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The role of cognitive limitations and heterogeneous expectations for aggregate production and credit cycle

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The behavioural model of De Grauwe and Macchiarelli (2015) is extended to include financial frictions on the (aggregate) supply side. The result is a tight and sustained feedback loop between animal spirits on one hand, and supply of credit, capital purchase and production on the other. During phases of optimism, credit is abundant, access to production capital is easy, the cash-in-advance constraint is lax, risks are undervalued, and production is booming. Upon reversal in market sentiment, the contraction is quick and deep. Moreover, the model is capable of replicating the stylized fact of a long and sustained simultaneous growth in credit, production and asset prices observed in the US since mid 1990's. Lastly, the behavioural model does a decent job in matching US data including multiple supply-side relations such as capital-firm credit and inflation-interest rate.

Keywords: Supply-side, beliefs, financial frictions, model validation
JEL: B41, C63, C68, E22, E23, E37

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1 Motivation

Several empirical studies over the past years have argued that a combination of impaired financial services, a sharp drop in productivity and lack of supply side reforms has generated severe contractions and sharp drops in asset prices observed across many advanced economies since the beginning of the Great Recession. Broadbent (2012) finds that the main reason for the most recent contraction in the UK business cycle has been a contraction in the supply side. He argues that a combination of uneven demand across sectors combined with an impaired financial system (due to its inability to effectively reallocate resources sufficiently quickly to respond to shocks) has led to a reduction in aggregate output per employee. Equally, in a business cycle accounting exercise, Chadha and Warren (2012) find that both the efficiency wedge as well as labour wedge have equally been key drivers in the most recent UK recession. However, while the contractions might show up in those wedges, the original shocks might not necessarily originate from the supply side of the economy. Using a standard financial frictions model, they show that an asset price shock might equally appear in the supply-side wedges and generate equivalent losses to a scenario where shocks originate in the supply side. Therefore they suggest that the supply side might be a propagator of shock originated elsewhere rather than being the key cause of contractions. The model results of Chiarella and De Guilmi (2011) point in the same direction. Equivalently, for Italy, Manasse (2013) has argued that the main cause behind the most recent recession (and some other southern Euroarea countries) has been a weak and anaemic production and financial sector. A lack of reform in the credit market combined with stagnation in the product-and labour markets has resulted in weak productivity and low competitiveness performance for more than a decade.

Yet the interaction between financial frictions and the supply side has not been extensively studied in the theoretical macroeconomics (or finance) literature. In the few studies where such interactions are captured, such as Christiano et al (2005), or Gertler et al (2011), the models do not manage to capture the tight (positive) evolution between asset prices on one hand, and credit, production and productivity on the other. Since product markets are complete, imperfections from the credit market do not translate into stagnation in productivity, and further down the line a sustained drop in asset prices. Instead, credit only affects asset prices insofar that agents face constraints in borrowing, and thus in their earnings. However, it is impossible to generate endogenous sustained drops in asset prices. Since beliefs are perfect and rational, these imperfections are corrected relatively quickly on the market unless new shocks emerge. This is because in these frameworks expectations adjust quickly and there is no discrepancy on the market regarding the future path of the economy, allowing a quick market readjustment.

In the current paper, on the other hand, we take a different path to the existing literature and examine the interaction between credit imperfections, production/productivity and asset prices under a framework of bounded rationality and heterogeneous expectations. In particular, we wish to examine how imperfect financial markets coupled with (imperfect) stock market beliefs affect the allocations on the production side of the economy. We do not claim that this is the only way to examine the interactions, but provide a novel and interesting avenue to analyse the role of limited cognitive abilities on the feedback mechanism between credit, production and asset prices. We will focus our analysis on three channels: the role of credit frictions for production and asset prices, the role of imperfect information for (expected) production and productivity, and the role of heterogeneous expectations for asset prices. To check the validity and consistency of the model, we perform the standard impulse response analysis, analysis in business cycle frequency, as well as statistical matching with US data using a sample of more than 60 years of macroeconomic and financial data.

Impulse response analyses show that (temporary) supply-side shocks do not only improve the fundamentals in the economy but cause a brief wave of market sentiment (or animal spirit), which in the case of a positive shock result in a more-than-proportional increase in output, capital supply, bank equity, and fall in interest rate. Moreover, credit supply to entrepreneurs is permanently increased, which means that firms can access greater external financing in the future. As a result, firm productivity is permanently improved.

The statistical validation of the model (including moment matching) show that the model is capable of replicating important characteristics found in the US data. This includes variables such as the (risk-free) interest rate, inflation, deposits, firm financing spread and net worth of banks. It is also successful in matching several supply-side relations (capital-firm credit, inflation-interest rate) as well as their autocorrelations (output, capital and inflation). Moreover, we find a strong co-movement between asset prices on one hand, and net worth and the financing spread on the other hand. We are capable of replicating long and sustained booms and subsequent sharp reversals in asset prices and net worth, which has been characterieristic of the US macro-financial cycles since mid-1990's.

2 Model set-up

To incorporate a supply side with an asset price bubble and financing constraints in a New-Keynesian macro framework, we apply the following adjustments to the model in De Grauwe and Macchiarelli (2015). The first modification is an extension of the financial accelerator mechanism onto input markets. We allow a firm's purchasing position on the input markets to directly depend on its financial state. A stronger financial (net worth) position allows a firm to borrow more, which in turn presses the marginal costs down, and allows it to buy capital inputs at a (relatively) lower price.

The second modification is a pay-in-advance constraint on the input market. We impose the condition that (a share of) the cost of capital must be paid in advance of purchase in order to insure capital good producers that they will sell what they produce. It is a kind of depository insurance. We make the pre-payment share time varying over the business cycle in order to capture the asymmetries in financial (or liquidity) positions over the cycle.

The third modification we introduce is a utilization rate of capital. Producing firms, apart from choosing the amount of capital to purchase, choose the rate at which capital will be used in production activities. Higher share increases the number of (intermediate) goods produced for a fixed amount of capital at the same time as it increases the rate of capital depreciation. More details are provided in subsection 2.2.

With respect to the original paper of De Grauwe and Macchiarelli (2015), we use most of their original model, but exclude the time-invariant leverage ratio at the same time as we extend the constant bank net worth and the external finance premium. Moreover, we include a production sector (CGP and entrepreneurs). Full list of equations is included in the appendix.

We will provide a detailed outline of the three modifications in the next subsection (including a description of the incomplete information-and learning framework). The model is then solved in section 3 and the simulated quantitative results are analysed in section 4. Section 5 concludes.

2.1 Supply side and financial frictions

In what follows, we will disentangle capital production from capital utilization rate, and introduce variable capital usage in an otherwise standard financial accelerator mechanism (augmented with stock market cycles) as in DeGrauwe and Macchiarelli (2015).[1] Capital good producers produce capital which they rent to entrepreneurs at a cost R_t^s. Entrepreneurs use the newly purchased capital and labour to produce intermediate goods. Whereas capital good producers and entrepreneurs operate in perfectly competitive goods markets, retailers are monopolistically competitive. Therefore they price discriminate, resulting in price frictions on the aggregate supply side (Phillips curve).

2.1.1 Capital good producers

Following Gerali et al (2010), perfectly competitive capital good producers (CGP) produce a homogeneous good called "capital services" using input of the final output from entrepreneurs $(1-\delta)k_{t-1}$ and retailers (i_t), and the production is subject to investment adjustment costs. They sell new capital to entrepreneurs at price Q_t. The objective of a CGP is to choose a K_t and I_t to solve:

$$\max_{K_t, I_t} E_0 \Sigma_{t=0}^{\infty} \Lambda_{0,t} [Q_t[K_t - (1-\delta)K_{t-1}] - I_t] \tag{1}$$

subject to:

$$K_t = (1-\delta)K_{t-1} + [1 - \frac{\kappa_i}{2}[\frac{i_t \epsilon_t^{qk}}{i_{t-1}} - 1]^2] I_t \tag{2}$$

where $[1 - \frac{\kappa_i}{2}(\frac{i_t \epsilon_t^{qk}}{i_{t-1}} - 1)^2] I_t$ is the adjustment cost function. κ_t denotes the cost for adjusting investment and ϵ_t^{qk} is a shock to the efficiency of investment. Including adjustment costs of investment in the production of capital solves the so-called "investment puzzle" and produces the hump-shaped investment in response to a monetary policy shock (Smets and Wouters, 2007 and Christiano et al, 2011).

2.1.2 Entrepreneurs

Perfectly competitive entrepreneurs produce intermediate goods using the constant returns to scale technology:

[1] For a discussion of the remaining model set-up, we refer to the aforementioned paper and DeGrauwe (2008, 2012).

$$Y_t = A_t[\psi(u_t)K_t]^\alpha L^{1-\alpha} \tag{3}$$

with A_t being the stochastic total factor productivity, u_t the capacity utilization rate, and K_t and L_t capital and labour inputs. Capital is homogeneous in this model.[2] We assume a fixed survival rate of entrepreneurs in each period γ, in order to ensure a constant amount of exit and entry of firms in the model. This assumption also assures that firms will always depend on external finances for their capital purchases, so they will never become financially self-sufficient.

We introduce a shock to productivity ϵ_t^z (calibrated to 0.5) to later examine the impact of a productivity increase in the macroeconomic performance. With the shock, equation 3 becomes:

$$Y_t = A_t[\epsilon_t^z\psi(u_t)K_t]^\alpha L^{1-\alpha} \tag{4}$$

Just as in the financial accelerator model (Bernanke, Gertler and Gilchrist, 1999) we will continue to work under the assumption that all earnings (after paying the input costs) from production are re-invested into the company such that a constant share is paid out to shareholders.[3] This is why entrepreneurs will maximize their value function rather than their production function.[4]

Entrepreneurs also choose the level of capacity utilization, $\psi(u_t)$. As is standard in the capital utilization literature, the model assumes that using capital more intensively raises the rate at which it depreciates.[5] The increasing convex function $\psi(u_t)k_t$ denotes the (relative) cost in units of investment good of setting the utilization rate to u_t. This is chosen before the realization of the production shock (see Auernheimer and Trupkin (2014) for similar assumption). This timing assumption is important because it separates the choice of the stock of productive factor K_t, taken before the revelation of the states of nature, from the choice of the flow of factor u_tK_t, taken during the production process.

The choice of the rate of capital utilization involves the following trade-off. On

[2] We could have made capital firm-specific, but the set-up would have to be much more complex without qualitatively altering the results. Using homogeneous capital assumption is standard in these type of models, see for instance Bernanke et al (1999), Gerali et al (2010), Gertler et al (2012). Chiarella and De Guilmi (2011) develop a model where firms hold heterogeneous capital structures, and examine the transmission of (micro-level) financial shocks to the production sector.

[3] In our exercises, we will set this share to 0, similar to Bernanke et al (1999).

[4] And so y_t is not a direct argument of the function.

[5] We could equally assume a fixed rate of capital depreciation and impose a cost as a function of output, as in Christiano et al (2005) or Gerali et al (2010).

the one hand, a higher u_t implies a higher output. On the other hand, there is a cost from a higher depreciation of the capital stock. Therefore this rate can be understood as an index that shows how much of the stock of capital is operated relative to the steady state, per unit of time, given a capital-labour services ratio.

Moreover we specify the following functional form for $\psi(u_t)$:

$$\psi(u_t) = \xi_0 + \xi_1(u_t - 1) + \frac{\xi_2}{2}(u_t - 1)^2 \tag{5}$$

in line with Schmitt-Grohe and Uribe (2006), Gerali et al (2010), and Auernheimer and Truphin (2014).[6] To examine the sensitivity of capital utilization on production, we will introduce a shock in utilization costs uc_t according to:

$$\psi(u_t) = \xi_0 + \xi_1(u_t - 1) + \frac{\xi_2}{2}(u_t - 1)^2 + uc_t \tag{6}$$

where uc_t has the following AR structure:

$$uc_t = \rho_{uc} uc_{t-1} + \epsilon_t^{uc} \tag{7}$$

and ϵ_t^{uc} is a white noise shock, which is calibrated to 0.5. In our simulations, we calibrate the AR component ρ_{uc} to 0.1 in order to strictly limit the possibility of the shock driving the model dynamics. However, a simple white noise utilization cost shock is excessively short-lived, and would not allow us to study the endogenous dynamics in full.

To understand how a firm's financial position influences its' purchasing power in the capital input market, we need to understand the costs it faces. A firm minimizes the following cost function:

$$S(Y_t) = \min_{k,l}[R_t^s K_t + w_t L_t] \tag{8}$$

The real marginal cost is therefore $s(Y_t) = \frac{\partial S(Y_t)}{\partial (Y_t)}$, which is:

$$s(Y_t) = \frac{1}{1-\alpha}^{1-\alpha} \frac{1}{\alpha}^{\alpha} (r_t^s)^{\alpha} (w_t)^{1-\alpha} \tag{9}$$

The gross return on capital is defined as $R_t^s = \frac{S_t - S_{t-1}}{S_{t-1}}$. Keeping the wage rate constant, an increase in the (stock) market value of capital reduces the (relative) cost of capital service inputs, purchased at today's capital price.[7]

[6] In the simulations, u_t will be normalized such that $\psi(u_t) = 0.80$.

[7] In line with the costs that intermediate firms face in the model of Christiano et al (2005).

This is easier to see in the entrepreneur's budget constraint:[8]

$$S_{t+1}K_{t+1} + w_tL_t + \psi(u_t)k_{t-1} + R_tB_{t-1} + (1-\vartheta)S_tK_t = \frac{Y_t}{X_t} + B_t + S_t(1-\delta)K_{t-1} \Rightarrow$$

$$S_{t+1}K_{t+1} + w_tL_t + \psi(u_t)K_t + R_t[S_tK_t - N_t] + (1-\vartheta)S_tK_t = \frac{Y_t}{X_t} + S_{t+1}K_{t+1} - N_{t+1}] + S_t(1-\delta)K_{t-1} \quad (10)$$

where

$$B_t = S_tK_t - N_t \quad (11)$$

or B_t is the amount entrepreneurs borrow, δ being the depreciation rate of capital, $\psi(u_t)K_{t-1}$ the cost of setting a level u_t of the utilization rate, ϑ is the share of capital purchases required to be paid in advance by CGP, and $\frac{P_t^f}{P_t} = \frac{1}{X_t}$ is the relative competitive price of the final good in relation to the capital good (i.e. mark-up).[9] An increase in the (stock) market price (right-hand side) has two effects.[10] First, it reduces the contemporaneous (relative) cost of capital purchases since firms can borrow more and pay a higher pre-payment share ϑ of capital. Second, a higher market price means that the probability of default of an entrepreneur drops (since the value of the firm is higher), CGP will expect entrepreneurs to be solvent in the next period and will therefore require a smaller pre-payment (i.e. ϑ on the left-hand side will fall). Let us elaborate on this second mechanism a bit further.

As a form of depository insurance, CGP will in some periods require entrepreneurs to pay in period t a share of the total capital produced and delivered to entrepreneurs in period *t+1*. In particular, when CGPs suspect that entrepreneurs will either face liquidity problems, a lower production, or a lower collateral value in the next period, they assume the firm to be less solvent. Because the default probability of entrepreneurs rises, CGP become suspicious of the entrepreneur's ability to pay for the entire capital purchase. Therefore, as an insurance mechanism, CGP will ask

[8]We assume that entrepreneurs borrow up to a maximum permitted by the borrowing constraint.

