Topography of Generically Folded Spacecapes[1]

Towards a Cognitive Metatheory in Architectural Design

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General Motivation of the Project

If we visualize philosophy as a science of the sciences (in the tradition of Hans Heinz Holz), then theories within the philosophical framework are always *meta-theories* of the respective field of interest in question with which philosophy is actually dealing. Hence, a philosophy of architecture in particular, shows up as a meta-theory of theories of architecture with respect to all the possible conditions that are necessary to actually constitute its characteristic field of interest (including the social modes of communication). As we can easily see, it is also a *cognitive* meta-theory in the first place (i.e. one of the human condition). If then, traditionally, philosophy can also be visualized as being primarily constituted by four components that turn to its topic, namely by means of historical analysis, scientific approach, aesthetical and ethical conceptualizations, respectively, we can recognize that the second and the third overlap somehow with

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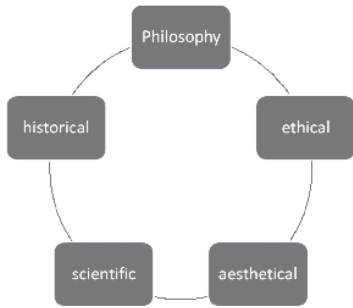
the architectural activities. This is certainly true since the establishment of architecture as a field of activity of its own, separated from building and engineering, as was achieved in the epoch of the Italian Renaissance by Brunelleschi, Alberti, and others.

But note that in this case, the type of conceptual (and in fact, methodological) overlapping also provides a general perspective of interdisciplinary kind in so far as «scientific» refers here explicitly to all the relevant sciences, including the sciences of nature which are usually formulated in mathematical terms. Obviously, as architecture has a lot to do with geometrical forms and shapes, there is a straightforward relationship to mathematics from the beginning on.

Nevertheless, the actual starting point refers to the field of the *social* sciences instead: To be more precise, we begin with a notion of *hodological space* as it has been introduced by Jean-Paul Sartre in his first main work on social philosophy, *L'Etre et le néant* of 1943, where he utilizes this concept to discuss Proust's famous novel *In Search of Lost Time*. The concept itself is going back to the Berlin school of Gestalt psychology of Lewin *et al.* who published their ideas in Berlin and New York between 1922 and 1936.

Another philosopher who has discussed this concept in a different setting is Walter Benjamin, especially in his 1932 radio essays on the *Berlinian Childhood around 1900*. Essentially, hodological space, as space of those paths in two-dimensional geographical space that carry an explicitly social connotation for an individual person (such that this space is a sub-space of geographical space) is the arena of the unfolding of a biography. It has thus an explicitly narrative form. This is the reason for talking of «topo-graphy» here, namely of a composition of «topology» and «graphism» which is so characteristic for human life.

Walter Benjamin tried to write a kind of social history of modern city architecture by concentrating on what he called «capital of the 19th century», referring to Paris. In his book on the *Arcades Project* which was not published (as a torso by then) before 1982 he also introduces the concept of the «flaneur», thus taking the position of a genuinely pedestrian perspective onto urban behaviour, originally inspired in fact, by Friedrich Engels and his essay: *The Condition of the Working Class in England* of 1845. This topic was later re-introduced by Michel de Certeau in his book: The Practice of Everyday Life. (1984) We have used this as an entry to the field when discussing the emergence of the *flaneur* in a more formal manner in an earlier essay on combining hermeneutic with formal techniques referring to self-organizing processes: «The Transition from Town to City.» (1986) For a more recent example of Benjamin's «private city map» associated with a personal hodological space we have chosen a living quarter in Berlin, the so-called «Schoeneberg Island», part of the former district no. 11 consisting of roughly two square kilometres with about 13.000 inhabitants (by the way, coincidentally comparable to the city size of the ancient *polis*, e.g. in the case of classical Athens).

