omega}{\omega_1} & for\ \omega\le\omega_1 \\ \sigma_1-\sigma_1\dfrac{\omega-\omega_1}{\omega_c-\omega_1} & for\ \omega>\omega_1\end{cases}$ Eq.1	$\sigma(\omega)=f_t\left[1-\left(\dfrac{\omega}{\omega_c}\right)^n\right]$ Eq.2
Fracture energy	$G_F=\dfrac{\alpha_1+\alpha_2}{2}f_t\omega_c$ Eq.3 Where: $\alpha_1=\sigma_1/f_t$ and $\alpha_2=\omega_1/\omega_c$ – relative coordinates of the kink point (point B, Figure 2b).	$G_F=\dfrac{n}{n+1}f_t\omega_c$ Eq.4
Centroid coordinates	$\sigma_{cg}=\left[\dfrac{\alpha_1(\alpha_1+\alpha_2)+\alpha_2}{\alpha_1+\alpha_2}\right]\dfrac{f_t}{3}$ Eq.5	$\sigma_{cg}=\dfrac{n}{2n+1}f_t$ Eq.6
	$\omega_{cg}=\left[\dfrac{\alpha_2(\alpha_1+\alpha_2)+\alpha_1}{\alpha_1+\alpha_2}\right]\dfrac{\omega_c}{3}$ Eq.7	$\omega_{cg}=\dfrac{n+1}{2(n+2)}\omega_c$ Eq.8

3. Detailed assessment and proposed estimation of each criterion

Table 2 summarizes 15 different concrete mixes from 6 studies contributing to the proposed approach in this paper. Among them, class N cement is the major focus beside other diversities (e.g. cement type, cement class, w/c ratio, maximum aggregate size and type, with- or without fibre addition, vibrating concrete or SCC etc.). All the mixes were under similar standard curing conditions at ~20-23^{o}C. The effect of different curing temperatures is therefore not considered, due to data deficiency.

3.1. Tensile strength f_t

For the estimation of tensile strength, the popular *maturity method* should be, as it has been, a priority. The method combines two important influencing factors on strength gain – temperature and age – into one term called "equivalent age" [16], from which further transformation facilitates strength prediction, with some level of reliability. However, despite its popularity, the concept has mostly been applied for compressive strength [17-22], while limited effort can be found for the case of tensile strength and other properties.

In this study, the recommendations in BS EN 1992 [23] is employed, considering its simplicity and reasonable accuracy in practice. Under the same curing temperature, tensile strength at a certain early age can be interpolated based on the strength at mature age, f_{ctm}, and an "aging" function, $\beta_{cc}(t)$. The provided function of $\beta_{cc}(t)$ is expressed in Eq.9 as follows:

$$\beta_{cc}(t) = e^{s\left[1-\sqrt{28\times 24/t}\right]} = \frac{f_{ctm}(t)}{f_{ctm}} = \frac{f_t(t)}{f_t(28d)} \qquad \text{Eq.9}$$

where:

f_{ctm}, $f_t(28d)$ – Mean value of uni-axial tensile strength of concrete at 672h (28 days);

$f_{ctm}(t)$, $f_t(t)$ – Mean value of uni-axial tensile strength of concrete at time t, in hours;

t – Concrete age, in hours, $2h \leq t \leq 672h$;

s – A constant, depending on cement class.

It should be noted that in Eq.9, $\beta_{cc}(t)$ depends roughly on concrete age and factor s. However, while assumed to be a constant over time, experimental data (Table 2) indicates tremendous variation of s, especially during the first 24 hours and 48 hours for concretes of class N and class S cement, respectively. Even worse, other factors such as temperature, w/c ratio, cement class and especially, the actual *hydration process* with its different stages unfortunately have not been accounted for. All of those disadvantages combined makes the prediction method less convincingly accurate, particularly during the first 3 days [24].

Table 2. Summarization of 15 different concrete mixes contributing to this study.

Author	No.	Test setup –	Cement	w/c	a_{max}(e)	Test age	Curing

		fracture area (mm^2)	type-class	ratio	(mm)		temperature (^{o}C)
Nguyen et al. [25, 26][(f)]	1	DTT[(a)] 70x100	I-N	0.63	20	3h-10h	22-24
	2			0.63[(c)]			
Dao et al. [11]	3	DTT 70x100	I-N	0.63	10	2h-10h	23
	4			0.42			
Abel-Hover [27]	5	DTT 102x101	I-N	0.30	12.5	2h-8h	21-27
Østergaard [12]	6	WST[(b)] 50x100	II-N	0.32	16	~10h-28d	20±2.5
	7			0.41			
	8			0.50			
	9		I-N	0.32			
	10		I-S	0.32			
Kim et al. [28]	11	WST 100x125	I-N	0.30	19	1d-28d	N/A
	12			0.55			
Kang et al. [14]	13	WST 105x120	I-N	0.37	9[(d)]	1d-28d	20
	14			0.33			
	15			0.31			

Note: (a) DTT – direct tensile test; (b) WST – wedge-splitting test;
(c) With micro fibre addition; (d) Self-consolidating concrete;
(e) Maximum aggregate size; (f) Recorded data from this current study.

