

An Comparative Analysis of Different Models of Belief Revision using Information from Multiple Sources

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Abstract

In this work we analyze the problem of knowledge representation in a collaborative multi-agent system where agents can obtain new information from others through communication. Namely, we analyze several approaches of belief revision in multi-agent systems. We will describe different research lines in this topic and we will focus on *Belief Revision using Information from Multiple Sources*. For this, we are going to accomplish a comparative analysis of different models of belief revision that use information from multiple sources.

Key words: Multi-Agent Systems, Revision, Plausibility, Comparison.

1 INTRODUCTION

The problem of belief revision (BR) has been widely discussed in the past twenty years [1, 2, 10]. In belief revision theory, new information must be adopted and some existing information will be dropped to accommodate it. However, many researchers argued that new information should not always have the priority over the existing information and some non-prioritized belief revision methods have been proposed in which new information is not necessarily accepted [6, 8, 11, 15].

An essential skill an autonomous agent should possess is the ability to revise its beliefs in a coherent and rational fashion when it receives new information. Most BR research, however, has been developed with a single agent in mind, *i.e.*, only one problem solver using the BR service.

Multi-Agent Systems (MASs) are distributed computing systems composed of a number of interacting computational entities. One important characteristic distinguishing MASs from traditional distributed systems is that both MAS and its components (agents) are intelligent. As MASs become increasingly attractive for solving larger and more complex problems, the need for adequate BR technology in the MAS paradigm arises. Only a few BR frameworks are known that claim to be suited for MAS applications. There exist different fashions to cope to the consistency maintenance in MAS. In Section 2, a BR hierarchy is presented to clarify the terminologies adopted in recent research on Multi-Agent Belief Revision (MABR).

In this work, namely we are interested in the stages where several deliberative agents are involved in a MAS and they can receive new information from others through communication. Belief Revision

in this type scene is called *Belief Revision using Information from Multiple Sources* (MSBR). Moreover, recently we have presented an article [16] where we defined a new operator of Belief Revision which it is based on a credibility order among agents.

In this paper, we are going to accomplish a comparative analysis between our approach [16] and different models of MSBR. Specifically, we will compare our approach with one proposed by Dragoni in [4] and other proposed by Cantwell in [3].

This paper is organized as follows. Next section introduces an exhaustive analysis of the MABR approaches. In the third section a brief description of our revision operator will be given. Then, in the fourth section we will compare our approach with the presented by Dragoni. In the fifth section, the comparison of our proposal will be done with the Cantwell's proposal. Finally, conclusions are included.

2 BELIEF REVISION IN MULTI-AGENT SYSTEMS

A variety of notations have been adopted by researchers investigating *Belief Revision in Multi-Agent Systems*. A good understanding of the relationships between these approaches is essential before carrying out any further research. In [13] an exhaustive analysis of these approaches is presented and a very interesting hierarchy is introduced (See Figure 1). Observe that in the hierarchy, *Multi-Agent Belief Revision* and *Belief Revision using information from Multiple Sources* are distinguished.

As stated in [13], BR could be considered as part of the agent's skills to maintain the consistency of its own epistemic state. In this case, an individual BR process is carried out in a multi-agent environment, where the new information may come from multiple sources and maybe conflict. BR in this sense is called MSBR by [5]. Cantwell [3] tries to resolve conflicting information by ordering the information sources on the basis of their trustworthiness. This could be served as a rational way of generating the new information credibility based on the source reliability using the terms of MSBR.

However, as discussed in [13], BR could also be used to achieve a society's or team's mutual belief goals (*e.g.* reaching consensus before carrying out plans). In this setting, more than one agent takes part in the process. In order to pursue the mutual goal, agents involved need to communicate, cooperate, coordinate and negotiate with one another. A MABR system is a MAS whose mutual goal involves BR.

MSBR studies individual agent revision behaviors, *i.e.*, when an agent receives information from multiple agents towards whom it has social opinions. MABR investigates the overall BR behavior of agent teams or a society. MSBR is one of the essential components of MABR.

The AGM paradigm [2] has been widely accepted as a standard framework for BR. But it is only capable of prescribing revision behaviors of a single agent. The BR process is more complex in multiple agent case. Besides the Principle of Minimal Change, there exist other requisites due to the sophisticated agent interactions.

An agent is capable of carrying out Individual Belief Revision (IBR), while an agent society or team is capable of MABR. IBR in a single agent environment (Single Belief Revision, SBR) could be achieved using classical BR satisfying or adapting AGM postulates. IBR in a multiple agent environment is MSBR, *i.e.*, a single agent will have to process information coming from more than one source.

