Operations on admissible attack scenarios

Diego C. Martínez, Alejandro J. García, and Guillermo R. Simari

Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET),
Artificial Intelligence Research and Development Laboratory LIDIA,
Department of Computer Science and Engineering, Universidad Nacional del Sur, Bahía Blanca,
ARGENTINA
Email: {dcm,ajg,grs}@cs.uns.edu.ar

Abstract. In classical abstract argumentation, arguments interact with each other through a single abstract notion of attack. However, several concrete forms of argument conflict are present in the literature, all of them of different nature and strength for a particular context. In this work we define an argumentation framework equipped with a set of abstract attack relations of varied strength. Using this framework, semantic notions dealing with the relative difference of strength are introduced. The focus is put on argument defense, and the study of admissible sets according to the quality of defenders.

1 Introduction

Abstract argumentation systems [1–4] are formalisms for argumentation where some components remain unspecified towards the study of pure semantic notions. Most of the existing proposals are based on the single abstract concept of *attack* represented as a binary relation, and according to several rational rules, extensions are defined as sets of possibly accepted arguments. The attack relation is basically a subordinate relation of conflicting arguments. For two arguments $\mathcal{A}$ and $\mathcal{B}$, if $(\mathcal{A}, \mathcal{B})$ is in the attack relation, then the status of acceptance of $\mathcal{B}$ is conditioned by the status of $\mathcal{A}$, but not the other way around. It is said that argument $\mathcal{A}$ attacks $\mathcal{B}$, and it implies a priority between conflicting arguments. One of the most important formalizations on abstract argumentation is the framework defined by Dung in [1], where the simplicity of the model allows practical definitions of sets of arguments as possible sets of acceptance. The attack relation in Dung's work is a binary relation between arguments as described previously. However, this relation is not always accurate to model some situations where more detail is needed. In several argumentation scenarios, not every argument conflict has the same weight, as they arise for underlying different reasons. Every argumentation system defines one or more notions of argument conflict, leading to an evolved concept of argument attack. For example, several systems use the notion of *rebut* and *undercut* conflict [5]. The first one is due to contradictory conclusions between arguments, while the second is due to a contradiction between a conclusion of an argument and a premise of the other. In Defeasible Logic Programming [6] two kinds of attack (defeat) relations are present. These relations are obtained by applying a preference criterion between conflicting arguments, thus obtaining *blocking* and *proper* attacks. An abstract framework capturing this dual interaction is defined in [4, 7, 8]. In [9] the aggregation of different

abstract attack relations over a common set of arguments is addressed. These attacks represent diversity of criteria on several rational agents willing to reach on agreement about argument conflicts.

In a previous paper [10], an argumentation framework equipped with a set of unquestionable abstract attack relations of varied intrinsic force was introduced. Using this novel framework with ordered attacks, the classic notions of acceptability of arguments and admissible sets are applied leading to new formalizations of similar ideas. An interesting structure presented in that work is called an *attack scenario*, a pair formed by a set of arguments and a set of restrictions on attacks. The attack scenario captures the difference of strength that are valid for defense on a certain set of arguments. An argument $\mathcal{A}$ defends another argument $\mathcal{B}$ if $\mathcal{A}$ attacks at least one attacker $\mathcal{C}$ of $\mathcal{B}$. This defense for $\mathcal{A}$ may be achieved by the use of an attack of different strength that the one $\mathcal{A}$ suffers from $\mathcal{C}$.

In this work, we explore *expansion*, *contraction* and *complement* of attack scenarios according to defense conditions, and its relation to Dung's admissibility semantics.

In the next section the abstract framework with varied strength attacks is introduced. Semantic notions dealing with this relative difference of strength are defined in subsequent sections. Finally, a simple operator to safely add arguments in an extension is presented.

2 Abstract Framework

Our framework includes a set of arguments and a finite set of binary attack relations denoting conflicts of different nature.

