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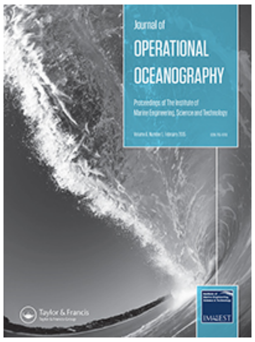

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An empirical approach to improving tidal predictions using recent real-time tide gauge data

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Harmonic tidal prediction methods are often problematic in estuaries owing to the distortion of tidal fluctuations in shallow water, causing disparity between predicted and observed sea levels. The UK National Tidal and Sea Level Facility attempted to reduce prediction errors for the short-term forecasting of High Water (HW) extremes using three alternative techniques to the Harmonic Method in the Bristol Channel, where prediction errors are relatively large. A simple procedure for correcting Harmonic Method HW predictions using recent observations (referred to as the Empirical Correction Method) proved most effective and was also successfully applied to sea-level records from 42 of the 44 UK Tide Gauge Network locations. It is to be incorporated into the operational systems of the UK Coastal Monitoring and Forecasting Partnership to improve UK short-term sea level predictions.

Introduction

The classical Harmonic Method of Tidal Analysis and Prediction is long-established, having been developed by Laplace, Lord Kelvin and George Darwin (Darwin 1911) and further advanced by Doodson (1921) and Cartwright and Tayler (1971) among others. It is based upon the principle that a series of tidal observations (T) at times (t) may be decomposed into a finite number of sinusoidal functions (or tidal constituents) of the form given in Equation (1), with angular speeds (σ) that are related to a number of known astronomical frequencies.

$$T(t) = Z_0 + \sum_{n=1}^{N} H_n f_n \cos\left[\sigma_n t - g_n + (V_n + u_n)\right] \quad (1)$$

The amplitudes (H) and phase lags (g, relative to the Equilibrium Tide at Greenwich) of these constituents are derived by least-squares regression. Variations in phase and amplitude that are caused by the 18.6-year cycle in maximum lunar monthly declination are accounted for by nodal factors f_n and u_n, while V_n is the equilibrium phase angle of the nth constituent.

The tidal constituents collectively define the tidal characteristics at a particular location and may be recombined to predict future tidal patterns at that site. The selection of constituents to be used in an Harmonic Analysis and Prediction is dependent upon the length and sampling interval of the observational time series, but as a general rule, in the deep water of the open ocean and for time series of 12 months or longer, the observed tide may be resolved using only 60 tidal constituents (Doodson 1921) – a practice that is generally referred to as the Standard Harmonic Method (SHM).

However, in the relatively shallow coastal seas, the progression of the tide is modified by non-linearities introduced by depth and bottom friction and this shallow water distortion of the tide requires the use of additional higher frequency tidal constituents. This Extended Harmonic Method (EHM) (Rossiter and Lennon 1967) typically involves the use of 114 constituents to resolve the observed tide for records of 1 year or longer. Even so, in certain environments (such as long, shallow estuaries) tidal distortion can be such that even EHM may fail to resolve the observed tide sufficiently. Consequently, the residual times series (i.e. the total observed water level minus the calculated tidal component) may still contain significant tidal energy so that any future tidal predictions based upon such an analysis will fail to capture all of the observed tidal variability.

Such tidal predictions are an integral part of the UK Coastal Monitoring and Forecasting (UKCMF) Service which provides a coordinated system of sea-level monitoring and forecasting at 44 locations around the UK coast (Figure 1). This network of tide gauges (the UK Tide Gauge Network, UKTGN) evolved in response to the 1953 North Sea storm surge event and transmits sea level observations from each location at 15 min intervals to the

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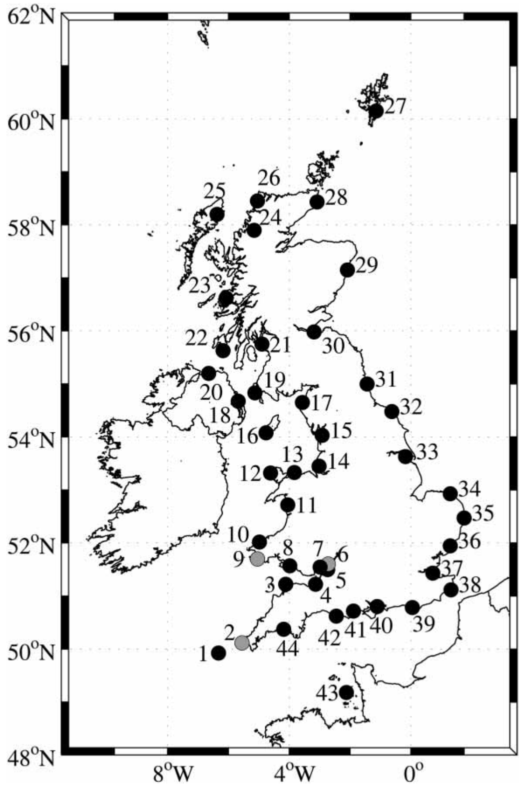

Figure 1. Locations of the 44 UKTGN stations. Grey circles show the Avonmouth pilot study site (No. 6) and the tide gauges at Newlyn and Milford Haven (No. 2 and No. 9 respectively, which were used as reference ports for the Species Concordance technique). All stations were used to devise the validation dataset. Ports numbers are used only for the purposes of this study and are allocated clockwise from the southwest tip of the UK.

UK Flood Forecasting Centre, where they are used in conjunction with EHM tidal predictions to monitor and plan for tidally and meteorologically induced extremes. Given the operational importance of these predictions, it is essential to minimize the associated prediction errors.

For the majority of UKTGN sites, the Mean Absolute Error (MAE) associated with EHM predictions is typically in the region of 5–6 cm, but in the Bristol Channel region, owing to a combination of shallow water effects and large tidal range of around 12 m, these errors more than double resulting, for example, in a MAE of 14.37 cm at Avonmouth and 15.83 cm at Newport Newport for the period 2006–2009 inclusive. Consequently, the National Tidal and Sea Level Facility (NTSLF) at the National Oceanography Centre (NOC) in Liverpool was commissioned by the Environment Agency to conduct a pilot study in the Bristol Channel to investigate the potential for alternative methods of tidal prediction or methods to improve the prediction accuracy associated with classical harmonic analysis.