[9]Note that $\vartheta S_{t+1}K_{t+1} \leq S_{t+1}K_{t+1}$.

[10]For an explanation of the evolution of stock prices, we refer to Annex I, or De Grauwe and Macchiarelli (2015).

the entrepreneur to pay in advance a share of its capital purchases.[11] The share to be paid is tightly linked to the amount that the entrepreneur can borrow on the credit market B_t.

Formally, the pay-in-advance constraint that entrepreneurs face in the input market is:

$$S_{t+1}K_{t+1} \leq \vartheta_t B_t \equiv \vartheta_t[S_{t+1}K_{t+1} - N_t] \tag{12}$$

We allow ϑ to vary over time in order to capture the variations in CGP's precautionary motive over the business cycle. In theory, a value of 1 means that the entrepreneur will need to use *all* of his external finances (loans) to pay for the capital purchases since CGP expect entrepreneur's financial (cash) position to sharply worsen in the next period. Conversely, a value of 0 means that no pre-payment is required as CGP expect the entrepreneur to be able to re-pay in full in the next period. As a result, the constraint will not be binding. Considering the entire parameter spectrum of ϑ, the constraint will almost always be binding, except for the extreme case when $\vartheta = 0$, or no pre-payment is required at all due to very sound financial conditions of the entrepreneur. In our long-run simulations, the constraint is mostly binding since slightly more than half of the time ϑ is above 0. This implies that in most cases CGPs require entrepreneurs to pay at least a share of their input purchases in advance.

Both the individual and aggregate capital stock evolves according to:

$$K_t = (1-\delta)K_{t-1} + \Psi(\frac{I_t}{K_t})K_{t-1} \tag{13}$$

where $\Psi(\frac{I_t}{K_t})K_{t-1}$ are the capital adjustment costs in the usage of capital. $\Psi(.)$ is increasing and convex, and $\Psi(0) = 0$.[12] Note that the micro-founded dynamic problem of entrepreneurs is very similar to the myopic expected utility maximization problem of agents in Chiarella et al (2006).

[11] We could equivalently assume that legal conditions/constraints stipulate that entrepreneurs need to pay in advance for their inputs as in Champ and Freedman (1990, 1994). Our approach is analogue to the one taken in Fuerst (1995) or Christiano and Eichenbaum (1992) for labour input costs.

[12] The log-linearized version of this expression is: $k_t = (1-\delta)k_{t-1} + \delta i_t$, as in Bernanke, Gertler and Gilchrist (1999) or Gerba (2017) and the one used in the simulations. δi_t is the steady state version of $\Psi(\frac{I_t}{K_t})K_{t-1}$.

2.2 Aggregate dynamics

Since we have introduced a production economy in the baseline behavioural model, we also need to adapt the aggregate system equations. First we need to link capital to real interest rate. Linking the investment demand equation from DeGrauwe and Macchiarelli (2015):

$$i_t = i(\rho)_t = e_1 \tilde{E}_t y_{t+1} + e_2(\rho - \tilde{E}_t \pi_{t+1});\ e_2 < 0 \tag{14}$$

to the aggregate capital accumulation 13, we find that the relation between capital and the real interest rate is:

$$k_t = (1-\delta)k_{t-1} + \Psi(\frac{i_t}{i_{t-1}})i(\rho)_t = (1-\delta)k_{t-1} + \Psi(\frac{i_t}{i_{t-1}})e_1 \tilde{E}_t y_{t+1} + e_2(r_t + x_t - \tilde{E}_t \pi_{t+1}); e_2 < 0 \tag{15}$$

Note that we have introduced $\tilde{E}_t$ in both expressions above. That is the behavioural extension of the rational expectations E_t operator, where the formation of these expectations is ruled by a particular learning process described in the next subsection.[13] Continuing by incorporating a supply side into the aggregate system of De Grauwe and Macchiarelli (2015) - by combining equations 14, 15 and 5 - gives:

$$y_t = a_1 \tilde{E}_t y_{t+1} + (1-a_1)y_{t-1} + a_2(r_t - \tilde{E}_t \pi_{t+1}) + (a_2+a_3)x_t + (a_1-a_2)\psi(u_t)k_t + Adj_t + \epsilon_t;\ (a_1-a_2) > 0 \tag{16}$$

Note that the first four terms are equal to the aggregate demand expression in De Grauwe and Macchiarelli (2015). Besides that, aggregate demand now also depends on the usable capital in the production, $u_t k_t$ but discounted for the cost of financing (x_t). Christiano et al. (2005), Smets and Wouters (2007), and Gerali et al. (2010) arrive at the same resource constraint expression in their models. There is an adjustment cost in investment given by equation 15, which we capture by $Adj_t = \Psi(\frac{i_t}{i_{t-1}}) \equiv \Psi(\frac{I_t}{K_t})$.[14] However, it will be calibrated in such a way to equal δ, as in standard DSGE models.

[13] The standard rational expectations (RE) E_t is a special case of the broader expectations operator we consider in this paper. In other words, if we reduce our system to a standard expectations-setting, then the model would only remain with credit-and supply side frictions. There would not be any information frictions anymore.

[14] These two are equivalent since, via the Cobb-Douglas production function, capital has a constant share in production over time and is homogeneous.

Monetary policy affects aggregate demand in two ways. First, it affects the opportunity cost of current consumption and investment, via the third term in equation 16. Second, it affects return on investment in expression 15, which in turn affects capital demand, captured by the fifth term in the aggregate demand expression. To examine the impact of changes in interest rate on the economy, we introduce a (negative) monetary policy shock (ϵ_t^{mp} calibrated to 0.5) with the following autoregressive process:

$$r_t = r_{t-1} + \gamma\pi_t + (1-\gamma)y_t + \epsilon_t^{mp} \tag{17}$$

We will examine the system's response to this shock in section 4.1.1.

The reader will notice that aggregate demand also depends on the external finance (or risk) premium x_t. This is a reduced form expression for investment, since investment is governed directly by this premium, and therefore it is the dependent variable (see DeGrauwe and Macchiarelli (2015) for a derivation of this term).

The aggregate supply (AS) equation is obtained from the price discrimination problem of retailers (monopolistically competitive):

$$\pi_t = b_1\tilde{E}_t\pi_{t+1} + (1-b_1)\pi_{t-1} + b_2y_t + \nu_t \tag{18}$$

As explained in DeGrauwe and Macchiarelli (2015), $b_1 = 1$ corresponds to the New-Keynesian version of AS with Calvo-pricing (Woodford (2003), Branch and McGaugh (2009)). Setting $0 < b_1 < 1$ we incorporate some price inertia in the vein of Gali and Gertler (1999). Equally, the parameter b_2 varies between 0 and ∞ and reflects the degree of price rigidities in the context of a Calvo pricing assumption (DeGrauwe, 2012). A value of $b_2 = 0$ corresponds to complete price rigidity and $b_2 = \infty$ to perfect price flexibility (firms have a probability of 1 of changing prices in period t).

2.3 Expectations formation and learning

Next, we wish to characterize the information friction and heterogeneous expectations used in this model. We will focus on two things. First, we will model the cognitive limitation that agents have regarding the aggregate states, which is out of direct control for them. Second, we will outline the learning framework that they use in order to gain a better understanding of those aggregate states. The learning framework incorporates heterogeneous expectations in its intrinsic dynamics.

Under rational expectations, the expectations term will equal its realized value in the next period, i.e. $E_t\Phi_{t+1} = \Phi_{t+1}$, denoting generically by Φ_t any variable in the model. However, as anticipated above, we depart from this assumption in this framework by considering bounded rationality as in DeGrauwe (2011, 2012). Expectations are replaced by a convex combination of heterogeneous expectation operators $E_t y_{t+1} = \tilde{E}_t y_{t+1}$ and $E_t\pi_{t+1} = \tilde{E}_t\pi_{t+1}$. In particular, agents forecast output and inflation using two alternative forecasting rules: *fundamentalist rule* vs. *extrapolative rule*. Under the fundamentalist rule, agents are assumed to use the steady-state value of the output gap - y^*, here normalized to zero against a naive forecast based on the gap's latest available observation (extrapolative rule). Equally for inflation, *fundamentalist* agents are assumed to base their expectations on the central bank's target - π^* against the *extrapolists* who naively base their forecast on a random walk approach.[15] We can formally express the fundamentalists in inflation and output forecasting as:

$$\tilde{E}_t^f \pi_{t+1} = \pi^* \tag{19}$$

$$\tilde{E}_t^f y_{t+1} = y^* \tag{20}$$

and the extrapolists in both cases as:

$$\tilde{E}_t^e \pi_{t+1} = \theta\pi_{t-1} \tag{21}$$

$$\tilde{E}_t^e y_{t+1} = \theta y_{t-1} \tag{22}$$

This particular form of adaptive expectations has previously been modelled by Pesaran (1987), Brock and Hommes (1997, 1998), and Branch and McGough (2009), amongst others. Setting $\theta = 1$ captures the "naive" agents (as they have a strong belief in history dependence), while a $\theta < 1$ or $\theta > 1$ represents an "adaptive" or an "extrapolative" agent (Brock and Hommes, 1998). For reasons of tractability, we set $\theta = 1$ in this model.

All variables here are expressed in gaps. Focusing on their cyclical component makes the model symmetric with respect to the steady state (see Harvey and Jaeger,

[15] The latest available observation is the best forecast of the future.

1993). Therefore, as DeGrauwe and Macchiarelli (2015) show, it is not necessary to include a zero lower bound constraint in the model since a negative interest rate should be understood as a negative interest rate gap. In general terms, the equilibrium forecast/target for each variable is equal to its' steady state value.

Next, selection of the forecasting rule depends on the (historical) performance of the various rules given by a publically available goodness-of-fit measure, the mean square forecasting error (MSFE). After time t realization is revealed, the two predictors are evaluated *ex post* using MSFE and new fractions of agent types are determined. These updated fractions are used to determine the next period (aggregate) forecasts of output-and inflation gaps, and so on. Agents' rationality consists therefore in choosing the best-performing predictor using the updated fitness measure. There is a strong empirical motivation for inserting this type of switching mechanism amongst different forecasting rules (see DeGrauwe and Macchiarelli (2015) for a brief discussion of the empirical literature, Frankel and Froot (1990) for a discussion of *fundamentalist* behaviour, and Roos and Schmidt (2012), Cogley (2002), Cogley and Sargent (2007) and Cornea, Hommes and Massaro (2013) for evidence of *extrapolative* behaviour, in particular for inflation forecasts). More recently, Chiarella et al (2012) performed an empirical validation of a reduced-form heterogeneous agent financial model with Markov chain-regime dependent expectations and showed that such a learning mechanism matches well the boom-bust cycle in the US stock market at the same time as it has good predictability power.

Just as in Chiarella and Khomin (1999), the aggregate market forecasts of output-and inflation gap is obtained as a weighted average of each rule:

$$\tilde{E}_t \pi_{t+1} = \alpha_t^f \tilde{E}_t^f \pi_{t+1} + \alpha_t^e \tilde{E}_t^e \pi_{t+1} \tag{23}$$

$$\tilde{E}_t y_{t+1} = \alpha_t^f \tilde{E}_t^f y_{t+1} + \alpha_t^e \tilde{E}_t^e y_{t+1} \tag{24}$$

where α_t^f is the weighted average of fundamentalists, and α_t^e that of the extrapolists. Following Chiarella and Khomin (1999), these shares are time-varying and based on the dynamic predictor selection. The mechanism allows to switch between the two forecasting rules based on MSFE / utility of the two rules, and increase (decrease) the weight of one rule over the other at each t. Assuming that the utilities of the two alternative rules have a deterministic and a random component (with a log-normal distribution as in Manski and McFadden (1981) or Anderson et al (1992)), the two weights can be defined based on each period utility for each forecast $U^{\phi}_{i,t}$, $i = (y, \pi)$, $\phi = (f, e)$ according to:

$$\alpha^{f}_{\pi,t} = \frac{exp(\gamma U^{f}_{\pi,t})}{exp(\gamma U^{f}_{\pi,t}) + exp(\gamma U^{e}_{\pi,t})} \tag{25}$$

$$\alpha^{f}_{y,t} = \frac{exp(\gamma U^{f}_{y,t})}{exp(\gamma U^{f}_{y,t}) + exp(\gamma U^{e}_{y,t})} \tag{26}$$

$$\alpha^{e}_{\pi,t} \equiv 1 - \alpha^{f}_{\pi,t} = \frac{exp(\gamma U^{e}_{\pi,t})}{exp(\gamma U^{f}_{\pi,t}) + exp(\gamma U^{e}_{\pi,t})} \tag{27}$$

$$\alpha^{e}_{y,t} \equiv 1 - \alpha^{f}_{y,t} = \frac{exp(\gamma U^{e}_{y,t})}{exp(\gamma U^{f}_{y,t}) + exp(\gamma U^{e}_{y,t})} \tag{28}$$

,where the utilities are defined as:

$$U^{f}_{\pi,t} = -\sum_{k=0}^{\infty} w_k [\pi_{t-k-1} - \tilde{E}^{f}_{t-k-2}\pi_{t-k-1}]^2 \tag{29}$$

$$U^{f}_{y,t} = -\sum_{k=0}^{\infty} w_k [y_{t-k-1} - \tilde{E}^{f}_{t-k-2}y_{t-k-1}]^2 \tag{30}$$

$$U^{e}_{\pi,t} = -\sum_{k=0}^{\infty} w_k [\pi_{t-k-1} - \tilde{E}^{e}_{t-k-2}\pi_{t-k-1}]^2 \tag{31}$$

$$U^{e}_{y,t} = -\sum_{k=0}^{\infty} w_k [y_{t-k-1} - \tilde{E}^{e}_{t-k-2}y_{t-k-1}]^2 \tag{32}$$

and $w_k = (\varrho^k(1 - \varrho))$ (with $0 < \varrho < 1$) are gemoetrically declining weights adapted to include the degree of forgetfulness in the model (DeGrauwe, 2012). γ is a parameter measuring the extent to which the deterministic component of utility determines actual choice. A value of 0 implies a perfectly stochastic utility. In that case, agents decide to be one type or the other by simply tossing a coin, implying a probability of each type equalised to 0.5. On the other hand, $\gamma = \infty$ imples a fully deterministic utility, and the probability of using the fundamentalist (extrapolative) rule is either 1 or 0. Another way of interpreting γ is in terms of learning from past performance: $\gamma = 0$ implies zero willingness to learn, while the willingness increases with the size of the parameter, i.e. $0 < \gamma < \infty$.

As mentioned above, agents will subject the performance of both rules to a goodness-of-fit measure and choose the one that performs best. In that sense, agents are "boundedly" rational and learn from their mistakes. More importantly, this discrete choice mechanism allows to endogenise the distribution of heterogeneous agents over time to the proportion of each agent using a certain rule (parameter $\alpha^{\phi}, \phi = (f, e)$). The approach is consistent with the empirical studies (Cornea et al, 2012) who show that the distribution of heterogeneous agents varies in reaction to economic volatility (Carroll (2003), Mankiw et al (2004)).