To include the mentioned factors for a more realistic s, a fitting function is proposed with its empirical constants determined for each mix. Further analysis shows good correlation of such empirical constants of all concrete made of Class N cement without fibre addition, under a stable curing temperature of ~20-23^{o}C. The fitting function can then be generalized in the following form:

$$s(t) \cong 0.270 + ke^{-\frac{0.062}{(w/c)}t} - 0.285e^{-\frac{0.008}{(w/c)}t} \qquad \text{Eq.10}$$

where t – Concrete age in hours, 2h ≤ t ≤ 672 h (28 days);

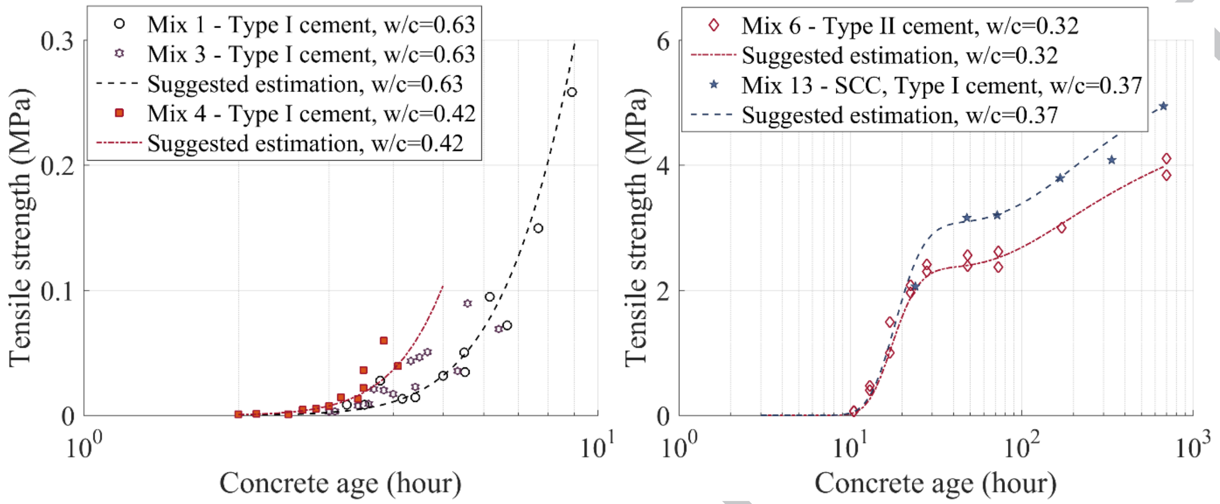
w/c – Ratio of water to cementitious materials, 0.30 ≤ w/c ≤ 0.70;

k – Empirical factor, depending on concrete age t and w/c ratio:

- For 0.30 ≤ w/c ≤ 0.60:
  - t ≤ 10h, k = 0.70
  - t > 10h, k = $32e^{-6(w/c)}$
- For w/c > 0.60, k = 0.70

From Eq.10, not only a single parameter is considered, but a combination of w/c ratio, concrete age and different stages of hydration has been reflected in the formula. The amount

0.270 represents the stabilized s after ~24 hours of age of class-N-cement concretes, which is relatively close to the suggested s from BS EN 1992 (0.25). The last 2 terms in the formula illustrate the initial fluctuation of s – swiftly down at first, then moderately up before stabilizing.



(a) (b)

Figure 3. Tensile strength estimation based on BS EN 1992 [23] and the revised s (Eq.10).

The revised s when applied back to Eq.9 gives accurate estimation for 10 out of 14 mixes made of class N cement, as illustrated in Figure 3 for mix 1, 3, 4, 6 and 13. It can be noted that both strength at infancy (less than 10 hours) and later age can be reliably- and conveniently determined. Among the 4 exceptions, the presence of micro fibre addition (mix 2) and irregular/questionable strength gain patterns (mix 12, 14 and 15) are possible reasons for the deviation.

3.2. Tensile strength f_t – fracture energy G_F multi-linear correlation

Despite large variation of mix designs and test setups, there appears to be a *strong linear relationship* between fracture energy and recorded tensile strength. From the illustration in Figure 4, the inter-relationship between the two seems strongly dependent on concrete age and cement type, if not mentioned the effect of micro fibres in mix 2 (Figure 4a).

- *Concrete age:* Different studies confirm that the ratio of G_F(J/m^2)/f_t(MPa) at earlier than 10 hours is at least 10 times larger than that at later age. This is possibly because of the countering effects of the hydration process, where:
  - During the dormant period (2-4 hours after mixing), strength gain is normally restrained at a negligible rate [29]. However, the influence actually remains effective for several hours longer before significant development of f_t is notable. Considering mixes 6-9 as

an example: At 24 hours of age, tensile strength was reported to reach approximately 50% of that at 28 days; while at 10 hours, recorded strength was only 1-5%;

- Concrete was reported to be more ductile during the first day of age, and the tail of the tensile softening curve was evidenced to be much longer compared to that at later age [11, 25, 26], leading to a considerable increase of the area under that curve (G_F). This explains relatively high fracture energy observed in mixes 1-4 before 10 hours of age.

▫ *Concrete type:*

- For the mixes of type I cement, a tri-linear model can be employed (with high confidence) according to 3 distinct stages of strength gain – (i) The first 10 hours, (ii) From 10 to 24 hours for class N and 48 hours for class S, and (iii) Later (Figure 4a, c);
- For the mixes of type II cement, one single linear model can be used to correlate fracture energy and tensile strength after 10 hours of age (Figure 4b). No evidence of such relationship can be found from the contributing mixes at earlier age, however.