Three alternative tools for Tidal Analysis and Prediction were evaluated in the pilot study, and were benchmarked against EHM tidal predictions. The three methods evaluated were (1) the use of Artificial Neural Network (ANN) models, (2) the Species Concordance technique, and (3) a simple empirical procedure for correcting Harmonic Method High Water (HW) predictions based upon a few recent observations, which is referred to hereafter as the Empirical Correction Method.

Data and methods

Sea-level data

Two test sea-level datasets were prepared for this study. The first consisted of quality-controlled sea level observations for the UK Tide Gauge Network gauge at Avonmouth, for the period January to December 2007 (inclusive). These data were obtained from the electronic archive of the British Oceanographic Data Centre (BODC) (http://www.bodc.ac.uk/data/online_delivery/ntslf/processed/) and were supplied in the form of 15 min averages, derived from 1 second sampling by a bubbler gauge. These data formed the basis of the pilot study to evaluate the alternative Tidal Analysis and Prediction techniques in the Bristol Channel and are therefore referred to hereafter as the Bristol Channel Pilot Study Dataset.

Certain of the prediction techniques tested in the pilot study demand the use of additional data for reference purposes. For example, the ANN model requires the use of training and validation datasets and consequently, year-long BODC sea level time series for the Avonmouth gauge for 2005 and 2006 were obtained for these respective purposes. Similarly, the Species Concordance Technique involves the use of a long observational tidal record at a reference location that is unaffected by tidal distortion. Therefore, BODC quality-controlled sea level observations were obtained for two potential reference ports: Milford Haven (for the period 1993–2009) and Newlyn (between 1915 and 2009) (see Figure 1). Other UKTGN ports adjacent to Avonmouth were discounted as reference locations since they are themselves affected by shallow water tidal distortion.

The second test dataset consisted of quality-controlled sea level data from the BODC electronic archive for the period 2006–2009 inclusive at 44 UK TGN sites (Figure 1). This dataset was used to evaluate the most successful technique from the pilot study in the context of the wider UK Tide Gauge Network. This is referred to hereafter as the UKTGN Validation dataset.

All datasets were downsampled to hourly intervals in order to reduce computational complexity and data values identified as ‘suspect’ by BODC were excluded from the study.

Storm surge data

Two datasets of the meteorologically induced surge component of sea level (*S*) were prepared, affording identical spatial and temporal coverage to the sea level datasets for the Bristol Channel Pilot Study and the UKTGN Validation dataset. These data were derived from hourly surge hindcasts which are produced and archived annually for model validation purposes by NTSLF, using the UK Operational Storm Surge Model (CS3x) (Horsburgh & Flowerdew 2014). This is a 12 km 2D hydrodynamic shelf model that is run four times daily at the UK Met Office and is forced by 10 m winds and sea level pressure (SLP) from the UK Met Office Global Atmospheric model.

Methods

In order to quantify the potential improvement afforded by the alternative prediction methods, two sets of tidal predictions were prepared using the classical Harmonic Method, which has traditionally been used by UKCMF for flood forecasting purposes. The first consisted of hourly tidal predictions for 2007 for the port of Avonmouth and was generated by applying the EHM to the Avonmouth sea level record for 1988–2007 (inclusive). This record length was selected to encompass an entire lunar nodal cycle, in order to capture the full extent of the Avonmouth tidal range.

The second set of hourly tidal predictions afforded identical spatial and temporal coverage to the UKTGN Validation Dataset and was produced by applying the EHM to sea-level observations at the 44 UKTGN sites using data for 1990–2009. For locations where 19 years' observations were not available, the longest available sea-level record was used to derive the harmonic predictions.

ANN

ANN models are adaptive numerical models that emulate biological neural networks, consisting of various elements (neurons) that operate in parallel and are interconnected by weightings. Such models employ a training dataset to iteratively identify and adjust patterns and relationships between neurons to minimize the error between the training data and the model output. At each iteration, the resulting model is tested using an authentication dataset.

To evaluate the potential for ANN models to improve on short-term water level predictions in the Bristol Channel, a simple ANN model was developed using the MATLAB neural network toolbox, version 6.0.1 in MATLAB R2008b1. Initially, the models were developed with a single output, for computational speed, so that each model was built with the aim of predicting water level at a given number of hours in advance. Models were tested for predictions up to 24 h in advance, using the Avonmouth pilot study sea level dataset. Two inputs were presented to the model; the observed water level at Avonmouth and the predicted tide level using the EHM, over periods of 7 days to 24 h.

A feed-forward back propagation (FFBP) ANN structure was chosen for the initial stage of work owing to its simplicity. The original FFBP method (Rumelhart et al. 1986) has been improved considerably since its first publication, with recent advances such as the introduction of the Levenberg-Marquardt algorithm in ANN training (Levenburg 1944; Marquardt 1963). This study used a Bayesian Regulation training algorithm, which employs the Levenberg–Marquardt algorithm to optimize the weights and biases of the model with respect to the mean squared error. This method incorporates an early stopping routine to avoid overfitting; a common problem with ANNs, where the model is optimized on the training data set with a subsequent loss of generality when applied to other data sets. The Bayesian Regulation training algorithm derives model weights and biases from the training data set, testing the resulting model on a validation data set at each iteration. When the mean squared error of the validation set deteriorates consistently over a given number of iterations, the training ceases. The default number of iterations for early stopping is six.

The internal structure of an ANN model can be made as simple or complex as necessary for any given problem. In this study, a range of ANN model structures were tested, including variations in the number of hidden layers, in the number of neurons and in transfer function types. In practice, the greatest success was achieved with a three-layer model, consisting of two hidden layers of 24 and six neurons respectively using tan sigmoid transfer functions and one output layer using a pure linear transfer function. The results described hereafter relate only to that combination of parameters.

The model predictions may vary for the same training and validation data sets because the weights and biases are initialized by a random number generator and therefore the optimization techniques may converge to different minima. Consequently, the model was reproduced three times to ensure consistency of ANN model performance.

