Reheating constraints in quintessential inflation

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We study the consequences of reheating in quintessential inflation. From simple inflationary quintessential models introduced in [1, 2], we show that when the reheating is due to the production of heavy massive particles conformally coupled with gravity, a viable model which matches with the current observational data [3–5] is only possible for reheating temperatures that range between 1 GeV and 10^4 GeV. On the other hand, when the universe reheats via the production of massless particles, the viability of the model is only possible when those particles are nearly conformally coupled with gravity, leading to a reheating temperature between 1 MeV and 10^4 GeV.

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1. INTRODUCTION

In two recent papers [1, 2] some families of quintessential inflation models have been obtained, coming from very simple polynomial potentials which fit well with the current observational data provided by Planck's team [3, 4] and the BICEP/Keck-Planck Collaboration [5]. The models unify the early inflationary period with the current cosmic acceleration and contain a phase transition from the inflationary phase to a kination regime, which is essential to produce enough particles to thermalize the universe with a reheating temperature compatible with the bounds coming from nucleosynthesis.

In the present work we explore the consequences and constraints that one obtains from the relation that exists between the spectral index and the reheating temperature. This relation comes from the fact that the number of e-folds could be obtained in two completely different ways: by definition through an expression that only depends on the spectral index or using the whole history of the universe that leads to a function of the reheating temperature and the spectral index. On the other hand, to obtain simple expressions of the reheating temperature one can consider either the production of heavy massive particles conformally coupled with gravity or massless particles nearly conformally coupled with gravity.

Therefore, for a given reheating temperature between 1 MeV and 10^9 GeV, which is required in order to have a successful nucleosynthesis [6], one obtains the corresponding value of the spectral index and, thus, the value of the ratio of tensor to scalar perturbations. Since this ratio must be less than 0.12 [5], this constrains the reheating temperature to range between 1 MeV and 10^4 GeV. Moreover, since in the case of creation of heavy particles the mass of these produced particles must be less than Planck's mass -in the opposite case these particles would become micro black holes-, the model only supports temperatures greater than 1 GeV and less than 10^4 GeV. Finally, dealing with massless particles, we have shown that the viability of the model is only possible when the coupling constant is very close to 1/6, i.e., the created particles have to be nearly conformally coupled with gravity.

The paper is organized as follows: In Section II, we depict all the characteristics of the simplest model of the family, which corresponds to a potential whose inflationary piece is a quadratic one and that, in order to have a phase transition to a kination regime, is matched with a small cosmological constant which is responsible for the current cosmic acceleration. Section III is devoted to relate the reheating temperature, obtained from different particle production after the phase transition, with the spectral parameters, which leads to a reheating temperature less than 10^4 GeV in order to match with the current observation data. Finally, we have introduced an Appendix where we prove one of the basic formulas of this work: the energy density of the heavy massive particles conformally coupled with gravity. In [1, 2] this quantity has been calculated using the concept of *number of particle produced in curved spacetimes*, which does not have a universal consensus [7], whereas here we obtain this quantity directly from the definition of energy density and using the "adiabatic prescription" (see [8] for a review of the method) to regularize it.

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The units used throughout the paper are $\hbar = c = 1$ and, with these units, $M_{pl} = \frac{1}{\sqrt{8\pi G}}$ is the reduced Planck's mass.

2. REVIEW OF THE MODEL

In a recent paper [2] one of the authors of the present work and his collaborators introduced a family of models depending on a parameter α, which matches, at 2σ C.L., with the current observational data provided by Planck's team [3, 4] when $\alpha \in [0, \frac{1}{2}]$. Here, we consider the simplest one which corresponds to $\alpha = 0$ and which describes a universe containing a small cosmological constant Λ and filled with a fluid with a very simple linear Equation of State (EoS), which has a sudden phase transition when the energy density is $\rho = \rho_E$,

$$P = \begin{cases} -\rho + 2\rho_E & \text{for } \rho \geq \rho_E \\ \rho & \text{for } \rho \leq \rho_E, \end{cases} \tag{1}$$