In the picture, the blue triangle indicates the shape of what is called «island» here, but is not really an island in the strict sense. This is because its boundary is outlined by three lines of the city railway (the «Wannsee Bahn», the line to Teltow, and the circle line which closes the triangle at its base). The orange line depicts the Federal Street no. 1 [B 1] (almost vertical on the left-hand side leading from the eastern boundary of Berlin to the far south where on the Glienicke bridge (= Bridge of Spies) the B 1 passes into the city of Potsdam), while it is continued to the East (horizontal) by Kolonnen-Street, an ancient axis leading forward to the former city airport of Tempelhof which can be spotted on the far right-hand-side. In the period of Schoenberg being a district of West

Berlin, these two streets gained an explicitly political connotation, because they also outlined the route of any international visitor to the city who came from Tempelhof airport on the way to the West Berlin city hall (far left-hand side) which was the residence then of the city's Governing Mayor. Hence, all the people living in one of the houses flanking this route, felt quite realistically the political impact of their biographical situation, especially at the occasion of prominent visits (by Queen Elizabeth or John F. Kennedy as to that). In the same sense, one could find more historical connotations when thinking of the B 1 as the former «Reichsstrasse 1» leading from Koenigsberg in East Prussia up to Aachen in the West of the country – being a kind of *synecdoche* of Prussian history with all its ups and downs over the centuries including its final catastrophe.

Different from this public context (which operates very much on the level of *chronotopoi* as introduced by Bachtin), there is also a completely private context that is relevant for individual persons living in this quarter. This refers to Benjamin's *private city map* as being the set of all relevant paths utilized by a pedestrian day by day: While passing through this hodological space, the characteristic morphology of the buildings, the flow characteristics of the street traffic, the implicit and explicit communication within the ruling discourse of the people around, all of this is loaded with social connotations due to the characteristic use the mentioned person makes of streets and buildings. For children this is particularly relevant and acts as an imprint onto their personality in the long run: When they have grown up much later, even when they went away to other places in the meantime, these first impressions still form their behaviour, e.g. by choosing a place to live in another city, because the location carries similar connotations as the early environment of their birth place. (Not necessarily will they realize this kind of choice.) The following diagram displays such a network of characteristic paths for a person within a

given interval of time (here: 1960 through 1970). We can recognize the island area again and the city airport on the right-hand side. The depicted hodological space is roughly that of one of the author's (R.E.Z.).

Obviously, for later periods, the shape of this hodological space is changing due to biographic events (choosing a larger flat, children going to school, and all that). By depicting the progressive evolution of the hodological space in question, one can gain a topological impression of a life. This is what Walter Benjamin pointed at in the first place.

Of course, this pedestrian viewpoint of urban living quarters has been discussed earlier within the field of urban architecture: Sigfried Giedion, in his book: Space, Time, and Architecture (1938-1939), points to similar aspects. He defines thus a city as the expression of the diversity of social relationships which have become fused into a single organism. (41) And it is from here where the concept of networks and the organic style of architecture come into play. A particular point to this was the declaration of the *Freedom for the Pedestrian* in the 1951 CIAM conference taking place in London. (Giedion, 842, 702) In Spain, Antoni Gaudí can be visualized as the originator of what is called *organic development* (particularly innovative in the case of Barcelona's *Guell Park*, developed from 1900 through 1914). Remember that the concept of urbanization itself has been invented in this very city (as *Urbanización* introduced 1867 by Ildefons Cerdà i Sunyer who was the architect of the Eixample district). Hence, architecture shows up here within its relation with the network of interacting agents in urban social space. Architecture is thus *folding the space(cape)*. Why *folding* rather than *filling*? Because space is not a container that would be filled with objects. Instead: It is a set of relations among objects. Consequently, if objects are constructed and produced, then a set of relations emerges, and each relation is essentially the representation of an edge
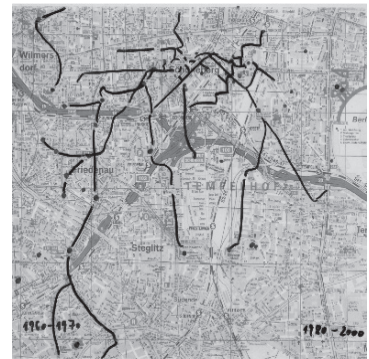
of a network of interacting agents. (From here we can recognize the straightforward connection to systems.) In particular, architectural objects are actually inserted into a bundle of relations that is already present. This is what urban planning is all about in the first place.