Those multi-linear fitting models, given that either tensile strength or fracture energy is reliably known, forms a basic- yet simple rule to determine the other. Table 3 summarizes such suggested model (i.e. for concrete without fibre addition).

Table 3. Suggested correlation factor λ for the multi-linear relationship between G_F and f_t.

Cement type	$\lambda = G_F(\text{J/m}^2)/f_t(\text{MPa})$		
I	$t \le 10$h	$10\text{h} < t \le 24$h (class N) $10\text{h} < t \le 48$h (class S)	> 24h (class N) > 48h (class S)
	600-700	75-80	5-15
II	$t \le 10$h	> 10h (class N)	
	N/A	60-70	

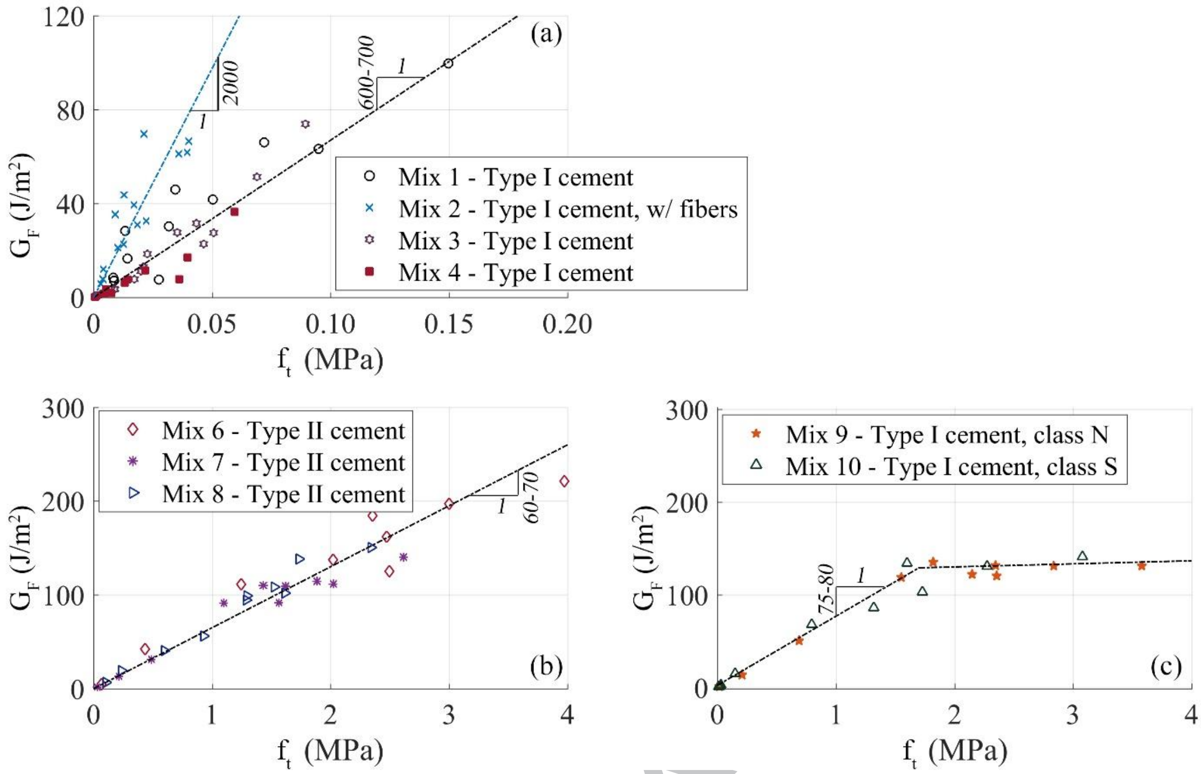


Figure 4. Multi-linear correlation model for concrete of different cement types: (a) Earlier than 10 hours and (b-c) Later age.

3.3. σ_{cg} – Centroid coordinate along σ axis

Examining a softening curve shown in Figure 2a, according to Nomura and Mihashi [30, 31], the first phase of stress reduction corresponds to micro-crack localization and extension. In the second phase, crack localization becomes more visible crack surfaces; and the ability to withstand a very low level of stress, before final separation, is mainly due to aggregate bridging.

Provided the second phase of the softening curve – aggregate bridging – has minor contribution to the whole area (i.e. fracture energy), the second phase itself as a result has minor effect to σ_{cg} (Figure 2b). That assumption also implies an important fact: *the variation of σ_{cg} over time is mostly in tune with that of its respective tensile strength*. It is therefore reasonable to consider the "aging" factor $\beta_{cc}(t)$ in Eq.9 with the revised s in Eq.10 into σ_{cg} estimation, as follows:

$$\sigma_{cg}(t) \cong \beta_{cc}(t) \times \sigma_{cg}(28d) \qquad \text{Eq.11}$$

where $\sigma_{cg}(28d)$ – Centroid coordinate along σ axis at mature age.

