A Comparison Between Non-Monotonic Formalisms

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ABSTRACT

It is interesting and important to compare, analyze and assess the alternative tools that could be used in the area of Knowledge Representation. In this paper we present a reasearch line associated to this goal: to formally establish the relation among Knowledge Representation formalisms in order to make a sensible use of them. As a part of this main task, we present a comparison between Normal Default Theory and Defeasible Logic Programming. This comparison is achieved introducing a DELP variant, called DELP$^{\emptyset}$, which allows to associate the answers of a DELP interpreter to the consequences, credulous and skeptical, of a Normal Default Theory.

Keywords: Knowledge Representation, Non-monotonic Reasoning, Default Logic, Defeasible logic programming.

1 INTERESTS AND MOTIVATION

In general, it is interesting and important to compare, analyze and assess the alternative tools that could be used to confront a specific problem. In particular, in the area of Artificial Intelligence there are several research lines dedicated to the development of formalisms and tools regarding Knowledge Representation. These formalisms are so diverse that many times it is difficult to recognize their advantages, disadvantages and differences in order to make a plausible use of them. For this reason, it is interesting to analyze the relation among knowledge representation formalisms to evaluate their differences and similarities. Several works relating diverse approaches of defeasible and non-monotonic reasoning have been developed [6, 5, 4, 3, 1, 2].

In this paper, we present, as a part of a major work, a preliminary analysis about the relation between Default Logic and Defeasible Logic Programming (DELP). The current goal is to establish the possibility of defining a set of conditions and transformation rules that allows us to study this relation in a formal way. In this first stage, we analyze several scenarios and specific examples that evidence the differences between these two formalisms, in order to establish their causes. This is the first step to clarify the relation under study.

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2 DEFAULT LOGIC AND DELP

In this section, we briefly describe the theoretical background of both Default Logic and DELP. We will also introduce a variant of DELP that will be used for comparison.

2.1 Default Logic

A Default Theory $T = \langle W, D\rangle$ consists of a set of facts W of ground sentences. Each default rule in D has the form $\frac{a:b_1,\ldots,b_n}{c}$ (sometimes written $a : b_1, \ldots, b_n/c$), where a is called the prerequisite, b_i are the justifications and c is the consequent of the default. The intuitive meaning of a default is: if a can be derived and it is possible to consistently assume each b_i, then conclude c. In this first work, we will consider default theories with the following characteristics:

($C1$) The theories are normal. The defaults have the form $a : c/c$; for short, we write $a \hookrightarrow c$.

($C2$) Only propositional sentences are allowed.

($C3$) For every default $a \hookrightarrow c$, the sentence c is a single literal.

2.2 DELP and DELP$^{\emptyset}$

Defeasible Logic Programming (DELP) is a formalism that combines Logic Programming and Defeasible Argumentation. DELP allows the representation of defeasible information in the form of two kinds of rules: *defeasible* rules and *strict* rules. The first ones, also called weak rules, are used in the representation of tentative information, and the second ones for representing strict knowledge. The language used allows strong negation, and DELP uses an argumentation formalism to deal with contradictory knowledge. In DELP, a literal h is *warranted* if there exists a non-defeated *argument* $\mathcal{A}$ supporting h. An argument $\langle \mathcal{A}, h \rangle$ for a literal h is a minimal consistent set of defeasible rules that infers h. In order to establish whether $\langle \mathcal{A}, h \rangle$ is a non-defeated argument, *argument rebuttals* or *counter-arguments* that could be *defeaters* for $\langle \mathcal{A}, h \rangle$ are considered, *i.e.,* counter-arguments that, by some criterion, are preferred to $\langle \mathcal{A}, h \rangle$. Since counter-arguments are arguments, there may exist defeaters for them, and defeaters for these defeaters, and so on. Thus, a sequence of arguments called *argumentation line* may appear, where each argument defeats its predecessor in the line. Usually, each argument has more than one defeater and more than one argumentation line exists. Therefore, a tree of arguments called *dialectical tree* is constructed, where the root is $\langle \mathcal{A}, h \rangle$ and each path from the root to a leaf is an argumentation line. A *dialectical analysis* of this tree is used for deciding whether h is warranted. This dialectical analysis is carried out labelling the arguments conforming the dialectical tree. The arguments in the leaves of the tree are considered undefeated. Every inner node with at least a child marked as undefeated, is considered and marked as a defeated argument. In the other case, it is undefeated. Following this analysis, a literal h is said warranted if there is a dialectical tree where the root is an argument for h that has been marked as undefeated.