Definition 1 (Framework) *An argumentation framework with varied strength attacks (AFV) is a triplet $\langle Args, Atts, R \rangle$ where $Args$ is a set of arguments, $Atts$ is a set of binary attack relations $\{\rightarrow_1, \rightarrow_2, \ldots, \rightarrow_n\}$ defined over $Args$, and R is a binary relation defined over $Atts$.*

The relation $R \subseteq Atts \times Atts$ denotes an order of strength between argument conflicts. Arguments are abstract entities that will be denoted using calligraphic uppercase letters. The set $Atts$ represents different abstract forms of conflicts between arguments, modeled by every $\rightarrow_i \subseteq Args \times Args, 1 \leq i \leq n$. For two arguments $\mathcal{A}$ and $\mathcal{B}$, if $\mathcal{A} \rightarrow_i \mathcal{B}$ then it is said that $\mathcal{A}$ *attacks* $\mathcal{B}$. In this work R it is only assumed to be reflexive.

Definition 2 (Relative strength) *Let $\langle Args, Atts, R \rangle$ be an AFV where $Atts = \{\rightarrow_1, \rightarrow_2, \ldots, \rightarrow_n\}$. For two attack relations $\rightarrow_i$ and $\rightarrow_j$, $1 \leq i, j \leq n$,*
- *if $(\rightarrow_i, \rightarrow_j) \in R$ and $(\rightarrow_j, \rightarrow_i) \notin R$ then it is said that $\rightarrow_i$ is a stronger attack than $\rightarrow_j$, denoted $\rightarrow_i \gg \rightarrow_j$. It may also be said that $\rightarrow_j$ is a weaker attack than $\rightarrow_i$, denoted $\rightarrow_j \ll \rightarrow_i$.*
- *if $(\rightarrow_i, \rightarrow_j) \in R$ and $(\rightarrow_j, \rightarrow_i) \in R$ then it is said that $\rightarrow_i$ and $\rightarrow_j$ are equivalent in force, denoted $\rightarrow_i \approx \rightarrow_j$.*
- *if $(\rightarrow_i, \rightarrow_j) \notin R$ and $(\rightarrow_j, \rightarrow_i) \notin R$ then it is said that $\rightarrow_i$ and $\rightarrow_j$ are of unknown difference force, denoted $\rightarrow_i \ ? \ \rightarrow_j$.*

An attack may be stronger, or equivalent in force, or incomparable to other attacks. Being R reflexive, then $\rightarrow_i \approx \rightarrow_i$ for any attack $\rightarrow_i$[1]. We will only explicitly mention those ?-pairs of attacks which are relevant for the particular case. Clearly, there may be more attacks not related under R, but they will be omitted for simplicity. Note that if $\rightarrow_i \approx \rightarrow_j$ for all $\rightarrow_i, \rightarrow_j \in Atts$, then the result is the Dung's classical abstract framework [1] $AF = \langle Args, At \rangle$ where $At = \rightarrow_1 \cup \rightarrow_2 \cup \ldots \cup \rightarrow_n$.

We will focus mainly on argument defense. We depict argumentation frameworks using graphs, where arguments are represented as black triangles and a labeled arc ($\leftarrow$) is used to denote attacks. An arc with label i denotes the attack $\rightarrow_i$. Consider the argumentation framework depicted in Figure 1. Arguments $\mathcal{C}$ and $\mathcal{D}$ are attacking $\mathcal{B}$, which in turn attacks $\mathcal{A}$. Thus, it is said that $\mathcal{C}$ and $\mathcal{D}$ are *defenders* of $\mathcal{A}$ against $\mathcal{B}$. Regarding argument $\mathcal{A}$, the attack $\rightarrow_i$ is an *offensive* attack, while attacks $\rightarrow_j$ and $\rightarrow_k$ are *defensive* attacks. Two kinds of attack comparison can be made. First, the defensive attack can be compared to the offensive attack, leading to a measure of strength of one particular defense. We call this an offense-defense comparison. In Figure 1, $\rightarrow_i$ can be compared to $\rightarrow_j$. Second, all the defensive attacks on a single argument can be compared to each other. We call this a defense-defense comparison. In Figure 1, $\rightarrow_j$ can be compared to $\rightarrow_k$.
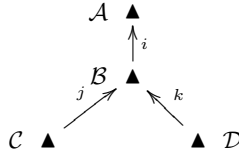

Fig. 1. $\mathcal{C}$ and $\mathcal{D}$ are defending $\mathcal{A}$ against $\mathcal{B}$.

