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**TITLE:**

An expert system to minimize operational costs in mixed-model sequencing problems with activity factor

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**ABSTRACT:**

One of the major issues in industrial environments is currently maximizing productivity while reducing manufacturing cost. This can be seen clearly reflected in mixed-model assembly lines based systems, where obtaining efficient manufacturing sequences is a key to be competitive in a dynamic and globalized market. However, this continuous cost reduction and productivity growth should not penalize the welfare of employees. This work is intended to address this lack of compatibility between the economic and social objectives through the study of the mixed-model sequencing problem from both the business and labor perspective. This is done by considering the possibility of reducing or increasing processing times of operations by varying the work pace of line's operators within the permissible legal boundaries. Thus, depending on this flexible activation time of operators, the amount of completed work and idle time will be one or the other and, consequently, the productivity of the line will also improve or get worse. In this regard, we propose new approach to the sequencing problem without incurring cost increases and providing a safe working environment, in accordance with applicable law. This new approach leads to obtain efficient manufacturing sequences, in terms of both productivity and labor conditions. Specifically, the objective of the new problem is minimizing the unproductive costs of the line by incorporating the possibility of increasing production through the variation of the work pace of line's operators. Increasing the work pace of operators, the amount of non-completed work or the preventable idle time can be reduced and therefore, their associated costs too. In addition, and without losing sight of the effort involved in working with a work pace above the normal, we propose several economic criteria to compensate the activation of workers where necessary.

**KEYWORDS:**

Activity factor; Idle time; Economic compensation; Mixed-model assembly lines; Sequencing; Work overload; Work pace.

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## **An expert system to minimize operational costs in mixed-model sequencing problems with activity factor**

One of the major issues in industrial environments is currently maximizing productivity while reducing manufacturing cost. This can be seen clearly reflected in mixed-model assembly lines based systems, where obtaining efficient manufacturing sequences is a key to be competitive in a dynamic and globalized market. However, this continuous cost reduction and productivity growth should not penalize the welfare of employees. This work is intended to address this lack of compatibility between the economic and social objectives through the study of the mixed-model sequencing problem from both the business and labor perspective. This is done by considering the possibility of reducing or increasing processing times of operations by varying the work pace of line's operators within the permissible legal boundaries. Thus, depending on this flexible activation time of operators, the amount of completed work and idle time will be one or the other and, consequently, the productivity of the line will also improve or get worse. In this regard, we propose new approach to the sequencing problem without incurring cost increases and providing a safe working environment, in accordance with applicable law. This new approach leads to obtain efficient manufacturing sequences, in terms of both productivity and labor conditions. Specifically, the objective of the new problem is minimizing the unproductive costs of the line by incorporating the possibility of increasing production through the variation of the work pace of line's operators. Increasing the work pace of operators, the amount of non-completed work or the preventable idle time can be reduced and therefore, their associated costs too. In addition, and without losing sight of the effort involved in working with a work pace above the normal, we propose several economic criteria to compensate the activation of workers where necessary.

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## 1. Introduction

Since the first assembly line, the automobile industry has undergone a constant evolution. This evolution has not based only on production methods but also on management ideologies. The mass production and the increase of flexibility thanks to the Toyota Method together with the Just in Time ideology (JIT), and the synchronous manufacturing thanks to the Douki-Seisan philosophy (DS), are some of the innovations that the sector has suffered in the last 100 years.

All these changes have shared the same basic objective: offering a wide range of products while reducing costs and increasing productivity.

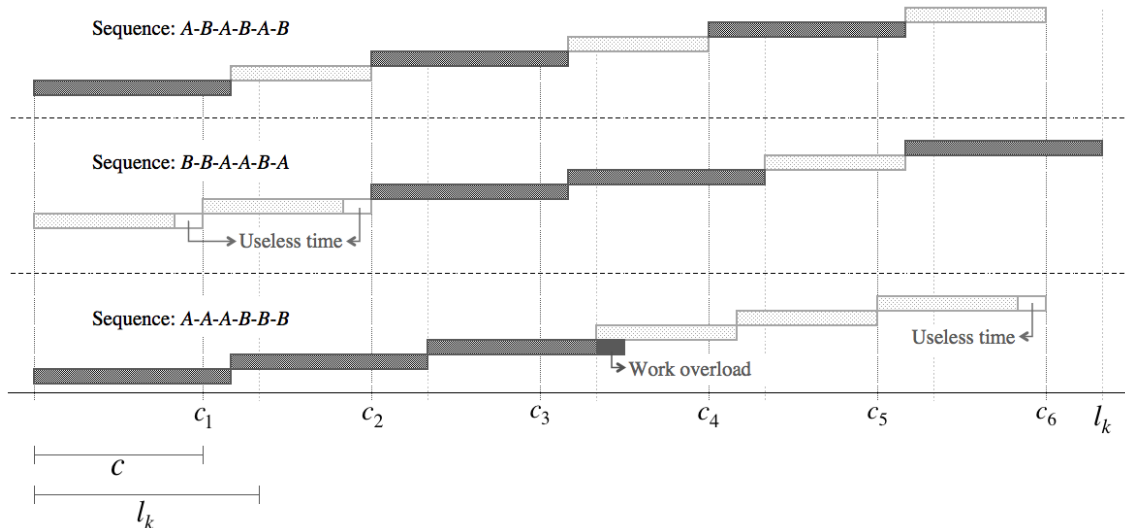
Even today, this objective continues to govern any improvement in production and management systems where flexibility is an essential requirement.

A clear example is the Mixed-Model Sequencing Problem (MMSP) in assembly lines, whose objective is to determine the best sequence of products in terms of some productive criterion, such as the amount of completed work or the idle time of the line. While it is true that almost any manufacturing sequence is technically feasible, differences between product types lead to not all sequences have the same economic impact. Indeed, workload distributions and the consumption of components will be one or another depending on the sequence (Boysen et al., 2009).

As an example (see Figure 1), sequencing product units with high workload, consecutively, might make workstations require a greater time the cycle time,  $c$ , to complete all workload. If this extra time is not available, workstations might not complete the work and generate work overload. This may occurs even though there is the time window,  $l_k$ , which allows an extra time ( $l_k - c \geq 0$ ) to complete the work on a product. Obviously, the consumption of this time by any workstation supposes the reduction of the available working time for the next product at the said station and for the said product at the next station.

On the opposite side, there is the idle time that appears when units with low workload are consecutively sequenced. In this case, processors of workstations might finish the work before the cycle time ends and, therefore, they have to wait for the next unit.

Fig.1. Impact of the sequence on the work overload and idle time or useless time.



Both situations, work overload and idle time, involve an extra cost for the assembly line. The first generated by the loss of non-completed products and the second one by the inefficiency of the line. Accordingly, reduction of both work overload and idle time has been the focus of numerous sequencing problems in literature (Yano and Rachamadugu, 1991; Scholl et al., 1998; Bautista et al., 2018).

Obtaining sequences with null work overload and idle time is obviously the ideal situation. However, this situation is hardly achievable due to characteristics of assembly lines and production mixes. Nevertheless, both negative factors may be minimized simultaneously by considering variable work pace of operators and, therefore, the unproductive cost may be reduced.

Processors of workstations are operators; therefore, they can work more or less quickly at certain times of the workday by means of the variation of their work pace. In this way, operators will be able to complete more or less workload in accordance with needs of products.

Accordingly, we propose, in this paper, a new variant for the MMSP, whose objective aims at minimizing the unproductive costs. The proposal addresses simultaneously the minimization of the work overload and the minimization of the idle time through the assessment of their costs. Thereby, the efficiency of the line and the amount of completed work will increase and, therefore, manufacturing sequences with the lowest possible cost in terms of loss of production will be obtained.

This minimization of unproductive costs is achieved by the relaxation of the determinism of processing times of operations that will vary in regard with the work pace of operators.

Specifically, we incorporate the flexibility concept into the processing times of operations. This flexibility is subject to the workers of the line, those who must regulate their work pace, within the legal limits, in order to reduce or increase the processing times and so reduce work overload concentrations or the idle time, respectively.

Contrary to other works on this topic (Bautista et al., 2015a,b), this paper does not consider a synchronized work pace for all workstations of the line.

The studied variant allows unsynchronized workstations. Processors will be able to modulate their work pace to their production needs, independently of the work pace of processors of other workstations. This ensures that no processor works more quickly if not necessary, thus avoiding unnecessary efforts.

Obviously, in order to ensure security and to prevent worker injuries, this flexible activation is subject to the legal limitations for the activation.

Furthermore, considering the effort that supposes the increase in work pace to workers, we also define different compensations metrics to ensure a good working environment in terms of job satisfaction. Specifically, we define different ways to compensate economically the excess effort of operators when they work at faster pace than the normal rate, in order to complete their workload and to avoid production loss or work overload.

Accordingly, the problem proposed in this paper does not only considers economic gains for the company, but also takes into account workers' rights and suggests compensating operators' effort from the company's gains, either through premiums or extra payments.

Briefly, the main originalities of our research are (i) the minimization of operational costs by work overloads and idle time in mixed-model sequences, (ii) the bounded but flexible activation of operators of assembly line in order to improve productivity, (iii) the economic compensation of excess effort of operators, (iv) the economic gains for the company and operators because of the recovery of production drop, and (v) the computational experience linked to the Nissan's powertrain plant in Barcelona.

Having regard to the above, this paper is organized as follows: section 2 describes briefly reference models for this research, and then, presents two mathematical models for the proposed variant of the MMSP. In section 3, we define the metrics to calculate the economic compensation of the excess effort of processors by increasing their activity factor and we expound some properties of these metrics. Section 4 addresses the computational experience and the result analysis, comparing the results of the proposed

models with the reference ones through a case study linked with Nissan's Engine Plant; and finally, section 5 shows the conclusions of this research.

## **2. Minimizing the costs of idle time and work overload**

Flexibility is a key feature in industrial environments where varying models of a common base product must be assembled or manufactured. However, this desire to implement mass customization leads to a large complexity arising from product variety on assembly lines. Indeed, differences between product requirements, such as different use of resources and component consumption, may result in productivity and quality losses, which can then be translated into economic losses.

Because of this issue and focusing on the automotive sector, next we present a small literature review on mixed-model sequencing and, specifically, on the MMSP. Finally, we present the reference models from literature and the variant proposed for the MMSP with the objective of obtaining manufacturing sequences that minimize the detrimental effects of product variety.

### **2.1. Preliminaries**

In literature, there are many variants of sequencing problems in regard with the optimization criteria. A classification of these criteria can be the following: (o.1) to maximize the productive time by completing the maximum number of product units and reducing the idle time (Yano and Rachamadugu, 1991); (o.2) to maximize the satisfaction degree of a set of constraints related to special components of products (Parrello et al., 1986; Siala et al., 2015; Bautista-Valhondo, 2016); (o.3) to maintain, as constant as possible, the rate of production for the different product types (Miltenburg, 1989) and the rate of component consumption (Aigbedo and Monden, 1997; Aigbedo, 2009; Monden, 2011) in order to reduce the maximum stock levels, and to assess the impact of maintaining constant the product manufacturing rate concerning the consumption components rate (Bautista et al., 2013).

Since the first work by Thomopoulos (1967), Macaskill (1973) and Okamura and Yamashina (1979), the variety of perceptions that exist on the controllable factors and production policies are shown. Specifically, in these researches it is possible to appreciate the range of criteria to define objectives: the minimization of costs of inefficiency, such as the idle time, the extra-effort, the workload concentration, the utility work minimization, the length of the line, are some examples. However, all of

these criteria pursue the same fundamental objective that is to maximize the line efficiency and the cost reduction.

Many authors formulate objective functions that are associated with some of the above optimization criteria but with an economic approach. Among them, Sarkera and Panb (1998) and Fattahi and Salehi (2009) introduced the minimization of the costs from the idle and utility times considering launch intervals. Giard and Jeunet (2010) formulated a cost function associated with hiring temporary/utility workers to avoid line stoppages and with setup times. Lin and Chu (2013, 2014) minimized the total cost, considering labour, warehouse capacity and order fulfilment rates. Avni and Tamir (2016) studied how to share costs between machines using game theory in scheduling problems.

Nevertheless, there are not many works, in literature, that consider human aspects in order to improve the line efficiency. Bautista et al., (2015a,b) formulated two equivalent models,  $M3 \cup 4_{\dot{\alpha}I}$  and  $M4 \cup 3_{\dot{\alpha}I}$ , whose objective is to minimize the overall work overload of the line or maximizing the completed work.

To achieve this objective, authors promote increasing the line productivity by means of the modulated activation of workers of workstations. Specifically, the authors relied on increasing the work pace of workers according to linear function consistent with the periods of adaptation, routine and fatigue typical of the workday. In contrast to these studies, this work relies on allowing workers a free activation in regard with the production needs.

The functions used in previous works (Bautista et al., 2015a,b) fixed the activity level of operators following different linear functions that were defined on the basis of the Yerkes-Dodson Law (Muse et al., 2003). In this way, a moderate activity was established at the beginning and end of the workday while a high activity was required at the intermediate periods. Therefore, the adaptation and fatigue periods -when operators present less level of stress-, and the routine periods -when operators increase their level of stress-, were respected for all operators at the same time.

While it is true that activation models proposed in previous works facilitate their implementation in real industrial environments, they can result in increasing the level of stress and fatigue of operators, injuries and production drops. The synchronous activation of workers of all workstations could be done through modifying the cycle time of the assembly line -that is by increasing or decreasing the speed of the conveyor belt. However, this type of activation can suppose an unnecessary extra effort by some operators.

This work aims to avoid the above problem. Specifically, we present a new activation model that allows operators to activate their work pace factor according to production needs. Obviously, this flexible activation is limited by the minimum enforceable and optimal work pace established by companies through collective agreements.

Companies, and more specifically automotive companies belonging to the Organization for Economic Cooperation and Development (OECD), determine the processing times of operations by the Method and Times Measurement system (MTM) by considering as normal activity level the one established in the collective agreement. This normal activity corresponds to the normal work pace factor ( $\alpha^N = 1.00$ ) that any operator can support throughout effective working hours without suffering any injury.