As presented in Figure 5, Eq.11 gives appropriate estimation of σ_{cg} for most contributing cases, both at infancy and at later age. Exceptions in this estimation are mix 2 (with micro fibres) and mixes 12, 14, 15 (with irregular/questionable strength gain patterns). In addition, mix 5 is excluded because of insufficient data. Also in Figure 5a, due to the lack of experimental data, σ_{cg}(28d) is determined based on the mature tensile strength f_t(28d) and the ratio of σ_{cg}(28d)/f_t(28d) extrapolated from Table 4.

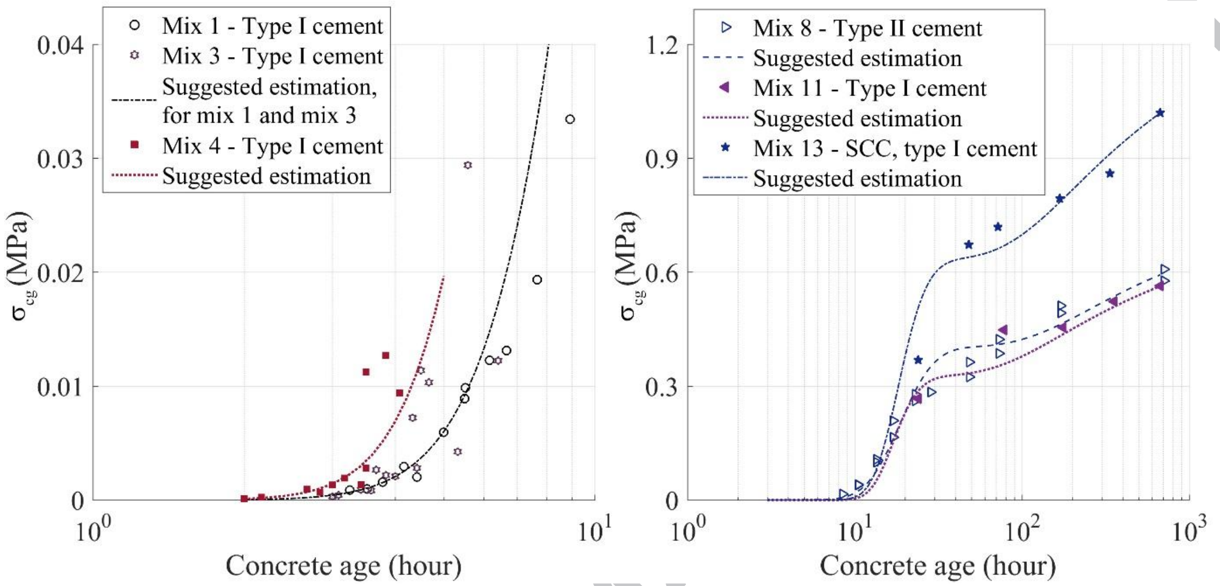


Figure 5. Estimation of σ_{cg} using Eq.11 and the revised s factor in Eq.10.

Table 4. Ratio of σ_{cg}(28d)/f_t(28d) for different mixes (obtained from test data).

Mix	6	7	8	9	10
w/c	0.32	0.41	0.50	0.32	0.32
Cement type-class	II – N	II – N	II – N	I – N	I – S
σ_{cg}(28d)/f_t(28d)	0.30	0.27	0.25	0.27	0.25

3.4. ω_{cg} – Centroid coordinate along ω axis

In Figure 6, ω_{cg} is shown to have larger variation in most cases compared to σ_{cg}. The development of ω_{cg} appears to be in tune with that of ω_c (Figure 1) with several important characteristics listed below:

- From ~24 hours and ~48 hours for the mixes of class N and class S cement, respectively, ω_{cg} stabilizes at a very low level. This characteristic is observed in all of Østergaard's data (WST – [12]) and agrees well with other studies with different test setups, including Jin-Li (direct tensile test – [32]), Kim et al. (WST – [28]), Abdalla-Karihaloo (WST and three-point bending test – [2]) and Kang et al. (WST – [14]). Further analyses of mixes 6-15 also gives evidence about the relationship of ω_{cg} with both maximum aggregate size in use and w/c ratio;

- At earlier age, in most cases ω_{cg} sharply decreases from a very high level (mixes 1, 3 and 4). Comparing mix 1 and mix 3, it is obvious that ω_{cg} is strongly dependent on the maximum aggregate size in use.