Different formalisms have been presented to deal with MABR [13, 14, 12]. In [13, 14] an ontology to solve MABR is defined. That is, in these papers three major categories of heterogeneity, namely social, semantic and syntactic heterogeneity are clarified. There, is shown that several issues posed by such heterogeneities are addressed in the context of BR. They also propose the use of ontology as

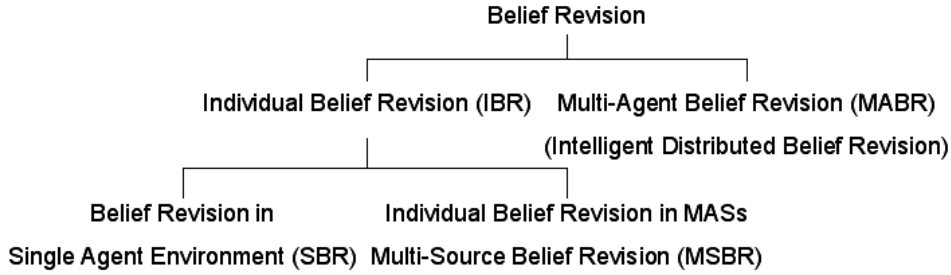


Figure 1: Belief Revision Hierarchy

a tool to handle the heterogeneity issues so as to achieve the necessary reliable communication and system interoperability required by MABR.

In [12] research on MABR in the context of heterogeneous systems is initiated. The Private Domains (PD_i) and the Shared Domain (SD) of the agent knowledge base are defined in order to capture a general setting where each agent has private beliefs as well as beliefs shared with other agents. Under such knowledge structure, each agent may have its own perspective of the world but needs to coordinate (*i.e.* agree on) its belief on shared elements. The shared domain also defines the communication language for the agents.

In contrast to these last two proposals [13, 14, 12], in our approach [16] we have focused on MSBR, where agents maintain the consistence of their belief bases. Since an agent can receive information that is contradictory with its own beliefs, in order to maintain its belief base consistent, it has to decide whether to accept or reject the new information. If an agent decides to accept a new belief that is contradictory with its belief base, then it has to select some beliefs from its belief base in order to withdraw them and avoid the contradiction.

Similarly to Dragoni [4] and Cantwel [3], in our approach, informant agents can have different credibility and this credibility is used to decide which information prevails when a contradiction arises. However, there are differences with these authors, which will be analyzed in this work.

In our approach, a credibility order among agents was defined, and this order is used by all the participants of the multi-agent system. In [16] we have assumed that the credibility order among agents is fixed. To decide whether to reject or accept a new belief, a comparison criterion among beliefs was defined. As it will be explained in detail below, this comparison criterion (that we call plausibility) is based on the credibility order among agents.

3 NON-PRIORITIZED REVISION USING PLAUSIBILITY

This section gives a brief description of an operator that we have defined for multi-source belief revision (recently presented in NMR 2008 [16]).

3.1 Epistemic Model

In our approach [16], we considered a finite set of agents identifiers that will be denoted as $Agents$. Since agents can obtain information from other agents, agents' beliefs are represented as tuples (α, A) , where $A \in Agents$ and α is a sentence of a propositional language \mathcal{L} .

Let $\mathcal{K} = 2^{\mathcal{L} \times Agents}$, each agent $A \in Agents$ has a belief base $K_A \in \mathcal{K}$. As stated above, in this approach informant agents are ranked by their credibilities. Hence, a Credibility Order over the set $Agents$ is introduced:

Definition 1 A *credibility order among agents*, denoted by infix ' \leq_{C_o} ', is a total order over *Agents*, where $A_1 \leq_{C_o} A_2$ means that A_2 is at least as credible as A_1 . The strict relation $A_1 <_{C_o} A_2$, representing A_2 is strictly more credible than A_1 , is defined as $A_1 \leq_{C_o} A_2$ and $A_2 \not\leq_{C_o} A_1$. Moreover, $A_1 =_{C_o} A_2$ means that A_1 is as credible as A_2 , and it holds when $A_1 \leq_{C_o} A_2$ and $A_2 \leq_{C_o} A_1$.

Example 1 Consider a set *Agents* = $\{A_1, A_2, A_3, A_4\}$ where the credibility order is $A_1 \leq_{C_o} A_2$, $A_2 \leq_{C_o} A_3$, $A_3 \leq_{C_o} A_2$, $A_3 \leq_{C_o} A_4$. Note that A_2 is as credible as A_3 . The belief base of the agent A_1 is $K_{A_1} = \{(\beta, A_1), (\alpha, A_2), (\alpha, A_3), (\alpha \rightarrow \beta, A_2), (\alpha \rightarrow \beta, A_4), (\omega, A_1), (\omega \rightarrow \beta, A_4), (\alpha \rightarrow \delta, A_2), (\delta \rightarrow \beta, A_1), (\gamma, A_2), (\gamma, A_3), (\gamma \rightarrow \epsilon, A_2), (\epsilon \rightarrow \beta, A_2), (\epsilon \rightarrow \beta, A_3), (\epsilon \rightarrow \beta, A_4)\}$. Observe that K_{A_1} has three tuples with the sentence $\epsilon \rightarrow \beta$. Although this can be considered as redundancy, each one comes from a different informant agent. The reasons for maintaining all of them will be explained below.

Besides, in [16] two auxiliary functions was introduced in order to obtain the set of sentences (resp. set of agents) that belong to a belief base $K \in \mathcal{K}$.

Definition 2 The *sentence function*, $Sen : \mathcal{K} \mapsto 2^{\mathcal{L}}$, is a function such that for a given belief base $K \in \mathcal{K}$, $Sen(K) = \{\alpha : (\alpha, A) \in K \text{ for any } A \text{ in } Agents\}$.