However, operators can work more or less quickly at some moments of their workday but respecting the minimum and maximum activity levels that are fixed by the collective agreement. In case of automotive companies, these values usually imply working at 90% and 120% of the normal activity, respectively, and therefore, they suppose a minimum work pace factor of 0.90 ( $\alpha^0 = 0.90$ ) and a maximum one of 0.120 ( $\alpha^* = 1.20$ ).

Having said that, the problem proposed in this paper allows workers to increase their work pace whenever required by the workload. However, they will work at normal work pace when not necessary to avoid work overload.

Furthermore, unlike models proposed by Bautista et al. (2015a), in this research, the problem has an economic approach. Specifically, the model is focused on minimizing the unproductive costs of the line, either by production drop (or work overload) or unproductive time (or idle time).

## **2.2. Mixed-Model Sequencing Problem with cost minimization and bounded activity factor**

With the goal of obtaining manufacturing sequences that reduce the economic impact of incomplete products or workloads and favor both the efficient use of the line and the performance of operators, we formulate two mathematical models from the  $M3 \cup 4_{\alpha I}$  and  $M4 \cup 3_{\alpha I}$  models (Bautista et al., 2015a). The models incorporate the possibility of increasing freely the work pace of processors of the line as far as the production needs require (e.g. to minimize the work overload). Notwithstanding, this



flexible activation must respect the minimum and maximum permissible limit values for the activity factor, which are fixed by the collective agreements. Obviously, this flexible activation will lead to unsynchronized workstations regarding the work pace and, therefore, processors shall be given all facilities necessary for them to work more or less quickly in accordance with the obtained sequence.

The parameters and variables used in the models for the Mixed Model Sequencing Problem with Cost minimization (MMSP<sub>Γ</sub>) are the following:

#### Parameters

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$K$	Set of workstations ( $k = 1, \dots,  K $ )
$b_k$	Number of homogeneous processors at station $k$ ( $k = 1, \dots,  K $ )
$I$	Set of product types ( $i = 1, \dots,  I $ )
$d_i$	Partial demand programmed for each product type $i$ ( $i = 1, \dots,  I $ )
$p_{i,k}$	Processing time required by type of product $i$ ( $i = 1, \dots,  I $ ), for each homogeneous processor of station $k$ ( $k = 1, \dots,  K $ ) (at normal activity, $\alpha^N = 1$ ).
$P_k$	Processing time (at normal activity, $\alpha^N = 1$ ) required by the demand plan programmed for each homogeneous processor of workstation $k$ ( $k = 1, \dots,  K $ ): $P_k = \sum_{i=1}^{ I } p_{i,k} d_i$
$T, D$	Total demand; it is equal to the number of productive cycles of any station. Obviously, $\sum_{i=1}^{ I } d_i = T \equiv D$
$t$	Position index in the sequence ( $t = 1, \dots,  T $ )
$c$	Cycle time; time (measured at normal activity) assigned to each workstation ( $k = 1, \dots,  K $ ) for processing any product unit.
$l_k$	Time window; maximum time that each processor at workstation $k$ ( $k = 1, \dots,  K $ ) can work on any product unit, let $l_k - c > 0$ be the maximum time that one product unit can be held at workstation $k$ , once the cycle time is over.
$L_k$	Physical presence time of processors at workstation $k$ ( $k = 1, \dots,  K $ ); it is equal to the workday of operators assigned to the processors of workstation $k$ : $L_k = c \cdot T + l_k - c$

- $\alpha_{k,t}^+$  Upper limit of dynamic activity factor associated with the  $t^{th}$  operation of the product sequence ( $t = 1, \dots, T$ ) at the station  $k$  ( $k = 1, \dots, |K|$ ). It must fulfil
- $$\alpha_{k,t}^+ \leq \alpha^* = 1.20 \quad (\forall k = 1, \dots, K; \forall t = 1, \dots, T)$$
- $\alpha_t^+$  Upper limit of dynamic activity factor associated with the  $t$  ( $t = 1, \dots, T + |K| - 1$ ) period of the extended workday. This extended workday includes  $T$  manufacturing cycles at the first station (total demand) and  $|K| - 1$  additional cycles that are needed to complete the required work at the last station. Here it is assumed all stations have the same upper limit and it must fulfil  $\alpha_t^+ \leq \alpha^* = 1.20$  ( $\forall t : 1 \leq t \leq T + |K| - 1$ ).
- $\alpha_{k,t}^-$  Lower limit of dynamic activity factor associated with the  $t^{th}$  operation of the sequence ( $t = 1, \dots, T$ ) at the station  $k$  ( $k = 1, \dots, |K|$ ).
- $\alpha_t^-$  Lower limit of dynamic activity factor associated with the  $t$  ( $t = 1, \dots, T + |K| - 1$ ) period of the extended workday. Here it is assumed all stations have the same lower limit ( $\alpha_{k,t}^- = \alpha_t^-$ ,  $\forall k = 1, \dots, K$ ) and it must fulfil  $\alpha_t^- \leq \alpha^0 = 0.90$  ( $\forall t : 1 \leq t \leq T + |K| - 1$ ).
- $\gamma_W$  Cost per work overload unit. It is associated with the production drop that is measured through the work overload.
- $\gamma_b$  Cost per unit of time of a processor.
- $\gamma_U$  Cost per unit of idle time. Here it is assumed  $\gamma_b = \gamma_U$ .

## Variables

- $x_{i,t}$  Binary variable that is equal to 1 if a product unit  $i$  ( $i = 1, \dots, |I|$ ) is assigned to the position  $t$  ( $t = 1, \dots, T$ ) of the sequence, and 0 otherwise.
- $s_{k,t}$  Absolute start instant for the  $t^{th}$  unit of the sequence of products at workstation  $k$  ( $k = 1, \dots, |K|$ ).
- $\hat{s}_{k,t}$  Relative start instant. Positive difference between the start instant and the earliest start instant of the  $t^{th}$  operation in station  $k \in K$ .  $\hat{s}_{k,t} = \max(0, s_{k,t} - (t + k - 2)c)$
- $\rho_{k,t}$  Processing time required (at normal activity) by the  $t^{th}$  unit of the product

- sequence at each homogenous processor of station  $k$  ( $k = 1, \dots, |K|$ ).
- $v_{k,t}$  Processing time applied to the  $t^{th}$  unit sequenced at workstation  $k$  ( $k = 1, \dots, |K|$ ) for each homogeneous processor (at normal activity). It is equivalent to the completed work with regard to the required work ( $\rho_{k,t}$ ).
- $\hat{v}_{k,t}$  Processing time applied to the  $t^{th}$  unit of the product sequence at workstation  $k$  ( $k = 1, \dots, |K|$ ) for each homogeneous processor (at activity,  $\dot{\alpha}_{k,t}$ ).
- $w_{k,t}$  Work overload generated for the  $t^{th}$  unit of the product sequence at station  $k$  for each homogeneous processor. It is measured in units of time (at normal activity).
- $V$  Total processing time applied at normal activity. Total completed work.
- $W$  Total work overload, or production drop.
- $U_k$  Idle time by each processor at station  $k$  ( $k = 1, \dots, |K|$ ), measured at normal activity. This time is considered and penalized in accordance with the presence time,  $L_k$ .
- $\dot{\alpha}_{k,t}$  Dynamic work pace factor associated with the  $t^{th}$  operation of the product sequence ( $t = 1, \dots, T$ ) at workstation  $k$  ( $k = 1, \dots, |K|$ ). This factor is calculated from the normal and actual processing times:  $\dot{\alpha}_{k,t} = \hat{v}_{k,t}/v_{k,t} \Rightarrow \hat{v}_{k,t} = v_{k,t}(\dot{\alpha}_{k,t})$ .
- $\tilde{v}_{k,t}$  Processing time recovered by each homogeneous processor on the  $t^{th}$  product unit sequenced at workstation  $k$  ( $k = 1, \dots, |K|$ ). This time is measured at normal activity.
- $\Gamma$  Total operational cost: costs by production losses resulting from both the total work overload ( $\Gamma_W$ ) and the idle time ( $\Gamma_U$ ).

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Note that, in this paper, we consider seconds as units of time. Therefore, the processing times, the work overload, the cycle time, and the idle time, are measured in seconds.

Model  $M1_\Gamma$ :

$$\text{Min } \Gamma = \Gamma_W + \Gamma_U = \gamma_W \sum_{k=1}^{|K|} \left( b_k \sum_{t=1}^T w_{k,t} \right) + \gamma_U \sum_{k=1}^{|K|} b_k U_k \quad (1)$$

Subject to:

$$\sum_{t=1}^T x_{i,t} = d_i \quad \forall i = 1, \dots, |I| \quad (2)$$

$$\sum_{i=1}^{|I|} x_{i,t} = 1 \quad \forall t = 1, \dots, T \quad (3)$$

$$\rho_{k,t} = \sum_{i=1}^{|I|} p_{i,k} \cdot x_{i,t} \quad \forall k = 1, \dots, |K|; \forall t = 1, \dots, T \quad (4)$$

$$v_{k,t} + w_{k,t} = \rho_{k,t} \quad \forall k = 1, \dots, |K|; \forall t = 1, \dots, T \quad (5)$$

$$v_{k,t} - \dot{\alpha}_{t+k-1}^+ \cdot \hat{v}_{k,t} \leq 0 \quad \forall k = 1, \dots, |K|; \forall t = 1, \dots, T \quad (6)$$

$$v_{k,t} - \dot{\alpha}_{t+k-1}^- \cdot \hat{v}_{k,t} \geq 0 \quad \forall k = 1, \dots, |K|; \forall t = 1, \dots, T \quad (7)$$

$$s_{k,t} \geq s_{k,t-1} + \hat{v}_{k,t-1} \quad \forall k = 1, \dots, |K|; \forall t = 2, \dots, T \quad (8)$$

$$s_{k,t} \geq s_{k-1,t} + \hat{v}_{k-1,t} \quad \forall k = 2, \dots, |K|; \forall t = 1, \dots, T \quad (9)$$

$$s_{k,t} + \hat{v}_{k,t} \leq (t + k - 2) \cdot c + l_k \quad \forall k = 1, \dots, |K|; \forall t = 1, \dots, T \quad (10)$$

$$s_{k,t} \geq (t + k - 2) \cdot c \quad \forall k = 1, \dots, |K|; \forall t = 1, \dots, T \quad (11)$$

$$U_k + \sum_{t=1}^T \hat{v}_{k,t} = L_k \quad \forall k = 1, \dots, |K| \quad (12)$$

$$U_k, v_{k,t}, \hat{v}_{k,t}, w_{k,t} \geq 0 \quad \forall k = 1, \dots, |K|; \forall t = 1, \dots, T \quad (13)$$

$$x_{i,t} \in \{0,1\} \quad \forall i = 1, \dots, |I|; \forall t = 1, \dots, T \quad (14)$$

In  $M1_\Gamma$  model, objective function (1) represents the minimization of total operational or unproductive costs arising from production drop (work overload) and idle time (unproductive work time). Constraints (2) – (5) and (8) – (11) are equal than the constraints from reference models. Specifically, constraints (2) force the demand satisfaction. Constraints (3) force the assignment of only one product unit to each position of the sequence. Constraints (4) establish the processing time required by each product unit sequenced at each workstation. Constraints (5) establish the relation between the applied processing time, the generated work overload, and the processing time required by each product unit sequenced at each workstation. Finally, constraints (8) – (11) define the start instants of each product unit sequenced at each workstation. On the other hand, the new equation (12) determine the idle time at each workstation, considering the total available processing time and the total applied processing time. Meanwhile, the new constraints (6) and (7) limit the maximum and minimum work pace allowed for processors of stations at each cycle, considering that the limitative profiles of the activity factor are synchronized between workstations.

Finally, constraints (13) and (14) force the non-negative and binary conditions for variables, respectively.

It should be noted that  $M1_\Gamma$  model is useful in determining the most appropriate activity factor to reduce the production drop at each station and period (15), as well as the processing time recovered by each activated homogeneous processor (16).

$$\dot{\alpha}_{k,t} = v_{k,t} / \hat{v}_{k,t} \quad \forall k = 1, \dots, |K|; \forall t = 1, \dots, T \quad (15)$$

$$\tilde{v}_{k,t} = v_{k,t} - \hat{v}_{k,t} \quad \forall k = 1, \dots, |K|; \forall t = 1, \dots, T \quad (16)$$

An immediate variant of the model  $M1_\Gamma$  is the one that considers the  $\hat{s}_{k,t}$  variables (relative start instants) instead of the  $s_{k,t}$  variables (absolute start instants); we call this variant,  $M2_\Gamma$  model (see Bautista et al., 2018).

Objective function (1) as well as blocks (2) - (7) and (12) - (14) of constraints from the  $M2_\Gamma$  model coincide with the  $M1_\Gamma$  model. Nevertheless, the  $M2_\Gamma$  variant requires the following set of constraints, (17) - (20), to determine the relative start instants of each product unit sequenced at each workstation.

$$\hat{s}_{k,t} \geq \hat{s}_{k,t-1} + \hat{v}_{k,t-1} - c \quad \forall k = 1, \dots, |K|; \forall t = 2, \dots, T \quad (17)$$

$$\hat{s}_{k,t} \geq \hat{s}_{k-1,t} + \hat{v}_{k-1,t} - c \quad \forall k = 2, \dots, |K|; \forall t = 1, \dots, T \quad (18)$$

$$\hat{s}_{k,t} + \hat{v}_{k,t} \leq l_k \quad \forall k = 1, \dots, |K|; \forall t = 1, \dots, T \quad (19)$$

$$\hat{s}_{1,1} = 0 \quad (20)$$

### 3. Economic compensation by productivity increase

Obviously, increasing the work pace factor above the normal activity ( $\dot{\alpha}_{k,t} > \alpha^N = 1.0$ ) leads to reduce the cost due to production drop or work overload. In addition, the reduction of idle time, through the penalization of its cost ( $\Gamma_U$ ), should be reflected in a better balance of the work. This balance of workload throughout all workday can help prevent work-related injuries and disorders and therefore, costs arising from sick leaves, rehabilitation and training of replacement personnel may be eliminated.