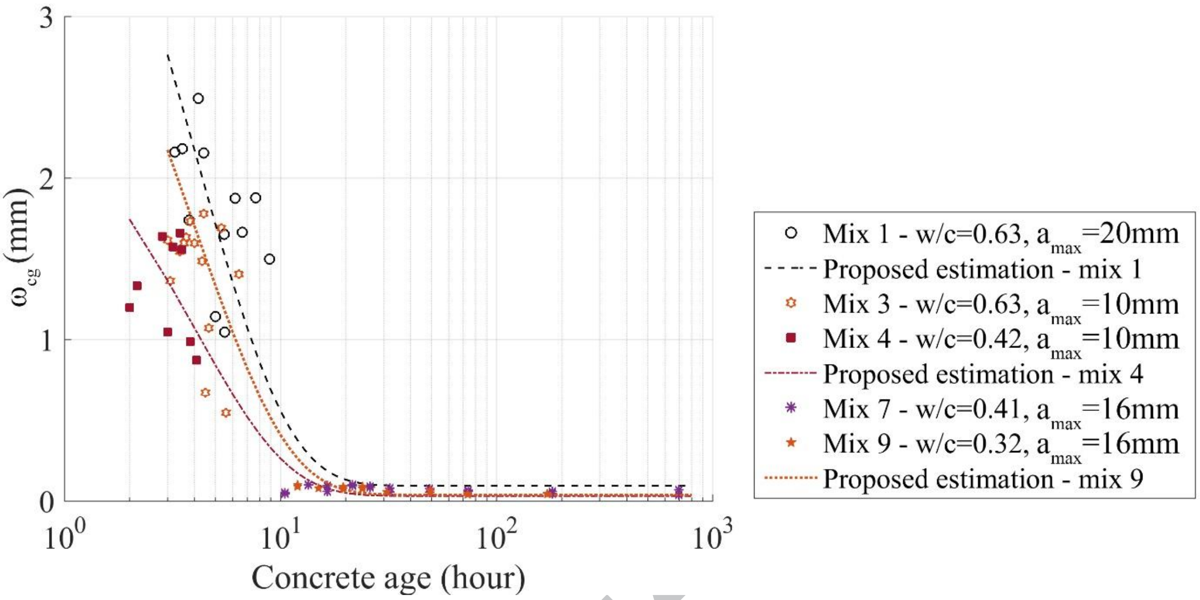


Figure 6. ω_{cg} estimation based on Eq.12.

To characterize the abovementioned evolution of ω_{cg}, a fitting function is proposed with the general trend shown in Figure 6. Fitting process contains the following stages:

- First, the stabilized ω_{cg} from 24 hours of age (class N cement) is considered on the basis of mixes 6-15. Important contributing factors in this case include w/c ratio and maximum aggregate size;
- Then, its evolution at earlier age is considered based on test data of mixes 1, 3, 4, as well as on the fitted ω_{cg} at later age. Major contributing factors in this stage consist of concrete age and maximum aggregate size. Other factors having minor effects are generally included in empirical constants. The final analytical function, in mm, then can be expressed in the following form:

$$\omega_{cg}(t) \cong \left[0.0075(w/c) + 0.22e^{0.26(1-t)}\right]a_{max} \qquad \text{Eq.12}$$

where t – Concrete age, in hours, 2h ≤ t ≤ 672h (28 days);

w/c – Ratio of water to cementitious materials, by mass, 0.30 ≤ w/c ≤ 0.65;

a_{max} – Maximum aggregate size, 9mm ≤ a_{max} ≤ 20mm.

The term 0.0075(w/c).a_{max} represents the stabilized ω_{cg} after 24h for concretes of class N cement, indicating its slight increase with increasing w/c ratio and/or maximum aggregate size.

Several examples of ω_{cg} estimation are illustrated in Figure 6, most of which agrees with its respective test data, although with less confidence compared to σ_{cg}.

4. Interpretation of the concept into practical situations

4.1. Example 1: When limited experimental data are available

4.1.1. Initial parameters

The whole estimation process relies on initial parameters supplied in the following table:

Cement type:	II	Concrete properties at a certain age (e.g. 28 days):					
Cement class:	N	f_t(28d):	2.61MPa	σ_{cg}(28d):	0.71MPa	ω_c(28d):	0.20mm
w/c:	0.41	G_F(28d):	140.22J/m^2	ω_{cg}(28d):	0.05mm	a_{max}:	16mm

Equivalent test results, for comparison, are adopted from mix 7 (Østergaard [12]). Both bi-linear- and power approximation models presented in Table 1 will be employed, by which their estimation outcomes can be finally compared.

4.1.2. Bi-linear approximation model (Figure 2)

- For concrete made of type II class N cement, fracture energy versus tensile strength can be approximated by a single linear trend, therefore according to Eq.3:

$$\frac{G_F(t)}{f_t(t)} = \frac{\alpha_1(t)+\alpha_2(t)}{2}\omega_c(t) \cong \frac{G_F(28d)}{f_t(28d)} \quad \text{Eq.13}$$

  Compatibility check for Eq.13: $60 \leq G_F(28d)[J/m^2]/f_t(28d)[MPa] \leq 70$ (Table 3).

- The "aging" factor $\beta_{cc}(t)$ with revised s is then applied to estimate $f_t(t)$ and $\sigma_{cg}(t)$ as follows:

$$f_t(t) = \beta_{cc}(t) \times f_t(28d) \quad \text{Eq.14}$$

$$\sigma_{cg}(t) \cong \beta_{cc}(t) \times \sigma_{cg}(28d) \quad \text{Eq.15}$$

- Applying Eq.14 and Eq.15 into Eq.5, the following form is obtained:

$$\beta_{cc}(t) \times \sigma_{cg}(28d) = \left[\frac{\alpha_1(t)[\alpha_1(t)+\alpha_2(t)]+\alpha_2(t)}{\alpha_1(t)+\alpha_2(t)}\right]\frac{\beta_{cc}(t) \times f_t(28d)}{3} \quad \text{Eq.16}$$

  Further transformation of Eq.16 gives:

$$\frac{\alpha_1(t)[\alpha_1(t)+\alpha_2(t)]+\alpha_2(t)}{\alpha_1(t)+\alpha_2(t)} = 3\frac{\sigma_{cg}(28d)}{f_t(28d)} \quad \text{Eq.17}$$

- For concrete made of type II class N cement, ω_{cg} (in mm) can be approximated by Eq.12.