Definition 3 The *agent identifier function*, $Ag : \mathcal{K} \mapsto 2^{Agents}$, is a function such that for a given belief base $K \in \mathcal{K}$, $Ag(K) = \{A : (\alpha, A) \in K \text{ for any } \alpha \text{ in } \mathcal{L}\}$.

As stated above, agents can receive new beliefs from other informants. This new information can be contradictory with their current beliefs. For instance, consider again the belief base (K_{A_1}) of Example 1, where $Sen(K_{A_1}) \vdash \beta$ (observe that there are several derivations for β). Suppose now that the agent A_1 receives the input $(\neg\beta, A_4)$. It is clear that adding $(\neg\beta, A_4)$ to K_{A_1} will produce an inconsistent belief base. Therefore, the agent has to revise its beliefs and decide whether it rejects $(\neg\beta, A_4)$ or it withdraws β . The credibility order is used to decide which information prevails. However, since there can be several derivations of β , then we have to “cut” all of them. For doing that, all the minimal subsets of K_{A_1} that entails β will be obtained, using an extension of *Kernel contractions*.

Kernel contractions introduced in [10] are based on a selection among the sentences that are relevant to derive the sentence to be retracted. In order to perform a contraction, kernel contractions use incision functions which cut into the minimal subsets that imply the information to be given up. We have adapted the notion of kernel contraction to our epistemic model. First, we will define the kernel set (Definition 4) and then we will present incision functions (Definition 11) that will cut beliefs according to their plausibility (Definition 8).

Definition 4 Let $K \in \mathcal{K}$ and $\alpha \in \mathcal{L}$. Then $H \in K^{\perp}\alpha$ if and only if

1. $H \subseteq K$.
2. $Sen(H) \vdash \alpha$.
3. if $H' \subset H$, then $Sen(H') \not\vdash \alpha$.

The set of minimal subsets of a belief base $K \in \mathcal{K}$ that imply α (denoted $K^{\perp}\alpha$) is called a *kernel set*. Note that each α -kernel ($H \in K^{\perp}\alpha$) is a set of tuples from K .

The information (α, A_p) that an agent A_i receives from A_p could be consistent with its current belief base K_{A_i} if $Sen(K_{A_i}) \not\vdash \alpha$ or $Sen(K_{A_i}) \vdash \alpha$. If $Sen(K_{A_i}) \not\vdash \alpha$, then it is clear that (α, A_p) is added to K_{A_i} . If $Sen(K_{A_i}) \vdash \alpha$ then (α, A_p) is also added to K_{A_i} because the plausibility of α may

increase (see Section 3.2). Therefore, a belief base $K \in \mathcal{K}$ may contain the same belief in two tuples with different agents identifiers (for instance, $\{(\alpha, A_1), (\alpha, A_2)\} \subseteq K$). Thus, we may say that K has redundant information or we may say that K is redundant. In Example 1 the sentence $\epsilon \rightarrow \beta$ is in three tuples. From the tuples point of view there is no redundancy, due to each tuple represent to a different informant.

In the following section we will show how the new operator uses the additional information (agents identifiers) in order to guide the revision process. The plausibility of the sentences will be defined by using the agents identifiers stored in the belief bases of the agents and the credibility order among agents.

3.2 Sentences Plausibility

The agents identifiers (which are in the second field of the tuples) represent the information that will be used to compute the plausibility of the beliefs. That is, each agent's belief will have an associated plausibility that will depend on the agent identifier and the *credibility order among agents*. The behavior of the plausibility is similar to the epistemic entrenchment defined in [9]. That is, if α and β are sentences in \mathcal{L} , the notation $\alpha \preceq_{K_A} \beta$ will be used as a shorthand for “ β is at least as plausible as α relative to the belief base K of the agent A ”.

One belief base $K \in \mathcal{K}$ may contain either explicit sentences or entailed sentences. As stated above, the explicit sentences are those contained in $Sen(K)$. The entailed sentences are those such that they are not in $Sen(K)$ but they are entailed by sentences in $Sen(K)$. In order to obtain the entailed sentences from a belief base K we are going to use the following function:

Definition 5 The *belief function*, $Bel : \mathcal{K} \mapsto 2^{\mathcal{L}}$, is a function such that for a given belief base $K \in \mathcal{K}$, $Bel(K) = \{\alpha : \alpha \in \mathcal{L} \text{ and } Sen(K) \vdash \alpha\}$.

Note that also K may contain explicit sentences that are entailed by $Bel(K)$. Thus we will have several proofs for the same sentence in K . Therefore, to calculate the plausibility of a sentence (β) we should analyze all its proofs. In order to achieve this, we are going to use the kernels set. We consider that this calculation should be cautious. That is, from each β -kernel, we desire to obtain the less plausible tuples. This plausibility gives us the plausibility of each proof. Then, the plausibility of a derived sentence β will be the greater plausibility among the plausibilities of each β -kernel. In order to define that, two functions will be given next.

Definition 6 The *less credible source function*, $min : \mathcal{K} \mapsto \mathcal{K}$, is a function such that for a given belief base $K \in \mathcal{K}$, $min(K) = \{(\alpha, A_i) : (\alpha, A_i) \in K \text{ and for all } (\delta, A_j) \in K, A_i \leq_{C_o} A_j\}$.