Definition 3 (Defense strength) *Let $\langle Args, Atts, R \rangle$ be an AFV. Let $\mathcal{A}, \mathcal{B}, \mathcal{C} \in Args$ such that $\mathcal{B} \rightarrow_i \mathcal{A}$ and $\mathcal{C} \rightarrow_j \mathcal{B}$. Then*
- *$\mathcal{C}$ is a strong defender of $\mathcal{A}$ against $\mathcal{B}$ if $\rightarrow_j \gg \rightarrow_i$.*
- *$\mathcal{C}$ is a weak defender of $\mathcal{A}$ against $\mathcal{B}$ if $\rightarrow_j \ll \rightarrow_i$.*
- *$\mathcal{C}$ is a normal defender of $\mathcal{A}$ against $\mathcal{B}$ if $\rightarrow_j \approx \rightarrow_i$.*
- *$\mathcal{C}$ is an unqualified defender of $\mathcal{A}$ against $\mathcal{B}$ if $\rightarrow_j ? \rightarrow_i$.*

As attacks are ordered by its force, strong defenders are considered better than normal defenders. In the same manner, normal defenders are considered better than unqualified defenders.

Argument $\mathcal{C}$ is said to dominate $\mathcal{D}$ as a defender if $\rightarrow_j \gg \rightarrow_k$.

Example 1 *Consider the AFV of Figure 2, where $\rightarrow_1 \gg \rightarrow_2$, $\rightarrow_2 \gg \rightarrow_3$, $\rightarrow_4 ? \rightarrow_2$ and $\rightarrow_4 \gg \rightarrow_3$. Every argument achieve its defense with different strength. Argument $\mathcal{E}$ is a strong defender of $\mathcal{A}$. Argument $\mathcal{C}$ is a weak defender of $\mathcal{A}$ while $\mathcal{F}$ is an unqualified defender of $\mathcal{A}$.*

In the following section, the classic notions of acceptability of arguments and admissible sets are applied to the abstract framework with varied-strength attacks.

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[1] For simplicity, we will omit reflexive cases when describing frameworks

$\mathcal{B}$

$\mathcal{A}$ ▲ ←—2— ▲ ←—3— ▲ $\mathcal{C}$

2↑   4

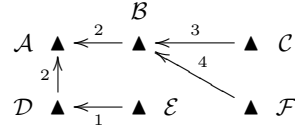$\mathcal{D}$ ▲ ←—1— ▲ $\mathcal{E}$ ▲ $\mathcal{F}$

Fig. 2. Defenses of varied strength

3 Admissibility semantics

Argumentation semantics is about argument classification through several rational positions of acceptance. A central notion in most argument extensions is *acceptability*.

Definition 4 (Classic Acceptability) *[1] An argument $\mathcal{A}$ is acceptable with respect to a set of arguments S if and only if every attacker $\mathcal{B}$ of $\mathcal{A}$ has an attacker in S.*

Acceptability is the basis of many argumentation semantics and leads to the notion of *admissibility*, which is applied to conflict-free sets.

Definition 5 (Conflict-free) *Let $\langle Args, Atts, R \rangle$ be an AFV. A set of arguments $S \subseteq Args$ is said to be conflict-free if for all $\mathcal{A}, \mathcal{B} \in S$ it is not the case that $\mathcal{A} \rightarrow_i \mathcal{B}$ for any $\rightarrow_i \in Atts$.*

Definition 6 (Classic Admissibility) *[1] A set of arguments S is said to be admissible if it is conflict-free and every argument in S is acceptable with respect to S.*

An admissible set is able to defend any argument included in that set. These widely accepted definitions are suitable for an AFV, where an attack is interpreted as any $\rightarrow_i \in Atts$. However, when a specific strength constraint is desired in argument defense, this global notion of acceptability is no longer sufficient. In that case, there is a need to capture defense under certain conditions, as shown in the following definitions.

Definition 7 (Attack scenario) *Let $\Phi = \langle Args, Atts, R \rangle$ be an AFV, $\mathcal{A} \in Args$, $S \subseteq Args$ and $P \subseteq \{\gg, \ll, \approx, ?\}$. The pair $p = [S, P]$ is called an (attack) scenario of Φ. The set P is called the defense condition of p.*

An attack scenario is formed by a set of arguments and a set of defense conditions. These conditions are capturing the difference of strength in defense that is take into consideration for acceptability purposes. For simplicity in notation, for any scenario $p = [S, p]$, we will say that $\mathcal{A} \in p$ if, and only if $\mathcal{A} \in S$.