In this formalism, several elements can be adjusted to define variants of DELP; for instance, the notions of attack and defeat, as well as the conditions required for acceptable argumentation lines. We will consider a DELP variant, that we call DELP$^{\emptyset}$, observing the following condition:

- The relation defining the comparison criterion is the empty set. This criterion turns attack into defeat.

In general, given two conflicting arguments A and B, they can be compared using some criterion. In that case, if A is better than B, A is a *proper defeater* for B. But, if neither of the two is better than the other, A is a *blocking defeater* for B, and vice versa. Note that, in DELP$^{\emptyset}$ every attack is a blocking defeat and there are no proper defeaters.

3 EXTENSIONS AND ANSWERS

Default Logic allows the existence of multiple extensions; therefore, in these cases, it is necessary to define what pieces of information (or beliefs) will be accepted. For instance, a reasoner may choose to believe only in those pieces of information present in every extension. Several semantics had been defined, but we will consider the classical notions of consequence: skeptical and credulous. If a literal l belongs to every extension of a default theory, l is said to be *skeptical*, denoted $l \in$ SKEP. Otherwise, if there exists an extension E such that literal l does not belong to E, we say that literal l is a *credulous* consequence, denoted $l \in$ CRED. In the general case, it is possible for a default theory to have no extensions, but, in the case of normal theories, the existence of at least one extension is ensured.

In a defeasible logic program (de.l.p.) P it is possible to associate each literal in P to one of the following three sets:

- YES $= \{L : L$ is warranted from $P\}$
- NO $= \{L : \overline{L}$ is warranted from $P\}$
- UND $= \{L : L, \overline{L}$ are not warranted from $P\}$

We want to determine the relation between Normal Default Theories and DELP$^{\emptyset}$. The de.l.p. associated to a default theory will be determined through an transformation from default rules (in the default theory) into defeasible rules (in the de.l.p.).

Other research works had established the relation between Logic Programs and Default Logic. For this, the standard translation considers: rules of the form

$$c \leftarrow a_1 \ldots, a_n, \textit{not } b_1, \ldots, \textit{not } b_m$$

can be viewed as the default rule

$$a_1, \ldots, a_n : \neg\, b_1, \ldots, \neg\, b_m / c$$

where *not* represents negation as failure.

We will show that, for our purpose, a default rule can be expressed as a *defeasible rule*. In both formalisms the existence of this kind of rules (defaults and defeasible) follow a similar goal: to represent a relation among pieces of knowledge that could be defeated if new or global information is considered. For instance, we can accept the general rules:
- *if it is Sunday and there is no reason against going shopping, we go.*
- *if we have homework and nothing is contrary to keep working at home, we stay at home.*
and the strict rule:
- *if we stay at home, we do not go shopping.*

This information could be represented by a normal default theory with the rules:
$(sunday \hookrightarrow go)$ $(homework \hookrightarrow stay)$
$(stay \rightarrow \neg\, go)$ $(sunday)$ $(homework)$
and through a de.l.p. with the rules:

$$(go \mathrel{-\!\!\prec} sunday) \quad (stay \mathrel{-\!\!\prec} homework)$$
$$(\neg\, go \leftarrow stay) \;\; (sunday \leftarrow) \;\; (homework \leftarrow)$$

Considering the fact *"we have homework"*, both formalisms conclude reasonably *"do not go shopping"*, but additional knowledge about the fact *"it is Sunday"* makes no conclusion drawn about going shopping or not.

Therefore, we use a defeasible rule:

$$b \mathrel{-\!\!\prec} a$$

to represent each default rule

$$a : b/b.$$

Then, given a normal default theory $T = \langle W, D\rangle$, such that $D = \{\delta_1, \ldots, \delta_n\}$, where $\delta_i = a_i \hookrightarrow b_i$, the *associated defeasible logic program* is

$$P_T = (\Pi, \Delta)$$

where $\Delta = \{b_i \mathrel{-\!\!\prec} a_i : \delta_i \in D\}$ and Π consists of strict rules originated from clauses in W.

In the rest of the article we will show that every literal belonging to the set SKEP of a normal default theory T will be in the set YES of the associated de.l.p. P_T, and vice versa.

4 EXAMPLES

In this section we present some examples in order to clarify the relation between Normal Default Logic and DELP$^{\emptyset}$.