where $\rho_E = (3H_E^2 - \Lambda)M_{pl}^2$, being H_E the parameter of the model. A simple calculation leads to the following dynamics

$$\dot{H} = \begin{cases} -3H_E^2 + \Lambda & \text{for } H \geq H_E \\ -3H^2 + \Lambda & \text{for } H \leq H_E, \end{cases} \tag{2}$$

which can be analytically solved leading to the following Hubble parameter

$$H(t) = \begin{cases} \left(-3H_E^2 + \Lambda\right)t + H_E & \text{when } t \leq 0 \\ \sqrt{\frac{\Lambda}{3}} \frac{3H_E + \sqrt{3\Lambda}\tanh(\sqrt{3\Lambda}t)}{3H_E\tanh(\sqrt{3\Lambda}t) + \sqrt{3\Lambda}} & \text{when } t \geq 0, \end{cases} \tag{3}$$

and the corresponding scale factor is

$$a(t) = \begin{cases} a_E e^{\left[\left(-3H_E^2 + \Lambda\right)\frac{t^2}{2} + H_E t\right]} & \text{when } t \leq 0 \\ a_E \left(\frac{3H_E}{\sqrt{3\Lambda}}\sinh(\sqrt{3\Lambda}t) + \cosh(\sqrt{3\Lambda}t)\right)^{\frac{1}{3}} & \text{when } t \geq 0. \end{cases} \tag{4}$$

Moreover, the effective EoS parameter, namely w_{eff}, which is defined as $w_{eff} \equiv \frac{P}{\rho} = -1 - \frac{2\dot{H}}{3H^2}$, for our model is given by

$$w_{eff} = \begin{cases} -1 + \frac{2}{3H^2}\left(3H_E^2 - \Lambda\right) & \text{when } H \geq H_E \\ 1 - \frac{2\Lambda}{3H^2} & \text{when } H \leq H_E, \end{cases} \tag{5}$$

which shows that for $H \gg H_E$ one has $w_{eff}(H) \cong -1$, meaning that we have an early inflationary quasi de Sitter period. As has been proved in [2], this model provides theoretical values of the spectral index and the ratio of tensor to scalar perturbations which enters, at 2σ C.L., in the 2-dimensional marginalized joint confidence contour in the plane (n_s, r), without the presence of running in the spectral index. At the phase transition, i.e. when $H \cong H_E$, the EoS parameter satisfies $w_{eff}(H) \cong 1$ and the universe enters in a kination or deflationary period [9, 10], and finally, for $H \cong \sqrt{\frac{\Lambda}{3}}$ one also has $w_{eff}(H) \cong -1$ depicting the current cosmic acceleration.

Remark 2.1 *The dynamics in* (2) *could also be obtained assuming that the universe is filled with a viscous fluid. Effectively, in this case the Raychauduri equation becomes [11]*

$$\dot{H} = -\frac{3}{2}(1 + w)H^2 + \frac{3\zeta}{2M_{pl}^2}H + \frac{1}{2}(1 + w)\Lambda. \tag{6}$$

Therefore, taking $w = 1$ and the following viscosity coefficient

$$\zeta = \begin{cases} 2M_{pl}^2\left(H - \frac{H_E^2}{H}\right) & H \geq H_E \\ 0 & H \leq H_E, \end{cases} \tag{7}$$

one obtains the dynamics (2) *after inserting it in* (6).

On the other hand, using the reconstruction method [1, 2], one can also calculate the quadratic potential that leads to this dynamics

$$V(\phi) = \begin{cases} \frac{9}{2}\left(H_E^2 - \frac{\Lambda}{3}\right)\left(\phi^2 - \frac{2}{3}M_{pl}^2\right) & \text{for} \quad \phi \leq \phi_E \\ \Lambda M_{pl}^2 & \text{for} \quad \phi \geq \phi_E, \end{cases} \tag{8}$$

where the value of the scalar field at the transition time is $\phi_E \equiv -\sqrt{\frac{2}{3}}\frac{H_E M_{pl}}{\sqrt{H_E^2 - \frac{\Lambda}{3}}} \cong -\sqrt{\frac{2}{3}}M_{pl}$ and we have included the cosmological potential into the potential.