2.4 Firm equity

To complete the model, we need to characterize the evolution of net worth. In DeGrauwe and Macchiarelli (2015), it is shown that:

$$n_t^{f,m} = \frac{1}{\tau_t}(L_{t-1}^D + i_t) \tag{33}$$

and

$$n_t^{f,m} = \bar{n}_t S_t \tag{34}$$

where $\bar{n}_t$ represents the number of (time-varying) shares of the firm and S_t is the current (stock) market price. Combining the two, we get that the number of shares is:

$$\bar{n}_t = \frac{\frac{1}{\tau_t}(L_{t-1}^D + i_t)}{S_t} \tag{35}$$

Inserting the investment demand equation $i(\rho)_t = e_1\tilde{E}_t(y_{t+1}) + e_2(r_t + x_t - \tilde{E}_t(\pi_{t+1}))$ from DeGrauwe and Macchiarelli (2015) into the expression above, we get:

$$S_t\bar{n}_t = \frac{1}{\tau_t}(L_{t-1}^D + e_1\tilde{E}_t(y_{t+1}) + e_2(r_t + x_t - \tilde{E}_t(\pi_{t+1}))) \tag{36}$$

We observe three things. First, net capital (or equity) the firm holds after repaying the cost of borrowing is scaled by the inverse leverage ratio. The more it borrows, the smaller will be its equity in the next period. Second, a higher (expected) production increases its revenues and therefore the capital level (via the capital accumulation function). However, a portion of the production is financed by external funds and thus it will need to pay a cost for those funds, represented by the risk-adjusted interest rate $r_t + x_t$. However, the more leveraged the firm is, the higher the down-payment on loans and therefore the more "exposed" the firm will be in recessions. Third, a higher expected inflation implies a reduction in the cost of external financing. For a given level of leverage, this reduces firm's debt exposure today and permits her, *ceteris paribus* to take on additional loans. Finally, note that the more leveraged the firm is, the higher is the impact on firm equity (or shares) from movements in (stock) market prices. This set-up is analogous to the state equation shown in Gerba (2017).

3 Behavioural model derivations

3.1 Model solution in the behavioural model

We solve the model using recursive methods (see DeGrauwe (2012) for further details). This allows for non-linear effects. The model has six endogenous variables, output gap, inflation, financing spread, savings, capital, and interest rate. The first five are obtained after solving the following system:

$$\begin{bmatrix} 1 & -b_2 & 0 & 0 & 0 \\ -a_2c_1 & 1-a_2c_2 & -(a_2+a_3) & 0 & (a_1-a_2)\psi(u_t) \\ -\psi\tau_{t-1}^{-1}e_2c_1 & -\psi\tau_{t-1}^{-1}e_2c_1 & (1-\psi\tau_{t-1}^{-1}e_2) & 0 & 0 \\ d_3c_1 & -(1-d_1-d_3c_2) & 0 & 1 & 0 \\ 0 & 0 & e_2 & 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ y_t \\ x_t \\ s_t \\ k_t \end{bmatrix} =$$

$$= \begin{bmatrix} b_1 & 0 & 0 & 0 & -e_2 \\ -a_2 & 1-a_1 & 0 & 0 & \Psi(\frac{i_t}{i_{t-1}})e_1 \\ -\psi\tau_{-1t}^{-1}e_2 & -\psi\tau_{t-1}^{-1}e_2 & 0 & 0 & 0 \\ d_3 & -d_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{E}_t[\pi_{t+1}] \\ \tilde{E}_t[y_{t+1}] \\ \tilde{E}_t[x_{t+1}] \\ \tilde{E}_t[s_{t+1}] \\ \tilde{E}_t[k_{t+1}] \end{bmatrix} +$$

$$+ \begin{bmatrix} 1-b_2 & 0 & 0 & 0 & 0 \\ 0 & 1-a_1^{'} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -(1-d_1-d_2) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & (1-\delta) \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ y_{t-1} \\ x_{t-1} \\ s_{t-1} \\ k_{t-1} \end{bmatrix} +$$

$$+\begin{bmatrix} 0 & 0 & 0 \\ a_2c_3 & 0 & 0 \\ -\psi\tau_{t-1}^{-1}e_2c_3 & \psi\tau_{t-1}^{-1} & \psi\tau_{t-1}^{-1} \\ d_2c_3 & 0 & 0 \\ e_2 & 0 & 0 \end{bmatrix}\begin{bmatrix} r_{t-1} \\ D_{t-1} \\ n_{t-1}^b \end{bmatrix}+\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & a_2 & 1 & (a_1-a_2) \\ 0 & \psi^{-1}\tau_{t-1}e_2 & 0 & 0 \\ 0 & -d_3 & -(1-d_1) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \eta_t \\ u_t \\ \epsilon_t \\ ucf_t \end{bmatrix}$$

Using matrix notation, we can write this as: $\boldsymbol{AZ_t = B\tilde{E}_tZ_{t+1} + CZ_{t-1} + DX_{t-1} + Ev_t}$. We can solve for $\boldsymbol{Z_t}$ by inverting: $\boldsymbol{Z_t = A^{-1}(B\tilde{E}_tZ_{t+1} + CZ_{t-1} + DX_{t-1} + Ev_t)}$ and assuring $\boldsymbol{A}$ to be non-singular.

Solution for the interest rate r_t is obtained by substituting y_t and π_t into the Taylor rule. Investments, utilization costs, bank equities, loans, labour and deposits are determined by the model solutions for output gap, inflation, financing spread, savings and capital.[16] Evolution in expectation of capital stock, which is one of the novelties in this model, follows a similar mechanism to that employed in the forecasting of output and inflation. However, since capital has a constant share in output, via Cobb-Douglas production technology in 3, and because capital is directly related to the inflation rate via the real rate relation in 15, *de facto* agents simultaneously forecast capital when they forecast output and inflation.[17] This follows the same argument used for asset prices in this paper, and in De Grauwe and Macchiarelli (2015). Besides, since capital and its price are driven by the same fundamentals, it is logical to assume that they follow the same learning process or heuristics evolution. Apart from being consistent with the reduced-form version, this also represents a micro-foundation for the entire heuristics set-up since agents do not hold imperfect information about the entire economic structure, but only regarding a very narrow subset of core model variables. However, imperfect information has implications for the entire model via its (contemporaneous, lagged or lead) relation to the entire system.

Expectation terms with a tilde $\tilde{E}_t$ implies that we do not impose rational expectations. Using the system of equations above, if we substitute the law of motion consistent with heterogeneity of agents (fundamentalists and extrapolators), then we can show that the endogenous variables depend linearly on lagged endogenous

[16]However, capital, savings, and the external financing spread do not need to be forecasted as these do not affect the dynamics of the model (i.e. there is no structure of higher order beliefs as LIE does not hold in the behavioural model). See section 3.1 in DeGrauwe and Macchiarelli (2015) for comparison of solutions under rational expectations and bounded rationality (”heuristics”).

[17]However, it needs to be discounted with the investment adjustment cost as this is the main friction in the accumulation of capital.

variables, their equilibrium forecasts and current exogenous shocks.

Note that for the forecasts of output and inflation gap, the forward looking terms in equations 15, I.1 and I.7 are substituted by the discrete choice mechanism in I.18. For a comparison of solutions in the "bounded rationality" model and rational expectations framework, see section 3.1 of De Grauwe and Macchiarelli (2015).

3.2 Calibration and simulations

To simplify the discussion, we will only present the calibration of the parameters that are new in this model. A full parameter list can be found in the Appendix.

In line with De Grauwe and Macchiarelli (2015), we calibrate the aggregate demand parameters (d_1, d_2, e_1) to $(0.5, 0.15, 0.1)$ which is consistent with standard macroeconomic simulation results. τ (or a firms' average leverage ratio) is again set to 1.43, following Pesaran and Xu (2013), and κ (or banks' equity ratio) is, following Gerali et al (2010), set to 0.09.

The parameters specific to this model are set to standard values in the literature. The share of capital in the production α is set to 0.30 as in Boissay et al (2013). Following Christiano et al (2005), Smets and Wouters (2003, 2007) and Gerali et al (2010), we set the capital depreciation rate δ to 0.025. The elasticity of the capital utilization adjustment cost function $\psi(i_t)$ is parametrised to 0.5 as in Smets and Wouters (2007).[18]

The sensitivity of capital (or investment) to changes in the real interest rate e_2 is, in line with the empirical evidence, set to $e_2 < 0$. To conclude, the parameters of the function determining adjustment costs for capacity utilization (ξ_0, ξ_1, ξ_2) are set to $(0.8, 0.3, 0.25)$ in order to capture the estimation results of Smets and Wouters (2005) who find that the capital utilization adjustment costs are between 0.14 and 0.38 (Euro Area 1983-2002) and 0.21 and 0.42 (US 1983-2002), with a mean of 0.25 (Euro Area) and 0.31 (US). If we normalize u_t to 1 (as in Christiano et al (2005), Miao et al (2013) or Auernheimer and Trupkin (2014)), then the cost for utilizing capital will be 0.20 $(1 - \xi_0)$, which is well within the estimated intervals of Smets and Wouters (2005).

All shocks, except for the capital utilization, are parametrised as white noise, which means that their autoregressive component is set to 0. Likewise the standard

[18] This is equivalent to setting a κ_i equal to the estimated range $(10.18 - 12.81)$ as in Gerali et al (2010).

deviations of shocks are set to 0.5 across the entire spectrum.[19]

4 Quantitative results

Our analysis consists of three parts. The first part is an analysis of (model consistent) impulse responses to a set of independent white noise shocks. Remember that all shocks were calibrated to 0.5. The second is an examination of the (model generated) second-, and higher-order moments to contrast the fit of the model to the US data. The final part consists of depicting and analysing the nature of the model variables over the business cycle. For future work, we wish to compare the quantitative results to a standard DSGE model with an equal mechanism.[20]

4.1 Impulse response analysis

Figure II.4 depicts the full impulse responses to a negative monetary policy shock, Figures II.5 to a technology shock, and Figures II.6 to a shock to utilization costs. Note that the numbers on the x-axis indicate number of quarters. All the shocks are introduced in t=100 and we observe the responses over a long period of 60 quarters (or 15 years). The black line represents the median impulse response, while the red intervals depict the full 95% distribution (or confidence interval) of the responses for different initializations. For the sake of clarity and focus, we will only concentrate on the median impulse response in the discussion, which is a good representation of the overall distribution.

4.1.1 Monetary policy shock

As is standard, an expansionary monetary policy (0.5% fall) leads to a fall in the external finance premium, which relaxes the credit that firms can access and therefore pushes up investment (0.3%). This pushes up capital accumulation (0.4%).

[19]The AR-component of the shock to capital utilization cost is set conservatively to 0.1, just enough to generate some persistence in the capital cost structure.

[20]Before we begin with the analysis, bear in mind that the behavioural model does not have one steady state that is time invariant for the same calibration (as is standard for the DSGE method). Therefore, following a white noise shock, the model will not necessarily return to a previous steady state. If not the same steady state, it can either reach a new steady state, or have a prolonged response to the initial shock. In other words, there is a possibility for the temporary shock to have permanent effects in the model (via the animal spirits channel). However, due to the methodological proximity to the DSGE analogue and because it is a standard evaluation (and comparison) tool in the literature, we will proceed analysing the impulse responses in the behavioural model.

Agents perceive this expansion as a period of positive outlook, which triggers the optimism (animal spirits up 0.2%). This optimism is translated into an increase in deposits (0.25%) and bank equity (0.3%). The expansion leads to an increase in output (0.20%) and a rise in inflation (0.01%), but with a lag of 1 quarter.[21]

However, this optimism is very brief as the monetary authority raises the interest rate (0.1%) to combat the rising inflation. Agents perceive this as the end of the expansionary phase, resulting in a reversal of the sentiment to pessimism (animal spirits fall by 0.05%). The consequence is a turn in the response of macroeconomic and financial aggregates, leading to return of these variables to the steady state.

Hence in the behavioural model, we see two waves of responses. The first, standard in the DSGE models, is driven directly by a monetary policy expansion. The second, on the other hand, is purely driven by animal spirits. The response of the monetary authority to the initial expansion kills and turns the initial optimism into a pessimism (or negative bubble on the financial market). This results in a reversal in the financial and macroeconomic aggregates, making the initial monetary expansion extremely short-lived. This type of market behaviour is difficult to capture in standard DSGE models (but frequently observed empirically).

4.1.2 Technology shock

Let us now turn to the first of the supply side shocks. An improvement in TFP (or equivalently, increase in productivity) of 0.5% results in an inflation reduction (1%) and a more than proportional output expansion (1.15%). This is a result from both an increase in capacity in the final goods market, as well as from an increase in investment (0.3%) following the heavy fall in interest rate (1.3%) as a response to the falling inflation. Following this general supply-side expansion, deposits and loans to firms also increase (1 and 1.3% respectively) since the value of firm net worth (i.e. collateral) has increased. As a consequence of the lower marginal cost of investment and higher marginal return on capital, capital accumulation increases significantly in the next period (0.5%). This results in a general market optimism (animal spirits rise by 0.1%).

However, as soon as the inflation starts recovering, interest rate react very rapidly to their increase and start rising (0.35%). Because of this rise in cost of capital, coupled with the fall in external financing for firms, investment-and output expansion

[21] Initially, output falls by 0.25% as well as inflation by 0.05%, but this is reverted after 1 period. This finding is frequent in the literature and denominated as the price puzzle.

reverts. However, unlike in the DSGE models, the model has eventually reached a new steady state, where bank loans, deposits and equity are permanently 1.1%, 0.7% and 0.1% above the previous pre-shock level. [22] Hence a technology improvement in the behavioural model will have long-lasting positive effects on the banking sector and financial efficiency.

4.1.3 Shock to utilization costs

The second of the supply side shocks is a 0.5% decrease in the cost of utilizing capital in production (i.e. a positive supply-side shock). This will therefore increase the marginal benefit (or return to capital), which will lead to an increase in the demand for capital. Hence, capital good producers will produce more, and so investment rises (0.02%). The level of capital will also rise significantly (0.2%) as a result of both capital demand-and supply expansion. Therefore, output will expand (0.1%). Because of the higher capital (and thus collateral) and the resulting fall in the financing spread, the quantity of credit to firms will expand (0.7%). Since this is an improvement on the supply side, inflation initially falls (0.03%), and the monetary authority reacts by reducing the interest rate (0.15%). This is reverted as soon as the monetary authority increases the interest rate (0.02%) because of the recovery in the inflation. Following 15 years after the shock, in the new steady state, firm credit and deposits are 0.6% and 0.2% above the pre-shock level. Again a temporary supply-side shock is having permanent effects on financial sector activity.

4.1.4 Impulse responses and learning

To test the sensitivity of our impulse responses to learning, which is a fundamental component of the behavioural model, we simulate the same impulse response but simply varying the learning parameter γ (keeping the model structure, shock process, and size of the shock equal across). As we described in the model outline (section 2.3), γ determines the willingness to learn from past performance, where $\gamma = 0$ means zero willingness, and $\gamma = N$ (where N is a large number) means a very strong willingness to learn. In other words, small values of γ denote slow learning, and large values denote fast learning. To evaluate this responsiveness, we depict the impulse response of output and inflation to a monetary policy shock for values of γ

[22] In standard DSGE models, this is only possible to achieve with permanent or continuous shocks.

from 0 (no learning) to 1000.[23] These are depicted in Figure II.3.

We find the same pattern for both impulse responses. The larger the γ parameter, the more hump-shaped is the impulse response as the cycles are extended. In particular, the initial drop is larger, longer-lasting, while also the subsequent (but smaller) rise is more pronounced. Intuitively, the stronger the learning mechanism, the more profound will be the transmission of a shock, keeping the shock size constant. Yet, when no learning occurs ($\gamma = 0$) the utilities in learning become stochastic, and so contemporaneous market sentiment dominates the cycles and determines the impulse responses. That is why we see very heavy swings in the two responses in Figure II.3.