Accordingly, we suggest compensating the excess effort from processors through gains that company may obtain. This compensation may be carried out by setting aside part of the gains from the recovered production, to premium fund or salary bonus for operators. Obviously, on the assumption that processors are homogeneous, we propose an egalitarian compensation system for all operators of the same workstation.

### 3.1 Metrics for the economic compensation

We present two metrics to calculate the economic compensation.

- (a) Sharing a common fund out proportionally, in regard with the excess effort of each workstation.
- (b) Establishing an economic value to the excess effort unit (e.g.  $\gamma_b = \gamma_U$ ) and thus, changing individual and/or collective effort in monetary units.

In addition, considering the second metric (b), we propose two ways to measure the excess effort of operators by workstation and cycle:

- (1) Activity above the normal.
- (2) Recorevered processing time (transformation of work overload in completed work).

In supporting the (b-1) option, the economic compensation by extra activity at the  $k$  ( $k = 1, \dots, |K|$ ) workstation and the  $t$  ( $t = 1, \dots, T$ ) cycle is determined as follows:

$$g_{k,t}^1 = \begin{cases} \gamma_b \cdot b_k (\dot{\alpha}_{k,t} - 1) \cdot c, & \text{if } t = 1, \dots, T-1 \\ \gamma_b \cdot b_k (\dot{\alpha}_{k,t} - 1) \cdot l_k, & \text{if } t = T \end{cases} \quad \forall k = 1, \dots, |K|; \forall t = 1, \dots, T \quad (21)$$

Therefore, the economic compensation, according to the (b-1) option, per workstation ( $k = 1, \dots, |K|$ ) and workday ( $T$ ) must be:

$$G_k^1 = \sum_{t=1}^T g_{k,t}^1 = \gamma_b \cdot b_k ((\bar{\alpha}_k - 1)c \cdot T + (\dot{\alpha}_{k,T} - 1)(l_k - c)) \quad \forall k = 1, \dots, |K| \quad (22)$$

Where  $\bar{\alpha}_k$  is the average of dynamic activity factors at station  $k \in K$ :

$$\bar{\alpha}_k = \frac{1}{T} \sum_{t=1}^T \dot{\alpha}_{k,t}.$$

On the other hand, the economic compensation by processing time recovered ( $\tilde{v}_{k,t} \equiv v_{k,t} - \hat{v}_{k,t}$ ) (b-2 option) at the  $k \in K$  station and the  $t$  ( $t = 1, \dots, T$ ) cycle is calculated as follows:

$$g_{k,t}^2 = \gamma_b \cdot b_k \cdot \tilde{v}_{k,t} = \left\{ \begin{array}{l} \gamma_b \cdot b_k (v_{k,t} - \hat{v}_{k,t}) \\ \gamma_b \cdot b_k (\hat{\alpha}_{k,t} - 1) \cdot \hat{v}_{k,t} \\ \gamma_b \cdot b_k (1 - 1/\hat{\alpha}_{k,t}) \cdot v_{k,t} \end{array} \right\} \quad \forall k = 1, \dots, |K|; \forall t = 1, \dots, T \quad (23)$$

Accordingly, the economic compensation by active workday at the workstation  $k \in K$  must be:

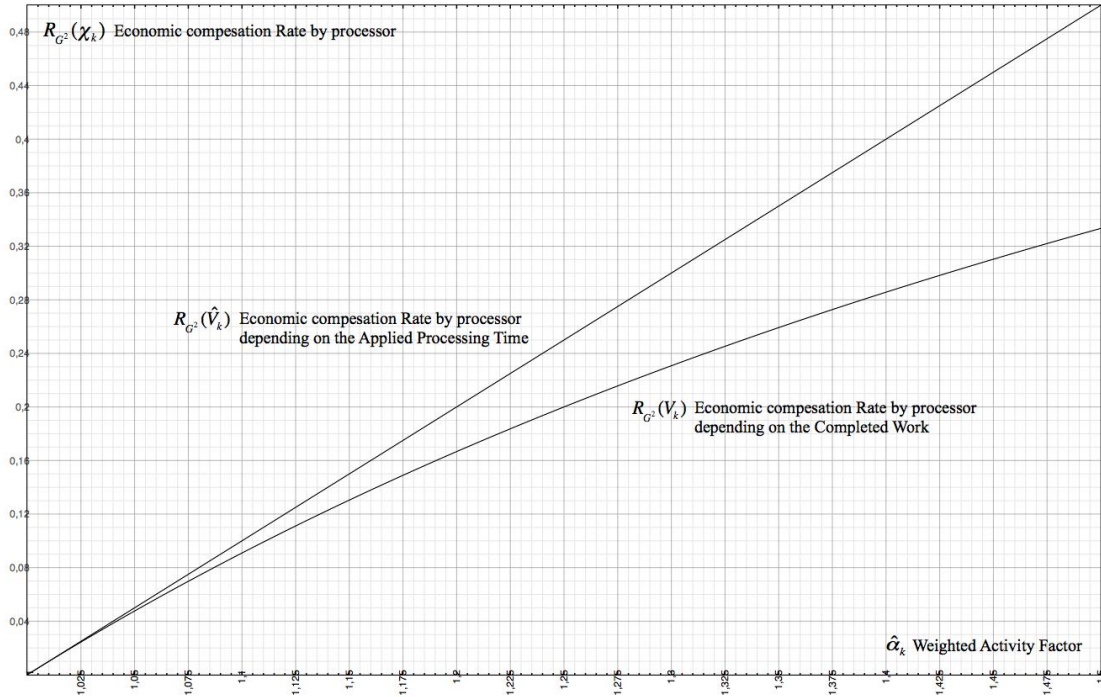
$$G_k^2 = \sum_{t=1}^T g_{k,t}^2 = \gamma_b \cdot b_k \sum_{t=1}^T \tilde{v}_{k,t} = \left\{ \begin{array}{l} \gamma_b \cdot b_k (V_k - \hat{V}_k) \\ \gamma_b \cdot b_k (\hat{\alpha}_k - 1) \cdot \hat{V}_k \\ \gamma_b \cdot b_k (1 - 1/\hat{\alpha}_k) \cdot V_k \end{array} \right\} \quad \forall k = 1, \dots, |K| \quad (24)$$

Where  $V_k$  is the completed work,  $\hat{V}_k$  is the processing time applied at the  $k \in K$  workstation, and  $\hat{\alpha}_k$  is the average of dynamic activity factors ( $\hat{\alpha}_{k,t}$ ) weighted by the applied processing times ( $\hat{v}_{k,t}$ ); that is,  $\hat{\alpha}_k = \left( \sum_{t=1}^T \hat{\alpha}_{k,t} \cdot \hat{v}_{k,t} \right) / \hat{V}_k$ .

Figure 2 shows the relationship between these variables and parameters together with the economic compensation rate assignable to one processor:

$$R_{G^2}(\chi_k) = G_k^2 / (\gamma_b \cdot b_k \cdot \chi_k); \chi_k \in \{V_k, \hat{V}_k\} \quad \forall k = 1, \dots, |K| \quad (25)$$

Fig. 2. Economic compensation rate  $R_{G^2}(\hat{V}_k)$  according to criterion (b-2) that corresponds with the processing time recovered depending on the average of dynamic activity factors ( $\hat{\alpha}_k$ ), the completed work ( $V_k$ ) and the applied processing time ( $\hat{V}_k$ ).



Obviously, economic compensations given by the criteria (b-1) and (b-2) will not be equal in all cases. Indeed, the (b-1) criterion takes into account the time of presence of operators of the line throughout a workday -without considering the penalization by idle time- and the activity levels at each production cycle. Meanwhile, (b-2) criterion only considers the active time of operators and the activity factor of operators.

### 3.2 Properties of economic compensation

It should be noted that the criteria defined for calculating economic compensations of excess efforts of operators present a set of properties.

*Theorem 1:* If the dynamic activity factor ( $\dot{\alpha}_{k,t}$ ) is homogeneous in time at one workstation ( $k \in K$ ), the economic compensation provided by the (b-1) criterion will be greater than, or equal to, the compensation provided by the (b-2) criterion.

*Proof:*

Let  $\dot{\alpha}_{k,t}$  be homogeneous in  $t$  ( $\forall t = 1, \dots, T$ ), i.e.,  $\dot{\alpha}_{k,t} = \dot{\alpha}_k$ , then  $\hat{\alpha}_k = \bar{\alpha}_k = \dot{\alpha}_k$



$$(i) \hat{\alpha}_k = \dot{\alpha}_k \Rightarrow G_k^2 = \gamma_b \cdot b_k \cdot (\hat{\alpha}_k - 1) \cdot \hat{V}_k = \gamma_b \cdot b_k \cdot (\dot{\alpha}_k - 1) \cdot \hat{V}_k$$

$$(ii) \hat{V}_k \leq (c \cdot T + l_k - c) \Rightarrow G_k^2 \leq \gamma_b \cdot b_k \cdot (\dot{\alpha}_k - 1) \cdot (c \cdot T + l_k - c)$$

$$(iii) \bar{\alpha}_k = \dot{\alpha}_k \Rightarrow G_k^1 = \gamma_b \cdot b_k \cdot ((\bar{\alpha}_k - 1)c \cdot T + (\dot{\alpha}_{k,T} - 1)(l_k - c)) \Rightarrow$$

$$G_k^1 = \gamma_b \cdot b_k (\dot{\alpha}_k - 1)(c \cdot T + l_k - c)$$

Therefore,  $G_k^1 \geq G_k^2$ .

*Theorem 2:* If the dynamic activity factor ( $\dot{\alpha}_{k,t}$ ) is homogeneous in time at one workstation ( $k \in K$ ) and the idle time ( $U_k$ ) is null at the said station, the compensations provided by (b-1) and (b-2) criteria will be equal.

*Proof:*

$$(i) U_k = 0 \Rightarrow \hat{V}_{k,t} = c \cdot T + l_k - c$$

$$(ii) \hat{\alpha}_k = \dot{\alpha}_k \Rightarrow G_k^2 = \gamma_b \cdot b_k \cdot (\dot{\alpha}_k - 1) \cdot \hat{V}_k = \gamma_b \cdot b_k \cdot (\dot{\alpha}_k - 1) \cdot (c \cdot T + l_k - c)$$

$$(iii) \bar{\alpha}_k = \dot{\alpha}_k \Rightarrow G_k^1 = \gamma_b \cdot b_k (\dot{\alpha}_k - 1)(c \cdot T + l_k - c)$$

Therefore,  $G_k^1 = G_k^2$ .

*Theorem 3:* If  $l_k = c$  at one workstation ( $k \in K$ ) (no time window), the compensation provided by (b-1) is greater than, or equal to, the compensation provided by (b-2) at the said station.

*Proof:*

$$(i) l_k = c \Rightarrow \hat{v}_{k,t} \leq c (\forall t = 1, \dots, T)$$

$$(ii) \sum_{t=1}^T (\dot{\alpha}_{k,t} - 1) \cdot c \geq \sum_{t=1}^T (\dot{\alpha}_k - 1) \cdot \hat{v}_{k,t}$$

$$(iii) \gamma_b \cdot b_k \cdot \sum_{t=1}^T (\dot{\alpha}_{k,t} - 1) \cdot c \geq \gamma_b \cdot b_k \cdot \sum_{t=1}^T (\dot{\alpha}_k - 1) \cdot \hat{v}_{k,t}$$

Therefore,  $G_k^1 \geq G_k^2$ .

*Theorem 4:* If  $l_k = c$  at one workstation ( $k \in K$ ) (no time window) and the idle time is null at the said station, the compensations provided by both criteria are the same.

*Proof:*

$$(i) (l_k = c) \wedge (U_k = 0) \Rightarrow \hat{v}_{k,t} = c (\forall t = 1, \dots, T)$$

$$(ii) G_k^1 = \gamma_b \cdot b_k \cdot \sum_{t=1}^T (\dot{\alpha}_{k,t} - 1) \cdot c = \gamma_b \cdot b_k (\bar{\alpha}_k - 1) \cdot c \cdot T$$

$$(iii) G_k^2 = \gamma_b \cdot b_k \cdot \sum_{t=1}^T (\hat{\alpha}_{k,t} - 1) \cdot \hat{v}_{k,t} = \gamma_b \cdot b_k \cdot \sum_{t=1}^T (\hat{\alpha}_{k,t} - 1) \cdot \hat{v}_{k,t} = \gamma_b \cdot b_k (\bar{\alpha}_k - 1) \cdot c \cdot T$$

Therefore,  $G_k^1 = G_k^2$ .

*Theorem 5:* If work overload ( $W_k$ ) at one workstation ( $k \in K$ ) is null, the maximum economic compensation, according to (b-2) criterion, will be reached whether the applied processing time ( $\hat{V}_k$ ) is minimum.

*Proof:*

$$(i) W_k = 0 \Rightarrow V_k = P_k$$

$$(ii) G_k^2 = \gamma_b \cdot b_k \cdot (V_k - \hat{V}_k) = \gamma_b \cdot b_k \cdot (P_k - \hat{V}_k)$$

$$(iii) G_k^2 = cte - \gamma_b \cdot b_k \cdot \hat{V}_k$$

Therefore,  $\max G_k^2 \Leftrightarrow \min \hat{V}_k$ .

*Theorem 6:* If work overload ( $W_k$ ) at one workstation ( $k \in K$ ) is null, the maximum economic compensation, according to (b-2) criterion, will be reached when the weighted average activity ( $\hat{\alpha}_k$ ) is maximum.

*Proof:*

$$(i) W_k = 0 \Rightarrow V_k = P_k$$

$$(ii) G_k^2 = \gamma_b \cdot b_k \cdot (1 - 1/\hat{\alpha}_k) \cdot V_k = \gamma_b \cdot b_k \cdot (1 - 1/\hat{\alpha}_k) \cdot P_k$$

$$(iii) G_k^2 = cte \cdot (1 - 1/\hat{\alpha}_k)$$

Therefore,  $\max G_k^2 \Leftrightarrow \max \hat{\alpha}_k$ .