Further contextual work on this topic has been provided, among others, by Henri Lefebvre in his 1974 book: *La production de l'espace*. Here he classifies the various *modi* of the social production of space and its interaction with the ruling discourse. This is also topical in Pierre Bourdieu: *Le sens pratique* of 1980. Bourdieu introduces the concept of *habitus* visualized as a product of relations in social space, where the important point is that as a rule, the Logic of Praxis is actually different from the Logic of Logic. Bourdieu employs what is called the structuralistic method (as derived from Lévi-Strauss); and the action of symbolic (especially linguistic) power is very crucial within this context that prescribes «to characterize each element by its relations which it entertains to other elements of a system». Hence, we have used this for developing what we call a *topology of communication*, essentially equivalent to an intrinsic *logic of space*. This implies a suitable formalization by qualitative mathematics, as has been shown earlier in the Bologna papers (2001, 2003, 2004) published together with Anna Soci and Giorgio Colacchio, and in a recent book of one of us (R.E.Z.): (Berlin, 2014) [in German] as well as in Metaphysics of Emergence (part 1) as of 2015.

2. Reference Work as a Starting Point

It is necessary now to list three items of reference work relevant for this ongoing project and very central for our special approach to urban space: The first one is that of Bill Hillier from University College London, as published in the two books by Hillier himself and Julienne

Hanson: *The Social Logic of Space* (1984), and by Hillier alone: *Space is the Machine* (1999). Essentially, the basic idea is here to introduce what is called space syntax dealing with particular spatial configurations such as the axiality, convexity, and permeability of spatial networks.

The second is the work of Michael Leyton (now at Rutgers, NJ) on what he calls a generative theory of shape (2001). As it turns out, space can be associated with an explicit process grammar that furnishes the basis of spatial morphology (2012). One of the main results is the so-called «Symmetry-Curvature Duality Theorem» (1987) which states that to every curvature extremum, there is a symmetry axis leading to, and terminating at, this extremum. Michael Leyton can thus demonstrate the significance of asymmetries (in terms of a generic symmetry breaking): hence, curvature variation (i.e. foldings) can be utilized as a memory storage in the process grammar. In other words: Architectural shape is equivalent to the memory that consists in the uncovering of the single components in the process of generating itself in the first place. Shape is frozen memory or storage of information. The various symmetry axes as observable in the present are visualized thus as the directions along which past processes are most likely to have acted.

The third approach is provided by Richard Buckminster Fuller in what he calls synergetics: See his book on the «Explorations in the Geometry of Thinking» as of 1975. Here, synergy is visualized as describing the behaviour of whole systems unpredicted by the behaviour of their parts taken separately (essentially, this is nothing else than what we call emergence). Synergy is represented most clearly in terms of what Buckminster Fuller also calls tensegrity: understood as a special type of tensional or geodesic integrity. His successor Norman Robert Foster is still conceptualizing architecture in his
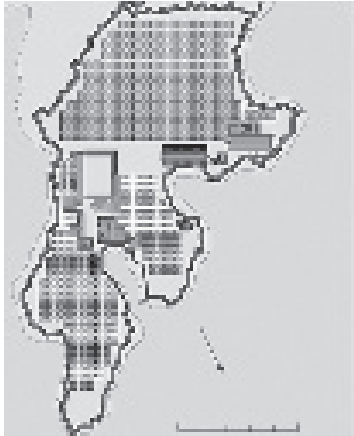
own works very much in the sense of Buckminster Fuller whom Chu Hsiao-Yun called a comprehensive anticipatory design scientist (in: Chu Hsiao-Yun, Roberto G. Trujillo, eds., New Views on R. Buckminster Fuller, Stanford UP, 2009, 99-124. See also the contribution of Jonathan Massey: Fuller's Sumptuary Aesthetic.)