Finally, the number of e-folds from when the pivot scale leaves the Hubble radius until the end of inflation, and the main slow-roll parameter when the pivot scale crosses the Hubble radius, in terms of the spectral index, can be calculated exactly in this model, giving as a result

$$N = \frac{1}{2}\left(\frac{4}{1-n_s} - 1\right), \qquad \epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{1-n_s}{4} \Longrightarrow r = 16\epsilon = 4(1-n_s). \tag{9}$$

We note also that the value of the parameter of the model H_E is obtained from the theoretical [12] and the observational [13] value of the power spectrum

$$\mathcal{P} \cong \frac{H^2}{8\pi^2 \epsilon M_{pl}^2} \sim 2 \times 10^{-9}, \tag{10}$$

using that for our model $H \cong \frac{H_E}{\left(\frac{\epsilon}{3}\right)^{\frac{1}{2}}}$ and $\epsilon = \frac{1-n_s}{4}$, obtaining

$$H_E \sim 7 \times 10^{-4}\left(\frac{1-n_s}{12}\right)M_{pl}. \tag{11}$$

3. REHEATING CONSTRAINTS

The model contains a phase transition from inflation to kination which is essential to create enough particles which reheat the universe [9, 14]. When the produced particles are very massive and conformally coupled with gravity satisfying $H_E \ll m \leq M_{pl}$, its energy density evolves like (see the Appendix)

$$\rho \sim 4 \times 10^{-3}\frac{H_E^6}{m^2}\left(\frac{a_E}{a}\right)^3, \tag{12}$$

where a_E is the scale factor at the transition time. These particles decay into lighter particles (see [15] for details) leading to a reheating temperature of the order [2]

$$T_R \sim 10^{-1}\left(\frac{H_E}{M_{pl}}\right)^2\left(\frac{H_E}{m}\right)M_{pl} \sim 3 \times 10^{-11}\left(\frac{1-n_s}{12}\right)^3\frac{M_{pl}}{m}M_{pl}, \tag{13}$$

and when one only considers massless particles nearly conformally coupled with gravity, its energy density decays as [14, 16]

$$\rho \sim \mathcal{N}\left(\xi - \frac{1}{6}\right)^2 H_E^4\left(\frac{a_E}{a}\right)^4, \tag{14}$$

where ξ is the coupling constant and $\mathcal{N} = \frac{1}{8\pi^2}\int_0^\infty s|g(s)|^2 ds$ where $g(s) = \frac{1}{H_E^4 a_E^4}\int_{-\infty}^\infty e^{-2is\tau}a^2(\tau)R(\tau)d\tau$. By integrating twice by parts it is straightforward to prove that $|g(s)|$ is of order $1/s^2$ for large values of s because $R(\tau)$ is continuous at the transition time. Hence, $\mathcal{N}$ is not UV-divergent. Moreover, proceeding analogously as in [1], we can compute an upper bound for $\mathcal{N}$ by using Cauchy-Schwarz inequality and Plancherel Theorem,

$$\mathcal{N} \leq \frac{1}{32\pi H_E^4 a_E^4}\sqrt{\int_{-\infty}^\infty a(t)\left(\frac{d(a^2(t)R(t))}{dt}\right)^2 dt}\sqrt{\int_{-\infty}^\infty a^3(t)R^2(t)dt} \approx 3.59, \tag{15}$$

showing that the value $\mathcal{N}$ is convergent. Moreover, we have been able to compute numerically a lower bound, i.e., $\mathcal{N} \geq \frac{1}{8\pi^2} \int_0^{20} s|g(s)|^2 ds \approx 2.12$. Therefore, we can state that $\mathcal{N}$ is of order 1. In this case the reheating temperature becomes

$$T_R \sim \mathcal{N}^{3/4} \left| \xi - \frac{1}{6} \right|^{3/2} \frac{H_E^2}{M_{pl}^2} M_{pl} \sim 5 \times 10^{-7} \left| \xi - \frac{1}{6} \right|^{3/2} \left(\frac{1-n_s}{12} \right)^2 M_{pl}. \tag{16}$$