## 4. Assessment of operational costs of a mixed-model assembly line.

To assess the new models,  $M1_\Gamma$  and  $M2_\Gamma$ , for the  $MMSP_\Gamma$  we use a case study linked with the Nissan's powertrain plant in Barcelona.

The variant proposed, in this paper, for the  $MMSP_\Gamma$  can be studied under two different perspectives. On the one hand, from the free perspective,  $MMSP_\Gamma(\text{free})$ , that considers the possibility of interrupting operations at any time between the end of the cycle and the limit given by the time window. And on the other hand, from a forced perspective,

MMSP\_ $\Gamma$ (forced), which only allows to interrupt operations when the time window ends (Bautista and Alfaro-Pozo, 2018).

As can be seen in the mathematical models proposed in section 2.2., in this paper, we study the MMSP\_ $\Gamma$ (free), which corresponds with the general case of interruptions of operations.

Referring the resolution procedures, there are many options described in the literature. Among them, we have the following five alternatives: the Mixed Integer Linear Programming (MILP), the Greedy and Randomized Adaptive Search Procedure (GRASP), the Bounded Dynamic Programming (BDP), the hybrid procedure that combines GRASP with the Linear Programming (GRASP-LP), and the hybrid procedure that combines the BDP with the Linear Programming (BDP-LP).

These procedures present a set of strengths and weaknesses in the following five qualities: (i) guarantee of achieving optimal solutions, (ii) memory requirement, (iii) ease of implementation, (iv) quality of solutions, and (v) the CPU time. Specifically:

- GRASP offers high quality solutions in reduced CPU times. It is an easily implementable procedure and requires low memory but it does not guarantee optimal solutions. It is efficient in the forced variant, while it requires the assistance of LP to solve the free variant (GRASP-LP).
- BDP offers high quality solutions with CPU times similar to those used by the MILP. Its implementation is very laborious and requires more memory than GRASP but less than MILP. In addition, it can guarantee optimum results and offer lower bounds. Like GRASP, the BDP is efficient for solving the MMSP\_ $\Gamma$ (forced) variants but it requires LP to solve the MMSP\_ $\Gamma$ (free) variants (BDP-LP).
- MILP offers high quality solutions in reasonable CPU times (two hours). It is the procedure easier to implement, but it requires more memory than GRASP and BDP. It can guarantee optimal solutions and give lower bounds for instances. It solves efficiently the free variant, while it is inefficient for solving the forced one.

For the above reasons, we decide to use MILP to solve the variant for the MMSP\_ $\Gamma$ (free) studied in this paper, leaving the others procedures to future works. This gives us reference solutions for future researches and lower bounds for the forced version.

## 4.1. Nissan's Case Study. Data set.

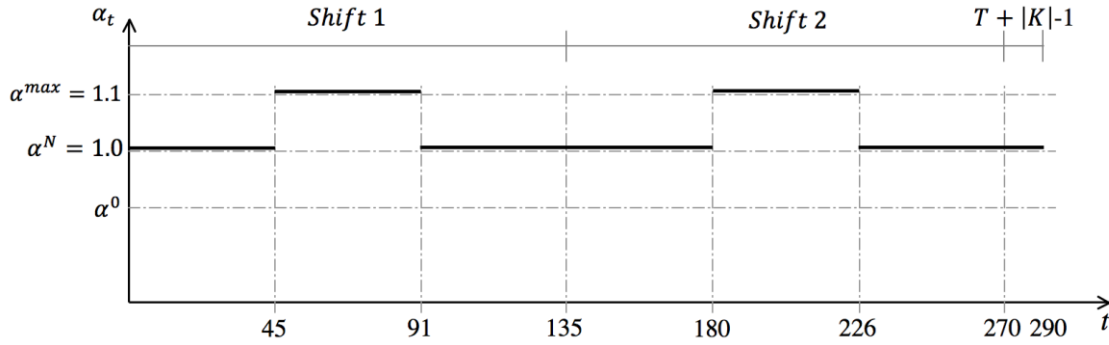
As already stated, the models proposed in this paper,  $M1_\Gamma$  and  $M2_\Gamma$ , are assessed through the Nissan-9Eng.I, which is a case study linked with the Nissan's powertrain plant in Barcelona.

The Nissan's case study is focused on sequencing different production plans of vehicle's engines that must be manufactured into an assembly line. The basic features for the powertrain line are the following:

- Assembly line composes by 21 workstations arranged in series ( $|K| = 21$ ).
- Each workstation has only one homogeneous processor ( $b_k = 1, \forall k \in K$ ). This homogeneous processor is equivalent to a team of two operators with the same abilities and tools and the same requests for auxiliary equipment.
- The effective cycle time is  $c = 175s$  and the time window is identical for all workstations,  $l_k = 195s$ .
- The Consolidated Operating Profit (COP) of the line is the 10% of the profit of one engine (i.e., the 10% of 4000€). Therefore, the loss of one engine supposes an economic cost of 400€. This together with the consideration that the line produces an engine each 175 seconds, allows us to know the cost by unit of work overload or production drop,  $\gamma_w = 2.28\text{€}/s$ .
- The production plant, where is the assembly line under study, is in Barcelona, therefore, the horary cost is around 20€/h per operator (considering the automotive sector in Spain). On the other hand, each processor of the assembly line consists of two operators; then, we establish the cost by idle time unit in  $\gamma_b = \gamma_U = 0.01\hat{1}\text{€}/s$ .
- For reference models with pre-fixed activity profiles, we use the stepped function ( $\alpha^S$ ) (Bautista et al., 2015a). This function adjusts the work pace of operators of the line to different moments of workday (adaptation – activation – fatigue) (see figure 3). Specifically, after an adaptation period ( $t_0 = 45$  for the first shift and  $t_0 = 180$  for the second one) working at normal work pace ( $\alpha^N = 1.0$ ), the activity factor is increased to 1.1 ( $\alpha^{\max} = 1.1$ ). This activation corresponds with the maximum activity factor considered in this computational experiment to ensure compliance with the maximum activity established in the

collective agreement ( $\alpha^{\max} = 1.1 \leq \alpha^* = 1.2$ ). The activation period finish in  $t_{\infty} = 91$  and  $t_{\infty} = 226$  for both shifts, respectively, and then the activity factor becomes normal again.

Fig. 3. Stepped function given the Nissan's data.



- Additionally, we consider the linear function equivalent to the stepped function, i.e., a constant function with value equal to the average activity that corresponds to the stepped function ( $\dot{\alpha}_{k,t} = \bar{\alpha}^S = 1.03\bar{3}, \forall k \in K; \forall t : 1 \leq t \leq T + |K| - 1$ ).
- In case of  $M1_\Gamma$  and  $M2_\Gamma$  models, where the activity level is free within an upper and lower limit, we consider the stepped function for the work pace factor. Thereby, the normal activity is considered as the minimum activity factor allowed ( $\dot{\alpha}_t^- = \alpha^N = 1.0, \forall t : 1 \leq t \leq T + |K| - 1$ ) and the upper limit corresponds with the maximum activity for the stepped function ( $\dot{\alpha}_t^+ = \alpha^{\max} = 1.1, \forall t : t_0 \leq t \leq t_{\infty}$ ). Therefore, the minimum work pace considered in this computational experience is greater the minimum prescribed by collective labor agreement ( $\alpha^0 = 0.90$ ) and the maximum one is less than the maximum activity prescribed by collective labor agreement ( $\alpha^* = 1.20$ ). These limits involve a maximum increase of operators' activity by 10% throughout one third of their effective workday, as it is determined by the stepped function.
- In case of the linear function, the upper limit is equal to the average value of the activity factor ( $\dot{\alpha}_t^+ = \bar{\alpha}^S = 1.03\bar{3}, \forall t : 1 \leq t \leq T + |K| - 1$ ).

Referring demand plans, the main features are the following:

- We have a set E of 23 different demand plans (see Block I of Table 6 in Bautista and Cano, 2011). All of them correspond with production for a workday, which

is divided into two shifts of 8 hours each one. This means 13.125 effective hours of work per day, after discounting the statutory breaks and rest periods.

- Each demand plan,  $\varepsilon$  ( $\varepsilon = 1, \dots, |E|$ ), consists of a total demand of  $T = 270$  engines. This demand is divided into nine types of engines that are grouped into three families, according to the type of vehicles: (1) crossovers and Sport Utility Vehicles (SUVs) ( $p_1, p_2$  and  $p_3$ ), (2) vans ( $p_4$  and  $p_5$ ), and (3) medium tonnage trucks ( $p_6, \dots, p_9$ ). In turn, each type of engine requires different processing time at workstations ( $p_{i,k}, \forall i = 1, \dots, |I|; \forall k = 1, \dots, |K|$ ), and it can vary between 89s and 185s (see Table 5 in Bautista and Cano, 2011). Figure 4 shows an engine that belongs to the SUVs - Sport Utility Vehicle family.
- The partial demands by types of engine ( $d_i, \forall i = 1, \dots, |I|$ ) differentiate the 23 demand plans. Thus, we have demand plans that are very balanced (30 engines per type) and plans more or less unbalanced (Bautista et al., 2012a).

Fig. 4. Nissan Pathfinder Engine. Characteristics: (i) 747 parts and 330 references, (ii) 378 elemental assembly tasks grouped in 140 production line tasks.



Following, you will find tables with the processing times of operations by type of engine and workstation (table 1), and with the detail of the 23 demand plans (table 2).

Table 1. Processing times (in seconds) at normal activity ( $p_{i,k}$ ) for the 9 engine types ( $i \in I$ ) in the 21 workstations ( $k \in K$ ) of the set of instances Nissan-9Eng.I.

$k/i$	M1	M2	M3	M4	M5	M6	M7	M8	M9
1	104	100	97	92	100	94	103	109	101
2	103	103	105	107	101	108	106	102	110
3	165	156	164	161	148	156	154	164	155
4	166	175	172	167	168	167	168	156	173
5	111	114	114	115	117	117	115	111	111
6	126	121	122	124	127	130	120	121	134
7	97	96	96	93	96	89	94	101	92
8	100	97	95	106	94	102	103	102	100
9	179	174	173	178	178	171	177	171	174
10	178	172	172	177	178	177	175	173	175
11	161	152	168	167	167	166	172	157	177
12	96	106	105	97	101	100	96	104	96
13	99	101	102	101	99	101	96	102	99
14	147	155	142	154	146	143	154	153	155
15	163	152	156	152	153	152	154	156	156
16	163	185	183	178	169	173	172	182	171
17	173	179	178	169	173	178	174	175	175
18	176	167	181	180	172	173	173	168	184
19	162	150	152	152	160	151	155	148	167
20	164	161	157	159	162	160	162	158	157
21	177	161	154	168	172	170	167	149	169

Table 2. Daily demands by product and for the 23 instances Nissan-9Eng.I ( $d_{i,\varepsilon}, \forall \varepsilon \in E$ ).

$\varepsilon \in E$	M1	M2	M3	M4	M5	M6	M7	M8	M9	SUV	Van	Truck	Total
#1	30	30	30	30	30	30	30	30	30	90	60	120	270
#2	30	30	30	45	45	23	23	22	22	90	90	90	270
#3	10	10	10	60	60	30	30	30	30	30	120	120	270
#4	40	40	40	15	15	30	30	30	30	120	30	120	270
#5	40	40	40	60	60	8	8	7	7	120	120	30	270
#6	50	50	50	30	30	15	15	15	15	150	60	60	270
#7	20	20	20	75	75	15	15	15	15	60	150	60	270
#8	20	20	20	30	30	38	38	37	37	60	60	150	270
#9	70	70	70	15	15	8	8	7	7	210	30	30	270
#10	10	10	10	105	105	8	8	7	7	30	210	30	270
#11	10	10	10	15	15	53	53	52	52	30	30	210	270
#12	24	23	23	45	45	28	28	27	27	70	90	110	270
#13	37	37	36	35	35	23	23	22	22	110	70	90	270
#14	37	37	36	45	45	18	18	17	17	110	90	70	270
#15	24	23	23	55	55	23	23	22	22	70	110	90	270
#16	30	30	30	35	35	28	28	27	27	90	70	110	270
#17	30	30	30	55	55	18	18	17	17	90	110	70	270
#18	60	60	60	30	30	8	8	7	7	180	60	30	270
#19	10	10	10	90	90	15	15	15	15	30	180	60	270
#20	20	20	20	15	15	45	45	45	45	60	30	180	270
#21	60	60	60	15	15	15	15	15	15	180	30	60	270
#22	20	20	20	90	90	8	8	7	7	60	180	30	270
#23	10	10	10	30	30	45	45	45	45	30	60	180	270

## 4.2. Nissan's Case Study. Model results.

To solve the set of instances (table 2) we have used MILP. Specifically, both models have been implemented on the LP Solver of the Gurobi Optimizer v5.0. and they have been run on an Apple Macintosh iMac computer with an Intel Core i7 2.93-GHz processor, 8 GB of RAM memory, and a MAC OS X 10.6.8 operating system, with a maximum CPU time of two hours.

After 92 executions (23 instances, two models and two upper limit functions for the work pace of operators) the unproductive costs from the obtained sequences have been the following (Table 3):

Table 3. Unproductive costs,  $\Gamma$ , given by  $M1_{\Gamma}$  and  $M2_{\Gamma}$  models and best bound found by the solver for each demand plan  $\varepsilon \in E$ . Optimal solutions are marked with this symbol \*.