To conclude this section, learning is a powerful propagator of shocks. In a set of experiments, we have shown that the quicker (or stronger) the learning (friction), the more pronounced will be the impulse responses since these generate higher persistence. However, if no learning takes place, then the responses become very curved, reflecting the stochastic utility in learning. Moreover, in a recent paper, Gerba and Zochowski (2017) have compared a very similar learning set-up to one with only rational expectations (keeping the rest of the model structure equal). They show that when beliefs and learning are removed, the impulse responses to the same shock become smaller and non-persistent.

4.2 Distributions and statistical moments over the business cycle

The second part of the model evaluation consists of analysing and validating the model-generated distribution and statistical moments over the business cycle. These are generated using the entire sample period of 2000 quarters. For our purposes, we will use the data on second and higher moments in Tables II.2 and II.3, the evolution of the model variables over the business cycle in Figures II.7 to II.8, as well as histograms of a selection of these variables in Figure II.9. Note for the graphs that we are plotting the business cycles over a sub-sample period of 100 quarters.

[23] We tried also larger values, but after $\gamma = 1000$ the IRFs do not change significantly, which we take as 1000 being an upper limit on the speed of learning.

4.2.1 Macroeconomic aggregates

The short-term cycles of output, inflation and the interest rate are asymmetric. While the amplitude of expansions is in general higher for output, the duration of recessions is longer. This is further confirmed by the histogram for output, which is asymmetric and skewed to the right, with a higher probability mass on the left of the mean of the distribution. Moreover, the autocorrelation of output is very high (0.86), as is the volatility (2.17) and it is leptokurtic (kurtosis=10.91).

The opposite applies to inflation. The amplitude of deflationary periods is in general higher, while the duration of inflationary periods is longer. From histogram, the distribution of inflation is slightly skewed to the left. In line with the data, inflation is three times less volatile than output but has a very similar kurtosis to output. Further, inflation is very persistent over time ($\rho = 0.74$) and countercyclical (-0.42), exactly as in the US data (-0.43).

Turning to the (risk-free) interest rate, it is mostly positive and remains above the trend for a longer period over the cycle. It is also highly correlated with the business cycle (0.39) as well as with inflation (0.57), indicating a firm inflation target on the part of the monetary authority. It is almost as volatile as output (0.95), but highly skewed to the left (-4.29) compared to the general business cycle.

4.2.2 Firm and supply-side variables

Looking at Figure II.8, capital stock is mostly positive over the cycle, with a mean-reversion around 1. This is in line with the data on inventories, which shows it is positive mean-reverting. It is highly persistent ($\rho = 0.95$) and correlated with output (0.45). It is also highly positively correlated with animal spirits (0.34). Distribution-wise, it is less volatile than the business cycle (0.413), but heavily skewed to the right (3.48).

The first thing one notes from utilization costs is that while apparently more volatile, it oscillates within a much smaller interval compared to any of the other model variables. Hence, the volatility is 4 times smaller compared to output. In addition, it reverts around a mean of approx. 0.5. This is in line with the data, which points towards a largely non-negative cost in utilizing capital over the cycle. It is however weakly countercyclical (-0.1), and symmetric as well as mesokurtic.

The cash-in-advance constraint (or percent of external credit used for capital input purchases) is strictly non-negative and acyclical (0.02). It is also independent from the cycles of capital- (-0.01), and financing spread (0.01). In addition, the distribution of ϑ_t is highly volatile, skewed to the right and leptokurtic. Effectively, with 95% probability (or higher) ϑ_t is significantly above zero.

On the other hand, the financing spread for firms is highly countercyclical (-0.41), as well as negatively correlated with animal spirits (-0.12). This is consistent with the model set-up and data, which shows that during expansions both the real risk (via a higher collateral value) and the perceived risk (via the optimistic sentiment) of loan default falls, which pushes down the risk premium and so the spread. The opposite holds for recessions. That is why the spread is both negatively correlated with the business cycle (collateral value), and with the market sentiment (agents' risk perception). Statistically, the spread is as volatile as the general business cycle, but highly skewed to the left, meaning that for most of the time the spreads will be close to zero (or negative), This is further confirmed by the graph in Figure II.8. However, with some non-negligible probability, the spread can spike, causing a severe contraction in liquidity, and the banking market.[24] These results are also in line with the (model-generated) statistical moments on loan supply, which is procyclical (0.11), positively correlated with animal spirits (0.12) and capital-net worth (0.28), but negatively correlated with the financing spread (-0.1)

4.2.3 Market sentiment

An important driver of the business cycle is the market sentiment (or animal spirits). It is highly procyclical (0.84) throughout the entire sample period (see Figure II.7). Moreover, we observe a higher persistence during the pessimistic interval compared to the optimistic. This is in line with our previous observation on the general business cycle (or output) showing that recessions have a longer duration compared to expansions. Moreover, market sentiment has fat tails on the left and right of the mean, but is smoother than the general business cycle.

4.2.4 Moment matching

The next step in model validation consists of matching the (model generated) moments to the US data. For that, we have calculated the statistical moments for all

[24] However, the spread is not persistent ($\rho = 0.01$) implying an RBC type of frictionless financial sector, and non-staggered price setting. That is not a surprise for the current model since the financial market is modelled in reduced form. However, future work should try to extend the model by modelling a more complex and empirically consistent financial price setting mechanism.

variables using the longest data sample period available from 1953:I - 2014:IV.[25] Following Stock and Watson (1998), we choose 1953:I as the starting year of our sample since the (post-war) quarters prior to 1953 include noise and inaccuracies in the data recording. The sample includes 247 quarters (or 62 years) which is the closest approximation available for the long-run (cyclical) moments that is generated by the model. During this period, the US economy experienced 10 cycles (using NBER business cycle dates), and the average GDP increase (quarter-on-quarter) during expansions was 1.05% while it was -0.036% during recessions. The data were downloaded from Flow of Funds at the Fed St Louis database. These were de-trended using a standard two-sided HP-filter before their moments were calculated.[26] A full list of variables and other details can be found in Table II.

The behavioural model matches precisely the correlations of many supply-side and financial variables. This includes credit to firms, deposits, the (risk-free) interest rate, inflation, and firm financing spread. It is also very successful in reproducing the autocorrelations of output, capital, and inflation, as well as the correlations between capital and credit to firms, and inflation and the (risk-free) interest rate. However, there is room for improvement in matching stock variables, such as firm and bank net worth, some macroeconomic aggregates (investment mainly) as well as the autocorrelation of firm financing spread. While in the model they are all acyclical and lack persistence, in the data they are highly procyclical and persistent.

Turning to (relative) second-, third-, and fourth moments, the model is highly successful in reproducing the moments of inflation, the (risk-free) interest rate, credit to firms, deposits, and net worth of banks. It is also successful in making net worth of firms more skewed and more leptokurtic than output. However, the moments of the latter are higher in the model compared to US data. On the other hand, capital and investment are smoother in the model.

Another strength of the model lies in reproducing irregular business cycles. In contrast to standard first-, second-, or even third order approximated DSGE models. the behavioural model generates substantial asymmetries between expansions and recessions as well as produces non-Gaussian probability distribution functions for most variables. That is much more in line with the observed pattern in the US cyclical data. Nonetheless, for some variables (net worth, consumption, savings, (risk free) interest rate, and credit to firms) the model generates excessive skewness

[25]The most recent data recorded is for 2014:IV using Fed St Louis database on March 2, 2015.

[26]This is in order to allow for a smoother comparison with the model generated (cyclical) moments.

and/or kurtosis.

To sum up, the model matches most of the US data. This includes supply-side and financial variables such as the (risk-free) interest rate, inflation, credit to firms, deposits, firm financing spread and net worth of banks. It is also successful in matching several supply-side relations (capital-firm credit, inflation-interest rate) as well as their autocorrelations (output, capital and inflation) There is, however, some scope for improvement in matching demand-side variables (such as consumption, savings, investment) as well as stocks (net worth of firms).

4.2.5 Persistence in the behavioural model

An alternative way to evaluate the persistence a model generates is to look at the full spectrum of autocorrelations generated in the model. In other words, a model may be able to generate high autocorrelation of order one, but this may decay quickly for orders larger than 1. This is usually the case for DSGE models with standard shocks. Therefore, a more robust way to evaluate the (endogenous) persistence that a model generates is to look at much larger lag lengths. To do so, we depict the autocorrelation function (ACF) of order ten of the key variables in the model. We report the ACFs of output, inflation, interest rate, capital, investment, utilization costs, deposits, and animal spirits in Figures II.1 and II.2.

The first thing to note from the figures is that some variables are very persistent over time, while others are not. The model is capable of generating high (endogenous) persistence in inflation, capital, deposits, (risk-free) interest rate, output and animal spirits, and a much lower persistence (lower autocorrelation parameter) for investment and utilization costs. Second, output, inflation, and interest rate have a rapid decaying (low ρ) autoregressive process, while stock variables such as capital or deposits have a slow (high ρ) one. This is because the second category of variables has an (autoregressive) evolutionary process that they follow, which makes them exhibit high time-inertia. Third, some variables, such as investment and utilization costs exhibit a periodic behaviour (since their autoregressive process ρ is periodic) and so may have an AR(2) process.

In parallel, the standard errors bands of the third and fourth model moments in Table II.4 are relatively tight. We report the 66% bands in our exercises. Even for investment, which has a high kurtosis and skewness, the confidence band is tight around the mean estimate of moments. This means that there is low uncertainty regarding the moment estimates, and thus they are precise.

In short, the model is capable of generating diverse levels of persistence in the model. While many variables are highly persistent with a high autocorrelation parameter, others have lower persistence but with a periodic behaviour, just as an AR(2) process. Moreover, the model-generated moments reported in the tables are precise and a good reflection of the diverse and asymmetric dynamics that this model can generate.

4.2.6 The nature of business cycles

Next, we wish to understand to what extent the model is capable of generating inertias in the business and financial cycles.

As discussed in Milani (2012) and DeGrauwe and Macchiarelli (2015), business cycle movements in a rational expectations environment arise as a result of exogenous shocks (including the autoregressive structure of shocks), leads and lags in the endogenous transmission of shocks (such as lagged or expected output), habit formation, interest rate smoothing, or nominal rigidities (price and wage stickiness). One could therefore call this "exogenously created" business cycle fluctuations. The behavioural model, on the other hand, generates inertia and business cycle fluctuations even in the absence of endogenous frictions, lags in endogenous transmissions, and autocorrelation shock structures, as shown in De Grauwe (2012).[27] In the current case, however, we have introduced supply-side and financial market frictions, as well as leads and lags in the output, inflation and capital transmission mechanisms.[28] This is in order to set the behavioural model at par with a standard DSGE model, so to facilitate the comparison between the two frameworks.

The evolution of the different model variables over the business cycle are reported in figures II.7 and II.8. The time period covered is 100 quarters, which is enough to cover multiple cycles.[29] The first thing to note is that with this "snapshot" of the business cycle, we have managed to capture one long cycle (with a high amplitude) followed by several shorter cycles. Not only is the business cycle peak the highest during those 25 years ($t = 295$), but the amplitude is also the widest (between $t = [280 : 300]$ counting from trough to trough). Moreover, the subsequent bust

[27] De Grauwe (2012) analyses only 3 variables in his paper: output, inflation and animal spirits. On the other hand, in the current paper we will analyse and contrast many more variables in order to get a holistic view of the business cycle performance of the model.

[28] Note that capital only has lagged transmission structure, no leads are incorporated. That is standard in the macroeconomics literature.

[29] The model is simulated over 2000 quarters, so data and figures for the longer time period are available upon request.

following the high boom is the sharpest, since it takes the economy more than 40 quarters to return to a level above the long-run trend (or above the zero line). In addition, the subsequent expansions are significantly weaker, somewhat implying that some fundamental (or structural) changes occurred in the economy following the preceding boom and bust.[30] Compare that to the boom preceding the Great Recession and the subsequent bust in the US.

Closely related to above observations, we find that the other variables experience similar cycles (inflation, interest rate, capital and the financing spread). Because the model concentrates on the supply side (with a weak demand-side) and we only employ supply side shocks, inflation falls when output rises (and vice versa).[31] So during the period of sharpest expansion of the business cycle, inflation experiences its sharpest decline. In contrast to output, however, inflation oscilliates relatively evenly around zero (i.e. we don't observe any temporal shifts in the trend).

As expected, the interest rate responds elastically to the evolution of inflation (see Figure II.8). Nevertheless, it is smoother than inflation since we have included an interest rate lag in the Taylor rule (see De Grauwe and Macchiarelli, 2014), which smoothens the reaction of the interest rate to current inflation. We also observe a lag in the response of inflation to monetary policy over the cycle, in line with observations from the data.

Capital, on the other hand, is positively skewed and is mostly above the zero line during the entire period. Since it is a stock variable, that is to be expected and in line with the US data (see table II.3). In addition, capital accumulates the most during the long expansionary period discussed above, and contracts under the proceeding episode. Just as the general business cycle, the subsequent capital accumulations are weaker, and the stock of capital is still below it's pre-crisis level 40 quarters (or 10 years) after the bust. Contrast that to the Great Recession episode.

In the same vein, utilization costs are also positively skewed (see Figure II.9), but more volatile than output. This is to be expected since utilization cost function is of second order (see equation 5) and depends directly on the production capacity. Therefore the volatility of production will be squared, which increases the fluctuations in the cost. Also, as Figure II.8 shows, the more capital is accumulated and used in production, the higher utilization costs the producer will face (due to the inherent trade-offs explained in subsection 2.1.2). The correlation between the two

[30] However, to confirm this fact one would need to perform a structural breaks analysis on the full data, which includes the trend.

[31] See Figure II.8 for the correlation between output and inflation during the entire period.

is positive throughout the entire period.

The next thing to note is the high degree of asymmetry over the business cycle. As histograms in Figure II.9 confirm, all variables are skewed. Meanwhile output and asset prices are positively skewed (skewed to the right), inflation, capital and (in particular) utilization costs are negatively skewed (skewed to the left). Taking into account the distribution of animal spirits in Figure II.9 and asset prices in Figure II.9, this implies that pessimistic phases (or busts) dominate optimistic ones (or booms).

Just as in the DeGrauwe (2011,12) and DeGrauwe and Macchiarelli (2015) models, output is highly correlated with animal spirits throughout the entire period. It's correlation with animal spirits is 0.83 (see table II.2). We can interpret the role of animal spirits in the model as follows. When the animal spirits index clusters in the middle of the distribution we have tranquil periods. There is no particular optimism or pessimism, and agents use a fundamentalist rule to forecast the output gap. At irregular intervals, however, the economy is gripped by either a wave of optimism or of pessimism. The nature of these waves is that beliefs get correlated. Optimism breeds optimism; pessimism breeds pessimism. This can lead to situations where everybody has become either optimist or pessimist. The index then becomes 1 respectively 0. These periods are characterized by extreme positive or extreme negative movements in the output gap (booms and busts).