Species Concordance technique

The Species Concordance Method of Tidal Prediction (Simon 1981; George & Simon 1984) is a variant of the Response Method (Munk & Cartwright 1966) and was specifically designed to overcome the effects of both shallow-water distortion and variations in fluvial discharge that arise in estuaries. The full mathematical derivation of this technique is given by George and Simon (George & Simon 1984), but in essence it involves the derivation of a number of complex spherical harmonics, each of which represents a frequency band of the tidal potential, the theory being that the oceanic response to gravitational forcing (including tidal distortion in shallow water) will be the same at all frequencies within each tidal species. Complex amplitudes (C) are derived for each frequency band (or tidal species, k) at a problematic port (m) and at a nearby reference location (R) at which the tidal patterns are not distorted. Least-squares regression is then used to deduce a magnification factor (B) between the complex amplitudes at the two ports. So, for even species, the complex amplitude at location m and time $t + \tau$ (where τ is the mean time lag of the M2 tide between ports m and R) is described by:

$$C_{mk}(t+\tau) = B_{mk}(t)C_{Rk}^{k/2}(t) \tag{2}$$

and for odd species, by:

$$C_{mk}(t+\tau) = B_{mk}(t)C_{Rk}(t) \tag{3}$$

The resulting complex amplitudes C_{mk} can then be used to predict water levels $H_m(t)$ at the problematic site from:

$$H_m(t) = \frac{1}{2}\sum_{k=0}^{k-1}[C_{mk}(t)e^{jkq_1 t} + C'_{mk}(t)e^{-jkq_1 t}] \tag{4}$$

where $q_1 = 2\pi$ radians per lunar day, and C'_{mk} is the complex conjugate of C_{mk}.

The technique has been successfully applied to the Gironde and Loire Estuaries (George & Simon 1984) and to a number of ports in Brittany and Normandy (Simon 1989). In the latter study, for example, the non-tidal residuals at Le Conquet for a 12-month analysis period obtained using the EHM exhibited a standard deviation of 5.3 cm, while the predictions of the Species Concordance Method for the same analysis period generated residuals with a standard deviation of 4.4 cm.

The present study used the MAS (MArvé Simon) Species Concordance suite of programs supplied by the French Service Hydrographique et Océanographique de la Marine (SHOM), who use this method routinely for tidal analysis of short time series (of ~1–12 months' duration) and for ports experiencing shallow water distortion. Complex amplitudes (C_{mk}) were derived for each tidal species at two reference locations that were unaffected by shallow-water distortion, using the longest available time series of hourly sea level heights at each site. In the case of Avonmouth, two potential reference ports were identified: (1) Milford Haven, which is situated on the South Wales coast across the Bristol Channel from Avonmouth and affords continuous sea level records since 1993 and (2) Newlyn, which is situated over 200 km from Avonmouth but provides a long sea level record (observations commenced in 1915) and is a deep water port experiencing little shallow water distortion. These complex amplitudes were then used as data inputs, in conjunction with hourly observed sea level heights from Avonmouth for an overlapping period, in order to derive tidal magnification factors (B_{mk}) between the reference and problematic ports and thereafter produce tidal predictions for Avonmouth using Equation (4). Of the two reference ports, optimum results were obtained using the Newlyn record, and so only those results are reported in this paper.

The technique was tested using various periods of overlapping observations at Avonmouth and the Newlyn reference record, but in practice the success of the technique displayed little dependence upon the length of the concurrent observation period at the two ports. The results reported here are based upon an overlapping period of 1 year's observations at Avonmouth and Newlyn.

Empirical Correction Method

This simple empirical procedure was developed by Hisscott (Hisscott 2009) to adjust harmonic tidal predictions by a correction factor that is based upon differences between observations and predictions during a few recent HW, when the accuracy of tidal predictions is of greatest concern from a flood forecasting perspective. The technique has previously been successfully applied by the Isle of Man Meteorological Office to HW levels at Douglas, halving the RMS error of EHM predictions from ~8 cm to ~4 cm for the period 1 June 2009 to 31 May 2010 (Hisscott 2013, personal communication).

Since this technique is applied solely to HW predictions and observations and not to the whole tidal curve, both the Bristol Channel Pilot Study time series and the UKTGN Validation time series of (a) sea-level observations, (b) harmonic analysis predictions and (c) storm surge hindcasts were sub-sampled to HW only.

To exclude the influence of diurnal inequality (a disparity in the height of adjacent HW caused by the lunar declination), the correction process was made using predictions and observations for alternate HW, producing the following algorithm:

$$\begin{aligned} H_{(N)} = A_{(N)} + S_{(N)} &+ C_1[H_{(N-2)} - S_{(N-2)} - A_{(N-2)}] \\ &+ C_2[H_{(N-4)} - S_{(N-4)} - A_{(N-4)}] \\ &+ C_3[H_{(N-6)} - S_{(N-6)} - A_{(N-6)}] + C_4 \end{aligned} \tag{5}$$

where $H_{(N)}$ is the observed HW level for tide number N (see Figure 2), $A_{(N)}$ is the astronomical HW prediction derived by the EHM, and $S_{(N)}$ is the meteorologically induced 'surge' component for the same tide, while C_1, C_2, and C_3 are prediction correction coefficients, derived by least-squares regression of the prediction error for $H_{(N)}$ against the predictions errors in the preceding $H_{(N-2\,\ldots\,N-6)}$. Correction coefficients ($C_1 \ldots C_N$) were calculated for preceding alternate HW between N – 2 and N – 8, but optimum results were obtained for N – 2 to N – 6, resulting in the use of three coefficients ($C_1 \ldots C_3$) and a constant term (C_4). Consequently, predictions made using this method can be prepared up to 24 h in advance.

It should be noted that the Empirical Correction Method ignores differences in the predicted and observed arrival time of HW, so that $A_{(N)}$ and $H_{(N)}$ represent a semi-diurnal maximum in time series of astronomical predictions and sea level observations, even though the timing of that maximum may differ slightly in the two records.

Results and discussion

For each alternative prediction method (j), the associated error (err) at time (i) was estimated exclusive of the surge sea level component and was defined thus:

$$err_{(ij)} = Ht_{(i)} - S_{(i)} - P_{(ij)} \tag{6}$$

where Ht is the observed water level i, S is the hindcast surge, and P is the prediction.

Several criteria were used to evaluate the efficacy of each prediction method compared with classical harmonic analysis, including the use of spectral analysis and various statistical measures. Such comparisons were made for hourly interval data for the EHM, the Species Concordance Technique and the ANN model. The residuals of these three alternative techniques were also sub-sampled to HW intervals, to facilitate 'like-for-like' comparisons with the Empirical Correction Method.