Definition 7 The *more credible source function*, $max : \mathcal{K} \mapsto \mathcal{K}$, is a function such that for a given belief base $K \in \mathcal{K}$, $max(K) = \{(\alpha, A_i) : (\alpha, A_i) \in K \text{ and for all } (\delta, A_j) \in K, A_j \leq_{C_o} A_i\}$.

Example 2 Consider $Agents = \{A_1, A_2, A_3\}$ where $A_1 \leq_{C_o} A_2 \leq_{C_o} A_3$. Let $K_{A_1} = \{(\alpha, A_1), (\alpha, A_2), (\beta, A_1), (\gamma, A_1), (\alpha \rightarrow \gamma, A_3)\}$ be the belief base of A_1 . Then,

- $Bel(K_{A_1})$ will contain the sentences from $Sen(K_{A_1})$ plus the sentences derived by “ \vdash ”. For instance, $\alpha, \beta, \gamma, \alpha \vee \beta, \alpha \vee \beta \vee \gamma, \dots$, and so on.
- $min(K_{A_1}) = \{(\alpha, A_1), (\beta, A_1), (\gamma, A_1)\}$.
- $max(K_{A_1}) = \{(\alpha \rightarrow \gamma, A_3)\}$.

Next, we will introduce a function that returns the plausibility of a sentence that can be explicitly in K or inferred from K .

Definition 8 *The **Plausibility function**, $Pl : \mathcal{L} \times \mathcal{K} \mapsto Agents$, is a function such that for a given sentence $\alpha \in \mathcal{L}$ and a belief base $K \in \mathcal{K}$, $Pl(\alpha, K)$ returns lexicographically the lowest agent identifier of $Ag(max(\bigcup_{X \in K \perp \alpha} min(X)))$.*

It is important to note that if $(\gamma, A_1) \in K_{A_1}$ then $Pl(\gamma, K_{A_1})$ could be different from A_1 . For instance, consider the Example 2, then $Pl(\alpha, K_{A_1}) = A_2$, $Pl(\beta, K_{A_1}) = A_1$ and $Pl(\gamma, K_{A_1}) = A_2$.

Definition 9 ***Plausibility Criterion.** Let $K_A \in \mathcal{K}$ be the belief base of the agent A and let $\{\alpha, \beta\} \subseteq Bel(K_A)$, then $\alpha \preceq_{K_A} \beta$ if and only if $Pl(\alpha, K_A) \leq_{C_o} Pl(\beta, K_A)$.*

The strict relation $\alpha \prec_{K_A} \beta$, representing “ β is more plausible than α ”, is defined as “ $\alpha \preceq_{K_A} \beta$ and $\beta \not\preceq_{K_A} \alpha$ ”. Moreover, $\alpha \simeq_{K_A} \beta$ means that α is as plausible as β , and it holds when $\alpha \preceq_{K_A} \beta$ and $\beta \preceq_{K_A} \alpha$. From the previous definition we can observe that the plausibility of the sentences inherits the properties of the *credibility order among agents* (“ \preceq_{K_A} ” is a total order on \mathcal{L}). Furthermore, note that the relation “ \preceq_{K_A} ” is only defined with respect to a given K_A (different belief bases may be associated with different ordering of plausibility, in Example 3 this situation is shown).

Example 3 *Consider a set $Agents = \{A_1, A_2, A_3\}$ where the credibility order is $A_1 \leq_{C_o} A_2$, $A_2 \leq_{C_o} A_3$. Suppose that the agent A_2 has the following belief base $K_{A_2} = \{(\alpha, A_1), (\beta, A_2), (\gamma, A_3)\}$, and suppose that the agent A_3 has the following belief base $K_{A_3} = \{(\alpha, A_1), (\beta, A_3), (\gamma, A_2)\}$. Then, for both agents, β is more plausible than α (i.e., $\alpha \preceq_{K_{A_2}} \beta$ and $\alpha \preceq_{K_{A_3}} \beta$). However, for A_2 , γ is more plausible than β ($\beta \preceq_{K_{A_2}} \gamma$) whereas for A_3 , β is more plausible than γ ($\gamma \preceq_{K_{A_3}} \beta$).*

In the following section, we will define a non-prioritized revision operator that uses the sentences plausibility in order to guide the revision process.

3.3 Non Prioritized Revision Operator Using Plausibility

In this section, the behavior of the revision operator presented in [16] will be shown. This operator is based on the sentences plausibility. In case that a belief base $K \in \mathcal{K}$ is revised by a tuple (α, A_i) we will have two cases:

- α is consistent with $Bel(K)$. This is the most simple case of characterizing from the logical point of view because it consists only in the addition of new tuples. In the limit case in which $\alpha \in Bel(K)$ then this operation could increase the plausibility of α .
- α is inconsistent with $Bel(K)$, that is $\neg\alpha \in Bel(K)$. This case requires a deeper analysis because: a) it is necessary to determine when the sentence will be accepted; and b) if the input is accepted then it is necessary to erase some tuples of K . For the second case we need to define an incision function on each α -kernel.

We will adapt the incision function definition proposed by [10] to our framework.