Remark 3.1 *Formula* (13) *is only valid for masses smaller than the reduced Planck mass, because for $M_{pl} \lesssim m$ the produced particles become micro black holes. Effectively, an elementary particle of mass m has a Compton wavelength $1/m$ and the corresponding Schwarzschild radius is $\frac{m}{4\pi M_{pl}^2}$. Then for $m \geq \sqrt{4\pi}M_{pl}$ the Compton wavelength is smaller than its Schwarzschild radius [17]. For these micro black holes, Hawking's formulas about evaporation are not applicable because in this case the emitted particles have masses of the order of Planck's mass or greater [18], which means that the thermodynamic description breaks down at these scales and it is unknown whether or how they radiate [19].*

On the other hand, the number of e-folds can also be calculated using the well-known formula [20]

$$\frac{k}{a_0 H_0} = e^{-N} \frac{H}{H_0} \frac{a_{end}}{a_E} \frac{a_E}{a_R} \frac{a_R}{a_M} \frac{a_M}{a_0} \tag{17}$$

being k the pivot scale and where "end", R, M and 0 respectively symbolize the end of inflation, the beginning of radiation era, the beginning of matter domination era (when the energy density of radiation and matter is the same) and the value of any quantity at the current time.

For our model one obtains

$$N = 70.94 - \frac{1}{6} \left[2 - \ln \left(\frac{1-n_s}{12} \right) \right] - \frac{1}{3} \ln \left(\frac{g_R^{\frac{1}{4}} T_R}{\text{GeV}} \right), \tag{18}$$

where $g_R = 107$, 90 and 11 for $T_R \geq 175$ GeV, 175 GeV $\geq T_R \geq 200$ MeV and 200 MeV $\geq T_R \geq 1$ MeV, respectively.

This quantity has to be equal to the number of e-folds calculated in formula (9), obtaining a relation between the spectral index and the reheating temperature

$$\frac{1}{2} \left(\frac{4}{1-n_s} - 1 \right) = 70.94 - \frac{1}{6} \left[2 - \ln \left(\frac{1-n_s}{12} \right) \right] - \frac{1}{3} \ln \left(\frac{g_R^{\frac{1}{4}} T_R}{\text{GeV}} \right); \tag{19}$$

hence, for a given temperature between 1 MeV and 10^9 GeV one obtains the value of the spectral index for our model. And once one has this value for a given temperature, the mass of the quantum field can be computed using formula (13) or the coupling constant with (16) when one deals with massless particles.

Defining $Y = \frac{12}{1-n_s}$, and $F(Y) = Y + \ln Y$ one obtains the equation

$$Y + \ln Y = 426.64 - 2\ln \left(\frac{g_R^{\frac{1}{4}} T_R}{\text{GeV}} \right). \tag{20}$$

which, given T_R, only has a solution, because the function $Y + \ln Y$ is monotone.

3.1. Heavy massive particles conformally coupled with gravity

Then, for heavy massive particles, if one chooses

1. $T_R = 1$ MeV, one obtains

$$Y + \ln Y = 439.25, \tag{21}$$

which leads to the following value of the spectral index $n_s \cong 0.9723$, and a ratio of tensor to scalar perturbations equal to $r = 0.1108$, where we have used that $r = 4(1 - n_s)$.

Further, using (13) one obtains the following mass $m \sim 10^3 M_{pl}$. The value of the Hubble parameter at the transition time will be $H_E \sim 10^{-6} M_{pl}$, and its value when the pivot scale leaves de Hubble radius is $H \sim 2 \times 10^{-5} M_{pl}$. Finally the number of e-folds is $N = 72$.