Example 1
Consider the default theory $T_1 = \langle W_1, D_1\rangle$, where

$$W_1 = \{a, b\}$$
$$D_1 = \{(a \hookrightarrow x), (b \hookrightarrow \neg\, x), (b \hookrightarrow w)\}$$

The corresponding de.l.p. will be $P_{T_1} = (\Pi_1, \Delta_1)$, where

$$\Pi_1 = \{(a \leftarrow), (b \leftarrow)\},$$
$$\Delta_1 = \{(x \mathrel{-\!\!\prec} a), (\neg\, x \mathrel{-\!\!\prec} b), (w \mathrel{-\!\!\prec} b)\}$$

Theory T_1 has two extensions. Literal x belongs to one of these extensions, whereas $\neg\, x$ belongs to the other, and w is in both extensions. For this reason, we say that $\{x, \neg\, x\} \subseteq$ CRED and $w \in$ SKEP. In P_{T_1}, literal w is *warranted*, but literals x and $\neg\, x$ are *undecided*. That is, $w \in$ YES and $\{x, \neg\, x\} \subseteq$ UND.

Example 2
Consider the following theory $T_2 = \langle W_2, D_2\rangle$ where

$$W_2 = \{a, b, c\}$$
$$D_2 = \{a \hookrightarrow x, b \hookrightarrow x, c \hookrightarrow \neg\, x\}$$

The corresponding de.l.p. is $P_{T_2} = (\Pi_2, \Delta_2)$, where

$$\Pi_2 = \{(a \leftarrow), (b \leftarrow), (c \leftarrow)\}$$
$$\Delta_2 = \{(x \mathrel{-\!\!\prec} a), (x \mathrel{-\!\!\prec} b), (\neg\, x \mathrel{-\!\!\prec} c)\}$$

In example this example, we can also see that for theory T_2, $\{x, \neg\, x\} \subseteq$ CRED and for P_{T_2}, $\{x, \neg\, x\} \subseteq$ UND.

Note that, in example 2 there are two default derivations for literal x, one using the default rule $a \hookrightarrow x$ and the other using $b \hookrightarrow x$, whereas there is only a single derivation for $\neg\, x$. Each default derivation in T_2 can be associated to a defeasible derivation in the associated de.l.p. P_2. Then,there are two arguments for x and only one argument for $\neg\, x$. However, since in DELP$^{\emptyset}$ every attack is a defeat, and all defeaters are blocking defeaters, there is no chance of *defense*. That is, if an argument A is defeated by argument B there is no chance of defeating B using a third argument C. This is because of the condition that avoids blocking-blocking situations in the formation of argumentation lines.

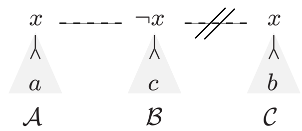

Figure 1: Blocking situation en DELP$^{\emptyset}$

It is important to note that, in normal default theories, the existence of two applicable conflicting default rules makes their conclusions to be in different extensions. Since DELP$^{\emptyset}$ avoids any opportunity of defense among arguments (see figure 1), the literals supported by conflicting arguments are *undecided*.

The next example shows that the concept of *"defense among arguments"* arises just when a preference criterion among arguments can be established. If an argument A is preferred over an argument B, A could *defend* an argument C from being attacked by B.

Example 3
Given the theory $T_3 = \langle\{x, w, z\}, \{\delta_1, \delta_2, \delta_3\}\rangle$ where

$$\delta_1 = x \hookrightarrow b,$$
$$\delta_2 = x, w \hookrightarrow \neg\, b,$$
$$\delta_3 = x, w, z \hookrightarrow b$$

The associated de.l.p. is $P_{T_3} = (\Pi_3, \Delta_3)$, where

$$\Pi_3 = \{(x \leftarrow), (w \leftarrow), (z \leftarrow)\}$$
$$\Delta_3 = \{(b \mathrel{-\!\!\prec} x), (\neg\, b \mathrel{-\!\!\prec} x, w), (b \mathrel{-\!\!\prec} x, w, z)\}$$

Considering the normal default theory T_3, $\{b, \neg\, b\} \subseteq$ CRED, and considering DELP$^{\emptyset}$, we have $\{b, \neg\, b\} \subseteq$ UND. Nevertheless, suppose that the information supplied by the rule δ_3 is preferred to the information given by δ_2, and this is preferred to δ_1. Normal Default Logic and DELP$^{\emptyset}$ cannot express this preference.