Bristol channel Pilot Study dataset

The MAE, the root mean squared error (RMSE), and the coefficient of determination ($r^2 = 1 - \mathrm{var(err)}/\mathrm{var}(H)$) were derived for each prediction method using the Bristol Channel Pilot Study dataset. Since the Empirical Correction Method is based solely upon HW prediction errors, these statistics are presented for the ANN, Species Concordance and Harmonic Analysis methods for hourly intervals [Table 1(a)] and, for comparison with the Empirical Correction Method, sub-sampled to HW intervals [Table 1(b)]. In addition, histograms of the errors associated with each method, together with the skewness (s) and kurtosis (k) of each error distribution, are presented in Figure 3 for hourly sampled data and in Figure 5 for HW intervals only. The skewness (s) and kurtosis (k) are defined as:

$$s = \frac{\sum_{i=1}^{n} (xi - \bar{x})^3}{sd^3} \tag{7}$$

and

$$k = \frac{\sum_{i=1}^{n} (xi - \bar{x})^4}{sd^4} \tag{8}$$

(where sd is the standard deviation) and their respective standard errors (se_s and se_k) are defined as follows:

$$se_s = \sqrt{\frac{6}{n}} \tag{9}$$

and

$$se_k = \sqrt{\frac{24}{n}} \tag{10}$$

Consequently, skewness or kurtosis values that are smaller than their respective standard errors are not significantly different from zero.

Panel (a) of Table 1 shows that, of the three methods tested at hourly sampling at Avonmouth, the ANN model shows the greatest improvement upon the classical EHM, exhibiting a markedly lower RMSE and MAE and a higher r^2 value. Indeed, Table 1(a) indicates that the average magnitude of the prediction error (MAE) associated with the ANN model is 5.84 cm smaller than that of the EHM. In contrast, the Species Concordance Method performs relatively poorly, resulting in larger prediction errors than the alternative methods. For example, the MAE associated with the Species Concordance Method is 2.26 cm larger than that of the EHM. Additionally, the r^2 value of the Species Concordance predictions is lower than those derived by other methods, suggesting that they explain a smaller proportion of the observed variance.

Histograms of the errors associated with each method using hourly sampled data [Figure 3(a)–3(c)] show that the error relating to the EHM is positively skewed, relative to a mean of 5.21 cm [shown as a thick dashed line in Figure 3(a)]. A positive skew indicates a tendency for over-prediction, while the mean itself is indicative of under-prediction and perhaps reflects the extreme under-prediction errors to the right of the distribution. In contrast, Figures 3(b) and 3(c) show that the error distributions associated with the Species Concordance and ANN methods are both negatively skewed relative to mean errors of 2.16 cm and 0.82 cm respectively. This is indicative of a tendency towards under-prediction. As regards kurtosis, the EHM error distribution [Figure 3(a)] is platykurtic (broad), suggesting that the errors are relatively widespread about the mean, while the distributions of the alternative methods exhibit greater kurtosis, meaning that there are fewer outliers and that errors are generally smaller. In particular, the ANN model error distribution is strongly leptokurtic (narrow), exhibiting a more acute peak, with a greater concentration of errors close to zero. The relative success of applying this method to the hourly sampled data is confirmed by spectral analysis [Figure 4(a)–4(c)], which shows that the residual time series of the EHM and the Species Concordance techniques

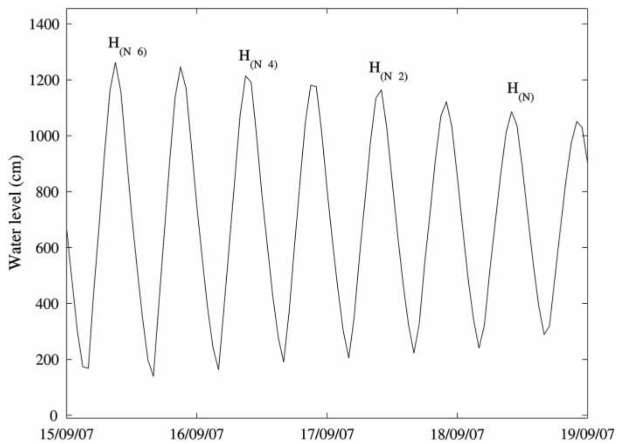

Figure 2. Example of the indexing of HW employed by the Empirical Correction Method, using hourly sampled data from the Avonmouth tide gauge.

contain more energy at tidal frequencies than the ANN model residuals, particularly at fourth-diurnal and higher frequencies, although tidal energy is not entirely eliminated by the ANN model as spectral peaks are still obvious in Figure 4(c) at diurnal and semi-diurnal frequencies.

When comparisons between the techniques are made solely at HW [Table 1(b)], the improvement afforded by the ANN model is less clear. While the MAE and RMSE are 1.71 cm and 2.01 cm lower respectively than those associated with the EHM approach, the r^2 value derived for the EHM (0.9862) is higher than that relating to the ANN model predictions (0.9804), meaning that the harmonic predictions explain a greater proportion of the observed variance than those of the ANN model. This is explained by examination of histograms of the errors associated with these methods at HW [Figure 5(a) and 5(c)], which show that the EHM errors exhibit fewer outliers than those of the ANN model, meaning that the EHM predictions are more frequently closer to observations that the ANN model predictions. However, Figures 5(a)–5(d) also show, that while all error distributions display a negative skew (reflecting a tendency towards under-prediction), the skewness of the ANN model error distribution is relative to a mean of only 2.36 cm, while the means of the EHM and Species Concordance error distributions are 12.74 cm and 13.67 cm respectively, indicating that this under-prediction bias is smaller in the ANN model.

Table 1. Statistical comparison of prediction methods at Avonmouth (a) for hourly sampling intervals and (b) for HWs only.

	RMSE (cm)	MAE (cm)	r^2
(a) Hourly sampling			
EHM	26.30	21.24	0.9939
ANN model	19.80	15.40	0.9964
Species Concordance	29.69	23.50	0.9920
(b) HW only			
EHM	19.55	16.30	0.9862
ANN model	17.84	14.29	0.9804
Species Concordance	22.03	18.77	0.9813
Empirical correction	11.59	9.05	0.9916

Note: The prediction error is defined as the observed water level minus the model hindcast surge and tidal prediction [Equation (6)]. RMSE and MAE are the root mean squared error and the mean absolute error, respectively.

As regards the Species Concordance Method, Table 1 indicates that it performs better at HW [panel (b)] than at hourly intervals [panel(a)], but it still fails to improve upon the results obtained by the EHM. Figure 5(a) and (b) confirm this, since the error distribution associated with the Species Concordance Method shows that errors are more widespread about the mean than those of the EHM and that they display a larger negative skew relative to a higher mean value, reflecting a greater bias towards under-prediction than the EHM. This comparatively poor performance may be attributable to the limited availability of suitable nearby reference ports, being constrained by both the spatial coverage of the UK Tide Gauge Network and by the large physical scale of the Bristol Channel, which means that only distant ports are unaffected by shallow water tidal distortion.