Let us continue by examining one of the novelties of this model, the share of loans used for capital input pre-payment. It is clear from Figure II.8 that when the economy expands and the stock market booms, the share of loans required by CGP for the capital pre-payment is very low, and often zero. This is because of the stock market booms implying a low probability of default for entrepreneurs (since it's collateral value is high, or loan-to-value ratio low). Because of this low probability of default, entrepreneurs will be able to borrow more, increasing their (expected) cash positions and so CGP will not require a pre-payment. In contrast during busts, and in particular during a sharp contraction (as in $t = [295, 300]$) CGP become wary of the entrepreneur's ability to pay their capital purchases in the next period (because of an expected lower cash position of entrepreneurs, or reduced production), and therefore require a high share to be pre-paid. The higher the contraction, the higher the share required to be pre-paid (see lower graph in Figure II.8). The model is capable of generating these asymmetries over the cycle, in particular in relation to the cash position of entrepreneurs.

To conclude, we see a strong co-movement between asset prices on one hand, and net worth and the financing spread. During stock market booms, net worth rises which increases the firm's collateral value and reduces its probability of default, and so it reduces the external financing spread (as it is less risky for banks to lend to firms).

4.2.7 Comparison to a benchmark model

In order to appreciate the added value of this additional model feature, we compare the statistical moments and distributions to a benchmark version where supply side financial frictions are not included, as in De Grauwe and Macchiarelli (2015). The results for this model version are included in the third column of Table 1, and the distributions and evolution of a selection of variables are depicted in Figures II.10 and II.11. The column on the left depicts the figures in the full model, while those on the right depict those of the benchmark model. Note that the scales and axes in both set of figures are the same, so to facilitate a quick visual comparison.

Beginning with the table, the full model matches the data much better than the benchmark version. Not only are the moments of the full model much closer to the data, but in several cases, the moment of the full model is very close to the empirical counterpart and contrary to those of the benchmark model. This means that including supply side financial friction indeed improves the moment fit of the data.

Turning to the (cyclical) evolution of key model variables, in general we find that the model with supply side financial frictions generates additional volatility in all three variables (output, inflation and spread). Since in general, more frictions lead to higher cyclical amplitude, this is in line with the intuition, and proves the consistency of this model. At the same time, while benchmark model just generates modest business cycle movement, the full model is capable of generating both small and larger swings.

Lastly, turning to distributions, we observe two things. First, those of benchmark model are more skewed to the left. Second, the distribution is more disperse, with higher probability mass away from the mean. In addition, for animal spirits we find that there is less mass in the middle, and more at the extremes, in particular on the right-hand side (or optimist region). The underlying reason is that animal spirits drive more of the model dynamics in the benchmark version, since there are less of other frictions. On the other hand, in the full model, supply-side financial frictions,

Table 1: Model correlations

Correlations	Full behavioural model	Benchmark behavioural model	US data
$\rho(y_t, y_{t-1})$	0.86	0.65	0.85
$\rho(y_t, \pi_t)$	-0.42	-0.12	-0.43
$\rho(y_t, d_t)$	0.17	0.30	0.32
$\rho(y_t, l_t^s)$	0.11	0.27	0.18
$\rho(y_t, x_t)$	-0.41	-0.69	-0.49
$\rho(y_t, n_t^b)$	-0.01	0.02	0.45
$\rho(y_t, n_t^f)$	-0.02	0.07	0.22
$\rho(\pi_t, \pi_{t-1})$	0.74	0.69	0.93
$\rho(\pi_t, r_t)$	0.57	0.64	0.34
$\rho(x_t, x_{t-1})$	0.01	-0.007	0.68
$\rho(l_t^s, x_t)$	-0.09	-0.17	0.26

banking frictions and animal spirits all interact so that less variability comes from animal spirits only, and the economy is less driven by market sentiment.[32]

5 Discussion and concluding remarks

Including credit frictions on the supply side is a novel way of thinking about financial frictions in the macroeconomics literature. Sharp rises in stock prices do not only allow firms to increase their credit and capital demand, but can equally reduce the input costs for firms, or their input-output ratio. Conversely, a sharp drop in asset prices can restrict the supply of credit to firms, which will increase the production costs for firms, reduce the supply of capital, and (over time) reduce their production capacity (or productivity). Including this mechanism in a behavioural model has significantly improved the fit of the model to the data.

Impulse response analyses show that (temporary) supply-side shocks do not only improve the fundamentals in the economy but cause a brief wave of market sentiment (or animal spirit), which in the case of a positive shock result in a more-than proportional increase in output, capital supply, bank equity, and fall in interest rate. Moreover, credit supply to entrepreneurs is permanently increased, which implies that firms can access a higher external financing in the future. This means that productivity of firms is permanently improved.

Statistical validation of the model (including moment matching) show that the

[32]Note, however, that this is all in relative terms, and that in the full model animal spirits still matter a lot for driving the cycles. Yet, because of other frictions, it now on its own matter less, but interacts more with other "fundamental" frictions.

model is capable of capturing many of the supply-side relations found in the data. This includes supply-side and financial variables such as the (risk-free) interest rate, inflation, deposits, firm financing spread and net worth of banks. It is also successful in matching several supply-side relations (capital-firm credit, inflation-interest rate) as well as their autocorrelations (output, capital and inflation). Moreover, we find a strong co-movement between asset prices on one hand, and net worth and the financing spread. During stock market booms, net worth rises which increases the firm's collateral value and reduces its probability of default, and so it reduces the external financing spread (as it is less risky for banks to lend to firms).

There is, nevertheless, scope for improvement in matching demand-side variables (such as consumption, savings, investment) as well as stocks (net worths of firms). Net worth of banks and firms are more volatile and asymmetric in the model compared to the data.

There are multiple ways in which the current work can be extended. First, it would be highly interesting to contrast the agent-based framework to a DSGE model *at par*. In particular, it would be of high relevance to quantify the proportion of the results that are directly and exclusively generated by the learning framework . Hence, a rigorous comparison with a rational expectations model is necessary to extract this share.

Taking into account the (global) capital market disruptions of 2008-09 and more recently the sovereign fund disruptions in the Eurozone, a second important extension would be to study the type of market (agent) behaviour or (size of) shock that is necessary within this theoretical set-up in order to generate the financial market disruption that was observed in the Eurozone in 2012.

A methodological extension would be to make use of the growing literature in forecast evaluation of agent-based models, and test the forecast performance of this model, in particular with respect to relevant competing models.

Lastly, we calibrate our parameters in the model. An interesting exercise would be to estimate the parameters of the model in order to get a more accurate representation of the business cycles.

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Appendices

I System of equations

Aggregate Demand:

Aggregate Demand:

$$y_t = a_1\tilde{E}_t y_{t+1} + (1-a_1)y_{t-1} + a_2(r_t - \tilde{E}_t\pi_{t+1}) + (a_2+a_3)efp_t + (a_1-a_2)\psi(u_t)k_t + Adj_t + \epsilon_t \tag{I.1}$$

Investment

$$i_t = e_1\tilde{E}_t[y_{t+1}] + e_2[r_t + efp_t - \tilde{E}_t[\pi_{t+1}]] \tag{I.2}$$

External Finance Premium

$$efp_t = \phi\bar{n}_t S_t \tag{I.3}$$

Consumption

$$c_t = 1 - s_t \tag{I.4}$$

Aggregate Supply:

Cobb-Douglas Production Function

$$y_t = a_t(k_t\psi(u_t))^\alpha h_t^{\omega 1-\alpha} \tag{I.5}$$

Utilization cost function

$$\psi(u_t) = \xi_0 + \xi_1(u_t - 1) + \frac{\xi_2}{2}(u_t - 1)^2 \tag{I.6}$$

Approximated Philips Curve:

$$\pi_t = b_1 \tilde{E}_t \pi_{t+1} + (1 - b_1)\pi_{t-1} + b_2 y_t + \nu_t \tag{I.7}$$

Capital evolution

$$k_t = (1 - \delta)\psi(u_t)k_{t-1} + \Psi i_t \tag{I.8}$$

Cash-in-advance constraint

$$S_{t+1}K_{t+1} \leq \vartheta_t[S_{t+1}K_{t+1} - N_t] \tag{I.9}$$

Labour market

$$y_t = \frac{l_t w_t}{1 - \alpha} \tag{I.10}$$

Financial market:

Bank net worth

$$n_t^b = \kappa(l_t^s + \bar{\bar{n}} S_t) \tag{I.11}$$

Evolution of bank leverage

$$\tau_t = \tau_{t-1} + \frac{l_t^d}{n_t^f} \tag{I.12}$$

Stock market price

$$S_t = \frac{E_t[\bar{\Lambda}_{t+1}]}{R_t^s} \equiv \frac{f[\tilde{E}_t y_{t+1} + \tilde{E}_t \pi_{t+1}]}{R_t^s} \tag{I.13}$$

Firm net worth

$$n_t^f = S_t \bar{n}_t = \frac{1}{\tau}(L_{t-1}^D + e_1 \tilde{E}_t y_{t+1} + e_2(r_t + efp_t - \tilde{E}_t \pi_{t+1})) \tag{I.14}$$

Deposits

$$d_t = d_{t-1} + s_t \tag{I.15}$$

Loan demand

$$l_t^d = l_{t-1}^d + i_t \tag{I.16}$$

Credit market equilibrium

$$l_t^d = l_t^s \tag{I.17}$$

Learning environment:

Inflation learning

$$\tilde{E}_t \pi_{t+1} = \alpha_t^f \tilde{E}_t^f \pi_{t+1} + \alpha_t^e \tilde{E}_t^e \pi_{t+1} \tag{I.18}$$

Output learning

$$\tilde{E}_t y_{t+1} = \alpha_t^f \tilde{E}_t^f y_{t+1} + \alpha_t^e \tilde{E}_t^e y_{t+1} \tag{I.19}$$

Learning rules:

$$\tilde{E}_t^f \pi_{t+1} = \pi^* \tag{I.20}$$

$$\tilde{E}_t^f y_{t+1} = y^* \tag{I.21}$$

$$\tilde{E}_t^e \pi_{t+1} = \theta \pi_{t-1} \tag{I.22}$$

$$\tilde{E}_t^e y_{t+1} = \theta y_{t-1} \tag{I.23}$$

Weights

$$\alpha_{\pi,t}^f = \frac{exp(\gamma U_{\pi,t}^f)}{exp(\gamma U_{\pi,t}^f) + exp(\gamma U_{\pi,t}^e)} \tag{I.24}$$

$$\alpha_{y,t}^f = \frac{exp(\gamma U_{y,t}^f)}{exp(\gamma U_{y,t}^f) + exp(\gamma U_{y,t}^e)} \tag{I.25}$$

$$\alpha_{\pi,t}^e \equiv 1 - \alpha_{\pi,t}^f = \frac{exp(\gamma U_{\pi,t}^e)}{exp(\gamma U_{\pi,t}^f) + exp(\gamma U_{\pi,t}^e)} \tag{I.26}$$

$$\alpha_{y,t}^e \equiv 1 - \alpha_{y,t}^f = \frac{exp(\gamma U_{y,t}^e)}{exp(\gamma U_{y,t}^f) + exp(\gamma U_{y,t}^e)} \tag{I.27}$$

Utilities:

$$U_{\pi,t}^f = -\sum_{k=0}^{\infty} w_k [\pi_{t-k-1} - \tilde{E}_{t-k-2}^f \pi_{t-k-1}]^2 \tag{I.28}$$

$$U^f_{y,t} = -\sum_{k=0}^{\infty} w_k [y_{t-k-1} - \tilde{E}^f_{t-k-2} y_{t-k-1}]^2 \tag{I.29}$$

$$U^e_{\pi,t} = -\sum_{k=0}^{\infty} w_k [\pi_{t-k-1} - \tilde{E}^e_{t-k-2} \pi_{t-k-1}]^2 \tag{I.30}$$

$$U^e_{y,t} = -\sum_{k=0}^{\infty} w_k [y_{t-k-1} - \tilde{E}^e_{t-k-2} y_{t-k-1}]^2 \tag{I.31}$$

Shocks

Monetary policy shock:

$$r_t = r_{t-1} + \gamma \pi_t + (1-\gamma) y_t + \epsilon_t^{mp} \tag{I.32}$$

Technology shock

$$Y_t = A_t [\epsilon_t^z \psi(u_t) K_t]^{\alpha} L^{1-\alpha} \tag{I.33}$$

Shock to utilization costs

$$\psi(u_t) = \xi_0 + \xi_1 (u_t - 1) + \frac{\xi_2}{2}(u_t - 1)^2 + uc_t \tag{I.34}$$

$$uc_t = \rho_{uc} uc_{t-1} + \epsilon_t^{uc} \tag{I.35}$$

Evolution of stock prices: Just as in De Grauwe and Macchiarelli (2015), the share price is derived from the stable growth Gordon discounted dividiend model: $S_t = \frac{E_t[\bar{\Lambda_{t+1}}]}{R_t^s}$ where $\bar{\Lambda_{t+1}}$ are expected future dividends net of the discount rate, R_t^s. Agents in this set-up assume that the 1-period ahead forecast of dividends is a fraction f of the nominal GDP one period ahead, and constant thereafter in *t+1, t+2*, etc. Since nominal GDP consists of a real and inflation component, agents make forecast of future output gap and inflation according to the specification in subsection 2.3. This forecast is reevaluated in each period. As a result, in order to get the expected (stock) market price, the expected output gap and inflation needs to be defined.