$\varepsilon$	$M1_{\Gamma}$				$M2_{\Gamma}$			
	$\dot{\alpha}^S$		$\overline{\alpha}^S$		$\dot{\alpha}^S$		$\overline{\alpha}^S$	
	$\Gamma$	Best Bound	$\Gamma$	Best Bound	$\Gamma$	Best Bound	$\Gamma$	Best Bound
#1	1090.7	1023.4	1024.8	1023.4	1231.7	1023.5	1024.8	1023.4
#2	1294.5	1024.9	1025.9	1024.9	1267.2	1024.9	1026.0	1024.9
#3	1080.5	1026.6	1026.9	1026.5	1135.4	1026.6	1026.8	1026.5
#4	1257.2	1024.0	1025.0	1023.9	1243.1	1024.0	1025.0	1023.9
#5	1539.5	1081.0	1027.8	1026.7	1445.5	1080.5	1027.7	1026.7
#6	1397.1	1024.4	1026.0	1024.3	1408.8	1024.4	1025.8	1024.3
#7	1313.6	1130.8	1028.6	1028.1	1336.3	1130.7	1028.5	1028.1
#8	1254.3	1023.8	1024.8	1023.7	1201.3	1023.8	1024.8	1023.7
#9	1645.5	1093.0	1026.8	1025.7	1652.5	1087.6	1026.8	1025.7
#10	2081.6	2051.7	1031.7	1031.6	2084.0	2051.7	1031.6*	1031.6
#11	1164.1	1023.7	1024.4	1023.7	1111.6	1023.7	1024.4	1023.7
#12	1277.9	1024.9	1025.7	1024.8	1255.3	1024.9	1025.9	1024.8
#13	1295.9	1024.2	1025.4	1024.1	1321.8	1024.2	1025.4	1024.1
#14	1224.5	1025.0	1026.3	1025.0	1389.2	1025.0	1026.2	1025.0
#15	1263.0	1026.0	1026.7	1025.9	1309.1	1026.0	1026.7	1025.9
#16	1250.0	1023.9	1025.3	1023.9	1129.3	1023.9	1025.2	1023.9
#17	1320.5	1026.0	1027.0	1026.0	1336.5	1026.0	1027.0	1026.0
#18	1599.7	1072.2	1026.3	1025.1	1599.2	1071.3	1026.5	1025.1
#19	1595.6	1538.6	1029.9	1029.8	1600.0	1538.8	1029.9	1029.8
#20	1105.9	1023.4	1024.3	1023.4	1164.9	1023.4	1024.3	1023.4
#21	1482.5	1025.0	1026.0	1025.0	1473.1	1025.0	1026.0	1025.0
#22	1685.2	1628.9	1030.1	1029.9	1710.5	1629.2	1030.1	1029.9
#23	1055.1	1024.1	1024.7	1024.0	1155.5	1024.1	1024.6	1024.0
Avg.	1359.8	1130.0	1026.5	1025.6	1372.3	1129.7	1026.5	1025.6
Min.	1055.1	1023.4	1024.3	1023.4	1111.6	1023.4	1024.3	1023.4
Max.	2081.6	2051.7	1031.7	1031.6	2084.0	2051.7	1031.6	1031.6



Likewise, in order to evaluate the quality of solutions, table 4 shows the average, maximum and minimum values of the gap between the costs given by the sequences obtained with each model and the best bound for the solution found by the solver.

Table 4. Gap between solutions given by  $M1_\Gamma$  and  $M2_\Gamma$  models, and the best bound found by the solver for each demand plan  $\varepsilon \in E$  and for each activity profile.

$\varepsilon$	$M1_\Gamma$		$M2_\Gamma$	
	$\dot{\alpha}^S$	$\overline{\dot{\alpha}^S}$	$\dot{\alpha}^S$	$\overline{\dot{\alpha}^S}$
Avg.	16.68%	0.09%	17.67%	0.09%
Min.	1.43%	0.01%	1.55%	0.00%
Max.	33.57%	0.16%	34.18%	0.14%

The analysis of results on unproductive costs (table 3 and table 4) confirms the following points:

- Given the CPU limit of 7200 seconds, the solver can only reached the optimal solution for the demand plan number #10 when the maximum possible activation is equal to 1.033 throughout all workday (average stepped function).
- When the stepped function is considered for establishing the maximum permissible work pace factor,  $M1_\Gamma$  overtakes slightly  $M2_\Gamma$  in the average gap (16.68% versus 17.67%), although difference between both models is quite insignificant.
- Considering the average stepped function as the maximum possible activity, both models are equal in terms of average gap (0.088% versus 0.087%).
- It is very difficult to improve solutions given by  $M1_\Gamma$  and  $M2_\Gamma$ , for the 23 demand plans and for the average stepped function. Indeed, ranges of gap values for  $M1_\Gamma$  and  $M2_\Gamma$  are [0.006%, 0.162%] and [0.004%, 0.144%] respectively.

Accordingly, solutions are very close to optimal solutions.

- Solutions given by  $M1_\Gamma$  for the demand plans #3, #10, #19, #22 and #23 in case of stepped function are acceptable for us because their values are at less than 5% of the best bound. Similarly, solutions given by  $M2_\Gamma$  for the demand plans #10, #19 and #22 are also acceptable.
- Obviously, solutions not commented in the above point are susceptible of improvement in future researches. Our proposal is based on using others resolution procedures, such as GRASP-LP. Accordingly, first the MMSP\_Γ(forced) version will be solved through a GRASP and, then, we will use LP in order to reduce the work overload, the activity factor and the cost, allowing the free interruption of operations (MMSP\_Γ(free) version).

On the other hand, in order to quantify the effect of the new considerations (the idle time and the flexible and non-synchronous activation of operators) the obtained results are assessed in terms of work overload and idle time (table 5).

Table 5. Work overload values,  $W$ , and idle time,  $U$ , given by  $M1_\Gamma$  and  $M2_\Gamma$  models, for each demand plan,  $\varepsilon \in E$ , and considering the stepped function  $\dot{\alpha}^S$  and its linear function  $\overline{\dot{\alpha}^S}$  for establishing limits of work pace factor of operators.

$\varepsilon$	Work overload: $W$				Idle time: $U$			
	$\dot{\alpha}^S$		$\overline{\dot{\alpha}^S}$		$\dot{\alpha}^S$		$\overline{\dot{\alpha}^S}$	
	$M1_\Gamma$	$M2_\Gamma$	$M1_\Gamma$	$M2_\Gamma$	$M1_\Gamma$	$M2_\Gamma$	$M1_\Gamma$	$M2_\Gamma$
#1	28.0	90.0	0.0	0.0	185804.0	185705.0	185695.4	185689.4
#2	117.0	105.0	0.0	0.0	185911.0	185894.0	185890.4	185907.4
#3	23.0	47.0	0.0	0.0	186043.0	186075.0	186064.4	186043.4
#4	101.0	95.0	0.0	0.0	185760.0	185724.0	185711.6	185724.0
#5	223.0	182.0	0.0	0.0	186369.0	186323.0	186203.8	186218.4
#6	162.0	167.0	0.0	0.0	185819.0	185921.0	185902.5	185862.4
#7	124.0	134.0	0.0	0.0	186426.0	186387.0	186380.2	186350.2
#8	100.0	77.0	0.0	0.0	185662.0	185626.0	185684.5	185674.2
#9	270.0	273.0	0.0	0.0	186117.0	186124.0	186051.5	186025.3
#10	459.0	460.0	0.0	0.0	186810.0	186836.0	186915.8	186918.6
#11	61.0	38.0	0.0	0.0	185526.0	185514.0	185617.7	185619.8
#12	110.0	100.0	0.0	0.0	185807.0	185817.0	185844.3	185886.4
#13	118.0	129.0	0.0	0.0	185759.0	185852.0	185787.8	185797.5
#14	86.0	158.0	0.0	0.0	186041.0	186059.0	185948.4	185950.5
#15	103.0	123.0	0.0	0.0	185997.0	186049.0	186032.5	186031.6
#16	98.0	45.0	0.0	0.0	185722.0	185811.0	185773.6	185764.5
#17	128.0	135.0	0.0	0.0	186044.0	186048.0	186081.6	186088.4
#18	250.0	250.0	0.0	0.0	186045.0	185962.0	185959.6	185994.6
#19	247.0	249.0	0.0	0.0	186576.0	186544.0	186613.4	186601.8
#20	35.0	61.0	0.0	0.0	185706.0	185632.0	185591.4	185601.4
#21	199.0	195.0	0.0	0.0	185945.0	185960.0	185901.4	185906.3
#22	286.0	297.0	0.0	0.0	186643.0	186660.0	186638.5	186639.6
#23	13.0	57.0	0.0	0.0	185632.0	185606.0	185668.4	185653.4

The results show how models cannot eliminate the work overload completely when the maximum allowed activation is in line with the stepped function. However, when operators can work freely with an activity factor between 1.0 and 1.0333 ( $\dot{\alpha}_t^- = 1.0$  and  $\dot{\alpha}_t^+ = 1.033$ ) throughout all workday, the work overload is null. Regarding idle time, both bounded functions for the activity factor offer similar values. Indeed, the maximum difference between the idle time given by considering one or the other bounded function for the activity factor, is 165.2s ( $M1_\Gamma$ , demand plan #5).

When the maximum activity follows the stepped function, model with absolute start instants,  $M1_{\Gamma}$ , gets slightly better results than model with relative start instants,  $M2_{\Gamma}$  (i.e.  $M1_{\Gamma}$  achieves better work overload values in 14 of all 23 demand plans and it ties in plan #18). Despite this, we cannot state any conclusion about it, because the average difference between the values of work overload from both models is negligible (5.48 seconds in favor of the  $M1_{\Gamma}$  model).

Finally, taking into account solutions from reference models (shown in tables 12 and 13 of Appendix): (1) models without both activity increase and idle time penalization,  $M3 \cup 4$  and  $M4 \cup 3$  (Bautista et al., 2012b); and (2) models with pre-fixed activation profiles but without idle time penalization,  $M3 \cup 4_{\dot{a}I}$  and  $M4 \cup 3_{\dot{a}I}$  (Bautista et al., 2015a), we can determine the following:

- (i) the work overload reduction (given as a fraction of unity) achieved by models with activation ( $M3 \cup 4_{\dot{a}I}$ ,  $M4 \cup 3_{\dot{a}I}$ ,  $M1_{\Gamma}$  and  $M2_{\Gamma}$ ) in comparison with the work overload values given by original reference models ( $M3 \cup 4$  and  $M4 \cup 3$ ) without activation of operators (table 6);
- (ii) the increase of idle time that supposes the activation of operators in the  $M3 \cup 4_{\dot{a}I}$ ,  $M4 \cup 3_{\dot{a}I}$ ,  $M1_{\Gamma}$  and  $M2_{\Gamma}$  models, versus the non-activation of  $M3 \cup 4$  and  $M4 \cup 3$  models. In this way, we can denote the positive effect of activating operators freeform but controlled, and the effect of considering the idle time in the optimization (table 6).

Table 6. Work overload reduction ( $\Delta W/W$ ) and idle time increase ( $\Delta U/U$ ) given by  $M3 \cup 4_{\dot{\alpha}I}$  and  $M1_{\Gamma}$  versus  $M3 \cup 4$  ( $\alpha^N$ ), and  $M4 \cup 3_{\dot{\alpha}I}$  and  $M2_{\Gamma}$  versus  $M4 \cup 3$  ( $\alpha^N$ ). V.gr. plan #5 and  $\dot{\alpha}^S$  stepped function:  $\Delta W/W = (W_{M3 \cup 4} - W_{M1_{\Gamma}}) / W_{M3 \cup 4} = (709 - 223) / 709 = 0.69$ .

$\varepsilon$	$\Delta W/W$						$\Delta U/U$				
	$M3 \cup 4_{\dot{\alpha}I}$		$M4 \cup 3_{\dot{\alpha}I}$		$M1_{\Gamma}$		$M2_{\Gamma}$		$M(\cdot)_{\dot{\alpha}I}$		$M(\cdot)_{\Gamma}$
	$\dot{\alpha}^S$	$\bar{\alpha}^S$	$\dot{\alpha}^S$	$\bar{\alpha}^S$	$\dot{\alpha}^S$	$\bar{\alpha}^S$	$\dot{\alpha}^S$	$\bar{\alpha}^S$	$\dot{\alpha}^S$	$\bar{\alpha}^S$	$\dot{\alpha}^S, \bar{\alpha}^S$
#1	1.00	1.00	1.00	1.00	0.91	1.00	0.71	1.00	-0.13	-0.14	-0.00
#2	0.96	1.00	0.97	1.00	0.73	1.00	0.75	1.00	-0.13	-0.14	-0.00
#3	1.00	1.00	1.00	1.00	0.95	1.00	0.91	1.00	-0.13	-0.14	-0.00
#4	1.00	1.00	1.00	1.00	0.75	1.00	0.78	1.00	-0.13	-0.14	-0.00
#5	0.74	1.00	0.71	1.00	0.69	1.00	0.77	1.00	-0.13	-0.14	-0.00
#6	0.83	1.00	0.84	1.00	0.70	1.00	0.68	1.00	-0.13	-0.14	-0.00
#7	0.85	1.00	0.87	1.00	0.84	1.00	0.83	1.00	-0.13	-0.14	-0.00
#8	1.00	1.00	1.00	1.00	0.61	1.00	0.67	1.00	-0.13	-0.14	-0.00
#9	0.75	1.00	0.71	1.00	0.67	1.00	0.67	1.00	-0.13	-0.14	-0.00
#10	0.62	1.00	0.62	1.00	0.62	1.00	0.62	1.00	-0.13	-0.13	-0.00
#11	1.00	1.00	1.00	1.00	0.64	1.00	0.78	1.00	-0.13	-0.14	-0.00
#12	0.98	1.00	1.00	1.00	0.70	1.00	0.73	1.00	-0.13	-0.14	-0.00
#13	0.93	1.00	0.95	1.00	0.74	1.00	0.67	1.00	-0.13	-0.14	-0.00
#14	0.89	1.00	0.91	1.00	0.83	1.00	0.69	1.00	-0.13	-0.14	-0.00
#15	0.95	1.00	0.98	1.00	0.81	1.00	0.75	1.00	-0.13	-0.14	-0.00
#16	1.00	1.00	0.98	1.00	0.71	1.00	0.86	1.00	-0.13	-0.14	-0.00
#17	0.92	1.00	0.90	1.00	0.77	1.00	0.74	1.00	-0.13	-0.14	-0.00
#18	0.66	1.00	0.70	1.00	0.63	1.00	0.62	1.00	-0.13	-0.14	-0.00
#19	0.74	1.00	0.75	1.00	0.74	1.00	0.74	1.00	-0.13	-0.13	-0.00
#20	1.00	1.00	1.00	1.00	0.84	1.00	0.74	1.00	-0.13	-0.14	-0.00
#21	0.86	1.00	0.85	1.00	0.70	1.00	0.71	1.00	-0.13	-0.14	-0.00
#22	0.72	1.00	0.70	1.00	0.72	1.00	0.70	1.00	-0.13	-0.13	-0.00
#23	1.00	1.00	1.00	1.00	0.93	1.00	0.70	1.00	-0.13	-0.14	-0.00
Avg.	0.89	1.00	0.89	1.00	0.75	1.00	0.73	1.00	-0.13	-0.14	-0.0009

From Table 6, we can comment on the following observations:

- When the maximum activity factor is fixed in accordance with the stepped function, reference models,  $M3 \cup 4_{\dot{\alpha}I}$  and  $M4 \cup 3_{\dot{\alpha}I}$ , get a greater reduction in work overload than  $M1_{\Gamma}$  and  $M2_{\Gamma}$ . Specifically, reference models with fixed activity profile reduce the work overload by 89% on average, while reduction from the proposed models is by 75 - 73%, on average.
- When activity factor follows a linear function with value equal to the average activation of the stepped function, all models are able to complete all required work.