Definition 10 *An **incision function** σ for $K \in \mathcal{K}$ is a function such that for all α*

1. $\sigma(K \perp \alpha) \subseteq \cup(K \perp \alpha)$.
2. if $\emptyset \neq X \in K \perp \alpha$, then $X \cap \sigma(K \perp \alpha) \neq \emptyset$

The incision function selects sentences to be discarded. Contracting α from K should be equal to all the elements of the original set K that are not removed by the incision function.

In the definition of *incision function* of Hansson's work is not specified how the function selects the sentences that will be discarded of each α -kernel. This can be solved with the sentences plausibility that we have defined above. The incision function σ will select the less plausible sentences of each α -kernel. Hence, the new operator differs of the kernel revision operator defined by Hansson in the following issues:

1. The new operator will do an analysis to determine if the revision is necessary.
2. The sentences selection for the incision function will be defined.

According to 1, the new operator permits two options, completely accepts all the input, or completely rejects all the input. For this reason the new operator is non prioritized. Some non prioritized operators of the literature that completely accept or reject the input are Semi-Revision [11] and Screened Revision [15]. Another operators may partially accept the new information, for instance Revision by a Set of Sentences [7] and Selective Revision [8].

Next we will define a specific incision function, based on the beliefs plausibility, that will select the less plausible sentences of each α -kernel (following the principle of minimal change).

Definition 11 σ_{\downarrow} is a **bottom incision function** for K if σ_{\downarrow} is an incision function such that, $\sigma_{\downarrow}(K \perp \alpha) = \{(\delta, A_i) : (\delta, A_i) \in X \in K \perp \alpha \text{ and for all } (\beta, A_j) \in X \text{ it holds that } A_i \leq_{C_o} A_j\}$.

Example 4 Consider a set Agents = $\{A_1, A_2, A_3\}$ where the credibility order is $A_1 \leq_{C_o} A_2, A_2 \leq_{C_o} A_3$. Suppose that the agent A_2 has the following belief base $K_{A_2} = \{(\alpha, A_3), (\beta, A_2), (\beta \rightarrow \alpha, A_1), (\omega, A_1), (\omega \rightarrow \alpha, A_3), (\delta, A_1)\}$. Then, $K_{A_2} \perp \alpha = \{H_a, H_b, H_c\}$ where $H_a = \{(\alpha, A_3)\}$, $H_b = \{(\beta, A_2), (\beta \rightarrow \alpha, A_1)\}$, $H_c = \{(\omega, A_1), (\omega \rightarrow \alpha, A_3)\}$. $\sigma_{\downarrow}(K_{A_2} \perp \alpha) = \{(\alpha, A_3), (\beta \rightarrow \alpha, A_1), (\omega, A_1)\}$.

Now that we have given the necessary background on the behavior of the new operator, the *Non-Prioritized Revision Using Plausibility* will be defined.

Definition 12 Let $K \in \mathcal{K}$, let $\alpha \in \mathcal{L}$, let *TopBase* be a maximum belief base function, and let $K \perp \alpha$ be the set of α -kernels of K . Let σ_{\downarrow} be a bottom incision function for K . The operator " \circ ", called **Non-Prioritized Revision Using Plausibility**, is defined as follow:

$$K \circ (\alpha, A_i) = \begin{cases} K \cup \{(\alpha, A_i)\} & \text{if } \neg\alpha \notin \text{Bel}(K) \\ K & \text{if } \neg\alpha \in \text{Bel}(K) \text{ and } A_i \leq_{C_o} \text{Pl}(\neg\alpha, K) \\ (K \setminus X) \cup \{(\alpha, A_i)\} & \text{if } \neg\alpha \in \text{Bel}(K) \text{ and } \text{Pl}(\neg\alpha, K) <_{C_o} A_i \end{cases}$$

where: $X = \{(\omega, A_j) : \omega \in \text{Sen}(\sigma_{\downarrow}(K \perp \neg\alpha)) \text{ and } (\omega, A_j) \in K\}$.

Example 5 Consider a set Agents = $\{A_1, A_2, A_3, A_4, A_5\}$ where the credibility order is $A_1 \leq_{C_o} A_2, A_2 \leq_{C_o} A_3, A_3 \leq_{C_o} A_2, A_3 \leq_{C_o} A_4, A_4 \leq_{C_o} A_5$. Suppose that the agent A_1 has the following belief base $K_{A_1} = \{(\beta, A_1), (\alpha, A_2), (\alpha, A_3), (\alpha \rightarrow \beta, A_2), (\alpha \rightarrow \beta, A_4), (\omega, A_1), (\omega \rightarrow \beta, A_4), (\alpha \rightarrow \delta, A_2), (\delta \rightarrow \beta, A_1), (\gamma, A_2), (\gamma, A_3), (\gamma \rightarrow \epsilon, A_2), (\epsilon \rightarrow \beta, A_2), (\epsilon \rightarrow \beta, A_3), (\epsilon \rightarrow \beta, A_4)\}$. Furthermore, we suppose A_1 receives the tuple $(\neg\beta, A_5)$. Then, A_1 should revise K_{A_1} by $(\neg\beta, A_5)$. Next we will describe the behavior of the new operator step by step.