Table 1(b) shows that the greatest improvement upon the EHM tidal predictions at HW is afforded by the Empirical Correction Method, which reduces the RMSE and MAE by 7.96 cm and 7.25 cm respectively and improves the r^2 statistic from 0.9862 to 0.9916. Thus, not only is the average magnitude of the error reduced, but the improved r^2 statistic indicates that the Empirical Correction Method predictions are more frequently closer to observed HW levels than is true of the EHM HW predictions. Further examination of Figure 5(a)–(d) show that despite the general susceptibility of all methods to under-prediction at HW, such bias is minimized using the Empirical Correction Method, as the skewness of the error distribution is smallest and is relative to a zero mean. Figure 5 (a)–(d) also indicates that statistically, the kurtosis of the error distributions association with EHM, Species Concordance and the ANN model are not significantly different from zero, but even so, it is clear from visual examination of these histograms that the errors associated with the Empirical Correction Method are smaller than those of the alternative methods. Indeed, the error distribution of Empirical Correction Method predictions is clearly leptokurtic, exhibiting a greater concentration about the mean value (in this case, 0.00 cm) and fewer outliers. This confirms that the HW predictions of this latter technique are generally closer to observed HW levels than is true of the alternative methods.

Given that (a), from a flood forecasting perspective, the accuracy of tidal predictions is most crucial at HW and (b), the Empirical Correction Method outperformed the alternative techniques at such intervals using the Pilot Study

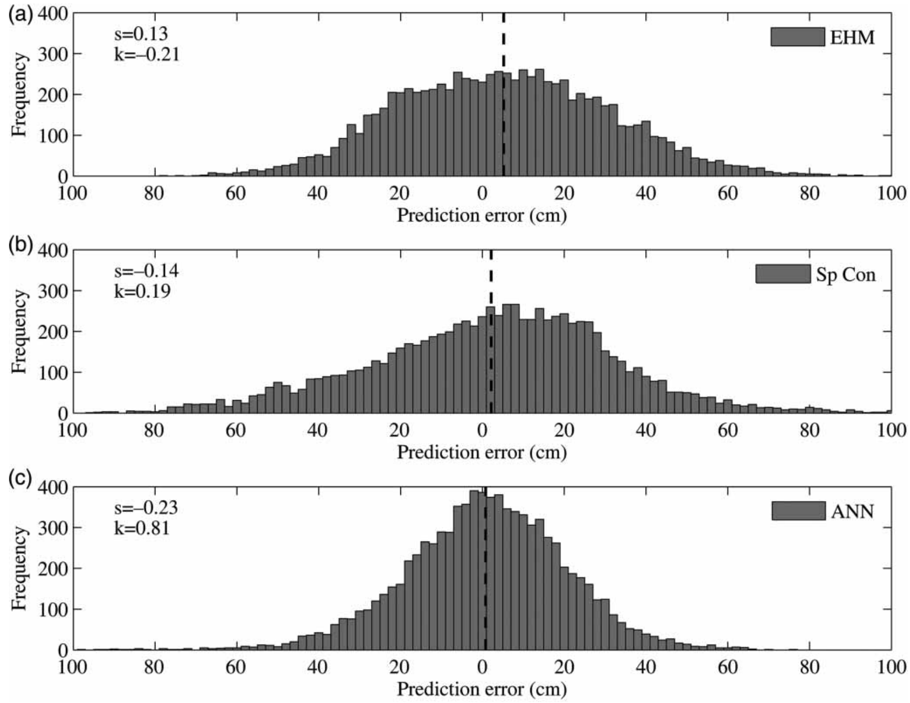

Figure 3. Histogram of prediction errors using hourly sampled data at Avonmouth for 2007 associated with (a) the EHM (b) the Species Concordance Method, and (c) the ANN Model. The error bin interval is 2 cm. The skewness (s) and kurtosis (k) of each distribution are shown on the upper left of each panel, and the associated standard errors are 0.026 and 0.053 respectively. The mean value of each distribution is represented by the black dashed line.

dataset, the latter technique was then applied to the UKTGN Validation dataset.

UKTGN validation dataset

Table 2 shows the RMSE, the MAE and the r^2 statistic at each of the 44 UKTGN Validation Dataset locations for (a) the EHM using residuals that were sub-sampled to HW intervals and for (b) the Empirical Correction Method. These data are also presented graphically in Figure 6(a)–(c).

For ports numbered 22 and 41 in Table 2 and Figure 6 (Bournemouth and Port Ellen), the Empirical Correction Method could not be applied, since these locations experience multiple high and low waters each day owing to non-linear shallow water effects, which are enhanced in the vicinity of amphidromic points where the M2 amplitude is low (Pugh 1987). This renders the computational identification of HW through the extraction of turning points in the tidal curve extremely complex. However, both ports exhibit relatively small tidal ranges of ~1 m (at Port Ellen) and ~2 m (at Bournemouth) and the associated EHM prediction error is therefore similarly low. For example, MAEs relating to the EHM at hourly intervals for 2006–2009 were 6.85 cm and 5.36 cm respectively. Given that prediction errors have been shown to be smaller over HW than at hourly intervals at Avonmouth, it is likely that any potential improvement afforded by the Empirical Correction Method over HW at these ports would also be small.

Table 2 and Figure 6(a)–6(c) both clearly show that for all other locations, the Empirical Correction Method results consistently in a lower MAE and a lower RMSE than is the case for the EHM. The consistently higher r^2 statistic also shows that empirically corrected predictions explain a larger proportion of the observed variance than is the case for the EHM predictions. In particular, relatively large reductions in error of around 5 cm [panels (a) and (b)] and improved r^2 statistics are seen in the high tidal range area of the Bristol Channel (port numbers 3–9), which is an important development from the perspective of coastal flood forecasting. Comparison of the results using the 12 month Pilot Study dataset at Avonmouth [Table 1(b)] with those of the 48 month UKTGN Validation Dataset for the same location (Table 2) shows that the Empirical Correction Method achieves remarkably similar results, irrespective of the length of the observational time series used. For the shorter time series, the RMSE and MAE of the EHM prediction error are several cm higher than for the UKTGN Validation dataset, but despite this the Empirical Correction technique yields RMSEs of 11.59 cm and 11.61 cm for the Pilot Study and Validation datasets respectively, MAEs of 9.05 cm and 8.94 cm and r^2 statistics of 0.9916 and 0.9914.