Compatibility check for Eq.12: $0.0075(w/c)a_{max} \approx \omega_{cg}(28d)$.

- Substituting Eq.12 into Eq.7, the formula becomes:

$$\left[\frac{\alpha_2(t)[\alpha_1(t)+\alpha_2(t)]+\alpha_1(t)}{\alpha_1(t)+\alpha_2(t)}\right]\omega_c(t) = 3\left[0.0075(w/c)+0.22e^{0.26(1-t)}\right]a_{max} \quad \text{Eq.18}$$

- Eq.13, Eq.17 and Eq.18 altogether shape up an equivalent approximation of the stress-crack opening curve at any age. By solving those 3 equations, coordinates of the kink point (α_1 and α_2) and the ultimate separation point (ω_c) are determined, from that a prediction of time-dependent fracture energy and tensile post-crack behaviour can be clearly characterized.

4.1.3. Power approximation model (Figure 7)

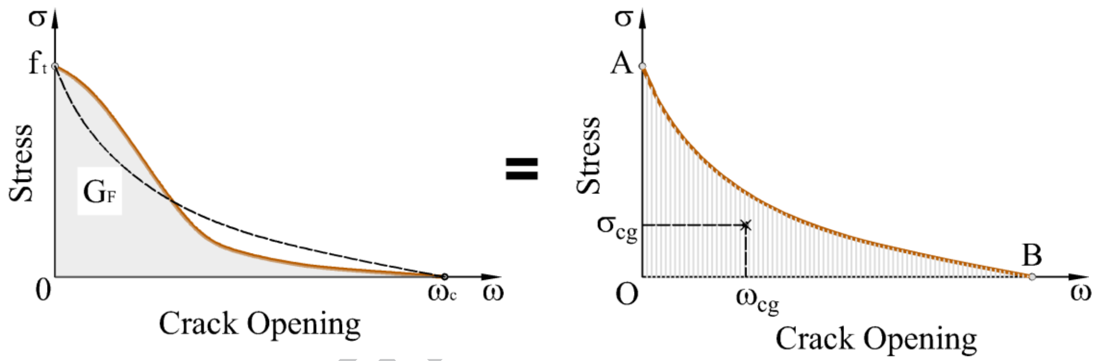


Figure 7. Power model to approximate tensile softening curve and fracture energy.

- Similarly, for concrete made of type II class N cement and from Eq.4:

$$\frac{G_F(t)}{f_t(t)} = \frac{n(t)}{n(t)+1}\omega_c(t) \cong \frac{G_F(28d)}{f_t(28d)} \quad \text{Eq.19}$$

- Combining Eq.6, Eq.14 and Eq.15 gives:

$$\sigma_{cg}(t) = \beta_{cc}(t)\times\sigma_{cg}(28d) = \frac{n(t)}{2n(t)+1}\beta_{cc}(t)\times f_t(28d) \quad \text{Eq.20}$$

Further transformation of Eq.20 can be made as follows:

$$\frac{n(t)}{2n(t)+1} = \frac{\sigma_{cg}(28d)}{f_t(28d)} = constant \;\rightarrow\; n(t) = \frac{\sigma_{cg}(28d)}{f_t(28d)-2\sigma_{cg}(28d)} \quad \text{Eq.21}$$

- For concrete made of type II class N cement, ω_{cg} (in mm) in Eq.12 can be employed to approximate its development over time. By substituting Eq.12 into Eq.8, the third condition is achieved:

$$\omega_{cg}(t) = \frac{n(t)+1}{2[n(t)+2]}\omega_c(t) = \left[0.0075(w/c) + 0.22e^{0.26(1-t)}\right]a_{max} \quad \text{Eq.22}$$

- With this model, only Eq.19 and Eq.22 combined are sufficient to determine basic parameters of a power-typed softening curve at any age (n and ω_c). The constant n obtained from Eq.21 reflects the stabilized value at relatively more mature age, which therefore can be used for verification purposes.