Definition 8 (Constrained Acceptability.) *Let Φ be an AFV, and let $[S, P]$ be an attack scenario in Φ. An argument $\mathcal{A}$ is acceptable with respect to $[S, P]$ if, and only if, for any argument $\mathcal{X}$ such that $\mathcal{X} \rightarrow_i \mathcal{A}$, there is an argument $\mathcal{Y} \in S$ such that $\mathcal{Y} \rightarrow_j \mathcal{X}$ and $\rightarrow_j \rho \rightarrow_i$ where $\rho \in P$.*

The set $P \subseteq \{\gg, \ll, \approx, ?\}$ is a form of defense condition that must be satisfied for every defender of an argument. In this way we are modelling conditions regarding difference of strength between offensive and defensive attacks.

Example 2 *Consider the framework of Figure 2. Argument $\mathcal{A}$ is acceptable with respect to $[\{\mathcal{E}, \mathcal{F}\}, \{\gg, ?\}]$ but not with respect to $[\{\mathcal{C}, \mathcal{E}\}, \{\gg, ?\}]$.*

Having Definition 8, the derived notion of constrained admissibility is straightforward.

Definition 9 (Constrained Admissibility) *Let Φ be an AFV, and let $[S, P]$ be an attack scenario in Φ. The pair $[S, P]$ is called an admissible scenario if, and only if, S is conflict free and every argument $\mathcal{X} \in S$ is acceptable with respect to $[S, P]$.*

In the abstract framework depicted in Figure 2, the pair $[\{\mathcal{A}, \mathcal{E}, \mathcal{F}\}, \{\gg, ?\}]$ is an admissible scenario. In [10] it has been proved that if $[S, P]$ is an admissible scenario then S is (classical) admissible.

In the following section we introduce several operations on attack scenarios.

4 Working with attack scenarios

Defense conditions are specially interesting in dialectical processes. Deciding on the acceptance of an argument requires the analysis of its direct and indirect attackers and defenders. The bigger the set of arguments, the harder the process of acceptance. Defense conditions are naturally bounding this analysis.

4.1 Admissible expansion of scenarios

An admissible set $p = [S, \{\gg\}]$ includes only strongly defended arguments. An admissible scenario with defense condition P can be safely expanded into another admissible scenario using a different defense condition Q. This is achieved by the identification of acceptable arguments according to Q

Definition 10 (Expansion) *Let $p = [S, P]$ be an admissible attack scenario, and let Q be a set of defense conditions. The expansion of p according to Q is defined as $p \oplus Q = [S' \cup S, P \cup Q]$, where $S' = \{\mathcal{A} \in Args : \mathcal{A} \text{ is acceptable with respect to } [S, Q]\}$*

The admissible scenario $p \oplus Q$ is constructed over p by the inclusion of acceptable arguments under defense condition Q.

Example 3 *Consider the AFV of Figure 3 where $\rightarrow_2 \gg \rightarrow_1$, $\rightarrow_3 \gg \rightarrow_2$ and $\rightarrow_4 \approx \rightarrow_3$ The scenario $p_1 = [\{\mathcal{A}, \mathcal{E}, \mathcal{I}\}, \{\gg\}]$ is admissible. Then $p_2 = p_1 \oplus \{\approx\} = [\{\mathcal{A}, \mathcal{E}, \mathcal{I}, \mathcal{H}, \mathcal{C}, \mathcal{D}\}, \{\gg, \approx\}]$. and $p_3 = p_1 \oplus \{\gg\} = [\{\mathcal{A}, \mathcal{E}, \mathcal{I}, \mathcal{C}, \mathcal{D}\}, \{\gg\}]$. Note that arguments $\mathcal{C}$ and $\mathcal{D}$ are included in any expansion, because these arguments are acceptable with respect to the empty set, and therefore no defense condition is needed.*