II Tables and Figures

Table II.1: Parameters of the behavioural model and descriptions

Parameter	Description	Value
Calibrated		Calibration
π^*	The central bank's inflation target	0
d_1	Marginal propensity of consumption out of income	0.5
e_1	Coefficient on expected output in investment eq.	0.1
d_2	Coefficient on expected output in consumption eq. to match $a_1 = 0.5$	$0.5 * (1 - d_1) - e_2$
d_3	Coefficient on real rate in consumption eq.	-0.01
e_2	Coefficient on real rate in investment eq. to match $a_2 = -0.5$	$(-0.5) * (1 - d_1) - d_3$
a_1	Coefficient of expected output in output eq.	$(e_1 + d_2)/(1 - d_1)$
$a_1^{'}$	Coefficient of lagged output in output eq.	$d_2/(1 - d_1)$
a_2	Interest rate elasticity of output demand	$(d_3 + e_2)/(1 - d_1)$
a_3	Coefficient on spread term in output eq.	$-d_3/(1 - d_1)$
b_1	Coefficient of expected inflation in inflation eq.	0.5
b_2	Coefficient of output in inflation eq.	0.05
c_1	Coefficient of inflation in Taylor rule eq.	1.5
ψ	Parameter on firm equity	-0.02
τ	Firms' leverage	1.43
κ	Banks' inverse leverage ratio	0.09
e	Equity premium	0.05
α^d	Fraction of nominal GDP forecast in expected future dividends	0.2
$\bar{n}$	Number of shares in banks' balance sheets	40
$\tilde{n}$	Initial value for number of firms' shares	60
β	Bubble convergence parameter	0.98
c_2	Coefficient of output in Taylor equation	0.5
c_3	Interest smoothing parameter in Taylor equation	0.5
δ	Depreciation rate of capital	0.025
α	Share of capital in production	0.3
Ψ	Adjustment cost function in investment	0.5
γ	Switching parameter in Brock-Hommes (or intensity of choice parameter)	1
ρ	Speed of declining weights in memory (mean square errors)	0.5
z	Technological development parameter	0.5
ω	Share of earnings paid out to shareholders	0
ξ	Parameter 1 in the utilization cost function	0.8
ξ_1	Parameter 2 in the utilization cost function	0.3
ξ_2	Parameter 3 in the utilization cost function	0.25
$\psi(.)$	Utilization cost function	0.80
ϵ_t^z	Std. deviation of technology shock	0.5
ϵ_t^{mp}	Std. deviation of nom. interest rate shock	0.5
ϵ_t^{uc}	Std. deviation of shock in the utilization cost function	0.5
ρ^k	AR process of shock to utilization cost function	0.1

Table II.2: Model correlations - comparisons

Correlations	Value - behavioural model	Value - US data
$\rho(y_t, y_{t-1})$	0.86	0.85
$\rho(y_t, k_t)$	0.45	0.15
$\rho(y_t, \pi_t)$	-0.42	-0.43
$\rho(y_t, as_t)$	0.84	-
$\rho(y_t, AD_t)$	0.17	-
$\rho(y_t, AS_t)$	-0.11	-
$\rho(y_t, \psi(u_t))$	-0.01	-
$\rho(y_t, d_t)$	0.17	0.32
$\rho(y_t, l_t^s)$	0.11	0.18
$\rho(y_t, r_t)$	0.39	0.45
$\rho(y_t, i_t)$	0.23	0.90
$\rho(y_t, c_t)$	0.21	0.32
$\rho(y_t, s_t)$	0.26	-0.28
$\rho(y_t, x_t)$	-0.41	-0.49
$\rho(y_t, \vartheta_t)$	0.02	-
$\rho(k_t, k_{t-1})$	0.96	0.88
$\rho(k_t, as_t)$	0.32	-
$\rho(k_t, \vartheta_t)$	0.02	-
$\rho(k_t, r_t)$	0.08	0.31
$\rho(l_t^s, as_t)$	0.12	-
$\rho(l_t^s, k_t)$	0.28	0.38
$\rho(l_t^s, x_t)$	-0.09	0.26
$\rho(\pi_t, \pi_{t-1})$	0.74	0.93
$\rho(\pi_t, as_t)$	-0.38	-
$\rho(r_t, r_{t-1})$	0.83	0.96
$\rho(\pi_t, r_t)$	0.57	0.34
$\rho(\pi_t, r_{t-1})$	0.49	0.34
$\rho(x_t, x_{t-1})$	0.01	0.68
$\rho(x_t, as_t)$	-0.12	-
$\rho(x_t, k_t)$	-0.24	0.09
$\rho(x_t, \vartheta_t)$	0.01	-
$\rho(\vartheta_t, as_t)$	0.007	-
$\rho(y_t, n_t^b)$	-0.01	0.45
$\rho(y_t, n_t^f)$	-0.02	0.22

Note: GDP deflator was used as the inflation indicator, 3-month T-bill for the risk-free interest rate, the deposit rate as the savings indicator and the Corporate lending risk spread (Moody's 30-year BAA-AAA corporate bond rate) as the counterpart for the firm borrowing spread in the models. The variables that are left blank do not have a direct counterpart in the data sample. These are also called 'deep variables'. The only way is to estimate a structural model (using for instance Bayesian techniques) and to derive a value based on a (theoretical) structure. Alternatively, one could also approximate values using micro data. However, this is outside the scope of this paper.

Table II.3: Second and higher moments

Variable	Symbol	Standard deviation **model**	Standard deviation **data**	Skewness **model**	Skewness **data**	Kurtosis **model**	Kurtosis **data**
Output	y_t	2.17	0.93	0.21	1.45	10.91	-0.44
Inflation	π_t	0.35	0.50	-1.81	-0.66	0.36	3.54
Capital	k_t	0.42	1.50	1.24	0.82	0.37	-1.66
Financing spread	x_t	1	0.18	20.9	-5.8	27.9	58.6
Animal spirits	as_t	0.15	-	0.19	-	0.18	-
Deposits	d_t	3.72	1.36	-0.52	1.36	0.17	4.54
Loans	l_t^s	5.07	3.55	1.90	-0.61	0.17	3.75
Interest rate	r_t	0.95	0.76	-4.29	-1.27	1.1	2.38
Investment	i_t	0.24	3.08	-7.81	1.18	9.54	0.71
Utilization costs	$\psi(u_t)$	0.24	-	-0.05	-	0.05	-
Pre-payment share	ϑ_t	73.89	-	3.89	-	53.5	-
Consumption	c_t	0.24	0.81	7.05	0.37	9.79	0.14
Savings	s_t	0.24	8	-7.1	0.49	9.82	8.39
Net worth banks	n_t^b	4.45	1.32	-4.43	-2.34	52.66	9.39
Net worth firms	n_t^f	73.9	2.21	-3.86	-0.34	53.52	16.37
Asset prices	S_t	1.23	6.8	-3.33	1.27	53.75	2.11

Note: The moments are calculated taking output as the denominator. Following a standard approach in the DSGE literature, this is in order to examine the moments with respect to the general business cycle. These are calculated using the full sample of US data stretching from 1953:I - 2014:IV. During this period, the US economy experienced 10 cycles (using NBER business cycle dates), and the average GDP increase per quarter during expansions was 1.05% while it was -0.036% during recessions. The data were de-trended using a standard two-sided HP filter before the moments were calculated in order to facilitate comparison with the model generated (cyclical) moments. The variables that are left blank do not have a direct counterpart in the data sample. These are also called 'deep variables'. The only way is to estimate a structural model (using for instance Bayesian techniques) and to derive a value based on a (theoretical) structure. Alternatively, one could also approximate values using micro data. However, this is outside the scope of this paper.

Table II.4: Third and fourth moments

Variable	Kurtosis	Confidence interval	Skewness	Confidence interval
y_t	141	[137:145]	6.89	[3:10.8]
π_t	9.63	[8.6:10.7]	0.91	[-0.11:1.9]
k_t	25.9	[24:27.6]	2.88	[1.2:4.5]
x_t	649	[642:657]	-22.7	[-30:-15.3]
as_t	1.82	[1.47:2.16]	0.02	[-0.33:0.37]
d_t	2.47	[-4.4:9.4]	-0.59	[-7.5:6.3]
l_t^s	2.47	[-10:8.2]	-0.97	[-10:8.2]
r_t	70	[66.5:73.7]	4.4	[0.77:8]
i_t	469	[467:470]	17.3	[15.8:18.8]
$\psi(u_t)$	2.50	[2:3.5]	0.006	[-0.49:0.5]

Note: The moments are reported in absolute terms and not with respect to output. The confidence interval includes +/- one standard error from the mean moment estimate.

Table II.5: US variables and sources

Variable	US data name	Frequency	Source	Period
y_t	Real GDP	Quarterly	Fed St Louis database	1953:I-2014:IV
c_t	Real Personal Consumption Expenditure	Quarterly	Fed St Louis database	1953:I-2014:IV
i_t	Real Investment	Quarterly	Fed St Louis database	1953:I-2014:IV
k_t	Capital Stock	Annually	Fed St Louis database	1953:I-2011:I
d_t	Total Savings and Time Deposits of Households	Quarterly	Fed St Louis database	1953:I-2014:IV
s_t	Net Private Savings Households	Quarterly	Fed St Louis database	1953:I-2014:IV
l_t^s	Credit Market Instruments for Firms	Quarterly	Fed St Louis database	1953:I-2014:IV
r_t	Effective Federal Funds Rate	Monthly	Fed St Louis database	1954:II-2014:IV
x_t/efp_t	Moody's (30 year) BAA - AAA Corporate Bond Spread	Monthly	Fed St Louis database	1953:I-2014:IV
n_t^b	Financial Business Total Liabilities and Net Worth	Quarterly	Fed St Louis database	1953:I-2014:IV
n_t^f	Nonfinancial Corporate Business Net Worth	Quarterly	Fed St Louis database	1953:I-2014:IV
π	GDP Deflator	Quarterly	Fed St Louis database	1953:I-2014:IV

Note: All variables were downloaded on March 2, 2015. The latest recorded observation for each variable was 2014:IV (except for capital stock).

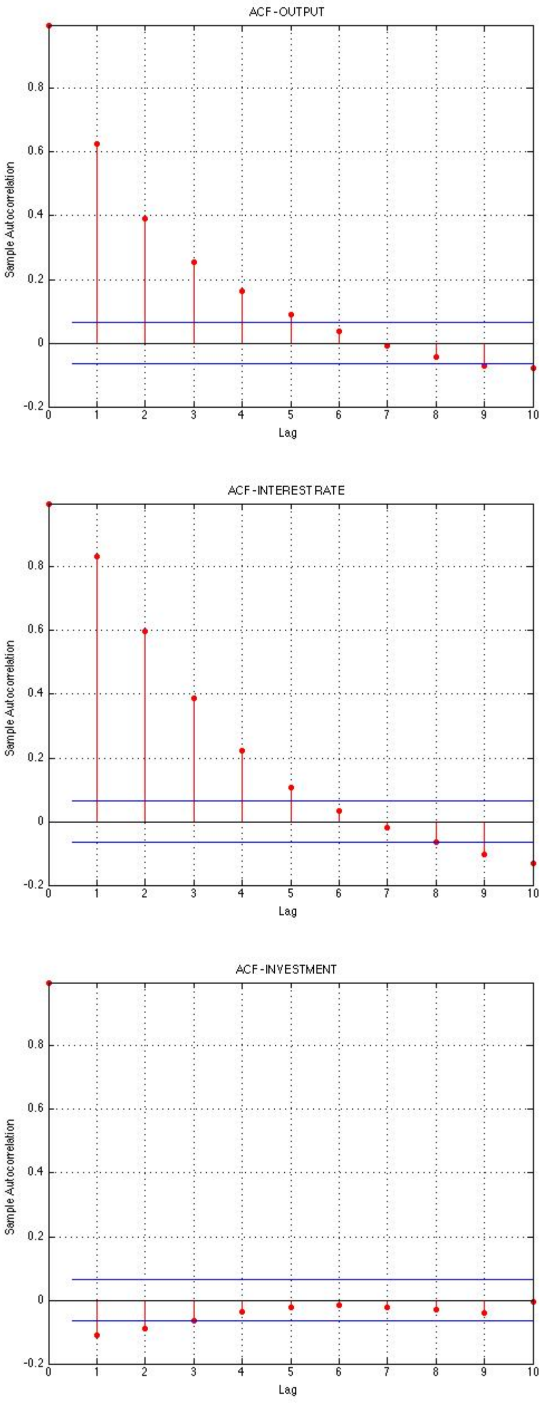

Figure II.1: Model autocorrelation functions (ACF) of lag length 10 of (from left to right and top to down): output, inflation, interest rate, capital, investment, and utilization costs

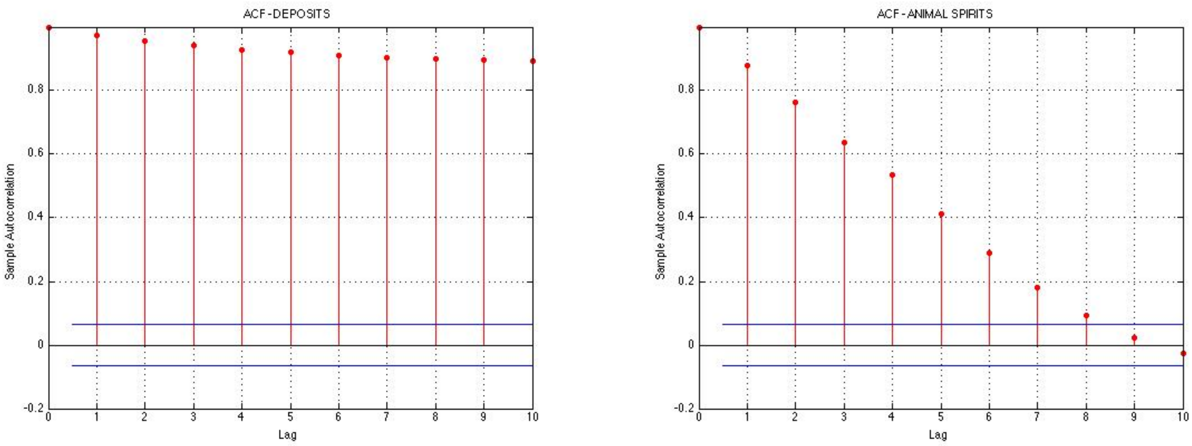

Figure II.2: Model autocorrelation functions (ACF) of lag length 10 of (from left to right): deposits and animal spirits

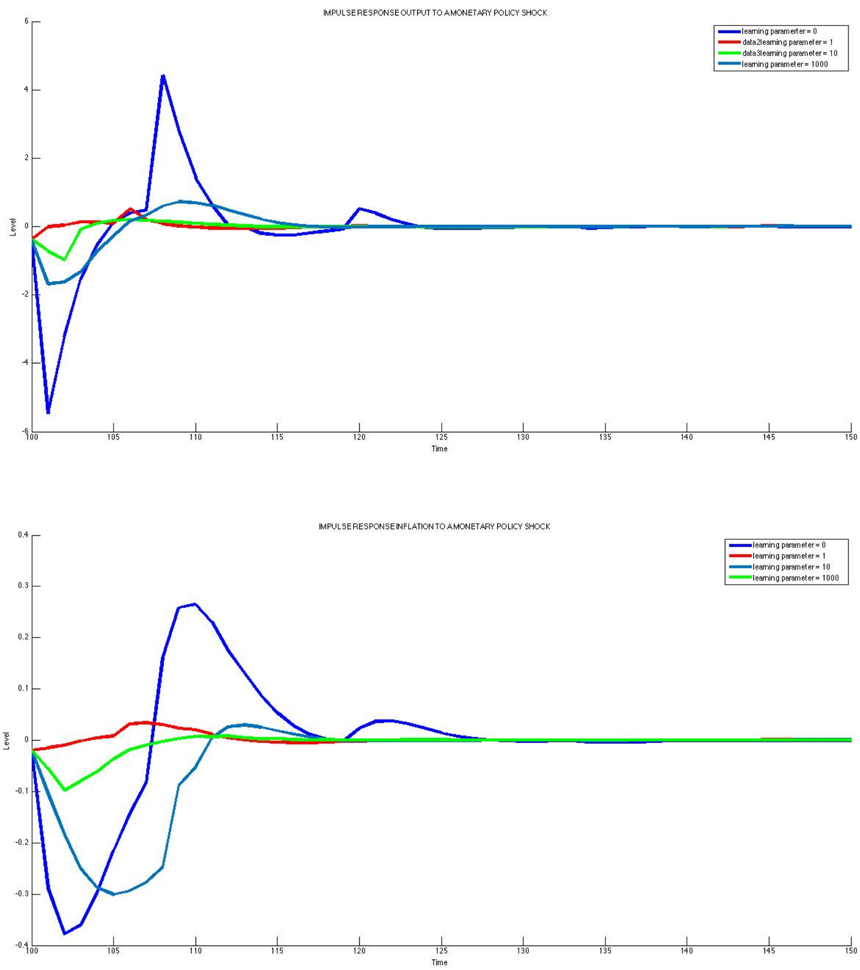

Figure II.3: Impulse responses of output (top) and inflation (down) to a (negative) monetary policy shock for different values of the learning (in switching) parameter γ. $\gamma = 0$ implies no learning at all, while the bigger the γ, the quicker the learning.