Within all of these approaches the relevance of mathematics becomes readily apparent. This is also true for the arts (see e.g. Salvador Dalí with his tesseract forms in the crucifixion scenes). Hence, the idea of introducing design science as a kind of meta-science, namely as visualized in terms of the modeling of modeling. (Our institute is referring directly to this aspect and has been founded to put forward related conceptions: www.designscience.de.) The methodology utilized by projects of this institute, especially within the research project «language & space» of which the section «urban social space as emergent complex system» is a part, refers also to scientific work of Ilya Prigogine and his school (self-organization, formation of structure, chaos), as well as to the Santa Fe school as represented by Stuart Kauffman (self-organized criticality), and to the work of Louis Kauffman (knot theory).

3. Architecture and Urban Social Space

Coming now to the applications derived from the above-mentioned approaches, we find the following: The archetypal space structures in urban settlements go back to Mesopotamia and Greece, namely to the city of Uruk (3500-2800 BC) which can be confronted with later urban conceptions as introduced in the city of Miletos (479 BC) by the architect Hippodamos. While Uruk presents itself as a straightforward generalization and extension of palace architecture, Miletos shows up as a study in geometry with a characteristic checkerboard structure:

The important difference between the feudal urbanization of Uruk and the (pre-)democratic urbanization of Miletos is in the function of two centres of public space: in the market place (agorá), here visualized from a re-construction of the Berlin Pergamon museum,and in the theatre (theátron) – both of these forming a polar tension within which city life unfolded characteristically.

Note that an elementary rule of geometry, called *golden section*, is extremely relevant here. In fact, as it turns out, it is relevant for most aspects of geometry observable within everyday life: This can be seen by starting from complex numbers: Given the Möbius transform (which is a bijective conformal map of the Riemann sphere plus infinity: $C^* = C \cup \{\infty\}$), then it can be shown that it establishes a generic isomorphism between the group of automorphic mappings and the projective group of special linear matrices: Aut $C^* \approx$ PSL (2, C). A subgroup of this is actually the modular group of fractal geometries which is defining the self-similar symmetries within the larger group of generally linear matrices with n real entries GL(n, R). Again, a special case of this is the well-known Fibonacci group of maps defined by the number series (0, 1, 1, 2 , 3, 5, 8, 13, ...) such that lim(n $\rightarrow \infty$) f(n+1)/f(n) = $\Phi \approx$ 1.618 [...].

Alternatively, by the common classical approach (which we can use as a thumb rule), we have the following: Given some distance x + y, then the golden section can be formed by the relationship (x + y) / x = x / y $\rightarrow x^2 - x\,y - y^2 = 0$. If y = 1 $\rightarrow x^2 - x - 1 = 0$. For this case, we readily find two solutions $x_{1,2} = \frac{1}{2} \pm \sqrt{(1/4 + 1)} = 0.5 \pm 1.118 = (1.618, -0.618)$. Because we deal with distances, we can neglect the negative solution, and we are left with the ratio of the golden section again. For the usual plane, this can depicted as in the following:
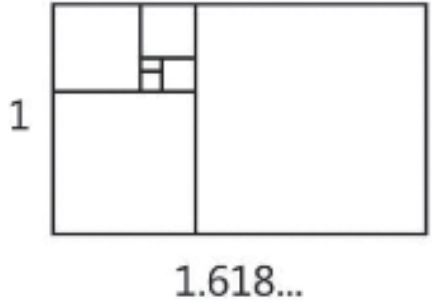
This becomes more complex, if applying it to what we call *biomorphism* in architecture. For this, Antoni Gaudí's organic architecture can be taken as an example. Beyond too simple structures of the Golden Section this leads straightforwardly towards fractal patterns which are relevant for chaos theory. In fact, the Hippodamian pattern can be visualized as an especially simple case of an overall *generically* harmonic whole. In other words: Here, the concept of genericity shows up as an intrinsically cognitive property of human beings rather than as a property of natural forms and shapes. This can be quickly recognized when depicting the branching of a tree in a computer-generated manner:

In the abstract visualization of computer graphics, this can be mapped as an isomorphic shape of the following type:
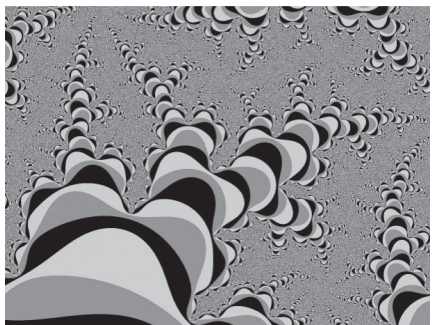
Hence, it is chiefly the cognitive impression plus the formal conceptualization that results in the qualitative mathematics. In particular, the tree-structure indicated above can be readily identified with the fractal approach of Gaudì's when looking at the interior of the «Sagrada Familia». We progress here in the strict sense of *topo-graphy*.

This also implies a very useful isomorphism to (mathematical) categories: A category C can be defined thus as a class of *objects* ob (C) and a class of *morphisms* hom (C), with unique source object a, and target object b, a, b $\in$ ob (C), f: a ® b. Then, Hom (a, b) is said to be the class of all morphisms. Also, for every three objects a, b, c, there is a binary operation called *composition* such that Hom (a, b) x hom (b, c) $\rightarrow$ hom (a, c); g o f, f: a $\rightarrow$ b, g: b $\rightarrow$ c, and such that the law of associativity is valid: if f, g, h, then h o (g o f) = (h o g) o f, and also the law of identity is valid: for every object x, there is a morphism 1(x): x $\rightarrow$ x such that for every other morphism f: a $\rightarrow$ x and g: x $\rightarrow$ b, 1(x) o f = f and g o 1(x) = g.

Then a category is called *small*, if both ob (C) and hom (C) are sets. Otherwise it is called large. It is said to be locally small, if for all a, b: hom (a, b) is a set called *homset*. In particular, we call Cat the category of small categories and their functors (which is itself a large category, so that there is no formal paradox). And this is also a 2-category. Note that in particular, a *topos* is a category plus an additional logical structure (where we replace the common Boolean algebra by a more general Heyting algebra). The idea is then as follows: «A topos acts like a 'Lindenbaum algebra' for a logical theory whose models are the points of a space.» (Steve Vickers, 2007) For social spaces, we formulate the subsequent conjecture: «A *weak* topos acts like that for a pre-scientific, everyday (weak) theory whose models are the points of *social* space.»

First of all, we realize the *isomorphism between catego-ries and mathematical graphs*: A *network* as represented in terms of a graph is also isomorphic to a category, where the objects are the vertices of the graph (or knots of the network, respectively), and the morphisms are the edges of the graph (or links of the network). This is a simple consequence of the «Rosetta stone results» origi-nally presented by John C. Baez and Mike Stay in their paper: «Physics, Topology, Logic, and Computation: A Rosetta Stone.» (2009) These results can be summa-rized in the following table:

Categories	Physics	Topology	Logic	Computation
object	Hilbert space	manifold	proposition	data type
morphism	operator	cobordism	proof	program
tensor product of objects	space of joint systems	disjoint union of manifolds	conjunction	product
tensor product of morphisms	parallel processes	disjoint union of cobordisms	parallel proofs	parallel programs
internal homo-morphisms	space of anti-X and Y	disjoint union of orientation reversed X, Y	conditional proposition	function type

This can be generalized to other fields of research such as:

Game theory	Social games	Social space
position (utility space)	pro-position	pro-position
strategies (rules)	communication	strategic interac-tions (linguistic)
product of moves	sequence	sequence
communication =	strategic mapping of pro-positions to propositions	translations