- On the other hand, reference models,  $M3 \cup 4_{\dot{\alpha}I}$  and  $M4 \cup 3_{\dot{\alpha}I}$ , get worse values for the idle time than reference models without activation. Indeed, a pre-fixed activation profile increases the idle time by 13 – 14% on average. However, when the activation is flexible between an upper and lower limit, this increase does not exceed the 0.2% in any case.
- The variation in the product mix has an insignificant effect on the new models. However, models with fixed activation ( $M3 \cup 4_{\dot{\alpha}I}$  and  $M4 \cup 3_{\dot{\alpha}I}$ ) achieve lower reduction of work overload when the presence of engines for trucks is minority (demand plans #5, #9, #10, #18, and #22).

It is important to highlight the low average activation required by sequences given by  $M1_{\Gamma}$  and  $M2_{\Gamma}$  models. Specifically, the average activation of the assembly line given by all sequences from the new models does not exceed under any case the 0.15%. This does not occur with reference models,  $M3 \cup 4_{\dot{\alpha}I}$  and  $M4 \cup 3_{\dot{\alpha}I}$ , where the average activity is 3.33% in all cases. Therefore, we can say that new models encourage the non-increase of idle time.

### 4.3. Comparative analysis of gains for company

Once it has been checked that new models may generate gains for company, as compared to the original sequencing models ( $M3 \cup 4$  and  $M4 \cup 3$ ), we calculate next the magnitude of these gains in terms of costs by both production loss ( $\Gamma_w$ ) and idle time ( $\Gamma_U$ ). In this way, we can compare the results obtained by the new models with those obtained with reference models, which present a synchronous activation of processors. Then, considering the operational costs ( $\gamma_w = 2.28 \text{€}/s$ ,  $\gamma_b = \gamma_U = 0.011 \text{€}/s$ ) and taking as reference the total costs ( $\Gamma = \Gamma_w + \Gamma_U$ ) from the models without activation ( $M3 \cup 4$  and  $M4 \cup 3$ ) we calculate the cost reduction achieved by models with pre-fixed and free activation (Table 7).

Table 7. Reduction (fraction of unity) of total operational costs ( $\Gamma = \Gamma_w + \Gamma_U$ ) given by  $M3 \cup 4_{\dot{\alpha}I}$  and  $M1_{\Gamma}$  versus  $M3 \cup 4$  ( $\alpha^N$ ), and  $M4 \cup 3_{\dot{\alpha}I}$  and  $M2_{\Gamma}$  versus  $M4 \cup 3$  ( $\alpha^N$ ). V.gr. plan #10,

$$\overline{\dot{\alpha}^S} \text{ linear function: } \Delta\Gamma/\Gamma = (\Gamma_{M4 \cup 3} - \Gamma_{M2_{\Gamma}}) / \Gamma_{M4 \cup 3} = (4836.07 - 2076.87) / 4836.07 = 0.571.$$

$\varepsilon$	Reduction of total operational costs (fraction of unity): $\Delta\Gamma/\Gamma$							
	$M3 \cup 4_{\dot{\alpha}I}$		$M4 \cup 3_{\dot{\alpha}I}$		$M1_{\Gamma}$		$M2_{\Gamma}$	
	$\dot{\alpha}^S$	$\overline{\dot{\alpha}^S}$	$\dot{\alpha}^S$	$\overline{\dot{\alpha}^S}$	$\dot{\alpha}^S$	$\overline{\dot{\alpha}^S}$	$\dot{\alpha}^S$	$\overline{\dot{\alpha}^S}$
#1	0.152	0.145	0.156	0.150	0.225	0.249	0.178	0.253
#2	0.226	0.233	0.224	0.227	0.238	0.326	0.241	0.320
#3	0.259	0.253	0.271	0.266	0.326	0.343	0.320	0.354
#4	0.224	0.219	0.232	0.226	0.236	0.313	0.248	0.319
#5	0.254	0.363	0.259	0.389	0.300	0.439	0.353	0.461
#6	0.227	0.286	0.222	0.276	0.260	0.372	0.245	0.363
#7	0.327	0.392	0.344	0.401	0.390	0.464	0.394	0.472
#8	0.119	0.113	0.100	0.093	0.134	0.221	0.136	0.203
#9	0.293	0.407	0.277	0.410	0.321	0.477	0.323	0.480
#10	0.299	0.514	0.300	0.514	0.354	0.571	0.353	0.571
#11	0.049	0.042	0.050	0.043	0.102	0.159	0.125	0.160
#12	0.190	0.190	0.201	0.195	0.201	0.288	0.214	0.292
#13	0.219	0.238	0.193	0.203	0.243	0.330	0.199	0.300
#14	0.239	0.273	0.245	0.273	0.299	0.360	0.248	0.360
#15	0.270	0.283	0.260	0.262	0.297	0.369	0.262	0.350
#16	0.179	0.173	0.159	0.160	0.194	0.273	0.224	0.261
#17	0.268	0.294	0.244	0.277	0.291	0.379	0.268	0.363
#18	0.208	0.348	0.220	0.343	0.267	0.426	0.261	0.421
#19	0.314	0.445	0.320	0.446	0.377	0.510	0.378	0.512
#20	0.086	0.079	0.104	0.097	0.159	0.191	0.153	0.207
#21	0.290	0.342	0.287	0.349	0.293	0.421	0.303	0.427
#22	0.321	0.465	0.307	0.462	0.379	0.528	0.370	0.525
#23	0.072	0.065	0.065	0.058	0.167	0.179	0.121	0.173
Avg.	0.221	0.268	0.219	0.266	0.263	0.356	0.257	0.354
Min.	0.049	0.042	0.050	0.043	0.102	0.159	0.121	0.160
Max.	0.327	0.514	0.344	0.514	0.390	0.571	0.394	0.571

Based on the obtained gains, we can state the following:

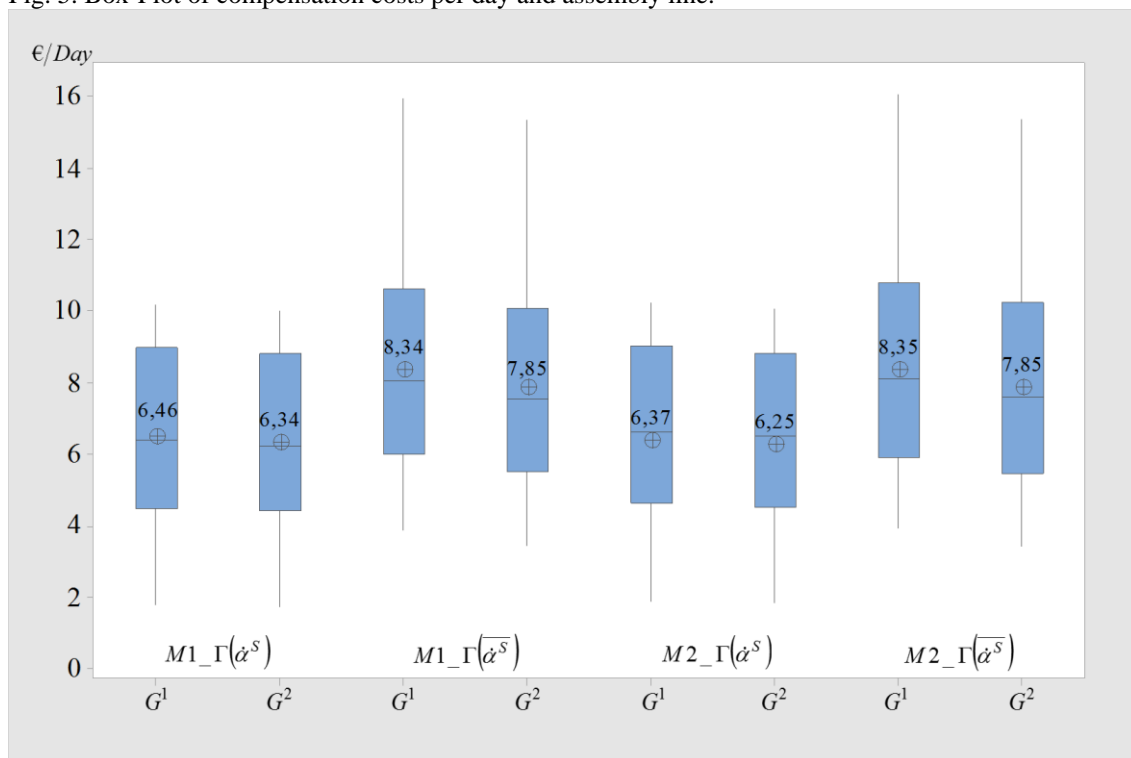
- Synchronous activation of operators throughout all workday with  $\overline{\dot{\alpha}^S}$  (linear function) reduces non-productive costs (plan #11: minimum reduction of 4.22% that means a daily minimum saving of 103.38€/day; i.e.  $\Delta\Gamma = \Gamma_{M3 \cup 4} - \Gamma_{M3 \cup 4_{\dot{\alpha}I}}$ :  $2451.76 - 2348.37 = 103.38$ ). These savings are mainly due to reduction of work overload. Indeed, the increase of costs by idle time is offset by the improvement in cost by production loss.

- $M1_\Gamma$  and  $M2_\Gamma$  models get slightly better results than reference models,  $M3 \cup 4_{\dot{\alpha}I}$  and  $M4 \cup 3_{\dot{\alpha}I}$  in all cases. Obviously, difference is in cost by idle time because models proposed in this paper only increase the activity factor of operators when it is necessary to avoid work overload.

As we can see in Table 7, the new models have similar results to  $M3 \cup 4_{\dot{\alpha}I}$  and  $M4 \cup 3_{\dot{\alpha}I}$  models (average saving of 36% versus 27%). However, the activity levels are not equal. Therefore, if we maintain our idea of economically rewarding the excess effort of operators when they work with an activity greater than the normal, the compensation costs should be added to the operational costs. For that purpose, we calculate the metrics defined in section 3, b-1 and b-2, formulas (22) and (24) respectively, for each production plan (see tables 14 and 15 in Appendix).

Figure 5 shows the distribution of compensation costs from each model, in accordance with the two profiles for the maximum work pace and the two defined metrics.

Fig. 5. Box-Plot of compensation costs per day and assembly line.



Briefly, tables 8 and 9 show the average, maximum and minimum cost that the company would have in case of compensating the excess effort of operators of the assembly line in accordance with the b-1 metric and the b-2 one, respectively.

Table 8. Daily cost by compensating the excess effort of operators ( $G^1 = \sum_{\forall k} G_k^1$ ). Average, minimum and maximum values considering all demand plans studied.

	Daily cost of the line by compensating the excess effort: $G^1$							
	$M3 \cup 4_{\dot{\alpha}I}$		$M4 \cup 3_{\dot{\alpha}I}$		$M1_{\Gamma}$		$M2_{\Gamma}$	
	$\dot{\alpha}^S$	$\overline{\dot{\alpha}^S}$	$\dot{\alpha}^S$	$\overline{\dot{\alpha}^S}$	$\dot{\alpha}^S$	$\overline{\dot{\alpha}^S}$	$\dot{\alpha}^S$	$\overline{\dot{\alpha}^S}$
Avg.	367.50	367.62	367.50	367.62	6.46	8.34	6.37	8.35
Min.	367.50	367.62	367.50	367.62	1.76(#11)	3.88(#11)	1.86(#11)	3.90(#11)
Max.	367.50	367.62	367.50	367.62	10.21(#22)	15.98(#10)	10.27(#22)	16.07(#10)

Table 9. Daily cost per line by compensating the work recovered by operators when they work with an average activity greater than the normal activity ( $G^2 = \sum_{\forall k} G_k^2$ ). Average, minimum and maximum values considering all demand plans studied.