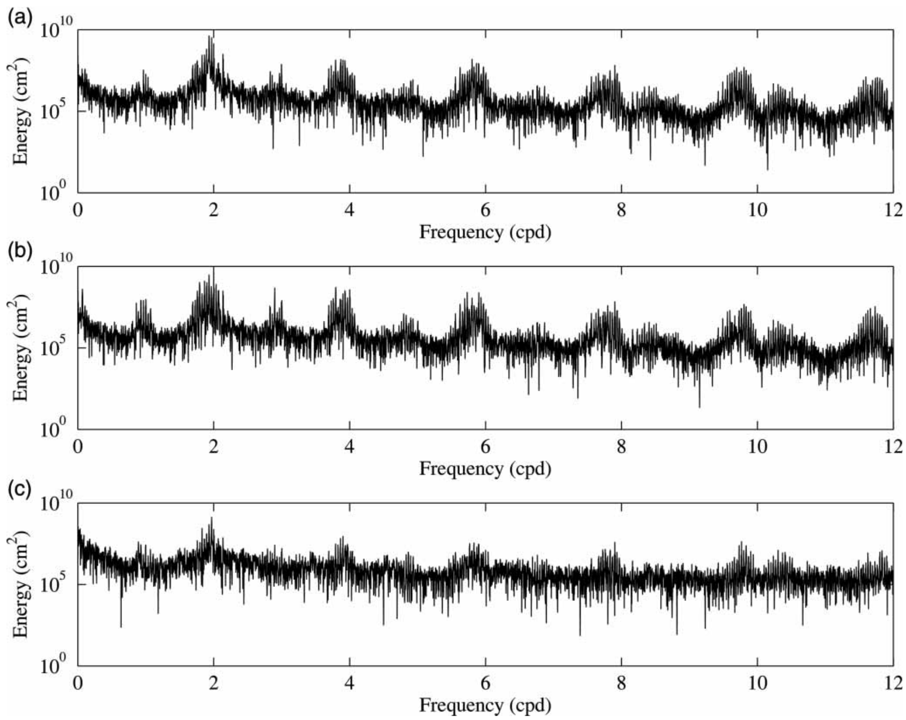

Figure 4. Power spectra of non-tidal residual time series for January–December 2007 for the Avonmouth tide gauge using (a) the EHM, (b) the Species Concordance Method, and (c) the ANN model.

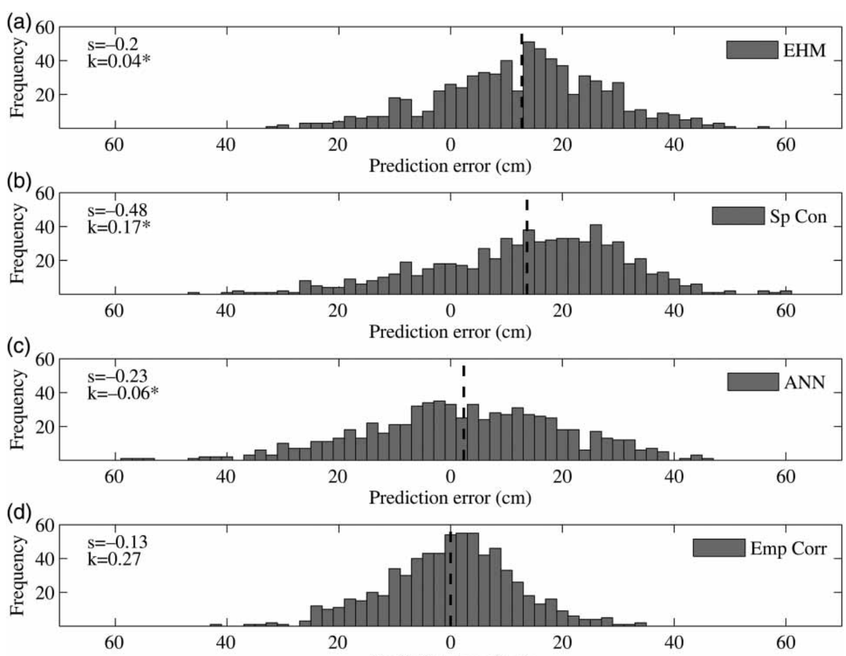

Figure 5. Histogram of prediction errors at HW at Avonmouth for 2007 associated with (a) the EHM, (b) the Species Concordance Method, (c) the ANN Model, and (d) the Empirical Correction Method. The error bin interval is 2 cm. The skewness (s) and kurtosis (k) of each distribution are shown on the upper left of each panel and the associated standard errors are 0.093 and 0.186 respectively. *Skewness or kurtosis is smaller than the associated standard error and is therefore not statistically different from zero. The mean value of each distribution is represented by the black dashed line.

Table 2. Statistical comparison of HW prediction errors for 2006–2009 (inclusive) at 42 of the 44 UKTGN stations, using (a) the classical Extended Harmonic Method (EHM) and (b) the Empirical Correction Method.