4. Structurally Stable Regions
of Communication

What is the practical use now of all of this? Remember that we visualize urban social space as a state space under the influence of a *polarized tension*. This is an idea going back to Richard Sennett: With Sennett we characterize this tension in terms of the afore-mentioned intrinsic polarity of the regions of the *market* (agorá) and the *theatre* (theátron). (Zimmermann, 2014, 2015[4]) So we have the market (agorá) on the one hand: signifying places of variety, flexibility, change of perspectives, and of chaotic forms of organized discourse (i.e. taking difference in sight). We have the theatre (theátron) on the other hand: referring to places of concentration, reflexion, and ordered forms of organized discourse (i.e. deciding on the background of difference by creating identity).[5]

Important is here the fundamental assumption that forms of conflict, competition, and dispute cannot be actually absent from a social system: Instead, it is the permanent mediation of *stásis* (interior conflict) and *pólemos* (exterior conflict) in the sense of Massimo Cacciari's approach to modern geo-philosophy[6] that has to be taken into account representing the metaphor introduced once by Heraclitos in his example of stirring the *kykewn*. Hence, the structure of the social system visualized as a set of social groups is basically one that is essentially governed by competing subsets. Consequently, the social system is a system of systems.

It is straightforward then to model social interaction by diagrams derived from the isomorphism between

4. Cf. for details:
   http://www.ocg.at/sites/ocg.at/files/sozverm_06062014.pdf
5. Richard Sennett: The Spaces of Democracy. In: Robert A. Beauregard, Sophie Body-Gendrot (eds.), The Urban Moment. Sage, Thousand Oaks, London, New Delhi, 1999, 273-285.
6. Geophilosophia dell'Europa, 1994.

systems and mathematical categories. This can be achieved by referring back to the terminology of chemical reaction networks: If we choose a general network with interacting species A, B, C, D, E and their complexes, of the form:

$$A \longrightarrow B,$$

$$B \longrightarrow A,$$

$$A + C \longrightarrow D$$

$$B + E \longrightarrow A + C$$

$$B + E \longrightarrow D$$
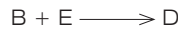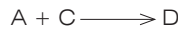
then we can treat the species as groups of persons, and the complexes as groups of groups in the sense of appropriate coalitions. The hyper-complexes are treated accordingly. Now, the region in social space which is described such that within it all available groups can interact with minimal losses whilst conserving the structure of the social space itself is called *harmonic*. It is thus *structurally stable*, if the social process remains in that region for substantial time intervals. Alternatively, the situation can be represented in diagrammatic form in terms of an appropriate Petri net of the following type:
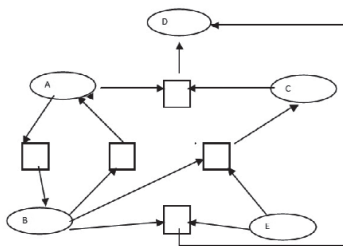
In the technical jargon, this afore-mentioned region of stability is called stoichiometric subspace (of the complete space of possible interactions): This is the space of free play within the boundary conditions of the given system pre-defining the set of possible interactions, usually called Stoch $\subseteq R^s$, such that for any solution of the social process X(t), we have X(t) $\in$ X(0) $\pm$ Stoch for t $\rightarrow$ ∞. The structural stability of Stoch is thus *robust* in the sense that every solution that starts within it, also ends within it. In fact, to actually determine this region is all what we can do with a view to the ongoing

social process (rather than trying to steer this process in minute detail.)

5. Example from the Past

In order to clarify somewhat the range of possible applications of the theoretical aspects mentioned above, we discuss here an example from the time between World Wars I and II: As a consequence of the treaty of Lausanne following the war between Greece and Turkey, in 1923, the disentanglement of Greek and Turkish communities was undertaken resulting in roughly 1.2 million Greek refugees arriving at the Greek mainland. About half of them settled between Athens and Piraeus. A prominent location here is a place called Nea Kokkiniá (which is Nikaia nowadays). The main periods of refugee settlement at the time can be classified according to the three stages of the building process constructing a completely new region, in fact.