2. $T_R = 1$ GeV, one finds

$$Y + \ln Y = 424.39, \tag{22}$$

which leads to the following value of the spectral index $n_s \cong 0.9713$, and a ratio of tensor to scalar perturbations equal to $r = 0.1148$.

Further, using (13) one obtains the following mass $m \sim M_{pl}$. The value of the Hubble parameter at the transition time will be $H_E \sim 10^{-6} M_{pl}$, and its value when the pivot scale leaves de Hubble radius is $H \sim 2 \times 10^{-5} M_{pl}$. Finally the number of e-folds is $N = 69$.

3. $T_R = 10^4$ GeV, one finds

$$Y + \ln Y = 405.88, \tag{23}$$

which leads to the following value of the spectral index $n_s \cong 0.9700$, and a ratio of tensor to scalar perturbations equal to $r = 0.1200$.

Further, using (13) one obtains the following mass $m \sim 10^{-4} M_{pl}$. The value of the Hubble parameter at the transition time will be $H_E \sim 10^{-6} M_{pl}$, and its value when the pivot scale leaves de Hubble radius is $H \sim 2 \times 10^{-5} M_{pl}$. Finally the number of e-folds is $N = 66$.

4. $T_R = 10^9$ GeV, one finds

$$Y + \ln Y = 382.85, \tag{24}$$

which leads to the following value of the spectral index $n_s \cong 0.9682$, and a ratio of tensor to scalar perturbations equal to $r = 0.1272$.

Further, using (13) one obtains the following mass $m \sim 10^{-9} M_{pl}$. The value of the Hubble parameter at the transition time will be $H_E \sim 2 \times 10^{-6} M_{pl}$, and its value when the pivot scale leaves de Hubble radius is $H \sim 4 \times 10^{-5} M_{pl}$. Finally the number of e-folds is $N = 62$.

In fact, in Figure 1 we show the corresponding ratio of tensor to scalar perturbations r and mass m obtained from a range of temperatures between 1 MeV and 10^9 GeV. From these results we can conclude that:

- The theoretical value of the spectral index provided by the model ranges from 0.9682 to 0.9723 and, since Planck2015 data obtained the observational value $n_s = 0.968 \pm 0.006$, all the theoretical values of n_s enter in the 1-dimensional marginalized 2σ C.L.

- Since $Y(T_R)$ is a decreasing function with the reheating temperature, and $r(T_R) = \frac{48}{Y(T_R)}$, one concludes that the tensor/scalar ratio is an increasing function with the reheating temperature. Therefore, for temperatures greater than 10^4 GeV, our model does not support the observational data given by the joint analysis of BICEP2/ Keck Array and Planck Data, where the B-mode polarization constrains the ratio of tensor to scalar perturbations to $r < 0.12$ at 2σ C.L. [5]. Moreover, since our formula (13) only holds for masses smaller than the Planck's one, one can conclude that, when the reheating is due to the production of heavy massive particles, our model only supports reheating temperatures between 1 GeV and 10^4 GeV.
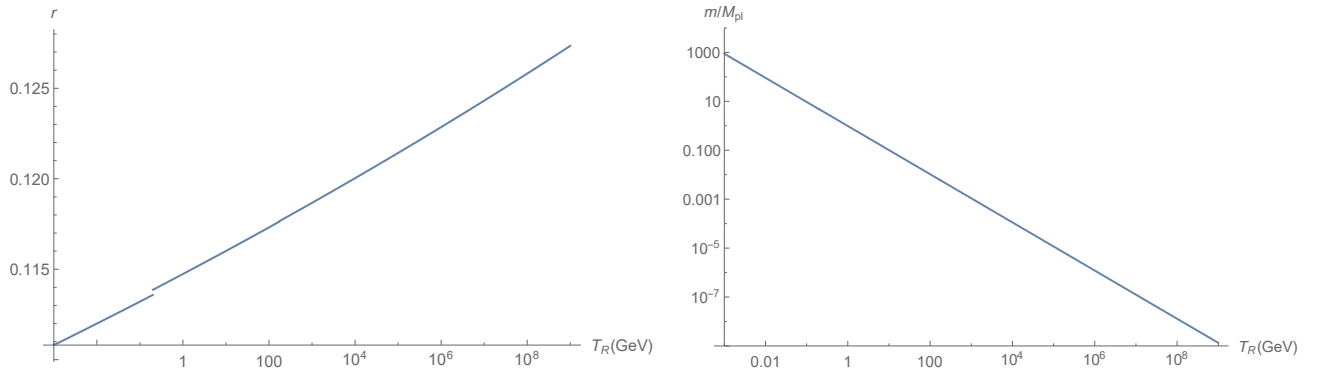
FIG. 1: Evolution of the tensor/scalar ratio r (left) and of the mass of the produced particles m (right) versus the reheating temperature T_R.