		RMSE (cm)		MAE (cm)		r^2 (cm)	
Port number	Port name	(a) EHM	(b) Emp Corr	(a) EHM	(b) Emp Corr	(a) EHM	(b) Emp Corr
1	St Marys	7.19	3.41	5.94	2.57	0.9892	0.9953
2	Newlyn	10.30	3.72	9.09	2.85	0.9864	0.9932
3	Ilfracombe	9.87	5.87	8.14	4.50	0.9923	0.9953
4	Hinkley Point	14.12	10.28	11.50	7.88	0.9865	0.9919
5	Portbury	18.11	11.50	14.25	8.84	0.9834	0.9909
6	Avonmouth	17.80	11.61	14.37	8.94	0.9857	0.9914
7	Newport	19.26	12.89	15.83	9.71	0.9821	0.9884
8	Mumbles	8.47	6.42	6.79	4.90	0.9895	0.9938
9	Milford Haven	12.22	4.34	10.92	3.35	0.9915	0.9955
10	Fishguard	7.37	4.25	5.89	3.22	0.9806	0.9925
11	Barmouth	8.32	6.55	6.47	4.89	0.9763	0.9853
12	Holyhead	7.12	4.58	5.61	3.46	0.9757	0.9897
13	Llandudno	8.15	6.55	6.46	4.85	0.9852	0.9902
14	Liverpool	10.00	5.76	8.40	4.47	0.9904	0.9940
15	Heysham	9.14	6.28	7.33	4.75	0.9900	0.9944
16	Port Erin	7.23	4.50	5.94	3.44	0.9784	0.9886
17	Workington	9.01	6.75	6.76	4.97	0.9855	0.9908
18	Bangor	6.30	4.54	4.76	3.40	0.9285	0.9620
19	Portpatrick	7.65	5.27	6.22	3.93	0.9355	0.9665
20	Portrush	7.06	4.49	5.40	3.36	0.9264	0.9658
21	Millport	8.33	6.15	6.45	4.64	0.8945	0.9410
22	Port Ellen[a]	–	–	–	–	–	–
23	Tobermory	7.69	4.72	5.87	3.55	0.9725	0.9887
24	Ullapool	9.59	4.82	7.36	3.70	0.9694	0.9896
25	Stornoway	7.53	4.01	6.01	3.09	0.9733	0.9921
26	Kinlochbervie	9.40	4.45	7.40	3.42	0.9699	0.9898
27	Lerwick	6.71	3.53	5.28	2.77	0.8920	0.9680
28	Wick	6.78	4.19	5.31	3.23	0.9521	0.9806
29	Aberdeen	6.21	4.49	5.01	3.42	0.9715	0.9834
30	Leith	8.06	6.26	6.35	4.81	0.9731	0.9805
31	North Shields	8.71	5.39	7.06	4.07	0.9730	0.9831
32	Whitby	15.79	5.57	14.14	4.20	0.9709	0.9833
33	Immingham	11.55	8.30	8.77	6.07	0.9701	0.9778
34	Cromer	8.35	6.08	6.47	4.53	0.9696	0.9792
35	Lowestoft	6.90	5.54	5.30	4.18	0.8687	0.9059
36	Harwich	17.38	6.93	15.33	5.28	0.9050	0.9408
37	Sheerness	12.94	6.98	10.47	5.35	0.9484	0.9695
38	Dover	9.56	6.46	7.74	4.83	0.9758	0.9835
39	Newhaven	7.47	5.60	5.95	4.16	0.9858	0.9902
40	Portsmouth	7.93	5.54	6.38	4.15	0.9591	0.9721
41	Bournemouth[a]	–	–	–	–	–	–
42	Weymouth	6.98	4.34	5.60	3.23	0.9486	0.9794
43	Jersey	9.38	6.73	7.53	4.92	0.9936	0.9963
44	Plymouth	7.30	4.08	5.89	3.14	0.9796	0.9902

Note: The prediction error is defined as the observed water level minus the model hindcast surge and tidal prediction [Equation (6)].
[a]Ports that experience multiple HW per day, to which the Empirical Correction Technique could not be applied.

Interestingly, while ports situated around the North Channel of the Irish Sea (Bangor, Portpatrick, Portrush and Millport, port numbers 18–21) exhibit relatively low MAE and RMSE for the EHM predictions, the r^2 statistic is also relatively low, implying that the prediction error is large relative to the observed tidal variability and that there is frequent disparity between observations and predictions. It is likely that this is because the primary semi-diurnal tidal constituents are not so dominant in this region (Pugh 1987). Comparatively low predictions errors and r^2 statistics are also found at Lerwick and Lowestoft (port number 27 and 35 respectively), which are again, areas of small M2 amplitude and enhanced shallow water distortion.

Several other locations exhibit comparatively high EHM prediction errors, most notably Whitby, Harwich and Sheerness (port numbers 32, 36 and 37 respectively) and in the latter two cases, the r^2 statistic is also relatively

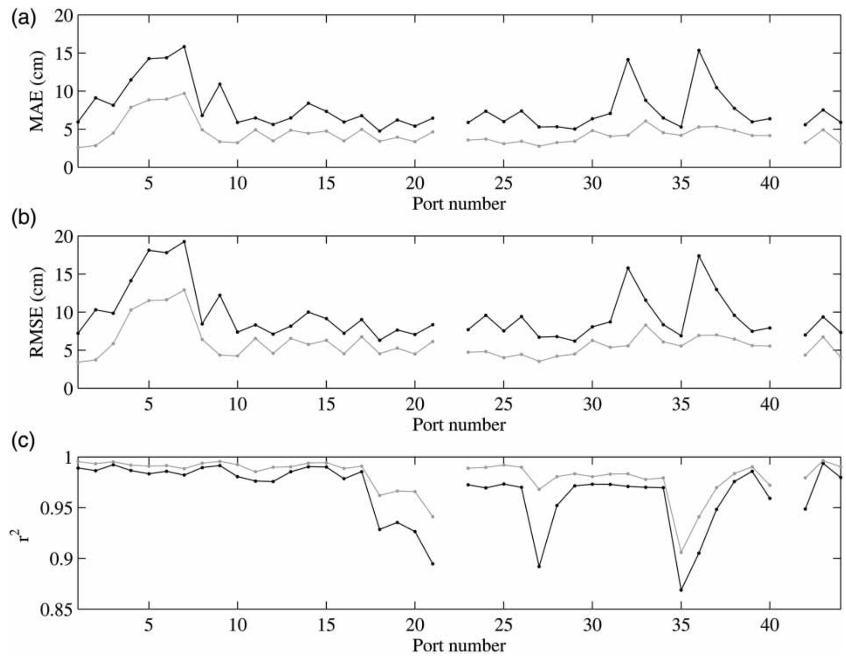

Figure 6. Comparison of (a) MAE, (b) RMSE, and (c) r^2 statistic of prediction errors using the EHM (black line) and the Empirical Correction Method (grey line) for 42 of the 44 UK Ports used in this study. Ports are numbered clockwise from the Southwest tip of the UK (see Figure 1).