4.1.4. Estimation outcomes and comparison between two approximation models

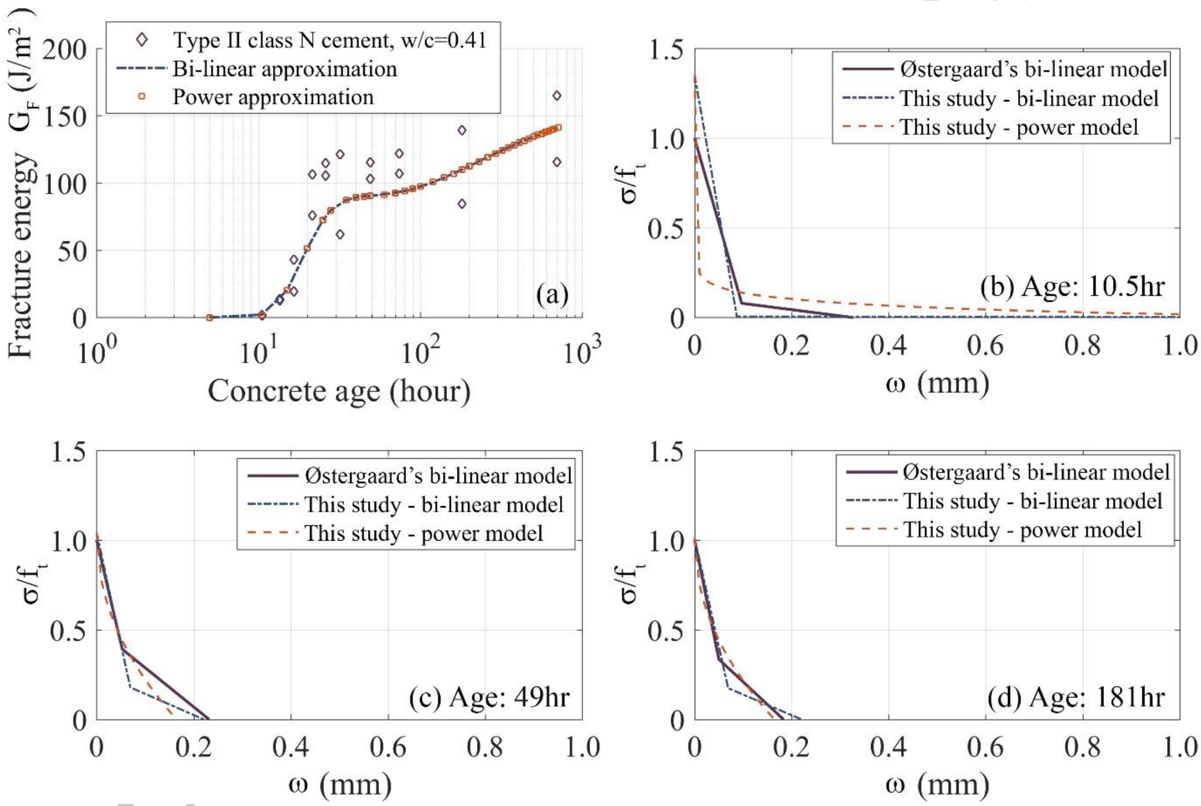


Figure 8. Estimation results compared to experimental data (mix 7, Østergaard's [12]).

Calculated results are illustrated in Figure 8 with some brief comparisons between both models summarized in the following table.

Criteria	Approximation method	
	Bi-linear model	Power model
Fracture energy	Both models result in unique estimated values and agrees well with test data (Figure 8a). Also, power model proves its better simplicity for calculation.	

<table>
<tr><td>Ultimate strain ω_c
▫ $t > 1$day
▫ $t \leq 1$day</td><td colspan="2">Varies with time – decreasing with increasing maturity.
Agrees well with Østergaard's model (Figure 8c, d).
Much larger compared to Østergaard's model, but shows good agreement with other studies [11, 25, 26] (Figure 8b).</td></tr>
<tr><td>Tensile strength</td><td colspan="2">Shows good agreement with recorded data, although still contains some inaccuracy at $t \leq 1$day (Figure 8b, c, d).</td></tr>
<tr><td>Softening curve
▫ $t > 1$day
▫ $t \leq 1$day</td><td>
Agrees with Østergaard's model.
Agrees with Østergaard's model.</td><td>
Agrees with Østergaard's model.
Sharp decrease of stress at the beginning of the curve seems unrealistic (Figure 8b).</td></tr>
</table>

4.2. Example 2: When no experimental data is present

4.2.1. Initial parameters

Cement type:	I	Cement class:	N	w/c:	0.63
Design strength, f_{ck}:	32MPa	a_{max}:	10/20mm		

With no experimental data available, more assumptions and approximations are necessary, with the primary focus on the first 10 hours after placement. Equivalent experimental data, for comparison, are from mix 1 (recorded in this research program [26]) and mix 3 (from Dao et al. [11]). Both bi-linear- and power models are considered.

4.2.2. Basic assumptions and approximations

Fundamental formulae are exactly as provided in Example 1. In addition, the following assumptions should be considered to facilitate the calculation process:

- For concrete made of type I class N cement, fracture energy versus tensile strength can be approximated by a multi-linear model suggested in Table 3;
- The ratio of $\sigma_{cg}(28d)/f_t(28d)$ is based on Table 4, where an obvious decreasing trend with increasing w/c ratio can be observed for concrete made of type II class N cement. Assuming a similar trend for type I class N cement and starting from the value of 0.27 for a

w/c of 0.32, such ratio is extrapolated to range within *0.18-0.20* for a w/c of 0.63 in this example.

- For concrete made of type I class N cement, mature tensile strength is estimated based on current standards, for example:
  - BS EN 1992 [23] and CEB-FIP 1990 [15]:

$$f_t(28d) = f_{ctm} = 0.3{f_{ck}}^{2/3} = 0.3 \times 32^{2/3} = 3.02MPa \quad \text{Eq.23}$$

where: f_{ck} - Characteristic compressive cylinder strength at 28 days.

  - AS 3600 [33]:

$$f_t(28d) = 1.4f'_{ct} = 1.4 \times 0.36{f'_c}^{1/2} = 1.4 \times 0.36 \times 32^{1/2} = 2.85MPa \quad \text{Eq.24}$$

where: f'_c - Characteristic compressive cylinder strength at 28 days.

The value provided by BS EN 1992 is chosen in this example.