- **Step 1.** Obtain the minimal subsets that derive β from belief base K_{A_1} .

$K_{A_1}^{\perp\perp}\beta = \{H_a, H_b, H_c, H_d, H_e, H_f, H_g, H_h, H_i, H_j, H_k, H_l, H_m, H_n\}$ where

$H_a = \{(\beta, A_1)\}$, $H_b = \{(\alpha, A_2), (\alpha \rightarrow \beta, A_2)\}$,

$H_c = \{(\alpha, A_2), (\alpha \rightarrow \beta, A_4)\}$, $H_d = \{(\alpha, A_3), (\alpha \rightarrow \beta, A_2)\}$,

$H_e = \{(\alpha, A_3), (\alpha \rightarrow \beta, A_4)\}$, $H_f = \{(\omega, A_1), (\omega \rightarrow \beta, A_4)\}$,

$H_g = \{(\alpha, A_2), (\alpha \rightarrow \delta, A_2), (\delta \rightarrow \beta, A_1)\}$, $H_h = \{(\alpha, A_3), (\alpha \rightarrow \delta, A_2), (\delta \rightarrow \beta, A_1)\}$,

$H_i = \{(\gamma, A_2), (\gamma \rightarrow \epsilon, A_2), (\epsilon \rightarrow \beta, A_2)\}$, $H_j = \{(\gamma, A_2), (\gamma \rightarrow \epsilon, A_2), (\epsilon \rightarrow \beta, A_3)\}$,

$H_k = \{(\gamma, A_2), (\gamma \rightarrow \epsilon, A_2), (\epsilon \rightarrow \beta, A_4)\}$, $H_l = \{(\gamma, A_3), (\gamma \rightarrow \epsilon, A_2), (\epsilon \rightarrow \beta, A_2)\}$,

$H_m = \{(\gamma, A_3), (\gamma \rightarrow \epsilon, A_2), (\epsilon \rightarrow \beta, A_3)\}$ and $H_n = \{(\gamma, A_3), (\gamma \rightarrow \epsilon, A_2), (\epsilon \rightarrow \beta, A_4)\}$.

- **Step 2.** Apply the bottom incision function “ σ_{\perp} ” to the set of minimal subsets of K_{A_1} to obtain the less plausible tuples set of each β -kernel.

$\sigma_{\perp}(K_{A_1}^{\perp\perp}\beta) = \{(\beta, A_1), (\alpha, A_2), (\alpha \rightarrow \beta, A_2), (\alpha, A_3), (\omega, A_1), (\delta \rightarrow \beta, A_1), (\gamma, A_2), (\gamma \rightarrow \epsilon, A_2), (\epsilon \rightarrow \beta, A_2), (\epsilon \rightarrow \beta, A_3), (\gamma, A_3)\}$.

- **Step 3.** Obtain from the tuples of the previous item, those tuples of greater plausibility with the more credible source function “ max ”($max(\sigma_{\perp}(K_{A_1}^{\perp\perp}\beta))$).

$max(\{(\beta, A_1), (\alpha, A_2), (\alpha \rightarrow \beta, A_2), (\alpha, A_3), (\omega, A_1), (\delta \rightarrow \beta, A_1), (\gamma, A_2), (\gamma \rightarrow \epsilon, A_2), (\epsilon \rightarrow \beta, A_2), (\epsilon \rightarrow \beta, A_3), (\gamma, A_3)\}) = \{(\alpha, A_2), (\alpha \rightarrow \beta, A_2), (\alpha, A_3), (\gamma, A_2), (\gamma \rightarrow \epsilon, A_2), (\epsilon \rightarrow \beta, A_2), (\epsilon \rightarrow \beta, A_3), (\gamma, A_3)\}$.

- **Step 4.** Compare the agent identifier of the input tuple with the agent identifier of any tuple obtained from previous item (lets suppose (α, A_j)). If $A_5 \leq_{C_0} A_j$ the operation has no effect, the input is rejected, i.e., $K \circ (\neg\beta, A_5) = K$. On the other hand, if $A_j <_{C_0} A_5$ then $K \circ (\neg\beta, A_5) = K \setminus \{(\omega, A_j) : \omega \in Sen(\sigma_{\perp}(K^{\perp\perp}\neg\alpha)) \text{ and } (\omega, A_j) \in K\} \cup \{(\alpha, A_i)\}$.

$K_{A_1} \circ (\neg\beta, A_5) = \{(\omega \rightarrow \beta, A_4), (\alpha \rightarrow \delta, A_2), (\neg\beta, A_5)\}$.

Remark 1 Note that, since the belief base may be redundant, in step 4 of Example 5 if the revision gives rise to a contraction then we will discard from K all those tuples whose sentences were selected by the bottom incision function without regarding the respective informants. Besides, note that our operator will never discard more plausible sentences than the input. This control can be seen in Step 4 of Example 5.