3.2. Massless particles non-conformally coupled with gravity

As we have seen above using equation (19), for our model the temperatures that lead to a ratio of tensor to scalar perturbations less than 0.12 range between 1 MeV and 10^4 GeV. Therefore, using formula (16) it is not difficult to check that, for these temperatures, the coupling constant ξ must satisfy

$$3 \times 10^{-7} \lesssim \left| \xi - \frac{1}{6} \right| \lesssim 10^{-2}. \tag{25}$$

On the other hand, dealing with massless particles, which are far from the conformal coupling with gravity, by using a toy model where there is a phase transition from de Sitter phase to radiation regime, it has been shown that the energy density of these produced particles is approximately [21]

$$\rho \sim 10^{-2} H_E^4 \left(\frac{a_E}{a} \right)^4, \tag{26}$$

which leads, after equating it to the reheating time with the energy density of the background, to the following reheating temperature

$$T_R \sim 3 \times 10^{-2} \frac{H_E^2}{M_{pl}^2} M_{pl} \sim 10^{-8} \left(\frac{1 - n_s}{12} \right)^2 M_{pl}, \tag{27}$$

which together with equation (19) defines a system of two equations where the spectral index and the reheating temperature are the unknown variables.

Unfortunately, the system is only compatible, i.e. has solutions, for temperatures greater than 10^5 GeV, leading to a tensor/scalar ratio greater than 0.12, which disagrees with the recent observational data [5] meaning that the model does not work for particles with a conformal coupling far from 1/6. Consequently, we can conclude that our model only supports the production of massless particles with a coupling constant very close to 1/6, i.e. nearly conformally coupled with gravity, leading to a reheating temperature ranging between 1 MeV and 10^4 GeV.

4. CONCLUSIONS

In the present work we have obtained formula (19), which relates the spectral index with the reheating temperature. Since for the quintessential inflation model studied here the spectral index (n_s) and the ratio of tensor to scalar perturbations (r) are related through $r = 4(1 - n_s)$, from the observational constraint $r \leq 0.12$ [5] one can deduce using (19) that the reheating temperature must be below 10^4 GeV.

On the other hand, the simplest way to reheat the universe is via the production, at the phase transition from inflation to kination, of very heavy massive particles conformally coupled with gravity or massless particles. In the

former case, since elementary particles with masses around the Planck's mass become micro black holes, whose physics is unknown, one has to consider the production of particles with mass smaller than the Planck's one, which leads to a reheating temperature greater than 1 GeV. Moreover, the constraint for the mass of the produced particles, $10^{-4}M_{pl} \leq m \leq M_{pl}$, has also been found by using expression (12), which refers to the energy density of massive produced particles as is extensively proved in the Appendix.

Finally, dealing with the creation of massless particles, we have shown that the viability of the model is only possible when these particles are nearly conformally coupled with gravity, because when the coupling constant is far from 1/6 one obtains reheating temperatures greater than 10^5 GeV, which is incompatible with the fact that the tensor/scalar ratio must be smaller than 0.12.