4.2.3. Some comments over estimated values

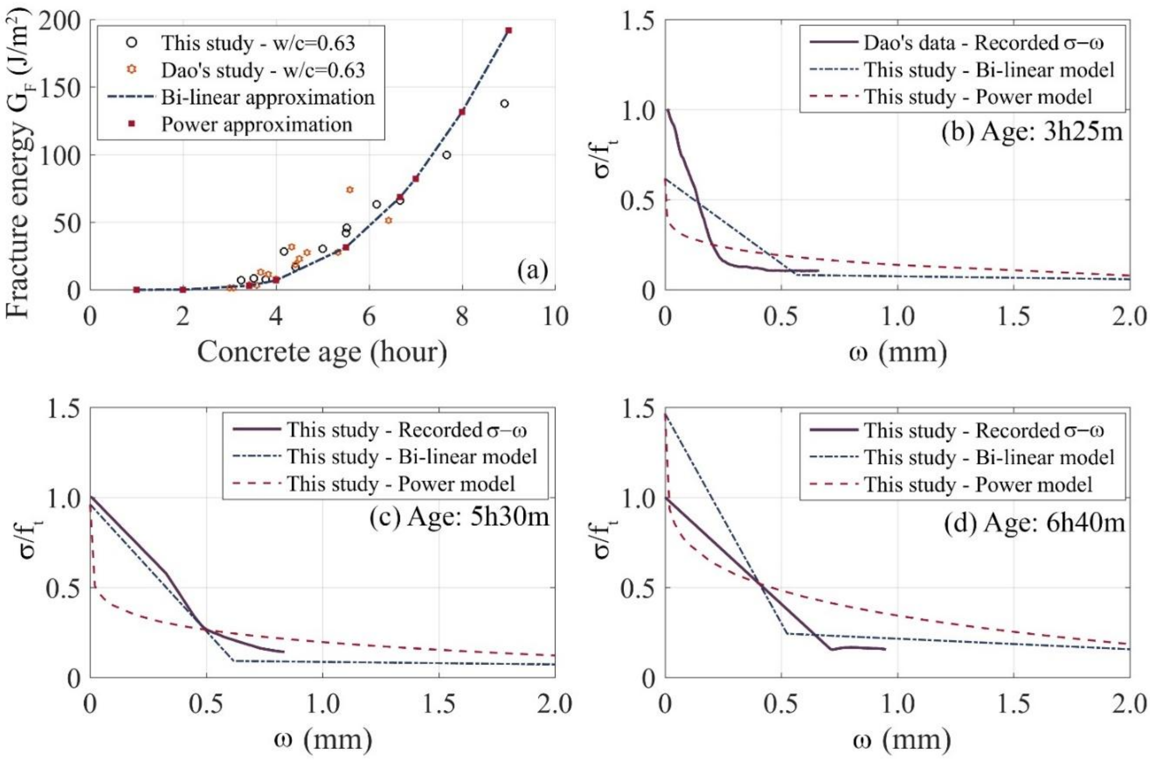


Figure 9. Proposed estimation compared to experimental data (mixes 1 [26] and 3 [11]).

Calculated results of both models are shown in Figure 9. It is notable that due to larger variation of tensile strength at this very early age, estimated strength, although reflecting the actual evolution (Figure 3), show some differences when each specific case is considered (Figure 9b, d).

From both examples, it can be seen among the two, power model shows its better simplicity in the calculation process. However, while both models give good estimation of fracture energy, their appropriateness to reflect tensile post-peak behaviour are different. Sudden sharp decrease of stress at the beginning of the power curve shown during the first day of age (Figure 8b and Figure 9b, c, d) is somewhat *unrealistic*. Bi-linear curve, on the other hand, appears to better represent the actual tensile softening curve, from infancy to more mature age.

5. Conclusions

Proper estimation of concrete fracture energy and tensile softening behaviour, as well as their evolutions from very early age, is significant for both practical design-construction and modelling/simulation. Despite extensive past research effort, virtually no available method with confirmed reliability and with a systematic approach can be found.

In this paper, a novel approach established for the stated purpose is presented:

- First, two centroid coordinates of the area under a tensile softening curve are introduced as a fundamental basis for estimation, based on their temporal evolutions with visibly more consistent- and predictable trend, compared to other factors;
- The approach then relies on 3 criteria: (i) A better prediction of tensile strength development with more convincing accuracy, (ii) Confirmed relationship between fracture energy and tensile strength over time and (iii) Respected centroid coordinates, regardless of the approximation curve employed. The approach with these 3 criteria together is shown to be sufficient and applicable for both bi-linear- and power approximation models;
- Next, based on experimental data from 6 different studies with a total of 15 contributing concrete mixes, each criterion is assessed in detail with respective estimation formulae suggested afterward;
- Finally, practical applications of the approach are illustrated for both bi-linear- and power approximation models. Further comparisons from 2 examples indicate that: Although both models work similarly well for concrete after the first day of age, bi-linear curve appears to better reflect softening behaviour of younger concrete.

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Highlights:

- A novel approach for estimating of the evolution of fracture energy and tensile softening curve of concrete from early age is proposed;
- The proposed approach is based on three fundamental and logical criteria, each of which is thoroughly assessed on the basis of a rich collection of test data from different studies;
- The capability and reliability of the approach are demonstrated through practical examples.