Remark 2 In this approach, we have assumed that the credibility order among agents is fixed. However, this order may be replaced and this will not affect the behavior of the operator. If the credibility order among agents changes, then the plausibility of all sentences may also change without changing the belief bases of the agents. This feature was one of the motivations for using agent identifiers instead of representing explicitly the plausibility of sentences as a number. For instance, consider a set $Agents = \{A_1, A_2\}$ where the credibility order is $A_1 \leq_{C_0} A_2$, $K_{A_1} = \{(\alpha, A_1), (\beta, A_2)\}$ and $K_{A_2} = \{(\omega, A_2), (\gamma, A_1)\}$. Hence, $\alpha \preceq_{K_{A_1}} \beta$ and $\gamma \preceq_{K_{A_2}} \omega$. If the credibility order changes to $A_2 \leq_{C_0} A_1$ then $\beta \preceq_{K_{A_1}} \alpha$ and $\omega \preceq_{K_{A_2}} \gamma$. Note that the tuples in K_1 and K_2 remain unchanged.

3.4 Retransmission of Information

As stated above, each agent $A \in Agents$ will have a belief base $K_A \in \mathcal{K}$, where $\mathcal{K} = 2^{\mathcal{L} \times Agents}$. Hence, the agents store each belief with an agent identifier in tuples. When an agent sends information to other agent, it sends tuples. Consider for example an agent set $\{A_1, A_2, A_3\} \subseteq Agents$ where $A_1 \leq_{C_o} A_2 \leq_{C_o} A_3$. Suppose that $K_{A_1} = \{(\alpha, A_3)\}$ then if A_1 wants to send α to A_2 , it has to send a tuple with an agent identifier. This identifier may be:

- the proper sender identifier (e.g., A_1), or
- the agent identifier stored with the belief in the sender's base (e.g., A_3).

Here, we adopt the latter option. That is, A_1 will send the tuple (α, A_3) to A_2 . Thus, the receiver agent A_2 will know the source from where the sender A_1 has obtained the information.

From the receiver point of view, when it receives from A_1 the tuple (α, A_3) it may store:

- (α, A_3) , i.e., the agent identifier stored with the belief in the sender's base, or
- (α, A_i) where A_i is the agent identifier more credible (according to the credibility order among agents, Definition 1) that results of comparing the A_1 and A_3 .

Here, we adopt the latter option. Thus, the agents will hold their beliefs with the more credible informant known. In this case, since $A_1 \leq_{C_o} A_3$, the receiver agent A_2 will apply the new revision operator over the tuple (α, A_3) .

4 DRAGONI'S APPROACH VS. OUR APPROACH

In this section a brief summary of the proposal of Dragini in [4, 5] will be given. Besides, we are going to do a comparison of this proposal with our approach.

In [4, 5] is considered that agents detect and store in tables the *nogoods*. The *nogoods* are the minimally inconsistent subsets of the agents' knowledge bases. In contrast to this, a *good* is a subset of the knowledge base such that: it is not inconsistent (it is not a superset of a *nogood*), and if augmented with whatever else assumption in knowledge base it becomes inconsistent. Here, we can note the first difference. In contrast to our approach, they do not remove beliefs to avoid a contradiction, but, quite more generally, to choose which is the new preferred *good* among them in knowledge base. In our model, we obtain the kernel sets to cut some sentences, thus we broke the contradictions if it is necessary.

That is, their strategy is similar to the proposal by AGM, "Maxichoice revision" [2]. The contractions of the type maxichoice are contractions based on a selection operator that chooses one and only one element of the remains set. In [1], Alchourron and Makinson tried to give a more explicit construction of the contraction process, and hence also of the revision process via the *Levi identity*. Their basic idea was to choose $A - x$ as a maximal subset of A that fails to imply x . Contraction functions defined in this way were called "choice contractions" in [1].

Other difference can be noted in the tuples. Like us, they propose to store additional information with each sentence. However, their tuples contain 5 elements: <Identifier, Sentence, OS, Source, Credibility>, where Origin Set (OS) records the assumption nodes upon which it really ultimately depends (as derived by the theorem prover). In contrast to them, in our model a tuple only store a sentence and a source, but a tuple does not store the credibility. That is, in our model the plausibility of a sentence is not explicitly stored with it, as [4] does. Thus, when the plausibility of some sentence

is needed the *plausibility function* should be applied. As is shown in Example 6, given a sentence α its plausibility depend on its proofs (α -kernels). Therefore, if one of the sentences of these proof changes, then the plausibility of α may change. Hence, if the credibility order is replaced, then the sentence plausibility may change without changing the belief base.

Example 6 Consider a set $Agents = \{A_1, A_2\}$ where the credibility order is $A_1 \leq_{C_0} A_2$, $K_{A_1} = \{(\alpha, A_1), (\alpha \rightarrow \beta, A_2)\}$ and $K_{A_2} = \{(\alpha, A_2)\}$. By Definition 8, $Pl(\beta, K_{A_1}) = A_1$. Now, suppose that A_1 receives from A_2 the belief α . Now $K_{A_1} = \{(\alpha, A_1), (\alpha, A_2), (\alpha \rightarrow \beta, A_2)\}$ and A_1 has two derivations for β , hence $Pl(\beta, K_{A_1}) = A_2$. Observe that plausibility of β is increased.