The expansion operator preserves admissibility, as it was proved in [10]. This means that for any admissible scenario $p = [S, P]$, and a set of defense conditions Q, the scenario $p \oplus Q$ is also admissible.
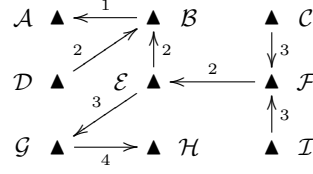
Fig. 3. Framework of Example 3

Although simple, this form of expansion allows the construction of a bigger set of admissible scenarios. Classical notions as preferred and grounded extensions can be captured by the use of the $\oplus$ operator. Note that when no arguments are acceptable with respect to an admissible scenario $p = [S, P]$, then $p \oplus P = p$. In fact, when any additional defense condition does not expand an admissible scenario $[S, P]$, then the set S is maximal admissible [10].

The characterization of skeptical acceptance under a defense condition P can be achieved by gradually expanding admissible scenarios. Naturally, an argument without attackers does not need defense as it is acceptable with respect to the empty set. For any defense condition P, the scenario $[\emptyset, P] \oplus P = [ND, P]$ where ND is the set of arguments free of attackers in $Args$ [10]. The repeated application of operator $\oplus$ with defense condition P will subsequently add new arguments to an already admissible scenario, where every defense fulfills P.

Definition 11 (Grounded scenario) *Let Φ be an AFV and let $P \subseteq \{\gg, \ll, \approx, ?\}$. The scenario $\Phi\uparrow^+ P$, called the grounded scenario according to P, is defined as the least fixpoint of $\oplus$ using defense condition P.*

Example 4 *Consider the AFV Φ of Figure 4 where $\to_2 \gg \to_1$ and $\to_2 \approx \to_3$. The following is the grounded scenario obtained by considering defense condition $\{\gg\}$.*

$p_0 = [\emptyset, \{\gg\}] \oplus \{\gg\} = [\{\mathcal{A}, \mathcal{B}, \mathcal{D}, \mathcal{H}\}, \{\gg\}]$
$p_1 = p_0 \oplus \{\gg\} = [\{\mathcal{A}, \mathcal{B}, \mathcal{D}, \mathcal{H}, \mathcal{I}\}, \{\gg\}]$
$p_2 = p_1 \oplus \{\gg\} = [\{\mathcal{A}, \mathcal{B}, \mathcal{D}, \mathcal{H}, \mathcal{I}\}, \{\gg\}] = p_1$
Then $\Phi\uparrow^+\{\gg\} = [\{\mathcal{A}, \mathcal{B}, \mathcal{D}, \mathcal{H}, \mathcal{I}\}, \{\gg\}]$

In addition, note that $\Phi\uparrow^+ \{\approx\} = [\{\mathcal{A}, \mathcal{B}, \mathcal{D}, \mathcal{H}\}, \{\gg\}]$. It includes only arguments without attackers, as the defense condition is very restrictive for Φ.

The grounded scenario $\Phi\uparrow^+ P$ is admissible. The term *grounded* is used because it captures the skeptical acceptance modeled by the classical grounded extension [1], but limited to certain defense conditions. In fact, if all defense conditions are taken into account, then the result is an attack scenario equivalent to the classical grounded extension. In [10] it has been proved that the grounded scenario according to all defense conditions is the classical grounded extension.

An argumentation framework with varied-strength attacks allows several skeptical sets of acceptance, depending on the permitted defense conditions. In Example 4, two grounded scenarios are shown.

The following proposition states that by considering more defense conditions, the grounded extension allows the inclusion of more arguments. Proofs are omitted for space reasons.
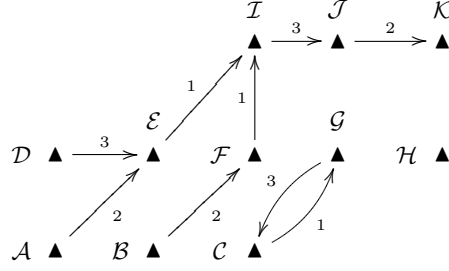
Fig. 4. Framework of Example 4

Proposition 1 *If $P \subseteq Q$ then $\Phi\!\uparrow^+ P \subseteq \Phi\!\uparrow^+ Q$.*

4.2 Contraction of attack conditions

The expansion of an admissible scenario is a safe operation in the sense that it preserves admissibility, by adding only acceptable arguments. In some way, the expansion corresponds to a *flexibility of conditions* for defense: new defenses are now available, in the original scenario as the starting point. On the other hand, the removal of defense conditions from a given attack scenario, is called *contraction*. In order to properly define contraction, we need to introduce a restricted version of expansion operator.