5. APPENDIX

In this appendix we will calculate the one-loop energy density due to a heavy massive quantum field, namely χ, conformally coupled with gravity. This quantity is given by [22]

$$\rho_\chi(\tau) = \frac{1}{4\pi^2 a^4(\tau)} \int_0^\infty (|\chi_k'|^2 + (k^2 + m^2 a^2(\tau))|\chi_k|^2)k^2 dk, \tag{28}$$

where χ_k is the k-mode associated to the vacuum. If initially, at some initial time namely τ_i, the quantum field is in the vacuum state, then the modes must satisfy at that time

$$\chi_k(\tau_i) = \frac{e^{-i\int^{\tau_i}\omega_k(\tau)d\tau}}{\sqrt{2\omega_k(\tau_i)}}, \quad \chi_k'(\tau_i) = -i\sqrt{\frac{\omega_k(\tau_i)}{2}}e^{-i\int^{\tau_i}\omega_k(\tau)d\tau}, \tag{29}$$

where $\omega_k(\tau) = \sqrt{k^2 + m^2 a^2(\tau)}$ is the frequency of the k-mode.

Inserting this quantity in the one-loop energy density one obtains

$$\rho_\chi(\tau_i) = \frac{1}{4\pi^2 a^4(\tau_i)} \int_0^\infty \omega_k(\tau_i)k^2 dk, \tag{30}$$

which is the zero-point energy density of the vacuum. This divergent quantity could be removed from different ways: the simplest one is to subtract to the one loop energy density the zero-point energy density, obtaining the following convergent quantity (see [23])

$$\rho_\chi^{conv}(\tau) = \frac{1}{4\pi^2 a^4(\tau)} \int_0^\infty (|\chi_k'|^2 + (k^2 + m^2 a^2(\tau))|\chi_k|^2 - \omega_k(\tau))k^2 dk. \tag{31}$$

In order to compute this integral, we use that the χ_k mode can be approximated by the WKB solution of 2n-th order, namely $\chi_{k,WKB}(\tau) = \frac{1}{\sqrt{W_{2n,k}(\tau)}}e^{-i\int^\tau W_{2n,k}(\eta)d\eta}$, defined by the recurrence [24]

$$W_{2n,k}(\tau) = \begin{cases} \omega_k(\tau), & n = 0 \\ \omega_k(\tau) - \frac{1}{2\omega_k}\left[\frac{W_{2(n-1),k}''(\tau)}{2W_{2(n-1),k}(\tau)} - \frac{3}{4}\left(\frac{W_{2(n-1),k}'(\tau)}{W_{2(n-1),k}(\tau)}\right)^2\right], & n \in \mathbb{N}. \end{cases} \tag{32}$$

Then, the integral in (31) in order zero becomes $\rho_\chi^{conv} \approx \frac{m^2 H^2}{96\pi}$. However, this prescription only holds for conformally coupled fields. When one deals with non-conformally coupled fields one of the most populars ways to renormalize the energy density is to use the adiabatic regularization, which consists in subtracting the zero, second and fourth order adiabatic expressions of the energy density (see for instance [25]). In this way, as has been showed in [26], when $m \gg H$ the renormalized energy density, namely $\rho_\chi^{ren}(\tau)$ is of the order $\frac{H^6}{m^2}$, then for values of the Hubble parameter satisfying $H \leq M_{pl}$ and $m \gg H$, since $\frac{H^6}{m^2} \ll H^2 M_{pl}^2$ one can conclude that back-reaction does not affect the dynamics of the background.

This condition is broken at the phase transition, because there the WKB approximation cannot hold and the negative and positive frequencies mix. Hence, the χ_k mode becomes approximately of the form $\alpha_k\chi_{k,WKB} + \beta_k\chi_{k,WKB}^*$, where

the β-Bogoliubov coefficient is given by $\beta_k = -i\mathcal{W}[\chi_k(\eta_E^-), \chi_k(\eta_E^+)]$ and verifying that $|\alpha_k|^2 - |\beta_k|^2 = 1$. Thus, the energy density becomes

$$
\begin{aligned}
\rho_\chi^{ren}(\tau) = \frac{1}{4\pi^2 a^4(\tau)} \int_0^\infty & \left\{ \left[|\chi'_{k.WKB}|^2 + \omega_k^2(\tau)|\chi_{k,WKB}|^2 \right] (1 + 2|\beta_k|^2) \right. \\
& \left. + 2\mathcal{R}e(\alpha_k \beta_k^* \left[(\chi'_{k,WKB})^2 + \omega_k^2(\tau)(\chi_{k,WKB})^2 \right]) \right\} k^2 dk - \rho_4^{adia},
\end{aligned}
\tag{33}
$$

where ρ_4^{adia} contains all the adiabatic terms up to order 4.