The communication policy among agents is other difference that we can note with respect to us approach. The communication policy that we have showed in Section 3.4 differs from the one [4] where the agents do not communicate the sources of the assumptions, but they present themselves as completely responsible for the knowledge they are passing on; receiving agents consider the sending ones as the sources of all the assumptions they are receiving from them. In Figure 2 the difference is shown. In (a) is our approach and in (b) the Dragoni's approach.

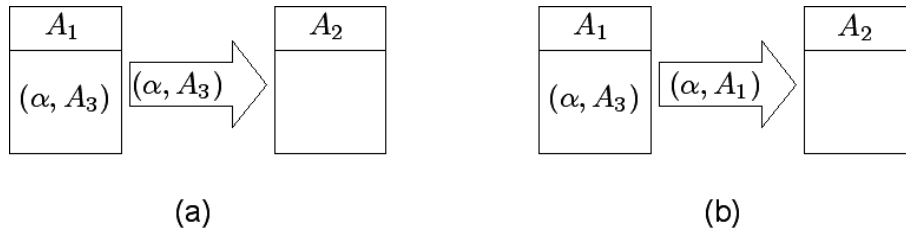


Figure 2: Communication policy

In (b) we can see that if an agent retransmits one belief, the original source is lost. In contrast to this, in our approach the agents hold their beliefs with the more credible informant known.

5 CANTWELL'S APPROACH VS. OUR APPROACH

In [3] Cantwell shows how a support ordering on the information can be generated and how it can be used to decide what information to accept and what not to accept. In this work, in contrast to our approach [16], a *scenario* (set of incoming information) presented by a source is treated as a whole and not sentence by sentence, and therefore, it can be inconsistent.

Besides, in [3] a relation of *trustworthiness* is introduced over sets of sources and not between single sources. Moreover, if two sources give the same piece of information ϕ , and a single agent gives $\neg\phi$, then ϕ will be preferred, that is, the decision is based on majority. This is shown in the following example.

Example 7 Let A_1, A_2 and A_3 be three sources that are more or less equally trustworthy, now if A_1 and A_2 both give the piece of information ϕ while A_3 gives the information $\neg\phi$ one would probably be inclined towards accepting ϕ rather than $\neg\phi$ as the joint trustworthiness of A_1 and A_2 would typically be greater than that of A_3 . To capture this Cantwell introduced a relation of trustworthiness not between single sources, but between sets of sources.

In Cantwell's approach the order in which the evidence is considered does not seem to be important. However, in the following example it is shown that in our work the order in which beliefs are considered is important.

Example 8 Consider a set $Agents = \{A_1, A_2, A_3\}$ where the credibility order is $A_1 \leq_{Co} A_2$, $A_2 \leq_{Co} A_3$, $A_3 \leq_{Co} A_2$. Note that A_2 is as credible as A_3 . Suppose that the belief base of A_1 is empty. If A_1 receives the tuple (α, A_2) and then receives $(\neg\alpha, A_3)$, then $(\neg\alpha, A_3)$ will be rejected. However, if A_1 first receives the tuple $(\neg\alpha, A_2)$ and then receives (α, A_3) , then (α, A_3) will be rejected.

6 CONCLUSION

In this paper we have introduced an analysis of the different research lines of MABR based on the hierarchy presented in [13]. Namely, we have focused in MSBR and briefly we have described our approach recently presented [16]. Then, we have compared our approach with two other models (Dragoni's approach [4, 5] and Cantwell's approach [3]) that cope with Multiple Sources Belief Revision (MSBR). Like us, both consider that the reliability of the source affects the credibility of incoming information, and this reliability is used to decide whether a received formula is accepted or rejected. However, these two approaches differs from ours in several issues.

In [4, 5] is considered that agents detect and store in tables the *nogoods* and the *good*. In contrast to our approach, they do not remove beliefs to avoid a contradiction, but, quite more generally, to choose which is the new preferred *good* among them in knowledge base. This strategy is similar to "Maxichoice revision" of AGM. In our model, rather than that, we obtain the kernel sets to cut some sentences, thus we broke the contradictions if it is necessary.

Like us, they propose to store additional information with each sentence. However, their tuples contain 5 elements: $\langle \text{Identifier, Sentence, OS, Source, Credibility} \rangle$. In contrast to them, in our model a tuple only store a sentence and a source, but a tuple does not store the credibility. That is, in our model the plausibility of a sentence is not explicitly stored with it, as [4] does. Thus, when the plausibility of some sentence is needed the *plausibility function* should be applied.

The communication policy that we have showed in Section 3.2 differs from the one [4] where the agents do not communicate the sources of the assumptions, but they present themselves as completely responsible for the knowledge they are passing on; receiving agents consider the sending ones as the sources of all the assumptions they are receiving from them.

In [3], a *scenario* (set of incoming information) presented by a source is treated as a whole and not sentence by sentence, and therefore, it can be inconsistent. A relation of *trustworthiness* is introduced over sets of sources and not between single sources. Besides, if two sources give the same piece of information ϕ , and a single agent gives $\neg\phi$, then ϕ will be preferred, that is, the decision is based on majority. In his approach, the order in which the evidence is considered does not seem to be important. However, in our work, the order in which beliefs are considered is important: If an agent receives α and then receives $\neg\alpha$ and both have the same plausibility, then $\neg\alpha$ will be rejected.

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