Definition 12 (Restricted expansion) *Let $\langle Args, Atts, R \rangle$ be an AFV. Let $p = [S, P]$ be an admissible attack scenario, let $T \subseteq Args$ be a set of arguments and let Q be a set of defense conditions. The expansion of p in T according to Q is defined as $p \oplus_T Q = [S' \cup S, P \cup Q]$, where $S' = \{\mathcal{A} \in T : \mathcal{A}$ is acceptable with respect to $[S, Q]\}$*

The restricted expansion only consider arguments for expansion in a specific set T. Note that the original expansion operator is equivalent to restricted expansion $\oplus_{Args}$.

Definition 13 (Contraction) *Let $p = [S, P]$ be an admissible attack scenario, and let Q be a set of defense conditions such that $Q \subset P$. The contraction of p according to Q, denoted $p \ominus Q$, is defined as the least fixpoint of function F:*
$F^0 = [\{\}, \{\}]$
$F^i = F^{i-1} \oplus_S (P \setminus Q)$

The contraction of a scenario $p = [S, P]$ according to Q is an admissible scenario where only the defense conditions in $P \setminus Q$ are used. In Definition 13 the contraction is built from the empty scenario $[\{\}, \{\}]$ by progressively expanding it using the rest of defense conditions. As it only adds acceptable arguments, it leads to an admissible scenario.

Example 5 *Consider the AFV of Figure 5 where $\rightarrow_2 \gg \rightarrow_1$. The scenario $p_1 = [S, \{\gg, \approx\}]$ where $S = \{\mathcal{A}, \mathcal{D}, \mathcal{C}, \mathcal{E}\}$ is an admissible scenario. Then,*
$p_2 = p_1 \ominus \{\approx\} = [\{\mathcal{E}, \mathcal{D}, \mathcal{C}\}, \{\gg\}]$, *because*
$F^1 = [\{\}, \{\}] \oplus_S \{\approx\} = [\{\mathcal{D}, \mathcal{C}\}, \{\gg\}]$,
$F^2 = F^1 \oplus_S \{\approx\} = [\{\mathcal{E}, \mathcal{D}, \mathcal{C}\}, \{\gg\}]$,

$F^3 = F^2 \oplus_S \{\approx\} = F^2$.
Also, $p_3 = p_1 \ominus \{\gg\} = [\{\mathcal{A}, \mathcal{C}, \mathcal{D}\}, \{\approx\}]$, because
$F^1 = [\{\}, \{\}] \oplus_S \{\gg\} = [\{\mathcal{D}, \mathcal{C}\}, \{\approx\}]$,
$F^2 = F^1 \oplus_S \{\gg\} = [\{\mathcal{A}, \mathcal{D}, \mathcal{C}\}, \{\approx\}]$,
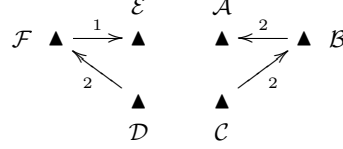$F^3 = F^2 \oplus_S \{\gg\} = F^2$.

Fig. 5. Framework of Example 5

The removal of defense conditions may lead to arguments without a valid defense, which are dropped off the final contraction. Clearly, the contraction of scenarios is not the inverse of the expansion, which adds arguments in one step of acceptability.

Definition 14 *Let $q = [S, P]$ be an admissible scenario. A defense condition $\rho \in P$ is said to be relevant in q if $q \ominus \{\rho\} = [S', P \setminus \{\rho\}]$ is such that $S' \subset S$. An admissible scenario $q = [S, P]$ is said to be tight if every condition in P is relevant in q.*

A tight admissible scenario needs all the specified defense conditions. Any subset of these will drop the defenses of every argument suffering an attack. In the framework of Example 5, the scenarios p_1, p_2 and p_3 are tight, because by discarding any condition some argument become defenseless.

4.3 Complement

The complement of an admissible scenario p is obtained by the complement of its defense conditions. It considers every defense not valid in p. This also needs a recalculation of acceptable arguments according to the new defense conditions, always limited to the original set.