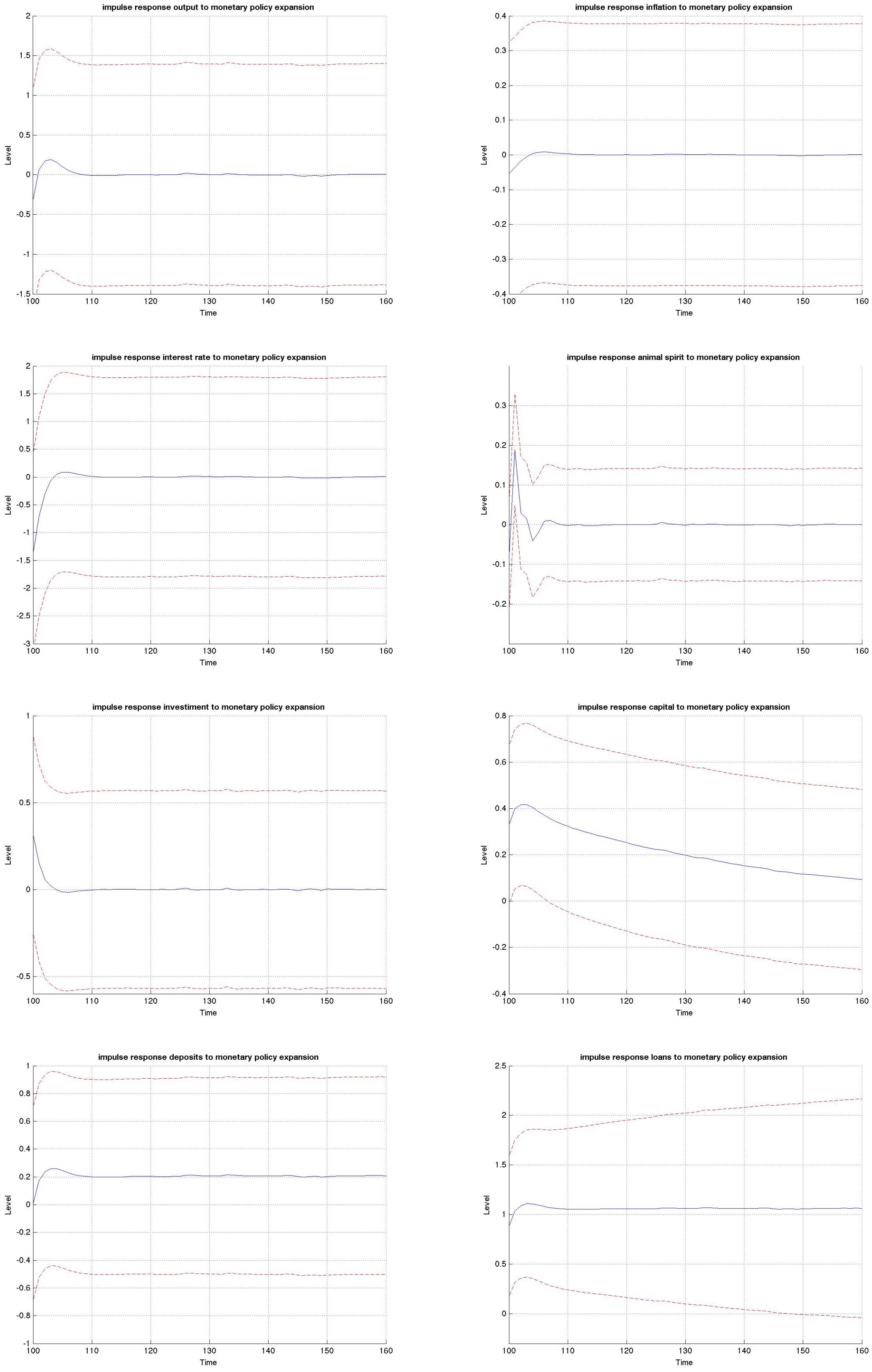

Figure II.4: Full impulse responses to an expansionary monetary policy shock with 95% confidence interval. The impulse responses reported are (from left to right, and top to down): output, inflation, interest rate, animal spirits, investment, capital, deposits, and loans.

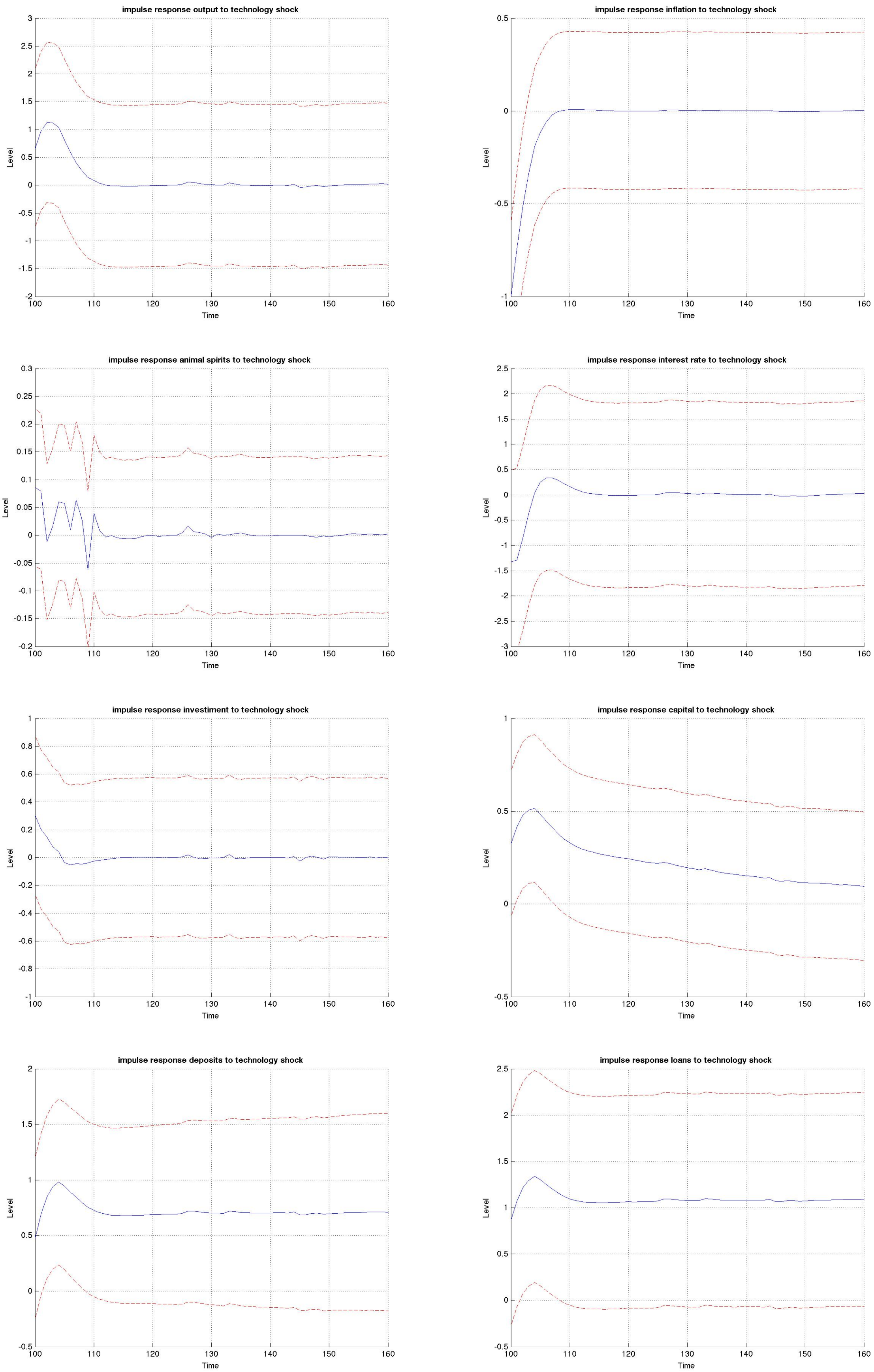

Figure II.5: Full impulse responses to an expansionary technology shock with 95% confidence interval. The impulse responses reported are (from left to right, and top to down): output, inflation, animal spirits, interest rate, investment, capital, deposits, and loans.

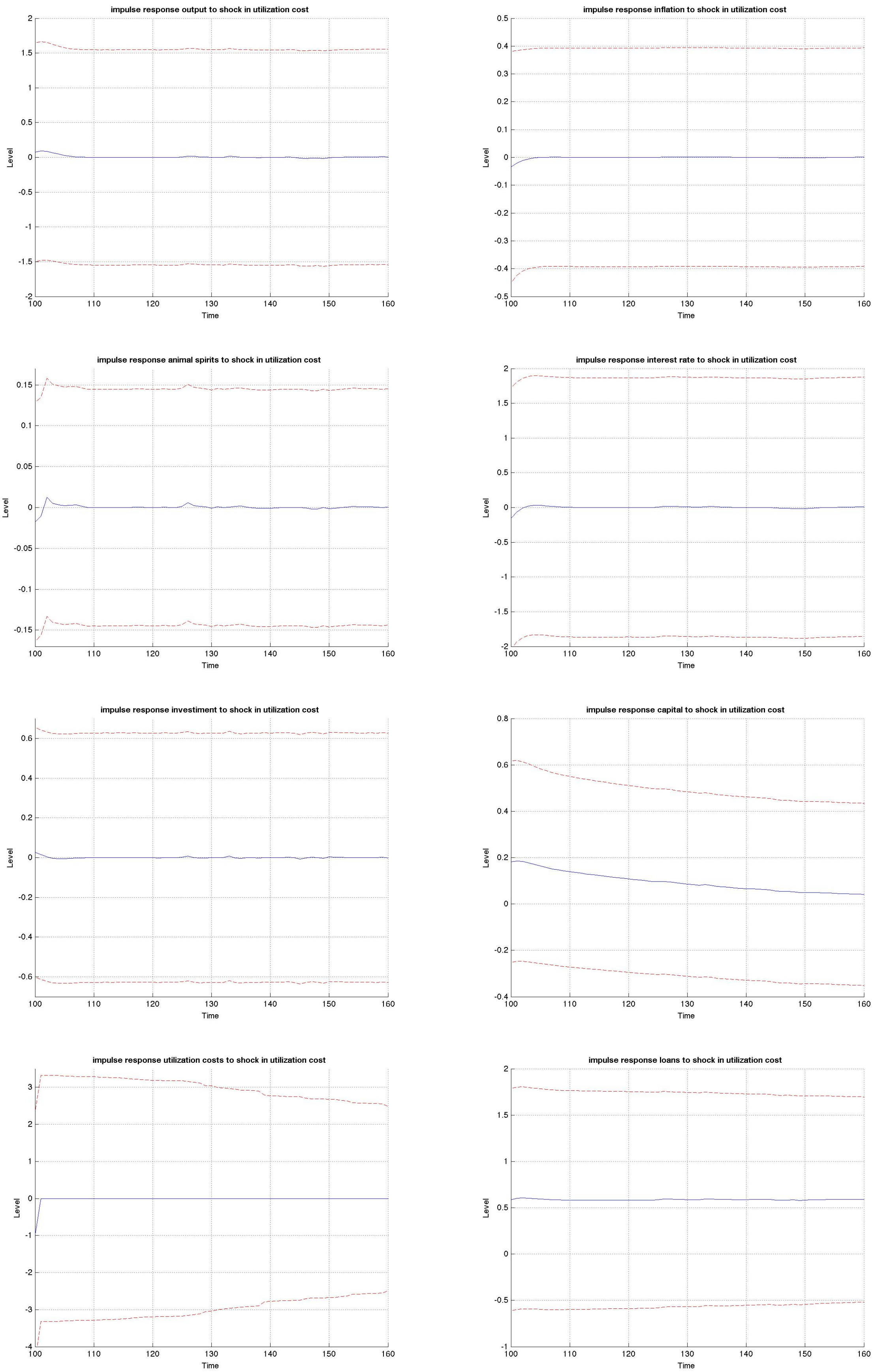

Figure II.6: Full impulse responses to shock in utilization cost with 95% confidence interval. The impulse responses reported are (from left to right, and top to down): output, inflation, animal spirits, interest rate, investment, capital, deposits, and loans.

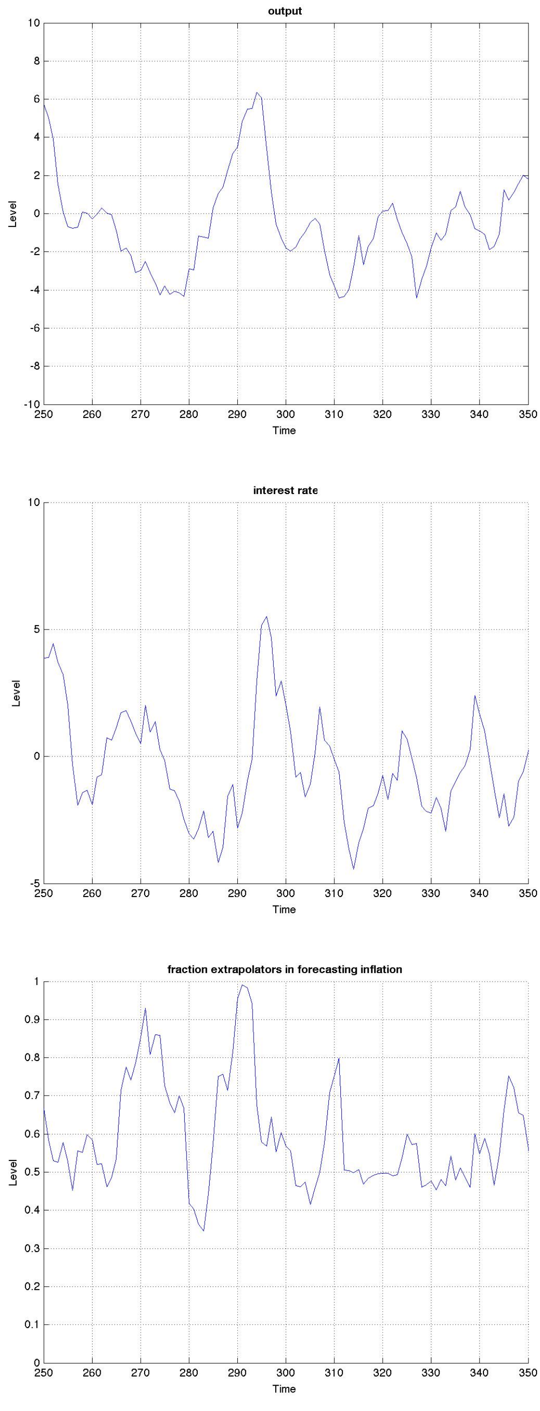

Figure II.7: Evolution of the key aggregate variables and agent behaviour. The figures depict the evolution of (from left to right, and top to down): output, inflation, interest rate, fraction of extrapolators in forecasting output, fraction of extrapolators in forecasting inflation, and animal spirits between t=[250:350] or 25 years.