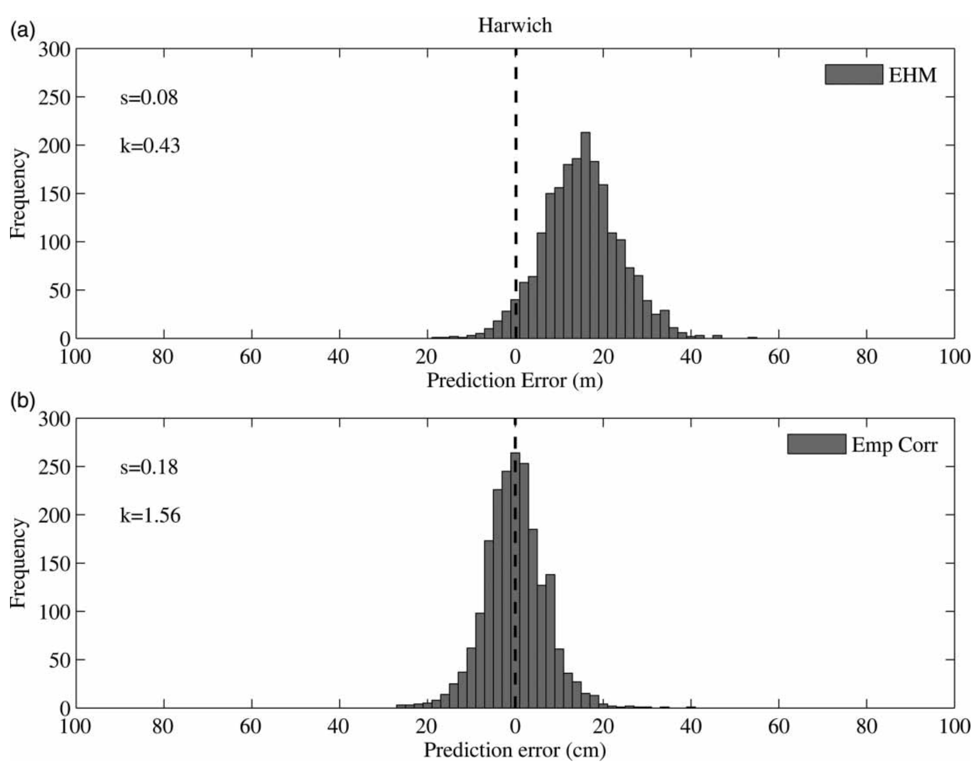

Figure 7. Histogram of prediction errors at HW at Harwich for 2006–2009 (inclusive) associated with (a) the EHM and (b) the Empirical Correction Method. The error bin interval is 2 cm. The skewness (s) and kurtosis (k) of each distribution are shown on the upper left of each panel, and the associated standard errors are 0.046 and 0.092, respectively. The mean value of each distribution is represented by the black dashed line.

low, showing that the predictions explain less of the observed variability than at most other locations.

Regardless of this apparent regional variability in the performance of the EHM, what clearly emerges from Figure 6(a)–6(c) is that the application of the Empirical Correction Method to the EHM predictions results in a noticeable improvement in the r^2 statistic at these locations and a reduction in prediction errors. Histograms of the error distributions associated with the two methods at each of these ports (see example for Harwich in Figure 7), show a general tendency of the EHM to underpredict HW, but the application of the Empirical Correction Method increases the Kurtosis and decreases the skewness of the error distribution in each case, reducing the disparity between observations and predictions.

Since the interactions of the lunar perigeal and nodical cycles induce an amplitude variation in tidal extremes over a period of ∼4.5 years, it was considered that the UKTGN Validation dataset duration of four years would be sufficient to capture the full extent of the tidal range at each site, thereby ensuring that the empirical correction coefficients would be pertinent to tidal predictions errors at other times. To verify this, the correction coefficients were validated by testing the correction algorithm at all 42 locations using EHM predictions and sea-level observations for 2010 and were found to reduce EHM prediction errors to a similar extent as was the case for the UKTGN Validation dataset covering the period 2006–2009.

In a further test, the Empirical Correction Method was applied solely to HW Springs using the UKTGN Validation dataset, but the results were not materially different to those presented here for all HW.

Conclusions

This study was motivated by the problems associated with classical harmonic methods of tidal prediction in shallow waters, particularly in estuaries like the Bristol Channel which experience a large tidal range and therefore commensurately large prediction errors. The minimization of such errors is of particular importance where these predictions are used in the context of coastal flood forecasting. Indeed, such forecasting will become increasingly demanding in terms of accuracy given that Mean Sea Level Rise will increase the risk of any given storm event.

Of the alternative methods tested, the Species Concordance Method was least successful, yielding larger prediction errors than the EHM – a result that may be attributed to the requirement by this technique for tidal observations from a nearby port that does not experience tidal distortion, since the large scale of the Bristol Channel means that only remote ports are unaffected by shallow water effects. In contrast, both the ANN model and the Empirical Correction Method offer considerable scope for improvement upon the classical EHM for certain forecast times into the future (and here we have focused upon operational forecast times). From this perspective, the two methods are comparable in that they both allow predictions to be made up to 24 h in advance.

At hourly intervals, the ANN model performed best, but when all four techniques were compared solely at HW, it offered less potential for improvement upon the harmonic method, which is currently adopted by the UKCMF partnership as the standard method of tidal prediction. From the perspective of operational flood forecasting, the accuracy of tidal predictions is of greatest importance at HW and this flaw in the ANN model is therefore a drawback to its use for these purposes. However, it is important to note that the ANN model results at HW were based upon subsampling of hourly predictions and not on the application of the model solely to HW intervals, which is an area for future study.

Clearly, the advantage of the ANN model is that it allows predictions to be made for the whole tidal curve and is not restricted to HW. Where such high frequency predictions are required, the ANN model may offer potential for improvement upon harmonic methods, if errors can be reduced at HW. It is certainly possible that experimentation with alternative model structures, model inputs and training algorithms might lead to improved predictions. For example, if use is made of additional input data such as residual water level and/or SLP datasets together with tidal predictions, this might enable the model to discriminate between tidal, atmospheric and other forcings and thereby improve the predictability of these components. In addition, consideration could be given to the use of recurrent inputs, i.e. using the predictions of the ANN Model as inputs to improve the convergence between predictions and observations.

Nevertheless, it is evident that for HW predictions, the relatively simple Empirical Correction Method is most successful, reducing prediction errors (RMSE and MAE) by between ∼2 and ∼10 cm at UKTGN locations. Since this technique can be considered an enhancement to the present method of tidal prediction used by the UKCMF partnership, it is relatively simple and quick to implement, which is clearly of benefit given the recent UK incidence of coincident storm surges and HW during winter of 2013/2014. Evidently, the technique cannot be applied everywhere since the identification and indexing of alternate HW is problematic for locations that experience multiple High and Low Waters. However, it should be noted that the disparity between tidal predictions and observations was reduced at all remaining locations for which the technique was tested, irrespective of whether they experienced strong tidal distortion. Thus, the Empirical Correction Method offers the potential to improve HW prediction at many locations and has advantages in that it is both simple and remarkably effective in reducing tidal prediction error. Consequently, it is to be incorporated into the operational systems of the UKCMF Partnership in order to improve short-term sea level predictions for the UK and in particular, the accurate estimation of HW extremes.

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