	Daily cost of the line by compensating the excess effort: $G^2$							
	$M3 \cup 4_{\dot{\alpha}I}$		$M4 \cup 3_{\dot{\alpha}I}$		$M1_{\Gamma}$		$M2_{\Gamma}$	
	$\dot{\alpha}^S$	$\overline{\dot{\alpha}^S}$	$\dot{\alpha}^S$	$\overline{\dot{\alpha}^S}$	$\dot{\alpha}^S$	$\overline{\dot{\alpha}^S}$	$\dot{\alpha}^S$	$\overline{\dot{\alpha}^S}$
Avg.	272.02	289.38	272.01	289.38	6.34	7.85	6.26	7.85
Min.	271.62	289.29	271.63	289.29	1.72(#11)	3.42(#11)	1.84(#11)	3.44(#11)
Max.	272.38	289.46	272.39	289.46	10.02(#22)	15.34(#10)	10.09(#22)	15.37(#10)

Based on tables 8 and 9, it is worth noting the following points:

- Obviously, reference models,  $M3 \cup 4$  and  $M4 \cup 3$ , do not mean any compensation cost because they do not consider the activation of operators.
- Models with a pre-fixed activity profiles,  $M3 \cup 4_{\dot{\alpha}I}$  and  $M4 \cup 3_{\dot{\alpha}I}$ , involve greater costs of workers' compensation in all cases. These models force all processors of workstations to work with the same activity regardless an activity greater than the normal is necessary. This leads to a greater effort by operators even though the activation is not required to reduce the work overload. In addition, this activation generates idle time.
- The new models,  $M1_{\Gamma}$  and  $M2_{\Gamma}$ , reduce costs of workers' compensation significantly. With these models, operators only work with an activity above the normal one when necessary. Indeed, the average activation, considering all production mixes, does not exceed 0.1%. Furthermore, when maximum activity follows the stepped function, the new models only activate the five workstations ( $k \in \{9,10,16,17,18\}$ ) where work overloads were concentrated in previous works (see section 5 from Bautista et al., 2012a).



- Taking into account the  $M3 \cup 4_{\dot{\alpha}I}$  and  $M4 \cup 3_{\dot{\alpha}I}$  models and the linear function,  $\overline{\dot{\alpha}^S} = 1.033(\forall k, \forall t)$ , the conditions for the Theorem 1 are met and, therefore, the theorem is demonstrated ( $G_k^1 \geq G_k^2$ ).

Finally, taking into account the overall costs (costs by production loss, costs by non-productive time and costs of workers' economic compensations) of the assembly line per each manufacturing sequence, we can conclude that models proposed in this paper achieves the best results (minimal costs).

Allowing activation inside a minimum and maximum limit favors the reduction of idle time of the assembly line, as well as reduction of excess effort of operators. Thus, new models result in a decrease of costs and, therefore, in an increase of savings for company.

As an example, analyzing the first production plan, #1, we can see how the new models,  $M1_{\Gamma}$  and  $M2_{\Gamma}$ , get the best results regarding overall costs (see table 10) when the maximum activity factor is constant and equal to the average value of the stepped function (approximate cost of 2068 €/day).

Table 10. Plan #1: Work overload,  $W$ , useless time,  $U$ , cost by production loss  $\Gamma_w$ , cost by non-productive time  $\Gamma_U$ , compensation costs according to  $G^1$  and  $G^2$  metrics, overall costs considering both compensation metrics  $\Gamma + G^1$  and  $\Gamma + G^2$ .

Model	Activity	$W$	$U$	$\Gamma_w$	$\Gamma_U$	$G^1$	$G^2$	$\Gamma + G^1$	$\Gamma + G^2$
$M3 \cup 4$	$\alpha^N$	300.0	185550.0	685.7	2061.7	0.0	0.0	2747.4	2747.4
$M4 \cup 3$	$\alpha^N$	306.0	185556.0	699.4	2061.7	0.0	0.0	2761.2	2761.2
$M3 \cup 4_{\dot{\alpha}I}$	$\dot{\alpha}^S$	0.0	209745.0	0.0	2330.5	367.5	272.2	2698.0	2602.7
	$\overline{\dot{\alpha}^S}$	0.0	211295.5	0.0	2347.7	367.6	289.4	2715.3	2637.1
$M4 \cup 3_{\dot{\alpha}I}$	$\dot{\alpha}^S$	0.0	209734.4	0.0	2330.4	367.5	272.0	2697.9	2602.4
	$\overline{\dot{\alpha}^S}$	0.0	211295.5	0.0	2347.7	367.6	289.4	2715.2	2637.1
$M1_{\Gamma}$	$\dot{\alpha}^S$	28.0	185804.0	64.0	2064.5	6.0	5.8	2134.4	2134.3
	$\overline{\dot{\alpha}^S}$	0.0	185695.4	0.0	2063.3	5.4	4.9	2068.7	2068.2
$M2_{\Gamma}$	$\dot{\alpha}^S$	90.0	185705.0	205.7	2063.4	4.1	4.1	2273.2	2273.2
	$\overline{\dot{\alpha}^S}$	0.0	185689.4	0.0	2063.2	5.3	4.9	2068.6	2068.1

Finally, to briefly analyze the effect of operators' compensation, we calculate the reduction of overall costs achieved by models with activation, regarding original models without activation ( $M3 \cup 4$  and  $M4 \cup 3$ ). For that purpose, we consider the possible compensation metrics. Therefore, considering all demand plans, table 11 shows the

average and extreme values of overall cost reductions (operational costs and compensation costs).

Table 11. Overall cost reduction  $\Delta(\Gamma + G)/(\Gamma + G), G \in \{G^1, G^2\}$  achieved by  $M3 \cup 4_{\dot{\alpha}I}$  and  $M1_{\Gamma}$  versus  $M3 \cup 4$ , and  $M4 \cup 3_{\dot{\alpha}I}$  and  $M2_{\Gamma}$  models versus  $M4 \cup 3$ . Negative values mean that reference models ( $M3 \cup 4$  and  $M4 \cup 3$ ) get better results.

		Overall cost reduction: $\Delta(\Gamma + G)/(\Gamma + G), G \in \{G^1, G^2\}$							
		$M3 \cup 4_{\dot{\alpha}I}$		$M4 \cup 3_{\dot{\alpha}I}$		$M1_{\Gamma}$		$M2_{\Gamma}$	
	$G$	$\dot{\alpha}^S$	$\overline{\dot{\alpha}^S}$	$\dot{\alpha}^S$	$\overline{\dot{\alpha}^S}$	$\dot{\alpha}^S$	$\overline{\dot{\alpha}^S}$	$\dot{\alpha}^S$	$\overline{\dot{\alpha}^S}$
Avg.	$G^1$	0.11	0.15	0.10	0.15	0.26	0.35	0.26	0.35
Min.	$G^1$	-0.10	-0.11	-0.10	-0.11	0.10	0.16	0.12	0.16
Max.	$G^1$	0.24	0.44	0.25	0.44	0.39	0.57	0.39	0.57
Avg.	$G^2$	0.14	0.18	0.13	0.18	0.26	0.35	0.26	0.35
Min.	$G^2$	-0.06	-0.08	-0.06	-0.07	0.10	0.16	0.12	0.16
Max.	$G^2$	0.26	0.45	0.27	0.45	0.39	0.57	0.39	0.57

Results reinforce the performance of the models proposed in this research. Although both the flexible or forced activation of operators decreases, on average, the overall costs, new models get more savings. Moreover, in some cases, (plans #8, #11, #20 and #23) gains obtained by  $M3 \cup 4_{\dot{\alpha}I}$  and  $M4 \cup 3_{\dot{\alpha}I}$  models, in terms of work overload, do not offset extra costs by idle-time increase or by compensation of activations of operators. Therefore, allowing operators of the assembly line to increase their work pace when necessary, in order to complete the required work, and within limits established by law, brings greater profits to the company and maintains the operators' working conditions.

### 4.3. Conditions for the implementations of solutions

To implement a solution that is represented by a manufacturing sequence with its attributes at all workstation  $k (k \in K)$  and all cycle  $t (t = 1, \dots, T)$ , the following conditions must be fulfilled:

- C1. Solution must be legal: the manufacturing sequence must meet the standards established pursuant to the collective agreement between the employee and the company. Thus, no operator shall be subject to an activity factor above the maximum factor legally allowed. This condition should be given in all workstation  $k (k \in K)$  and all cycle  $t (t = 1, \dots, T)$ , throughout the two working shifts.  $M1_{\Gamma}$  and  $M2_{\Gamma}$  models meet this condition through the set (6) of constraints.

- C2. Operators must be economically compensated in accordance with their productivity: we think that it is important to compensate the overexertion of operators when they work with an activity above the normal established by company. We have two alternatives,  $G_k^1$  and  $G_k^2$ , to calculate the compensation of each workstation  $k (k \in K)$  of the assembly line.
- C3. Operators must be trained for met the requirements from Production Management: operators shall be aware the activity factor,  $\dot{\alpha}_{k,t}$ , needed at their workstations and each manufacturing cycle.
- C4. Operators must be kept informed about the rhythm and the progress of production at their workstations: an operator assigned to workstation  $k (k \in K)$  shall know at all manufacturing cycle  $t (t = 1, \dots, T)$  the following main data: (i) the type of product  $i (i \in I)$  that reaches the workstation; (ii) the subset of tasks  $J(k, t)$  that makes up the operation in progress  $(k, t)$ ; (iii) the start instant  $(s_{k,t})$  of the operation in progress; (iv); the processing time  $(\rho_{k,t})$  required to complete the operation in progress with a normal activity; (v) the time available to carry out the operation in progress, that coincides with the applied time,  $\hat{v}_{k,t}$ , at activity  $\dot{\alpha}_{k,t}$ ; and (vi) tasks that can remain unfinished whether it is desirable to generate work overload  $(w_{k,t} > 0)$ .

In the Nissan-9Eng's case, we can state the following:

1. Compliance with C1 condition is ensured in Spain and it is a common practice in the member states of OECD.
2. C2 condition is easy to meet through collective bargaining in member states of the European Union and other western countries.
3. C3 condition is a common practice in the western automotive industry. In addition, taking into account that the cycle time for the engines' assembly line is 3 minutes and one operation is composed, on average, by 6 elemental tasks, operators have enough time to adapt their activity at each cycle in accordance with the requirements of Production Management.
4. C4 condition can be easily achieved using technologies of Internet of Things (IoT) in the context of industry 4.0.

Our proposal consists of applying the industry 4.0 through implanting an information system. This system will be assisted by wireless connection

between the central computer from production management and the set of customized tablets (42 tablets to cover the 21 workstations). Thereby, tablets will report, visual and acoustically, on the production on progress to each workstation.

Accordingly, each operator will have all production information at any moment. Operator will receive through its tablet and for each manufacturing cycle, the following automatic signals: (i) audible and visual warning that indicates the beginning of an operation; (ii) accelerated audible and visual warning when the time available to complete an operation is ending (dynamic takt time), (iii) visual warning of the dynamic list of pending tasks on a manufacturing cycle, with the possibility that operator validates the concluded tasks and actualizes the list of tasks; and (iv) visual warning of the list of tasks that can be ignored by the operator whenever it is convenient to generate local work overload.

## 5. Conclusions

We have proposed two equivalent mathematical models for the MMSP-W, the  $M1_{\Gamma}$  and  $M2_{\Gamma}$ . Unlike reference models presented in previous works, the new models are focused on minimizing the operational costs of a production sequence of mixed products. Specifically, models minimize costs by both production drop (or work overload) and idle time (or non-productive time) simultaneously. Additionally, models allow processors of workstations to increase their activity factor in order to complete more amount of required work in less time. Indeed, operators' work pace is not subjected to a specific profile, and it can vary over time and from one workstation to another within a maximum and minimum limits. These upper and lower limits for the activity factor of operators are in accordance with the maximum and minimum performance values allowed in the pertinent collective labor agreement.

Operators' activation leads to complete more required work and therefore, company obtains profits. Accordingly, we have provided two metrics to compensate the excess effort of operators. Obviously, these compensations are in line with the amount of recovered work and with the average activity of processors. For this reason and with the objective of reducing the excess effort of operators, the proposed models also consider the idle time concept. Specifically, the effective work time not used by processors to

work on any product unit is economically penalized. This favors that operators' activation is the minimum necessary and therefore the injury risk is reduced.

To assess the new models, a case study linked with the Nissan's powertrain plant in Barcelona has been used, such as was used at previous papers (Bautista et al., 2012a,b and 2015a,b). This allowed us to compare the performance of new models against the reference ones with pre-fixed profiles for activity ( $M3 \cup 4_{\dot{a}I}$  and  $M4 \cup 3_{\dot{a}I}$ ) and those without activation ( $M3 \cup 4$  and  $M4 \cup 3$ ). Thus, we have been able to measure the effect of new contributions, the incorporation of variable activity factor and the penalization of idle time, as well as, the impact of both concepts on costs by unproductive work time, production drop and economic compensation of excess effort of processors.

Results show how models without activation,  $M3 \cup 4$  and  $M4 \cup 3$ , lead to greater values of work overload but less idle time. Obviously, these models do not suppose any compensation cost. However, the production sequences given by these reference models involve higher operational costs because the cost by production drop is greater than the cost by non-productive time. Thus, we can state that allowing or forcing activation to processors leads to increasing productivity and therefore, gives rise to an economic saving for the company.

Regarding reference models with activation profiles,  $M3 \cup 4_{\dot{a}I}$  and  $M4 \cup 3_{\dot{a}I}$ , and the new models with bounded activation,  $M1_{\Gamma}$  and  $M2_{\Gamma}$ , although both get equal values for the completed work when the maximum activity is linear and constant (work overload equal to 0 in all cases), the firsts involve greater idle time and greater average activation. Therefore, the cost by operators' compensations and the costs by idle time are higher and sometimes, costs are not offset by the work overload reduction. In short, models proposed in this paper reduce costs by production drop, idle time and operators' compensation, simultaneously. Thus, new models obtain the lower overall costs of the production sequences. Indeed, in terms of average values,  $M1_{\Gamma}$  and  $M2_{\Gamma}$  models achieve an approximate reduction by 26% of overall costs in regard with the  $M3 \cup 4$  and  $M4 \cup 3$  models, when the maximum activity factor only is greater than the normal throughout a third of workday (stepped function); and by 35% when the maximum activation is linear and constant throughout all workday. This supposes savings greater than 250 €/day in the worst case, reaching even 2759 €/day for the demand plan number #10. Similarly, the  $M1_{\Gamma}$  and  $M2_{\Gamma}$  reduce costs from

$M3 \cup 4_{\dot{\alpha}I}$  and  $M4 \cup 3_{\dot{\alpha}I}$  by 14% and 21% on average, considering the minimum compensation ( $G^2$ ).