Since, as has been showed in [26], the term

$$
\frac{1}{4\pi^2 a^4(\tau)} \int_0^\infty \left[|\chi'_{k.WKB}|^2 + \omega_k^2(\tau)|\chi_{k,WKB}|^2 \right] k^2 dk - \rho_4^{adia},
\tag{34}
$$

is of the order H^6/m^2, using that $\alpha_k \simeq 1$ and the approximation to zero order $\chi_{k,WKB}(\eta) \approx \frac{e^{-i\int^\eta \omega_k(\tau)d\tau}}{\sqrt{2\omega_k(\eta)}}$, we obtain

$$
\rho_\chi^{ren}(\tau) \approx \frac{1}{4\pi^2 a^4(\tau)} \int_0^\infty \left[\frac{(\omega'_k(\tau))^2}{4\omega_k^3(\tau)} |\beta_k| + 2\omega_k(\tau)|\beta_k|^2 \right] k^2 dk.
\tag{35}
$$

In [2], it was already computed that $|\beta_k| \approx \frac{9m^2 a_E^5 H_E^3}{8(k^2 + m^2 a_E^2)^{5/2}}$. Therefore, we obtain

$$
\rho_\chi^{ren} \approx \frac{81 H_E^6}{128\pi^2 m^2} \left(\frac{a_E}{a} \right)^4 \int_0^\infty \frac{x^2 \sqrt{x^2 + \left(\frac{a}{a_E} \right)^2}}{(x^2 + 1)^5} dx + \frac{9 H^2 H_E^3}{128\pi^2 m} \left(\frac{a}{a_E} \right)^2 \int_0^\infty \frac{x^2 dx}{\left(x^2 + \left(\frac{a}{a_E} \right)^2 \right)^{5/2} (x^2 + 1)^{5/2}}.
\tag{36}
$$

First of all we analyze the second term, which can be bounded by

$$
\frac{9 H^2 H_E^3}{128\pi^2 m} \left(\frac{a_E}{a} \right)^3 \int_0^\infty \frac{x^2 dx}{(x^2 + 1)^2} = \frac{9 H^2 H_E^3}{512\pi m} \left(\frac{a_E}{a} \right)^3.
\tag{37}
$$

Since the energy density of the background evolves as $H^2 M_{pl}^2$ and $H_E < m < M_{pl}$ one can easily check that (37) is subdominant with respect to the background.

To deal with the first term, we note that after the phase transition the universe enters into a kination regime, whose dynamics are given by $\dot{H} \cong -3H^2$. Then, with a simple calculation, we find that the time needed after the phase transition to have $a = 10^2 a_E$ is of the order $\frac{10^6}{H_E} \sim 10^{-32} s$. This is a very small time compared with the time after the phase transition to reheat the universe, which is of the order $10^{-10} s$ [27]. Then, one can assume that from the phase transition to the end of the reheating process $a/a_E \gg 1$, and following the calculations performed in [28] one can make the approximation $\sqrt{x^2 + \left(\frac{a}{a_E} \right)^2} \cong \frac{a}{a_E}$, to finally obtain

$$
\rho_\chi^{ren} \approx \frac{405 H_E^6}{32768\pi m^2} \left(\frac{a_E}{a} \right)^3 \sim 4 \times 10^{-3} \frac{H_E^6}{m^2} \left(\frac{a_E}{a} \right)^3.
\tag{38}
$$

Acknowledgments

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