Definition 15 (Complement) *Let $Cond = \{\gg, \ll, \approx, ?\}$. Let $q = [S, P]$ be an attack scenario. The complement of q, denoted q^C, is the scenario obtained as the least fixpoint of function G:*
$G^0 = [\{\}, \{\}]$
$G^i = F^{i-1} \oplus_S (Cond \setminus P)$

The complement of an admissible scenario p is the maximal admissible scenario that can be constructed using the same set of arguments but with the remaining defense conditions, *i.e.*, those not considered in p.

Example 6 *Consider the AFV of Figure 6 where $\rightarrow_2 \gg \rightarrow_1$ and $\rightarrow_3 ? \rightarrow_1$. The scenario $q = [S, \{\gg\}]$ where $S = \{\mathcal{A}, \mathcal{D}, \mathcal{C}, \mathcal{F}\}$ is an admissible scenario. Then, $q^C = [\{\mathcal{D}, \mathcal{C}, \mathcal{F}\}, \{\approx, \ll, ?\}]$, because*

$F^1 = [\{\}, \{\}] \oplus_S \{\approx, \ll, ?\} = [\{\mathcal{D}, \mathcal{F}\}, \{\approx, \ll, ?\}],$
$F^2 = F^1 \oplus_S \{\approx, \ll, ?\} = [\{\mathcal{D}, \mathcal{C}, \mathcal{F}\}, \{\approx, \ll, ?\}],$
$F^3 = F^2 \oplus_S \{\approx\} = F^2.$
Note that $\mathcal{C}$ has a double defense against $\mathcal{E}$.
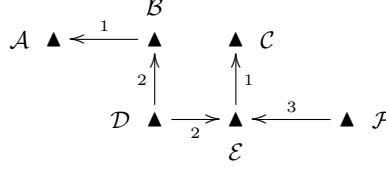
Fig. 6. Framework of Example 6

Clearly, for an argument $\mathcal{X}$ included in an admissible scenario p, the necessary condition to be included in the complement of p is to be defended by more than one argument, using attacks of different relative strength.

Proposition 2 *Let $p = [S, P]$ be an admissible scenario and let $p^C = [S', Cond \setminus P]$ its complement. Let $\mathcal{A} \in S$ be an argument with only one defender. Then $\mathcal{A} \notin S'$.*

Any argument free of attackers does not need defense, and then it belongs to the grounded scenario according to any defense condition.

Proposition 3 *Let P be a defense condition and let $\mathcal{A}$ be an argument free of attackers. Then $\mathcal{A} \in \Phi\uparrow^+P$ and $\mathcal{A} \in (\Phi\uparrow^+P)^C$.*

In the framework of Example 6, argument $\mathcal{C}$ is also included in the complement scenario. This is a good property for an argument, as its defense is also available when changing the selected defense condition.

Definition 16 *Let P be a defense condition. Let $\mathcal{A} \in \Phi\uparrow^+P$. Argument $\mathcal{A}$ is said to be full-defended in $\Phi\uparrow^+P$ if $\mathcal{A} \notin (\Phi\uparrow^+P)^C$.*

5   Conclusions and future work

An argumentation framework with varied-strength attacks is equipped with a set of abstract attack relations of varied strength. This improvement of Dung's classical framework is a pathway to new elaborations about arguments and preferences, specially the definition of new semantics extensions. An *admissible scenario* is formed by a set of arguments S fulfilling a set of defense conditions P. The attack scenario captures the difference of strength (specified in the set P) that are valid for defense on a certain set of arguments.

Attack scenarios are the main structure in which several semantics notions are built. The quality of admissible sets in attack scenarios is strongly determined by the set of available defense conditions. Changing conditions may lead to different admissible sets,

even within the same set of arguments. In this work we explored three operations on admissible scenarios: the expansion (introduced in [10]), the contraction and the complement of a given scenario. In order to satisfy admissibility, the contraction and the complement requires the recalculation of the admissible scenario according to updated defense conditions. Future work is centered in the formalization of new operators, such as the combination of different scenarios. We are also interested in attack-tolerant semantics, where some conflicts between arguments are permitted. In our framework, this can be achieved by stating which attacks are tolerant, and thus ignored in admissible constructions.

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