Furthermore, results have also allowed us for observing the following: (i) the economic compensation obtained from the average activity level of processors is more likely to favor operators, while compensation obtained from recovered work time, favors to company; and (ii) models with activation must reduce the work overload such a way as to offset the extra costs by both compensation of activation and increase of idle time.

Logically, reference models,  $M3 \cup 4_{\dot{\alpha}I}$  and  $M4 \cup 3_{\dot{\alpha}I}$ , require more amount of recovered work to bring profits to the company, favoring the excess effort of operators.

Indeed, solutions from  $M3 \cup 4$  and  $M4 \cup 3$  for some production plans involve less work overload, and therefore, forcing the activation of operators leads to greater costs.

However, this does not occur when a free but bounded activation is allowed.  $M1_{\Gamma}$  and  $M2_{\Gamma}$  models require a negligible minimum reduction of work overload.

Accordingly, it is clearly shown that  $M1_{\Gamma}$  and  $M2_{\Gamma}$  models bring more profits to the company and operators simultaneously, because company can compensate their workers and still get profits from the initial situation.

However, although we have obtained good results from the new models through MILP, in future works we want to develop the following lines: (L1) design and implementation of new procedures to solve the  $MMSP_{\Gamma}$ , such as GRASP and GRASP-LP; (L2) design and explorations of new models and procedures for the  $MMSP_{\Gamma}$  with the bi-objective of maximizing productivity and minimizing operators' activation.

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## Appendix 1

Table 12. Work overload values ( $W$ ) for all the demand plans ( $\varepsilon \in E$ ) given by the reference models,  $M3 \cup 4$ ,  $M4 \cup 3$ ,  $M3 \cup 4_{\dot{\alpha}I}(\dot{\alpha}^S)$ ,  $M4 \cup 3_{\dot{\alpha}I}(\dot{\alpha}^S)$ ,  $M3 \cup 4_{\dot{\alpha}I}(\overline{\dot{\alpha}^S})$  and  $M4 \cup 3_{\dot{\alpha}I}(\overline{\dot{\alpha}^S})$ . Values with the symbol ‘\*’ are optimal solutions.

$\varepsilon$	Work overload for all the demand plans: $W$					
	Normal activity: $\alpha^N$		Stepped function ( $\dot{\alpha}^S$ )		Linear function ( $\overline{\dot{\alpha}^S}$ )	
	$M3 \cup 4$	$M4 \cup 3$	$M3 \cup 4_{\dot{\alpha}I}$	$M4 \cup 3_{\dot{\alpha}I}$	$M3 \cup 4_{\dot{\alpha}I}$	$M4 \cup 3_{\dot{\alpha}I}$
#1	300.0	306.0	0.0 *	0.0 *	0.0 *	0.0 *
#2	437.0	426.0	17.0	12.0	0.0 *	0.0 *
#3	473.0	496.0	0.0 *	0.0 *	0.0 *	0.0 *
#4	412.0	424.0	0.0 *	0.0 *	0.0 *	0.0 *
#5	709.0	776.0	182.0	225.0	0.0 *	0.0 *
#6	536.0	515.0	93.0	84.0	0.0 *	0.0 *
#7	785.0	810.0	116.0	105.0	0.0 *	0.0 *
#8	256.0	231.0	0.0 *	0.0 *	0.0 *	0.0 *
#9	827.0	837.0	204.0	239.0	0.0 *	0.0 *
#10	1208.0 *	1208.0 *	459.0	458.0	0.0 *	0.0 *
#11	171.0	172.0	0.0 *	0.0 *	0.0 *	0.0 *
#12	366.0	374.0	7.0	0.0 *	0.0 *	0.0 *
#13	446.0	387.0	33.0	21.0	0.0 *	0.0 *
#14	510.0	509.0	55.0	46.0	0.0 *	0.0 *
#15	530.0	489.0	27.0	10.0	0.0 *	0.0 *
#16	340.0	320.0	0.0 *	8.0	0.0 *	0.0 *
#17	552.0	517.0	46.0	53.0	0.0 *	0.0 *
#18	672.0	659.0	228.0	198.0	0.0 *	0.0 *
#19	945.0 *	951.0	249.0	242.0	0.0 *	0.0 *
#20	214.0	236.0	0.0 *	0.0 *	0.0 *	0.0 *
#21	657.0	673.0	89.0	104.0	0.0 *	0.0 *
#22	1014.0	1004.0	282.0	301.0	0.0 *	0.0 *
#23	197.0	189.0	0.0 *	0.0 *	0.0 *	0.0 *

Table 13. Useless time ( $U$ ) for all the demand plans ( $\varepsilon \in E$ ) given by the reference models,  $M3 \cup 4$ ,  $M4 \cup 3$ ,  $M3 \cup 4_{-\dot{\alpha}I}(\dot{\alpha}^S)$ ,  $M4 \cup 3_{-\dot{\alpha}I}(\dot{\alpha}^S)$ ,  $M3 \cup 4_{-\dot{\alpha}I}(\overline{\dot{\alpha}^S})$  and  $M4 \cup 3_{-\dot{\alpha}I}(\overline{\dot{\alpha}^S})$ . Values with the symbol ‘\*’ are optimal solutions.

$\varepsilon$	Useless time for all the demand plans: $U$					
	Normal activity: $\alpha^N$		Stepped function ( $\dot{\alpha}^S$ )		Linear function ( $\overline{\dot{\alpha}^S}$ )	
	$M3 \cup 4$	$M4 \cup 3$	$M3 \cup 4_{-\dot{\alpha}I}$	$M4 \cup 3_{-\dot{\alpha}I}$	$M3 \cup 4_{-\dot{\alpha}I}$	$M4 \cup 3_{-\dot{\alpha}I}$
#1	185550.0	185556.0	209744.96	209734.4	211295.5	211295.5
#2	185737.0	185726.0	209798.34	209773.8	211343.9	211343.9
#3	185883.0	185906.0	209884.65	209878.6	211450.4	211450.4
#4	185582.0	185594.0	209675.91	209657.7	211218.0	211218.0
#5	186004.0	186071.0	209986.29	210019.5	211339.0	211339.0
#6	185701.0	185680.0	209744.85	209721.2	211213.2	211213.2
#7	186190.0	186215.0	210022.12	210012.5	211445.5	211445.5
#8	185561.0	185536.0	209774.39	209820.1	211348.8	211348.8
#9	185882.0	185892.0	209705.00	209740.7	211106.6	211106.6
#10	186743.0	186743.0	210472.36	210470.9	211571.3	211571.3
#11	185481.0	185482.0	209770.15	209771.8	211353.7	211353.7
#12	185676.0	185684.0	209830.99	209786.1	211353.6	211353.6
#13	185689.0	185630.0	209774.70	209743.2	211288.7	211288.7
#14	185778.0	185777.0	209798.70	209804.4	211312.9	211312.9
#15	185865.0	185824.0	209833.17	209824.9	211377.8	211377.8
#16	185615.0	185595.0	209745.50	209748.9	211319.7	211319.7
#17	185877.0	185842.0	209840.83	209873.6	211368.1	211368.1
#18	185807.0	185794.0	209833.58	209810.3	211184.1	211184.1
#19	186430.0	186436.0	210204.63	210205.9	211523.0	211523.0
#20	185469.0	185491.0	209734.52	209749.8	211300.4	211300.4
#21	185742.0	185758.0	209625.27	209648.5	211135.7	211135.7
#22	186469.0	186459.0	210238.12	210259.3	211493.9	211493.9
#23	185532.0	185524.0	209825.84	209810.3	211377.8	211377.8

Table 14. Daily cost by compensating the excess effort of operators working with an average activity greater than the normal activity ( $G^1 = \sum_{k=1}^{|K|} G_k^1$ ) given by the models:  $M3 \cup 4_{-\dot{\alpha}I}$ ,  $M4 \cup 3_{-\dot{\alpha}I}$ ,  $M1_{-\Gamma}$  and  $M2_{-\Gamma}$ .

$\varepsilon$	Daily cost of the line by compensating the excess effort: $G^1$							
	$M3 \cup 4_{-\dot{\alpha}I}$		$M4 \cup 3_{-\dot{\alpha}I}$		$M1_{-\Gamma}$		$M2_{-\Gamma}$	
	$\dot{\alpha}^S$	$\overline{\dot{\alpha}^S}$	$\dot{\alpha}^S$	$\overline{\dot{\alpha}^S}$	$\dot{\alpha}^S$	$\overline{\dot{\alpha}^S}$	$\dot{\alpha}^S$	$\overline{\dot{\alpha}^S}$
#1	367.50	367.62	367.50	367.62	5.96	5.40	4.11	5.34
#2	367.50	367.62	367.50	367.62	5.59	7.05	5.62	7.25
#3	367.50	367.62	367.50	367.62	7.01	7.80	7.09	7.56
#4	367.50	367.62	367.50	367.62	5.47	6.47	5.14	6.60
#5	367.50	367.62	367.50	367.62	9.65	10.61	9.61	10.82
#6	367.50	367.62	367.50	367.62	5.53	8.64	6.62	8.13
#7	367.50	367.62	367.50	367.62	10.19	11.31	9.58	11.09
#8	367.50	367.62	367.50	367.62	2.91	4.68	2.76	4.56
#9	367.50	367.62	367.50	367.62	8.96	11.57	9.01	11.19
#10	367.50	367.62	367.50	367.62	9.19	15.98	9.52	16.07
#11	367.50	367.62	367.50	367.62	1.76	3.88	1.86	3.90
#12	367.50	367.62	367.50	367.62	4.45	6.43	4.63	6.90
#13	367.50	367.62	367.50	367.62	4.45	6.53	5.40	6.62
#14	367.50	367.62	367.50	367.62	7.79	8.03	7.21	8.08
#15	367.50	367.62	367.50	367.62	6.40	8.21	6.74	8.26
#16	367.50	367.62	367.50	367.62	3.93	6.00	5.52	5.89
#17	367.50	367.62	367.50	367.62	6.75	8.94	6.66	8.99
#18	367.50	367.62	367.50	367.62	7.43	9.61	6.47	9.99
#19	367.50	367.62	367.50	367.62	9.53	13.16	9.15	13.06
#20	367.50	367.62	367.50	367.62	4.67	4.19	3.53	4.30
#21	367.50	367.62	367.50	367.62	7.44	9.48	7.67	9.56
#22	367.50	367.62	367.50	367.62	10.21	13.67	10.27	13.78
#23	367.50	367.62	367.50	367.62	3.24	4.19	2.43	4.00
Avg.	367.50	367.62	367.50	367.62	6.46	8.34	6.37	8.35
Min.	367.50	367.62	367.50	367.62	1.76	3.88	1.86	3.90
Max.	367.50	367.62	367.50	367.62	10.21	15.98	10.27	16.07

Table 15. Daily cost by compensating the recovered work by operators working with an average activity greater than the normal activity ( $G^2 = \sum_{k=1}^{|K|} G_k^2$ ) given by the models:  $M3 \cup 4_{\dot{\alpha}I}$ ,  $M4 \cup 3_{\dot{\alpha}I}$ ,  $M1_{\Gamma}$  and  $M2_{\Gamma}$ .

Daily cost of the line by compensating the excess effort: $G^2$								
$\varepsilon$	$M3 \cup 4_{\dot{\alpha}I}$		$M4 \cup 3_{\dot{\alpha}I}$		$M1_{\Gamma}$		$M2_{\Gamma}$	
	$\dot{\alpha}^S$	$\overline{\dot{\alpha}^S}$	$\dot{\alpha}^S$	$\overline{\dot{\alpha}^S}$	$\dot{\alpha}^S$	$\overline{\dot{\alpha}^S}$	$\dot{\alpha}^S$	$\overline{\dot{\alpha}^S}$
#1	272.17	289.39	272.05	289.39	5.84	4.95	4.06	4.88
#2	272.01	289.38	271.80	289.38	5.49	6.56	5.43	6.75
#3	271.94	289.34	271.87	289.34	6.78	7.27	6.87	7.04
#4	272.29	289.42	272.09	289.42	5.43	6.02	5.10	6.16
#5	272.33	289.38	272.22	289.38	9.46	10.10	9.40	10.26
#6	272.08	289.42	271.91	289.42	5.47	8.19	6.54	7.75
#7	272.23	289.34	272.25	289.34	9.97	10.84	9.42	10.50
#8	271.88	289.38	272.39	289.38	2.86	4.22	2.71	4.10
#9	271.62	289.46	271.63	289.46	8.80	11.07	8.84	10.78
#10	271.98	289.29	271.98	289.29	9.07	15.34	9.34	15.37
#11	271.78	289.37	271.80	289.37	1.72	3.42	1.84	3.44
#12	272.38	289.37	271.96	289.37	4.30	5.94	4.52	6.40
#13	272.21	289.40	271.99	289.40	4.42	6.05	5.33	6.16
#14	271.95	289.39	272.12	289.39	7.63	7.56	7.03	7.58
#15	271.90	289.36	272.00	289.36	6.21	7.75	6.57	7.74
#16	271.89	289.39	271.84	289.39	3.88	5.54	5.46	5.44
#17	271.89	289.37	272.17	289.37	6.57	8.41	6.53	8.48
#18	271.90	289.43	271.97	289.43	7.33	9.16	6.41	9.55
#19	271.90	289.31	271.99	289.31	9.38	12.54	9.00	12.41
#20	271.99	289.39	272.16	289.39	4.62	3.74	3.51	3.85
#21	271.68	289.45	271.77	289.45	7.34	9.07	7.56	9.13
#22	272.23	289.32	272.26	289.32	10.02	13.15	10.09	13.16
#23	272.12	289.36	271.95	289.36	3.16	3.70	2.38	3.54
Avg.	272.02	289.38	272.01	289.38	6.34	7.85	6.26	7.85
Min.	271.62	289.29	271.63	289.29	1.72	3.42	1.84	3.44
Max.	272.38	289.46	272.39	289.46	10.02	15.34	10.09	15.37