Observe that the de.l.p. of example 3 allows the construction of arguments $A = \{b \prec x, w, z\}$, $B = \{(\neg\, b \prec x, w)\}$, and $C = \{(b \prec x)\}$. Suppose that we are considering a DELP variant with a preference criterion based on a relation establishing that A is better that B, and B is better than C. In others words, argument A is a proper defeater for B, and B is a proper defeater for C. This situation allows the construction of an acceptable argumentation line $C - B - A$ (see figure 2), and the literal b, which is supported by C, is finally *warranted*.

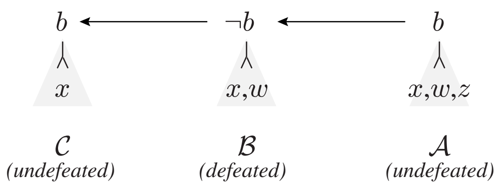

Figure 2: Preference in DELP

In Default Logic, conflict between default rules appears either directly or indirectly. A direct conflict occurs when two applicable defaults rules conclude complementary literals, which will end up in separate extensions. Regarding the associated de.l.p., one argument for each literal is built and since these arguments are in conflict, both literals will be *undecided*. The other kind of conflict we are considering is the indirect conflict. This one occurs when the strict information of the theory concludes a literal that blocks the application of a default rule.

Next, we describe an example that evidences the occurrence of an indirect conflict in a normal default theory, and we show how DELP$^{\emptyset}$ models this conflict, and gives the expected answers.

Example 4 (Buy)
Consider the theory $T_4 = \langle W_4, D_4 \rangle$ where W_4 consists of the facts and rules:

$$(\textit{pricy})$$
$$(\textit{studyBook})$$
$$(\textit{mandatory} \rightarrow \textit{buy})$$

and D_4 consists of the default rules:

$$\delta_1 = \textit{interesting} \hookrightarrow \textit{buy}$$
$$\delta_2 = \textit{pricy} \hookrightarrow \neg\, \textit{buy}$$
$$\delta_3 = \textit{studyBook} \hookrightarrow \textit{mandatory}$$

Figure 3 shows that the associated DELP$^{\emptyset}$ determines a mutual attack between arguments for $\neg$ *buy* and *buy*. However, the attack is indirectly determined by literals *buy* and *mandatory*.

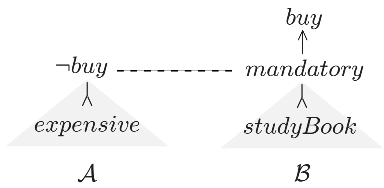

Figure 3: Indirect conflict

Example 5
Given the theory $T_5 = \langle W_5, D_5 \rangle$ where

$$W_5 = \{p, x, z\}$$
$$D_5 = \{p \hookrightarrow a, x \hookrightarrow \neg\, c, z \hookrightarrow c, c \hookrightarrow \neg\, a, \},$$

The corresponding de.l.p. has the rules:

$$\Pi_5 = \{(p \leftarrow), (x \leftarrow), (z \leftarrow)\}$$
$$\Delta_5 = \{(a \prec p), (\neg\, c \prec x), (c \prec z), (\neg\, a \prec c)\}$$

The theory T_5 has three extensions and the sets of conclusions $\{c, a\}$, $\{c, \neg\, a\}$ and $\{\neg\, c, a\}$ are subsets of CRED. In DELP$^{\emptyset}$ literals $a, \neg\, a, c$ and $\neg\, c$ are *undecided*. Figure 4 depicts extensions and arguments defined by this theory.

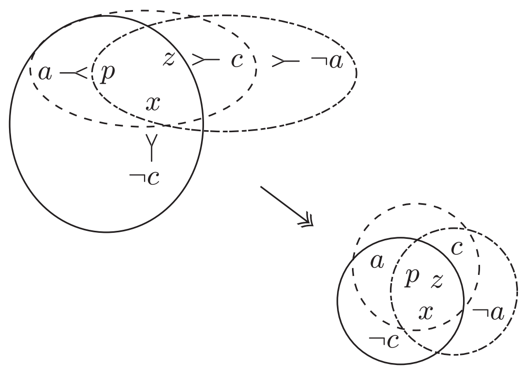

Figure 4: Extensions and Arguments

Example 6
Given the theory $T_6 = \langle W_6, D_6 \rangle$ where W_6 consists of the following rules and facts:

$$\begin{array}{c} \textit{anniversary} \\ \textit{rain} \\ \textit{noon} \\ (\textit{overcast} \rightarrow \neg\, \textit{sunny}) \\ (\textit{anniversary} \rightarrow \textit{winter.day}) \end{array}$$

and D_6 consists of the default rules:

$$\begin{array}{l} \delta_1 = \textit{noon} \hookrightarrow \textit{sunny} \\ \delta_2 = \textit{winter.day} \hookrightarrow \textit{overcast} \\ \delta_3 = \neg\, \textit{sunny} \hookrightarrow \textit{cold} \\ \delta_4 = \textit{rain} \hookrightarrow \neg\, \textit{sunny} \\ \delta_5 = \textit{overcast}, \textit{cold} \hookrightarrow \textit{take.a.taxi} \end{array}$$