The following pictures display the regions from where the refugees originated (upper and lower regions on the right-hand-side), and the new settlement area in the south of Athens.

As to the geometrical structure of the settlement, we note a pattern that is strongly reminiscent of the Hippodamian structure mentioned earlier.

Looking at this structure more closely, noting also the interior space of the respective building blocks, we can easily verify an explicitly fractal structure.

The important point is that during the three main stages of the building time, from the beginning on, stone houses are erected according to the given layout of the original scheme. In other words: Intermediate stages of erecting tents or similar temporary structures persist
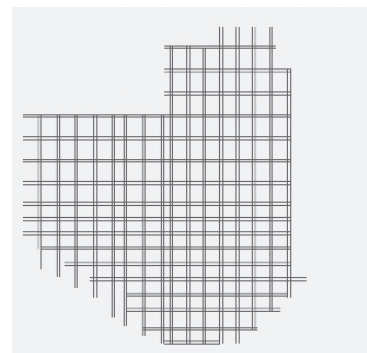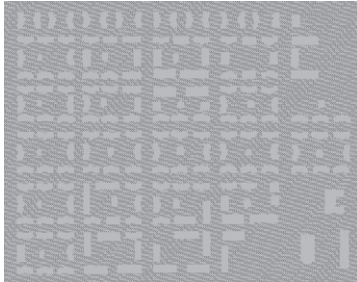
for only a very short time, but then regular buildings are constructed. If we compare this with the present situation of refugees which is topical in many political controversies in countries of the EU, we notice that the emphasis in the past had been laid on concepts of permanence and endurance rather than on artificial tentativeness and temporary arrangements.[7] In the following the respective stages are depicted as examples:

What are the chief characteristics of these formations? We note first of all a dense network of communal open spaces, of connecting courtyards, streets, shared staircases, narrow corridors, and wooden roofs. Hence, the structures concentrated on the social connectivity in terms of a mediation among encounter, festivities, and play. At the same time, the social dynamics was governed by a collection of *intrinsic dualities* reflecting the dialectics of a variety of living styles in terms of an essentially topological layout:

limited interior – ample exterior
closed spaces – open spaces
private spaces – shared spaces
family - neighbourhood

The long-term result is the evolution of a structurally stable region within the urban area of Athens and living styles that cannot be called otherwise than «integrated».

7. The argument put forward today very often, that in the 1920s this concerned immigration of Greeks into Greece and not people from different cultural backgrounds is not quite cogent after all, because on the one hand, there was indeed a cultural difference despite the formal affiliation to the same country, and on the other hand, refugees from countries with different cultural background have been integrated already at another occasion by Germany some time later – namely in the case of Vietnam's so-called «boat people» in the late 1970s and early 1980s.

6. Preliminary Conclusion

The important insight we gain by this kind of approach is that ethics comes prominently into play here whilst discussing primarily scientific as well as aesthetical topics: This implies that it is a kind of concrete normativity that serves as a catalogue of criteria for adequacy. Indeed, if we remember the conceptual origin of the modern interpretation of normativity, we are referred back to an ancient Stoic Principle of adequacy which is contained in the concept of *kátà physin* which means «according to nature» (or to be more precise: according to human nature or the anthropological condition). Hence, to visualize architecture as a folding of the spacecape means also to visualize its intrinsic aesthetics as expression of the cognitive as well as communicative capacity of human beings who actually construct their social space. So that it is mainly the explicitly *spatial mode of reflexion* that serves as the basis for architectural design after all. And we find what we can call *genericity* (of this design) in the *fractal* structure of space which at the same time incorporates the likewise cognitive aspects of ongoing competition: as to *stability vs. instability,* or *symmetric harmony vs. deformation* (entársis). Hence, we find the above-mentioned isomorphism between the representation of social space and that of urban architectural space. And the spatial structure is always discursive!