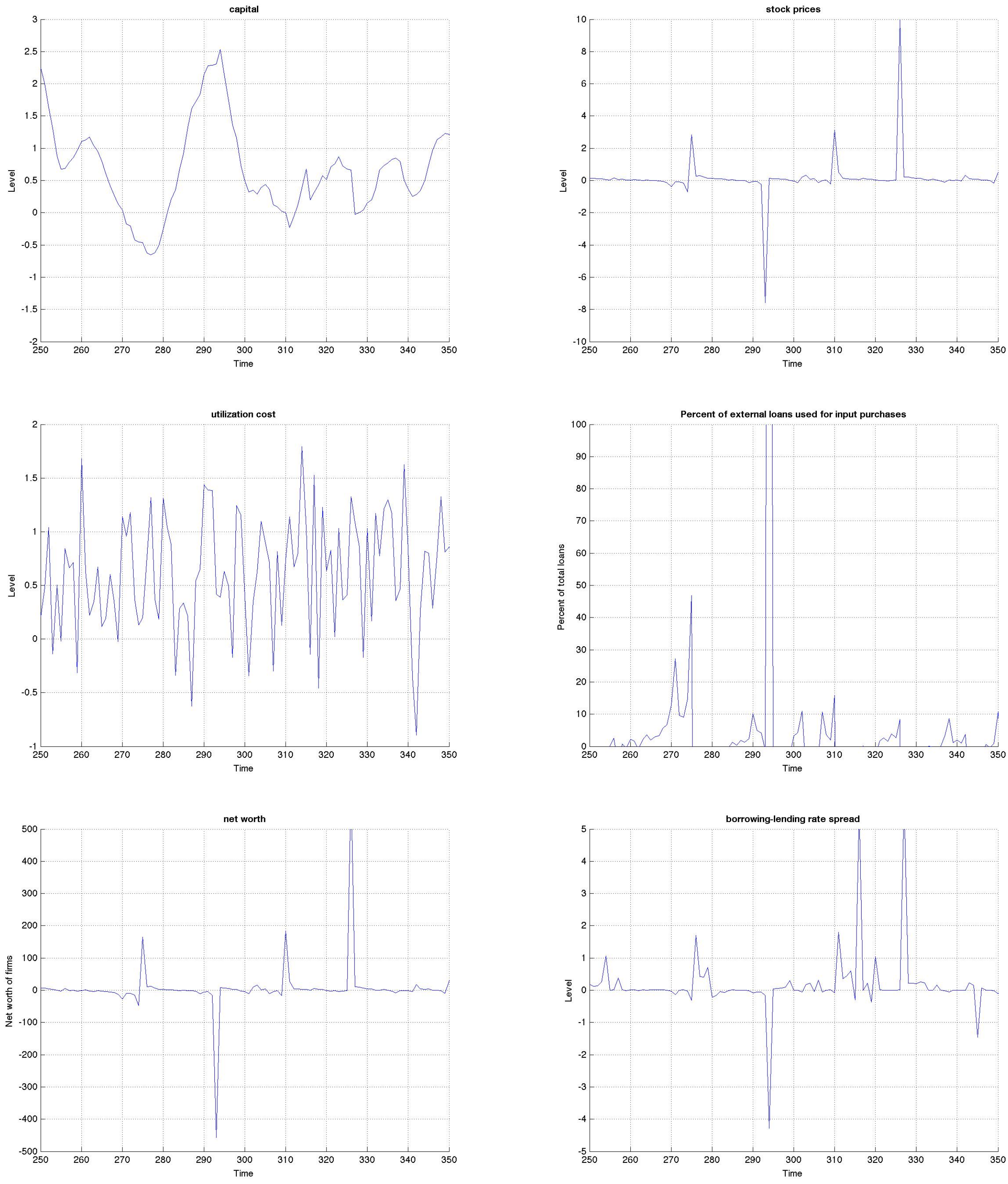

Figure II.8: Evolution of the key aggregate variables 2. The figures depict the evolution of (from left to right, and top to down): capital, stock prices, utilization costs, percent of external loans used for input purchases, net worth of firms, and external financing spread between t=[250:350] or 25 years

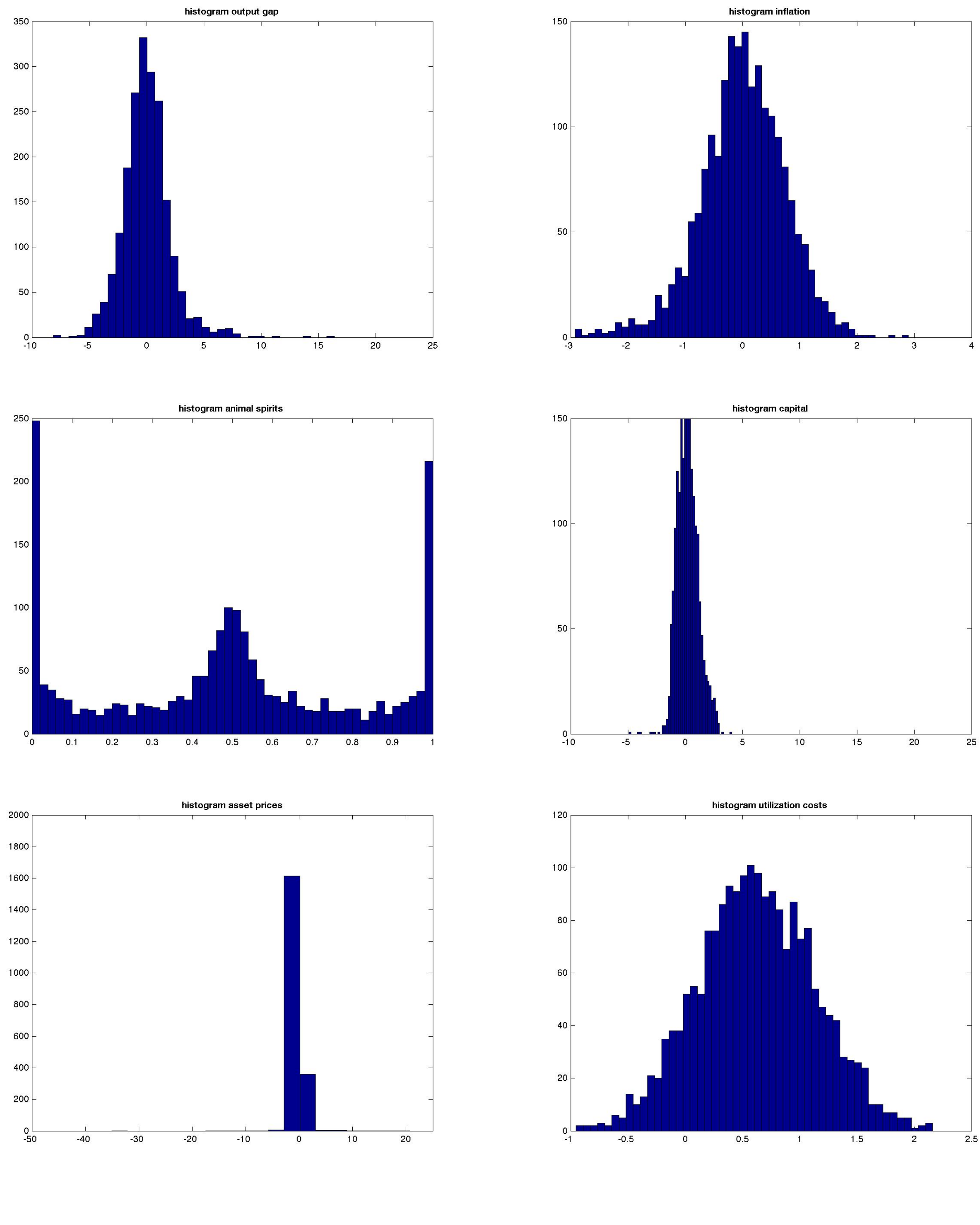

Figure II.9: (Ergodic) distributions for the full sample (t=2000 quarters) of the following variables: output, inflation, animal spirits, capital, stock prices, and utilization costs.

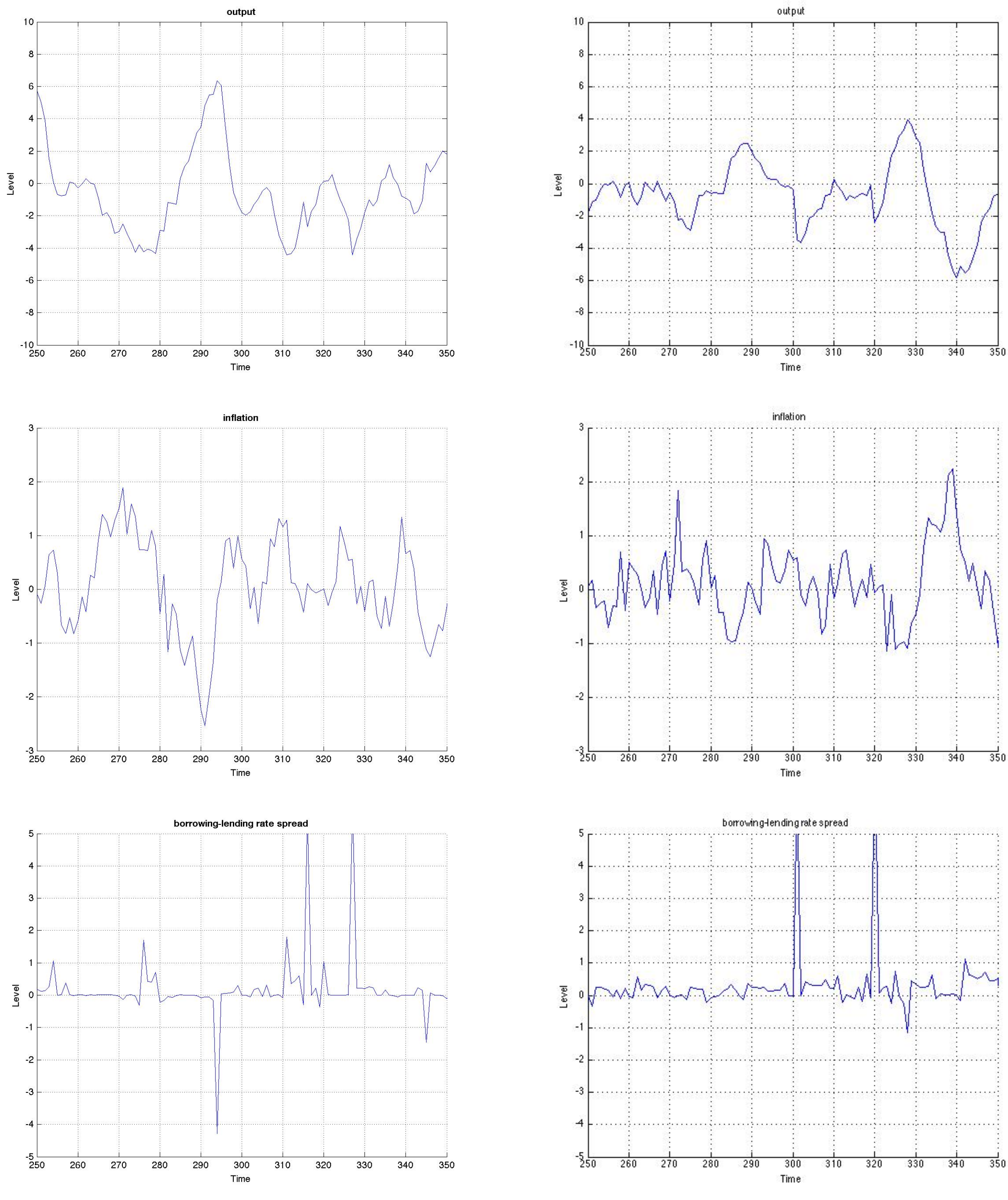

Figure II.10: Comparison between the (cyclical) evolution of key aggregate variables in the full model version (left) and the benchmark version (right) where the supply side financial friction is not present for the time interval t=[250:350] or 25 years. The reported variables in both columns are (from top-down): output, inflation, and external financing spread.

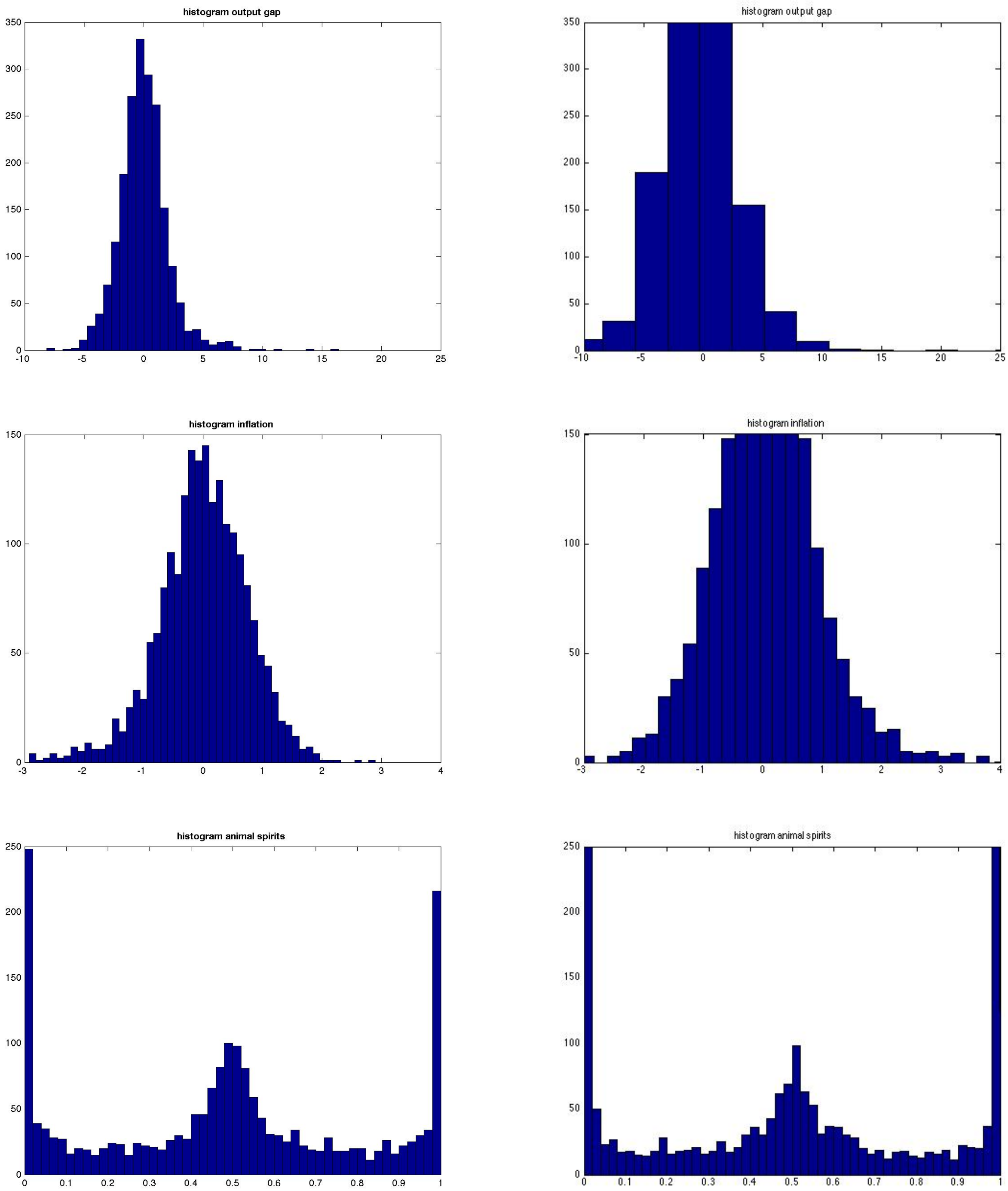

Figure II.11: Comparison of (ergodic) distributions with the full sample (t=2000) between the full model version (left) and the benchmark version (right) where the supply side financial friction is not present. The reported variables in both columns are (from top-down): output, inflation, and animal spirits. The scales are the same in the left and right column to facilitate a quick visual inspection.