The associated de.l.p. determines, for instance, that *take.a.taxi* $\in$ UND, because of the presence of an argument for *sunny* that attacks (and defeats) the argument for *take.a.taxi*. The default theory T_6 in a similar way, sanctions *take.a.taxi* $\in$ CRED, because there is an extension E such that *sunny* $\in E$ and both literal are incompatible, since *take.a.taxi* is derived from $\neg$ *sunny*. Hence, *take.a.taxi* $\in E$ and it is a credulous consecuence.

Example 7 (A classical example)
Given the theory $\langle W_7, D_7 \rangle$ where:

$$W_7 = \{\textit{evidenceA}, \textit{evidenceB}\}$$

$$\begin{array}{c} D_7 = \{\hookrightarrow \neg\, \textit{guilty}, \\ \textit{evidenceA} \hookrightarrow \textit{responsible}, \\ \textit{responsible} \hookrightarrow \textit{guilty}, \\ \textit{evidenceB} \hookrightarrow \neg\, \textit{responsible}\} \end{array}$$

In this final example, we can see again that default theory sanctions literals *guilty* and *responsible* as credulous conclusions, and the de.l.p. associated to this theory classifies both literals as *undecided*.

Note that the default rule ($\hookrightarrow \neg$ *guilty*) states the presumtion of innocence. This rule is associated to the defeasible rule ($\neg$ *guilty* $\prec$.)

5 DELP$^\emptyset$ AND DEFAULT THEORY: FUTURE WORK

In this article, we have presented a variant of DELP, which we call DELP$^\emptyset$, which allows to model the behavior of a Normal Default Logic via defeasible argumentation. The relation under study maps the different kinds of consequences from the default logic to the different answers given by DELP$^\emptyset$. In this way, every skeptical consequence of a particular normal default theory will be a warranted literal in DELP$^\emptyset$, whereas every credulous consequence of a default logic will be an undecided literal in DELP$^\emptyset$. The variant we are considering defines the argument preference criterion as the empty relation. This decision implies that no argument is better than any other. Thus, there are no proper defeats since every defeat is a blocking defeat. The definition of DELP$^\emptyset$ follows the behavior of the normal default theories. In this formalism, the application of a default rule *blocks* the derivation of a literal that conflicts with the conclusion of the rule being applied. In default logic, whenever this *blocking* situation arises, a new extension is generated. In DELP$^\emptyset$, this situation turns the conflicting literals into *undecided*.

Future work in this research line includes the study of the relation between DELP and general default theories, prioritized default theories, and other extensions.

References

[1] ANTONIOU, G., MAHER, M., BILLINGTON, D., AND GOVERNATORI, G. Comparison of sceptical naf-free logic programming approaches. *Logic Programming and Non-monotonic Reasoning 1730* (1999), 347–356.

[2] ANTONIOU, G., MAHER, M. J., AND BILLINGTON, D. Defeasible logic versus logic programming without negation as failure. *Journal of Logic Programming 42*, 1 (2000), 47–57.

[3] BREWKA, G. Well-founded semantics for extended logic programs with dynamic preferences. *Journal of Artificial Intelligence Research 4* (1996), 19–36.

[4] BREWKA, G. On the relation between defeasible logic and well-founded semantics. *LPNMR* (2001), 121–132.

[5] BREWKA, G., AND GOTTLOB, G. Well-founded semantics for default logic. *Fundamenta Informaticae 31*, 3/4 (1997), 221–236.

[6] CHESÑEVAR, C. I., DIX, J., STOLZENBURG, F., AND SIMARI, G. R. Relating defeasible and normal logic programming through transformation properties. *Theoretical Computer Science 290*, 1 (2002), 499–529.

[7] GARCÍA, A., AND SIMARI, G. Defeasible logic programming, an argumentative approach. *Theory and Practice of Logic Programming 4(1)* (2004), 95–138.

[8] REITER, R. A logic for default reasoning. *AI 13* (1980), 81–132.